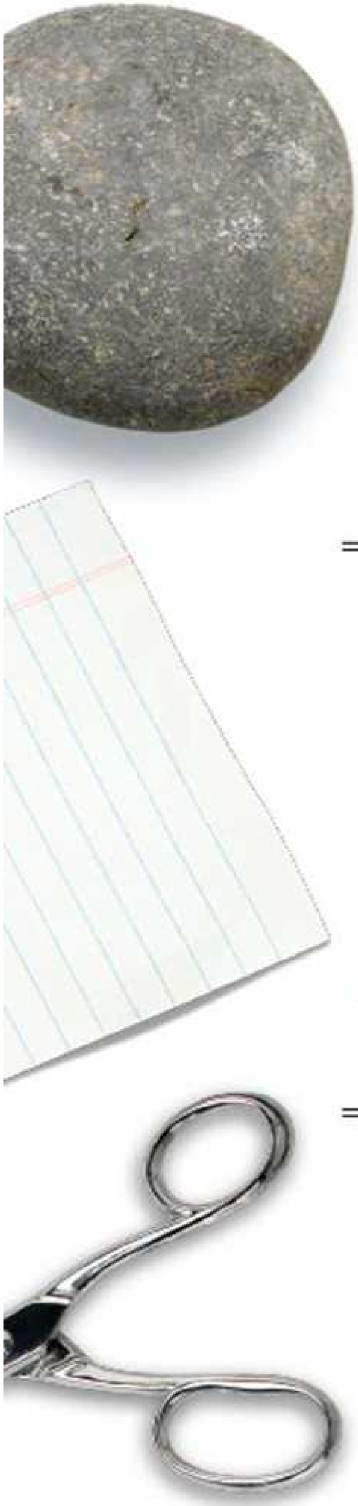

"A model of popular economics."
—Tim Harford, *Financial Times*

A Game Theorist's Guide to Success in Business and Life

The Art *of* Strategy

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Barry J. Nalebuff

authors of the best-selling *Thinking Strategically*

CHAPTER 2

Games Solvable by Backward Reasoning

IT'S YOUR MOVE, CHARLIE BROWN

In a recurring theme in the comic strip *Peanuts*, Lucy holds a football on the ground and invites Charlie Brown to run up and kick it. At the last moment, Lucy pulls the ball away. Charlie Brown, kicking only air, lands on his back, and this gives Lucy great perverse pleasure.

Anyone could have told Charlie that he should refuse to play Lucy's game. Even if Lucy had not played this particular trick on him last year (and the year before and the year before that), he knows her character from other contexts and should be able to predict her action.

At the time when Charlie is deciding whether or not to accept Lucy's invitation, her action lies in the future. However, just because it lies in the future does not mean Charlie should regard it as uncertain. He should know that of the two possible outcomes—letting him kick and seeing him fall—Lucy's preference is for the latter. Therefore he should forecast that when the time comes, she is going to pull the ball away. The logical possibility that Lucy will let him kick the ball is realistically irrelevant. Reliance on it would be, to borrow Dr. Johnson's characterization of remarriage, a triumph of hope over experience. Charlie should disregard it, and forecast that acceptance will inevitably land him on his back. He should decline Lucy's invitation.

TWO KINDS OF STRATEGIC INTERACTIONS

The essence of a game of strategy is the interdependence of the players' decisions. These interactions arise in two ways. The first is *sequential,* as in the Charlie Brown story. The players make alternating moves. Charlie, when it is his turn, must look ahead to how his current actions will affect the future actions of Lucy, and his own future actions in turn.

The second kind of interaction is *simultaneous*, as in the prisoners' dilemma tale of chapter 1. The players act at the same time, in ignorance of the others' current actions. However, each must be aware that there are other active players, who in turn are similarly aware, and so on. Therefore each must figuratively put himself in the shoes of all and try to calculate the outcome. His own best action is an integral part of this overall calculation.

When you find yourself playing a strategic game, you must determine whether the interaction is simultaneous or sequential. Some games, such as football, have elements of both, in which case you must fit your strategy to the context. In this chapter, we develop, in a preliminary way, the ideas and rules that will help you play sequential games; simultaneous-move games are the subject of chapter 3. We begin with really simple, sometimes contrived, examples, such as the Charlie Brown story. This is deliberate; the stories are not of great importance in themselves, and the right strategies are usually easy to see by simple intuition, allowing the underlying ideas to stand out much more clearly. The examples get increasingly realistic and more complex in the case studies and in the later chapters.

The First Rule of Strategy

The general principle for sequential-move games is that each player should figure out the other players' future responses and use them in calculating his own best current move. This idea is so important that it is worth codifying into a basic rule of strategic behavior:

RULE 1: Look forward and reason backward.

Anticipate where your initial decisions will ultimately lead and use this information to calculate your best choice.

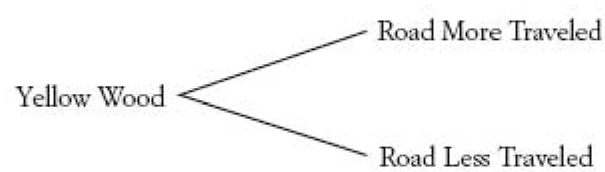
In the Charlie Brown story, this was easy to do for anyone (except Charlie Brown). He had just two alternatives, and one of them led to Lucy's decision between two possible actions. Most strategic situations involve a longer sequence of decisions with several alternatives at each. A tree diagram of the choices in the game sometimes serves as a visual aid for correct reasoning in such games. Let us show you how to use these trees.

DECISION TREES AND GAME TREES

A sequence of decisions, with the need to look forward and reason backward, can arise even for a solitary decision maker not involved in a game of strategy with others. For Robert Frost in the yellow wood:

> Two roads diverged in a wood, and I—
> I took the road less traveled by,
> And that has made all the difference.[1]

We can show this schematically.

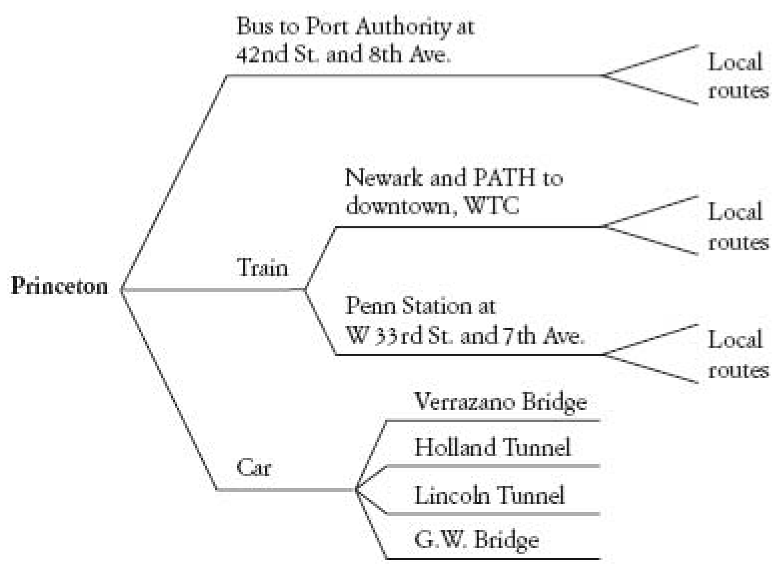


This need not be the end of the choice. Each road might in turn have further branches. The road map becomes correspondingly complex. Here is an example from our own experience.

Travelers from Princeton to New York have several choices. The first decision point involves selecting the mode of travel: bus, train, or car. Those who drive then have to choose among the Verrazano-Narrows Bridge, the Holland Tunnel, the Lincoln Tunnel, and the George Washington Bridge. Rail commuters must decide whether to switch to the

PATH train at Newark or continue to Penn Station. Once in New York, rail and bus commuters must choose among going by foot, subway (local or express), bus, or taxi to get to their final destination. The best choices depend on many factors, including price, speed, expected congestion, the final destination in New York, and one's aversion to breathing the air on the New Jersey Turnpike.

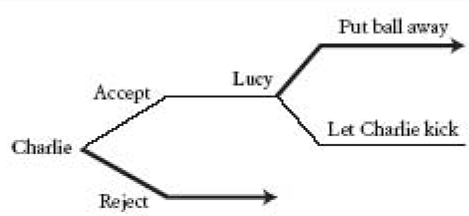
This road map, which describes one's options at each junction, looks like a tree with its successively emerging branches—hence the term. The right way to use such a map or tree is not to take the route whose first branch looks best—for example, because you would prefer driving to taking the train when all other things are equal—and then "cross the Verrazano Bridge when you get to it." Instead, you anticipate the future decisions and use them to make your earlier choices. For example, if you want to go downtown, the PATH train would be superior to driving because it offers a direct connection from Newark.



We can use just such a tree to depict the choices in a game of strategy, but one new element enters the picture. A game has two or more players. At various branching points along the tree, it may be the turn of different players to make the decision. A person making a choice at an earlier point must look ahead, not just to his own future choices but to those of others. He must forecast what the others will do, by putting himself figuratively in their shoes, and thinking as they would think. To remind you of the difference, we will call a tree showing the decision sequence in a game of strategy a *game tree,* reserving *decision tree* for situations in which just one person is involved.

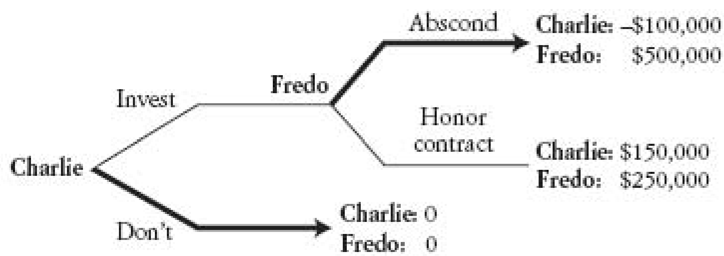
Charlie Brown in Football and in Business

The story of Charlie Brown that opened this chapter is absurdly simple, but you can become familiar with game trees by casting that story in such a picture. Start the game when Lucy has issued her invitation, and Charlie faces the decision of whether to accept. If Charlie refuses, that is the end of the game. If he accepts, Lucy has the choice between letting Charlie kick and pulling the ball away. We can show this by adding another fork along this road.



As we said earlier, Charlie should forecast that Lucy will choose the upper branch. Therefore he should figuratively prune the lower branch of her choice from the tree. Now if he chooses his own upper branch, it leads straight to a nasty fall. Therefore his better choice is to follow his own lower branch. We show these selections by making the branches thicker and marking them with arrowheads.

Are you thinking that this game is too frivolous? Here is a business version of it. Imagine the following scenario. Charlie, now an adult, is vacationing in the newly reformed formerly Marxist country of Freedonia. He gets into a conversation with a local businessman named Fredo, who talks about the wonderful profitable opportunities that he could develop given enough capital, and then makes a pitch: "Invest $100,000 with me, and in a year I will turn it into $500,000, which I will share equally with you. So you will more than double your money in a year." The opportunity Fredo describes is indeed attractive, and he is willing to write up a proper contract under Freedonian law. But how secure is that law? If at the end of the year Fredo absconds with all the money, can Charlie, back in the United States, enforce the contract in Freedonian courts? They may be biased in favor of their national, or too slow, or bribed by Fredo. So Charlie is playing a game with Fredo, and the tree is as shown here. (Note that if Fredo honors the contract, he pays Charlie $250,000; therefore Charlie's profit is that minus the initial investment of $100,000—that is, $150,000.)



What do you think Fredo is going to do? In the absence of a clear and strong reason to believe his promise, Charlie should predict that Fredo will abscond, just as young Charlie should have been sure that Lucy would pull the ball away. In fact the trees of the two games are identical in all essential respects. But how many Charlies have failed to do the correct reasoning in such games?

What reasons can there be for believing Fredo's promise? Perhaps he is engaged in many other enterprises that require financing from the United States or export goods to the United States. Then Charlie may be able to retaliate by ruining his reputation in the United States or seizing his goods. So this game may be part of a larger game, perhaps an ongoing interaction, that ensures Fredo's honesty. But in the one-time version we showed above, the logic of backward reasoning is clear.

We would like to use this game to make three remarks. First, different games may have identical or very similar mathematical forms (trees, or the tables used for depictions in later chapters). Thinking about them using such formalisms highlights the parallels and makes it easy to transfer your knowledge about a game in one situation to that in another. This is an important function of the "theory" of any subject: it distills the essential similarities in apparently dissimilar contexts and enables one to think about them in a unified and therefore simplified manner. Many people have an instinctive aversion to theory of any kind. But we think this is a mistaken reaction. Of course, theories have their limitations. Specific contexts and experiences can often add to or modify the prescriptions of theory in substantial ways. But to abandon theory altogether would be to abandon a valuable starting point for thought, which may be a beachhead for conquering the problem. You should make game theory your friend, and not a bugbear, in your strategic thinking.

The second remark is that Fredo should recognize that a strategic Charlie would be suspicious of his pitch and not invest at all, depriving Fredo of the opportunity to make $250,000. Therefore Fredo has a strong incentive to make his promise credible. As an individual businessman, he has little influence over Freedonia's weak legal system and cannot allay the investor's suspicion that way. What other methods may be at his disposal? We will examine the general issue of credibility, and devices for achieving it, in chapters 6 and 7.

The third, and perhaps most important, remark concerns comparisons of the different outcomes that could result based on the different choices the players could make. It is not always the case that more for one player means less for the other. The situation where Charlie invests and Fredo honors the contract is better for both than the one where Charlie does not invest at all. Unlike sports or contests, games don't have to have winners and losers; in the jargon of game theory, they don't have to be zero-sum. Games can have win-win or lose-lose outcomes. In fact, some combination of commonality of interest (as when Charlie and Fredo can both gain if there is a way for Fredo to commit credibly to honoring the contract) and some conflict (as when Fredo can gain at Charlie's expense by absconding after Charlie has invested) coexist in most games in business, politics, and social interactions. And that is precisely what makes the analysis of these games so interesting and challenging.

More Complex Trees

We turn to politics for an example of a slightly more complex game tree. A caricature of American politics says that Congress likes pork-barrel expenditures and presidents try to cut down the bloated budgets that Congress passes. Of course presidents have their own likes and dislikes among such expenditures and would like to cut only the ones they dislike. To do so, they would like to have the power to cut out specific items from the budget, or a line-item veto. Ronald Reagan in his State of the Union address in January 1987 said this eloquently: "Give us the same tool that 43 governors have—a line-item veto, so we can carve out the boon-doggles and pork, those items that would never survive on their own."

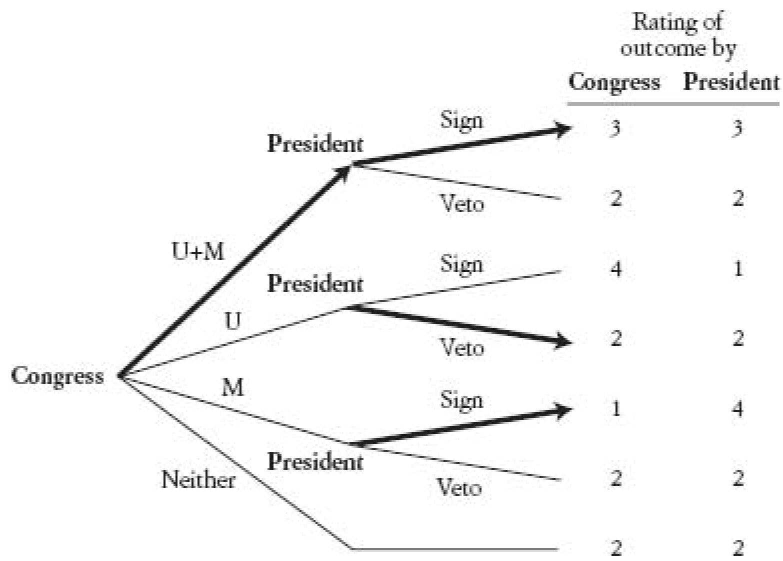
At first sight, it would seem that having the freedom to veto parts of a bill can only increase the president's power and never yield him any worse outcomes. Yet it is possible that the president may be better off without this tool. The point is that the existence of a line-item veto will influence the Congress's strategies in passing bills. A simple game shows how.

For this purpose, the essence of the situation in 1987 was as follows. Suppose there were two items of expenditure under consideration: urban renewal (U) and an antiballistic missile system (M). Congress liked the former and the president liked the latter. But both preferred a package of the two to the status quo. The following table shows the ratings of the possible scenarios by the two players, in each case 4 being best and 1, worst.

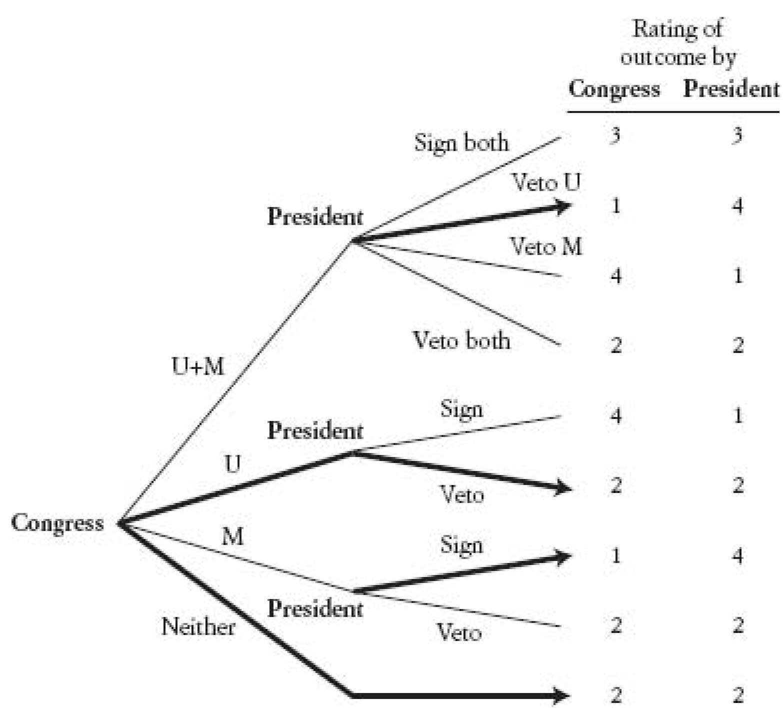
Outcomes	Congress	President
Both U and M	3	3
U only	4	1
M only	1	4
Neither	2	2

The tree for the game when the president does not have a line-item veto is shown on the following page. The president will sign a bill containing the package of U and M, or one with M alone, but will veto one with U alone. Knowing this, the Congress chooses the package. Once again we show the selections at each point by thickening the chosen branches and giving them arrowheads. Note that we have to do this for all the points where the president might conceivably be called upon to choose, even though some of these are rendered moot by Congress's previous choice. The reason is that Congress's actual choice is crucially affected by its calculation of what the president would have done if Congress had counterfactually made a different choice; to show this logic we must show the president's actions in all logically conceivable situations.

Our analysis of the game yields an outcome in which both sides get their second best preference (rating 3).



Next, suppose the president has a line-item veto. The game changes to the following:



Now Congress foresees that if it passes the package, the president will selectively veto U, leaving only M. Therefore Congress's best action is now either to pass U only to see it vetoed, or pass nothing. Perhaps it may have a preference for the former, if it can score political points from a presidential veto, but perhaps the president may equally score political points by this show of budgetary discipline. Let us suppose the two offset each other, and Congress is indifferent with respect to the two choices. But either gives each party only their third-best outcome (rating 2). Even the president is left worse-off by his extra freedom of choice.[2]

This game illustrates an important general conceptual point. In single-person decisions, greater freedom of action can never hurt. But in games, it can hurt because its existence can influence other players' actions. Conversely, tying your own hands can help. We will explore this "advantage of commitment" in chapters 6 and 7.

We have applied the method of backward reasoning in a game tree to a very trivial game (Charlie Brown), and extended it to a slightly more complicated game (the line-item veto). The general principle remains applicable, no matter how complicated the game may be. But trees for games where each player has several choices available at any point, and where each player gets several turns to move, can quickly get too complicated to draw or use. In chess, for example, 20 branches emerge from the root—the player with the white pieces can move any of his/her eight pawns forward one square or two, or move one of his two knights in one of two ways. For each of these, the player with the black pieces has 20 moves, so we are up to 400 distinct paths already. The number of branches emerging from later nodes in chess can be even larger. Solving chess fully using the tree method is beyond the ability of the most powerful computer that exists or might be developed during the next several decades, and other methods of partial analysis must be sought. We will discuss later in the chapter how chess experts have tackled this problem.

Between the two extremes lie many moderately complex games that are played in business, politics, and everyday life. Two approaches can be used for these. Computer programs are available to construct trees and compute solutions.[3] Alternatively, many games of moderate complexity can be solved by the logic of tree analysis, without drawing the tree explicitly. We illustrate this using a game that was played in a TV show that is all about games, where each player tries to "outplay, outwit, and outlast" the others.

STRATEGIES FOR "SURVIVORS"

CBS's *Survivor* features many interesting games of strategy. In the sixth episode of *Survivor: Thailand*, the two teams or tribes played a game that provides an excellent example of thinking forward and reasoning backward in theory and in practice.[4] Twenty-one flags were planted in the field of play between the tribes, who took turns removing the flags. Each tribe at its turn could choose to remove 1 or 2 or 3 flags. (Thus zero—passing up one's turn—was not permitted; nor was it within the rules to remove four or more at one turn.) The team to take the last flag, whether standing alone or as a part of a group of 2 or 3 flags, won the game.[5] The losing tribe had to vote out one of its own members, thus weakening it in future contests. In fact the loss proved crucial in this instance, and a member of the other tribe went on to win the ultimate prize of a million dollars. Thus the ability to figure out the correct strategy for this game would prove to be of great value.

The two tribes were named Sook Jai and Chuay Gahn, and Sook Jai had the first move. They started by taking 2 flags and leaving 19. Before reading on, pause a minute and think. If you were in their place, how many would you have chosen?

Write down your choice somewhere, and read on. To understand how the game should be played, and compare the correct strategy with how the two tribes actually played, it helps to focus on two very revealing incidents. First, each tribe had a few minutes to discuss the game among its own members before the play started. During this discussion within Chuay Gahn, one of the members, Ted Rogers, an African American software developer, pointed out, "At the end, we must leave them with four flags." This is correct: if Sook Jai faces 4 flags, it must take 1 or 2 or 3, leaving Chuay Gahn to take the remaining 3 or 2 or 1, respectively, at its next turn and win the game. Chuay Gahn did in fact get and exploit this opportunity correctly; facing 6 flags, they took 2.

But here is the other revealing incident. At the previous turn, just as Sook Jai returned from having taken 3 flags out of the 9 facing them, the realization hit one of their members, Shii Ann, a feisty and articulate competitor who took considerable pride in her analytical skills: "If Chuay Gahn now takes two, we are sunk." So Sook Jai's just-completed move was wrong. What should they have done?

Shii Ann or one of her Sook Jai colleagues should have reasoned as Ted Rogers did but carried the logic of leaving the other tribe with 4 flags to its next step. How do you ensure leaving the other tribe with 4 flags at its next turn? By leaving it with 8 flags at its previous turn. When it takes 1 or 2 or 3 out of eight, you take 3 or 2 or 1 at your next turn, leaving them with 4 as planned. Therefore Sook Jai should have turned the tables on Chuay Gahn and taken just 1 flag out of the 9. Shii Ann's analytical skill kicked into high gear one move too late! Ted Rogers perhaps had the better analytical insights. But did he?

How did Sook Jai come to face 9 flags at its previous move? Because Chuay Gahn had taken 2 from 11 at *its* previous turn. Ted Rogers should have carried his own reasoning one step further. Chuay Gahn should have taken 3, leaving Sook Jai with 8, which would be a losing position.

The same reasoning can be carried even farther back. To leave the other tribe with 8 flags, you must leave them with 12 at their previous turn; for that you must leave them with 16 at the turn before that and 20 at the turn before that. So Sook Jai should have started the game by taking just 1 flag, not 2 as it actually did. Then it could have had a sure win by leaving Chuay Gahn with 20, 16,…4 at their successive turns.*

Now think of Chuay Gahn's very first turn. It faced 19 flags. If it had carried its own logic back far enough, it would have taken 3, leaving Sook Jai with 16 and already on the way to certain defeat. Starting from any point in the middle of the game where the opponent has played incorrectly, the team with the turn to move can seize the initiative and win. But Chuay Gahn did not play the game perfectly either.*

The table below shows the comparison between the actual and the correct moves at each point in the game. (The entry "No move" means that all moves are losing moves if the opponent plays correctly.) You can see that almost all the choices were wrong, except Chuay Gahn's move when facing 13 flags, and that must have been accidental, because at their next turn they faced 11 and took 2 when they should have taken 3.

Tribe	No. of flags before move	No. of flags taken	Move to put team on path to sure victory
Sook Jai	21	2	1
Chuay Gahn	19	2	3
Sook Jai	17	2	1
Chuay Gahn	15	1	3
Sook Jai	14	1	2
Chuay Gahn	13	1	1
Sook Jai	12	1	No move
Chuay Gahn	11	2	3
Sook Jai	9	3	1
Chuay Gahn	6	2	2
Sook Jai	4	3	No move
Chuay Gahn	1	1	1

Before you judge the tribes harshly, you should recognize that it takes time and some experience to learn how to play even very simple games. We have played this game between pairs or teams of students in our classes and found that it takes Ivy League freshmen three or even four plays before they figure out the complete reasoning and play correctly all the way through from the first move. (By the way, what number did you choose when we asked you to initially, and what was your reasoning?) Incidentally, people seem to learn faster by watching others play than by playing themselves; perhaps the perspective of an observer is more conducive to seeing the game as a whole and reasoning about it coolly than that of a participant.

TRIP TO THE GYM NO. 1

Let us turn the flag game into hot potato: now you win by forcing the other team to take the last flag. It's your move and there are 21 flags. How many do you take?

To fix your understanding of the logic of the reasoning, we offer you the first of our Trips to the Gym—questions on which you can exercise and hone your developing skills in strategic thinking. The answers are in the Workouts section in the end of the book.