Probability Theory, pt. III

ECON 3640–001

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Motivation

Housekeeping

Notes based on `Blitzstein & Hwang (2019)`, ch. 2

- sections 2.3 & 2.4.

Motivation

Last time, we saw that **conditional probabilities** are the "*soul*" of Statistics.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Although being an extremely **simple** definition, it has far-reaching *applications* and *possibilities*.

An application

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Based on survey data, `CNBC ran a study last Summer regarding vaccine mandates`.

With a sample size (n) of 802 individuals, the survey found that:

- 68% of Americans had been vaccinated;
- Among those who had been vaccinated, 63% approved of vaccine mandates;
- Among unvaccinated interviewees, 17% supported these mandates.

As a **first task**, set up a **contingency table** for these data.

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Secondly, given that an individual **supports** a vaccine mandate, what is the probability that they **are vaccinated**?

The law of total probability

The law of total probability

Without explicitly calling for it, in the previous exercise, you have applied the **Law of Total Probability**.

It directly follows from the definition of **conditional probability** that

$$P(A \cap B) = P(B)P(A|B) = P(A)P(B|A)$$

Then, suppose the set of events $\{A_1, A_2, A_3, \ldots, A_k\}$ partition the sample space S. For any event $B \subseteq S$

$$B = \bigcup_{i=1}^{k} (B \cap A_i) = (B \cap A_1) \ \cup \ (B \cap A_2) \ \cup \ \ldots \ \cup \ (B \cap A_k)$$

For pairwise **disjoint** events,

$$P(B) = \sum_{i=1}^{k} P(B \cap A_i) = \sum_{i=1}^{k} P(B|A_i)P(A_i)$$

The law of total probability

In case we have the simple partition $\{A, A^C\}$, the **Law of Total Probability** looks like

$$P(B) = P(B \cap A) + P(B \cap A^C) = P(A)P(B|A) + P(A^C)P(B|A^C)$$

Therefore, the **LTP** is useful when we want to compute an **unconditional** probability, such as $P(B)$, and the only available information are conditional probabilities, $P(B|A_i)$.

Bayes' Theorem

Bayes' Theorem

Another thing that you have done was to use **Bayes' Theorem** without calling for it.

It is defined by

$$P(A|B) = \frac{P(A)\,P(B|A)}{P(B)}$$

It tells us that the **posterior** probability of A, in light of information B, $P(A|B)$, is given by

- The **prior** probability of A, $P(A)$;
- The **chances** of observing data B if A occurs, $P(B|A)$[1];
- The **overall** chance of observing B, $P(B)$.

1: This part is also called the **likelihood**.

Bayes' Theorem

$$P(A|B) = \frac{P(A)\,P(B|A)}{P(B)}$$

This theorem is extremely useful when we want to know the *conditional* probability $P(A|B)$, but only have knowledge of the *reverse conditional*, $P(B|A)$.

As we will explore in detail in future lectures, the above theorem is the foundation of **Bayesian Statistics**.

Next time: Random variables and probability distributions