Coordination, Geometrization, Unification. An Overview of the Reichenbach–Einstein Debate on the Unified Field Theory Program

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The quest for a 'unified field theory,' integrating gravitational and electromagnetic fields into a single field structure, spans most of Einstein's professional life from 1919 till his death in 1955. Seldom, it is noted that Hans Reichenbach was possibly the only philosopher who could find his bearings within the technical intricacies of the various unification attempts. By analyzing published writings and private correspondences, the paper aims to provide an overview of the Einstein-Reichenbach relationship from the point of view of their evolving attitude toward the program of unifying electricity and gravitation. The paper will conclude that the Einstein-Reichenbach relationship is more complex than it is usually portrayed. Reichenbach was not only the indefatigable 'defender' of relativity theory but also the caustic 'attacker' of Einstein's and others' attempts at unified field theory. Over the years, Reichenbach managed to provide the first, and possibly only, overall philosophical reflection on the unified field theory program. Thereby, Reichenbach was responsible for having brought to the debate, often for the first time, some of the central issues of the philosophy of space-time physics: (a) the relation between a theory's abstract geometrical structures (metric, affine connection) and the behavior of physical probes (rods and clocks, free particles and so on) (b) the question of whether such association should be regarded as a geometrization of physics or a physicalization of geometry (c) the interplay between geometrization and unification in the context of a field theory.

Keywords: Reichenbach • Einstein • Weyl • Unified Field Theory • General Relativity • Geometrization • Unification • Coordination

Introduction

Most of Einstein's published work from 1919 till his death in 1955 (Einstein and Kaufman, 1955)¹ is dominated by the search for a unified field theory, which would combine both gravitational field and electromagnetic field into a single mathematical structure and at the same time integrate the field with its sources (Tonnelat, 1966). The history of Einstein's engagement with such a program has been rightly described as a rapid succession of hopes and disappointments (Vizgin, 1994, 183ff.)² (Vizgin, 1994, 187). Einstein was aware that, without an analogon of the equivalence principle, the choice of the basic field structure to represent the combined electromagnetic/gravitational field could not be empirically motivated from the outset, as in the case of metric in his theory of gravitation. Thus, in the last resort, Einstein had to rely on the criterion of mathematical simplicity that was arbitrary to a large degree.

¹For the history of the unified field theory-project I draw freely from the standard historical literature on the subject Goenner, 2004, Goldstein and Ritter, 2003, Vizgin, 1994. For an overview of Einstein's work on unified field theory-project, see Sauer (2014); for the philosophical background of Einstein's search for a unified field theory see Dongen (2010); on Einstein's philosophy of science Ryckman (2017).

²During his Berlin years, after using a trace-free equation (Einstein, 1919c), Einstein explored the so-called conformal theory (Einstein, 1921b), Kaluza's (1921) five-dimensional theory, and then moved to Eddington's (1921a) purely affine approach in late 1922 (Einstein, 1923b,c,d). By 1925 Einstein proposed his first original approach, the so-called metric-affine theory (Einstein, 1925a), which he soon abandoned to return to the trace-free equations after a correspondence with G. Y. Rainich (Einstein, 1927a). Inspired by the work of O. Klein, moved to the five-dimensional approach (Einstein, 1927b,c). The decade closed with the *Fernparallelismus*-field theory (Einstein, 1928b,d, 1929d, Einstein and Mayer, 1930), the use five-dimensional vectors attached at four-dimensional points Einstein and Mayer, 1931, 1932a, and finally the semi-vector theory before leaving for the United States in 1933 (Einstein and Mayer, 1932b, 1933a,b, 1934). During his Princeton years Einstein returned to the five-dimensional approach (Einstein and Bergmann, 1938), and after the interlude the bi-vector theory (Einstein, 1944a, Einstein and Bargmann, 1944), pursed again the metric-affine approach until his death in 1955 (Einstein, 1945, Einstein and Kaufman, 1955, Einstein and Straus, 1945).

It has not been fully appreciated that Hans Reichenbach was probably the only philosopher who, alongside his acclaimed work the foundation of relativity theory (Reichenbach, 1920b, 1924, 1928a), possessed the technical skills to make head or tail of the variety of the unification attempts. Indeed, Reichenbach followed the historical development of the unified field theory-project firsthand in a way that is inextricably entangled with his personal and intellectual relationship with Einstein. In the late 1910s, Reichenbach witnessed the rise of the program when he attended the Berlin lectures and was confronted with Einstein's skeptical reaction to Hermann Weyl's (1918b) early attempt. In the mid-1920s, he was confronted with Einstein's sudden change of attitude towards the unification program (Vizgin, 1994, 188). When he returned to Berlin as a professor by the end of the decade (see Hecht and Hoffmann, 1982), Reichenbach was directly involved in the journalistic craze surrounding Einstein's latest theory (Sauer, 2006). Thereby, Reichenbach not only closely trailed the technical meanders of the field-theoretical approach to unification, but he was a privileged witness of the progressive transformation of Einstein's philosophical outlook, from the cautious empiricism of his youth to the extreme rationalism of his later years.

This paper aims to revisit the Einstein-Reichenbach relationship from the point of view of their evolving attitude toward the program of unifying electricity and gravitation. In particular, it considers Reichenbach not in his capacity of a staunch defender of relativity theory Hentschel, 1982, Reichenbach, 2006, but in his less-known role of indefatigable attacker of the unified field theory-project. Most of Reichenbach's critical remarks on this topic appeared in published writings. However, the personal and philosophical motivations of his mistrust towards the unification program emerge more straightforwardly in his private correspondence. For a general overview, the paper organizes Reichenbach's reflections on this topic around three correspondences, which roughly seem to revolve around three different conceptual issues:

Coordination: The Reichenbach-Weyl correspondence (1920-1922). In his 1920 habilitation, Reichenbach, although somewhat in passing, accused Weyl of attempting to deduce physics from geometry by reducing physical reality to 'geometrical necessity' (Reichenbach, 1920b, 73). On the contrary, Reichenbach considered the greatest achievement of general relativity was to have shifted the question of the truth of geometry from mathematics to physics (Reichenbach, 1920b, 73). Reichenbach insisted on what he thought was the core message of Einstein's epistemology: spacetime geometry is in itself neither true nor false, it acquires a physical meaning only when it is coordinated with the behavior of physical probes, like rods and clocks. After a correspondence with Weyl, Reichenbach (1922a, 367–368), Reichenbach accepted Weyl's (1921d)'s defense that he did not mean to derive physics from mathematics. However, Weyl also insisted that abstract spacetime geometry has nothing to do with behavior similar physical measuring devices. However, Reichenbach objected that, in this way, Weyl's theory becomes overly formal, losing its convincing power (Reichenbach, 1922a, 367).

Geometrization: Reichenbach-Einstein correspondence (1926-1927). Reichenbach became convinced that, in spite of the initial failure of Weyl's approach, Weyl's style of doing physics was prevailing. Physicists seemed to be convinced that after the geometrization of the gravitational field, further physical insight could be obtained by geometrizing the electromagnetic field. By the end of 1922 Einstein's (1923a) himself started to pursue more aggressively the unification program, when he pursed Eddington's (1921a) approach. In March 1926, after making some critical remarks on Einstein's newly published metric-affine theory (Einstein, 1925a), Reichenbach sent Einstein a 10-page 'note' (Reichenbach, 1926b). In it, he constructed a toy unification of the gravitational and electricity in a single geometrical framework, thereby showing that the 'geometrization' of a physical field was a mathematical trickery rather than a physical achievement. After a back and forth, Einstein seemed to agree (Lehmkuhl, 2014). The note was later included as section §49 in a lengthy technical Appendix to the *Philosophie der Raum-Zeit-Lehre* (Reichenbach, 1928a, SS46-50) in which general relativity was presented as a 'physicalization of geometry' rather than a 'geometrization' of gravitation' (Giovanelli, 2021).

Unification: Reichenbach-Einstein correspondence (1928-1929). A few months after the publication of the Philosophie der Raum-Zeit-Lehre (Reichenbach, 1928a), Einstein (1928b,d) launched yet another attempt at a unified field theory, the so-called Fernparallelismus-field theory. Reichenbach, now back in Berlin sent him once again a manuscript with some comments (Reichenbach, 1928c) and discussed the new theory in person with Einstein. This exchange of letters marked the cooling of their personal friendship but also the end of their philosophical kinship. In the late 1920s, Reichenbach (1929b,c,d) came to realize that, in Einstein's mind, the actual goal of the unified field theory-project was not

the geometrization, but as the *unification* of two different fields, an undertaking for the sake of which Einstein was ready to embrace a strongly speculative approach to physics (Dongen, 2010). The heuristic of mathematical simplicity gradually gained prominence in Einstein's scientific practice overshadowing the separation of mathematics and physics, on which the Einstein-Reichenbach philosophical alliance was based

The present paper does not have the ambition to provide new documentary material. The recognition of the importance of ?? has been a significant result of the Reichenbach-scholarship of the last decades (Ryckman, 1995, 1996). The other two correspondences ?? and ??, has been published and analyzed in detail only recently (Giovanelli2022, Giovanelli, 2016). However, the paper makes an attempt to build a coherent narrative out of these episodes that have been considered only separately. Thereby, it hopes to shed new light on each of them. On the other hand, the Reichenbach-Weyl debate will be inserted into the larger context of Reichenbach's hostile attitude towards the unification program; at the same time, it will be shown how Reichenbach launched the same line of attack against Einstein that he had previously used against Weyl. In Reichenbach's view, the great achievement of general relativity was the separation between mathematics and physics. Mathematics teaches what is physically permissible but never what is physically true. Reichenbach was disappointed that relativists had started to believe that mathematical simplicity in itself could provide an insight into physical reality. In this sense, Reichenbach's role as a 'defender' of relativity and that of a 'critic' of further unification are two sides of the same coin. Reichenbach's disapproval of the unified field theory-project, including Einstein's contribution to the field, was at the same a vindication philosophical achievement of Einstein's theory: "The general theory of relativity by no means turns physics into mathematics. Quite the opposite: it brings about the recognition of a physical problem of geometry" (Reichenbach, 1929a, 11).

In this manner, Reichenbach, somewhat unwittingly, was able to formulate a sort of 'theory of spacetime-theories' (Lehmkuhl, 2017). He attempted to unravel the key to Einstein's success in formulating a field theory of gravitation by uncovering the reasons for the failure of subsequent unification attempts. Thereby, Reichenbach was responsible for having brought to the debate, often for the first time, some of the central issues of the philosophy of space-time physics: (a) the relation between a theory's abstract geometrical structures (metric, affine connection) and the behavior of physical probes (rods and clocks, free particles, etc.); (b) the question whether such association should be regarded as a geometrization of physics or a physicalization of geometry (c) the interplay between geometrization and unification in the context of a field theory; .

1 Coordination. The Weyl-Reichenbach Correspondence (1920–1921)

After serving in World War I, from 1917 until 1920, Reichenbach worked in Berlin as an engineer specializing in radio technology to support himself after the death of his father. Nevertheless, in his spare time, he managed to attend Einstein's lectures on special and general relativity in winter term 1917–1918 and in summer term 1919. We possess three sets of Reichenbach's undated student notes (HR, 028-01-04, 028-01-03, 028-01-01). One set of notes (HR, 028-01-01) appears to be very similar to Einstein's own lecture notes from 1919 (Einstein, 1919a). In presenting general relativity, Einstein's lectures followed roughly the corresponding sections of his previous published presentations of relativity theory (Einstein, 1916, 1914). The mathematical apparatus of Riemannian geometry is introduced by starting from the metric $g_{\mu\nu}$ as the fundamental concept, that is from the formula to calculate the squared distance $ds^2 = g_{\mu\nu} dx_{\mu} dx_{\nu}$ between any two neighboring points x_{ν} and $x_{\nu} + dx_{\nu}$ independently of the coordinate system. From the $g_{\mu\nu}$, one can calculate the so-called Christoffel symbols ${\mu\nu \brace \tau}$, which enters the geodesic equation, and the Riemann tensor $R^{\tau}_{\mu\nu\sigma}$ which generalized the Gaussian notion of curvature.

However, both Reichenbach's (see fig. 1) and Einstein's notes show that in the lectures May-June 1919, Einstein used for the first time the interpretation of the curvature in terms of the parallel displacement of vectors, which was introduced by Tullio Levi-Civita (1916) and applied to relativity theory by Hermann Weyl (1918b). Both names are mentioned explicitly (HR, 028-01-03, 33). Instead of using the metric as a fundamental concept, it is more convenient to introduce a coordinate-independent criterion of parallelism of vectors at neighboring points x_{ν} and $x_{\nu} + dx_{\nu}$ $dA^{\mu} = \Gamma^{\tau}_{\mu\nu}A^{\nu}x_{\nu}$ (HR, 028-01-03, 33). The $\Gamma^{\tau}_{\mu\nu}$, which is supposed to be symmetrical in the lower indexes ($\Gamma^{\tau}_{\mu\nu} = \Gamma^{\tau}_{\mu\nu}$), is the so-called affine connection or displacement. The metric could be introduced at a later stage by defining the notion of scalar

³Throughout the paper, the notation used by Reichenbach (1928a) which, in turn is based on Eddington (1923, 1925) is used.

 $^{^4}$ The affine geometry is the study of parallel lines, Weyl (1918c) hence the expression 'affine connection' (affiner Zu-



Figure 1: Reichenbach's student notes. Einstein introduces the notion of parallel transport of vectors

product in a manner independent of the choice of the coordinate system. A vector's squared length is the scalar product of this vector with itself: $l^2 = g_{\mu\nu}A^{\mu}A^{\nu}$. By imposing the condition that the length of vectors does not change under parallel transport, the coefficients of the $\Gamma^{\tau}_{\mu\nu}$ happen to have the same numerical values of the Christoffel symbols $\Gamma^{\tau}_{\mu\nu} = -\left\{ \begin{matrix} \mu\nu \\ \tau \end{matrix} \right\}$ (up to a sign). The structure of the Einstein-Riemann geometry could be then recovered without any reference to the metric $g_{\mu\nu}$. It differs from the Euclidean structure by the fact that, when a vector is transported along a closed curve, it will acquire a rotation whose amount is determined by the Riemann tensor $R^{\tau}_{\mu\nu\sigma}(g)$. Only when the latter vanishes, the parallelism of vectors can be defined at a distance.

Weyl's technical innovation in differential geometry played a fundamental role, not only in successive presentations of general relativity (see Einstein et al., 1922, 45ff.), but more prominently in the development of the unified field theory-project. If one starts with a symmetric $g_{\mu\nu}$ as fundamental variables, the road is, so to speak, marked. The Christoffel symbols are the only possible destination. However, if one defines the displacement $\Gamma^{\tau}_{\mu\nu}$ independently of the metric $g_{\mu\nu}$, the Riemannian connection $\Gamma^{\tau}_{\mu\nu} = -\begin{Bmatrix} \mu\nu \\ \tau \end{Bmatrix}$ appears only as a special case that has been achieved by introducing a series of conditions, that turned out not be necessary. By dropping some of these conditions, additional mathematical degrees of freedom emerge that might be used to accommodate the electromagnetic field alongside the gravitational field in a unified 'geometrical' description.

As it is well-known, Weyl (1918a,c, 1919a) was bothered by a conceptual asymmetry characterizing Riemannian geometry. The comparison of direction of vectors is path-dependent, whereas the comparison of their lengths remains distant-geometrical. To compensate for this 'mathematical injustice' (Afriat, 2009), he introduced a more general affine connection depending not only on the metric/gravitational tensor $g_{\mu\nu}$ but also on the four-vector φ_{ν} . The latter could be identified with electromagnetic four-potential. However, in the absence of an analogon of the equivalence principle, the justification of such identification was merely formal.⁵ Nevertheless, Weyl could conclude that, just like general relativity represented a geometrization of gravitational phenomena, his theory represented a unified geometrization of both gravitational and electromagnetic phenomena. Weyl did not hesitate to declare that "Descartes' dream of a purely geometrical physics" had been finally fulfilled (Weyl, 1919b, 263).

Einstein had repeatedly criticized Weyl's attempt.⁶ Nevertheless, by the spring of 1919, after a

sammenhang), where 'connection' refers to the possibility of comparison of vectors at close points. However, because it is a relation of 'sameness' rather than parallelism that is relevant, others, such as Reichenbach (1928b), prefer to speak of the operation of 'displacement' (Verschiebung), where the latter indicates the small coordinate difference dx_{ν} along which the vector is transferred. The word "displacement" also refers to the vector dx_{ν} . In order to avoid confusion, the world "transfer" $\ddot{U}bertragung$ was also used, e.g. by Schouten (1922).

⁵In Weyl's (1918a) theory the vector field φ_{ν} determines the change of length of vectors; the curl of φ_{ν} is the length-curvature tensor $F_{\mu\nu}$, which satisfy satisfy an identity which looks a lot like Maxwell-Minkowski equations in empty space. Thus, it was very suggestive to interpret φ_{ν} as the electromagnetic four-potential and its curl $F_{\mu\nu}$ as the electromagnetic tensor.

⁶Einstein raised at least four objections against Weyl's theory: (1) the so-called 'measuring rod objection' (Maßstab-Einwand) (Einstein, 1918) is most famous. Weyl's theory predicts that the clocks' ticking rate should depend on the

corresponding with Theodore Kaluza (Wünsch, 2005)⁷ he had started to show increasing interest in the unification program (Einstein, 1919b). The question fell into the background after the success of the eclipse expedition was announced in November 1919 (Dyson, Eddington, and Davidson, 1920). By the end of the year, Einstein was turned into an international celebrity, leaving him little time to work (Einstein to Fokker, Dec. 1, 1919; CPAE, Vol. 9, Doc. 187, Einstein to Hopf, Feb. 2, 1920; CPAE, Vol. 9, Doc. 295). As a trained physicist with a doctorate in philosophy (Reichenbach, 1916), Reichenbach was in a privileged position to deal with the philosophical implications of the theory. By following Einstein's lectures, he had acquired a detailed technical knowledge of the new theory that probably had no equal among the philosophers of his time. In February or March 1920, Reichenbach, who has just moved to Stuttgart, decided to write his habilitation on this topic. According to his later recollections, in the preceding months, he had further worked on the theory "also according to Weyl" (HR, 044-06-23) that is, probably studying Weyl's textbook Raum-Zeit-Materie (Weyl, 1918b). The Kapp-Pusch coup on March of 1920 gave Reichenbach a few days of leave from the Huth radio industry, where he was employed (HR, 044-06-23). Thus, he could work without interruptions, and, in ten days, he completed an early draft. The manuscript was then typed and shown among others to Einstein. Thanks to the mediation of Arnold Berliner, the influential editor of the Naturwisseschaften, Reichenbach obtained a publication agreement with Springer (HR, 044-06-23).

1.1 Reichenbach's Habilitation and his critique of Weyl Theory

According to young Reichenbach, one the main philosophical merits of the theory of relativity was to revelation of the physical character of geometry. The possibility of non-Euclidean geometries had already indicated that the Euclidean character of physical space could no longer be taken for granted (Reichenbach, 1920b, 3; tr. 1969 3). According to Reichenbach, "the theory of relativity embodies an entirely new idea" (Reichenbach, 1920b, 3; tr. 1969 4). Relativity theory claims that the theorems of Euclidean geometry do not apply to the physical space, that Euclidean geometry is simply false (Reichenbach, 1920b, 3; tr. 1969 4). In this way, relativity theory has made it inevitable to distinguish between pure geometry as an uninterpreted, formal system and applied geometry as an empirical theory of physical space (Reichenbach, 1920b, 73; tr. 1969 76). The propositions of pure geometry are neither true nor false. The question of the truth of physical geometry pertains to physics alone. In order to emphasize the importance of this philosophical achievement, rather in passing, Reichenbach indicated Weyl's recent theory as a glaring example of how easy it was to slip into old habits. Weyl once again believed to have found a particular geometry that, for its intrinsic mathematical appeal, must have been 'true' for physical reality: "In this way, the old mistake is repeated" (Reichenbach, 1920b, 73; tr. 1969 76).

Reichenbach's brief outline of Weyl's theory is sufficient to grasp the gist of his argument. As Reichenbach's put it, "Weyl's generalization of the theory of relativity [...] abandons altogether the concept of a definite length for an infinitesimal measuring rod" (Reichenbach, 1920b, 73; tr. 1969 76). In Euclidean geometry, a vector can be shifted parallel to itself along a closed curve so that upon its return to the point of departure it has the same direction and the same length. In the Einstein-Riemannian geometry, it has merely the same length, no longer the original direction, after its return. In Weyl's theory, it does not even retain the same length. As we have seen, in this way, in addition to the 'metric tensor' $g_{\mu\nu}$, a

clocks' prehistory. However, the spectral lines of atoms used as clocks are well-defined; (2) the geodesic equation in Weyl's theory contains terms proportional to the vector potential φ_{ν} . Thus, the electromagnetic four-vector potential affects the motion of uncharged particles; (3) the representation of the Lagrangian is the mere sum of electromagnetic and gravitational components, thus Weyl's theory does not achieve a proper unification; (4) The field equations derived from this Lagrangian were of the fourth-order in the $g_{\mu\nu}$ which, even in the absence of an electromagnetic field, did not reduce to the generally relativistic equations of gravitation, violating the correspondence principle.

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⁷Einstein to Kaluza, Apr. 21, 1919; CPAE, Vol. 9, Doc. 26; Einstein to Kaluza, Apr. 28, 1919; CPAE, Vol. 9, Doc. 30; Einstein to Kaluza, May 5, 1919; CPAE, Vol. 9, Doc. 35; Einstein to Kaluza, May 14, 1919; CPAE, Vol. 9, Doc. 40; Einstein to Kaluza, May 29, 1919; CPAE, Vol. 9, Doc. 48).

⁸These autobiographical notes HR, 044-06-23 were written in 1927.

⁹Reichenbach's habilitation has recently attracted renewed attention (Friedman, 2001). Reichenbach borrowed from Schlick (1918) that idea that physical knowledge is, ultimately (Zuordnung), the process of relating an axiomatically defined mathematical structure to concrete empirical reality (Padovani, 2009). However, Reichenbach attempted to give this insight a 'Kantian' twist. According to Reichenbach, in a physical theory, besides the 'axioms of connections' (Verknüpfungsaxiome) encoding the mathematical structure of a theory, one needs a special class of physical principles the 'axioms of coordination' (Zuordnungsaxiome) to ensure the univocal coordination of that structure to reality. For the young Reichenbach, the latter axioms are a priori in the sense that they are 'constitutive' of the object of a physical theory. However, they are not apodeictic or valid for all time. As it is well known, Reichenbach would soon abandon the project of a constitutive but relativized a priori. However, he would firmly maintain the separation between the mathematical framework of a theory (the 'defined side') and the way it related to empirical reality (the 'undefined side') (Reichenbach, 1920b, 40; tr. 1969 42) as an essential feature of his philosophy.

'metric vector' φ_{ν} is introduced that formally behave like the electromagnetic four-potential. Reichenbach conceded that Weyl's theory represented a possible generalization of Einstein's conception of spacetime that, "although not yet confirmed physically, is by no means impossible" (Reichenbach, 1920b, 76; tr. 1969 79).

Reichenbach seemed to have been aware of Einstein's main objection to Weyl's proposal (see Einstein, 1918). In general relativity, the length ds of the time-like vector dx_{ν} is measured by a physical clock, e.g., by the crests of waves of radiation were emitted by an atom. If we maintain this interpretation, then Weyl's theories implies that "the frequency of a clock is dependent upon its prehistory" (Reichenbach, 1920b, 77; tr. 1969 80). It particular, it is affected by the electromagnetic potentials φ_{ν} it has encountered. Thus, two atomic clocks, at one place, will, in general, not tick at the same rate when they are separated brought back together. This result appears to contradict a vast amount of spectroscopic data that shows that all atoms of the same type have the same systems of stripes in their characteristic spectra independently of their past history. Reichenbach conceded to Weyl that these effects might "compensate each other on the average" (Reichenbach, 1920b, 77; tr. 1969 80). Thus, the fact that "the frequency of a spectral line under otherwise equal conditions is the same on all celestial bodies" could he interpreted as an approximation, rather than being a consequence of the Riemannian nature of space-time (Reichenbach, 1920b, 77; tr. 1969 81). According to Reichenbach, Weyl seems to imply that his non-Riemannian geometry must be true physically because it is mathematically superior to Riemannian geometry, being the true realization of the principle of locality.

As we have seen, in Weyl geometry, a vector moving around a closed loop would have the same length but a different direction in Riemannian geometry, different length, and different direction in Weyl geometry. Thus, Weyl geometry eliminated the last distant-geometrical treatment of Riemannian geometry. Weyl geometry seems to be the most 'general geometry,' a purely infinitesimal geometry. As a consquence, there would be no reason to assume that a more special geometry applies to reality from the outset. However, Reichenbach had already surmised that this generalization could be continued. In Weyl geometry, lengths can be compared at the same point in different directions but not at distant points. "The next step in the generalization would be to assume that the vector changes its length upon turning around itself" (Reichenbach, 1920b, 76; tr. 1969 85). Probably, more complicated generalizations could be thought of. "Nothing may prevent our grandchildren from being confronted some day by a physics that has made the transition to a line element of the fourth degree" (Reichenbach, 1920b, 76; tr. 1969 79). Thus, there is no "'most general' geometry" that in and by itself must be physically true (Reichenbach, 1920b, 76; tr. 1969 80). No matter one pushes further the level of mathematical abstraction, the "difference between physics and mathematics" (Reichenbach, 1920b, 76; tr. 1969 80) cannot be erased; geometry alone can never be sufficient to establish the reality of physical space (Reichenbach, 1920b, 76; tr. 1969 80).

In this way, Reichenbach accused Weyl of neglecting the main philosophical lesson of general relativity, the unbridgeable difference between physics and mathematics. A mathematical axiom system is indifferent with regard to the applicability, and "never leads to principles of an *empirical theory*" (Reichenbach, 1920b, 73; tr. 1969 76). On the contrary, "[o]nly a physical theory [can] answer the question of the validity" (Reichenbach, 1920b, 73; tr. 1969 76) of a particular geometry for physical space:

[Thus] it is incorrect to conclude, like Weyl¹¹ and Haas,¹² that mathematics and physics are but one discipline. The question concerning the validity of the axioms for the physical world must be distinguished from that concerning possible axiomatic systems. It is the merit of the theory of relativity that it has removed the question of the truth of geometry from mathematics and relegated it to physics. If now, from a general geometry, theorems are derived and asserted to be a necessary foundation of physics, the old mistake is repeated. This objection must be made to Weyl's generalization of the theory of relativity [...] Such a generalization is possible, but whether it is compatible with reality does not depend on its significance for a general local geometry. Therefore, Weyl's generalization must be investigated from the viewpoint of a physical theory, and only experience can be used for a critical analysis. Physics is not a 'geometrical necessity'; whoever asserts this returns to the pre-Kantian point of view where it was a necessity given by reason (Reichenbach, 1920b, 73; tr. 1969 76).

 $^{^{10}}ds^4 = g_{\mu\nu\sigma\tau}dx_{\mu}dx_{\nu}dx_{\sigma}dx_{\tau}$ instead of $ds^2 = g_{\mu\nu}dx_{\mu}dx_{\nu}$ as in Riemannian geometry.

¹¹Weyl, 1918b, 227. In the 1919 edition of the *Raum–Zeit–Materie*, Weyl included a presentation of his unified field theory. Thus, the 'Conclusion' of the book was characterized by even more inspired rhetoric: "physics and geometry coincide with each other" (Weyl, 1919b, 263). The tendency of physicalizing geometry that has prevailed among the leading protagonists of the 19th century from Gauss to Helmholtz seemed to be superseded by the project of geometrizing physics that run from Crifford to Einstein: "geometry has not been physics but physics has become geometry" (Weyl, 1919b, 263). ¹²Haas, 1920.

To a certain extent, this objection contains the backbone of Reichenbach's criticism of the unified field theory-project in the following decade. Weyl seems to have misunderstood the fundamental lesson of Einstein's theory. The question of the "validity of axioms for the physical world" (Reichenbach, 1920b, 73; tr. 1969 76) must be distinguished from that concerning possible axiomatic systems happened to be in reality.

It is true that it is "a characteristic of modern physics to represent all processes in terms of mathematical equations", and, one might add, progressively more abstract mathematics. Still, "the close connection between the two sciences must not blur their essential difference" (Reichenbach, 1920b, 33; tr. 1969 34). The truth of mathematical propositions depends upon internal relations among their terms; the truth of physical propositions, on the other hand, depends on the coordination (*Zuordnung* to something external, on a connection with experience. "This distinction is due to the difference in the objects of knowledge of the two sciences" (Reichenbach, 1920b, 33; tr. 1969 34). The mathematical object of knowledge is uniquely determined by the axioms and definitions of mathematics. The definitions indicate how a term is related to that Schlick (1918) had called "implicit definitions" (Reichenbach, 1920b, 33; tr. 1969 36). For this reason mathematics is absolutely certain and necessary. On the contrary, "the physical object cannot be determined by axioms and definitions. It is a thing of the real world, not an object of the logical world of mathematics" (Reichenbach, 1920b, 34; tr. 1969 37). For this reason physical knowledge always implies a certain degree of approximation.

As it is well-known, Reichenbach will abandon the Kantian framework in which this uncoupling of mathematics and physics was initially presented. However, he never abandoned the idea that this clear-cut division of labor was of paramount epistemological importance: mathematical necessity must be sharply distinguished from physical reality. This separation was the irreversible conceptual shift that relativity theory had forced upon philosophy. On June 24, 1920, Einstein praised Reichenbach's Habilitationschrift in a letter to Schlick (Einstein to Schlick, Apr. 19, 1920; CPAE, Vol. 9, Doc. 378). A few days later, Reichenbach asked Einstein to dedicate the book to him, insisting on the philosophical significance of relativity theory: "very few among tenured philosophers have the faintest idea that your theory performed philosophical act and that your physical conceptions contain more philosophy than all the multivolume works by the epigones of the great Kant" (Reichenbach to Einstein, Jun. 13, 1920; CPAE, Vol. 10, Doc. 57). Einstein conceded that the theory might have had philosophical relevance: "The value of the th. of rel. for philosophy seems to me to be that it exposed the dubiousness of certain concepts that even in philosophy were recognized as small change [Scheidemünzen]" (Einstein to Reichenbach, Jun. 30, 1920; CPAE, Vol. 10, Doc. 66). Alleged a priori principles are like those parvenus that are ashamed of their humble origin and try to deny it: "[c]oncepts are simply empty when they stop being firmly linked to experience" (Einstein to Reichenbach, Jun. 30, 1920; CPAE, Vol. 10, Doc. 66). Einstein's remark, which Reichenbach would later quote in published writing (Reichenbach, 1922b, 354), seals a sort of philosophical alliance between them. Against Weyl's speculative style of doing physics which reduced physical reality to geometrical necessity, Einstein defended a clear-cut separation between geometrical necessity and physical reality. Reichenbach's booklet ultimately provided an articulated philosophical the problem of the 'coordination' (Zurordnung) between mathematics and reality. As we shall see, this philosophical covenant will be broken less than a decade later.

1.2 The Reichenbach-Weyl Correspondence

Reichenbach's book was timely published a few months later in September 1920, on the occasion of the 86th Versammlung der Gesellschaft Deutscher Naturforscher und Ärzte in Bad Nauheim. This meeting has been of fundamental importance for in the history of relativity theory, not least for the famous debate between Einstein and Philipp Lenard on general relativity (Dongen, 2007). Reichenbach met Weyl there for the first time Weyl, who gave a talk on his unified theory (Weyl, 1920a). Reichenbach

¹³Einstein used a similar wording by commenting on the manuscript of Cassirer's 'Kantian' booklet on relativity (Cassirer, 1921). "Conceptual systems appear empty to me, if the manner in which they are to be referred to experience is not established" (Einstein to Cassirer, Jun. 6, 1920; CPAE, Vol. 10, Doc. 44). In particular, "[w]ith the interpretation of the ds as a result of measurement, which is obtainable by means of measuring rods and clocks the general theory of relativity as a physical theory stands or falls" (Einstein to Cassirer, Jun. 6, 1920; CPAE, Vol. 10, Doc. 44). The gravitational redshift, can be taken as an empirical confirmation of general relativity only because different atoms of the same substance can be regarded as identically constructed clocks reproducing the identical unit of time. For this reason, it is possible to 'normalize' the absolute value of ds by counting an atom's wave crests. Accordingly, Weyl's theory deprived the ds of any physical meaning. However, real rods and clocks behave differently than predicted by Weyl's theory, forcing Weyl to assume an inconsistent position. According to Einstein, like general relativity, Weyl's theory "is based on a measuring rods geometry", that is, it presupposes the comparability of lengths. However, it pertains only "thought measuring rods [nur gedachte Massstäbe]" that behave differently from the real ones. "This is repugnant" (Einstein to Besso, Aug. 26, 1920; CPAE, Vol. 10, Doc. 85; m.e.).

might have assisted at the debate that followed Weyl's talk, in which Einstein rehearsed his objections against Weyl's theory, ¹⁴ and the same time defended the possibility of a field theory of matter against Pauli's attacks. ¹⁵ Einstein's famous lecture on 'geometry and experience' of the end January of 1921 *Geometrie und Erfahrung* was probably meant to address the epistemological issues that had emerged at Bad Nauheim. ¹⁶

Reichenbach sent around copies of his *Relativitätstheorie und Erkenntnis apriori* (Reichenbach, 1920b). Schlick, who did not attend Bad Nauheim received the book in those days. Writing to Einstein, he praised it but complained about his critique of conventionalism (Schlick to Einstein, Sep. 23, 1920). The five letters that Reichenbach exchanged with Schlick between October and November 1920¹⁷ turned out to be of fundamental importance in his intellectual biography, inducing him to abandon his early Kantianism in favor of a form conventionalism with empiricist traits. Reichenbach must have sent a copy to Weyl as well despite the rather severe criticisms he had expressed in the book (Rynasiewicz, 2005). Weyl replied with some delay in February of 1921. He did not appear to be upset by Reichenbach's objections and replied rather amicably to some issues "which concern less the philosophical than the physical" (Weyl to Reichenbach, Feb. 2, 1921; HR, 015-68-04). In particular, Weyl denied having ever claimed that physics has been absorbed into mathematics:

It is certainly not true, as you say on p. 73, that, for me, mathematics (!!, e.g. theory of the ζ -function?) and physics are growing together into a single discipline. I have claimed only that the concepts in geometry and field physics have come to coincide [...] As for my extended theory of relativity, I cannot admit that the epistemological situation is in any way different from that of

¹⁵Einstein showed a more flexible attitude replying to Pauli's remarks during the same discussion. Pauli reiterated his objection based on his 'observability' criterion. Just as the field strength in the interior of the electron is meaningless because there is no smaller test particle than the electron, "one could claim something similar concerning spatial measurements, since there are no infinitely small measuring-rods" (Einstein et al., 1920, 650). Einstein replied to Pauli that "with the increasing refinement of the system of scientific concepts, the manner and procedure of associating the concepts with experiences becomes increasingly more complicated" (Einstein et al., 1920, 650). In particular, he recognized that in cases such as that of the continuum theories, "one finds that a definite experience cannot be associated any longer with a concept" (Einstein et al., 1920, 650). According to Einstein, there is an alternative: one can abandon 'continuum theories' for the sake of Pauli's observability criterion, or replace such a "system of associating concepts [with experiences] with a more complicated none" (Einstein et al., 1920, 650). Einstein in his contributions to the discussion that followed Max von Laue's (1920)'s Bad Nauheim paper. Einstein, however, in the very same sentence, did not hesitate to admit that "[it] is a logical shortcoming of the theory of relativity in its present form to be forced to introduce measuring rods and clocks separately instead of being able to construct them as solutions to differential equations" (Einstein's reply to Laue, 1920, 662; m.e.).

¹⁶On January 14, 1921 Einstein, while in Vienna, released the following declaration: "A theoretical system can only claim completeness if the relationships of the concepts to the facts that can be experienced are clearly established. It is not enough, for example, to base the theory of relativity on a mathematical fundamental invariant [ds]. It must also be clear how this invariant is related to the observable facts, as [it happenes] for the fundamental concepts of Maxwell's theory by Heinrich Hertz. If one disregards this point of view, one can only arrive at unrealistic systems" (CPAE, Vol. 7[13], Doc. 50a) A few days later on January 27, 1921, Einstein held his celebrated Geometrie und Erfahrung in Berlin. The lecture ultimately meant to address precisely this issue, although in a popular form. (a) the invariant ds is measured by ideal rods and clocks, like the electric field strengths are measured by charged test particles (b) ideal rods and clocks do not exist in nature (as pointed out by Poincaré). Conclusion: sub specie aeterni geometry cannot be tested separately from the rest of physics (the famous G + P formula). The choice of a particular geometry is ultimately justified by its success in leading to a good physical theory. The connection of the ds with rods-and-clocks measurements is only provisional. Ultimately it assumed that solutions to theory's dynamical equations exist that at least arbitrarily closely approximated the postulated behavior of rods and clocks that measure the ds. In Weyl's theory, however, real rods and clocks that appear as solutions were supposed to behave differently from the ideal rods and clocks described by Weyl geometry. This inconsistency was for Einstein unbearable. Thus, in March 1921, he preferred to suggest a theory in which there were no transportable ideal rods at all and only ds = 0 has physical meaning (Einstein, 1921b).

¹⁷Schlick to Reichenbach, Sep. 25, 1920; HR, 015-63-23 Schlick to Reichenbach, Nov. 26, 1920; HR, 015-63-22; Schlick to Reichenbach, Dec. 11, 1920; HR, 015-63-19; Reichenbach to Schlick, Nov. 29, 1920; Reichenbach to Schlick, Sep. 10, 1920.

 18 Reichenbach was confronted with Schlick's objection that his 'axioms of coordination' were nothing but 'conventions.' Reichenbach initially opposed some resistance. If the coordinating principles are fully arbitrary, he feared, geometry would be empirically meaningless. In Poincaré's conventionalism, Reichenbach missed claim that "the arbitrariness of the principles is constrained, if the principles are combined" Reichenbach to Schlick, Nov. 26, 1920; HR, 015-63-22. Einstein's famous lecture on 'geometry and experience' of the end January of 1921. The publication of Einstein (1921a) seemed to have tipped the scale in Schlick's favor. Reichenbach (1922b) seems to have turned the Einstein's G+P formula into his G+F formula, where F is a 'metric' or universal force force affecting all bodies in the same way. By setting F=0, geometry becomes empirically testable. Thus, Reichenbach could embrace conventionalism without accepting that the propositions of geometry are empirical meaningless.

¹⁴Commenting on Weyl's talk, he pointed out once again that in the "arrangement of [a] conceptual system," "it has become decisive [massgebend] to bring elementary experiences into the language of signs [Zeichensprache]" (Einstein et al., 1920, 650). For Einstein, "temporal-spatial intervals are physically defined with the help of measuring rods and clocks", under the assumption that "their equality is empirically independent of their prehistory" (Einstein et al., 1920, 650). Einstein insisted that precisely upon this assumption rests "the possibility of coordinating [zuzuordnen] a number ds to two neighboring world points"; if this were impossible, general relativity would be robbed of "its most solid empirical support and possibilities of confirmation" (Einstein et al., 1920, 650).

Einstein. [...] Experience is in no way anticipated by the assumption of that general metric; that the laws of nature, to which the propagation of action in the ether is bound, can be of such a nature that they do not allow any curvature. [...] What I stand for is simply this: The integrability of length transfer (if it exists, but I don't think so, because I don't see the slightest dubious reason for it) does not lie in the nature of the metric medium, but can only be based on a special law of action.¹⁹ If the historical development had been different, it seems that no one would have thought of considering the Riemannian case from the outset. As far as the notorious 'dependence on the previous history' is concerned, I probably expressed my opinion clearly enough in Nauheim (Weyl to Reichenbach, Feb. 2, 1921; HR, 015-68-04).

In Bad Nauheim, Weyl outlined a now well-known speculative explanation for the discrepancy between the behavior of 'ideal' and 'real' rods. Roughly, Weyl suggested that the atoms we use as clocks might not preserve their size if transported, but adjust it every time to some constant field quantity. It was natural to identify with the constant radius of the spherical curvature of every three-dimensional slice of the world (Weyl, 1920b). The geometry read off from the behavior of material bodies would appear different from the actual geometry of spacetime, because of the 'distortion' due to the mechanism of the adjustment. In 1921, the 'pivotal year' for unified field theories (Vizgin, 1994, ch. 4), Weyl (followed to some extent by Eddington, 1921a,b) reacted by expanding his strategy of 'doubling of the geometry,' the real 'aether geometry' and the 'body geometry' distorted by the mechanism of adjustment,²⁰ in three papers intended for different audiences, February (Weyl, 1921f), May (Weyl, 1921c) and July (Weyl, 1921e). In the July paper, Weyl also addressed Reichenbach's criticism publicly:

From different sides,²¹ it has been argued against my theory that it would attempt to demonstrate in a purely speculative way something a priori about matters on which only experience can actually decide. This is a misunderstanding. Of course from the epistemological principle [aus demerkenntnistheoretischen Prinzip] of the relativity of magnitude does not follow that the 'tract' displacement [Streckenübertragung] through 'congruent displacement' [durch kongruente Verpflanzung] is not integrable; from that principle that no fact can be derived. The principle only teaches that the integrability per se must not be retained, but, if it is realized, it must be understood as the outflow [Ausfluß] of a law of nature (Weyl, 1921b, 475; last emphasis mine).

As Weyl explains in this passage, he never claimed that his geometry entails in its mathematical structure alone the $a\ priori$ justification of its physical truth. On the contrary, he questioned the alleged $a\ priori$ status of the assumption that the comparison of lengths is path-independent. For this reason, Weyl did not deny the well-established empirical fact that two atoms of the same chemical substance placed identically in the same conditions is independent of their prehistory. However, he insists that, in principle, the physical behavior of atoms does not have anything to do with the abstract notion of parallel transport of vectors. Einstein assumed as an empirical fact that the ratio of the wave lengths of two spectral lines is a physical constant that can be used to normalize the ds. Weyl, on the contrary, claims that the wave lengths of two spectral lines are always a multiple of a certain field quantity of dimension of a length that can be used to normalize the ds.

1.3 The Weyl-Reichenbach Appearement

Weyl's paper referencing Reichenbach appeared at the beginning of September (Weyl, 1921e). A few weeks later, Reichenbach and Weyl met again in Jena on occasion of the first *Deutsche Physiker- und Mathematikertag*, the first national scientific meeting held independently from the meetings of the *Gesellschaft Deutscher Naturforscher und Ärzte*. Weyl gave a talk in which he tried to provide a mathematical justification for the quadratic or Pythagorean nature of the metric (Weyl, 1921a). Reichenbach presented a

¹⁹That is on the field equations of the theory which, in turn, can be derived from an 'action principle'.

²⁰A different variation of this strategy of 'doubling the geometry' was suggested by Eddington (1921a) at about the same time. He considered non-Riemannian geometries as mere 'graphical representations' that might serve to organize different theories into a common mathematical framework. The "natural geometry" remains exactly Riemannian (Eddington, 1921a).
²¹The reference is to Reichenbach, 1920b and Freundlich, 1920 who, however, refers to Haas, 1920.

²²In September 1921, Pauli's (1921) encyclopedia article on relativity theory was published as part of the fifth volume of the *Enzyklopädie der Mathematischen Wissenschaften*. In the chapter dedicated to Weyl's theory, Pauli suggested that Weyl provided two different versions of the theory. In its first version, Weyl's theory sought to make predictions on the behavior of rods and clocks, just like Einstein's theory. From this point of view, the theory is empirically meanigfull, but inadequate because of the existence of atoms with sharp spectral lines. Later, Weyl renounced this interpretation. The ideal process of the congruent displacement vectors has nothing to do with the real behavior of rods and clocks (Pauli, 1921, 763; tr. 1958, 196). However, in this way, the theory furnishes only "formal, and not physical, evidence for a connection between [the] world metric and electricity" (Pauli, 1921, 763; tr. 1958, 196). In this form, Pauli argues, the theory loess its "convincing power [Uberzeugunggkraft]" (Pauli, 1921, 763; tr. 1958, 196).

report of his work on the axiomatization of relativity (Reichenbach, 1921). This report is the first written testimony of the development of Reichenbach's philosophy after the Schlick-correspondence. Reichenbach suggested that in a physical theory one should distinguish the *axioms* as an empirical proposition about light rays, rods and clocks, etc. and the *definitions* that establish the conceptual framework of the theory (Reichenbach to Einstein, Dec. 5, 1921; CPAE, Vol. 12, Doc. 266). After the paper came out by the end of the year (Reichenbach, 1921), Reichenbach must have sent a copy to Weyl in a lost letter of January 8, 1922. He might have included a personal retraction of his criticisms. However, the letter, which is no longer extant, reached Weyl only months later (Weyl to Reichenbach, Mar. 3, 1922; HR, 015-68-03), since he was in Barcelona, where he was giving his Catalonian Lectures (Weyl, 1923).

However, Reichenbach soon issued a public retraction. In those months, he was working on a lengthy review article about philosophical interpretations of relativity that he finished in Spring 1922. In March, Erwin Freundlich sent the proofs of the paper to Einstein (Freundlich to Einstein, Mar. 24, 1922; CPAE, Vol. 13, Doc. 109), who expressed his general agreement with Reichenbach's analysis (Einstein to Reichenbach, Mar. 27, 1922; CPAE, Vol. 13, Doc. 119). The paper reviewed the most significant philosophical interpretations of relativity. However, it also included a last section on Weyl's unified field theory: "One cannot conclude an exposition of relativistic philosophy", Reichenbach wrote, "without considering the important extension that Weyl bestowed on the problem of space three years ago" (Reichenbach, 1922b, 365).

Reichenbach appears to be now fully converted to conventionalism. The choice between Euclidean and non-Euclidean geometries rests upon a *convention* about which rods are to be considered rigid (Reichenbach, 1922b, 366). This convention is arbitrary, but can be fixed postulating that metrical forces should be eliminated (Reichenbach, 1922c). However, both Euclidean and non-Euclidean geometries tacitly presuppose the validity of an axiom based on an *empirical fact*: rods that are of equal length in one place can be obtained in one place, it will be possible at any other places, no matter how the prehistory of each rod might have been. If this were not the case, a different definition of the unit of length would have to be given for every space point. Reichenbach labeled this tacit assumption the "axiom of the Riemann class" (Reichenbach, 1922b, 366). The merit of Weyl is to have shown that this axiom, although quite natural, is not necessary and can be questioned.

From this point of view, what Weyl achieved is a purely mathematical result: "Weyl's great discovery is that he uncovered a more general type of manifold, of which Riemann's space is only a special case" (Reichenbach, 1922b, 365). The fact that he tried to follow this path, regardless of its empirical correctness, was a "genial advance [genialer $Vorsto\beta$]" in the philosophical foundation of the relations between geometry and physics (Reichenbach, 1922a, 367f.). Concerning the application of this mathematical apparatus to reality, Reichenbach embraces what might be called the two-theory interpretation 23 :

- W-I In Weyl geometry, like in Riemannian geometry, the length of vectors $l^2 = g_{\mu\nu}A^{\nu}A^{\mu}$ can be compared at the same point in different directions. Weyl dropped the assumption that l remains unchanged under parallel transport at a distant point. If a vector of length l is displaced from x_{ν} to $x_{\nu} + dx_{\nu}$, it will in general have a new length l + dl, so that $dl/l = \varphi_{\nu}dx_{\nu}$. "The change in scale is measured by 4 quantities φ_{μ} forming a vector field". As Reichenbach pointed out, "this procedure is a purely mathematical discovery" (Reichenbach, 1922b, 366), and as such is neither true nor false. It acquires a physical meaning if one coordinates the length l as readings of some physical measuring instruments. In general relativity, the length l of the time-like vector l is measured by a clock, e.g., the spectral lines of an atomic clock. Weyl considered natural to maintain this interpretation, so that "still possible to measure also in this case" (Reichenbach, 1922b, 366). However, the existence of atoms with the same spectral lines shows that clocks, even in the presence of the electromagnetic field, behave differently than predicted by Weyl's theory. Thus, it turned out that this axiom "is quite well fulfilled in reality, so the first way of generalization seems unsuitable. The latter was therefore rejected by Weyl" (Reichenbach, 1922b, 366)
- W-II Weyl adopted a different strategy. He "defines an ideal process of scale transfer, which however has nothing to do with the behavior of real scales" (Reichenbach, 1922b, 367). He needs this "transplantation process" only becouse, he "he wants to identify the vector field φ_{ν} with the electromagnetic potential", like in general relativity the $g_{\mu\nu}$ were identified with the gravitational potentials (Reichenbach, 1922b, 367). Once one has individuated the basic geometrical field-quantities, the next step is to find the field equations "then obvious forms for the most general physical equations arise" via "the 'action principle' [Wirkungsprinzip]" (Reichenbach, 1922b, 367)—a variational principle applied to the invariant integral $\int \mathfrak{W} dx$ for a specific Lagrangian \mathfrak{W} . According to Reichenbach,

 $^{^{23}}$ Reichenbach might have been inspired by Pauli (1921). However, his name is not mentioned.

however, in this way the "theory loses its convincing character [überzeugenden Charakter] and comes dangerously close to a mathematical formalism" ²⁴ (Reichenbach, 1922b, 367). For this reason, according to Reichenbach, "Weyl's theory is viewed very cautiously by physicists (especially by Einstein)" (Reichenbach, 1922b, 367)

Ultimately, Reichenbach seems to imply that both strategies led to a dead end. From the point of view of W-I, Weyl's infinitesimal geometry is physically inadequate from that of W-II, it is physically empty. Nevertheless, Reichenbach conceded that his objection against Weyl's theory in his 1920 booklet missed the point. Neither W-I nor W-II can be considered attempts to prove *a priori* that Weyl's non-Riemannian geometry must be true for reality because it is mathematically preferable as a truly infinitesimal geometry:

However, I have to retract my earlier objection [Reichenbach, 1920a, 73] that Weyl wants to deduce physics from reason, after Weyl has cleared up this misunderstanding [Weyl, 1921b, 475]. Weyl takes issue with the fact that Einstein simply accepts the unequivocal transferability of the standards. He does not wish to dispute the Riemann-class axiom for natural standards, but only to demand that the validity of this axiom, since it is not logically necessary, should be understood as 'a consequence of a law of nature.' I can only agree with Weyl's demand; it is the importance of mathematics that they are. I can only agree with Weyl's demand; it is the importance of mathematics that, in uncovering more general possibilities, it marks the special facts of experience as special and thus preserves physics from naivity [Simplizität]. Admittedly, Weyl succeeds in explaining the unambiguous transferability of natural standards only very imperfectly. But the fact that Weyl tried to go this way, regardless of the empirical correctness of his theory, remains an ingenious advance towards the philosophical foundation of physics (Reichenbach, 1922b, 367f.).

Weyl's point was not that the axiom of Riemann class is necessarily false for a priori reasons, but, on the contrary, that is not a priori true as it was previously assumed. It cannot be a coincidence if two measuring rods placed next to each other are of the same length regardless of their location; this coincidence cries out for an explanation. Weyl's explanation of the apparent Riemannian behavior of "trough the adaptation to radius of 'curvature of the world'" (Reichenbach, 1922b, 368; fn. 1) only means posing a problem rather than providing an answer. The problem would be solved only by developing a proper theory of matter. However, even if this theory could be provided, Reichenbach, like Einstein, found the idea of deducing the Riemannian behavior of real clocks from a theory based on the non-Riemannian behavior of geometrical lengths awkward. In this way, the "congruent transplantation [...] remains physically empty" (Reichenbach, 1922b, 368; fn. 1). If the non-Riemannian congruent transplantation of vectors must be, Reichenbach argues, then the real rods should better behave in a non-Riemannian way.

Thus, Reichenbach concluded, the main achievement of Weyl was mathematical and not physical. As it has often happened in the past, mathematics enlarges the range of possibilities among which physicists are allowed to choose. This process, however, is far from being concluded with Weyl's rather special affine connection:

The philosophical significance of Weyl's discovery consists in the fact that it proved that the problem of space cannot be considered concluded even with Riemann's concept of space. If today epistemology tried to update Kant's transcendental aesthetics by claiming that the geometry of experience must, in any case, at least have a Riemannian structure, it would be refuted by Weyl's theory. That Weyl geometry is at least possible for reality cannot be denied. One must not even believe that Weyl's theory has reached the highest level of generality. Einstein (1921b) has shown that Weyl's requirement of the relativity of magnitude can also be satisfied without making use of Weyl's method of measurement. After that, Eddington (1921a) developed a further generalization of which Weyl's space [Raumklasse] is only a special case. Eddington's space [Raumklasse] is again included as a special case in a more general one found by Einstein (1921b). The merit of Schouten's theory is that it gives the conditions under which a space [Raumklasse] ca be considered as the most general; they are very general conditions, like differentiability and the like. But, of course, there is no absolutely most general space; and the history of the mathematical problem of space may teach epistemology never to make general claims of this kind. There are no most general concepts (Reichenbach, 1922b, 368; fn. 1).

This passage essentially repeats the argument that Reichenbach had used in his habilitation: there is nothing special in Weyl geometry. At the same time, it shows that, in the meantime, Reichenbach had closely followed the development of the unified field theory-project. He was familiar with Einstein's

²⁴This choice of words is similar to that of Pauli, who claimed that Weyl's theory in the second form lost his *Uberzeugunggkraft* (Pauli, 1921, 763; tr. 1958, 196).

(1921b) 'conformal' theory, in which distances can be compared only at a single point with light rays, but the comparison at distant points with transportable rods was not defined. Reichenbach also knew Eddington's (1921a) purely affine approach in which lengths go vectors cannot be compared even not at the same place in different directions. Moreover, it is quite impressive that he was even acquainted with Schouten's (1922) recent systematic classification of connections. Thus, Reichenbach was already aware that, in principle, also the natural assumption of the symmetry of the $\Gamma^{\tau}_{\mu\nu}$ could be dropped. In general, by further relaxing the constraints on the symmetry and on the relationship between the connection and the metric, one could open a great number of possibilities to incorporate the electromagnetic field into the geometrical structure of spacetime.

In Reichenbach's view, physicists should be completely free to choose among all these mathematical possibilities; however, mathematics alone cannot provide a criterion of choice for which possibility is realized in nature. Mathematics is the science of possibility, physics only is the science of reality. However, once the choice has been made, it is essential to 'coordinate' the structure chosen with the behavior of various idealized physical entities used as probes. Only in this way, the choice of the geometrical structure of spacetime can be tested experimentally. E.g., in general relativity, the claim that in the presence of real gravitational fields, the spacetime geometry is non-flat can be confirmed or disconfirmed by rods-and-clocks measurements. Weyl had initially proceeded following the same epistemological model W-I. In the presence of the electromagnetic field rods and clocks should behave in a non-Riemannian way. However, this geometrical prediction turned out to be empirically inadequate. As a response, Weyl embraced W-II. He introduced a sort of conspiratorial distortion of all measuring instruments. However, in this way, he deprived the geometrical setting of any empirical content.

However, Weyl disagreed with Reichenbach's historical reconstruction, but for reasons that reveal a completely different frame of mind. In a letter to Reichenbach written when the latter's review article was already in press, Weyl confessed that he actually never abandoned W-I in favor of W-II. As a matter of fact, he never adopted W-I in the first place: "I never gave up the plan to identify rigid rods with my transplantation, because I've never had that plan"; on the contrary, "I was surprised when I said that physicists had interpreted that into my words" (Weyl to Reichenbach, May 20, 1922; HR, 015-68-02). The atoms that we use as clocks are physical systems like any other and do not have in principle any privileged relation with the abstract mathematical behavior of vectors. It is the theory that decides whether we should use them as reliable clocks or not. In general, the physical interpretation of the theory's mathematical structure in terms of the behavior of idealized physical entities, like rods and clocks can only be provisional. Ultimately, one has to find the field equations governing that structure and require that solutions to these equations exist exhibiting the postulated behavior of rods and clocks. This reasoning applies to Einstein and Weyl's theory: "Einstein has to show that from the dynamics of the rigid body, it follows that the rod always has the same length, measured in his ds. Similarly, I have to show that the rod has always has the same length normalized ds normalized by R = const" (Weyl to Reichenbach, May 20, 1922; HR, 015-68-02). In both cases, the behavior of rods and clocks comes out as a byproduct of the theory.²⁵ However, in Weyl's theory, the Riemannian behavior of rods and clocks that came out at end contradicts the non-Riemannian length connection on which the theory was based (see e.g., Du Pasquier to Einstein, Dec. 13, 1921; CPAE, Vol. 12, Doc. 379).

2 Geometrization: The Reichenbach-Einstein Correspondence (1926–1927)

Up to this point, Reichenbach had good reasons to believe that his criticisms of Weyl's approach were broadly in agreement with Einstein's point of view. Thus, Einstein continued to express skepticism towards Weyl's 'Hegelian' approach to physics (Einstein to Zangger, Jan. 1, 1921; CPAE, Vol. 12, Doc. 5). He lamented a lack of "physical clues" for these attempts at unification (Einstein to Lorentz, Jun. 30, 1921; CPAE, Vol. 12, Doc. 163) that were therefore still too speculative (Einstein to Weyl, Jun. 6, 1922; CPAE, Vol. 13, Doc. 219; Einstein to Zangger, Jun. 18, 22; CPAE, Vol. 13, Doc. 241). However, the situation changed by the end of 1922, when Einstein, during a trip to Japan, started to realize that Eddington's theory had potentialities that had not been fully exploited.

On the shipboard, he jotted down a five-page manuscript dated January 1923 from Singapore (CPAE, Vol. 13, Doc. 417). Ironically, the third, fourth, and fifth pages were written on the back of the typescript of Reichenbach's Jena's talk (Reichenbach, 1922a). Eddington (1921a) had extended Weyl's approach

²⁵The unit of time should define a certain number of spacing between the atoms of a cubic crystal system; each of atom, in turn, consists of electrons and protons arranged according to a specific law. A specific solution to the field equations must provide information about all the details of this arrangement. The unit of time is a certain multiple of the vibration in a hydrogen atom, which, in turn, corresponds to a solution of the field equations.

by using only the coefficients of an affine connection $\Gamma^{\tau}_{\mu\nu}$, rather than metrical quantities $g_{\mu\nu}$ and φ_{ν} , as fundamental variables. In this context, vectors' lengths are not comparable even not at the same place; thus, in Einstein's view, the theory avoided Weyl's inconsistency of having geometrical lengths behaving differently from real rods and clocks. On February of 1923, Planck presented Einstein's attempt to derive a set of field equations to the Prussian Academy of Sciences (Einstein, 1923b). After he returned to Berlin, Einstein published two further integrations on the same approach in May Einstein, 1923b,c

In May of 1923, Reichenbach requested a copy of Einstein's paper "on Eddington's extension [Erweiterung" (Einstein, 1923b) (Reichenbach to Einstein, May 2, 1923; EA, 20 080). He did not comment on Einstein's unification attempt at this point.²⁶ In his correspondence with Einstein, Reichenbach was rather concerned with more mundane matter of finding a publisher for his work on the axiomatization of special relativity that he had just finished (Reichenbach to Einstein, Apr. 19, 1923; EA, 20 079, Reichenbach to Einstein, May 2, 1923; EA, 20 080, Einstein to Reichenbach, Jun. 9, 1923; EA, 20 081, Reichenbach to Einstein, Jul. 10, 1923; EA, 20 082). Due to lack of founding, Reichenbach managed to publish the book only a year later in March 1924. With the Axiomatik der relativistischen Raum-Zeit-Lehre Reichenbach (1924) Reichenbach's philosophy started to assume a more recognizable contour. In particular, Reichenbach introduced, for the first time, his celebrated distinction between "conceptual definitions" used in mathematics and "coordinate definitions" used in physics which relate the concept of a theory to a "piece [Ding] of reality" (Reichenbach, 1924, 5; tr. 1969, 8). There is little doubt that Reichenbach believed that this epistemological model was Einsteinian in spirit. However, at about that time, Einstein explicitly confessed that he changed his mind on the topic (Einstein, 1924, 1692, see Giovanelli, 2014). In particular, he denied that every individual concept of a theory should receive a measurement-operational justification.²⁷ Ultimately only geometry and physics taken together could be compared with experience, 28 a claim that seems to have a quite different meaning than Reichenbach had initially surmised.

The Axiomatik der relativistischen Raum-Zeit-Lehre (Reichenbach, 1924) received a lukewarm reception from philosophers, who probably found the book overly technical. However, it was Weyl's (1924) the negative review that was a hard blow for Reichenbach and put an end to their previously amicable relationship. Reichenbach felt that Weyl had used his authority as a mathematician to attack his 'empiricist' reading of relativity (Reichenbach, 1925). What was worse, Reichenbach must have sensed that Weyl's 'geometrical' reading of relativity had taken over relativistic research. Einstein's latest works seemed to reveal that he had also fall under its spell.²⁹ It is not surprising that Reichenbach might have felt necessary to make a case for a different interpretation of relativity theory in a more accessible form. At about the same time, he started work on a two-volume book with the ambitious title Philosophie der exakten Naturerkenntnis. Only the first volume on space and time will be published. He wrote the first chapters in March 1925 (HR, 044-06-25).

During those same months, Reichenbach, despite the support of Max Planck, was struggling to obtain his Umhabilitation³⁰ from Stuttgart to Berlin in order to be appointed to a chair of natural philosophy

²⁶As one might infer from Reichenbach's later writings, his point of view might have been again similar to that of Pauli. In a long latter to Eddington of September 1923, Pauli insisted that a good theory should start "with the definition of the used field quantities, of how this quantities can be measured" (Pauli to Eddington, Sep. 23, 1923; WPWB, Doc. 45). One of the great achievements of relativity theory was that the coefficients $g_{\mu\nu}$ could be measured with rods and clocks. Pauli explained Weyl attempted to pursue this strategy again but then abandoned this approach. (Pauli to Eddington, Sep. 23, 1923; WPWB, Doc. 45). In this way, he produced, what Eddington had rightly called a 'graphical representation' of the two fields in unified formalism, but not a 'natural geometry' found experimentally as in general relativity (see Eddington, 1923, 197). Similarly, in Einstein-Eddington new theory "[t]he quantities $[\Gamma^{\tau}_{\mu\nu}]$ cannot be measured directly" (Pauli to Eddington, Sep. 23, 1923; WPWB, Doc. 45). The measurable quantities $g_{\mu\nu}$ and $F_{\mu\nu}$ can be calculated from the $\Gamma^{\tau}_{\mu\nu}$ only through complicated calculations. Thus, not not only we do not have a "'natural geometry' but also not a 'natural theory' (Pauli to Eddington, Sep. 23, 1923; WPWB, Doc. 45).

²⁷ In particular, I would like to mention that criticism was rightly aimed against one statement by the reviewer: that a concept should only be permissible in physics when it can be established whether or not it applies in concrete cases of observation; it is objected that, in general, it is not to an individual concept that possible experiences must correspond but to the system as a whole" (Einstein, 1924, 1691).

²⁸In general relativity, the $g_{\mu\nu}$ had a physical meaning ex ante; they were supposed to be measurable with rods and clocks with respect to a given coordinate system. In his recent theory, Einstein introduced the 40 coefficients $\Gamma^{\tau}_{\mu\nu}$ as fundamental field variables without giving this quantity any physical meaning. If using the $\Gamma^{\tau}_{\mu\nu}$ leads to the right set of field equations, then the initial choice would turn out to be justified post facto (see Einstein, 1924). In this sense, only geometry $(g_{\mu\nu}, \Gamma^{\tau}_{\mu\nu}, \Gamma^{\tau}_{\mu\nu})$ etc.) and physics (field equations) together could be compared with experience (e.g., by finding appropriate exact solutions corresponding to electrons) "From this point of view is the whole content of geometry conventional. Which geometry one should chose depends on how 'simple' the physics can be brought in harmony with experience" (Einstein, 1926, 19).

²⁹As Weyl himself ironically remarked, Einstein undertook "the same purely speculative paths which [he was] earlier always protesting against" (Weyl to Einstein, May 18, 23; CPAE, Vol. 13, Doc. 30; cf. Weyl to Seelig, May 19, 1952, cit. in Seelig, 1960, 274f.).

30 The process of obtaining the *venia legendi* at another university.

that had been created there (Hecht and Hoffmann, 1982). Reichenbach had been attacked for his pacifist positions during the war. After the situation seemed to have turned for the better, in October 1925, he started to work more consistently on his book project. He interrupted the drafting of the manuscript to follow the emerging quantum revolution at the turn of 1926, and must have started again a few months later: "March-April 1926 Weyl's theory was worked on, and the peculiar solution of §49 was found. The entire Appendix was also written at that time. (Correspondence with Einstein)" (HR, 044-06-25). The correspondence with Einstein mentioned in this passage has been preserved. It testifies about Reichenbach's concerns with Einstein's style of doing physics becoming progressively more speculative, forgoing the solid empirical foundation of his old theory of gravitation.

2.1 Reichenbach's Geometrization of the Electromagnetic Field

During a trip to South America in 1925, Einstein became interested in the rationalistic and realistic reading of relativity proposed by Émile La déduction relativiste (Meyerson, 1925) CPAE, Vol. 14, Doc. 455, 6; March 12 who could provide a more adequate philosophical support for the search of a unified field theory than Schlick's or Reichenbach's 'positivism'. However, he also realized that the Weyl-Eddington-Schouten line had dried up (CPAE, Vol. 14, Doc. 455, 9; March 17). By returning from South America, he embraced what he considered to be a new approach. He introduced non-symmetric $\Gamma^{\tau}_{\mu\nu}$ and the $g_{\mu\nu}$ to be treated as independent fields in the variation. The antisymmetric part of the $g_{\mu\nu}$ was the natural candidate for representing of the electromagnetic field, at least for infinitely small fields. The physical test depended, as usual, on the construction of exact regular solutions corresponding to elementary particles. The paper was published in September of 1925 with the ambitious title Einheitliche Feldtheorie von Gravitation und Elektrizität (Einstein, 1925a). However, by that time, Einstein seemed to have already lost his confidence in that approach and moved on.³³

At the turn of the year, after working on the new quantum mechanics, Reichenbach must have read Einstein's new paper. On March 16, 1926, Reichenbach sent a letter to Einstein in which, after discussing his academic misfortunes, he made some critical remarks (Einstein, 1925a). Reichenbach was quite skeptical of the viability of Einstein's current style of doing physics:

I have read your last work on the extended Rel. Th. ³⁴ more closely, but I still can't get rid of a sense of artificiality that characterizes all these attempts since Weyl. The idea, in itself very deep, to ground the affine connection independently of the metric on the $\Gamma^{\tau}_{\mu\nu}$ alone, serves only as a calculation crutch here in order to obtain differential equations for the $g_{\mu\nu}$ and the φ_{ν} and the modifications of the Maxwell equations which allow the electron as a solution. If it worked, it would, of course, be a great success; have you achieved something along these lines with Grommer? However, the whole thing does not have the beautiful convincing power [*Ueberzeugungskraft*] of the connection between gravitation and the metric based on the equivalence principle of the previous theory (Reichenbach to Einstein, Mar. 16, 1926; CPAE, Vol. 15, Doc. 224).

. Reichenbach's objections are quite sensible and not dissimilar to those of professional physicists.³⁵ In general relativity, the choice of the $g_{\mu\nu}$ as fundamental variables is anchored in the principle of equivalence.

³¹Meyerson rationalistic realism offered a philosophical alternative to both empiricists and positivists. Against the positivists, he defended the logical independence of concepts from sensory experiences, against the Kantians that that the non-empirical 'ideal' conceptual constructions were mere conventions. Against both, Meyerson justified that the physicists are justified to assume that certain conceptual construction exists in reality, say electromagnetic field, electron independently of observation.

³²These doubts became a certainty when Einstein returned to Europe. "On June 1, I got back from South America," Einstein wrote to Besso, "I am firmly convinced that the whole line of thought Weyl-Eddington-Schouten does not lead to anything useful from a physical point of view and I found a better trail that is physically more grounded" (Einstein to Besso, Jun. 5, 1925; CPAE, Vol. 15, Doc. 2).

³³In July, he was still convinced that he had "really found the relationship between gravitation and electricity" (Einstein to Millikan, Jul. 13, 1925; CPAE, Vol. 15, Doc. 20). However, during the summer, Einstein had already started to nurture some skepticism (Einstein to Ehrenfest, Aug. 18, 1925; CPAE, Vol. 15, Doc. 49; Einstein to Millikan, Jul. 13, 1925; CPAE, Vol. 15, Doc. 20; Einstein to Ehrenfest, Sep. 18, 1925; CPAE, Vol. 15, Doc. 71). The paper was published at the beginning of September, and by that time, Einstein probably already moved on (Einstein to Rainich, Sep. 13, 1925; CPAE, Vol. 15, Doc. 106; see Einstein, 1927a).

 $^{^{34}}$ Einstein, 1925a.

³⁵In a review of the German translation (Eddington, 1925) of Eddington's relativity textbook (Eddington, 1923) that came out a few weeks later, Pauli expressed similar concerns Pauli (1926). Without the equivalence principle the entire geometrization program appeared to Pauli unjustified: "An attempt at an analogous geometrical interpretation of the electromagnetic field faces the difficulty that there is no empirical fact corresponding to the equality of heavy and inert mass, which would make such an interpretation appear 'natural' " (Pauli, 1926). The solution was to avoid any connection between geometry and behavior of rods and clocks. However, in this way one could at most produce what Eddington called a 'graphical representation.' According to Pauli, similar objections can be raised against Einstein's last work (Einstein, 1925b).

The latter justified the double meaning of the $g_{\mu\nu}$, as determining the behavior rods and clocks, as well as the gravitational field. On the contrary, Einstein's new theory introduces the non-symmetric affine connection $\Gamma^{\tau}_{\mu\nu}$ independently of the metric $g_{\mu\nu}$ without giving to these field variables any physical motivation. The separate variation of the metric and connection was nothing more than a 'calculation device' to find the desired field equations. Only in hindsight, for formal reasons, the symmetric part was identified of $g_{\mu\nu}$ was identified with the gravitational field and antisymmetric with the electromagnetic field. In this form, the theory has little he 'convincing power' (Uberzeugungskraft)—the same expression that Reichenbach (1922a, 367) had used for characterizing Weyl's theory in his second form. Reichenbach would have been ready to retract his criticism, if Einstein's theory delivered the 'electron.' This concession, however, barely hides his skepticism that a field-theory of matter was a concrete possibility.

Einstein replied on March 20 that he agreed with Reichenbach's 'Γ-Kritik': "I have absolutely lost hope of going any further using these formal ways"; "without some real new thought" he continued, "it simply does not work" (Einstein to Reichenbach, Mar. 20, 26; CPAE, Vol. 15, Doc. 230). Einstein's reaction reflects his disillusion with the attempts with an approach based on the generalization of Riemannian geometry by weakening the relations by $g_{\mu\nu}$ and $\Gamma^{\tau}_{\mu\nu}$. He would have probably been less ready to embrace the actual implications of Reichenbach's Γ-critique, the requirement that the operation of parallel displacement of vectors should receive a 'coordinative definition' from the outset. At any rate, Reichenbach took the opportunity of Einstein's positive reaction, and on March 31, 1926 sent him a note (Reichenbach, 1926b), in which he developed the Γ-critique in detail (Reichenbach to Einstein, Mar. 24, 1926; CPAE, Vol. 15, Doc. 235).

A point by point commentary of Reichenbach's note has been provided elsewhere (Giovanelli, 2016). Reichenbach introduced a non-symmetric $\Gamma^{\tau}_{\mu\nu}$ in order to define an operation of displacement expressing the effect of both the gravitational and electromagnetic fields. In Riemannian geometry, the straightest lines is also the shortest lines between two points. If the connection is non-symmetric, the straightest lines generally do not coincide with the shortest. Charged mass points of unit mass move (or their velocity four-vector is parallel-transported) along the straightest lines, and uncharged particles move on the straightest lines that are at the same time the shortest ones (or rather, the line of extremal length) (Reichenbach, 1926b).

Einstein was not impressed (Einstein to Reichenbach, Mar. 31, 1926; CPAE, Vol. 15, Doc. 239). Thus, Reichenbach rushed to point out that Einstein had misunderstood the spirit of the typescript. As Reichenbach explained, he was working on a book on the philosophy of space and time. "Thereby I wondered what the geometrical presentation of electricity actually means" (Reichenbach to Einstein, Apr. 4, 1926; CPAE, Vol. 15, Doc. 244). Reichenbach wanted to challenge the idea that geometrizing a field is per se a useful heuristic strategy: "If one succeeds in establishing unified field equations that admit the electron as a solution, this would be something new." (Reichenbach to Einstein, Apr. 4, 1926; CPAE, Vol. 15, Doc. 244). To this purpose, however, the field Maxwell and Einstein equations should be modified. "This is the problem on which you are working and of course also what Weyl and Eddington meant" (Reichenbach to Einstein, Apr. 4, 1926; CPAE, Vol. 15, Doc. 244). However, the geometrical representation of electricity in itself does not lead to this goal. "It can at most be an aid [Hilfsmittel] to guessing the right equations"; it might be that "what looks most simple from the standpoint of Weyl geometry also happens to be correct. But this would be only a coincidence" (Reichenbach to Einstein, Apr. 4, 1926; CPAE, Vol. 15, Doc. 244).

With his theory Reichenbach "wanted to turn against the notion that something had already been gained with the geometrical presentation of electricity" (Reichenbach to Einstein, Apr. 4, 1926; CPAE, Vol. 15, Doc. 244). In comparison with Eddington or Einstein's last theories, Reichenbach insisted, his approach had even the advantage "that the operation of displacement possesses a physical realization [Realisierung]" (Reichenbach to Einstein, Apr. 4, 1926; CPAE, Vol. 15, Doc. 244), namely, the velocity-vector of charged mass particles. In this way, the notion of straightest and shortest lines are physically meaningful. For this reason, in Reichenbach's view, his geometrization was comparable to that provided by general relativity. Nevertheless, differently from Einstein's theory of gravitation, Reichenbach's theory did not lead to any new physical prediction. Thus, Reichenbach concluded, a successful geometrization does not necessarily lead to a successful physical theory.

Although Einstein probably continued to find the technical details of Reichenbach's attempt questionable, his philosophical point clearly resonated with Einstein:

You are completely right. It is incorrect to believe that 'geometrization' means something essential. It is instead a mnemonic device $[Eselsbr\"{u}cke]$ to find numerical laws. If one combines geometrical representations [Vorstellungen] with a theory, it is an inessential, private issue. What is essential in Weyl is that he subjected the formulas, beyond the invariance with respect to [coordinate] trans-

formation, to a new condition ('gauge invariance'). However, this advantage is neutralized again, since one has to go to equations of the 4. order, ³⁷ which means a significant increase of arbitrariness (Reichenbach to Einstein, Apr. 8, 1926; CPAE, Vol. 15, Doc. 249).

Einstein seems to endorse Reichenbach's claim that a 'geometrization' is not an essential achievement of general relativity. However, it is worth noticing that Einstein goes beyond Reichenbach and claims that the very notion of 'geometry' is meaningless (Lehmkuhl, 2014). The $g_{\mu\nu}$, $\Gamma^{\tau}_{\mu\nu}$, etc. are ultimately multi-components mathematical objects characterized by their transformation properties under change of coordinates. There is nothing 'geometrical' about those quantities. Thus, Einstein's point is only superficially similar to that of Reichenbach. Einstein declared that the difference between geometry and the rest of mathematics was inessential. On the contrary, as we shall see, Reichenbach intended to show that the difference between geometry and physics was essential. Einstein's argument was meant to provide a support of the unified field theory-project. Against those that believed that the geometrization program could not be extended beyond the gravitational field, he could argue that geometrization has never been the issue in the first place.³⁸ Reichenbach's argument was on the opposite an attack on the unified field theory-project, which was based on the idea the geometrization in itself would have led to physical results.

The Appendix to the Philosophie der Raum-Zeit-Lehre

Strengthened by Einstein's endorsement in May Reichenbach presented the note in Stuttgart at the regional meeting of the German physical Society (Reichenbach, 1926a). In the following months, Reichenbach must have further work on the manuscript of his book and by the end of the year, he could write to Schlick that "[t]he first volume that deals with space and time [was] finished" (Reichenbach to Schlick, Dec. 6, 1926; SN). Reichenbach hoped to publish the book in the forthcoming Springer series 'Schriften zur wissenschaftlichen Weltauffassung' directed by Schlick and Philipp Frank. However, Springer rejected the book as being too long. By July, Reichenbach could announce to Schlick that he had reached a publication arrangement with de Gruyter (Reichenbach to Schlick, Jul. 2, 1927; SN). The publisher agreed to publish only the first volume under the title Philosophie der Raum-Zeit-Lehre. According to Reichenbach's recollections, the manuscript was not changed significantly after February 1927 (HR, 044-06-25). The drafts were finished in September, and the preface was dated October 1927. The note that Einstein had sent to Einstein in Spring 1926 became, with few changes, the §49 of a longer Appendix dedicated to the modern development of differential geometry and to the problem of geometrical interpretation of electricity. If read with the inclusion of the Appendix 39 the Philosophie der Raum-Zeit-Lehre appears as a much more complex book. It was not only as a defense of a 'conventionalist' reading of the foundations of geometry, as it is usually claimed; it was at the same time an attack on the widespread of interpretation of general relativity as a 'geometrization' of the gravitational field Reichenbach, 1928a, 294; tr. 1958, 256.

The Appendix to the *Philosophie der Raum-Zeit-Lehre* was nothing but the continuation of this line of argument, which only partially developed in the last chapter of the book. "The geometrical interpretation of gravitation", Reichenbach wrote using an effective analogy "is merely the visual cloak in which the factual assertion" encoded by the equivalence principles "is dressed" (Reichenbach, 1928a, 354; tr. [493]). The cloak might be conceived as an inextensible network of rods and clocks, that have to be tailored to the body of the gravitational field. However, "[i]t would be a mistake to confuse the cloak with the body which it covers; rather, we may infer the shape of the body from the shape of the cloak which it wears. After all, only the body is the object of interest in physics" (Reichenbach, 1928a, 354; tr. [493]). The fact that a Euclidean cloak, so to speak, does not fit the body of a real gravitational field allows knowing something new about the shape of the body, that is to make the new predictions about the behavior of free-falling mass particles, light rays, clocks, etc. Unfortunately, according to Reichenbach, recent relativistic research seemed to have confused the cloak for the body itself. "The great success, which Einstein had attained with his geometrical interpretation of gravitation" led many "to believe that similar success might be obtained from a geometrical interpretation of electricity" (Reichenbach, 1928a, 352; tr. [491]).

After the physics community accepted general relativity as a theory of gravitation, the search for a suitable geometrical cloak that could cover the naked body of the electromagnetic field began. The

That is, invariance by the substitution of g_{ik} with λg_{ik} where λ is an arbitrary smooth function of position (cf. Weyl, 1918b, 468). Weyl introduced the expression 'gauge invariance' (Eichinvarianz) in Weyl, 1919a, 114.

³⁷Cf. Weyl, 1918b, 477. Einstein regarded this as one of the major shortcomings of Weyl's theory; see Einstein to Besso, Aug. 20, 1918; CPAE, Vol. 8b, Doc. 604, Einstein to Hilbert, Jun. 9, 1919; CPAE, Vol. 9, Doc. 58.

38 Pauli's (1926) review of the German translation of Eddington (1925) is a typical example of this type of criticism.

 $^{^{39}\}mathrm{The}$ Appendix was not included in the English translation Reichenbach, 1958.

separation of the 'operation of displacement of vectors' $\Gamma^{\tau}_{\mu\nu}$ from the operation of comparison of length at a distance $g_{\mu\nu}$ gave physicists new mathematical degrees of freedoms that could be exploited to accommodate the electromagnetic field alongside the gravitational field. "However, the fundamental fact which would correspond to the principle of equivalence is lacking" (Reichenbach, 1928a, 354; tr. [493]). Thus, physicists needed to proceed by trial and error in the search for suitable geometrical-field variables. Initially, attempts were made to identify these geometrical structures with 'true' the geometry of spacetime. The latter was supposed to be endowed with a more general affine structure. To give this claim empirical content, Weyl initially provided a "realization of the process of displacement" in terms of the behavior of rods and clocks. Weyl's project failed because rods and clocks, in the presence of an electromagnetic field, did not behave as predicted by the theory. "This means that we have found a clock in which we can dress the new theory, but we do not have the body that this new cloak would fit" (Reichenbach, 1928a, 353; tr. [493]).

Nevertheless, physicists did not abandon the geometrization program. Instead, they came to the conclusion that "such 'tangible' [handgreifliche] realizations does not lead to the desired field equations" (Reichenbach, 1928a, 371; tr. [517]). Thus, theories were proposed by "Weyl, Eddington and Einstein" which "renounced such a realization of the process of displacement" (Reichenbach, 1928a, 371; tr. [517]). The geometrical structure chosen, say the $\Gamma^{\tau}_{\mu\nu}$, φ_{ν} , the $\Gamma^{\tau}_{\mu\nu}$ and $g_{\mu\nu}$, etc. did not have any physical meaning from the outset, i.e., the values of the coefficients of those were not the results of measurements. "Einstein, in particular, has devised several new formulations in which the geometrical interpretation is reduced to the role of a mathematical tool [Rechenhilfsmittels]" (Reichenbach, 1928a, 369; tr. [516]). The trick was to find right dynamical variables from which the right action can be constructed from which the desired equations could be derived. However, since the fundamental variables do not have any physical meaning, the field equations thus obtained could not be compared with experience directly, as it happens in the case of general relativity.

The field equations could be confronted with reality only by integrating them in the hope that they deliver the 'electron.' On the one hand, in Maxwell's electrodynamics the cohesion of the electron's charge has always been attributed to a 'foreign force'; namely, the force of cohesion that keeps the Coulomb forces from exploding. Einstein's theory of gravitation, on the other hand, does not imply any effect of gravitation on charge and cannot, therefore, yield the cohesive force. To find a solution to the problem of matter, Maxwell's and Einstein's field equations should be valid to "a high degree of approximation" to recover the success of previous theories in the case of weak fields; yet they should be "changed, because, otherwise, they would never give us the electron as a solution" (Reichenbach, 1928a, 370; tr. [517]). Physicists have to guess what kind of change has to be put forward. Without an analogon of the equivalence principle, they have become convinced that, "[[i]n this 'guessing,' the geometrical interpretation of electricity is supposed to be the guide" (Reichenbach, 1928a, 371; tr. [517]). The point of departure in this approach was "the (unwritten) assumption that whatever looks simple and natural from the viewpoint of the geometrical interpretation will lead to the desired changes in the equations of the field" (Reichenbach, 1928a, 370; tr. [517]).

"The many ruins along this road", Reichenbach pointed out, should have suggested physicists "that solutions should be sought in an entirely different direction" (Reichenbach, 1928a, 370; tr. [517]). Why did they still persist? Reichenbach quite perceptively grasped their psychological motivation: "It is not the geometrical interpretation of electricity" but a deeper assumption which lies at the basis of all these attempts; namely, "the assumption that the road to a simple conception, in the sense of a geometrical interpretation, is also the road to true relationships in nature" (Reichenbach, 1928a, 370; tr. [517]). The geometrical interpretation provided by general relativity was based on a physical hypothesis, the equivalence principle, which, in turn, was based on an empirical fact, the identity of gravitational and inertial mass. The unified field theory-project is based on a different physical hypothesis of a more speculative nature, the hypothesis that the world is geometrically simple. Indeed, by reading papers on the unified field theory, one is struck by the fact that they are full of expressions like 'most natural assumption,' 'simplest invariant', etc. (Reichenbach, 1928a, 370; tr. [517])

Needless to say, the idea that the 'simplicity' of geometrical setting could have bearing for its physical truth appeared Reichenbach the consequence of a severe conceptual mistake (Reichenbach, 1928a, 372; tr. [519]). The final decision if the unified field theory-project is worth pursuing, he conceded, "must be left to the physicist, whose physical instinct provides the sole illumination" (Reichenbach, 1928a, 372; tr. [519]). Ultimately, the scientists' "physical instinct", their deep conviction that the world is mathematically simple pertains to the realm of the logic of discovery and thus lies outside the competence of epistemology. However, Reichenbach made no mysteries that, by denouncing once again physicists' neverending temptation to blur mathematics and physics, he hoped that his epistemological analysis could tie

scientists to the mast of empiricism protecting them from "the sirens' enchantment [Sirenenzauber] of a unified field theory" (Reichenbach, 1928a, 373).

3 Unification: Reichenbach-Einstein Correspondence (1929–1930)

In October 1927, Reichenbach moved back to Berlin, where he took the position of an "unofficial associate professor" (Hecht and Hoffmann, 1982). At about the same time, Einstein read the manuscript of the *Philosophie der Raum-Zeit-Lehre* (Einstein to Elsa Einstein, Oct. 23, 1927; CPAE, Vol. 16, Doc. 34). Soon after that, he wrote a short book review. Einstein was quite perceptive in pointing out the two themes that Reichenbach had treated in the Appendix: (1) "In the Appendix, the foundation of the Weyl-Eddington theory is treated in a clear way and in particular the delicate question of the *coordination* of these theories to reality" (Einstein, 1928c, 20; m.e.). As we have seen, Reichenbach had insisted that, as in any other theory, also in unified field theory based on the affine connection is a fundamental variable, one should give physical meaning to the operation of displacement from the outset. Einstein did not comment further on this issue since he realized that this requirement was too strict over the years. However, Einstein seemed to be in full agreement with the second point made by Reichenbach: (2) In the Appendix, "in my opinion quite rightly—it is argued that the claim that general relativity is an attempt to reduce physics to geometry is unfounded" (Einstein, 1928c, 20; m.e.). As we have seen, Reichenbach and Einstein had already discussed this topic in a private correspondence less than two years earlier (Giovanelli, 2016).

At about the same time, Einstein published a more extensive review of Meyerson's La déduction relativiste (Meyerson, 1925) in the Revue philosophique de la France et de l'étranger. The review reveals how Einstein's perspective had become quite different from that of Reichenbach on both issues. Einstein embraced Meyerson's rationalist philosophy, insisting on the deductive-speculative nature of physics' enterprise, implicitly disavowing the operational-empirical rhetoric that seemed to have dominated his early philosophical pronouncements. However, Einstein strongly disagreed with Meyerson's insistence that Weyl's and Eddington's theories were the crowning moment of a long process of geometrization of physics. He insisted again that geometry in this context is "devoid of meaning" (Einstein, 1928a, 165; m.e.). However, he also clarified the motivations against his critique of the geometrization program: "The essential point of the theories of Weyl and Eddington", was not to geometrize the electromagnetic field, but to "represent gravitation and electromagnetic under a unified point of view, whereas beforehand these fields entered the theory as logically independent structures" (Einstein, 1928a, 165; m.e.). Einstein's further attempts at unified field theory in the immediately following months reveal more clearly the reasons behind Einstein's philosophical turnabout.

In Spring 1928, during a period of illness, Einstein came up with a new proposal for a unified field theory. On June 7, 1928 Planck presented a note to the Prussian Academy on a 'Riemannian Geometry, Maintaining the Concept of Distant Parallelism' (Einstein, 1928d), a flat space-time that is nonetheless non-Euclidean since the connection $\Gamma^{\tau}_{\mu\nu}$ is non-symmetrical. He introduced a new formalism, based on the concept of n-Bein (or n-legs), n unit orthogonal vectors representing a local coordinate system attached to a point of an n dimensional continuum. Vectors at distance points considered as equal and parallel if they have the same local coordinates with respect to their n-bein. The vierbein-field h^{ν}_a defines both the metric tensor $g_{\mu\nu}$ and the electromagnetic four-potential φ_{μ} . Its sixteen components can be considered as the fundamental dynamical variables of the theory. The question arises as to the field equations that determine the vierbein-field. On June 14, 1928 he submitted a second paper in which the field equations are derived from a variational principle (Einstein, 1928b).

A few months later, when the paper appeared in print, Reichenbach managed to prepare a type-scripted note (Reichenbach, 1928b) with some comments that he send Einstein for feedback:

Dear Herr Einstein,

I did some serious thinking on your work on the field theory and I found that the geometrical construction can be presented better in a different form. I send you the ms. enclosed. Concerning the physical application of your work, frankly speaking, it did not convince me much. If geometrical interpretation must be, then I found my approach simply more beautiful, in which the straightest line at least means something. Or do you have further expectations for your new work? (Reichenbach to Einstein, Oct. 17, 1928 EA, 20-92; m.e.).

In this passage, Reichenbach makes two unrelated points, which, however, seem to be part of a single two-pronged argument. In the note sent to Einstein, Reichenbach (1928c) had shown that, if one lets aside from the n-bein formalism, Einstein's new geometrical settings could be easily inserted into the

Weyl–Eddington–Schouten lineage, as a special case of metric space in which the connection is flat, but non-symmetric.⁴⁰ If so, Reichenbach could raise the same objection he had raised against Einstein's previous theories.

According to Reichenbach, a "real physical achievement is obtained only if, moreover, the operation of displacement is filled with physical content" (Reichenbach, 1928c, 7). Einstein's geometry, being flat, implies the existence of a straight line, a line of which all elements are parallel to each other, which is nevertheless not identical with a geodesic (Einstein, 1928b, 224). However, as Reichenbach reported, the latter has no physical meaning in Einstein's theory. "If geometrical interpretation must be", Reichenbach concluded, then his §49-theory was preferable since the straightest lines and shortest line correspond to the motion of charged and uncharged test particles under the influence of the combined gravitational-electromagnetic field. Once again, Einstein's goal was to use this geometrical apparatus as a starting point to find the right 'action' from which a set of field equations. However, Reichenbach commented, nothing new came out of it: "[T]he derivation of the Maxwellian and gravitational equation from a variational principle was already achieved by other approaches" (Reichenbach, 1928c, 6), like, say, Einstein-Eddington purely affine theory.

In the subsequent letter, Einstein defended his classification of geometries but did not comment on Reichenbach's objection. However, he invited Reichenbach and his first wife Elisabeth for a cup of tea on November 5, 1928. On that occasion, Einstein might have informed Reichenbach about his plan to abandon the variational strategy to find the field equations (Sauer, 2006). However, it is quite probable that Einstein might have also explained to Reichenbach that his goal was not to provide a geometrization of the electromagnetic field, but to provide their unification of both fields. Thus, Einstein's choice of the field-structure was not motivated by geometrical considerations nor had a geometrical meaning. The goal was to recover a set of field equations yielding the classical equations of gravitation and electromagnetism only to first order. That is, the theory should predict new effects in the case of strong fields. To obtain this result, Einstein was ready to adopt a whatever-it-takes strategy. Not only he was ready to forgo any physical interpretation of the fundamental variables of the theory; he was even ready to abandon the variational approach as in the paper he was working on.

It is hard to imagine that the divergence of their philosophical views did not emerge during those discussions. In a semi-popular paper, Einstein had submitted a few weeks later (Einstein, 1929c, 131). Einstein insisted on the speculative nature of the new theory. One starts from this mathematical structure and then searches for the simplest and most natural field equations that the vierbein-field can satisfy (Einstein, 1929c, 131). The physical soundness of the field equations can be confirmed only by integrating them, finding particle solutions, and the laws governing their motions in the field. However, this was usually a challenging task. Einstein warned his readers of the dangers of proceeding "along this speculative road" (Einstein, 1929c, 127). "Meyerson's comparison with Hegel's program [Zielsetzung]" Einstein put it in a footnote, "illuminates clearly the danger that one here has to fear" (Einstein, 1929c, 127).

3.1 Reichenbach's Articles on Fernparallelismus field theory

In the late 1920s, Reichenbach was a regular contributor to the *Vossische Zeitung*, at that time Germany's most prestigious newspaper; not surprisingly, he was asked for a comment on Einstein's theory, which had started attracting irrational attention in the daily press (see Pais, 1982, 346). With the advantage of having personally discussed the topic with Einstein a few weeks earlier, Reichenbach published a brief didactic paper on Einstein's theory on January 25, 1929 (Reichenbach, 1929c). Reichenbach seems to have indeed profited from the conversation with Einstein. In particular, it is revealing that Reichenbach did not present Einstein *Fernparallelismus*—as it would been more natural in a popular writing—as an attempt of a *geometrization* of the electromagnetic field on par with the previous geometrization gravitational field achieved by general relativity. On the contrary, he decided to present *Fernparallelismus* in terms of an attempt of *unification* of two separate fields on par with a similar unifications operated by special and general relativity.

Although the article was fairly innocuous, Einstein was distraught by Reichenbach's decision to leak a private conversion to the press Einstein to Vossische Zeitung, Jan. 25, 1929; EA, 73-229. The exchange of the letters that ensued (Reichenbach to Einstein, Jan. 27, 1929; CPAE, Vol. 16, Doc. 384, Einstein to Reichenbach, Jan. 30, 1929; CPAE, Vol. 16, Doc. 390, Reichenbach to Einstein, Jan. 31, 1929; CPAE, Vol. 16, Doc. 391) put a serious strain in their personal relationships. This quarrel on a rather mundane

⁴⁰One starts from a general non-symmetric affine connection $\Gamma^{\tau}_{\mu\nu}$ and imposes the condition that the length of vectors does not change under parallel transport. Then, Einstein and Riemann space could be obtained via the "exchangeability of the specializations" (Reichenbach, 1928c, 5). If one imposes that the Riemann tensor vanishes, one obtains Einstein space; if one imposes that the connection is symmetric, one obtains Riemannian space.

matter seemed to have been coupled with a sense of a deeper intellectual estrangement. If Reichenbach's private letters expressed Reichenbach's despondency for Einstein's betrayal of their personal friendship, his published writings point to his disappointment for Einstein's betrayal of their shared philosophical ideals. By the time of the article's publication for the Vossische Zeitung, Reichenbach had already written two papers on the Fernparallelismus that are both dated January 22, 1929 that were published in the following months (Reichenbach, 1929b,d). These articles are Reichenbach's last contribution to issues related to relativity theory and spacetime theories. On the one hand, Reichenbach attempted to make his previous reflections about the unified field theory-project in the Appendix to the Philosophie der Raum-Zeit-Lehre to bear fruit (Reichenbach, 1928a, §46). On the other hand, he added new elements of clarification by clearly distinguishing the 'geometrization program' and the 'unification program'.

In the first paper for the Zeitschrift für Angewandte Chemie, Reichenbach introduced the history of the unified field theory in an entirely different manner than he had done before. In the Appendix to the Philosophie der Raum-Zeit-Lehre, the history of the unified field theory-project program was ultimately presented as a linear evolution of the geometrization program that had progressively become more abstract. Now Reichenbach—probably following the discussion he had with Einstein in November—describes the history of the unified field theory as the progressive downfall of the geometrization program and the concurrent rise of the unification one. After the failure of Weyl's first attempts, most physicists, including Einstein (1923, 1925), considered it preferable to sacrifice the geometrical interpretation—i.e., to relinquish the coordination of geometrical notion of parallel transport of vectors with the behavior rods and clocks—and then to use the geometrical variables ($\Gamma^{\tau}_{\mu\nu}$, φ_{ν} and so on) as 'calculation device' for the greater good of finding the field equations. Reichenbach had come to understand that, in Einstein's view, the aim of the unified field theory-project was not the geometrization of the electromagnetic field alongside the gravitational field; it was the unification of the electromagnetic and gravitational fields.

Thus, Reichenbach's concern became to explain what 'unification' means in this context. The problem was addressed in detail in the more technical paper, which grew out of the manuscript that Reichenbach had sent to Einstein (Reichenbach, 1929d). In the second part, Reichenbach introducded the distinction between (a) a formal unification and (b) an inductive unification. This distinction seems to be an application of Reichenbach's more famous classification of two types of simplicity (Reichenbach, 1924, 9, 1929a, §11) to the case of unified field theories. In this way, Reichenbach could describe two opposite approaches to the unified field theory-project. In this setting, his §49-theory came in handy. Reichenbach's theory uses a similar geometrical setting as Einstein's theory. Both use a non-symmetric affine connection. In Einstein's approach, the further conditions that the geometry is flat is imposed, allowing for distant parallelism.

According to Reichenbach, his §49-theory was able to provide a *proper* geometrical interpretation of the combined gravitational/electromagnetic field. However, the theory could achieve only a *formal* unification item a because no new testable predictions were made:

The author [Reichenbach] has shown that the first way can be realized in the sense of a combination of gravitation and electricity to one field, which determines the geometry of an extended Riemannian space; it is remarkable that thereby the operation of displacement receives an immediate geometrical interpretation, via the law of motion of electrically charged mass-points. The straightest line is identified with the path of electrically charged mass points, whereas the shortest line remains those of uncharged mass points. In this way, one achieves a certain parallelism to Einstein's equivalence principle. By the way, [the theory introduces] a space cognate to the one used by Einstein, i.e., a metric space with non-symmetrical $\Gamma^{\tau}_{\mu\nu}$. The aim was to show that the geometrical interpretation of electricity does not mean a physical value of knowledge per se (Reichenbach, 1929d, 688; m.e.).

If one wants to give a geometrical interpretation of a combined gravitational/electromagnetic field using the affine connection $\Gamma^{\tau}_{\mu\nu}$ as a fundamental variable, then one should at least provide a coordinate definition of the operation of parallel displacement of vectors before starting to search for the field equations. Otherwise, it is hard to understand how one could test whether the latter made correct predictions or not. Reichenbach's theory was meant to show that a successful geometrical interpretation of this kind can always be achieved with some mathematical trickery. However, a successful geometrization is insufficient to achieve a substantive unification. For Reichenbach, this should have been a warning that the very hope that the geometrical interpretation of a physical field itself was the key to new physical insights was misplaced.

Einstein Fernparallelismus-field theory is an instance of a second approach, which claims to achieve item b, an inductive unification, by forgoing the geometrical interpretation, that is, without providing a physical meaning of the $\Gamma^{\tau}_{\mu\nu}$ in terms of the motion of test particles:

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On the contrary, Einstein's approach of course uses the second way, since it is a matter of increasing physical knowledge; it is the goal of Einstein's new theory to find such a concatenation of gravitation and electricity, that only in first approximation it is split in the different equations of the present theory, while is in higher approximation reveals a reciprocal influence of both fields, which could possibly lead to the understanding of unsolved questions, like the quantum puzzle. However, it seems that this goal can be achieved only if one dispenses with an immediate interpretation of the displacement, and even of the field quantities themselves. From a geometrical point of view this approach looks very unsatisfying. Its justification lies only on the fact that the above mentioned concatenation implies more physical facts that those that were needed to establish it (Reichenbach, 1929d, 688; m.e.).

In Reichenbach's view, Fernparallelismus appeared not only as a formally satisfying unification but as a real advance over the available theories. It entails some coupling between the electromagnetic and gravitational fields that was not present in the given individual field theories. However, Reichenbach argues that Einstein could only achieve this result at the expense of a physical interpretation of the fundamental geometrical variables. As we have seen, Einstein's flat affine connection $\Gamma^{\tau}_{\mu\nu}$ defines a set of straight lines as privileged paths; however, these lines are not interpreted as paths of particles (Einstein, 1930e, 23). The laws governing the latter are unknown before integrating the field equations (Einstein to Cartan, Jan. 7, 1930; Debever, 1979, A-XVI). As consequence, the theory cannot be confirmed or disproved experimentally by observing the behavior of suitable indicators.

In Reichenbach's diagnoses, the stagnation of the unified field theory-project depended on the presence of a sort of trade-off between geometrization and unification of which physicists were only partially aware. General relativity was the only theory that was able to combine both virtues: (1) the theory provided a proper geometrical interpretation of the gravitational field because it introduced a coordinative definition of the field variables $g_{\mu\nu}$, in terms of the behavior of those that were traditionally considered geometrical measuring instruments, such as rods and clocks, light rays, free-falling particles (2) the theory provided a proper unification by predicting that the gravitational field had specific effects on such measuring instruments that were not implied by previous theories of gravitation—such as gravitational time dilation (Reichenbach, 1928a, 350). Successive attempts to include the electromagnetic field in the framework of general relativistic field theory failed to uphold this standard.

According to Reichenbach, the reason for this failure was ultimately not hard to pinpoint. The effective interplay between geometrization and unification did not seem reproducible without a proper analogon of equivalence principle. Without the equivalence principle, a further geometrization of electromagnetic fields was not worth pursuing since it had no physical justification.⁴¹ Einstein could counter these objections by claiming that geometrization had never been the goal. The achievement general relativity was have combined inertial and gravitational just like special relativity has combined magnetic and electric field as components of a unified field structure. However, without an analogon of the equivalence principle, there seems to be also no physical justification for searching for further unification of the electromagnetic and gravitational field. Nevertheless, Einstein clearly considered the separation between the two fields as theoretically unbearable (Einstein, 1930e, 24). However, we do not have any physical clue as to what the more comprehensive mathematical structure may be, in which the electromagnetic and gravitational fields will be appear as two sides of the same field. Einstein was forced to resort to the criterion of mathematical simplicity which is not free from arbitrariness.

To Reichenbach's dismay, Einstein had abandoned the *physical heuristic* that leads him to general relativity in the name of a *mathematical heuristic* that was not different from Weyl's speculative approach that he had dismissed a decade earlier. As we have seen, as early as in his habilitation, he considered the great achievement of relativity theory the *separation of mathematical necessity and physical reality*. Reichenbach had always perceived this separation as nothing more than a philosophical distillation of Einstein's scientific practice. However, in the search for a unified field theory, Einstein had come implicitly to question this distinction, coming close to a plea for a *reduction of physical reality to mathematical necessity*. Einstein put it candidly in his Stodola-*Festschrift*'s contribution—that he sent for publication toward the end of January (Einstein to Honegger, Jan. 30, 1929; CPAE, abs. 864). The ultimate goal of understanding reality is achieved when one could prove that "even God *could not have established these connections otherwise* than they actually are, just as little as it would have been in his power to make the number 4 a prime number" (Einstein, 1929c, 127).

⁴¹see section 1.2.

4 Conclusion

Just after the publication of the new derivation of the Fernparallelismus-field equations at the end of January 1929 (Einstein, 1929d), Einstein wrote a popular account of the theory on the first page of their Sunday supplement of the New York Times on February 3 and in The Times of London in two installments on February 4 and 5 (Einstein, 1929a,b, also published as Einstein, 1930d). Einstein insisted on the highly speculative nature of unified field theory-project, without being afraid of endorsing even Meyerson's somewhat outrageous comparison with Hegel. It is hard to deny that Einstein's decision to mention Meyerson rather than Reichenbach as a philosophical interlocutor in an article with such a vast readership has a symbolic significance. After a decade of personal friendship and intellectual collaboration that had shaped the history of 20th-century philosophy of science, Einstein seems to have put into question the very core of his philosophical alliance with Reichenbach. Whereas Reichenbach considered the separation between mathematics and physics the great achievement of the relativity theory, Einstein saw in mathematical simplicity itself the key to access the fundamental structure of reality.

Although Einstein's Fernparallelismus attracted the attention of mathematicians, Reichenbach's skepticism was shared by the physics community. Einstein was fully aware of the marginality of his position, but, throughout 1929, continued to express his confidence in the Fernparallelismus-program, defending the theory in public talks (Einstein, 1930a,b,c), as well as in as well in private correspondence. However, only a few months later, Einstein and Walther Mayer presented a new approach (Einstein and Mayer, 1931), generalizing the n-bein formalism to five dimensions. The optimism faded away quickly again since the theory was unable to solve the matter problem. In a popular talk given in Vienna towards mid-October of 1931, Einstein could only describe his field-theoretical work since general relativity as a "cemetery of buried hopes" (Einstein, 1932, 441).

However, Einstein's philosophical motivation for continuing on this path has not changed ⁴⁵ Many of his former philosophical allies considered this attitude hard to fathom. ⁴⁶ However, Einstein's 1933 Oxford lecture address leaves no room for doubt. Einstein's quest for unification, he insisted, was motivated by the

⁴²Weyl, whom Einstein had always scolded for his speculative style of doing physics, could relaunch the accusation in a paper (Weyl, 1929) in which he had uncovered the gauge symmetry of the Dirac theory of the electron (Dirac, 1928a,b). "The hour of your revenge has come", Pauli wrote to Weyl in August: "Einstein has dropped the ball of distant parallelism, which is also pure mathematics and has nothing to do with physics and *you* can scold him" (Pauli to Weyl, Aug. 26, 1929; WPWB, Doc. 235). As Pauli complained, writing to Einstein's close friend Paul Ehrenfest, "God seems to have left Einstein completely!" (Pauli to Ehrenfest, Sep. 29, 1929; WPWB, Doc. 237).

⁴³Pauli did not hesitate to describe Einstein's presentation at the Berlin Colloquium as a "terrible rubbish" (Pauli to Jordan, Nov. 30, 1929; WPWB, Doc. 238). When he received the drafts of Einstein's Annalen paper, he wrote only slightly more politely that he did not find the derivation of the field equations convincing (Pauli to Einstein, Dec. 19, 1929; WPWB, Doc. 239). Pauli knew that Einstein would not have changed his mind, but he was ready to "make any bet" that "after a year at the latest you will have given up all the distant parallelism, just as you had given up the affine theory before" (Pauli to Einstein, Dec. 19, 1929; WPWB, Doc. 239).

 $^{^{44}}$ It is interesting to notice that one of the reasons that induced Einstein to abandon the theory was not dissimilar to Reichenbach's criticism: "The main reason for the uselessness of the distant parallelism construction lies, I feel, in that one can attribute absolutely no physical meaning to the 'straight lines' of the theory, while the physically meaningful (macroscopic) equations of motion cannot be obtained from it 3 . In other words, the h_{sv} give rise to no useful representation of the electromagnetic field" (Einstein to Cartan, May 21, 1932; Debever, 1979, A XXXV). Thus, for Einstein, it was legitimate to abandon the physical interpretation of the straight line from the outset if the theory provided the laws of motion of the electrons at the end.

⁴⁵At the end of 1931, Lanczos wrote a semi-popular presentation of the distant parallelism approach (Lanczos, 1931), in which he effectively described the positions on the field. He distinguished between (a)a positivist-subjectivist interpretation of relativity theory as a theory that makes predictions about the behavior of rods and clocks (b) metaphysical-realistic interpretation of relativity as a theory that aims to uncover the mathematical structure of the field. Many readers might have easily recognized someone like Pauli in Lanczos's 'positivist'; however, others were baffled to find out that Einstein was classified among the 'metaphysicians'. However, Pauli, by reviewing Lanczos's article, did not fully recognize himself in the portrait of the 'positivist' (Pauli, 1932). Such labels, he argued, "are highly subjective and arbitrary". Pauli mocked the Naturwissenschaften for having published Lanczos' paper in series entitled 'Results in exact sciences' (Ergebnisse der Exakten Wissenschaften). Indeed, Einstein published this sort of theory at a rhythm of one each year, and in every case, he claims that it is the definitive solution: "Einstein's new field theory is dead, long live Einstein's new theory!" (Pauli, 1932).

⁴⁶Philipp Frank read Lanczos's (1931) article with some bewilderment, as he reports in his Einstein's biography (Frank, 1947). Frank was "quite astonished" to find the theory of relativity characterized as the expression of a realist program "since I had been accustomed to regarding it as a realization of Mach's program" (Frank, 1947, 215). However, when Frank met Einstein in Berlin at around the same time, he found out that Lanczos had indeed characterized Einstein's point of view accurately (Frank, 1947, 215f.). According to his recollection, Einstein complained that "[a] new fashion" had arisen in physics according to which quantities that in principle cannot be measured does not exist, and that to "to speak about them is pure metaphysics" (Frank, 1947, 216). Frank objected that this was the same philosophical attitude that led to relativity theory. By contrast, Einstein insisted, the essential point of relativity theory is to "regard an electromagnetic or gravitational field as a physical reality, in the same sense that matter had formerly been considered so" (Frank, 1947, 216). The theory of relativity teaches us the connection between different descriptions of one and the same reality.

deep-seated conviction that "nature is the realization of the most simple mathematical ideas" (Einstein, 1933a). Einstein conceded that experience remains the sole criterion of the physical adequateness of a mathematical construction. However, he insisted that the true creative role belongs to mathematics: "I hold it to be true that pure thought is competent to comprehend the real, as the ancients dreamed" (Einstein, 1933a, 167). After all, he now claims, the search field theories has always followed the same heuristic pattern: "the theorist's hope of grasping the real in all its depth" lies "in the limited number of the mathematically existent simple field types, and the simple equations possible between them" (Einstein, 1933a, 168). Maxwell's equations are the simplest laws for an antisymmetric tensor field which is derived from a vector; Einstein's equations are the simplest equations for the metric tensor, etc. This strategy applies to Einstein's last attempt at a unified field theory on a theory based on semi-vectors (Einstein and Mayer, 1932b, 1933a,b, 1934). After ordinary vectors, the latter are the simplest mathematical fields that are possible in four dimensions and seem to describe certain elementary particles' properties (Dongen, 2004). One has to search for the simplest laws these semi-vectors satisfy (Einstein, 1933a, 168).

In September 1933, three months after the Oxford lecture, Einstein left Europe for Princeton. Reichenbach started to teach at the University of Istanbul in the fall of the same year. He tried to obtain a position in Princeton a few years later (Verhaegh, 2020). However, Reichenbach was concerned about Weyl's possible opposition: "He is my adversary since a long time," he wrote to Charles W. Morris; in the latter, Weyl was described as a supporter of a form a "mathematical mysticism" that was "very much opposed to my empiricist interpretation of relativity" (Reichenbach to Morris, Apr. 12, 1936; HR, 013-50-78). Thus, in April 1936, Reichenbach turned to Einstein to ask for his support. "More than 10 years ago", he explained, "Herr Weyl spoke out very negatively about my work on the theory of relativity". Reichenbach feared "Weyl's opposition persists to these days" (Reichenbach to Einstein, Apr. 12, 1936; EA, 20-107). Reichenbach might have good reasons for turning to Einstein's help against Weyl in academic matters. However, it is worth noticing that, by that time, Reichenbach might have been closer to Weyl than to Einstein in scientific matters. In the late 1930s, Weyl had utterly lost confidence in the "geometrical leap *Luftsprünge*" of the early 1920s,⁴⁷ and felt the need to "return to the solid ground of physical facts" (Weyl, 1931, 343), to the vast amount of experimental data provided by spectroscopy. On the contrary, gravitational research had turned Einstein into a 'believing rationalist' (Ryckman, 2014), convinced that physical truth lies in mathematical simplicity (Einstein to Lanczos, Jan. 24, 1938; EA, 15-268)

Only in 1938 Reichenbach managed to move to the United States (Verhaegh, 2020). Soon after that, Reichenbach and Einstein came into epistolary contact again to support Bertrand Russell, who had been dismissed from the City College of New York because of his anti-religious stance (Reichenbach to Einstein, Aug. 14, 1940; EA, 20-127, Einstein to Reichenbach, Aug. 22, 1940; EA, 20-110). Later both contributed (Einstein, 1944b, Reichenbach, 1944) to a volume in Russell's honor for the series *Library of Living Philosophers* edited by Paul Schilpp (1944). Reichenbach was asked to contribute to a similar volume in honor of Einstein a few years later (Schilpp, 1949). In some unpublished notes about Reichenbach's (1949) contribution, Einstein (1949b) praised him his rare ability for combing breath of knowledge with clarity (Einstein, 1949b). However, Einstein ultimately disagreed with many of Reichenbach's philosophical tenets. In particular, Reichenbach's claim that "'the meaning of a statement is reducible to its verifiability' appeared to Einstein problematic; in particular, he found "dubious whether this conception of 'meaning' can be applied to the single statement of Einstein, 1949b).

As it is well-known, in the so-called 'Reply to criticisms' (Einstein, 1949a) included in the Schilpp-volume, Einstein reformulated this line of argument by staging a dialogue between Reichenbach-Helmholtz, Poincaré, and an anonymous non-positivists, who claims that geometry and physics can be compared with experience only as a whole (Einstein, 1949a, 676f.). Although this dialogue has become enormously famous, its meaning has been ultimately misunderstood. Einstein was not making a philosophical digression on the 19th-century debate on the foundation of geometry. The question at stake, as Einstein put it jokingly, was nothing but Pilates's famous question 'What is truth?' (Einstein, 1949a, 676). This question was ultimately motivated by his tireless pursuit of the theory of the 'total field.' At that time, Einstein had returned to his 1925 metric-affine approach introducing non-symmetric $g_{\mu\nu}$ and $\Gamma^{\tau}_{\mu\nu}$ as fundamental variables (Einstein, 1945, Einstein and Straus, 1945). In private correspondence, Einstein's long-life friend Michele Besso raised against Einstein objections similar to those that Reichenbach had advanced over twenty years earlier against the same theory. The symmetric part of the $g_{\mu\nu}$ and the

⁴⁷In a way not dissimilar to Reichenbach Weyl considered early unified field theories as "merely geometrical dressings (geometrische Einkleidungen) rather than as proper geometrical theories of electricity". (Weyl, 1931, 343).

⁴⁸In English in the text.

⁴⁹In English in the text.

⁵⁰In English in the text.

corresponding $\Gamma^{\tau}_{\mu\nu}$, Besso claimed, should define the straightest line, which is also the shortest. Do these lines represent the trajectories of test particles? What is their physical meaning? (Besso to Einstein, Apr. 11, 1950; Speziali, 1972, Doc. 171). Einstein's reply reveals his fundamental philosophical conundrum:

Your questions are fully legitimate, but it is not answerable for the time being [...]. This is because there is no real definition of the field in a consistent field theory. This indeed puts you in a Don Quixotic situation, in that you have absolutely no guarantee whether it ever possible to know if the theory is 'true.' A priori there is no bridge to empiricism. For example, there isn't a 'particle' in the strict sense of the word because the existence of particles doesn't fit the program of representing reality by everywhere continuous, even analytic functions. For example, in the theory, there is a symmetric $g_{\mu\nu}$ [...] and then a geodesic line. But from the outset, one has no clue that these lines have any physical meaning, not even approximately [...] It boils down to the fact that a comparison with what is empirically known can only be expected from the fact that strict solutions of the system of equations can be expected found, that reproduce the behavior of empirically 'known' structures and their interactions. Since this is extremely difficult, the skeptical attitude of contemporary physicists is probably is completely understandable. In order to really grasp this conviction of mine, you must read again and again my answer in the anthology [Sammelband] [51] (Einstein to Besso, Apr. 15, 1950; Speziali, 1972, Doc. 172).

This passage summarizes many of the issues that Reichenbach and Einstein have discussed over the years. It explains Reichenbach's legitimate concern that geometrical concepts of the theory, like that of the straightest lines, should receive a physical interpretation from the outset in terms of the motion of test particles. However, it also explains why Einstein did not find this approach viable in pursuing a theory in which there were no particles but only regions of the field, the equations of motions of which should be derived from the field equations. In the current state of the theory, it was not possible to judge whether the interpretation of the fundamental quantities representing the field is correct or not. Usually, a field is defined in the first place by the forces that it exerts on test particles. However, discovering this force law requires the integration of the field equations, which have not yet been carried out. It is in this context that question of the 'truth' of a theory of this kind could not be avoided. It was ultimate this question that Reichenbach and Einstein had discussed for over 30 years.

Abbreviations

CPAE Albert Einstein (1987–). The collected papers of Albert Einstein. Ed. by John Stachel et al. 15 vols. Princeton: Princeton University Press, 1987–.

EA The Albert Einstein Archives at the Hebrew University of Jerusalem.

ECW Ernst Cassirer (1998–). Gesammelte Werke. Hamburger Ausgabe. Ed. by Birgit Recki. 26 vols. Hamburg: Meiner, 1998–.

HR Archives of Scientific Philosophy (1891–1953). The Hans Reichenbach Papers. 1891–1953.

SN $Schlick\ Nachlass.$ Noord-Hollands Archief, Haarlem.

WPWB Wolfgang Pauli (1979–). Wissenschaftlicher Briefwechsel mit Bohr, Einstein, Heisenberg u.a. Ed. by Karl von Meyenn. 4 vols. Berlin/Heidelberg: Springer, 1979–.

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⁵¹Einstein, 1949a.

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