Machine Learning Worksheet Solution 2

Julius Jankowski

1 Basic Probability

Problem 1:

The problem describes a scenario which can be described by two random variables $X = T, C$ and $Y = ST, SC$, where X states whether a person is a terrorist (T) or a (upstanding) citizen (C). $p(X)$ and the conditional probabilty $p(Y \mid X)$ are given by the problem description.

The desired solution for the stated problem is the conditional probability $p(X = T \mid Y = ST)$. With Bayes theorem:

$$p(X = T \mid Y = ST) = p(Y = ST \mid X = T) \times \frac{p(X = T)}{p(Y = ST)} \tag{1.1}$$

Where $p(Y = ST)$ is unknown in the equation. It can be computed by marginalizing the joint probability distribution $p(X, Y)$. Expressing the joint probabilities by the known quantities results in:

$$p(Y = ST) = \sum_X p(X, Y = ST) = p(Y = ST \mid X = T)p(X = T) + p(Y = ST \mid X = C)p(X = C) \tag{1.2}$$

Fusing equation 1.1 and 1.2 yields $p(X = T \mid Y = ST) \approx 0.161$.

Problem 2:

The solution for problem 2 is the conditional probability $p(Y = 2 \mid X = 3)$, where X relates to the number of red balls randomly drawn from the bowl and Y the number of red balls in the bowl as a result from two coin tosses.

By intuiton, one can build up a joint probability table from the fair coin toss and the probability distribution of red balls according to the number of red balls in the bowl:

		Drawn red balls			
		0	1	2	3
Heads	0	$\frac{1}{4}$	0	0	0
	1	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$
	2	0	0	0	$\frac{1}{4}$

Based on the joint probabilities the solution can be computed:

$$p(Y = 2 \mid X = 3) = \frac{p(X = 3, Y = 2)}{p(X = 3, Y = 0) + p(X = 3, Y = 1) + p(X = 3, Y = 2)} = 0.8 \qquad (1.3)$$

following the same ideas as for problem 1.

2 Probability Inequalities

Problem 6: