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Exercises related to Discrete Sequences And Systems:

Es1:

1.1 This problem gives us practice in thinking about sequences of numbers. For centuries mathematicians have developed clever ways of computing π. In 1671 the Scottish mathematician James Gregory proposed the following very simple series for calculating π:

$$\pi \approx 4 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} \ldots\right).$$

Thinking of the terms inside the parentheses as a sequence indexed by the variable n, where $n = 0, 1, 2, 3, \ldots, 100$, write Gregory's algorithm in the form

$$\pi \approx 4 \cdot \sum_{n=0}^{100} (-1)^{?} \cdot ?$$

replacing the "?" characters with expressions in terms of index n.

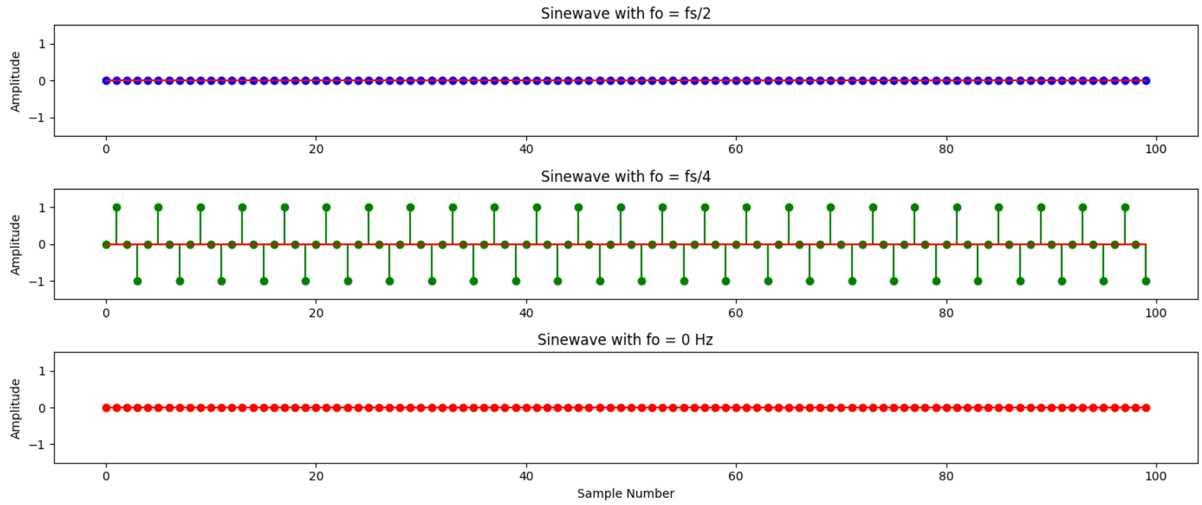
Answer:

$$\pi \approx 4 \sum_{n=0}^{n=100} (-1)^n \frac{1}{2n+1}$$

Es2:

1.2 One of the ways to obtain discrete sequences, for follow-on processing, is to digitize a continuous (analog) signal with an analog-to-digital (A/D) converter. A 6-bit A/D converter's output words (6-bit binary words) can only represent 2^6=64 different numbers. (We cover this digitization, *sampling*, and A/D converters in detail in upcoming chapters.) Thus we say the A/D converter's "digital" output can only represent a finite number of amplitude values. Can you think of a continuous time-domain electrical signal that only has a finite number of amplitude values? If so, draw a graph of that continuous-time signal.

Answer:



Es3:

1.3 On the Internet, the author once encountered the following line of C-language code

$$PI = 2*asin(1.0);$$

whose purpose was to define the constant π. In standard mathematical notation, that line of code can be described by

$$\pi = 2 \cdot \sin^{-1}(1).$$

Under what assumption does the above expression correctly define the constant π?

Answer:

The above expression will correctly yield π if the computation of asin() function is indeed in radians, which is the standard in most programming languages including C.

If, instead the angle was returned in degrees, the same formula would not yield π but rather a numerical values corresponding to 180 degrees, requiring conversion back to radians by multiplying by π/180.

Es4:

1.4 Many times in the literature of signal processing you will encounter the identity

$$x^0 = 1.$$

That is, x raised to the zero power is equal to one. Using the Laws of Exponents, prove the above expression to be true.

Answer:

From the product of powers rule:

$$x^a x^b = x^{a+b}$$

X is a nonzero number.

$$x^n x^0 = x^{n+0}$$

$$x^{n+0} = x^n$$

$$x^n x^0 = x^n$$

By dividing both sides by x^n:

$$\frac{x^n x^0}{x^n} = \frac{x^n}{x^n}$$

$$x^0 = 1$$

Es5:

1.5 Recall that for discrete sequences the t_s sample period (the time period between samples) is the reciprocal of the sample frequency f_s. Write the equations, as we did in the text's Eq. (1–3), describing time-domain sequences for unity-amplitude cosine waves whose f_o frequencies are

(a) $f_o = f_s/2$, one-half the sample rate,
(b) $f_o = f_s/4$, one-fourth the sample rate,
(c) $f_o = 0$ (zero) Hz.

Answer:

a) $x[n] = \cos\left(\frac{2\pi \, n\frac{fs}{2}}{fs}\right) = \cos(\pi \, n)$

This expression will alternate between +1 and -1 with each sample for integer values of n.

b) $x[n] = \cos\left(\frac{2\pi \, n\frac{fs}{4}}{fs}\right) = \cos\,(n\,\pi/2)$

This sequence describes a cosine wave that completes one full cycle over four samples, repeating this pattern indefinitely.

c) $x[n] = \cos(0) = 1$

For this case, the sequence is a costant value of 1 for all n, representing a DC signal.

Es6:

1.6 Draw the three time-domain cosine wave sequences, where a sample value is represented by a dot, described in Problem 1.5. The correct solution to Part (a) of this problem is a useful sequence used to convert some lowpass digital filters into highpass filters. (Chapter 5 discusses that topic.) The correct solution to Part (b) of this problem is an important discrete sequence used for *frequency translation* (both for signal *down-conversion* and *up-conversion*) in modern-day wireless communications systems. The correct solution to Part (c) of this problem should convince us that it's perfectly valid to describe a cosine sequence whose frequency is zero Hz.

Answer:

For a sine wave, the general formula for a time domain sequence sampled at a frequency fs is:

X[n] = sin(2 π f0 n ts)

Given ts=1/fs, the formula becomes:

X[n] = sin(2 π f0 n /fs)

Substituting f0=fs/2:

X[n] = sin(π n)

For integer n, the function returns zero for all n because sine of any integer multiple of π is zero. This results in a costant zero signal.

Substituing f0=fs/4:

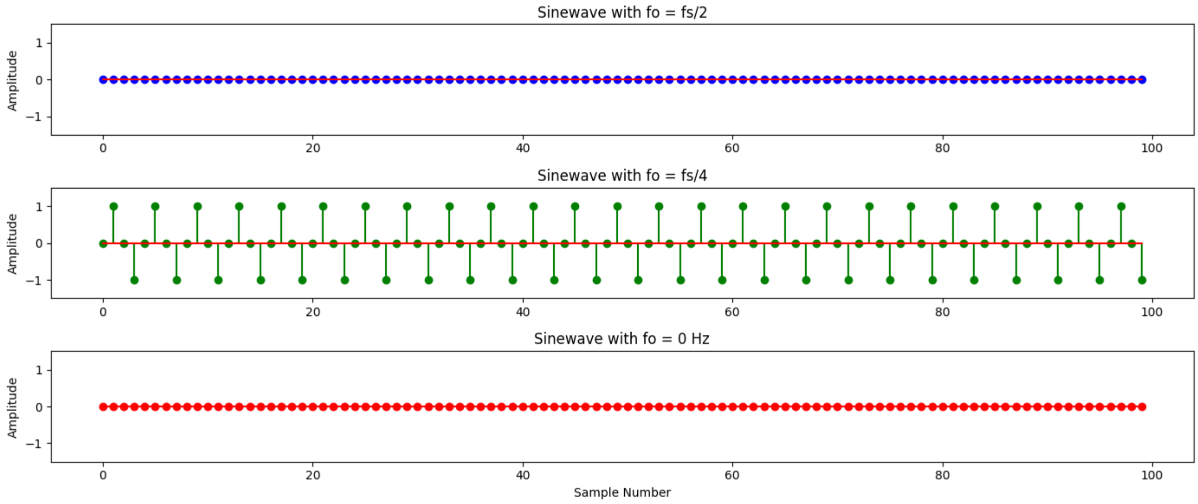
$X[n] = \sin(\pi n/2)$

This sequence will complete one full cycle every four samples.

Substituing f0=0:

$X[n] = \sin(0) = 0$

The result is a constant zero signal because there is no frequency component, representing a DC level at zero amplitude.



Es7:

1.8 Consider the infinite-length time-domain sequence $x(n)$ in Figure P1–8. Draw the first eight samples of a shifted time sequence defined by

$$x_{\text{shift}}(n) = x(n+1).$$

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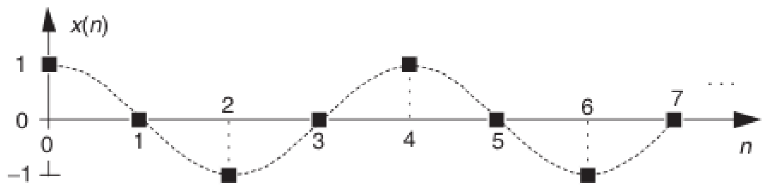
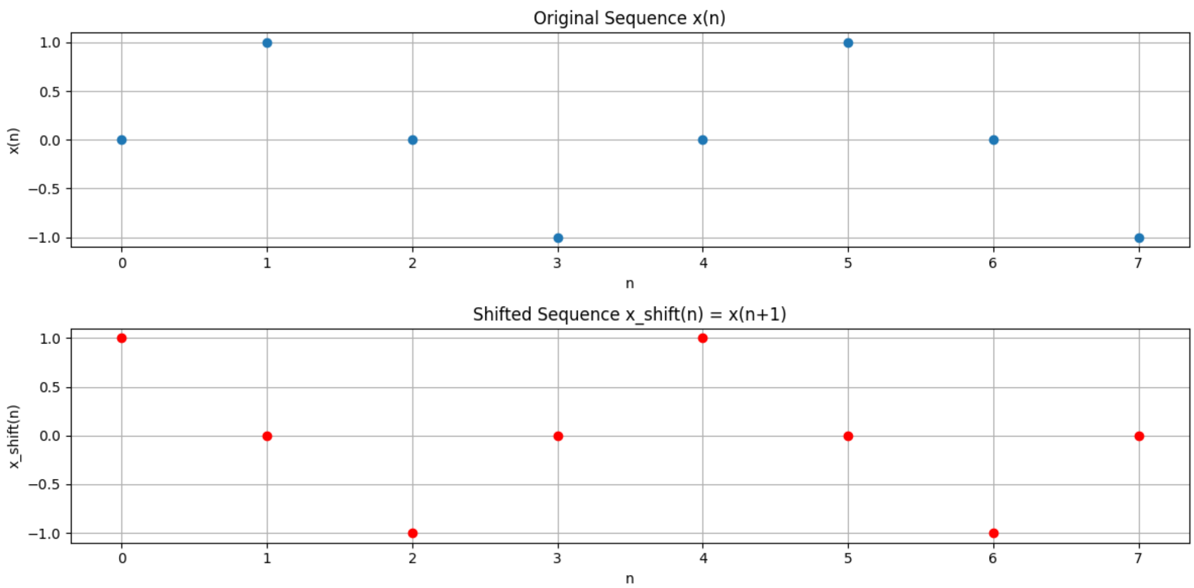


Figure P1–8

Answer:

The shifted version of sequence $x_{shift}(n) = x(n+1)$, corresponds to the sequence x(n) with all values shifted one place to the left.

This means each value of x(n) at index n will now be at index n-1 in $x_{shift}(n)$.



Es8:

1.9 Assume, during your reading of the literature of DSP, you encounter the process shown in Figure P1–9. The $x(n)$ input sequence, whose f_s sample rate is 2500 Hz, is multiplied by a sinusoidal $m(n)$ sequence to produce the $y(n)$ output sequence. What is the frequency, measured in Hz, of the sinusoidal $m(n)$ sequence?

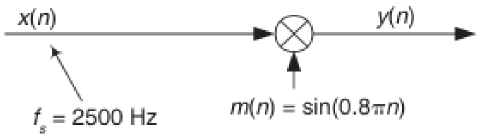


Figure P1-9

Answer:

The generic expression for a discrete sinusoidal function is:

m(n) = $\sin(2\pi\ f_0\ n\ t_s)$ = $\sin(2\pi\ f_0 n/f_s)$

By equalizing the arguments:

$2\pi\ f_0 n\ t_s = 0.8\ \pi\ n$

Substituting: $t_s = 1/f_s$

$2\pi\ f_0 n\ /f_s = 0.8\ \pi\ n$

$f_0 = 0.8\ f_s\ /\ 2 = 1000$ Hz

The corrected frequency of the sinusoidal sequence m(n) for a sampling rate of fs=2500 Hz is exactly 1000 Hz.

Es9:

1.10 There is a process in DSP called an "N-point running sum" (a kind of digital lowpass filter, actually) that is described by the following equation:

$$y(n) = \sum_{p=0}^{N-1} x(n-p).$$

Write out, giving the indices of all the $x()$ terms, the algebraic expression that describes the computations needed to compute $y(9)$ when N=6.

Answer:

The algebraic expression needed to compute y89) when N=8 is simply the sum of the input signal x(n) at indeces 9,8,7,6,5 and4. Tis represents a simple moving average of lowpass filterwhere each output sample y(n) is the average of the last N samples of the input x(n) assuming a unitary weight for each sample.

Y(9) = x(9-0) + x(9-1) + x(9-2) + x(9-3) + x(9-4) + x(9-5)

Y(9) = x(9) + x(8) + x(7) + x(6) + x(5) + x(4)

Es10:

1.13 Let's say you are writing software code to generate an $x(n)$ test sequence composed of the sum of two equal-amplitude discrete cosine waves, as

$$x(n) = \cos(2\pi f_o n t_s + \phi) + \cos(2\pi f_o n t_s)$$

where t_s is the time between your $x(n)$ samples, and ϕ is a constant phase shift measured in radians. An example $x(n)$ when $\phi = \pi/2$ is shown in Figure P1–13 where the $x(n)$ sequence, represented by the circular dots, is a single sinusoid whose frequency is f_o Hz.

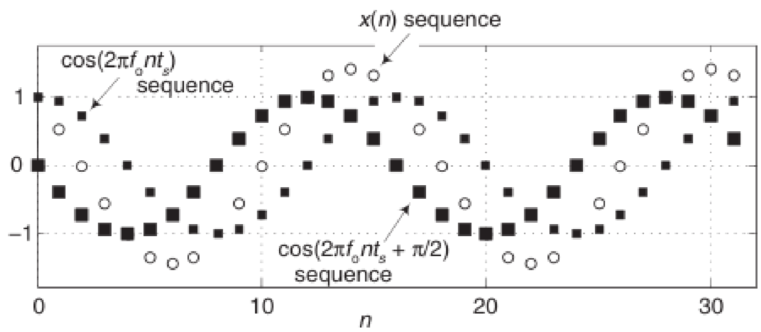


Figure P1-13

Using the trigonometric identity $\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos(\alpha)\cos(\beta)$, derive an equation for $x(n)$ that is of the form

$$x(n) = 2\cos(\alpha)\cos(\beta)$$

where variables α and β are in terms of $2\pi f_o n t_s$ and ϕ.

Answer:

To express x(n) in the form of x(n) = 2cos(α)cos(β), it is needed to identify α and β from the given expression:

$$\cos\,(2\pi\, f_0 n t_s +\ \varphi) + \cos\,(2\pi f_0 n t_s)$$

$$\cos\,(\alpha + \beta) + \cos\,(\alpha - \beta)$$

$$\alpha = 2\pi f_0 n t_s$$

$$\beta =\ \varphi$$

Applying the trigonometric identity:

$\cos(\alpha+\beta)+\cos(\alpha-\beta) = 2\cos(\alpha)\cos(\beta)$

Substituting α and β gives:

$$\cos(2\pi\, f_0 n t_s + \varphi) + \cos(2\pi f_0 n t_s) = \cos(2\pi\, f_0 n t_s)\cos(\varphi)$$

The expression for x(n) becomes:

$$\mathrm{x(n)} = 2\cos(2\pi\, f_0 n t_s)\cos(\varphi)$$

The result shows that the sequence x(n) is essentially a cosine wave at frequency f0 modulated in amplitude by cos(φ), a constant factor determined by the phase shift φ.