Decision Making under Model Uncertainty

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  (reasoning and decision making under uncertainty)

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- Postdoc with Thomas Augustin at LMU Munich
  (SNSF Research Fellowship, October 2007 – March 2009):
  Decision making on the basis of a probabilistic-possibilistic hierarchical description of uncertain knowledge

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- **minimax** decision criterion:

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- **Bayesian** decision criterion with prior probability measure π on $\mathcal{M}$:

$$\text{minimize} \quad \int d(m)\, l(m)\, \mathrm{d}\pi(m)$$

nonadditive measures and integrals

- in the likelihood approach to statistics, the likelihood function is usually extended to the nonadditive measure λ on $\mathcal{M}$ defined by

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- λ is a completely maxitive measure (**possibility** measure):

$$\lambda\left(\bigcup_{M \in \mathcal{S}} M\right) = \sup_{M \in \mathcal{S}} \lambda(M) \quad \text{for all } \mathcal{S} \subseteq 2^{\mathcal{M}}$$

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- **likelihood-based** decision criterion:

$$\text{minimize} \quad \int d(m) \, \mathrm{d}\lambda(m)$$

likelihood-based statistical decisions

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  - asymptotic optimality (consistency)
  - asymptotic efficiency
  - parametrization invariance
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- a simple property (sure-thing principle) characterizes the likelihood-based decision criterion based on the Shilkret integral: the **MPL** (Minimax Plausibility-weighted Loss) decision criterion:

$$\text{minimize} \quad \int^{\mathrm{S}} d(m)\,\mathrm{d}\lambda(m) = \sup_{m \in \mathcal{M}} d(m)\, l(m)$$

example: estimation of variance components

- estimation of the variance components in the 3×3 random effect one-way layout, under normality assumptions and weighted squared error loss

$$X_{ij} = \mu + \alpha_i + \varepsilon_{ij} \quad \text{for all } i, j \in \{1, 2, 3\}$$

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- normality assumptions:

$$\alpha_i \sim \mathcal{N}(0, v_a), \;\; \varepsilon_{ij} \sim \mathcal{N}(0, v_e), \;\; \text{all independent}$$

$$\Rightarrow \;\; X_{ij} \sim \mathcal{N}(\mu, v_a + v_e) \;\text{ dependent}, \;\; \mu \in \mathbb{R}, \;\; v_a, v_e \in (0, \infty)$$

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- probabilistic models:

$$\mathcal{M} = \{m_{\mu, v_a, v_e} : \mu \in \mathbb{R},\, v_a, v_e \in (0, \infty)\}$$

example: estimation of variance components

- estimates $\widehat{v_e}$ and $\widehat{v_a}$ of variance components v_e and v_a are functions of

$$SS_e = \sum_{i=1}^{3} \sum_{j=1}^{3} (x_{ij} - \bar{x}_{i\cdot})^2 \quad \text{and} \quad SS_a = 3 \sum_{i=1}^{3} (\bar{x}_{i\cdot} - \bar{x}_{\cdot\cdot})^2,$$

where

$$\bar{x}_{i\cdot} = \frac{1}{3} \sum_{j=1}^{3} x_{ij}, \quad \bar{x}_{\cdot\cdot} = \frac{1}{9} \sum_{i=1}^{3} \sum_{j=1}^{3} x_{ij},$$

$$\frac{SS_e}{v_e} \sim \chi^2_6, \quad \text{and} \quad \frac{\frac{1}{3}\, SS_a}{v_a + \frac{1}{3}\, v_e} \sim \chi^2_2$$

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- decisions (estimates $\widehat{v_e}, \widehat{v_a} \in (0, \infty)$ with invariant loss):

$$d_{\widehat{v_e}}(m_{\mu, v_a, v_e}) = 3\, \frac{(v_e - \widehat{v_e})^2}{{v_e}^2} \quad \text{and} \quad d_{\widehat{v_a}}(m_{\mu, v_a, v_e}) = \frac{(v_a - \widehat{v_a})^2}{(v_a + \frac{1}{3}\, v_e)^2}$$

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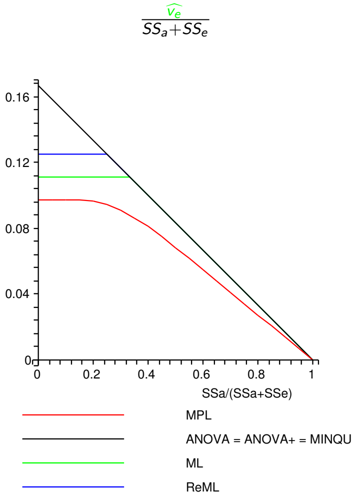

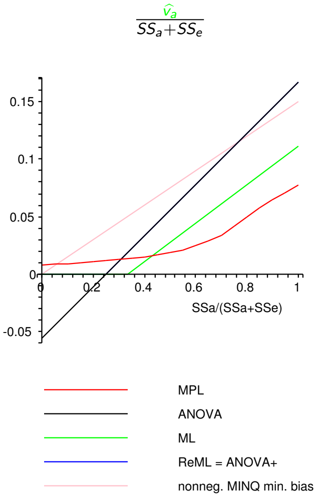

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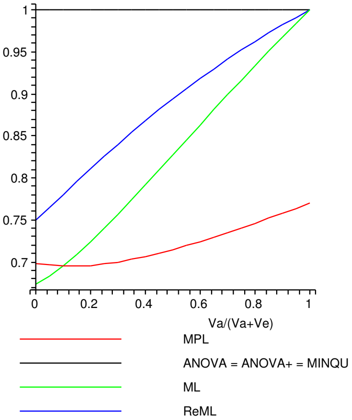
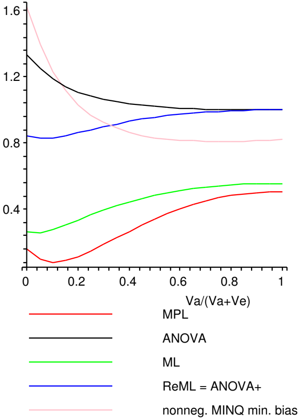
$$3\ \frac{E[(\widehat{v_e} - v_e)^2]}{{v_e}^2}$$



$$\frac{E[(\widehat{v_a} - v_a)^2]}{(v_a + \frac{1}{3}\ v_e)^2}$$



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- application to graphical models
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- application to financial risk measures
  (derivation and interpretation of convex risk measures)