Model Description

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Exogenous Parameters. We model two types of players, the *experienced* (E) and the *inexperienced* (I). The population of players evolves over time, from period $t = 0$ to period T. In the sequel, we refer to each player category using the superscript symbol $\bullet$, with $\bullet \in \{E, I\}$.

At period $t = 0$, we assume the size of the experienced population to be fixed and equal to $n_0^E = 1000$. On the other hand, the population size for group I, i.e., n_0^I, is defined as a proportion of n_0^E, such that the ratio between the two population sizes changes in a prespecified set. More precisely, we run simulations with varying initial conditions, such that:

$$\frac{n_0^E}{n_0^I} \in \{0.5, 0.6, \cdots, 1.4, 1.5\} \tag{1}$$

We assume the existence of an exogenous survival probability p, which is kept constant over time. During the study, we modeled the game assuming different values of p, produced using the following function:

$$p = \sqrt{x}, \quad x \in \{0.025, 0.05, \cdots, 0.25\} \tag{2}$$

To model the phenomenon of over-and-underestimation of survival probabilities for different types of players, we assume the following:

- When the exogenous survival probability p is low (e.g., $p \leq 0.2$), then players of type I tend to overestimate their individual chance of surviving, while players of type E tend to underestimated the probability of success;
- When the exogenous survival probability p is high (e.g., $p > 0.2$), then players of type I tend to undererestimate their individual chance of surviving, while players of type E tend to overestimated the probability of success;

This type of behaviour is modeled using a correcting factor $\delta^\bullet$ as follows:

$$\delta^E = \begin{cases} 0.8, & \text{if } p \leq 0.2 \\ 1.2, & \text{if } p > 0.2 \end{cases} \tag{3}$$

$$\delta^I = \begin{cases} 1.2, & \text{if } p \leq 0.2 \\ 0.8, & \text{if } p > 0.2 \end{cases} \tag{4}$$

We also assume the existence of an exogenous expected return, i.e., the average return a generic player is expected to achieve. Such return is modeles as a percentage value and, therefore, can be seen as proportional to the objective return of the investment, defined as follows:

$$s = \min\left\{\delta^E, \delta^I\right\} \times p \tag{5}$$

Characteristics of each player. Each player is defined by a set of subjective estimates, used to face the following decisions:

- At period $t = 0$, a player decides whether to enter the market (*entering* or *forfeiting* decision);

- In periods $t = 1, \cdots, T$, a player decides whether to remain in the market (*remaining* or *quitting* decision.)

Each player is modeled via a subjective set of estimates for the following parameters:

- Individual survival probability. This probability is proportional to the exogenous survival probability p, and adjusted to embed subjective mechanisms of over-and-underestimation of the likelihood of survival via the correction factor $\delta^\bullet$, defined via Equations (3) and (4). More precisely, at period $t = 0$, each player's individual survival probability is defined as follows:

$$p_{i0} = \delta^\bullet \times p + r_i^\bullet \tag{6}$$

where $r_i^\bullet \sim N(0, \sigma^\bullet)$ is a random number generated under a normal distribution with mean $\mu = 0$ and standard deviation $\sigma^\bullet = 0.05$ when $\bullet = E$, and $\sigma^\bullet = 0.10$ when $\bullet = I$. Note that we are assigning a higher variability in the estimation of individual survival probabilities to the inexperienced group.

- Individual threshold value. This value is an estimate of the expected return, and is proportional to the exogenous expected return s defined via Equation (5). Each player estimates an expected threshold value s_i as follows:

$$s_i = s + r_i^\bullet \tag{7}$$

where $r_i^\bullet \sim N(0, \sigma^\bullet)$ is a random number generated under a normal distribution, as described above. Note that s_i does not depend on the time period t and remains unchanged during the entire time horizon.

- Individual experience update. At each period $t \geq 1$, we account for the gained experience of players of type I, by bringing their subjective probabilities closer to the values of the subjective probabilities of players of type E. Therefore, each player of type I revises its individual probability using the following correction:

$$p_{it} = \begin{cases} (1+\gamma)\, p_{it}, & \text{if } p > 0.2 \\ \dfrac{1}{1+\gamma} p_{it}, & \text{if } p \leq 0.2 \end{cases} \tag{8}$$

where γ indicates the correction factor and is defined as follows:

$$\gamma = \frac{|\delta^E - \delta^N|}{T} \tag{9}$$

- Bayesian learning process. At the end of each period $t \geq 1$, each player updates its beliefs with respect to the probability of survival of its specific class. Let us indicate with $n_t^\bullet$ the number of players of class $\bullet$ still remaining in the game at the end of period t. An objective probability of survival, for each of the two classes, is thus computed as follows:

$$p_B^\bullet = \frac{n_t^\bullet}{n_{t-1}^\bullet} \tag{10}$$

Thus, each player updates its individual beliefs applying the following smoothed learning approach:

$$p_{it} = \alpha p_{it-1} + (1-\alpha) p_B^\bullet \tag{11}$$

New Entrants. At the end of each period $t \geq 1$, we model the existence of potential new entrants. They observe the situation of the game and estimate their individual p_{it} and s_i. More precisely, s_i is estimated using Equation (7) while p_{it} is set as the average p_{it} computed over all the players of its class. The number of potential entrants at the end of period t is fixed at $0.2 \times n_t^\bullet$.

Player's Decision Rule. Each player's decision is made based on the following rule:

$$d_{it} = \mathbb{1}_{p_{it} \geq s_i} \tag{12}$$

where $\mathbb{1}_A$ is the indicator function, which takes value 1 if A is true, and 0 otherwise. Finally, in each period $t = 0, 1, \cdots, T$, a player i selects decision *entering* or *remaining* if $d_{it} = 1$ and *forfeiting* or *quitting* if $d_{it} = 0$.

The Algorithm. Let us now sketch the overall algorithm. Assume a set of parameters $n_0^\bullet, p, T$ has been given. A high level description of the algorithm is provided below.

Algorithm 1 : `Simulation`

Require: $n_0^\bullet, p, T$

Set $t = 0$ and compute $\delta^\bullet$, s $\{$`initialization`$\}$

Generate $n_0^\bullet$ players: Define s_i and p_{it} $\{$`Eqs.(6)-(7)`$\}$

Make decision d_{it} for $i = 1, \ldots, n_0^\bullet$ $\{$`entering or forfeiting`$\}$

for all $t = 1, \ldots, T$ **do**

  Select pool of surviving players $\{$`survival probability` $p\}$

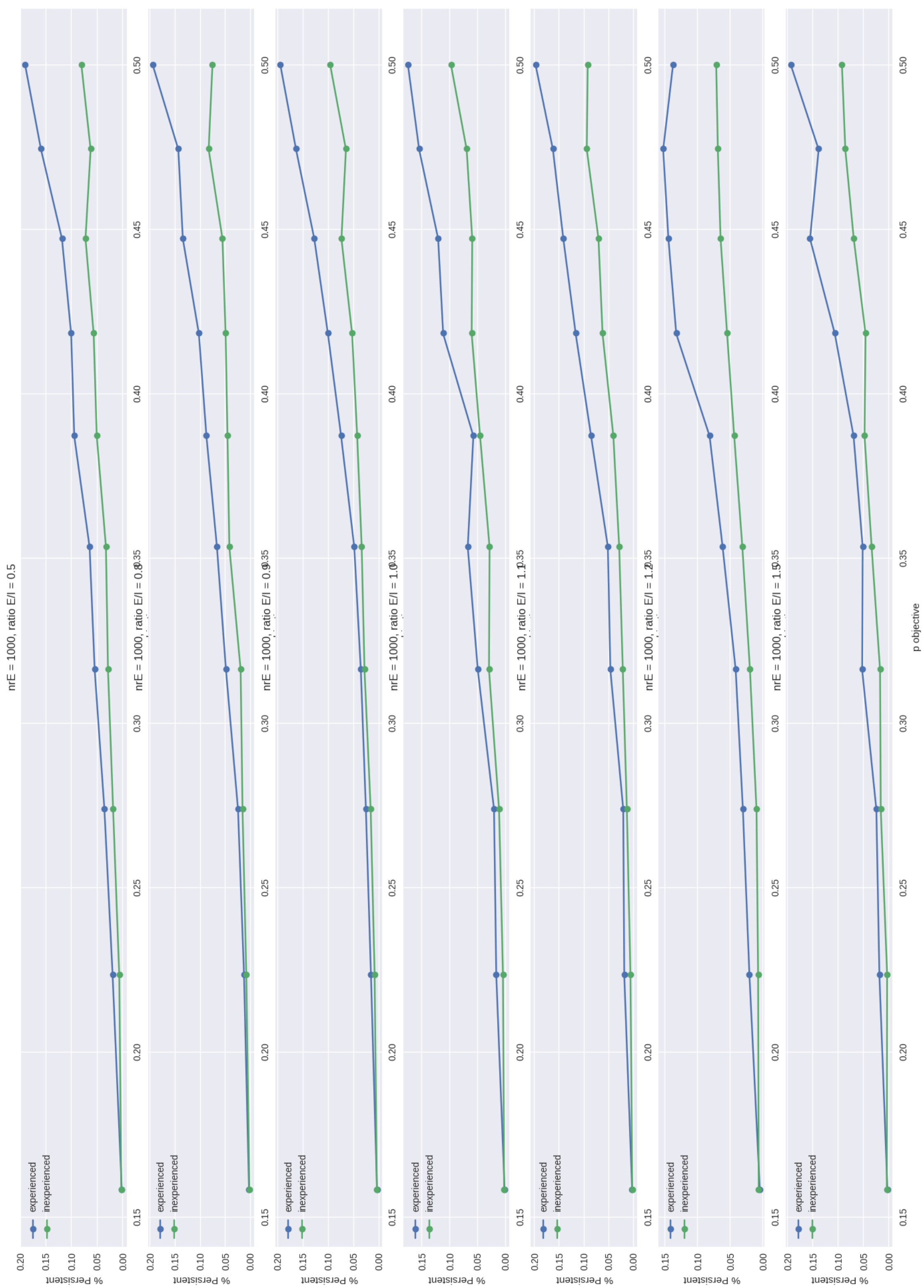
  Apply individual experience update $\{$`Eq.(8)`$\}$

  Apply Bayesian learning process and update p_{it} $\{$`Eqs.(10)-(11)`$\}$

  Make decision d_{it} for $i = 1, \ldots, n_t^\bullet$ $\{$`Eq.(12)`$\}$

  Generate potential new entrants and select the entering ones $\{$`get` s_i, p_{it}`, and` $d_{it}\}$

end for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nrE = 1000, ratio E/I = 0.8
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experienced
inexperienced
% Persistent
p objective