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On the dimensionality of inference in credal nets with interval probabilities

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Statement

The number of decision variables (or dimensionality) required to compute the **inference in two-state credal networks** with interval probabilities grows at most **linearly with the number of nodes** directly connected to the queried variable.

Strategy and proof

We prove this statement by means of the *interval gradient* on a *vacuous credal* network. A vacuous credal network is a network whose probabilities are in the open interval (0, 1). The interval gradient is obtained from the derivatives of the independent inputs over the open interval.

$$\left.\frac{\partial P_{infer}(x_k)}{\partial x_k}\right|_{(0,1)} < 0, \quad x_k \in X^{\downarrow} \qquad (1)$$

$$\left.\frac{\partial P_{infer}(x_k)}{\partial x_k}\right|_{(0,1)} > 0, \quad x_k \in X^{\uparrow} \qquad (2)$$

$$X^{\{M\}} = X^{\downarrow} \cup X^{\uparrow}, \qquad k = 1, \dots, D \qquad (3)$$

$$k^{\{M\}} = \{k : x_k \in X^{\{M\}}\}, \qquad R = D - \#k^M \qquad (4)$$

Where x_k is the k^{th} independent input variable. The proof needs specialisation on the network under study, however coefficients can be stored upfront on recurring architectures. In (4) the integer R is the reduced dimension of the credal network.

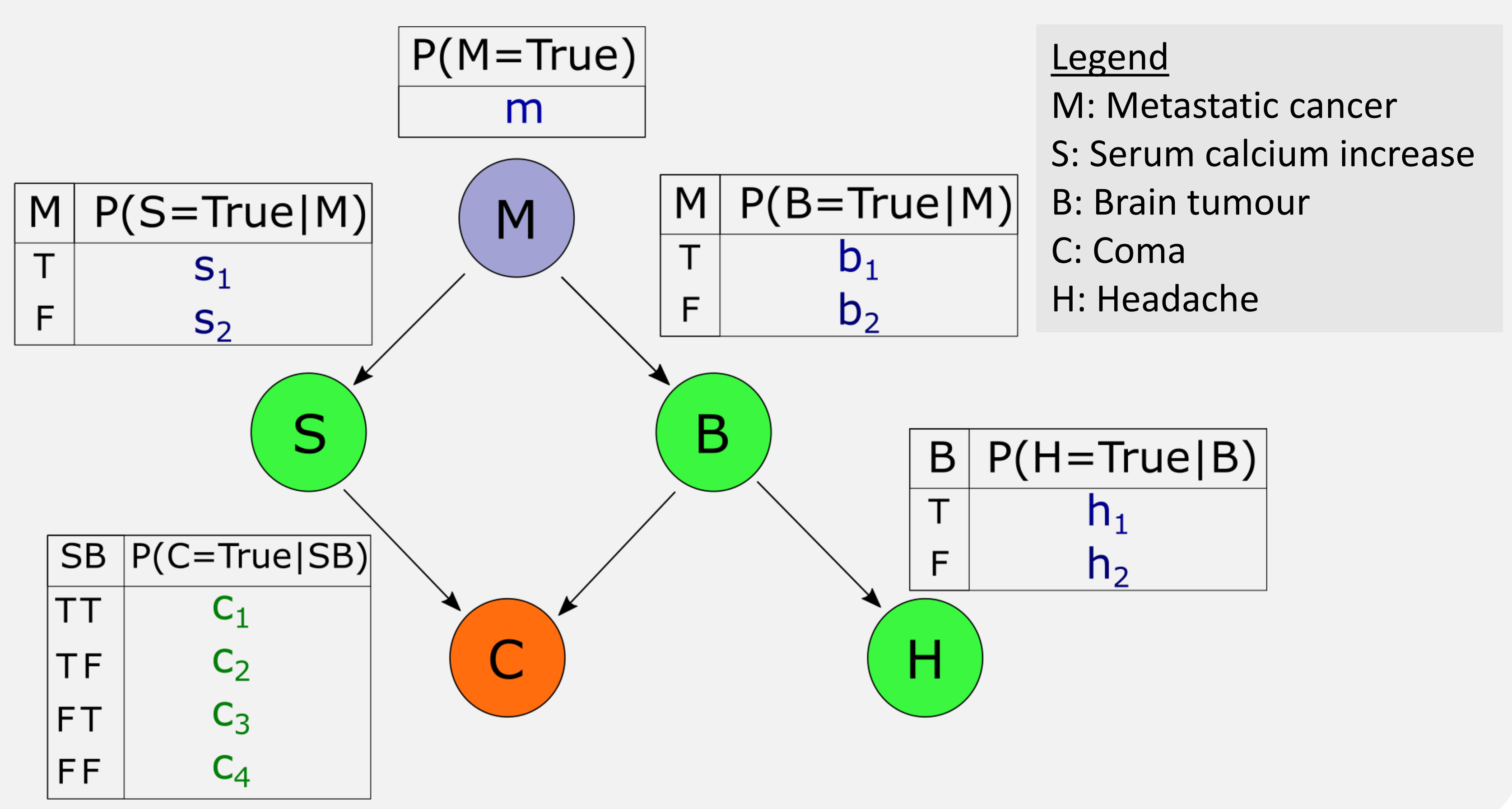
Example: *Multiply-connected network*

Queries:

$\overline{P}(C|H)$ $\quad x_{k^{\{M\}}} = \{\mathbf{c}_{1:4}\}$ $\quad x_{\neg k^{\{M\}}} = \{\mathbf{m}, \mathbf{s}_{1:2}, \mathbf{b}_{1:2}, \mathbf{h}_{1:2}\}$

$\overline{P}(S|C)$ $\quad x_{k^{\{M\}}} = \{\mathbf{s}_{1:2}, \mathbf{c}_{1:4}\}$ $\quad x_{\neg k^{\{M\}}} = \{\mathbf{m}, \mathbf{b}_{1:2}\}$

$\overline{P}(H)$ $\quad x_{k^{\{M\}}} = \{\mathbf{h}_1, \mathbf{h}_2\}$ $\quad x_{\neg k^{\{M\}}} = \{\mathbf{m}, \mathbf{b}_{1:2}\}$



Query

$$\overline{P}(G_j|C) =$$
$$= \overline{P}_j(x_k) = ?$$

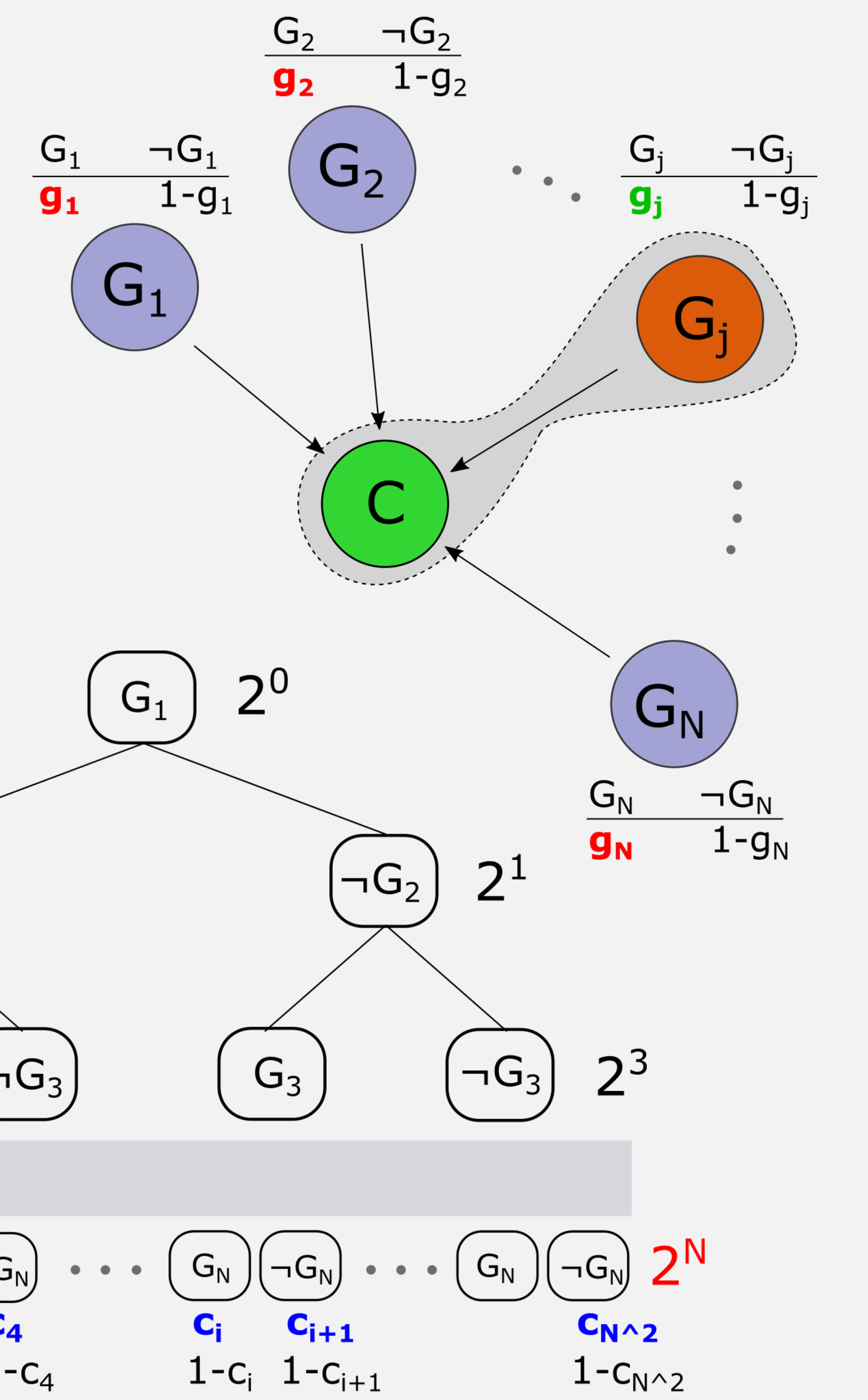
Ex.: $x_{k=1} = g_1$

$g_j = P(G_j = True)$

Active variable

Query variable

Hidden variable



Algorithm

$$P(G_j|C) = \frac{\sum_{\{g_1,\dots,g_N\}\setminus g_j} P(G_1,\dots,G_N,C)}{P(C)} = \frac{P(G_j,C)}{P(C)} =$$

$$= P(X_k) = \frac{P(G_j,C)}{P(C)}$$

1. $x_k = \{g_1, g_2, \dots, g_j, \dots, g_N, c_1, c_2, \dots, c_{2^N}\}$

2. $k^{\{M\}} = \left\{ k\colon \left.\frac{\partial P_j(x_k)}{\partial x_k}\right|_{(0,1)} \setminus \{0\} \right\}$

3. $x_k = \{g_1, g_2, \dots, \mathbf{g_j}, \dots, g_N, \mathbf{c_1}, \mathbf{c_2}, \dots, \mathbf{c_{2^N}}\}$

4. $x_{k^{\{M\}}} = \{\mathbf{g_j}, \mathbf{c_1}, \mathbf{c_2}, \dots, \mathbf{c_{2^N}}\}$ $\; x_{\neg k^{\{M\}}} = \{g_1, g_2, \dots, g_N\}$

5. $\underline{P}(X_k) = \min_{k \in k^{\{M\}}} P(x_k) \qquad \overline{P}(X_k) = \max_{k \in \neg k^{\{M\}}} P(x_k)$

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