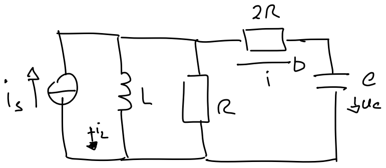
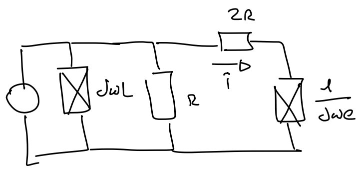
Signals and Systems 2

2025. 9. 10 Tyauma



whats the transfer characteristic?



$$H(j\omega) = \frac{R \times j\omega L}{R \times j\omega L + 2R + \frac{1}{j\omega C}}$$

$$\frac{\frac{(R \cdot j\omega L)}{R + j\omega L}}{\frac{R \cdot j\omega L}{R + j\omega L} + \frac{2Rj\omega C + 2R}{j\omega C}} = \frac{Rj\omega L}{Rj\omega L + 2R^2 + \frac{R}{j\omega C} + 2Rj\omega L + \frac{j\omega L}{j\omega C}}$$

$$= \frac{(j\omega)^2 RLC}{3(j\omega)^2 LRC + 2R^2 j\omega C + j\omega L} = \frac{\frac{1}{3}(j\omega L)^2}{(j\omega)^2 + j\omega\left(\frac{2R}{3L} + \frac{1}{3RC}\right) + \frac{1}{3LC}}$$

HW: check SVD

$\dot{u}_C = ?$ $\varphi = L i_L$

$\dot{i}_L = ?$

$i = ? \cdot (C\dot{u}_C)$

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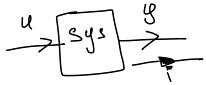
Example 1

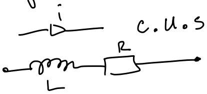
$H(j\omega) = \dfrac{3j\omega + 6}{(j\omega)^2 + 4j\omega + 9}$



[V, kΩ, μS → mA; MH, nF → Mrad; mW]

RE DO at home

$y(t) =$



c.u.s

RL

$L = 0.4 \quad R = 5\,\Omega$

$\bar{S} = ?$

$u(t) = [10 + 4\cdot\cos(t + 15^\circ) + 3\cdot\cos(5t - 10^\circ)]$

ω	$\bar{U}$	$\bar{H}$	$\bar{Y}$
0	10	$\frac{6}{9}$	$\frac{60}{9}$
1	$4e^{j15}$	$\frac{3}{4}$	$3e^{j15}$
5	$3e^{-j10}$	$0.6307e^{-j60}$	$1.89e^{-j70.5}$

$H(1) = \dfrac{3j + 6}{j^2 + 4j + 9} \qquad H(5) = \dfrac{15j + 6}{25(j)^2 + 20j + 9}$

$y(t) = \left[\frac{60}{9} + 3\cos(t + 15^\circ) + 1.89\cos(5t - 70.5^\circ)\right]$ mA

$\bar{Z}_0 =$

$\bar{S}_0 = (i)^2\bar{Z} = i_0^2 \cdot R = \left(\frac{60}{9}\right)^2 \cdot 5 = \frac{2000}{9}$

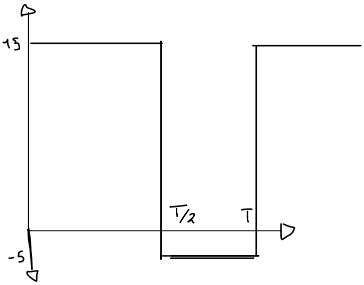
$\bar{S}_1 = \frac{1}{2}(i)^2\bar{Z} = \frac{1}{2}(3)^2 \cdot (5 + j0.4) = \frac{9\sqrt{629}}{10}e^{j4.57}$

$\bar{S}_1 = \frac{1}{2}(i)^2\bar{Z} = \frac{1}{2}(1.89)^2 \cdot (5 + j0.4\cdot5) = 9.618e^{j21.8}$

$\bar{S} = \frac{2000}{9} + \frac{9\sqrt{629}}{10}\cos(t + 4.57^\circ) + 9.618\cos(5t + 21.8)^\circ$

$I_{RMS} = \sqrt{\left(\frac{60}{9}\right)^2 + \frac{3^2 + 1.89^2}{2}}$

$I_{RMS}^2 \cdot 5 = A + B + C$



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Example 2

re do at home

$$\sqrt{\frac{1}{T}\int_0^T (u(t))^2\,dt} = \sqrt{\frac{1}{T}\left[\int_0^{T/2} 15^2\,dt + \int_{T/2}^{T} (-5)^2\,dt\right]} = \sqrt{\frac{1}{T}\cdot 125T}$$

$$= \sqrt{125} = 11.18$$

$$u_0 = \frac{1}{T}\int_0^T u(t)\,dt = \frac{1}{T}\left[\int_0^{T/2} 15\,dt + \int_{T/2}^{T} -5\,dt\right] = 5$$

$$\overline{U}_1 = \frac{1}{T}\left[\int_0^{T/2} 15e^{-ji\omega_0 t}\,dt + \int_{T/2}^{T} -5e^{-ji\omega_0 t}\right]$$

$$= \frac{1}{T}\left(\frac{15}{-ji\omega_0}\left[e^{-ij\omega_0\frac{T}{2}} - 1\right] + \frac{5}{ji\omega_0}\left[e^{-ji\omega_0 T} - e^{-ji\omega_0\frac{T}{2}}\right]\right)$$

$$= \frac{1}{Tji\frac{2\pi}{T}}\left[\left(15(1 - \underset{\pm 1}{e^{-ji\pi}}) + 5(\underset{1}{e^{-ji2\pi}} - \underset{\mp 1}{e^{-ji\frac{2\pi}{T}\frac{T}{2}}})\right)\right]$$

$$= \frac{1}{ji2\pi}\left(20 - 20^{-ji\pi}\right)$$

$$\overline{U}_1 = \frac{1}{j2\pi}(20 + 20) = \frac{20}{j\pi}$$

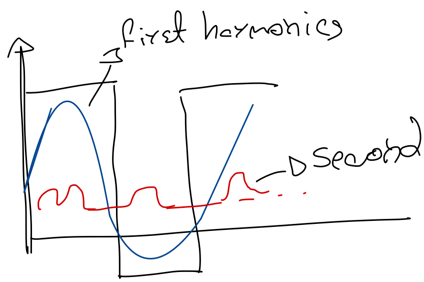
$$\overline{U}_2 = \frac{1}{j4\pi}(20 - 20) = \emptyset$$

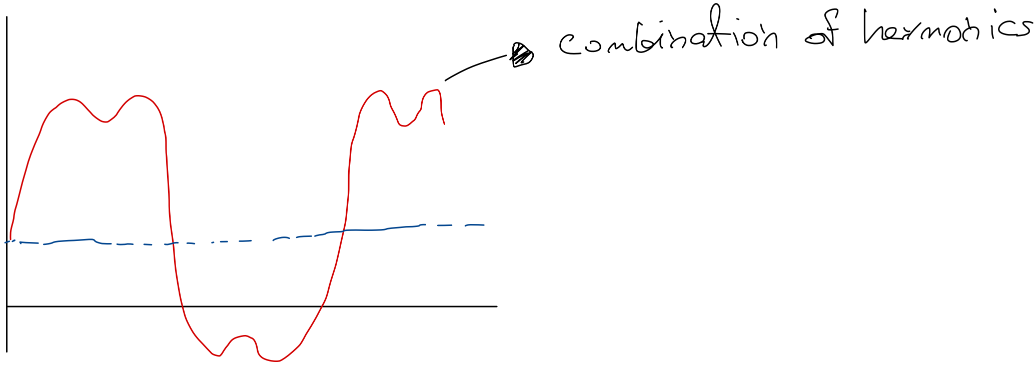
$$\overline{U}_3 = \frac{1}{j6\pi}(20 + 20) = \frac{20}{3j\pi}$$

$$u(t) = 5 + \frac{40}{\pi}\cos\left(\omega_0 t - \frac{\pi}{2}\right) + 0 + \frac{40}{3\pi}\cos\left(3\omega_0 t - \frac{\pi}{2}\right)$$

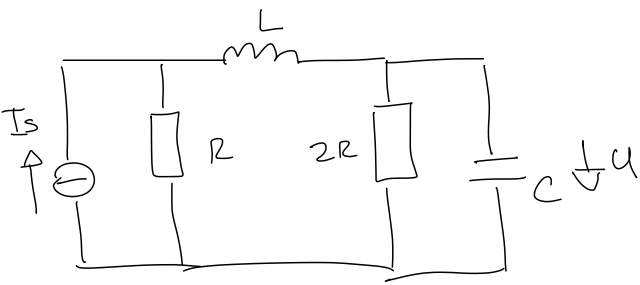
Third order approx of fourier series

$$u_{RMS} = \sqrt{3^2 + \frac{\left(\frac{40}{\pi}\right)^2 + \left(\frac{40}{3\pi}\right)^2}{2}} = 11.535$$





Example 3 2023.9.10



$$H(j\omega) = I \cdot \frac{R}{\frac{1}{j\omega C} \times 2R + j\omega L + R} \cdot \frac{2R}{2R + \frac{1}{j\omega C}} \cdot \frac{1}{j\omega C}$$

$$H(j\omega) = \frac{R}{\dfrac{\frac{1}{j\omega C} \cdot 2R}{\frac{1}{j\omega C} + 2R} + j\omega L + R} \cdot \frac{2R}{\left(2R + \frac{1}{j\omega C}\right)} \frac{1}{j\omega C} =$$

$$\frac{\dfrac{2R^2}{j\omega C}}{\dfrac{2R}{j\omega C} + 2R j\omega L + 2R^2 + \dfrac{j\omega L}{j\omega C} + \dfrac{R}{j\omega C}} =$$

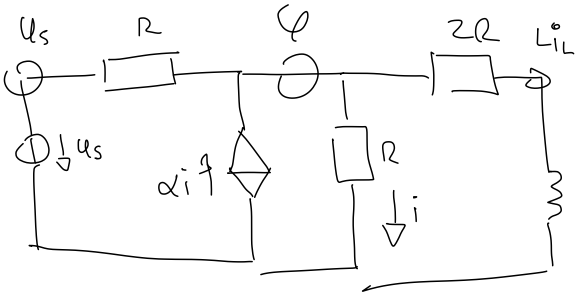
$$\frac{2R^2}{2R + 2R(j\omega)^2 LC + 2R^2(j\omega C) + j\omega L + R} =$$

$$= \left(\frac{2R^2}{3R + j\omega L + (j\omega)^2 2RLC + 2R^2 j\omega C} \right) = \left[\frac{\frac{R}{LC}}{(j\omega)^2 + j\omega\left(\frac{1}{2RC} + \frac{R}{L}\right) + \frac{3}{2LC}} \right]$$

correct transfer characteristic

$$H(j\omega) = \left[\frac{\frac{R}{LC}}{(j\omega)^2 + j\omega\left(\frac{1}{2RC} + \frac{R}{L}\right) + \frac{3}{2LC}} \right]$$

$u(t) = ?$



$H(j\omega) = ?$

$i = \frac{\varphi}{R}$

$Ri = \varphi$

$$\varphi: \quad \frac{\varphi - U_s}{R} + \frac{\varphi}{R} + \frac{\varphi - L i_L'}{2R} - \alpha i = \emptyset$$

$$2\varphi - 2U_s + 2\varphi + \varphi - L i_L' - 2R\alpha i$$

$$5Ri - 2U_s - L i_L' - 2R\alpha i = \emptyset$$

$$\frac{L i_L' - Ri}{2R} + i_L = \emptyset$$

$$L i_L' = Ri - 2R i_L$$

$$i = \frac{L i_L'}{R} + 2 i_L$$

$$5R\left(\frac{L i_L'}{R} + 2 i_L\right) - 2U_s - L i_L' - 2R\alpha\left(\frac{L i_L'}{R} + 2 i_L\right)$$

$$= 5L i_L' + 10R i_L - 2U_s - L i_L'$$

$$- \frac{2\cancel{R}\alpha L i_L'}{\cancel{R}} - 4R\alpha i_L = \emptyset$$

$$i_L(10R - 4R\alpha)$$

$$= 4L i_L' - 2\alpha L i_L' - 2U_s + 10R i_L - 4R\alpha i_L =$$

$$L i_L'(4 - 2\alpha) = i_L(4R\alpha - 10R) + 2U_s$$

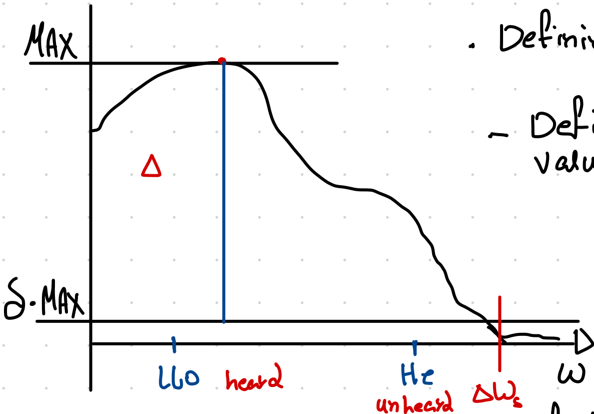
$$i_L' = \frac{i_L(4R\alpha - 10R)}{(4-2\alpha)L} + \frac{2U_s}{(4-2\alpha)L}$$

$$H(j\omega) = c^T (j\omega I - A)^{-1} B + D$$

$$\frac{-1}{2-\alpha} \quad \frac{1}{j\omega + \frac{5R \times R}{(2-\alpha)L}} \quad \frac{1}{(2-\alpha)L} + \frac{1}{(2-\alpha)R}$$

2025.9.15

Fourier Transform



- Defining signal bandwidth
  - Define some % of max value

$\Delta\omega_s$ signal BW

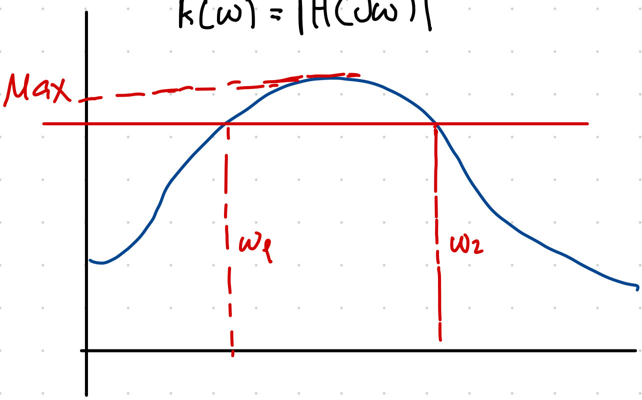
$\delta \leq 0.05$

$\Delta \geq 30$ dB

Network vs Signal BW

How do we determine network BW?

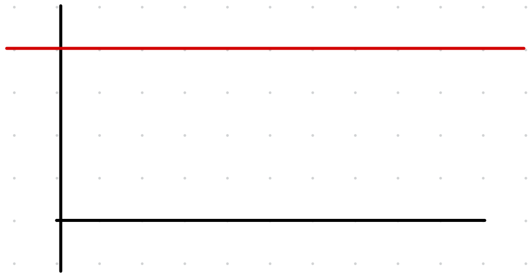
$k(\omega) = |H(j\omega)|$



$\delta \geq 0.7$

$\Delta \leq 3$ dB

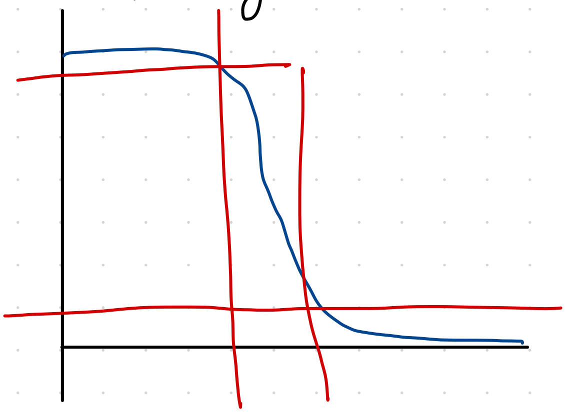
limit: $\dfrac{1}{\sqrt{1+\varepsilon^2}}$ $\varepsilon \leq 1$

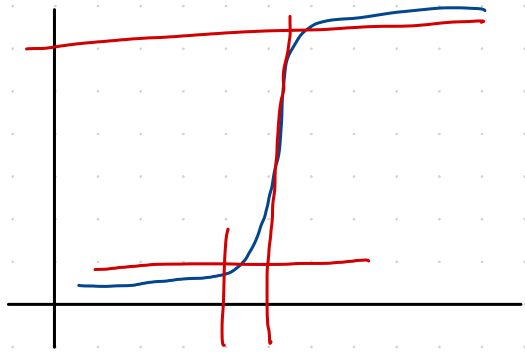


All pass system

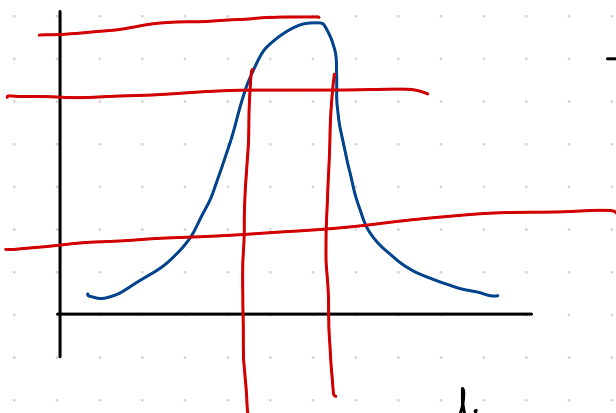
- All frequencies transferred through the network.

Low p. system





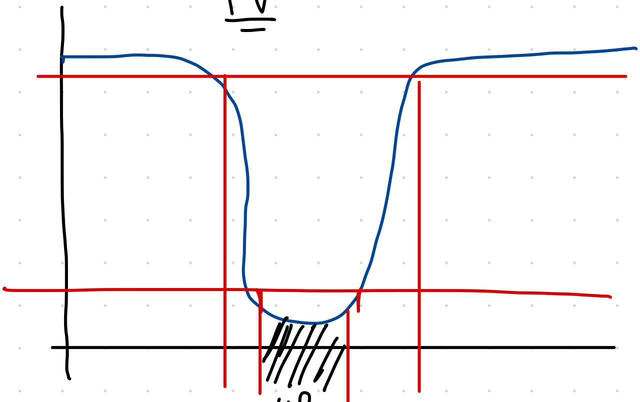
H pass



B p.

radios

IV



Bend closed

$u_s(t) = 10 \cdot e^{-4t}\, \varepsilon(t)$

BW of Signal ?

Suppose signal Spectrum

$\delta = 0.02$

1. Determine spectrum

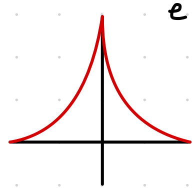
$$F\{u_s(t)\} = \int_{-\infty}^{\infty} u_s(t)\, e^{-j\omega t}\, dt = \int_{0}^{\infty} 10 e^{-4t} \cdot e^{-j\omega t}\, dt =$$

$$\frac{10\left(e^{-4\infty\, \infty j\omega} - e^{-4\cdot 0 j\omega}\right)}{-4 - j\omega} = \frac{10}{j\omega + 4}$$

$$\left|\frac{10}{j\omega+4}\right| = \frac{10}{\sqrt{\omega^2 + 4^2}}\Bigg|_{\omega=0} = \frac{10}{4}$$ Max Value

$f(t)$	$F(j\omega)$
$\delta(t)$	1
$\varepsilon(t)$	$\frac{1}{j\omega} + \pi\,\delta(j\omega)$
$\varepsilon(t)\cdot e^{-\alpha t}$, $\lvert\alpha\rvert > 0$	$\frac{1}{j\omega + \alpha}$
$t\cdot\varepsilon(t)e^{-\alpha t}$	$\frac{1}{(j\omega + \alpha)^2}$
$e^{-\alpha\lvert t\rvert}$	$\frac{2\alpha}{\alpha^2 + \omega^2}$

$\varepsilon(t)$ has no fourier transform because it is not absolute integrable



$$\left(\frac{1}{\alpha - j\omega}\right)$$

$$e^{-\alpha|t|} = \varepsilon(-t)\cdot e^{-\alpha t} + \varepsilon(t)e^{-\alpha t}$$

$$\left(\frac{1}{\alpha + j\omega}\right)$$

$$\frac{\alpha + j\omega + \alpha - j\omega}{\alpha^2 + \omega^2} = \frac{2\alpha}{\alpha^2 + \omega^2}$$

• if u have an even function the fourier of the system will always be a real function

2. everything below 0.02 is neglected

$$\delta.\,Max = \frac{10}{\sqrt{\omega_s^2 + 4^2}} = 0.02 \cdot \frac{\cancel{10}}{4} = \frac{\cancel{10}}{\sqrt{\omega_s^2 + 4^2}}$$

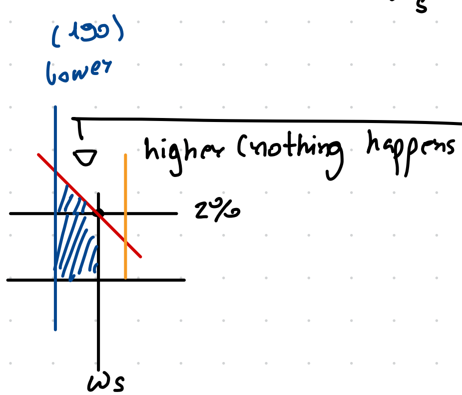
$$\sqrt{\omega_s^2 + 4^2} = \frac{4}{0.02} = 200$$

$$\omega_s^2 = 40000 - 16 = 39984$$

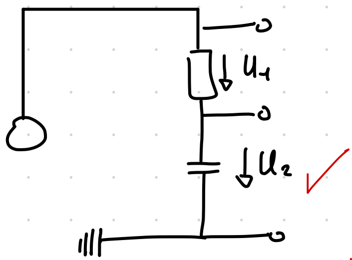
$$\omega_s = \sqrt{39984} = 199.96$$

199 or 200

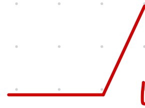
which? $\boxed{200}$



3. Transfer system to a complicated one



$$H_1(j\omega) = \frac{R}{R+\frac{1}{j\omega C}} = \frac{j\omega RC}{j\omega RC+1} = \frac{j\omega}{j\omega+\frac{1}{RC}}$$



Bode diagram

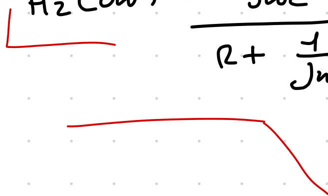
high pass system

$R = 2\Omega$

$\Delta = 1\,dB$

$C = ?$

$$H_2(j\omega) = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{1}{Rj\omega C+1} = \frac{1/RC}{j\omega + 1/RC}$$



Low pass system

$$|H|_{max} = ? = \left.\frac{\frac{1}{RC}}{\sqrt{\omega^2 + (1/RC)^2}}\right|_{\omega=1}$$

$\to = 1 = 0\,dB$

$(20\log(1))$

$1 \cdot 10^{-\frac{1}{20}} = 0.89$

How to calculate $\Delta = 1$?

$$\frac{\begin{matrix}0\\-1\end{matrix}}{-1\,dB} \to \text{amp} \Rightarrow 10^{\frac{-1}{20}} = 0.891$$

$$0.89 \le \frac{\frac{1}{2C}}{\sqrt{(200)^2 + (\frac{1}{2C})^2}}$$

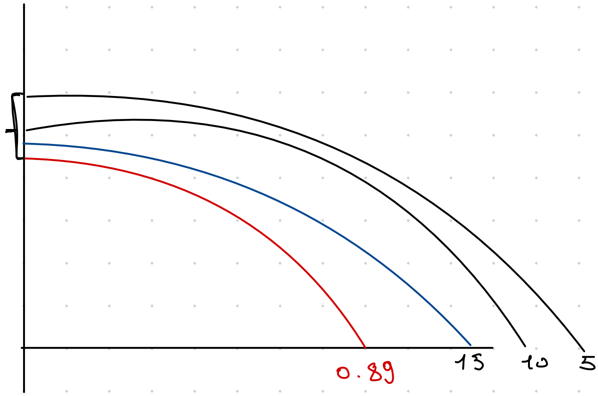
$$(0.89)\sqrt{(200)^2 + (\frac{1}{2C})^2}$$

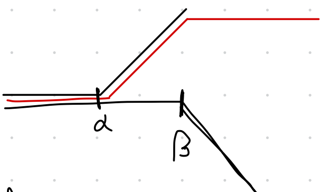
$C = ?$

$\to 17,26$

~~15~~
~~20~~
13
10
5

$$\frac{\frac{1}{\alpha}}{S_M + \frac{1}{a}}$$





$$\frac{j\omega + \alpha}{j\omega + \beta}$$

if $\frac{\alpha}{\beta} > 1$, LP V

if $1 > \frac{\alpha}{\beta}$, HP Λ

$\omega = \emptyset$

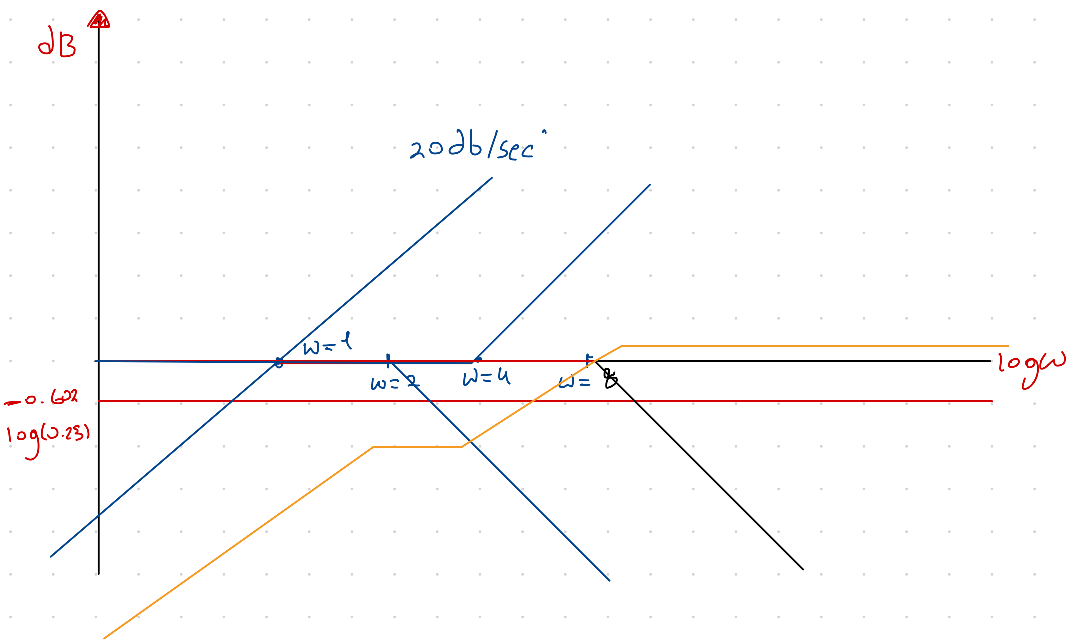
Third approx

$$H(j\omega) = \frac{j\omega^2 + 4j\omega}{(j\omega+2)(j\omega+8)}$$

$$\left(1 + j\frac{\omega}{\omega_0}\right)$$

log(0.25)

$$\frac{j\omega\left(\frac{j\omega}{4}+1\right)\cdot 4}{\left(1+j\frac{\omega}{2}\right)\left(1+j\frac{\omega}{8}\right)16}$$



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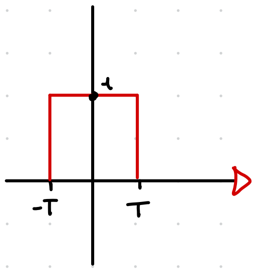
fourier transform Examples

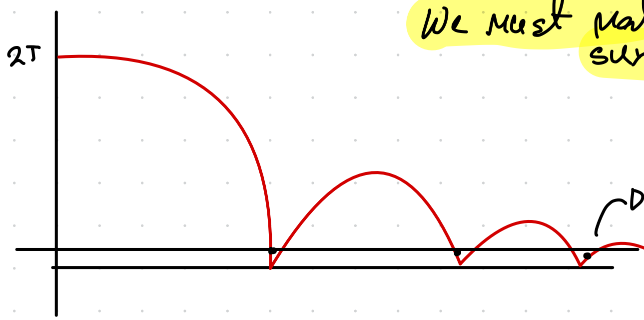
. $\varepsilon(t)$ is not integrable, $\varepsilon(t) \longrightarrow \frac{1}{j\omega} + \pi\delta(t)$



$$\int_{-T}^{T} e^{-j\omega t}\,dt = \frac{e^{-j\omega T} - e^{j\omega T}}{-j\omega} = \frac{e^{j\omega T} - e^{-j\omega T}}{2j\omega T}\,2T$$

$$= 2T\,\frac{2\sin\omega T}{\omega T}$$

$$\frac{1}{j\omega}\cdot e^{j\omega T} - \frac{1}{j\omega}\,e^{-j\omega T}$$

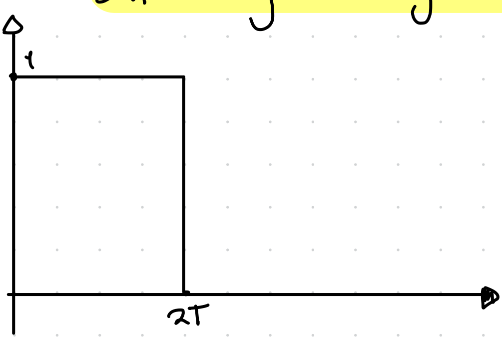


We must make sure that the BW has ↑ to be at a less value

→ use last value

shift using shifting Theorem

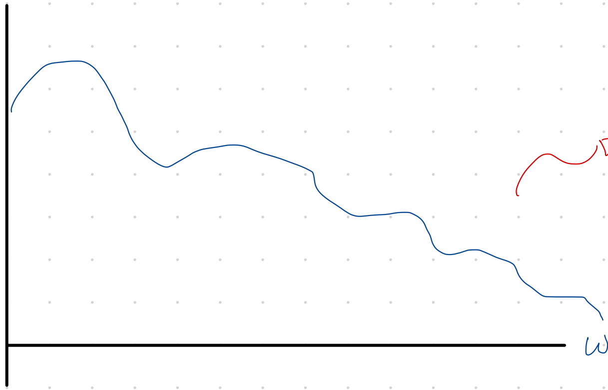
$$\mathcal{F}\{x(t-T)\} = x(j\omega)e^{-j\omega t}$$



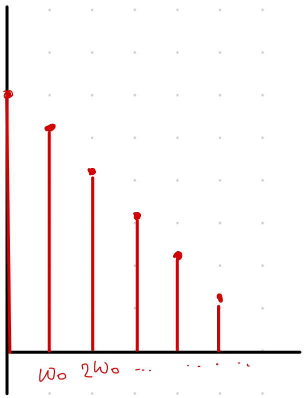
$$2T\cdot\frac{\sin(\omega T)}{\omega T}\cdot e^{-j\omega t}$$

$$\int_{0}^{2T} e^{-j\omega T}\cdot dt = \frac{e^{-2j\omega t} - 1}{-j\omega}$$

$$= \frac{1}{j\omega} - \frac{e^{-2j\omega t}}{j\omega} =$$



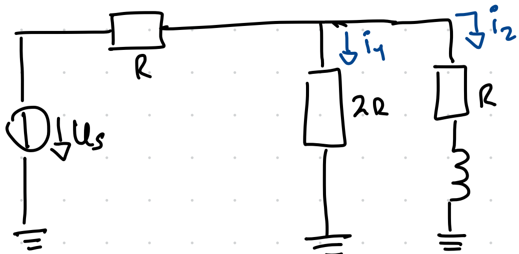
Non periodic function



Periodic

use to determine if we need a HPass / LPass

$U_s(t) = 10\,e^{-\alpha t}\,\varepsilon(t)$



$R = 5$

$L = 0.4$

$\Delta_N = 1\,dB$

$\delta = 0.01$

$\omega = 0 \quad \frac{1}{5R}$

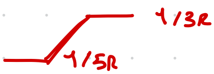
$\omega = \infty \quad \frac{1}{3R}$

Bode

$$H_1(j\omega) = \frac{2R \times (R + j\omega L)}{R + 2R \times (R + j\omega L)} \cdot \frac{1}{2R} = \frac{2R + 2j\omega L}{5R + 3j\omega L} \cdot \frac{1}{2R}$$

(U)

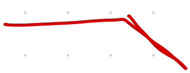
$$= \frac{2R + 2j\omega L}{2R(5R + 3j\omega L)}$$



$$H_2(j\omega) = \frac{2R \times (R + j\omega L)}{R + 2R \times (R + j\omega L)} \cdot \frac{1}{R + j\omega L} = \frac{2R + 2j\omega L}{5R + 3j\omega L} \cdot \frac{1}{R + j\omega L}$$

$$= \frac{\frac{2}{3L}}{j\omega + \frac{5R}{3L}}$$

Bode



$\omega = 0 \;,\; \frac{2}{5R}$

Why we need a low B_p ?

$$\frac{10}{j\omega+\alpha} \quad MAX\Big|_{\omega=\phi} = \frac{10}{\alpha}$$

$$\frac{10}{\sqrt{\omega^2+\alpha^2}}$$

2 / 2.5/3

use to determine

$$0.01 \cdot \frac{10}{\alpha} \leq/= \frac{10}{\sqrt{\omega_H + \alpha^2}}$$

$\Rightarrow \alpha$

use to find α

Suppose

2 ?

$\alpha = 2.5$

3 ?

$\Rightarrow$ 2 is the answer in this case

$$k(\omega) = \frac{\frac{2}{3L}}{\sqrt{\omega^2 + \left(\frac{5R}{3L}\right)^2}}$$

$$MAX_H = \frac{2}{25}$$

$$10^{-\frac{1}{20}} \cdot Max = \frac{\frac{2}{1.2}}{\sqrt{\omega_0^2 + \left(\frac{25}{1.2}\right)^2}}$$

ω?

$$\omega_0 = \sqrt{\left(\frac{\frac{2}{1.2}}{10^{-\frac{1}{20}} \cdot \frac{2}{25}}\right)^2 - \left(\frac{25}{1.2}\right)^2}$$

$= 23.28$

W_0

2025.9.17

$$U_s(t) = 4 \cdot e^{-2|t|}$$

$$A = \begin{vmatrix} -2 & 1 \\ -3 & -4 \end{vmatrix}$$

2025. 9. 17

Laplace Transform vs Fourier Transform

Mathematical equation

· → time domain

complex domain

Fourier Transform → signal spectrum

→ transform time domain

signal domain

rep : $\int f(t) e^{-j\omega t} dt$

$\sigma + j\omega \to s$

$$F(s) = \int_{-0}^{\infty} f(t) \cdot e^{-st} \cdot dt$$

bc

$\Delta t \to -\phi \to +\phi$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{F}(j\omega) e^{j\omega t} d\omega$$

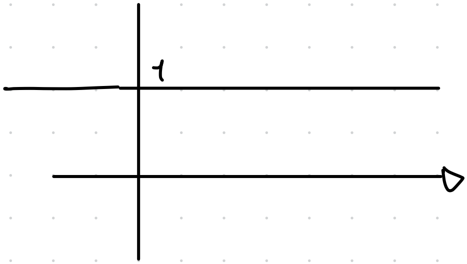
· Professor hasn't used his entire life

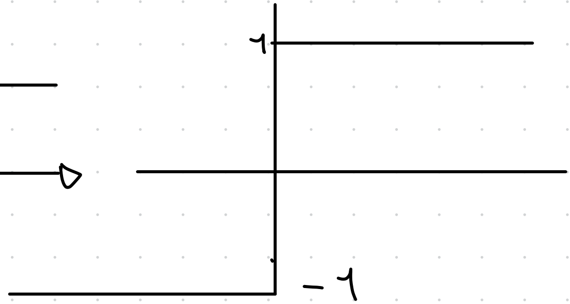
$$\frac{1}{2\pi j} \int_{\sigma - j\omega}^{\sigma + j\omega} F(s) e^{st} ds$$

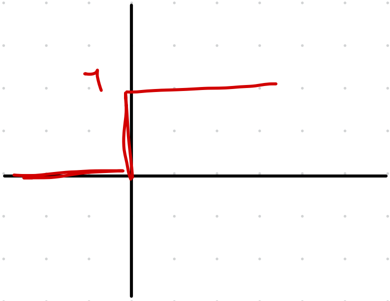
· function can be any kind of function

· doesn't have to be integrable

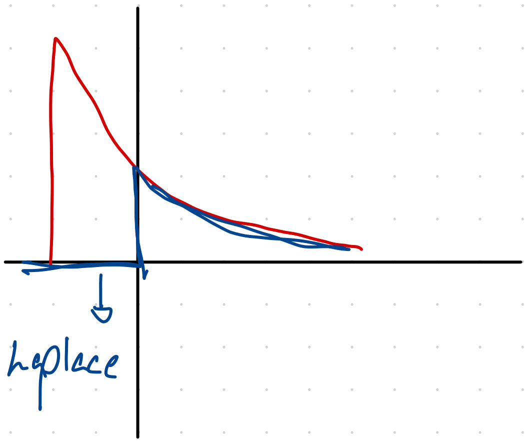
Riemann Mellin







- if function is caused we can't recover it
- all functions have a laplace transform
- cuts everything before ϕ



$f(t)$	$F(s)$
$\delta(t)$	1
$\varepsilon(t)$	$\frac{1}{s}$
$t\,\varepsilon(t)$	$1/s^2$
$t^n\,\varepsilon(t)$	$n!/s^{n+1}$
$\varepsilon(t)e^{-\alpha t}$	$1/s+\alpha$
$\varepsilon(t)\cos\omega_0 t$	$\frac{s}{s^2+\omega_0^2}$
$\varepsilon(t)\sin\omega_0 t$	$\frac{\omega_0}{s^2+\omega_0^2}$
$\varepsilon(t-T)x(t-T)$	$X(s)\cdot e^{-sT}$

2025 9.18

· Laplace transform ·

I. $c\,\alpha\{x(t)+y(t)\} = \alpha\{c\,x(t)\} + \alpha\{c\,y(t)\}$

II. $\alpha\{x(t)\cdot e^{-\alpha t}\} = x(s+\alpha)$ $\quad \int_{-0}^{\infty} x(t)\cdot e^{-st}$

III. $\alpha\{x(t)\cdot e^{-j\omega t}\} = x(s+j\omega_0)$ $\quad \int_{-0}^{\infty} \varepsilon(t)\cdot e^{-st}\,dt = \frac{e^{-s\infty}-e^{-s\emptyset}}{-s} = \frac{1}{s}$

IV. $\alpha\{\dot{x}(t)\} = s\cdot x(s) + x(-\emptyset)$ $\quad \varepsilon(t)\cdot\cos\omega_0 t = \frac{\varepsilon(t)\cdot e^{j\omega t} + e^{-j\omega_0 t}}{2}$

V. $\alpha\left\{\int_{0}^{\infty} x(t)\,dt\right\} = \frac{x(s)}{s}$ $\quad \frac{1}{2}\left(\frac{1}{s-j\omega_0} + \frac{1}{s+j\omega_0}\right) = \frac{s}{s^2+\omega_0^2}$

VI. $x(t=\pm 0) = \lim_{s\to\infty} s\,x(s)$

VII. $x(t\to\infty) = \lim_{s\to\emptyset} s\,x(s)$

VIII. $\alpha\{\varepsilon(t-T)\,x(t-T)\} = x(s)\,e^{-sT}$

IX. $\alpha\{h(t) * u(t)\} = \alpha\{h(t)\}\cdot\alpha\{u(t)\}$

t	R	L	C	
$j\omega$	R	$j\omega L$	$\frac{1}{j\omega C}$	CD
s	R	sL	$\frac{1}{sC}$	LD

* in complex domain frequency is not changed

* eddy currents on wires shift resistivity a bit

Rewrite Notes at home

$u_L = L\cdot\frac{di}{dt}$ $\quad i_c = C\dot{u}_c$

$U_L(s) = L\cdot s\, I(s)$

- $a(t) = \varepsilon(t)(3e^{-4t} + 6e^{-7t}) \Rightarrow \frac{3}{s+4} + \frac{6}{s+7} \cancel{=} \frac{9s+45}{s^2+11s+28}$

$$\int_0^\infty 3e^{-4t}\cdot e^{-st}\,dt = 3\int e^{-(s+4)t}\,dt$$

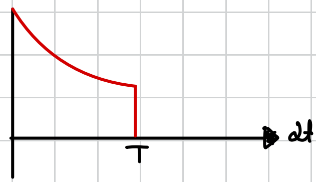
write this way ↓

$(s+4)\cdot(s+7)$

- $b(t) = \varepsilon(t)\cdot 3\cdot t\cdot e^{-4t}$ (use table)

Whats the Laplace ?

$$B(s) = \frac{3}{(s+4)^2}$$



- $c(t) = \left[\varepsilon(t) - \varepsilon(t-T)\right]3\cdot e^{-4t} = 3e^{-4t}\varepsilon(t) - \varepsilon(t-T)3e^{-4t}$

$$3e^{-4t}\varepsilon(t) - \varepsilon(t-T)3e^{-4(t-T+T)} =$$

$$= \varepsilon(t)3e^{-4t} - \varepsilon(t-T)3\cdot e^{-4T}\cdot e^{-4(t-T)}$$

$$= \frac{3}{s+4} - \frac{3e^{-4t}}{s+4}e^{-sT} =$$

$$\frac{1}{s+4}\left(3 - 3e^{-4T}e^{-sT}\right)$$

↳ implies function will be shifted.

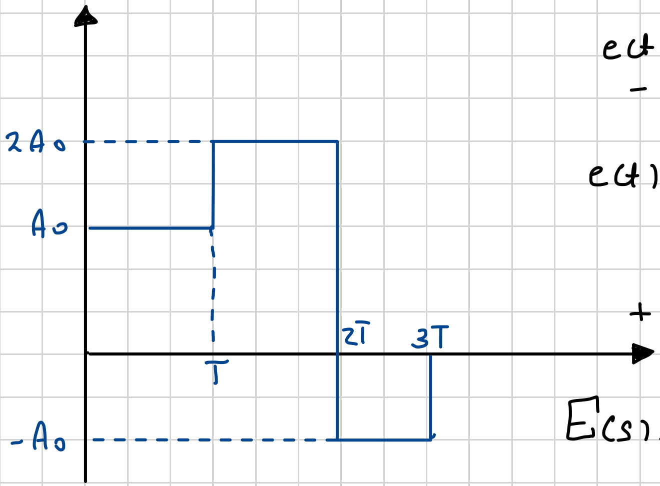
REWRITE Notes on new doc !!!

$$d(t) = [\varepsilon(t) - \varepsilon(t-T)]\, 3t \cdot e^{-4t} = 3t e^{-4t} \varepsilon(t) - \varepsilon(t-T)\, 3t \cdot e^{-4t}$$

$$= \varepsilon(t)\, 3t e^{-4t} - \varepsilon(t-T)\, 3(t-T+T)\, e^{-4(t-T+T)} =$$

$$\varepsilon(t)\, 3t e^{-4t} - \varepsilon(t-T)\, 3(t-T)\, e^{-4T} \cdot e^{-4(t-T)} - \varepsilon(t-T)\, 3T\, e^{-4T} \cdot e^{-4(t-T)}$$

$$3/(s+4)^2 - \frac{3 e^{-4T} \cdot e^{-sT}}{(s+4)^2} - \frac{3T e^{-4T}}{s+4} e^{-sT}$$



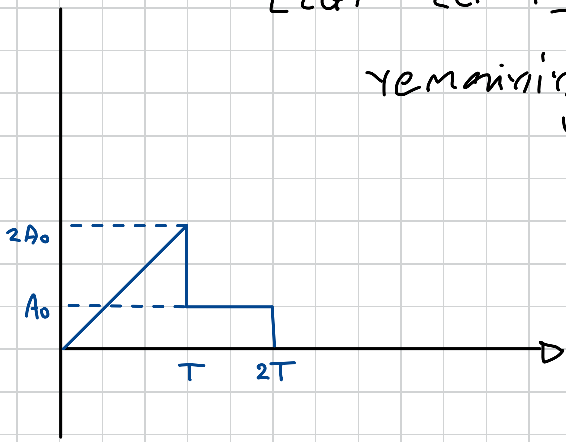
$$e(t) = A_0 \varepsilon(t) + A_0 \varepsilon(t-T) - 3A_0 \varepsilon(t-2T) + A_0 \varepsilon(t-3T)$$

$$e(t) = [\varepsilon(t) - \varepsilon(t-T)] A_0 + [\varepsilon(t-T) - \varepsilon(t-2T)]\, 2A_0 + [\varepsilon(t-2T) - \varepsilon(t-3T)](-A_0)$$

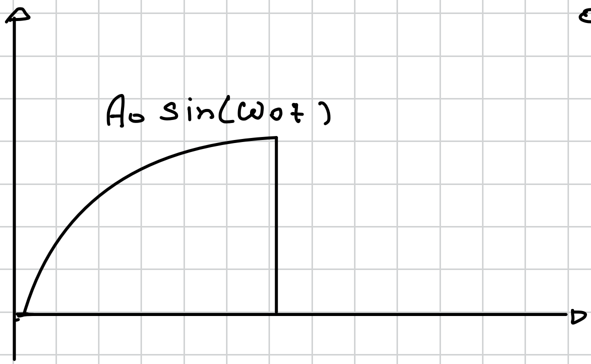
$$E(s) = \frac{1}{s}\left(A_0 + A_0 e^{-sT} - 3A_0 e^{-s2T} + A_0 e^{-s3T}\right)$$

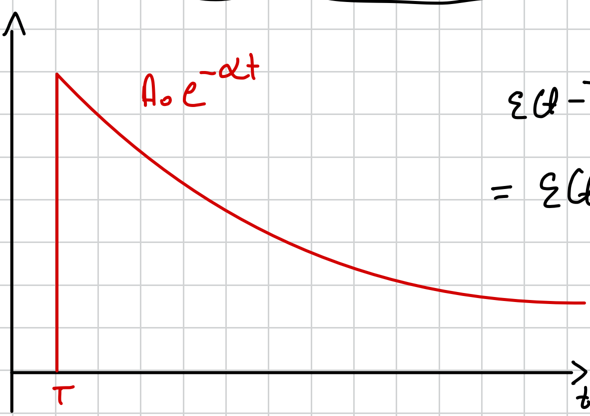
$$[\varepsilon(t) - \varepsilon(t-T]\, \frac{6A_0}{T} + [\varepsilon(t-T$$

remaining is in gallery



on gallery

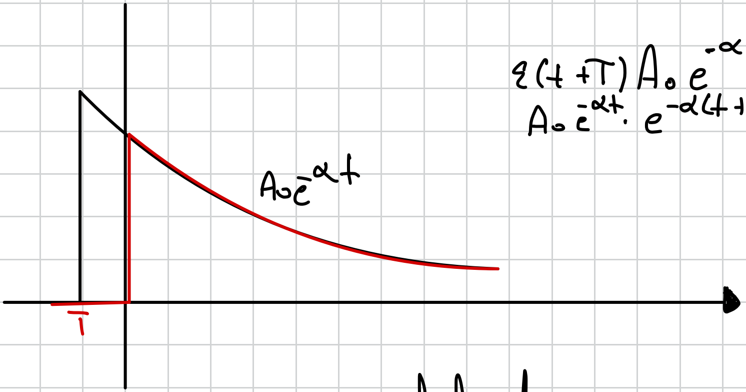




$$\varepsilon(t-T)\, A_0 e^{-\alpha(t-T+T)}$$

$$= \varepsilon(t-T) \cdot A_0 \cdot e^{-\alpha T} \cdot e^{-\alpha(t-T)}$$

$$\frac{A_0 e^{-\alpha T}}{s+\alpha} e^{-sT}$$



$$\varepsilon(t+T)\, A_0\, e^{-\alpha(t-T+T)} = \varepsilon(t+T)\cdot$$

$$A_0 e^{-\alpha t} \cdot e^{-\alpha(t+T)} = \varepsilon(t) \cdot A_0 e^{\alpha T} \cdot e^{-\alpha t}$$

$$= \frac{A_0 e^{\alpha T}}{s+\alpha}$$

Non real fraction polynomial division

$$F(s) = \frac{3s^2+4s+6}{s^2+8+12} = 3s^2+4s+6 : (s^2+8s+12) =$$

$$-(3s^2+24s+36) \quad \cdot 3$$

$$-20s - 30$$

$$-20s - 160 - \frac{240}{s}$$

$$130 + \frac{240}{s}$$

$$3 - \frac{20}{s} + \frac{130}{s^2} \ldots$$

$$3\delta(t) - 20\varepsilon(t) + 130t\, \varepsilon(t) T \ldots$$

$$3 + \frac{-20s-30}{s^2+8s+12}$$

$$\frac{-20s-30}{(s+6)(s+2)} = 3 + \frac{A}{s+6} + \frac{B}{s+2}$$

$$A(s+2) + B(s+2) = -20s - 30$$

$$A + B = -20$$

$$2A + 6B = -30$$

$$A = \frac{-90}{4}$$

$$B = \frac{10}{4}$$