ECS644U/P Microwave and Millimetrewave Electronics
Assignment 5: Final design project

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1 Introduction

We worked together as a group on this report, the main contributors to the parts are mentioned in the text.

However, this report was not produced by simply dividing up the task and then putting the document together, the process by which this report was compiled should rather be called synthesis, because all parts of this report has been discussed in detail and agreed by all the group members.

2 Part A: Theory: Unilateral Amplifier stability

2.1 μ test theoretical equivalences

We are going to prove the equivalence of two statements:

$$|S_{11}| < 1 \qquad |S_{22}| < 1 \tag{1}$$

$$\frac{1-|S_{11}|^2}{|S_{22}(1-|S_{11}|^2)|} > 1 \tag{2}$$

Inequality 2 is the μ test formula for amplifier stability in the *unilateral* case as it is shown in section 2.1.1. Inequalities 1 are the $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ conditions given the unilateral condition.

We interpreted the assignment statement in two different ways in the group. Therefore we are providing proof for both directions of the equivalence.

In section 2.1.1 we prove that assuming inequality 2 then inequalities in line 1 follow.

Then in return in section 2.1.5 we prove the other direction of the equivalence. We prove that assumming inequalities 1 then inequality 2 follows.

2.1.1 Assuming $\mu > 1$, $|S_{11}| < 1$ and $|S_{22}| < 1$ follows

According to the μ test a device is unconditionally stable if

$$\frac{1-|S_{11}|^2}{|S_{22}-S_{11}^*\Delta| + |S_{12}S_{21}|} > 1 \tag{3}$$

Where $\Delta = S_{11}S_{22} - S_{12}S_{21}$. The condition for an unilateral device is $S_{12} = 0$, this means $\Delta = S_{11}S_{22}$. Therefore the μ test equation 3 takes the following form:

$$\frac{1-|S_{11}|^2}{|S_{22}-S_{11}^*S_{11}S_{22}|} > 1 \tag{4}$$

Since $S_{11}^*S_{11} = |S_{11}|^2$ equation 4 takes the form:

$$\frac{1-|S_{11}|^2}{|S_{22}-|S_{11}|^2S_{22}|} > 1 \tag{5}$$

Which then can be rewritten noticing that S_{22} is a common term in the denominator as the following equation:

$$\frac{1-|S_{11}|^2}{|S_{22}(1-|S_{11}|^2)|} > 1 \tag{6}$$

Using the multiplicative associativity of the absolute value, equation 6 is the same as:

$$\frac{1}{|S_{22}|}\frac{1-|S_{11}|^2}{|1-|S_{11}|^2|} > 1 \tag{7}$$

For convenience of referencing we defined the following quantities. Expression A contains the S_{22} parameter and expression B contains the S_{11} parameter only.

$$A = \frac{1}{|S_{22}|} \qquad B = \frac{1 - |S_{11}|^2}{|1 - |S_{11}|^2|} \tag{8}$$

With the newly defined symbols the μ test for the unilateral amplifier takes the following form: $AB > 1$.

Let's take the following cases:

2.1.2 Case 1

$|S_{11}|^2 = 1$ In this case $lim_{|S_{11}|^2 \to 0} B$ doesn't exist with the left limit being 1 and the right limit -1. Therefore the expression can not be evaluated, so $|S_{11}| \neq 1$.

2.1.3 Case 2

$|S_{11}|^2 > 1$ in this case $B < 0$, see 8 and since $|S_{22}| > 0$, $AB < 0$, therefore we are only left with the option of $|S_{11}|^2 < 1$ which is the first condition specified in the exercise.

2.1.4 Case 3

So far we have proven that $\mu > 1$ can only be satisfied if $|S_{11}|^2 < 1$. $|S_{11}|^2 < 1$ means that $B = 1$, therefore equation 7 takes the following form:

$$\frac{1}{|S_{22}|} > 1 \tag{9}$$

Which means that $|S_{22}| < 1$ which is the second condition in the exercise. ∎

2.1.5 Assuming $|S_{11}| < 1$ and $|S_{22}| < 1$, $\mu > 1$ follows

We know:

$$|S_{22}| < 1 \tag{10}$$

We also know that $|S_{11}| < 1$, from this later inequality it follows that $1 - |S_{11}|^2 = |1 - |S_{11}|^2|$. It should be noted that this statement is not true for any other values of $|S_{11}|$, (see section 2.1.4). This equation means that the number 1 can be written in the form of:

$$1 = \frac{1 - |S_{11}|^2}{|1 - |S_{11}|^2|} \tag{11}$$

Then we combine equations 10 and 11.

$$|S_{22}| < \frac{1 - |S_{11}|^2}{|1 - |S_{11}|^2|} \tag{12}$$

Which then by dividing by $|S_{22}|$ leads to the expression of:

$$1 < \frac{1 - |S_{11}|^2}{|S_{22}||1 - |S_{11}|^2|} \tag{13}$$

Which is the μ test formula for the unilateral case, see equation 6. ∎

2.2 Low noise amplifier design method

In the design of amplifiers an important concern is the Noise figure. This can be defined as

$$F = F_{min} + \frac{R_N}{G_S}|Y_S - Y_{OPT}|^2 \tag{14}$$

where Y_{OPT} is a values specified on the amplifier's data sheet: the optimum source admittance, which results in the minimum noise figure F_{min}. To do so, noise is modelled as a resistance, visible in the above equation as R_N. According to RF design fashion, admittances will be expressed as reflection coefficients, as seen many times throughout the module. Hence the above equation can be expressed as

$$F = F_{min} + \frac{4RN}{Z_0}\frac{|\Gamma_s - \Gamma_{OPT}|^2}{(1-|\Gamma_s|^2)|1+\Gamma_{OPT}|^2} \tag{15}$$

Having set Γ_S the now fixed noise figure F can be defined by the *Fixed Noise Figure Parameter* N as

$$N = \frac{F - F_{MIN}}{\frac{4R_N}{Z_0}}|1+\Gamma_{OPT}|^2 \tag{16}$$

This yields a *Noise Circle* in the Γ_S plane on a smith chart, which can be defined with a center C_F of

$$C_F = \frac{\Gamma_{OPT}}{N+1} \tag{17}$$

and a radii R_F of

$$R_F = \frac{\sqrt{N(N+1-|\Gamma_{OPT}|^2)}}{N+1} \tag{18}$$

Once the noise circle has been defined, the intersection point with constant gain circles will determine the length of the stubs to be implemented at source and load end.
In order to achieve gains of magnitude smaller than the maximum available gain, mismatches between source and input load of the amplifier (load and output port) have to be introduced. The inclusion of mismatches is equivalent to introducing new section which reduce the overall gain by a factor of G_S and G_L. This can be seen in the below figure.

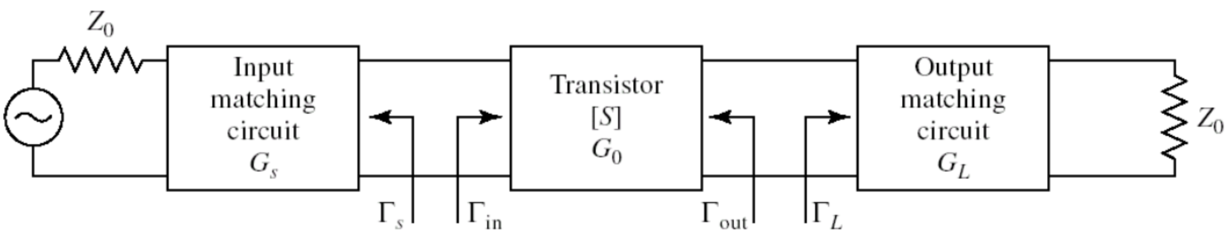


Figure 1: General transistor amplifier circuit.

To simplify calculations this design process is usually carried out after having verified the unilateral assumption holds at least at operating frequency. Later in the report the methodology to evaluate whether the assumption holds is explained in detail. Under this assumption G_S and G_L can be evaluated as follows

$$G_S = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_S|^2} \tag{19}$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \tag{20}$$

Designing input and output sections for the amplifier to deliver maximum gain can be done applying the *Maximum Power Transfer Theorem*. Briefly recapping, this states that maximum power is delivered from a source network to a load when its impedance is equal to the complex conjugate of the

source's internal impedance.
This because:

1. The real parts of the complex impedances would be identical, preventing reflections to occur (hence minimizing power loss)

2. The imaginary parts of the complex impedances would be opposite, hence the series of these two elements would figure as a short circuit from the source network.

It becomes clear that $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$ which allows us to express G_S, max and G_L, max. With these values, normalised gain factors

$$g_s = \frac{G_S}{G_S, max} \tag{21}$$

$$g_l = \frac{G_L}{G_L, max} \tag{22}$$

These parameters (defined for $0 \leq g_s, g_l \leq 1$) allow us to design for a specific gain (smaller than the maximum one, obviously). Fixing values for g_s, g_l results in constant gain circles in the Γ_S and Γ_L planes. Centers and radii can be evaluated as follows

$$C_S = \frac{G_S S11^*}{1-(1-g_s)|S_{11}|^2} \tag{23}$$

$$R_S = \frac{\sqrt{1-g_s}(1-|S_{11}|^2)}{1-(1-g_s)|S_{11}|^2} \tag{24}$$

$$C_L = \frac{G_L S22^*}{1-(1-g_L)|S_{22}|^2} \tag{25}$$

$$R_S = \frac{\sqrt{1-g_L}(1-|S_{22}|^2)}{1-(1-g_L)|S_{22}|^2} \tag{26}$$

Having to evaluate the maximum gain for a set noise level, the above equations show how the constant gain circle should have a radius of the minimum length which allows there to be an intersection point with the noise level circumference. In order to evaluate the values from the constant gain circles, which are going to be chosen for source and load reflection, the closest value to the center will be the one yielding minimum mismatch, hence maximising bandwidth.

Expressing the gain parameters in dB scale the overall gain G_T can be evaluated as

$$G_T = G_S + G_O + G_L \tag{27}$$

2.3 Low noise amplifier calculations

(10 marks) Device C is a GaAs FET, that has the following noise parameters at $6GHz$ ($\mathrm{Z}_0 = 50\Omega$) :$\mathrm{F}_{min} = 2.0dB$, $\Gamma_{opt} = 0.62\angle 100°$, and $R_N = 20\Omega$. Design an amplifier for a noise$figure$ of 2.5 dB and the maximum gain that can be achieved with this noise figure. Use open-circuited shunt stubs in the matchin

$$S_{11} = 0.6\angle-60 = 0.3 - 0.52j, S_{22} = 0.7\angle-60 = 0.35 - 0.60j$$

$$S_{12} = 0, S_{21} = 2.0\angle 81 = 0.31 + 1.98j$$

g First step is to calculate K and Δ in order to check whether the device is unconditionally stable even without the approximation of a unilateral device:

$$|\Delta| = |S_{11}S_{22} - 0| = 0.42 < 1$$

Substituting the values for the S parameter absolute values that can be read off the Euler representation we ind that

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = frac1 - 0.36 - 0.49 + 0.180 =]$$

We have shown that $K > 1$ and $\Delta < 1$ therefore the device is unconditionally stable even without the approximation of a unilateral device. The next step is to compute the unilateral figure of merit U:

$$U = \frac{|S_{11}||S_{21}||S_{12}||S_{22}|}{(1 - |S_{11}|^2)(1 - |S_{22}|^2)}$$

For the case of the unilateral transistor, $|S12|$ is zero therefore we do not expect an error in the transistor gain, the ratio G_T/G_{TU} is bounded by 1s as shown by:

$$\frac{1}{1 + U^2} < \frac{G_T}{G_{TU}} < \frac{1}{1 - U^2}$$

The next step is to find the centre and the radius of the 2.5 dB noise figure circle. To do this we use equations in Pozar (12.57) and (12.58) to find the noise figure given the minimum noise figure F_{min}, and (12.60) to find the centre and radius of the circle. Since the device is unilateral, $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$ as shown in (12.38).

$$F = F_{min} + \frac{4RN}{Z_0}\frac{|\Gamma_s - \Gamma_{OPT}|^2}{(1 - |\Gamma_s|^2)|1 + \Gamma_{OPT}|^2}$$

Substituting the values we find that:

$$F = 2.0 + 1.6\left(\frac{0.59}{1.01}\right) = 2.93dB$$

$$N = \frac{F - F_{MIN}}{\frac{4R_N}{Z_0}}|1 + \Gamma_{OPT}|^2$$

Substituting the values into the formula above:

$$N = \frac{0.93}{1.6}|0.89 + 0.61j| = 0.68$$

$$C_F = \frac{\Gamma_{opt}}{N + 1} = 0.37\angle 100^\circ$$

$$R_F = \frac{\sqrt{N(N + 1 - \Gamma_{opt}^2}}{N + 1} = 0.56$$

$$G_S = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_S|^2} = 1.56dB \quad (28)$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} = 1.94dB \quad (29)$$

$$G_{TU} = G_S + G_0 + G_L = 9.54dB$$

$$G_{S,max} = 1.56dB \quad G_{L,max} = 1.92dB$$

The next step is to find the the normalized gain factors g_s and g_l. We need to find the input matching circuit G_S and output matching circuit G_L and the maximized values of both parameters, which occurs when $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$.

$$g_s = \frac{G_S}{G_S, max} = 0.67 \tag{30}$$

$$g_l = \frac{G_L}{G_L, max} = 0.52 \tag{31}$$

The data for the gain circles are now calculated. For the output section, we have $\Gamma_L = S_{22}^*$ for a maximum G_L of:

$$G_L = 1.92 = 2.84dB \quad G_0 = |S_{21}| = 4.02 = 6.04dB$$

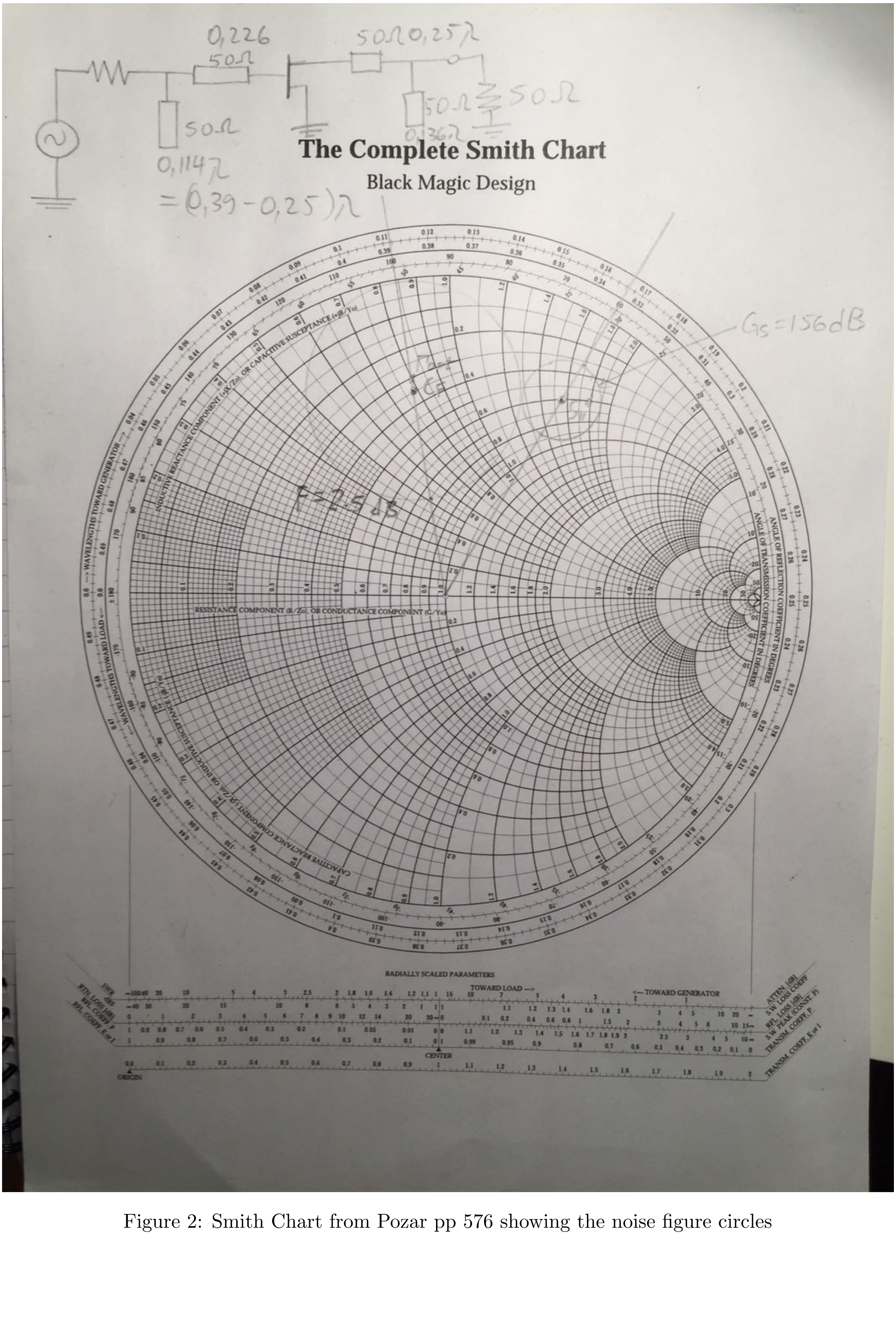


Figure 2: Smith Chart from Pozar pp 576 showing the noise figure circles

2.4 μ parameter testing

Looking at the parameter values in the assignment brief we can see that all devices are unilateral, therefore we can use the simplified formula for the μ test with $S_{12} = 0$ and $\Delta = S_{11}S_{22}$. For the derivation see section 2.1.1.

$$\frac{1 - |S_{11}|^2}{|S_{22} - |S_{11}|^2 S_{22}|} > 1$$

Therefore we only need the S_{11} and S_{22} parameters. Because there are only 3 values to compute these can be calculated using Wolfram Alpha[1] with the following expression:

```
(1-abs(s11)^2)/abs(s22*(1-abs(s11)^2)
```

An example calculation for device A would look like the following

```
(1-abs(0.34*exp(i*(-170)))^2)
/abs(0.45*exp(i*(-25))*(1-abs(0.34*exp(i*(-170)))^2)
```

A link to the calculation of parameter A can be found here[2]. The results are summarized in the following table.

Device	μ
A	2.22
B	-1.43
C	1.43

This means that Device A and C are stable, with A being more stable than C. Device B is not unconditionally stable.

2.5 Amplifier calculations

3 Part B: Simulation: Amplifier design

In this section we interpret the factory given S parameters of the amplifier using numerical methods. In two section we are going to use Python and Matplotlib, since we are reusing the script that was developed for Lab 2, then we perform a μ test, and then to show a different method with a MATLAB script developed to determine the stability in second way and available gains.

The full python script is in section 8.

Marco carried out the simulation for the HMC636ST89 amplifier using MATLAB. In order to do so the Touchstone file of the device is read into MATLAB in the following way

Listing 1: Matlab code to read data from the $HMC636ST89.s2p$ Touchstone file of the amplifier.

```
% read data
fid = fopen('HMC636ST89.s2p','r');
data = textscan(fid,'%f %f %f %f %f %f %f %f %f','HeaderLines',36);
data = cell2mat(data);
f = data(:,1);
% S matrix
S = [data(:,2), data(:,3), data(:,4), data(:,5), ...
        data(:,6), data(:,7), data(:,8), data(:,9)];
[X,Y] = pol2cart(degtorad(S(:,2)),S(:,1));
% S param vectors
S11 = complex(X,Y);
```

[1] www.wolframalpha.com
[2] https://tr.im/1X0fM

```
[X,Y] = pol2cart(degtorad(S(:,4)),S(:,3));
S21 = complex(X,Y);
[X,Y] = pol2cart(degtorad(S(:,6)),S(:,5));
S12 = complex(X,Y);
[X,Y] = pol2cart(degtorad(S(:,8)),S(:,7));
S22 = complex(X,Y);
```

Achieving column vectors for each S-parameter covering the whole frequency range (stored in a different column vector).

Stability To verify the stability of the device over the whole frequency bandwidth, the $K - \Delta$ test is performed applying the following equations. The condition over K is known as *Rollet's condition*, and Δ is the determinant of the S-Matrix.

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1 \tag{32}$$

$$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| < 1 \tag{33}$$

This is performed in MATLAB using the following code.

Listing 2: Matlab code to perform $K - \Delta$ stability test.

```
%%
% determinant
D = S11.*S22-S21.*S12;
% rollet condition
K = (1-abs(S11).^2-abs(S22).^2+abs(D).^2)./(2*abs(S12.*S21));
i=1:length(S11);
for i=1:length(S)
    if(K(i)>1 && abs(D(i))<1) %K-test
        stab(i)=1;
    else
        stab(i)=0;
    end
end
```

A stem graph is plotted: on the y-axis values 1 or 0 indicate whether the $K - \Delta$ test succeeded or failed, respectively.

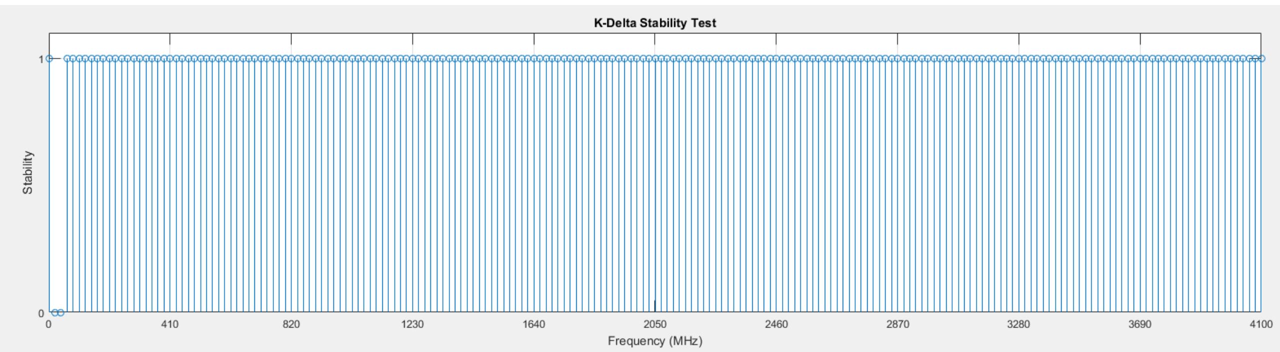


Figure 3: Results of the $K - \Delta$ test over the whole frequency bandwidth.

It's visible the device is unconditionally stable for all frequencies except $f_1 = 20.6MHz$ and $f_2 = 41.1MHz$.

4 μ parameter test of stability

The HMC636ST89.s2p vile was converted to a .csv to use it with the already written python script for Lab 2 by Mihaly. Please find it in section 8. Then a function for the mu parameter test was defined on the S matrix and the results plotted with matplotlib.

```
def mu_test_stability(S):
        #mu test
        delta = S.flat[0]*S.flat[3]-S.flat[2]*S.flat[1]
        mu = (1-abs(S.flat[0])**2)/(abs(S.flat[3]-delta*S.flat[0].conjugate())\
        +abs(S.flat[2]*S.flat[1]))
        return mu
```

We can use the μ test to compare the stability of the device over the whole frequency range, too. This test also confirmed the result that the device is unconditionally stable over the whole frequency range, except for two frequencies: 20 and 40 MHz. The device is most stable above 2 GHz with reaching a peak of stability around 2.6 GHz.

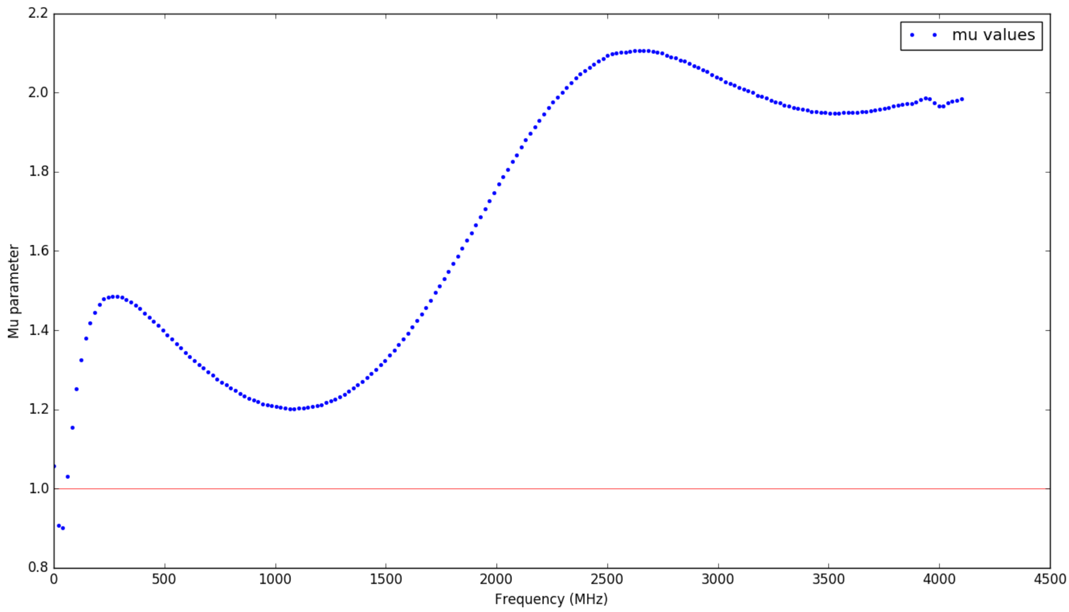


Figure 4: The μ parameter test of the amplifier

4.1 Unilateral assumption

To calculate the unilateral figure of merit over the whole frequency range, the transducer gain should be calculated two ways: once without $\left(|\Gamma_{in}| = \left|S_{11}\frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}\right|\right)$ and once with the unilateral assumption ($|\Gamma_{in}| = S_{11}$). Then we should evaluate whether $\frac{1}{1+U^2} < \frac{G_T}{G_{TU}} < \frac{1}{1-U^2}$ holds or not, where $U = \frac{|S_{11}||S_{21}||S_{12}||S_{22}|}{(1-)}$

We are assuming the source and loads are matched as provided by the datasheet, therefore $|\Gamma_S| = S_{11}^*$, $|\Gamma_L| = S_{22}^*$.

The following function calculated the relevant parameters, then it was evaluated in a branch statement within the main data processing loop.

```
def unilateral(S):
        s11u = S.flat[0]
        s21u = S.flat[1]
        s12u = S.flat[2]
        s22u = S.flat[3]

        #assuming matched conditions
        gamma_l = s22u.conjugate()
        gamma_s = s11u.conjugate()
        gamma_in = s11u+s12u*s21u*gamma_l/(1-s22u*gamma_l)

        #calculating the gains
        g0 = abs(s21u)
        gl = (1-abs(gamma_l)**2)/abs(1-s22u*gamma_l)**2
        gs = (1-abs(gamma_s)**2)/abs(1-gamma_in*gamma_s)**2
        gsu = (1-abs(gamma_s)**2)/abs(1-s11u*gamma_s)**2

        #because of matched conditions gl and g0 is the same
        #so they cancel
        err = gs/gsu
        U = (abs(s11u)*abs(s21u)*abs(s12u)*abs(s22u))/\
                ((1-abs(s11u))**2*(1-abs(s22u))**2)

        return [err, 1/(1+U)**2, 1/(1-U)**2, U]
```

The fraction of the output of this script can be found below:

Unilateral doesn't hold for: 0.1 MHz
Unilateral holds for: 20.5995 MHz
...
Unilateral holds for: 3915.5 MHz
Unilateral holds for: 3936 MHz
Unilateral holds for: 3956.5 MHz
Unilateral holds for: 3977 MHz
Unilateral holds for: 3997.5 MHz
Unilateral holds for: 4018 MHz
Unilateral holds for: 4038.5 MHz
Unilateral holds for: 4059 MHz
Unilateral holds for: 4079.5 MHz
Unilateral holds for: 4100 MHz

Therefore we can conclude that the unilateral condition holds for all reasonable frequencies of operation. This can be seen as well utilising the following MATLAB script.

To evaluate whether the unilateral assumption holds over the whole frequency bandwidth,holds, the *unilateral figure of merit*

$$U = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)} \tag{34}$$

should have a decibel (dB) value

$$U_{dB} < 0.5dB \tag{35}$$

This is performed in MATLAB using the following code.

Listing 3: Matlab code to verify unilateral assumption.

```
% Unilateral figure of merit
U = (abs(S12).*abs(S21).*abs(S11).*abs(S22))./((1-(abs(S11)).^2).*(1-abs(S22)).^2);
U_db = 20.*log10(U);
for i=1:length(S)
    if(U_db(i)<0.5) %condition (dB)
        uni(i)=1;
    else
        uni(i)=0;
    end
end
```

Again, a stem graph with 1 and 0 a only possible values for the y axis indicates whether the the unilateral assumption holds for a particular frequency or not.

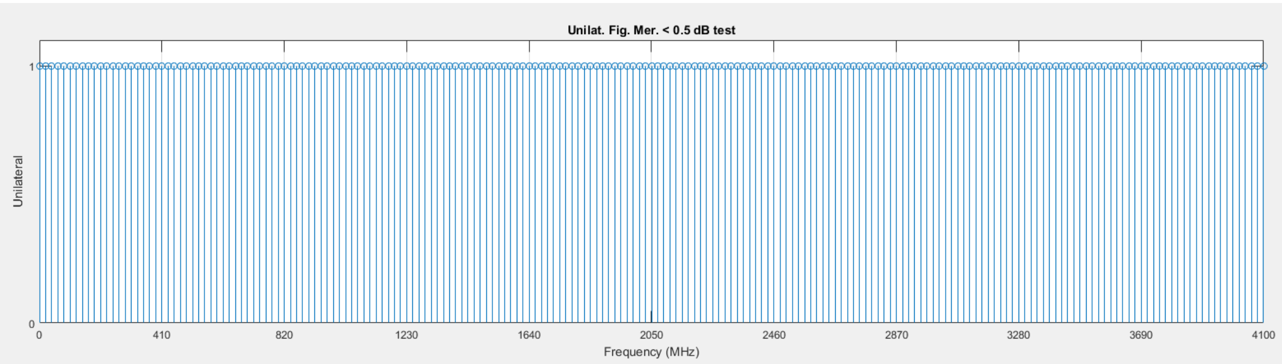


Figure 5: Results of the $K-\Delta$ test over the whole frequency bandwidth.

It can be seen that the assumption holds throughout the whole frequency bandwidth.

4.2 Available gain

To evaluate the maximum available gain, over the whole frequency bandwidth, $G_{T,max}$ is evaluated.

$$G_{Tmax} = \frac{1}{1-|\Gamma_s|^2}|s_{21}|^2\frac{1-\Gamma_l|^2}{|1-S_{22}\Gamma_L|^2} \tag{36}$$

Despite having seen the device can be considered unilateral, for the sake of completeness and precision, the maximum gain available for both *bilateral* and unilateral assumptions will be evaluated.
Bilateral: a simultaneous match match between $\Gamma_{in} = \Gamma_S^*$ and $\Gamma_{out} = \Gamma_L^*$ is required, as these two quantities affect one the other.
We can see

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \tag{37}$$

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} \tag{38}$$

where

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \tag{39}$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \tag{40}$$

$$C_1 = S_{11} - \Delta S_{22}^* \tag{41}$$

$$C_2 = S_{22} - \Delta S_{11}^* \tag{42}$$

Given the $\pm$ sign present in 37 and 38 - once again, for sake of completeness - maximum gain for both scenarios $G_{t,max,p}$ and $G_{t,max,n}$ are evaluated in MATLAB using the following code.

Listing 4: Matlab code to evaluate the maximum available gain.

```
%%
% source/load reflection coefficients
b1_=1+abs(S11).^2-abs(S22).^2-abs(D).^2;
b2_=1+abs(S22).^2-abs(S11).^2-abs(D).^2;
c1_=S11-D.*conj(S22);
c2_=S22-D.*conj(S11);
r_s_p = (b1_+sqrt(b1_.^2-4.*abs(c1_).^2))./(2.*c1_);
r_l_p = (b2_+sqrt(b2_.^2-4.*abs(c2_).^2))./(2.*c2_);
r_s_n = (b1_-sqrt(b1_.^2-4.*abs(c1_).^2))./(2.*c1_);
r_l_n = (b2_-sqrt(b2_.^2-4.*abs(c2_).^2))./(2.*c2_);
% maximum gain
G_t_max_p = abs(S21).^2./(1-abs(r_s_p).^2) .* ...
            (1-abs(r_l_p).^2)./(abs(1-S22.*r_l_p).^2);
G_max_p_db = 20*log10(G_t_max_p);
G_t_max_n = abs(S21).^2./(1-abs(r_s_n).^2) .* ...
            (1-abs(r_l_n).^2)./(abs(1-S22.*r_l_n).^2);
G_max_n_db = 20*log10(G_t_max_n);
```

If instead the device was to be considered unilateral over the whole bandwidth, the source and load reflection coefficients would be

$$\Gamma_S = S_{11}^* \tag{43}$$

$$\Gamma_L = S_{22}^* \tag{44}$$

And the maximum availble gain could be evaluated using the following

$$G_{TU,max} = \frac{1}{1 - |S_{11}|^2} |S_{21}|^2 \frac{1}{1 - |S_{22}|^2} \tag{45}$$

The MATLAB implementation of such equations is shown below

Listing 5: Matlab code to evaluate the maximum available gain considering the device unilateral over tho whole bandwidth.

```
G_max_u = (abs(S21).^2)./(1-abs(S11).^2) .* ...
       1/(1-abs(S22).^2);
G_max_u_db = 20*log10(G_max_u);
```

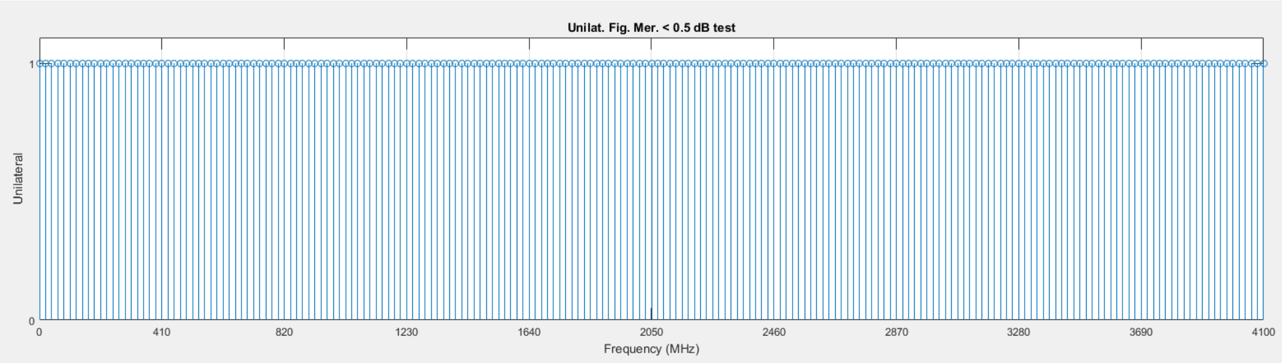


Figure 6: Results of the $K - \Delta$ test over the whole frequency bandwidth.

4.2.1 Python simulation for available gain G_A

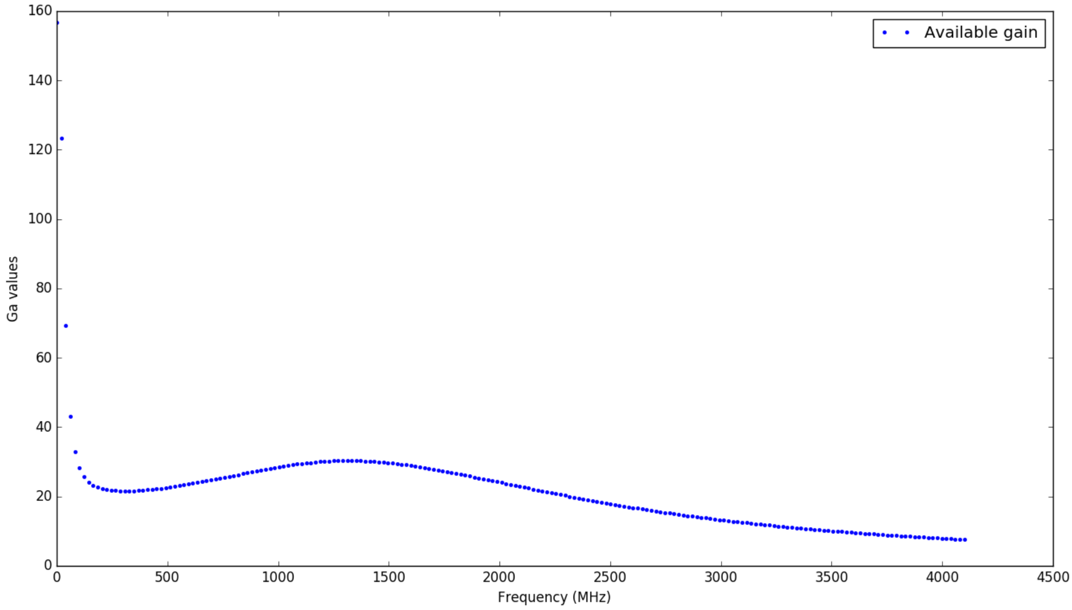
For the sake of completeness we calculated an plotted the available gain G_A.

```
def available_gain(S):
        s11u = S.flat[0]
        s21u = S.flat[1]
        s12u = S.flat[2]
        s22u = S.flat[3]

        #assuming matched conditions
        gamma_l = s22u.conjugate()
        gamma_s = s11u.conjugate()
        gamma_in = s11u+s12u*s21u*gamma_l/(1-s22u*gamma_l)
        gamma_out = s22u+s12u*s21u*gamma_s/(1-s11u*gamma_s)

        ga = (1-abs(gamma_s)**2)/abs(1-s11u*gamma_s)**2*abs(s21u)**2*1/\
                (1-abs(gamma_out)**2)

        return ga
```



Available gain
Ga values
Frequency (MHz)
0
20
40
60
80
100
120
140
160
500
1000
1500
2000
2500
3000
3500
4000
4500

Figure 7: G_A

4.3 Impedance matching

The datasheet of the amplifier indicated that the amplifier is matched for both the input and output for a 50Ω load or source impedance.

5 Part C: Measurements: Amplifier Design Boards

Once the design had been finalised and fabricated, we soldered the components on the board. The two port components were of the size of 0402, therefore we used a microscope to solder these parts.

For the 0402 SMD parts we used a reflow method with a soldering iron and solder paste. We put the paste on the pads, then placed the component on the board and heated up the pads at the same time allowing the solder to cover both terminals of the component. Then the since the molten solder will take the form of the minimum surface area for a given volume, thereby minimising the surface energy component, the two port SMD component will move to its correct place using this phenomenon. If the soldering is done with the excess amount of solder, the same physics explained above results in the forming of little solder balls, since the sphere is the geometrical figure with the minimum surface area for a given volume.

Once the small components were soldered, we soldered the amplifier, the SMA connectors and wires.

Then we connected the device to the VNA and measured all S parameters. The amplifier met the expectations of the bandwidth, less so for the gain.

6 Group report

6.1 Roles

Apart from the roles shared by all group members,

- Imran: stability calculations for the amplifier and introduction.
- Marco: impedance matching and modelling of the amplifier.
- Klajd: calculation of the impedances of the coupled lines filter.
- Mihaly: realizing and optimizing the filter using microstrip lines and editing, parts of conclusion.

6.2 Introduction - Imran

For this group project we have decided to get together and design a coupled lined filter operating at S band. We chose this one specifically over other designs because we thought it was the most challenging and it is used widely in broadband and telecommunications. The technique is based on the use of coupled lined filters and quarter wave transformer for variation and stabilisation of the amplifier. We have chosen a centre frequency of 2GHz, and a bandwidth of 10%. This report includes the stability of the amplifier, the conjugate matching of the filter, coupled lined filter design and tuning and optimisation of the filter.

6.3 Amplifier Stability - Imran

From the transistor amplifier circuit we've seen so many times in this module and is inserted into the report under methodology, we know that oscillation is likely to happen if the input or output port impedance has negative real part. From our measurements of our amplifier we can see that the real part is negative, this shows that $|\Gamma_{in}| > 1$ and $|\Gamma_{out}| > 1$. We know that Γ_{in} and Γ_{out} varies according to the source and load matching network. Leading to two types of stability:

1. Unconditional stability $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$
2. Conditional stability $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$

We have to keep in mind that stability condition of an amplifier is frequency dependent. Our amplifier was designed at 2GHz, at this frequency the amplifier is stable, this may change once the frequency is altered.

$$|\Gamma_{in}| = \left| S_{11} \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1 \tag{46}$$

$$|\Gamma_{out}| = \left| S_{22} \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \right| < 1 \tag{47}$$

Since the device is unilateral $S_{12} = 0$ which only leaves us with

$$|S_{11}| < 1 \quad |S_{22}| < 1 \tag{48}$$

From the calculations we did with the parameters obtained using the VNA we find that the device is unconditionally stable and this is also evident from the graphs drawn by the mu parameter test of the amplifier. The problem with our measurements of the device is that we couldn't measure the device without oscillations with respect to S_{22}. See figure 8.

Therefore, the stability criteria could not be used appropriately. Also as mentioned above we know that the following discussion of stability is limited to 2 ports amplifier circuits, this is shown where the scattering parameters of the active device can be measured without oscillations.

Amplifier are designed to control the gain. During maximum gain the conjugate matching $\Gamma_{in} = \Gamma_S^*$ and $\Gamma_{out} = \Gamma_L^*$. For specified gain we need input and output constant gain circles, which can be found from the input and output impedance matches. For low noise amplifier design, there will be constant noise circles. Transducer Gain happens when source and load are conjugately matched to the transistor.

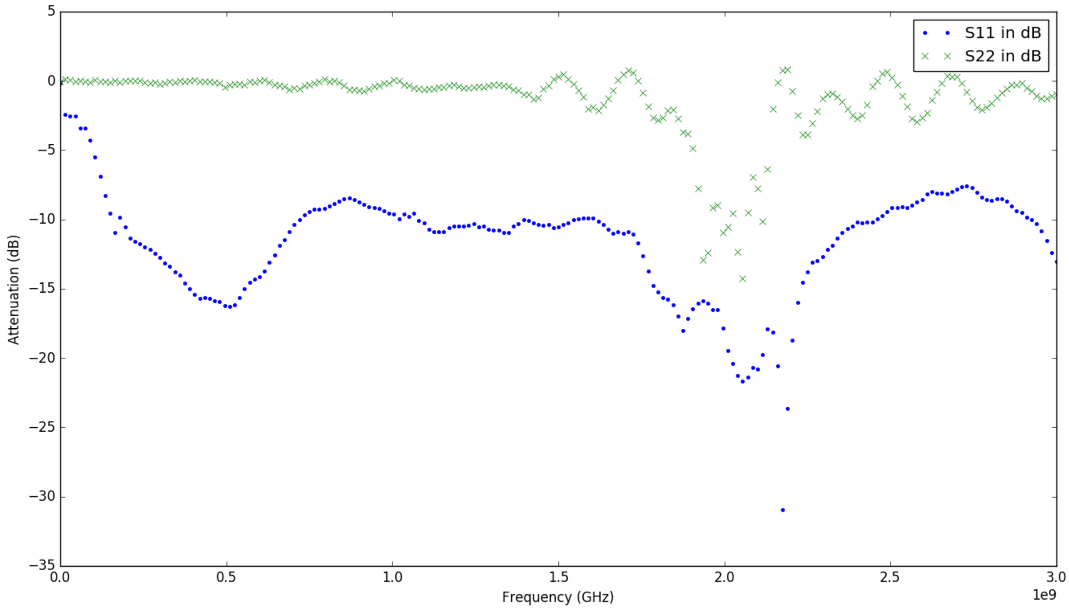


Figure 8: Measured S_{11} and S_{22} parameters.

6.4 Impedance matching - Marco

The importance of impedance matching in RF circuits has been discussed several times throughout this module. Reflections, due to the mismatches, cause power loss. This is why the, in the general schematic of a transistor, a gain G_S and G_L can be associated with the two matching sections.

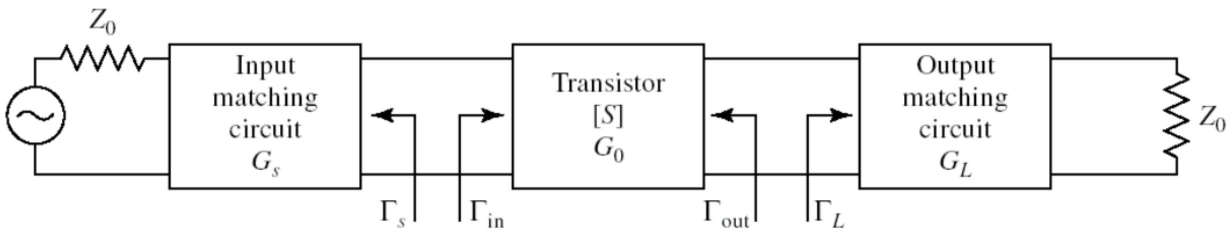


Figure 9: General transistor amplifier circuit.

Having verified the unilateral assumption holds for all the frequency bandwidth, therefore also at the operating frequency, no load matching circuit is expected to be required for the amplified signal to be delivered at the load effectively.
The transistor's gain G_0 is fixed, therefore the design of source and load matching sections determines the overall operation of the amplifier. Having decided as a group that we would have implemented a design for maximum gain, conjugate matching was required. Recalling what previously mentioned: Designing input and output sections for the amplifier to deliver maximum gain can be done applying the *Maximum Power Transfer Theorem.* Briefly recapping, this states that maximum power is delivered from a source network to a load when its impedance is equal to the complex conjugate of the source's internal impedance.
This because:

1. The real parts of the complex impedances would be identical, preventing reflections to occur (hence minimizing power loss)

2. The imaginary parts of the complex impedances would be opposite, hence the series of these two elements would figure as a short circuit from the source network.

For sake of completeness, calculations for the *bilateral* case will be carried out. It will be then shown that the length of the stubs to be implemented at loads end, would be so small to make their presence negligible.
To evaluate values for Γ_S and Γ_L equations 36, 37, 38 will be used. These result being

$$\Gamma_S = 0.29\underline{/173.4^\circ} \tag{49}$$

$$\Gamma_L = 0.19\underline{/-96.4^\circ} \tag{50}$$

Plotting the complex values on the Smith Chart, performing rotations for the reflection coefficient to land on the 1+j circumference first, the center of the Smith chart secondly, it's possible to evaluate length of the stubs utilising single stub matching technique.

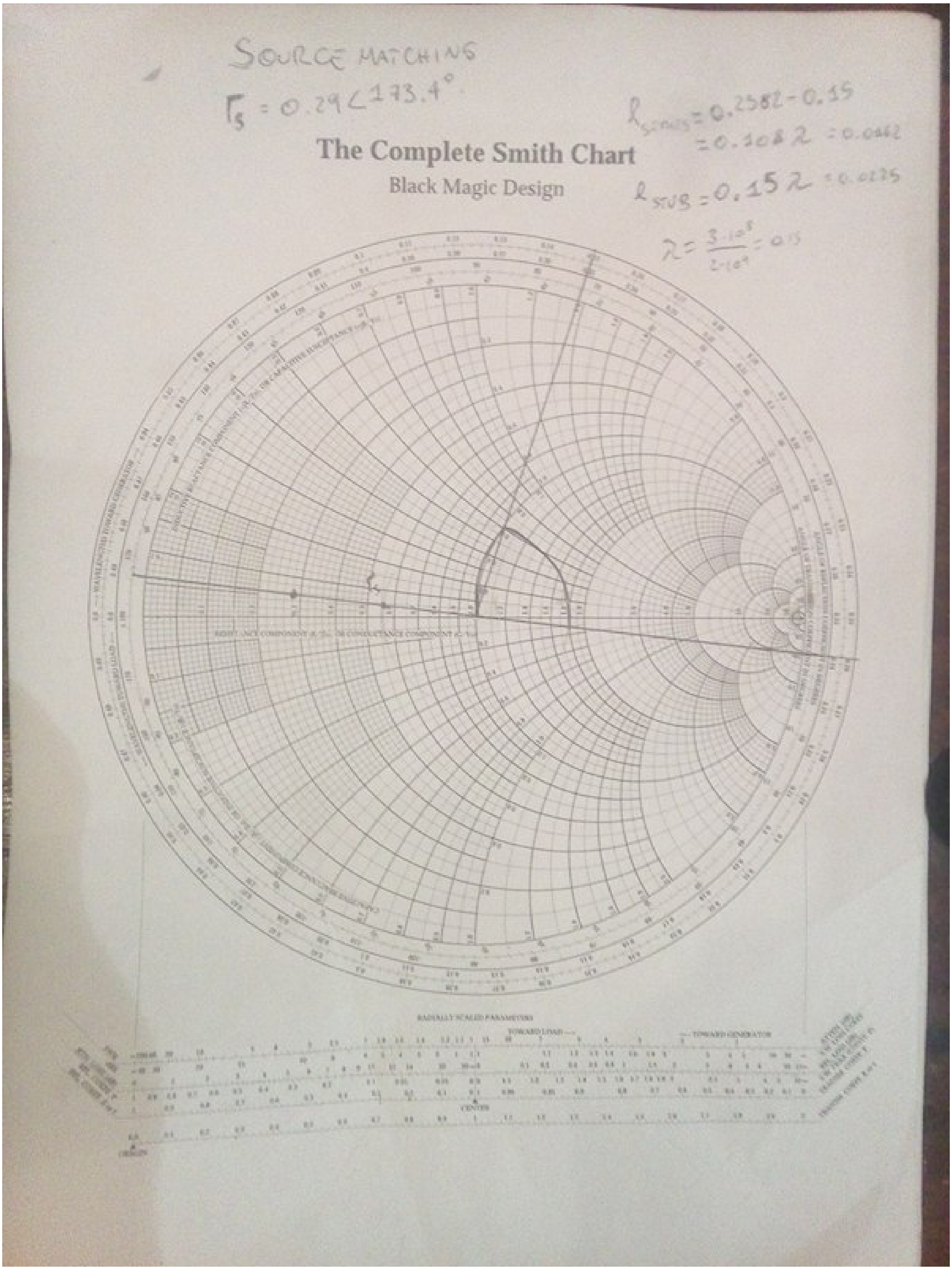


Figure 10: Source Impedance matching smith chart.

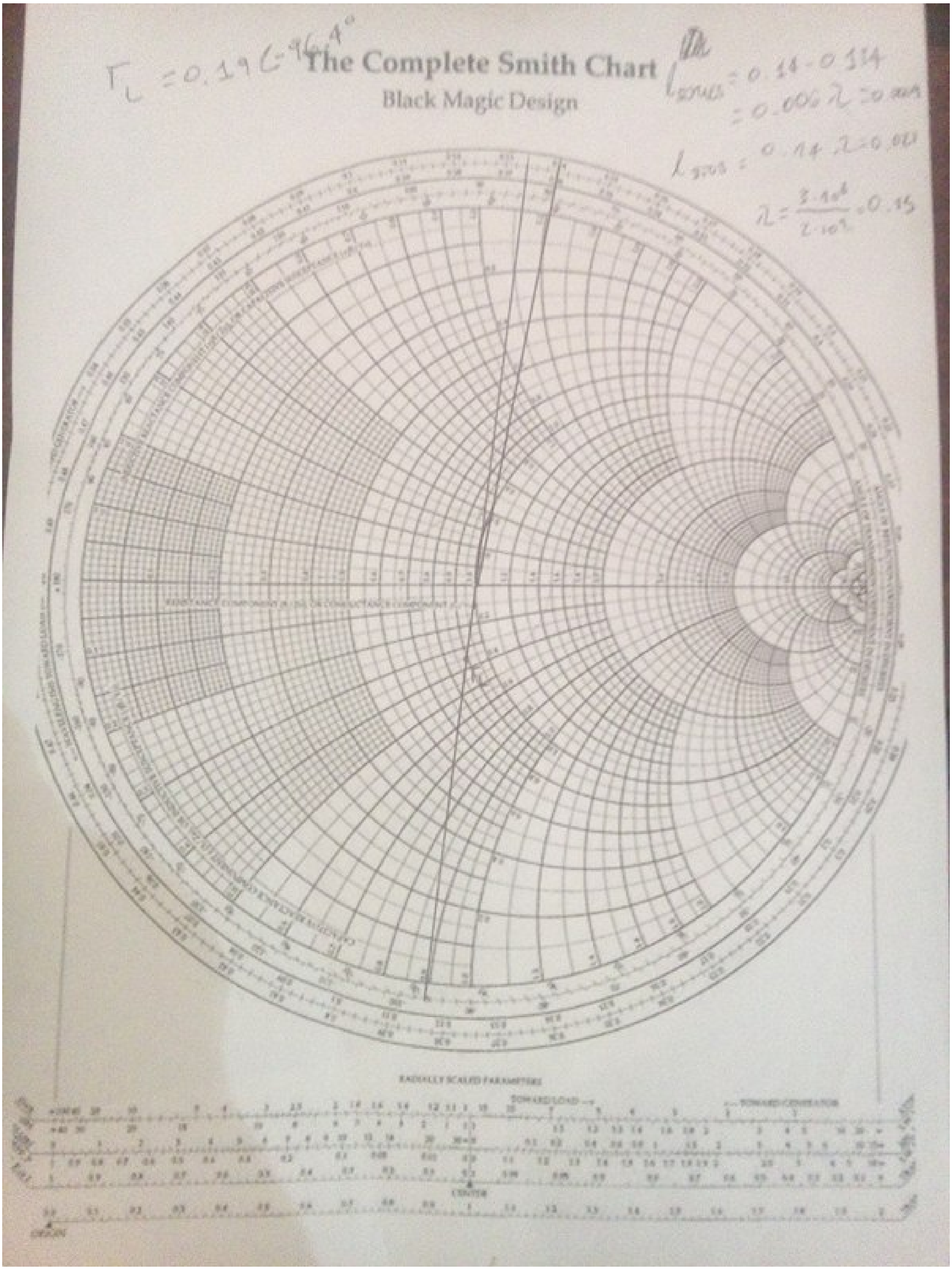


Figure 11: Load Impedance matching smith chart.

Carrying out calculations to evaluate the length of the stubs (with $\lambda = \frac{c}{f} = 0.15$ for f = 2GHz) it can be seen that for the source matching network

$$l_{series} = 0.2582 - 0.15 = 0.108\lambda = 0.0162m \tag{51}$$

$$l_{stub} = 0.15\lambda = 0.0225m \tag{52}$$

For the Load matching network instead

$$l_{series} = 0.14 - 0.134 = 0.006\lambda = 0.0009mequation \tag{53}$$

$$l_{stub} = 0.14\lambda = 0.021m \tag{54}$$

It can be seen from the length of the series stub and more easily form its smith chart that the rotation on the load reflection coefficient are so small that the relative stubs can be omitted from the design. For maximum gain to be delivered by the amplifier, source matching though should be implemented. At the time of the design, despite having performed these calculations, not having as good understanding of it as now, led me to take what was said in the data sheet for granted. In fact, it can be seen in the below figure that "no matching component" is said to be required.

HMC636ST89 / 636ST89E

GaAs PHEMT HIGH LINEARITY Gain Block, 0.2 - 4.0 GHz

Features

Low Noise Figure: 2.2 dB

High P1dB Output Power: +22 dBm

High Output IP3: +40 dBm

Gain: 13 dB

50 Ohm I/O's - No External Matching

Industry Standard SOT89 Package

General Description

The HMC636ST89(E) is a GaAs pHEMT, High Linearity, Low Noise, Wideband Gain Block Amplifier covering 0.2 to 4.0 GHz. Packaged in an industry standard SOT89, the amplifier can be used as either a cascadable 50 Ohm gain stage, a PA Pre-Driver, a Low Noise Amplifier, or a Gain Block with up to +23 dBm output power. This versatile Gain Block Amplifier is powered from a single +5V supply and requires no external matching components The internally matched topology makes this amplifier compatible with virtually any PCB material or thickness.

6.5 Coupled lines filter calculations - Klajd

We chose to use the coupled line bandpass filter for some reasons: firstly when compared to other filters, the coupled line filter produces higher power rating. Secondly, it permits a more desired and broader bandwidth then other filters and finally, this was our group choice even because it was a challenging filter to work with.

In this section we have included the procedure and all calculations needed to be processed in order to change a low pass lumped elements filter into a band pass filter. Additionally, the admittance inverter constant and even and odd mode characteristic impedances have been calculated in order to build the coupled line bandpass filter.

Data: Centre frequency: 2GHz, Type of filter response: 0.5dB equal ripple response, Bandwidth: 10% Impedance Z0: 50 Ohms N = 5

Given the filter is 0.5 dB Equal Ripple Filter and N = 5, by looking at the Equal Ripple (Chebyshev) Filter Table (0.5 dB)

n	1	2	3	4	5	6
g	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000

Table 1: g values

The second step is to design a prototype low pass filter. We chose to build a T network filter with the following values beginning with a shunt inductor as shown in the figure below

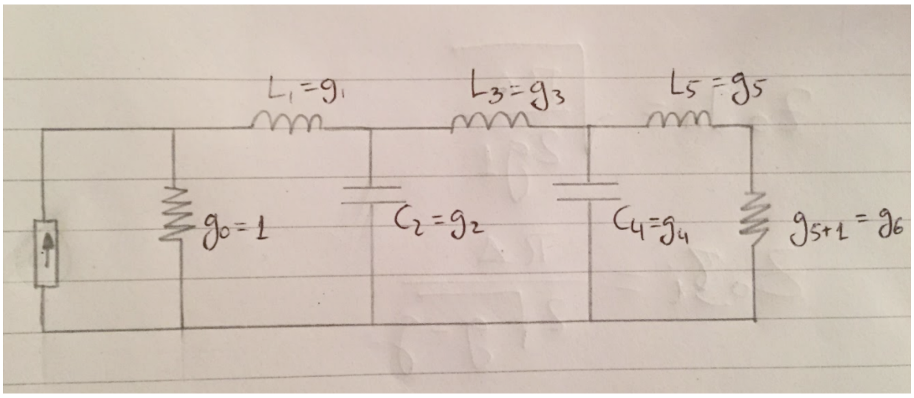


Figure 12: Low pass filter prototype

The next step is the rescaling of the lumped elements with a 50 Ohm resistance following the formulae:

$$C_n = \frac{g_n}{2\pi f_c R_s} \quad L_n = \frac{g_n R_s}{2\pi f_c}$$

As the g values indicate, inductors 1 and 5 have the same values, and capacitors 2 and 4 have the same values also.

n	1	2	3	4	5	6
$C(pF)$		1.96		1.96		
$L(nH)$	6.79		10.11		6.79	

Table 2

Transforming the filter into a Bandpass filter: The ω_1 or ω_{lower} and ω_2 or ω_{lower} determine the passband bandwidth. The centre frequency of the filter is denoted with ω_0. Having ω_1, ω_2 which can be found by using the bandwidth 10%, and ω_0, then a bandpass response is found following:

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0} = 0.1$$

Using the transformation table, the series inductor Lk is transformed into a series LC circuit with element values as below

$$L_k' = \frac{L_k R_0}{\Delta \omega_0}$$

$$C_k' = \frac{\Delta}{L_k \omega_0 R_0}$$

Likewise, the shunt capacitor, Ck, is converted to a shunt LC circuit with the following element values:

$$L_k' = \frac{\Delta R_0}{C_k \omega_0}$$

$$C_k' = \frac{C_k}{\Delta \omega_0 R_0}$$

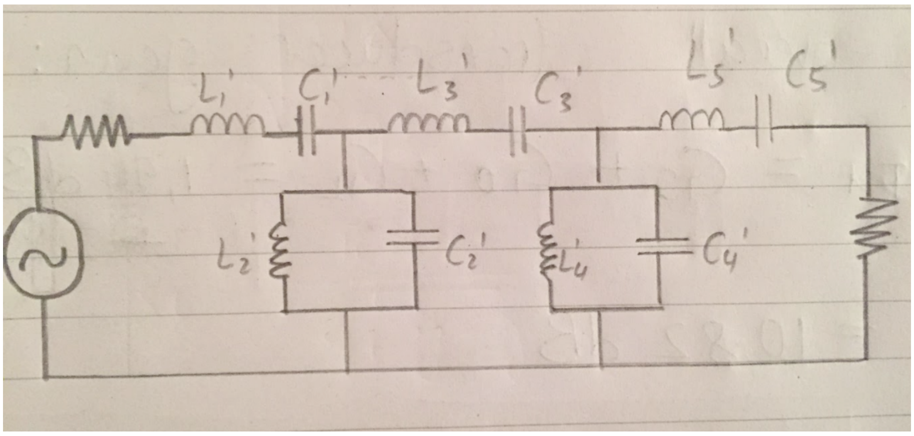


Figure 13: Band pass filter prototype

n	1	2	3	4	5	6
$L(nH)$	67.87	0.32	101.1	0.32	67.89	
$C(pF)$	0.093	19.57	0.063	19.57	0.093	

Table 3: Bandpass prototype values

Next we need to compute the admittance inverter constants. Next we need to compute the admittance inverter constants. The design equations for a bandpass filter with N+1 coupled line sections are:

$$Z_0 J_1 = \sqrt{\frac{\pi\Delta}{2g_1}}$$

$$Z_0 J_n = \frac{\pi\Delta}{2\sqrt{g_{n-1}g_n}} \quad (n = 2, 3, ..., N)$$

$$Z_0 J_{n+1} = \sqrt{\frac{\pi\Delta}{2g_n g_{N+1}}}$$

The even and odd mode characteristic impedance for each section of the coupled line bandpass filter can be found using the following formulae:

$$Z_{0e} = Z_0[1 + JZ_0 + (JZ_0)^2]$$

$$Z_{0o} = Z_0[1 - JZ_0 + (JZ_0)^2]$$

$$Z_0 J_1 = \sqrt{\frac{\pi 0.1}{2 \cdot 1.7058}}$$

$$Z_0 J_2 = \frac{\pi 0.1}{2\sqrt{1.7058 \cdot 1.2296}} = 0.1085$$

$$...$$

$$Z_0 J_{n+1} = \sqrt{\frac{\pi 0.1}{2 \cdot 1.7058 \cdot 1.0000}} = 0.3035$$

$$Z_{0e,1} = 50[1 + 0.3035 + (0.3035)^2] = 69.78\Omega$$

$$Z_{0o,1} = 50[1 - 0.3035 + (0.3035)^2] = 39.43\Omega$$

The final results can be seen in the table below:

n	1	2	3	4	5	6
Z_{0e}	69.38	55.64	54.87	54.87	55.64	69.38
Z_{0o}	39.82	44.64	45.81	45.81	44.64	39.82
$Z_0 J_n$	0.3035	0.1085	0.0889	0.00889	0.1085	0.3035
$J_0(\cdot 10^{-3})$	6.069	2.169	1.777	1.777	2.169	6.069

Table 4: ADmittance inverter constants and even and odd mode line impedances.

6.6 Realizing the filter using microstrips and optimization - Mihaly

The filter was realized using microstrip lines. The challenges of designing such structure can be summarised by the following points.

1. There are multiple theoretical models for a coupled lines filter realized with microstrips. Different models lead to designs having different characteristics.
2. The required even and odd mode line impedances calculated for a given microstrip line do not correspond to exactly to a given physical structure, only approximately.
3. Some of the line lengths or spacings having desirable characteristics such as low insertion loss or narrower bandwidth sometimes require hardly realizable line gaps, widths or lengths.
4. Even if the physical parameters of the line sections are realizable, sometimes the lines overlap around the connection points, therefore the connection is not realizable.

6.6.1 TXLine coupled lines

The model used for the coupled lines filter was the model used by the TX line calculator given in the AWR Microwave suite. Since there are multiple physical parameters to achieve a given characteristic impedance either in the odd or the even mode, the physical parameters have to be entered into the tool first then to find a given even or odd mode characteristic impedance. The values found for the calculated parameters are in table 5.

The global parameters used for the microstrips were the following. $\epsilon_r = 4.3, f_0 = 2GHz, tan\delta = 0.02, T = 0.015mm, H = 1.6mm$

The meaning of the symbols can be found in figure 14. The parameter definitions are the following: Z_{0e} is the even mode characteristic impedance, Z_{0o} is the odd mode characteristic impedance, Θ is the electrical length of the transmission line sections $\lambda/4$, L_{0e} is the length of the section in even mode, L_{0o} is the length of the section in the odd mode, W_{0e} is the width of the section in the even mode, W_{0o} is the width of the section in the odd mode, S_{0e} is the gap between the sections in the even mode and S_{0o} is the gap between the sections in the odd mode. The quantities without the index were calculated as the mean of the even and odd quantities, and the quantities with the opt index are the quantities after optimization.

n	1	2	3	4	5	6
Z_{0e}	69.38	55.64	54.87	54.87	55.64	69.38
Θ	89.6	89.7	89.9	89.9	89.7	89.6
L_{0e}	20.0	20.0	20.1	20.1	20.0	20.0
W_{0e}	2.4	2.8	2.8	2.8	2.8	2.4
S_{0e}	0.5	2.6	3.0	3.0	2.6	0.5
Z_{0o}	39.82	44.64	45.81	45.81	44.64	39.82
Θ	90.6	89.4	90.2	90.2	89.4	90.6
L_{0o}	22.0	21.3	21.4	21.4	21.3	22.0
W_{0o}	2.5	3.1	3.1	3.1	3.1	2.5
S_{0o}	0.5	2.6	3.0	3.0	2.6	0.5
L	21.0	20.7	20.8	20.8	20.7	21.0
W	2.5	3.0	3.0	3.0	3.0	2.5
S	0.5	2.6	3.0	3.0	2.6	0.5
L_{opt}	20.4	20.1	20.2	20.9	19.5	21.1
W_{opt}	0.8	2.4	2.9	3.4	2.4	0.9
S_{opt}	1	1.4	1.5	1.4	1.6	1

Table 5: The microstrip coupled line parameters calculated.

The coupled lines schematic can be found in figure 15 with the calculated parameters and figure 16 contains the optimized parameters.

6.6.2 Optimization

The design was optimized as a 2D layout since the parameters could be only chosen to use the optimization algorithms this way. The fine tune parameter in terms of the characteristic impedance was found to be the distance between the two lines (S) and the other parameters such as the width (W) affected the impedance in a much more significant way.

The optimizer was run with different parameters with constraints that came from the fact that the filter should be realizable in a physical form. The goal of the optimizer was to have minimal insertion loss and narrow enough bandpass band.

The physical constraints that were built into the optimizer were the the lines should be at least 0.5 mm thick, the distance between the lines should be at least 0.5 mm and the difference between the width of the sections should be so that the next section should be able to join the previous one.

After the optimizer ran, I found a way to manually tune the device a bit further by looking at the meshes. The off centre connections of the lines improved the attenuation performance of the filter compare figures 17 and 18. The S parameter graphs prove the this see 20 and 21 below.

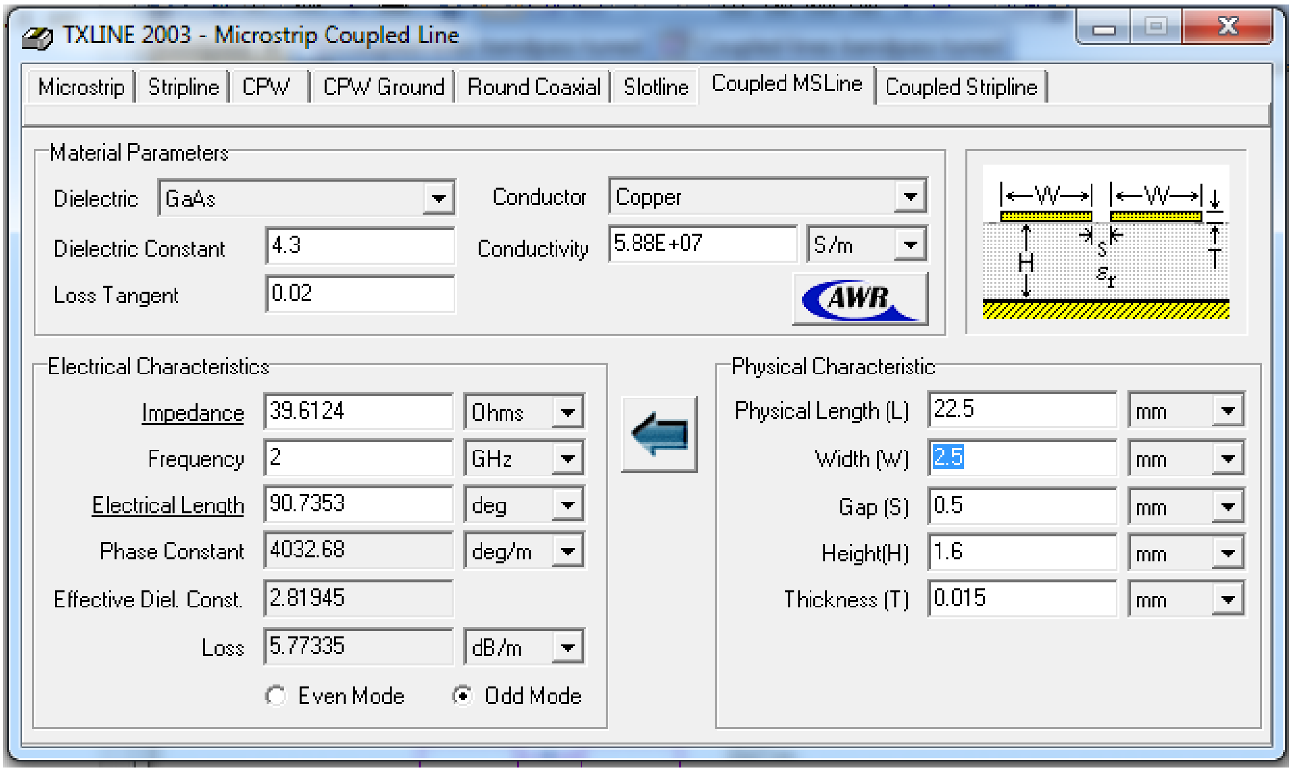


Figure 14: The coupled line parameters as calculated by the TXline tool.

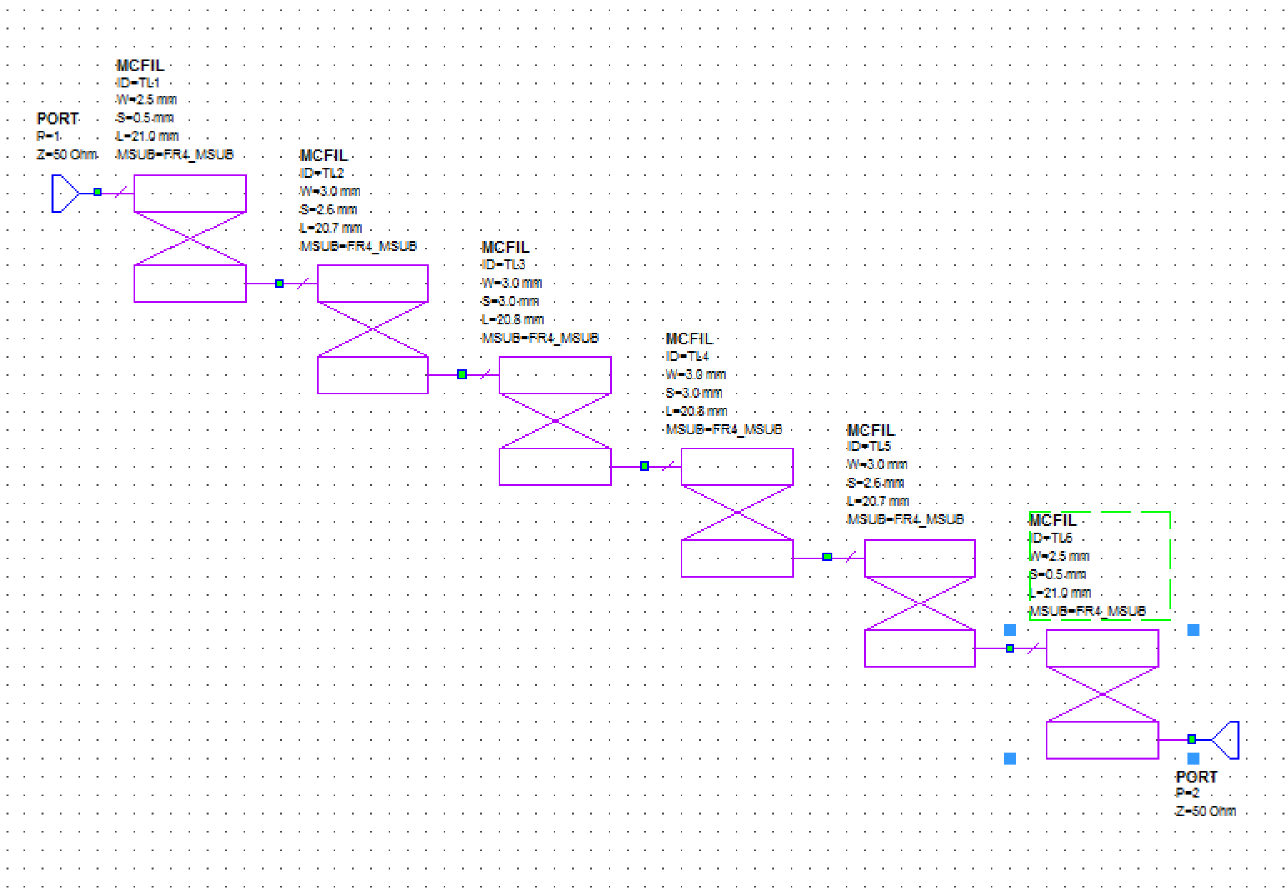


Figure 15: Coupled lines schematic with the TX line calculated parameters.

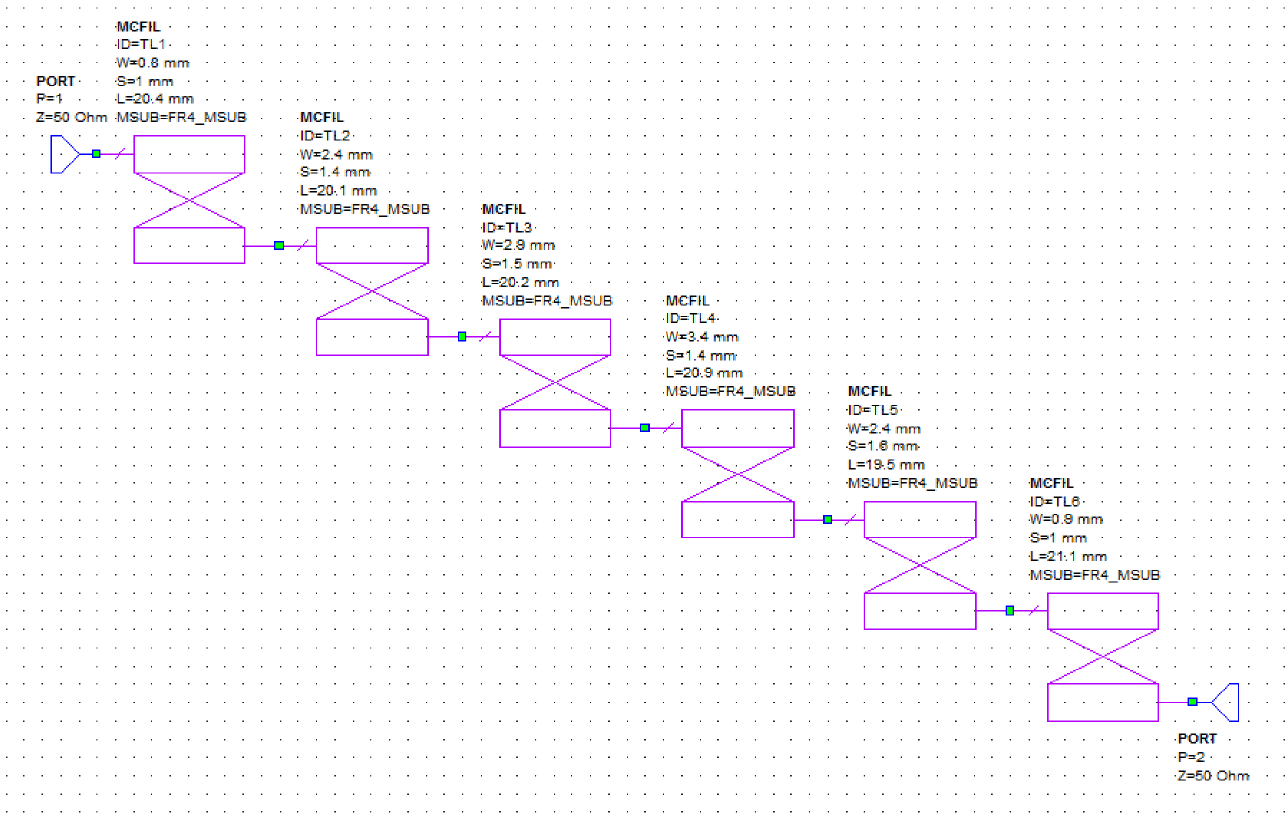


Figure 16: Coupled lines schematic with the optimized parameters.

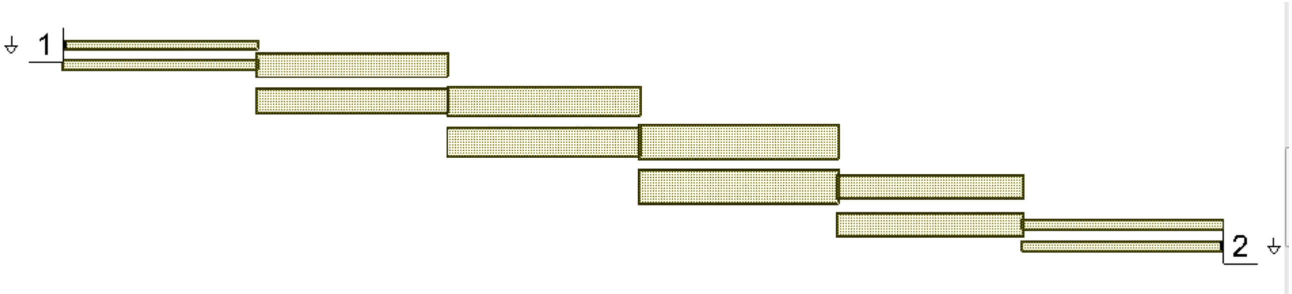


Figure 17: The coupled lines structure with the optimized parameters.

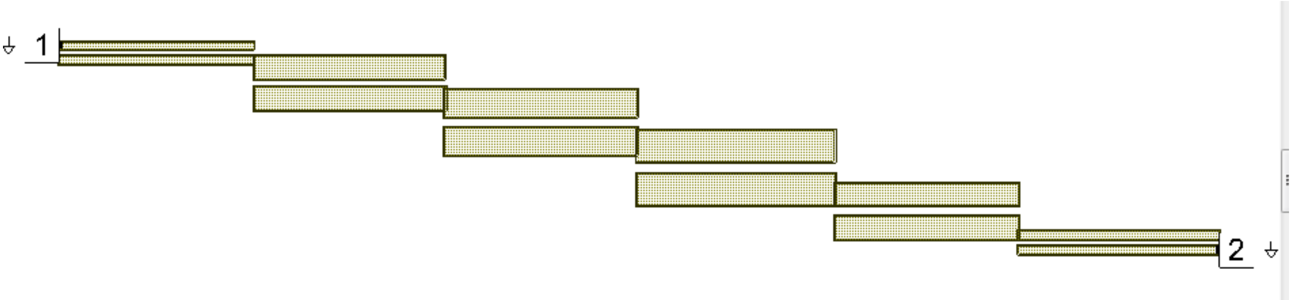


Figure 18: The coupled lines structure after manual tuning.

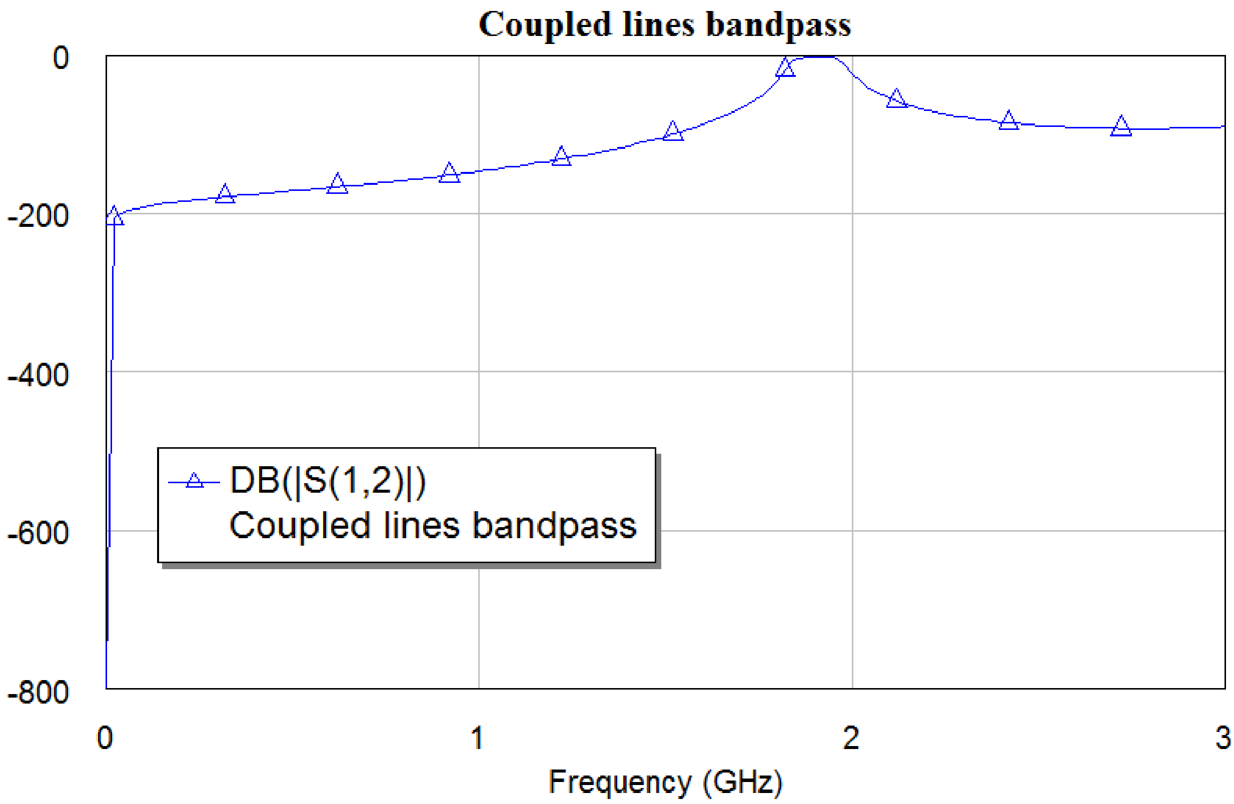


Figure 19: Coupled lines bandpass S_{12} parameters.

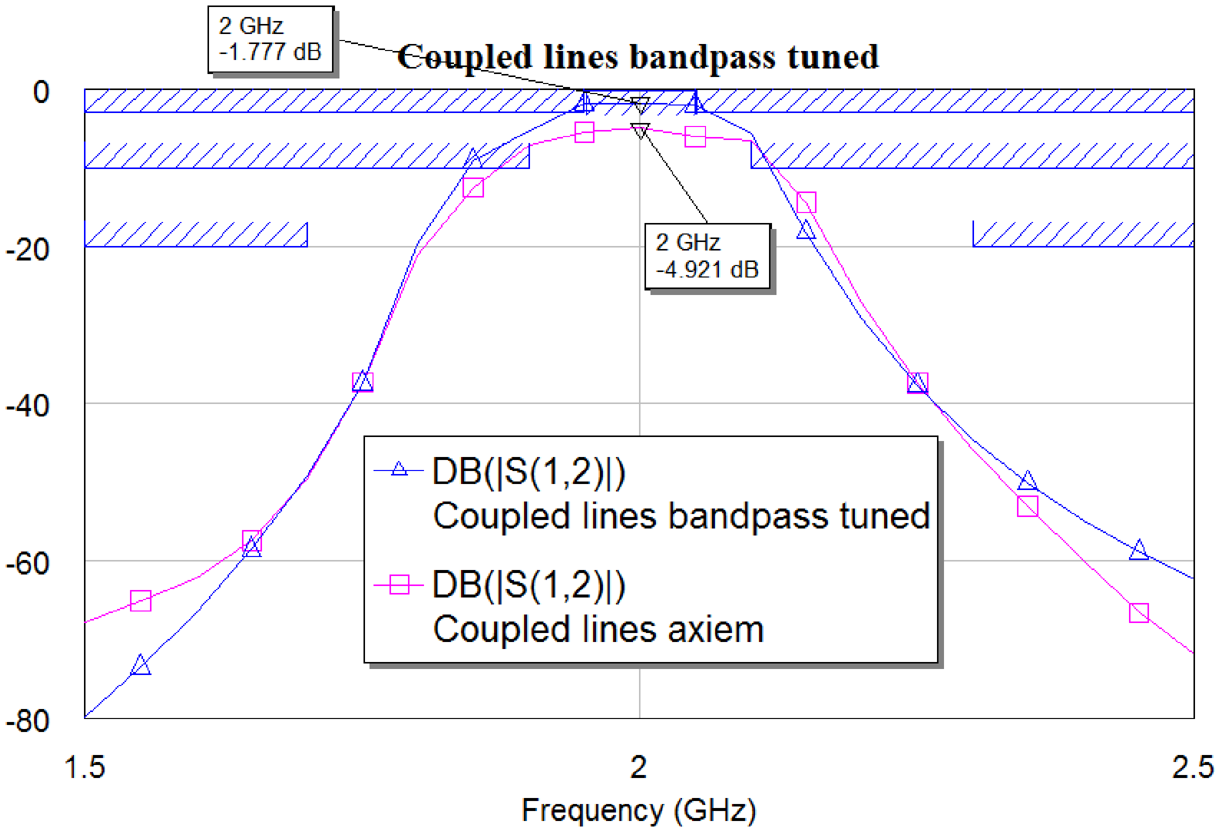


Figure 20: Coupled lines bandpass S_{12} parameters after optimization.

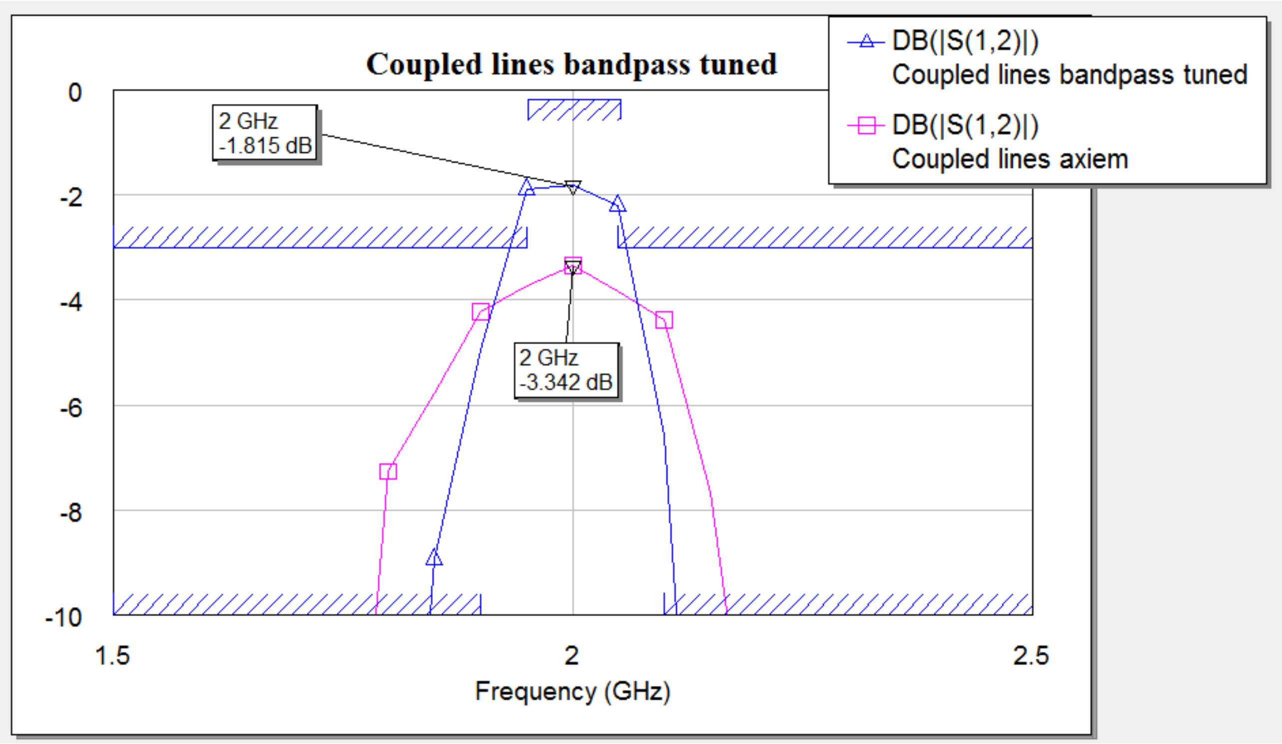


Figure 21: Coupled lines bandpass S_{12} parameters after optimization.

6.7 Design schematic and simulations - Mihaly

After receiving the schematic with the amplifier from Hassan, the matched filter was inserted into the design see figure 22 and 24.

Figure 24 shows the peak around f_0 and $2f_0$, too, as it is expected for a coupled lines structure.

All S parameters simulated can be seen in figure 24. The oscillation described in section 6.3 could not be observed on the simulations.

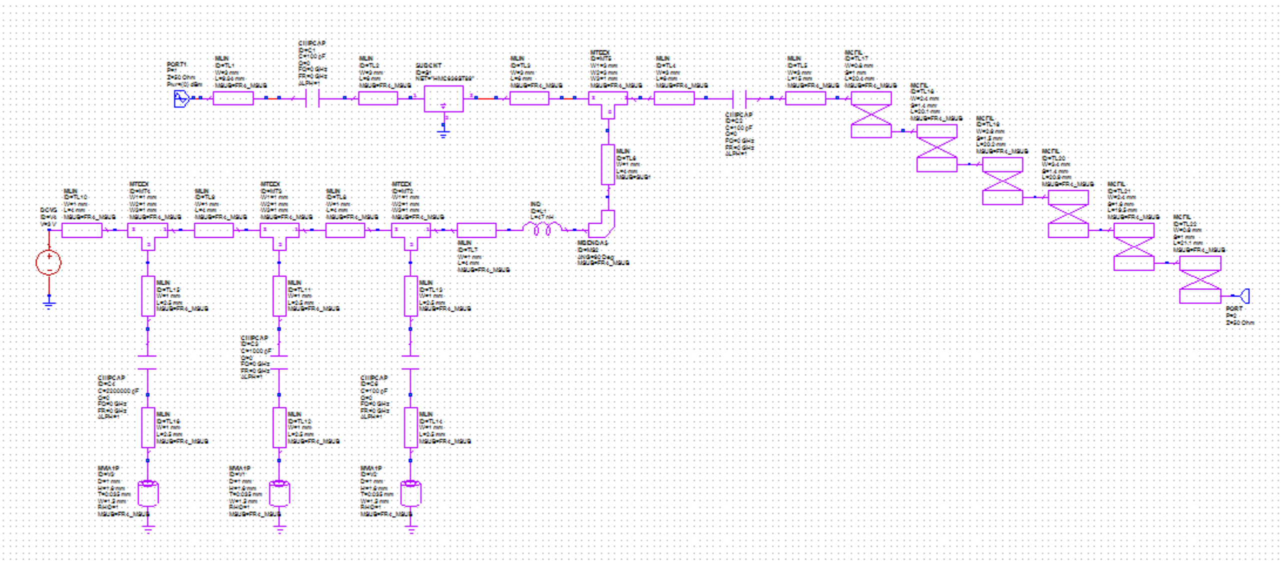


Figure 22: The complete circuit schematics.

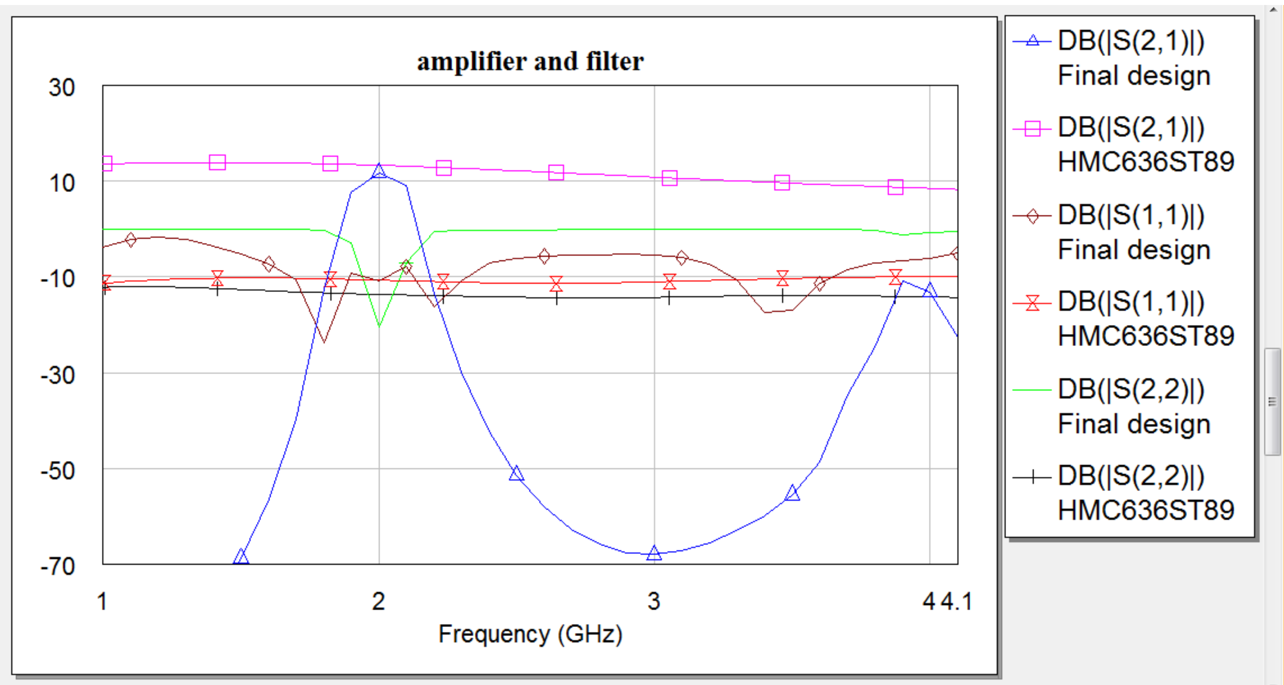


Figure 23: Simuating the amplifier and the whole structure.

6.8 Measured results

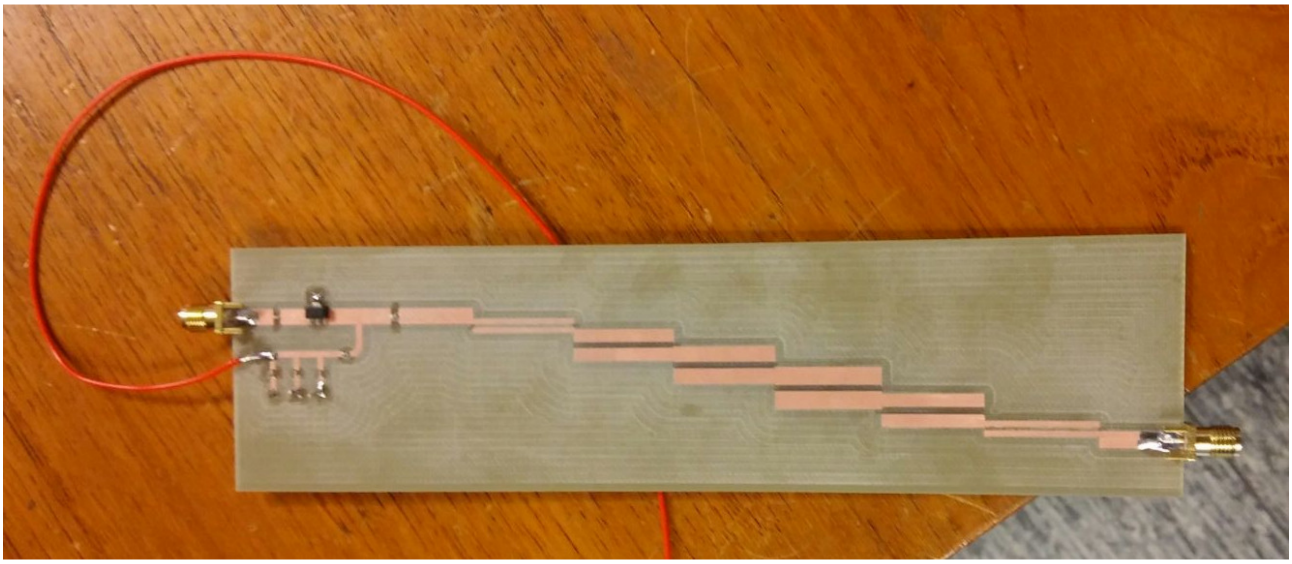


Figure 24: The final, soldered design. The filter matching sections can be seen on the PCB, too.

6.8.1 Plotting the results - Mihaly

To produce graphs for the measured S parameters, the VNA output file was fed into a python script that was developed for Lab 2 of this course by Mihaly. In this assignment we only used the graphing function of the script, since the reciprocal and lossless properties are not relevant in this case. The graphs were constructed using matplotlib. The script can be found in section 8.

6.8.2 Conclusions 1: bandwidth, gain 2D and 3D simulations - Mihaly

The bandwidth of approximately 10% can be confirmed $f_1 = 1.98GHz$ and $f_2 = 2.10GHz$. The centre frequency is somewhat shifted to $2.04GHz$. The unilateral nature of the amplifier can be seen by the S_{12} parameter following the S_{21} by approximately $30dB$ lower.

The gain is less than in the simulated results it peaks around 0 dB, instead of the expected 10 dB. This is probably due to higher than expected losses on the filter section. We couldn't simulate the filter section and the amplifier together in the 3D simulator, only in the 2D simulator which verified the expected gain of the circuit. However, the design could not have been validated in a more accurate simulation that can lead to significantly different results as it can be seen in figure 20 and figure 21. It can be clearly seen that the blue lines corresponding to the 2D simulation show much lower losses than the 3D simulations. Therefore we can assume that the final design will have similar problems.

The coupled lines filters are sensitive to the manufacturing process, too, for instance if the drill that removes the copper digs a bigger well between the coupled line sections that can change the effective permittivity of the coupled lines, by removing or leafing more FR4 material, thereby changing the even and odd mode line impedances and ultimately the insertion loss of the filter.

6.8.3 Conclusions 2: impedance matching

The oscillation in the S_{22}.

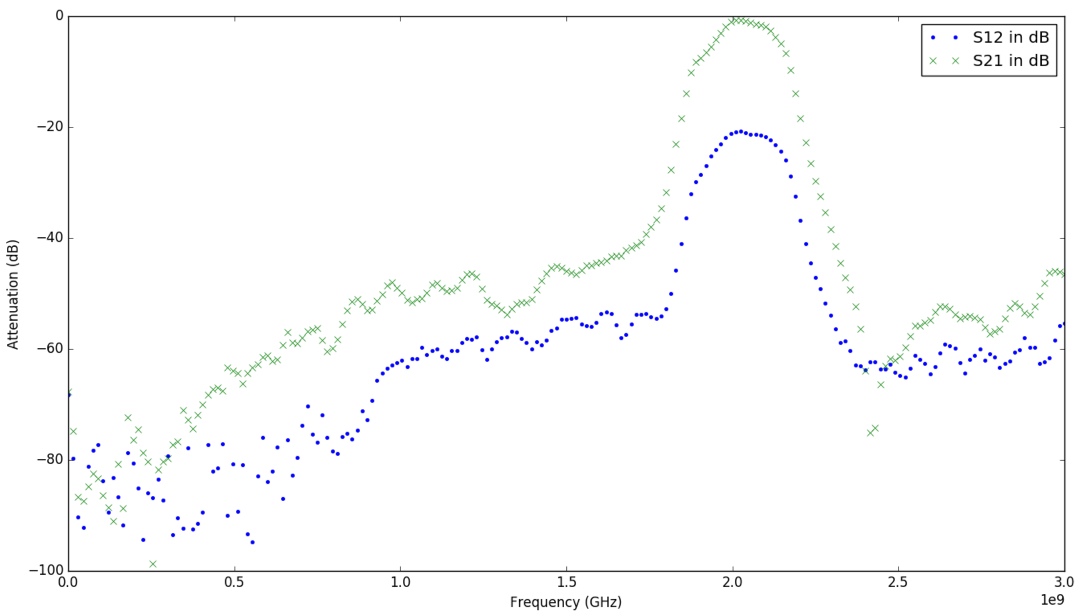


Figure 25: S_{12} and S_{21} parameters.

7 References

1. Pozar, David M.: Microwave Engineering, Fourth Edition, John Wiley & Sons, Inc., 2012

8 Appendix A

```
# s-params.py
#
# Assignment 2C @ QMUL ECS644P
# Author: Mihaly Vadai
#
# Tests for reciprocal and lossless networks by analysing the S matrix.
# For usage information type: python s-params.py -h
#
# Created: 23/10/2016
# Last modified: 17/12/2016

from optparse import OptionParser
import csv
import numpy as np
import matplotlib.pyplot as plt
import sys

#lists for error calcualtions
err1 = []

#lists for S parameter magnitude plots
f = []
S11 = []
S12 = []
S21 = []
S22 = []
S11a = []
S12a = []
S21a = []
S22a = []
muparams = []
ga = []

parser = OptionParser()
parser.add_option("-f", "--file", dest="filename",
        help="Input file containing all S parameters in the following format: \
        frequency Sij magnitude, Sij angle (degrees)\
        eg.  9000; 10; 134; 0.5; 134; 1e5; -60; 1; 14", metavar="FILE")
parser.add_option("-t", "--threshold", dest="thres", metavar="THRESHOLD",
        help="Specify the error threshold to be used with the S matrix. Can\
         be in float format, too. Eg. 1e-8.")
parser.add_option("--db", action="store_true", dest="db", \
        help="Specify --db if the magnitudes supplied are in decibel.")
parser.add_option("-v", action="store_true", dest="verbose", \
        help="Print all S matrices.")
parser.add_option("-l", action="store_true", dest="lossless", \
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        help="Print only lossless cases.")
parser.add_option("-r", action="store_true", dest="reciprocal", \
        help="Print only reciprocal cases.")

args = parser.parse_args()[0]
filename = args.filename
thres = args.thres
decibel = args.db
verbose = args.verbose
rec = args.reciprocal
ll = args.lossless

if thres == None and filename == None:
        sys.exit("\nPlease use the -h option to see a usage help.")

if thres == None:
        thres = 0
else:
        thres = float(thres)

print "\nUsing threshold: {}".format(float(thres))

if filename == None:
        sys.exit("\nPlease specify a file name.")

#if the data is in decibels
#it has to be converted back for the test
def inv_decibel_voltage(db):
        return 10**(-float(db)/20)

def inv_decibel_pwr(db):
        return 10**(-float(db)/10)

def to_decibel_pwr(mag):
        return -10*np.log10(float(mag))

def to_decibel_voltage(mag):
        return -20*np.log10(float(mag))

#takes angles in radians and returns the complex number in numpy format
def to_complex(r, theta):
        return r*np.complex(np.cos(theta), np.sin(theta))

def available_gain(S):
        s11u = S.flat[0]
        s21u = S.flat[1]
        s12u = S.flat[2]
        s22u = S.flat[3]

        #assuming matched conditions
        gamma_l = s22u.conjugate()
        gamma_s = s11u.conjugate()
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        gamma_in = s11u+s12u*s21u*gamma_l/(1-s22u*gamma_l)
        gamma_out = s22u+s12u*s21u*gamma_s/(1-s11u*gamma_s)

        ga = (1-abs(gamma_s)**2)/abs(1-s11u*gamma_s)**2*abs(s21u)**2*1/\
                (1-abs(gamma_out)**2)

        return ga

def unilateral(S):
        s11u = S.flat[0]
        s21u = S.flat[1]
        s12u = S.flat[2]
        s22u = S.flat[3]

        #assuming matched conditions
        gamma_l = s22u.conjugate()
        gamma_s = s11u.conjugate()
        gamma_in = s11u+s12u*s21u*gamma_l/(1-s22u*gamma_l)

        #calculating the gains
        g0 = abs(s21u)
        gl = (1-abs(gamma_l)**2)/abs(1-s22u*gamma_l)**2
        gs = (1-abs(gamma_s)**2)/abs(1-gamma_in*gamma_s)**2
        gsu = (1-abs(gamma_s)**2)/abs(1-s11u*gamma_s)**2

        #because of matched conditions gl and g0 is the same
        #so they cancel
        err = gs/gsu
        U = (abs(s11u)*abs(s21u)*abs(s12u)*abs(s22u))/\
                ((1-abs(s11u))**2*(1-abs(s22u))**2)

        return [err, 1/(1+U)**2, 1/(1-U)**2, U]


def mu_test_stability(S):
        #mu test
        delta = S.flat[0]*S.flat[3]-S.flat[2]*S.flat[1]
        mu = (1-abs(S.flat[0])**2)/(abs(S.flat[3]-delta*S.flat[0].conjugate())+abs
        return mu


with open(filename, 'r') as csvfile:
        sreader = csv.reader(csvfile, delimiter=',')
        avg_error = 0
        k = 0
        for row in sreader:
                if verbose:
                        print "\nThe data in the row:"
                        print ', '.join(row)

                if decibel:
                        for i in [1,3,5,7]:
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                    row[i] = inv_decibel_pwr(row[i])
                f.append(row[0])
                S11.append(row[1])
                S21.append(row[3])
                S12.append(row[5])
                S22.append(row[7])
        else:
#               row[3] = float(row[3])*float(row[3])
#               for i in [1,3,5,7]:
#                   row[i] = to_decibel_pwr(row[i])
                f.append(row[0])
                S11.append(row[1])
                S21.append(row[3])
                S12.append(row[5])
                S22.append(row[7])



        S = np.matrix([\
        [to_complex(float(row[1]), np.radians(float(row[2]))), \
        to_complex(float(row[3]), np.radians(float(row[4])))], \
        [to_complex(float(row[5]), np.radians(float(row[6]))), \
        to_complex(float(row[7]), np.radians(float(row[8])))]])
        '''
        if test_lossless(S):
                print "Frequency: {} is lossless".format(float(row[0]))
        else:
                if not ll:
                        print "Frequency: {} is not lossless".format(float
                pass

        if test_reciprocal(S,row[0]):
                print "Frequency: {} is reciprocal".format(float(row[0]))
        else:
                if not rec:
                        print "Frequency: {} is not reciprocal".format(flo
                pass
        '''
        mu = mu_test_stability(S)
        muparams.append(mu)
        #print f[k], mu

        output = unilateral(S)
        ufm = output[0]
        lower_bound = output[1]
        upper_bound = output[2]
        if lower_bound < ufm and upper_bound > ufm:
                print "Unilateral holds for: {0} MHz \\\\".format(f[k])
        else:
                print "Unilateral doesn't hold for: {0} MHz\\\\".format(f[

        ga_ = available_gain(S)
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        ga.append(ga_)

        k = k + 1

plt.xlabel('Frequency (GHz)')
plt.ylabel('% error')
plt.legend(handles=[mag], loc=1)
plt.show()
plt.xlabel('Frequency (MHz)')
plt.ylabel('Gain (dB)')
str1 = "S12"
s12, = plt.plot(f, S12, '.', label=str1)
str2 = "S21"
s21, = plt.plot(f, S21, 'x', label=str2)
plt.legend(handles=[s12, s21], loc=0)
plt.show()
plt.xlabel('Frequency (MHz)')
plt.ylabel('Gain (dB)')
str1 = "S11"
s11, = plt.plot(f, S11, '.', label=str1)
str2 = "S22"
s22, = plt.plot(f, S22, 'x', label=str2)
plt.legend(handles=[s11, s22], loc=0)
plt.show()
plt.xlabel('Frequency (MHz)')
plt.ylabel('Mu parameter')
str1 = "mu values"
s11, = plt.plot(f, muparams, '.', label=str1)
str2 = "stability limit"
plt.axhline(y=1,xmin=0,xmax=3,c="red",linewidth=0.5,zorder=0)
plt.legend(handles=[s11], loc=0)
plt.show()
plt.xlabel('Frequency (MHz)')
plt.ylabel('Ga values')
str1 = "Available gain"
s11, = plt.plot(f, ga, '.', label=str1)
plt.legend(handles=[s11], loc=0)
plt.show()
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