4) $$I = \int_0^{+\infty} \frac{e^{-x/2}}{\sqrt{x}}\, dx$$

$$x = -\ln(u) \qquad dx = -\frac{1}{u}\, du$$

$$\hookrightarrow \quad I = \int_1^0 \frac{e^{\ln(u)/2}}{\sqrt{-\ln(u)}} \left(-\frac{1}{u}\right) du$$

$$= \int_0^1 \sqrt{\frac{1}{-u\ln(u)}}\, du$$

$$x = \frac{1}{1-u} - 1 = \frac{u}{1-u}$$

$$dx = \frac{1-u+u}{(1-u)^2}\, du = \frac{1}{(1-u)^2}\, du$$

$$I = \int_0^1 \frac{\exp\left\{\frac{u}{1-u}\right\}}{\sqrt{\frac{u}{1-u}}} \cdot \frac{1}{(1-u)^2}\, du$$

$$= \int_0^1 \frac{\exp\left\{\frac{u}{1-u}\right\}}{\sqrt{u(1-u)^3}}\, du$$

5) $I := \int_{-\infty}^{+\infty} \exp\left\{-\frac{x^2}{2} - x\right\} dx$

$$x = \tan\left(\pi\left(u - \frac{1}{2}\right)\right)$$

$$dx = \frac{\pi}{\cos^2\left(\pi\left(u - \frac{1}{2}\right)\right)}$$

alternativa:

$$x = \ln\left(\frac{u}{1-u}\right)$$

$$dx = \frac{\cancel{1-u}}{u} \cdot \frac{1-\cancel{u}+\cancel{u}}{(1-u)^{\cancel{2}}}\, du = \frac{du}{u(1-u)}$$

↳ $I = \int_0^1 \exp\left\{-2\ln\left(\frac{u}{1-u}\right)\right\} \cdot \frac{1}{u(1-u)}\, du$

$$= \int_0^1 \frac{(1-u)^{\cancel{2}}}{u^2} \cdot \frac{1}{u\cancel{(1-u)}}\, du$$

8) $I := \int_0^1 \int_{-2}^2 x^2 \cos(xy)\, dx\, dy$

$\hookrightarrow x = 4u - 2 \quad , \quad u \in U_{[0,1]}$

$\hookrightarrow y = v \quad , \quad v \in U_{[0,1]}$

$h : [0,1]^2 \to [-2,2] \times [0,1]$

$\hookrightarrow h(u,v) = (4u-2, v)$

$$J_h = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \rightsquigarrow \det = 4$$

$$\Rightarrow I = \int_0^1 \int_0^1 4(4u-2)^2 \cos\left((4u-2)v\right) du\, dv$$

9) $$\int_1^{+\infty}\int_3^{+\infty} 2x e^{(1-x^2)y^2}\,dx\,dy$$

$\hookrightarrow h: [0,1]^2 \longmapsto [3,+\infty) \times [1,+\infty)$

$$u \longmapsto x = 3/u$$

$$t \longmapsto y = 1/t$$

$$J_h(u,v) = \begin{pmatrix} -\frac{3}{u^2} & 0 \\ 0 & -\frac{1}{v^2} \end{pmatrix}$$

$$\det J_h(x,y) = \frac{3}{x^2 y^2}$$

$$\int_0^1 \int_0^1 \frac{18}{u^3 t^2} \exp\left\{\left(1-\frac{9}{u^2}\right)\frac{1}{t^2}\right\} du\,dt$$

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b)

$$\left.\begin{aligned} x &= u_1 \\ y &= 2u_2 \\ z &= u_3 + 1 \\ u &= u_4 \\ v &= u_5 + 2 \end{aligned}\right\} \quad J = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\det J = 2$$

$$xy^2 + xzv + yv^2$$

$$\frac{1}{2}y^2 + \frac{1}{2}zv + yv^2$$

$$\frac{1}{6}\cdot 8 + zv + 2v^2$$

$$\frac{1}{6}\cdot 8 + \frac{3}{2}v + 2v^2$$

$$\frac{1}{6}8 + \frac{3}{2}v + 2$$

$$\frac{4}{3} + \frac{15}{4} + 2 = \frac{16+45+24}{12} = \frac{85}{12}$$

11) $F^{-1}_{(\frac{1}{60})} \rightsquigarrow q_{pois}$

[q representa el punto donde cortamos la distr.]

$U \in \left(0, F^{-1}_{(\frac{1}{60})}(q+1)\right)$

14) MUESTREO POR IMPORTANCIA

$$f_1(\underline{x}) = \prod_{i=1}^{n} \left[\frac{1}{\sqrt{2\pi}\,\sigma_1} \exp\left\{ -\frac{(x_i - \mu_1)^2}{2\sigma_1^2} \right\} \right]$$

$$= \left(\frac{1}{\sqrt{2\pi}}\right)^n \cdot \frac{1}{\sigma_1^n} \cdot \exp\left\{ -\frac{1}{2\sigma_1^2} \sum_{i=1}^{n} (x_i - \mu_1)^2 \right\}$$

$$f_2(\underline{x}) = (\ldots)$$

$$\hookrightarrow L(\underline{x}) = \frac{f_1(\underline{x})}{f_2(\underline{x})}$$

$$= \left(\frac{\sigma_2}{\sigma_1}\right)^n \exp\left\{ \frac{1}{2} \left(\frac{1}{\sigma_2^2} \sum_i (x_i - \mu_2)^2 - \frac{1}{\sigma_1^2} \sum_i (x_i - \mu_1)^2 \right) \right\}$$