Celestial Mechanics / Computational Astrodynamics

Spring 2024

HW No. 5

Due Friday, 10 May 2024, 24:00

1. Write a (set of) function(s) to compute Stumpff's functions $c_n(z), n=1,2,\ldots,5$ and their first derivatives $c'_n(z), n=1,2,3$ valid for any z. Hint: For low values of the order n, the expressions of the Stumpff's functions in term of (hyperbolic) trig functions can be used when the argument is sufficiently far from zero, otherwise the series expansions should be used for better numerical accuracy. (10/100)

2. Using the function(s) of the previous exercise, write a function to solve the Universal Kepler Equation

$$t-t_0 = r_0 s + r_0 \dot{r}_0 s^2 c_2\left(-2Cs^2\right) + \left(\mu + 2r_0 C\right) s^3 c_3\left(-2Cs^2\right)$$

   (20/100)

3. Using the Lagrange coefficients formulation of the TBP in terms of universal variables and Stumpff's functions, propagate for 36 hours the following Venus-centric dynamical state

$$
\begin{aligned}
x_0 &= -267733.084163 \text{ km} \\
y_0 &= 199426.194677 \text{ km} \\
z_0 &= 254709.414665 \text{ km} \\
\dot{x}_0 &= 4.168950 \text{ km/s} \\
\dot{y}_0 &= -2.598877 \text{ km/s} \\
\dot{z}_0 &= -3.925639 \text{ km/s}
\end{aligned}
$$

   (this is the Galileo Orbiter dynamical state on 1990 FEB 9, 12:00:00 UTC). Plot the orbit and find its final dynamical state. (20/100)

4. Using the function(s) developed in Ex No. 1 to compute Stumpff's functions, write a function to solve the Universal Lambert Equation for the time of flight $t-t_0$,

$$t-t_0 = A\sqrt{s^2 c_2(z)} + \mu s^3 c_3(z),$$

   with

$$s^2 = \frac{1}{\mu c_2(z)}\left(r_0 + r - A\frac{c_1(z)}{\sqrt{c_2(z)}}\right), \qquad A = \frac{\sqrt{r_0 r}\sin\Delta f}{\sqrt{1-\cos\Delta f}},$$

   where μ is the gravitational parameter, r_0 the distance at time t_0, r the distance at time t, and Δf the change in true anomaly. (30/100)

5. Given the following initial and final Venus-centric position vectors

$$\mathbf{r}_0 = \begin{pmatrix} -267733.084163 \\ 199426.194677 \\ 254709.414665 \end{pmatrix}, \qquad \mathbf{r}_1 = \begin{pmatrix} 177071.935393 \\ -334448.764629 \\ -184024.725921 \end{pmatrix}$$

   expressed in units of km, solve the associated Lambert problem to find the initial velocity $\mathbf{v}_0$ in units of km/s so that position $\mathbf{r}_1$ (the actual position of the Galileo Orbiter on 1990 FEB 11, 00:00:00 UTC) is reached after 36 hours. Find the final velocity $\mathbf{v}_1$. Find the difference between the initial and final dynamical states just found and those computed in Problem 3. What might be the reasons for the differences? (20/100)

<u>Useful data</u>

Gravitational parameter of Venus: GM = 324859.2139518842890 km^3/s^2