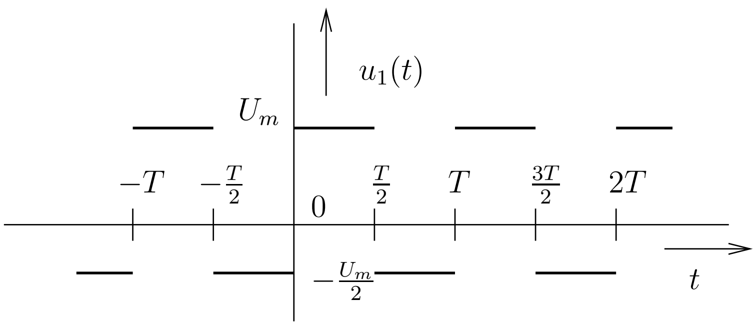
1. The input signal and the response of a system represented by a network are voltages. The frequency response of the system is: $H(j\omega) = \frac{j\omega T}{j\omega T+2}$ where T is a parameter. The input signal of the system is the periodic voltage signal given in the figure.



a) Find the three-order Fourier polynomial of the input signal!

b) Find the response of the system in three-order Fourier polynomial approximation!

Solution

a) $U_p^C = \frac{1}{T}\int\limits_{t=\langle T\rangle} u_1(t)e^{-jp\omega_0 t}\mathrm{d}t, \qquad \langle T\rangle = \left\{-\frac{T}{2};\ \frac{T}{2}\right\}.$

$U_0^C = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} u_1(t)\mathrm{d}t = \frac{1}{T}\left(\int_{-\frac{T}{2}}^{0}\left(-\frac{U_m}{2}\right)\mathrm{d}t + \int_0^{\frac{T}{2}} U_m \mathrm{d}t\right) = \frac{1}{T}\left(-\frac{U_m T}{4} + \frac{U_m T}{2}\right) = \frac{U_m}{4},$

$U_0 = U_0^C = \frac{U_m}{4}.$

For $p > 0$: $U_p^C = \frac{1}{T}\left(\int_{-\frac{T}{2}}^{0}\left(-\frac{U_m}{2}\right)e^{-jp\frac{2\pi}{T}t}\mathrm{d}t + \int_0^{\frac{T}{2}} U_m e^{-jp\frac{2\pi}{T}t}\mathrm{d}t\right) =$

$= \frac{U_m}{T}\left(-\frac{1}{2}\frac{jT}{2\pi p}\left[e^{-jp\frac{2\pi}{T}t}\right]_{-\frac{T}{2}}^{0} + \frac{jT}{2\pi p}\left[e^{-jp\frac{2\pi}{T}t}\right]_0^{\frac{T}{2}}\right) = \frac{jU_m}{2\pi p}\left(-\frac{1}{2}\left(1-e^{j\pi}\right) + e^{-j\pi} - 1\right).$

$e^{-j\pi p} = e^{j\pi p} = (-1)^p$, so $U_p^C = \frac{jU_m}{2\pi p}\left(-\frac{3}{2} + \frac{3}{2}(-1)^p\right) = \frac{j3U_m}{4\pi p}\left(-1 + (-1)^p\right).$

$p = 1 \qquad U_1^C = \frac{-j3U_m}{2\pi}, \qquad U_1 = 2\left|U_1^C\right| = \frac{3}{\pi}U_m, \qquad \rho_1 = angle\left(U_1^C\right) = -\frac{\pi}{2}.$

$p = 2 \qquad U_2^C = 0, \qquad U_2 = 0.$

$p = 3 \qquad U_3^C = \frac{-jU_m}{2\pi}, \qquad U_3 = 2\left|U_3^C\right| = \frac{1}{\pi}U_m, \qquad \rho_3 = angle\left(U_3^C\right) = -\frac{\pi}{2}.$

The three-order Fourier polynomial of $u_1(t)$ is:

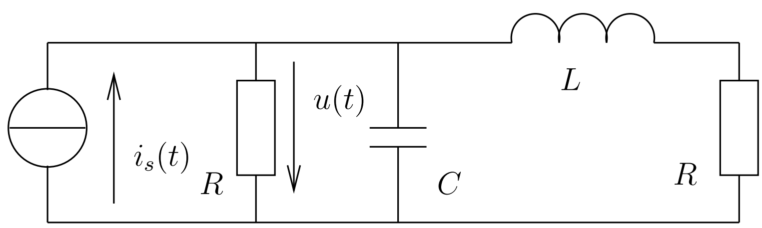
$u_1(t) \simeq U_m\left(0,25 + 0,9549\cos\left(\frac{2\pi}{T}t - \frac{\pi}{2}\right) + 0,3183\cos\frac{6\pi}{T}t - \frac{\pi}{2}\right).$

b) $H(j\omega)\big|_{\omega=0} = 0, \qquad H(j\omega)\big|_{\omega=\frac{2\pi}{T}} = \frac{j2\pi}{2+j2\pi} = 0,9529e^{j0,3082}$

$H(j\omega)\big|_{\omega=\frac{6\pi}{T}} = \frac{j6\pi}{2+j6\pi} = 0,9944e^{j0,1057}.$

$u_2(t) = U_m\left[0,9099\cos\left(\frac{2\pi}{T}t - 1,2626\right) + 0,3165\cos\left(\frac{6\pi}{T}t - 1,4651\right)\right].$

2. The source current of the current source and the parameters of the network components are given: $i_s(t) = \left[20 + 10\cos\omega_0 t + 5\cos\left(2\omega_0 t - \frac{\pi}{6}\right)\right] mA, \quad \omega_0 = 2krad/s, \quad R = 2k\Omega,$ $L = 0,25H,\ C = 250nF.$



a) Find the rms value of the source current!

b) Find the time function of the noted $u(t)$ voltage!

c) Find the average power of the current source!

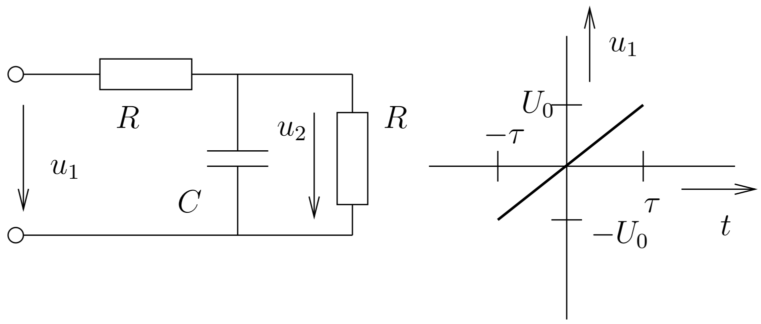
Solution

a) $I_s = \sqrt{20^2 + \frac{10^2}{2} + \frac{5^2}{2}} \simeq 21,5058mA.$

b) The suitable coherent unit system: V, mA, mW, $k\Omega$, H, ms, μF, $krad/s$.
At ω angular frequency the impedance of the two-pole connected to the source is:
$Z(j\omega) = R \times \frac{1}{j\omega C} \times (R + j\omega L) = 2 \times \frac{1}{0,25j\omega} \times (2 + 0,5j\omega) = \frac{4+j\omega}{2+0,5j\omega+j\omega+0,25(j\omega)^2+2} =$
$= \frac{j\omega+4}{0,25(j\omega)^2+1,5j\omega+4} = \frac{4j\omega+16}{(j\omega)^2+6j\omega+16},$ $\qquad Z(j\omega)|_{\omega=0} = 1,$
$Z(j\omega|_{\omega=\omega_0=2} = \frac{16+j8}{12+j12} = 1,0541e^{-j0,3218},$ $\qquad Z(j\omega|_{\omega=2\omega_0=4} = \frac{16+j16}{j24} = 0,9428e^{-j\frac{\pi}{4}},$
$u(t) = [20 + 10,541\cos(\omega_0 t - \underset{-18,43^o}{0,3218}) + 4,714\cos(2\omega_0 t - \underset{-75^o}{1,3090})]V.$

c) $P_s = \left(-20 \cdot 20 - \frac{1}{2} \cdot 10 \cdot 10,541 \cdot \cos(-0,3218) - \frac{1}{2} \cdot 5 \cdot 4,714 \cdot \cos(-1,309 + 0,5236)\right) mW =$
$= -458,33mW.$

3. The input signal and the response of the system represented by the given network are the u_1 and the u_2 voltages, respectively. The input signal is the voltage impulse drawn in the figure.



a) Find the frequency response of the system represented by the network!

b) Find the Fourier transform of the response signal!

Solution

a) $\bar{U}_2 = \bar{U}_1 \frac{R \times \frac{1}{j\omega C}}{R + R \times \frac{1}{j\omega C}},$ $\qquad H(j\omega) = \frac{R \times \frac{1}{j\omega C}}{R + R \times \frac{1}{j\omega C}} = \frac{\frac{R}{1+j\omega CR}}{R + \frac{R}{1+j\omega CR}} = \frac{R}{R + j\omega CR^2 + R} = \frac{\frac{1}{CR}}{j\omega + \frac{2}{CR}}.$

b) We will find the spectrum of the input signal first.
$U_1(j\omega) = \int_{-\infty}^{\infty} u_1(t)e^{-j\omega t}\mathrm{d}t = \int_{-\tau}^{\tau} \frac{U_0}{\tau} t e^{-j\omega t}\mathrm{d}t.$
In the partial integration $(\int uv' = uv - \int u'v)$ $u = \frac{U_0}{\tau}t,$ $u' = \frac{U_0}{\tau},$ $\qquad v' = e^{-j\omega t},$
$v = \frac{1}{-j\omega}e^{-j\omega t}.$
$U_1(j\omega) = \left[\frac{U_0}{\tau}t\frac{1}{-j\omega}e^{-j\omega t}\right]_{-\tau}^{\tau} - \frac{U_0}{-j\omega\tau}\int_{-\tau}^{\tau} e^{-j\omega t}\mathrm{d}t = \frac{U_0}{-j\omega\tau}\left(\tau e^{-j\omega\tau} - (-\tau)e^{j\omega\tau}\right) -$
$-\frac{U_0}{(j\omega)^2\tau}\left[e^{-j\omega t}\right]_{-\tau}^{\tau} = \frac{j2U_0}{\omega}\cos\omega\tau + \frac{U_0}{\omega^2\tau}\left(e^{-j\omega\tau} - e^{j\omega\tau}\right) = \frac{j2U_0}{\omega}\cos\omega\tau + \frac{U_0}{\omega^2\tau}(-2j\sin\omega\tau) =$
$= j\frac{2U_0}{\omega}\left(\cos\omega\tau - \frac{\sin\omega\tau}{\omega\tau}\right).$
The Fourier transform of the response signal is: $U_2(j\omega) = j\frac{2U_0}{\omega}\left(\cos\omega\tau - \frac{\sin\omega\tau}{\omega\tau}\right)\frac{\frac{1}{CR}}{j\omega + \frac{2}{CR}}.$