Multicollinearity

EC 339

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Motivation

Linear relationships

Let us recall **CLRM Assumption VI**:

> *No explanatory variable is a perfect linear function of any other explanatory variable.*

This assumption implies a **deterministic** relationship between two independent variables.

$$x_1 = \alpha_0 + \alpha_1 x_3$$

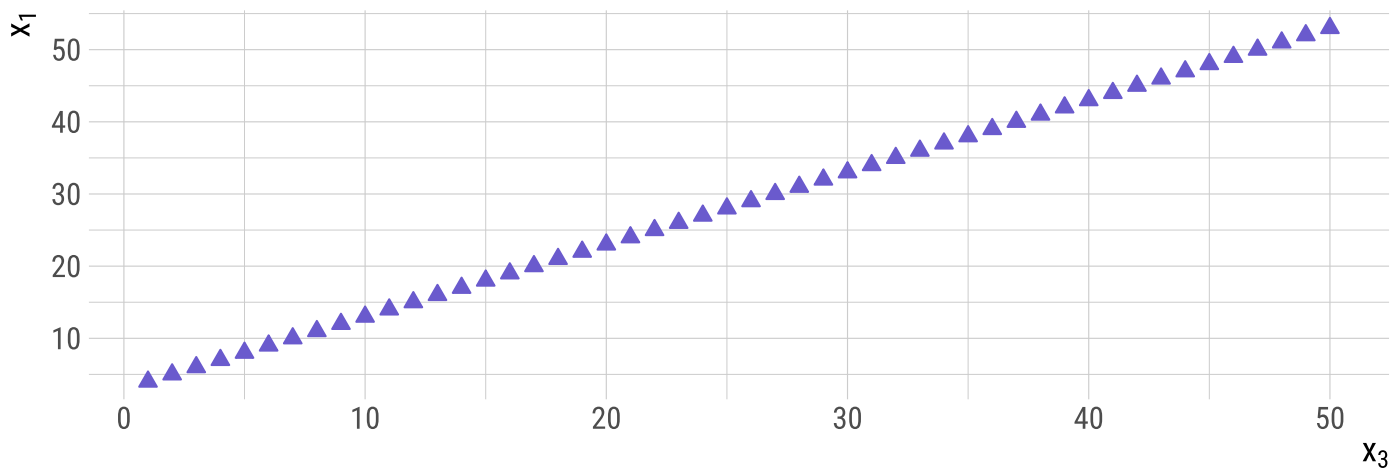
However, in practice we should worry more about strong **stochastic** relationships between two independent variables.

$$x_1 = \alpha_0 + \alpha_1 x_3 + \epsilon_i$$

Linear relationships

What does a linear relationship between two independent variables mean in practice?

- If two variables (say, x_1 and x_3) move **together**, then how can OLS **distinguish** between the effects of these two on y?
    - It **cannot**!



Perfect multicollinearity

Perfect multicollinearity

CLRM Assumption VI only refers to **perfect** multicollinearity.

With its presence, OLS estimation is **indeterminate**.

- Why?

How to disentangle the effect of each independent variable on y?

The *ceteris paribus* assumption no longer holds.

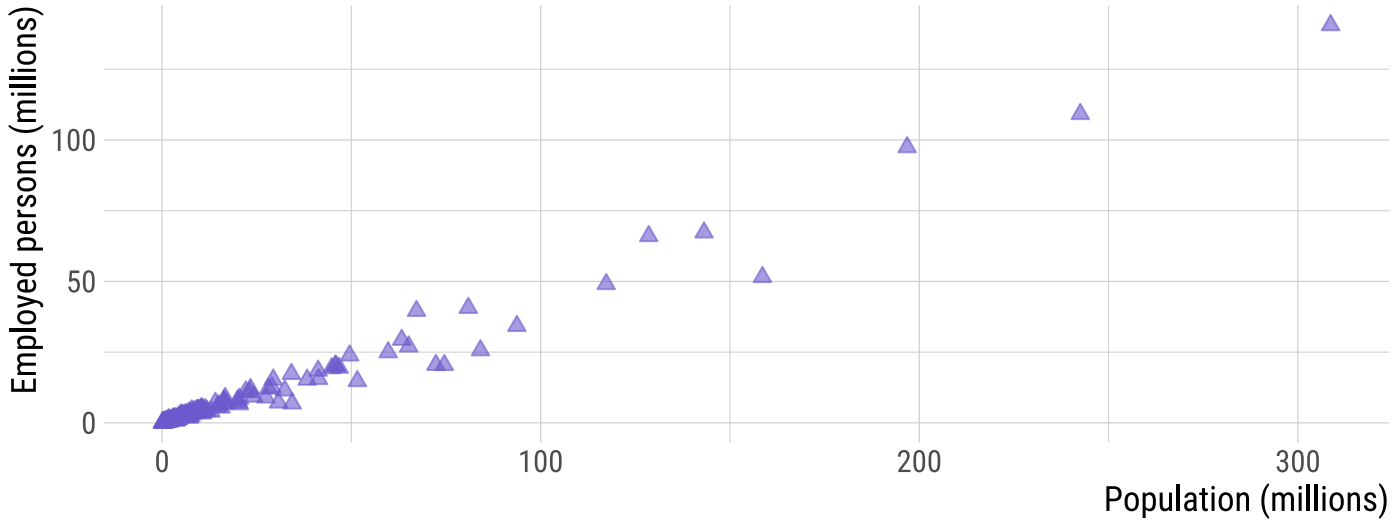
- **Good news**: *rare* to occur in practice.

Imperfect multicollinearity

Imperfect multicollinearity

Even though CLRM Assumption VI **does not** contemplate this version of multicollinearity, it is an actual problem within OLS estimation.

Strong **stochastic** relationships imply strong **correlation coefficients** between two independent variables.



Imperfect multicollinearity

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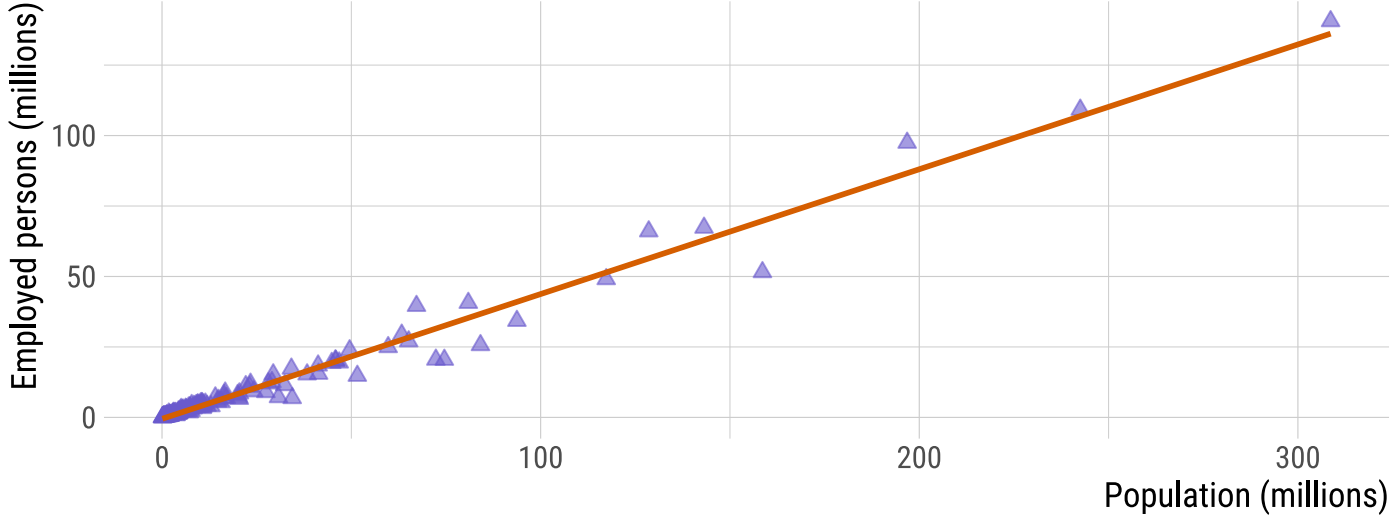

Consequences of multicollinearity

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By itself, multicollinearity **does not** cause **bias** to OLS β coefficients.

However, it affects OLS **standard errors**.

Recall that standard errors are part of the **t-test formula**:

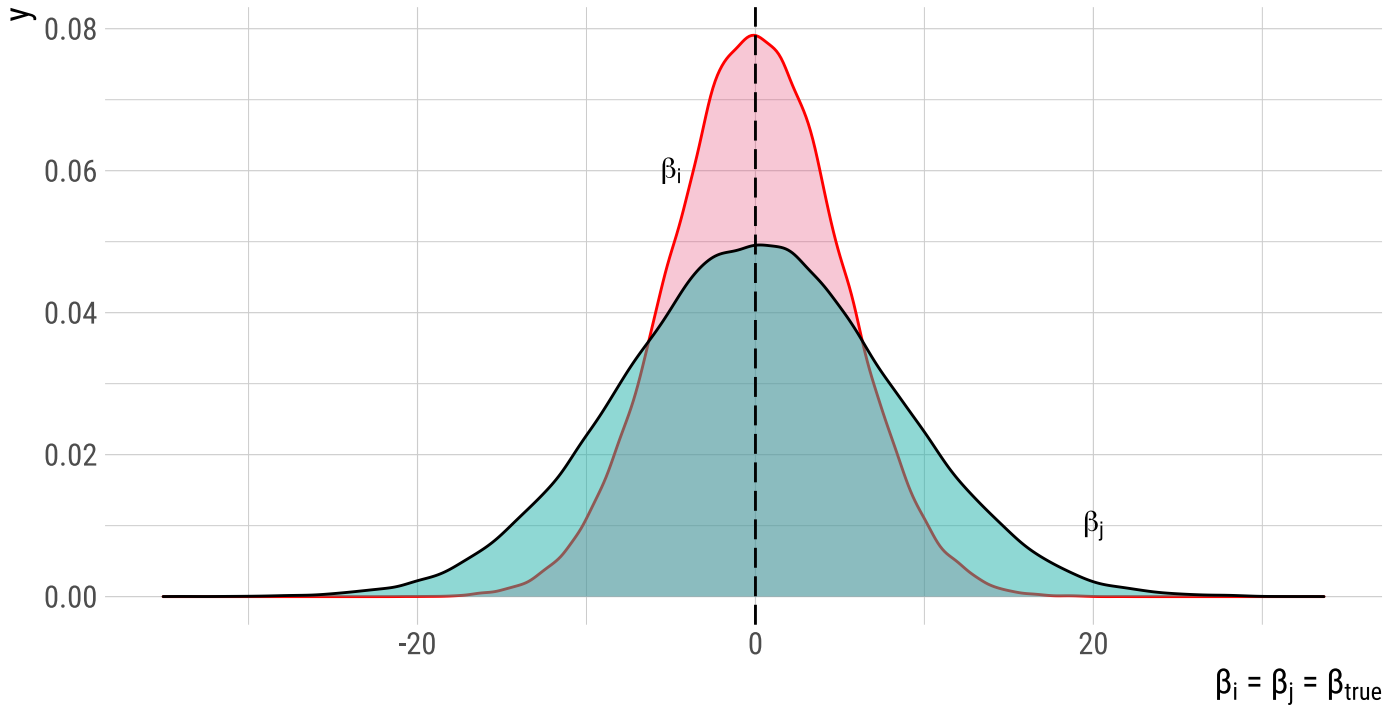
$$t = \frac{\hat{\beta}_k}{SE(\hat{\beta}_k)}$$

Therefore, it affects OLS **inference**.

Consequences of multicollinearity

Visually:

- Which estimate is *relatively more efficient*?



Dealing with multicollinearity

Dealing with multicollinearity

Consider the following model:

$$log(rgdpna_i) = \beta_0 + \beta_1 pop_i + \beta_2 emp_i + \beta_3 ck_i + \beta_4 ccon_i + u_i$$

where (for each country *i*):

- `rgdpna`: real GDP (millions 2011 USD)
- `pop`: population (millions)
- `emp`: number of employed persons (millions)
- `ck`: capital services levels (index, USA = 1)
- `ccon`: real consumption (households and government)

Dealing with multicollinearity

```
#>
#> ================================================
#>                          Dependent variable:
#>                      ---------------------------
#>                              log(rgdpna)
#> ------------------------------------------------
#> pop                           0.050***
#>                               (0.018)
#> emp                           -0.069
#>                               (0.042)
#> ck                            26.632***
#>                               (6.518)
#> ccon                          -0.00000***
#>                               (0.00000)
#> Constant                      10.785***
#>                               (0.145)
#> ------------------------------------------------
#> Observations                    130
#> R2                             0.478
#> Adjusted R2                    0.461
#> Residual Std. Error       1.404 (df = 125)
#> F Statistic           28.605*** (df = 4; 125)
#> ================================================
#> Note:               *p<0.1; **p<0.05; ***p<0.01
```

Dealing with multicollinearity

A little modification:

$$log(rgdpna_i) = \beta_0 + \beta_1 log(emp_i) + \beta_3 ck_i + \beta_4 log(ccon_i) + u_i$$

Dealing with multicollinearity

```
#>
#> ================================================
#>                          Dependent variable:
#>                      ---------------------------
#>                              log(rgdpna)
#> ------------------------------------------------
#> log(emp)                      -0.059**
#>                                (0.029)
#> ck                             -0.206
#>                                (0.288)
#> log(ccon)                     1.076***
#>                                (0.027)
#> Constant                       -0.487*
#>                                (0.275)
#> ------------------------------------------------
#> Observations                     130
#> R2                              0.979
#> Adjusted R2                     0.979
#> Residual Std. Error       0.277 (df = 126)
#> F Statistic          2,001.826*** (df = 3; 126)
#> ================================================
#> Note:                *p<0.1; **p<0.05; ***p<0.01
```

Dealing with multicollinearity

Checking **correlation** coefficients:

- $Corr(pop_i, emp_i) = 0.987$
- $Corr(ccon_i, emp_i) = 0.980$
- $Corr(log(ccon_i), emp_i) = 0.584$

Dealing with multicollinearity

A recommended procedure is to always check out the **correlation coefficient** among the chosen independent variables.

- In addition, we can calculate **Variance Inflation Factors** (VIFs):

$$VIF(\hat{\beta}_i) = \frac{1}{(1 - R_i^2)}$$

where R_i^2 is the coefficient of determination of the *auxiliary regression* models.

- The procedure is to estimate one auxiliary regression model for *each* independent variable.
- Then, store the R^2 for each regression.
- A *VIF* greater than 5 is already sufficient to imply high multicollinearity.

Dealing with multicollinearity

In `Stata`...

```
reg lrdgpna pop emp ck ccon

vif
```

Variable	VIF	1/VIF
emp	48.52	0.020608
pop	42.69	0.023425
ck	30.44	0.032854
ccon	**27.30**	**0.036626**
Mean VIF	37.24	

- What do we conclude?

Dealing with multicollinearity

In `Stata`...

```
reg lrdgpna lemp ck lccon

vif
```

Variable	VIF	1/VIF
lccon	4.24	0.236040
lemp	3.72	0.268975
ck	**1.52**	**0.659385**
Mean VIF	3.16	

- What do we conclude?

Next time: Multicollinearity in practice