

07_homework_svm

December 8, 2017

1 Programming assignment 7: SVM

```
In [1]: import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline

from sklearn.datasets import make_blobs

from cvxopt import matrix, solvers
```

1.1 Your task

In this sheet we will implement a simple binary SVM classifier.

We will use CVXOPT <http://cvxopt.org/> - a Python library for convex optimization. If you use Anaconda, you can install it using

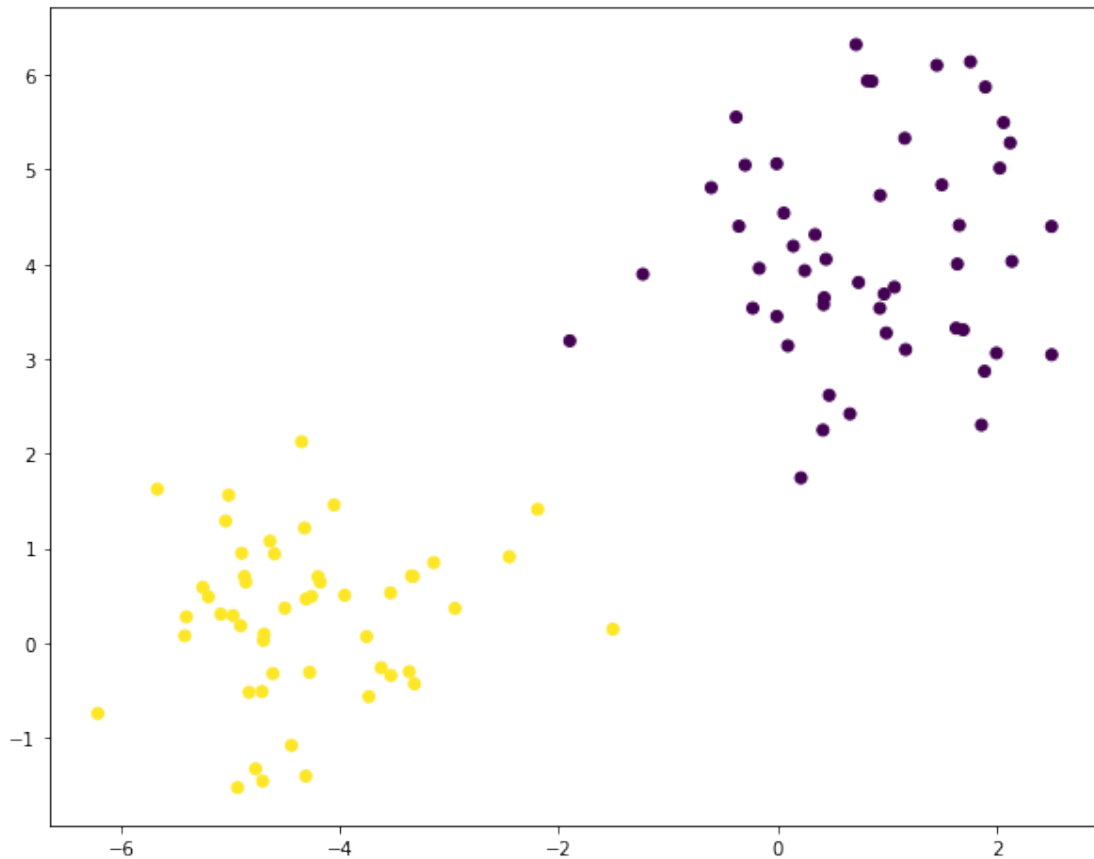
```
conda install cvxopt
```

As usual, your task is to fill out the missing code, run the notebook, convert it to PDF and attach it to your HW solution.

1.2 Generate and visualize the data

```
In [2]: N = 100 # number of samples
D = 2 # number of dimensions
C = 2 # number of classes
seed = 3 # for reproducible experiments

X, y = make_blobs(n_samples=N, n_features=D, centers=2, random_state=seed)
y[y == 0] = -1 # it is more convenient to have {-1, 1} as class labels (instead of {0, 1})
y = y.astype(np.float)
plt.figure(figsize=[10, 8])
plt.scatter(X[:, 0], X[:, 1], c=y)
plt.show()
```



1.3 Task 1: Solving the SVM dual problem

Remember, that the SVM dual problem can be formulated as a Quadratic programming (QP) problem. We will solve it using a QP solver from the CVXOPT library.

The general form of a QP is

$$\min_{\mathbf{x}} \frac{1}{2} \mathbf{x}^T \mathbf{P} \mathbf{x} - \mathbf{q}^T \mathbf{x}$$

$$\text{subject to } \mathbf{G} \mathbf{x} \preceq \mathbf{h}$$

$$\text{and } \mathbf{A} \mathbf{x} = \mathbf{b}$$

where \preceq denotes "elementwise less than or equal to".

Your task is to formulate the SVM dual problems as a QP and solve it using CVXOPT, i.e. specify the matrices \mathbf{P} , \mathbf{G} , \mathbf{A} and vectors \mathbf{q} , \mathbf{h} , \mathbf{b} .

```
In [3]: def solve_dual_svm(X, y):
        """Solve the dual formulation of the SVM problem.

        Parameters
```

```

-----
X : array, shape [N, D]
    Input features.
y : array, shape [N]
    Binary class labels (in {-1, 1} format).

Returns
-----
alphas : array, shape [N]
    Solution of the dual problem.
"""
# TODO
# These variables have to be of type cvxopt.matrix
N = len(y)
D = y[:, None] * X
D = np.dot(D, D.T)
P = matrix(D)
q = matrix(-np.ones((N, 1)))
G = matrix(-np.eye(N))
h = matrix(np.zeros(N))
A = matrix(y.reshape(1, -1))
b = matrix(np.zeros(1))
solvers.options['show_progress'] = False
solution = solvers.qp(P, q, G, h, A, b)
alphas = np.array(solution['x'])
return alphas

```

1.4 Task 2: Recovering the weights and the bias

```

In [4]: def compute_weights_and_bias(alpha, X, y):
    """Recover the weights w and the bias b using the dual solution alpha.

    Parameters
    -----
    alpha : array, shape [N]
        Solution of the dual problem.
    X : array, shape [N, D]
        Input features.
    y : array, shape [N]
        Binary class labels (in {-1, 1} format).

    Returns
    -----
    w : array, shape [D]
        Weight vector.
    b : float
        Bias term.
    """

```

```

N = len(y)
w = 0
for i in range(N):
    w += alpha[i] * y[i] * X[i, :]

ind = (alpha > 1e-10).reshape(-1)
b = np.sum(y[ind] - np.dot(X[ind, :], w))/len(ind)
return w, b

```

1.5 Visualize the result (nothing to do here)

```

In [5]: def plot_data_with_hyperplane_and_support_vectors(X, y, alpha, w, b):
        """Plot the data as a scatter plot together with the separating hyperplane.

        Parameters
        -----
        X : array, shape [N, D]
            Input features.
        y : array, shape [N]
            Binary class labels (in {-1, 1} format).
        alpha : array, shape [N]
            Solution of the dual problem.
        w : array, shape [D]
            Weight vector.
        b : float
            Bias term.
        """

        plt.figure(figsize=[10, 8])
        # Plot the hyperplane
        slope = -w[0] / w[1]
        intercept = -b / w[1]
        x = np.linspace(X[:, 0].min(), X[:, 0].max())
        plt.plot(x, x * slope + intercept, 'k-', label='decision boundary')
        # Plot all the datapoints
        plt.scatter(X[:, 0], X[:, 1], c=y)
        # Mark the support vectors
        support_vecs = (alpha > 1e-4).reshape(-1)
        plt.scatter(X[support_vecs, 0], X[support_vecs, 1], c=y[support_vecs], s=250, marker='x')
        plt.xlabel('$x_1$')
        plt.ylabel('$x_2$')
        plt.legend(loc='upper left')

```

The reference solution is

```

w = array([[ -0.69192638],
          [-1.00973312]])

```

```

b = 0.907667782

```

Indices of the support vectors are

[38, 47, 92]

```
In [6]: alpha = solve_dual_svm(X, y)
        w, b = compute_weights_and_bias(alpha, X, y)
        plot_data_with_hyperplane_and_support_vectors(X, y, alpha, w, b)
        plt.show()
```

