particle. physics





Radiation length values

What is the X0 of Pb?

- Three options
 - Search tabulated value on PDG!
 - 2. Compute from full formula

$$X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \ln \frac{183}{Z^{\frac{1}{3}}}}$$

3. Compute from approximated formulas...

 $X_0 = \frac{180A}{Z^2} \frac{g}{cm^2}$

$$egin{split} X_0 &= 716.4 ext{ g cm}^{-2} rac{A}{Z(Z+1) \ln rac{287}{\sqrt{Z}}} \ X_0 &= 1433 ext{ g cm}^{-2} rac{A}{Z(Z+1)(11.319 - \ln Z)} \end{split}$$

Cherenkov counter exercise!



- A threshold Cherenkov detector is used to separate muons from pions in a beam with momentum |p| = 150 MeV.
 - ✓ What values of the refraction index can be used?

A simple shower model

Simple shower model: [from Heitler]

Only two dominant interactions: Pair production and Bremsstrahlung ...

γ + Nucleus → Nucleus + e⁺ + e⁻ [Photons absorbed via pair production]

e + Nucleus → Nucleus + e + γ
[Energy loss of electrons via Bremsstrahlung]

Shower development governed by X₀ ...

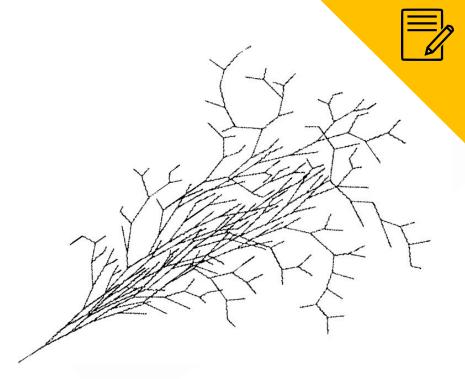
After a distance X_0 electrons remain with only $(1/e)^{th}$ of their primary energy ...

Photon produces e^+e^- -pair after $9/7X_0 \approx X_0 ...$

Assume:

 $E > E_c$: no energy loss by ionization/excitation

 $\mathsf{E} < \mathsf{E}_{c}$: energy loss only via ionization/excitation



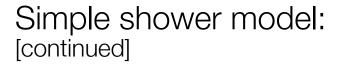
Use Simplification:

 $E_{\gamma} = E_e \approx E_0/2$ [E_e looses half the energy]

 $E_e \approx E_0/2$ [Energy shared by e^+/e^-]

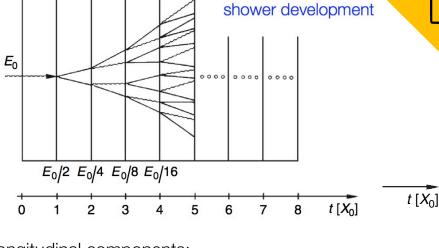
... with initial particle energy E₀

A simple shower model



Shower characterized by:

Number of particles in shower Location of shower maximum Longitudinal shower distribution Transverse shower distribution



Longitudinal components; measured in radiation length ...

... use:
$$t = \frac{x}{X_0}$$

Sketch of simple

Number of shower particles after depth t:

$$N(t) = 2^t$$

Energy per particle after depth t:

$$E = \frac{E_0}{N(t)} = E_0 \cdot 2^{-t}$$

$$\rightarrow$$
 $t = \log_2(E_0/E)$

Total number of shower particles with energy E_1 :

$$N(E_0, E_1) = 2^{t_1} = 2^{\log_2(E_0/E_1)} = \frac{E_0}{E_1}$$

Number of shower particles at shower maximum:

$$N(E_0,E_c)=N_{
m max}=2^{t_{
m max}}=rac{E_0}{E_c}$$
 Shower maximum at: $\propto E_0$

$$t_{
m max} \propto \ln(E_0/E_c)$$

A simple shower model



Simple shower model: [continued]

Longitudinal shower distribution increases only logarithmically with the primary energy of the incident particle ...

Some numbers:
$$E_c \approx 10$$
 MeV, $E_0 = 1$ GeV \rightarrow $t_{max} = ln \ 100 \approx 4.5; N_{max} = 100$ $E_0 = 100$ GeV \rightarrow $t_{max} = ln \ 10000 \approx 9.2; N_{max} = 10000$

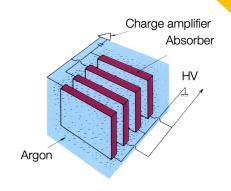
$$t_{\max}[X_0] \sim \ln \frac{E_0}{E_c}$$

A sampling calorimeter...

- The ATLAS electromagnetic calorimeter is made from roughly 2 mm thick layers of lead (Pb), interleaved by 2 mm wide gaps filled with liquid Argon (LAr).
 - ✓ Pb: Z = 82, A = 206, density = 11.34 g/cm³
 - ✓ LAr: Z = 18, A = 40 density = 1.4 g/cm³.
- At $\eta = 0$ the depth of the ATLAS electromagnetic calorimeter is ~22 X_0
- What is the calorimeter depth in cm?
 - \checkmark Hint; compute $X_0(Pb)$ and $X_0(LAr)$
- What would it be is if it was a homogeneous calorimeter (i.e. all made of LAr)?
- And if it was all made of Pb?



A sampling calorimeter...





A sampling electromagnetic calorimeter is composed of series of lead layers about 2 mm thick layers of lead (Pb)¹. Between the lead layers are 2 mm wide gaps filled with liquid Argon (LAr). Lead has a Z=82, A=206 and a density of 11.34 g/cm³. Liquid argon has a Z=18, A=40 and a density of 1.4 g/cm³.

- 1. At $\eta=0$ the depth of the ATLAS electromagnetic calorimeter is about 22 radiation lengths X_0 . What would be the depth of the detector in cm if it was all made of LAr? And if it was all made of lead?
- 2. An electron of 5 GeV generated an electromagnetic shower in the calorimeter. Assuming that the detector was all made of LAr, at what depth would the shower reach its maximum?
- 3. How much energy does a minimum-ionizing-particle (mip) deposit in 22 X_0 of LAr, assuming:

$$\frac{1}{\rho_{\rm LAr}} \left(\frac{dE}{dx}\right)_{\rm mip} = 1.52 \,\text{MeV/(g} \cdot \text{cm}^{-2}) \tag{1}$$

4. How deep in cm is the *real* ATLAS electromagnetic calorimeter at $\eta = 0$, assuming a perfect succession of lead and liquid argon layers of the same thickness?