Università degli Studi di Padova

MODELS OF THEORETICAL PHYSICS

DI

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Introduction

The aim of this document is to collect the notes of *Models of Theoretical Physics* course, held by professors Marco Baiesi and Amos Maritan for the "Physics of data" curriculum in the academic year 2018-2019 (which is the first year of this new curriculum), to have them written in a neater way.

As just told, this document is far from pretendig to be perfect and his goal is to help studying in a neater and better way. For this reason, there may be some errors among it.

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I have tried to be as much ruthless as possible in finding and correcting errors and mistakes, and I apologize if some have survived.

I hope that the overall result will anyway be satisfactory.

Padua, October 2018
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Chapter 1

Methods to compute usefull integrals

In this course we will use many kind of particular integrals. For this reason, in this first chapter we will see a brief introduction to all we need to calculate them.

1.1 Gaussian integrals

Let us consider the following integral:

$$Z(A) = \int d^n x \cdot e^{-A_2(\vec{x})} \tag{1.1}$$

where $Z(A)$ is the integral we want to calculate, $A_2(\vec{x})$ is a x quadratic form as the following:

$$A_2(\vec{x}) = \frac{1}{2} \sum_{i,j=1}^{n} x_i A_{ij} x_j \tag{1.2}$$

Here A is a matrix that can be identified as a metric matrix. In the case we consider A is symmetric with (generally) complex coefficients; furthermore, it has non-negative real parts and non-vanishing eigenvalues a_i.

$$Re(a_i) \geq 0 \qquad \text{and} \qquad a_i \neq 0_{\mathbb{C}}$$

(Note: if these conditions are not true the integral would be divergent...).

Let us bring $A \in \mathbb{R}$. In this case it can be digonalized with orthogonal transformation O, for which it holds:

$$\sum_i O_{ij} x_j = x'_j \qquad \text{and} \qquad |det(O)| = 1$$

This implies that the Jacobian matrix of the transormation J (which is O itself) has $|det(J)| = 1$: so when we use it to change coordinates in the integral, we can say that "it has no effect" (we multiply by 1). Furthermore, with that transformation the non-diagonal coefficients of the new matrix become all 0 (the matrix after the transformation is diagonal), so the new coordinates x'_j are indipendents: this mean that we can evaluate $Z(A)$ by dividing it in the integrals of every single x'_j.
We obtain:

$$Z(A) = (2\pi)^{\frac{n}{2}} \prod_{i=1}^{n} a_i^{-\frac{1}{2}} = (2\pi)^{\frac{n}{2}} \, (det(A))^{-\frac{1}{2}} \tag{1.3}$$

(Note that for a diagonal matrix the product of the eigenvalues is equal to the determinant. Moreover, for Binet's theorem it holds $det(A \cdot O) = det(A) \cdot det(O)$ (true if A and O are square matrix with same dimensions)).
(Note: I think that what just written in the previous note modifies the conditions on $det(O)$, which I think would be $det(O) = +1$ and not $|det(O)| = 1$ otherwise the last formula is meaningless.
This have to be investigated.
However, the important key is that the integral with that tranformation does not become divergent.)