Celestial Mechanics / Computational Astrodynamics

Spring 2024

HW No. 2

Due Friday, 22 March 2024, 24:00

1. Write a (in Matlab, or Python, …) function that implements the classical RK method of the fourth order defined by

$$
\begin{aligned}
\mathbf{x}_{n+1} &= \mathbf{x}_n + \frac{1}{6}\left(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4\right) \\
\mathbf{k}_1 &= h_n \mathbf{f}\left(t_n, \mathbf{x}_n\right), \\
\mathbf{k}_2 &= h_n \mathbf{f}\left(t_n + \frac{1}{2}h_n, \mathbf{x}_n + \frac{1}{2}\mathbf{k}_1\right), \\
\mathbf{k}_3 &= h_n \mathbf{f}\left(t_n + \frac{1}{2}h_n, \mathbf{y}_n + \frac{1}{2}\mathbf{k}_2\right), \\
\mathbf{k}_4 &= h_n \mathbf{f}\left(t_n + h_n, \mathbf{x}_n + \mathbf{k}_3\right),
\end{aligned}
$$

which applies to the IVP system of first-order ODE's

$$
\frac{d\mathbf{x}}{dt} = \mathbf{f}\left(t, \mathbf{x}\right), \qquad \mathbf{x}\left(t_0\right) = \mathbf{x}_0,
$$

and where the time integration step is h_n. (25/100)

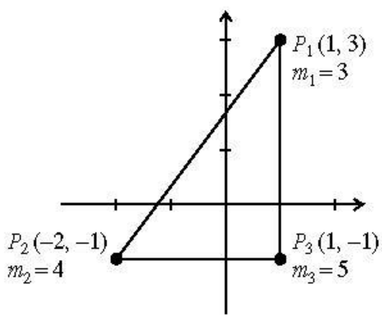
2. Numerically integrate the differential equations of the inertial motion of two bodies, each having the mass of the Earth, subject to their mutual gravitational interaction. Use (a) an RK method available in Matlab (or Python, …), and (b) the RK4 method developed in Exercise no. 1. Try several settings of the Matlab RK integrator and comment on the difference in the results obtained using the two methods.

Assume the following initial conditions (IC)

$$
\mathbf{x}_1(0) = \begin{pmatrix} x_1 \\ y_1 \\ \dot{x}_1 \\ \dot{y}_1 \end{pmatrix} = \begin{pmatrix} 0.0 & \text{km} \\ 0.0 & \text{km} \\ 0.0 & \text{km/s} \\ 0.0 & \text{km/s} \end{pmatrix}, \qquad
\mathbf{x}_2(0) = \begin{pmatrix} x_2 \\ y_2 \\ \dot{x}_2 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} 384400.0 & \text{km} \\ 0.0 & \text{km} \\ 0.91647306922544 & \text{km/s} \\ 0.91647306922544 & \text{km/s} \end{pmatrix}.
$$

Plot their trajectories and that of their CM over a time interval not less than 3×10^6 s. Discuss your results. (35/100)

3. Consider Burrau's problem, also known as the Pythagorean Three-Body Problem, where three bodies of masses 3, 4, 5 are at rest and located at the vertices of a triangle of sides 3, 4, 5. The masses face corresponding sides.



(A) Write the equations of motion of this system of three bodies, choosing units such that the constant of universal gravitation *G* has the value 1.

(B) Numerically integrate the equations of motion up to t = 70.

Try to use the numerical integrator you developed in Problem 1, but rely on a Matlab-provided numerical integration tool (or any reliable numerical integration code in your preferred coding language). (40/100)

Beware that the time step used in the numerical integration must be a very small fraction of unity and that it must be appropriately modified when close encounters of the bodies occur. You may try using a variable step size h according to

$$h = \frac{h_0}{\dfrac{1}{r_{12}^2} + \dfrac{1}{r_{23}^2} + \dfrac{1}{r_{31}^2}},$$

where h_0 is your initially selected time step.

As a safety measure, keep the energy and angular momentum integrals under control while performing the integration.

Useful data

The gravitational parameter of the Earth: $GM_\oplus$ = 398600.4415 km^3/s^2

Note - Remember to show your work clearly (including appropriate graphics when applicable) in your write-ups and provide descriptions of what you are doing and why. This will maximize my chances to understand your derivations and follow your reasoning.