Evolving Bids for a Fantasy-Football Auction

Metaheuristics and Inverse Optimization in a Multi-Manager Setting

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2024/25

Agenda

Problem Statement

Goal

Optimise a multi–manager fantasy-football auction by evolving a vector of **bids** that maximises total team score while satisfying *all* hard constraints.

Notice (CLI)

B	current budget of *one* manager
b_i	bid placed by that manager for player i
$N_{\max}$	maximum squad size (user input)
N_{rem}	empty slots still to be filled
m_r, M_r	min / max players for role r
r	role index: P (GK), D, M, F

Context

- Fantasy football cast as an *inverse-optimisation* task
- 12 managers, each selects **PSO**, **DE** or **ES**
- Conflicts resolved by deterministic rebids
- Fitness = real score – penalties

Inputs (CLI)

- Initial budget per manager B ($500\,$cr default)
- Max squad size $N_{\max}$
- Role quotas (m_r, M_r) for $r \in \{P, D, M, F\}$
- Data set: 600+ Serie A players (23/24 stats)

Hard constraints

- Bid domain $b_i \in \{0\} \cup [1, \infty)$
- Turn budget $\sum_i b_i \leq B$
- Per-player cap $b_i \leq 0.4B$
- Reserve credits $B - \sum b_i \geq N_{\text{rem}}$
- Squad size $|\text{team}| \leq N_{\max}$
- Role quotas enforced every turn

Genotype vs Phenotype

Genotype

- Continuous bid vector: $\mathbf{b} = (b_1, b_2, \ldots, b_n)$
- Index i is bound to a fixed player
- Crossover / mutation touch numerical values only
- *Example:* $(30.5, 0, 7.8, \ldots, 0)$

Phenotype

- Simulated squad obtained from $\mathbf{b}$
- Includes roles, spent budget, expected score
- Graded via fantasy-football scoring rules

Algorithmic Formulas & Parameters

Particle Swarm Optimization (PSO)

$$\mathbf{v}_i^{t+1} = \omega \mathbf{v}_i^t + c_1 r_1 (\mathbf{p}_i - \mathbf{x}_i^t) + c_2 r_2 (\mathbf{g} - \mathbf{x}_i^t)$$

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{v}_i^{t+1}$$

Differential Evolution (DE)

$$\mathbf{y} = \mathbf{x}_a + F(\mathbf{x}_b - \mathbf{x}_c)$$

$$z_j = \begin{cases} y_j & \text{if } r_j < CR \text{ or } j = j_{\text{rand}} \\ x_{i,j} & \text{otherwise} \end{cases}$$

Evolution Strategies (ES)

$$\mathbf{x}_{\text{child}} = \mathbf{x}_{\text{parent}} + \sigma \mathcal{N}(0, I)$$

(best μ of parents + offspring survive)

Algorithm Parameters

Alg.	Param.	Desc.	Val.
PSO	ω	inertia	0.7
PSO	c_1	cog.	1.8
PSO	c_2	soc.	1.8
DE	F	diff. w.	0.5–1.0
DE	CR	cross.	0.7
ES	μ	parents	40
ES	λ	offspr.	80

Fitness Function

$$\mathcal{F}(\mathbf{b}) = \mathrm{Penalty}(\mathbf{b}) - \sum_{i \in \mathcal{A}(\mathbf{b})} w_i \mathrm{Score}_i, \qquad \mathcal{A}(\mathbf{b}) = \{i \mid b_i \geq \mathsf{thr}\}$$

- Penalty mixes *budget leftover*, missing roles, squad size errors
- w_i doubles if the role is currently under-represented
- Minimisation problem: lower $\mathcal{F} \leftrightarrow$ stronger squad, fewer violations
- $thr = 1$

```
def score_player(player):
        goals = getattr(player, 'goals_scored', 0)
        ....
        matches = getattr(player, 'matches_played', 0)
        return (0.5 * goals + 0.2 * assists - 0.05 * yellow - 0.1 * red + 0.2 * rating + 0.2
             * pens - 0.5 * conceded + 0.5 * saved + 0.5 * matches)
```

Listing: Function score

Auction Conflict Heuristic

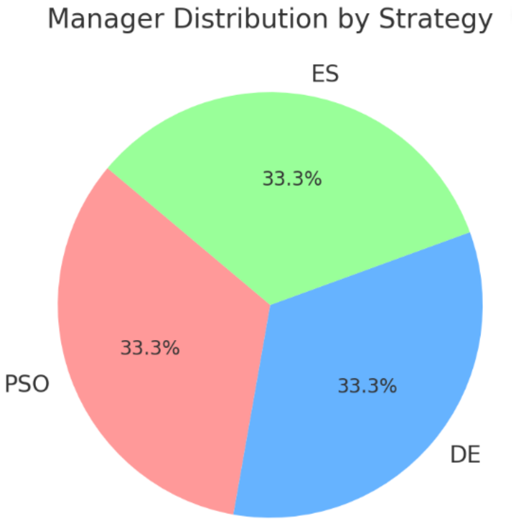
1. Gather all bids (mgr, player, b)
2. Group by player
3. Single bidder $\Rightarrow$ immediate assignment
4. Otherwise:
   - Sort bids $b_1 \geq b_2 \geq \dots$
   - If $b_1 - b_2 > g_{\text{trigger}} \Rightarrow$ highest wins
   - Else launch up to 5 dynamic rebids

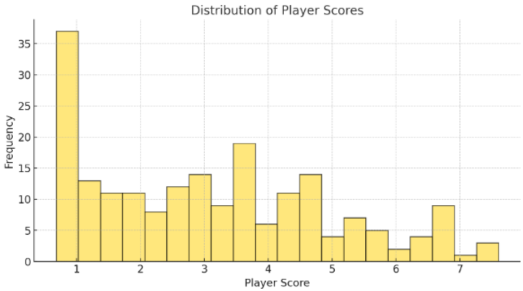
   (rebids = recompute bids with small noise or fallback threshold)

```
# Inputs: b1 = top bid, b2 = second bid, B = second manager's remaining budget, n = number
    of players still needed
ratio = B / n
gap = b1 - b2
#Compute dynamic rebid increment
dynamic_inc = max(1, int(round(gap / 2 * ratio))) + 1
# Apply rebid
b2 += dynamic_inc
```

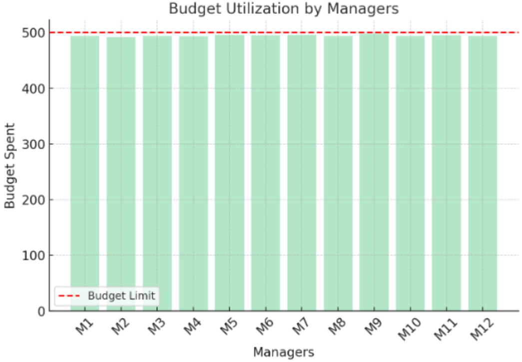
Listing: Semplified Dynamic rebid heuristic

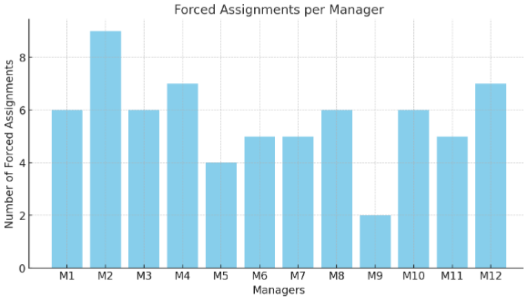
Example: Manager and Player Distributions





Example: Budget and Forced Assignments





Example: Tabular Analyses

Performance by Strategy

Str.	Mgr	Avg Score	Avg Forced
PSO	4	49.9	6
DE	4	**51.8**	7
ES	4	44.8	**4**

Manager Recap

Mgr	Forced	Spent	Score
1	6	494	52.9
2	9	492	51.5
3	6	494	44.5
4	7	493	53.6
5	4	496	49.2
6	5	495	46.0
7	5	496	47.0
8	6	494	55.3
9	2	498	44.2
10	6	494	46.6
11	5	495	40.0
12	7	494	53.2

Player Score Summary

	Best	Worst	Avg
Score	12.9	0.68	2.84

Hyper-parameter Tuning – Design

Methodology

- Same 25-player pool, fixed seed, 60 auction turns
- Test manager + 1 random rival (guarantees bidding pressure)
- Fitness = sum of final squad scores

Cartesian products $\Rightarrow$ 24 runs

- PSO = 2 inertia weights $\times$ 2 swarm sizes = **4** runs
- DE = 3 population sizes $\times$ 2 F ranges $\times$ 2 CR = **12** runs
- ES = 4 $(\mu + \lambda)$ pairs $\times$ 2 generation counts = **8** runs

Why these ranges?

- Values are the *standard defaults* most cited in the literature (Clerc and Kennedy for PSO, Storn and Price for DE, $\lambda > \mu$ rule for ES)
- Wide enough to capture meaningful variance yet small enough to keep the grid computationally feasible.

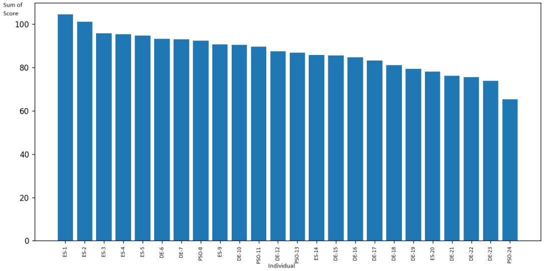
Hyper-parameter Tuning – Results

Best configuration for algorithm

Alg.	Best h-p	Sum Score
ES	$\mu = 20,\ \lambda = 40,\ ngen = 50$	**104.7**
DE	pop$= 15,\ F = (0.5, 1.0),\ CR = 0.7$	93.3
PSO	swarm$= 60,\ w = 0.9,\ c_1 = c_2 = 1.49445$	92.4

Parameter grid

PSO	$w \in \{0.9, 0.5\}$, $c_1 = c_2 = 1.49445$, swarm $\{30, 60\}$
DE	pop $\{10, 15, 20\}$, $F \in \{(0.5, 1.0), (0.7, 1.2)\}$, $CR \in \{0.7, 0.9\}$
ES	$(\mu + \lambda) \in \{(15, 30), (15, 40), (20, 30), (20, 40)\}$, $ngen \in \{50, 80\}$



Sum of score for every configuration (higher = better).

Inverse multi–tuning/1

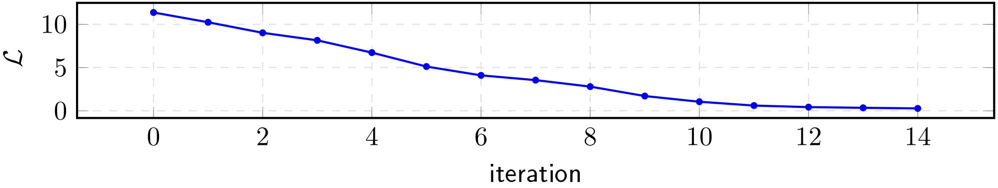
Goal

Fit the hyper–parameters of **PSO, DE & ES** so that each auctioned roster reproduces a target triple $(\text{score}, \text{forced picks}, \text{leftover}) = (100, 4, 0)$.

$$\mathcal{L}(\theta) = |\,\text{score} - 100\,| + |\,\text{forced} - 4\,| + |\,\text{leftover} - 0\,| \qquad \longrightarrow \min_{\theta}$$

- **Outer optimiser:** 30-particle PSO (40 iterations, $\omega = 0.7,\ c_1 = c_2 = 1.5$).
- **Search space:** *algo_id* $\in \{$0:PSO, 1:DE, 2:ES$\}$ + 4 real h-params.
- **Best configuration found:**
  DE (pop $= 10,\ F = [0.7, 1.2],\ CR = 0.7) \Rightarrow \mathcal{L}_{\min} = 0.278$.

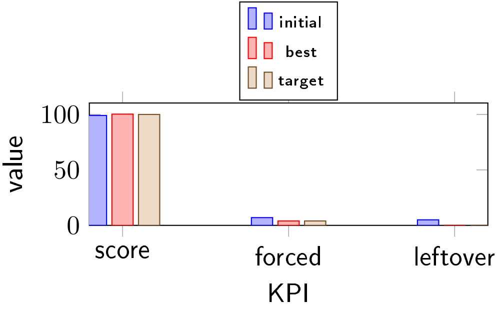
Convergence of the inverse tuning process: the total loss $\mathcal{L}$ progressively approaches zero.



Top-5 trials

Alg.	Pop./Swarm / $(\mu+\lambda)$	$F/(\omega)$	$CR/c_{1,2}$	$\mathcal{L}$
DE	10	0.7–1.2	0.7	0.278
ES	(20,30)	–	–	1.86
ES	(20,40)	–	–	1.98
PSO	60	0.5	1.49	4.08
DE	20	0.7–1.2	0.7	4.20

Best configurations found during inverse tuning: DE and ES performed best in reproducing the target profile.



KPI comparison: the best configuration closely matches the target (score 100, 4 forced picks, 0 leftover credits).