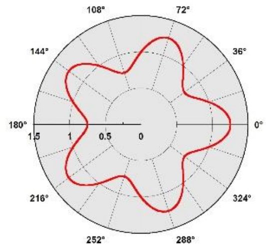
Celestial Mechanics / Computational Astrodynamics

Spring 2024

HW No. 3

Due Tuesday, 8 April 2024, 24:00

1. A particle is attracted to a fixed center. As a result, it follows a closed orbit which can be described as a sinusoidally perturbed circular orbit. Assuming that the average radius vector r is R and that the sinusoidal perturbation has amplitude a, find (A) the general form of the



attracting force $f(r)$ and (B) discuss the notable cases associated with special values of the parameters. It may be useful to recall Binet's equation

$$\frac{d^2u}{d\theta^2}+u^2=\frac{1}{h^2u^2}f\left(\frac{1}{u}\right),$$

where $u=1/r$, θ is the anomaly and h is the constant of areas. (25/100)

2. A projectile is launched due East with inertial speed 5 km/s at an inclination of 45° with respect to the horizon from a location on the equator. Find the range (travel distance computed on the spherical Earth when the projectile hits the ground) and the associated time of flight. (25/100)

3. Study the paper *Stability of planetary systems: A numerical didactic approach* by Pakter and Levin published in American Journal of Physics, **87**, 2019. You will find the paper on the course's moodle page. Report on the main ideas of Section II and Section III of the paper. Also use the numerical integrator you developed in the previous homework as well as an externally-provided numerical integration algorithm (like the integrator you have previously used in the context of the same homework) to reproduce the numerical results of Sections II and III of the paper, providing the necessary analytical background adopted, like, e.g., the equations of motion.

   Provide your assessment of the results obtained and compare them with those of the paper, also giving your evaluation of the final conclusions of the authors and your own.

   In particular, (a) state clearly how you define the initial dynamical state and the reasoning behind it and (b) clearly define any conserved quantity you may be using to check your computation. (50/100)

Useful data

The gravitational parameter of the Earth: $GM_\oplus$ = 398600.4415 km^3/s^2

Note - Remember to show your work clearly (including appropriate graphics when applicable) in your write-ups and provide descriptions of what you are doing and why. This will maximize my chances to understand your derivations and follow your reasoning.