PIEMONTE
FONDI STRUTTURALI E DI INVESTIMENTO EUROPEI 2014/2020
fondo sociale europeo
ITS TECNOLOGIE DELL'INFORMAZIONE E DELLA COMUNICAZIONE

Diffie-Hellman Key Exchange

Sicurezza informatica

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Prof. Marco Farina

marco.farina@its-ictpiemonte.it

t.me/marcofarina

in collaborazione con:
ISTITUTI TECNICI SUPERIORI
Ministero dell'Istruzione
UNIONE EUROPEA
REGIONE PIEMONTE
per una crescita intelligente, sostenibile ed inclusiva
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INIZIATIVA CO-FINANZIATA CON FSE

1

Le chiavi crittografiche

Proprietà delle chiavi crittografiche

Casuale

Protetta

Condivisa

La potenza di un sistema crittografico dipende dalla tecnica usata per condividere la chiave.

Come si può condividere la chiave?

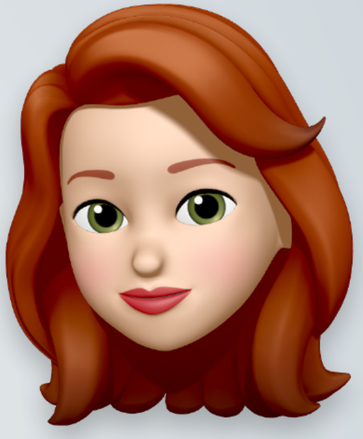
A mano

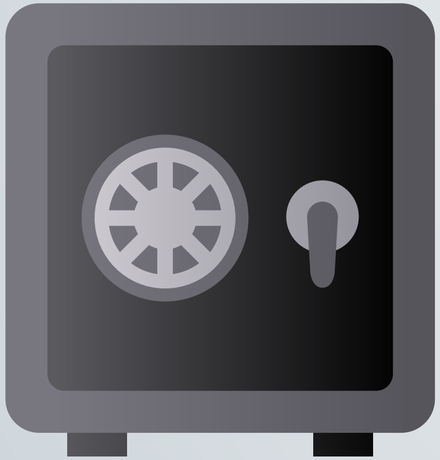
Terza parte

Canale sicuro

I primi due metodi non sono utilizzabili in epoca informatica; il terzo presuppone che una chiave sia già stata scambiata.

Il metodo dei lucchetti





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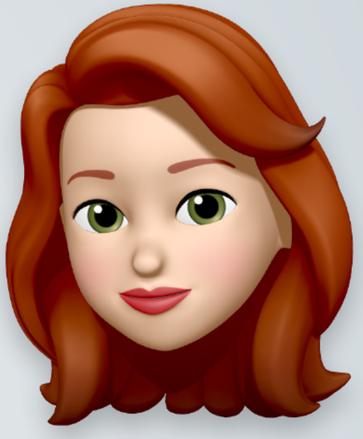

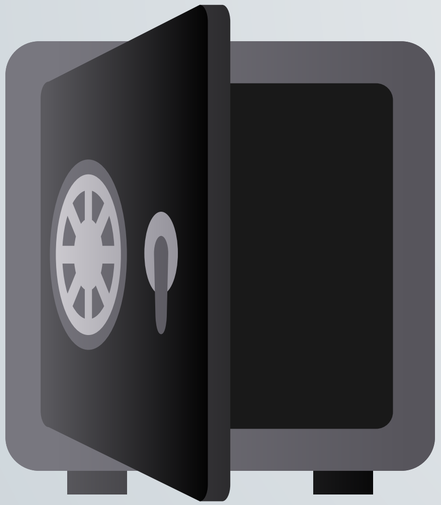

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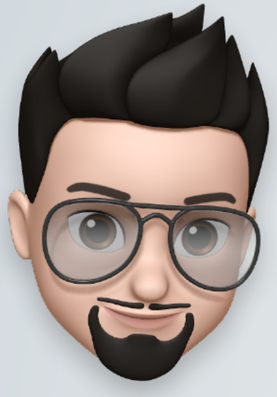

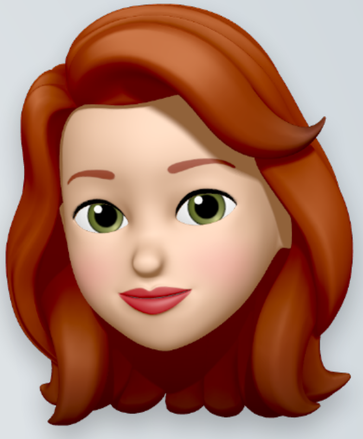

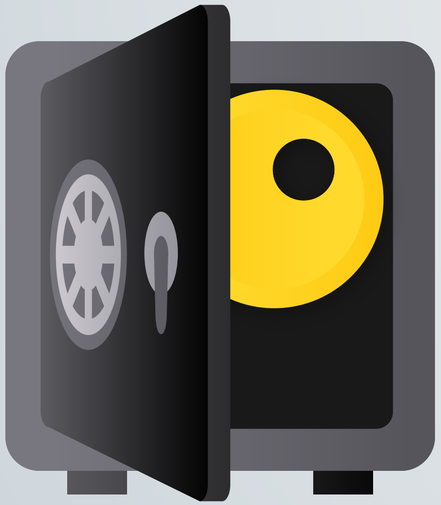

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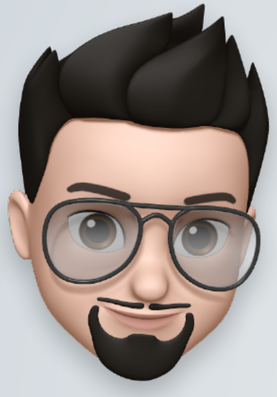

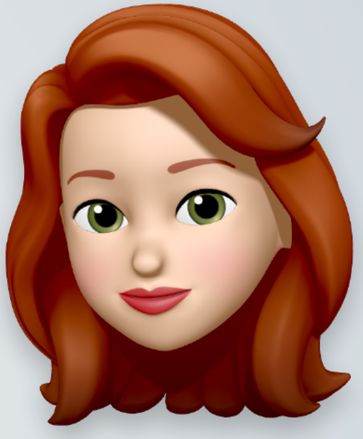

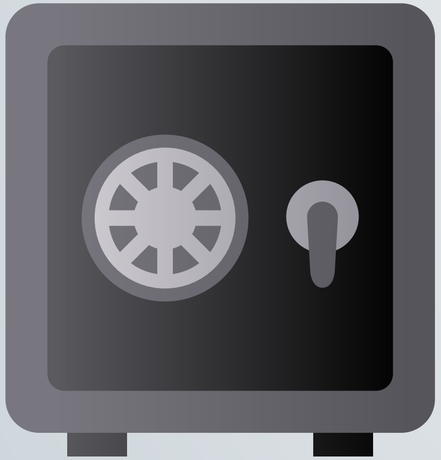

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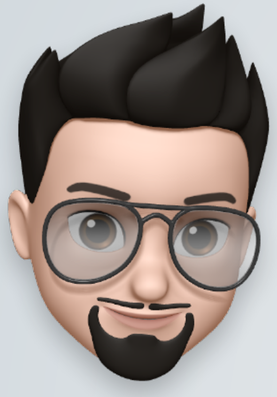

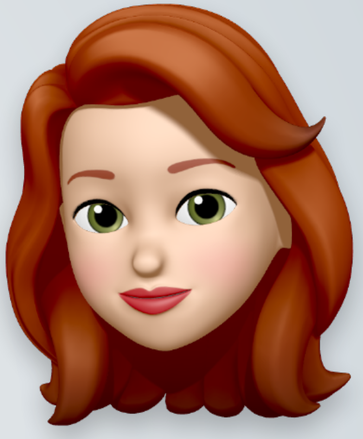

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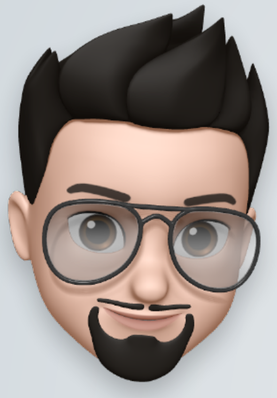

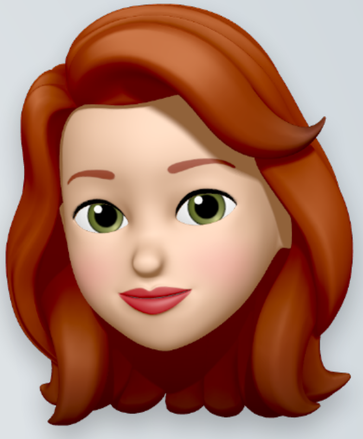

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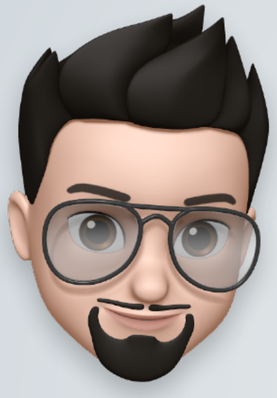

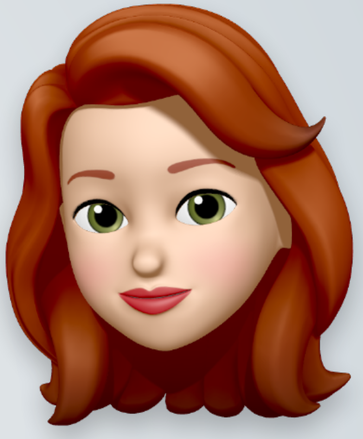

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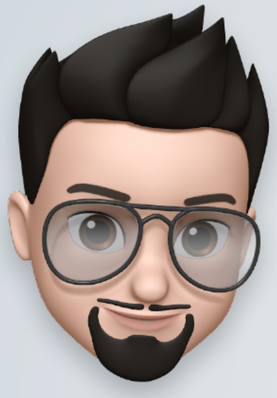

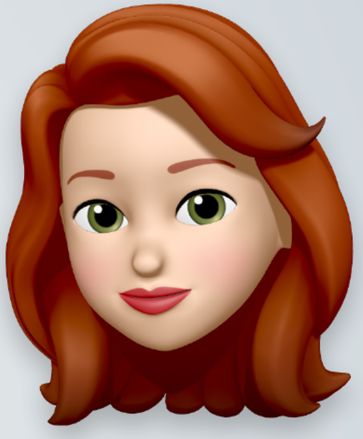

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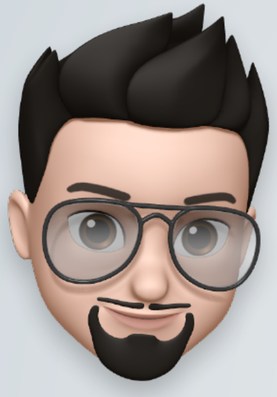

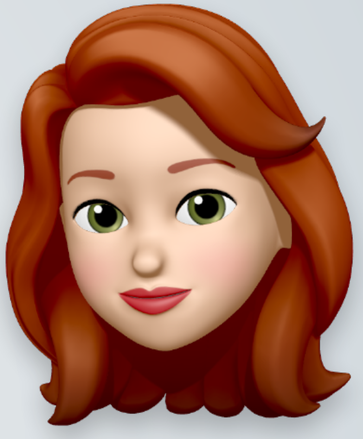

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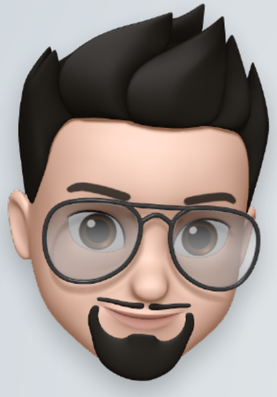

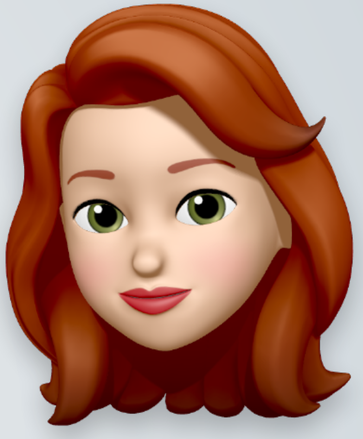

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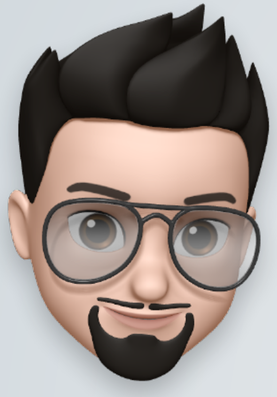

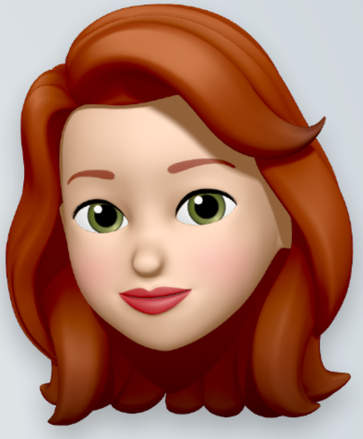

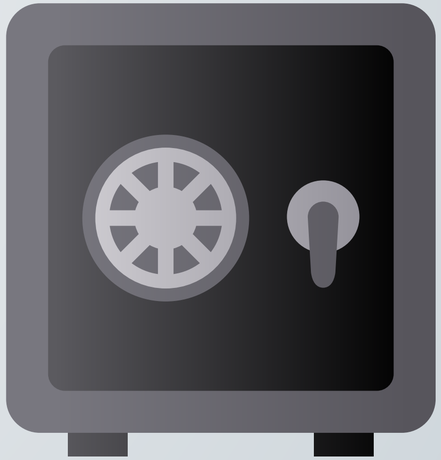

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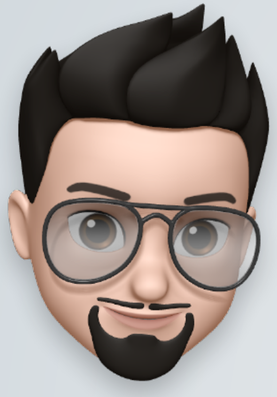

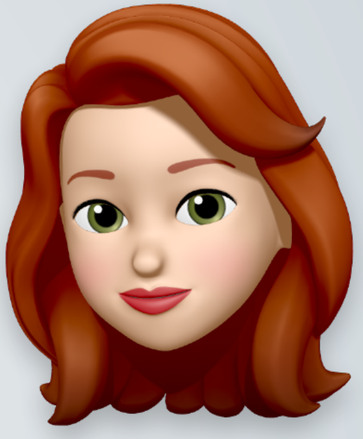

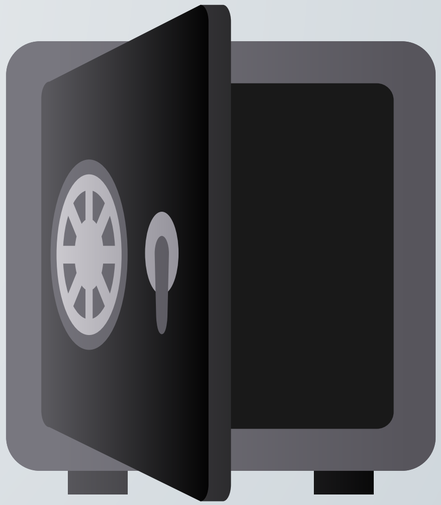

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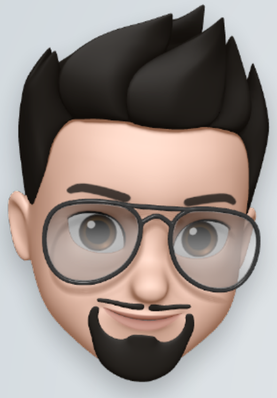
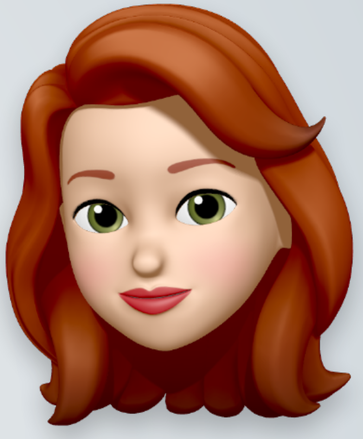
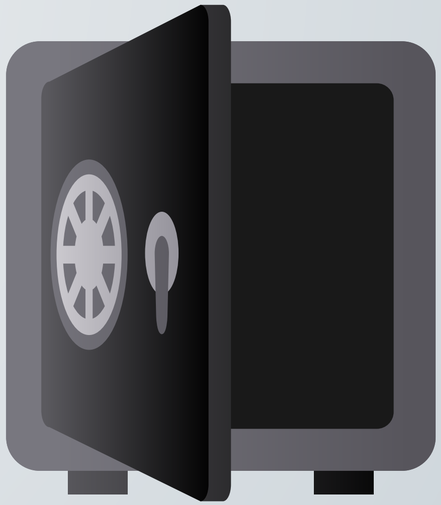




Perché questo metodo
non può funzionare?



Composizione di cifrature

Chiave di Bob

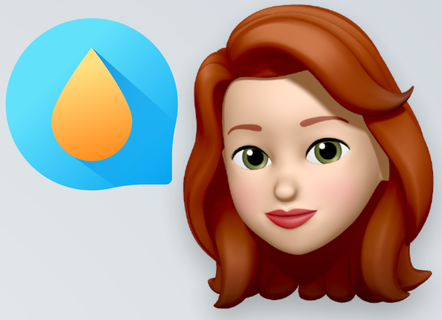
```
abcdefghijklmnopqrstuvwxyz
HFSUGTAKVDEOYJBPNXWCQRIMZL
```

Chiave di Alice

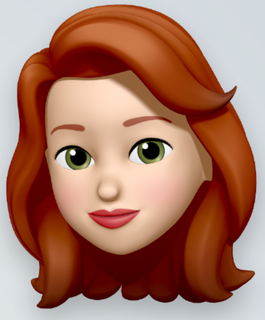
```
abcdefghijklmnopqrstuvwxyz
CPMGATNOJEFWIQBURYHXSDZKLV
```

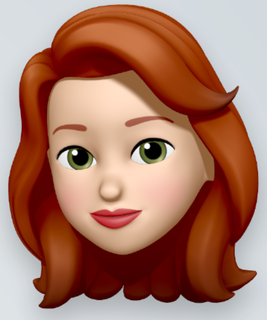
Messaggio	ci vediamo all una
Cifrato da Bob	SV RGUVHYB HOO QJH
Cifrato da Alice	HD YNSDOLP OBB REO
Decifrato da Bob	AJ MQCJLZP LOO VKL
Decifrato da Alice	ei cnaiywb yhh zxy

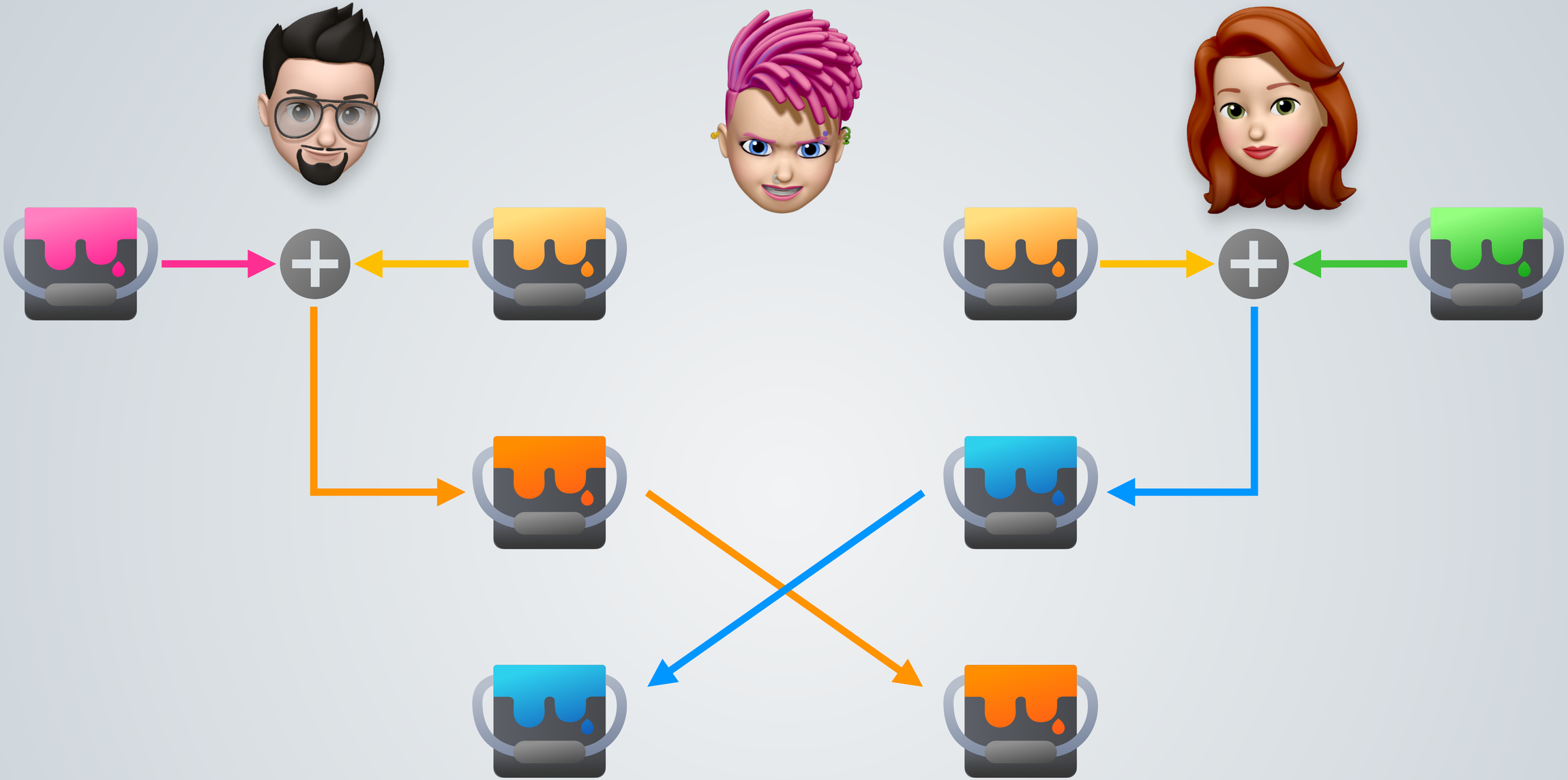
Il metodo dei colori

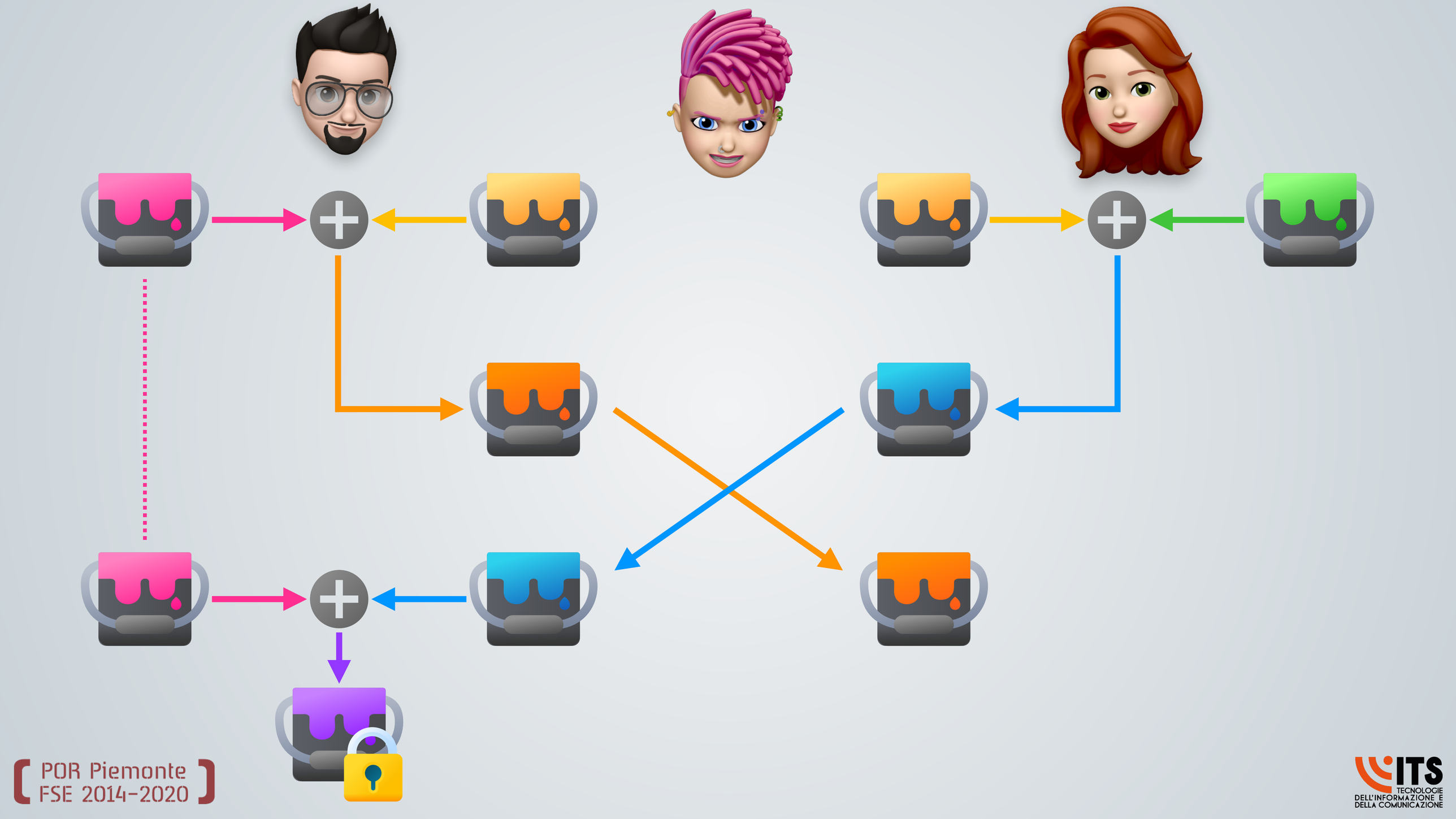


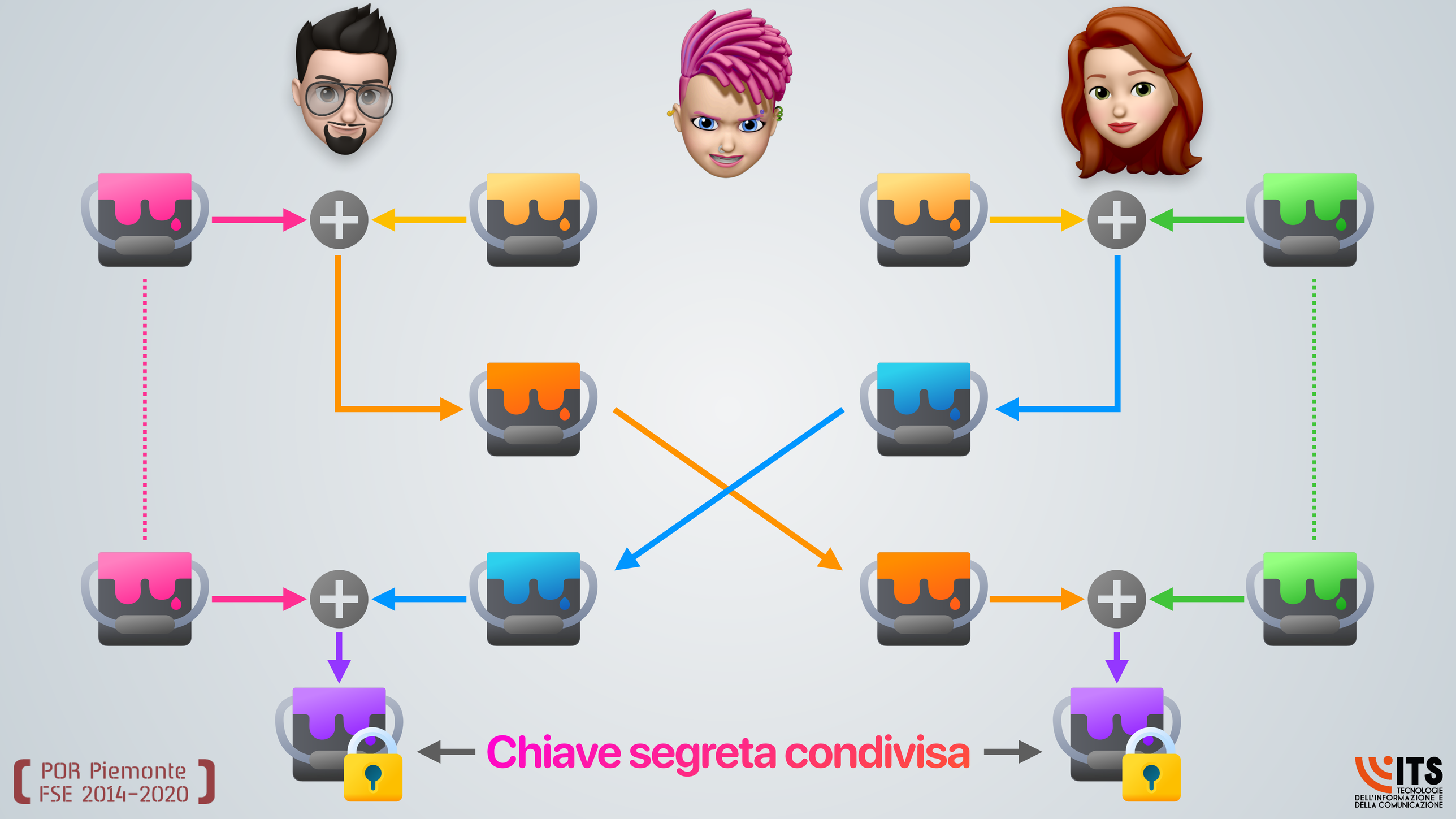












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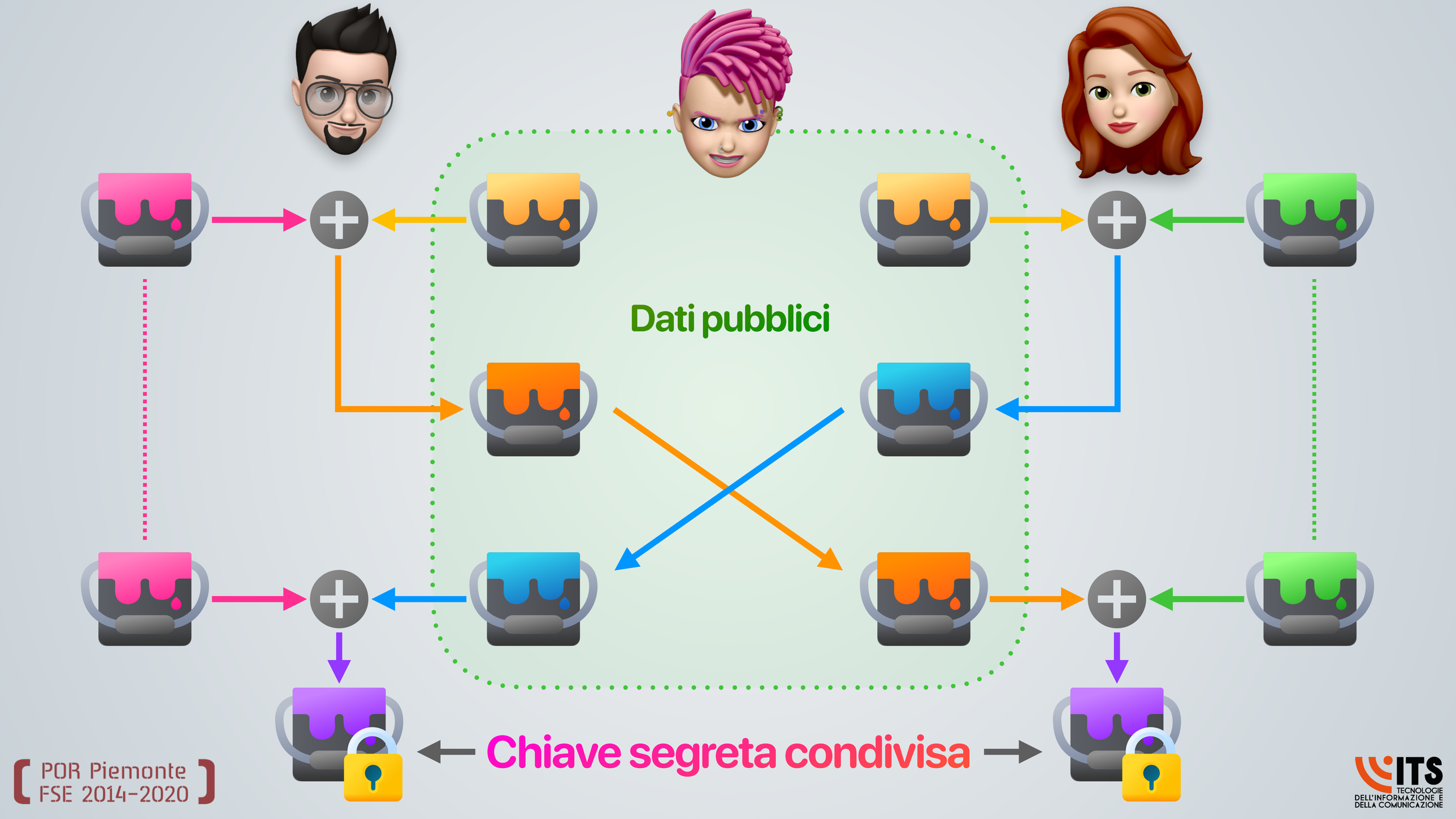

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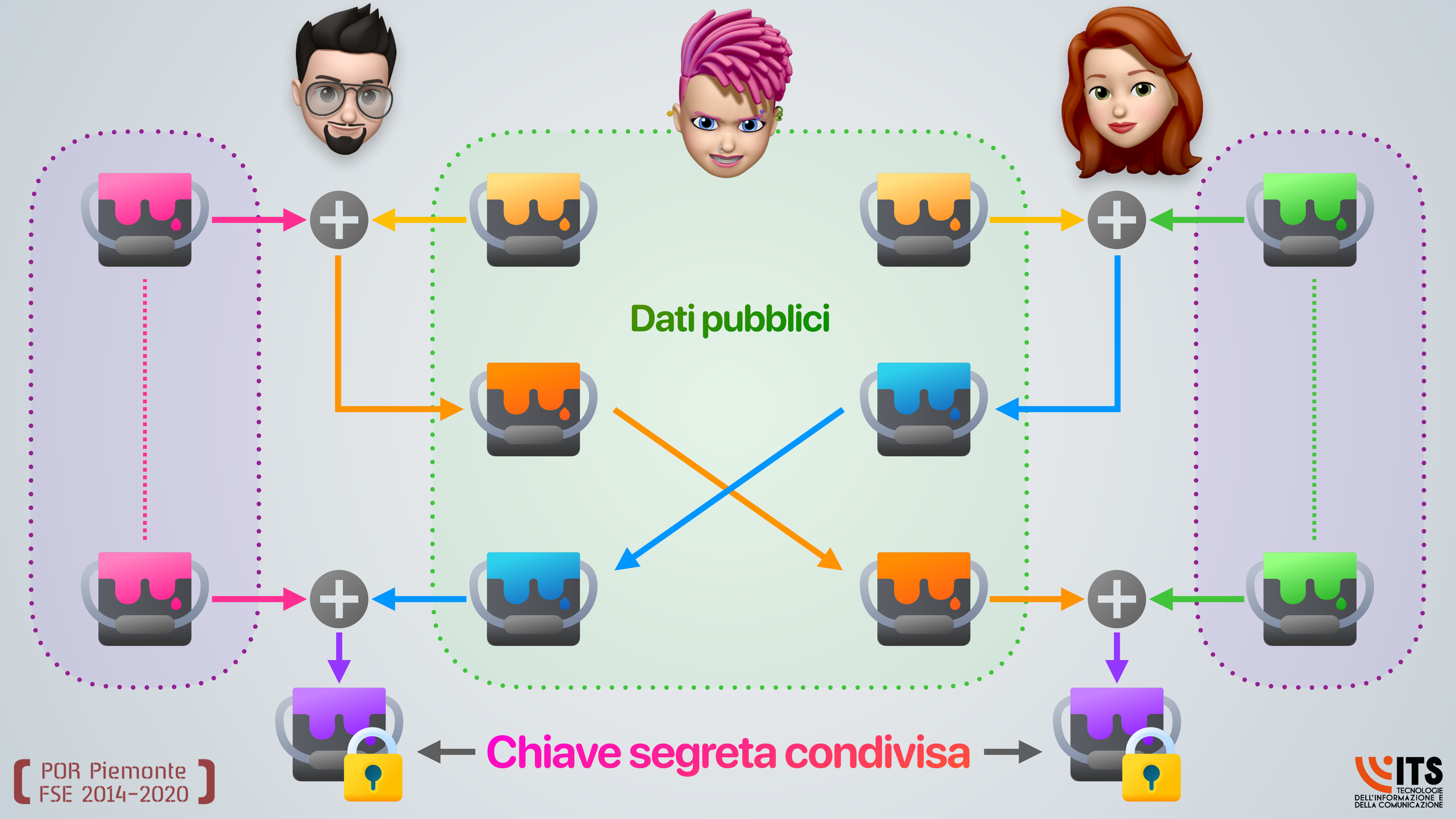

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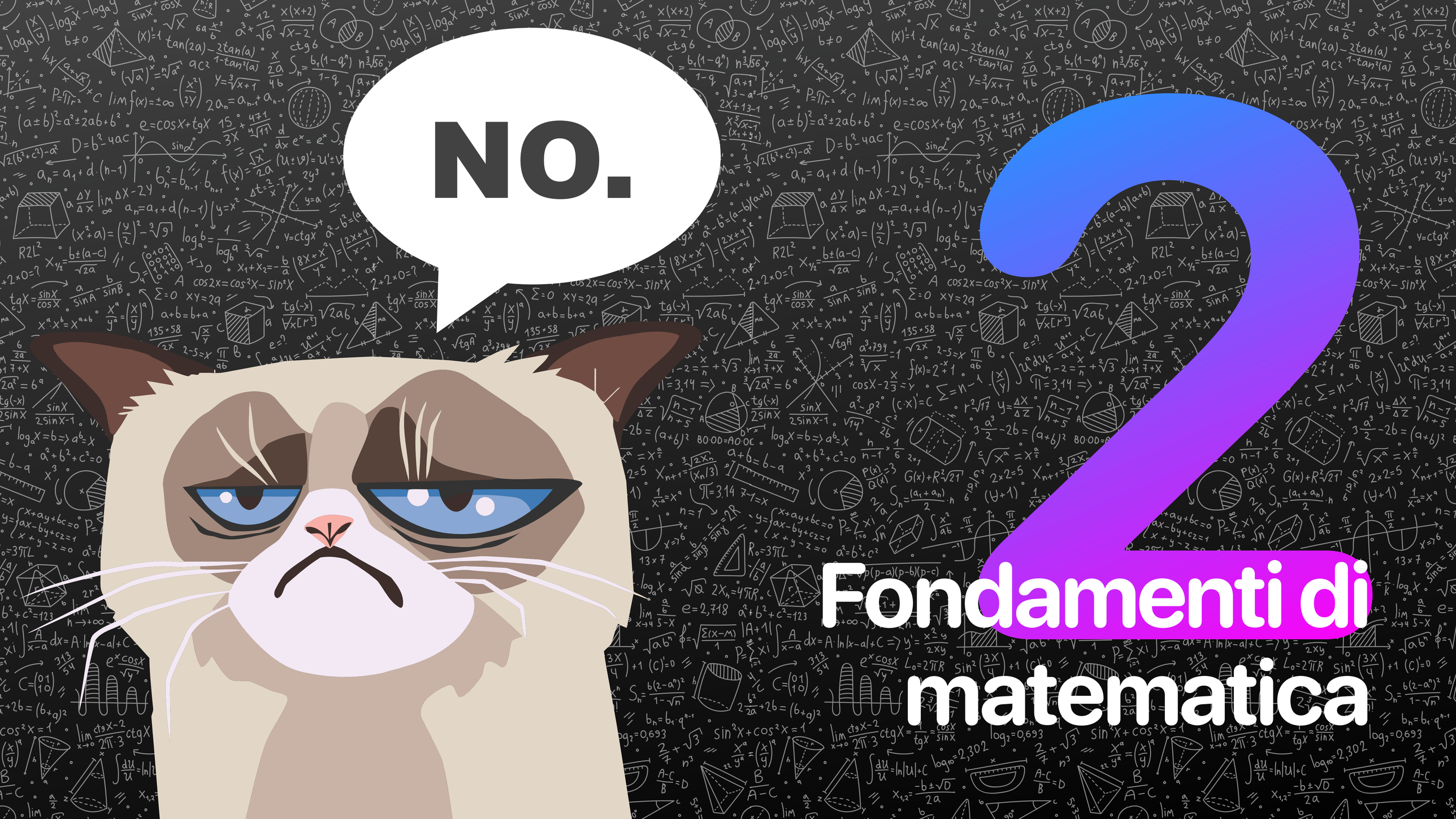
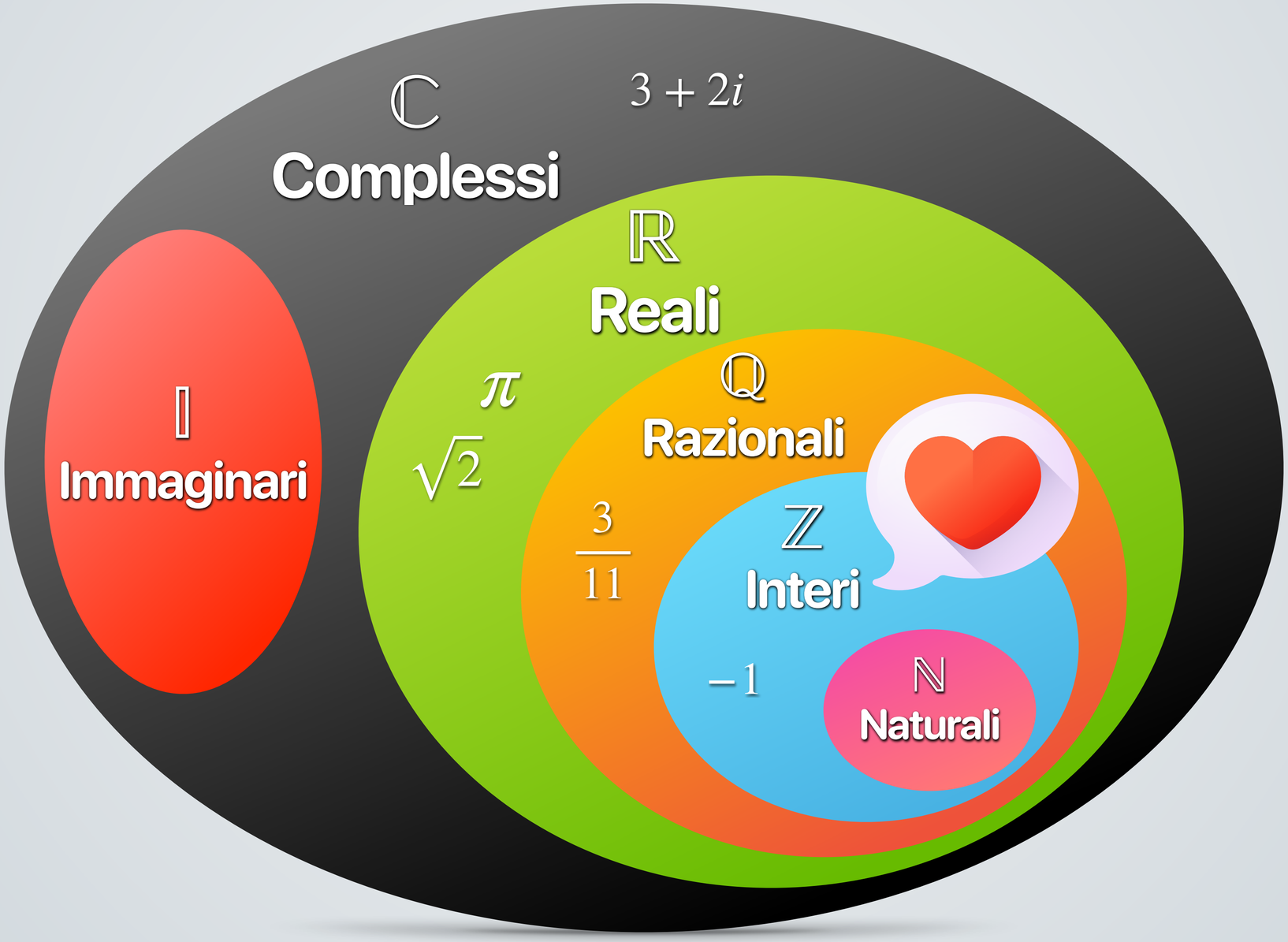


2 Fondamenti di matematica

Insiemi numerici

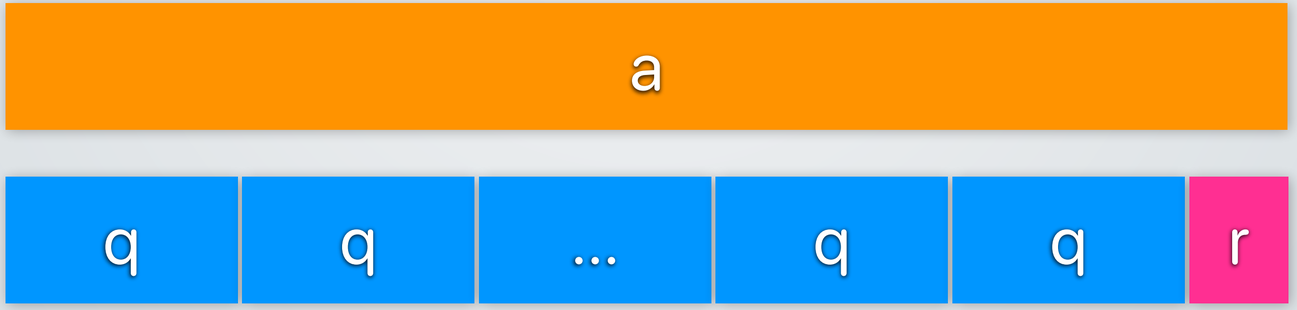


Teorema della divisione

Se $a \geq 0$ e $b > 0$ sono interi, esistono e sono univocamente determinati gli interi $q \geq 0$ ed r tali che

$$a = bq + r \quad \text{e} \quad 0 \leq r < |b|$$

q ed r sono detti rispettivamente il quoziente e il resto della divisione.



b blocchi grandi q più un certo resto r.

I numeri primi

Un numero intero p si dice primo se:

- $p \neq \{-1, 1, 0\}$;
- per ogni $a \in \mathbb{Z}$, se a divide p, allora $a \in \{-1, 1, p, -p\}$.

In altre parole, un intero è un primo se è diverso da $\{-1, 1, 0\}$ e non ha divisori propri.

PRIME AWARDS

POR Puglia FSE 2014-2020

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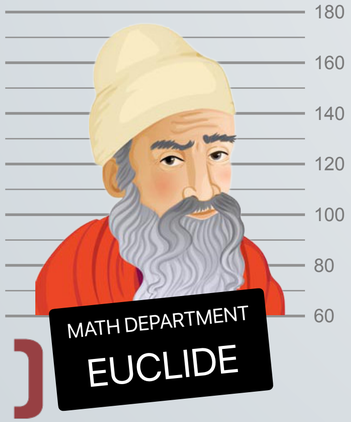
Teorema fondamentale dell'aritmetica

Fattorizzazione

Sia $z \in \mathbb{Z}$ un intero diverso da $\{-1, 0, 1\}$. Allora esistono numeri primi $p_1, p_2, \ldots, p_n$ tali che

$$z = p_1 \cdot p_2 \cdot \ldots \cdot p_n$$

Inoltre tale fattorizzazione è unica, a meno del segno dei numeri primi e del loro ordine nel prodotto.



Euclide (Grecia, -III/-V sec.), negli *Elementi*, insieme all'esistenza della fattorizzazione, aveva dimostrato una proposizione, oggi nota come *lemma di Euclide*, dalla quale si ricava la proprietà di fattorizzazione unica.

Il teorema fu poi dimostrato esplicitamente per la prima volta da Gauss nelle *Disquisitiones Arithmeticae*.

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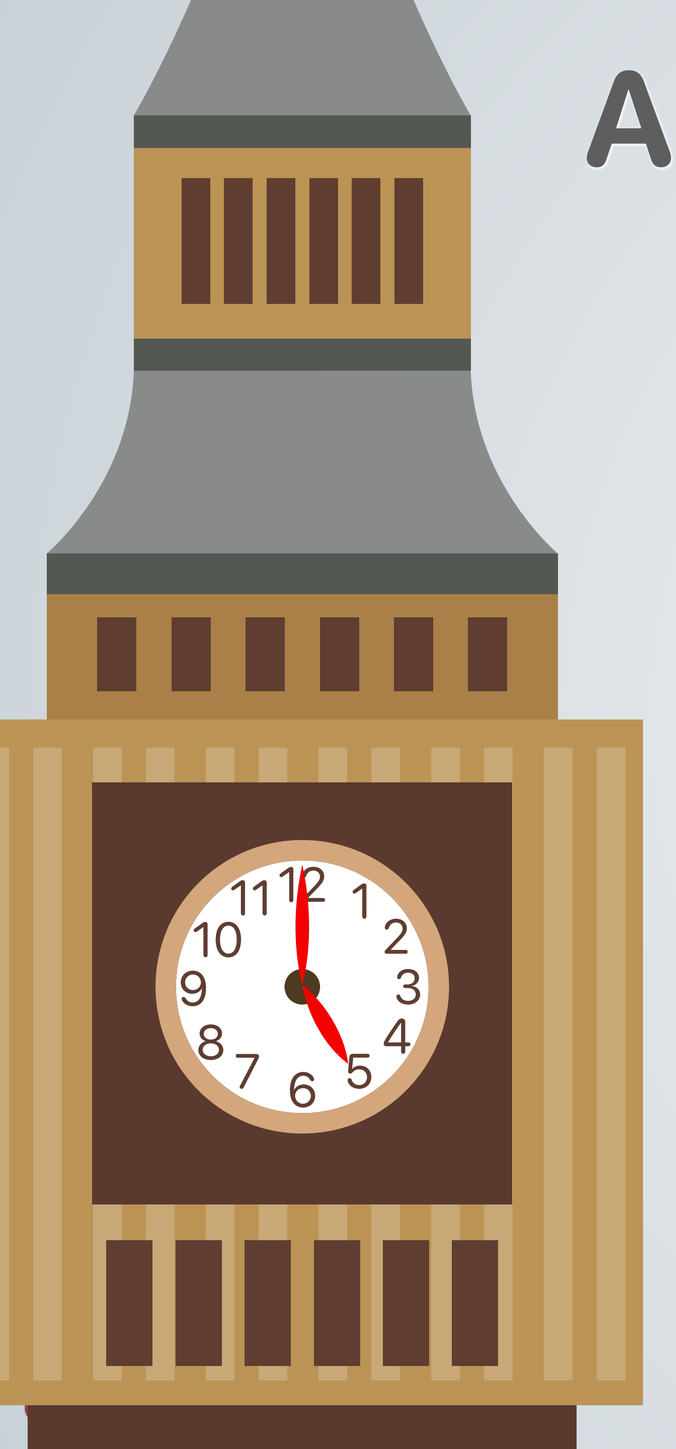
Aritmetica dell'orologio

Siamo a Londra e sono le 9 del mattino. Abbiamo un appuntamento tra 8 ore. A che ora abbiamo l'appuntamento?



Aritmetica dell'orologio

Siamo a Londra e sono le 9 del mattino. Abbiamo un appuntamento tra 8 ore. A che ora abbiamo l'appuntamento?

Qual è il rapporto tra il 5 e il 17?

5 è il resto della divisione intera tra 17 e 12!

- Possiamo definire tutte le operazioni aritmetiche nel "mondo dell'orologio".
- Possiamo astrarre l'idea di orologio e immaginarlo con n ore sul quadrante.

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Relazioni di congruenza

Fissiamo un intero n, con $n \geq 2$. I due numeri $a, b \in \mathbb{Z}$ sono detti congruenti modulo n se $n \mid (b - a)$. In questo caso si scrive

$$a \equiv b \mod n$$

o, equivalentemente, $a \mod n = b$.

Esempio (orologio).

Fissiamo $n = 12$ e scegliamo $a = 3$ e $b = 15$.

Questi numeri sono congruenti in modulo 12 se $12 \mid (15 - 3)$, che è vero. Dunque:

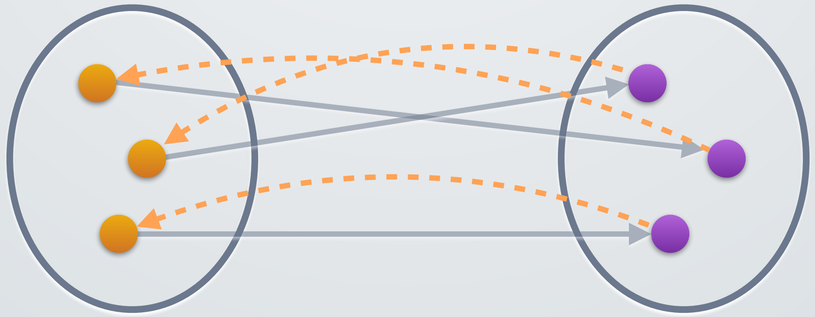
$$3 \equiv 15 \mod 12$$

Invertibilità di una funzione

Se una funzione $f\colon X \to Y$ è sia iniettiva che suriettiva, allora esiste una corrispondenza biunivoca tra i due insiemi X e Y.
Si può in questo caso definire la funzione inversa

$$f^{-1}(y) \iff f(x) = y$$

Una funzione viene detta unidirezionale (*one-way function*) se è computazionalmente difficile da invertire.



Diffie-Hellman-Merkle Key Exchange

Diffie, Hellman e Merkle

Ralph Merkle

Martin Hellman

Whitfield Diffie

Dopo due millenni di buio il problema dello scambio delle chiavi viene risolto nel 1976.

È il primo metodo pratico per scambiare chiavi crittografiche in un canale insicuro ed è tutt'oggi comunemente usato.

A caccia di funzioni unidirezionali

$$a^n = b \qquad b = \overbrace{a \cdot a \cdot \ldots \cdot a}^{n \text{ volte}}$$

$$a^n \equiv b \mod c \qquad b = \overbrace{a \cdot a \cdot \ldots \cdot a}^{\text{Quante volte?}}$$

Non esiste un modo facile per calcolarlo!
Bisogna provare tutti i numeri!

Esempio

Problema: $3^x \bmod 7 = 1$ **Soluzione:** $x = 6n,\ n \in \mathbb{Z},\ n \geq 0$

$3^1 \bmod 7 = 1$	$3 = 7 \cdot 0 + 3$	Nope.
$3^2 \bmod 7 = 1$	$9 = 7 \cdot 1 + 2$	Nope.
$3^3 \bmod 7 = 1$	$27 = 7 \cdot 3 + 6$	Nope.
$3^4 \bmod 7 = 1$	$81 = 7 \cdot 11 + 4$	Nope.
$3^5 \bmod 7 = 1$	$243 = 7 \cdot 34 + 5$	Nope.
$3^6 \bmod 7 = 1$	$729 = 7 \cdot 104 + 1$	F**k yeah!

La funzione "logaritmo discreto"

$$a^x = b \mod n$$

$$\downarrow$$

$$g^i = b \mod p$$

Il modulo è un numero primo p e la base dell'esponente è un generatore di p.

Ma... cosa diavolo è un generatore di p?

Alla ricerca dei generatori perduti

- Scegliere un numero intero g come candidato;
- Calcolare $g^i \mod p$, dove i sono tutti i numeri da 1 a $p-1$;
- Se i risultati sono una certa permutazione senza ripetizioni dei numeri da 1 a $p-1$, allora g è un generatore di p.

Esempio: scegliamo $g = 6$ e $p = 17$. Calcoliamo $6^i \mod p$ con $i = [1, p-1]$

$i = 1 \qquad 6^1 \mod 17 = 6$
$i = 2 \qquad 6^2 \mod 17 = 2$
$i = 3 \qquad 6^3 \mod 17 = 12$
...
$i = (p-1) \qquad 6^{16} \mod 17 = 1$

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$6^i \mod 17$	6	2	12	4	7	8	14	16	11	15	5	13	10	9	3	1

Possiamo quindi verificare che $g = 6$ è un generatore di $p = 17$ poiché è una permutazione senza ripetizioni.

POR Piemonte FSE 2014-2020

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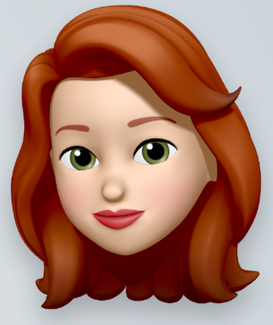
L'algoritmo Diffie-Hellman

Di comune accordo vengono scelti g e p.



$X_B \in [1, p-1]$

$X_A \in [1, p-1]$

$Y_B = g^{X_B} \pmod{p}$

$Y_A = g^{X_A} \pmod{p}$

Y_A

Y_B

$K = Y_A^{\ X_B} \pmod{p}$

K è la chiave segreta condivisa

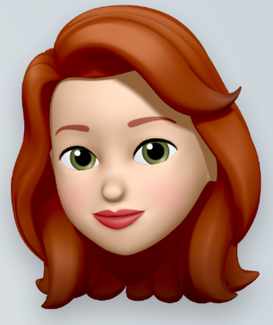
$K = Y_B^{\ X_A} \pmod{p}$

[POR Piemonte FSE 2014-2020]

ITS TECNOLOGIE DELL'INFORMAZIONE E DELLA COMUNICAZIONE

L'algoritmo Diffie-Hellman

Di comune accordo vengono scelti $g = 6$ e $p = 17$.



$X_B = 10 \in [1, 16]$

$X_A = 15 \in [1, 16]$

$Y_B = 6^{10} \pmod{17} = 15$

$Y_A = 6^{15} \pmod{17} = 3$

$Y_A = 3$

$Y_B = 15$

$K = 3^{10} \pmod{17} = 8$

$K = 15^{15} \pmod{17} = 8$

Dimostrazione

Supponiamo per assurdo che $K_A \neq K_B$.

Per definizione sappiamo che $Y_A = g^{X_A} \pmod p$ e che $Y_B = g^{X_B} \pmod p$.

Sappiamo inoltre che:

$$K_A = {Y_B}^{X_A} \pmod p \qquad K_B = {Y_A}^{X_B} \pmod p$$

Ora sostituiamo nelle equazioni precedenti le definizioni di Y_A e di Y_B:

$$K_A = \left(g^{X_B} \pmod p\right)^{X_A} \pmod p \qquad K_B = \left(g^{X_A} \pmod p\right)^{X_B} \pmod p$$

$$K_A = g^{X_B X_A} \pmod p \qquad K_B = g^{X_A X_B} \pmod p$$

Otteniamo che $K_A = K_B$, che contraddice l'ipotesi per assurdo rendendola falsa.

Crittoanalisi

Eve conosce il numero primo $p = 17$, il generatore $g = 6$ e i valori pubblici $Y_A = 3$ e $Y_B = 15$.

A partire da questi dati può calcolare la chiave condivisa K?

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$6^i \mod 17$	6	2	12	4	7	8	14	16	11	15	5	13	10	9	3	1

Eve deve calcolare tutte le equazioni $g^i \mod p$ finché non trova Y_A oppure Y_B.

A questo punto il logaritmo discreto è invertito poiché viene individuata la componente segreta X_A oppure X_B.

K può quindi essere facilmente calcolata con la sua equazione.

Perché la procedura è sicura?

Perché nelle implementazioni reali si usano numeri di

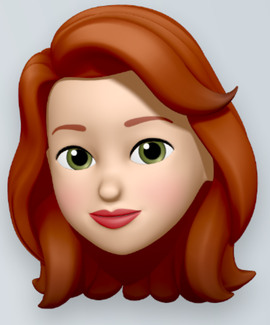
300

cifre

4

Man-in-the-middle attack

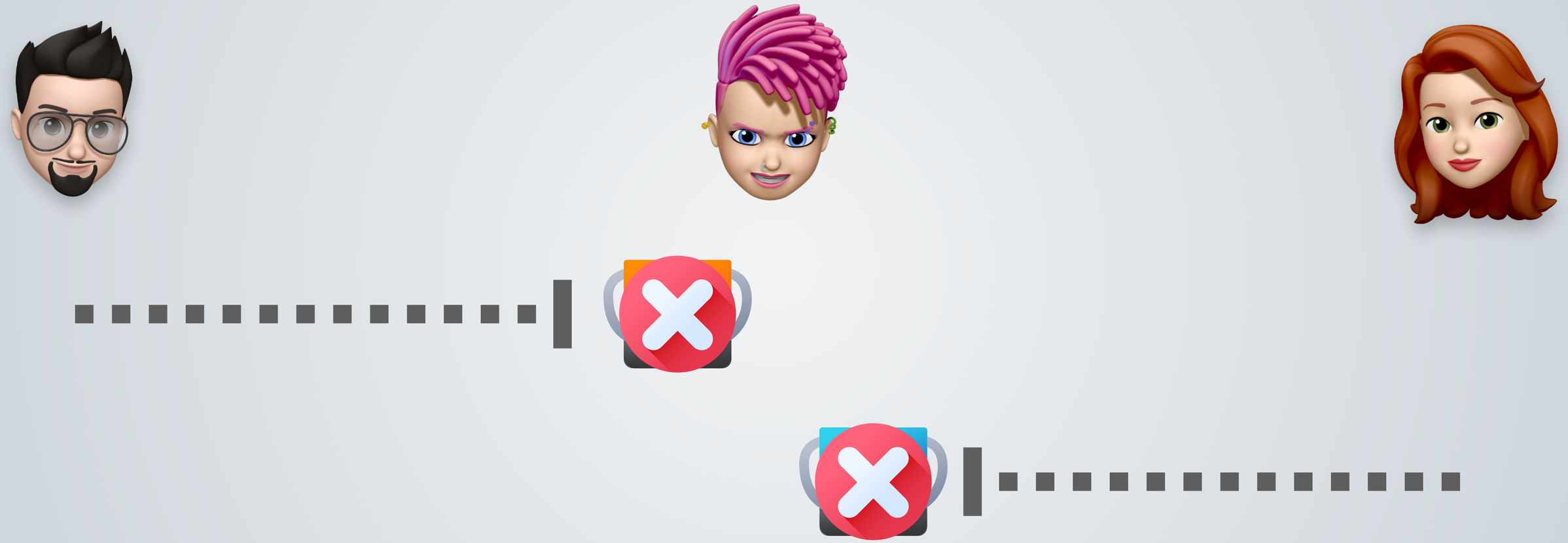
Man-in-the-middle attack



Alice e Bob devono scambiarsi l'informazione pubblica.

Eve controlla il canale di comunicazione.

Man-in-the-middle attack

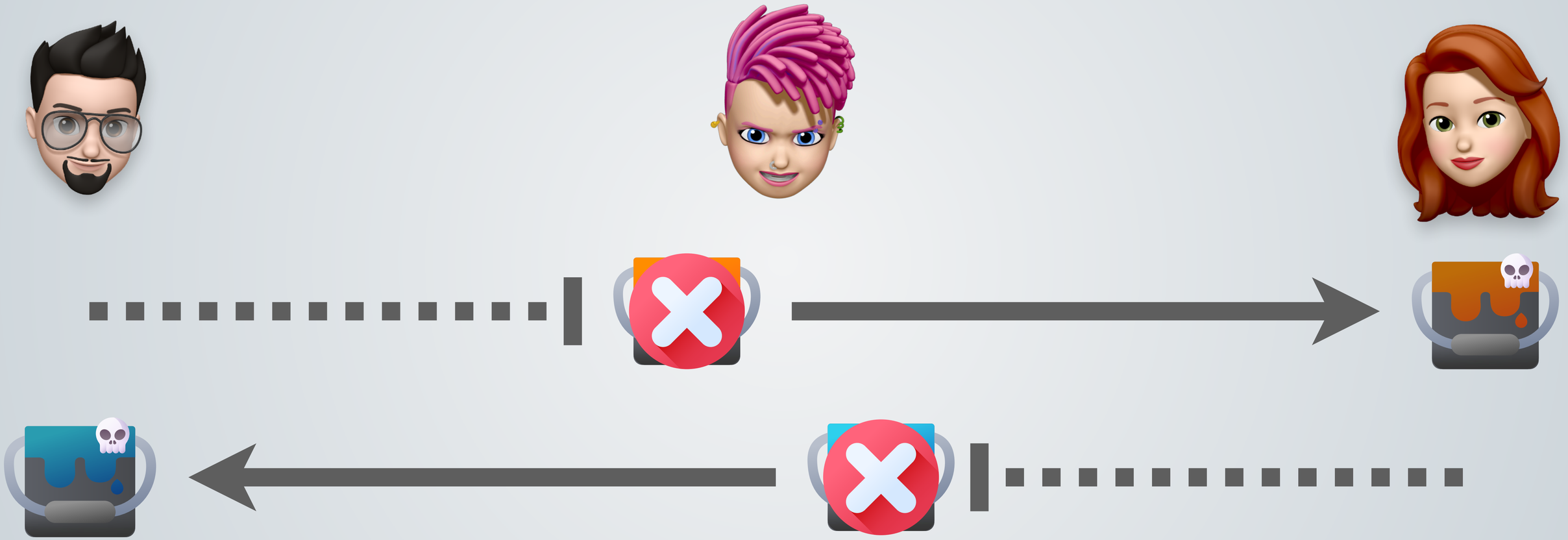


Alice e Bob inviano l’informazione, ma Eve le intercetta.

Man-in-the-middle attack

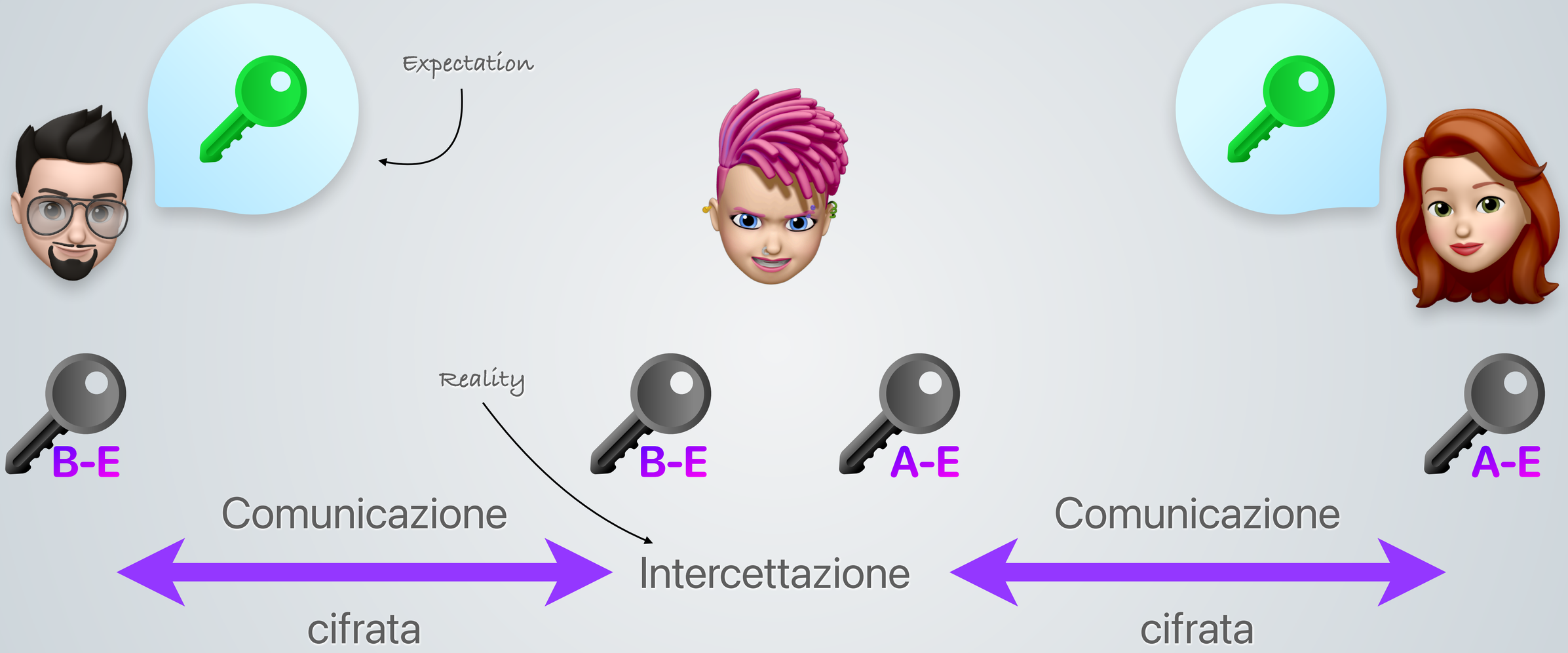
Eve crea delle versioni fraudolente delle informazioni pubbliche di Bob e Alice.

Man-in-the-middle attack



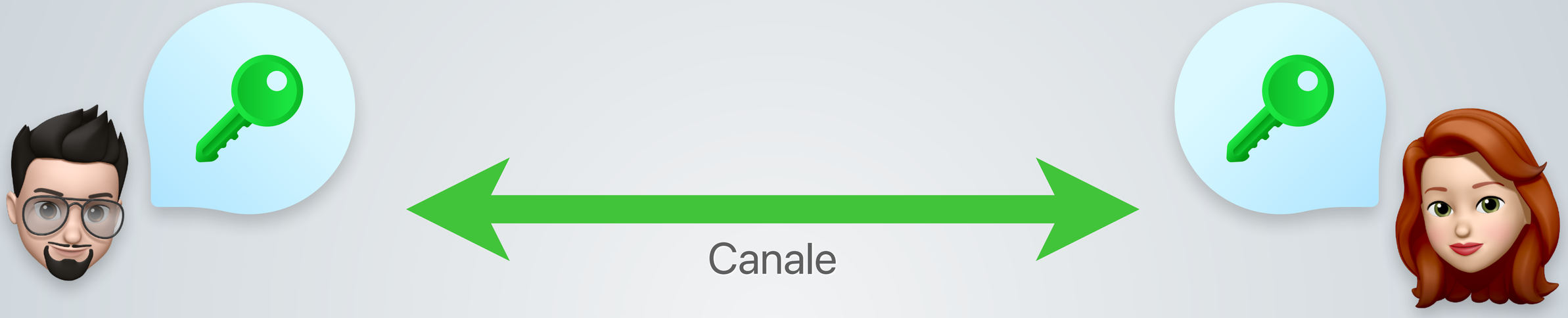
Eve spedisce le versioni fraudolente ingannando entrambi.

Man-in-the-middle attack



Alice e Bob credono di aver condiviso una chiave, ma l'hanno in realtà condivisa con Eve, che ora sta in mezzo alla conversazione e può intercettare tutto.

Risolvere il Man-in-the-middle



Come possiamo verificare che nessuno si sia messo in mezzo alla comunicazione?

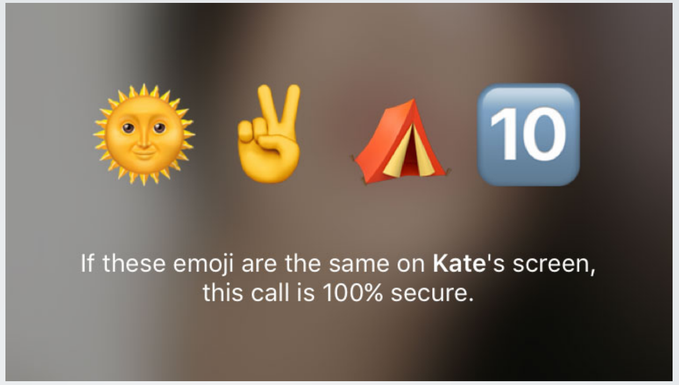
Verifica manuale

Certificazione

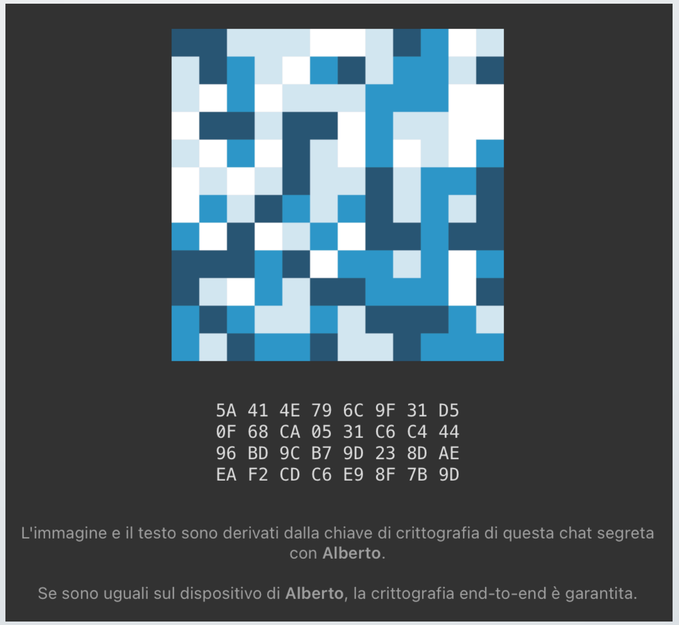
(tra due lezioni...)

Contromisure: Telegram

@marcofarina

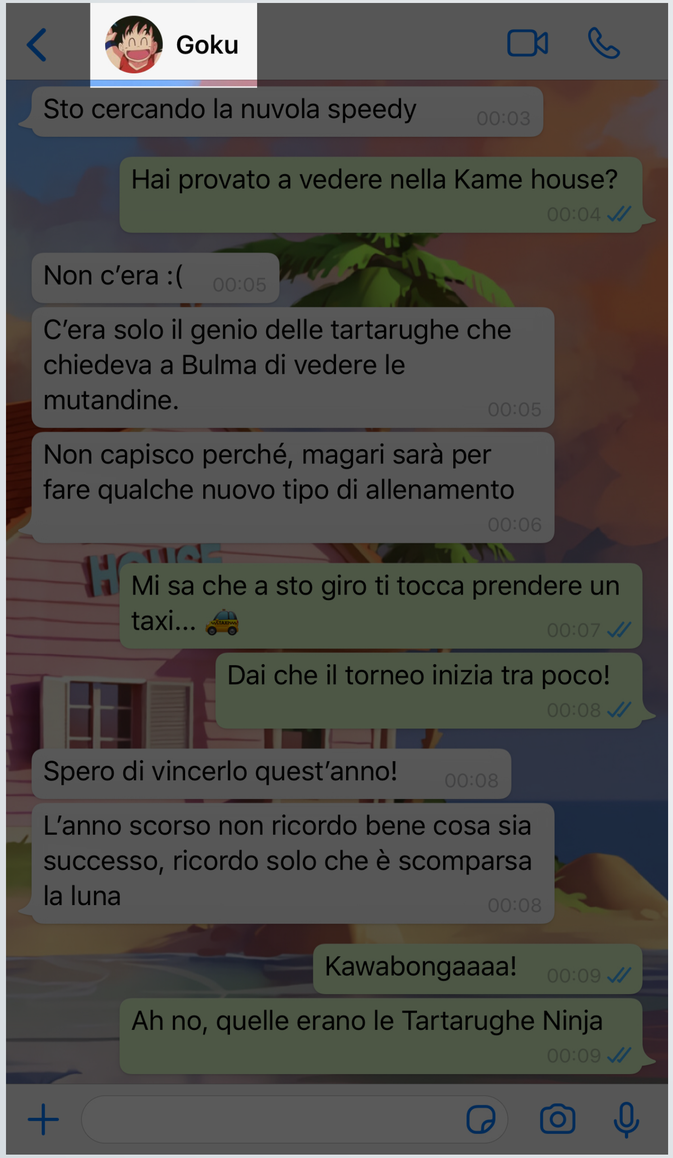


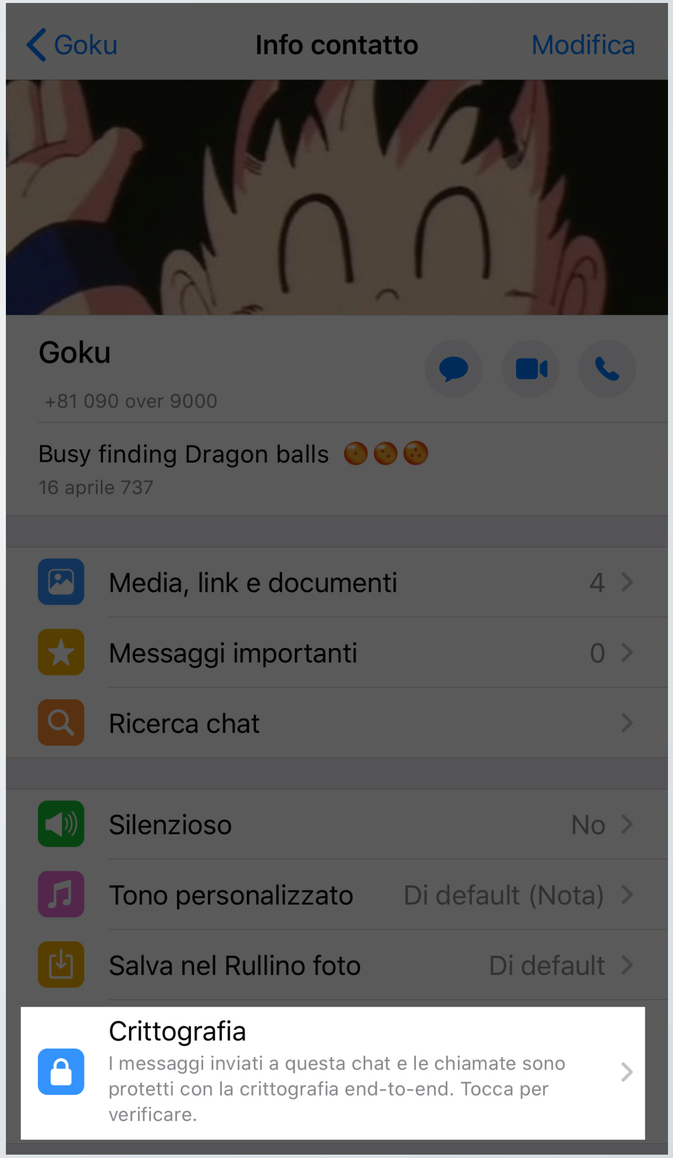
Video call



Secret chats

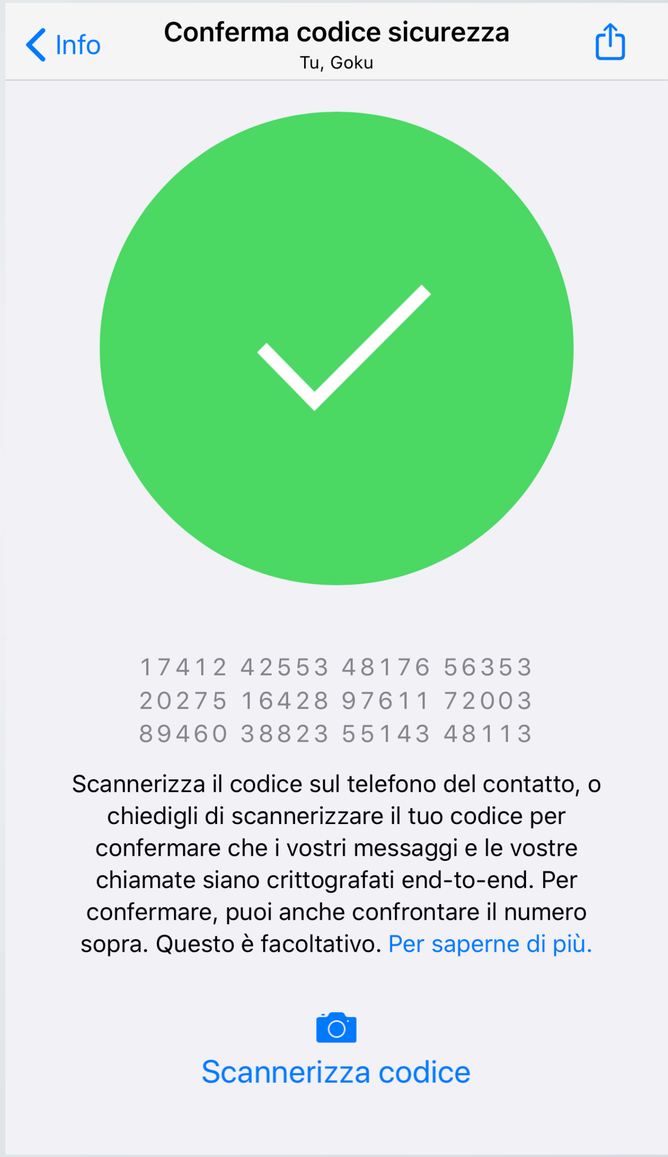
Contromisure: WhatsApp





Contromisure: WhatsApp

Contromisure: WhatsApp



Quindi nel mondo reale si usa DH?

Telegram Voice Call authentication

“Keys for end-to-end encrypted calls are generated using the Diffie-Hellman key exchange. Users who are on a call can ensure that there is no MitM by comparing key visualizations.”

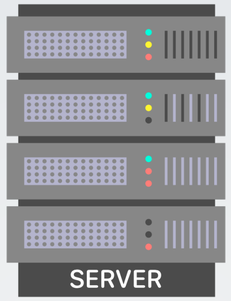
Signal key agreement protocol

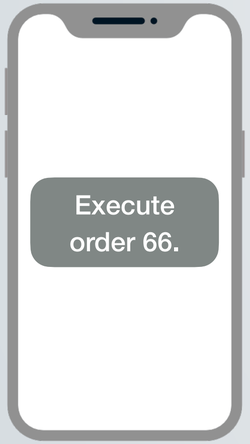
“This document describes the "X3DH" (or "Extended Triple Diffie-Hellman") key agreement protocol. X3DH establishes a shared secret key between two parties who mutually authenticate each other based on public keys. X3DH provides forward secrecy and cryptographic deniability.”

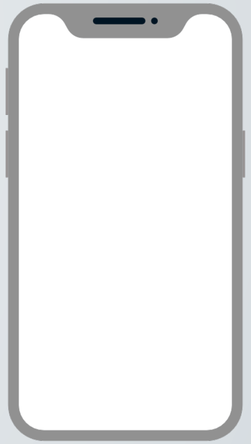
5

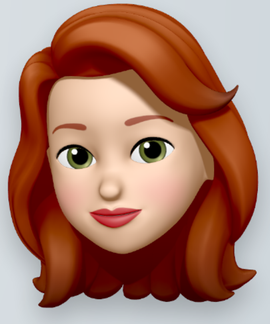
Crittografia end-to-end

Instant messaging: plain text

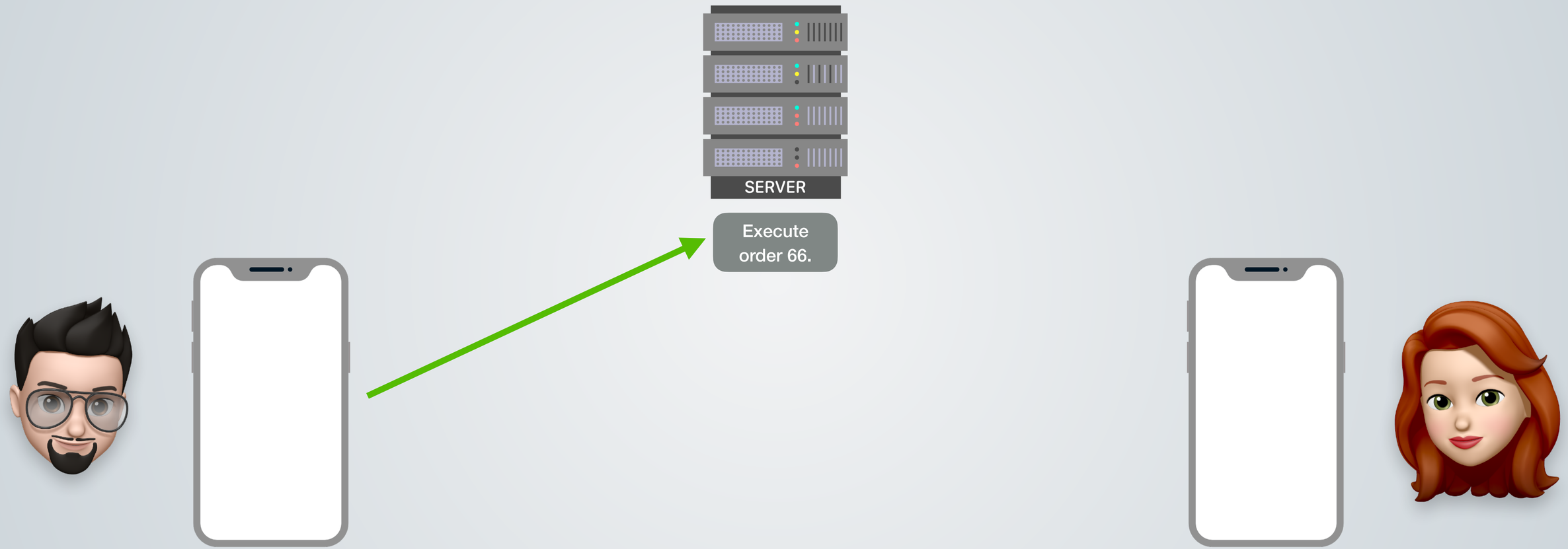








Instant messaging: plain text



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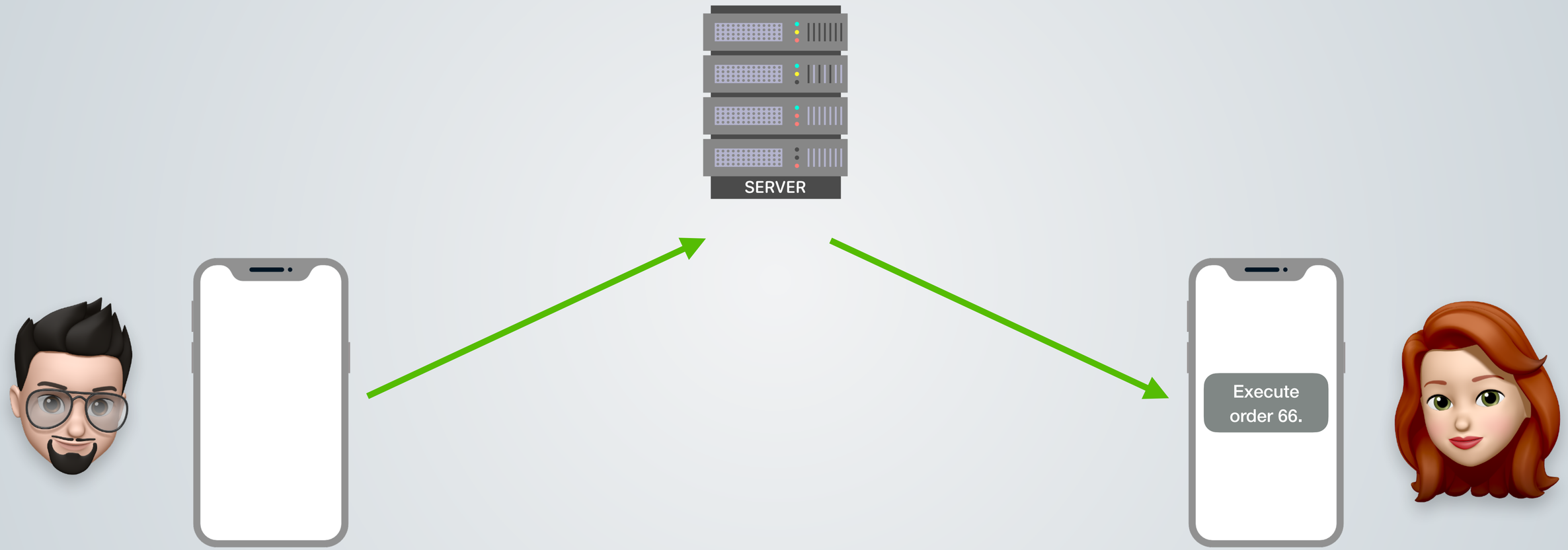
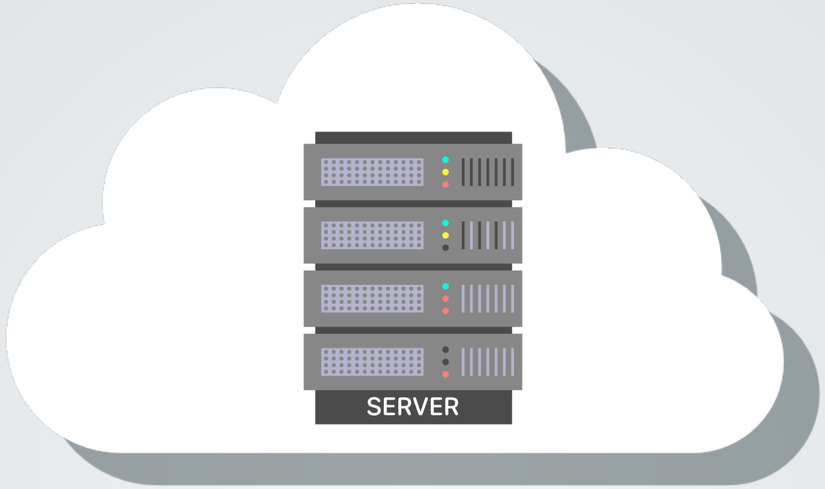
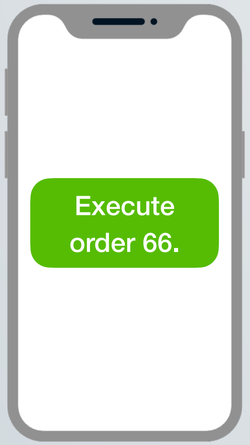
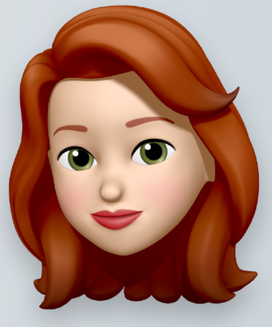


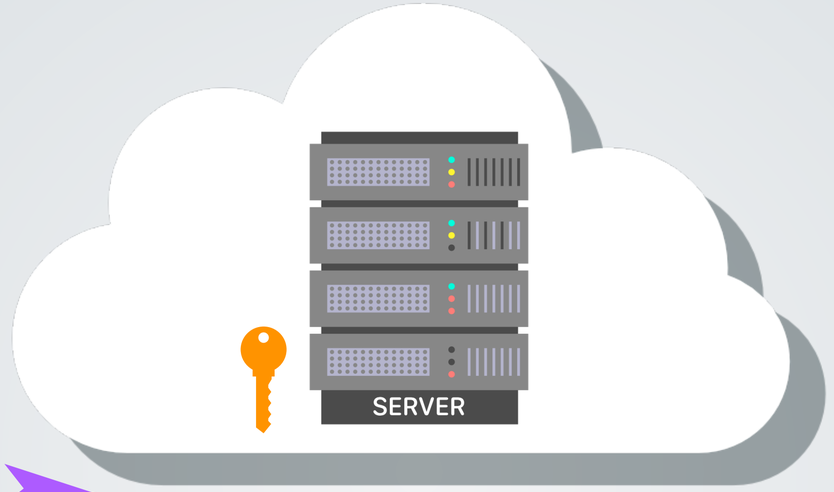
Instant messaging: encryption





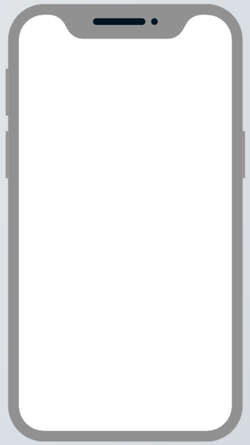


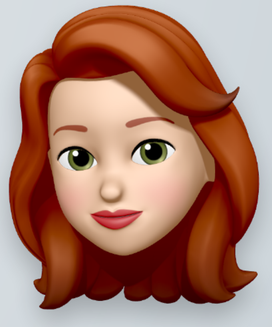
Instant messaging: encryption



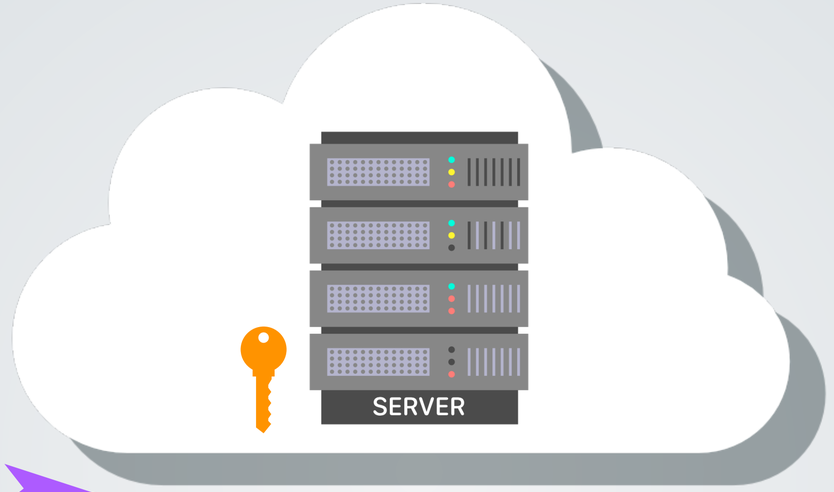
DHKE

1. Diffie-Hellman Key Exchange

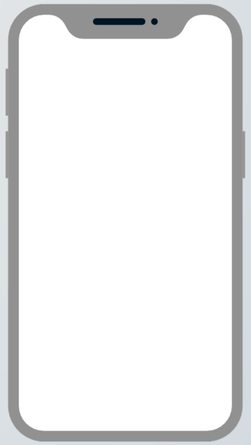


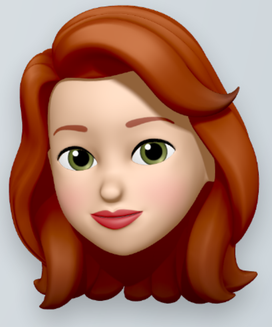


Instant messaging: encryption



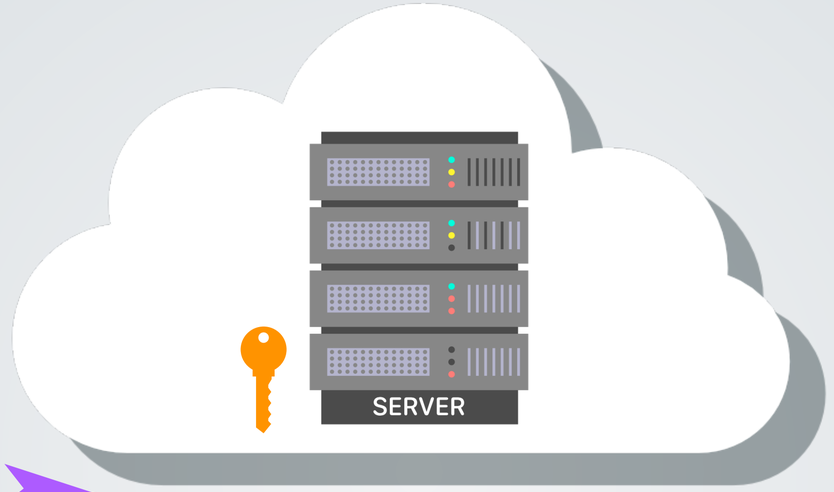
AES Enc





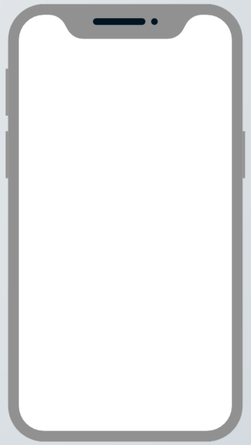
1. Diffie-Hellman Key Exchange
2. AES Encryption

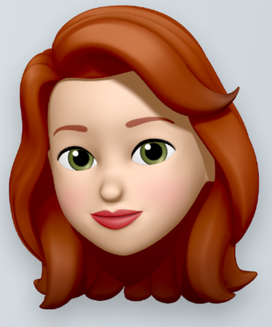
Instant messaging: encryption



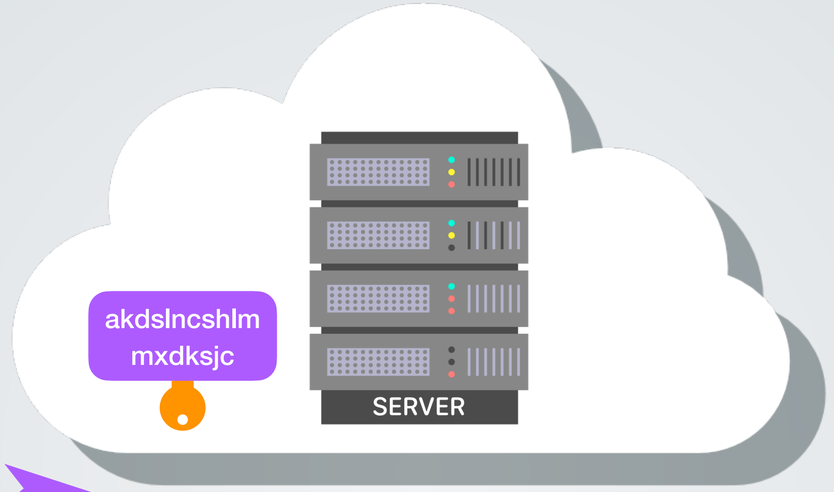
AES Enc

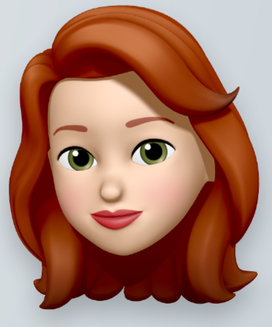
1. Diffie-Hellman Key Exchange
2. AES Encryption



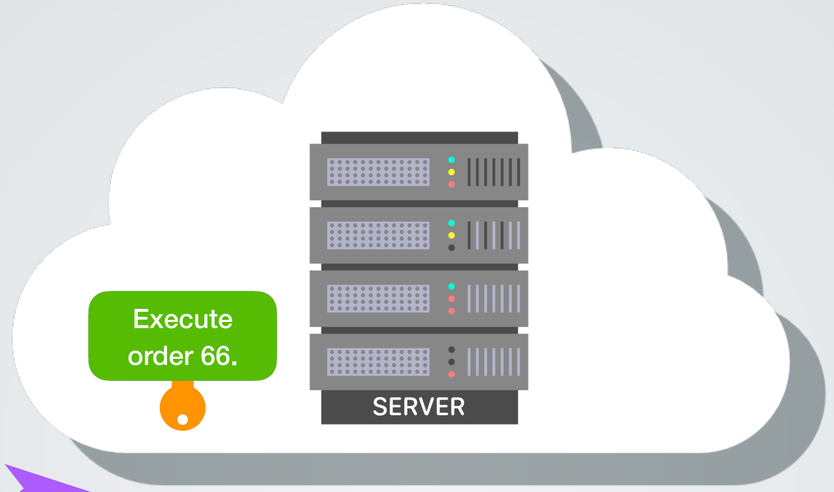


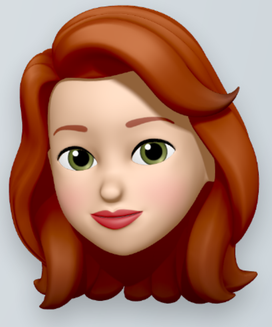
Instant messaging: encryption



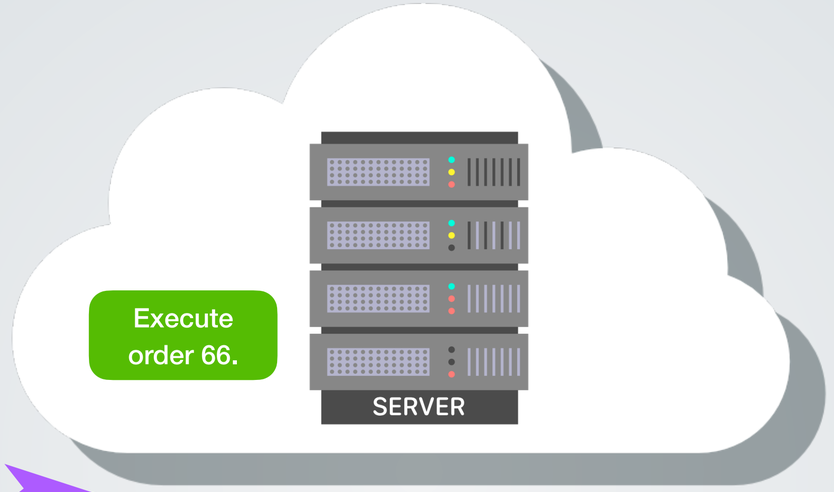


Instant messaging: encryption

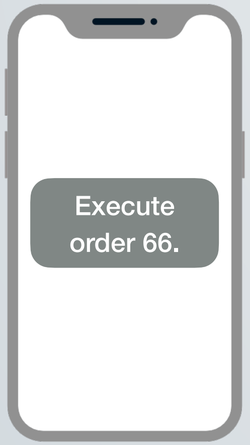


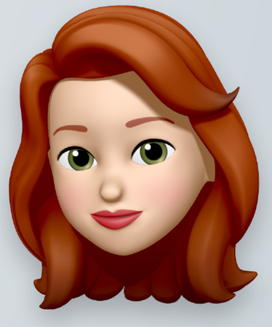


Instant messaging: encryption

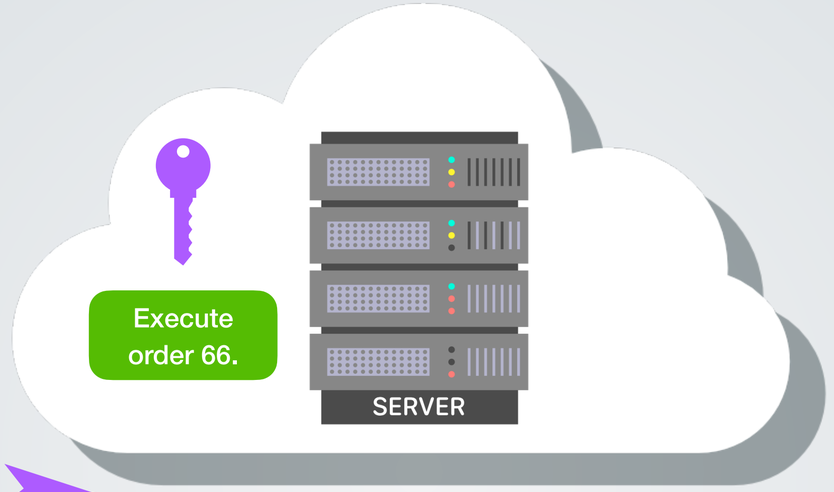


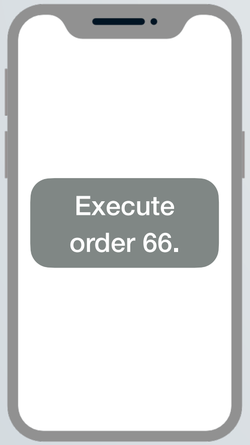
AES Enc

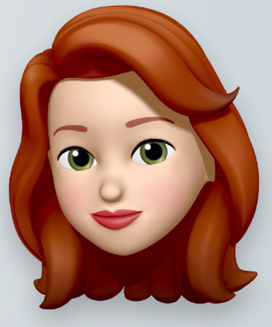




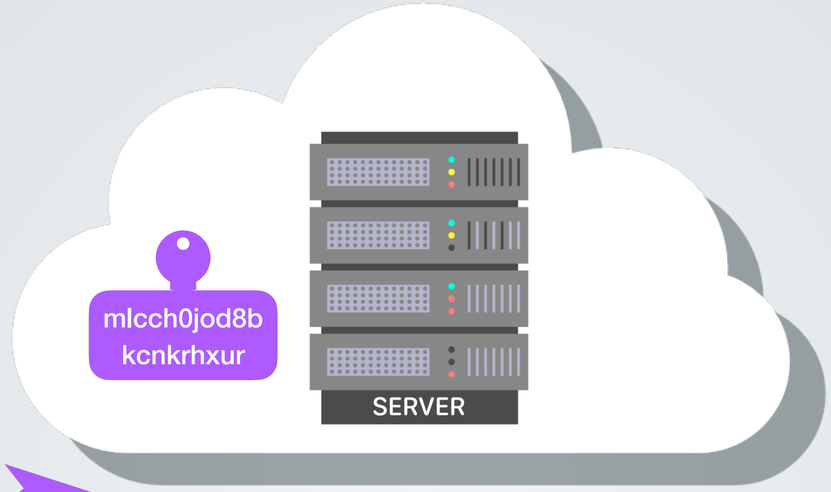
Instant messaging: encryption

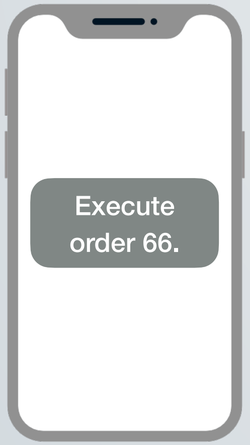


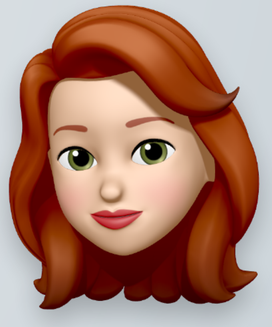




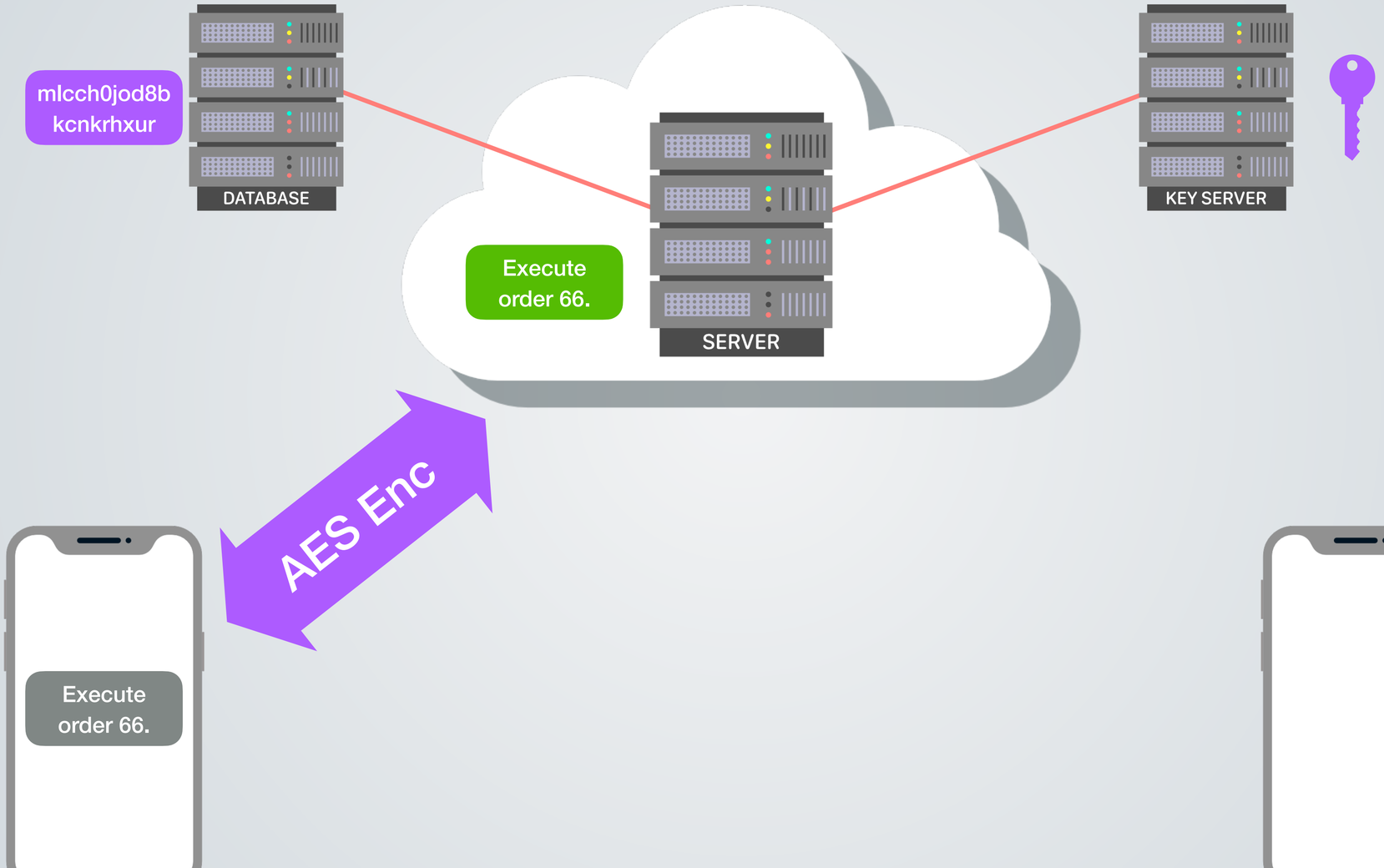
Instant messaging: encryption



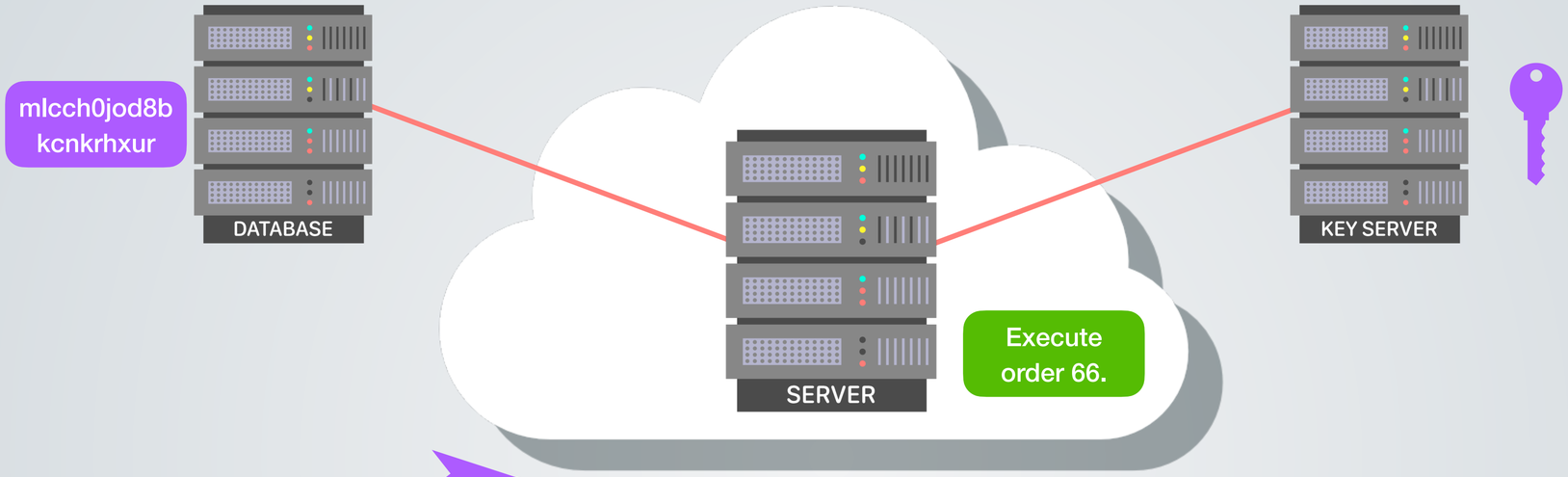


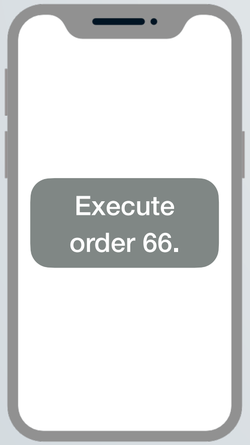


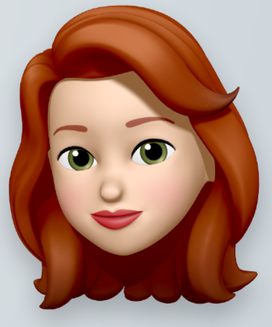
Instant messaging: encryption



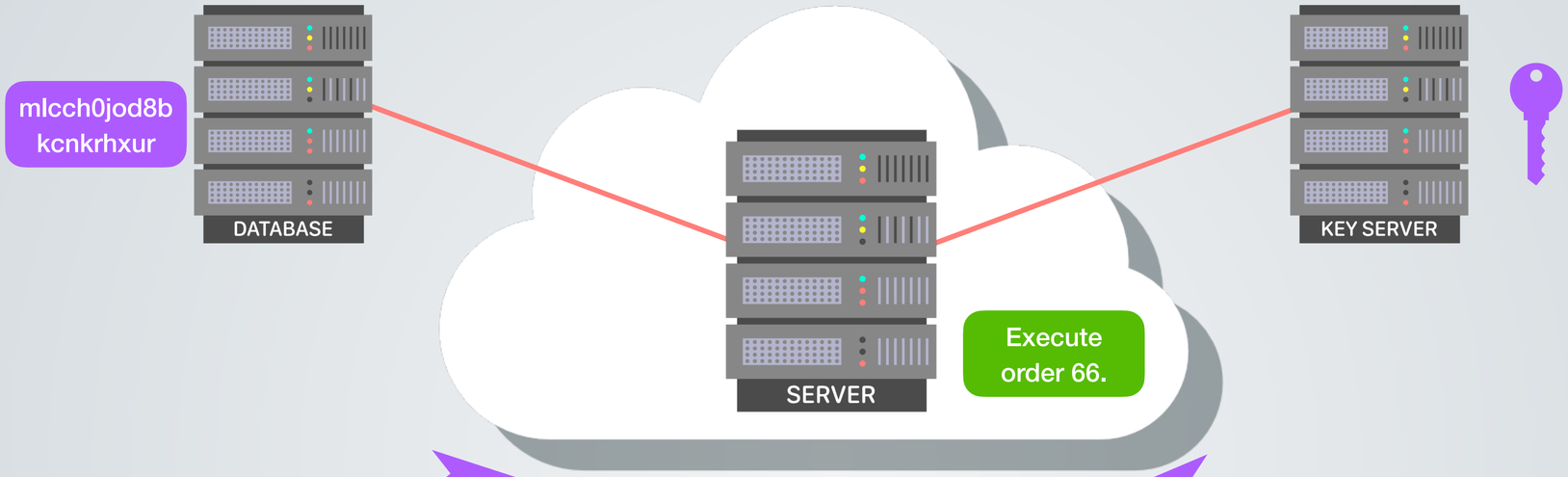
Instant messaging: encryption

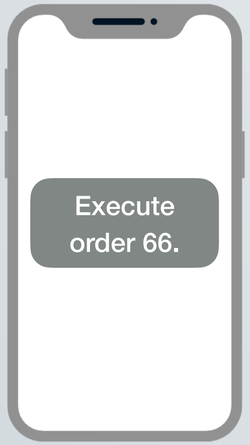






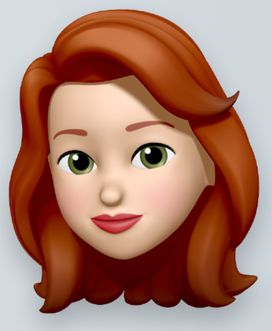
Instant messaging: encryption





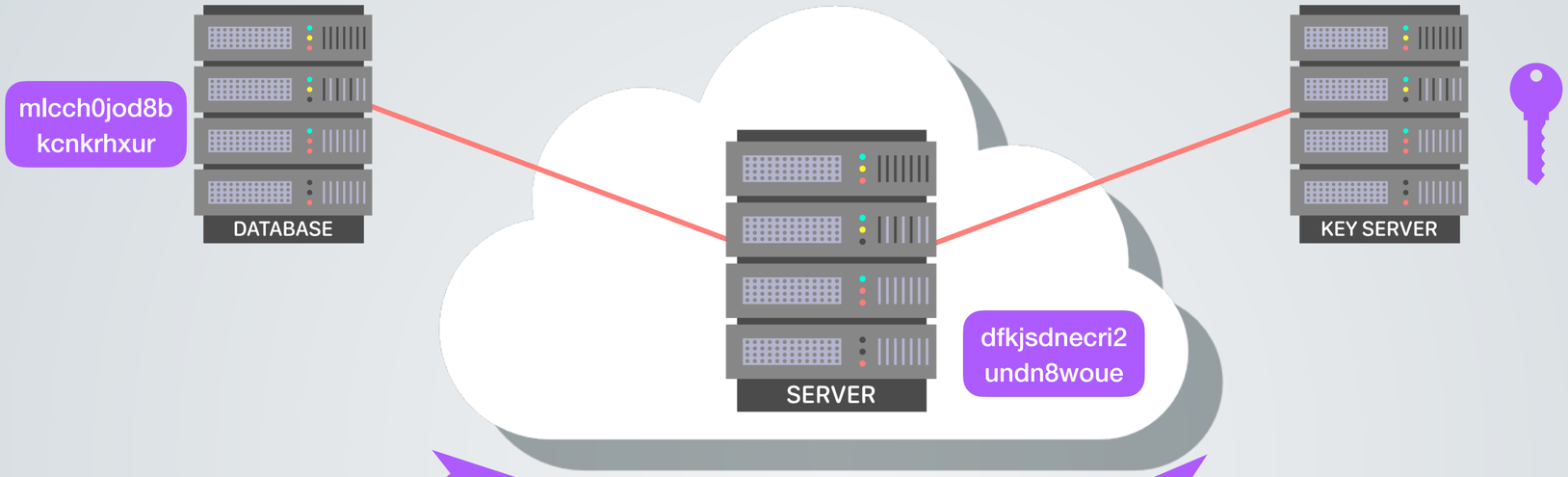
AES Enc

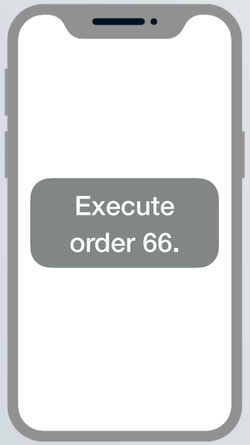
DHKE



1. Diffie-Hellman Key Exchange

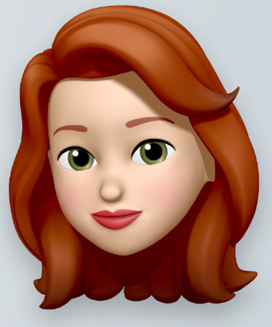
Instant messaging: encryption





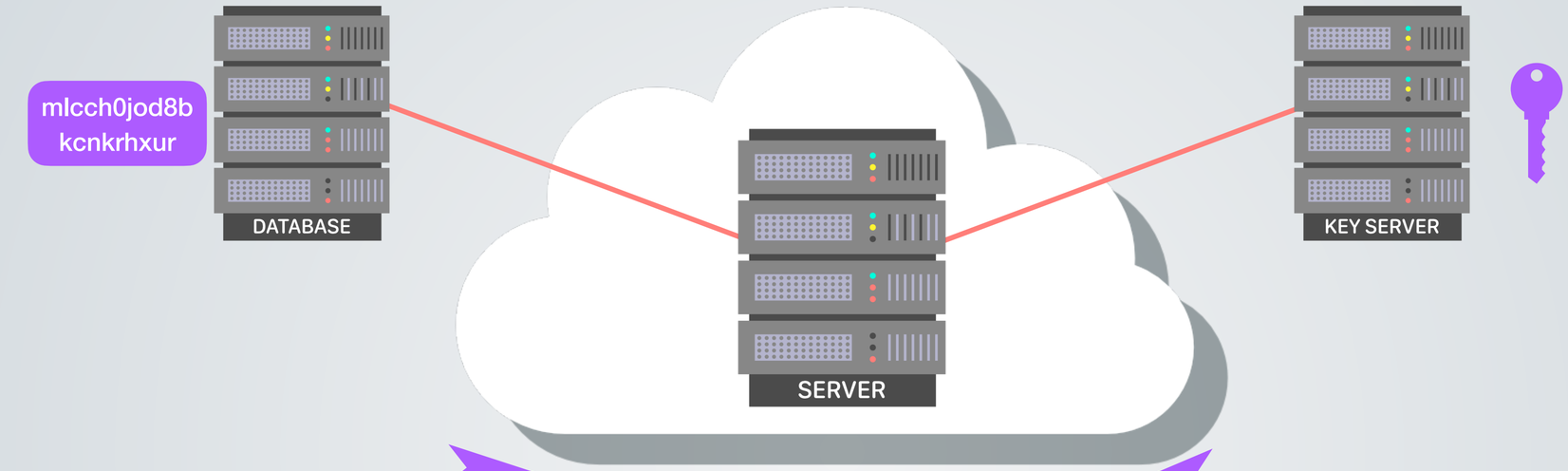
AES Enc

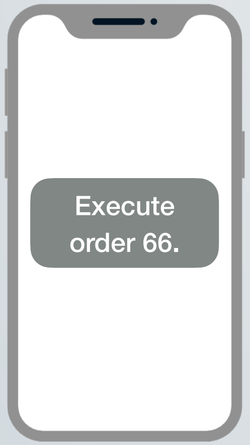
AES Enc



1. Diffie-Hellman Key Exchange
2. AES Encryption

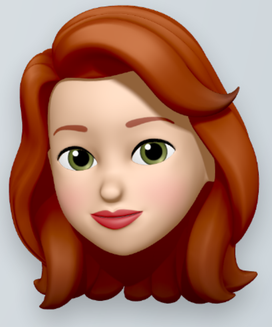
Instant messaging: encryption



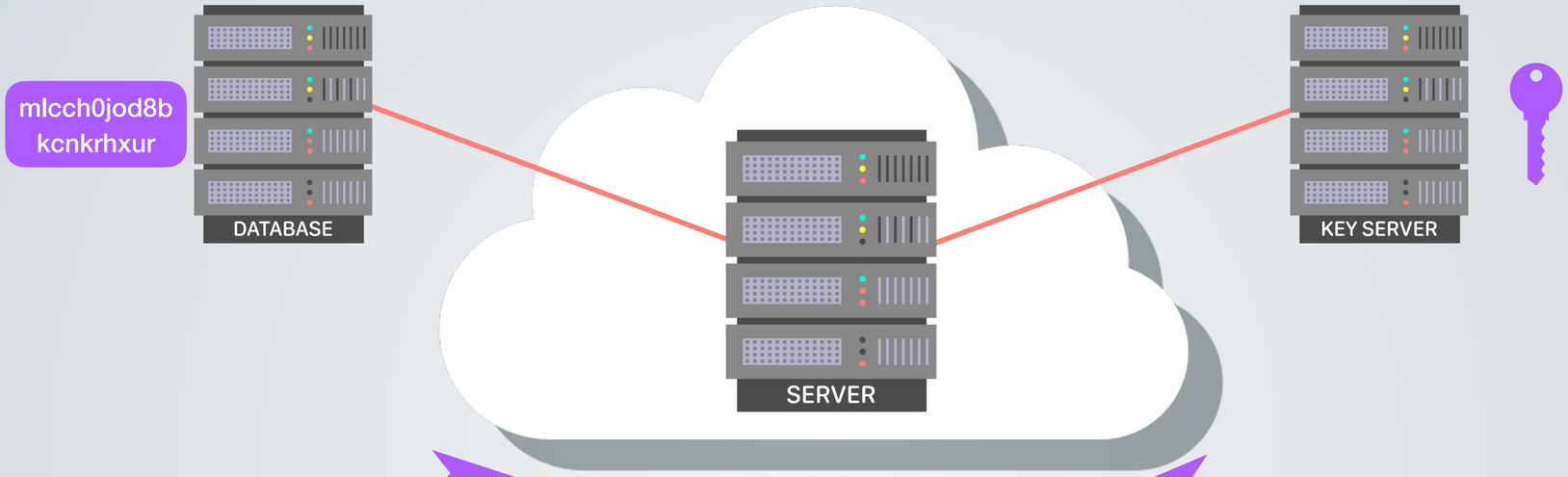


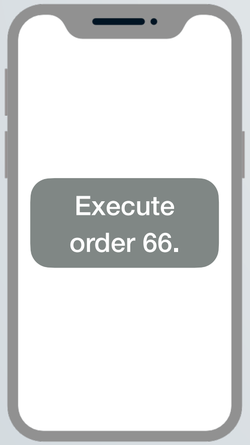
AES Enc

AES Enc



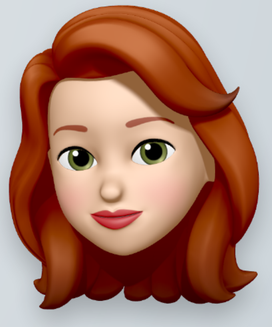
Instant messaging: encryption



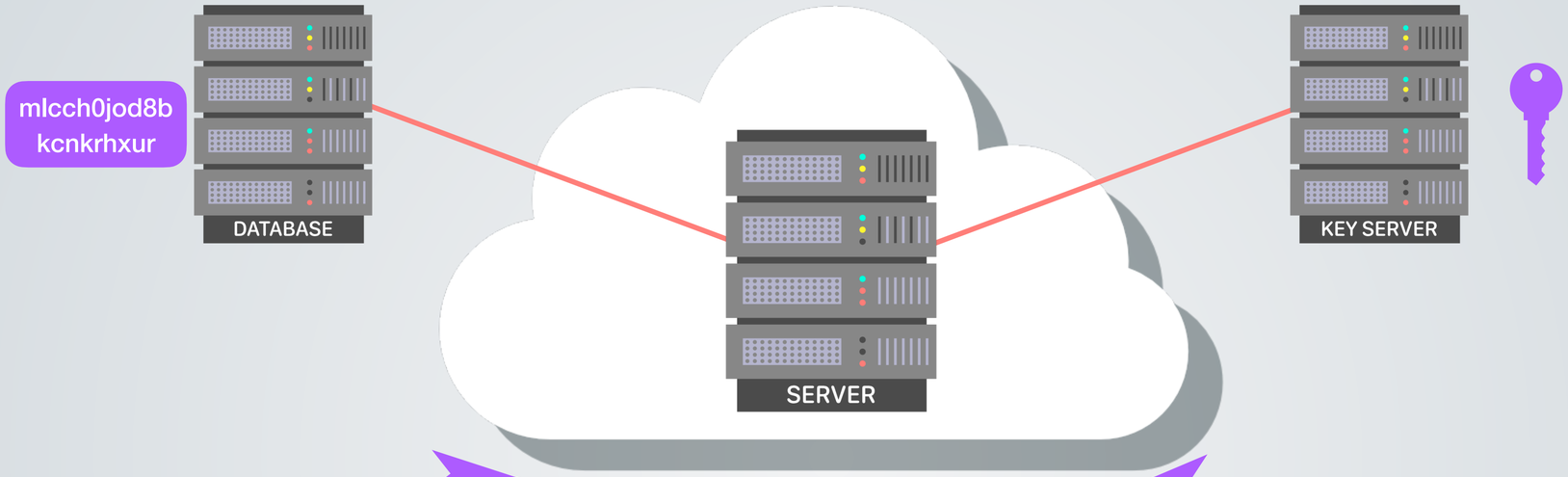


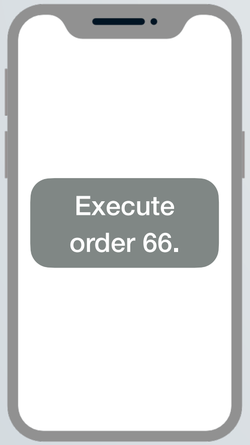
AES Enc

AES Enc



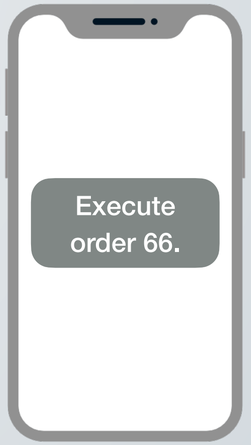
Instant messaging: encryption

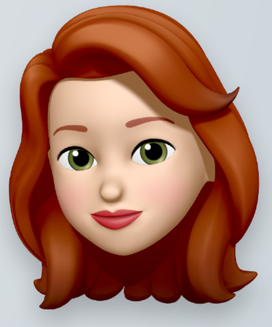




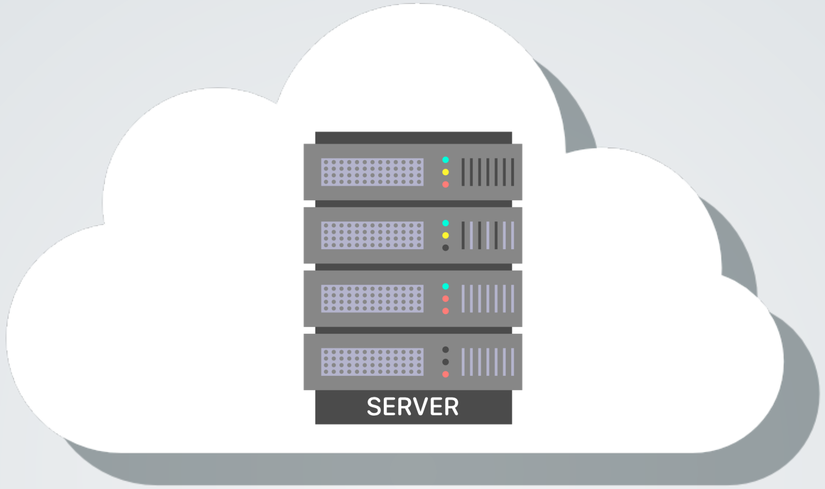
AES Enc

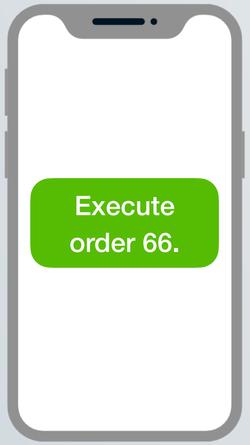
AES Enc

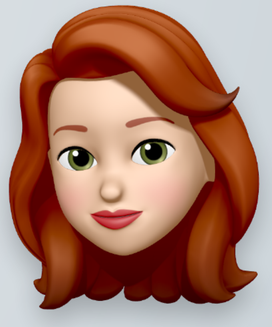




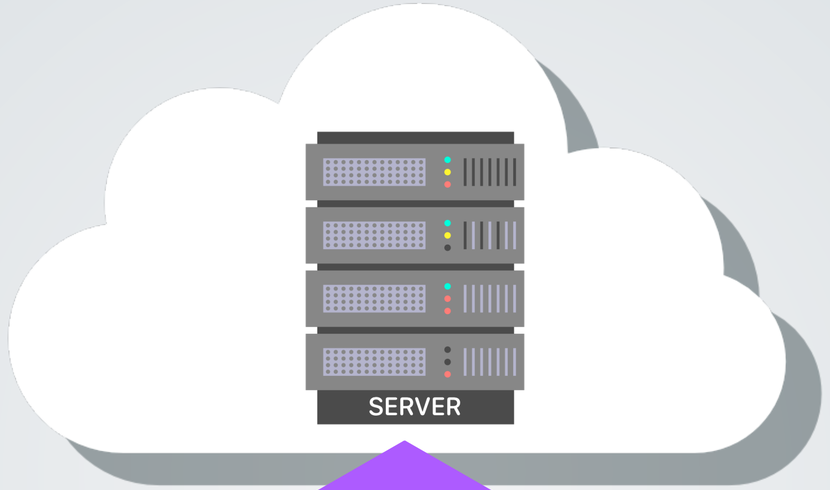
Instant messaging: end-to-end encryption

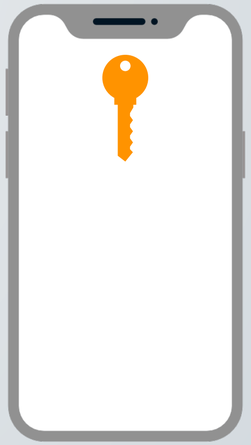


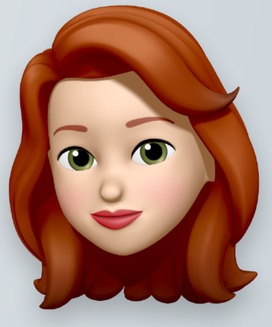




Instant messaging: end-to-end encryption

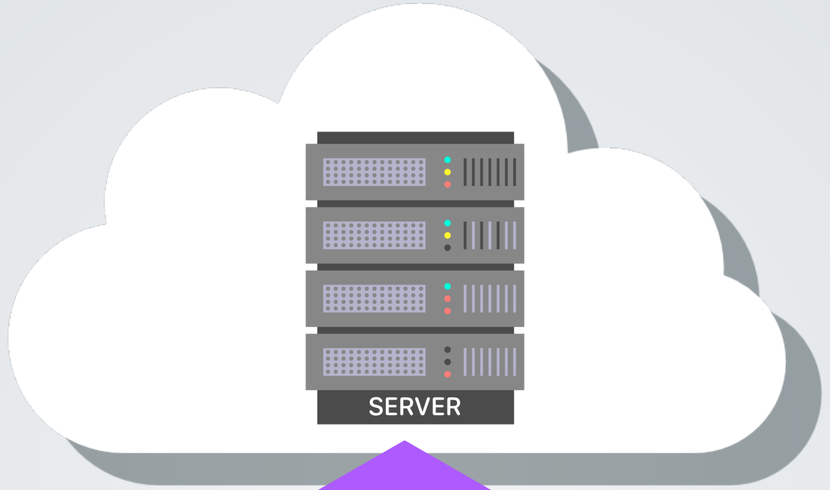


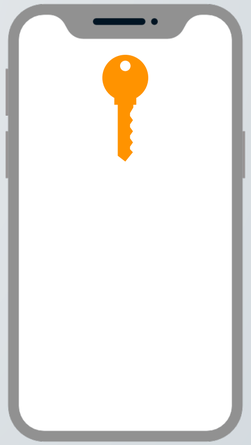


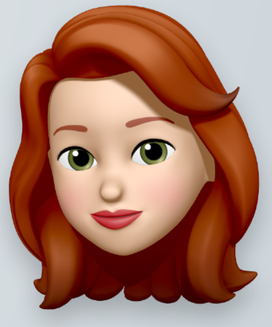


1. Diffie-Hellman Key Exchange

Instant messaging: end-to-end encryption

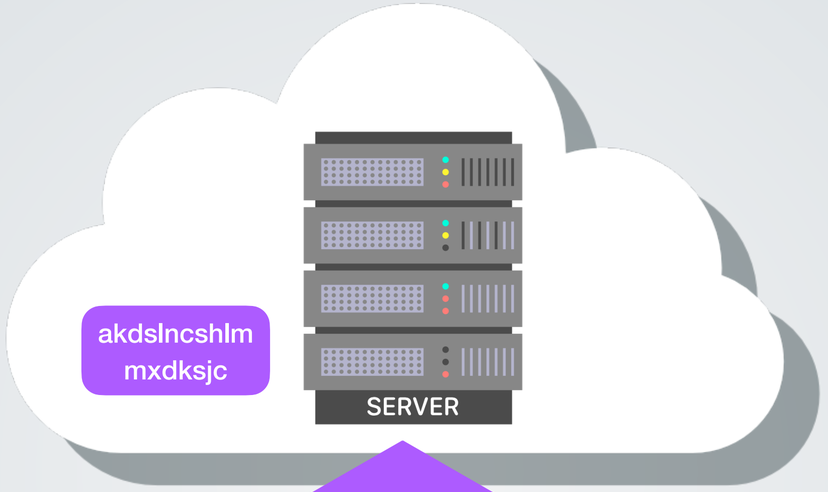


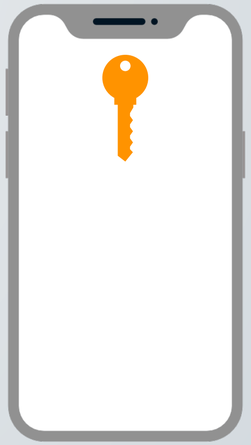


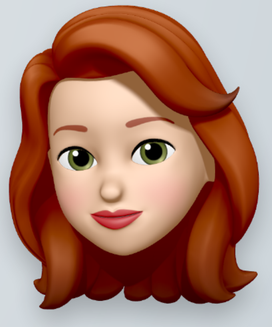


1. Diffie-Hellman Key Exchange
2. AES Encryption

Instant messaging: end-to-end encryption

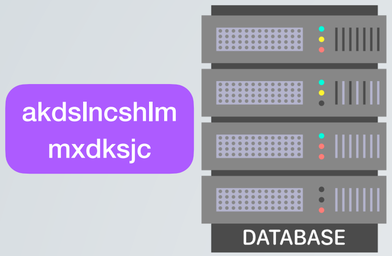


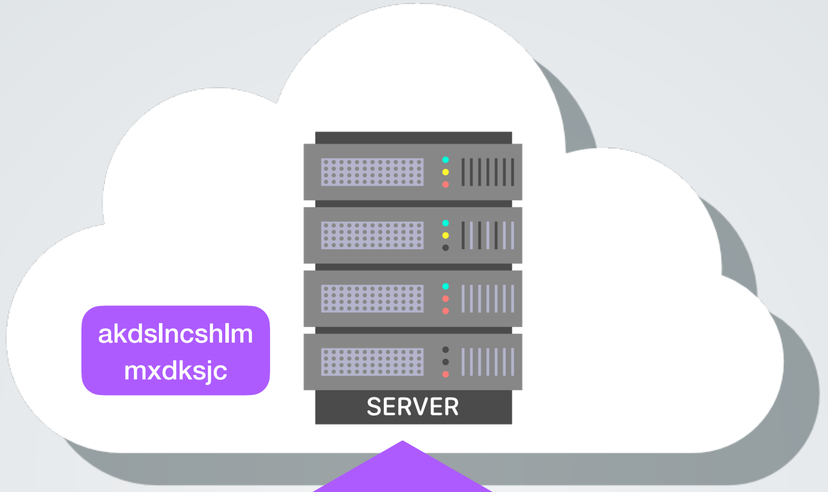


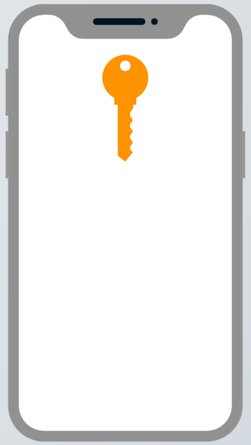


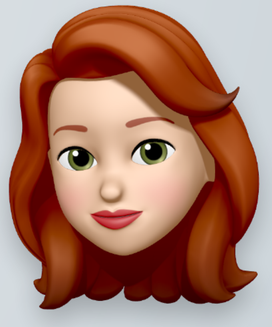
1. Diffie-Hellman Key Exchange
2. AES Encryption

Instant messaging: end-to-end encryption





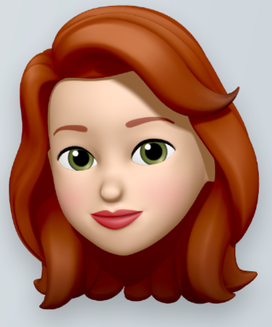




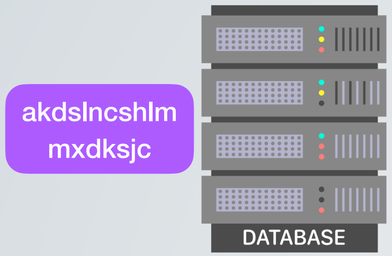
1. Diffie-Hellman Key Exchange
2. AES Encryption

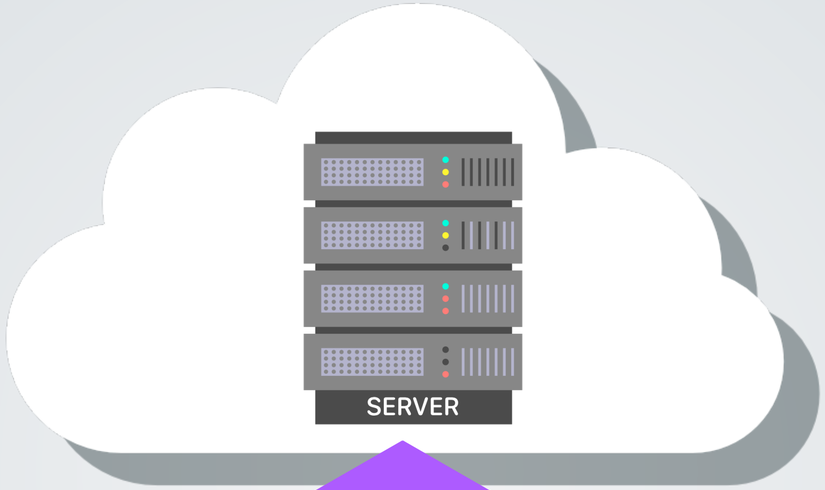
Instant messaging: end-to-end encryption

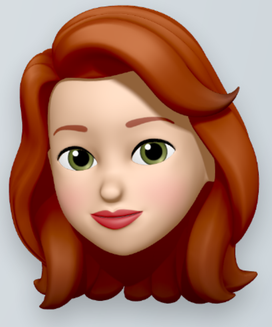
1. Diffie-Hellman Key Exchange
2. AES Encryption



Instant messaging: end-to-end encryption







1. Diffie-Hellman Key Exchange
2. AES Encryption