

Multicollinearity

EC 339

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Motivation

Linear relationships

Let us recall **CLRM Assumption VI**:

*No explanatory variable is a **perfect linear function** of any other explanatory variable.*

This assumption implies a **deterministic** relationship between two independent variables.

$$x_1 = \alpha_0 + \alpha_1 x_3$$

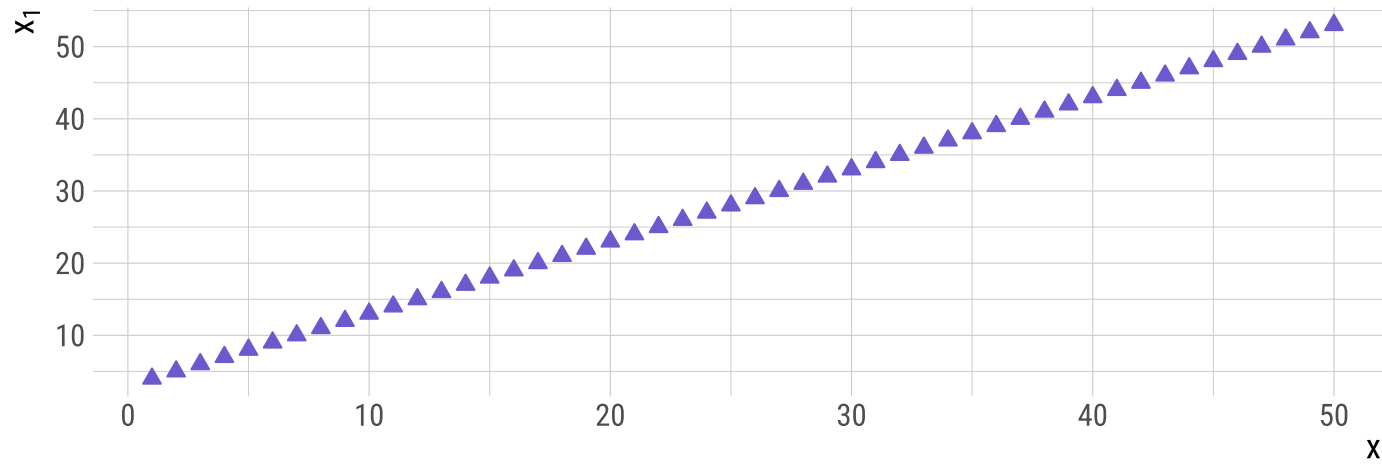
However, in practice we should worry more about strong **stochastic** relationships between two independent variables.

$$x_1 = \alpha_0 + \alpha_1 x_3 + \epsilon_i$$

Linear relationships

What does a linear relationship between two independent variables mean in practice?

- If two variables (say, x_1 and x_3) move **together**, then how can OLS **distinguish** between the effects of these two on y ?
 - It **cannot**!



Perfect multicollinearity

Perfect multicollinearity

CLRM Assumption VI only refers to **perfect** multicollinearity.

With its presence, OLS estimation is **indeterminate**.

- Why?

How to **disentangle** the effect of each independent variable on y ?

The *ceteris paribus* assumption no longer holds.

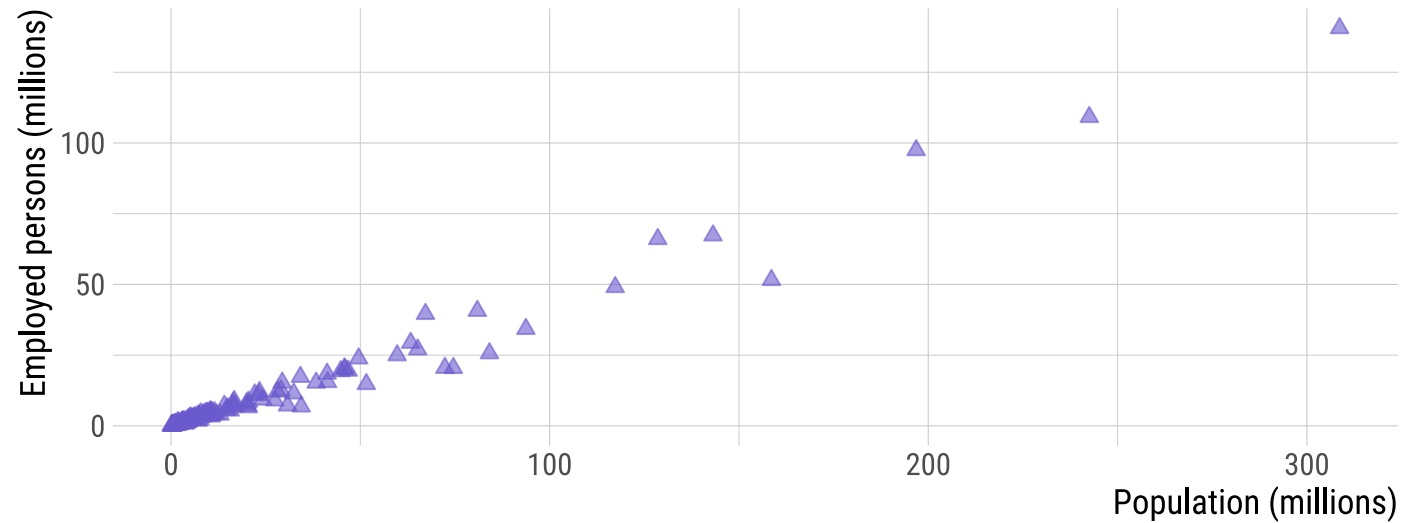
- **Good news**: *rare* to occur in practice.

Imperfect multicollinearity

Imperfect multicollinearity

Even though CLRM Assumption VI **does not** contemplate this version of multicollinearity, it is an actual problem within OLS estimation.

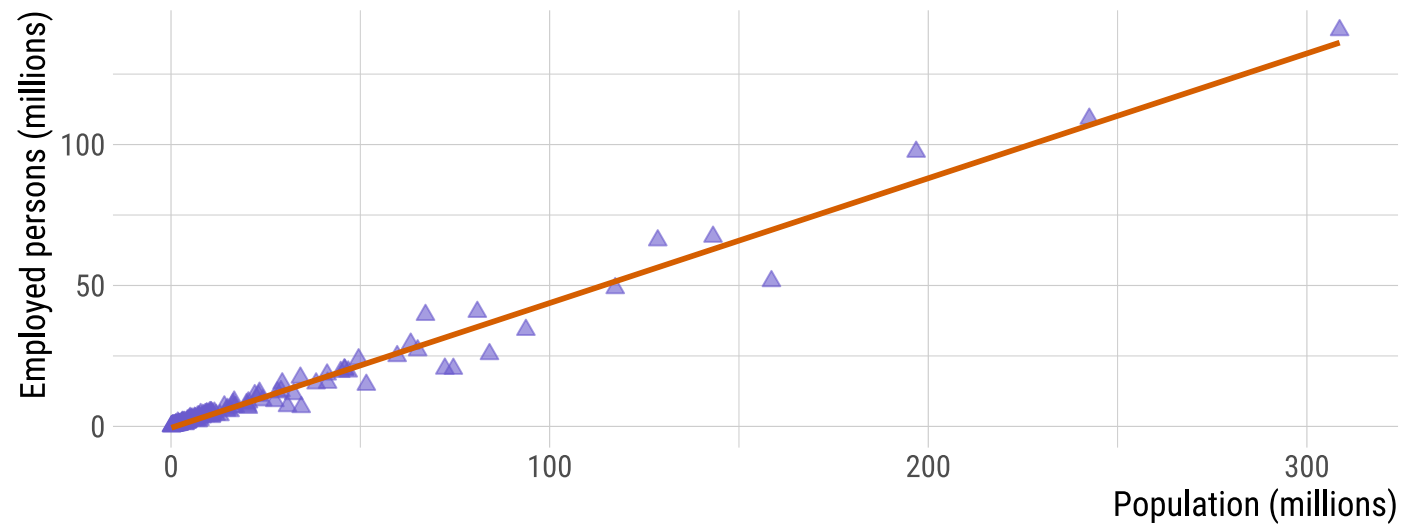
Strong **stochastic** relationships imply strong **correlation coefficients** between two independent variables.



Imperfect multicollinearity

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Consequences of multicollinearity

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By itself, multicollinearity **does not** cause **bias** to OLS β coefficients.

However, it affects OLS **standard errors**.

Recall that standard errors are part of the **t-test formula**:

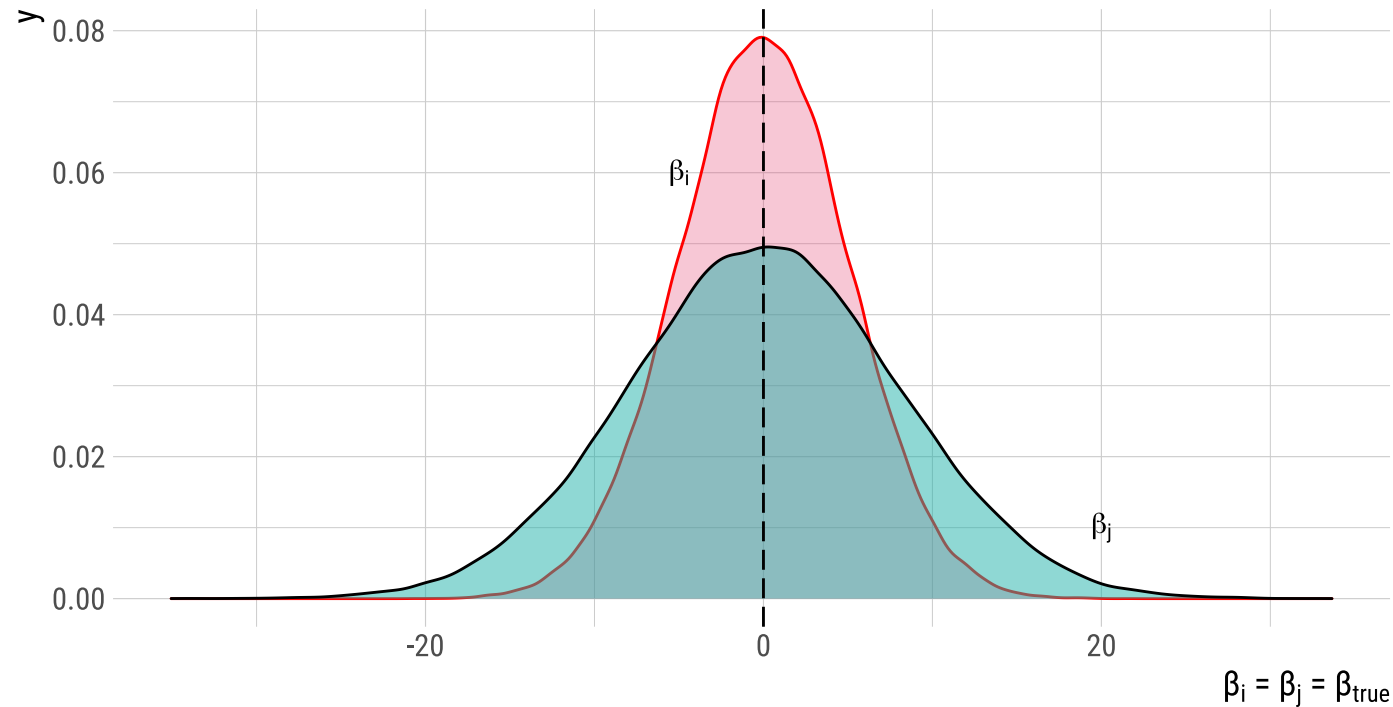
$$t = \frac{\hat{\beta}_k}{SE(\hat{\beta}_k)}$$

Therefore, it affects OLS **inference**.

Consequences of multicollinearity

Visually:

- Which estimate is *relatively more efficient*?



Dealing with multicollinearity

Dealing with multicollinearity

Consider the following model:

$$\log(\text{rgdpna}_i) = \beta_0 + \beta_1 \text{pop}_i + \beta_2 \text{emp}_i + \beta_3 \text{ck}_i + \beta_4 \text{ccon}_i + u_i$$

where (for each country i):

- `rgdpna`: real GDP (millions 2011 USD)
- `pop`: population (millions)
- `emp`: number of employed persons (millions)
- `ck`: capital services levels (index, USA = 1)
- `ccon`: real consumption (households and government)

Dealing with multicollinearity

```
#>
#> =====
#>                               Dependent variable:
#>                               -----
#>                               log(rgdpna)
#> -----
#> pop                          0.050***
#>                               (0.018)
#> emp                          -0.069
#>                               (0.042)
#> ck                           26.632***
#>                               (6.518)
#> ccon                         -0.00000***
#>                               (0.00000)
#> Constant                     10.785***
#>                               (0.145)
#> -----
#> Observations                  130
#> R2                           0.478
#> Adjusted R2                   0.461
#> Residual Std. Error          1.404 (df = 125)
#> F Statistic                   28.605*** (df = 4; 125)
#> =====
#> Note:                        *p<0.1; **p<0.05; ***p<0.01
```

Dealing with multicollinearity

A little modification:

$$\log(\text{rgdpna}_i) = \beta_0 + \beta_1 \log(\text{emp}_i) + \beta_3 \text{ck}_i + \beta_4 \log(\text{ccon}_i) + u_i$$

Dealing with multicollinearity

```
#>
#> =====
#>                               Dependent variable:
#>                               -----
#>                               log(rgdpna)
#> -----
#> log(emp)                      -0.059**
#>                               (0.029)
#> ck                            -0.206
#>                               (0.288)
#> log(ccon)                     1.076***
#>                               (0.027)
#> Constant                      -0.487*
#>                               (0.275)
#> -----
#> Observations                   130
#> R2                            0.979
#> Adjusted R2                   0.979
#> Residual Std. Error           0.277 (df = 126)
#> F Statistic                   2,001.826*** (df = 3; 126)
#> =====
#> Note:                         *p<0.1; **p<0.05; ***p<0.01
```

Dealing with multicollinearity

Checking **correlation** coefficients:

- $\text{Corr}(\text{pop}_i, \text{emp}_i) = 0.987$
- $\text{Corr}(\text{ccon}_i, \text{emp}_i) = 0.980$
- $\text{Corr}(\log(\text{ccon}_i), \text{emp}_i) = 0.584$

Dealing with multicollinearity

A recommended procedure is to always check out the **correlation coefficient** among the chosen independent variables.

- In addition, we can calculate **Variance Inflation Factors** (VIFs):

$$VIF(\hat{\beta}_i) = \frac{1}{(1 - R_i^2)}$$

where R_i^2 is the coefficient of determination of the *auxiliary regression* models.

- The procedure is to estimate one auxiliary regression model for *each* independent variable.
- Then, store the R^2 for each regression.
- A *VIF* greater than 5 is already sufficient to imply high multicollinearity.

Dealing with multicollinearity

In Stata...

```
reg lrdgpna pop emp ck ccon
```

```
vif
```

Variable	VIF	1/VIF
emp	48.52	0.020608
pop	42.69	0.023425
ck	30.44	0.032854
ccon 	27.30	0.036626
Mean VIF	37.24	

- What do we conclude?

Dealing with multicollinearity

In Stata...

```
reg lrdgpna lemp ck lccon
```

```
vif
```

Variable	VIF	1/VIF
-----+-----		
lccon	4.24	0.236040
lemp	3.72	0.268975
ck 	1.52	0.659385
-----+-----		
Mean VIF	3.16	

- What do we conclude?

Next time: Multicollinearity in practice