- ## CÁLCULO DE LA ESPERANZA

$X \sim GaI(a, \lambda)$

$$f_X(x; a, \lambda) = \frac{\lambda^a}{\Gamma(a)} \frac{1}{x^{a+1}} e^{-\lambda/x}, \quad \begin{array}{l} a>0 \\ \lambda>0 \\ x>0 \end{array}$$

$$E(X) = \int_0^{+\infty} x \cdot \frac{\lambda^a}{\Gamma(a)} \cdot \frac{1}{x^{a+1}} e^{-\lambda/x} \, dx$$

$$= \frac{\lambda^a}{\Gamma(a)} \cdot \int_0^{+\infty} \frac{1}{\underbrace{x^a}_{x^{a-1+1}}} e^{-\lambda/x} \, dx \quad ; \; a-1>0$$

$$= \frac{\lambda^a}{\Gamma(a)} \cdot \frac{\Gamma(a-1)}{\lambda^{a-1}} = \frac{\lambda}{a-1}$$

$$var(X) = E(X^2) - (E(X))^2$$

$$E(X^2) = \int_0^{\infty} x^2 \cdot \frac{\lambda^a}{\Gamma(a)} \cdot \frac{1}{x^{a+1}} e^{-\lambda/x} \, dx$$

• DISTRIBUCIÓN NGaI(m, c; a, b)

$$\underset{N}{X}, \underset{GaI}{Y} \sim NGaI(\underbrace{m, c; a, b})$$

$$f_{(X,Y)}(x,y) = f_Y(y) \cdot f_{X|Y=y}(x)$$

$$\propto \frac{1}{y^{a+1}} e^{-\lambda/y} \cdot \frac{1}{y^{1/2}} e^{-\frac{(x-m)^2}{2yc}}$$

• FUNCIÓN DE VEROSIMILITUD

Sea $x_1, \ldots, x_n \sim N(\mu, \sigma^2)$ con ambos parámetros desconocidos.

$$\ell(\mu, \sigma^2 | \mathbf{x}) = \prod_{i=1}^{n} f_{X_i}(x_i | \mu, \sigma^2)$$

$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$= (2\pi)^{-\frac{n}{2}} (\sigma^2)^{-n/2} \cdot e^{-\sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2}} ;$$

Se simplifica el sumatorio:

$$\sum_{i=1}^{n} (x_i-\mu)^2 = \sum_{i=1}^{n} (x_i - \bar{x} + \bar{x} - \mu)^2$$

$$= \sum_{i=1}^{n} \left[(x_i-\bar{x})^2 + (\bar{x}-\mu)^2 + 2(x_i-\bar{x})(\bar{x}-\mu) \right]$$

$$= \underbrace{\sum_{i=1}^{n} (x_i-\bar{x})^2} + \underbrace{\sum_{i=1}^{n} (\bar{x}-\mu)^2} + \underbrace{\sum_{i=1}^{n} 2(x_i-\bar{x})(\bar{x}-\mu)}$$

$$= n s^2 + n(\bar{x}-\mu)^2 + 2(\bar{x}-\mu) \cdot \underbrace{\sum_{i=1}^{n} (x_i-\bar{x})}_{\sum_1^n x_i - n\bar{x} = n\bar{x} - n\bar{x} = 0}$$

$$\left(s^2 = \frac{1}{n}\sum_1^n (x_i-\bar{x})^2\right)$$

$$\sum_{i=1}^{n} (x_i-\mu)^2 = n s^2 + n(\bar{x}-\mu)^2$$

$$\ell(\mu, \sigma^2 \mid \cancel{x}) = (2\pi)^{-n/2} \underbrace{(\sigma^2)^{-n/2}} e^{-\frac{1}{2\sigma^2}\left[n s^2 + n(\bar{x}-\mu)^2\right]}$$

DISTRIBUCION A POSTERIORI

CASO 1 : A priori no informativa

$$\pi(\theta, \phi) \propto \frac{1}{\phi}$$

$$\ell(\theta, \phi \mid \underset{\sim}{x}) \propto \frac{1}{\phi^{n/2}} \cdot e^{-\frac{1}{2\phi}\left[ns^2 + n(\bar{x} - \mu)^2\right]}$$

$$\pi(\theta, \phi \mid \underset{\sim}{x}) \propto \underbrace{\frac{1}{\phi} \cdot \frac{1}{\phi^{n/2}} \cdot e^{-\frac{ns^2}{2\phi}}} \cdot \underbrace{e^{-\frac{n(\bar{x}-\theta)^2}{2\phi}}}$$

formato; Ga I · Normal

$\Downarrow$

$$\underbrace{\frac{1}{\phi^{1/2}} \cdot \frac{1}{\phi^{n/2}} \; e^{-\frac{ns^2}{2\phi}}}_{GaI\left(\frac{n-1}{2}, \frac{ns^2}{2}\right)} \cdot \underbrace{\frac{1}{\phi^{1/2}} e^{-\frac{(\bar{x}-\theta)^2}{2\phi/n}}}_{N\left(\bar{x} ; \phi/n\right)}$$

$$\frac{1}{\phi^{1/2}} \cdot \frac{1}{\phi^{n/2}} = \frac{1}{\phi^{\frac{n+1}{2}}} \equiv \frac{1}{\phi^{a+1}} \equiv \frac{1}{\phi^{\frac{n-1}{2}+1}}$$

$$(\theta, \phi \mid \mathbf{x}) \sim NGaI\left(\bar{x}, \frac{1}{n} ; \frac{n-1}{2}, \frac{ns^2}{2}\right)$$

CASO 2 : A priori conjugada

$$(\theta, \phi) \sim NGaI(m, c ; a, b)$$

$$\pi(\theta,\phi) \propto \frac{1}{\phi^{a+1}} e^{-b/\phi} \cdot \frac{1}{\phi^{1/2}} \; e^{-\frac{(\theta-m)^2}{2c\phi}}$$

$$\ell(\theta,\phi \mid \mathbf{x}) \propto \frac{1}{\phi^{n/2}} e^{-\frac{ns^2}{2\phi}} \cdot \; e^{-\frac{n(\theta-\bar{x})^2}{2\phi}}$$

$$\pi(\theta,\phi \mid \mathbf{x}) \propto \frac{1}{\phi^{a+1+\frac{n}{2}+\frac{1}{2}}} \; e^{-\left[b + \frac{ns^2}{2} + \frac{1}{2}\left(\frac{m^2+nc\bar{x}^2}{1+nc}\right)\right]/\phi} \cdot e^{-\frac{(\theta-\mu^*)}{2\sigma^{*2}}}$$

Simplificación de la parte exponencial:

$$\frac{(\theta-m)^2}{c\phi}+\frac{(\theta-\bar{x})^2}{\phi/n} \equiv \underbrace{\left(\frac{1}{c\phi}+\frac{n}{\phi}\right)}_{A}\theta^2-2\theta\underbrace{\left(\frac{m}{c\phi}+\frac{n\bar{x}}{\phi}\right)}_{B}+\underbrace{\left(\frac{m^2}{c\phi}+\frac{n\bar{x}^2}{\phi}\right)}_{C}$$

$$\sigma^{*2}=\frac{1}{\frac{1}{c\phi}+\frac{n}{\phi}}=\frac{c\phi}{1+nc}=\frac{c}{1+nc}\phi$$

$$\mu^{*}=\left(\frac{m}{c\phi}+\frac{n\bar{x}}{\phi}\right)\cdot\frac{c\phi}{1+nc}=\frac{m+nc\bar{x}}{1+nc}$$

$$=\underbrace{\frac{1}{1+nc}}_{w_1}m+\underbrace{\frac{nc}{1+nc}}_{w_2}\bar{x}$$

$$A\theta^2-2\theta B+C \equiv A\left(\theta-\frac{B}{A}\right)^2+\left(C-B^2/A\right)$$