# Celestial Mechanics / Computational Astrodynamics

## Spring 2024

## HW No. 6

### Supplementary information on Reference Systems

The reference system (RS) in which the geopotential model $U$ (consisting of $GM$, $a_e$, Stokes coefficients $C_{nm}$, $S_{nm}$) is expressed is the International Terrestrial RF (ITRS), a form of body-fixed RS. The gradient of $U$ along these axes provides the gravitational acceleration $\boldsymbol{a}_b$. The numerical integration of the equations of motion occurs in the Geocentric Celestial RS (GCRS), a form of inertial RS, fixed at some epoch, usually J2000.0, where the gravitational acceleration is denoted as $\boldsymbol{a}_i$. The transformation of the representation of the acceleration between the two RS's is accomplished through the action of a matrix $\mathbf{R}(TT)$, where $TT$ is the terrestrial date and time

$$\boldsymbol{a}_b = \mathbf{R}\,\boldsymbol{a}_i .$$

The matrix $\mathbf{R}(TT)$ can be computed using the IAU SOFA libraries available at http://www.iausofa.org/ with python version to be found at https://pypi.org/project/pysofa2/. More specifically, the matrix **R** is composed of the rotation of Precession+Nutation **P** and Earth Rotation **E**

$$\mathbf{R} = \mathbf{EP}.$$

The matrix **P** can be computed as `RBPN as` shown in following lines (in Fortran)

```
CALL iau_PNM06A ( DATE1, DATE2, RBPN )
```

where the input date is given by the two TT (Terrestrial Time) arguments `DATE1` and `DATE2`.

The matrix **E** can be computed as `RC2TI as` shown in following lines (in Fortran)

```
UT11 = UTC1
UT12 = UTC2
ERA = iau_ERA00 ( UT11, UT12 )
CALL iau_CR ( RC2I, RC2TI )
CALL iau_RZ ( ERA, RC2TI )
```

where the input date is given by the two arguments `UT11` and `UT12`, which are here taken to be equal to `UTC1` and `UTC2`, respectively. This simplification means that we are ignoring the difference between the UTC and the UT1 timescales, which would more properly call for setting

```
UT1 = UTC + ΔUT1,
```

where `ΔUT1` is to be read from file (it is a measured quantity).

The date and time can be computed using the call

```
CALL iau_DTF2D ( 'UTC', IY, MO, ID, IH, IM, SEC, UTC1, UTC2, J )
```

where IY,MO,ID,IH,IM,SEC are the year, month, etc. and `UTC1, UTC2` are the UTC output date and time arguments, to be transformed to `TT1, TT2` in the TT timescale by the call

```
CALL iau_UTCTAI ( UTC1, UTC2, TAI1, TAI2, J )
CALL iau_TAITT ( TAI1, TAI2, TT1, TT2, J )
```