Physical insights from machine learning tools

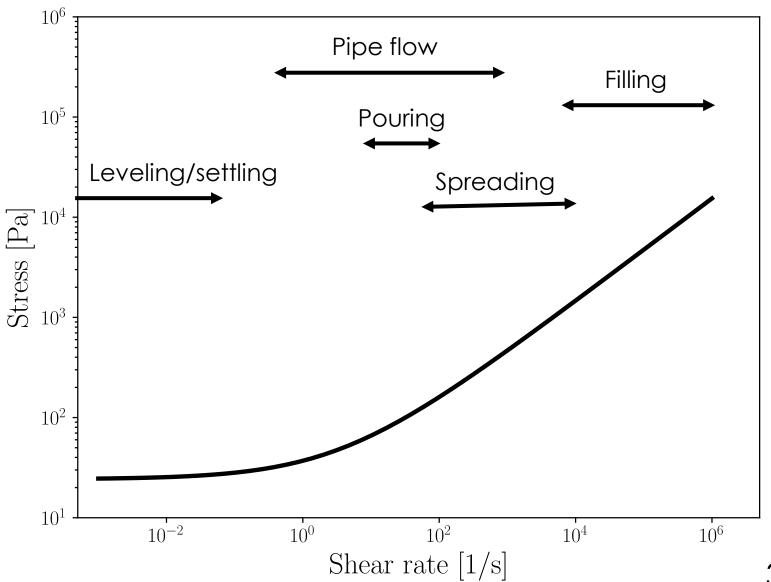
Marco Caggioni, William Hartt, Julie Hipp, Seth Lindberg, Emilio Tozzi

93rd Annual Society of Rheology Meeting 10/10/2022

Outline

- Motivations
- Mastercurves (MC)
 - Model based MC
 - Machine Learning Based MC
- Physical insights
- Challenges for the future

Motivation

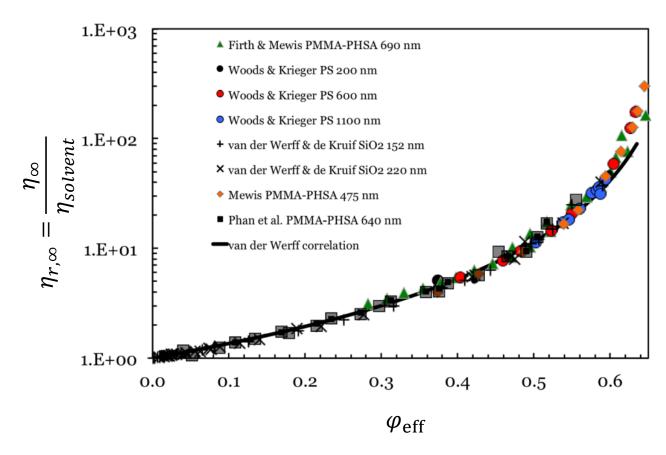


Master Curve Introduction

- Typically used to gain physical insight into behavior by plotting two meaningful dimensionless parameters against one another.
- Suspension rheology often uses Maron-Pierce type equations:

$$\eta_{r,\infty} = \frac{\eta_{\infty}}{\eta_{solvent}} = \left(1 - \frac{\varphi}{0.71}\right)^{-2}$$

Russel, et al. J. Rheol. 57 (2013): 1555-1567.



Mastercurve Example

Dekker, Riande I., et al. "Scaling of flow curves: Comparison between experiments and simulations." J. Nonnewton. Fluid Mech. 261 (2018): 33-37.

- Oil Emulsion in SDS solution
- Variable oil volume fraction
- Equilibrium flow curve
- Herschel-Bulkley (HB) model fit → yield stress

Master curve showing the collapse of the flow curves into one when plotted:

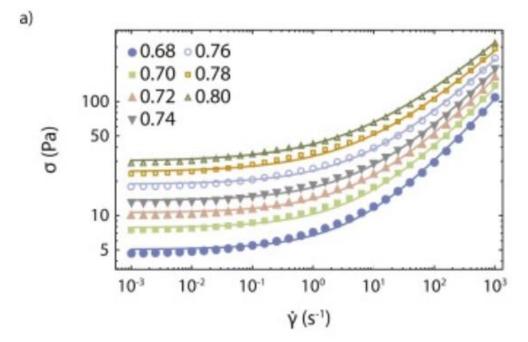
 $|\Delta \phi| = \phi - \phi_c$ distance from jamming ($\phi_c = 0.64$)

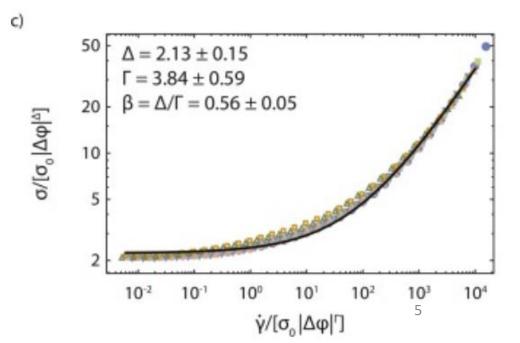
Reduced stress:
$$\frac{\sigma}{\sigma_o |\Delta\phi|^{\Delta}}$$
 with $\Delta=2.13\pm0.15$

Reduced shear rate: $\frac{\dot{\gamma}}{\sigma_o |\Delta \phi|^{\Gamma}}$ with $\Gamma = 3.84 \pm 0.59$

Mastercurve + HB fit $\beta = \Delta / \Gamma = 0.56$ and K=0.19

$$\sigma = \sigma_y + K \dot{\gamma}^\beta$$

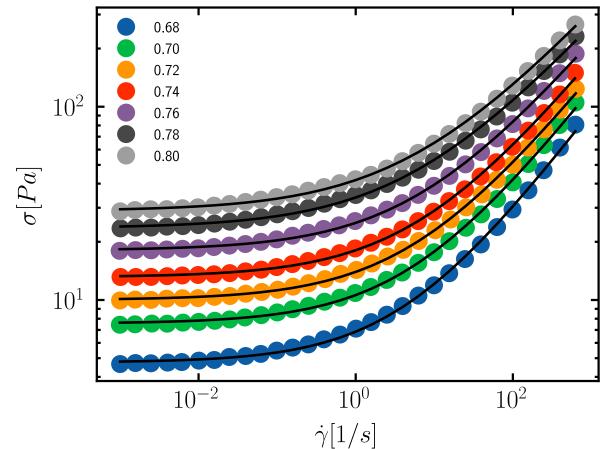


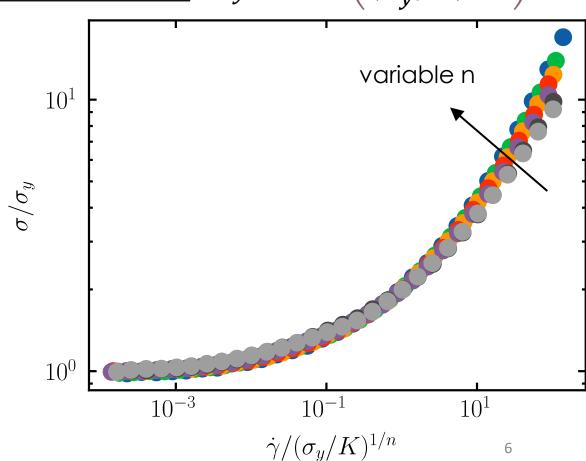


Natural scaling from rheological model: HB

$$\sigma(\dot{\gamma}) = \sigma_y + K\dot{\gamma}^n$$

Ф	K (Pa.s ⁿ)	n	σ_y (Pa)	
0.68	2.10	0.54	4.75	
0.70	3.04	0.53	7.56	
0.72	3.97	0.51	10.01	
0.74	4.94	0.51	13.14	
0.76	6.99	0.49	18.07	σ
0.78	10.36	0.46	23.55	$\frac{1}{1} = 1 + 1$
0.80	12.83	0.45	28.90	$\sigma_{\!\scriptscriptstyle \mathcal{Y}}$
				•





Three-component model

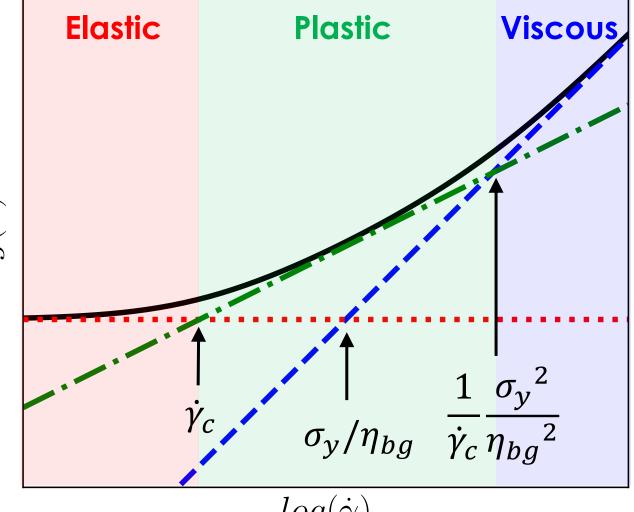
TC:
$$\sigma = \sigma_y + \sigma_y \cdot \left(\frac{\dot{\gamma}}{\dot{\gamma}_c}\right)^{\frac{1}{2}} + \eta_{bg} \cdot \dot{\gamma}$$

 $\sigma_{\rm v}$ = yield stress

 $\dot{\gamma_c}$ = critical shear rate

 η_{bg} = background viscosity





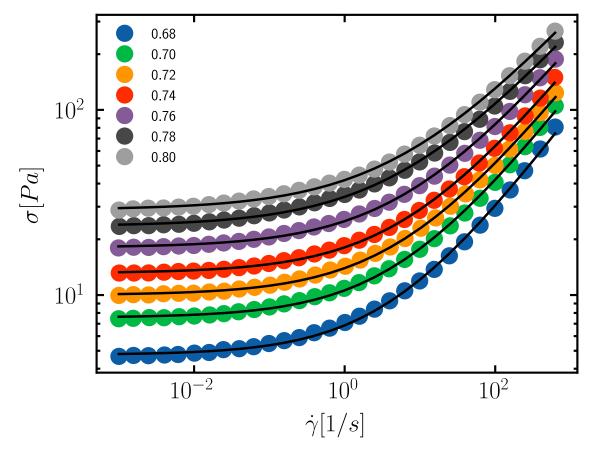
 $log(\dot{\gamma})$

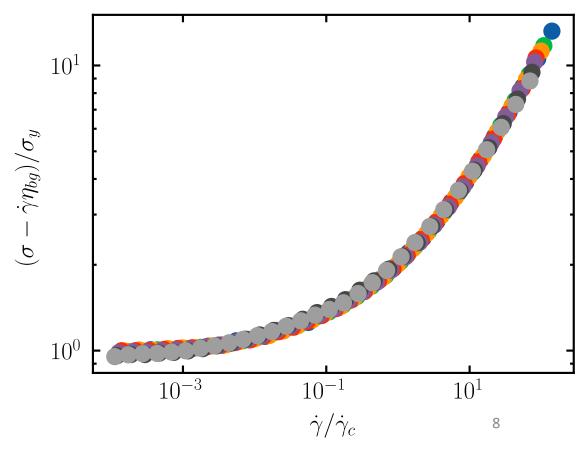
Natural scaling from rheological model: TC

$$\sigma(\dot{\gamma}) = \sigma_y + \sigma_y \left(\frac{\dot{\gamma}}{\dot{\gamma}_c}\right)^{0.5} + \eta_{bg} \dot{\gamma}$$

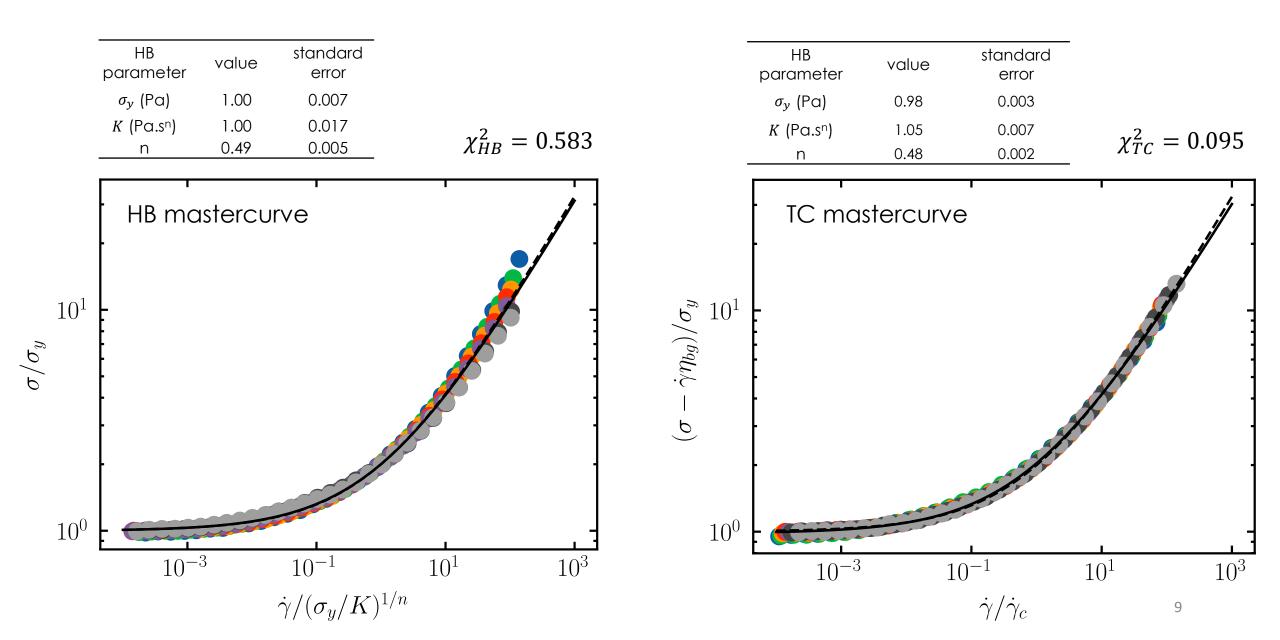
Ф	η_{bg} (Pa.s)	$\dot{\gamma}_c$ (S ⁻¹)	σ_y (Pa)
0.68	0.03	4.44	4.69
0.70	0.03	5.77	7.51
0.72	0.02	6.37	10.00
0.74	0.02	7.30	13.16
0.76	0.00	7.78	18.28
0.78	0.00	8.47	24.46
0.80	0.00	8.98	30.19

$$\frac{\sigma(\dot{\gamma}) - \eta_{bg}\dot{\gamma}}{\sigma_y} = 1 + \left(\frac{\dot{\gamma}}{\dot{\gamma}_c}\right)^{0.5}$$





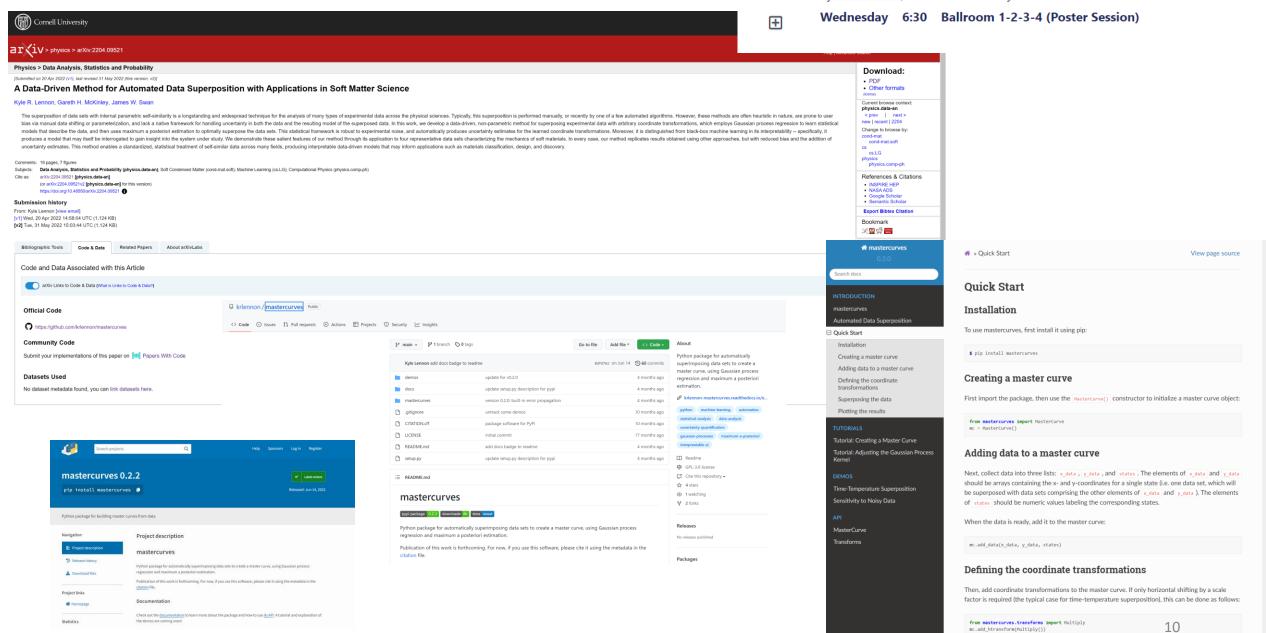
Judgement of Mastercurve Quality



Mastercurve ML tool

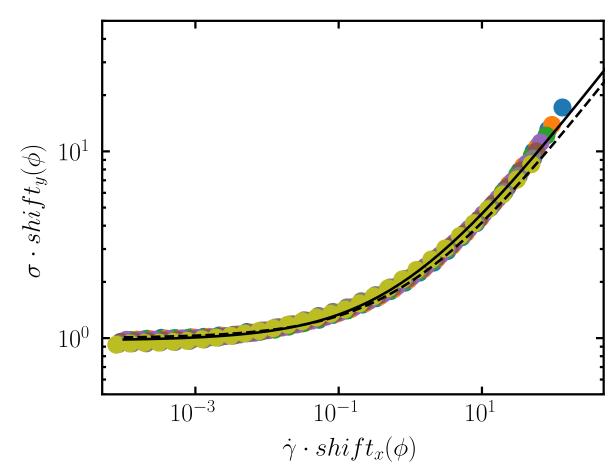
PO39 Automatic construction of rheological master curves

Kyle R. Lennon, Gareth H. McKinley and James W. Swan



Mastercurve with just X and Y multiplication

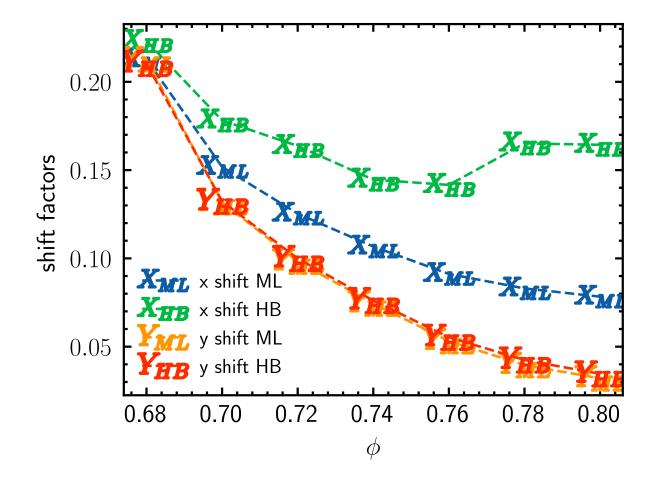
HB parameter	value	standard error	
σ_y (Pa)	1.00	0.007	
K (Pa.s ⁿ)	1.00	0.017	$\chi^2 = 0.2108$
n	0.49	0.005	λ - 0.2100



```
mc=MasterCurve()
# Build a master curve
mc.clear()
mc.add data(gdots, sigmas, phi)
# Add transformations
mc.add_htransform(Multiply())
mc.add_vtransform(Multiply())
# Superpose
loss = sum(mc.superpose())
mc.change ref(0.68, 4.7, 4.7)
print(loss)
fig1, ax1, fig2, ax2, fig3, ax3 = mc.plot(log=True)
```

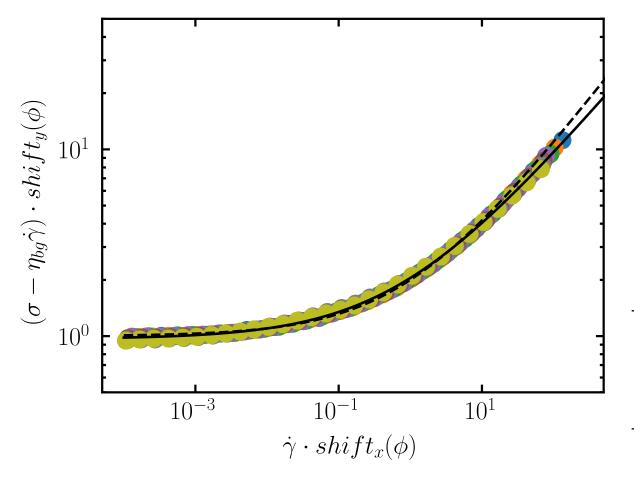
$\chi_{HB}^2 = 0.583$ 10^{1} 10^{0} 10^{-1} 10^{1} $\dot{\gamma}/(\sigma_y/K)^{1/n}$ $\chi_{ML}^2 = 0.2108$ $\sigma[Pa] \cdot shift_y(\phi)$ 0 0 10^{0} 10^{-3} 10^{1} 10^{-1} $\dot{\gamma}[1/s] \cdot shift_x(\phi)$

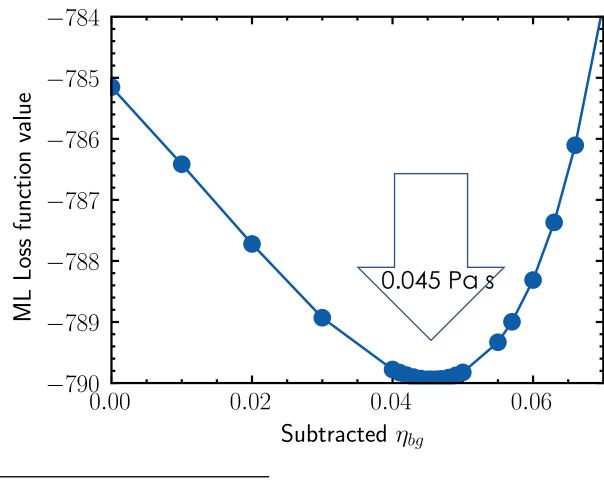
Can we gain physical insights from the shift factors?



Subtraction of a common background

Subtract η_{bg} $\dot{\gamma}$ from the stress data before passing to ML Mastercurve optimization tool





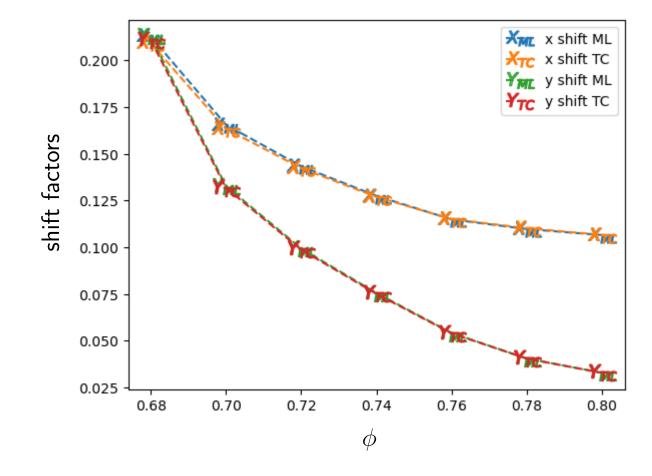
HB parameter	value	standard error
$\sigma_{\!\scriptscriptstyle \mathcal{Y}}$ (Pa)	0.96	0.002
K (Pa.s ⁿ)	1.08	0.005
n	0.45	0.001

 $\chi_{ML}^2 = 0.048$

$\chi_{TC}^2 = 0.048$ Subtracting fixed $-\dot{\gamma}\eta_{bg})/\sigma_{y}$ η_{bg} =0.045 Pas 10^{-3} 10^{-1} 10^{1} 10^{3} $\chi_{ML}^2 = 0.048$ - $\eta_{bg}\dot{\gamma})\cdot shift_y(\phi)$ Subtracting fixed η_{bg} =0.045 Pa s 10^{-3} 10^{1} 10^{-1} $\dot{\gamma} \cdot shift_x(\phi)$

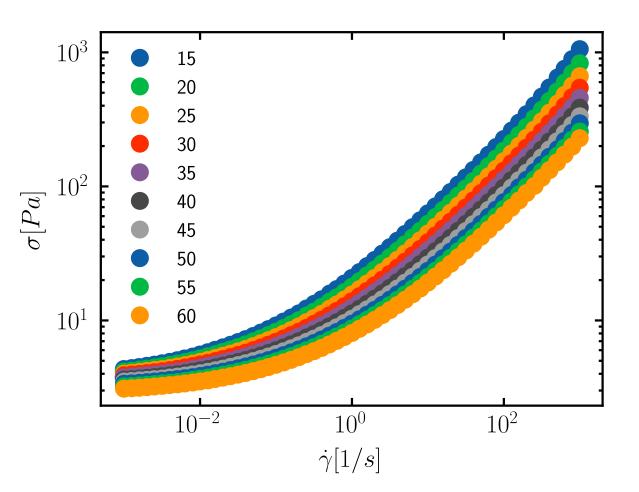
Physical insights from shift factors

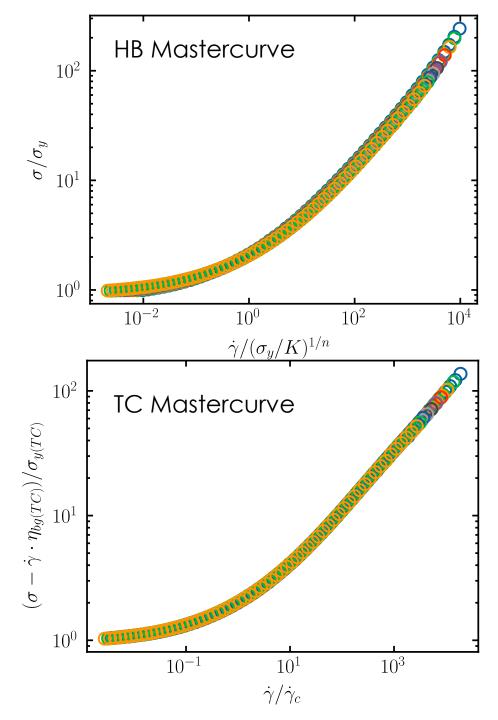
Model based mastercurve (TC) and ML mastercurve support model validity



More, challenging, cases

Carbopol microgel in Propylene Glycol Multiple datasets as a function of temperature

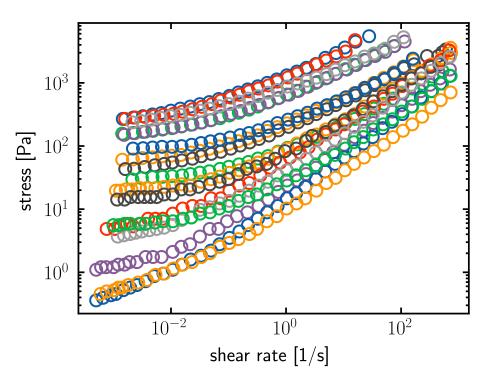


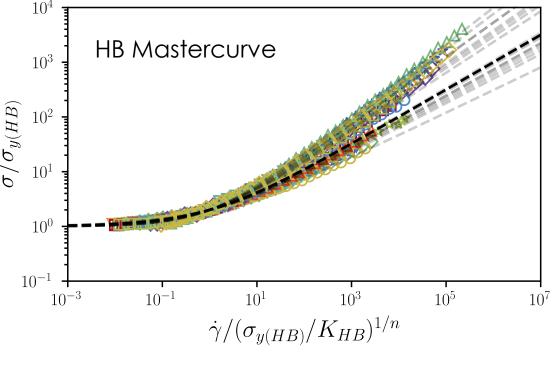


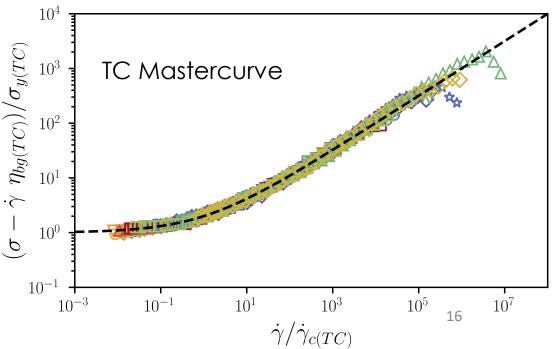
More, challenging, cases

Migliozzi, Simona, et al. "Investigation of the swollen state of Carbopol molecules in non-aqueous solvents through rheological characterization." Soft Matter 16.42 (2020): 9799-9815.

Carbopol Glycerin, PEG, and PEG/Gly Multiple datasets as a function Carbopol concentration 1-8% And solvent



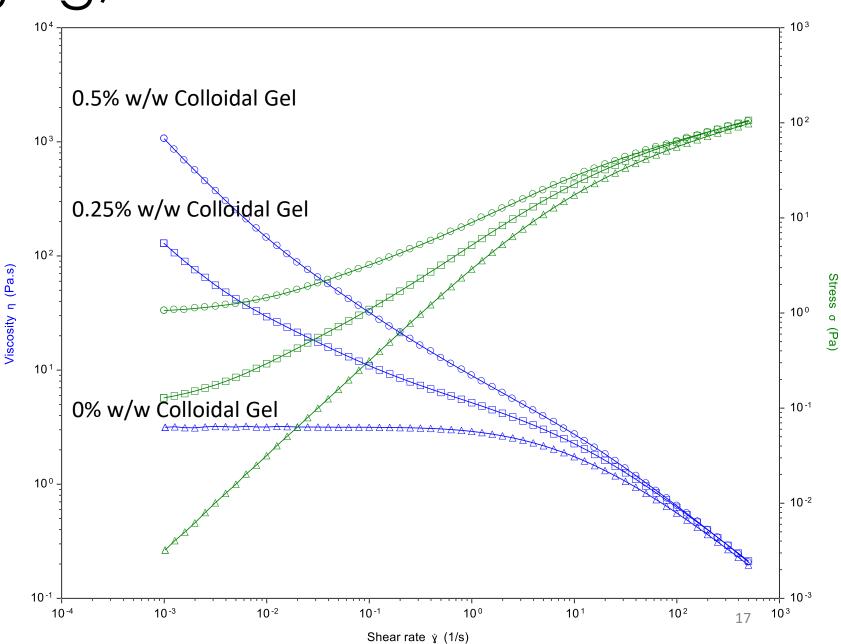




More, challenging, cases

Colloidal gel added to non-Newtonian fluid

Subtraction of a non-Newtonian background



Conclusions

- Mastercurves provide important physical insights: fundamental and practical
- ML tool represent objective ways to explore and judge different model/approaches: TC seems more reasonable than HB
- Open-source tool-chain: interesting new way to develop and share code
- Many opportunities to extend ML mastercurve to more complex cases