

# Progettazione di una Struttura Dati per Rappresentare e Analizzare Collezioni di Sogni

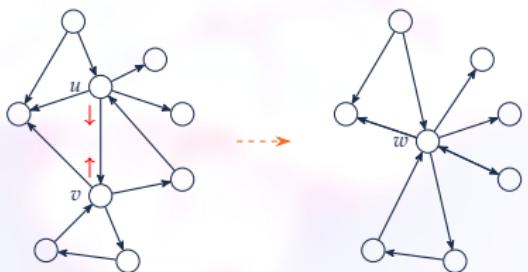
Marco Caputo  
[marco.caputo@studenti.unicam.it](mailto:marco.caputo@studenti.unicam.it)



22 Luglio 2024

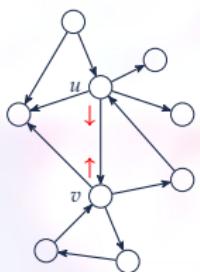
# CONTRAZIONE DI GRAFI

## Contrazione di archi

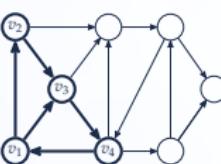


# CONTRAZIONE DI GRAFI

Contrazione di archi

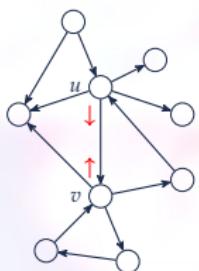


Contrazione di sottografi

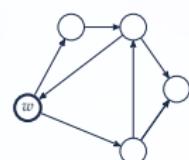
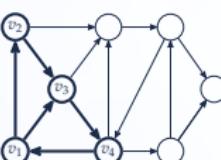


# CONTRAZIONE DI GRAFI

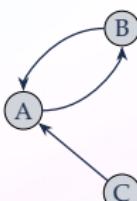
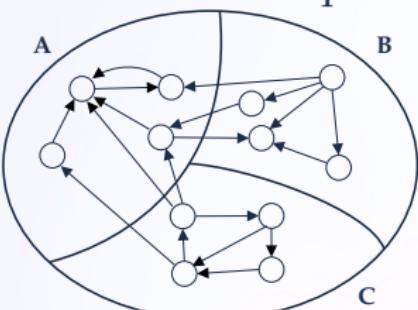
Contrazione di archi



Contrazione di sottografi



Grafo quoziante



# GRAFO MULTI-LIVELLO

## Definizione (Grafo multi-livello)

Un **grafo multi-livello**  $M$  è una coppia  $(G, \Gamma)$  dove:

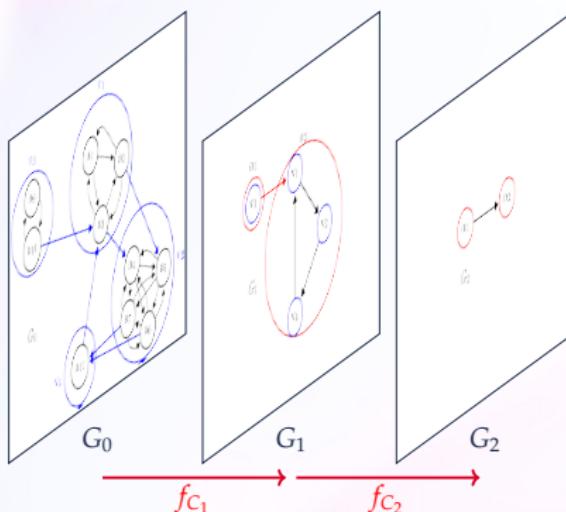
- ▶  $G = (V, E)$  è un grafo;
- ▶  $\Gamma = \langle f_{C_1}, f_{C_2}, \dots, f_{C_k} \rangle$  è una sequenza di funzioni di contrazione.

# GRAFO MULTI-LIVELLO

## Definizione (Grafo multi-livello)

Un **grafo multi-livello**  $M$  è una coppia  $(G, \Gamma)$  dove:

- $G = (V, E)$  è un grafo;
- $\Gamma = \langle f_{C_1}, f_{C_2}, \dots, f_{C_k} \rangle$  è una sequenza di funzioni di contrazione.



# GRAFO DECONTRAIBILE

## Definizione (Grafo decontraibile)

Un **grafo decontraibile** è una quadrupla  $G = (V, E, dec_V, dec_E)$  dove:

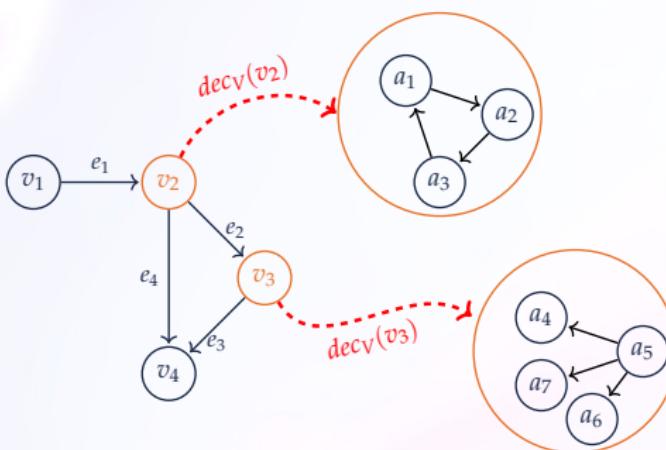
- ▶  $V$  è un insieme di elementi detti **supernodi**;
- ▶  $E \subseteq V \times V$  è un insieme di coppie ordinate di supernodi, dette **superarchi**;
- ▶  $dec_V : V \rightarrow \mathcal{G}_D$  è una funzione tale per cui  $dec_V(v) = (\mathcal{V}_v, \mathcal{E}_v, dec_{\mathcal{V}_v}, dec_{\mathcal{E}_v})$  è un grafo decontraibile rappresentato dal supernodo  $v$ ;
- ▶  $dec_E : E \rightarrow (\mathcal{V} \times \mathcal{V})$  con  $\mathcal{V} = \bigcup_{v \in V} \mathcal{V}_v$ , è una funzione tale per cui  $\forall e = (u, v), dec_E(e) = \mathcal{E}_e \subseteq \{(a, b) \mid a \in \mathcal{V}_u \wedge b \in \mathcal{V}_v\}$  è un insieme di archi rappresentati dal superarco  $e$ .

# GRAFO DECONTRAIBILE

## Definizione (Grafo decontraibile)

Un **grafo decontraibile** è una quadrupla  $G = (V, E, dec_V, dec_E)$  dove:

- ▶  $V$  è un insieme di elementi detti **supernodi**;
- ▶  $E \subseteq V \times V$  è un insieme di coppie ordinate di supernodi, dette **superarchi**;
- ▶  $dec_V : V \rightarrow \mathcal{G}_D$  è una funzione tale per cui  $dec_V(v) = (\mathcal{V}_v, \mathcal{E}_v, dec_{\mathcal{V}_v}, dec_{\mathcal{E}_v})$  è un grafo decontraibile rappresentato dal supernodo  $v$ ;
- ▶  $dec_E : E \rightarrow (\mathcal{V} \times \mathcal{V})$  con  $\mathcal{V} = \bigcup_{v \in V} \mathcal{V}_v$ , è una funzione tale per cui  $\forall e = (u, v), dec_E(e) = \mathcal{E}_e \subseteq \{(a, b) \mid a \in \mathcal{V}_u \wedge b \in \mathcal{V}_v\}$  è un insieme di archi rappresentati dal superarco  $e$ .

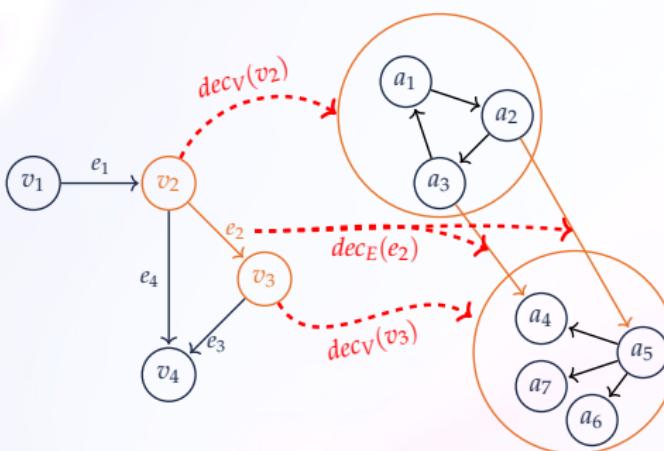


# GRAFO DECONTRAIBILE

## Definizione (Grafo decontraibile)

Un **grafo decontraibile** è una quadrupla  $G = (V, E, dec_V, dec_E)$  dove:

- ▶  $V$  è un insieme di elementi detti **supernodi**;
- ▶  $E \subseteq V \times V$  è un insieme di coppie ordinate di supernodi, dette **superarchi**;
- ▶  $dec_V : V \rightarrow \mathcal{G}_D$  è una funzione tale per cui  $dec_V(v) = (\mathcal{V}_v, \mathcal{E}_v, dec_{\mathcal{V}_v}, dec_{\mathcal{E}_v})$  è un grafo decontraibile rappresentato dal supernodo  $v$ ;
- ▶  $dec_E : E \rightarrow (\mathcal{V} \times \mathcal{V})$  con  $\mathcal{V} = \bigcup_{v \in V} \mathcal{V}_v$ , è una funzione tale per cui  $\forall e = (u, v), dec_E(e) = \mathcal{E}_e \subseteq \{(a, b) \mid a \in \mathcal{V}_u \wedge b \in \mathcal{V}_v\}$  è un insieme di archi rappresentati dal superarco  $e$ .



# SCHEMI DI CONTRAZIONE

SCHEMA

ALGORITMO

COMPLESSITÀ

# SCHEMI DI CONTRAzione

SCHEMA

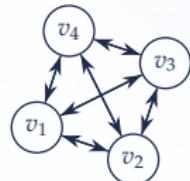
Cricche

ALGORITMO

Algoritmo di  
Bron-Kerbosch

COMPLESSITÀ

$O(3^{\frac{n}{3}})$



# SCHEMI DI CONTRAzione

## SCHEMA

Cricche

## ALGORITMO

Algoritmo di  
Bron-Kerbosch

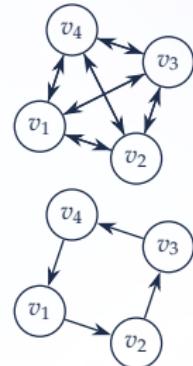
## COMPLESSITÀ

$$O(3^{\frac{n}{3}})$$

Circuiti  
semplici

Algoritmo dei  
circuiti semplici  
di Johnson

$$O((n + m) c)$$



# SCHEMI DI CONTRAzione

## SCHEMA

Cricche

Circuiti semplici

Componenti fortemente connesse

## ALGORITMO

Algoritmo di Bron-Kerbosch

Algoritmo dei circuiti semplici di Johnson

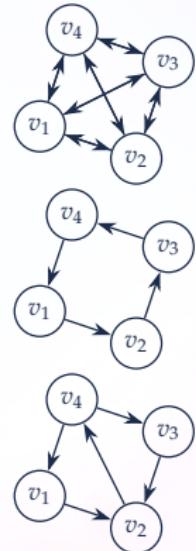
Algoritmo di Kosaraju-Sharir

## COMPLESSITÀ

$$O(3^{\frac{n}{3}})$$

$$O((n + m) c)$$

$$O(n + m)$$



# SCHEMI DI CONTRAzione

## SCHEMA

Cricche

Circuiti semplici

Componenti fortemente connesse

Stelle

## ALGORITMO

Algoritmo di Bron-Kerbosch

Algoritmo dei circuiti semplici di Johnson

Algoritmo di Kosaraju-Sharir

Algoritmo custom

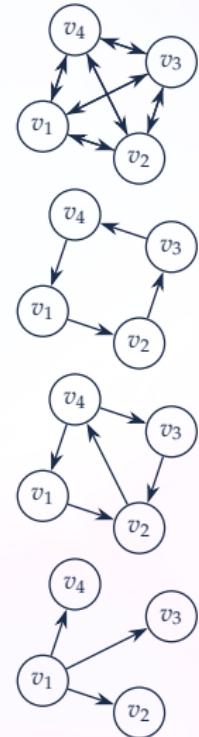
## COMPLESSITÀ

$$O(3^{\frac{n}{3}})$$

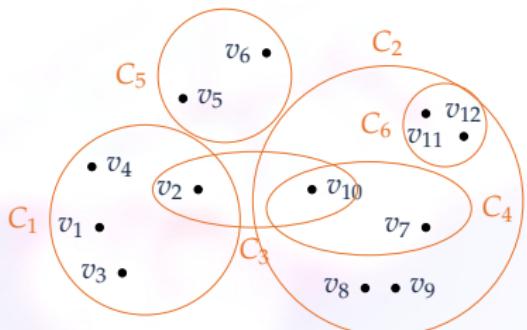
$$O((n + m)c)$$

$$O(n + m)$$

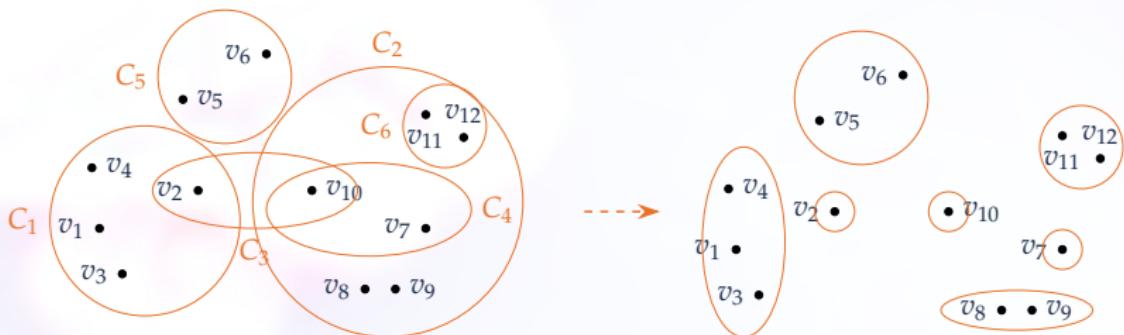
$$O(n + m)$$



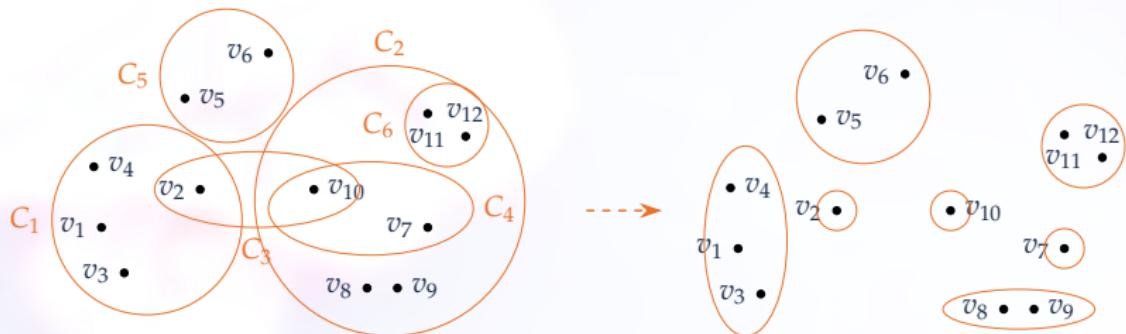
# CONTRAZIONE DI SOTTOINSIEMI



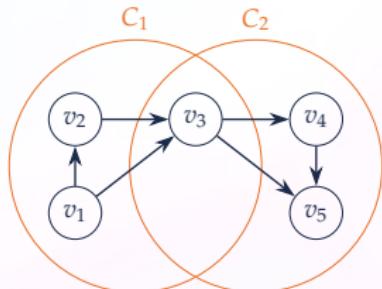
# CONTRAZIONE DI SOTTOINSIEMI



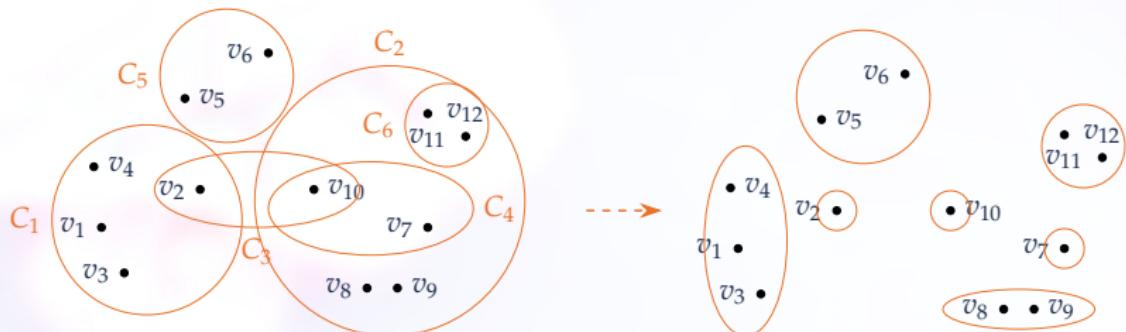
# CONTRAZIONE DI SOTTOINSIEMI

 $T$ 

$v_1$	$\{C_1\}$
$v_2$	$\{C_1\}$
$v_3$	$\{C_1, C_2\}$
$v_4$	$\{C_2\}$
$v_5$	$\{C_2\}$



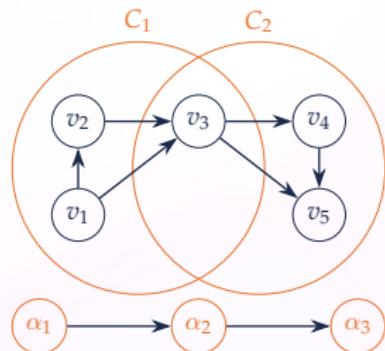
# CONTRAZIONE DI SOTTOINSIEMI

 $T$ 

$v_1$	$\{C_1\}$
$v_2$	$\{C_1\}$
$v_3$	$\{C_1, C_2\}$
$v_4$	$\{C_2\}$
$v_5$	$\{C_2\}$

 $T'$ 

$\{C_1\}$	$\alpha_1$
$\{C_1, C_2\}$	$\alpha_2$
$\{C_2\}$	$\alpha_3$



# UN SOGNO DI ARLIE

*"I am at a lake in my hometown. Something is going on there and we are in a hurry to get away. We get in a station wagon and have a hard time getting two pet deer, with the same names as my son and daughter, corralled. Finally we get them into the vehicle and we are almost all the way out when the wheel goes off one side of the road and the vehicle is stuck and the deer is about halfway out. At this point I notice my mother-in-law is cutting off a Christmas tree which is growing in the water at the end of the dock."*

# UN SOGNO DI ARLIE

*"I am at a lake in my hometown. something is going on there and we are in a hurry to get away. We get in a station wagon and have a hard time getting two pet deer, with the same names as my son and daughter, corralled. finally we get them into the vehicle and we are almost all the way out when the wheel goes off one side of the road and the vehicle is stuck and the deer is about halfway out. At this point I notice my mother-in-law is cutting off a christmas tree which is growing in the water at the end of the dock."*

RIMOZIONE  
STOPWORDS

# UN SOGNO DI ARLIE

*"I am at a lake in my hometown. something is go on there and we are in a hurry to get away. We get in a station wagon and have a hard time get two pet deer, with the same name as my son and daughter, corral. finally we get them into the vehicle and we are almost all the way out when the wheel go off one side of the road and the vehicle is stick and the deer is about halfway out. At this point I notice my mother-in-law is cut off a christmas tree which is grow in the water at the end of the dock."*

RIMOZIONE  
STOPWORDS  
LEMMATIZ-  
ZAZIONE

# UN SOGNO DI ARLIE

*"I am at a lake in my hometown. something is go on there and we are in a hurry to get away. We get in a station wagon and have a hard time get two pet deer, with the same name as my son and daughter, corral. finally we get them into the vehicle and we are almost all the way out when the wheel go off one side of the road and the vehicle is stick and the deer is about halfway out. At this point I notice my mother-in-law is cut off a christmas tree which is grow in the water at the end of the dock."*

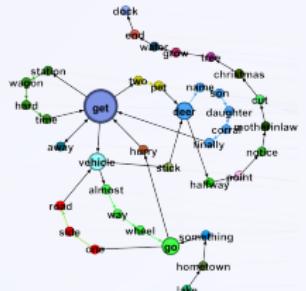
RIMOZIONE  
STOPWORDS  
LEMMATIZ-  
ZAZIONE



# UN SOGNO DI ARLIE

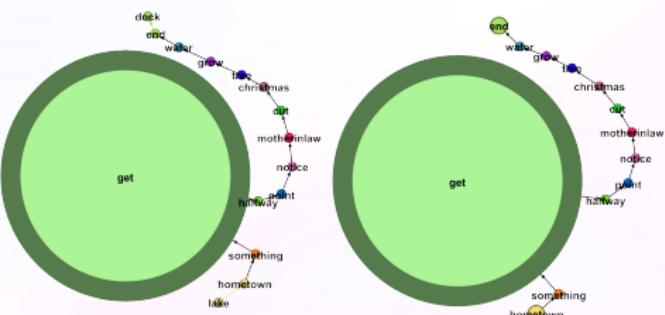
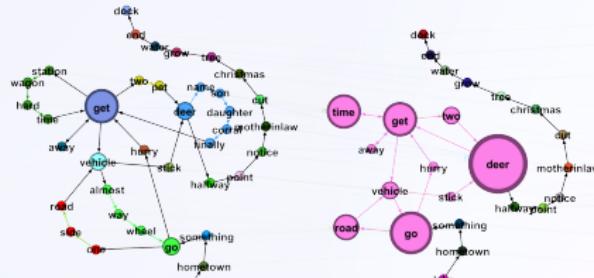
*"I am at a lake in my hometown. something is go on there and we are in a hurry to get away. We get in a station wagon and have a hard time get two pet deer, with the same name as my son and daughter, corral. finally we get them into the vehicle and we are almost all the way out when the wheel go off one side of the road and the vehicle is stick and the deer is about halfway out. At this point I notice my mother-in-law is cut off a christmas tree which is grow in the water at the end of the dock."*

RIMOZIONE  
STOPWORDS  
LEMMATIZ-  
ZAZIONE



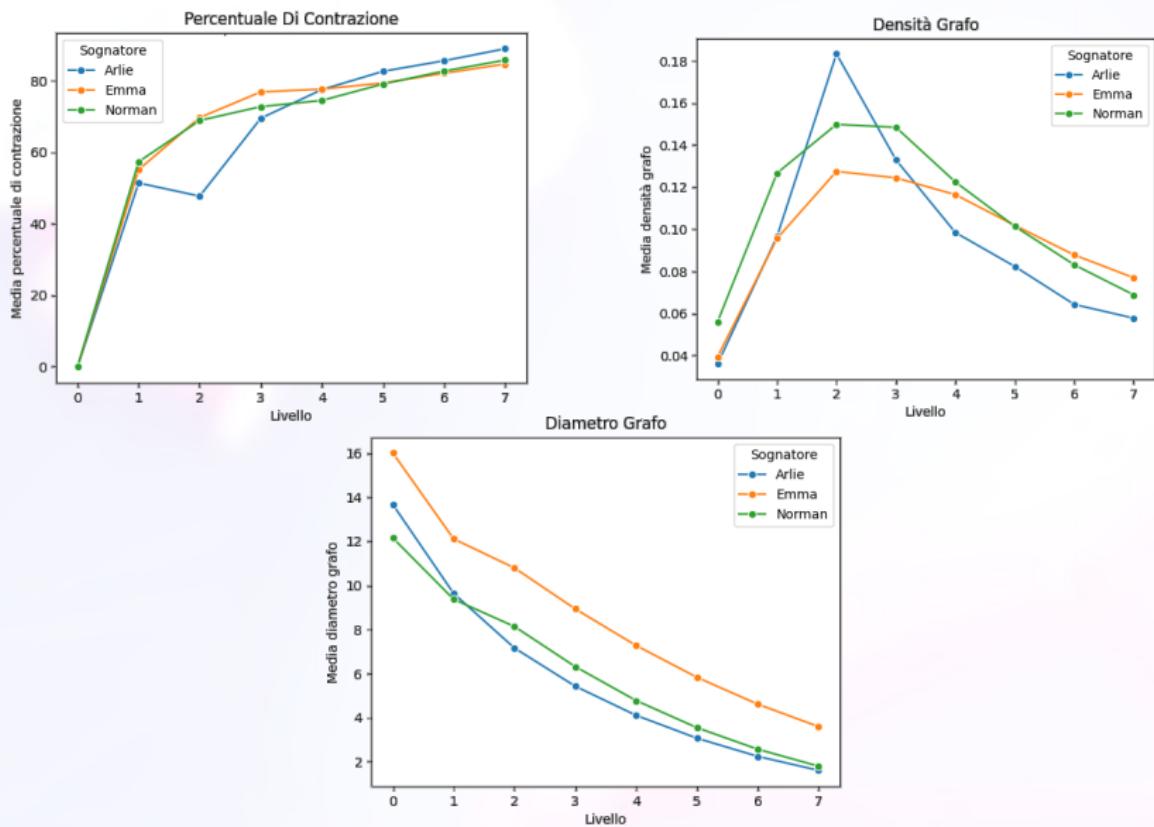
# UN SOGNO DI ARLIE

*"I am at a lake in my hometown. something is go on there and we are in a hurry to get away. We get in a station wagon and have a hard time get two pet deer, with the same name as my son and daughter, corral. finally we get them into the vehicle and we are almost all the way out when the wheel go off one side of the road and the vehicle is stick and the deer is about halfway out. At this point I notice my mother-in-law is cut off a christmas tree which is grow in the water at the end of the dock."*

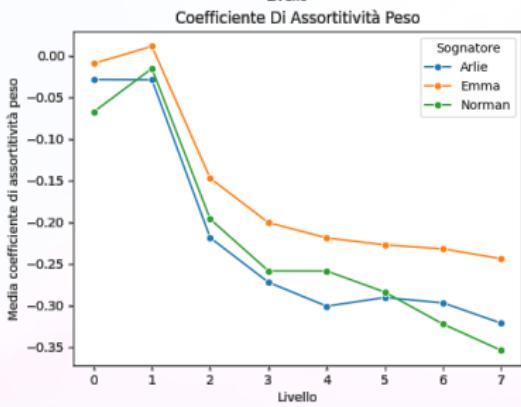
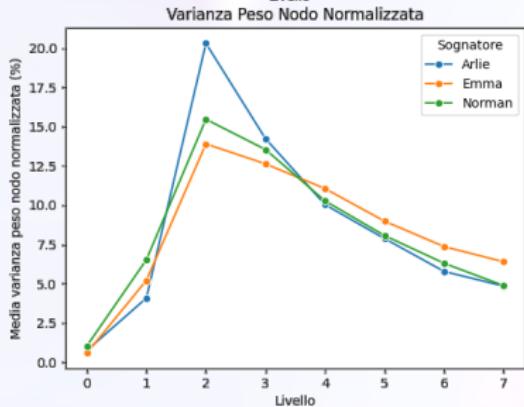
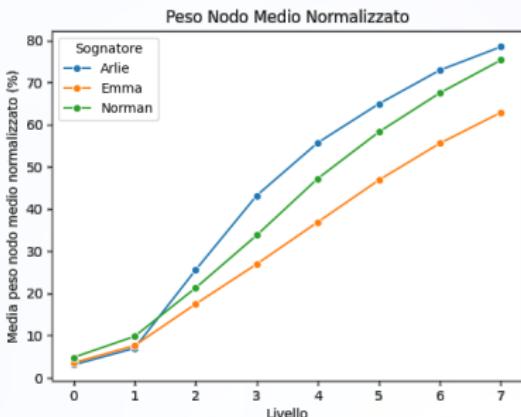
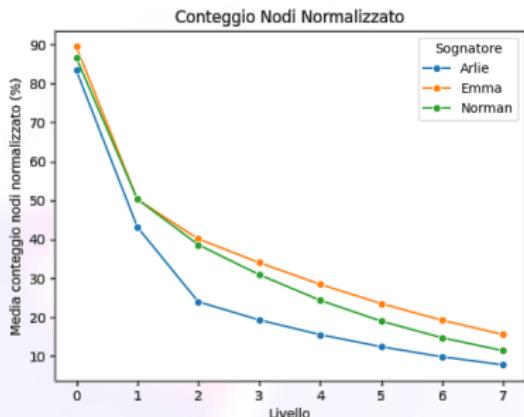


RIMOZIONE  
STOPWORDS  
LEMMATIZ-  
ZAZIONE

# RISULTATI DI ARLIE, EMMA E NORMAN



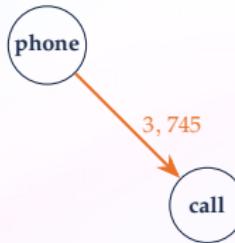
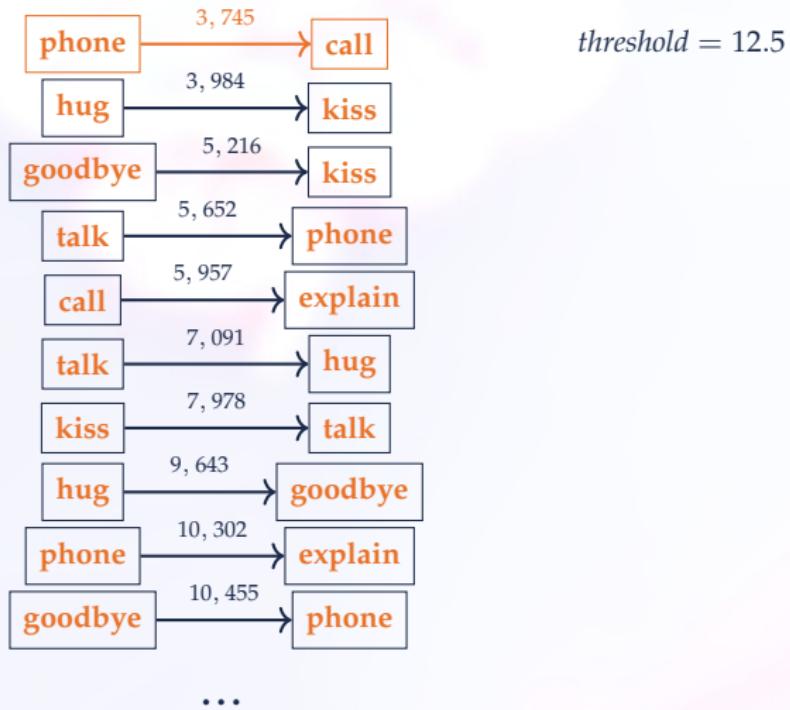
# RISULTATI DI ARLIE, EMMA E NORMAN



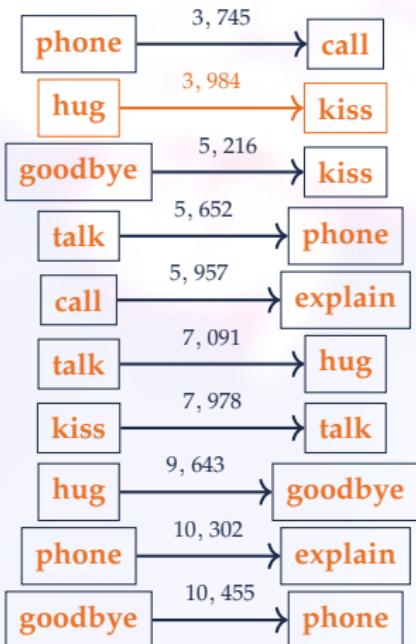
# RIDUZIONE DELLA CONNETTIVITÀ



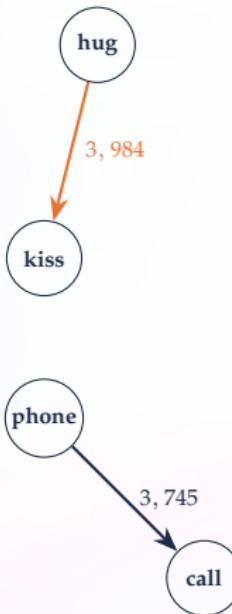
# RIDUZIONE DELLA CONNETTIVITÀ



# RIDUZIONE DELLA CONNETTIVITÀ



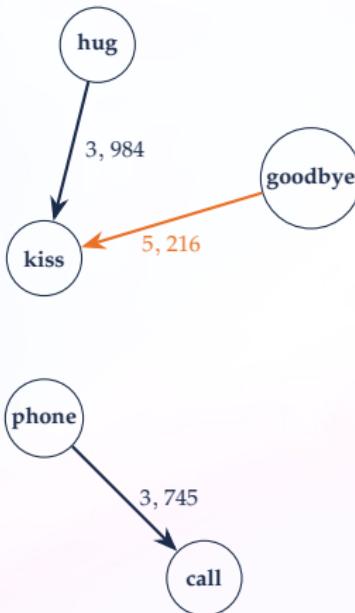
*threshold = 12.5*



# RIDUZIONE DELLA CONNETTIVITÀ



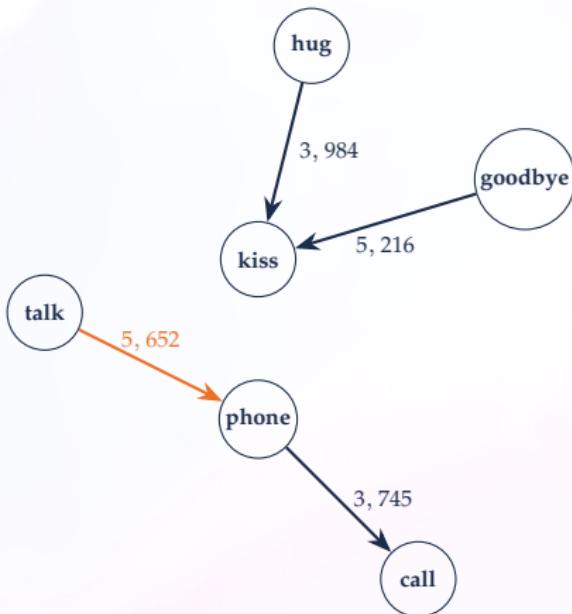
*threshold = 12.5*



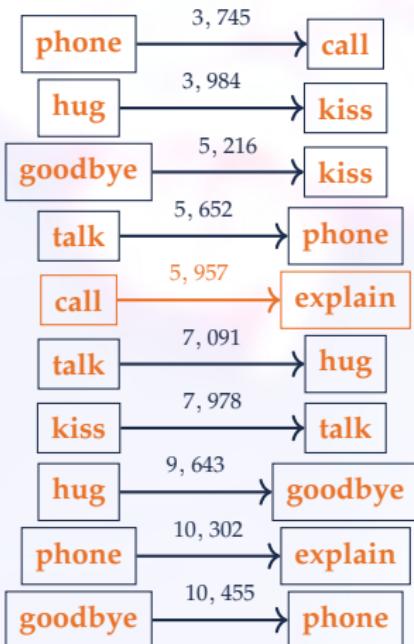
# RIDUZIONE DELLA CONNETTIVITÀ



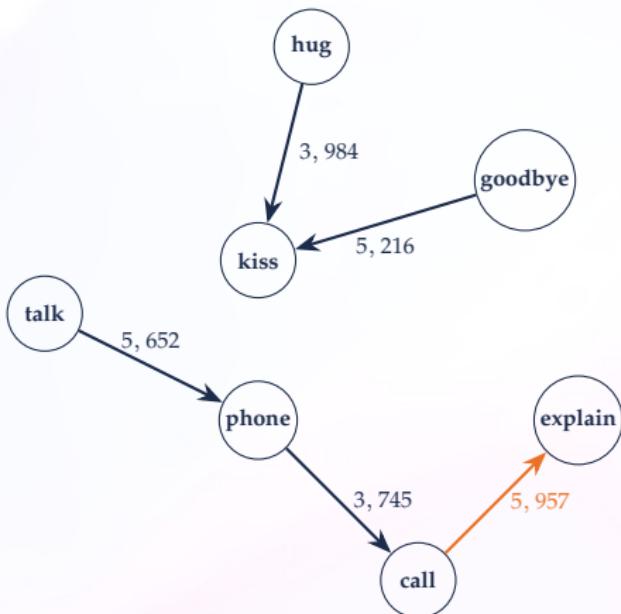
*threshold = 12.5*



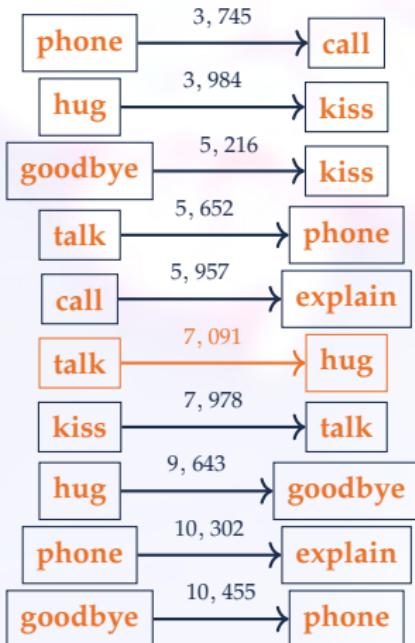
# RIDUZIONE DELLA CONNETTIVITÀ



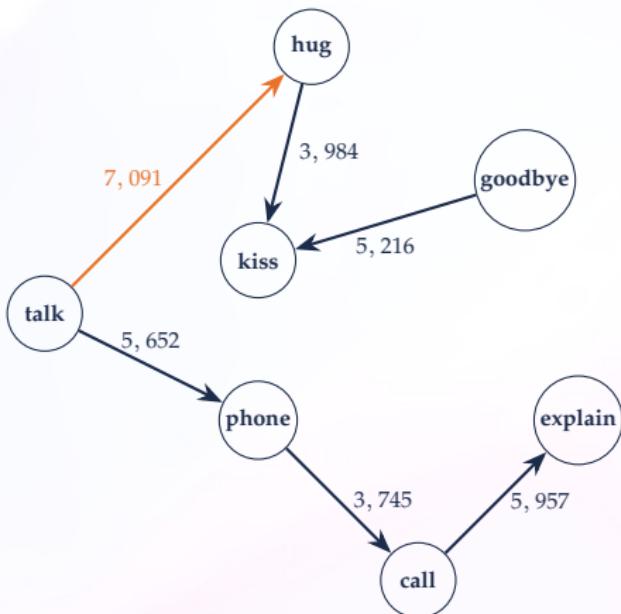
*threshold = 12.5*



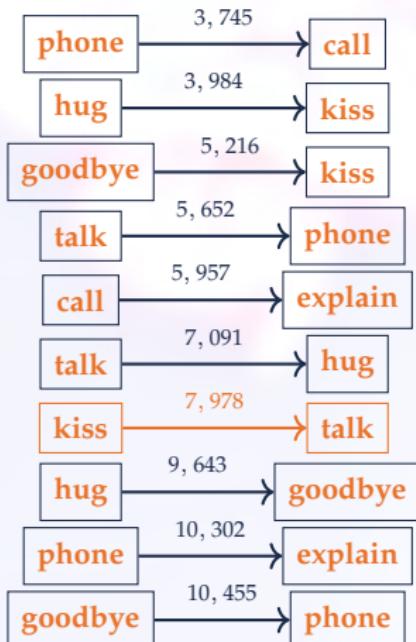
# RIDUZIONE DELLA CONNETTIVITÀ



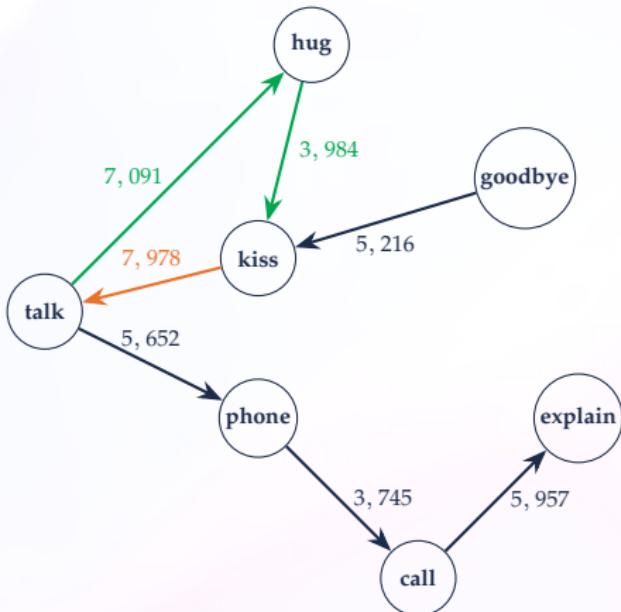
*threshold = 12.5*



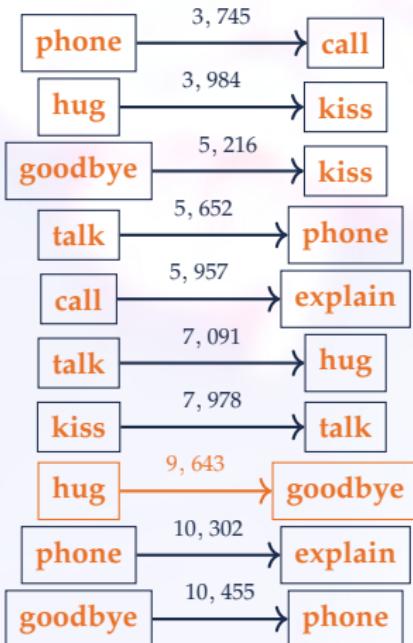
# RIDUZIONE DELLA CONNETTIVITÀ



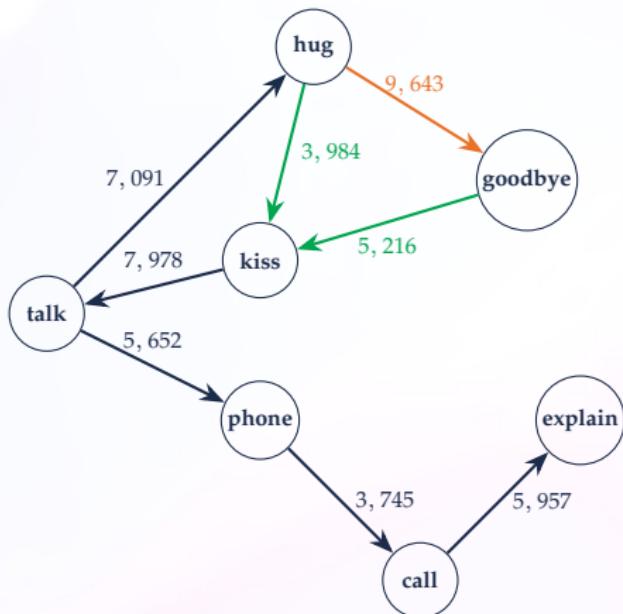
*threshold = 12.5*



# RIDUZIONE DELLA CONNETTIVITÀ



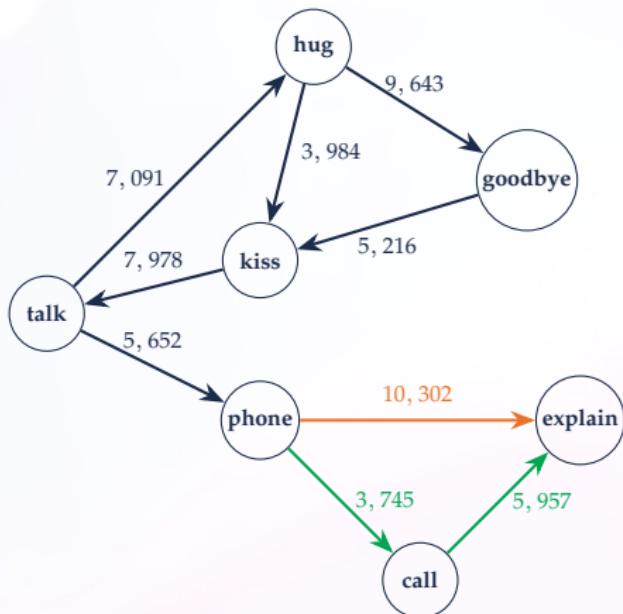
*threshold = 12.5*



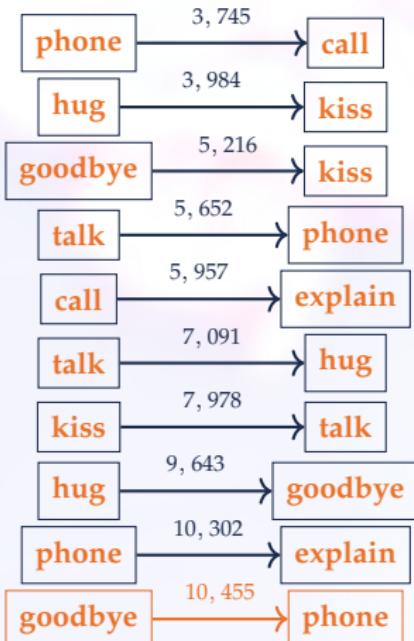
# RIDUZIONE DELLA CONNETTIVITÀ



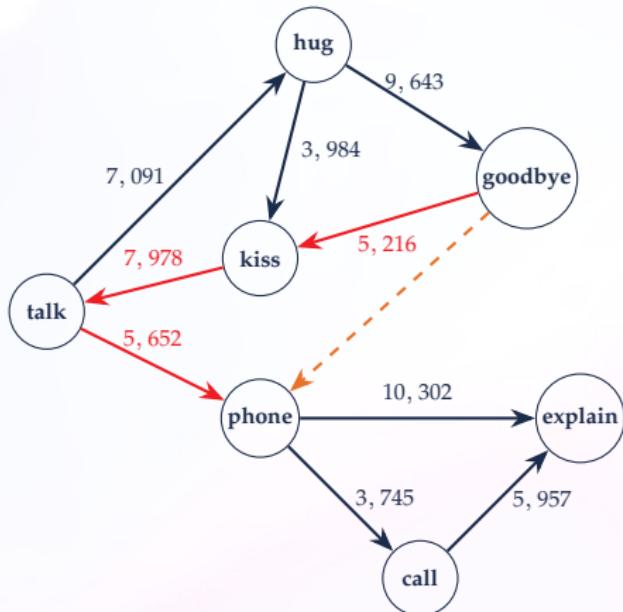
*threshold = 12.5*



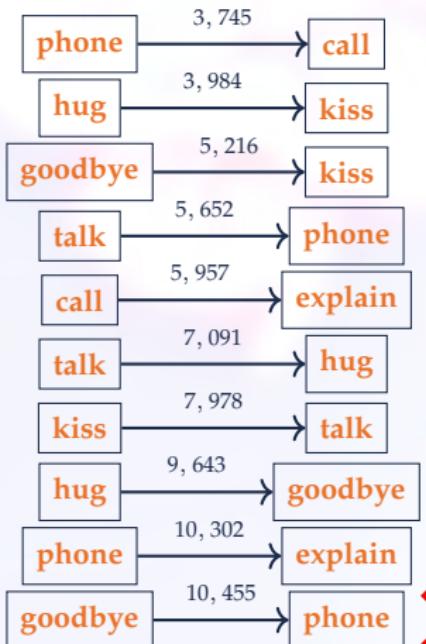
# RIDUZIONE DELLA CONNETTIVITÀ



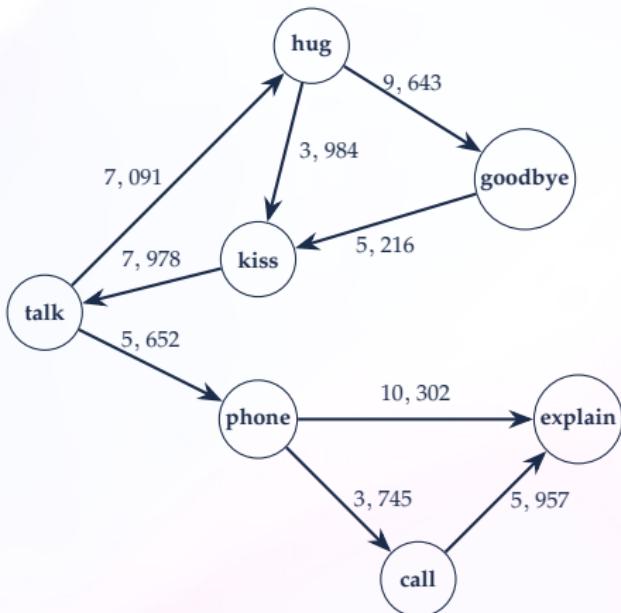
*threshold = 12.5*



# RIDUZIONE DELLA CONNETTIVITÀ

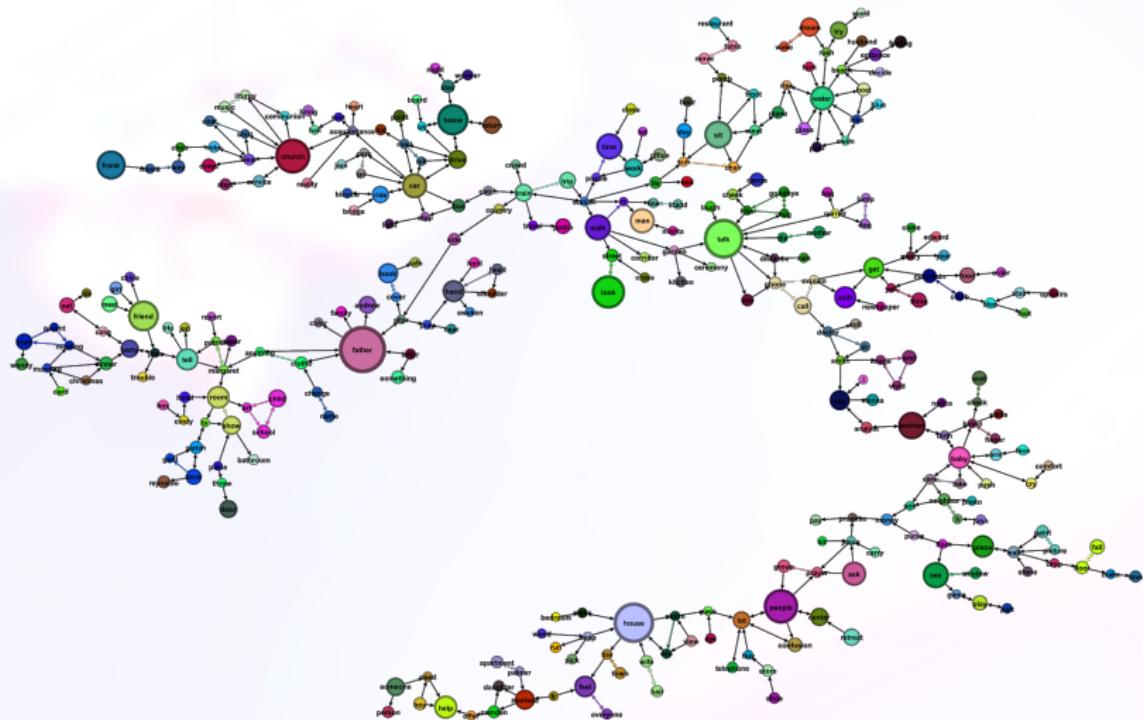


*threshold = 12.5*

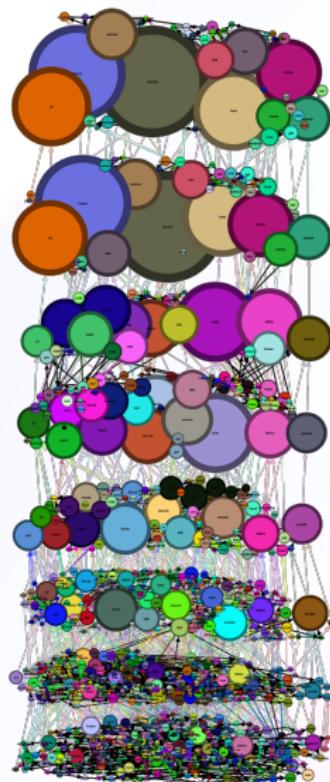


...

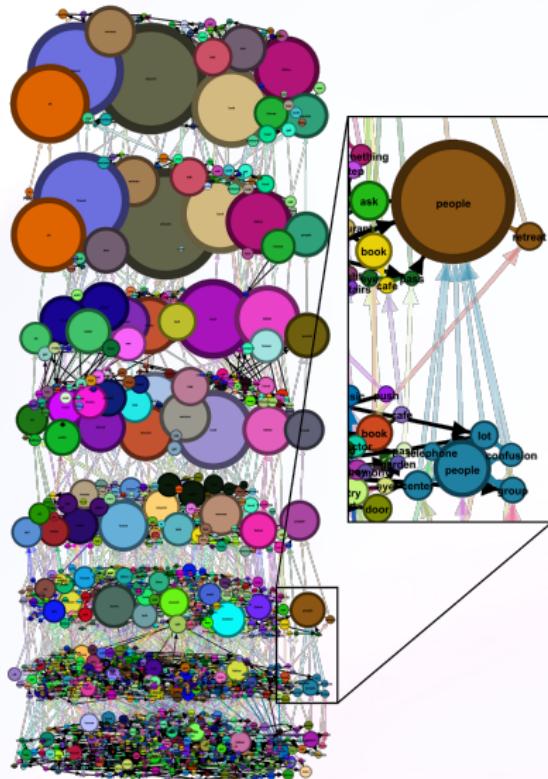
# GRAFO DI EMMA



# GRAFO MULTI-LIVELLO DI EMMA



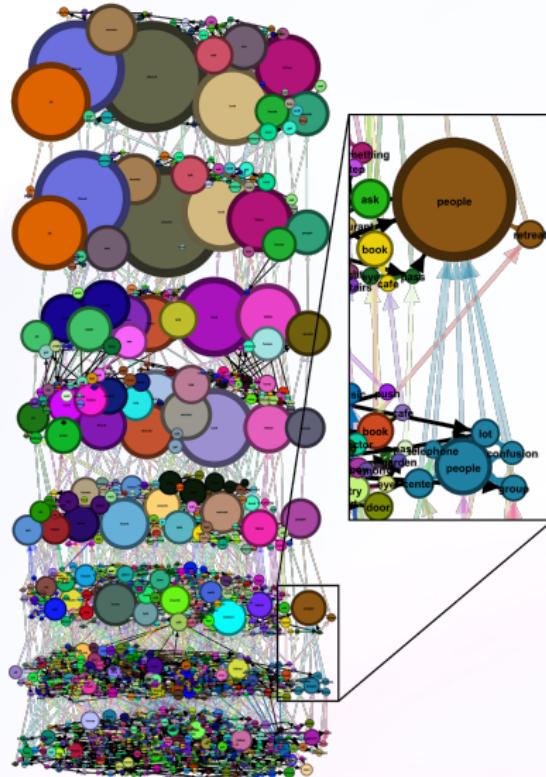
# GRAFO MULTI-LIVELLO DI EMMA



# GRAFO MULTI-LIVELLO DI EMMA

## ESEMPI DI CIRCUITI

- ▶ “cake”-“party”
- ▶ “boat”-“lake”-“water”
- ▶ “trip”-“train”-“station”
- ▶ “road”-“car”-“drive”-“hill”
- ▶ “liturgy”-“church”-“music”



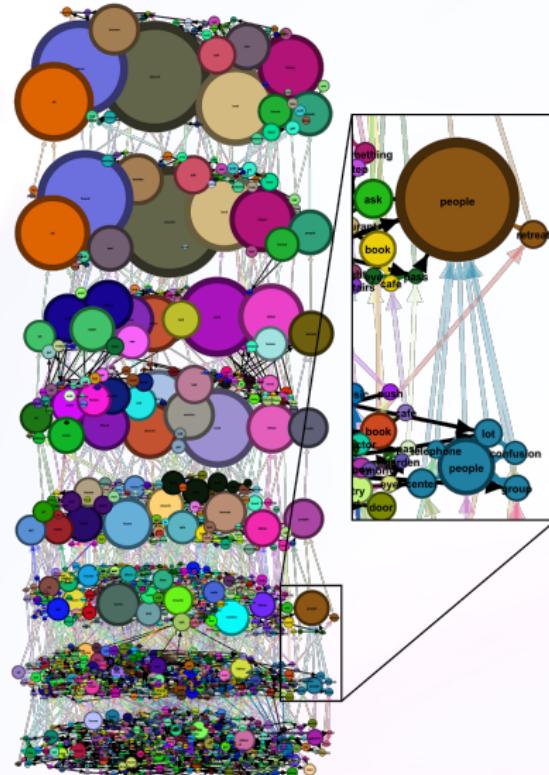
# GRAFO MULTI-LIVELLO DI EMMA

## ESEMPI DI CIRCUITI

- ▶ "cake"-“party”
- ▶ "boat"-“lake”-“water”
- ▶ “trip”-“train”-“station”
- ▶ “road”-“car”-“drive”-“hill”
- ▶ “liturgy”-“church”-“music”

## ESEMPI DI SCC

- ▶ “sit”-“chair”-“front”-  
“porch”-“seat”-“plane”
- ▶ “lake”-“boat”-“water”-“rise”-  
“pool”-“swim”-“rush”





**Grazie**  
per l'attenzione!