ECS644U Microwave and Millimetrewave Electronics Lab 2

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Contents

1 Aim

The aim of this third lab of ECS644U is to understand the purpose of impedance matching, apply different matching techniques and determine the value of resistive and reactive components for an antenna to be correctly matched using the Vector Network Analyser present in the lab.

2 Method

2.1 Maximum Power Transfer and Impedance Matching

The *Maximum Power Transfer Theorem* states that maximum power is delivered from a source network to a load when its impedance is equal to the complex conjugate of the source's internal impedance. This because:

1. The real parts of the complex impedances would be identical, preventing reflections to occur (hence minimizing power loss)
2. The imaginary parts of the complex impedances would be opposite, hence the series of these two elements would figure as a short cirucit from the source network.

In order to evaluate the impedance Z_m a matching circuit should have, the complex load impedance Z_L has to be evaluated. Provided the matching impedance is in series with the load impedance, the sum of the two should be equal to the internal resistance of the generator network (this is usually equal to the characteristic impedance $Z_0 = 50\Omega$). The reactive element of the load is expressed according to the SI of the reactive element: H Henry for inductors, F Farad for capacitors. Given the well known equations

$$Z_L = jwL,\ Z_C = \frac{1}{jwC} \tag{1}$$

and having the knowledge of the operating frequency the network is stimulated at, it's possible to evaluate the impedance for the reactive components. For ideal components (and for the scope of Lab 3 Part A) infact, we can ignore the parasitic elements in reactive components which would introduce resistivity in them. Hence, their impedance is going to be purely imaginary: capacitors yield negative reactance, inductors positive.
As impedance is easy to evaluate when dealing with components in series, in order to match loads to a generator network by carrying out calculations based on impedance we assume the matching impedance will be in series with both generator and load networks. It becomes clear why a load with

- Negative reactance load impedance would be matched to the generator by a network where an inductor is present: the inductance would be chosen for the inductor's impedance to be equal (in absolute value) to the capacitor's impedance, at the operating requency
- Unequal resistive load impedance would be matched by a network where a resitance equal is present: the value of such resistance would be equal to the difference between the source resistance and the load resistance $R_M = R_S - R_L$. This can lead to scenarios where $R_S \gg R_L$ which would cause most of the power to be dissipated by R_M. To prevent this introducing a reactive element to modify the resistivity of a load can solve the issue, although this will naturally change the reactivity. This must be taken into account by adding an opposite reactive component to cancel out the effect.

- Positive reactance load impedance would be matched to the generator by a network where a condutor is present: the capcitance would be chosen for the capacitors's impedance to be equal (in absolute value) to the inductors's impedance, at the operating requency.

- Unmatched load impedance would be matched by applying the above properties in order to equate generator and load impedance.

2.2 Impedance matching

2.2.1 L-section impedance matching

This technique is utilised in order to provide the correct matching without wasting power over resistive components in the matching network. This implies only reactive components are present in a L-section network: the two possible configurations for the device are presented below. The first one fig. 1 is to be utilised when $Z_L > Z_0$: a shunt connection will effectively lower the impedance by draining more current from the source, without changing the voltage. The second one fig 2 is to be utilised when $Z_L < Z_0$: a series connection will effectively increase the impedance by reducing the amount of current drawn from the generator.
In both configurations, in order not to introduce phase shifts in the generator signal, a second reactive component has to be present, cancelling the phase shift the first element had introduced.

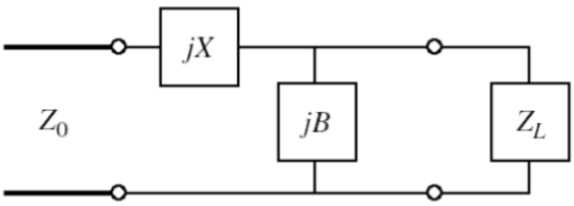

Figure 1: Shunt arrangement for L-section matching network.

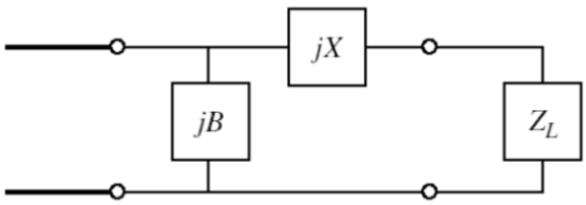

Figure 2: Series arrangement for L-section matching network.

In the excercise provided $Z_L > Z_S$ hence, the shunt configuration will be adopted.

Logically, the equivalent impedance of the arrangement has to equate the characteristic impedance. This is shown in the below eqution, where B is a susceptance.

$$Z_0 = jX + \frac{1}{jB + \frac{1}{R_L + jX_L}} \tag{2}$$

From this equation it's possible to derive values for the reactive components in a way such that, at operating the operating frequnecy, they will resonate, one canceling the 'lag' of the other (or vice versa). Given this techinque utlisies two components, from the above equation, two sets of results can be achieved - as shown below.

$$B = \frac{X_L \pm \sqrt{\frac{R_L}{Z_0}}\sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2} \tag{3}$$

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{BR_L} \tag{4}$$

When the susceptance B has a positive values, then the shunt element will be a capacitor, the series element an inductor. The second solution yields opposite arrangements with an inductor as shunt and a capacitor as series element. Desipite having the same behaviour at operating

(resonance) frequency, the two different soutions provide either a Low Pass filter behaviour (capacitor as shunt) or a High Pass filter - DC blocking - when the capacitor is the series component[1].
These equations could be implemented in a matlab function, which also considers the case $Z_L < Z_0$.

```
function [X,B,arr] = lsection(Zl,Z0)
if (real(Zl) > real(Z0))
    a = imag(Zl) + (real(Z0)/Z0)^1/2 *
                ((real(Zl))^2 + (imag(Zl))^2-Z0*real(Zl))^1/2;
    B(1) = a / ((real(Zl))^2 + (imag(Zl))^2);
    a = imag(Zl) - (real(Z0)/Z0)^1/2 *
                ((real(Zl))^2 + (imag(Zl))^2-Z0*real(Zl))^1/2;
    B(2) = a / ((real(Zl))^2 + (imag(Zl))^2);
    X(1) = 1/B(1) + imag(Zl)*Z0/real(Zl) - Z0/(B(1)*real(Zl));
    X(2) = 1/B(2) + imag(Zl)*Z0/real(Zl) - Z0/(B(2)*real(Zl));
    arr = 'p';
else
    B(1) = ((Z0-real(Zl)/real(Zl))^1/2)/Z0
    B(2) = - ((Z0-real(Zl)/real(Zl))^1/2)/Z0
    X = (real(Zl)*(Z0-real(Zl)))^1/2 - imag(Zl)
    arr = 's';
end
```

This function could then be used in a matlab script which returns inductance and capacitance for the two possible solutions.

```
clear all;
f = 2*10^9;
Z0 = 50;
Zl = 100 - 50i;
[X,B,arr] = lsection(Zl,Z0);
if (arr == 'p')
    str = sprintf('Parallel␣configuration')
    if (B(1) > 0 && B(2) <0) %shunt config
        for i = 1 : 2
            if (i == 1) %capacitor in shunt
                C(i) = B(i)/(2*pi*f*Z0);
                L(i) = (X(i)*Z0)/(2*pi*f);
            else       %inductor in shunt
                C(i) = -1/(2*pi*f*X(i)*Z0);
                L(i) = -Z0/(2*pi*f*B(i));
            end
            str= sprintf('Configuration␣%d:\t␣C=%.3f␣\t␣L=%.3f␣\n',i,C(i),L(i))
        end
    else
        str = sprintf('fucntion␣lsection␣has␣returned␣these␣weird␣values:
␣␣␣␣␣␣␣␣C(1)=%.3f␣\t␣L(1)=%.3f␣\n␣C(2)=%.3f␣\t␣L(2)=%.3f␣\n',C(1),L(1),C(2),L(2)
    end
```

[1]Fig 4.3-8 Conquer RF

```
else
    str = sprintf('Series configuration')
    if (X(1) > 0 && X(2) <0) %series config
        for i = 1 : 2
            if (i == 1) %capacitor in shunt
                C(i) = B(i)/(2*pi*f*z0);
                L(i) = (X(i)*Z0)/(2*pi*f);
            else        %inductor in shunt
                C(i) = -1/(2*pi*f*X(i)*Z0);
                L = -Z0/(2*pi*f*B(i));
            end
            str = sprintf('Configuration %d:\t C=%.3f \t L=%.3f \n', i, C(i), L(
        end
    else
        str = sprintf('fucntion lsection has returned these weird values:
        C(1)=%.3f \t L(1)=%.3f \n C(2)=%.3f \t L(2)=%.3f \n' , C(1), L(1), C(2),
    end
end
```

Similar results can be acheived by utilising a Smith Chart. This graphical tool allows to plot reflection coefficients Γ, and quickly convert to normalized impedance (or admittance) [2]. These normalized quantities can be defined as

$$z = Z/Z_0 \tag{5}$$

$$y = Y/Y_0 \tag{6}$$

and their relationship with the reflection coefficient is

$$\Gamma = \frac{z_L - 1}{z_L + 1} = |\Gamma| e^{j\omega} \tag{7}$$

Because of their nature, and their relationship with Γ, the axis of a Smith Chart represent lines of constant normalised resistance and lines of normalised reactance (bent in a way to fit the unity radius polar plot of the reflection coefficient) [3]. The above equations show why

$$\Gamma = 0 \leftrightarrow Re(z_L) = r_L = 1, Im(z_L) = x_L = 0 \tag{8}$$

and knowledge of the relation between reflection coefficient and microwave networks, allows us to know that a network is matched when $\Gamma = 0$. Furthermore we understand why, when considering impedance, the top half of the Smith Chart, represents inductive reactance components ($X_L > 0$) and the bottom half represents capacitive reactance components ($X_L < 0$).
Given the relationship between impedance and admittance, it's possible to construct a Smith Chart which relates Γ to the normalized admittance of the load y_L. This will be symmetrical to the impedance Smith chart, with respect to the $x = b = 0$ imaginary axis, which implies inductive and capacitive reactive components are flipped. Despite this, by applying the property of a $\frac{\lambda}{4}$ line (eq. 9), it's possible to easily convert from admittance to impedance (and vice versa), by simply reflecting a value across the center of the smith chart.

$$z_{in} = Z_0 \frac{z_L + jZ_0 tan\beta l}{Z_0 + jz_L tan\beta l} = \frac{1}{z_L} \tag{9}$$

[2] Pozar 2.4
[3] Conquer RF 4.4.1

These considerations allow us to define a specific method to design a matching network according to L-section technique using a Smith Chart:

1. Evaluate $|Z_L| \overset{?}{>} Z_0$ in order to pick either the shunt or the series arrangement.
2. Normalize $Z_L \rightarrow z_L = \frac{Z_L}{Z_0}$ and plot it's position on smith chart. This will be the intesection of the circumference of radius equal to the normalized resisitvity and the curved axis of value equal to the normalized reactivity.

Having set a “starting point” (z_l), knowing the arrangement of the matching network(shunt/series) and the “arrival point”($\Gamma = 0$), rotations which will lead to of the $1 + j$ circumference on the Smith Chart have to be performed. This “intermediate arrival point” is the point at which the resistive part of the load appears to match the resistive part of the network, thanks to the first reactive component.

3. Rotating impedances (or admittances) along the curved lines is equivalent to summing reactance (or susceptance). For shunt connections, susceptance will be summed, hence, a transformation from impedance to admittance is necessary. According to 9 this can be performed simply by reflecting z_l across $\Gamma = 0$ point.
4. Moving along the reactance/susceptance curved lines we rotate z_L to the $1 + j$ circumference

Once on the $1 + j$ circumference, although the resistivity might be matching, reactance in the load introduces phase shifts in the generator's signal. To cancel these another rotation, from the $1 + j$ circumference to the “arriving point” $\Gamma = 0$ has to be performed. Again, if necessary, a reflection can be performed to convert admittance in impedance (or viceversa) according to the matching network's arrangement.

5. Rotate along the curved lines towards $\Gamma = 0$ adding reactance (or susceptance) to complete the network's design.
6. Knowledge of operating frequency, matching network arrangement and characteristic impedance allow us to convert reactance and susceptance values into physical values (Henries and Farads) for the inductive and capacitive components.

As per the analytical discussion, there's two sets of possible values. In the smith chart this is possible by rotating towards the 1+j circumference down one direction or another (along the same reactive -curved- line).

2.2.2 Single-stub tuning

Lengths of transimssion lines left either open or short circuited can figure as a matching network. This technique is to be preferred in case of high frequencies, when the parasitic nature of reactive elements becomes non ignorable, causing L-section techinques to disperse a lot of power.
Given microstrip lines adopt open circuit stubs, and in the exercise it's specified to implement a shunt configuration, the following discussion will treat only this configuration (for time-management issues).
The aim of this technique is to reduce the reflection coefficient to $\Gamma = 0$ (load matching). This is achieved generating a reflection, by connecting a stub to the transmission line hence, sending some voltage down along it. The generated reflection will have to be equal in amplitude but

opposite in phase to the mismatched-load reflection, for maximum power to be delivered to the load. This explains why the two parameters that will have to be set for the single stub are:

1. Distance from the load (d): distance at which the normalized resistive load is equal to 1 (hence the resistive part of the load equals to Z_0).

2. Length of the stub (l): length which shifts the signal in the stub by half of the phase shift introduced by the load reactance. Reflecting at the open circuit, the signal will travel across the stub again, hence it will have travelled a distance that generates a phase shift equal in magnitude, but opposite in sign, to the one generated by the load.

The maths behind these concepts is shown below. The starting point is the input impedance felt at the input of a transmission line (which in this case would be the stub). We can consider the open circuit [4] as an infinite impedance. This yields

$$Z_{OC} = \lim_{Z_L \to \infty} Z_0 \frac{Z_L + jZ_0 tan\theta_{OC}}{Z_0 + jZ_L tan\theta_{OC}} = -\frac{Z_0}{jtan\theta_{OC}} \tag{10}$$

which shows how, depending on the value of θ_{OC} , a stub terminanted by an open circuit, could figure either as a capacitive [5] or as an inductive [6]component.
The mathematical implementation of the above points is shown below.

1. As we are dealing with shunt configurations it's ideal to carry out calculations utilising admittance, as it adds up in parallel. We can rewrite

$$Y_L = \frac{1}{Z_L} = \frac{1}{R_L + jX_L} = G_L + jB_L \tag{11}$$

As we are trying to determine at what distance from the load the stub should be set, the below equation allows us to express the load impedance, as felt from a distance d

$$Z = Z_0 \frac{(R_L + jX_L) + jZ_0 t}{Z_0 + j(R_L + jX_L)t} \tag{12}$$

where $t = tan\theta = tan\beta d$. Combining these two equations it's possible to achieve equations for reactance and susceptance of the unmatched load at a distance d.

$$G = \frac{R_L(1 + t^2)}{R_L^2 + (X_L + Z_0 t)^2} \tag{13}$$

$$B = \frac{R_L^2 t - (Z_0 - X_L t)(X_L + Z_0 t)}{Z_0[R_L^2 + (X_L + Z_0 t)^2]} \tag{14}$$

again, $t = tan\theta = tan\beta d$.
The aim is to acheive a reactance equal to Y_0, hence $G = Y_0$ is imposed and the resulting quadratic equation in terms of t is solved. As expected, this yields two different values for d

$$\frac{d}{\lambda} = \begin{cases} \frac{1}{2\pi} tan^{-}1(\frac{-X_L}{2Z_0}), & \text{for} \frac{-X_L}{2Z_0} \geq 1 \\ \frac{1}{2\pi} tan^{-}1(\pi + \frac{-X_L}{2Z_0}), & \text{otherwise} \end{cases} \tag{15}$$

[4]Again the maths will only represent the open-circuit case proposed in the lab.
[5]$0° \leq \theta \leq 90°$
[6]$90° \leq \theta \leq 180°$

2. In order to evaluate the length l of the stub the susceptance of the stub has to be opposite to the susceptance of the load at distance d.

$$B_s = -B \tag{16}$$

Utilising 10, where $\theta_{OC} = \beta l$, substituting $t = tan\beta d$ in 14, it's possible to derive

$$\frac{l}{\lambda} = \frac{1}{2\pi} tan^{-}1(\frac{B}{Y_0}) \tag{17}$$

where $\frac{\lambda}{2}$ can always be added, thanks to the properties of transmission lines.

As seen for L-section, this analytical process can be carried out using a Smith Chart.
This is possible because on the external rings of the Smith Chart, conductive and inductive reactance is matched to the length of a transmission line via the wavelength and, as seen in equation 17 this is sufficient to determine the length of the stub.
These considerations allow us to define a specific method to design a matching network according to Signle-stub (parallel, open circuit configuration) Tuning technique using a Smith Chart:

1. Normalize $Z_L \rightarrow z_L = \frac{Z_L}{Z_0}$ and plot it's position on smith chart. This will be the intesection of the circumference of radius equal to the normalized resisitvity and the curved axis of value equal to the normalized reactivity. Draw the SWR circumference (center in $\Gamma = 0$ and radius equal to the magnitude of the normalized impedance).

2. A transformation from impedance to admittance is necessary. According to 9 this can be performed simply by reflecting z_l across $\Gamma = 0$ point.

3. Plot the two intersections with the 1+j circumference

4. Count the Wavelengths Towards Generator that are present between the two intersections and the normalized admittance y_L. At the intersections it's also possible to read the susceptance felt at these points.

5. To cancel out this element, the stubs are required to have an opposite susceptance. To acheive this, plot the desired susceptance on the Smith Chart and count the Wavelengths Towards Generator from point y=0 (open circuit). This procedure, as the analytical one, yields the two possible results for the single stub matching techinque.

2.2.3 Quarter Wave Transformer

Intuitively, from what was seen in Signle-stub tuning, a quarter wave transformer (QWT) will only match real loads. Adapting equation 12 to the situation visible in image 3 ,it's possible to evaluate the input impedance felt from the generator as

$$Z_i n = Z_1 \frac{R_L + jZ_1 tan(\beta l)}{Z_1 + jR_L tan(\beta l)} \tag{18}$$

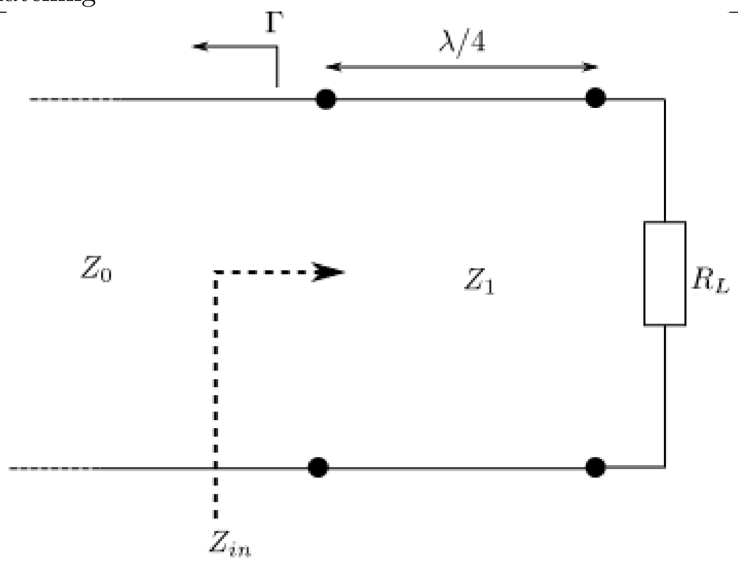

Figure 3: Schematic of a Quarter Wave Transformer.

Plucking $\beta l = \frac{2\pi}{\lambda}\frac{lambda}{4} = \frac{\pi}{2}$ into 18, solving by taking the limit $\beta l \rightarrow \frac{pi}{2}$ and setting, as required, $Z_in = Z_0$ it's possible to determine that

$$Z_1 = \sqrt{Z_0 R_L} \tag{19}$$

This value will be acheived solely at design frequency, hence to evaluate $\Gamma(f)$, the reflection coefficient should be expressed in terms of 18 .

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0 + j2t\sqrt{Z_0 Z_L}} \tag{20}$$

Once again $t = tan\beta l$ which has the frequency term inside it. Using matlab it's possible to evaluate the magnitude of this complex term easily and plot the relevant graph. Below the utilised code.

```
clear all;
Z0 = 50;
Zl = 100 - 50i;
bl = [0:0.01:pi];
t = tan(bl);
Gamma = (Zl-Z0)./(Zl+Z0+i*2.*t*sqrt(Z0*Zl));
figure;
plot(bl, abs(((Gamma))));
grid on;
set(gca,'Xtick',[0:pi/4:pi],'Xticklabel',{'0','pi/4','pi/2','3pi/4','pi'});

f0 = 2*10^9;
f = [0:10^6:3*10^9];
lambda0 = 3*10^8/f0;
lambda = 3*10^8./f;
b = 2*pi./lambda;
l = lambda0/4; %length is set for f0
t = tan(b*l);
```

```
Z1 = sqrt(Z0*Zl);
Zin = Z1*(Zl+i*Z1.*t)./(Z1+i*Zl.*t);
figure;
plot(f,real(Zin));
ylabel('Re(Zin)');
grid on;

figure;
plot(f,imag(Zin));
ylabel('Im(Zin)');
grid on;


Gamma = (Zl-Z0)./(Zl+Z0+i*2.*t*sqrt(Z0*Zl));
figure;
plot(f,abs(Gamma));
ylabel('abs(Gamma)');
grid on;
```

2.3 AWR Simulations

In order to efficiently carry out the following simulations, it's important to set the Global Project Units as a first step. For metric, capacitance and inductance units, these will be mm, pF, nH.

2.3.1 L-Section

The logic behind the schematic utilised in AWR (Fig. **??**) is presented.

- The Generator Network can be modelled, either with a voltage source, a source resistance, a volt meter and an amp meter (which will provide us the value of voltage and current felt by this network); or it can be modelled using a port, of which, characteristic impedance can be easily specified. Given the much greater simplicity to plot graphs and perform measurements utilising the port, this solution will be adopted. Hence, the port's impedance should be set to be equal to the characteristic impedance that has to be matched: 50Ω.
- The load $Z_L = 100 + j50\Omega$ could be modelled either combining resistors and reactive elements, to match the Z_L. In order to correctly choose inductance or capacitance values for the reactive component, frequency has to be present in our known terms. A better solution could be using the IMPED component AWR provides, where resistivity and reactance can be specified.
- In order to acheive a graph where the impedance felt by the generator at operating frequency is clearly visible, first set the operating frequency in the Project Options as a single point (2 GHz), then add a new graph plotting the Z11 parameter (impedance measured at Port1 when power is delevered from Port1). As this is a complex number, both the imaginary and real part of it will be plotted.
- Given $|Z_L| > Z_0$, the matching network's arrangement will yield an equivalent(matching +load) impedance where the load is in parallel with a susceptive element *and then* in series with a reactive one, rather than the opposit. Hence a a positive susceptive shunt component (capacitor) is placed in parallel with the load.

- In order to evaluate the capacitive value this element should have in order for the normalized resistance to be 1, it's either possible to use a tuner and change the capacitance value, observing at the impedance graph. Naturally, the selected value will generate a real part euqal to the characteristic impedance. Alternatively it's possible to go on Element Options, tick Opt and set upper and lower limits for the component. A new Optimization Goal has to be set: to implement this, select the measurement that has to be optimized (in this case $Re(z_{11})$) and specify the untiless value of the Goal (in this case, 50). Pressing F7 it's possible to select the optimization method to Discrete local Search. After pressing start it's visible a value has been assigned to the optimized component (in this case, capcitance). Checking the Z_{11} graph, it's visible $Re(Z_{11})$ is now fixed at 50Ω as desired.

- In order to remove the remaining reactance an inductor has to be added (given the selected arrangement). In order to select the correct inductance value again, it would be possible to observe how the Z_{11}graph changes whilst tuning the impedance to the correct value. But it's also possible (and preferred) to utilise the optimization process. In order to do so, the optimization tick has to be removed from the capacitor's Options (as its value already was optimized) and placed in the inductor's Options, along with upper and lower limits (and step). The previous goal can be modified to the imaginary part of Z_11, by selecting it as a measurement with a target goal of 0. Once again, running the optimization changes the value of inductance.

- In order to retrieve a frequency response for the optimized design, the frequencies with which the nework is simulated can't be a signle point value (2GHz), so in Project Options a new frequency range is set, from 1 to 10 GHz, with a step of 0.1GHz . The impedance is then switched for another Port2, to allow measurement of S12 and plot it in AWR. Switching between IMPED and Port2 is possible as Port2 can have a complex impedance (which is set to equal IMPED).

Below circuit schematics regarding the described procedure.

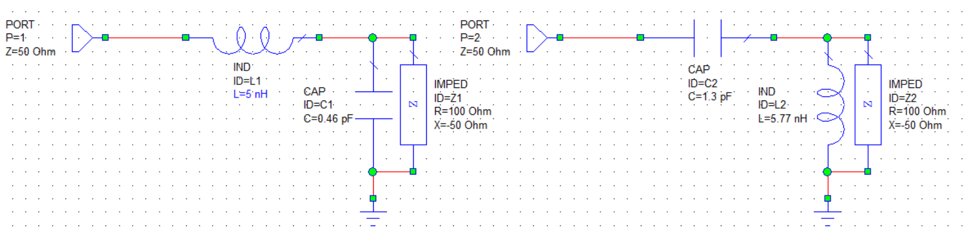

Figure 4: Circuit schematic of the AWR simulation for a L-Section matching network.

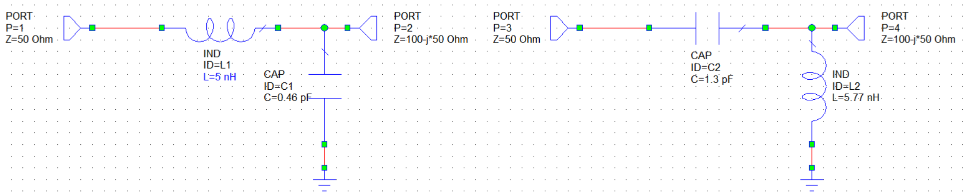

Figure 5: Circuit schematic of the AWR simulation for a L-Section matching network with a complex Port.

2.3.2 Single Stub Tuning [parallel open circuit configuration]

The logic behind the schematic utilised in AWR (Fig. **??**) is presented.

- The Generator Network can be modelled, either with a voltage source, a source resistance, a volt meter and an amp meter (which will provide us the value of voltage and current felt by this network); or it can be modelled using a port, of which, characteristic impedance can be easily specified. Given the much greater simplicity to plot graphs and perform measurements utilising the port, this solution will be adopted. Hence, the port's impedance should be set to be equal to the characteristic impedance that has to be matched: 50Ω.

- The load $Z_L = 100 + j50\Omega$ could be modelled either combining resistors and reactive elements, to match the Z_L. In order to correctly choose inductance or capacitance values for the reactive component, frequency has to be present in our known terms. A better solution could be using the IMPED component AWR provides, where resistivity and reactance can be specified.

- As seen in equation 2.2.2 , to normalise the resistivity felt from the generator, the distance at which the parallel open circuit stub should be placed at, has to be determined. Hence, a MLIN element has to be placed in series with the load.

- To represent the stub, a second MLIN element is placed with an open circuit termination, in shunt configuration.

- The electrical characteristics and the material parameters of the transmission lines are set by MSUB. The values are those utilisied in the previous labs, as we assume the same transimssion line is being used.

- In order to evaluate the physical Length (L) of the two MLINs, the TXLine tool can be utilised. In order to do so, the wavelengths found on the Smith Chart have to be converted into electrical lengths, used as inputs for the TXLine to provide us with the required data. $\theta_{d1,2} = d_{1,2}360°$, $\theta_{l1,2} = l_{1,2}360°$

- In order to retrieve a frequency response for the optimized design, the frequencies with which the nework is simulated can't be a signle point value (2GHz), so in Project Options a new frequency range is set, from 1 to 3 GHz, with a step of 0.1GHz . S_{11} measurements are then plotted against frequency for both of the single stub solutions and for the unmatched case.

Below circuit schematics regarding the described procedure.

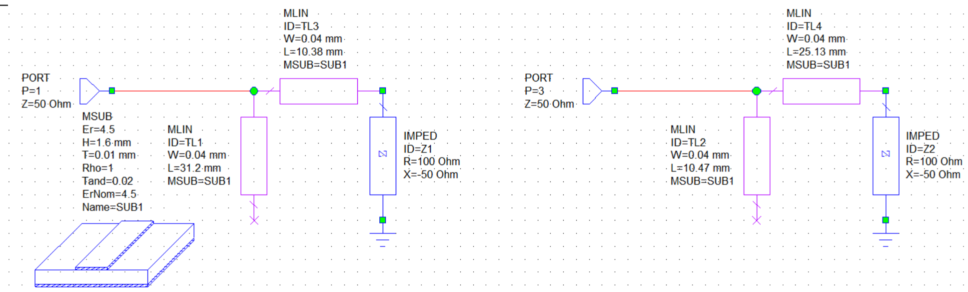

Figure 6: Circuit schematic of the AWR simulation for a Single Stub Tuning network.

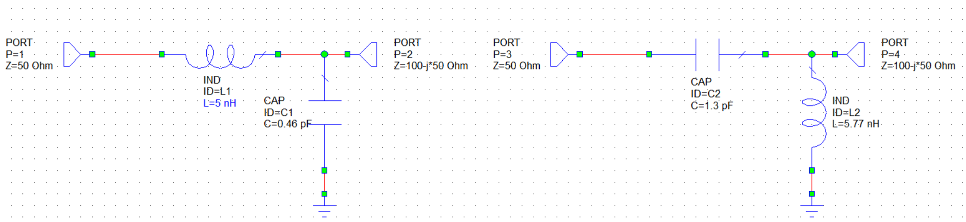

Figure 7: Circuit schematic of the AWR simulation for a Single Stub Tuning network with a complex Port.

2.3.3 Quarter Wavelength Transformer

The logic behind the schematic utilised in AWR (Fig. **??**) is presented.

- The Generator Network can be modelled, either with a voltage source, a source resistance, a volt meter and an amp meter (which will provide us the value of voltage and current felt by this network); or it can be modelled using a port, of which, characteristic impedance can be easily specified. Given the much greater simplicity to plot graphs and perform measurements utilising the port, this solution will be adopted. Hence, the port's impedance should be set to be equal to the characteristic impedance that has to be matched: 50Ω.

- The load $Z_L = 100 + j50\Omega$ could be modelled either combining resistors and reactive elements, to match the Z_L. In order to correctly choose inductance or capacitance values for the reactive component, frequency has to be present in our known terms. A better solution could be using the IMPED component AWR provides, where resistivity and reactance can be specified.

- In order to represent a Quarter Wavelength transmission line a MLIN lement is inserted in series with the load. The length of this transmission line is set to be equal to a quarter of the wavelength, evaluated via equations in AWR circuit schematic view.

- In order to retrieve a frequency response for the optimized design, the frequencies with which the nework is simulated can't be a signle point value (2GHz), so in Project Options a new frequency range is set, from 1 to 3 GHz, with a step of 0.1GHz . S_{11} measurements are then plotted against frequency both when the QWT matching network is present, and when the load in unmatched.

Below circuit schematics regarding the described procedure.

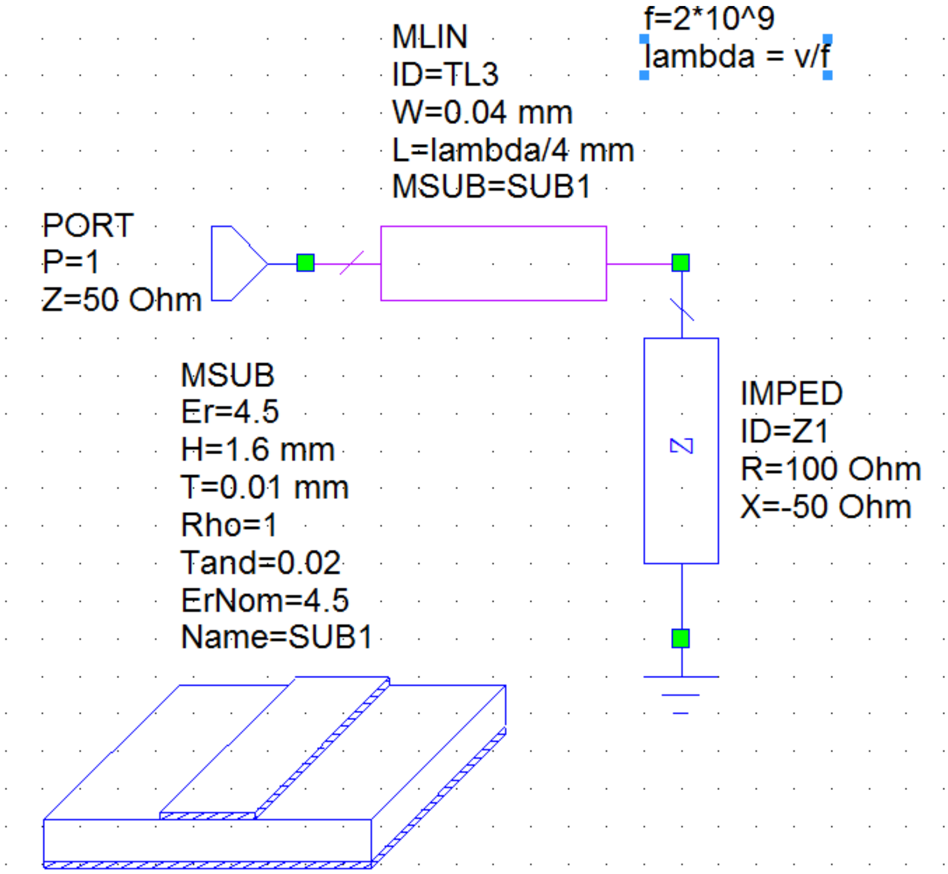

Figure 8: Circuit schematic of the AWR simulation for a Quarter Wavelength Transformer matching network.

2.4 Measurement: Smith Charts

Using the Rohde and Schwarz Vector Network Analyser, impedance matching for an antenna was carrying out during the lab. The VNA transmits a small amount of power to the antenna and measures how much power is reflected back to the VNA. The S-parameter is the magnitude of the reflected coefficient. This measures how close to 50ohms the antenna impedance is. In order to do so, the Smith Chart setting has to be selected to show the trace and frequency on the chart's Real and imaginary axis: the top half of the trace is for +j and the bottom half is for -j. The centre of the Smith Chart represents zero reflection coefficient, therefore the load is perfectly matched to the VNA. The perimeter of the Smith Chart represents the reflection coefficient with a magnitude of 1 which shows that the antenna is poorly matched to the VNA. The magnitude of the reflection coefficient depends on how far from the centre of the Smith Chart the point is. The antenna was taken as the load and a marker is present on the smith chart, indicating the antenna's impedance. Moving this marker chenges the values of real and imaginary components. The aim is to move the marker as close to the center as possible: this will indicate a 50Ω impedance, hence a perfect match. The frequency value for which we could approach a $\Gamma = 0$ reflection coefficient value was $2.4GHz$.

3 Results

3.1 Maximum Power Transfer and Impedance Matching

In the exercise provided in the lab sheet the load has an unmatched impedance. By combining the properties for unequal resistive loads and positive reactive loads, it's possible to state:

$$Z_L + Z_M = Z_G \tag{21}$$

which yields

$$Z_M = 30 - j\omega L_L \tag{22}$$

where $\omega = 2\pi f$ and $f = 2GHz$.

3.2 Impedance Matching

3.2.1 L-section

The results produced by the matlab script are:

1. Configuration 1: L: 4.87 nH C: 0.46 pF
2. Configuration 2: L: 5.77 nH C:1.30 pF

Below a scan of the Smith Chart solution for the same problem

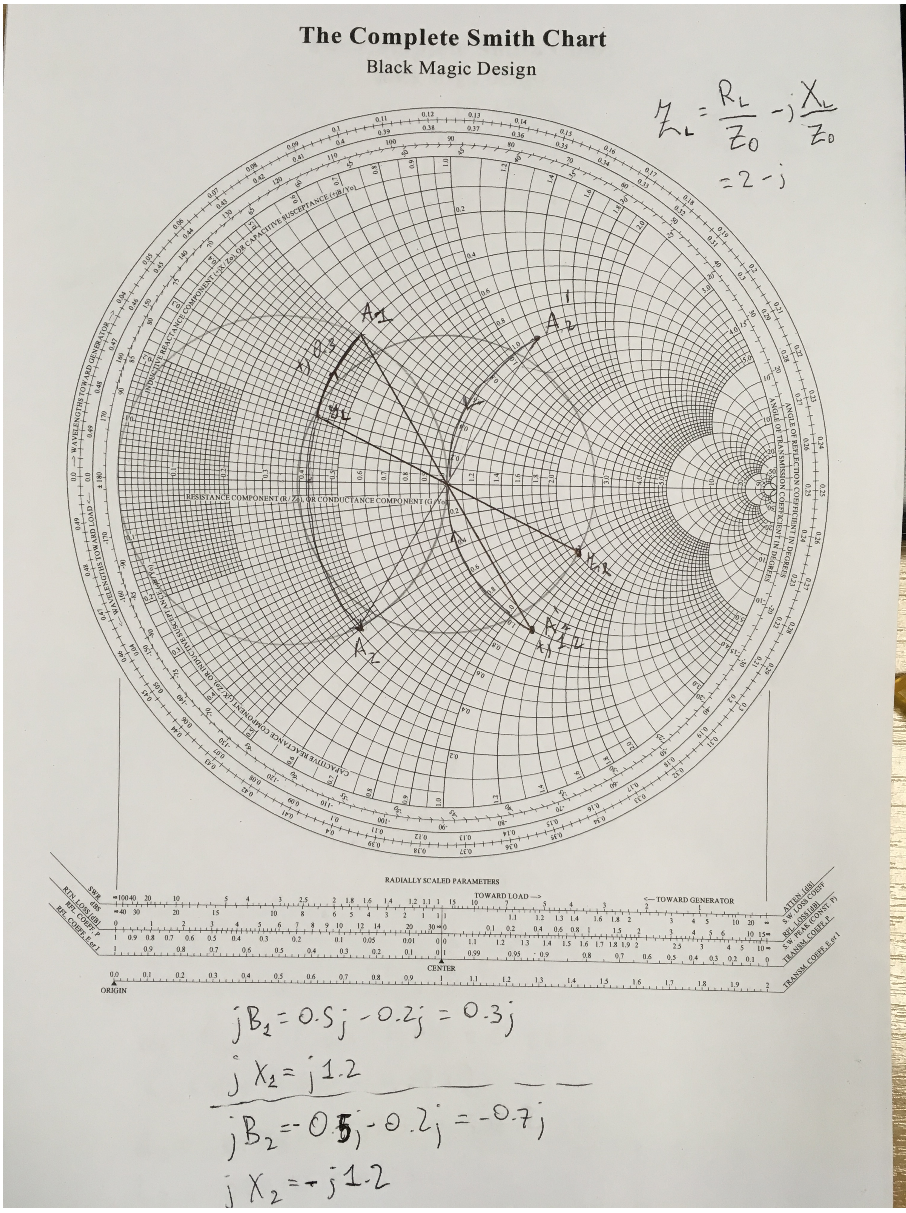

Figure 9: L section matching technique using a Smith Chart.

The results are:

1. Configuration 1: L: 4.77 nH C: 0.477 pF
2. Configuration 2: L: 5.65 nH C: 1.11 pF

They can be acheived using the following formulas

$$C = \frac{b}{2\pi f Z_0} \tag{23}$$

$$L = \frac{x Z_0}{2\pi f} \tag{24}$$

3.2.2 Single-Stub tuning

Below a scan of the Smith Chart solution for Single-Stub impedance matching technique:

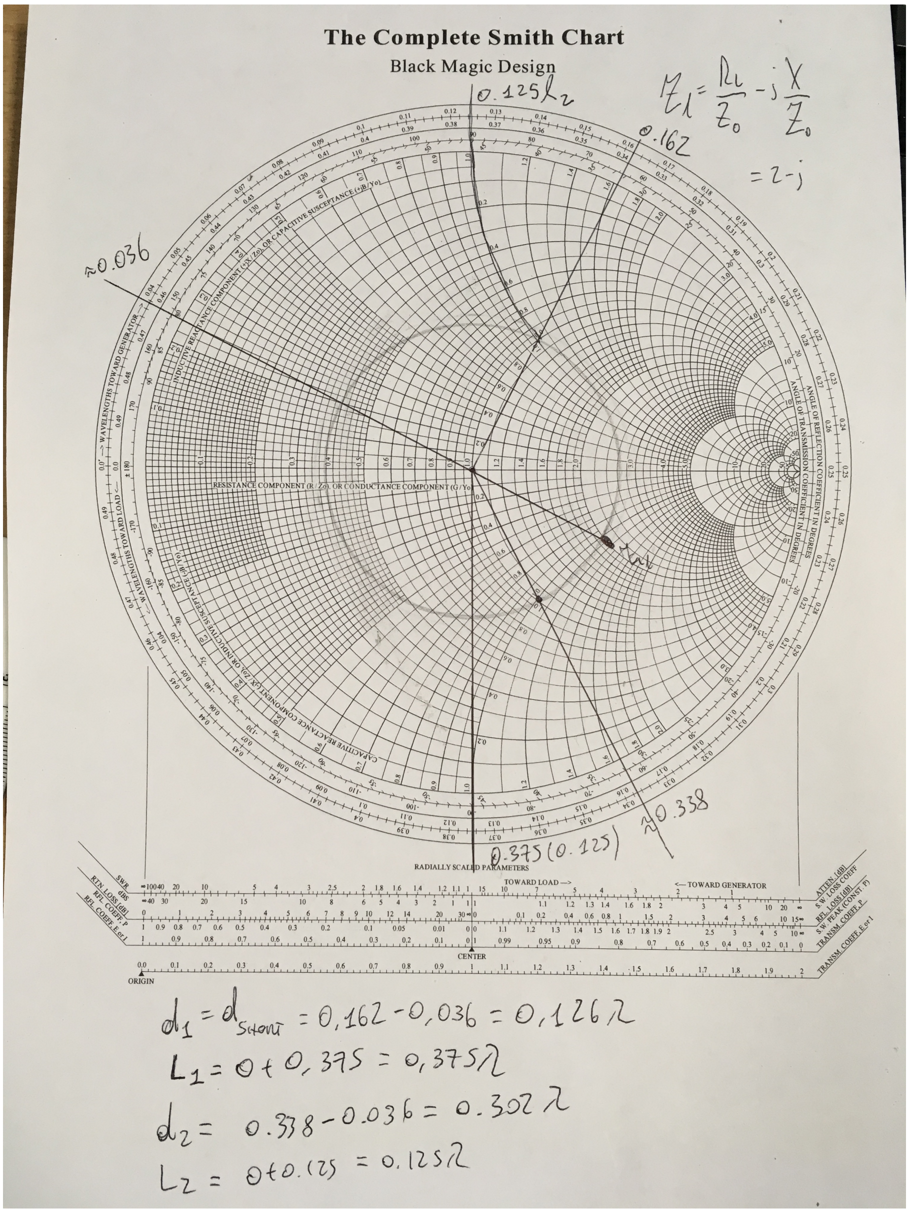

Figure 10: Single Stub tuning technique using a Smith Chart.

The results are visible in figure

3.2.3 Quarter Wave Transformer

Below are the graphs produced with the matlab script

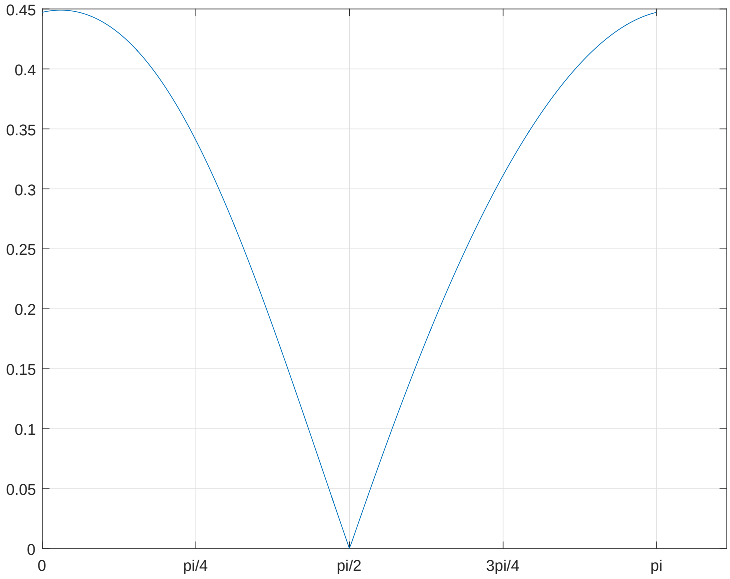

Figure 11: Theoretical response for a QWT matching network.

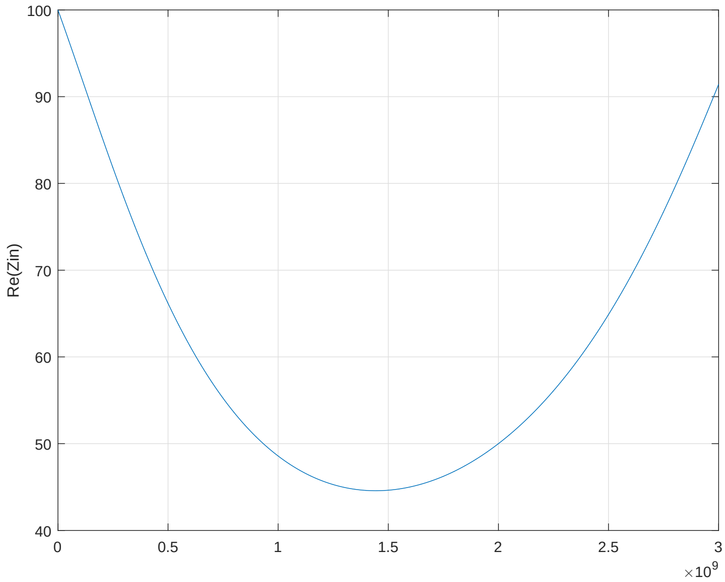

Figure 12: Frequency response of the real part of the input inpedance felt by the generator.

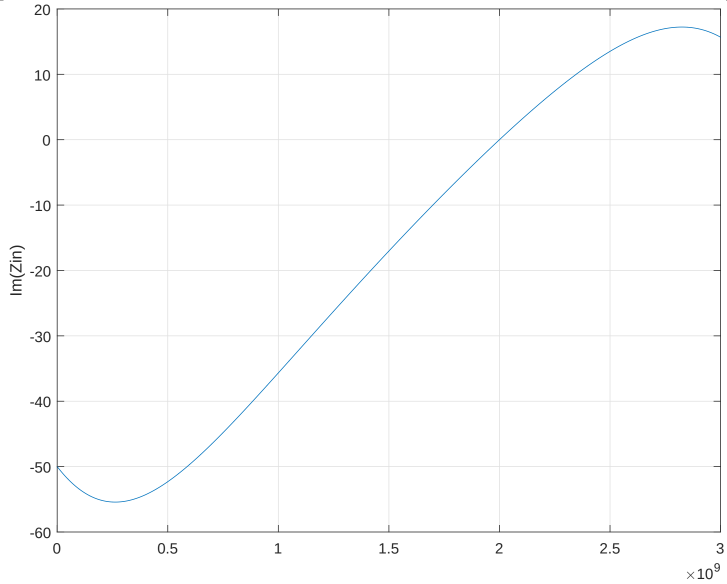

Figure 13: Frequency response of the imaginary part of the input inpedance felt by the generator.

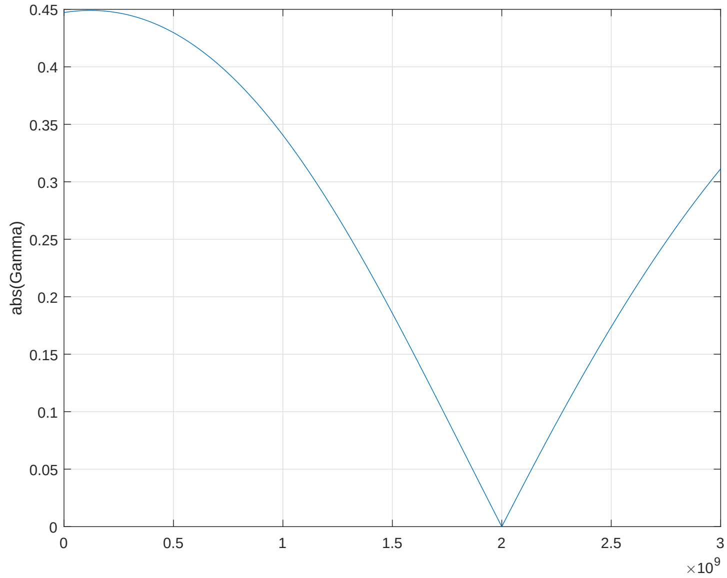

Figure 14: Frequency response of the magnitude of the reflection coefficient.

3.3 Simulations

Presented below are the graphs produced in AWR with the circuit Schematics described in 2.3

3.3.1 L-Section

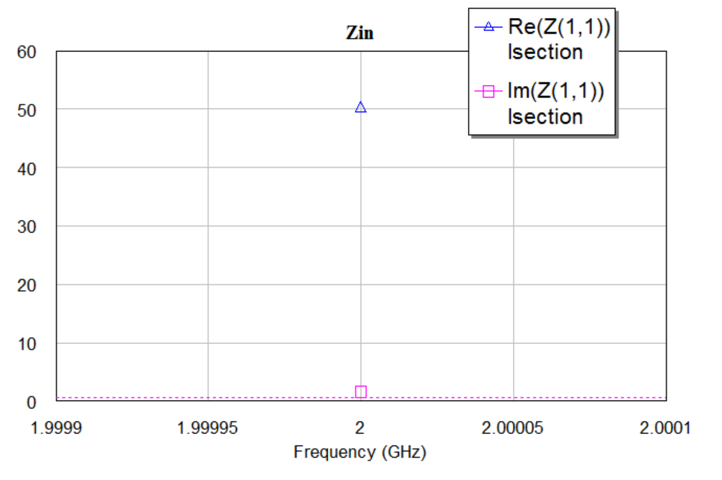

Figure 15: Graph for measured impedance felt by generator (Port1) @ 2GHz.

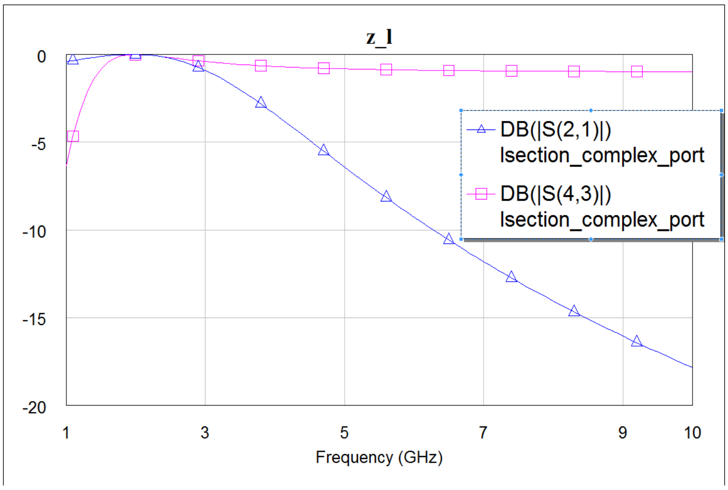

Figure 16: Frequency response of the L-Section network.

3.3.2 Single Stub Tuning

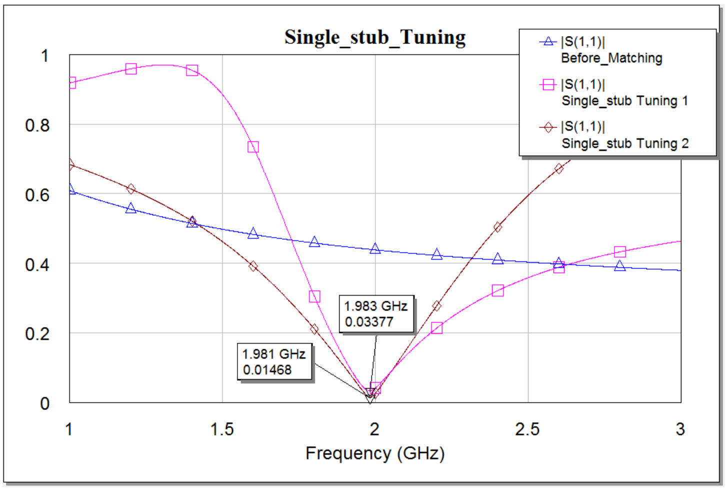

Figure 17: Frequency response of the single stub tuning network.

3.3.3 Quarter Wave Transformer

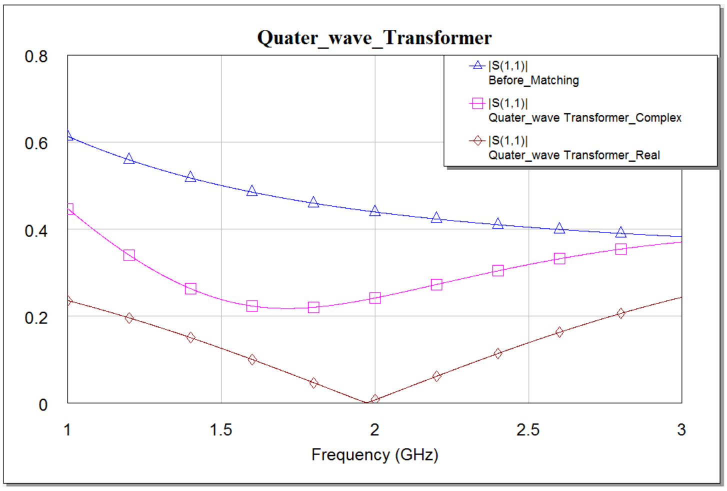

Figure 18: Frequency response of the Quarter Wavelength Transformer network.

4 Discussion

4.1 Maximum Power Transfer and Impedance Matching

As the result has a negative imaginary part, we understand the reactive component will either have a positive susceptance or a negative reactance. This means it will either be an inductor in parallel with the load, or a capacitor in series with it. As the resistive component of the matching circuit will have greater resistance than the load, power dissipation will be affected as maximum effeciency isn't met.

4.2 Impedance matching

4.2.1 L-Section

Desipite having the same behaviour at operating (resonance) frequency, the two different soutions provide either a Low Pass filter behaviour (capacitor as shunt) or a High Pass filter - DC blocking - when the capacitor is the series component[7]. The results achieved are very close to those evaluated with the analytical approach, and the mismatch is due to slight imperfections in carrying out the graphical approach.

4.2.2 Single Stub Tuning

Despite having the same behaviour at operating frequency, the two solutions have different behaviour. Looking at the frequency response it becomes clear that one should be chosen to allow more lower frequencies to pass, the other for higher frequencies.

4.2.3 Quarter Wavelength Transfomer

To solve a matching network two degrees of freedom are required. Given the only parameter the QWT only solves one of them a perfect match won't be implementable. The graphs plotted with matlab don't show the impedance mismatch, infact it's visible that at 2GHz $\text{Im}(Z_{in}) = 0$ and $Re(Z_{in}) = 50$. In order to evaluate the impedance mismatch and the correct reflection coeficient Γ the following code is utilised.

```
clear all;
Z0 = 50;
Zl = 100 - 50i;
Z1 = sqrt(Z0*real(Zl));
Z = Z1 + Zl; %total impedance of series combination for TX line and load
Gamma = (Z - Z0)/(Z+Z0) %reflection coefficient felt at generator's end
abs(Gamma)
```

By evaluating the total impedance of the series combination of the transmission line and the load, we manage to evaluate the impedance felt at the generator's end, and the reflection coefficient can be evaluated by comparing this value with the characteristic impedance. This code yields:

- Gamma = 0.5690 - 0.0976i
- abs(Gamma) = 0.5774

which proves the Quarter Wavelength Transformer isn't sufficient to match the complex load to the generator's network.

5 Resources

1. David M. Pozar *MICROWAVE ENGINEERING* Chapters 2, 5
2. Dr Francesco Fornetti *CONQUER RADIO FREQUENCY* Chapters 2,3,4

[7] Fig 4.3-8 Conquer RF