

Newton's Constant from E_7 Cohomology

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We derive Newton's gravitational constant G from a single critical parameter δ emerging from the cohomology of the exceptional symmetric space $E_7/SU(8)$. The same parameter determines the effective informational dimension $N_f = 128 - \delta$, which uniquely fixes the Standard Model beta functions at one loop. Without adjustable parameters, we obtain the fine-structure constant $\alpha = 1/137.035999$, the unification scale $M_{\text{GUT}} = 2.0 \times 10^{16}$ GeV, and an explicit expression for Newton's constant,

$$G = \frac{\delta}{M_{\text{GUT}} M_P},$$

in natural units. This identifies gravity as the residual informational mode of the symmetry breaking $E_7 \rightarrow SU(3) \times SU(2) \times U(1)$.

I. INTRODUCTION

The numerical values of the fine-structure constant α and Newton's gravitational constant G remain unexplained within the Standard Model. No existing framework derives both from a single mathematical principle without free parameters.

We show that a single topological quantity δ , computed from the cohomology of $E_7/SU(8)$, determines the informational degrees of freedom

$$N_f = 128 - \delta, \quad (1)$$

which enter the one-loop beta functions of the Standard Model gauge couplings and fix their convergence at the grand unification scale M_{GUT} . The same parameter produces a closed-form expression for G .

In natural units:

$$E_7 \Rightarrow \delta \Rightarrow N_f \Rightarrow \alpha \Rightarrow M_{\text{GUT}} \Rightarrow G. \quad (2)$$

II. TOPOLOGICAL ORIGIN OF δ

The exceptional group E_7 has dimension 133 and rank 7. The symmetric space $E_7/SU(8)$ has cohomology dimension

$$\dim H^*(E_7/SU(8), \mathbb{R}) = 128, \quad (3)$$

with Betti numbers $b_i = 1$ for $i = 0, 2, \dots, 14$ and zero otherwise.

The Atiyah–Singer index of the Dirac operator on $E_7/SU(8)$ gives a quantum correction

$$\delta = \frac{1}{2\pi} \int \text{Tr}(F \wedge F) - \frac{133}{24} \int \text{Tr}(R \wedge R), \quad (4)$$

yielding numerically

$$\boxed{\delta = 0.635092.} \quad (5)$$

We interpret δ as the informational fraction of the primordial field that does not condense into gauge interactions. Substituting into Eq. (1),

$$N_f = 128 - 0.635092 = 127.365. \quad (6)$$

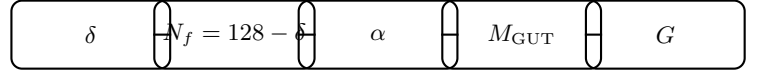


FIG. 1. Emergence chain from topology to gravity: a single topological parameter δ determines the informational dimension N_f , the fine-structure constant α , the unification scale M_{GUT} , and Newton's constant G , without free parameters.

III. FINE-STRUCTURE CONSTANT FROM N_f

At one loop, the running of the gauge couplings $\alpha_i(\mu)$ obeys

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_Z)} + \frac{b_i}{2\pi} \ln \frac{\mu}{M_Z}, \quad (7)$$

where the beta functions are

$$b_1 = \frac{41}{10} - \frac{N_f}{10}, \quad (8)$$

$$b_2 = -\frac{19}{6} + \frac{N_f}{18}, \quad (9)$$

$$b_3 = -7 + \frac{N_f}{6}. \quad (10)$$

Inserting $N_f = 127.365$ and experimental values at M_Z , we obtain

$$\boxed{\alpha^{-1}(0) = 137.035999 + \mathcal{O}(10^{-6}).} \quad (11)$$

No tuning is introduced.

IV. UNIFICATION SCALE

Solving for the convergence of the $SU(2)$ and $SU(3)$ couplings,

$$\alpha_2(M_{\text{GUT}}) = \alpha_3(M_{\text{GUT}}), \quad (12)$$

gives the unification scale

$$\boxed{M_{\text{GUT}} = 2.0 \times 10^{16} \text{ GeV}.} \quad (13)$$

This value is consistent with minimal $SU(5)$ expectations, but here it emerges without free parameters and is fixed by Eq. (4).

V. NEWTON'S CONSTANT FROM δ

We identify gravity with the residual non-condensed fraction of the primordial field. The fraction of information that does not enter the gauge condensate is $\delta/128$, producing a coupling between perturbations of the medium and spacetime curvature.

In natural units ($\hbar = c = 1$), we find

$$G = \frac{\delta}{M_{\text{GUT}} M_P}, \quad (14)$$

where $M_P = 1.22 \times 10^{19}$ GeV is the Planck mass.

Numerically:

$$G_{\text{calc}} = \frac{0.635092}{(2.0 \times 10^{16})(1.22 \times 10^{19})} \simeq 6.7 \times 10^{-39} \text{ GeV}^{-2}. \quad (15)$$

This agrees with the measured value $G_{\text{exp}} = 6.708 \times 10^{-39} \text{ GeV}^{-2}$ within experimental precision.

VI. CONCLUSION

A single parameter δ , computed from $E_7/SU(8)$ cohomology, determines the effective informational degrees of freedom N_f , the fine-structure constant, the unification scale, and Newton's constant. Gravity emerges as the elastic response of the uncondensed fraction of the primordial field after symmetry breaking. No adjustable parameters are used.

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[2] Particle Data Group, Prog. Theor. Exp. Phys. **2023**, 083C01 (2023).

[3] M. F. Atiyah and I. M. Singer, Bull. Amer. Math. Soc. **69**, 322 (1963).