A robust 3D Interest Points Detector based on Harris operator

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Plan of the presentation

I) Basic Harris IDP for images

II) Theoretical description of the proposed method

- A) General procedure
- B) How to define a neighbourhood ?
- C) How to define the best fitting plane ?
- D) How to fit a quadratic surface ?
- E) How to select interest points ?

III) Numerical experiments and the obtained results

- A) Some examples of IPD
- B) How to evaluate the quality of the detection ?
- C) Evaluation of the invariance to rotation
- D) Evaluation of the invariance to scaling
- E) Evaluation of the invariance to noise addition
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I) Basic Harris IDP for images

Autocorrelation function:

$$e(x, y) = \Sigma_{(u,\, v)\, \in K} W(u, v)[I(u+x, v+y) - I(u, v)]^2$$

with K the pixel neighbourhood and I the pixel intensity.

Taylor development:

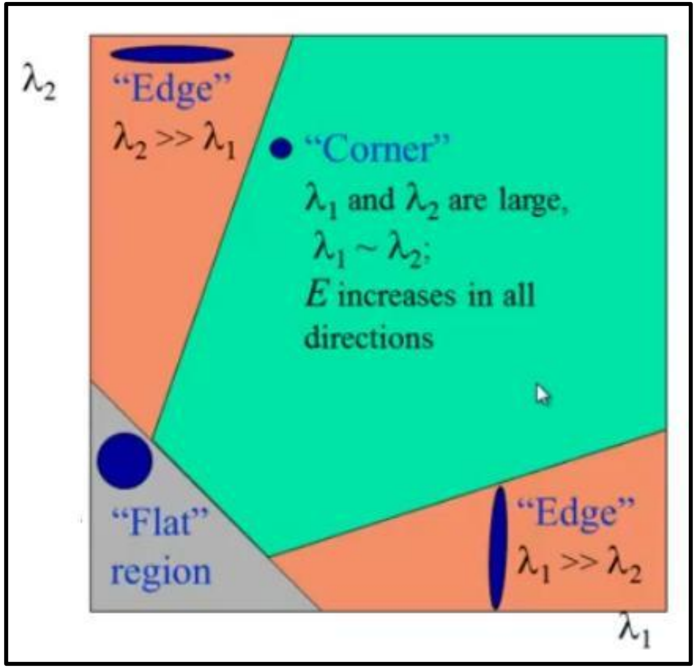
$$e(x, y) = \begin{pmatrix} x & y \end{pmatrix} \underbrace{\begin{pmatrix} \sum_{u,v} W(u,v) \cdot I_x^2 & \sum_{u,v} W(u,v) \cdot I_x \cdot I_y \\ \sum_{u,v} W(u,v) \cdot I_x \cdot I_y & \sum_{u,v} W(u,v) \cdot I_y^2 \end{pmatrix}}_{E} \begin{pmatrix} x \\ y \end{pmatrix}$$

Indicator:

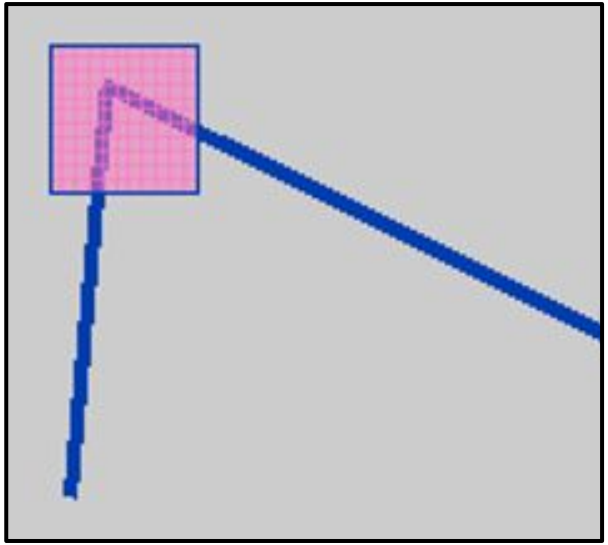
$$det(E) - h\, Tr(E)^2$$

with *h* a constant to tune

I) Basic Harris IDP for images

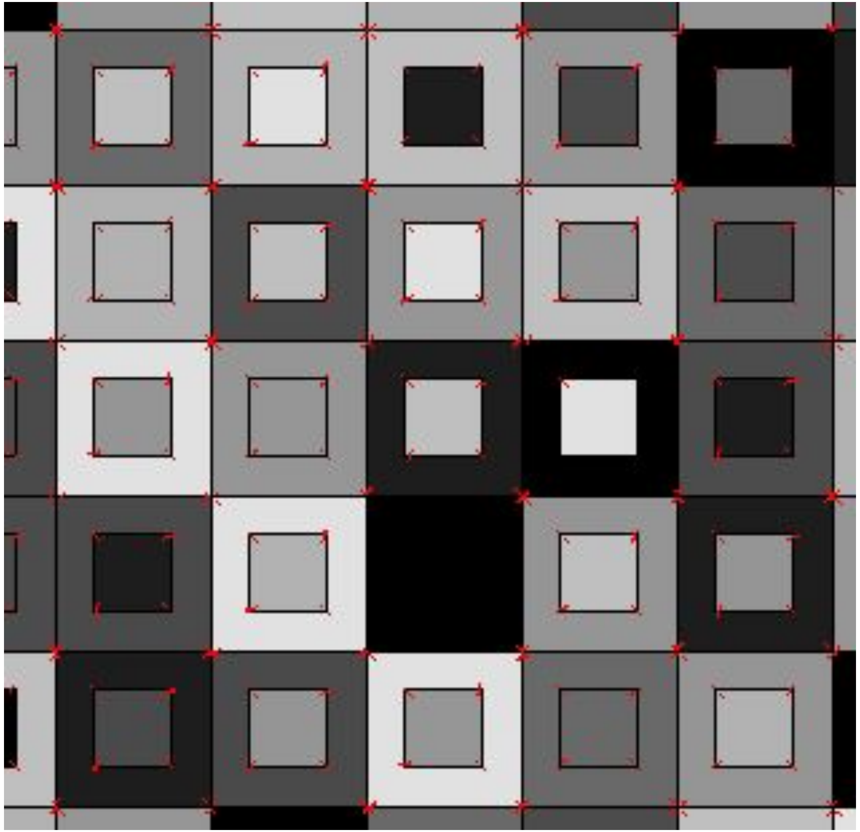


Shifting the window in any direction should yield a large change in appearance.



$\det(E) = \lambda_1 \lambda_2$
$Tr(E) = \lambda_1 + \lambda_2$

I) Basic Harris IDP for images



Advantages:

- Invariant to rotation
- Invariant to scale variation
- Invariant to illumination conditions
- Invariant to noise

Issue:

- Derivative definition for point clouds

II) Theoretical description of the proposed method

General procedure:

1. To define a neighbourhood of a vertex v
2. To translate the point cloud (centroid = origin)
3. To compute the best fitting plane
4. To rotate the point cloud (normal of the fitting plane = axis Z)
5. To translate the point cloud (v = origin)
6. To fit a quadratic surface
7. To compute the derivatives on this surface
8. To compute the hessian matrix E and the vertex response det(E) - h Tr(E)
9. To keep interest points according to the response values

II) Theoretical description of the proposed method

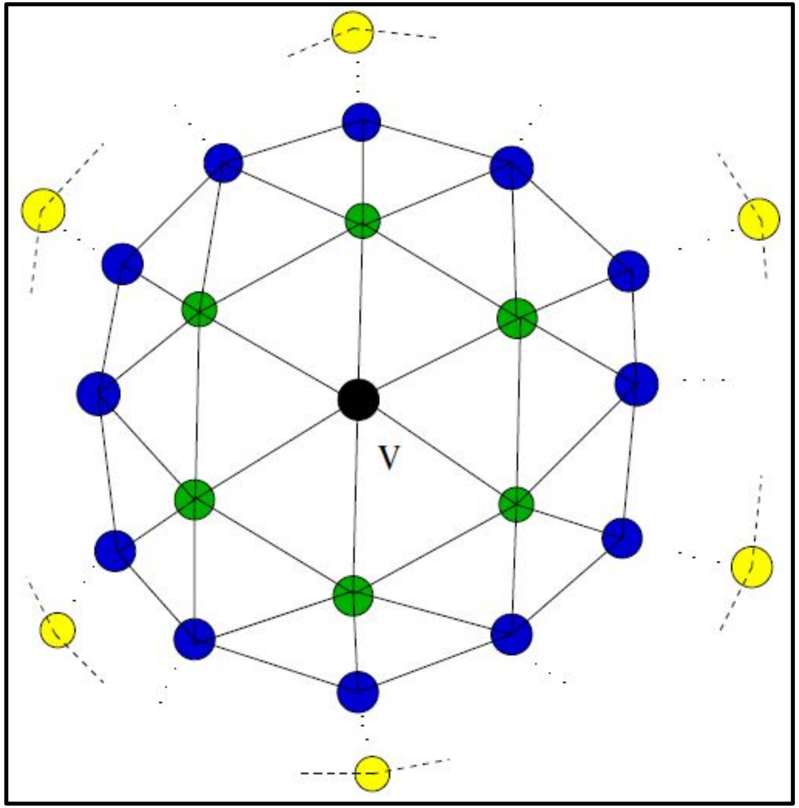
How to define the neighbourhood ?

- Spherical neighbourhood
- K-Nearest Neighbours (KNN)
- K-closest rings :

$ring_k(v)=\{w \mid sorthest\_path(v, w)=k\}$

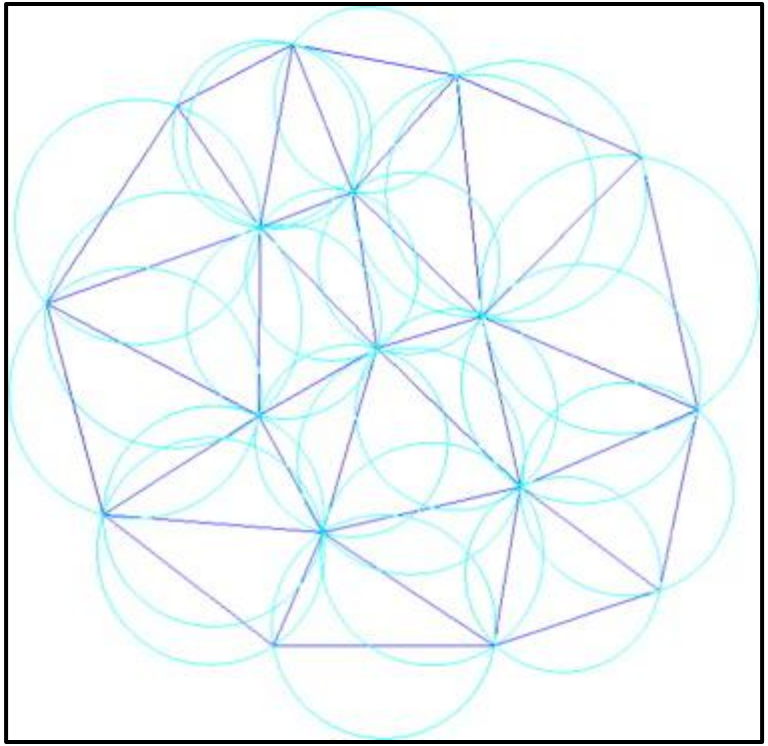
⇒ Need of tessellation

⇒ Delaunay’s triangulation ?



II) Theoretical description of the proposed method

A Delaunay's triangulation is such that no point is inside the circumcircle of any triangle in the triangulation. It maximizes the minimum angle of all the angles of the triangles in the triangulation.



II) Theoretical description of the proposed method

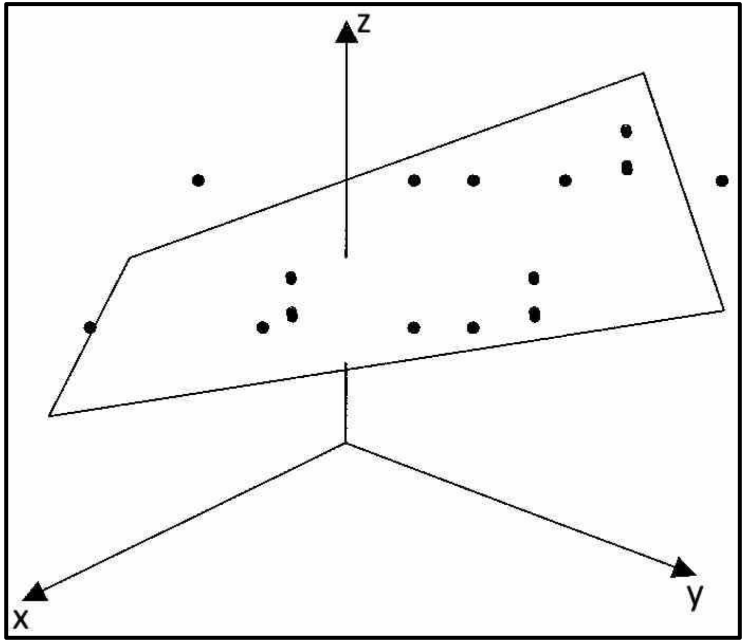
The adaptive technique

$$d_{ring}(v, ring_k(v)) = \max_{w \in ringk(v)} \|v-w\|^2$$

$$radius(v) = \{k \in \mathbb{N} \mid d_{ring}(v, ring_k(v)) \geq \delta \ \ \& \ \ d_{ring}(v, ring_{k-1}(v)) < \delta\}$$

with δ the fraction of the diagonal of the object bounding rectangle

II) Theoretical description of the proposed method

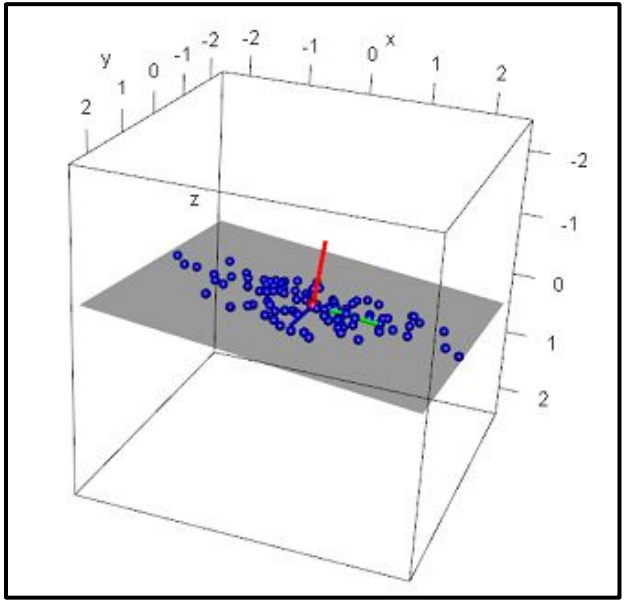


How to define the best fitting plane ?

- To apply Principal Component Analysis (PCA)
- To choose the eigenvector associated to the lowest eigenvalue

II) Theoretical description of the proposed method

Principal Component Analysis (PCA) is a technique to study the linear relationship of variables by converting a set of observations into a smaller set of linearly uncorrelated variables. It computes the eigenvalues and eigenvectors of the covariance matrix of the data. The eigenvector associated with the smallest eigenvalue represents the axis with the least amount of variance.



II) Theoretical description of the proposed method

How to fit a quadratic surface ?

- Least square method

$$f(x,y) = \frac{p_1}{2} \cdot x^2 + p_2 \cdot x \cdot y + \frac{p_3}{2} \cdot y^2 + p_4 \cdot x + p_5 \cdot y + p_6$$

- Integration of the derivatives with a Gaussian function

$$\boxed{\begin{aligned} A &= \frac{1}{\sqrt{2\pi}\sigma} \int_{R^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}} \cdot f_x(x,y)^2 dx \cdot dy \\ B &= \frac{1}{\sqrt{2\pi}\sigma} \int_{R^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}} \cdot f_y(x,y)^2 dx \cdot dy \\ C &= \frac{1}{\sqrt{2\pi}\sigma} \int_{R^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}} \cdot f_x(x,y) \cdot f_y(x,y) dx \cdot dy \end{aligned}}$$

$$\mathrm{E} = \begin{pmatrix} A & C \\ C & B \end{pmatrix}$$

II) Theoretical description of the proposed method

$$A = 2 \cdot p_1^2 + 2 \cdot p_2^2 + p_4^2$$

$$B = 2 \cdot p_2^2 + 2 \cdot p_3^2 + p_5^2$$

$$C = 2 \cdot p_1 \cdot p_2 + 2 \cdot p_2 \cdot p_3 + p_4 \cdot p_5$$

Implementation:

- numpy.linalg.lstsq
- Coefficient matrix :

$$\begin{pmatrix} 1 & y^1 & y^2 \\ x^1 & x^1 \cdot y^1 & x^1 \cdot y^2 \\ x^2 & x^2 \cdot y^1 & x^2 \cdot y^2 \end{pmatrix}$$

- Dependent variable : altitude of points

II) Theoretical description of the proposed method

How to select interest points ?

- Non-Maximum Suppression
- Select a proportion of points with highest Harris responses

$\Rightarrow$ not homogeneous distribution

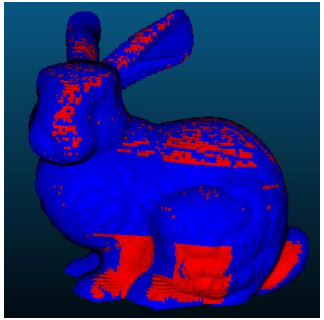
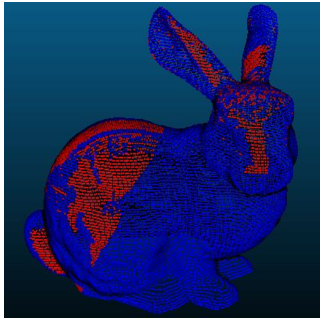
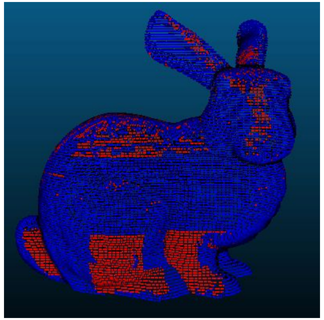
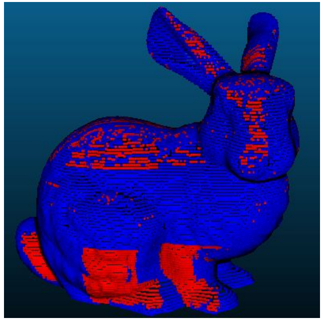
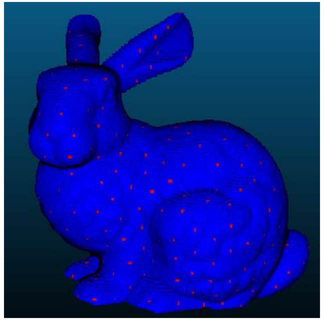
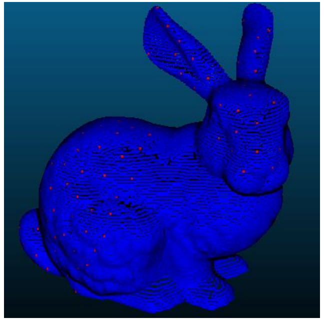
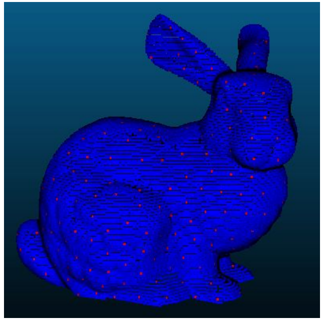
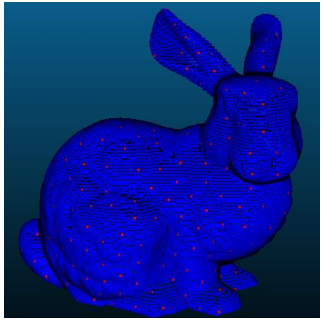
- Adaptive Non-Maximum Suppression

III) Numerical experiments and the obtained results

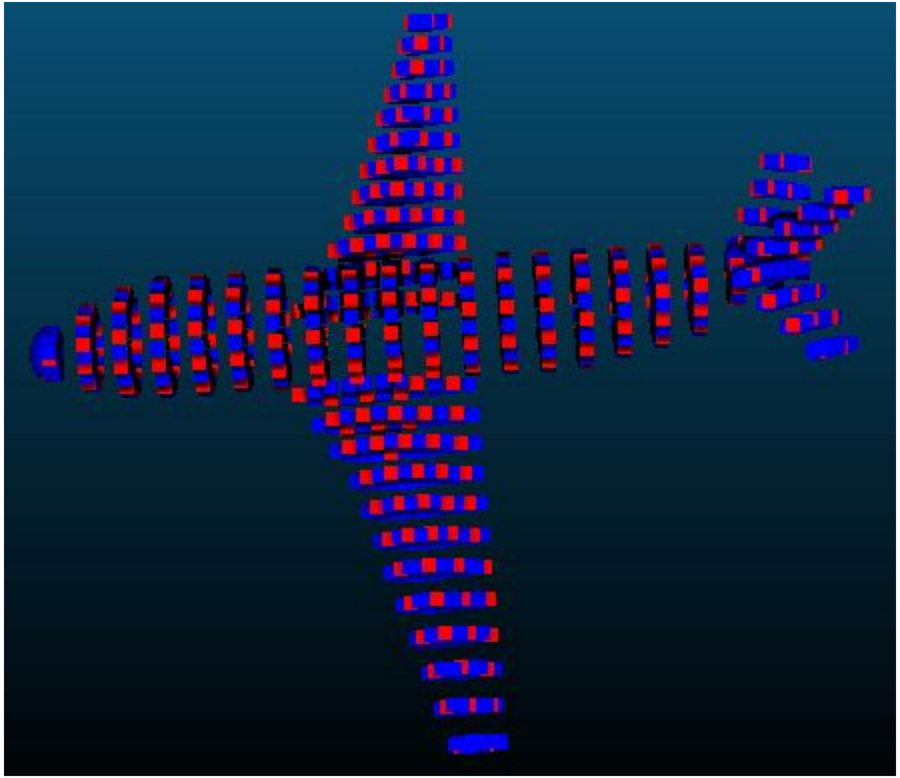
Parameters of the experiments:

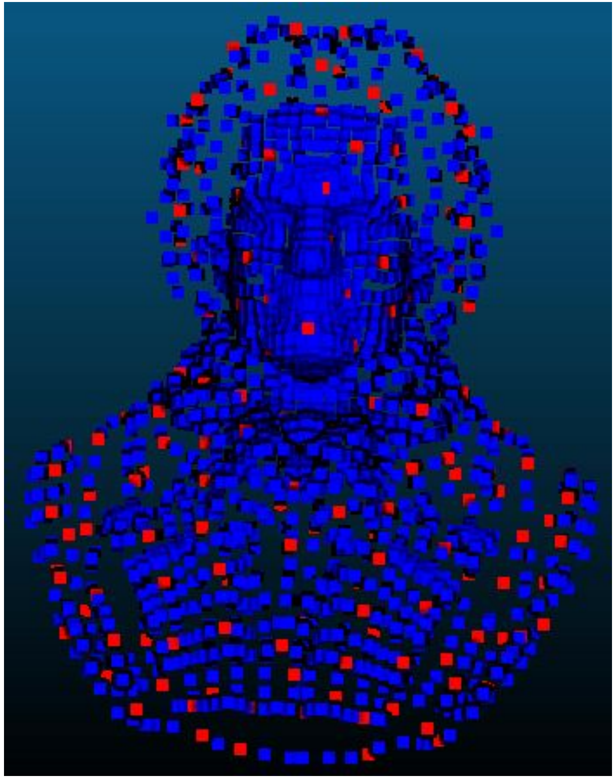
- The Harris constant parameter : h = 0.04
- The fraction of the diagonal of the object bounding rectangle : δ = 0.025
- The fraction of selected points : 1%
- The threshold for ANMS : 0.01

III) Numerical experiments and the obtained results

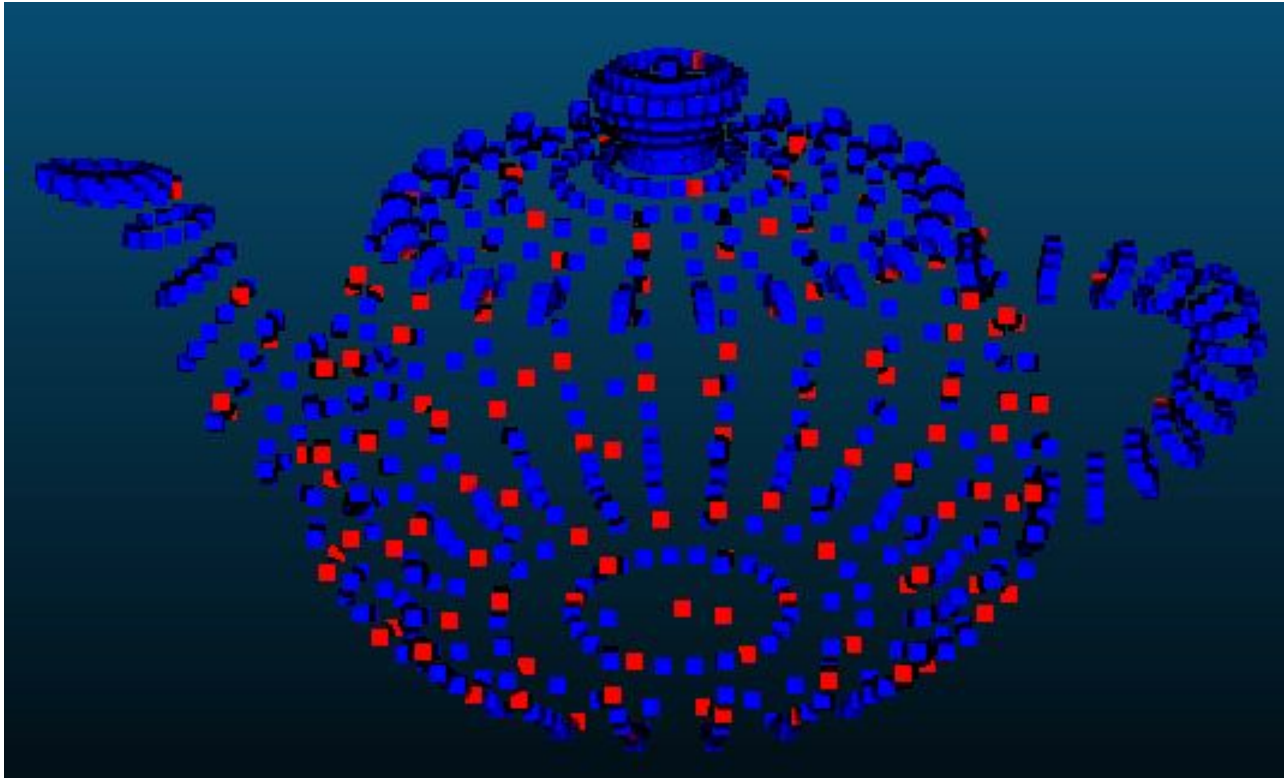
	KNN (3)	Spherical (0.1)	K-ring (3)	K-ring adaptive
Fraction				
ANMS				

III) Numerical experiments and the obtained results





III) Numerical experiments and the obtained results



III) Numerical experiments and the obtained results

How to evaluate the quality of the detection ?

Repeatability against several transformations (translation, scaling, rotation, noise addition…) :

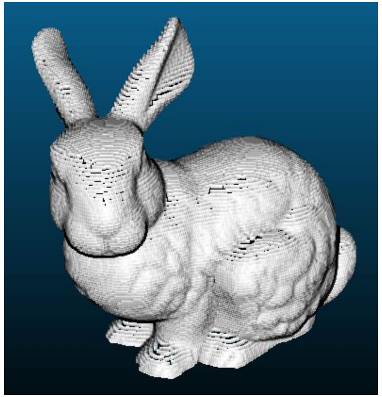
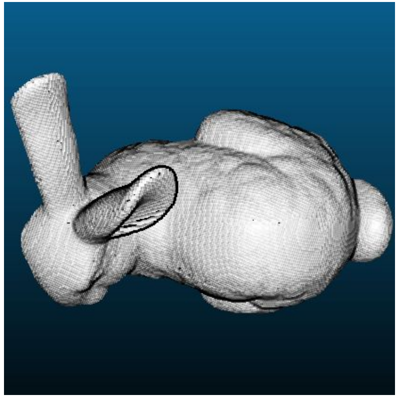
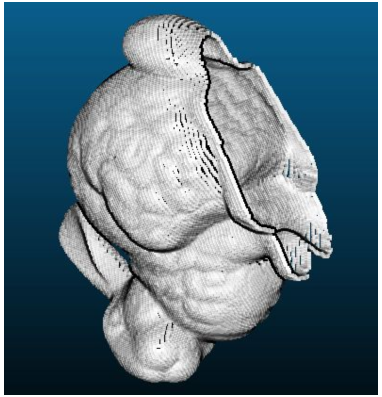
$$\boxed{R_{O,T(O)} = \frac{\#(P_O \cap P_{T(O)})}{\#P_O}}$$

with O the 3D object, P_O the detected interest points, T a transformation function and # the cardinal

III) Numerical experiments and the obtained results

Evaluation of the invariance to rotation:

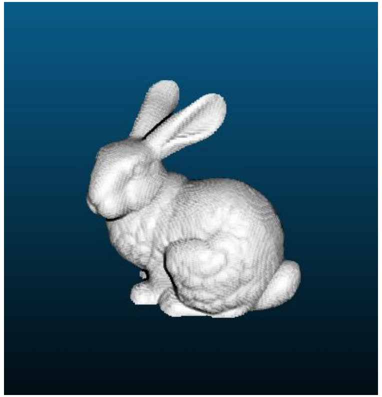
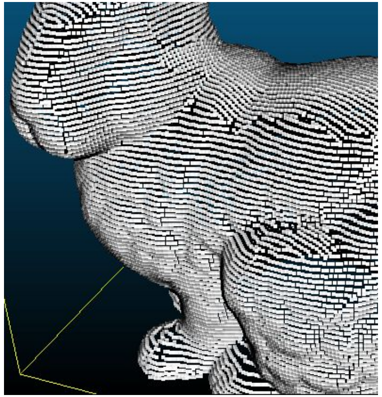
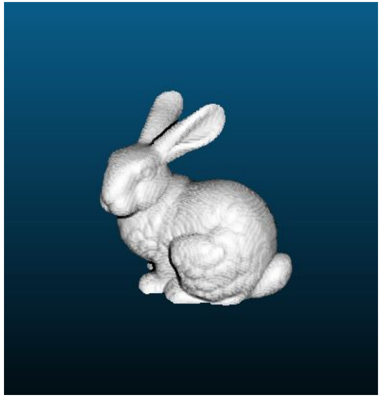
- Repeatability : 0.997 (article : 0.8745)

Point cloud			
Angle	0.8007 (axis y)	1.3470 (axis x)	2.0758 (axis z)

III) Numerical experiments and the obtained results

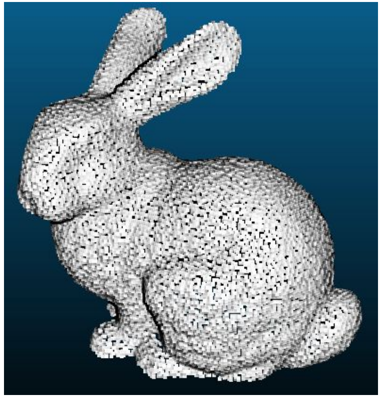
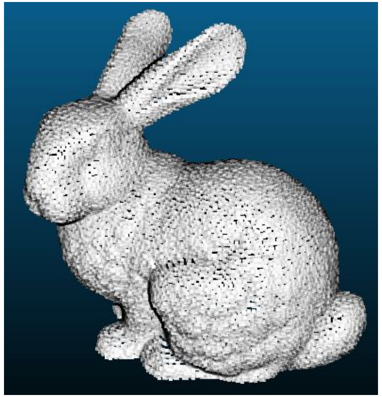
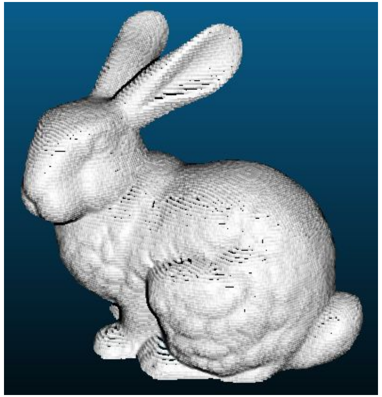
Evaluation of the invariance to scaling:

- Repeatability : 1 (article : 1)

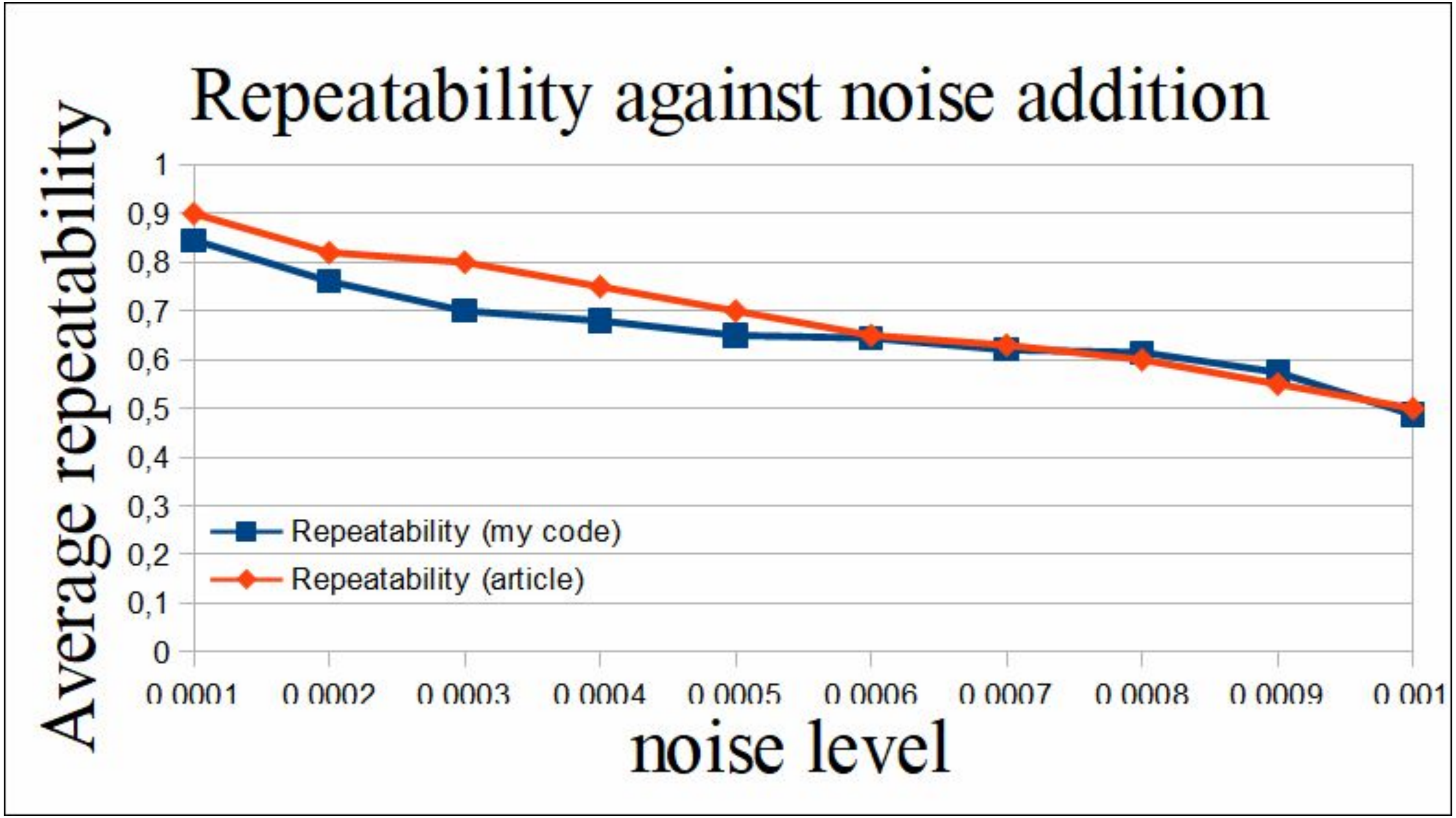
Point cloud			
scale	1.5878	0.6096	1.7704

IV) Numerical experiments and the obtained results

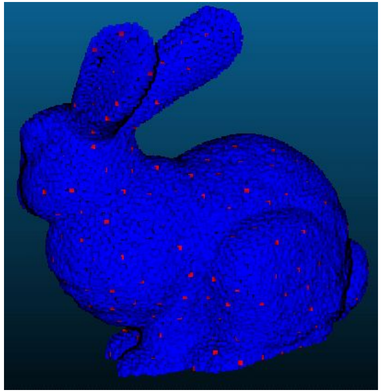
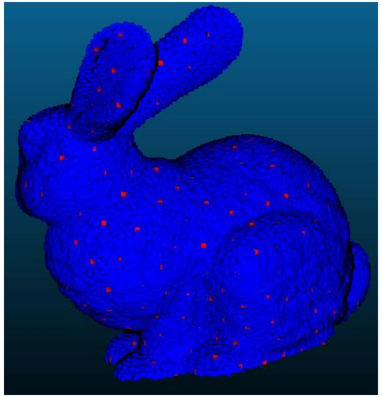
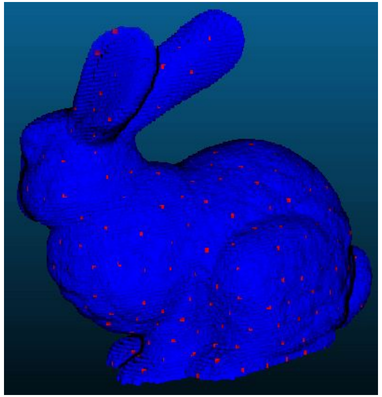
Evaluation of the invariance to noise addition:

Point cloud			
Noise level	0.00082	0.00055	0.00010

III) Numerical experiments and the obtained results



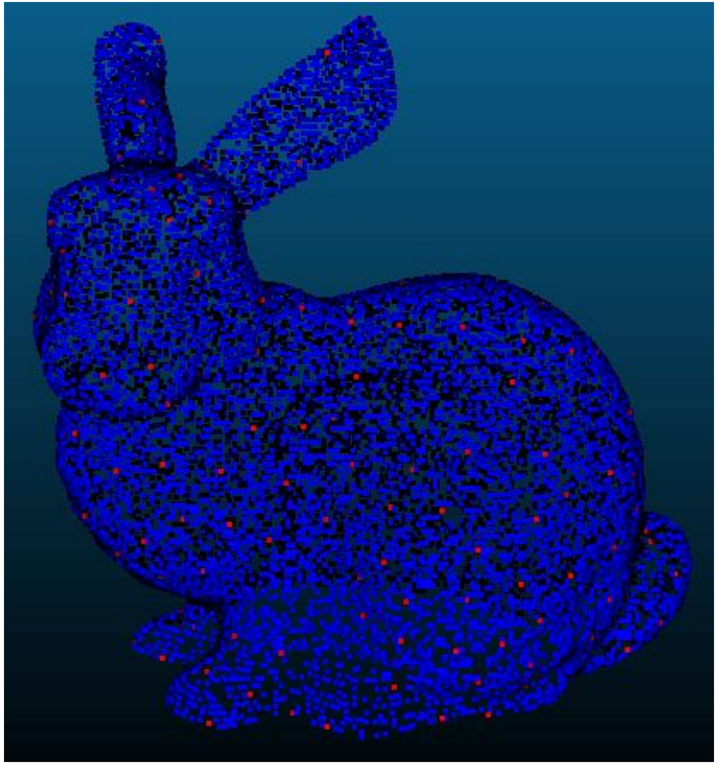
III) Numerical experiments and the obtained results

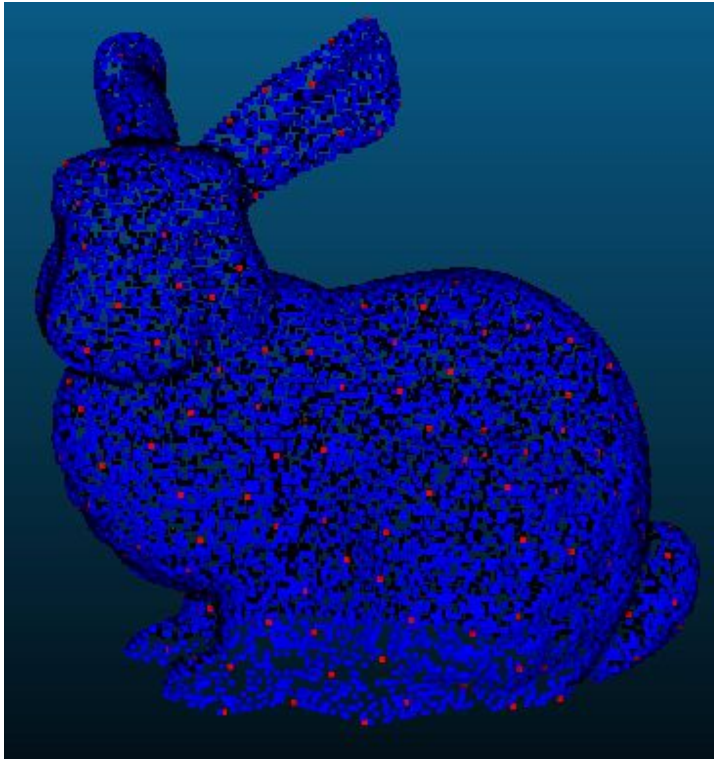
Point cloud			
Noise level	0.00073	0.00055	0.00028

⇒ Same distribution

III) Numerical experiments and the obtained results

Evaluation of the invariance to resolution change:

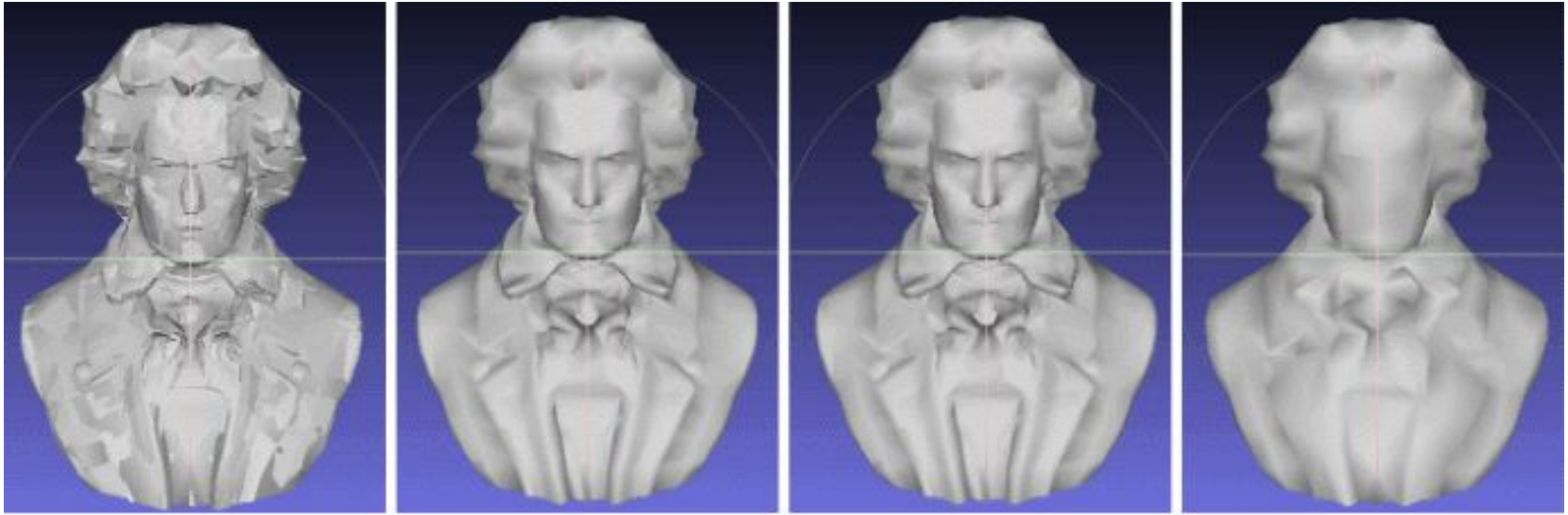




III) Numerical experiments and the obtained results

Evaluation of the invariance to resolution change:

Meshlab → Simplification: Quadric Edge Collapse Decimation



(a) Original mesh (b) Faces: 3000 | Points: 75% (c) Faces: 2000 | Points: 75% (d) Faces: 2000 | Points: 25%

III) Numerical experiments and the obtained results

Garland's Method: This surface simplification algorithm is based on quadric error metrics that measure how far a vertex is from a ideal spot. Each vertex v has an associated set of planes P_v (faces of the mesh).

The error is given by: $v^T \cdot (\Sigma_{p \in P_v} K_p) \cdot v = v^T \cdot Q \cdot v$

with the quadric K_p:
$$\begin{pmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{pmatrix}$$

III) Numerical experiments and the obtained results

Garland's Method:

INITIALISATION

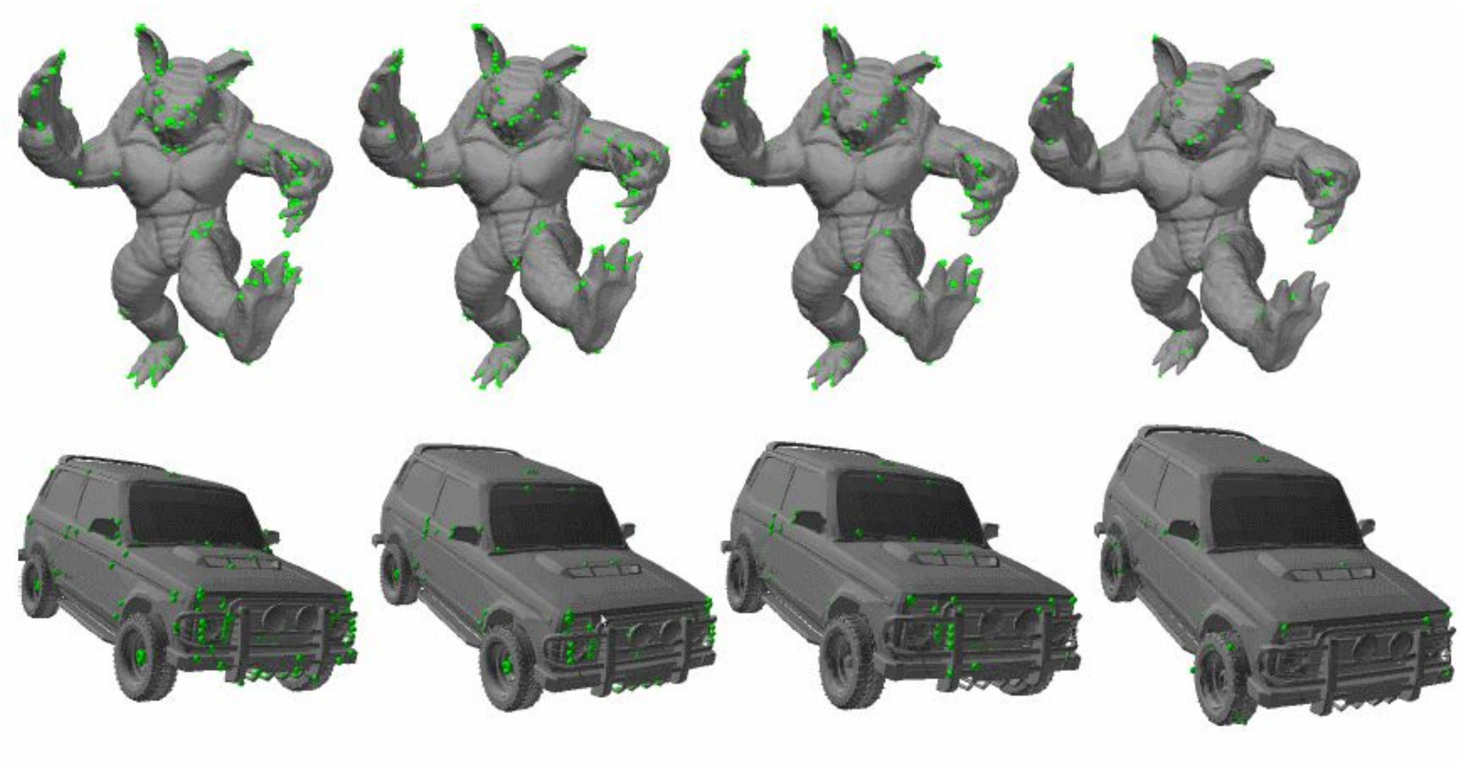
- To compute the quadrics for all initial vertices.
- To select all pairs of points whose distance is smaller than a given threshold.
- To compute minimal cost candidate for each pair.

ITERATIONS

- To select lowest cost pair (v_1, v_2).
- To contract (v_1, v_2) into a new vertex v associated to $Q = Q_1 + Q_2$.
- To update all pairs involving v_1 and v_2.

III) Numerical experiments and the obtained results

Evaluation of the invariance to resolution change:



Conclusion

Advantages

- Quite robust to noise, rotation and scaling
- Homogeneous distribution of interest points

Inconveniences

- Not robust against the resolution of an object
- Need of parameter tuning