Data Structures and Algorithms with Python

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Lecture 5

We've covered a lot

Toolkit:

- Computational and algorithmic thinking
- Complexity analysis
- Data structures and OOP
- Python for problem solving

Applications:

- Square roots, palindromes, etc
- Searching and sorting
- Monster fights
- Shortest paths in social networks (Kevin Bacon): BFS; Dijkstra's algorithm (extra)
- Greedy algorithms and heuristics, knapsack problem (today)

Heron's recipe

How to find the square root of x:

- Start with a guess g
- If $g * g$ is close to x, stop and return g as the answer
- Otherwise make new guess as the average of g and x/g
- Repeat process using new guess

```python
eps = 0.01
x = 50
guess = x/2
while abs(guess**2-x) >= eps:
    guess = (guess + x/guess)/2
print("The square root of ", x, " is approximately:", guess)
```

Analysing algorithm complexity

Principle 0: measure amount of work the computer does: count basic operations as function of input size

Principle 1: focus on worst-case analysis

Principle 2: ignore constant factors and lower-order terms

Principle 3: only care about large inputs

Formal way to describe this approach:

- Big-Oh notation: upper bound on worst-case running time

Binary search

Algorithm for finding x in sorted list L:

- Pick an index i roughly dividing L in half
- If $L[i] == x$, return True (if nothing left to search return False)
- If not:
  - If $L[i] > x$, recursively search left half of L
  - Otherwise recursively search right half

Find number 24 in a list $L = [9, 24, 32, 56, 57, 61, 59, 99]$

First iteration

9	24	32	**56**	57	59	61	99

9	24	32	56	57	59	61	99

$L[i] = 56 > 24$ —> discard right half and recursively call binary search on left half

Second iteration

9	**24**	32	56	57	59	61	99

$L[i] = 24$ —> return True

Complexity classes

Fast algorithm: worst-case running time grows slowly with input size

- $O(1)$: constant running time — basic operations
- $O(\log n)$: logarithmic running time — binary search
- $O(n)$: linear running time — linear search
- $O(n \log n)$: log-linear running time — merge sort
- $O(n^c)$: polynomial running time — selection sort
- $O(c^n)$: exponential running time — ??

Everything is an `object`

An **object** has:

- A type: `int`, `str`, `list` (`L=[0,1]` is an **instance** of a `list`)
- An internal representation of data
- A set of functions that operate on that data

```
L = [1,1999,0,-2,9]
L.append(8)
L.insert(2,1000)
t = L.pop()
L.remove(1)
help(L)
```

The point:

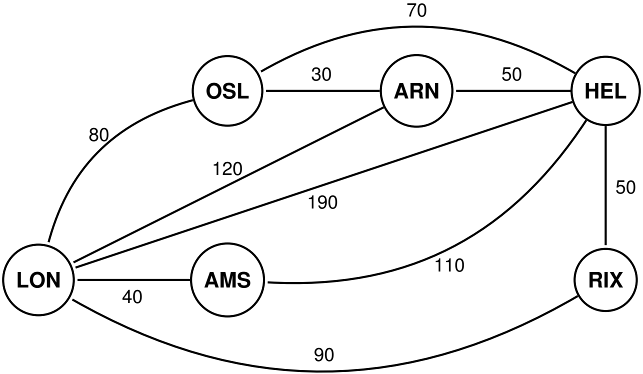
- Interface: the user knows what she can do with a list
- Abstraction barrier: the user does not need to know the details of what goes on under the hood (similarly to functions)
- Invaluable in managing complexity of programs

How to organize data?

What is the right data structure for my problem?

- What operations do I need to perform on data?
- Is a built-in data structure enough?
- Is algorithm efficiency important?

Graphs



A set of **nodes** (or vertices) connected by **edges**

Edges may or may not be directed -> **directed** or **undirected** graph (Twitter vs Facebook?)

Edges may have **weights** (eg prices): weighted graph

Breadth-first search (BFS)

Find shortest-distance paths between people in social networks?

- Path: a way to get from node v to node w via graph edges
- Distance: how many steps you need to take

Idea: explore graph in layers

- Choose starting node
- Explore all nodes connected to the starting node (1st degree of separation)
- Explore all nodes connected to these nodes (2nd degree of separation)
- Repeat...

Breadth-first search (BFS)

Algorithm: BFS(unweighted graph G, starting node s)

- **Initialize**: mark all nodes unexplored except s explored
- Use Q = queue data structure, add s to Q
- **Main loop**: While Q is not empty:
  - Remove the first node of Q and call it v
  - For each edge (v, w): if w unexplored:
    - Mark w explored
    - Add w to Q

What is a **queue**?

- First in, first out (just like in a café)
- Remove from front, insert to back in constant time $O(1)$
- Here we keep a queue of nodes to process next

Knapsack problem

Problem: fill a bag with the most valuable items available.

Input:

- Set of n items, with values v_i and sizes w_i (integer)
- Capacity W

Output: subset S of items that maximizes the sum of values subject to a capacity constraint:

- max $\sum_{i \in S} v_i$
- subject to $\sum_{i \in S} w_i \leq W$

Knapsack problem applications

Many problems with budget constraints are versions of knapsack

- Selecting portfolios (eg projects to invest in)

Two-item example

Items to pack

- **Shirt**: value 5, weight 5
- **Bottle**: value 10, weight 5

Subsets:

- {},{**Shirt**},{**Bottle**},{**Shirt**,**Bottle**}

Knapsack size limits feasible solutions

- $W < 5$: can pick neither
- $5 \leq W < 10$: neither or just one
- $W \geq 10$: all subsets feasible

Go through all possibilities?

Input:

- Set of n items, with values v_i and sizes w_i (integer)
- Capacity W

Output: subset S of items that maximizes the sum of values subject to capacity constraint:

- max $\sum_{i \in S} v_i$
- subject to $\sum_{i \in S} w_i \leq W$

Exhaustive (brute-force) search?

- Go through all subsets of $\{1, 2, 3, ..., n\}$
- Suppose we have 50 items: $O(2^n)$ subsets...

Greedy approaches for knapsack problem

Input:

- Set of n items, with values v_i and sizes w_i
- Capacity W

Output: subset S of items that maximizes the sum of values subject to capacity constraint:

- max $\sum_{i \in S} v_i$
- subject to $\sum_{i \in S} w_i \leq W$

Greedy approaches? — Pick myopically without worrying about future choices

- **Highest-value** item first?
- **Lowest-size** item first?
- Some easy way of **combining value and size**?

Greedy algorithms for knapsack

Greedy approaches?

- **Highest-value** item first?
- **Lowest-size** item first?

Example: capacity $W = 20$, three item with values $v = [10, 10, 11]$, weights $w = [10, 10, 20]$.

- Picking highest value first is **bad**...

Example: capacity $W = 20$, three item with values $v = [10, 10, 20]$, weights $w = [10, 10, 11]$.

- Picking lowest weight first is **bad**...

Some easy way of **combining value and size**?

- Eg **sort items by unit weight** v_i / w_i and pick them in this order

Greedy algorithm for knapsack

Greedy algorithm:

- Sort items in decreasing v_i/w_i
- Pick items until capacity full

Running time?

- Sorting?
- Loop?
- Total $O(n \log n)$

Is the algorithm correct?

It would be if we assumed that we **can divide items into fractions**

- But we cannot...

Example: capacity W=510, three items with values $v = [10, 10, 500]$, weights $w = [10, 10, 501]$.

- Greedy picking is **bad**...

We would need a different approach to find correct solution — dynamic programming

- Exploit problem structure to loop through all items and possible capacities
- Correct solution, $O(nW)$ time ("pseudo-polynomial")

Many important problems are intractable

Tractable problem = Solvable in polynomial time $O(n^k)$ for some k

Finding the **shortest route to a Pikachu nest** (Dijkstra) vs. finding the **shortest tour of all Pokemon nests**

Example of intractability: **traveling salesman problem** (TSP)

- **Input**: undirected graph with non-negative edge costs
- **Goal**: find minimum cost tour visiting every node
- **Conjecture**: no polynomial-time algorithm [**sidebar: P vs NP**]
- Many other important problems too...

My problem is intractable!

What can you do?

1. There may be **tractable special cases** (small knapsack DP)
2. Get an **approximate solution using heuristics** — fast but not "correct" (next slide)
3. Solve in exponential time but try to **improve on brute force** (large knapsack DP)

Greedy knapsack heuristic

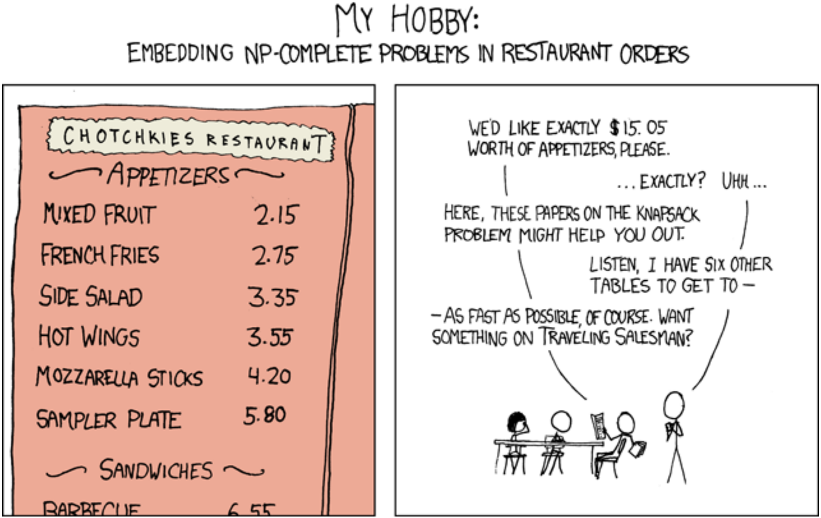
Greedy knapsack was incorrect but very fast: $O(n \log n)$

Greedy algorithm:

- Sort items in decreasing bang-for-buck v_i/w_i
- For each item, pick the item until reach total W

How bad is it?

- **Example**: capacity W=510, three items with values $v = [10, 10, 500]$, weights $w = [10, 10, 501]$.
- "Worst-case scenario": leave out (a single) extremely valuable object that would fit into knapsack
- **But often items are small** $\rightarrow$ the greedy choice cannot leave out many of them $\rightarrow$ the heuristic will be much better


MY HOBBY:
EMBEDDING NP-COMPLETE PROBLEMS IN RESTAURANT ORDERS
CHOTCHKIES RESTAURANT
APPETIZERS
MIXED FRUIT 2.15
FRENCH FRIES 2.75
SIDE SALAD 3.35
HOT WINGS 3.55
MOZZARELLA STICKS 4.20
SAMPLER PLATE 5.80
SANDWICHES
WE'D LIKE EXACTLY $15.05 WORTH OF APPETIZERS, PLEASE.
...EXACTLY? UHH...
HERE, THESE PAPERS ON THE KNAPSACK PROBLEM MIGHT HELP YOU OUT.
LISTEN, I HAVE SIX OTHER TABLES TO GET TO –
– AS FAST AS POSSIBLE, OF COURSE. WANT SOMETHING ON TRAVELING SALESMAN?

Pic: xkcd

Paris Where to eat for under €10

travel →

Morgana Robinson TV's top mimic

guide →

America and the gun by Gary Younge

weekend →

New exclusive collection of 20 simple recipes

Free Magazine

Easy Ottolenghi

Plus how to survive a sexless marriage

family →

theguardian

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One in seven takeaways fail hygiene tests

Guardian analysis also reveals one in 13 restaurants fall short of basic standards

Is this the most powerful woman in fashion?

Warning that damage to Labour could be 'terminal'

Be coldhearted.

Remember Greenpeace in your Will and help keep the Arctic as it should be. Cold.

Find out how to make space for a polar bear or two in your Will:

www.greenpeace.org.uk/polarbears

GREENPEACE

https://www.theguardian.com/world/2016/sep/23/revealed-uk-takeaways-fail-food-hygiene-tests-restaurants-takeaways

Workshop

After the break...

Knapsack problem and fantasy football

Extra: Python for your safety — South Ken food hygiene

Algorithms, Python, and your future