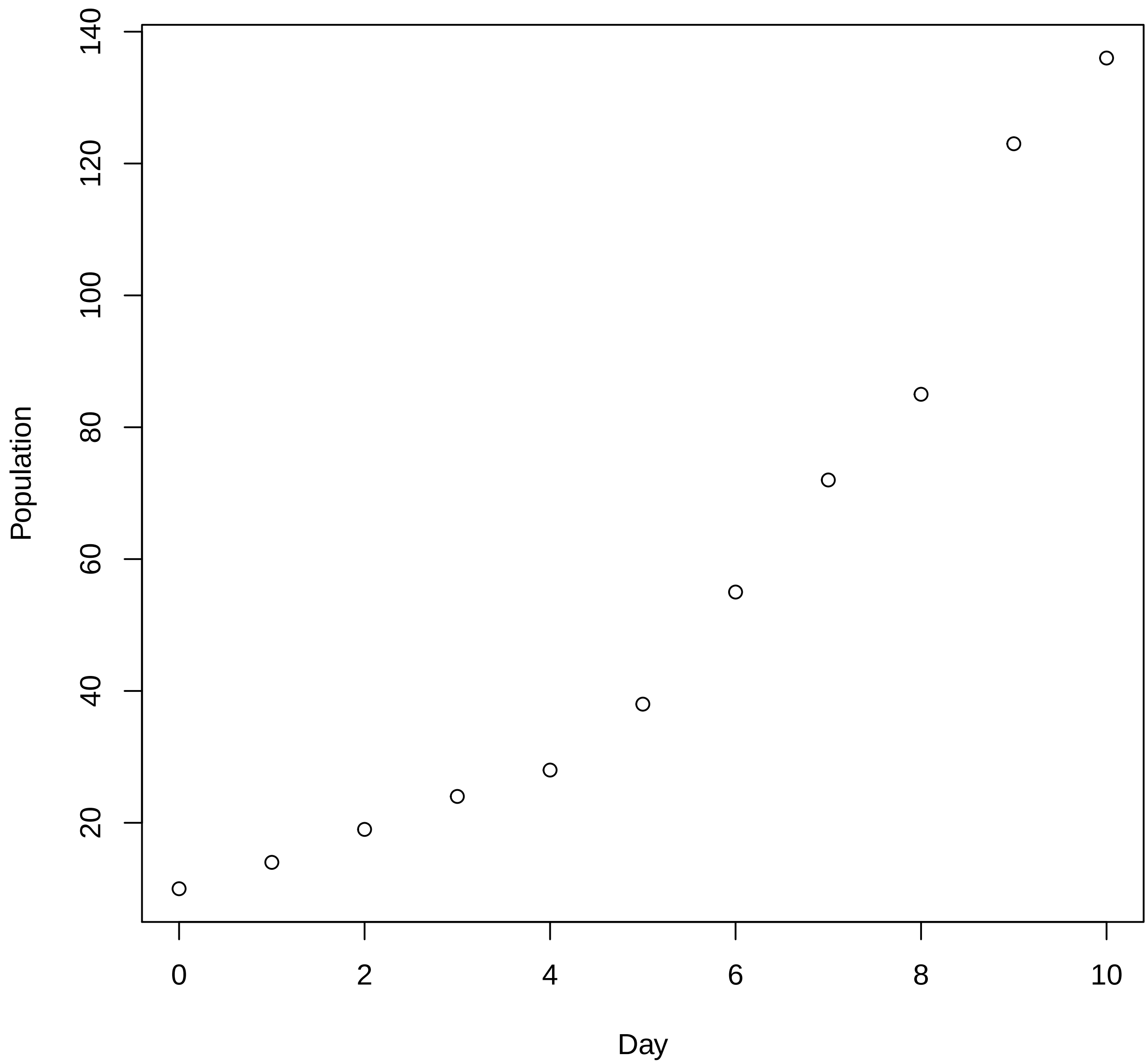
0.1 Population Growth Models

```r
day = c(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)
N = c(10, 14, 19, 24, 28, 38, 55, 72, 85, 123, 136)
q1.df = data.frame(Day = day, Population = N)
```

```r
plot(Population ~ Day, data=q1.df)
```



b) For more data, use **linear regression**: given $(t_0, P_0), (t_1, P_1), \ldots, (t_{N-1}, P_{N-1})$; find $r, \ln(K)$ to minimize $\sum_{n=0}^{N-1}(\ln(P_n) - \ln(K) - rt_n)^2$.

POPULATION GROWTH MODELS CONT.

Example Fruit fly data:

Day	0	1	2	3	4	5	6	7	8	9	10
Pop	10	14	19	24	28	38	55	72	85	123	136

Computer solution: $r \approx .26463$, $K \approx 10.563$

POPULATION GROWTH MODELS CONT.

Logistic Growth Model: $P' = r(1 - P/K)P$.
Modeling considers b and d to be dependent on size of P;
K is **carrying capacity** for population environment.

- Qualitative analysis:
  Equilibrium solutions?
  Stable or Unstable?

  Logistic Equation Solution:

$$P(t) = K/\big(1 + (K/P_0 - 1)e^{-r(t-t_0)}\big).$$

POPULATION GROWTH MODELS CONT.

Logistic Model Examples:

- $P_0 = 1000$, $P(8) = 1200$, eventual $P(t)$ is 20000.
  Find r, and t when $P(t)$ is 75% of K.

- More Fruit Fly Data

Day	0	3	7	9	12	15	18	21	24	28	32
Pop	6	10	21	52	67	104	163	226	265	282	319

Estimate r, K; and check model.

$$P(t) = K/\big(1 + (K/P_0 - 1)e^{-r(t-t_0)}\big).$$

For r, K, use 2 times, e.g. $t = 12, 24$, so

$67 = K/(1 + (K/6 - 1)e^{-12r})$, and

$265 = K/(1 + (K/6 - 1)e^{-24r})$.

Eliminate K and solve for $a = e^{-12r}$;

$K = 67(1 - a)/(1 - 67a/6)$,

$K = 265(1 - a^2)/(1 - 265a^2/6)$,

$(1 - 265a^2/6) = 265(1 + a)(1 - 67a/6)/67$;

$265/67 - 1 = 265(61)a/(67 \cdot 6)$; $a \approx .0735$,

$r \approx .218$, $K \approx 260$?, not consistent with data.

Try other data to find $K,\ r$?

Need to solve nonlinear equation for r.

Could use nonlinear least-squares fit.

US Population Modeling?

Can try to fit a logistic model, but predictions not good.

Need a model that includes immigration: e.g.

$$P' = r(1 - P/K)P + I$$

for immigration rate I, or

$$P' = r(t)(1 - P/K)P + I(t).$$