

# Álgebra Linear

## Transformações lineares e matrizes Pt.1

Condições de existência da transformação linear

**Def.** Sejam  $U$  e  $V$ , espaços vetoriais reais, uma função

$T : U \rightarrow V$  é uma **Transformação Linear** se:

(i) Para quaisquer  $u_1, u_2 \in U, T(u_1 + u_2) = T(u_1) + T(u_2)$

(ii) Dados  $\beta \in R, u \in U, T(\beta \cdot u) = \beta \cdot T(u)$

**Obs.** Quando  $U = V$ ,  $T$  é chamado um **operador linear** do espaço vetorial  $U$ .

**Prop.** Sejam  $U$  e  $V$  espaços vetoriais. Se  $T : U \rightarrow V$  é uma transformação linear, então  $T(\vec{0}_u) = \vec{0}_v$   
Uma transformação linear sempre leva o vetor nulo de um espaço ao vetor nulo do outro. Se isso não acontecer, então não é uma transformação linear. No entanto, o fato dessa condição ser satisfeita não garante que é uma transformação linear.

Dado  $u = (x_1, x_2, \dots, x_n) \in R^n$ , podemos associar o vetor  $u$  a uma matriz coluna com  $n$  linhas:

$$u = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

e vice versa.

Um exemplo desse tipo de utilização é no caso de uma operação linear do tipo  $T : R^2 \rightarrow R^2$  definida como

$$T(x, y) = A \cdot \begin{bmatrix} x \\ y \end{bmatrix} \text{ onde a matriz } A \in M_2(R)$$

Toda aplicação de uma matriz multiplicada por um elemento do  $R^n$  é uma aplicação linear.

Toda função  $T : R^n \rightarrow R^m$  dada por

$$T(x_1, x_2, \dots, x_n) = A \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \text{ onde a matriz } A \in M_{m \times n}(R) \text{ é}$$

uma transformação linear.

**Prop.** Toda transformação linear  $T : R^n \rightarrow R^m$  pode ser expressa na forma acima, para alguma matriz  $A \in M_{m \times n}(R)$

**Exemplo** Seja  $/T : R^3 \rightarrow R^2$  dada por

$T(x, y, z) = (x + y, x - z)$ . Encontre uma matriz  $A \in M_{2 \times 3}(R)$  tal que  $T$  se escreva na forma acima:

$$T(1, 0, 0) = (1, 1)$$

$$T(0, 1, 0) = (1, 0)$$

$$T(0, 0, 1) = (0, -1)$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

**Def.** Esta matriz  $A$  é chamada **matriz da transformação linear**  $T$  em relação às bases canônicas de  $R^3$  e  $R^2$ .

**Notação:**  $A = [T]_{Can}$  ou simplesmente  $A = [T]$

Sejam  $U$  e  $V$  espaços vetoriais  $\dim U = n$  e  $\dim V = m$ ,  $B$  e  $F$  bases de  $U$  e  $V$  respectivamente, então para toda transformação linear  $T : U \rightarrow V$  podemos encontrar uma matriz  $A \in M_{m \times n}(R)$  tal que:

$$[T(u)]_F = A \cdot [u]_B, \forall u \in U$$

*lê-se: as coordenadas da imagem do vetor  $u$  na base  $F$  é igual a matriz  $A$  vezes o vetor  $u$  na base  $B$*

Onde  $[T(u)]_F$  é a matriz coluna das coordenadas de  $T(u)$  e  $[u]_B$  é a matriz coluna das coordenadas de  $u$  na base  $B$ .

Notação:  $A = [T]_{BF}$ .  $A$  é chamada matriz de  $T$  com relação às bases  $B$  e  $F$

**Exemplo:** Seja  $T : P_2(R) \rightarrow P_1(R)$ , com  $T(p) = p(0) + p(1)t$ . Encontre a matriz de  $T$  em relação às seguintes bases  $B = \{1, t - 1, t^2\}$  e  $F = \{1, t - 2\}$  respectivamente de  $P_2(R)$  e  $P_1(R)$

Vamos calcular  $T$  nos vetores da base  $B$ .

$T(1) = 1 + T = (3, 1)_F$  primeiro na base  $B$  depois converte pra base  $F$

$$T(t - 1) = -1 = (-1, 0)_F$$

$$T(t^2) = 1 = (2, 1)_F$$

$$\text{Portanto } A = [T]_{BF} = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & -1 \end{bmatrix}$$

Usando a matriz obtida acima calcule a imagem do polinômio  $p(t) = 2 + 3t - 2t^2$

Temos  $B = \{1, t - 1, t^2\}$  e  $F = \{1, t - 2\}$

Vimos que  $[T(p)]_F = [T]_{BF} \cdot [p]_B$

Primeiro convertamos o polinômio para a base  $B$

$$2 + 3t - 2t^2 = a + b(t - 1) + ct^2 \text{ portanto } a = 5, b = 3 \text{ e } c = -2$$

$$\text{ou ainda } (5, 3, -2)_B \Rightarrow [p]_B = \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}, \text{ logo}$$

$$[T(p)]_F = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \end{bmatrix}, \text{ assim, convertendo}$$

para a base  $F$  temos

$$T(p) = (8, 3)_F = 8 \cdot (1) + 3 \cdot (t - 2) = 2 + 3t$$

## Transformações lineares e matrizes Pt.2

**Exemplo** Seja  $T : P_3R \rightarrow P_2R$  dada por  $T(p) = p'$ . Encontre a matriz de  $T$  em relação às bases canônicas.

Sejam  $B = \{1, t, t^2, t^3\}$  e  $F = \{1, t, t^2\}$  as bases canônicas de  $P_3(R)$  e  $P_2R$  respectivamente.

Primeiro vamos calcular  $T$  usando os vetores da base  $B$

$$T(1) = 1' = 0 = (0, 0, 0)_F$$

$$T(t) = t' = 1 = (1, 0, 0)_F$$

$$T(t^2) = t^{2'} = 2t = (0, 2, 0)_F$$

$$T(t^3) = t^{3'} = 3t^2 = (0, 0, 3)_F$$

$$\text{Portanto } [T] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

Vamos verificar o uso da matriz obtida no exemplo para calcular a derivada do polinômio  $p(t) = 2 + 3t - 2t^2 + 4t^3$

Lembrando que  $[T(p)]_F = [T]_{BF} \cdot [p]_B$  e que

$$p = (2, 3, -2, 4) \Rightarrow [p]_B = \begin{bmatrix} 2 \\ 3 \\ -2 \\ 4 \end{bmatrix}$$

$$[T(p)]_F = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 12 \end{bmatrix}$$

$$\text{Logo } T(p) = (3, -4, 12)_F \Leftrightarrow p'(t) = 3 \cdot 1 - 4 \cdot t + 12 \cdot t^2$$

**Def.** Seja  $U$  um espaço vetorial, o operador linear  $I : U \rightarrow U$  dado por  $I(u) = u, \forall u \in U$  será chamado operador identidade de  $U$ .

Se  $B$  é uma base de  $U$  então a matriz de  $I$  com relação a base  $B$  é a matriz identidade de ordem  $n = \dim U$  ou seja

$$\begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} = I_n$$

Seja  $F$  uma outra base de  $U$  então a matriz do operador  $I$  com relação às bases  $B$  e  $F$  tem a seguinte propriedade:

$[I(u)]_F = [I]_{BF} \cdot [u]_B$ , como  $I(u) = u$  a expressão acima se

transforma em:  $[u]_F = [I]_{BF} \cdot [u]_B$

Assim, a matriz  $[I]_{BF}$  relaciona as coordenadas do vetor  $u$  na base  $B$  com suas coordenadas na base  $F$

**Exemplo:** Sejam  $B = (1, 0)(0, 1)$  e  $F = (1, 1), (1, -1)$  bases de  $R^2$ . Determine a matriz do operador identidade em relação às bases  $B$  e  $F$ . Se  $u = (2, 3)_B$  quais são suas coordenadas na base  $F$ ?

$$I(1, 0) = (1, 0) = \frac{1}{2} \cdot (1, 1) + \frac{1}{2}(1, -1) = (\frac{1}{2}, \frac{1}{2})_F$$

$$I(0, 1) = (0, 1) = \frac{1}{2} \cdot (1, 1) - \frac{1}{2}(1, -1) = (\frac{1}{2}, -\frac{1}{2})_F$$

$$\text{Portanto a matriz } [I]_{BF} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} = \frac{1}{2} \cdot \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$[u]_F = [I]_{BF} \cdot [u]_B, u = (2, 3) \Rightarrow [u]_B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\text{Logo, } [u]_F = \frac{1}{2} \cdot \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \frac{1}{2} \cdot \begin{bmatrix} 5 \\ -1 \end{bmatrix} \Rightarrow u = (\frac{5}{2}, -\frac{1}{2})_F$$

Sejam  $T$  um operador linear de um espaço vetorial  $U$  e  $B$  uma base qualquer de  $U$ . Então  $T$  é invertível se, e só se, a matriz  $[T]_B$  for invertível, neste caso  $T^{-1}$  é um operador linear e vale:  $[T^{-1}]_B = ([T]_B)^{-1}$

**Exemplo:** Seja  $T : R^3 \rightarrow R^3$  o operador linear dado por  $T(x, y, z) = (2x + y, x - z, y + 2z)$ ,  $T$  é invertível?

$$T(1, 0, 0) = (2, 1, 0)$$

$$T(0, 1, 0) = (1, 0, 1)$$

$$T(0, 0, 1) = (0, -1, 2)$$

$$[T] = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}, \det[T] = 2 - 2 = 0 \text{ Para ser invertível o}$$

determinante tem que ser diferente de zero. Neste caso,  $T$  não é invertível.

**Exemplo:** Seja  $T : R^2 \rightarrow R^2$  o operador linear dado por  $T(x, y) = (x - 2y, 2x + y)$ . Determine a matriz de  $T^{-1}$  em relação à base canônica de  $R^2$ .

$T(1, 0) = (1, 2)$   
 $T(0, 1) = (-2, 1)$

$[T] = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}, \det[T] = 5 \neq 0$ , Logo  $T$  é invertível

Para achar a matriz inversa  $M_{2 \times 2}$ , basta pegar  $\frac{1}{\det}$  e multiplicar pela matriz da seguinte forma: trocar os elementos da diagonal principal e trocar o sinal da diagonal secundária. Portanto:

$[T^{-1}] = [T]^{-1} = \frac{1}{5} \cdot \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$

**Exemplo:** Calcule  $T^{-1}(2, 5)$  onde  $T$  é o operador do exemplo anterior.

$[T^{-1}(2, 5)] = [T]^{-1} \cdot [(2, 5)], [(2, 5)] = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ , portanto:

$[T^{-1}(2, 5)] = \frac{1}{5} \cdot \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \frac{1}{5} \cdot \begin{bmatrix} 12 \\ 1 \end{bmatrix}$ , logo

$T^{-1}(2, 5) = (\frac{12}{5}, \frac{1}{5})$

Para verificar fazemos

$T(\frac{12}{5}, \frac{1}{5}) = \frac{1}{5}T(12, 1) = \frac{1}{5}(10, 25) = (2, 5)$

Sejam  $T$  e  $S$  operadores lineares de um espaço vetorial  $U$  e  $B$  uma base qualquer de  $U$  então:

- (i) Dado  $\beta \in R, \beta \cdot T : U \rightarrow U$  é linear e  $[\beta \cdot T]_B = \beta[T]_B$
- (ii)  $T + S : U \rightarrow U$  é linear e  $[T + S]_B = [T]_B + [S]_B$
- (iii)  $T \circ S : U \rightarrow U$  é linear e  $[T \circ S]_B = [T]_B \cdot [S]_B$

**Exemplo:** Sejam  $T, S : R^2 \rightarrow R^2$  operadores lineares definidos por  $T(x, y) = (2x, x + y)$  e  $S(x, y) = (y - x, 3x)$ . Determine a matriz do operador  $G = 2T - 3(T \circ S)$  em relação a base canônica de  $R^2$

$[G] = [2T - 3(T \circ S)] = 2[T] - 3[T][S]$

$T(1, 0) = (2, 1), T(0, 1) = (0, 1)$  logo  $[T] = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$

$S(1, 0) = (-1, 3), S(0, 1) = (1, 0)$  logo  $[S] = \begin{pmatrix} -1 & 1 \\ 3 & 0 \end{pmatrix}$

Portanto

$[G] = 2[T] - 3[T][S] = 2 \cdot \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} - 3 \cdot \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 \\ 3 & 0 \end{pmatrix}$

Logo  $[G] = \begin{bmatrix} 10 & 0 \\ -4 & 1 \end{bmatrix}$

**Exemplo:** Sejam  $T, I : R^2 \rightarrow R^2$  operadores lineares tais que  $[T] = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$  e  $I(x, y) = (x, y)$

Determine a matriz do operador  $G = T - 3I$  em relação à base canônica de  $R^2$ .

$[G] = [T - 3I] = [T] - 3[I] = [T] - 3I_2$

Assim  $[G] = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 3 & -4 \end{bmatrix}$

## Núcleo e Imagem

book	Default is two-sided.
report	No \part divisions.
article	No \part or \chapter divisions.
letter	Letter (?).
slides	Large sans-serif font.

Used at the very beginning of a document:

`\documentclass{class}`. Use `\begin{document}` to start contents and `\end{document}` to end the document.

### Common documentclass options

10pt/11pt/12pt	Font size.
letterpaper/a4paper	Paper size.
twocolumn	Use two columns.
twoside	Set margins for two-sided.
landscape	Landscape orientation. Must use dvips

`draft` Double-space lines.

Usage: `\documentclass[opt,opt]{class}`.

### Packages

`fullpage` Use 1 inch margins.

`anysize` Set margins: `\marginsize{l}{r}{t}{b}`.

`multicol` Use  $n$  columns: `\begin{multicols}{n}`.

`latexsym` Use L<sup>A</sup>T<sub>E</sub>X symbol font.

`graphicx` Show image: `\includegraphics[width=x]{file}`.

`url` Insert URL: `\url{http://...}`.

Use before `\begin{document}`. Usage: `\usepackage{package}`

### Title

`\author{text}` Author of document.

`\title{text}` Title of document.

`\date{text}` Date.

These commands go before `\begin{document}`. The declaration `\maketitle` goes at the top of the document.

### Miscellaneous

`\pagestyle{empty}` Empty header, footer and no page numbers.

`\tableofcontents` Add a table of contents here.

### Document structure

`\part{title}` `\subsubsection{title}`

`\chapter{title}` `\paragraph{title}`

`\section{title}` `\subparagraph{title}`

`\subsection{title}`

Use `\setcounter{secnumdepth}{x}` suppresses heading numbers of depth  $> x$ , where `chapter` has depth 0. Use a `*`, as in `\section*{title}`, to not number a particular item—these items will also not appear in the table of contents.

### Text environments

`\begin{comment}` Comment (not printed). Requires `verbatim` package.

`\begin{quote}` Indented quotation block.

`\begin{quotation}` Like `quote` with indented paragraphs.

`\begin{verse}` Quotation block for verse.

### Lists

`\begin{enumerate}` Numbered list.

`\begin{itemize}` Bulleted list.

`\begin{description}` Description list.

`\item text` Add an item.

`\item[x] text` Use  $x$  instead of normal bullet or number. Required for descriptions.

### References

`\label{marker}` Set a marker for cross-reference, often of the form `\label{sec:item}`.

`\ref{marker}` Give section/body number of marker.

`\pageref{marker}` Give page number of marker.

`\footnote{text}` Print footnote at bottom of page.

### Floating bodies

`\begin{table}[place]` Add numbered table.

`\begin{figure}[place]` Add numbered figure.

`\begin{equation}[place]` Add numbered equation.

`\caption{text}` Caption for the body.

The *place* is a list valid placements for the body. `t`=top, `b`=bottom, `p`=separate page, `!`=place even if ugly. Captions and label markers should be within the environment.

## Text properties

### Font face

Command	Declaration	Effect
<code>\textrm{text}</code>	<code>\rmfamily text</code>	Roman family
<code>\textsf{text}</code>	<code>\sffamily text</code>	Sans serif family
<code>\texttt{text}</code>	<code>\ttfamily text</code>	Typewriter family
<code>\textmd{text}</code>	<code>\mdseries text</code>	Medium series
<code>\textbf{text}</code>	<code>\bfseries text</code>	<b>Bold series</b>
<code>\textup{text}</code>	<code>\upshape text</code>	Upright shape
<code>\textit{text}</code>	<code>\itshape text</code>	<i>Italic shape</i>
<code>\textsl{text}</code>	<code>\slshape text</code>	<i>Slanted shape</i>
<code>\textsc{text}</code>	<code>\scshape text</code>	SMALL CAPS SHAPE
<code>\emph{text}</code>	<code>\em text</code>	<i>Emphasized</i>
<code>\textnormal{text}</code>	<code>\normalfont text</code>	Document font
<code>\underline{text}</code>		<u>Underline</u>

The command (`t`*tt*) form handles spacing better than the declaration (`t`*tt*) form.

### Font size

<code>\tiny</code>	<small>tiny</small>	<code>\Large</code>	Large
<code>\scriptsize</code>	<small>scriptsize</small>	<code>\LARGE</code>	LARGE
<code>\footnotesize</code>	<small>footnotesize</small>		
<code>\small</code>	<small>small</small>	<code>\huge</code>	huge
<code>\normalsize</code>	<small>normalsize</small>		
<code>\large</code>	<small>large</small>	<code>\Huge</code>	Huge

These are declarations and should be used in the form `\small ...`, or without braces to affect the entire document.

### Verbatim text

`\begin{verbatim}` Verbatim environment.

`\begin{verbatim*}` Spaces are shown as `_`.

`\verb!text!` Text between the delimiting characters (in this case ‘!’) is verbatim.

## Justification

<i>Environment</i>	<i>Declaration</i>
<code>\begin{center}</code>	<code>\centering</code>
<code>\begin{flushleft}</code>	<code>\raggedright</code>
<code>\begin{flushright}</code>	<code>\raggedleft</code>

## Miscellaneous

`\linespread{x}` changes the line spacing by the multiplier  $x$ .

## Text-mode symbols

### Symbols

<code>&amp;</code>	<code>\&amp;</code>	<code>-</code>	<code>\_</code>	<code>...</code>	<code>\ldots</code>	<code>•</code>	<code>\textbullet</code>
<code>\$</code>	<code>\\$</code>	<code>^</code>	<code>\^{}{}</code>	<code> </code>	<code>\textbar</code>	<code>\</code>	<code>\textbackslash</code>
<code>%</code>	<code>\%</code>	<code>~</code>	<code>\~{}{}</code>	<code>#</code>	<code>\#</code>	<code>§</code>	<code>\S</code>

### Accents

<code>ò \’o</code>	<code>ó \’o</code>	<code>ô \^o</code>	<code>õ \~o</code>	<code>ö \=o</code>
<code>ô \.o</code>	<code>ö \"o</code>	<code>q \c o</code>	<code>ö \v o</code>	<code>ő \H o</code>
<code>ç \c c</code>	<code>q \d o</code>	<code>q \b o</code>	<code>öo \t oo</code>	<code>œ \oe</code>
<code>Œ \OE</code>	<code>æ \ae</code>	<code>Æ \AE</code>	<code>å \aa</code>	<code>Å \AA</code>
<code>ø \o</code>	<code>Ø \O</code>	<code>ı \l</code>	<code>L \L</code>	<code>ı \i</code>
<code>j \j</code>	<code>i \’i</code>	<code>ı \l</code>	<code>ı \l</code>	<code>ı \i</code>

### Delimiters

<code>‘ ‘ ‘ ‘</code>	<code>{ \{</code>	<code>[ [ [ [</code>	<code>( ( ( (</code>	<code>&lt; \textless</code>
<code>’ ’ ’ ’</code>	<code>} \}</code>	<code>] ] ] ]</code>	<code>) ) ) )</code>	<code>&gt; \textgreater</code>

### Dashes

<i>Name</i>	<i>Source</i>	<i>Example</i>	<i>Usage</i>
hyphen	-	X-ray	In words.
en-dash	--	1–5	Between numbers.
em-dash	---	Yes—or no?	Punctuation.

### Line and page breaks

<code>\</code>	Begin new line without new paragraph.
<code>\*</code>	Prohibit pagebreak after linebreak.
<code>\kill</code>	Don’t print current line.
<code>\pagebreak</code>	Start new page.
<code>\noindent</code>	Do not indent current line.

### Miscellaneous

<code>\today</code>	April 28, 2019.
<code>\sim\$</code>	Prints $\sim$ instead of <code>\~{}{}</code> , which makes $\sim$ .
<code>~</code>	Space, disallow linebreak (W.J.~Clinton).
<code>\@.</code>	Indicate that the $\cdot$ ends a sentence when following an uppercase letter.
<code>\hspace{l}</code>	Horizontal space of length $l$ (Ex: $l = 20\text{pt}$ ).
<code>\vspace{l}</code>	Vertical space of length $l$ .
<code>\rule{w}{h}</code>	Line of width $w$ and height $h$ .

## Tabular environments

### tabbing environment

`\=` Set tab stop. `\>` Go to tab stop.

Tab stops can be set on “invisible” lines with `\kill` at the end of the line. Normally `\` is used to separate lines.

### tabular environment

`\begin{array}[pos]{cols}`  
`\begin{tabular}[pos]{cols}`  
`\begin{tabular*}{width}[pos]{cols}`

### tabular column specification

`l` Left-justified column.  
`c` Centered column.  
`r` Right-justified column.  
`p{width}` Same as `\parbox[t]{width}`.  
`@{decl}` Insert *decl* instead of inter-column space.  
`|` Inserts a vertical line between columns.

### tabular elements

`\hline` Horizontal line between rows.  
`\cline{x-y}` Horizontal line across columns  $x$  through  $y$ .  
`\multicolumn{n}{cols}{text}`  
A cell that spans  $n$  columns, with *cols* column specification.

## Math mode

For inline math, use `\(...\)` or `$...$`. For displayed math, use `\[...\]` or `\begin{equation}`.

Superscript $x$	<code>\^{}{x}</code>	Subscript $x$	<code>\_{}{x}</code>
$\frac{x}{y}$	<code>\frac{x}{y}</code>	$\sum_{k=1}^n$	<code>\sum_{k=1}^n</code>
$\sqrt[n]{x}$	<code>\sqrt[n]{x}</code>	$\prod_{k=1}^n$	<code>\prod_{k=1}^n</code>

### Math-mode symbols

$\leq$	<code>\leq</code>	$\geq$	<code>\geq</code>	$\neq$	<code>\neq</code>	$\approx$	<code>\approx</code>
$\times$	<code>\times</code>	$\div$	<code>\div</code>	$\pm$	<code>\pm</code>	$\cdot$	<code>\cdot</code>
$\circ$	<code>\circ</code>	$\circ$	<code>\circ</code>	$\prime$	<code>\prime</code>	$\cdots$	<code>\cdots</code>
$\infty$	<code>\infty</code>	$\neg$	<code>\neg</code>	$\wedge$	<code>\wedge</code>	$\vee$	<code>\vee</code>
$\supset$	<code>\supset</code>	$\forall$	<code>\forall</code>	$\in$	<code>\in</code>	$\rightarrow$	<code>\rightarrow</code>
$\subset$	<code>\subset</code>	$\exists$	<code>\exists</code>	$\notin$	<code>\notin</code>	$\Rightarrow$	<code>\Rightarrow</code>
$\cup$	<code>\cup</code>	$\cap$	<code>\cap</code>	$\mid$	<code>\mid</code>	$\Leftrightarrow$	<code>\Leftrightarrow</code>
$\dot{a}$	<code>\dot{a}</code>	$\hat{a}$	<code>\hat{a}</code>	$\bar{a}$	<code>\bar{a}</code>	$\tilde{a}$	<code>\tilde{a}</code>
$\alpha$	<code>\alpha</code>	$\beta$	<code>\beta</code>	$\gamma$	<code>\gamma</code>	$\delta$	<code>\delta</code>
$\epsilon$	<code>\epsilon</code>	$\zeta$	<code>\zeta</code>	$\eta$	<code>\eta</code>	$\varepsilon$	<code>\varepsilon</code>
$\theta$	<code>\theta</code>	$\iota$	<code>\iota</code>	$\kappa$	<code>\kappa</code>	$\vartheta$	<code>\vartheta</code>
$\lambda$	<code>\lambda</code>	$\mu$	<code>\mu</code>	$\nu$	<code>\nu</code>	$\xi$	<code>\xi</code>
$\pi$	<code>\pi</code>	$\rho$	<code>\rho</code>	$\sigma$	<code>\sigma</code>	$\tau$	<code>\tau</code>
$\upsilon$	<code>\upsilon</code>	$\phi$	<code>\phi</code>	$\chi$	<code>\chi</code>	$\psi$	<code>\psi</code>
$\omega$	<code>\omega</code>	$\Gamma$	<code>\Gamma</code>	$\Delta$	<code>\Delta</code>	$\Theta$	<code>\Theta</code>
$\Lambda$	<code>\Lambda</code>	$\Xi$	<code>\Xi</code>	$\Pi$	<code>\Pi</code>	$\Sigma$	<code>\Sigma</code>
$\Upsilon$	<code>\Upsilon</code>	$\Phi$	<code>\Phi</code>	$\Psi$	<code>\Psi</code>	$\Omega$	<code>\Omega</code>

## Bibliography and citations

When using BibTeX, you need to run `latex`, `bibtex`, and `latex` twice more to resolve dependencies.

### Citation types

<code>\cite{key}</code>	Full author list and year. (Watson and Crick 1953)
<code>\citeA{key}</code>	Full author list. (Watson and Crick)
<code>\citeN{key}</code>	Full author list and year. Watson and Crick (1953)
<code>\shortcite{key}</code>	Abbreviated author list and year. ?
<code>\shortciteA{key}</code>	Abbreviated author list. ?
<code>\shortciteN{key}</code>	Abbreviated author list and year. ?
<code>\citeyear{key}</code>	Cite year only. (1953)

All the above have an NP variant without parentheses; Ex. `\citeNP`.

### BibTeX entry types

<code>@article</code>	Journal or magazine article.
<code>@book</code>	Book with publisher.
<code>@booklet</code>	Book without publisher.
<code>@conference</code>	Article in conference proceedings.
<code>@inbook</code>	A part of a book and/or range of pages.
<code>@incollection</code>	A part of book with its own title.
<code>@misc</code>	If nothing else fits.
<code>@phdthesis</code>	PhD. thesis.
<code>@proceedings</code>	Proceedings of a conference.
<code>@techreport</code>	Tech report, usually numbered in series.
<code>@unpublished</code>	Unpublished.

### BibTeX fields

<code>address</code>	Address of publisher. Not necessary for major publishers.
<code>author</code>	Names of authors, of format ....
<code>booktitle</code>	Title of book when part of it is cited.
<code>chapter</code>	Chapter or section number.
<code>edition</code>	Edition of a book.
<code>editor</code>	Names of editors.
<code>institution</code>	Sponsoring institution of tech. report.
<code>journal</code>	Journal name.
<code>key</code>	Used for cross ref. when no author.
<code>month</code>	Month published. Use 3-letter abbreviation.
<code>note</code>	Any additional information.
<code>number</code>	Number of journal or magazine.
<code>organization</code>	Organization that sponsors a conference.
<code>pages</code>	Page range (2,6,9--12).
<code>publisher</code>	Publisher’s name.
<code>school</code>	Name of school (for thesis).
<code>series</code>	Name of series of books.
<code>title</code>	Title of work.
<code>type</code>	Type of tech. report, ex. “Research Note”.
<code>volume</code>	Volume of a journal or book.
<code>year</code>	Year of publication.

Not all fields need to be filled. See example below.

### Common BibTeX style files

<code>abbrv</code>	Standard	<code>abstract</code>	<code>alpha</code> with abstract
<code>alpha</code>	Standard	<code>apa</code>	APA
<code>plain</code>	Standard	<code>unsrt</code>	Unsorted

The L<sup>A</sup>T<sub>E</sub>X document should have the following two lines just before `\end{document}`, where `bibfile.bib` is the name of the B<sub>I</sub>B<sub>T</sub>E<sub>X</sub> file.

```
\bibliographystyle{plain}
\bibliography{bibfile}
```

### B<sub>I</sub>B<sub>T</sub>E<sub>X</sub> example

The B<sub>I</sub>B<sub>T</sub>E<sub>X</sub> database goes in a file called *file.bib*, which is processed with `bibtex` file.

```
@String{N = {Na\-ture}}
@Article{WC:1953,
  author = {James Watson and Francis Crick},
  title = {A structure for Deoxyribose Nucleic Acid},
  journal = N,
  volume = {171},
  pages = {737},
```

```
    year = 1953
}
```

## Sample L<sup>A</sup>T<sub>E</sub>X document

```
\documentclass[11pt]{article}
\usepackage{fullpage}
\title{Template}
\author{Name}
\begin{document}
\maketitle

\section{section}
\subsection*{subsection without number}
text \textbf{bold text} text. Some math:  $2+2=5$ 
\subsection{subsection}
text \emph{emphasized text} text. \cite{WC:1953}
discovered the structure of DNA.
```

```
A table:
\begin{table}[!th]
\begin{tabular}{|l|c|r|}
\hline
first & row & data \\
second & row & data \\
\hline
\end{tabular}
\caption{This is the caption}
\label{ex:table}
\end{table}
```

The table is numbered `\ref{ex:table}`.  
`\end{document}`

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<http://wch.github.io/latexsheet/>