

5 - Image Classification

Marco Willi

Introduction

Image classification is a core task of Computer Vision. In image classification, an image is assigned to a predefined set of classes. In multi-class classification, there are 2 classes; in binary classification, there are 2 classes; and in multi-label classification, there are several sets of classes. Figure 1 illustrates the problem in multi-class classification.

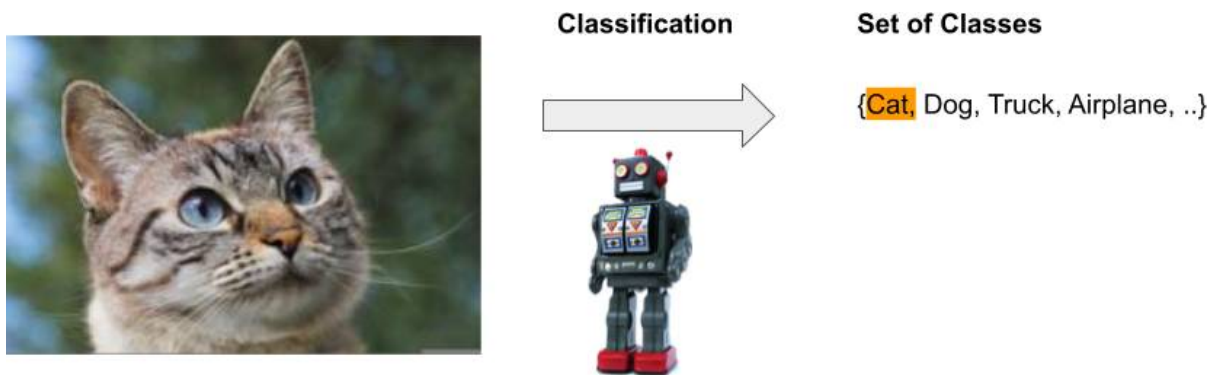


Figure 1: Image Classification example.

Figure 2 shows an example from the paper by Krizhevsky, Sutskever, and Hinton (2012), which achieved the best results in the 2012 ImageNet competition, demonstrating how well CNNs work.

Figure 3 illustrates the challenge with images taken by camera traps, which need to be classified along animal species.

Parametric Approach

In a parametric approach, we seek a model of the following form:

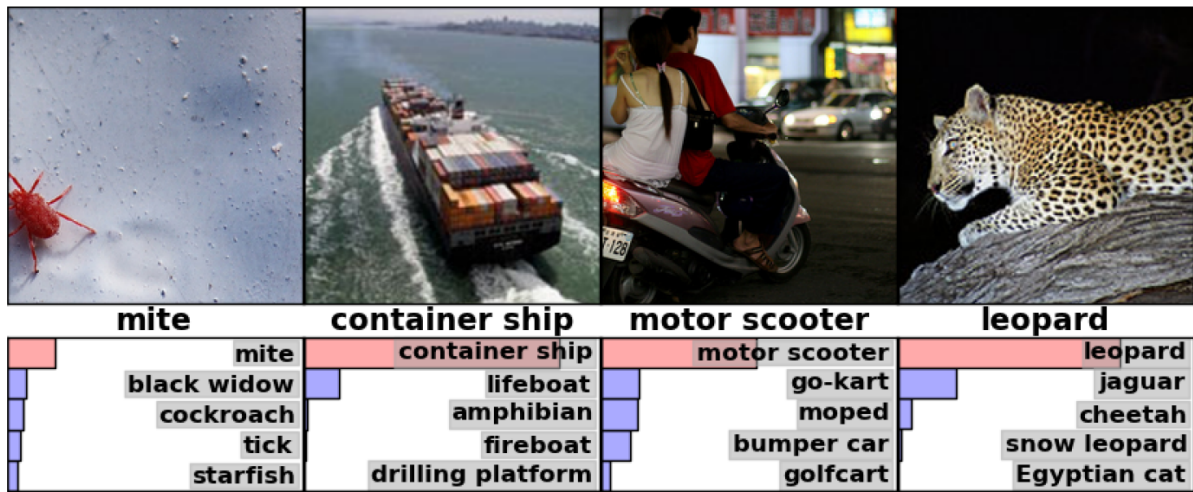


Figure 2: Image Classification example (from Krizhevsky, Sutskever, and Hinton (2012)).

Schwierigkeiten am Beispiel der Klassifikation von Bildern aus Kamerafallen

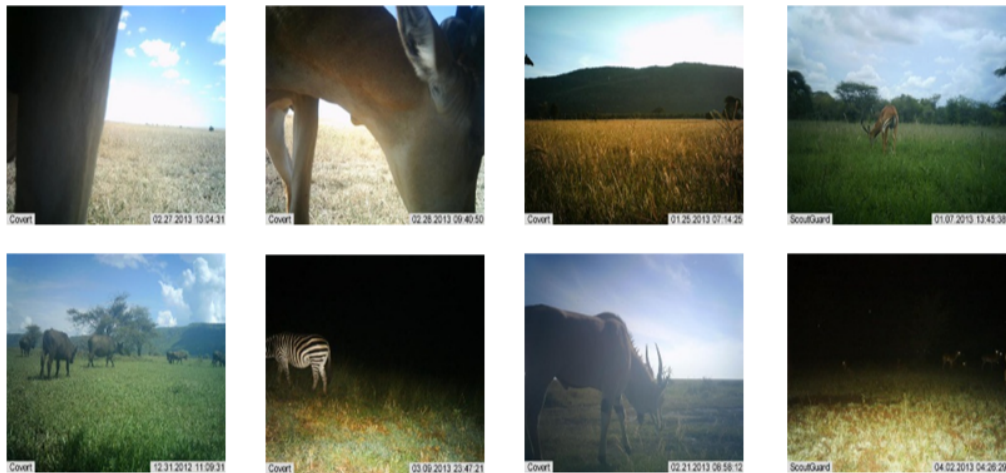


Figure 3: Example images from camera traps.

$$\hat{y}^{(i)} = f(\theta, \mathbf{x}^{(i)})$$

We want to find model parameters θ that output a score/prediction $\hat{y}^{(i)}$ for any data points $\mathbf{x}^{(i)}$ for each class $k \in K$. We then want to assess how good this score is with the help of a loss function.

Softmax Classifier

With a Softmax Classifier, we interpret model predictions/scores as probabilities of class memberships: $P(Y = \mathbf{y}^{(i)} | X = \mathbf{x}^{(i)})$. We interpret the output as a [Categorical Distribution](#) over all possible classes.

To obtain a valid probability distribution, the untransformed outputs \mathbf{z} , also called logits, of a model are transformed with the softmax function $\sigma(\mathbf{z})$:

$$P(Y = k | X = \mathbf{x}^{(i)}) = \sigma(\mathbf{z})_k = \frac{e^{z_k}}{\sum_i^K e^{z_i}}$$

Figure 4 shows an example of the effect of the softmax transformation.

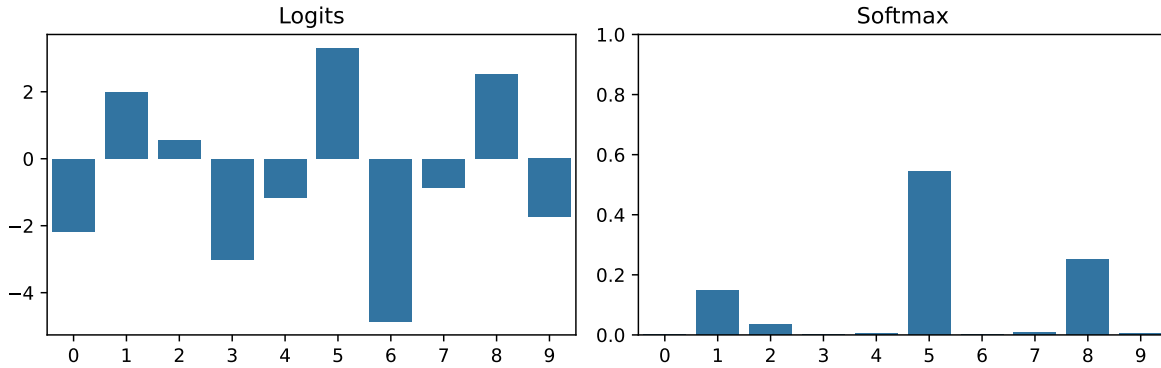


Figure 4: Logits (left) to probabilities with the Softmax function (right).

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Loss Function

Now we look at how to define the loss function $L(\mathbf{X}, \mathbf{y}, \theta)$ for a multi-class classification problem.

Likelihood

The likelihood of a data point $(\mathbf{x}^{(i)}, y^{(i)})$ is the probability of observing/realizing a data point, given a model with certain parameters:

$$P(Y = y^{(i)} | X = \mathbf{x}^{(i)}) = f(\theta, \mathbf{x}^{(i)})$$

This means we formulate a model with a probabilistic interpretation of predictions: $f(\theta, \mathbf{x}^{(i)}) : \mathbb{R}^n \mapsto [0, 1]$

For a multi-class classification, the label vector is one-hot encoded $\mathbf{y}^{(i)} \in \{0, 1\}^K$, where the true class is coded with 1 and the rest with 0. The likelihood of a data point is defined as:


$$\prod_{j=1}^K P(Y = j | X = \mathbf{x}^{(i)})^{y_j^{(i)}}$$

Since only one entry in $\mathbf{y}^{(i)}$ is 1, the likelihood is simply the prediction for the true class $P(Y = y^{(i)} | X = \mathbf{x}^{(i)})$.

[More Info](#)

Given $\mathbf{y} = [0, 1, 0, 1, 1]$ and the following $\hat{\mathbf{y}}$, calculate the likelihood.


For $\hat{\mathbf{y}} = [0.1, 0.8, 0.2, 0.7, 0.9]$:

 Click for result

```
y_pred = np.array([0.1, 0.8, 0.2, 0.7, 0.9])
display_likelihood(y_true, y_pred)
```

Total Likelihood: 0.362880


Does it get larger for $\hat{\mathbf{y}} = [0.1, 0.9, 0.2, 0.7, 0.9]$?

 Click for result

```
y_pred = np.array([0.1, 0.9, 0.2, 0.7, 0.9])
display_likelihood(y_true, y_pred)
```

Total Likelihood: 0.408240

What happens if we increase the dataset by copying the vector 10 times? $\hat{\mathbf{y}} = [0.1, 0.8, 0.2, 0.7, 0.9, 0.1, 0.8, \dots]$?

 Click for result

```
y_pred = np.array([0.1, 0.8, 0.2, 0.7, 0.9])

y_true = np.repeat(y_true, 10, axis=0)
y_pred = np.repeat(y_pred, 10, axis=0)

display_likelihood(y_true, y_pred)
```

Total Likelihood: 0.000040

Maximum Likelihood

The likelihood $P(\mathbf{y}|\theta, \mathbf{X})$ of observing our entire dataset (\mathbf{X}, \mathbf{y}) , given the parameters θ and assuming that the data points $(\mathbf{x}^{(i)}, y^{(i)})$ are independent and identically distributed, can be calculated as:

$$\prod_{i=1}^N \prod_{j=1}^K P(Y = j | X = \mathbf{x}^{(i)})^{y_j^{(i)}} \quad (1)$$

Under the Maximum Likelihood approach, we seek the parameters θ that maximize the likelihood of observing the dataset.

Negative Log-Likelihood

With Maximum Likelihood, we aim to choose the parameters θ such that $p(\mathbf{y}|\theta, \mathbf{X})$ is maximized. However, this function can be complex to handle, and we can use some mathematical tricks. We take the logarithm of the likelihood so that the product of probabilities becomes

a sum. Since the logarithm is a monotonically increasing function, we can maximize its logarithm instead of the function $p(\mathbf{y}|\theta, \mathbf{X})$. Finally, we take the negative of the function, allowing us to minimize it.

$$L(\mathbf{X}, \mathbf{y}, \theta) = -\log \prod_{i=1}^N \prod_{j=1}^K P(Y = j|X = \mathbf{x}^{(i)})^{y_j^{(i)}} L(\mathbf{X}, \mathbf{y}, \theta) = -\sum_{i=1}^N \sum_{j=1}^K y_j^{(i)} \log P(Y = j|X = \mathbf{x}^{(i)}) \quad (2)$$

Cross-Entropy

The loss function, derived with Maximum Likelihood, can also be viewed through the lens of cross-entropy between two discrete probability functions. Specifically, we can calculate and minimize the cross-entropy between the true distribution $\mathbf{y}^{(i)}$ and the predicted $\hat{\mathbf{y}}^{(i)}$. Cross-entropy comes from information theory and measures how many bits/nats on average are needed to describe an event of a probability distribution $p(x)$ when using the approximation $q(x)$.

$$CE = -\sum_{x \in X} p(x) \log q(x) CE = -\sum_{i=1}^N \sum_{j=1}^K y_j^{(i)} \log \hat{y}_j^{(i)} \quad (3)$$

It is evident that cross-entropy is identical to the negative log-likelihood.

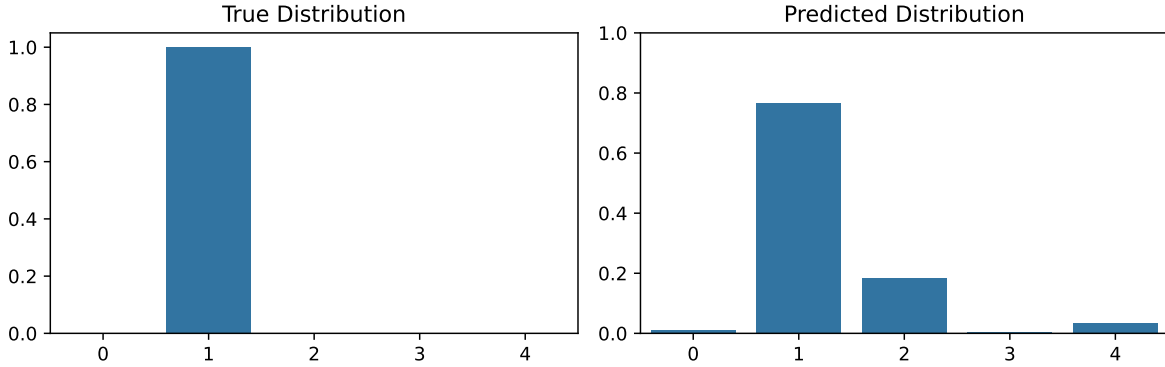


Figure 5: True Distribution (left) and Predicted Distribution (right).

Figure 5 shows an example with a cross-entropy value of: 0.266.

Architectures

Architecture refers to the complete definition of an untrained (deep-learning) model, i.e., the sequence and configuration of layers. The following are some important architectures that have been successfully used for image classification.

AlexNet

CNNs became extremely popular after winning the [ImageNet](#) Competition. Krizhevsky, Sutskever, and Hinton (2012) implemented a CNN with multiple layers, known as the AlexNet architecture, as shown in Figure 6. ImageNet is a large, hierarchical image dataset Deng et al. (2009), which enabled efficient training of CNNs for the first time.

AlexNet consists of 5 convolutional layers and 3 fully-connected layers. The last layer is a 1000-way softmax output to model the classes in ImageNet.

The model was trained with two GPUs (GTX 580) with 3GB memory each. Since 3GB was insufficient to train the model, the architecture was split across the GPUs. Some layers were split between the GPUs, allowing a larger network to be trained.

Figure 6 shows the detailed architecture, including kernel sizes, the number of filters per layer, activation map dimensions, and pooling layers. ReLU was used as the activation function. This representation is from the original paper and shows how the model was split across two GPUs.

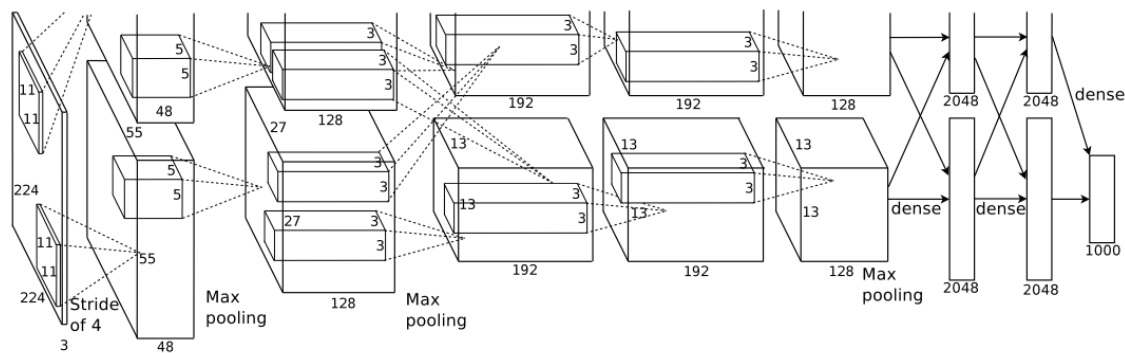


Figure 6: AlexNet Krizhevsky, Sutskever, and Hinton (2012).

Since the split across two GPUs is no longer necessary, the architecture is somewhat simplified. Modern implementations are shown in Figure 7.

Figure 8 presents the operations in AlexNet in tabular form.

We can also easily load AlexNet via torchvision.

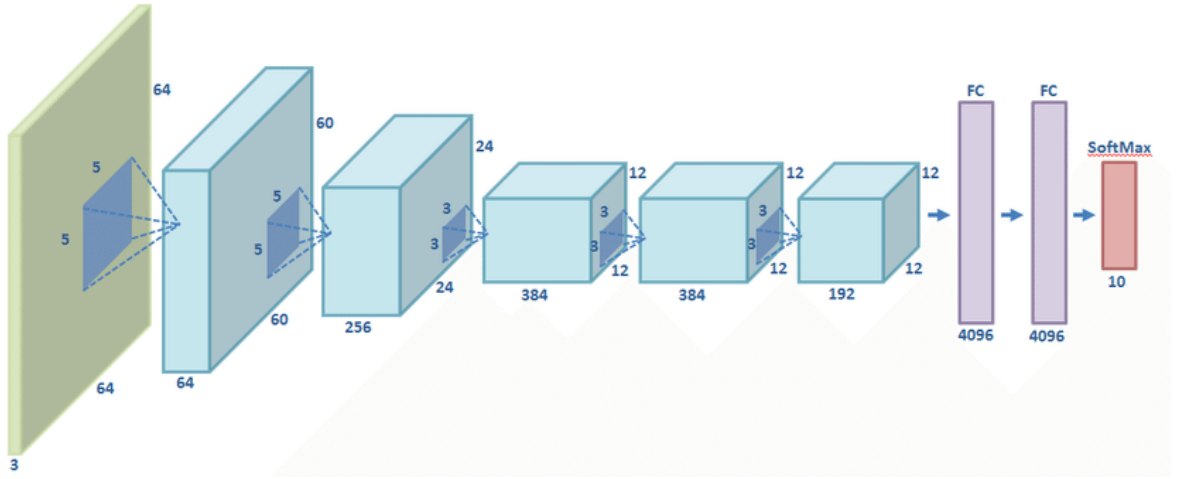


Figure 7: AlexNet Llamas et al. (2017).

	Input size			Layer			Output size				
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)	flop (M)
conv1	3	227	64	11	4	2	64	56	784	23	73
pool1	64	56		3	2	0	64	27	182	0	0
conv2	64	27	192	5	1	2	192	27	547	307	224
pool2	192	27		3	2	0	192	13	127	0	0
conv3	192	13	384	3	1	1	384	13	254	664	112
conv4	384	13	256	3	1	1	256	13	169	885	145
conv5	256	13	256	3	1	1	256	13	169	590	100
pool5	256	13		3	2	0	256	6	36	0	0
flatten	256	6					9216		36	0	0
fc6	9216		4096				4096		16	37,749	38
fc7	4096		4096				4096		16	16,777	17
fc8	4096		1000				1000		4	4,096	4

Figure 8: Source: Johnson (2019).


```

import torch
import torchvision.models as models
import torchinfo

alexnet = models.alexnet()
x = torch.zeros(1, 3, 224, 224, dtype=torch.float, requires_grad=False)
yhat = alexnet(x)

print(torchinfo.summary(alexnet, input_size=(1, 3, 224, 224)))

```

```

=====
Layer (type:depth-idx)                   Output Shape              Param #
=====
AlexNet                                  [1, 1000]                 --
  Sequential: 1-1                        [1, 256, 6, 6]           --
    Conv2d: 2-1                          [1, 64, 55, 55]          23,296
    ReLU: 2-2                           [1, 64, 55, 55]          --
    MaxPool2d: 2-3                      [1, 64, 27, 27]          --
    Conv2d: 2-4                          [1, 192, 27, 27]         307,392
    ReLU: 2-5                           [1, 192, 27, 27]         --
    MaxPool2d: 2-6                      [1, 192, 13, 13]         --
    Conv2d: 2-7                          [1, 384, 13, 13]         663,936
    ReLU: 2-8                           [1, 384, 13, 13]         --
    Conv2d: 2-9                          [1, 256, 13, 13]         884,992
    ReLU: 2-10                          [1, 256, 13, 13]         --
    Conv2d: 2-11                        [1, 256, 13, 13]         590,080
    ReLU: 2-12                          [1, 256, 13, 13]         --
    MaxPool2d: 2-13                    [1, 256, 6, 6]           --
  AdaptiveAvgPool2d: 1-2                [1, 256, 6, 6]           --
  Sequential: 1-3                        [1, 1000]                 --
    Dropout: 2-14                       [1, 9216]                 --
    Linear: 2-15                         [1, 4096]                 37,752,832
    ReLU: 2-16                          [1, 4096]                 --
    Dropout: 2-17                       [1, 4096]                 --
    Linear: 2-18                         [1, 4096]                 16,781,312
    ReLU: 2-19                          [1, 4096]                 --
    Linear: 2-20                        [1, 1000]                 4,097,000
=====
Total params: 61,100,840
Trainable params: 61,100,840
Non-trainable params: 0
Total mult-adds (Units.MEGABYTES): 714.68

```

```
=====
Input size (MB): 0.60
Forward/backward pass size (MB): 3.95
Params size (MB): 244.40
Estimated Total Size (MB): 248.96
=====
```

VGG

Simonyan and Zisserman (2015) won the ImageNet Challenge in 2014 with their VGG architecture. They showed that smaller 3x3 kernels work significantly better and that deeper networks with 16-19 layers can be trained. Figure 9 shows the architecture as presented in the original paper. Figure 10 visualizes the architecture.

VGG introduced a popular design element: A layer has the same number of filters as the previous layer unless the activation map dimensions are halved, in which case the number of filters is doubled. This was done to maintain the time complexity of the layers. VGG does not use normalization layers.

Figure 11 compares VGG with AlexNet.

ResNet

He et al. (2016) wondered whether CNNs could be improved simply by making them deeper, i.e., adding more layers. Their experiments showed that adding layers eventually stops being beneficial, and performance saturates and then rapidly degrades (see Figure 12). They noticed that the performance did not degrade due to overfitting (see Figure 13, which shows that training error is also poor), but because back-propagation becomes less effective and the weights do not optimize well. He et al. (2016) hypothesized that deeper networks could not perform worse than shallower ones because the additional layers could simply pass on activations unchanged.

They then hypothesized that passing on activations unchanged as a default behavior could avoid this problem. They introduced identity mappings. Figure 14 shows such a connection. These connections are also called residual connections because the network only needs to learn the change in activations from layer i to layer $i + 1$, the residue.

ResNet was otherwise inspired by VGG16. The convolutional layers consist of 3x3 kernels (except the first one), and the number of filters is doubled when the activation map dimensions are halved. ResNet uses convolutions with stride 2 for down-sampling and no max pooling. At the end, ResNet uses a global average pooling layer followed by a fully-connected layer with the number of classes. Variants of ResNet have 18, 34, 50, and over 100 layers. ResNet also uses batch normalization. Figure 15 shows the architecture.

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224×224 RGB image)					
conv3-64	conv3-64 LRN	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 conv1-256	conv3-256 conv3-256 conv3-256	conv3-256 conv3-256 conv3-256 conv3-256
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 conv1-512	conv3-512 conv3-512 conv3-512	conv3-512 conv3-512 conv3-512 conv3-512
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 conv1-512	conv3-512 conv3-512 conv3-512	conv3-512 conv3-512 conv3-512 conv3-512
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

Figure 9: VGG Simonyan and Zisserman (2015).

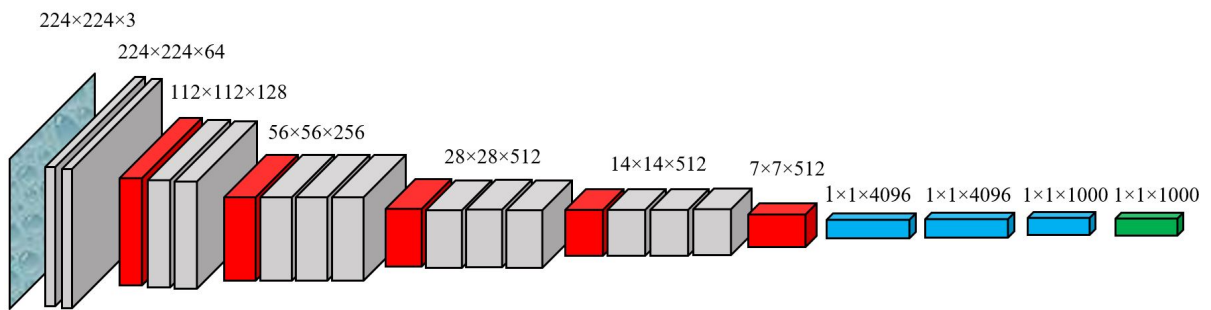


Figure 10: Source: [Link](#)

VGG: Deeper Networks, Regular Design

VGG Design rules:

All conv are 3x3 stride 1 pad 1

All max pool are 2x2 stride 2

After pool, double #channels

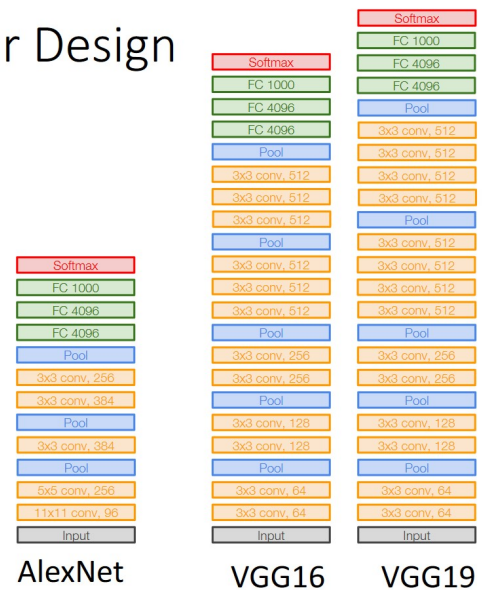


Figure 11: Source: Johnson (2019).

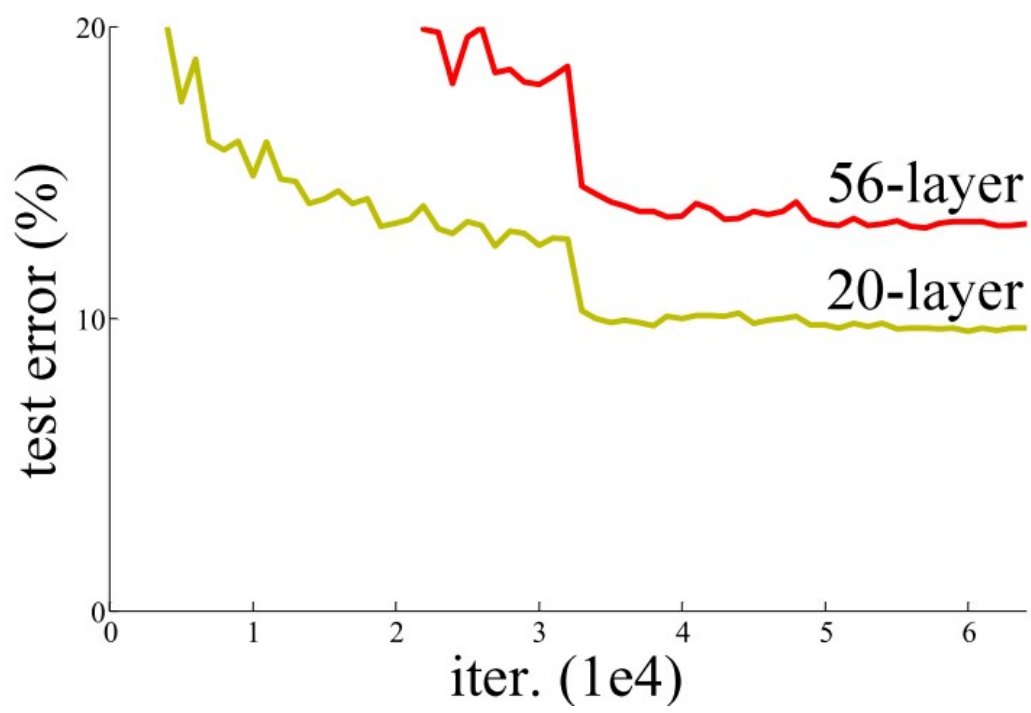


Figure 12: Source: He et al. (2016)

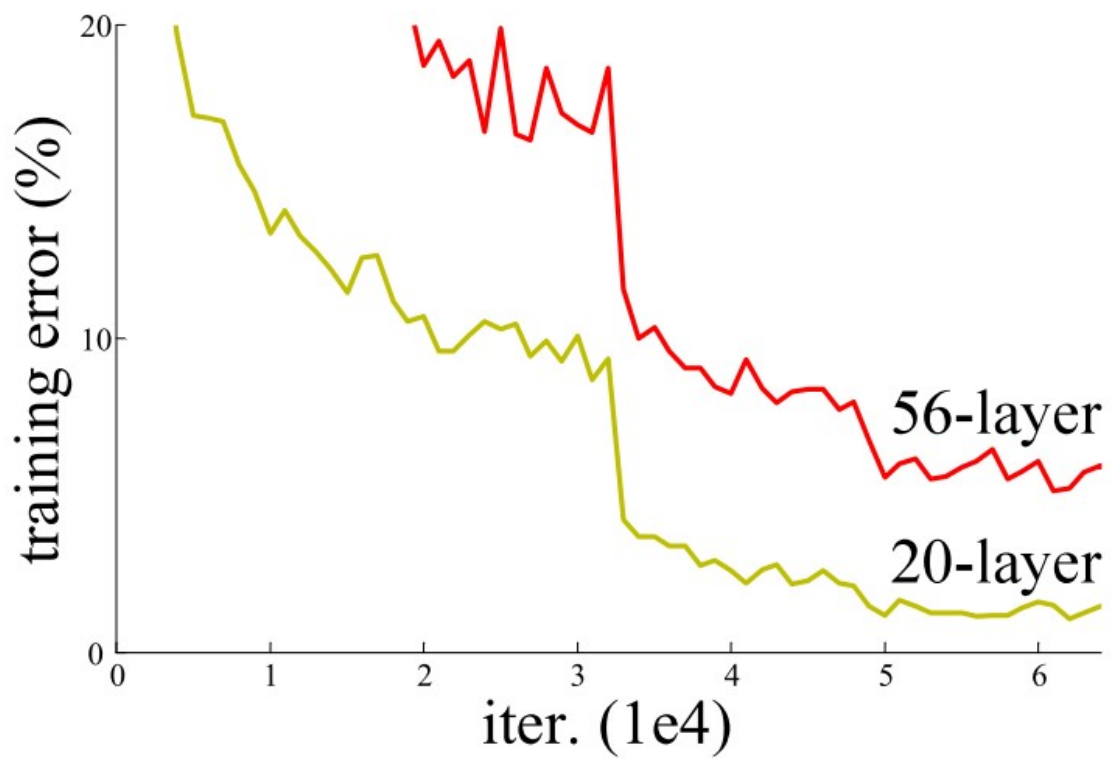


Figure 13: Source: He et al. (2016)

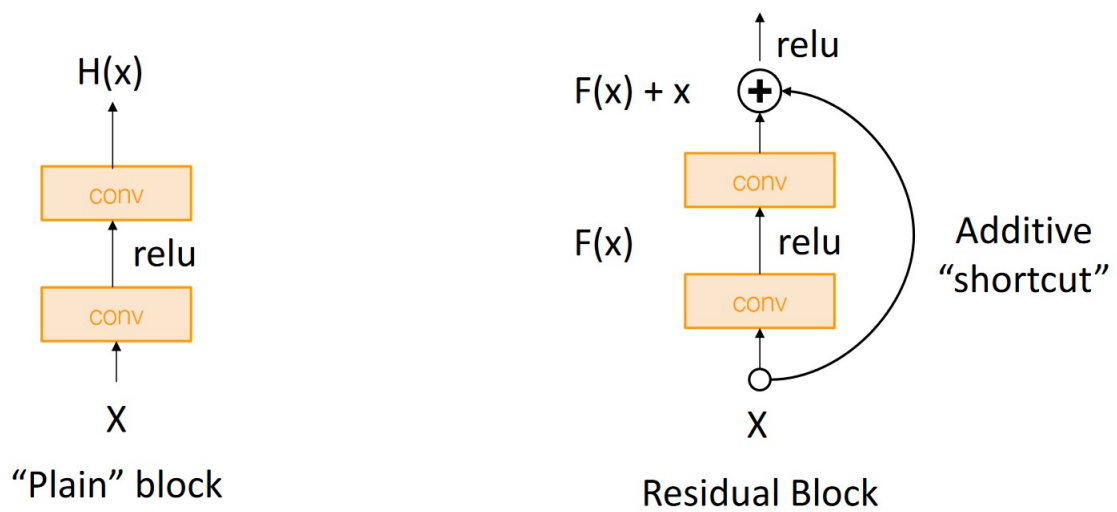


Figure 14: ResNet He et al. (2016) (Graphic from Johnson (2019).)

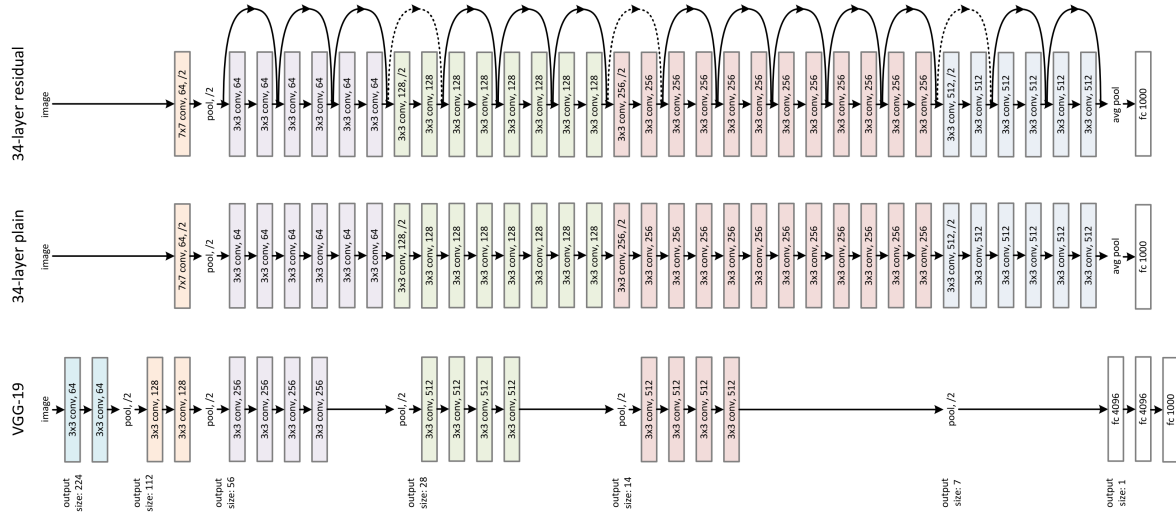


Figure 15: Source: He et al. (2016).

ResNet is very popular and is still widely used today (there are now more modern variants). Recent studies of identity mappings (or more generally skip connections) have shown that the gradient surface of the loss function becomes smoother, allowing the network to be optimized better. This is impressively illustrated in Figure 16.

ConvNext

One of the most modern CNN architectures was described in Liu et al. (2022). This architecture uses tricks and implementation ideas accumulated over decades from various architectures. Figure 17 shows, starting from a modern version of ResNet, what has been adjusted to define this state-of-the-art architecture. Examples include: larger kernels, different activation functions, layer normalization instead of batch normalization, and depthwise separable convolutions.

There is already a new version of this architecture Woo et al. (2023).

ImageNet Performance

Figure 18 shows the development of ImageNet performance. As of 2023, we are much closer to 100%, see [Link](#).

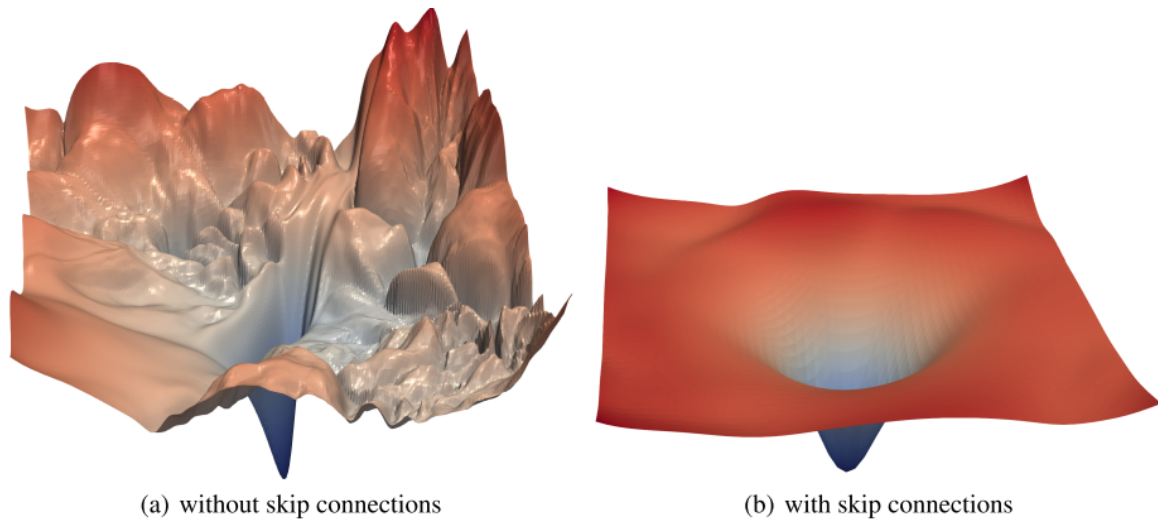


Figure 16: Source: Li et al. (2018).

Which Architecture?

Which architecture should be chosen for a specific problem? A common tip is: *Don't be a hero.*

One should rely on off-the-shelf architectures and not implement their own without a good reason.

Typically, ResNet-50 or ResNet-101 are good choices. However, there are also models that require significantly fewer parameters, such as Efficient Nets.

Important considerations are also the requirements regarding accuracy, performance (FLOPs), and model size (memory).

Further Variations

There are countless other architectures and especially specific layers or layer blocks that can be used in various architectures. The following are Squeeze/Excite Networks and the important class of normalization layers, which are often used and can significantly improve models or accelerate the optimization process.

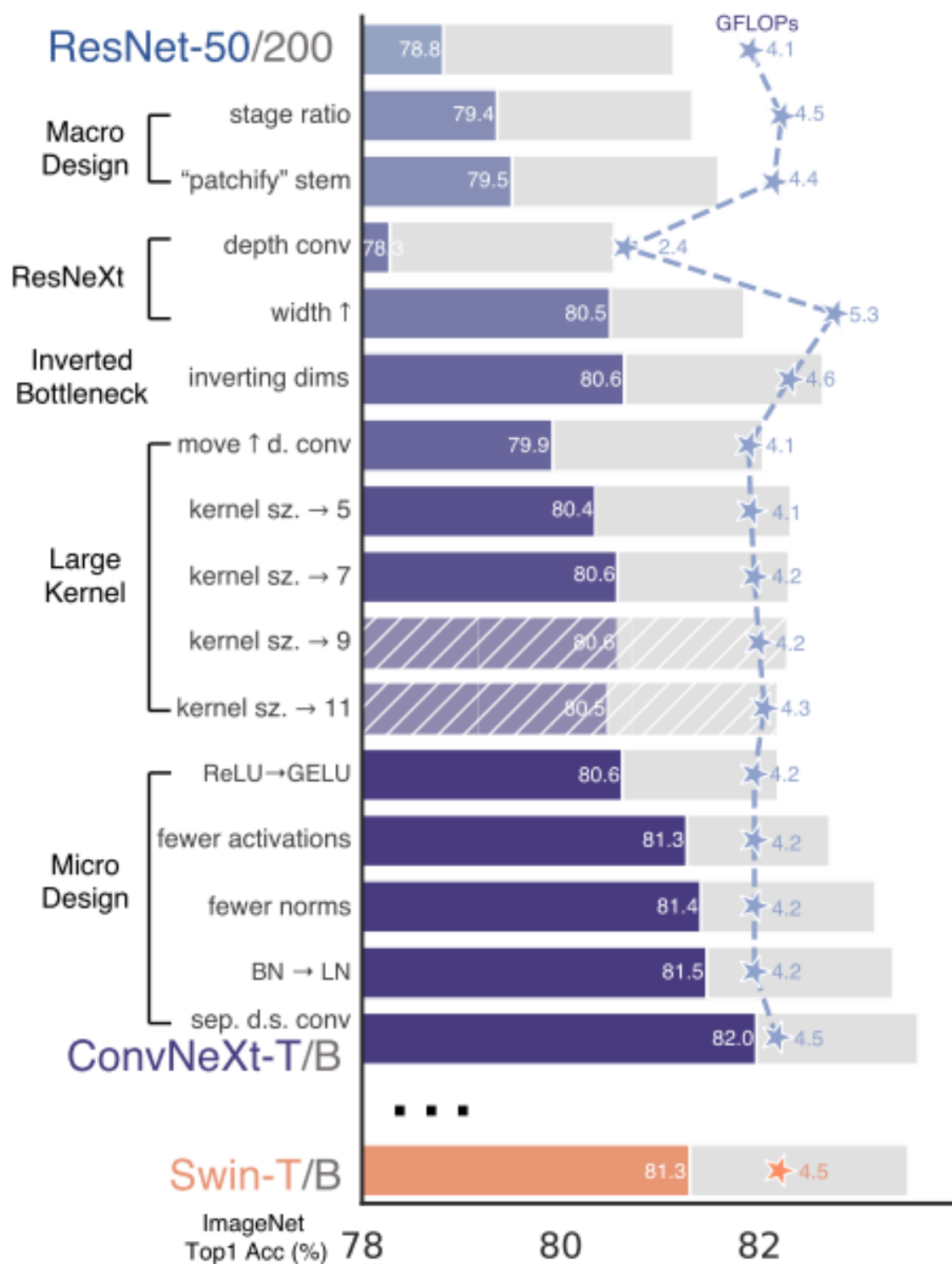


Figure 17: Convnext Liu et al. (2022).

ImageNet Classification Challenge

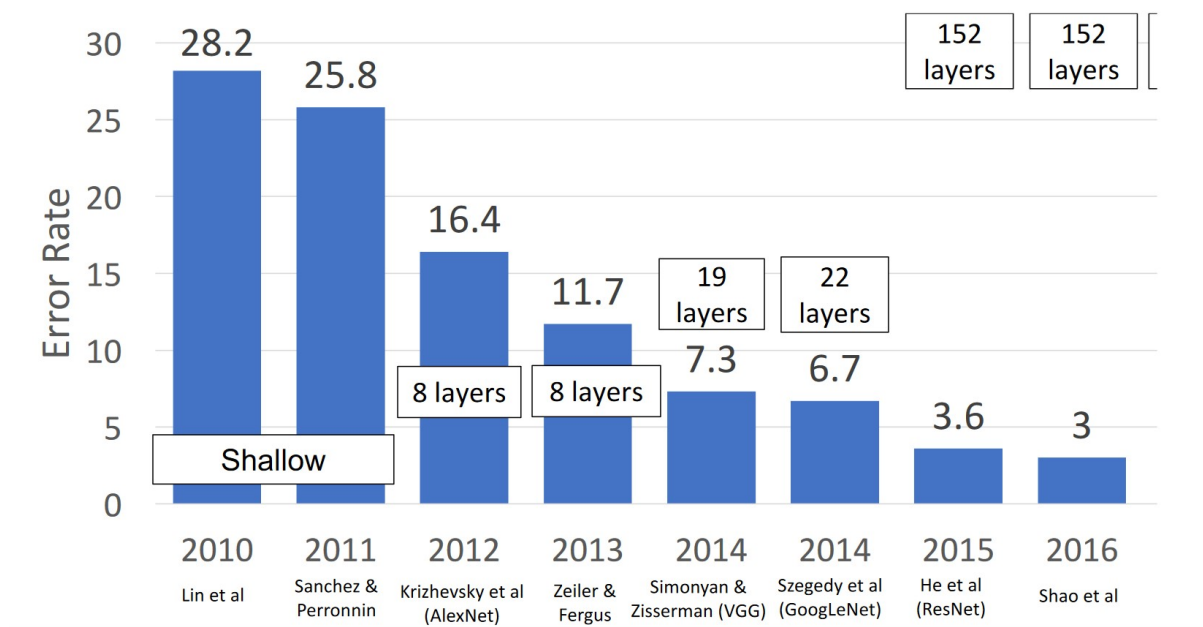


Figure 18: Graphic from Johnson (2019)

Squeeze/Excite Networks

Squeeze-and-Excite Networks (SE-Networks) were introduced in 2019 Hu et al. (2019). These include so-called Squeeze and Excite blocks (SE blocks), which allow the scaling of activation maps of a layer. This scaling is learnable through a few additional parameters. In practice, significant performance gains have been observed. Figure 19 shows an illustration.

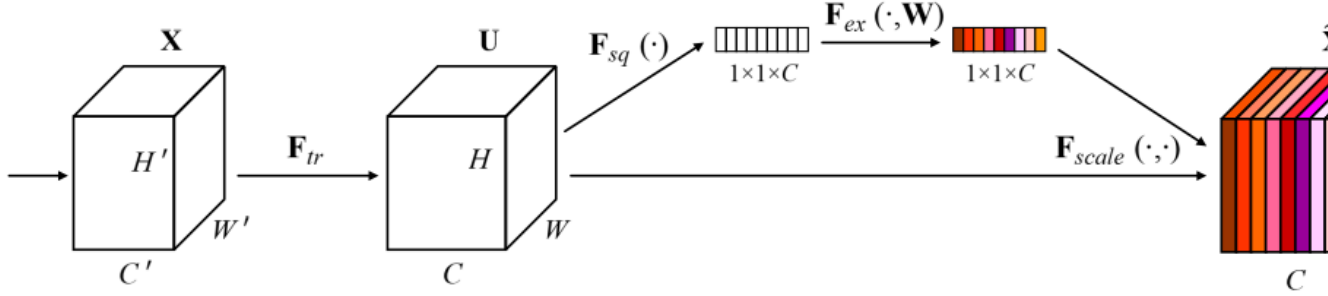


Figure 19: Source: Hu et al. (2019).

These SE blocks can be easily applied to arbitrary activation maps. Figure 19 shows an input feature map $\mathbf{X} \in \mathbb{R}^{H' \times W' \times C'}$ that is transformed with F_{tr} (e.g., with a convolutional layer). This results in the activation maps $\mathbf{U} \in \mathbb{R}^{H \times W \times C}$.

The transformation $F_{sq}(\cdot)$ applies the squeeze operation, a global average pooling, to generate a description of each channel by aggregating spatial information of the activation map \mathbf{U} . This results in a vector $\mathbf{z} \in \mathbb{R}^{1 \times 1 \times C}$.

The excitation operation $F_{ex}(\cdot, W)$ uses a gating mechanism with parameters W , implemented with two fully-connected layers and activation functions. The result is $\mathbf{s} \in \mathbb{R}^{1 \times 1 \times C}$, the channel weights. It is called gating because the weights range from $[0, 1]$ and thus control how much information of a channel flows through (gatekeeping).

Finally, $F_{scale}(\cdot)$ scales the activation maps \mathbf{U} with the channel weights.

The operation can be described as follows:

$$\mathbf{U} = F_{tr}(\mathbf{X}) \mathbf{z} = F_{sq}(\mathbf{U}) = \text{GlobalAvgPool}(\mathbf{U}) \mathbf{s} = F_{ex}(\mathbf{z}, W) = \sigma(\mathbf{W}_2 g(\mathbf{W}_1 \mathbf{z})) \hat{\mathbf{X}} = F_{scale}(\mathbf{U}, \mathbf{s}) = \mathbf{U} \odot \mathbf{s} \quad (4)$$

where $g()$ represents the ReLU function, and σ represents the sigmoid function.

Normalization Layers

Normalization layers normalize activation maps to improve parameter learning. There are many variations, as shown in Figure 20. In modern architectures, normalization layers are typically used. Popular is, for example, layer normalization (see Ba, Kiros, and Hinton (2016)). The general form of normalization is given in equation Equation 5. The parameters γ and β are learned, while the means $E[x]$ and variances $\sigma^2[x]$ are estimated from the activations. Why normalization layers work and which ones to prefer are still subjects of research and are often empirically tested in practice (considered as hyper-parameters). It is believed that the cost function becomes smoother overall, allowing the network to train faster and better Santurkar et al. (2019).

$$y = \frac{x - E[x]}{\sqrt{\sigma^2[x] + \epsilon}} * \gamma + \beta \quad (5)$$

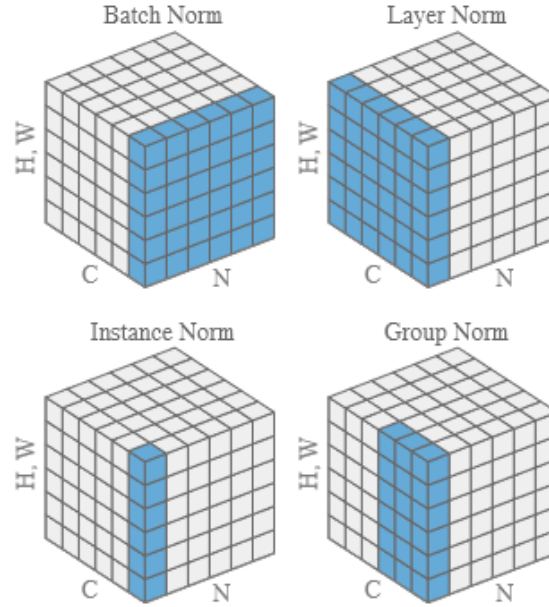


Figure 20: Source: Qiao et al. (2020).

Pre-Processing

Typically, images are pre-processed before being processed by the CNN. This is done to make the optimization process converge faster. Common steps include:

- Resizing/cropping to a fixed size, e.g., 224x224x3. This is necessary for the images to be processed in a batch (they must have the same dimensions).
- Scaling: Images are scaled from the interval [0, 255] to the interval [0, 1].
- Normalization: Normalization is often done along the color channels so that the mean value of the pixel values is 0.

In `torchvision`, the following pre-processing steps are used: [Link](#)

Transfer Learning

Transfer Learning refers to the process of adapting a trained model for Task A to Task B. Adapting pre-trained models often leads to better results and also reduces the number of training iterations. Deep learning frameworks often provide easy access to such models (also known as Model Zoo). For example, from PyTorch: <https://pytorch.org/vision/stable/models.html>.

```
from torchvision.models import resnet50, ResNet50_Weights
resnet50(weights=ResNet50_Weights.IMAGENET1K_V2)
```

PyTorch Example

Architecture

Example of a CNN architecture.

```
import torch
import torch.nn as nn
import torch.nn.functional as F

class Net(nn.Module):
    def __init__(self):
        super().__init__()
        self.conv1 = nn.Conv2d(3, 6, 5)
        self.pool = nn.MaxPool2d(2, 2)
        self.conv2 = nn.Conv2d(6, 16, 5)
        self.fc1 = nn.Linear(16 * 5 * 5, 120)
        self.fc2 = nn.Linear(120, 84)
        self.fc3 = nn.Linear(84, 10)

    def forward(self, x):
```

```

        x = self.pool(F.relu(self.conv1(x)))
        x = self.pool(F.relu(self.conv2(x)))
        x = torch.flatten(x, 1)
        x = F.relu(self.fc1(x))
        x = F.relu(self.fc2(x))
        x = self.fc3(x)
        return x

net = Net()

```

Loss Function

```

import torch.optim as optim

criterion = nn.CrossEntropyLoss()
optimizer = optim.SGD(net.parameters(), lr=0.001, momentum=0.9)

```

References

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