

# Celestial Mechanics / Computational Astrodynamics

Spring 2024

HW No. 6

## Supplementary information on the Associated Legendre Functions

The computation of the fully normalized Associated Legendre Functions can partially be verified by checking that these exact relations are satisfied

$$\sum_{n=0}^N \sum_{m=0}^n \left( \bar{P}_n^m(\mu) \right)^2 = (N+1)^2,$$
$$\sum_{n=0}^N \sum_{m=0}^n \left( \frac{d\bar{P}_n^m(\mu)}{d\mu} \right)^2 = \frac{N(N+2)(N+1)^2}{4}.$$

Note that if  $\mu = \sin \varphi$ , then

$$\frac{d\bar{P}_n^m(\mu)}{d\mu} = \frac{1}{\cos \varphi} \frac{d\bar{P}_n^m(\sin \varphi)}{d\varphi}.$$

Consider using the **EGM96** geopotential field model for your calculations. It is found at line 72 of the static models listed at [https://icgem.gfz-potsdam.de/tom\\_longtime](https://icgem.gfz-potsdam.de/tom_longtime).