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List of Figures

1 Seasonality function Nord Pool case . . . 5
2 The Gamma O-U process Y1(t) . . . 6
3 The spike O-U process Y2(t) . . . 6
4 Monte Carlo simulation of the daily spot price over one year . . . 6
5 Price comparison . . . 11
6 Residuals calibration . . . 12
7 Swap price with respect to Montecarlo number of simulations . . . 12
8 European put option prices with strikes in range [400;600]€ . . . 15
9 Price comparison 2 . . . 18
10 Residuals calibration 2 . . . 18
11 European put option prices with strikes in range [300;500]€ . . . 19
12 Swap comparison calibration 3 . . . 21
13 Residuals calibration 3 . . . 21

List of Tables

1 Nord Pool parameters values . . . 5
2 Initial conditions Nord Pool case . . . 10
3 Lower bounds parameters . . . 10
4 Upper bounds parameters . . . 10
5 Calibrated parameters values 1 . . . 11
6 Calibrated parameters values 2 . . . 17
7 Calibrated parameters values 3 . . . 21

Contents

1 Introduction **3**

2 Spot price in Arithmetic models case **4**
2.1. Modeling framework 4
2.2. Our specifics: Arithmetic model considering two O-U processes 4
2.3. Description of the processes Y_1 and Y_2 5

3 Future price **7**

4 Swap price **8**

5 Calibration of Arithmetic additive model on French power swap prices **10**
5.1. Choice of Swaps 10
5.2. Minimization & Constraints 10
5.3. Prices reproduction & performances 11
5.4. Computational Cost in MC 12

6 Pricing of European put options on the 2026 swaps **14**
6.1. Simulation of the Arithmetic model 14
6.2. Pricing of Plain Vanilla put options on the underlying swap 14

7 Modeling an Arithmetic model based on just a Gaussian O-U process **16**
7.1. Spot price 16
7.2. Forward price 16
7.3. Swap price 17
7.4. Calibration of the Arithmetic model on French swap prices 17
7.5. Pricing of European put options on the 2026 swaps 18

8 Conclusion **20**

A Appendix **21**

References **22**

1 Introduction

The goal of this project is to analyse the calibration of the arithmetic model on French electricity swap prices and the pricing of European options using a Monte Carlo simulation. This is done by considering the model driven by two inverse Gaussian processes.
$(p = m = 0, n = 2)$
The arithmetic model allows negative prices, a phenomenon which sounds odd in any normal market, since this means that the buyer of a commodity receives money rather than pays. For this reason this model is suitable in the electricity market and may have a simple explanation, since it can be more costly for a producer to switch off the generators than to pay someone to consume electricity in the case of more supply than demand. Thus, electricity is given away along with a payment. In fact, in almost all the liberalised electricity markets, negative prices occur from time to time, although very rarely.
Our working path can be summarized as follows:

1. Study of the arithmetic model, considering our case, for the spot price dynamics, and show the case in which the parameters are chosen in order to mimic the Nord Pool electricity spot prices (3);
2. Analysis of the future price and swap price computing analytically the integrals exploiting the Inverse Gaussian distribution (2);
3. Calibration of the additive model;
4. Simulation of the arithmetic model and pricing of European put options;
5. Analysis of an Arithmetic model based on just a Gaussian O-U process.

In the next section 2 we have briefly described the theoretical framework at the basis of our work, then we have proceeded with the presentation of the dataset used and the core of our project, respectively in 5 and 6 (where are presented the calibration, simulation and pricing). In the end, in section 7, we have concluded with the recalibration of the model based on just a Gaussian Ornstein-Uhlenbeck.

2 Spot price in Arithmetic models case

2.1. Modeling framework

We implement our work firstly writing the spot price, taking into account the Arithmetic model. Thus, we locate the stochastic process $S(t)$ in the Arithmetic case, generally defined as:

$$S(t) = \Lambda(t) + \sum_{i=1}^{m} X_i(t) + \sum_{j=1}^{n} Y_j(t) \tag{1}$$

where $\Lambda(t)$ is the seasonal function,

$$\begin{aligned} X_i(t) = X_i(0) \cdot e^{-\int_0^t \alpha_i(\nu)d\nu} + \int_0^t \mu_i(u) \cdot e^{-\int_u^t \alpha_i(\nu)d\nu} du + \\ + \sum_{k=1}^{p} \int_0^t \sigma_{ik}(u) \cdot e^{-\int_u^t \alpha_i(\nu)d\nu} dB_k(u), \qquad \forall i = 1, ..., m \end{aligned} \tag{2}$$

and

$$\begin{aligned} Y_j(t) = Y_j(0) \cdot e^{-\int_0^t \beta_j(\nu)d\nu} + \int_0^t \delta_j(u) \cdot e^{-\int_u^t \beta_j(\nu)d\nu} du + \\ + \int_0^t \eta_j(u) \cdot e^{-\int_u^t \beta_j(\nu)d\nu} dI_j(u), \qquad \forall j = 1, ..., n \end{aligned} \tag{3}$$

2.2. Our specifics: Arithmetic model considering two O-U processes

Since we are relying on arithmetic process driven by two Inverse Gaussian processes with m=p=0, n=2, Y_0=1, δ_1=δ_2=0, β_1, β_2, η_1, η_2 constant, we obtain the spot price $S(t)$ as follows:

$$S(t) = \Lambda(t) + Y_1(t) + Y_2(t) \tag{4}$$

where $\Lambda(t)$ is the seasonal function:

$$\Lambda(t) = Asin(2\pi t) + B + Ct \tag{5}$$

and

$$Y_j(t) = \beta_j \cdot e^{-\int_0^t dv} + \eta_j \cdot \int_0^t e^{-\beta_j \cdot \int_u^t dv} dI_j(u), \qquad \forall j = 1, 2. \tag{6}$$

2.3. Description of the processes Y_1 and Y_2

Let us consider the arithmetic spot price model (3.23) motivated by the study [Benth, Kallsen and Meyer-Brandis (2007)] relying on choosing parameters in order to mimic the Nord Pool electricity spot prices, however, not based on any rigorous empirical analysis.

β_1	β_2	η_1	η_2	A	B	C
0.085	1.10	0.1	0.1	30	100	0.025

Table 1: Nord Pool parameters values

We discuss the construction of such a process, and apply the algorithm discussed above to simulate price paths. Assume a seasonal floor:

$$\Lambda(t) = 100 + 0.025 \cdot t + 30sin\left(2\pi t/365\right) \tag{7}$$

The mean reversion speeds are set to $\beta_1 = 0.085$ and $\beta_2 = 1.1$ respectively and we assume that the stationary distribution of Y_1 is $\Gamma(\nu, \frac{1}{\mu_j})$, indeed Y_1 is an array of random numbers chosen from gamma distribution with parameters: $shape = 8.06$ and $scale = \frac{1}{7.7} = 0.1299$. On the other hand Y_2 is simulated directly by first simulating the (seasonal) occurrences of jumps and the corresponding jump sizes.
The process Y_1 models the "normal" variations in the market, while Y_2 accounts for the spikes. The innovators I_1 and I_2 are a subordinator and a time-inhomogeneous compound Poisson process, respectively. To have sample paths with spikes, Y_2 will have a fast speed of mean reversion, while Y_1 will revert to zero at a much slower rate.
Since Y_2 is modelling the spikes, it is natural to have a non-stationary jump intensity since spikes may be seasonally occurring, the Nord Pool market being a typical example where spikes are most often present during the winter.
The case study here is meant to give a flavour of the potential of an arithmetic model, and not intended as a complete study which would involve rather sophisticated methods.

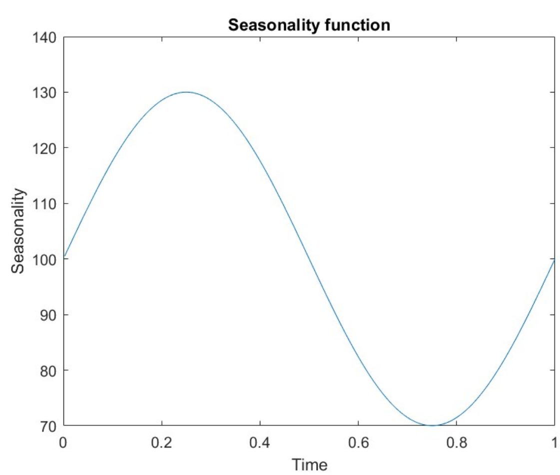

Figure 1: Seasonality function Nord Pool case

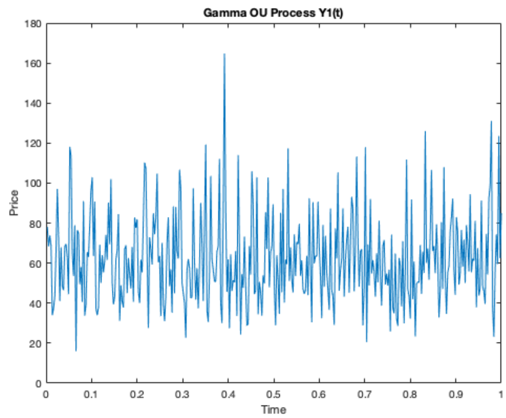

Figure 2: The Gamma O-U process Y1(t)

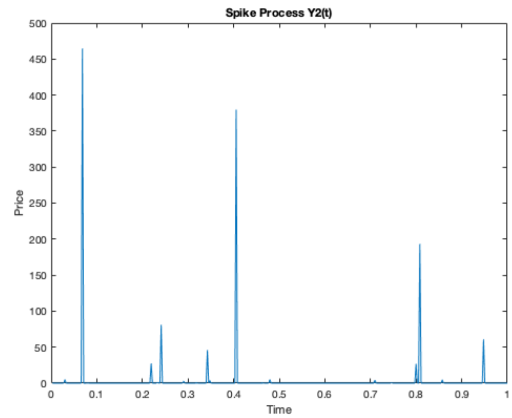

Figure 3: The spike O-U process Y2(t)

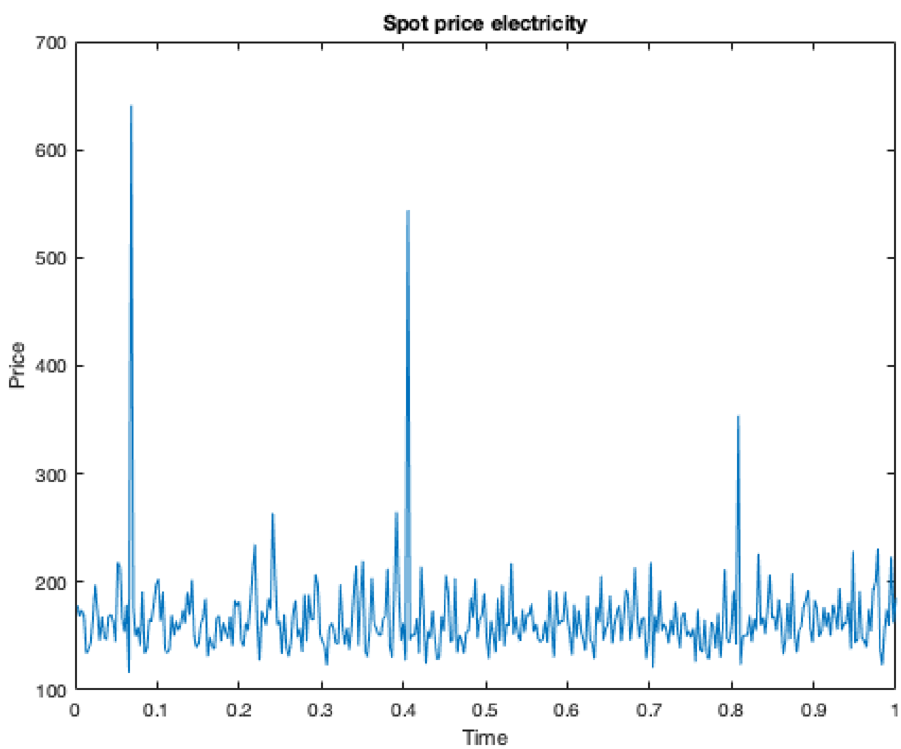

Figure 4: Monte Carlo simulation of the daily spot price over one year

3 Future price

Based on the arithmetic model of spot price defined above 2 we derive the forward price dynamics $f(t,\tau)$ at time τ , where $\tau \leq$ T :

$$f(t,\tau) = \Theta(t;\tau;\theta) + \Lambda(\tau) + \sum_{j=1}^{2} Y_j(t) e^{-\int_t^\tau \beta_j(s)ds}, \tag{8}$$

for $0 \leq t \leq \tau$.

Recalling that m = 0 since we do not assume any diffusional variations in the dynamics.

Taking $\theta = 0$ (The Esscher transform coefficient) (*1) we obtain that:

$$\begin{aligned}\Theta(t,\tau;0) = \sum_{j=1}^{n} \int_t^\tau \eta_j(\nu) e^{-\int_\nu^u \beta_j(s)ds} d\gamma_j(\nu) + \sum_{j=1}^{n} \int_t^\tau \int_{\mathbb{R}} \eta_j(\nu) e^{-\int_\nu^u \beta_j(s)ds} \\ \times z(e^{\tilde{\theta}_j}(\nu)z - \mathbb{I}_{|z|<1}) \cdot l_j(dz,d\nu).\end{aligned} \tag{9}$$

For n=2, thanks to the assumption that the driver of the additive OU is an Inverse Gaussian, we can compute analytically Θ obtaining:
(*2)

$$\Theta(t,\tau;0) = erfc\left(\frac{1}{\sqrt{2k}}\right)\left(\frac{1}{\beta_1}\right)(1-e^{-\beta_1}) + erfc\left(\frac{1}{\sqrt{2k}}\right)\left(\frac{1}{\beta_2}\right)(1-e^{-\beta_2}) \tag{10}$$

The analytical computation of $\Theta(t,\tau;0)$ grent us to reduce the computational cost and the calibration time.

(*1) We recall that the Esscher transform, used in derivatives pricing in many financial markets, is defined as follows:
$f(x;\theta) = e^{\theta x f(x)} / \int_{\mathbb{R}} e^{\theta y} f(y)dy$, where $f(y)$ is a probability density and θ a real number.

(*2) $erfc$:= complementary error function (also called the complementary Gauss error function). It is a complex function of a complex variable defined as: erfc(x) = 1 - erf(x).

4 Swap price

Whereas the Spot price process S is modelled as the arithmetic dynamics, the arithmetic property also holds for Swap contracts.
Considering our case the swap price is given by

$$
\begin{aligned}
F(t,\tau_1,\tau_2) = &\int_{\tau_1}^{\tau_2} \omega(u,\tau_1,\tau_2)\Lambda(u)du + \int_{\tau_1}^{\tau_2} \Theta(u,\tau_1,\tau_2)du + \\
&+ \sum_{j=1}^{n} Y_j(t) \int_{\tau_1}^{\tau_2} \omega(u,\tau_1,\tau_2)\eta_j(\nu)e^{-\int_\nu^u \beta_j(s)ds}
\end{aligned}
\tag{11}
$$

Where

$$
\begin{aligned}
\Theta(t,\tau_1,\tau_2) = &\sum_{j=1}^{n} \int_t^{\tau_2} \int_{max(\nu,\tau_1)}^{\tau_2} \omega(u,\tau_1,\tau_2)\eta_j(\nu)e^{-\int_\nu^u \beta_j(s)ds} du d\gamma_j(\nu) \\
&+ \sum_{j=1}^{n} \int_t^{\tau_2} \int_{\mathbb{R}} \int_{max(\nu,\tau_1)}^{\tau_2} \omega(u,\tau_1,\tau_2)\eta_j(\nu)e^{-\int_\nu^u \beta_j(s)ds} du \\
&\times z(e^{\tilde{\theta_j}}(\nu)z - \mathbb{I}_{|z|<1})du \cdot l_j(dz,d\nu)
\end{aligned}
\tag{12}
$$

$\Theta(t,\tau_1,\tau_2)$ can be solved analytically as in the previous point, and having $n=2$ we can rewrite the swap price as:

$$
\begin{aligned}
F(t,\tau_1,\tau_2) = &\int_{\tau_1}^{\tau_2} \omega(u,\tau_1,\tau_2)\Lambda(u)du + \int_{\tau_1}^{\tau_2} \Theta(u,\tau_1,\tau_2)du + \\
&+ Y_1(t) \int_{\tau_1}^{\tau_2} \omega(u,\tau_1,\tau_2)\eta_1(\nu)e^{-(\beta_1(u-\nu))} + \\
&+ Y_2(t) \int_{\tau_1}^{\tau_2} \omega(u,\tau_1,\tau_2)\eta_2(\nu)e^{-(\beta_2(u-\nu))}
\end{aligned}
\tag{13}
$$

Note the arithmetic structure of the swap price, inherited from the spot and forward price dynamics. Let us discuss the asymptoticity of the swap prices in the long end of the curve.

$$
\begin{aligned}
|F(t,\tau_1,\tau_2) - &\int_{\tau_1}^{\tau_2} \omega(u,\tau_1,\tau_2)\Lambda(u)du - \int_{\tau_1}^{\tau_2} \Theta(u,\tau_1,\tau_2)du| \leq \\
&|Y_1(t)|(1-e^{-\beta_1(\tau_2-\tau_1)})e^{-(\beta_1(\tau_1-t))} + \\
&+ |Y_2(t)|(1-e^{-\beta_2(\tau_2-\tau_1)})e^{-(\beta_2(\tau_1-t))}
\end{aligned}
\tag{14}
$$

where we have used the boundedness of $\omega(u,\tau_1,\tau_2)$. Indeed we know that ω is defined as follows:

$$
\omega(u,\tau_1,\tau_2) = \frac{\hat{\omega}(u)}{\int_{\tau_1}^{\tau_2} \hat{\omega}(v)dv}
\tag{15}
$$

Where $\hat{\omega}(u)$ being equal to one if the swap is settled at the end of the delivery period, which is our case. Then we have $\omega(u,\tau_1,\tau_2) = \frac{1}{\tau_2-\tau_1}$.

Letting $\tau_1 \to \infty$ and the length of the delivery period $\tau_2 - \tau_1$ be fixed, we obtain that

$$F(t, \tau_1, \tau_2) - \int_{\tau_1}^{\tau_2} \omega(u, \tau_1, \tau_2)\Lambda(u)du - \int_{\tau_1}^{\tau_2} \Theta(u, \tau_1, \tau_2)du \to_{\tau_1 \to \infty} 0 \tag{16}$$

Hence, the swap price behaves asymptotically as the weighted average seasonal function $\Lambda(t)$ and a risk-adjustment function Θ. This is in line with the asymptotics of forwards.

5 Calibration of Arithmetic additive model on French power swap prices

5.1. Choice of Swaps

To calibrate our model we take into account only swap prices of French power market with the following twenty maturities: 29-Nov-2022, 30-Dec-2022, 30-Jan-2023, 27-Feb-2023, 30-Mar-2023, 28-Apr-2023, 30-May-2023, 28-Dec-2022, 29-Mar-2023, 28-Jun-2023, 27-Sep-2023, 27-Dec-2023, 26-Mar-2024, 26-Jun-2024, 28-Dec-2022, 27-Dec-2023, 27-Dec-2024, 29-Dec-2025, 29-Dec-2026, 29-Dec-2027, thus obtaining eight parameters calibrated on twenty components vectors.

5.2. Minimization & Constraints

Having the set of initial Swap prices for each maturity, we minimize the sum of the squared distances between the market prices and the ones given by the closed Arithmetic model formula in the Inverse Gaussian case as below:

$$\begin{aligned} F(t,\tau_1,\tau_2) = \int_{\tau_1}^{\tau_2} \omega(u,\tau_1,\tau_2)\Lambda(u)du + \int_{\tau_1}^{\tau_2} \Theta(u,\tau_1,\tau_2)du + \\ + Y_1(t)\int_{\tau_1}^{\tau_2} \omega(u,\tau_1,\tau_2)\eta_1(\nu)e^{-(\beta_1(u-\nu))} + \\ + Y_2(t)\int_{\tau_1}^{\tau_2} \omega(u,\tau_1,\tau_2)\eta_2(\nu)e^{-(\beta_2(u-\nu))} \end{aligned} \tag{17}$$

We proceed in this way: maturity by maturity using `lsqcurvefit` as minimizer, thanks to which we are able to impose the constraints on parameters range and initial conditions described in 2. Furthermore, we can solve analytically some integrals in the swaps formula in order to decrease the computational effort in calibration, resulting in a remarkable improvement in terms of time to find the function minimum.

β_1	β_2	η_1	η_2	A	B	C	k
0.085	1.1	0.1	0.1	30	100	0.025	0.5

Table 2: Initial conditions Nord Pool case

β_1	β_2	η_1	η_2	A	B	C	k
ϵ	ϵ	-5	-10	ϵ	ϵ	ϵ	ϵ

Table 3: Lower bounds parameters

β_1	β_2	η_1	η_2	A	B	C	k
+10	+10	+5	+10	+1000	+1000	+1000	+10

Table 4: Upper bounds parameters

5.3. Prices reproduction & performances

For every different maturity we can compare the swap prices of additive process and the ones of the market dataset, obtaining good results. The precision is particularly visible for long end maturities. The value parameters calibrated and the plots with respect to expiries are reported below, omitting multiple prices for every single maturity.

β_1	β_2	η_1	η_2	A	B	C	k
+1.0977	+1.1064	-4.1432	-9.0517	+366.302	+442.48	+0.0017	+0.0011

Table 5: Calibrated parameters values 1

As we can see from the value parameters obtained, the main drivers of the price are A and B values present in the seasonality function , followed by η_1 and η_2 that will be very useful in the swap price simulation part. Since the calibration parameters are very sensitive to their ranges, we have tried also other calibrations using `lsqcurvefit` as minimizer, moving lower and upper bounds, that can be seen in the Appendix A section.

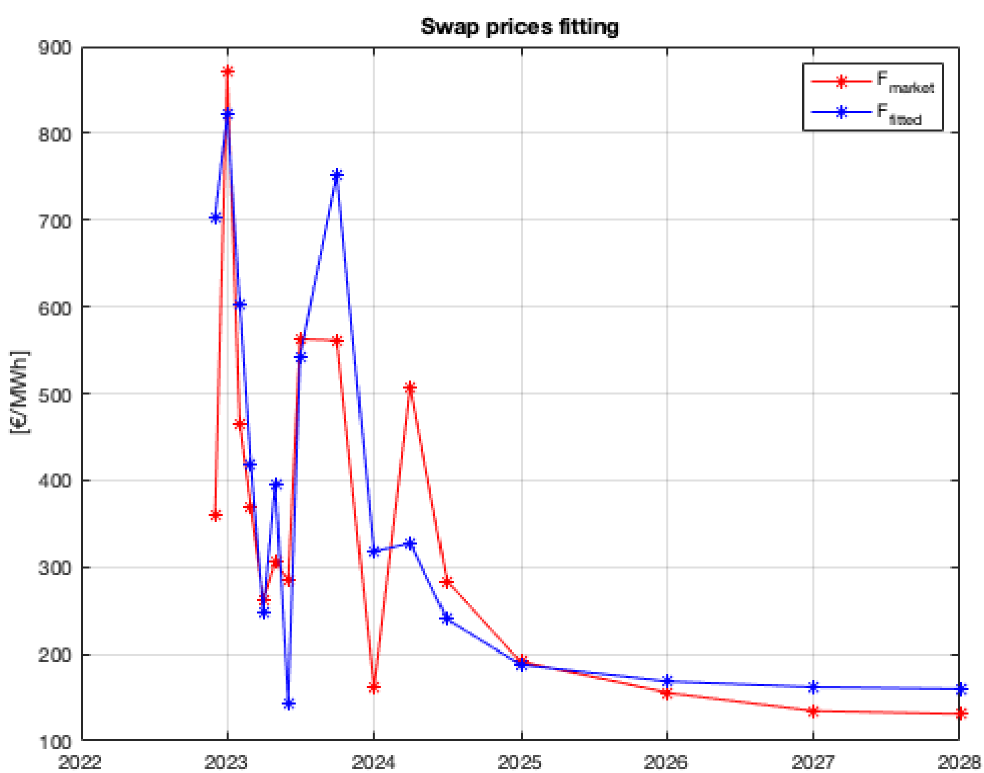

Figure 5: Price comparison

Moreover, we plot in a bar graph the residuals of the calibration for every price. As underlined before, the precision is particularly visible for middle and long end swap expiries. The calibration is not perfect for some expiries cause of the analytical formulas used in the calibration algorithm to increase the computational speed, but it is also due to the fluctuations in swap prices and futures that we have in the market nowadays.

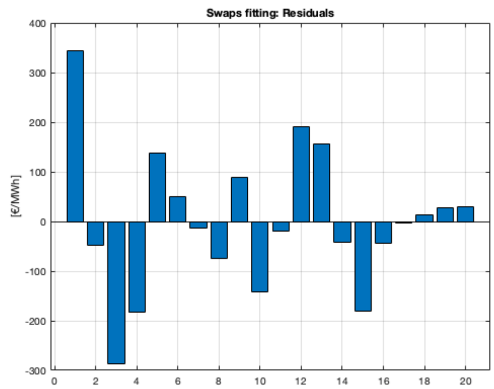

Figure 6: Residuals calibration

5.4. Computational Cost in MC

Using Montecarlo simulation methods, we can notice the convergence of the price with respect to increasing the number of simulations of a factor:

$$\frac{\sigma}{\sqrt{Nsim}} \tag{18}$$

Let us plot the obtained results varying $Nsim \in [10^2, 10^5]$ and considering three different simulations to show the same price reached up with respect to different simulation patterns.

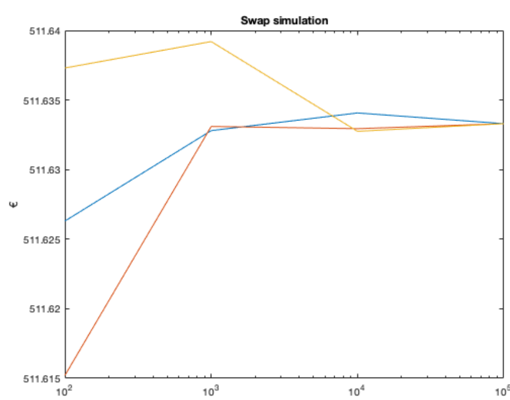

Figure 7: Swap price with respect to Montecarlo number of simulations

In the table below we can see the IC variation with respect to increasing the number of simulations exploiting the Central Limit Theorem:

Nsim	Price	CI 95 %
10^3	511.63617 €	[511.63093, 511.64142] €
10^4	511.63415 €	[511.63247, 511.63582] €
10^5	511.63335 €	[511.63282,511.63388] €

6 Pricing of European put options on the 2026 swaps

6.1. Simulation of the Arithmetic model

To simulate the Arithmetic model, firstly, we take into account the maturity of 2 years and we make a discretization of the time in order to obtain a vector of dates of length 720. Then, we use the parameters found previously to simulate the swap price at 2 years' maturity.

$$Y_{Q1} = Y_1 - \frac{\gamma}{\beta_1} \cdot \eta_1 \cdot (1 - e^{-\beta_1 \cdot t}) - \eta_1 \cdot erfc(\frac{1}{\sqrt{2k}}) \cdot \frac{1}{\beta_1} \cdot (1 - e^{-\beta_1}) \tag{19}$$

$$Y_{Q2} = Y_2 - \frac{\gamma}{\beta_2} \cdot \eta_2 \cdot (1 - e^{-\beta_2 \cdot t}) - \eta_2 \cdot erfc(\frac{1}{\sqrt{2k}}) \cdot \frac{1}{\beta_2} \cdot (1 - e^{-\beta_2}) \tag{20}$$

Therefore, we can derive the forward price dynamics:

$$f(t, T, Y_{Q1}, Y_{Q2}) = \Lambda(T) + \Theta(t, T) + e^{-\beta_1 \cdot (T-t)} \cdot Y_{Q1} + e^{-\beta_2 \cdot (T-t)} \cdot Y_{Q2} \tag{21}$$

The forward price is calculated as the conditional expectation of the spot with respect to the risk-neutral probability, so it follows that the forward price dynamics becomes positive in the arithmetic class of spot models defined by the book [Benth, Kallsen and Meyer-Brandis (2007)]. Then, using the vector of dates T_i, considering as initial and final dates T_1 and T_2, we find the vector of swap prices in this way:

$$F = F + \frac{f(t, T_i, Y_{Q1}, Y_{Q2})}{T_2 - T_1} \cdot (T_i - T_{i-1}) \tag{22}$$

In order to find the swap price, we get the mean of the vector F and we obtain that: $Swap_{2y} = 511.6333$€.

6.2. Pricing of Plain Vanilla put options on the underlying swap

A put plain swaption is a derivative contract that gives an entity the right to enter into a swap on a future date and at a predetermined fixed price. The pay-out from the option is thus no longer received immediately at expiration, but rather during the delivery period of the underlying swap (forward).
Put swaptions are generally used in Electricity market to hedge options positions, to aid in restructuring current positions, to alter portfolio's duration or to speculate.
For what concerns the prices of European put we have applied the formula below considering as underlying the swap price calculated with the simulation:

$$P = e^{-r \cdot (T-t)} \cdot \max(K - Swap_{2y}, 0) \tag{23}$$

In order to have in the money puts we have decided to select $Strikes \in [400, 600]$€.
In the graph below we can see the Put prices pattern with respect to the strikes chosen.

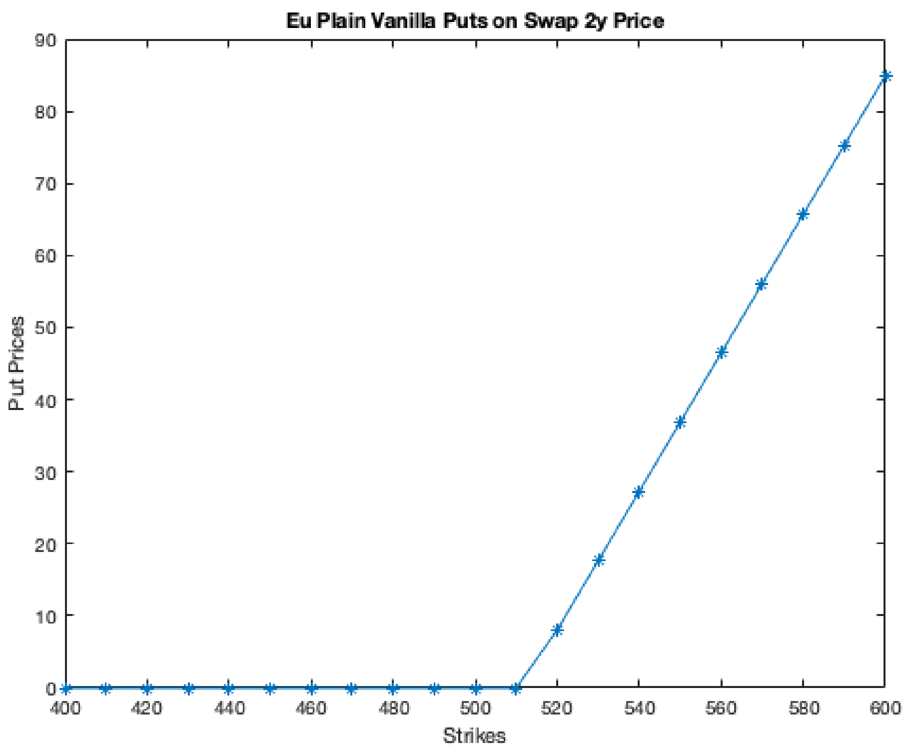

Figure 8: European put option prices with strikes in range [400;600]€

7 Modeling an Arithmetic model based on just a Gaussian O-U process

7.1. Spot price

In this case we refer again to the modelling framework of the arithmetic case explained in 2 section. Since we are relying on arithmetic model based on just a Gaussian O-U process with m=p=1, n=0, X_0=1, δ_1=δ_2=0, α, μ, σ constant, we obtain the spot price $S(t)$ as follows:

$$S(t) = \Lambda(t) + X(t) \tag{24}$$

where $\Lambda(t)$ is the seasonality function:

$$\Lambda(t) = Asin(2\pi t) + B + Ct \tag{25}$$

and

$$\begin{aligned} X_i(t) = X_i(0) \cdot e^{-\int_0^t \alpha_i(\nu)d\nu} + \int_0^t \mu_i(u) \cdot e^{-\int_u^t \alpha_i(\nu)d\nu} du + \\ + \sum_{k=1}^{p} \int_0^t \sigma_{ik}(u) \cdot e^{-\int_u^t \alpha_i(\nu)d\nu} dB_k(u), \qquad \forall i = 1, ..., m \end{aligned} \tag{26}$$

In our case, we consider α=0.5, μ=0, σ=0.2 and the parameters A=30, B=100, C=0.025 for the seasonality function as in 3:

$$X(t) = X(0) \cdot e^{-\alpha \cdot \int_0^t (\nu)d\nu} + \sigma \cdot \int_0^t e^{-\alpha \cdot \int_u^t (\nu)d\nu} dB_k(u), \tag{27}$$

$$\Lambda\left(t\right) = 100 + 0.025 \cdot t + 30sin\left(2\pi t/365\right) \tag{28}$$

7.2. Forward price

We derive again the forward price dynamics when m=1,p=1,n=0 :

$$f(t, \tau) = \Theta(t; \tau; \theta) + \Lambda(\tau) + X_1(t)e^{-\int_t^\tau \alpha_1(s)ds} + \int_t^\tau \mu_1(u)e^{-\int_u^\tau \alpha_1(v)dv} du, \tag{29}$$

where we have that:

$$\Theta(t, \tau; \theta) = \int_t^\tau \sigma_{11}(u)\hat{\theta}_1(u)e^{-\int_u^\tau \alpha_1(v)dv} du. \tag{30}$$

7.3. Swap price

Now analyzing the case of an arithmetic model based on just a Gaussian O-U model ($i.e. m = p = 1, n = 0$), the swap price is given by:

$$\begin{aligned} F(t,\tau_1,\tau_2) = \int_{\tau_1}^{\tau_2} \omega(u,\tau_1,\tau_2)\Lambda(u)du + \int_{\tau_1}^{\tau_2} \Theta(u,\tau_1,\tau_2)du + \\ + \sum_{i=1}^{m} \int_{t}^{\tau_2} \int_{max(\nu,\tau_1}^{\tau_2} \omega(u,\tau_1,\tau_2)\mu_i(\nu)e^{-\int_\nu^u \alpha_i(s)ds} dud\nu + \\ + \sum_{i=i}^{m} X_i(t)\omega(u,\tau_1,\tau_2)e^{-\int_\nu^u \alpha_i(s)ds} du \end{aligned} \tag{31}$$

Solving the intergral analytically we obtain:

$$\begin{aligned} F(t,\tau_1,\tau_2) = \int_{\tau_1}^{\tau_2} \omega(u,\tau_1,\tau_2)\Lambda(u)du + \int_{\tau_1}^{\tau_2} \Theta(u,\tau_1,\tau_2)du + \\ + \int_{t}^{\tau_2} \int_{max(\nu,\tau_1)}^{\tau_2} \omega(u,\tau_1,\tau_2)\mu_1(\nu)e^{-\alpha_1(u-\nu)} dud\nu + \\ + X_1(t)\omega(u,\tau_1,\tau_2)e^{-\alpha_1(u-t)} \end{aligned} \tag{32}$$

7.4. Calibration of the Arithmetic model on French swap prices

We have calibrated the model of an arithmetic model based on a Gaussian Ornstein-Uhlenbeck process comparing for every maturity the swap prices of the additive process and the ones from the market, obtaining good results in terms of residuals and computational effort. The value parameters calibrated and the plots with respect to expiries are reported below, omitting multiple prices for every single maturity.

a	θ_x	σ	A	B	C
0.8171	1.7e-06	0.1490	347.2739	510.0671	-82.7147

Table 6: Calibrated parameters values 2

As we can see from the value parameters obtained, the main drifts of the price are A, B and C present in the seasonality function, followed by a and the volatility σ.
As final remark, here we can notice a negative C with respect to the one described in 5 section, because we do not have the η dynamics in the O-U process.

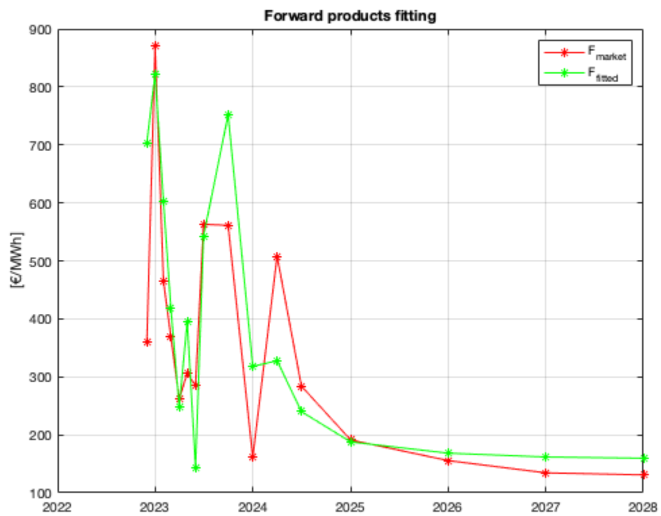

Figure 9: Price comparison 2

Furthermore, we plot in a bar graph the residuals of the calibration for every price.
The precision is remarkably visible for middle and long end swap expiries, obtaining positive outcomes with respect to the computational speed effort and the usage of analytical formulas to compute swap prices.

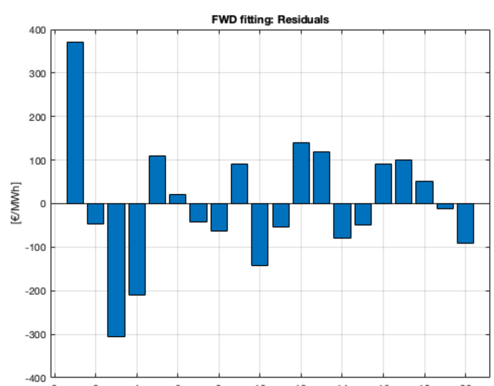

Figure 10: Residuals calibration 2

7.5. Pricing of European put options on the 2026 swaps

We have started considering the Arithmetic model taking into account the parameters found in the previous section and we have simulated the swap price at 2 years' maturity using the following formulas:

$$X_Q = X = X(0) \cdot e^{-\alpha \cdot t} + \frac{\mu}{\alpha} \cdot (1 - e^{-\alpha \cdot t}) + \sigma \cdot \sqrt{\frac{1 - e^{-2 \cdot a \cdot t}}{2 \cdot a}} \cdot G, \quad (*1) \tag{33}$$

Therefore, we can derive the forward price dynamics:

$$f(t,T,X_Q) = \Lambda(T) + \Theta(t,T) \cdot e^{\frac{\mu}{\alpha}\cdot(1-e^{-\alpha\cdot(T-t)})-\alpha\cdot(T-t)\cdot X_Q} \tag{34}$$

Then, using the vector of dates T_i, considering as initial and final dates T_1 and T_2, we find the vector of swap prices in this way:

$$F = F + \frac{f(t,T_i,Y_{Q1},Y_{Q2})}{T_2 - T_1} \cdot (T_i - T_{i-1}) \tag{35}$$

In order to find the swap price, we get the mean of the vector F and we obtain that: $Swap_{2y} = 342.5643$€.
After applying the European Put option formula, we have decided to select $Strikes \in [300, 500]$€ in order to have in the money puts.
In the graph below we can see the Put prices pattern with respect to the strikes chosen.

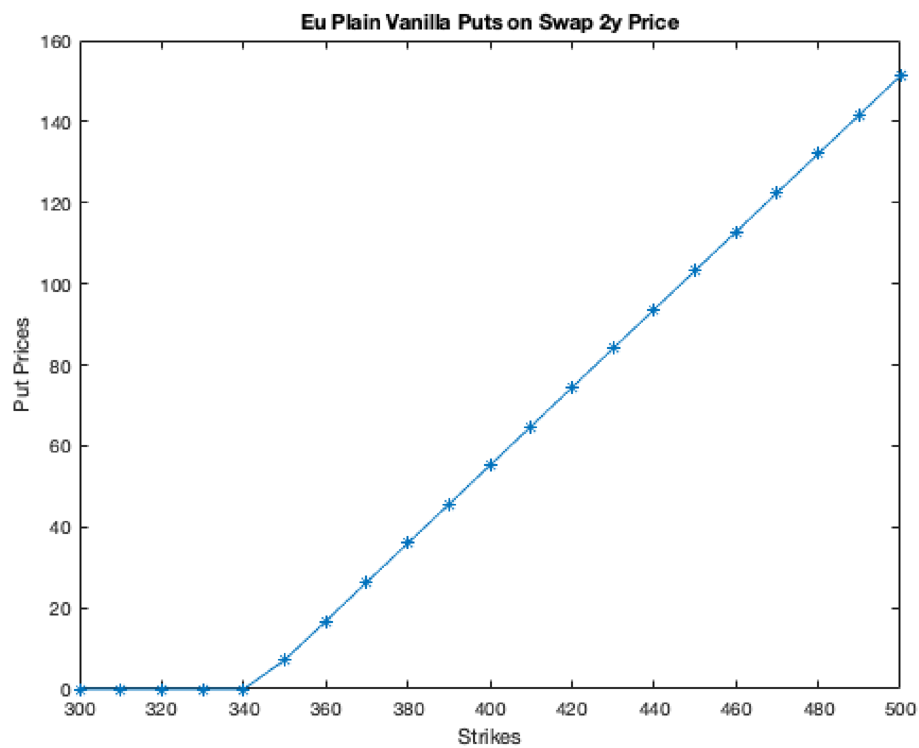

Figure 11: European put option prices with strikes in range [300;500]€

(*1) G is a vector of random values obtained from the standard normal distribution.

8 Conclusion

In this work we have investigated the properties of Arithmetic additive processes that, thanks to time-inhomogeneity, allowed us to model energy spots, forwards and swaps in a delivery period.

Representing the logarithmic prices, or the prices itself, by a series of OU processes allow us to model different speeds of mean reversion, and to incorporate a mixture of jump and diffusional behaviour of the prices peculiar to electricity markets. The models are multi-factor, driven by both Brownian motion and pure jump processes, with possible seasonally dependent jump size and intensity. The attractiveness of using an arithmetic model in the context of electricity and gas markets is clear from the explicitness of the swap dynamics.

We have calibrated the additive processes using swap power prices, highlighting the enhancements of this model type, as the improvements in computational speed making use of analytical formulas. This class of models have revealed to be really efficient also for pricing purposes enjoying a simple Monte Carlo scheme. Indeed, we have employed the former simulation in order to price European vanilla puts on swaps.

Combining an accurate reproduction of swaps curve making use of few parameters, with good applicability to fast and precise pricing techniques, arithmetic additive processes can be considered a very interesting modelling choice in quantitative finance.

A Appendix

We have reported in this appendix the results of the calibration on swap prices, which have been discarded due to parameter values η_1 and η_2 used in the simulation part of the project. The procedure followed is exactly the same of the one described in 5. As before we report the parameters calibrated, the prices comparison and the residuals obtained:

β_1	β_2	η_1	η_2	A	B	C	k
1.109	1.0975	-42.357	-49.258	368.062	440.643	0.5371	0.0529

Table 7: Calibrated parameters values 3

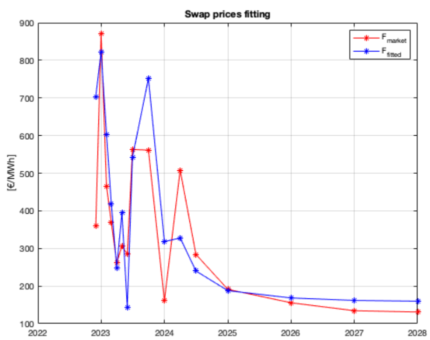

Figure 12: Swap comparison calibration 3

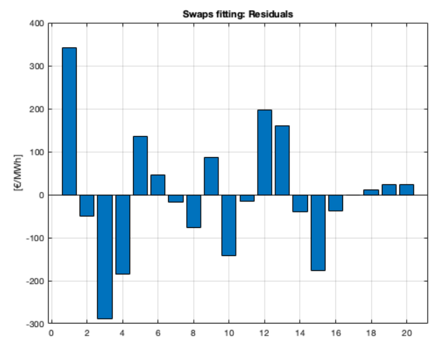

Figure 13: Residuals calibration 3

Below it is presented also a small summary of the performances of this calibration. Comparing with table in 5, it is possible to notice that η_1 and η_2 are remarkably negative, instead C has a bigger value. However we have decided to discard this parameters calibration driven by the fact that simulating the swap, it results in a relevant increment in variance, and consequently in an increasing price volatility.

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[1] [Benth, Kallsen and Meyer-Brandis (2007)] *A Non-Gaussian Ornstein-Uhlenbeck Process for Electricity Spot Price Modeling and Derivatives Pricing.*

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