The Past of an Electron: Young Popper's *Gedankenexperiment* Against the Indeterminacy Relations

Between 1934 and 1936, Popper wrote at least eight manuscripts on the interpretation of quantum mechanics, proposing a *Gedankenexperiment* in which reconstructing the past trajectory of an electron via momentum conservation would allow one to predict both position and momentum with arbitrary precision, thus apparently 'beating' the indeterminacy relations. During this period, Popper corresponded with leading physicists such as Weisskopf, Heisenberg, von Weizsäcker, and Einstein, who pointed out the flaws in his reasoning. In retrospect, Popper described these efforts as a "gross mistake". Yet Jammer (1974) suggested that this mistake may have influenced the 1935 Einstein–Podolsky–Rosen argument, while Margenau (1974) credited Popper with anticipating the later distinction between preparation and measurement. Drawing on unpublished material, this paper argues that both claims are doubtful. Popper's thought experiment is more closely related to the 1931 Einstein–Tolman–Podolsky setup, though he reached the opposite conclusion precisely because he misunderstood the link between preparation and measurement. The same 'mistake,' the paper concludes, also undermines the EPR-like thought experiment Popper proposed at the turn of the 1980s.

Introduction

Between 1934 and 1936, Karl R. Popper composed at least eight manuscripts revolving around a *Gedankenexperiment* intended to challenge the widespread interpretation of the so-called indeterminacy or uncertainty relations, which, in quantum mechanics, were thought to limit the joint *measurement* of conjugate quantities such as position and momentum, or time and energy:

1. *Zur Kritik der Ungenauigkeitsrelationen* — August 1934 (Popper 1934b)
2. *Ergänzung zu der vorstehenden kurzen Mitteilung* — before November 1934 (Popper 1934a)
3. *Bemerkungen zur Quantenmechanik* (chapter 2.VII of Popper 1935a) and Appendix V, VI, VII — Fall 1934
4. *Zur Kritik der Ungenauigkeitsrelationen, 2. Mitteilung [A]* — December 1934 (Popper 1934c)
5. *Zur Kritik der Ungenauigkeitsrelationen, 2. Mitteilung [B]* — December 1934 (Popper 1934d)
6. *Erwiderung auf die Kritik Heisenberg–Weizsäcker* — late 1934 to early 1935
7. *Meßanordnung* — February 1935
8. *Zur Kritik der Ungenauigkeitsrelationen [1935]* — August 1935 (Popper 1935b)
9. *Bemerkung zum Komplementaritätsproblem der Quantenmechanik* — 1936 (Popper 1936)

Only 1 and 3 were published during Popper's lifetime. Numbers 6 and 7 have been lost, although 6 may in fact be identical with 5. The remaining material was published only recently in a collection of Popper's early writings (Popper 2006). While drafting these texts, the young Popper—at that time a schoolteacher with only a single short publication to his name (1932–1933)—engaged in extensive correspondence with leading physicists of his time: (1) Victor Weisskopf, then assistant to Pauli in Zürich; (2) Heisenberg and his assistants Carl Friedrich von Weizsäcker and Hans Euler; and (3) Einstein. Some of this correspondence is preserved at the KPA and remains unpublished. Moreover, Popper discussed his ideas in person with Viennese physicists he could reach: Franz Urbach, his cousin, the crystallographer Käthe Schiff, Hans Thirring[1] and his assistant Eugene Guth, as well as Felix Ehrenhaft. Naturally, *verba volant*: we have little insight into the concrete content of these exchanges.

The late 1920s and early 1930s saw a proliferation of *Gedankenexperimente* as tools to probe conceptual issues *outside* the quantum-mechanical formalism—either to demonstrate its consistency or to refute it. Heisenberg (1927) formulated his famous γ-ray microscope thought experiment to show that the uncertainty relations reflect the mutual disturbance that arises from the experimental determination of position and momentum. By contrast, Einstein was regarded as having devised *Gedankenexperimente*, such as the light quantum box thought experiment, to challenge the uncertainty relations (Bohr 1931, 127f.). Popper, however, accepted the mathematical *derivation* of the uncertainty relations from the quantum formalism; he devised a *Gedankenexperiment* intended to expose the tension between two different *interpretations* of the uncertainty relations. Popper aimed to show that the indeterminacy relations impose a limitation on the *physical selection* of an ensemble of systems in which both position and momentum are sharply defined; however, *pace* Heisenberg, they do not impose any limitation on the accuracy of *measurements* of position and momentum on a single system.

Popper sought to exploit the fact that quantum physicists, following Heisenberg himself, generally conceded that *non-predictive measurements* of arbitrary sharpness are permitted by the uncertainty relations; nevertheless, they typically denied that *predictive measurements* were. According to Popper, the reason is that they tacitly assumed what he called the *coupling-hypothesis* (*Kopplungshypothese*)[2] between 'physical selections' and 'predictive measurements.' Popper, however, thought he could show that the coupling-hypothesis was inconsistent with the formalism of quantum mechanics. He devised a thought experiment based on a particle collision, beginning with a *non-predictive measurement* that reconstructed the past trajectory of a particle. By invoking momentum conservation, the setup was designed to permit arbitrarily precise *predictive measurements* on another particle after the collision, thereby avoiding any selection that would violate the statistical interpretation of the uncertainty relations.

In retrospect, Popper himself would later describe his thought experiment as "a gross mistake", for which he had been "deeply sorry and ashamed" (Popper 1982b, 15). Yet early commentators, writing at the height of Popper's fame, attributed some significance to his early engagement with quantum mechanics. Max Jammer (1974, 178f.) conjectured that Popper's mistaken thought experiment was at least instrumental

[1] According to Popper's recollection, Thirring invited Popper in 1934 (possibly 1935) to his seminar; see Schilpp (1974, 1125).

[2] Popper (1959, 238f.) later adopted the translation 'hypothesis of linkage,' possibly because 'coupling' in English typically implies a physical interaction. However, I prefer to retain a rendering closer to the original German, which better reflects its resonance with contemporary physical discourse.

in inspiring the 1935 Einstein–Podolsky–Rosen (EPR) thought experiment. Indeed, according to Jammer, Popper's setup bore a "striking resemblance" to the EPR setup, raising the question of a possible influence from the work of Einstein and his collaborators. Henry Margenau (1974, 757f.) credited Popper with anticipating the distinction between 'preparation' (which Popper called 'physical selections') and 'measurement,' a distinction he himself proposed around the same time (Margenau 1936, 1937) and which later became standard. In particular, according to Margenau, Popper was among the first philosophers to appreciate that the indeterminacy relations expressed a 'preparation uncertainty' rather than a 'measurement uncertainty' (see Popper 1974, 72f.).

This paper, based on little-known or unpublished material from the Karl Popper Archives (KPA), aims to demonstrate that both claims fail to withstand critical scrutiny. Popper was wary of implying that the "gross mistake made by a nobody (like myself)" might have influenced Einstein, noting only that "from a purely temporal standpoint" the possibility could not be ruled out (Popper to Jammer, Apr. 13, 1967, Jammer 1974, 178; fn. 30). However, it is precisely the chronology that seems to exclude any influence of Popper's thought experiment on the emergence of the 1935 EPR argument. What can be said is that Popper independently formulated a variant of the lesser-known 1931 Einstein–Tolman–Podolsky (ETP) thought experiment (Einstein, Tolman, and Podolsky 1931), though he arrived at the opposite conclusion. The paper argues that the reason was ultimately that Popper deeply misunderstood the relation between 'preparation' and 'measurement' that he allegedly anticipated (see Maxwell 2016, sec. 6). Popper's notion of 'physical selection' is akin to, but still not on par with, the modern notion of 'preparation.' In particular, the *coupling hypothesis* between preparations and measurements is not an 'additional hypothesis,' ultimately contradictory with the rest of the formalism, as he argued, but a central feature of quantum mechanics that distinguishes it from classical physics.

Against the background of the 1930s debate on the past trajectories of particles in quantum mechanics (section 1), the paper reconstructs the development of Popper's thought experiment (sections 2 to 4) and examines the criticisms raised by Heisenberg and his Leipzig group (section 5) as well as by Einstein (section 6). Popper ultimately, though only after considerable resistance, conceded that his thought experiment was a 'mistake.' The failure of the thought experiment should have convinced Popper of the validity of the coupling-hypothesis between preparations and measurements that he aimed to challenge. On the contrary, Popper concluded that it was the particular kind of ETP-like setup he had chosen to challenge the coupling-hypothesis that was at fault. Thus, Popper (1982b, 1982a, 1985, 1986) attempted to replace the ETP-like experiment with an EPR-like one. Recent historical literature on Popper's interpretation of quantum mechanics (Howard 2012; Shields 2012; Maxwell 2016, sec. 6; Del Santo 2018, 2019) mentions this episode only in passing, ultimately dismissing it as a youthful blunder and focusing instead on Popper's renewed engagement with the theory after the war (Popper 1967, 1982b). However, this setup too was ultimately based on the same mistake: using a perfect correlation, this time relative position, to circumvent the preparation–measurement coupling.

1 Setting the Stage: The Past Trajectory Problem

1.1 The Heisenberg-Schlick Dispute

In the spring of 1929, Heisenberg delivered a series of lectures on quantum mechanics at the University of Chicago. A written account of the lectures was completed in March 1930 and appeared later that summer in print, in both English and German editions (Heisenberg 1930a, 1930b). In presenting the indeterminacy relations, Heisenberg takes the opportunity to address, although rather in passing, the vexed question of the retrodictability of sharp joint values of position and momentum variables of, say, an electron. As already pointed out in Bohr's (1928, 583) Como lecture, if one performs two position measurements at subsequent times, then from the time-of-flight one can in principle reconstruct the electron's momentum with arbitrary sharpness between these two measurements[3]. Heisenberg discussed a similar thought experiment in his book, but replaced the first position measurement with a momentum measurement:

> [I]t should be noted that the uncertainty relations apparently do not apply to the past. For if the electron's *velocity* is initially known and then its *position* is measured precisely, one can also calculate the electron's positions for the *time before the position measurement* with precision; for this past, $\Delta p \Delta q$ is then smaller than the usual limit. Yet this *knowledge of the past* is purely speculative, since (because of the *change of momentum during the position measurement*) it in no way enters as an initial condition into any calculation concerning the electron's future and does not appear in any physical experiment at all. Whether one should ascribe any physical reality to the aforementioned calculation about the electron's past is therefore purely a *matter of taste* [*Geschmacksfrage*]. (Heisenberg 1930a, 15)

Heisenberg likely had in mind the measuring procedure he describes a few pages later (19f.). Electrons move along the $+x$ direction with a known longitudinal momentum p_x, while light quanta enter along the $-x$ axis (fig. 1). The Doppler shift $\delta\nu$ of a scattered light quantum at time t_0, assuming energy and momentum conservation, allows one to infer the electron's transverse momentum δp_y; the point where the electron later arrives on the detection screen at time t_1 determines its position δy.[4] This seems to permit reconstruction of the electron's 'past path' between t_0 and t_1 with arbitrary precision, $\delta y\, \delta p_y \approx 0$. Heisenberg's reference to the 'time before the position measurement' is somewhat ambiguous, seemingly implying the possibility of tracing the electron's path even *before* the first momentum measurement.

Heisenberg immediately pointed out that the reconstruction of the electron's past has no predictive significance for its future behavior, and therefore does not constitute a violation of the indeterminacy relations. However, the question of whether such reconstructions possess any 'physical meaning'—which Heisenberg dismissed as a 'matter of taste'—constitutes, as one might expect, an interesting philosophical puzzle. Indeed, the issue was immediately taken up by Moritz Schlick, who at that time was working on a manuscript on the problem of causality. At the end of November, Arnold Berliner (the editor of *Die Naturwissenschaften*) confirmed receipt of 'Die Kausalität in der gegenwärtigen Physik', which soon circulated among physicists, prompting correspondence with Einstein, Heisenberg, and Pauli. The paper identifies causality with predictability; thus, Schlick accepts that Heisenberg's relation concerns

[3] See also Ehrenfest to Goudsmit, Uhlenbeck, and Dieke, Nov. 3, 1927.

[4] In the following, we use Δq and Δp to denote the dispersions of the statistical distributions, and δq and δp to denote the measurement errors.

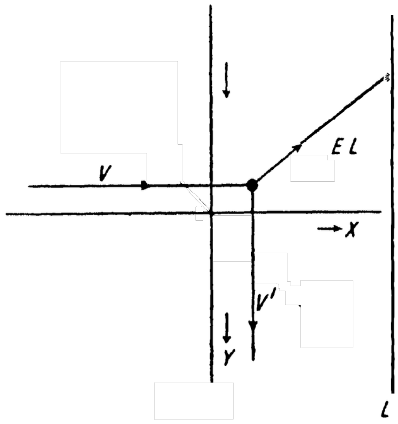

Figure 1: Adapted from Heisenberg (1930a, 19).

an indeterminacy of prediction, thereby limiting the applicability of causality on the microscopic scale. Schlick was aware that such indeterminacy does not apply to retrodictions, as Heisenberg (1930a, 15) himself points out. Nevertheless, Schlick remained unconvinced by Heisenberg's agnostic stance toward the physical reality of past trajectories:

> Heisenberg thinks that 'whether any physical reality should be assigned to the calculation concerning the past of the electron is purely a question of taste' [15]. But I should prefer to put it more strongly, in complete agreement with what I take to be the incontestable basic viewpoint of Bohr and Heisenberg themselves. If a statement about an electron's position is not *verifiable* within atomic dimensions, *we can attach no meaning to it*; it becomes impossible to speak of the 'path' of a particle between two points at which it has been observed. (This is not, of course, true of bodies of molar dimensions, for it is in principle possible to verify afterwards that the projectile was located at the intervening points). (Schlick 1931, 159)

Schlick argues that although quantum theory's equations allow one to calculate an electron's past positions after exactly measuring its velocity and then its position, such results are physically meaningless, since they are 'unverifiable' in principle: one cannot verify whether the electron was actually moving with that velocity at the calculated point, since any observation would disturb its path in an uncontrollable way. For Schlick, therefore, the very idea of a sharply defined past path of the electron lacked physical meaning, just like the notion of its future path.

1.2 ETP Experiment

It is not surprising that, in his comments on Schlick's article in late 1930, Einstein described Schlick's stance as 'too positivistic' (Einstein to Schlick, Nov. 28, 1930; EA, 21-603). Concluding his letter, Einstein mentioned that he would spend the coming winter in Pasadena. Just a fortnight after Schlick's (1931) paper appeared on February 13, 1931, Einstein, in collaboration with Richard Tolman and Boris Podolsky (ETP), submitted to the *Physical Review* a short letter reporting the results of his stay in California. The paper was published on March 15 (Einstein, Tolman, and Podolsky 1931). Neither Heisenberg nor Schlick are mentioned; however, Einstein and his collaborators address the same question of the past path of particles in quantum mechanics. They argue that *if* sharp retrodictions about one particle were possible, *then* one could, with the help of conservation laws, infer predictions about the future path of a second particle that would otherwise be excluded by the uncertainty relations.

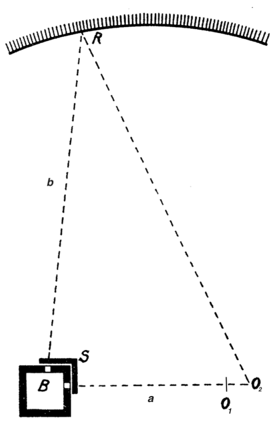

Figure 2: Adapted from Einstein, Tolman, and Podolsky (1931).

Thus, ETP conclude that retrodictions should *not* be allowed. The indeterminacy relations apply equally to the past and the future.

The proposed arrangement (fig. 2) resembles the one Einstein reportedly used in the 1930 debate with Bohr (1931, 127f.) in Brussels[5]. A box B contains identical particles. At time t_0, a shutter S releases two particles, a and b. By weighing the box before and after the emission, the observer can determine the total energy loss, $E_{\text{tot}} = E_a + E_b$. At O, particle a may undergo different measurements. Its momentum p_a (and energy E_a) can be determined, for example, by a Doppler-effect measurement at O_1, while its arrival position and time t_1 can be recorded at O_2 by some mechanical time-recording counter. From the measured position of a at t_1, together with its past momentum p_a, the shutter opening time t_0 can, in principle, be reconstructed. If E_a has been measured, then the energy of particle b can be inferred from $E_b = E_{\text{tot}} - E_a$. Once t_0 has been determined, the arrival time t_b at O_2 can be inferred from the geometrical arrangement of the setup, that is, from SRO.

At this stage, a backdoor appears to open, seemingly offering a way around the energy–time uncertainty relation $\delta E, \delta t \leq \frac{h}{4\pi}$. Yet ETP quickly close it again (Einstein, Tolman, and Podolsky 1931, 781). One might attempt to measure momentum via the Doppler shift of low-energy light quanta, that is, long-wavelength radiation (e.g., in the infrared range). Assuming conservation of momentum and energy, this method, in principle, allows one to infer the momentum of a particle before and after scattering from the frequency shift of the scattered photon. However, to determine the frequency with high precision, many oscillation periods must be observed, which blurs the exact time of the scattering event. As a result, the precise moment at which the shutter S was opened cannot be reconstructed. Conversely, if one tries to determine the scattering time very precisely, one must use high-frequency (short-wavelength) radiation, thereby losing information about the momentum before the collision. The authors summarized their conclusion as follows:

> Thus in our example, although the velocity of the first particle could be determined both before and after interaction with the infrared light, it would not be possible to determine the exact position along the path SO at which the change in velocity occurred as would be necessary to obtain the exact time at which the shutter was open.
> It is hence to be concluded that the principles of the quantum mechanics must involve an uncertainty in the description of *past* events which is *analogous* to the uncertainty

[5] Bacciagaluppi and Crull 2024, 1.4.3.

> in the prediction of *future* events. It is also to be noted that although it is possible to measure *the momentum of a particle and follow this with a measurement of position*, this will not give sufficient information *for a complete reconstruction of its past path*, since it has been shown that there can be *no method for measuring the momentum of a particle without changing its value.* (Einstein, Tolman, and Podolsky 1931, 781; my emphasis)

Thus, ETP point out the flaw in this kind of experiment: one can reconstruct the path of a particle *between* the momentum and the position measurements, but not *before* the momentum measurement at O_1. The uncertainty principle therefore not only restricts predictions about the future but also imposes limitations on the reconstruction of the past: the path reconstruction before the momentum measurement must be physically forbidden *if* the indeterminacy relations are to hold.

As Ehrenfest famously July 9, 1931 pointed out to Bohr, Einstein, contrary to common perception, was clearly not attempting to devise “new *perpetua mobilia* = machines against the uncertainty relation” (Ehrenfest to Bohr, Jul. 9, 1931, AHQP/BSC 18)[6] Rather, Einstein's point was that, *because of* the uncertainty relations, the experimental *choice* between determining the time or the momentum of a appeared to retroactively determine whether the emitted light quantum b possesses a definite energy or a definite emission time *after* it had already left the box (see also Einstein 1932). After a talk in Leiden in November 1931, Einstein might have been stimulated by Ehrenfest to replace the energy–time with the position–momentum uncertainty (Einstein to Ehrenfest, Apr. 5, 1932; EA, 10-231). Indeed, as later recalled by Léon Rosenfeld (1967, 127f.), Einstein, after a seminar in Brussels in the spring of 1933, apparently reformulated the argument without the aid of the box, considering two particles emitted simultaneously: a measurement of the momentum (or position) of the first particle allowed an inference of the corresponding momentum (or position) of the second. If Rosenfeld's testimony is accurate, Einstein possessed, in embryonic form, the gist of the EPR paper before permanently moving to the United States on October 17, 1933.

2 Popper's Turn to Quantum Theory. The Correspondence with Weisskopf

At the turn of 1934, Popper was reworking the manuscript of *Die Beiden Grundprobleme der Erkenntnistheorie* (Popper 1930–1933) into what would become *Logik der Forschung* (Popper 1935a). In the spring, Popper submitted a first version of the manuscript to Schlick, the editor of the series. At the request of the publisher Springer, the manuscript had to be shortened considerably (Hacohen 2010, 235–244). While Popper was adding two technical chapters on probability and one on quantum mechanics, much of the editing was carried out with the help of his uncle, Walter Schiff. Between May and June 1934, Popper likely began to devote special attention to the quantum physics chapter, discussing it with Viennese physicists like Urbach, Urbach's friend Weisskopf, and his cousin Käthe (237). At that time, Weisskopf was in Zurich as Pauli's assistant, so their exchange had to take place by mail, leaving us written evidence of Popper's thinking during that period. Although Popper's original letter has not survived, much of his position can be inferred from Weisskopf's reply.

Weisskopf seems to concede to Popper that the uncertainty relations do not exclude the possibility that the position and momentum of a single electron can be ‘known’ (*i.e.*, measured) with arbitrary accuracy. If the position is fixed by a very narrow slit,

[6] This brief overview is based on Bacciagaluppi and Crull 2024, Ch. 1.

the emerging beam no longer consists of electrons with a well-defined momentum or energy. The diffraction pattern on the screen reflects this momentum (or angular) distribution, which can also be inferred from the energy spread: when the electrons hit a target, some atoms are excited even though their excitation requires more energy than the mean kinetic energy of the electrons. Thus, once the position is fixed, the electron's path can be reconstructed with arbitrary precision. As Weisskopf put it, "[t]his consideration is of course not changed if, by some measurements, I determine which energies are present in this accumulation of electrons *without altering their spatial arrangement*. In this, I agree with you and admit my mistake" (Weisskopf to Popper, Jun. 26, 1934; KPA, 360-21).

However, Weisskopf immediately raises a counter-objection: "But now comes the point: if I can measure these energies without disturbing the spatial arrangement, then I have devices with which I can also *produce* [*herstellen*] beams (aggregations of electrons) that are 1.) spatially confined and 2.) possess a sharp energy (= momentum of free particles)" (Weisskopf to Popper, Jun. 26, 1934; 361-21). The possibility of producing such aggregates of electrons would indeed violate the indeterminacy relations, since a wave localized in space must contain infinitely many frequencies, and thus momenta and energies. Popper probably replied that instruments allowing *measurements* of sharp electron energy and position might exist without thereby enabling the *production* of an *aggregation* of such electron beams. However, Weisskopf doubted this, arguing that if we can *measure* electron energy, we should also be able to *separate* electrons according to predetermined values; that is, to produce an aggregate of electrons all with the same energy and the same position:

> (1) Apparatuses that can *measure* both position and energy at the same time are not necessarily able to *produce* electron aggregates with sharp position and energy. I consider this conclusion not entirely correct, since – in the case that one has the possibility of determining the energy of electrons at sharp position – it must surely also be technically possible somehow to remove them, for example, if they have too much energy.
> (2) You can probably anticipate my line of reasoning, which leads you to say that quantum mechanics is only capable of dealing with that world in which such *apparatuses* are not applied. But I would already regard that as a breakdown. After all, apparatuses are also part of nature and should therefore be valid within it.
> There would, however, have to exist accumulations of electrons whose position and energy are sharply determined, even though this is impossible according to wave theory. Thus the statement that electrons can in fact always be represented by waves would not always be correct, and in this lies perhaps the end of quantum mechanics. Weisskopf to Popper, Jun. 26, 1934; 360.21

Here the fundamental issues at stake are laid out quite clearly: (1) the distinction between (1a) *producing* an aggregate of electrons with arbitrarily sharp position and momentum, and (1b) *measuring* the position and momentum of a single electron with arbitrary precision; (2) the 'coupling' between the two operations, that is, the claim that the possibility of (1b) necessarily implies that of (1a), which in turn would certainly constitute a violation of the indeterminacy relations.

Weisskopf asked Popper to share his opinion on these ideas. However, regarding Popper's request to cite their discussions in his forthcoming book, he asked not to be mentioned by name. In the following weeks, Popper may have come up with a clever trick to counter Weisskopf's objection: a *Gedankenexperiment* in which an apparatus was laid out that could *measure* with arbitrary precision the position and momentum of a single particle without allowing the *production* of a super-pure case more homogeneous than the indeterminacy relations would permit. Already, on August 27, 1934, Popper submitted a short note, ‘Zur Kritik der Ungenauigkeitsrelationen’, to *Die Naturwissenschaften*, one of the most prestigious scientific journals of that time. As we shall see, Popper independently envisaged the same backdoor to the indeterminacy relations as the one ETP had already suggested, only using momentum and position instead of energy and time.

Popper seems to have soon realized that this backdoor could indeed be closed in the same way suggested by ETP. However, contrary to Einstein, Popper wanted to leave the backdoor open. Thus, he wrote a short supplement, *Ergänzung zu der vorstehenden kurzen Mitteilung*, to clarify the matter. By the autumn of 1934, Popper had completed the *Logik der Forschung*. Thus, by that time, chapter IX, titled *Bemerkungen zur Quantenmechanik*, together with the three appendices V, VI, and VII devoted to the same topic, can be regarded as complete. The manuscript of the *Ergänzung* was probably written after the paper had already been submitted to *Die Naturwissenschaften*, but before it was published.[7] It must also have been written prior to Appendices VI and VII, which appear to be Popper’s attempts to address problems raised in the *Ergänzung*.

3 Popper’s Interpretation of the Indeterminacy Relations

Popper starts from the well-known fact that, within the formalism of quantum mechanics, a central role is played by the so-called Heisenberg indeterminacy relations (*Unbestimmtheitsrelation*), that is, the inequality:

$$\Delta p_x \Delta x > h/4\pi \tag{1}$$

where x is the space coordinate of a point mass and the p_x the momentum component along the same direction. Popper’s aim was not to challenge the mathematical *derivation* of Heisenberg’s formula, but rather its physical *interpretation*: the indeterminacy relations are (a) ‘imprecision relations,’ (*Ungenauigkeitsrelationen*), which set a limit on the accuracy of simultaneous measurements of x and p_x for a *single* particle; and (b) ‘dispersion relations,’ (*Streuungsrelationen*), expressing the limit on the standard deviations of repeated x and p_x measurements on an *ensemble* of particles. It is undeniable that Popper was among the first to make this distinction, at least in the philosophical literature,[8] although similar views were defended around the same time by other Viennese scholars, such as Richard von Mises (1930, 1934), with whom Popper might have been familiar. However, Popper probably arrived at a similar conclusion independently, starting from von Mises’s (1928) frequency interpretation of probability, which he defends in Ch. VI.

According to Popper, Heisenberg’s *philosophical justification* for eq. (1) was based on the idea that measurement itself disturbs physical systems: precision in one variable (e.g., position) can be achieved only at the expense of precision in its conjugate (e.g., momentum), with eq. (1) establishing the bound. Against this view, Popper argued that the *mathematical derivation* of eq. (1) follows from Born’s interpretation of the ψ-function as a probability amplitude, with $|\psi(x)|^2 dx$ giving the probability of finding a particle along the x-axis within the infinitesimal interval between x and $x + dx$.

[7] No reference is made to von Weizsäcker’s response that was published alongside Popper’s (1934b) article; see below section 5.1.

[8] In the physics literature it was at least implied in Kennard (1927) and Weyl (1928, 67f.). Popper was familiar with the latter.

Using Fourier analysis, one can show that the position and momentum probability distributions are mathematically related in such a way that a narrower distribution in position entails a broader distribution in momentum, and vice versa. In this reading, eq. (1) regulates the dispersion relation between these distributions rather than limits the precision of measurement.

For Popper, the confusion between these two *interpretations* is ultimately the consequence of physicists not clearly distinguishing between (1) a *physical*[9] *selection* (*Aussonderung*), which occurs when, for example, a beam of particles is actively filtered so that only those passing through a narrow slit (a region Δx) remain, thereby selecting an 'aggregate' of particles all with the same position, a so-called 'pure case'[10] (2) a *measurement* (*Messung*) involves the detection of a particular system by means of a measuring instrument. Unlike separation, measurement produces an actual registration of a value—such as a spot on a screen or a pointer deflection—an irreversible change in a piece of macroscopic apparatus that constitutes a publicly observable record (Popper 1935a, 163)

Young Popper indeed appears to have articulated a distinction similar to the one between 'preparation' and 'measurement' formulated by Margenau (1936, 1937) around the same time.[11] In particular, he argues that (a) *not every physical selection amounts to a measurement*: for instance, when electrons are passed through a velocity filter, the procedure produces an ensemble of electrons with definite momentum (a pure case described by a monochromatic plane wave), but no value of the momentum is actually recorded. Conversely, Popper also makes the possibly more troublesome claim (b) *that not every measurement can be regarded as a physical selection*: electrons belonging to a monochromatic beam may be registered by a Geiger counter at a given location, yet in this case the electrons are not physically separated according to their position (Popper 1935a, 163f.). As we have seen, Weisskopf challenged point (b), arguing that once a measurement on single particles sharper than Heisenberg's 'imprecision relations' would allow is possible, the same setup could in principle be used to produce a super-pure ensemble of particles, thereby violating Heisenberg's 'variance relations' as well.

The two possible *interpretations* of eq. (1) can then be rephrased as follows: (a) the indeterminacy relations limit the degree of *statistical homogeneity* that can be achieved in the *physical selection* of an *ensemble* of identical systems; (b) the indeterminacy relations limit the *precision* of the *measurement* performed on a *single* system (164). According to (a), given an ensemble of particles with sharp momenta (pure case), we are not allowed to select a sub-ensemble that also has a sharp position (a super-pure case); (b) refers to the simultaneous measurement of position and momentum on a

[9]As opposed to merely 'mental' or 'imagined' selection (Popper 1935a, 163). These correspond to conditional selections, which define a subensemble after the fact, by conditioning on the value of a random variable obtained in a measurement (say, an ensemble of particles B that have the same momentum as A). Physical selections are controllable: the resulting subensemble can be reproduced at will by repeating the same filtering procedure. A conditional selection, by contrast, cannot be reproduced, since it depends on a previous physical selection and on specific measurement outcomes that cannot be controlled in advance.

[10]An aggregate (*Menge*) corresponds to what von Mises called a *Kollektiv*, in modern terms an 'ensemble.' As Popper (1959, 225; fn. *1) later recognized, at that time he did not clearly distinguish between an aggregate of co-present particles, such as a beam (spatial ensemble), and an aggregate of repetitions of an experiment performed with one particle or one system of particles (temporal ensemble).

[11]However, below I argue that Popper's notion of 'physical selection' is not identical to the modern notion of 'preparation'.

single particle, in which the two measurements hinder each other. Popper's program can then be articulated as follows: I. (a) follows directly from the formalism; II. (b) does not *logically* follow from (a); III. (b) follows from (a) only if one adds an *additional hypothesis* (c); IV. the combined system (a) + (c) turns out to be *contradictory* (Popper 1935a, 154f.).

With regard to point (I), Popper notes that it has rarely been observed that the mathematical derivability of the Heisenberg formulas from the fundamental equations of quantum mechanics must also be matched by a corresponding derivability of the interpretation of those formulas from the interpretation of the fundamental equations. Some, like Arthur March (1931, 1f., 57), regard the statistical dispersions as a mere consequence of measurement inaccuracies; others, like Weyl (1928, 68), seem at least to accept the reverse, namely that the limitations on single measurements follow from the statistical view. Popper rejects both options. Quantum formulas make *frequency predictions* (*Häufigkeitsprognosen*). The latter cannot imply prohibitions on individual measurements (apart from the trivial cases of probability 1 or 0) (Popper 1935a, 165f.). The uncertainty relations imply that if, say, 1000 position measurements show the electron to be at the same place, then it may be expected that 1000 momentum measurements would yield widely divergent results. A 'sharp' measurement of both x and p_x—that is, an exceptionally precise outcome for both variables—is not impossible but highly improbable. It represents an event analogous to a statistical fluctuation within the ensemble distribution.

In this way Popper addressed point (II) of his program: from a logical point of view all alleged 'proofs' that exact position and momentum *measurements* contradict the formalism of quantum mechanics are flawed. The indeterminacy relations do not *forbid* exact position–momentum measurements; they merely maintain that exact position–momentum *predictions* cannot be *derived* from them (166). Heisenberg himself could not conceal the tension between these two alternatives: (1) he argues that, since the indeterminacy relations *forbid* selecting an ensemble of systems with exact values of both position and momentum, *predictive measurements* of the future position–momentum trajectory of a particle with arbitrary precision are excluded; (2) he nevertheless concedes, almost in passing, that the indeterminacy relations *allow* arbitrary *non-predictive measurements* of position and momentum. These reconstructions can, in principle, be obtained by combining two predictive measurements: (a) position followed by position, (b) momentum followed by position, or (c) position followed by momentum (156f., 167).

Because predictive position–momentum measurements cannot be derived from quantum mechanics, physicists generally conclude that the very notion of a *future* trajectory has lost physical meaning in quantum mechanics (March 1931, 55). Yet the question of the physical meaning of the *past* trajectory between two measurements remains ambiguous. Popper could then enter the Heisenberg–Schlick debate presented in section 1.1. As we have seen, (a) Heisenberg dismissed the issue as a 'question of taste,' and (b) Schlick regarded it as a 'meaningless question.' However, both ultimately agreed that, since such reconstructions cannot be *verified*—that is, since no testable prediction can be derived from them—they pertain to the realm of 'metaphysics.' As one might expect, however, for Popper the question in this form is ill-posed. The point is that if sharp past trajectory reconstructions were not possible, the frequency predictions of quantum mechanics could not be *falsified*, and the theory would therefore belong to the realm of 'metaphysics.' Without the possibility of reconstructing the past paths of particles, one could not subject the theory to empirical control (*Nachprüfung*).

In, say, a diffraction experiment, the position is physically selected at x_1 by means of a screen containing a slit. The theory predicts that the particles' energy—and hence their speed—remains constant, while their direction, and thus the transverse component of momentum p_y, is spread. This can be subjected to empirical control by placing a photographic plate at x_2 and observing the distribution of spots. Each spot, corresponding to a single particle detected at position δy, allows one to infer the deflection angle $\delta\varphi$ and the corresponding past momentum component δp_y, so that the error product $\delta y \delta p_y$ can be made arbitrarily small. The observed distribution of these reconstructed paths (obtained through a position selection followed by a momentum measurement) must conform to the predicted distribution as the diffraction experiment is repeated many times; otherwise, the theory would have to be rejected (Popper 1935a, 169).

3.1 The Coupling-Hypothesis

As one can see, Popper articulated an early version of what might be called an 'objective statistical interpretation of quantum mechanics ' (Pechenkin 2022, 50.2), in which the collapse of the wave function is reinterpreted as the mere selection of a sub-ensemble. When a momentum measurement is performed, one in fact selects an ensemble of systems that have the same sharply defined momentum (Popper 1935a, 171). In this view, the indeterminacy relations imply that one cannot further select a sub-ensemble whose members also possess the same precisely defined position. By June 1934, Popper had managed to convince Weisskopf that the indeterminacy relations *forbid* physical selections of an ensemble more homogeneous than a pure case, but still *allow* arbitrarily precise measurements on single systems. However, Weisskopf may have prompted Popper to take a further step. As we have seen, Weisskopf countered that if it were possible to *measure* with arbitrary sharpness, then the same measuring instruments could be used to *produce* particle aggregates whose position and momentum spread would be smaller than that allowed by the indeterminacy relations. In this sense, the indeterminacy relations, as a prohibition of a super-pure case, also imply the prohibition of arbitrarily sharp predictive measurements.

In *Logik der Forschung*, Popper addressed an objection that an unnamed physicist might have raised—without any doubt Weisskopf. Indeed, Popper (1935a, 173–174) reproduces several passages from Weisskopf's letter almost verbatim, though without explicitly naming him as the source, as the latter had requested. Popper now offers a more articulated reply to Weisskopf's objection. According to Popper, Weisskopf's argument does not *prove* that sharp predictions contradict quantum mechanics. Rather, it tacitly introduces an *additional hypothesis*, which Popper calls the *coupling-hypothesis* (*Kopplungshypothese*). The latter maintains that predictive measurements and physical selections are inevitably linked, so that any predictive measurement is also a physical selection—or, operationally speaking, that any instrument capable of measuring position and momentum with arbitrary precision could also be used to select a corresponding super-pure case, at least in principle:

> But a strict proof of the contention [...] has not been given [...] None of these arguments prove that the precise predictions would contradict the quantum theory. They all introduce an *additional hypothesis*. The statement (which corresponds to Heisenberg's view) that exact single predictions are impossible turns out to be equivalent to the hypothesis that predictive measurements and physical selections are inseparably coupled.

> With this new theoretical system—the conjunction of quantum theory with this auxiliary *coupling-hypothesis*—my conception must indeed clash. What remains to be shown is that the system, consisting of statistically interpreted quantum mechanics (together with the laws of momentum and energy conservation), combined with the coupling hypothesis, is contradictory. (Popper 1935a, 174)

Popper had now realized that the most deep-seated prejudice of quantum physicists lay in the unexamined belief that physical selections and measurements are necessarily coupled. With this, he considered point (III) of his program accomplished. What remained to be addressed was point (IV): to demonstrate that the system—statistically interpreted quantum mechanics, together with the conservation laws of momentum and energy, when combined with the coupling hypothesis—leads to a contradiction. Popper deserves credit for having identified with great precision the central issue on which his entire criticism of quantum mechanics hinges. For this very reason, it also becomes clear where his mistake lies.

4 Popper's ETP-like Thought Experiment

The coupling-hypothesis ultimately takes the form of a *non-existence statement* (Popper 1935a, 33): no device can be built that measures x and p_x with arbitrary sharpness without allowing the corresponding super-pure ensemble. Thus, to disprove the coupling-hypothesis, Popper needed to demonstrate the existence of such a device. In particular, he aimed to show that a device could be constructed which, on the basis of a *non-predictive measurement*, would yield a *predictive* one, but could not be used to perform a *physical selection* producing a super-pure case (§77). As we have mentioned (section 3), there are three possible sequences of predictive measurements that yield a non-predictive measurement of a particle's trajectory: (a) position followed by position, (b) momentum followed by position, and (c) position followed by momentum. The case considered by Popper—momentum selection followed by position measurement—belongs to type (b). The choice is not accidental. Indeed, in Popper's reading, this is "just that case which, according to Heisenberg […], permits 'a calculation about the past of the electron'[12]" (177).

4.1 The Experimental Setup

As we have discussed above (section 1.1), Heisenberg (1930a, 15) uses a sequence of momentum followed by position measurements (b) to argue for the possibility of a 'calculation of the past' of the electron before the second position measurement. Popper seems to read this claim as implying that (b) allows one to calculate the past trajectory also *before* the first momentum measurement, whereas (a) and (c) allow it only *between* the two measurements.[13] As will become clear, this is the key but shaky premise on which Popper's argument hinges. On this basis, Popper proposed a thought experiment involving a particle collision. After the collision, by performing a *non-predictive* measurement on one particle (momentum followed by position), with the help of momentum conservation, one could in principle determine an arbitrarily sharp *predictive* measurement of position and momentum for another particle. As one can see, Popper arrived independently at a thought experiment analogous to the

[12] See section 1.1.

[13] In my view, there is little evidence to support this interpretation. Heisenberg's remark is simply somewhat careless.

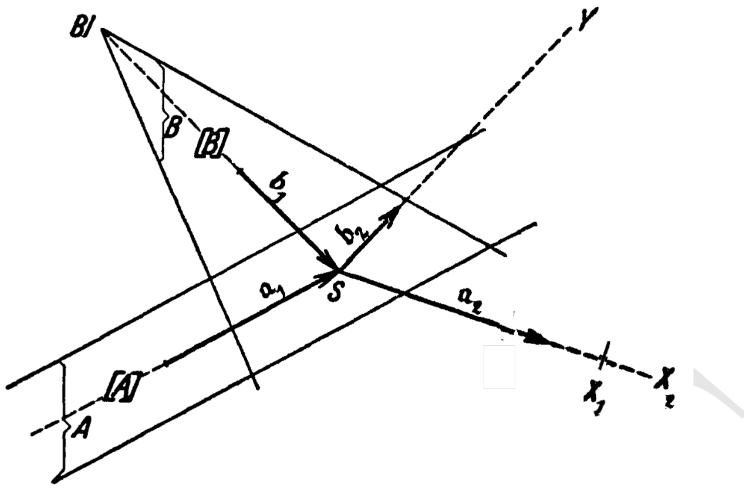

Figure 3: Slightly modified, by adding the labels X_1 and X_2

ETP scenario (see section 1.2 above), except that energy and time are replaced by position and momentum. Of course, Popper's goal was the opposite of ETP's: to show that the asymmetry between past and future does indeed allow one to overcome the indeterminacy relations when interpreted as 'inaccuracy relations.'

The idealized experimental setup consists (fig. 3) of two beams intersecting at S (*Schnittpunkt*). Each beam is treated as if it were a 'pure case.' The electron beam A is monochromatic, carrying a sharp known momentum $\mathfrak{a}_1$, while the light quantum beam B is slit-selected, with a sharp position at Bl, and thus momentum of unsharp direction but known magnitude $\mathfrak{b}_1$. One can then perform a *post hoc* mental selection focusing on the partial rays $[A]$ and $[B]$ that collide at S, in a manner similar to the Compton–Simon and Bothe–Geiger experiments. An electron from beam A emerges at detector X with momentum $\mathfrak{a}_2$, while a light quantum from beam B emerges at detector Y with momentum $\mathfrak{b}_2$. The process is governed by momentum conservation, such that $\mathfrak{a}_1 + \mathfrak{b}_1 = \mathfrak{a}_2 + \mathfrak{b}_2$. Since the light quantum has a broad momentum-direction distribution, in some runs of the experiment the electron 'keeps' its original momentum, while in others it 'inherits' the light quantum's momentum. If one considers only the electrons, disregarding the light quanta with which they interacted, the electron beam is a non-pure case after the collision.

Popper suggests placing an apparatus at X to measure the momentum $\mathfrak{a}_2$ of the electron after the collision and its position at the time t_A. In the Note for *Die Naturwissenschaften*, as well as in the main body of *Logik der Forschung*, he proposes using a moving photographic film (*Filmanstreifen*). When a particle strikes, it leaves a mark at a specific spatial coordinate (position) on the strip, and because the strip is moving, this also indicates a time coordinate (moment of impact). In this setup, the photographic film is also supposed to measure the energy of the electron indirectly through the energy transferred to the atoms of its sensitive material.[14] From the energy one can calculate the electron's momentum $\mathfrak{a}_2$ in the SX direction. Popper contends that "[t]he *accuracy* of this calculation is, for the SX direction (with a suitable arrangement), subject to no fundamental limitation of the kind implied by the uncertainty relations. It is assumed here that the magnitude of momentum and the

[14]See next section for a criticism of this setup.

time of an incoming particle—'non-predictive' measurements—can be measured with arbitrary precision" (Popper 1934b, 807).

Popper does not discuss this premise at this stage; he only argues that, *if* this is the case, from t_A and the momentum $\mathfrak{a}_2$ the time of collision t_S can be reconstructed. Via momentum conservation, one can infer the light quantum's momentum after the collision $\mathfrak{b}_2$ from the electron's momentum, $\mathfrak{b}_2 = (\mathfrak{a}_1 + \mathfrak{b}_1) - \mathfrak{a}_2$. Consequently, from the time of the collision and the light quantum's momentum $\mathfrak{b}_2$, one can make a predictive measurement sharper than allowed by the indeterminacy relations: the light quantum from beam B will reach Y at time t_B with momentum $\mathfrak{b}_2$. This prediction can, in principle, be tested empirically. Reconstructing the *past of an electron* via an arbitrarily sharp non-predictive measurement together with a conservation law appears to allow an arbitrarily predictive measurement of the *future of a light quantum*.

Popper clarifies that the apparatus "does *not* permit the assignment of a definite position and a definite momentum arbitrarily to *particular* particles" (807; my emphasis). One can infer something about B once one measures A, but one cannot control or reproduce that same condition on B in the next trial without measuring A again. Repeating the experiment many times would yield a statistical distribution of AB particle pairs, all with *different* but correlated positions and momenta, not an ensemble of B particles all having the *same* position and momentum: "The experiment therefore provides no indication for the production of a collection of particles that is more homogeneous than a *pure case*" (807). Popper can then conclude that the thought experiment shows that one can determine a predictive measurement of the position and momentum of a single B-particle without allowing for the selection of an ensemble of B-particles having both sharp momentum and sharp position.[15] While fully complying with Heisenberg's inequalities understood as statistical dispersion relations (*Streuungsrelationen*), Popper believed that one could nevertheless produce a case that violates Heisenberg's inequalities understood as limits of measurement accuracy (*Ungenauigkeitsrelationen*).

4.2 The Issue of the First Momentum Measurement

According to Popper, the possibility of making an arbitrarily sharp predictive measurement should suggest that each particle in the ensemble possesses a *pre-assigned, yet unknown* momentum and position, thus opening the possibility that deterministic 'precision laws' might underlie the 'quantum frequency' laws (Popper 1935a, §78). As already noted, Popper was aware that the device underlying the thought experiment works only if the momentum of A remains unaffected by the selection at X, allowing the entire path of A to be reconstructed up to the collision at S. Popper was probably initially confident that this was the case, relying on Heisenberg's (1930a, 15) somewhat elliptical remark (see section 1 above). An unpublished *Ergänzung* to the Note, written sometime between summer and fall 1934, shows that Popper soon realized a legitimate objection could be raised against this assumption: "One could, namely, take the following view: Non-predictive measurements allow us to make exact calculations only for certain intervals, namely for the interval *between* two measurements. For the time

[15] According to Redhead (1995, 166), Popper is "confusing" conditional and predictive selection. I would rather say that Popper, more or less consciously, is *exploiting* this very difference to infer $\mathfrak{b}_2$, given $\mathfrak{a}_2$, without physically selecting a new ensemble of B-particles with momentum $\mathfrak{b}_2 = (\mathfrak{a}_1 + \mathfrak{b}_1) - \mathfrak{a}_2$. Popper should perhaps have use the label $[\mathfrak{b}_2]$ to emphasize that this is only an epistemic selection, and not a physical one.

before the first or *after* the second measurement, however, the uncertainty relation holds, whose validity is therefore (contrary to Heisenberg's remark) symmetrical for past and future" (Popper 1934a, 397f.).

It may be recalled (section 1.2) that this was precisely the upshot of the ETP paper, which Popper, without any doubt, did not know of. Yet Popper sought to reach the opposite conclusion. He therefore had to offer at least a clarification concerning the measurement of momentum: (a) one can make the interaction region S arbitrarily narrow, so that the angle $\Delta\mathfrak{S} \to 0$, and still deduce $\mathfrak{b}_2$ exactly from momentum conservation—so that $\Delta\mathfrak{b}_2 \to 0$, once the magnitude of the momentum $\mathfrak{a}_2$ at S is reconstructed; this analysis was expanded in Appendix VII; (b) the path of the A-particles can be reconstructed even *before* the first momentum measurement, up to S. This discussion was later incorporated into Appendix VI. I will consider here only point (2). Indeed, with some good will, one might concede (1), since $\Delta\mathfrak{S}$ and $\Delta\mathfrak{b}_2$ are not conjugate variables. However, (2) proves to be the true Achilles' heel of Popper's setup.

As mentioned, Popper first proposed placing the film strip at X to measure not only position and time but also energy, and hence momentum, of the incoming particle. He soon realized that such an apparatus would violate the indeterminacy relations (400).[16] In the *Ergänzung*, Popper therefore outlined a revised setup, later included in Appendix VI of *Logik der Forschung*, described as follows (fig. 3): (1) At X_1, an electrostatic analyzer with parallel plates producing a uniform electric field E perpendicular to the beam direction[17] filters a sub-ensemble of electrons with the same *predetermined* momentum $\mathfrak{a}_2$ *without changing it* (Popper 1935a, 220f.). Since, for Popper, filtering momentum leaves both momentum and position unchanged, though the latter remains unknown; (2) the unknown position may later be revealed by a second measurement at X_2, where a Geiger counter (or clock-driven moving film strip) records position x and arrival time t. As the first measurement leaves the electron's state unchanged, one might think it possible to reconstruct the past of the electron not only *between* the two measurements—momentum followed by position—but also *before* the first measurement at X_1[18]

Popper realized that two possible objections can be raised against the possibility of a momentum measurement as described:

– *The filter leaves momentum unchanged but alters position.* Popper argues that, unlike a position selection (a narrow slit) which spreads the trajectory, a momentum selection (an electron spectrometer) allows particles of a certain momentum to pass along the same path while blocking others. Assuming that momentum selection unpredictably disturbs position would imply a discontinuous (superluminal) jump to another point, contradicting quantum mechanics, which permits discontinuities only for bound, not free, particles. Popper calls any theory postulating such position disturbances an 'imprecision theory' (*Ungenauigkeitstheorie*); such a theory would be logically possible but empirically indistinguishable from standard quantum mechanics . He concludes that, according to the statistical interpretation of the indeterminacy relations, momentum selection leaves positions *unknown* but unchanged (Popper 1934b, 401).

[16] See also Einstein's objection, section 6.1.

[17] See below fig. 4.

[18] Popper did not distinguish explicitly between X_1 and X_2, but I follow Redhead's (1995, 164) suggestion.

- *If the filter left momentum unchanged, one could produce a 'super-pure case'.* By reversing the order of selection—first localizing the particle through a narrow slit or short 'momentary shutter' (*Momentverschluß*), then selecting momentum with a filter[19]—one might expect, after many runs, to obtain an ensemble with sharp x and p_x. Popper replies that, unlike momentum selection, position selection spreads momentum: the sharper the position, the greater the diffraction and the fewer particles pass the filter. Only rare detections occur, revealing merely the *statistical distribution* of position and momentum in an 'anonymous' ensemble, not sharp individual trajectories (Popper 1935a, 221f.).

Popper's reasoning rests on an alleged asymmetry between position and momentum selection that, as will be shown below,[20] is only apparent. What is relevant here is that Popper clearly sees that the question of the momentum measurement decides the destiny of his thought experiment: "[a] Assuming that the x-coordinates of the particles are not disturbed by the momentum measurement, then the exact determination of position and momentum also extends to the time *before* the momentum selection [at X_1]. [b] Assuming that the momentum selection disturbs the x-coordinates, then we can calculate the trajectory exactly only for the time *between* the two measurements [at X_1 and X_2]" (220). As we shall see, Popper was compelled to respond to several interlocutors—some of the greatest physicists of all time—who challenged the legitimacy of assumption [a] and argued that [b] was the case. He eventually gave it up, but not without a fight.

5 The Leipzig Response

Logik der Forschung was completed and sent to the publisher in the fall, and printed at the end of December 1934 (despite the colophon bearing the date 1935). In the intervening months, Berliner, the editor of the *Naturwissenschaften*, had sent Heisenberg the galley proofs of the short Note that Popper had submitted in August. Heisenberg wrote to Popper on November 23, 1934, explaining that, at Berliner's request, he discussed the Note with his assistant von Weizsäcker. They subsequently sent Berliner von Weizsäcker's brief rejoinder, asking him to forward it to Popper: "You will then see what kind of criticism it is. Essentially, it concerns the concept of the 'non-prognostic measurement,' which in our view appears to be misunderstood in your work" (Heisenberg to Popper, Nov. 23, 1934; KPA, 305-32). Let us not forget that Popper was virtually unknown at the time, and it is quite remarkable that Heisenberg (newly appointed to a chair at Leipzig) and his group chose to engage in a detailed refutation of various variants of his thought experiment rather than dismiss him as a crank.

5.1 Von Weizsäcker's Objection and Popper's Reply

In his brief response, von Weizsäcker, like Heisenberg, suggested using a scattering method (the Doppler effect) to measure the electron's momentum at X_1, followed by a Geiger counter to record its position and time at X_2. With this setup, von Weizsäcker reached precisely the conclusion Popper sought to avoid: the electron 'trajectory' A can be reconstructed only for the interval *between* X_1 (momentum determination) and X_2 (position and time recording), but not *before* X_1. He conceded that one may know the

[19] I mention this detail since Einstein referred to this apparatus; see below section 6.3.
[20] See section 6.3.

momentum before and after the measurement exactly; however, measuring momentum via the Doppler effect requires determining the frequency ($\Delta\nu$) of the scattered radiation over a finite observation time (Δt); improving one worsens the other. Thus, one cannot know when the impact between the apparatus and the particle occurred, or for how long it had the 'momentum before the impact' or 'after the impact.' This uncertainty blurs the knowledge of position X before the impact; hence, the collision time at S remains indeterminate, and the trajectory [B] cannot be predicted. von Weizsäcker concluded:

> This trajectory, however, is *in principle* uncontrollable. It is valid only for the time interval between the end of the momentum measurement and the beginning of the position measurement, during which the particle has no interaction whatsoever with its surroundings, and it cannot be extended into the period before the momentum measurement, since the latter itself destroys the knowledge of the position in accordance with the uncertainty relation. [...] Thus, even if the position [at X_2] after the momentum measurement is exactly known, one still cannot infer the position *before* the momentum measurement [at X_1] with an accuracy greater than the error $h/4\pi\Delta p$. That is, our knowledge of the trajectory of particle A before the momentum measurement at X, and thus also of the trajectory of particle B after the collision at S, is in accordance with the uncertainty relation. (von Weizsäcker in Popper 1934b, 808)

Von Weizsäcker concedes that the proof applies only to the *specific* experimental setup he described. However, he had no reason to doubt that *any* other combination of measuring devices at X would yield the same result. The problem is that Popper does not distinguish 'non-predictive measurements' and verifiable measurements concerning the past. The former escape the indeterminacy relations since they are not physical measurements at all; the latter, by contrast, do satisfy the indeterminacy relations, which are symmetric for the past and future.

On November 26, Popper replied by sending the galley proofs of Appendices VI and VII of the *Logik* to Heisenberg (Popper to Heisenberg, Nov. 26, 1934; KPA, 305-52). Popper, somewhat prone to paranoia, felt that von Weizsäcker had not taken him seriously and reacted rather aggressively, emphasizing two points: 1. in the Note he did not *prove* the legitimacy of calculating the past of the electron, but merely *assumed* it, citing Heisenberg's own remark on p. 15 of his book as justification; 2. von Weizsäcker had refuted only one method of *momentum measurement*—that by scattering. Popper maintained instead that *momentum selection* of an aggregate of electrons (*Teilchenmenge*) by means of filters could determine momentum p_x *without* disturbing the x-coordinate, thus allowing for a non-predictive retrodiction before X_1. Popper wrote to Heisenberg:

> You will now ask why I did not elaborate further in the note on what I am writing here. The reason is that I assumed that the so-called 'non-prognostic measurement' (momentum selection followed by a position measurement) and the possibility of calculating the 'past' prior to the momentum measurement were already known. This assumption was based on a remark of yours (*Phys. Prinzipien*, p. 15) concerning case (b) and the 'calculation of the past of the electron.' In my note, however, I had to be brief and restrict myself to what was essential, that is, above all, to what was new. Only from Mr. Weizsäcker's note did I realize that at least he was not familiar with this case. (Popper to Heisenberg, Nov. 26, 1934; 305-52)

Since he mentions p. 15 of the German edition of Heisenberg's 1930 book, Popper probably refers to Heisenberg's claim that the 'calculation of the past of the electron' can be reconstructed *before the position measurement*—without specifying whether it can also be reconstructed before the momentum selection. At any rate, the time was too short for Popper to prepare a rejoinder. On November 30, 1934, the Note appeared in volume 22 of *Die Naturwissenschaften*, together with von Weizsäcker's critical comment.

On October 6, Popper sent Heisenberg a second communication (Popper 1934c) in response to von Weizsäcker, which he hoped to publish. Popper also attached a copy of the *Logik der Forschung* that had been just printed but still not distributed. Popper's counterargument is revealing of his way of thinking at that time.[21] He observes that von Weizsäcker had used momentum *measurement* at X_1 via the Doppler effect, that is, by using light quantum collision. The latter, however, has the shortcoming that it "disturbs the particle's momentum (recoil) and that, since the exact instant of the disturbance is not known" (406). Popper concedes that no reconstruction of the momentum before the scattering event is possible in this way. However, following the same argument he had developed in Appendix VI, Popper suggests that one can resort to "a momentum *selection* (electron spectrometer, light filter)", which "unlike the momentum measurement of individual particles, for example by means of the Doppler effect—has the property of not affecting the momenta or momentum components of the selected particles" (406). Thus, if $[A]$ is an electron beam, "only electrons with a certain *predetermined* amount of momentum enter the Geiger counter", and if $[A]$ is a light beam, the filter lets through only those light quanta that *already* possessed the required momentum before entering it (406).

One might object, Popper points out, that we do not really know in detail what a light filter does, and that its operation might itself disturb the light quantum's state. However, Popper argues, a filter produces not only light but also sharp images (*Bilder*)—thus, it does not, in any case, disturb the directions of the quanta. If the filter did disturb the light quanta, one would have to assume some mechanism that restores their direction after passage: either (1) the quanta pass through the filter but are unpredictably displaced along their paths, with their previous direction suddenly reestablished (*Zurückversetzung auf der Bahn*), or (2) the filtering process involves absorption and directed re-emission (*gerichtete Emission*) in the same direction (407). Yet both options appear problematic: the first contradicts the continuity of paths still required by quantum theory; the second assumes a kind of 'directed fluorescence' not known to occur[22] (407). Popper therefore concluded that the most plausible hypothesis is that, also in the case of the filter, the quanta with the 'right' momentum are allowed to pass through unchanged.

5.2 Heisenberg's and von Weizsäcker's Counter-Replies

On December 6, the same day as Popper's letter, von Weizsäcker drafted a detailed response (von Weizsäcker to Popper, Dec. 6, 1934; KPA, 360-21), which Heisenberg forwarded to Popper on December 10. Von Weizsäcker considered the setup suggested by Popper in Appendix VI, an electrostatic analyzer (see fig. 4), and showed that the very geometry of the deflection experiment already contains the limits expressed

[21] As discussed below (see section 6.3), his reconstruction of the event twenty years later seems to me quite different.

[22] The emission of electromagnetic radiation by atoms or molecules after absorbing radiation, occurring when they return to a lower energy state. The emission is generally spontaneous and isotropic, not directed.

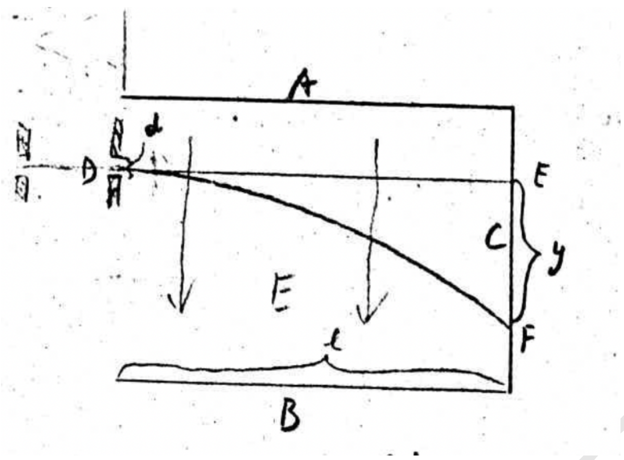

Figure 4: from von Weizsäcker to Popper, Dec. 6, 1934; KPA, 360-21

by the uncertainty principle. To determine the momentum of the particle from its impact point on the screen EF, the beam must pass through diaphragms that fix its path length precisely. But narrowing the slit to improve accuracy inevitably produces diffraction, so that the transverse spread $\Delta y = EF$ of the beam at the detector cannot be made smaller than a definite bound. In other words, the position on the screen, which is supposed to measure momentum, is blurred by the same diffraction effects introduced by the diaphragms (von Weizsäcker to Popper, Dec. 6, 1934; KPA, 360-21).

Since the vertical displacement y depends on the flight time $t = L/v$, this transverse uncertainty also propagates into the longitudinal motion. An uncertainty in y translates into an uncertainty in the velocity v, and thus into both the longitudinal momentum p_x and the coordinate $x = vt$. When these effects are consistently taken into account, one finds that the product $\Delta p_x \Delta x$ is likewise bounded below by Planck's constant. Figure 4 illustrates the intuition: the more tightly the beam is collimated at D, the more it spreads out when it reaches the screen at $E{-}F$, so that no arrangement of diaphragms or fields can evade the uncertainty relation. The situation was not dissimilar to the one described by Heisenberg (1930a, 21f.) in his book, where he used a homogeneous magnetic field instead of an electric field to measure the momentum (von Weizsäcker to Popper, Dec. 6, 1934; KPA, 360-21).

In his reply of December 10, Heisenberg enclosed von Weizsäcker's criticism of the electrostatic analyzer and also addressed Popper's suggestion of using a light filter instead of an electron spectrometer (Heisenberg to Popper, Dec. 10, 1934; 305-32). Heisenberg suggested that one might use reflective resonance filters, such as sodium or mercury vapor, following Wood's classical residual-ray experiments (*Reststrahlenverfahren*). When light strikes such a medium, radiation at the resonance frequencies is strongly absorbed and re-emitted (or reflected), while other frequencies pass through unaffected. Thus, a broad-spectrum beam incident on sodium vapor is filtered and yields only the reflected sodium D-lines—a doublet of closely spaced spectral lines in the yellow region of the visible spectrum. Heisenberg invoked Wood's experiment to show that the filtering process always involves finite line widths, not infinitely sharp spectra of a single frequency ν. When an atom absorbs a light quantum at its resonance frequency, it is excited to a higher state and, after a short lifetime t, decays by re-emitting a light quantum at the same frequency. Because the excited state exists only for a finite time, its energy cannot be perfectly sharp, in compliance with the time–energy uncertainty relation.

Even when light is 'filtered' into sharp momentum (frequency) states, the finite lifetime of the excited atoms ensures a residual spread $\Delta\nu$ (and hence Δp). This constitutes a momentum selection, since light quantum momentum is $p = h/\lambda$, but its precision is limited by the finite resonance linewidth, determined by the lifetime of the excited state, $\Delta t = 1/\Delta\nu$. Thus, light quanta cannot be prepared with arbitrarily sharp momentum. Even if the light quantum's later position is known precisely, its position before reflection cannot be reconstructed, because the time spent in the reflecting atom is indeterminate. Hence there is an uncertainty Δx such that $\Delta x \cdot \Delta p \approx \frac{h}{4\pi}$. The filter selects not an exact frequency, but a narrow band, whose spread is intrinsic to quantum mechanics through the lifetime–linewidth relation.

In his reply of December 16, Popper sent Heisenberg a revised version of the second communication summarizing the debate's main points (Popper 1934d). He conceded that von Weizsäcker's argument against the electrostatic selector was correct but defended his idea of using a light quantum filter against Heisenberg's objections. He noted that Heisenberg's remarks on Wood's experiment concerned only absorbed and re-emitted light quanta at the resonance line, whereas "the light of other frequencies *passes through*, and it is only to this transmitted light that my considerations apply" (Popper to Heisenberg, Dec. 16, 1934; KPA, 305-32). Only by assuming that transmitted light also underwent absorption and re-emission would the thought experiment lose validity. He further dismissed the claim that the transmitted light was too inhomogeneous: "It is in fact possible, by means of a set of filters, or even with a green filter alone, to isolate a finite range of transmitted frequencies $\Delta\nu$, i.e. a finite Δp interval" (Popper to Heisenberg, Dec. 16, 1934; 305-32). Once the light quantum had passed through the filter, its position could later be measured with arbitrary accuracy $\Delta x \to 0$, so that "the *product* of the two uncertainties could approach zero, even for imperfect filters" (Popper to Heisenberg, Dec. 16, 1934; 305-32).

Popper did not relent. He proposed a new idea for an apparatus, which Heisenberg left to his assistant Hans Euler to examine (293-29). On February 4, 1935, Euler sent Popper a detailed refutation. On February 12, Popper drafted a somewhat impatient response that was never sent, and another on April 14, 1935. I will not go into this exchange, since Popper's original proposal has been lost, and he seems to have later abandoned the idea. At this point, however, the Leipzig group was starting to become mildly annoyed by Popper's insistence. In a mid-March letter to Grete Hermann, von Weizsäcker, at her request for clarification, reconstructs the entire series of experimental conjectures and refutations, concluding somewhat humorously that it has become "a parlor game for us in Leipzig, to refute Popper's setups" (von Weizsäcker to Hermann, Mar. 13, 1935, in Herrmann 2019, Brief 15).[23] It was Heisenberg who ultimately decided that it was time to bring the game to an end a few days later.

In his final letter to Popper, on March 19, Heisenberg explained that all the considerations of his previous letter apply to transmitted light as well (Heisenberg to Popper, Mar. 19, 1935; KPA, 305-32). He considered a substance opaque over nearly

[23]The correspondence between von Weizsäcker and Hermann, and Hermann's (1935a, 1935b, 55; fn. 10) on Popper's interpretation of quantum mechanics would require a separate investigation (see Frappier 2016). However, it is worth noting that Hermann goes to the heart of the matter, challenging Heisenberg's concession that the reality of past trajectories is merely a *matter of taste* (Hermann 1935a, 55f.). According to Hermann, such reality is, so to speak, a *matter of context.* In Popper's parlance, each sequence of a selection and a measurement defines a complete and self-consistent 'trajectory.' Yet this trajectory does not exist in itself. Different measurement sequences (momentum-position vs. position-momentum) yield different reconstructed paths.

the entire spectrum due to strong absorption bands, yet transparent within a narrow region between two band heads (*Bandköpfe*). Light quanta with frequencies inside this window pass quickly, while those near the band heads are absorbed and re-emitted after a delay comparable to the lifetime of the excited states. As the interval between the band heads narrows, more quanta experience delayed re-emission, lengthening and blurring the passage time. In the limit of an infinitesimally narrow band, the mean delay grows without bound, consistent with the 'principle of harmonic resolving power' or Fourier relation $\Delta\nu\Delta t$ or $\Delta x\Delta k$: a sharp frequency (or momentum) definition entails an indeterminacy in time (or position) (Heisenberg to Popper, Mar. 19, 1935; KPA, 305-32). At this stage, Popper might have sought support from Weisskopf. Yet the latter confessed that he had met with Heisenberg and had come to side with him and von Weizsäcker (Weisskopf to Popper, Jan. 21, 1934). The game had run its course—but Popper still had one card up his sleeve.

6 The Popper–Einstein Correspondence

On March 25, 1935, Einstein, Podolsky, and Rosen submitted the eponymous EPR paper to the *Physical Review*; it was published on May. In the intervening weeks, Popper managed to get a copy of *Logik der Forschung* to Einstein. Popper's friend, the pianist Rudolf Serkin, performed with the Busch Quartet and had recently married the daughter of Frieda and Adolf Busch, who knew Einstein. Frieda sent him a copy of Popper's book on April 28, 1935 (Busch to Einstein, Apr. 28, 1935; EA, 34–338). As one can see, Einstein received Popper's work after the EPR paper had already been submitted. Jammer's (1974) hypothesis that Popper's paper might have served as an intermediary between the ETP and the EPR paper is intriguing. As mentioned above, Popper's experiment is essentially a version of the ETP experiment in which energy and time are replaced by momentum and position, as in the EPR argument. It therefore seems natural to suggest that he may have served as a bridge between ETP and EPR. However, this hypothesis is ultimately unlikely for chronological reasons[24]. However, it is plausible that Einstein immediately recognized that Popper's experimental setup was essentially a variant of his own ETP scheme and thus immediately dismissed it.

6.1 Einstein's Comment on the Logik der Forschung

Einstein replied to Popper on June 15, endorsing his general philosophy of science but criticizing the proposed thought experiment as flawed: "It is not correct that position and momentum of the particle at Y (p. 179) can be predicted. To this end you would have to measure with the help of your 'Film' time and momentum of the particle [at X] (that is, time and energy) which is impossible. You will see this easily if you think a little more about it" (Einstein to Popper, Jun. 15, 1935; EA, 19-124). The objection is probably the consequence of Popper's unfortunate choice of measuring apparatus on p. 179 of *Logik der Forschung*, a 'film' at X measuring momentum and energy, and position and time of arrival.

However, as discussed above (section 4.2), in Appendix VI of *Logik der Forschung* Popper had already proposed a different two-stage arrangement, a momentum filter at X_1 (electron spectrometer or light filter) and a position–time measuring apparatus (a film) at X_2. Still, in his lengthy reply on July 18, he had by that time already accepted

[24] I could not find any indication that Popper had already sent an offprint of the 1934 Note to Einstein in December, as claimed by Jammer (1974, 178).

that even this setup could not work:

> First, the question of the thought experiment (in the book p. 179). Unfortunately, regarding the measurement of the particle arriving at X—as you rightly noted—there are quite a few inaccuracies in the text of the book (for example, the use of the electric field and practically the whole Appendix VI). I admit my mistakes, but after having discussed them thoroughly with all the physicists available to me in Vienna, among others with Professor Thirring and his assistant Guth (with Professor Heisenberg and his assistant Weizsäcker I discussed the matter in writing), it still seems to me that the issue is not yet settled. From the discussions so far (and in this view I am not alone, but, among others, also Guth), the impossibility of my thought experiment has not yet been demonstrated. That is, it seems possible to avoid the aforementioned errors and to maintain the thought experiment in agreement with quantum mechanics. I do not wish to be 'proven right'; I would only be very glad if this matter could be clarified — even if the final decision goes against me. I hope I am not abusing your time and patience too much by presenting the situation as it currently appears to me. (Popper to Einstein, Jul. 18, 1935; EA, 19-126)

Popper now tried to investigate the arrangement in which a light filter is placed at X_1. He acknowledged that this setup—the optical version of the one described in the book—would not yield the desired result by using known filters (as Heisenberg showed him). However, Popper suspected that a combination of filters might exist that selected photons of a sharply defined momentum (frequency) without having them interact for a long time.

Popper went into some detail (see next Section) but, not wanting to make an already long eight-page letter even longer, he promised to send Einstein a supplement he had written two months earlier at Felix Ehrenhaft's suggestion (Popper to Einstein, Jul. 18, 1935; 19-126). Throughout the spring of 1935, Popper had worked on a published response to the Leipzig objections, expanding it into a longer manuscript again titled *Zur Kritik der Ungenauigkeitsrelationen.* Apparently he sent it to Carnap, Ehrenhaft, Heisenberg, and Schrödinger, hoping Ehrenhaft would forward it to Einstein (Popper to Carnap, Jun. 10, 1935; Carnap Collection). On July 18, he decided to send it himself. Popper admitted he was beset by doubts but, after further discussion with Weisskopf, realized the latter could not fully refute his argument. Having already announced the manuscript, he felt compelled to send it despite lingering uncertainties, having given up hope of a prompt clarification. Moreover, Weisskopf had informed him of Einstein's recent critique of quantum mechanics (the EPR paper), which further encouraged him to forward the manuscript (Popper to Einstein, Aug. 29, 1935; 19-127).

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6.2 Popper's 1935 Manuscript

The first part of Popper's 1935 manuscript restates his stance toward the uncertainty relations with additional reference to some literature that he had not cited in the book, such as von (Neumann 1932)'s textbook. Thus, for example, Popper began to use the expression 'dispersion-free ensemble' to indicate what he had previously called a 'super-pure case.' There are surely differences in the presentation, but the main message remains the same: the prohibition of a dispersion-free ensemble does not imply the prohibition of arbitrarily sharp measurements. Popper presupposes that the reader already knows his November 1934 Note and once again clearly identified the issue with his original setup:

> One would have to apply a 'non-prognostic measurement' according to case (b)—a position measurement [at X_2] preceded by a momentum measurement at [at X_1]—arranged in such a way that one can calculate the trajectory not only in the interval *between* the two measurements, but also for the time *before* the momentum measurement. The position coordinate is measured precisely by the peak counter (the moment of incidence [at X_2]). *Question: Is there a method of momentum measurement [at X_1] that could precede this position measurement without destroying (blurring) the position coordinate of the particle?*. Weizsäcker disputes this[25]; the case he discusses (momentum measurement of the particle using the Doppler effect) is indeed unsuitable. In my book I propose the following arrangement: 1. Momentum selection by means of a grating filter, 2. Position measurement by means of the counter. There I defend the thesis that such a momentum measurement, carried out with a filter, should not disturb the position coordinates; this thesis should here be examined more closely. (Popper 1935b, 427)

In other terms, Popper believes he could devise a filter that would let light quanta with unchanged momentum—and therefore position—pass through. His reasoning seems to have been roughly as follows.

A *real* filter must satisfy the Fourier constraint: the narrower the transmitted range of frequencies $\Delta\nu$ or wavelengths $\Delta\lambda$—that is, the sharper the momentum selection—the longer the temporal response Δt and the greater the spatial extension Δx of the transmitted wave packet. However, Popper argues that one could in principle conceive of two different kinds of *ideal* filters, corresponding to the limiting cases of the Fourier relation (428):

– *Type I apparatus:* Filters capable of spectrally resolving arbitrarily short light flashes ($\Delta t \to 0 \Rightarrow \Delta\nu \to \infty$). Such filters must necessarily operate *with spatial blurring*, since decomposing a very short pulse without spreading its position Δx would contradict the Fourier relation:

– *Type II apparatus:* Filters that cannot resolve very short wave packets but act only on infinitely long wave trains ($\Delta\nu \to 0 \Rightarrow \Delta t \to \infty$). In principle, such filters could operate *without spatial blurring*, transmitting only radiation of a single wavelength (or momentum).

To construct a filter of type II, Popper suggests using colored glass filters followed by gas layers (429). The glass filters, say a green filter, are passive filters that work by transmission: they allow a relatively broad band of wavelengths centered around some mean wavelength to pass while blocking both the high-frequency and low-frequency edges. Gas layers act actively, through absorption and delayed re-emission, and provide a further selection (since each gas has sharp absorption lines), ideally centered well away from any resonance edges that cause re-emission or time delay.

Popper probably believed that the double selection would exclude the spectral region around the resonance band edges, which was the basis of Heisenberg's objection (see above section 5.2). This would ensure that the quanta that go through the multi-stage filter are transmitted directly without delay and not absorbed and re-emitted. For this reason, one might assume that their momentum (frequency) is left unchanged. This implies that no mechanism exists that could alter their positions. The filter, Popper concludes, selects a homogeneous ensemble of light quanta with the same momentum, whose maximally spread individual positions are completely *unknown* yet remain unchanged. They can then be reconstructed by a subsequent position measurement: "*A momentum-selection apparatus without spatial smearing would therefore not contradict*

[25] See above section 5.1.

quantum mechanics if it is of type (II)" (Popper 1935b, 429).

6.3 *Einstein's Comment on Popper's Manuscript*

Einstein replied only a few weeks later. In a letter of September 11, 1935, acknowledging the broad direction of Popper's argument, he rejected the details of the experimental setup Popper proposed:

> Dear Mr. Popper,
> I have looked at your paper, and I largely [weitgehend] agree. Only I do not believe in the possibility of producing a 'super-pure case' which would allow us to predict position and momentum (colour) of a pho- ton with 'inadmissible' precision. The means proposed by you (a screen with a fast shutter in conjunction with a selective set of glass filters) I hold to be ineffective in principle, for the reason that I firmly believe that a filter of this kind would act in such a way as to 'smear' the position, just like a spectroscopic grid. (Einstein to Popper, Sep. 11, 1935; EA, 19-130)

This passage is somewhat puzzling. Einstein seems to think that Popper was proposing a method to prepare a super-pure case using a slit with shutter followed by filters. However, just as he did in *Logik der Forschung*, in the manuscript Popper (1935b, 401–403) uses the example of the screen with fast shutter followed by a filter to *exclude* that his setup would allow the construction of a super-pure case by measuring position and momentum (see above section 4.2).

Popper's proposal was a light filter followed by a Geiger counter. Einstein may have gone through the manuscript rather quickly; yet the last part of his objection did hit the mark. The glass filter decomposes a short light pulse into quasi-monochromatic wave trains W_n. Absorbing filters cut out all colors W_n except one, W_1. The filter thereby *prepares* light quanta in a state with sharp momentum W_1, but at the cost of making position unsharp (Einstein to Popper, Sep. 11, 1935; EA, 19-130). As Popper realized early on, if the filter smears the position, his thought experiment collapses. The A path reconstruction stops at the momentum measurement; one cannot determine the time of collision at S, and thus one cannot determine arbitrarily sharp predictions of the B particles via a conservation law. This is exactly the result that ETP had obtained a few years earlier.

Referring to his joint paper with Podolsky and Rosen, Einstein, Podolsky, and Rosen (1935) explained that he did not have any copies at hand but could briefly summarize its content. The situation is indeed not dissimilar to that of Popper's thought experiment. After, say, a collision, two particles are described by the joint, nonfactorizable wave function[26] $\psi(x_1, x_2)$, with sharp relative position ($y_B - y_A = 0$) and sharp total momentum ($p_A + p_B = 0$)—but spread individual particle momenta. If one measures, say, the momentum p_A of particle A, its state collapses into the corresponding momentum eigenstate ψ_A; quantum mechanics then assigns to subsystem B a conditional momentum eigenstate ψ_B. For this state, however, the position is completely undetermined, which blocks any attempt to beat the uncertainty relations, as Popper had hoped. Einstein's point, rather, was that the specific form of ψ_B would have been different had one performed a position measurement on system A. This means that two different ψ-functions correspond to one and the same physical state of B, which itself has not changed physically: "It is therefore not possible to regard the

[26] As is well known, the term 'entanglement' was first introduced by **Schroedinger1935Schrodinger1935a** in the ensuing months.

ψ-function as the complete description of the state of the system." (Einstein to Popper, Sep. 11, 1935; EA, 19-130).

In Einstein's contemporary correspondence, the 'private' formulation of Einstein's argument ends here (Einstein to Schrödinger, Jun. 19, 1935; 22-047), without reference to the indeterminacy relations. Rather, it involves the measurement of a single variable together with a conservation law or geometrical correlation (Howard 1985; Fine 1986, 38). However, since Popper had asked him to summarize the published paper, Einstein adds that, since both position and momentum of B can be predicted with certainty, according to the infamous EPR 'criterion of reality,' *both* quantities must correspond to something that exists in physical reality. This situation, because of the indeterminacy relations, finds no counterpart in the formalism of quantum mechanics (Einstein to Popper, Sep. 11, 1935; EA, 19-130). Although the argument was again meant to convey the incompleteness of the theory, it also seems to conflict with the spirit of the indeterminacy relations. Indeed, Popper (1959, 244; fn. *4) would later describe the EPR argument, as presented in this letter, as a weaker (no prediction for both position and momentum simultaneously) but correct (both position and momentum are simultaneously part of reality) version of his own failed ETP-like thought experiment.

Conclusion

After the objection from Einstein, Popper's scientific hero, he gave up. Thanks to Weisskopf's mediation (Weisskopf to Popper, Oct. 17, 1936; KPA, 360-21), he had the opportunity to meet Bohr in Copenhagen in June 1936 (see Popper 1936), where he was invited, at the last minute, to the Second International Congress for the Unity of Science. However, at that time he felt defeated (see also Strauss 1936). Only at the turn of the 1950s did Popper return to the philosophy of quantum theory (Del Santo 2019)—however, this time in the name of realism rather than determinism (Howard 2012). The reflections on the topic written between 1950 and 1956 were meant to appear in a *Postscript* to the English edition of *Logik der Forschung*, but their publication was ultimately delayed.[27] Still, in the footnotes and appendices added to the new edition (marked by starred numbers) Popper made no secret that his old thought experiment was "based upon a mistake", as had been pointed out to him by von Weizsäcker, Heisenberg, and Einstein: the past of an electron cannot be reconstructed without blurring its position (Popper 1959, 217).

Popper concedes that von Weizsäcker was correct: "for the electric field perpendicular to the direction of a beam of electrons" used in Appendix VI does not act as he expected: "for the width of the beam must be considerable if the electrons are to move parallel to the x-axis, and as a consequence, *their position before their entry into the field cannot be calculated* with precision after they have been deflected by the field" (299; fn. *1). Moreover, Popper argues, "as Einstein shows" in his 1935 letter,[28] the same can be said "for a filter acting upon a light quantum" (299; fn. *1). Popper acknowledges that the conclusion is inescapable: "non-predictive measurements determine the path of a particle only *between* two measurements", such as a measurement of momentum followed by one of position (or vice versa): "it is not possible, according to quantum theory, to project the path further back, *i.e.*, to the region of time before the first of these measurements" (242; fn. *3). As we have seen, Popper realized early on that,

[27] See Popper 1982b.

[28] Reproduced and translated in Appendix *xii of Popper (1959).

if this is the case, his thought experiment—and, more in general, ETP-like thought experiments—"collapses" (Popper 1959, 242; fn. *3).

However, in a long footnote attached to Appendix VI, as well as in a new Appendix *XI, Popper seems to blame this mistake on the fact that he was misled by Heisenberg. According to Popper, Heisenberg "*fails to establish that measurements of position and of momentum are symmetrical*" (451). In Heisenberg's famous γ-ray microscope thought experiment, position is determined using high-frequency light, which strongly *disturbs* the electron's momentum. Conversely, momentum can be determined, in principle, using low-frequency (long-wavelength) radiation via the Doppler effect so as to *avoid* disturbance; but in this case, the position remains indeterminate (451). Popper claims that he had extended this asymmetry to physical selections. Position selection via a slit disturbs the momentum (producing the diffraction pattern); momentum selection via a velocity filter only makes the position *unknown*: "But I now believe that I was wrong in assuming that what holds for Heisenberg's imaginary 'observations' or 'measurements' would also hold for my 'selections'" (299; fn. *1). Popper even argues that his thought experiment "can be used to point out an inconsistency in Heisenberg's discussion of the observation of an electron" (299; fn. *1).

Popper's recollection of the events strikes me as rather self-serving and appears to be contradicted by the unpublished textual evidence from that period. As we have seen, in his reply to Heisenberg, as well as in his unpublished second communication, Popper (1934c) explicitly argues that the momentum determination should occur not through momentum *measurement* via the Doppler effect, as von Weizsäcker (following Heisenberg) had proposed, because the latter does disturb the position; on the contrary, a momentum *selection* via an electron spectrometer or a light filter does not. It leaves the given momentum unchanged, and therefore also the position—albeit the latter remains unknown. Popper was not misled by Heisenberg's notion of measurement,[29] but by the notion of 'physical selection' he adopted.[30] Every accusation Popper levels against Heisenberg's notion of measurement is, in fact, a confession about his own notion of physical selection. It was, indeed, Popper rather than Heisenberg who tried to exploit a non-existent asymmetry between position and momentum selections. Popper (1974, 72f. 1982b, 23; fn. 32) had some reason to claim that his notion of 'physical selection' was similar in spirit to the modern concept of 'preparation,' as outlined around the same time by (Margenau 1936, 1937). Yet it is clearly not equivalent to it.

Paraphrasing Popper, one can say that the failure of his thought experiment "can be used to point out an inconsistency" in his notion of physical selection. Popper seems to have conceived of the momentum 'filter' intuitively as analogous to a classical 'sieve' that merely *selects* an ensemble of particles that already possess the *old* preassigned sharp momentum state, and simply leaves the position unknown.[31] However, unlike such a classical device, the quantum-mechanical filter actually *prepares* an ensemble of systems such that each member is in a *new* momentum eigenstate, and thus completely spread

[29]At most, Popper was misled by Heisenberg's ambiguous phrasing on p. 15 of his 1930 lecture notes; see above section 1.

[30]One might argue that in Heisenberg's (1927) analysis, the position measurement constitutes a genuine 'measurement' (recording); by contrast, the momentum measurement is, in Popper's terminology, merely a 'physical selection' of an ensemble characterized by a definite momentum distribution, while the position remains indeterminate. However, the idea that 'undetermined' here means 'unchanged, but unknown' is Popper's responsibility.

[31]That momentum has a preassigned but unknown value is what Popper was supposed to prove, not to assume.

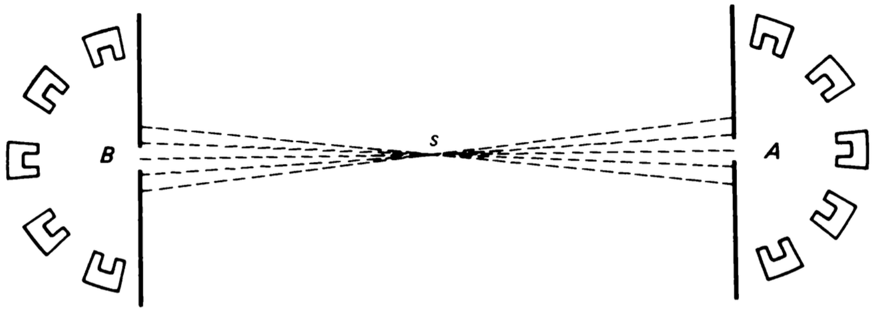

Figure 5: From Popper 1982b, 28

position. The apparent asymmetry between position and momentum preparations on which Popper relies is illusory. It arises from the fact that position is not an eigenstate of the Hamiltonian operator of a free particle, whereas momentum is. For this reason, momentum selection appears not to produce any analogous 'disturbance' to that occurring in a position measurement; it merely seems to allow particles to pass through the filter with unchanged momentum. Yet the point Popper missed is that a momentum preparation still projects the system into a momentum eigenstate, rendering its position completely undetermined, since position is not an eigenstate of the momentum operator. Consequently, there is no way to combine information from a preparation and a measurement to reconstruct an individual trajectory before the momentum selection, as Popper had hoped.

Popper did realize that his ETP-like arrangement was based on a 'mistake'; however, he ultimately does not seem to have appreciated the deeper conceptual reason behind it. In the second edition of the *Logic*, Popper complained that those "critics who rightly rejected the idea of this imaginary experiment appear to have believed that they had thereby also refuted the preceding analysis" (Popper 1959, 239; fn. *2). However, Popper was adamant that this was not so. He not only rightly continued to maintain that preparation and measurement should be *distinguished*, but also held on to the idea that, for this reason, the two can be *uncoupled*. To his credit, Popper made it unmistakably clear that this was the central point of his argument. However, the repeated failure of his experimental setups should have suggested that this point does not stand. Far from being an additional or auxiliary assumption, the coupling-hypothesis lies at the core of quantum mechanics, which is ultimately a theory of *transition* probabilities between prepared and measured states. Popper's repeated attempts to refute the 'preparation-measurement coupling hypothesis' inadvertently demonstrate its centrality to quantum mechanics: a measurement outcome is meaningful only relative to a prepared quantum state.

Even though he acknowledged the problem that affected his setup, he remained convinced that one could "replace [his] invalid experiment" (Popper 1972, 232; fn. **) of the ETP-type with a valid EPR-type experiment. After Popper (1967) returned to work on quantum physics, he outlined, around 1977 or 1978, a thought experiment of this kind, which he made public in the early 1980s Popper (1982b, 1982a, 1985, 1986). Popper was at the height of his fame, and his variant of the EPR argument sparked considerable public controversy (Del Santo 2018). However, on closer inspection, even this much more famous thought experiment rests on the same trick. Once again, Popper tries to exploit a perfect correlation (*i.e.*, relative position) inherent in a jointly prepared state to bypass the preparation–measurement coupling. He starts from an entangled state with sharp relative position ($y_B - y_A = 0$) and sharp total momentum ($p_A + p_B = 0$) for two particles emitted from a common source S (fig. 5). He argues that changing the preparation of system A (narrowing Δy_A via a slit) would alter the measurement statistics of the distant system B (increasing the spread Δp_{y_B}) *without changing its own preparation.* Once again, the thought experiment fails for the same 'mistake': perfect correlations belong to the joint preparation of the composite system and cannot be used to circumvent the preparation–measurement coupling for one of the subsystems.

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