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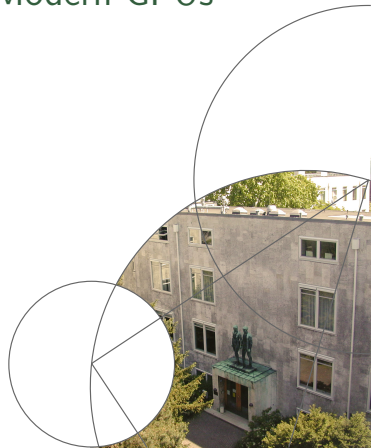
# Multireduce and Multiscan on Modern GPUs

Master's thesis defence

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April 15, 2014  
Slide 1/31



# Overview

- ➊ Introduction
- ➋ Sequential algorithms
- ➌ GPUs and CUDA
- ➍ Sort-based algorithms
- ➎ Adaptation of sequential algorithm
- ➏ Example of multireduce in shared memory
- ➐ Results and conclusion



# Reduce and scan

Works on any monoid  $M$  with binary function  $\odot$



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Examples with integers and addition



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Input vector

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0	4	5	8	17	17	23	28
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Scan: Sums of all prefixes

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Segmented reduce/scan: Partition input into contiguous segments and reduce/scan them separately





# Multireduce

Input *labels*

0	2	2	1	0	2	0	1	1	1	2	0
---	---	---	---	---	---	---	---	---	---	---	---

Input values

4	1	3	9	0	5	5	2	7	1	3	5
---	---	---	---	---	---	---	---	---	---	---	---



# Multireduce

Input *labels*

0	2	2	1	0	2	0	1	1	1	2	0
---	---	---	---	---	---	---	---	---	---	---	---

Input values

4	1	3	9	0	5	5	2	7	1	3	5
---	---	---	---	---	---	---	---	---	---	---	---

Output:

--	--	--



# Multireduce

Reduce the values for each label separately.

0	2	2	1	0	2	0	1	1	1	2	0
---	---	---	---	---	---	---	---	---	---	---	---

4	1	3	9	0	5	5	2	7	1	3	5
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--	--	--



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--	--	--



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14		
----	--	--



# Multireduce

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0	2	2	1	0	2	0	1	1	1	2	0
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4	1	3	9	0	5	5	2	7	1	3	5
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14	19	
----	----	--



# Multireduce

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0	2	2	1	0	2	0	1	1	1	2	0
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4	1	3	9	0	5	5	2	7	1	3	5
---	---	---	---	---	---	---	---	---	---	---	---

14	19	12
----	----	----



# Multiscan

Identical inputs:

0	2	2	1	0	2	0	1	1	1	2	0
4	1	3	9	0	5	5	2	7	1	3	5





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0	2	2	1	0	2	0	1	1	1	2	0
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4	1	3	9	0	5	5	2	7	1	3	5
---	---	---	---	---	---	---	---	---	---	---	---

Different output:

--	--	--	--	--	--	--	--	--	--	--	--



# Multiscan

*Scan* the values for each label separately

0	2	2	1	0	2	0	1	1	1	2	0
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---	---	---	---	---	---	---	---	---	---	---	---

0				4		4					9
---	--	--	--	---	--	---	--	--	--	--	---



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*Scan* the values for each label separately

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---	---	---	---	---	---	---	---	---	---	---	---

0			0	4		4	9	11	18		9
---	--	--	---	---	--	---	---	----	----	--	---



# Multiscan

*Scan* the values for each label separately

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---	---	---	---	---	---	---	---	---	---	---	---

0	0	1	0	4	4	4	9	11	18	9	9
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# Applications

Multiscan has been suggested as a primitive to build entire machines around.



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- Radix sort
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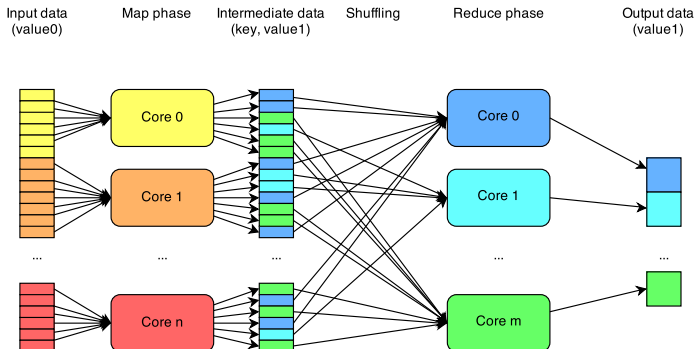
Multireduce generalizes important operations

- Histograms
- Scattering

General multireduce can be used for MapReduce framework



# Applications



# Sequential algorithms

Sequential algorithms for both operations are trivial.

**function** multiReduce(int  $n$ , int  $m$ , int  $indices[n]$ , int  $values[n]$ , int  $buckets[m]$ ) :

```
    for  $i = 0$  to  $m - 1$  do  
        |  $buckets[i] = 0$ ;  
    end  
    for  $i = 0$  to  $n - 1$  do  
        |  $buckets[indices[i]] += values[i]$ ;  
    end  
end
```



# Sequential algorithms

Sequential algorithms for both operations are trivial.

**function** multiScan(int  $n$ , int  $m$ , int  $indices[n]$ , int  $values[n]$ , int  $result[n]$ ) :

    int  $buckets[m]$ ;

**for**  $i = 0$  **to**  $m - 1$  **do**

$buckets[i] = 0$ ;

**end**

**for**  $i = 0$  **to**  $n - 1$  **do**

$result[i] = buckets[indices[i]]$ ;

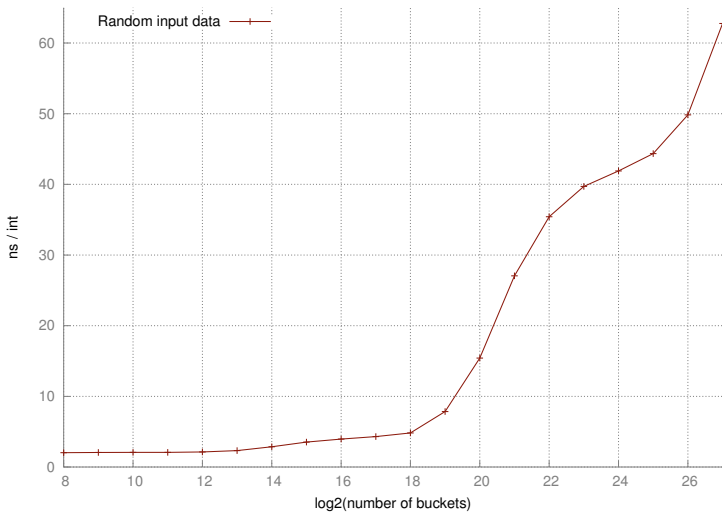
$buckets[indices[i]] += values[i]$ ;

**end**

**end**



# Performance problems



# Performance problems

- TLB misses
- Cache misses



# Solutions



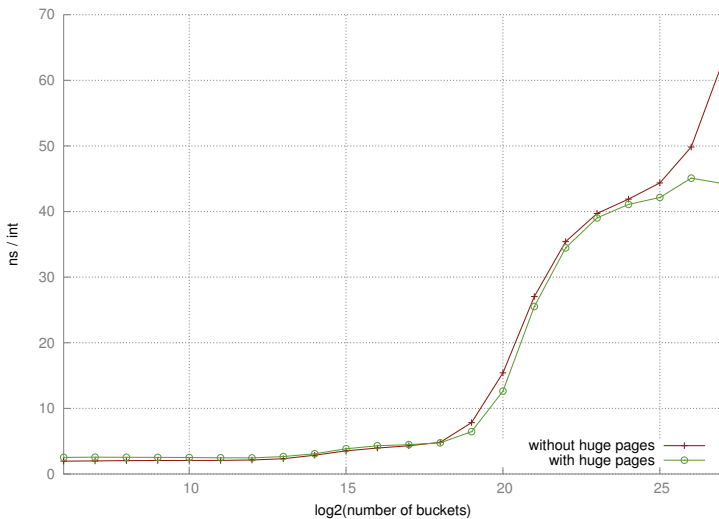
# Solutions

Huge pages can be used to mitigate the problem of TLB misses





# Solutions

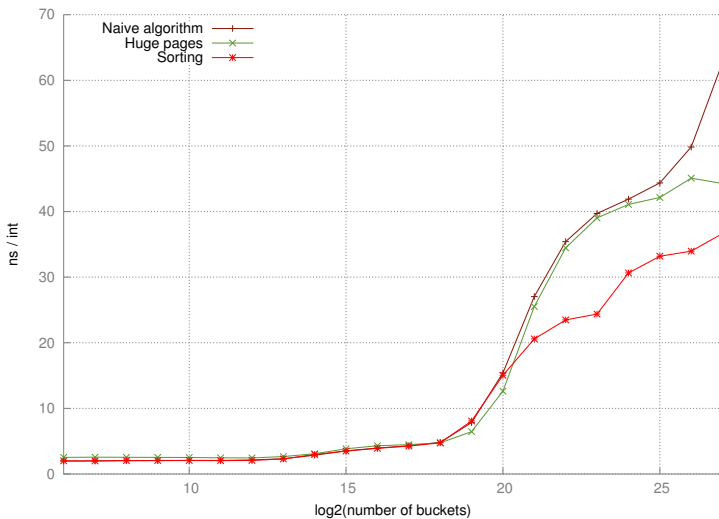


# Solutions

Sorting the input by labels turns random bucket accesses into sequential ones and avoids cache misses



# Solutions



# Applicability of sorting approach

Sorting *can* lead to a performance gain in some scenarios.  
Think accesses to disk, network, ...



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However...

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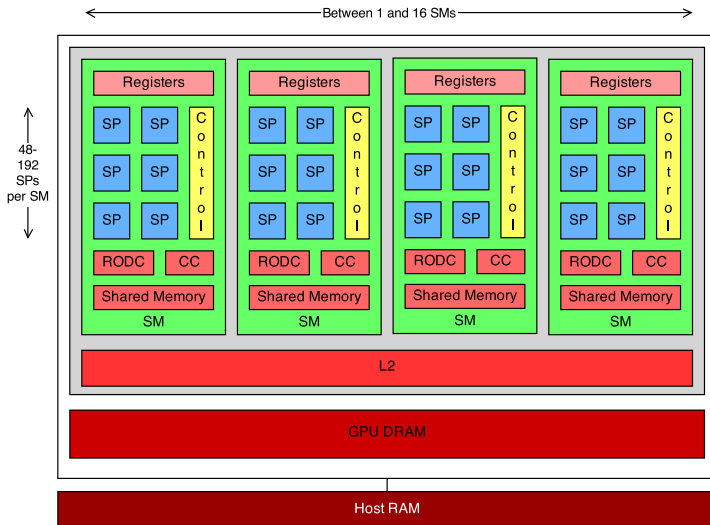
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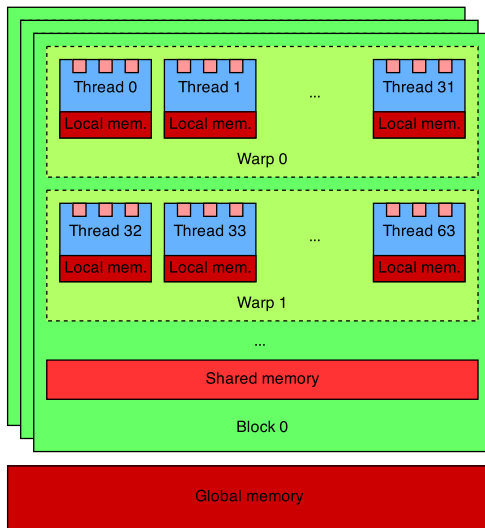
Developed a simulator to predict performance.



# GPUs



# CUDA architecture





# CUDA: Performance requirements



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- Hundreds of threads per SM



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- Atomic operations



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- Hundreds of threads per SM
- Limited memory bandwidth
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- Atomic operations
- Synchronization





# CUDA: PRAM algorithm performance



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# CUDA: PRAM algorithm performance

- Often not massively parallel, sometimes too much
- Usually assume SIMD
  - Require sync after each step...
  - ...which is either slow or impossible
- No concept of block-level shared memory
- Generally not much attention paid to memory layout
  - Often uncoalesced memory accesses
  - Often bank conflicts
  - Generally less focus on avoiding memory accesses



# Possibilities for algorithm design

Assume, a multireduce algorithm is given.  
There are two possibilities to use it:



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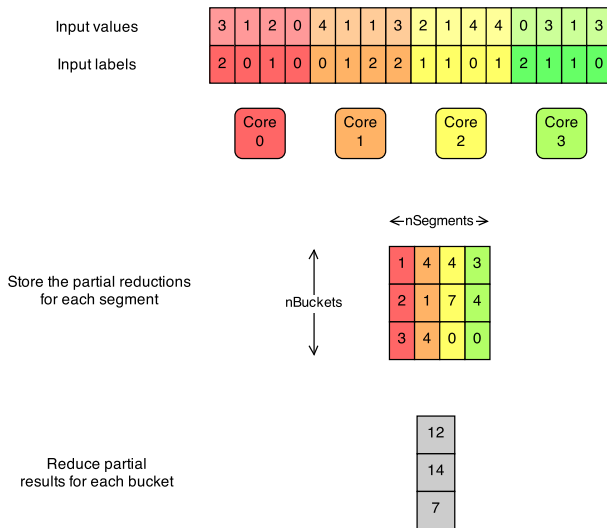
There are two possibilities to use it:

- ① Apply it directly to the entire input
- ② Partition the input into several segments, apply the algorithm to all segments in parallel, then combine partial results





# Possibilities for algorithm design



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  - High latency
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If partitioning is used, intermediate data can be stored...

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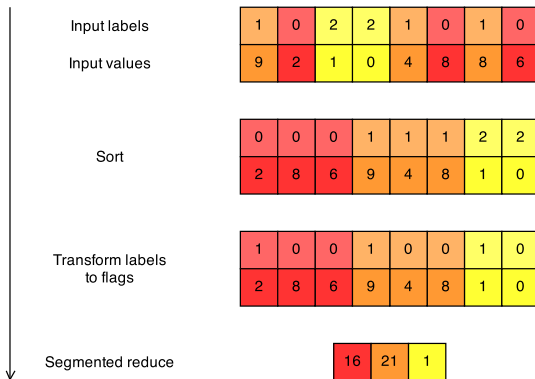
This results in three ways to implement (almost) any given algorithm.



# Sort-based algorithms

Idea:

- ① Sort input by labels, so that values for identical labels are adjacent
- ② Reduce/scan values for each label...
  - ... using a segmented reduce/scan



# Adapting the sequential algorithm

Idea: Apply the partitioning trick again on a block level.  
Let each thread work sequentially on its own segment of the input data, calculating its own partial result ("bucket set").



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Advantages:

- Arbitrary amounts of parallelism
- Basic algorithm is efficient

Problems:

- Random access reads/writes
- Input data is required in contiguous blocks, but fastest access is strided
- Many partial results have to be combined



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Many of the most frequently used operators are commutative.



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Many of the most frequently used operators are commutative.  
In this case:

- Input data can be processed in any order
- Several threads can share a bucket set
  - (as long as write conflicts are handled somehow)

Consequently

- Less partial results to process
- Optimal data access pattern without further effort
- Less cache misses
- Atomic operations needed



# Example of multireduce in shared memory

- Adaptation of sequential algorithm
- For commutative operators only
- Bucket sets stored in shared memory
- Derived from histogram algorithm



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- Adaptation of sequential algorithm
- For commutative operators only
- Bucket sets stored in shared memory
- Derived from histogram algorithm

Memory layout is the deciding factor.



# Example of multireduce in shared memory

Two approaches for histogram algorithms:

- ① Per-block bucket sets
  - Large amounts of parallelism
  - Bank and write conflicts
- ② Per-thread bucket sets
  - No conflicts at all
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My approach combines the best of both

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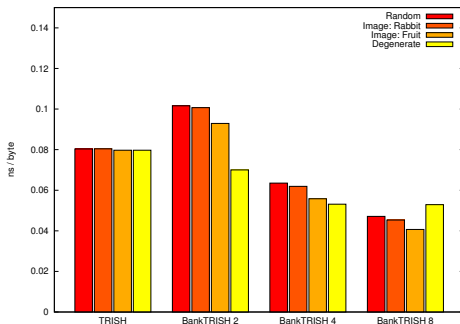
- Large amounts of parallelism
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Resulting multireduce is faster than the best histogram algorithm, but much more general.



# Application to histogramming

Algorithm can be transferred back to histogramming, resulting in a 40% speedup on average.



# Multiscan algorithms

Options for multiscan:



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Options for multiscan:

- 1 Sorting approach
  - More complicated



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Options for multiscan:

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  - Similar to non-commutative multireduce case



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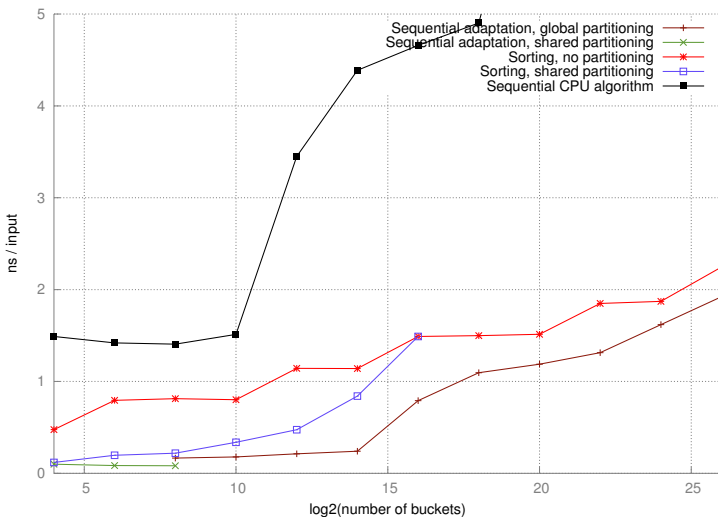
Options for multiscan:

- ① Sorting approach
  - More complicated
- ② Adaptation of sequential algorithm
  - Similar to non-commutative multireduce case
- ③ PRAM algorithm by Sheffler
  - Frequent memory accesses
  - Frequent synchronizations
  - Much data, few threads



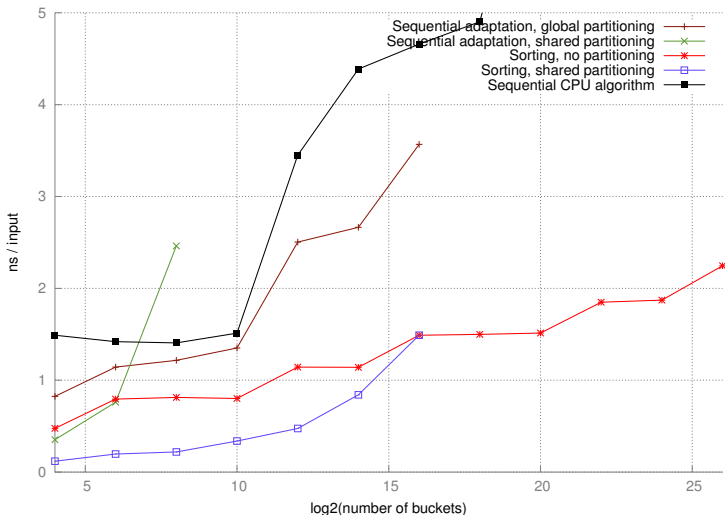
# Performance of multireduce

With commutative operator:



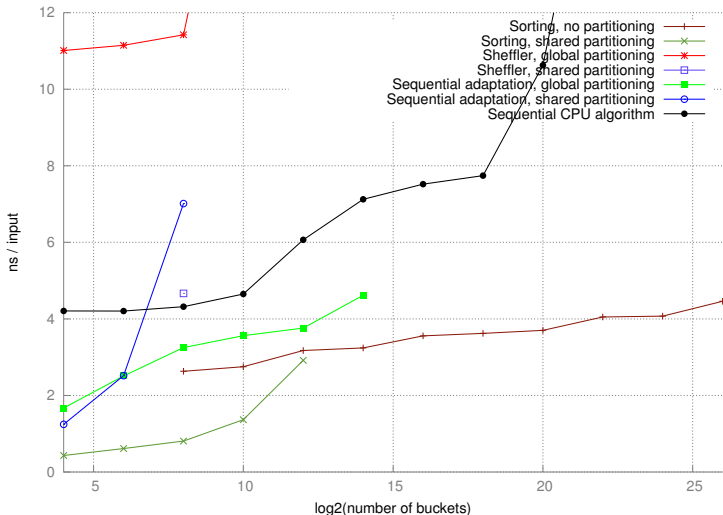
# Performance of multireduce

With non-commutative operator:





# Performance of multiscan



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  - Number of buckets and commutativity have big influence
- and can be recommended as parallel primitive
- Multiscan cannot



# Conclusion

- Theoretically optimal algorithm is not always the best in practice (caching problems)
- GPU always results in speedup over CPU
- Multireduce can be implemented efficiently
  - Number of buckets and commutativity have big influence
- and can be recommended as parallel primitive
- Multiscan cannot
- Multireduce algorithm can be applied directly to improve histogramming





# Future work



# Future work

- Explore sorting for better cache use
  - Improve simulator



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- Language integration



# Thank you for your attention!

## Questions?

