Celestial Mechanics / Computational Astrodynamics
Spring 2024
HW No. 4

Due Tuesday, 24 April 2024, 24:00

1. For which value of the eccentricity does the maximum angular deceleration on a Keplerian orbit occur at true anomaly $f = \pi / 4$? (20/100)

2. Write a (set of) Matlab function(s) to convert the orbit element set $(a, e, i, \Omega, \tilde{\omega} = \pi, L)$ to the Cartesian dynamical state (x, y, z, v_x, v_y, v_z) in the same inertial reference system where the elements are given. Orbit element sets are provided for each of the planets of the Solar System (Mercury—Neptune) in the attached Matlab m-file starting at epoch J2000.0 (1 January 2000, 12h UT, or JD = 2451545.0) in the mean (dynamical) ecliptic and equinox of date (MEcE) reference system. (20/100)

3. Using the Matlab® function(s) developed in the previous exercise, develop a function to compute (a) the geocentric Cartesian equatorial coordinates and (b) the right ascension and declination of Mercury, Venus and Mars for each day in 2024 at 23h and plot the path of the planet on the plane of the sky (i.e., in declination vs. right ascension). (20/100)

4. Write a Matlab® function to compute the azimuth *A* and elevation *h* for a specified Earth location. Run the function to compute an (*A*,*h*)—ephemeris of Mercury, Venus and Mars for each day in 2024 at 23h for Padua and plot its apparent motion on the plane of the sky. When is the planet visible? (20/100)

5. Plot the figure described in the sky by the Sun and Venus during 2024 as observed in Padua each day at 10h UT and at 20h UT, respectively. (20/100)

<u>Useful data</u>

Speed of light: c = 299792458 m/s

Astronomical Unit: AU = 149,597,870,700 m

The radius of the spherical Earth: $R_\oplus$ = 6,378,137 m

1 solar day = 1.002 737 909 350 795 sidereal days

Longitude and latitude of Padua: 11° 52.3' E, 45° 24' N

Greenwich Mean Sidereal Time

GMST at 0^h UT1 (i.e., at midnight) is defined as (Vallado 4th p. 187)

$$GMST_0 = 24110.54841 + 8640184.812866 \times T + 0.093104 \times T^2 - 0.0000062 \times T^3$$

in units of seconds, where the time T is in Julian centuries from J2000.0, or

$$T = \frac{JD - 2451545.0}{36525},$$

where JD is the Julian Date (referring to the midnight, i.e., of the form *.5).

GMST at a different time of day can then be obtained as

$$GMST = GMST_0 + \omega_\oplus \times UT1,$$

where UT1 is the time of day since midnight (in units of days) and

$$\omega_\oplus = 1.002737909350795$$

is the angular velocity of the Earth in rev/UT1-day.

Alternatively, for any JD, one can use

$$GMST = 67{,}310\overset{s}{.}54841 + (876{,}600^h + 8{,}640{,}184\overset{s}{.}812866) \times T + 0.093104^s \times T^2 - 0.0000062^s \times T^3$$

The mean obliquity $\bar{\varepsilon}$ according to the 1980 IAU Theory of Nutation is (Vallado 4th p. 225)

$$\bar{\varepsilon} = 23\overset{\circ}{.}439291 - 0.0130042 \times T - 0.00059 \times T^2 + 0.001813 \times T^3$$

in units of degrees.

NOTE

You will find a data package in the moodle page of the course which includes the files `Meeus___Classical Orbital Elements.m` and `Meeus___Classical Orbital Elements.pdf`. The first file contains a dataset of the orbital elements of the planets at J2000.0 in the MEcE reference system, including their time variations. Read the second file for information on how to use the dataset. To help convert between Julian and Gregorian dates see the folder `Time & Calendar info`.