

# Category Theory and Computational Complexity

Marco Larrea

Octavio Zapata

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A first-order dependence logic  $D$  is a class which consists of all  $D$ -definable properties where  $D := (FO + \mu.\bar{t})$  and  $\mu.\bar{t}$  denotes that term  $t_{|\bar{t}|}$  is functionally dependent on  $t_i$  for all  $i \leq |\bar{t}|$ . The model class  $FO$  is as always defined as the class of models of all first-order sentences (i.e.  $FO := \{S : (\exists \tau)(\exists \varphi \in L(\tau)) S = Mod(\varphi)\}$  where  $L(\tau)$  is a first-order language of type  $\tau$ ) and  $\mu.\bar{t}$  is interpreted as a recursively generated tuple of terms which we naturally identify with the set  $[[\bar{t}]] := \{1, 2, \dots, |\bar{t}|\}$ .  $D$  sentences are capable to characterise variable dependence and in general they are proven to be as expressive as the sentences of the second order  $\Sigma_1^1$  fragment. The intuitionistic dependence version  $ID$  has the same expressive power as full  $SO$ . It is a fact that  $MID$ -model checking is  $PSPACE$ -complete where  $MID$  is the intuitionistic implication fragment of the modal dependence logic  $MD$  which contains at least two modifiers. Hence,  $(FO + \mu.\bar{t}) = NP, ID = \Sigma_* P$  and  $MID = PSPACE$ . On the other hand,  $PSPACE = IP = QIP$  and so  $MID = QIP$  the quantum version of the interactive polytime class.

We shall try to cook up a purely algebraic definition for the class of structures  $MID$  and extend such categorical logic in order to capture other quantum and classical complexity classes.