Category Theory and Computational Complexity

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December 21, 2014

A first-order dependence logic D is a class which consists of all D-definable properties where $D := (FO + \mu.\bar{t})$ and $\mu.\bar{t}$ denotes that term $t_{|\bar{t}|}$ is functionally dependent on t_i for all $i \leq |\bar{t}|$. The model class FO is as always defined as the class of models of all first-order sentences (i.e. $FO := \{S : (\exists\tau)(\exists\varphi \in L(\tau))\ S = Mod(\varphi)\}$ where $L(\tau)$ is a first-order language of type τ) and $\mu.\bar{t}$ is interpreted as a recursively generated tuple of terms which we naturally identify with the set $[|\bar{t}|] := \{1, 2, \ldots, |\bar{t}|\}$. D sentences are capable to characterise variable dependence and in general they are proven to be as expressive as the sentences of the second order Σ^1_1 fragment. The intuitionistic dependence version ID has the same expressive power as full SO. It is a fact that MID-model checking is $PSPACE$-complete where MID is the intuitionistic implication fragment of the modal dependence logic MD which contains at least two modifiers. Hence, $(FO + \mu.\bar{t}) = NP, ID = \Sigma_* P$ and $MID = PSPACE$. On the other hand, $PSPACE = IP = QIP$, and so $MID = QIP$ which is the quantum version of the interactive polytime class IP.

We shall try to cook up a purely algebraic definition for the class of structures MID and extend such categorical logic in order to capture other quantum and classical complexity classes.