Student: Marco Defilippi

ES1

2.2 Assume we sampled a continuous $x(t)$ signal and obtained 100 $x(n)$ time-domain samples. What important information (parameter that we need to know in order to analyze $x(t)$) is missing from the $x(n)$ sequence?

Answer:

The most critical piece of missing information in this scenario is the sampling frequency or sampling rate.

Indeed, the sampling frequency (fs) determines how often the signal x(t) was sampled. Without knowing fs, you cannot accurately reconstruct the continuous signal from its samples because the time scale of the original signal is unknown.

Fs must be always greater than 2B (B is the signal's band) to separate spectral replications that occur at the folding frequencies of +/- fs/2.

The relationship of fs>= 2B is known as the Nyquist criterion.

ES2

2.3 National Instruments Corporation produces an analog-to-digital (A/D) converter (Model #NI-5154) that can sample (digitize) an analog signal at a sample rate of f_s=2.0 GHz (gigahertz).

(a) What is the t_s period of the output samples of such a device?

(b) Each A/D output sample is an 8-bit binary word (one byte), and the converter is able to store 256 million samples. What is the maximum time interval over which the converter can continuously sample an analog signal?

Answer:

a)

Given the sampling frequency fs= 2 GHz, the sampling period ts can be calculated as the inverse of the sampling frequency.

Ts=1/fs = 1/(2 x 10^9) = 0.5 ns

The period of the output samples of this device is 0.5 ns.

b)

The A/D converter can store up to 256 million samples, and each sample is acquired every 0.5 nanoseconds as calculated above.

To find the maximum time interval Tmax over which the converter can continuously sample an analog signal, the number of samples the converter can store must be multiplied by the sampling period per sample:

Tmax = nbr of samples x ts = 256 x10^6 samples x 0.5 x 10^-9 s/sample = 128 ms

Therefore, the maximum time interval over which the converter can continuously sample an analog signal is 128 milliseconds.

ES3

2.4 Consider a continuous time-domain sinewave, whose cyclic frequency is 500 Hz, defined by

$$x(t) = \cos[2\pi(500)t + \pi/7].$$

Write the equation for the discrete $x(n)$ sinewave sequence that results from sampling $x(t)$ at an f_s sample rate of 4000 Hz.

Note: This problem is not "busy work." If you ever want to model the $x(t)$ signal using software (MathCAD, MATLAB, Octave, etc.), then it is the desired $x(n)$ equation that you program into your software.

Answer:

To convert the continuous time-domain sinewave x(t) = cos(2pi*500*t + pi/7) into a discrete sinewave sequence x(n) when sampling at fs=4000hz, the following steps are needed:

F=500 hz is the frequency of the sinewave.

Phase shift = pi/7

The sampling period Ts is the inverse of the sampling frequency:

Ts=1/fs=1/4000 = 0.00025 s

The discrete signal x(n) can be derived by substituting t=nTs into the continuous signal equation where n is an integer representing the sample index.

X(n) = cos(2pi*500*nTs + pi/7)

X(n)=cos(2pi*500*n/4000 + pi/7)

X(n)= cos(pin/4 + pi/7)

The discrete-time sinewave sequence resulting from sampling the given continuous sinewave at 4000 Hz is:

X(n) = cos(pin/4 + pi/7)

ES4

2.5 If we sampled a single continuous sinewave whose frequency is f_o Hz, over what range must t_s (the time between digital samples) be to satisfy the Nyquist criterion? Express that t_s range in terms of f_o.

Answer:

To satisfy the Nyquist criterion, the sampling frequency must be at least twice the highest frequency present in the signal to accurately capture al the frequency content without aliasing. The range for t_s must be derived from the frequency f_0 of the signal.

Nyquist criterion is: $fs>=2f_0$

$1/fs<=1/(2f_0)$

$t_s <=1/(2f_0)$

There is no theoretical upper bound for fs as higher sampling frequencies than the minimum required 2 f_0 still satisfy the Nyquist criterion. Therefore, the lower bound for t_s is determined by the smallest value t_s can take without violating the Nyquist criterion, but within certain practical limitations.

To satisfy the Nyquist criterion t_s should be:

$t_s <=1/(2\ f_0)$

Thus, the time between digital samples t_s must be less than or equal to half the reciprocal of the frequency f0 of the sinewave to meet the Nyquist criterion. This condition ensures no aliasing occurs, and the signal is sampled adequately for accurate reconstruction.

ES5

2.6 Suppose we used the following statement to describe the Nyquist criterion for lowpass sampling: "When sampling a single continuous sinusoid (a single analog tone), we must obtain no fewer than N discrete samples per continuous sinewave cycle." What is the value of this integer N?

Answer:

The Nyquist criterion states that to adequately sample a signal without aliasing, the sampling frequency must be at least twice the highest frequency present in the signal. Translating this into the context of sampling a single continuous sinusoid, the statement means that you need to sample the sinusoid such that there are at least two samples per cycle of the sinusoid. This is the minimum requirement to capture the peak and the trough of the sinusoid, which are critical to reconstructing the waveform accurately.

Nyquist criterion is: fs>=2f0

This implies that the sampling frequency is at least twice the frequency of the sinusoid. Hence, within one cycle of the sinusoid, which takes 1/f0 seconds, the number of samples.

ES6:

2.7 The Nyquist criterion, regarding the sampling of lowpass signals, is sometimes stated as "The sampling rate f_s must be equal to, or greater than, twice the highest spectral component of the continuous signal being sampled." Can you think of how a continuous sinusoidal signal can be sampled in accordance with that Nyquist criterion definition to yield all zero-valued discrete samples?

Answer:

A continuous sinusoidal signal can indeed be sampled in accordance with the Nyquist criterion and yet yield all zero-valued discrete samples. This phenomenon can occur due to a specific phase and timing alignment between the signal and the sampling points.

The equation for a sinusoidal signal is:

X(t) = A cos(2pif0t + initPhase)

The discrete version is:

X[n] = s(nTs) = A cos(2pif0nTs + initPhase)

Ts is the sampling period, Ts=1fs

For the Nyquist criterion fs>=2f0.

For fs=2f0, Ts=1/(2f0).

Substituting into the sinuoisodal equation:

X[n] = A cos(2pi*f0*nF(1/2f0) + initPahse) = A cos(pi*n + initPhase)

For x[n] to be zero for all n, pi*n + initPhase must equal to:

pi*n + initPhase = pi/2 + k*pi, (where k is any integer).

This happen when:

Pi*n + initPhase = pi/2 + k*pi

initPhase = pi/2 + k*pi – pi*n

For example, if initPhase = pi/2, then at every sample point n, pi*n + pi/2 will be an odd multiple of pi/2, hence resulting in zero.

Thus, if you set the phase ϕϕ such that the sinusoid's zero crossings align exactly with the sampling points, all discrete samples can indeed be zero while still adhering strictly to the Nyquist criterion.

ES7:

2.9 Consider a continuous time-domain sinewave defined by

$$x(t) = \cos(4000\pi t)$$

that was sampled to produce the discrete sinewave sequence defined by

$$x(n) = \cos(n\pi/2).$$

What is the f_s sample rate, measured in Hz, that would result in sequence $x(n)$?

Answer:

The equation for a continuous time-domain sinewave is:

X(t) = cos(2pi*f0*t)

Equalizing the arguments:

2pi*f0*t = 4000*pi*t

f0=2000Hz

The equation of the discrete version for a continuous time-domain sinewave is:

X[n]=cos(2pi*f0*n*ts)

Equalizing the two arguments:

2pi*f0*n*ts = n*pi/2

Ts=1/fs

2pi*f0*n/fs = n*pi/2

2f0/fs=1/2

Fs=4*f0

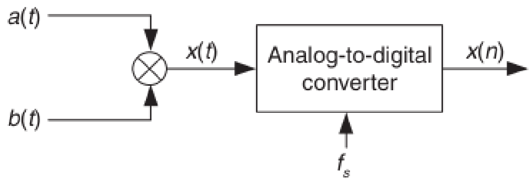
The sampling frequency fs that would result in the discrete sequence x[n] = cos(pi*n/2) from the continuous sinewave x(t)=cos(4000*pi*t) is 8000 Hz.

ES8:

2.10 Consider the two continuous signals defined by

$$a(t) = \cos(4000\pi t) \text{ and } b(t) = \cos(200\pi t)$$

whose product yields the $x(t)$ signal shown in Figure P2–10. What is the minimum f_s sample rate, measured in Hz, that would result in a sequence $x(n)$ with no aliasing errors (no spectral replication overlap)?



Answer:

To determine the minimum sampling frequency fs that would allow for aliasing free digitization of the signal x(t), which is the product of two continuous signals a(t) and b(t), it is needed to first analyze the product signal's frequency content.

A(t) = cos(4000*pi*t) corresponds to a frequency fa:

4000*pi*t = 2*pi*fa*t

Fa=2000 hz.

B(t) = cos(200*pi*t) corresponds to a frequency fb:

200*pi*t = 2*pi*fb*t

Fb= 100hz

Using the trigonometric identity for the product of cosines:

Cos(alpha)cos(beta) = ½[cos(alpha+beta) + cos(alpha-beta)]

Applying this to a(t) and b(t):

X(t) = cos(4000*pi*t)cos(200*pi*t) = ½[cos((4000*pi + 200*pi)*t) + cos((4000*pi – 200*pi)*t)]

X(t) = ½[cos(4200*pi*t) + cos(3800*pi*t)]

The frequencies components for the two cosines:

- Cos(4200*pi*t) has a frequency of 2100 hz
- Cos(3800*pi*t) has a frequency of 1900 hz

To avoid aliasing, the Nyquist criterion must be complied.

The highest frequency in x(t) is 2100 hz. Thus, the Nyquist frequency (f_N) is:

Fs>=2*2100 hz = 4200 hz.

The minimum fs to ensure no aliasing for x(t) is 4200 hz. This sampling rate guarantees that both the 1900 Hz and 2100 Hz components are accurately represented in the digital version of the signal x(n), without any overlap of spectral replicas, hence avoiding aliasing.

ES9:

2.11 Consider a discrete time-domain sinewave sequence defined by

$$x(n) = \sin(n\pi/4)$$

that was obtained by sampling an analog $x(t) = \sin(2\pi f_o t)$ sinewave signal whose frequency is f_o Hz. If the sample rate of $x(n)$ is $f_s = 160$ Hz, what are three possible positive frequency values, measured in Hz, for f_o that would result in sequence $x(n)$?

Answer:

The discrete sequence x(n) = sin(pi*n/4) implies a discrete frequency of:

Sin(pi*n/4)=sin(2*pi*f0*n*ts)

Equalizing the two arguments:

Pi*n/4 = 2*pi*f0*n*/fs

¼ = 2*f0/fs

F0=fs/8 = 160/8 = 20 hz.

Aliasing occurs when a signal is sampled and the sampling frequency is not high enough to capture its frequency content uniquely. The aliases for a given frequency f0f0 can be found using:

F0=f+k*fs, where k is any integer.

This formula will give all possible frequencies f0f0 that could map to the same sampled frequency due to aliasing.

Given fs=160 hz and the base frequency f = 20 hz, the possible analog frequencies f0 for positive k can be:

- K=0: f0=20 hz.
- K=1: f0= 20 + (1x160) = 180 hz
- K=2: f0=20 + (2*160) = 340 hz.

ES10:

2.13 Consider the simple analog signal defined by $x(t)=\sin(2\pi 700t)$ shown in Figure P2–13. Draw the spectrum of $x(n)$ showing all spectral components, labeling their frequency locations, in the frequency range $-2f_s$ to $+2f_s$.

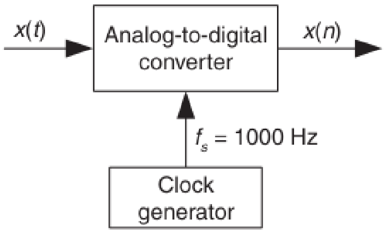


Figure P2–13

Answer:

The frequency of the analog signal is f0=700Hz.

The sampling frequency is fs=1000 Hz.

The f0 of 700 hz is above the Nyquist frequency, causing aliasing.

The aliased frequency fa can be calculated as:

F0=fa+k*fs

Fa = f0 -k*fs. For k=-1, fa=|700 – 1000| = 300 Hz.

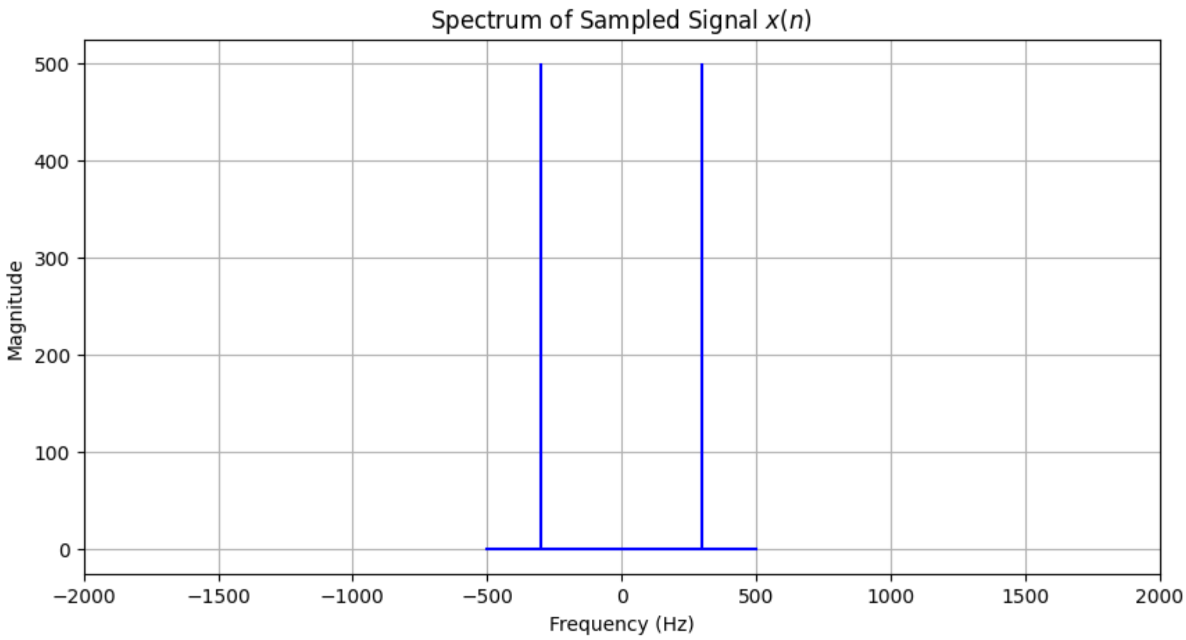
The signal will have its original frequency component aliased to 300 Hz. Because the frequency spectrum of a real sinusoid is symmetrical about zero, there will be frequency components at both +300 Hz and -300 Hz.

Further aliases appear at:

300 +/- k* fs.

Here the list of frequency components:

- -300 Hz
- 1300 Hz (second positive alias: 300 + 1000)
- -1300 Hz (second negative alias: -300 -1000)
- 2300 Hz (Third ositive alias: 300 + 2x1000)
- -2300 Hz (Third negative alias: -300 – 2x1000)


Spectrum of Sampled Signal $x(n)$
Magnitude
500
400
300
200
100
0
−2000
−1500
−1000
−500
0
500
1000
1500
2000
Frequency (Hz)