Study of Quantum Convolutional Neural Network (QCNN) applied to the MINST dataset

TensorFlow Quantum

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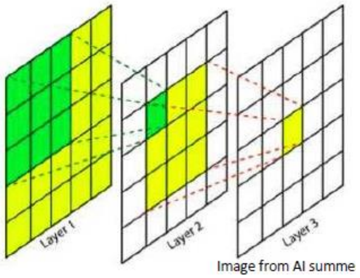
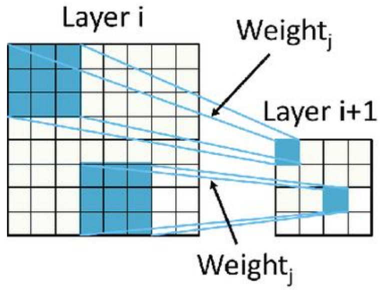
14/07/2023

Convolutional Neural Network

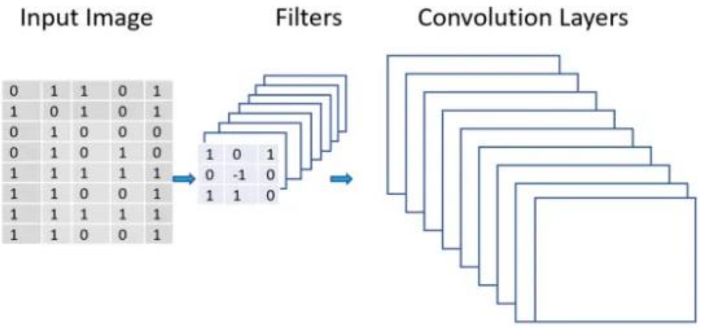
Local connectivity



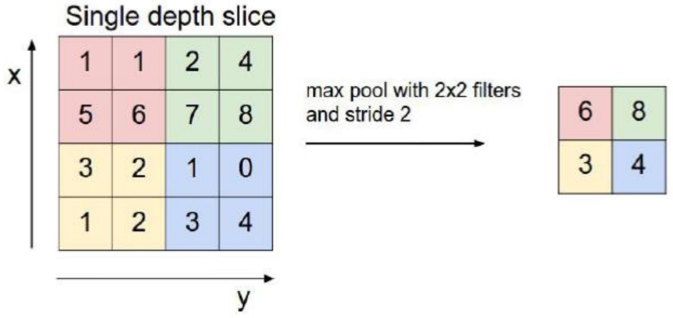
Shared weights



Multiple feature maps

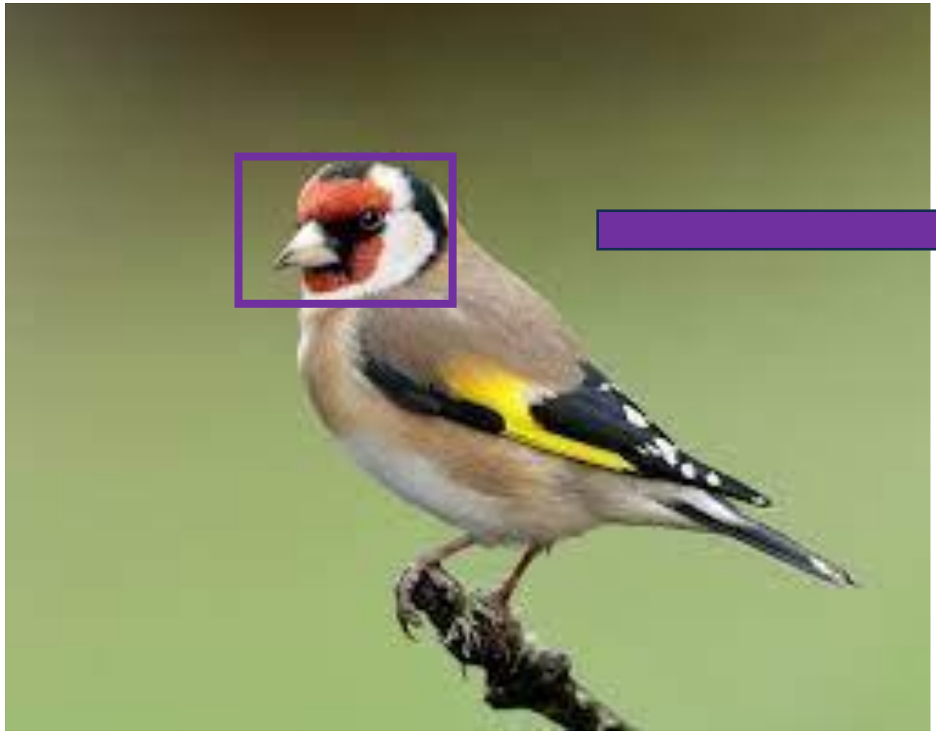


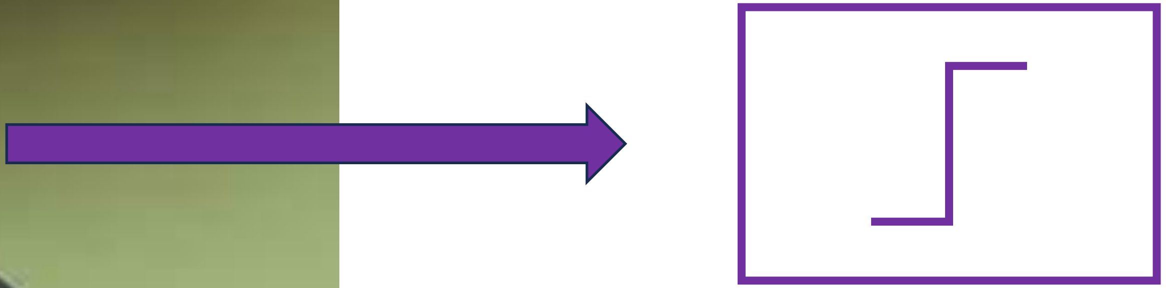
Max Pooling



Example: «beak detector»

Some patterns are much smaller than the whole image:





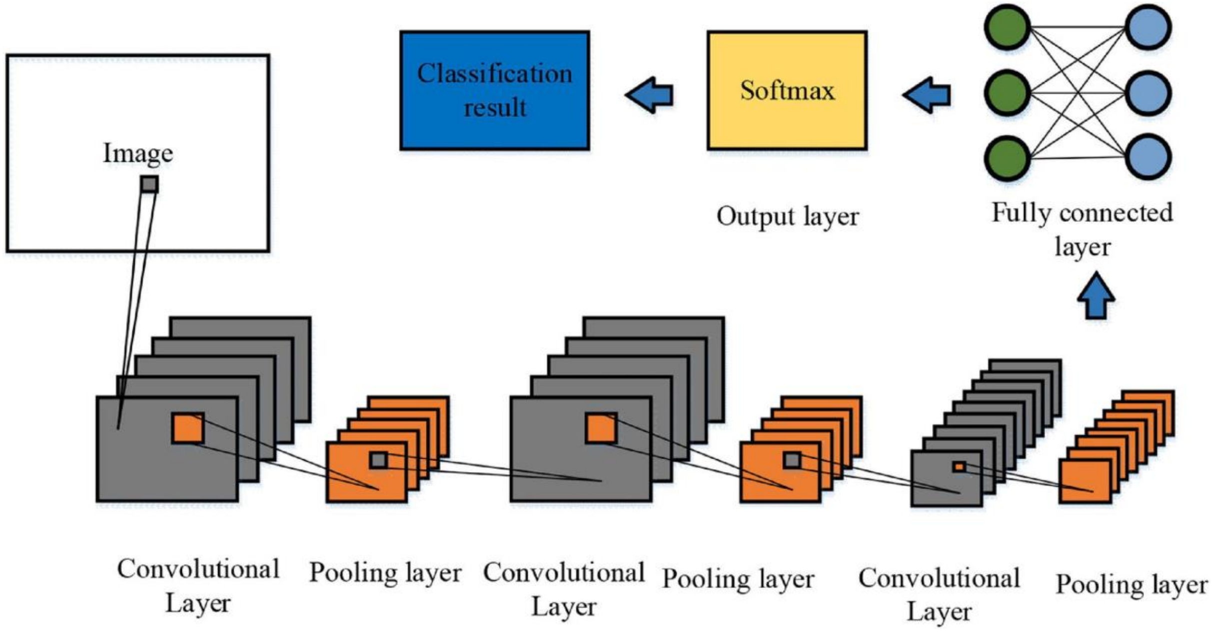
«Beak» signal

Convolutional NN vs Fully Connected NN

Hierarchical Representation

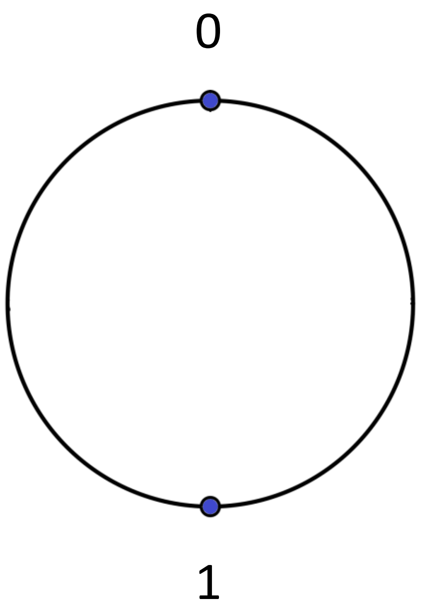
Compress the number of Connection

Reduce complexity

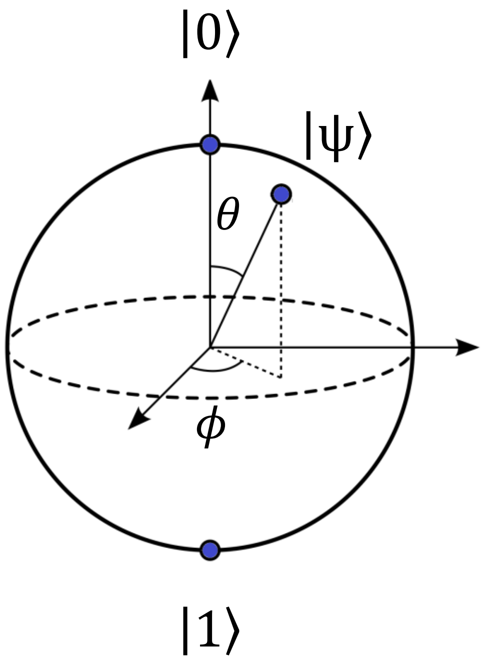


Quantum Computing

Bit



Qubit

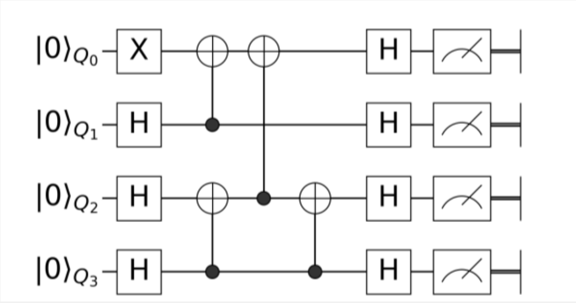


$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + \mathrm{sen}\left(\frac{\theta}{2}\right) e^{i\phi}|1\rangle$$

Some quantum algorithms are demonstrated to be **more efficient** than classical ones (Shor's algorithm).

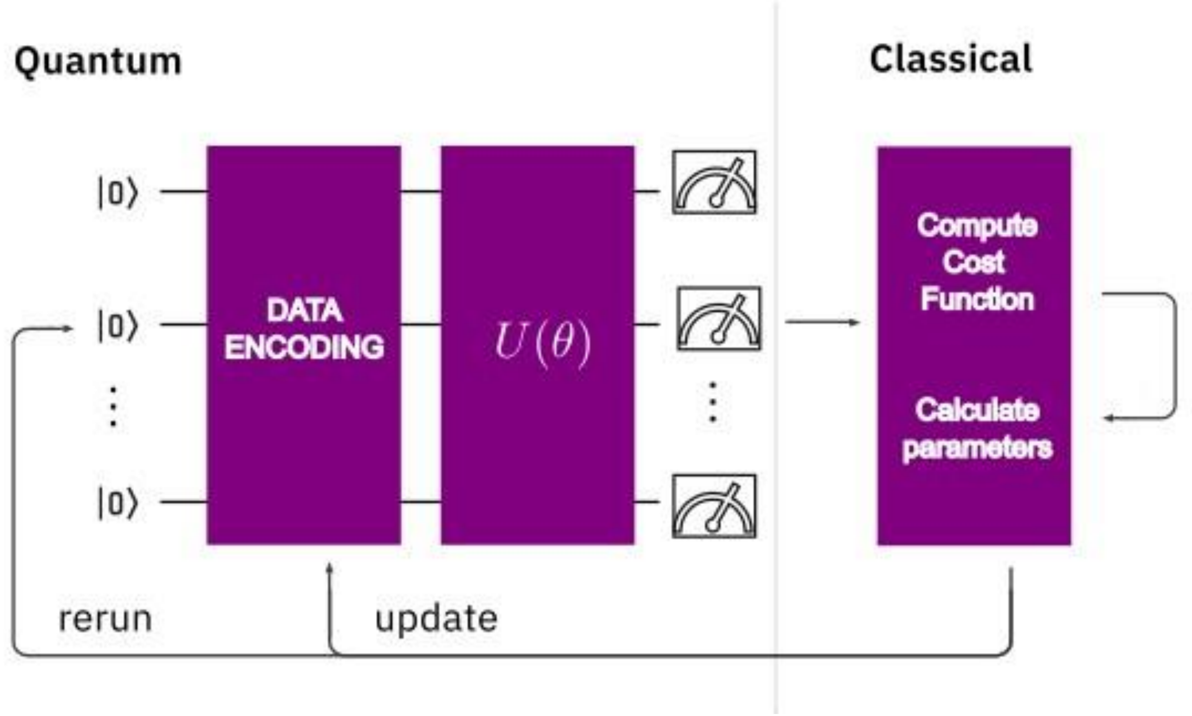
Every quantum algorithm can be decomposed with a finite set of gates (**universal set of gates**):

- Single qubit gate
- Multi-qubits gate



Quantum Neural Network

Theory

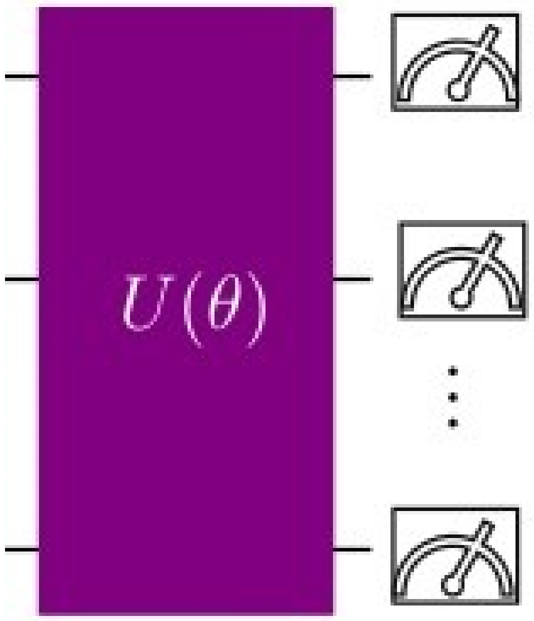


Why? Superposition and entanglement

- Quantum algorithms are able to **extract patterns** of a function (Deutsch–Jozsa algorithm)
- Entanglement is a **unique** feature of Quantum Systems
- Theoretically, can be stored more information in N qubits than in N bits

Quantum Neural Network

Parametrized quantum circuit (PQC)



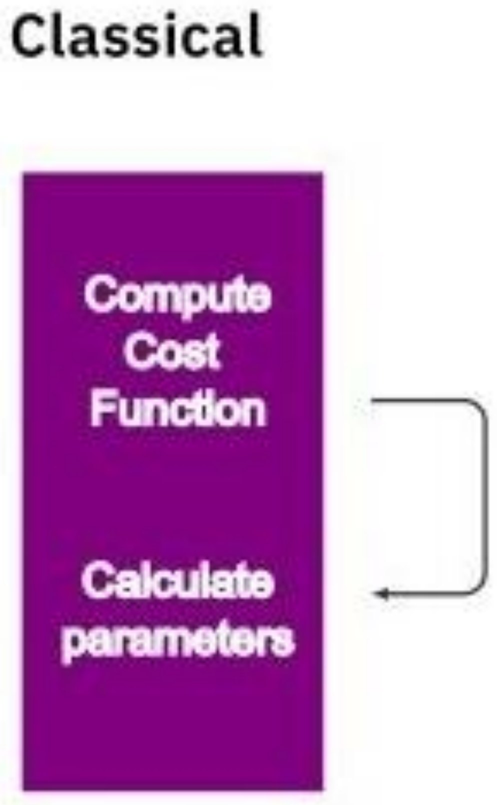
Non-linearity of the model comes from **measurements.**

Expressibility : extent to which a PQC can generate states within the Hilbert space.

Entangling capability : ability of a PQC to generate entangled states.

Quantum Neural Network

Classical Optimization



Parameter Shift Rule: Permits to analytically find the gradient of a linear function of expectation values of Pauli matrixes. $\nabla_\theta f(x;\theta) = \frac{1}{2}\left[f(x;\theta+\frac{\pi}{2}) - f(x;\theta-\frac{\pi}{2})\right].$

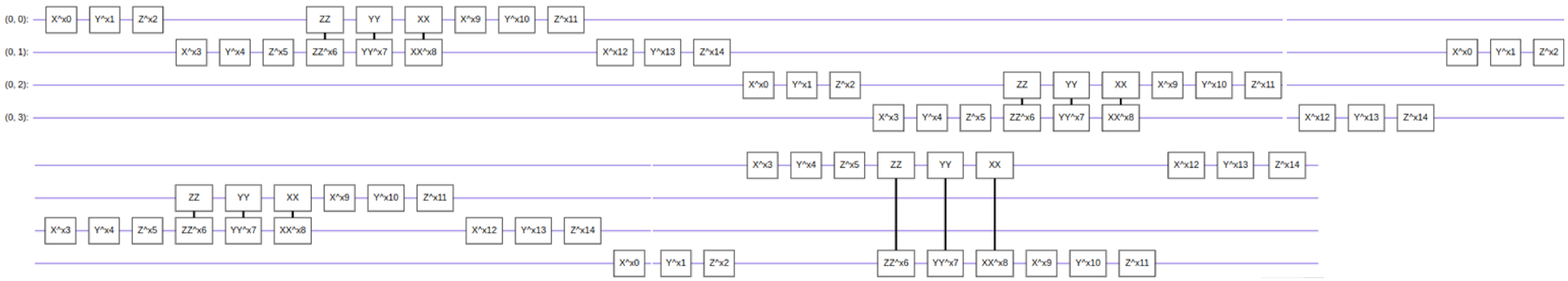
Minimize Mean Squared Error (MSE) between **<Z>** and true label.

Quantum Convolutional Neural Network

Convolutional Layer

Same Features as Classical CNN: Local Connectivity and Parameters Sharing.

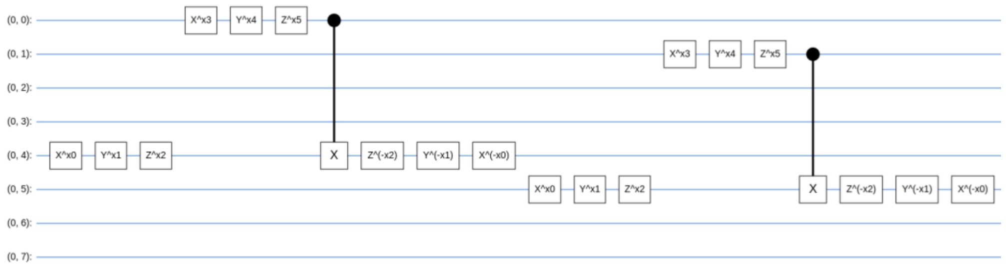
Consists of Two Qubit Unitary Operations U_i.

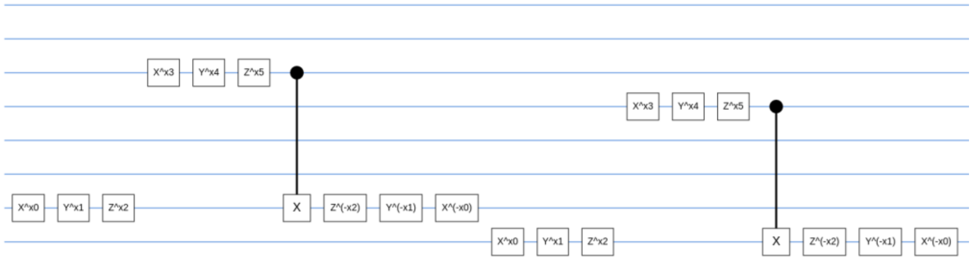


Y. Lu,et al., 40th CCC, 52363, (2021).

Quantum Convolutional Neural Network

Pooling Layer





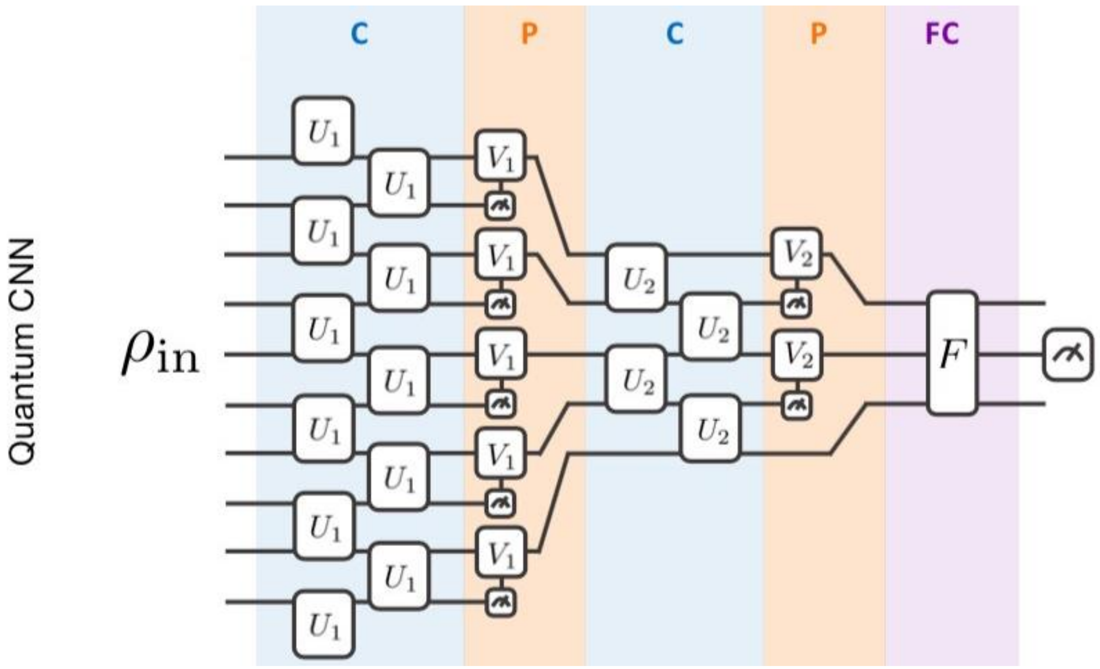
Qubits are **traced out** in the calculation of the expected value of Z in the end.

The outcome determines whether apply a single-qubit gate on neighbors' qubits.

Reduces dimensionality and introduces **nonlinearities**.

Y. Lu,et al., 40th CCC, 52363, (2021).

Quantum Convolutional Neural Network



FC layer is applied to perform classification on the extracted features.

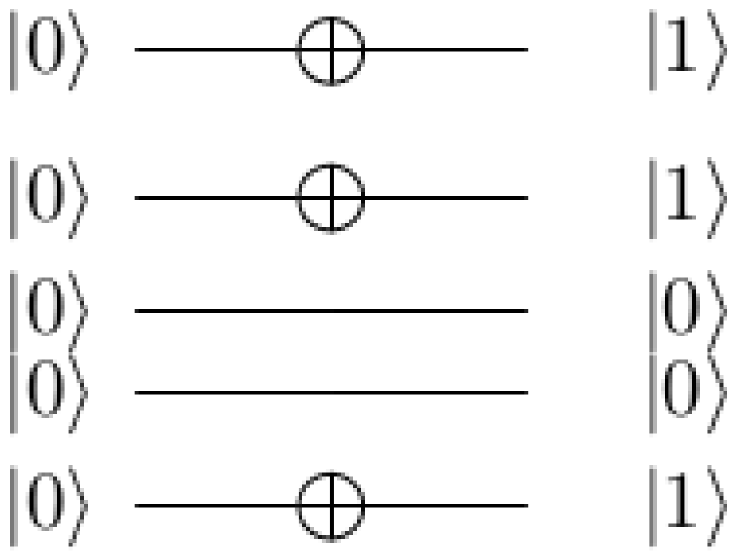
The output of the QCNN model for x_{in} is -> $f(\theta, x_{in}) \equiv \langle Z \rangle$.

Binary classification:
$\langle Z \rangle \geq 0$ -> one class
$\langle Z \rangle < 0$ -> other class

Y. Lu,et al., 40th CCC, 52363, (2021).

Basis encoding

Theory



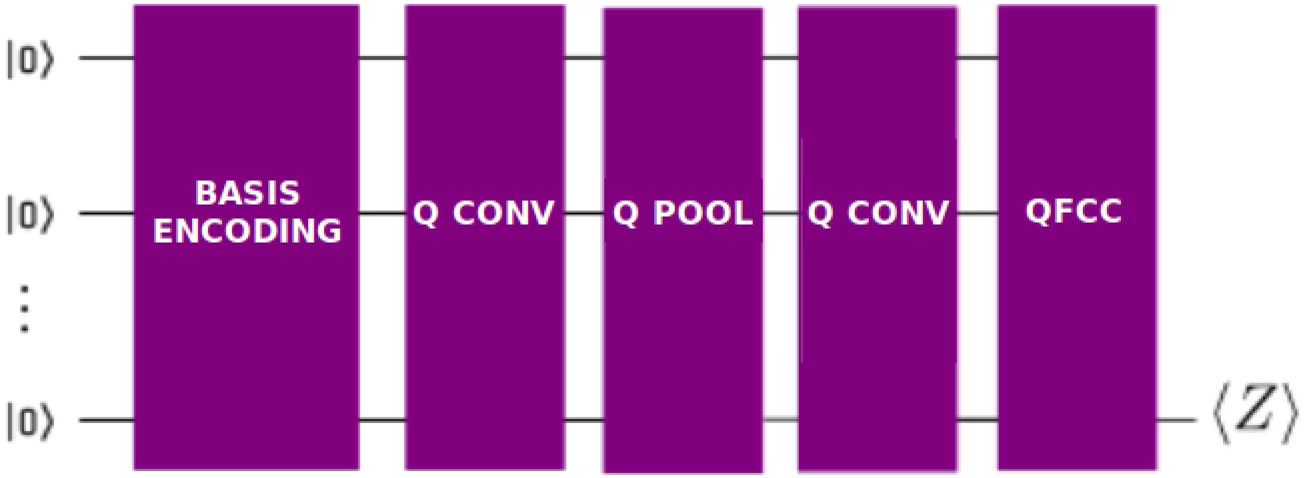
$$|\psi_x> = |i_x>$$

Advantages: Simple and straightforward approach. Low encoding circuit depth.

Disadvantages: At least N qubits per image of N pixels. The data input discretization produces information loss.

Basis encoding

Model

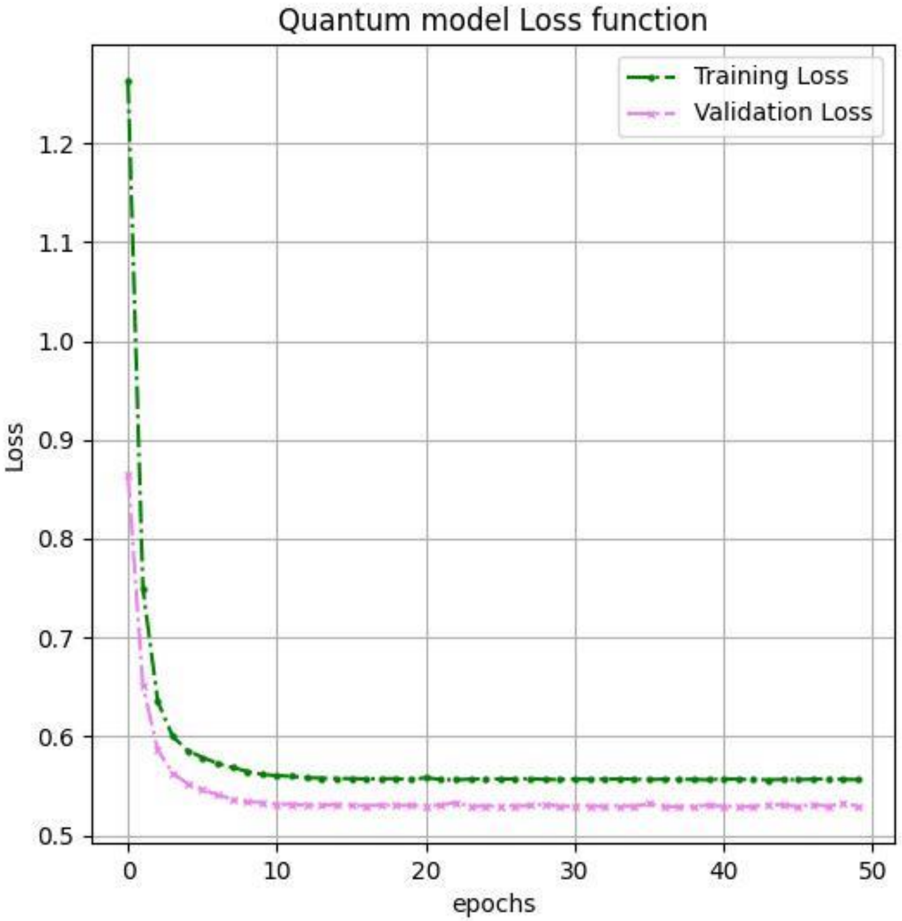


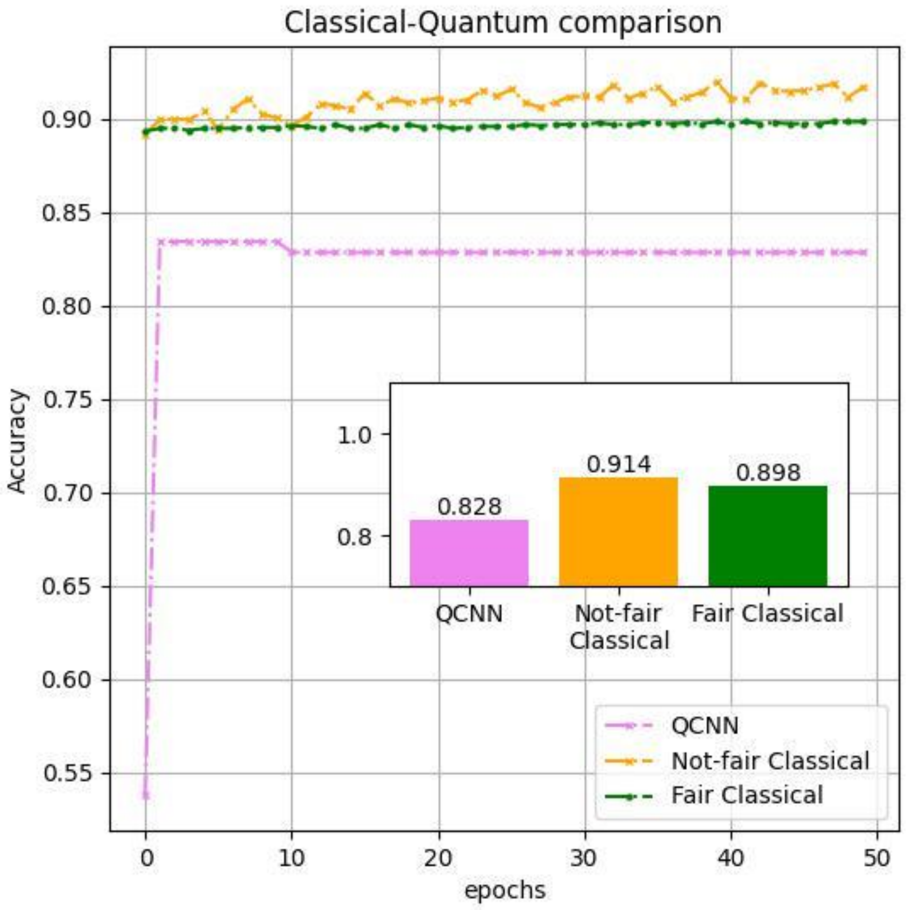
- 3x3 input images
- 9 qubits

- 30 parameters
- 21 sec/epoch

Basis encoding

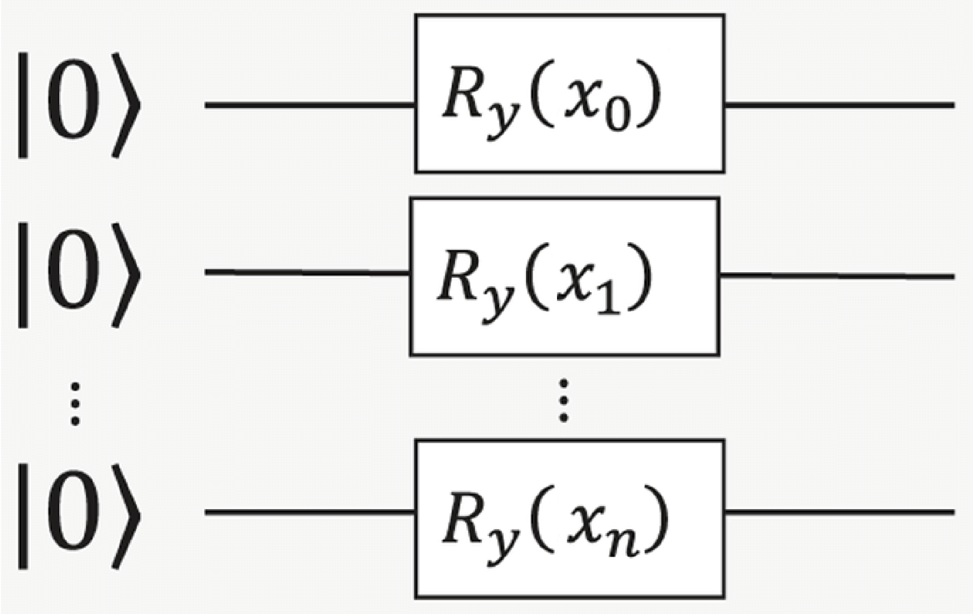
Model results





Angle encoding

Theory

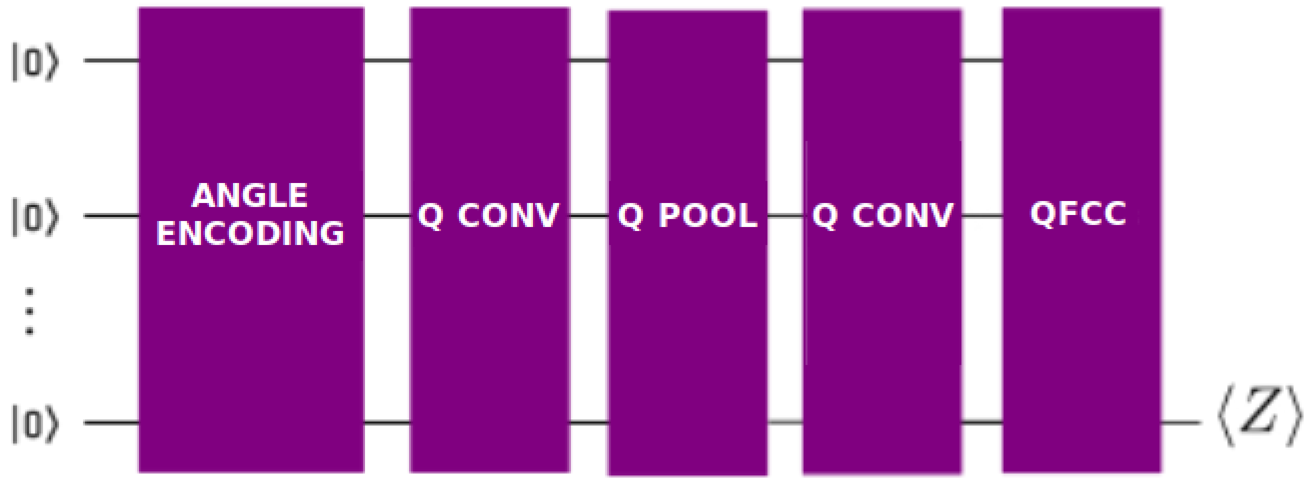


$$|\psi_x> = \cos\left(-\frac{x}{2}\right)|0> \; + \; \sin\left(-\frac{x}{2}\right)|1>$$

Advantages: Low depth of the encoding circuit. No information loss due to a continuous representation of the input.

Disadvantages: Use of N qubits per pixel. Susceptible to noise.

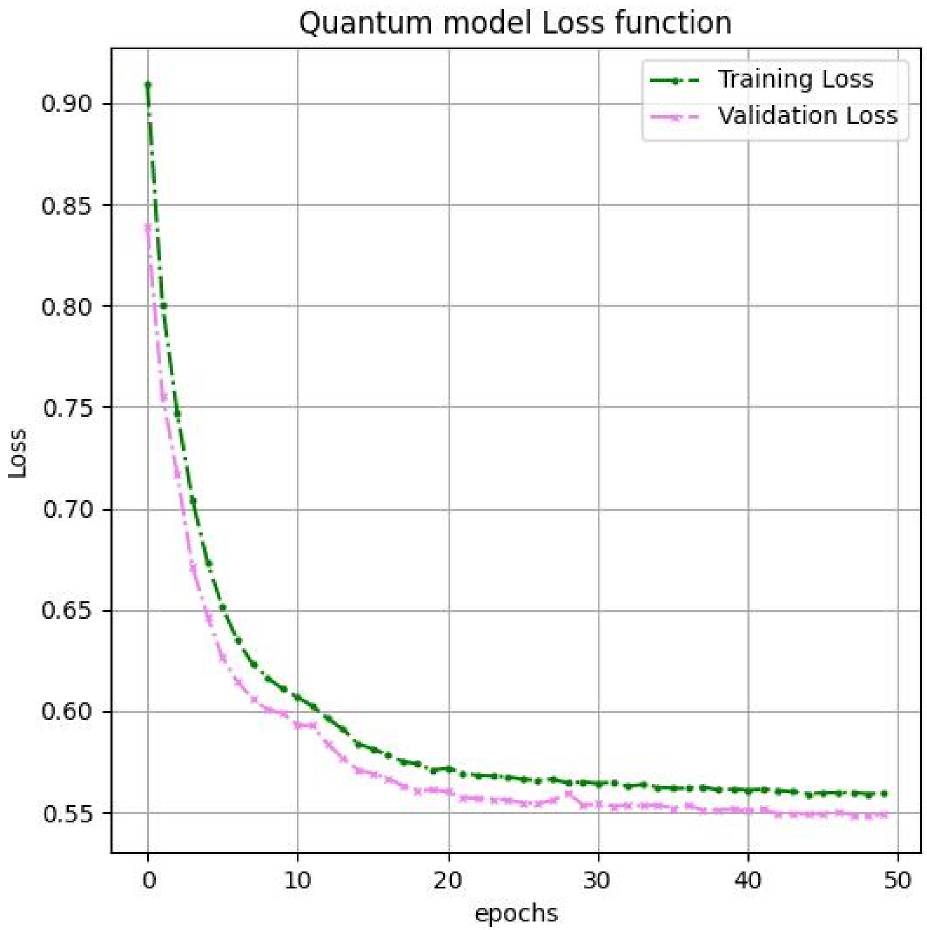
Angle encoding
Model

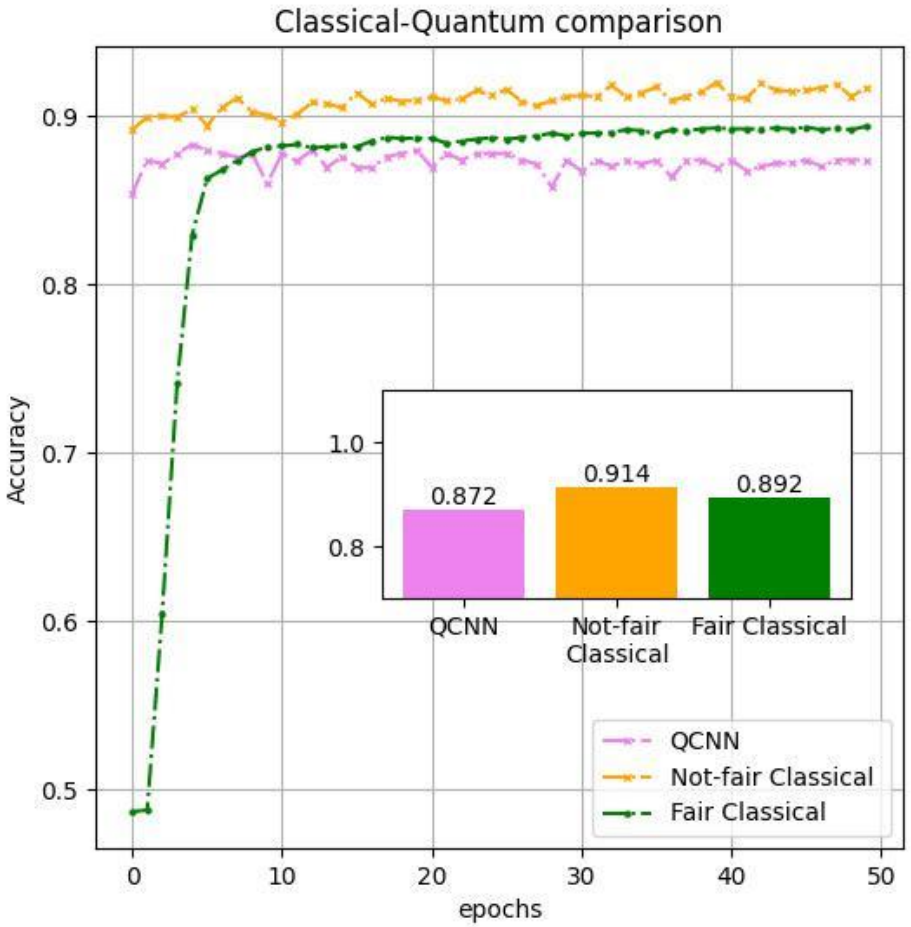


- 3x3 input images
- 9 qubits

- 57 parameters
- 40 sec/epoch

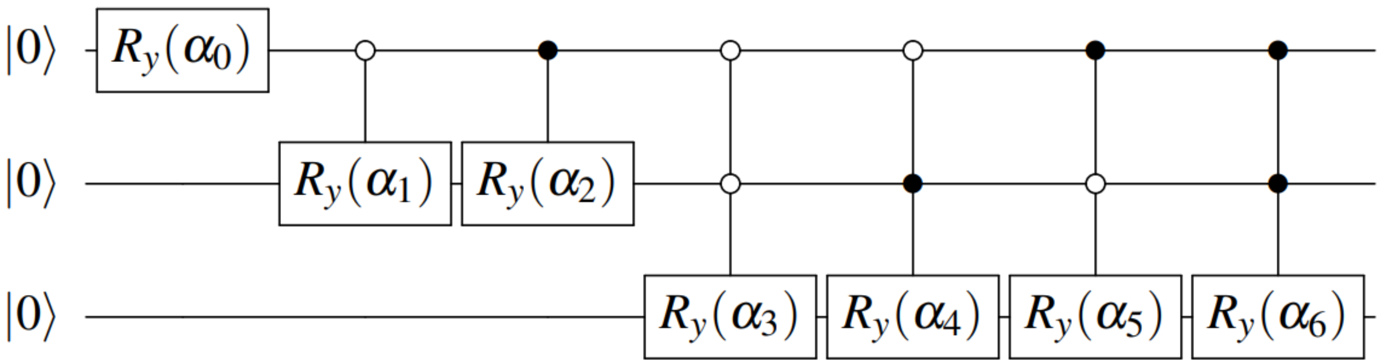
Angle encoding
Model results





Amplitude encoding

Theory

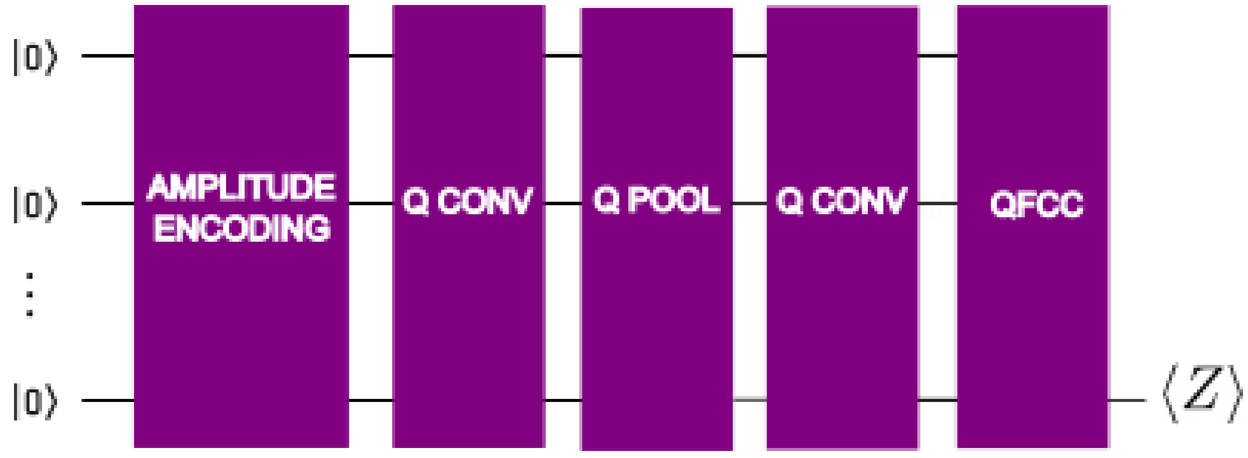


$$|\psi_x\rangle = \sum_{i=1}^{N} x_i |i\rangle$$

Advantages: number of data encoded scales exponentially with the number of qubits.

Disadvantages: depth of the encoding circuit scales exponentially with the number of qubits.

Amplitude encoding
First model

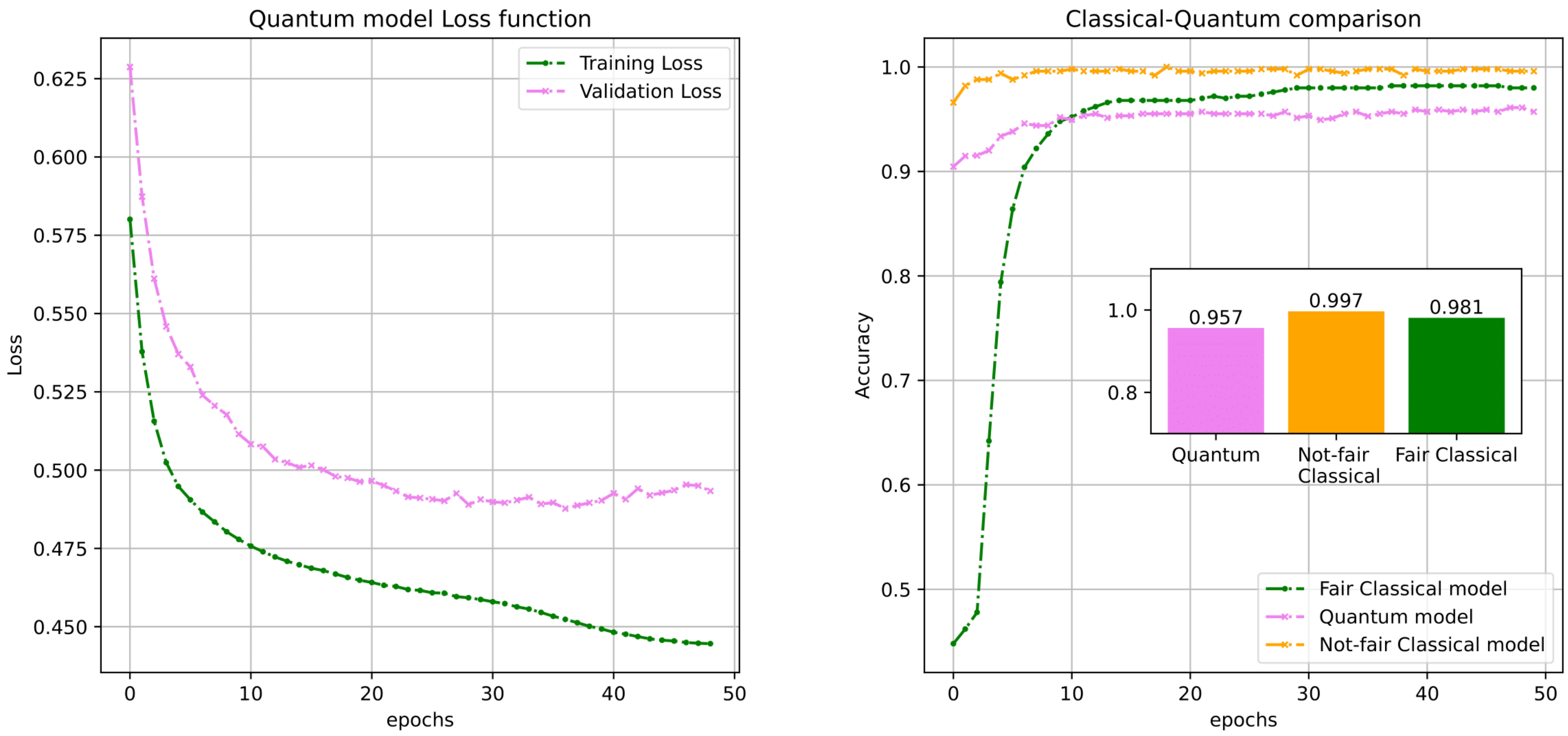


- 8x8 input images
- 6 qubits

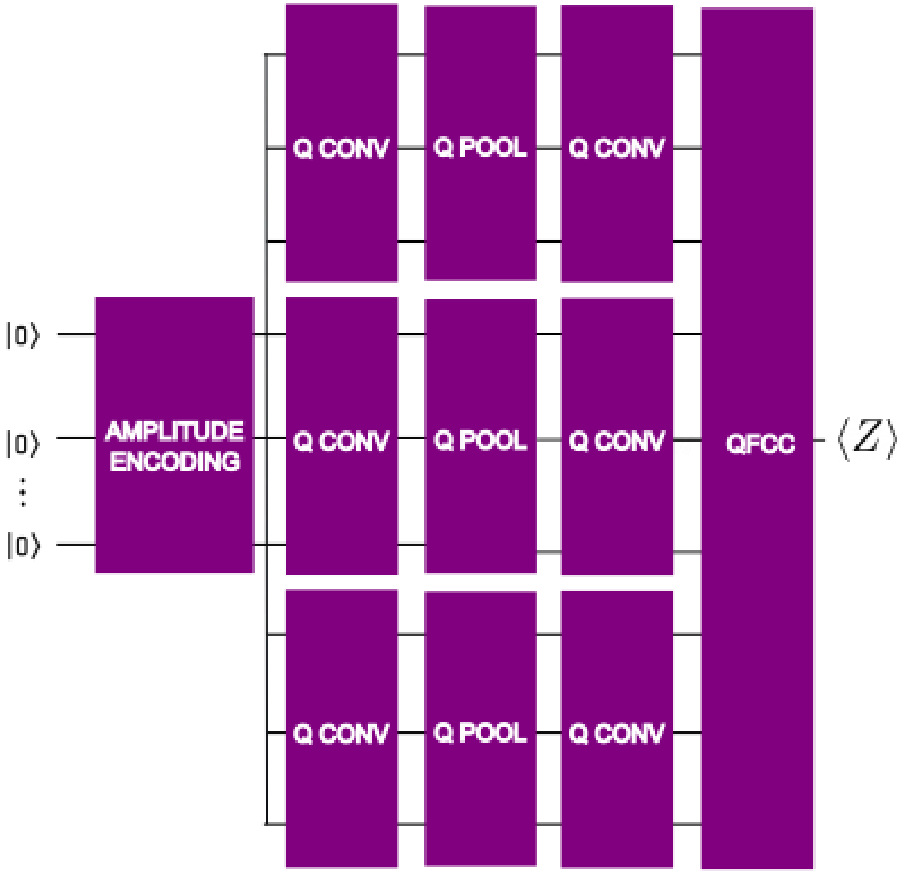
- 51 parameters
- 35 sec/epoch

Amplitude encoding

First model results



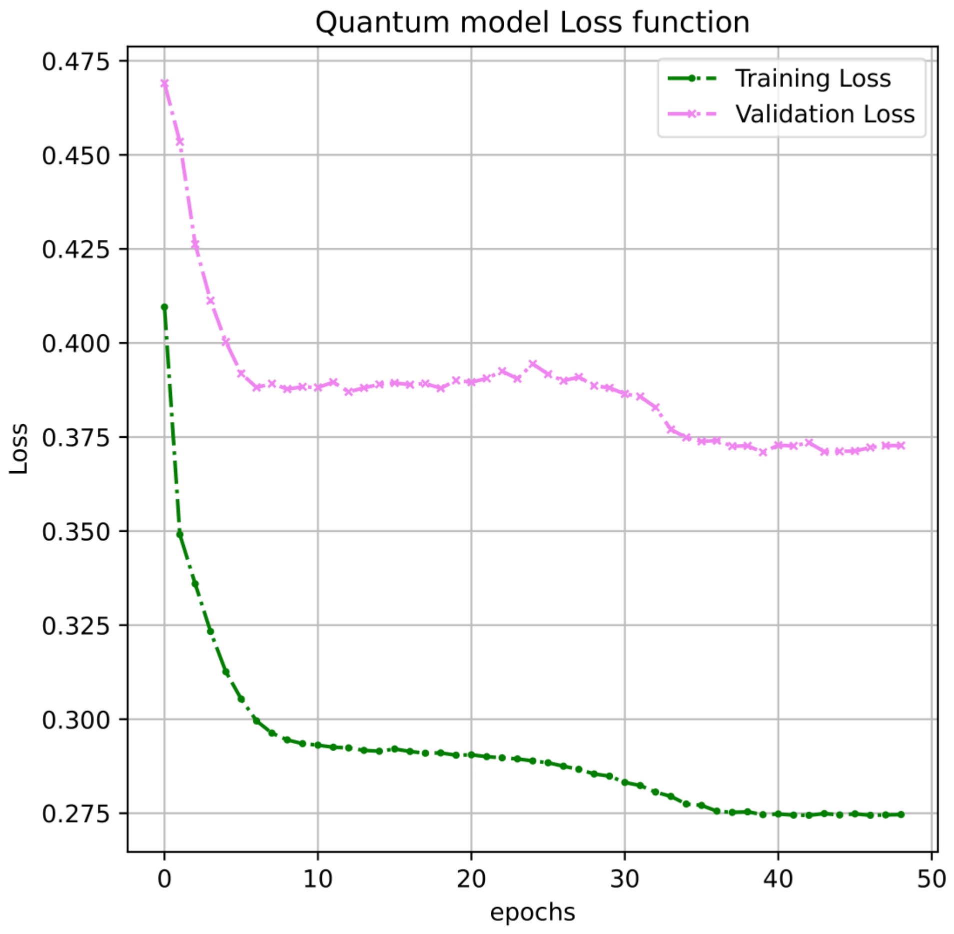
Amplitude encoding
Second model

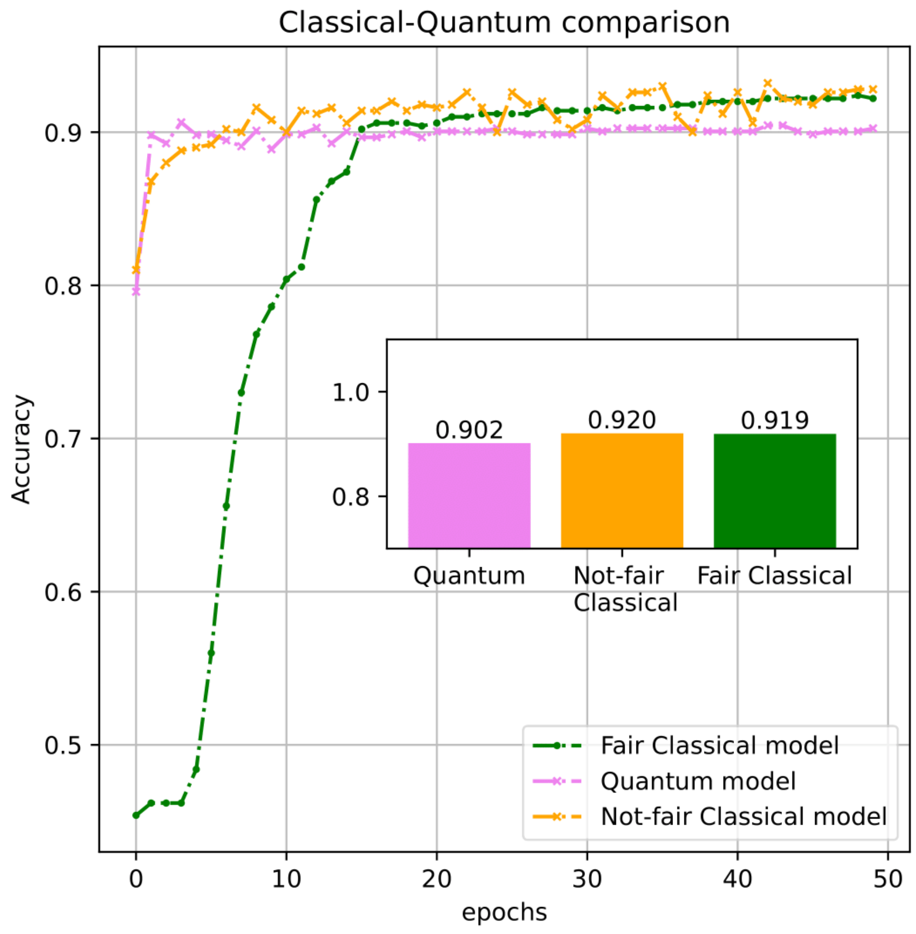


- 4x4 input images
- 4 qubits/filter (12qubits)

- 168 parameters
- 45 sec/epoch

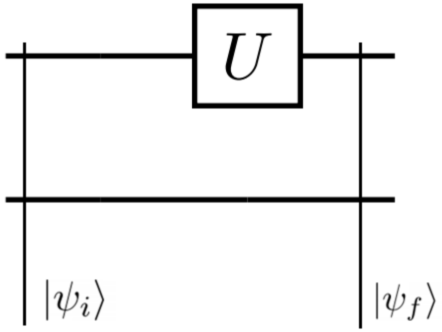
Amplitude encoding
Second model results





Amplitude encoding

Locality problem

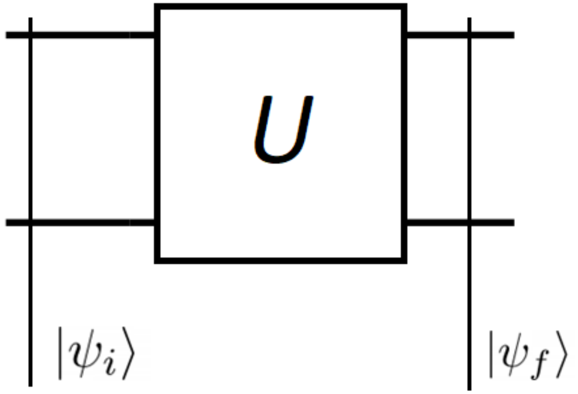


$$|\psi_i\rangle = x_{00}|00\rangle + x_{01}|01\rangle + x_{10}|10\rangle + x_{11}|11\rangle$$

$$|\psi_f\rangle = U|0\rangle \otimes (x_{00}|0\rangle + x_{01}|1\rangle) + U|1\rangle \otimes (x_{10}|0\rangle + x_{11}|1\rangle)$$

- Applying a local gate will mix information on amplitudes losing local connectivity.
- **We need unitary block matrixes to preserve local connectivity.**

Amplitude encoding
Locality problem



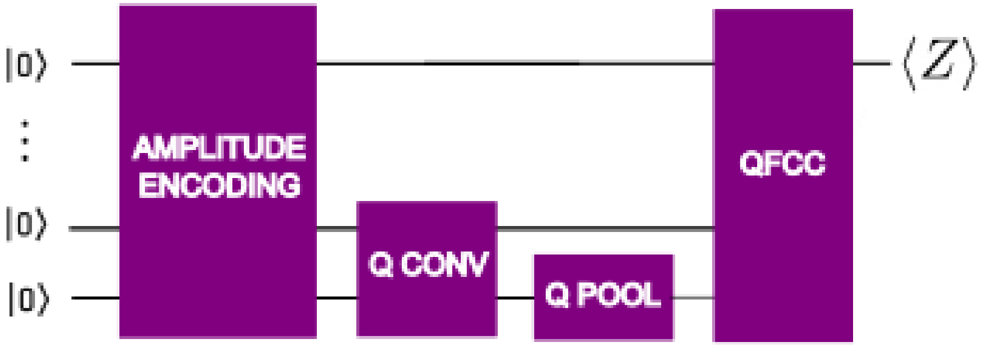
$$|\psi_i\rangle = x_{00}|00\rangle + x_{01}|01\rangle + x_{10}|10\rangle + x_{11}|11\rangle$$

$$U|\psi_i\rangle = \begin{bmatrix} \mathbf{U}' & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \vec{x}_{00,01} \\ \vec{x}_{10,11} \end{bmatrix} = \begin{bmatrix} \mathbf{U}'\vec{x}_{00,01} \\ \vec{x}_{10,11} \end{bmatrix}$$

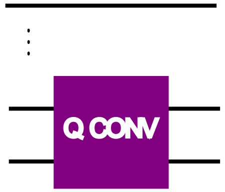
- Applying a local gate will mix information on amplitudes losing local connectivity
- **We need unitary block matrixes to preserve local connectivity**

Amplitude encoding

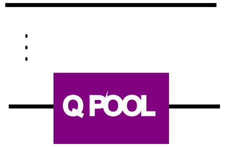
Third model



- 8x8 input images
- 6 qubits



$$= \mathbb{I}_{2x2} \otimes \mathbb{I}_{2x2} \otimes \cdots \otimes \mathbb{I}_{2x2} \otimes U_{4x4} = \begin{bmatrix} \mathbf{U}_{4x4} & 0 & \cdots & 0 \\ 0 & \mathbf{U}_{4x4} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{U}_{4x4} \end{bmatrix}$$

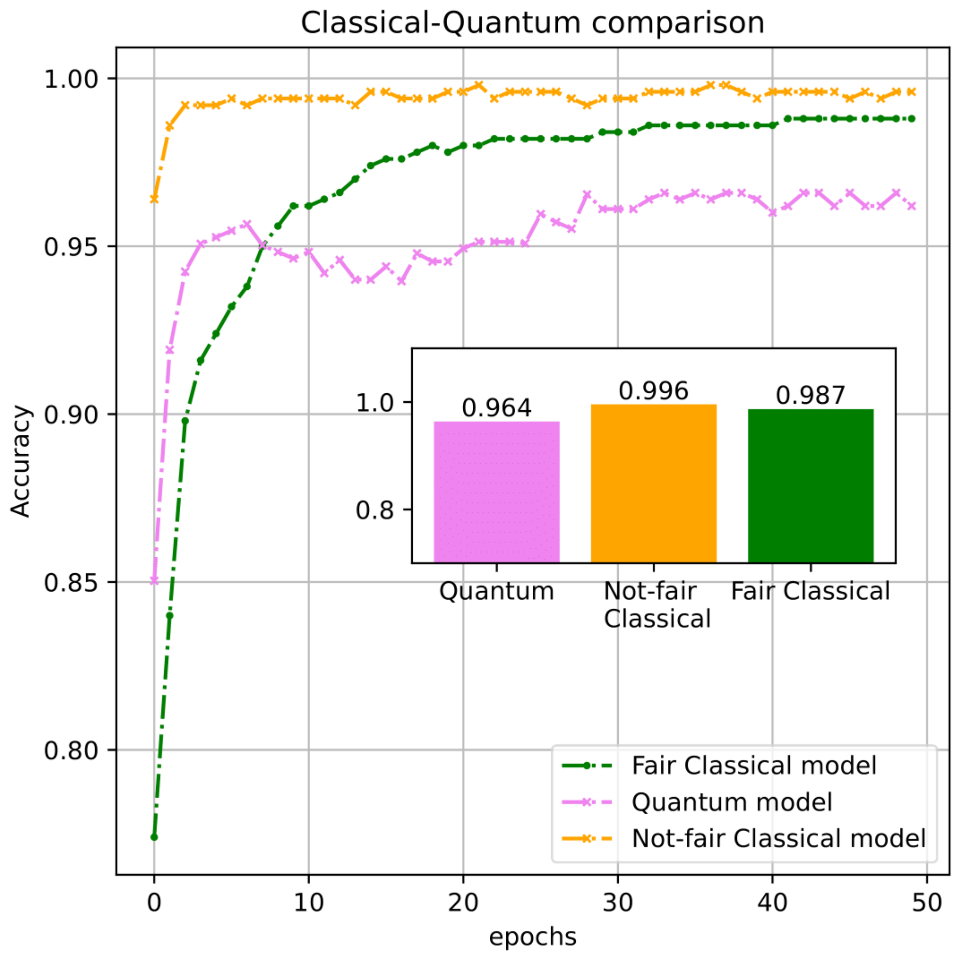


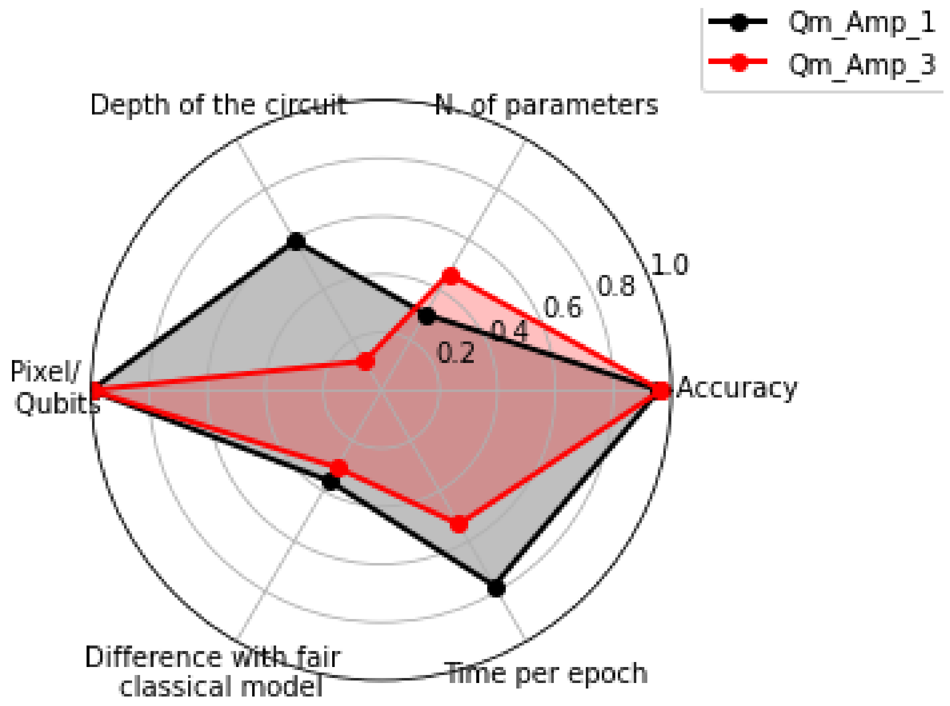
$$= \mathbb{I}_{2x2} \otimes \mathbb{I}_{2x2} \otimes \cdots \otimes \mathbb{I}_{2x2} \otimes U_{2x2} = \begin{bmatrix} \mathbf{U}_{2x2} & 0 & \cdots & 0 \\ 0 & \mathbf{U}_{2x2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{U}_{2x2} \end{bmatrix}$$

- 78 parameters
- 24 sec/epoch

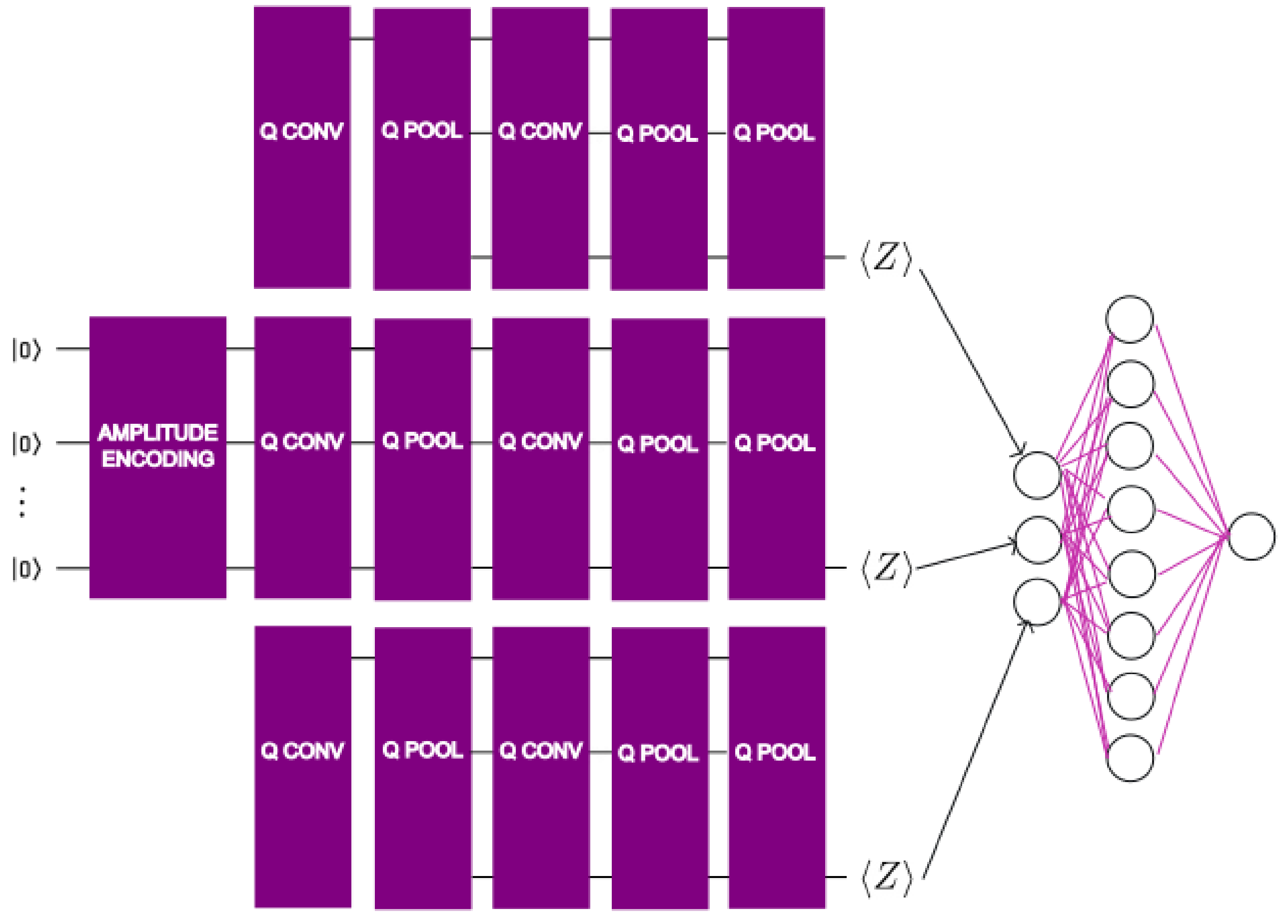
Amplitude encoding

Third model results and comparison





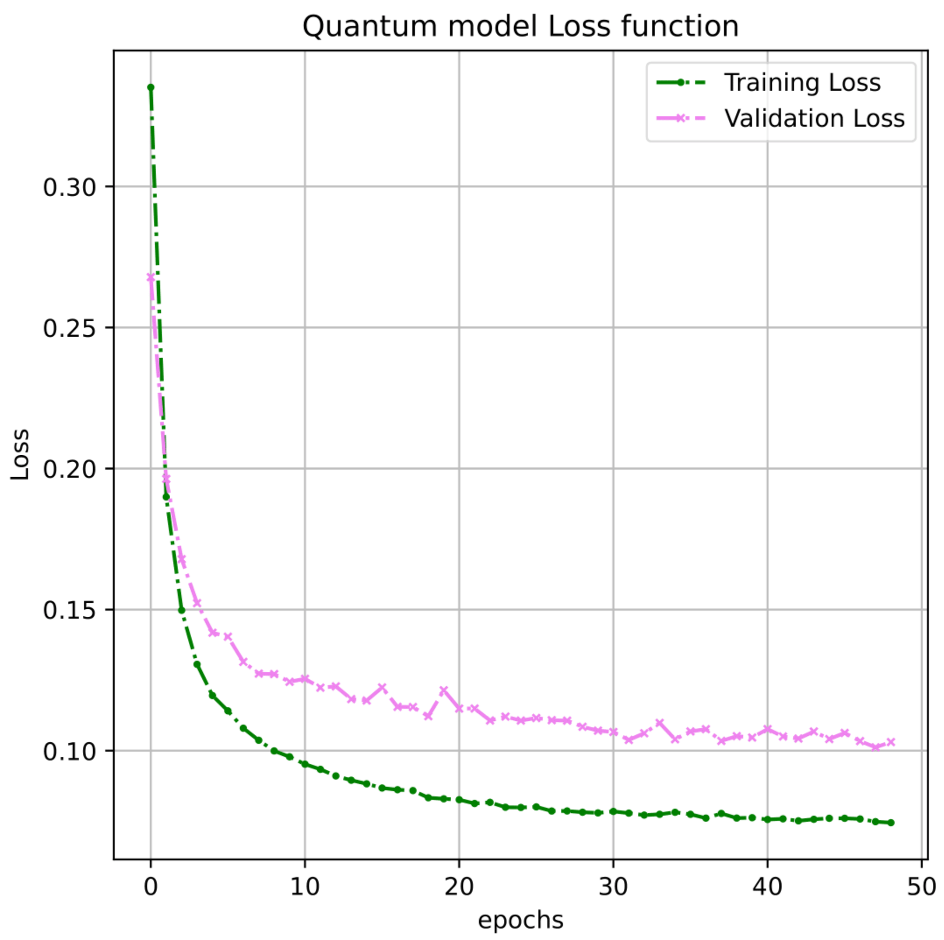
Amplitude encoding
Hybrid model

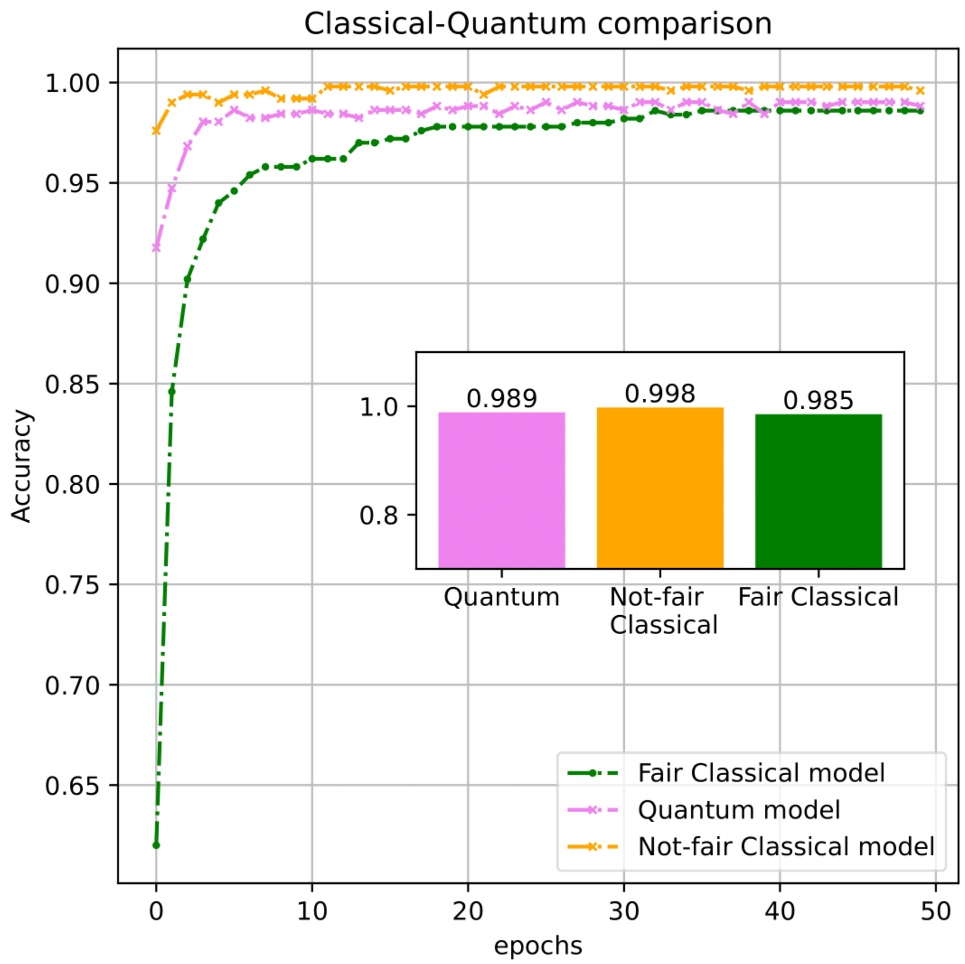


- 8x8 input images
- 6 qubits

- 168 parameters
- 45 sec/epoch

Amplitude encoding
Hybrid model results

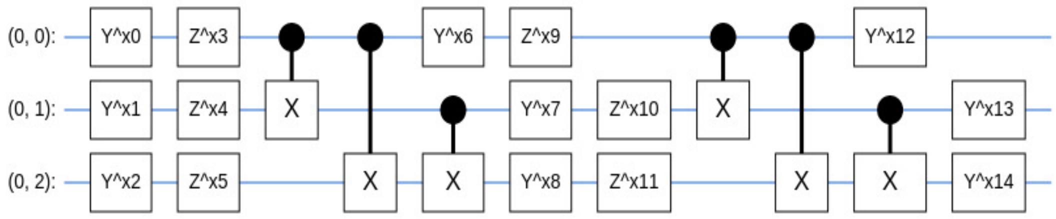




Convolutional Neural Network

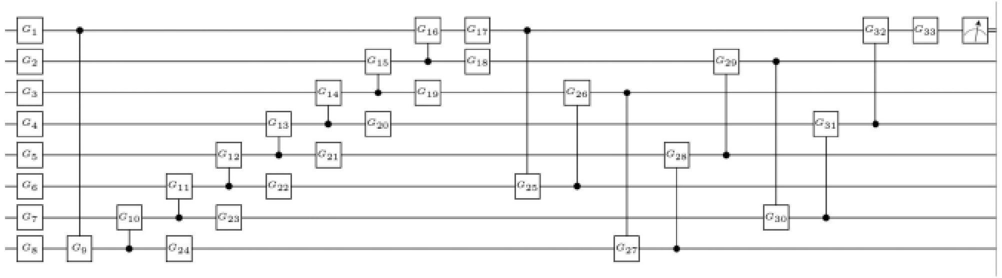
15 parameters,

our *QFCC*



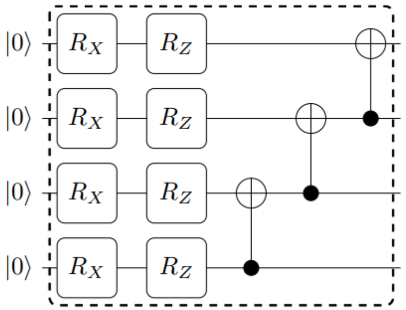
42 parameters,

Maria Schuld et al.,Phys. Rev. A 101, 032308 (2019)



6 parameters,

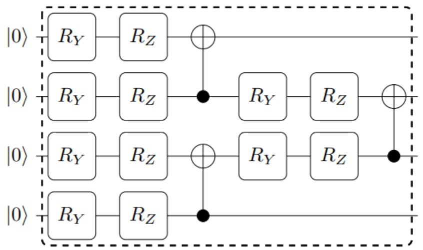
Sukin Sim et al., Adv. Quantum Technol. 2, 1900070 (2019)



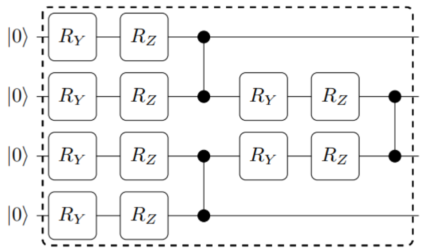
Circuit 2

10 parameters,

Sukin Sim et al., Adv. Quantum Technol . 2, 1900070 (2019)



Circuit 11



Circuit 12

Training parameters during the QFCC analysis

Each QFCC is adapted on 3 qubits to match the output of the QCNN

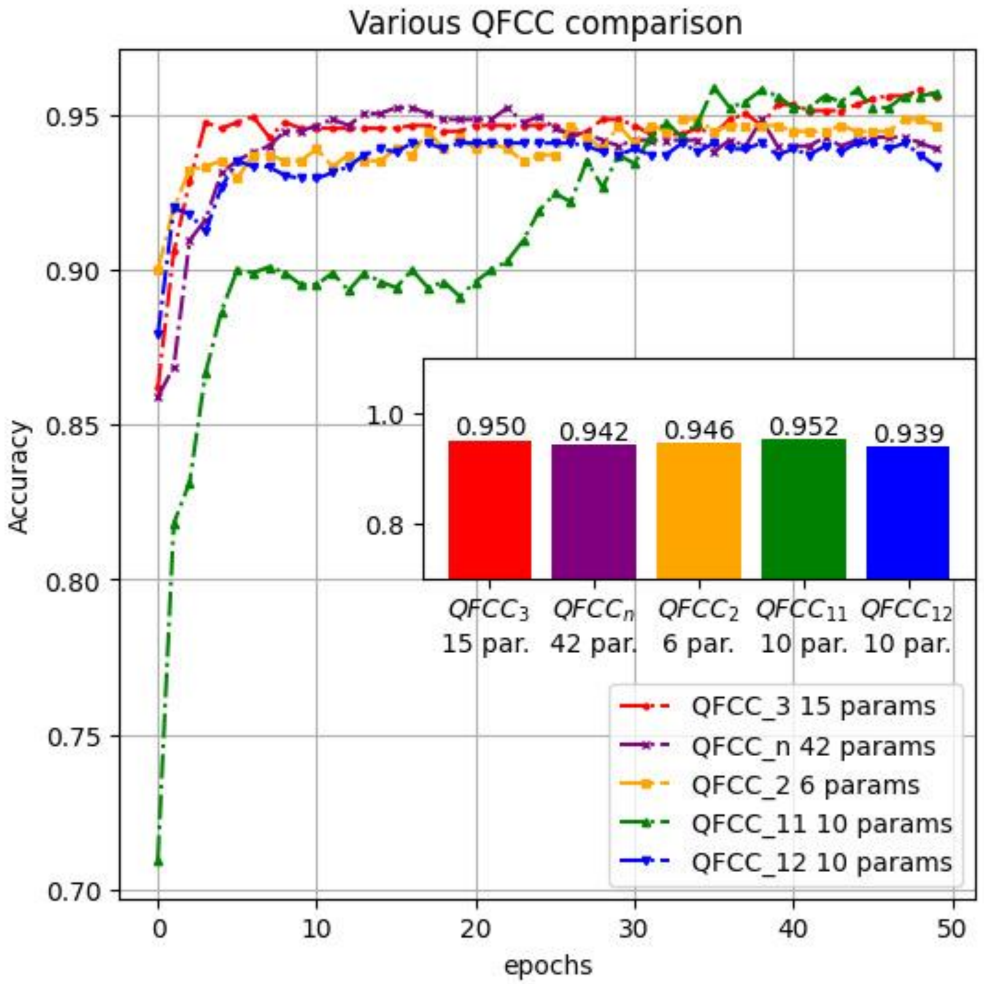
Training set size: 5000 samples
Test set size: 500 samples

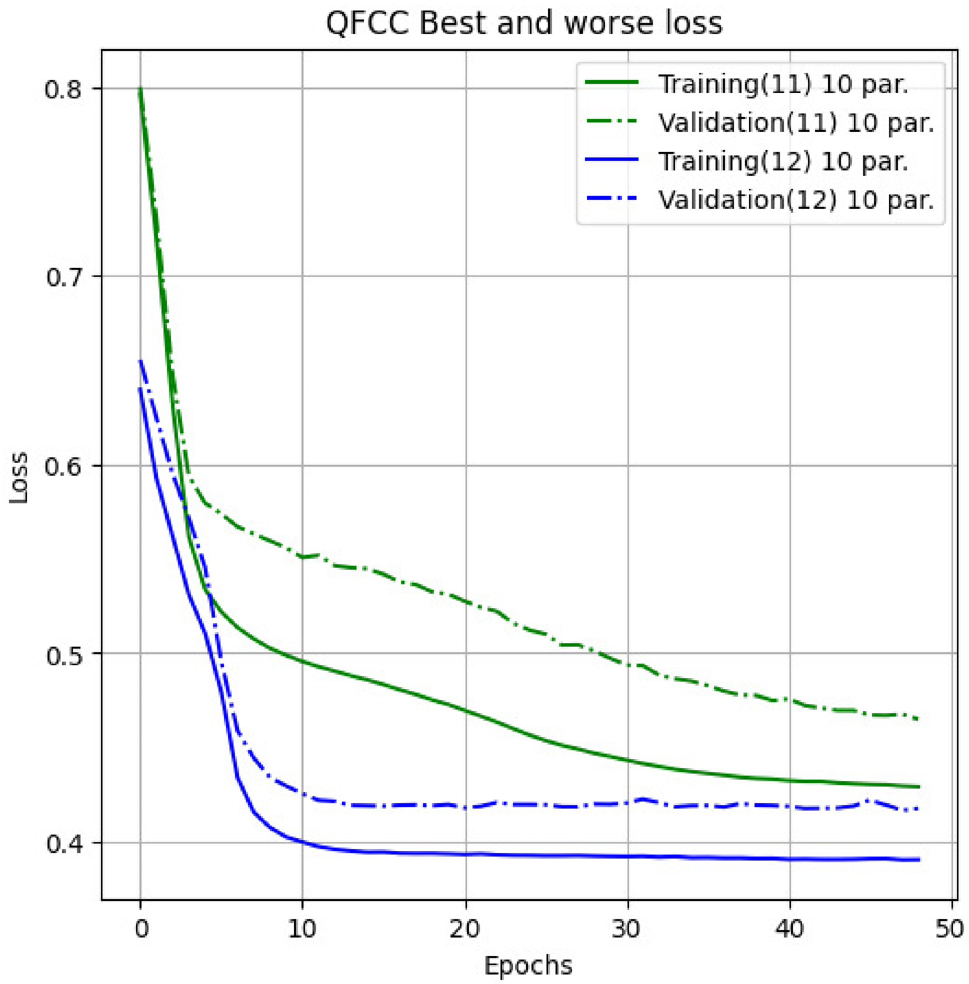
Gradient Descent: Adam optimizer
Loss function: MSE

Minibatch size: 75 samples
Epochs: 50

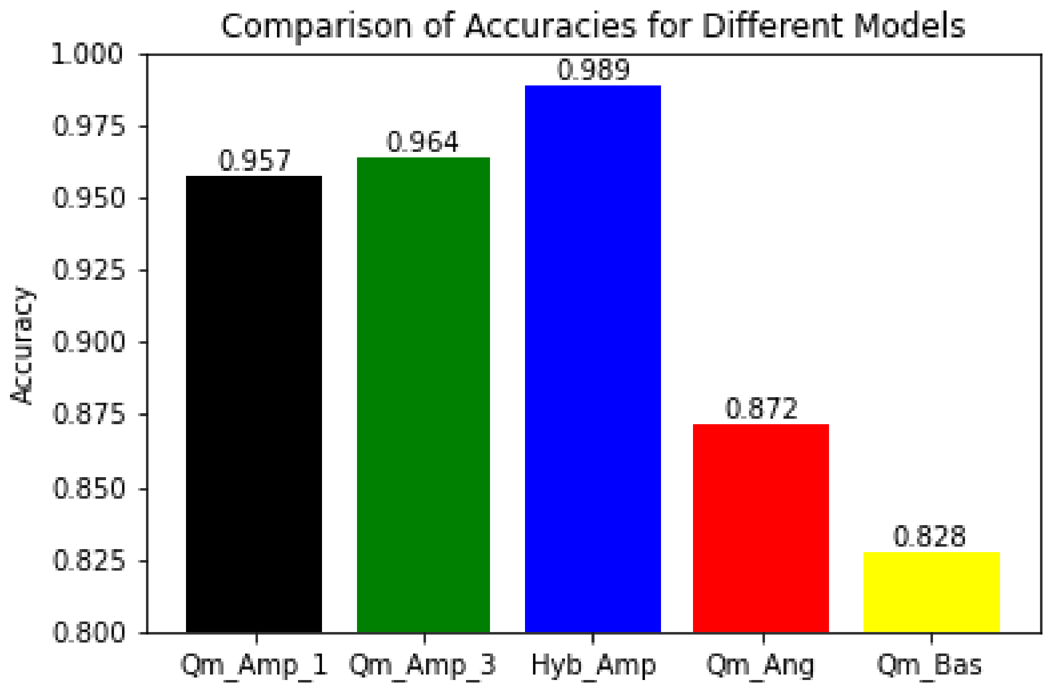
TensorFlow Quantum

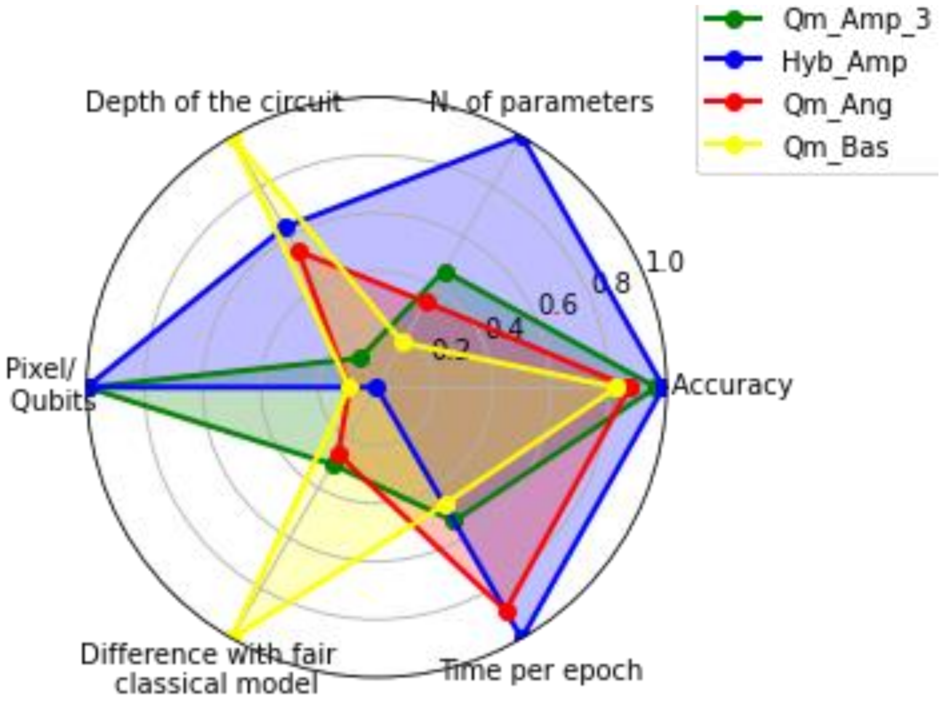
QFCC Results





Summary


Comparison of Accuracies for Different Models
Accuracy
1.000
0.975
0.950
0.925
0.900
0.875
0.850
0.825
0.800
0.957
0.964
0.989
0.872
0.828
Qm_Amp_1
Qm_Amp_3
Hyb_Amp
Qm_Ang
Qm_Bas


Qm_Amp_3
Hyb_Amp
Qm_Ang
Qm_Bas
Depth of the circuit
N. of parameters
Pixel/
Qubits
Accuracy
Difference with fair
classical model
Time per epoch
0.2
0.4
0.6
0.8
1.0

Conclusions

1 Y. Lu,et al., 40th CCC, 52363, (2021).

TensorFlow Quantum

We were not able to evaluate Basis and Angle encoding in QCNN accurately due to limitations in simulating a large quantum algorithm.

We reproduced similar models to 1 with amplitude encoding and found slightly smaller accuracies than classical fair models. However, thanks to amplitude encoding data capacity is higher in the quantum model.

We slightly improved that model's speed, depth, and accuracy (Qm_Amp_1) with a simpler model (Qm_Amp_3) by accurately considering local connectivity in amplitude encoding. By changing the final QFCC we have not found significant results.

With the Hybrid model with amplitude encoding, we improved the accuracy of the classical fair model.