Signals and Systems 2

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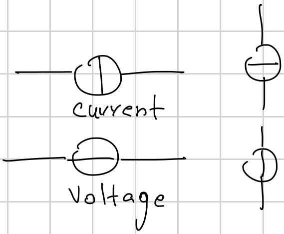

$$H(j\omega) = \frac{\text{output}}{\text{excitation (input)}} \longrightarrow \text{complex domain}$$

↓ transfer characteristic

Voltage split → $\frac{\text{own}}{\text{sum}}$

Current Split → $\frac{\text{Other}}{\text{sum}}$

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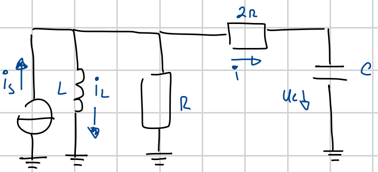
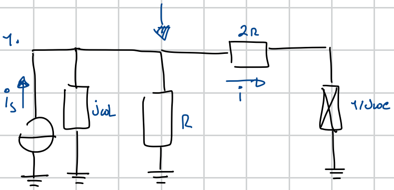
Example 1: revision

regular →	R	C	L
complex domain →	R	$1/j\omega C$	$j\omega L$



Whats the transfer characteristic ?

1. Assume circuit is in complex domain
2. Current Split
3. Simplify



parallel Notation $\parallel$ or $\times$

$$2.\quad H(j\omega) = \frac{i}{i_S} = \frac{\text{other}}{\text{sum}} = \frac{R \parallel j\omega L}{R \parallel j\omega L + 2R + 1/j\omega C}$$

$$= \frac{\frac{R \cdot j\omega L}{R + j\omega L}}{\frac{R \cdot j\omega L}{R + j\omega L} + 2R + 1/j\omega C} \overset{\text{Multiply}}{=} \frac{R \cdot j\omega L}{R \cdot j\omega L + 2R^2 + 2R j\omega L + \frac{R + j\omega L}{j\omega C}} =$$

$$\frac{(j\omega)^2 RLC}{3(j\omega)^2 RLC + 2R^2 j\omega C + R + j\omega L} = \frac{(j\omega)^2 RLC}{3(j\omega)^2 RLC + j\omega(2R^2C + L) + R}$$

$$= \frac{(j\omega)^2/3}{(j\omega)^2 + j\omega\left(\frac{2R^2C}{3RLC} + \frac{L}{3RLC}\right) + R/3RLC}$$

$$H(j\omega) = \frac{1/3 \cdot (j\omega)^2}{(j\omega)^2 + j\omega\left(\frac{2R}{3L} + \frac{1}{3RC}\right) + \frac{1}{3LC}}$$

Example 2: 1. find output

input

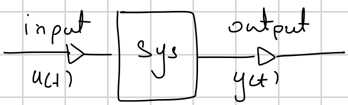
$$u(t) = [\underset{\omega_1 = 0}{10} + \underset{\omega_2 = 1}{4\cos(t + 15^\circ)} + \underset{\omega_3 = 5}{3 \cdot \cos(5t - 10^\circ)}]$$

$\quad \to 4e^{j15} \qquad \to 3e^{-j10^\circ}$

transfer characteristic

$$H(j\omega) = \frac{3j\omega + 6}{(j\omega)^2 + 4j\omega + 9}$$

System



coherent unit system

V, kΩ, μS → Mrad/s

mA mH/nF

mW

conversion table

ω	$\bar{u}$	$\cdot\ \bar{H}$	→ $\bar{Y}$
0	10	6/9	6.66
1	$4e^{j15}$	3/4	$3e^{j15}$
5	$3e^{-j10}$	$0.6307e^{-j60.4}$	$1.8921e^{-j70.4}$

$$H(j\omega)\Big|_{\omega=\phi} = \frac{6}{9}$$

$$H(j\omega)\Big|_{\omega=1} = \frac{3j + 6}{(j)^2 + 4j + 9} = \frac{3j + 6}{4j - 1 + 9} = \frac{3j + 6}{4j + 8} = \frac{3}{4}$$

use calc to find [substitute with $\underline{\omega}$]

output (solution)

$$y(t) = [6.66 + 3\cos(t + 15^\circ) + 1.8921\cos(5t - 70.4)]\ \text{mA}$$

2. find complex power $\bar{S}$ of this system 2025. 9. 10

$L = 0.4\,mH \quad R = 5k\,\Omega$

$$H(j\omega)\Big|_{\omega=3} = 0.6307e^{-j60.4}$$

$r\angle\phi$ form

$$\bar{S} = U I^{*} = (i)^2 \bar{Z} \rightarrow \text{impedance}$$

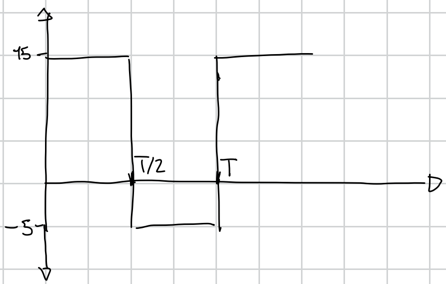
$$\bar{S}_0 = (i)^2 R = (6.66)^2 \cdot 5 = 221.778\ mW$$

$$\bar{S}_1 = \frac{(i)^2}{2}\bar{Z} = (6.66)^2 \cdot (5 + 0.4 \cdot \underbrace{\omega j}_{\omega_1})/2 = 111.243e^{4.57}\ mW$$

$$\bar{S}_2 = (6.66)^2 \cdot (5 + \underbrace{0.4 \cdot 5j}_{j\omega L}) = 110.889 + 44.355j$$

$$i_{rms} = \sqrt{(6.66)^2 + \frac{3^2 + 1.89^2}{2}} = \quad mA$$

Example 3: Fourier



$$\omega_0 = \frac{2\pi}{T} \leftarrow f = \frac{1}{T}$$

$$U_0 = \frac{1}{T}\int u(t)\,dt = \frac{1}{T}\left[\int_0^{T/2} 15\,dt + \int_{T/2}^{T} -5\,dt\right]$$

$$= \frac{1}{T}\left[15T/2 - (5T - 5T/2)\right] = 5$$

$$\bar{U}_i = \frac{1}{T}\left[\int_0^{T/2} 15e^{-j\omega_0 t i}\,dt + \int_{T/2}^{T} -5e^{-j\omega_0 t i}\,dt\right] =$$

$$\frac{15(e^{-j\omega_0 \frac{T}{2} i} - e^{-j\omega_0 0 i})}{-j\omega_0 i T} - \frac{5(e^{-j\omega_0 T i} - e^{-j\omega_0 \frac{T}{2} i})}{-j\omega_0 i T}$$

$$\frac{1}{-j2\pi i}\left[15(e^{-j\pi i} - 1) - 5(e^{-j2\pi i} - e^{-j\pi i})\right] =$$

$i = 1 \qquad e^{-j\pi} = \cos(\pi) - j\sin(\pi)$

$e^{-j\pi} = -1$

$$\frac{1}{-j2\pi}\left[15(\underbrace{e^{-j\pi}}_{-1} - 1) - 5(\underbrace{e^{-j2\pi}}_{1} - \underbrace{e^{-j\pi}}_{-1})\right] =$$

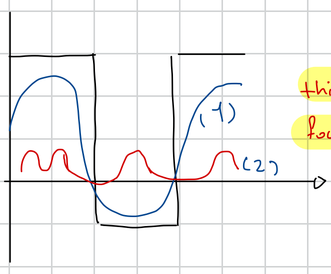
$$\frac{1}{-2\pi j}\left[15(-2) - 5(2)\right] = \frac{1}{j2\pi}\left[-30 - 10\right]$$

$i = 2$

$$\frac{1}{-j4\pi}\left[15(\underbrace{e^{-j2\pi}}_{+1} - 1) - 5(\underbrace{e^{-j4\pi}}_{1} - \underbrace{e^{-j2\pi}}_{+1})\right] =$$

iterate for $i = 3$

$$u(t) = \left[U_0 + \bar{U}_1 + \bar{U}_2 + \bar{U}_3\right]$$



third order fourier expression

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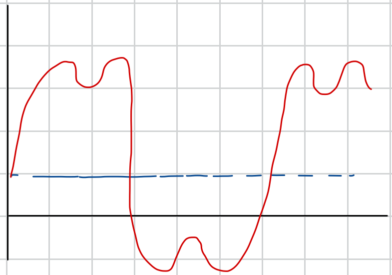
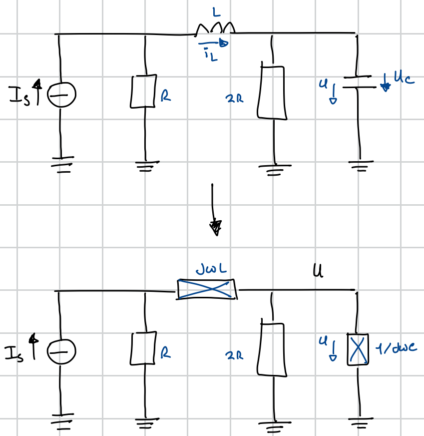


Harmonic Combination

Example 4: Find transfer characteristic of the system



$$H(j\omega) = \frac{u}{I_s} = \frac{R}{2R \times 1/j\omega C + R + j\omega L} \cdot \frac{2R}{\frac{(2R + 1/j\omega C)}{j\omega C}}$$

$$= \frac{R}{\frac{2R \cdot 1/j\omega C}{2R + 1/j\omega C} + R + j\omega L} \cdot \frac{2R}{\frac{(2R + 1/j\omega C)}{j\omega C}}$$

$$= \frac{R}{\frac{2R \cdot 1/j\omega C}{2R + 1/j\omega C} + R + j\omega L} \cdot \frac{2R}{\frac{(2R + 1/j\omega C)}{j\omega C}}$$

$$= \frac{2R^2}{\frac{2R \cdot 1/j\omega C}{j\omega C} + \frac{R(2R + 1/j\omega C)}{j\omega C} + \frac{j\omega L(2R + 1/j\omega C)}{j\omega C}}$$

$$= \frac{2R^2}{\frac{2R}{(j\omega C)^2} + \frac{R(2R + 1/j\omega C)}{j\omega C} + \frac{L(2R + 1/j\omega C)}{C}}$$

$\omega)^2 C$

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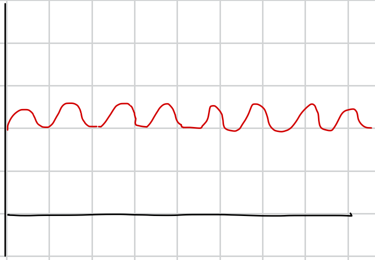
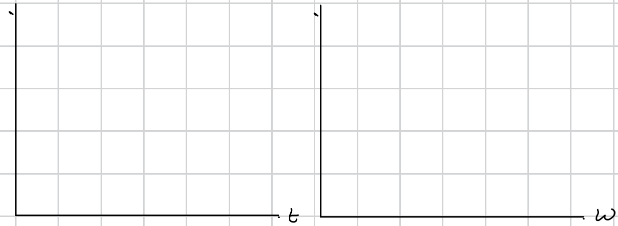
fourier transform



time / frequency domains



frequencies and their amplitudes

$$\tilde{F}(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

fourier series and transform can be used for different cases. Series for periodic functions. Transform for any function (integrable)

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{F}(j\omega)\, e^{j\omega t}\, d\omega$$

Inverse fourier transform

I. $c\mathcal{F}\{x(t)\} = \mathcal{F}\{c\,x(t)\}$

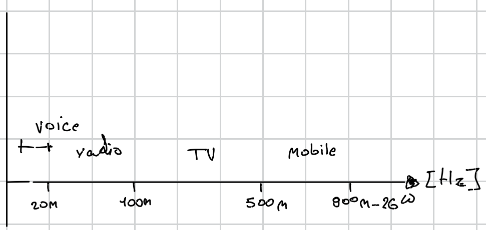
$\mathcal{F}\{x_1(t)\} + \mathcal{F}\{x_2(t)\} = \mathcal{F}\{x_1(t) + x_2(t)\}$

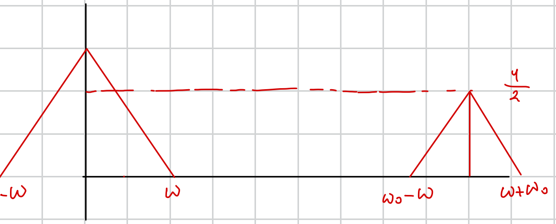
II. $\mathcal{F}\{x(t)\,e^{-\alpha t}\} = x(j\omega+\alpha)$

$\alpha > 0!$

III. Modulation theorem

$$\mathcal{F}\{x(t)\,e^{-j\omega_0 t}\} = x(j(\omega+\omega_0))$$





Spectrum of frequency

$x(jw)$

$x(t)e^{-jw^0 t}$

$x(t)\cos w_0 t = x(t)\dfrac{e^{jw_0 t} - e^{-jw_0 t}}{2}$

IV Derivative theory

$$f\left\{\int x(t)\,dt\right\} = -\frac{x(jw)}{jw} + \delta(x+\emptyset)$$

V -