

Production risk and the functional distribution of income in a developing economy: tradeoffs and policy responses

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Abstract

We develop a stochastic endogenous growth model to examine the relationship between the volatility of growth and the factor distribution of income. Our framework incorporates two important features of developing economies: the co-existence of a modern and a traditional sector and the fact that the income generated in the traditional sector can escape taxation. The relationship between volatility and factor distribution is complex, depending upon the source of risk and the elasticity of substitution between capital and labor in the formal sector. The policy options available to the government for counteracting changes in volatility are analyzed. The second best optimal tax structure is also characterized.

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1. Introduction

This paper addresses an important, but neglected, question, namely, the relationship between the volatility of growth and the distribution of factor income. The importance of

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this issue stems from the fact that who bears the cost of the volatility is likely to have important consequences for the overall performance of the economy, particularly in a developing economy where opportunities for insurance may be limited. The empirical evidence on the relationship between volatility and the distribution of factor income is sparse. Breen and García-Peñalosa (2004) obtain a positive relationship between a country's volatility (measured by the standard deviation of the rate of GDP growth) and income inequality. To the extent that greater inequality is likely to be associated with a higher share of income to capital, these findings suggest that more volatility will be associated with a smaller share of income being earned by labor.

A simple regression equation shows that this is indeed the case. Using a sample of 83 developed and developing countries, we measure volatility by the standard deviation of the annual growth rate of per capita GDP over the period 1960–1990 and compute the average labor share over the same period. Regressing labor share on volatility, we find that a 1 percentage point increase in volatility reduces labor share by 2.36 percentage points.¹ This is a sizable effect, with an increase of one standard deviation of volatility reducing the labor share by a third of its standard deviation, and raises the question of how risk affects the shares of output commanded by capital and labor.

In general, the distribution of factor income and growth volatility are endogenously determined and thus need to be analyzed within an integrated intertemporal general equilibrium framework. We employ an extension of the stochastic growth model developed by Grinols and Turnovsky (1993, 1998), Smith (1996), Corsetti (1997) and Turnovsky (2000). This is a one-sector growth model, in which aggregate equilibrium output evolves in accordance with a stochastic AK technology. Previous studies have been incapable of analyzing the impact of volatility on income distribution. This is because either they abstract from labor (Grinols and Turnovsky, 1993, 1998, Smith, 1996) or alternatively, are based on a Cobb–Douglas production function (Corsetti, 1997, Turnovsky, 2000), in which case the factor distribution of income remains fixed.

To allow volatility to influence distribution we need both endogenous employment levels *and* a production structure that allows for non-constant factor shares. One of the more striking facts when we examine the evidence on output volatility is that developing economies are subject to much greater fluctuations in their growth rates than are industrial countries. We therefore study a two-sector economy with a modern and a traditional sector, in which agents allocate their labor between the two sectors and where the overall factor shares depend, among other things, on the endogenous sizes of the two sectors. Adopting this framework, the equilibrium growth rate, its volatility and the distribution of income become jointly determined. The relationship between growth and its volatility has been subject to both theoretical and empirical investigation. The simplest stochastic growth model yields a negative tradeoff (as some of the more recent empirical evidence suggests)

¹ The labor share is defined as compensation of employees paid by resident producers, divided by GDP, both from the U.N. National Accounts. The data on volatility is from the Penn World Table. The sample contains 83 developed and developing economies for which both data were available. Regressing, LS, on volatility, S.D., we find $LS = 57.54 - 2.36S.D.$, with a *t*-statistic for the coefficient on S.D. of -3.40 , and $R^2 = 0.125$. Regressing the labor share in 1990 on volatility over the period 1960–1990 yields an even stronger negative relationship, although the sample is smaller.

if and only if the coefficient of relative risk aversion is less than unity, inconsistent with the evidence. Other more complex models, involving portfolio adjustments, are capable of generating a negative tradeoff under more plausible assumptions on preferences. The implications for income distribution provide a further dimension to this relationship, and indeed, the elasticity of substitution between the capital and labor is an important determinant of the growth-volatility tradeoff.

Our analysis has two aspects. First, we derive the equilibrium balanced growth path. The economy we consider has a modern sector, in which output is produced by a constant elasticity of substitution (CES) production function, using both private capital and labor, and a traditional sector in which individuals are self-employed and output is produced using only labor. In both sectors, the aggregate capital stock provides an externality that is consistent with an equilibrium of ongoing growth, as in [Romer \(1986\)](#).

The equilibrium we derive provides a framework for considering the options available to the policy maker to offset the effects of volatility. In doing so, we make the crucial assumption that the government cannot tax the traditional sector. Given our interest in the effects of risk in a developing economy, this seems a natural assumption. In such economies, the bulk of self-employment is found within what is often called the “informal sector”, and estimates of the proportion of the male non-agricultural labor force in that sector range between 15% and 90%, depending on the country.² It is often argued that the production structure of the economy and, in particular, the degree to which certain activities are commercialized as opposed to black-market or subsistence-oriented is a major determinant of the capacity of governments to raise tax revenue. To capture this feature of developing economies, we simply assume that all traditional sector employment is informal, taking place outside the formal labor market and consequently is non-taxable by the government.

This policy constraint allows us to address two questions. First, it makes the tax on labor income a tool that can be used to counteract the impact of increased volatility. Because only one of the sectors is taxed, changing the wage tax affects the allocation of labor across sectors and, consequently, partially offsets the impact of increased volatility on employment. Second, the policy constraint allows us to consider the effect of redistributing the tax burden from labor to capital on growth and welfare. This is an important question. In many developing countries, interest income, if taxed at all, is taxed at a rate below the labor income tax rate (see [Tanzi and Zee, 2000](#)). This not only implies a regressive tax structure³ but may also be inefficient in a representative agent framework. Moreover, as developing countries attempt to become fully integrated in the world economy, they need both a higher tax level and a reduction of their reliance on foreign trade taxation. This will require higher personal income tax rates and raises the question of the form that this increase in taxation should take.

Formal analysis is intractable and the second phase of our analysis is to calibrate the model to a developing economy. In this respect, the model is capable of replicating the equilibria of a range of such economies with relative ease. In general, we find that the

² See [United Nations \(2000\)](#) and [Thomas \(1992\)](#).

³ It is well documented that in many developing countries the tax structure is far from progressive. See [Jimenez \(1986\)](#).

relationship between volatility and factor shares is complex, depending upon both the sectoral source of the productive risk and the elasticity of substitution in production. Two main policy conclusions are obtained. First, attempting to stabilize aggregate volatility at its original level following an increase in risk is infeasible requiring a wage tax well in excess of 100%. The welfare loss can, however, be fully eliminated, and by a suitable adjustment of the tax rates on both labor and capital income, it is possible to maintain both factor shares and welfare at their original levels. Second, we find that the second best optimal tax policy response is to set the wage tax below the capital income tax. In other words, when the government is constrained in its capacity to tax all labor incomes, the standard first best result that taxing labor is preferable to taxing capital income no longer holds.⁴ Moreover, optimal policy exhibits a tradeoff between growth and welfare maximization.

The literature on this topic is sparse. The study of distribution in developing countries has been concerned mainly with examining Kuznets' hypothesis that as an economy grows, migration from agriculture to industry entails changes in the personal distribution of income. Recent work on this dual-economy model has shown the complexity of the relationship between development and inequality, but even when unemployment is introduced the assumption of risk neutrality has meant that risk and uncertainty has played no role.⁵ Our approach departs from this literature in various respects. First, our setup is not strictly a dual-economy model, as we do not consider migration, but rather the way in which individuals (or households) divide their time between two types of activities. Second, instead of examining how growth affects inequality, we argue that both distribution and the growth rate are endogenously determined by a number of factors, including the riskiness of the economy.⁶ Lastly, we focus on the factor rather than the personal distribution of income.

The paper closest to our work, at least in spirit, is [Aghion et al. \(1999\)](#), who find that greater inequality is associated with more volatility. They show how combining capital market imperfections with inequality in a two-sector model can generate endogenous fluctuations in output and investment. In their model, it is unequal access to investment opportunities and the gap between the returns to investment in the modern and the traditional sectors that cause fluctuations. We reverse the focus, examining how exogenous production uncertainty determines output volatility and distribution.

The remainder of the paper proceeds as follows. Section 2 sets out the components of the model, while Section 3 summarizes the implied macroeconomic equilibrium. We then provide some initial analytical results in Section 4. Section 5 undertakes the calibration,

⁴ See [Stokey and Rebelo \(1995\)](#) and the work they discuss for an analysis of the effects of shifting the tax burden in an endogenous growth model.

⁵ See, e.g., [Fields \(1980\)](#), [Bourguignon \(1990\)](#), and [Anand and Kanbur \(1993\)](#), for analyses of the Kuznets hypothesis where either inequality within sectors or relative prices change along the development path. [Temple \(1999\)](#) examined the evolution of inequality as an economy grows in a dual-economy model with unemployment in the modern sector.

⁶ [Eicher and García-Peñalosa \(2001\)](#) also address the question of how growth and distribution are simultaneously determined in developing economies, but the focus of that paper is innovation and the accumulation of human capital.

first computing the numerical impacts of increases in the exogenous risk on the key economic variables and then computing the appropriate policy responses. The next section considers second best optimal policy, obtaining the tax rates that maximize growth and welfare. Section 7 concludes, while the technical solution to the problem—itsself quite a challenging exercise—is relegated to Appendix A.

2. Elements of the economy

This section describes the analytical framework and the behavior of the relevant agents. In developing the model, we distinguish between quantities that pertain to and are chosen by the representative agent and corresponding economywide average quantities, denoted by bars, that the individual takes as given, but which in equilibrium are endogenously determined.

2.1. Technology and returns

We assume that the representative agent supplies a unit of labor inelastically. A fraction, $1-l$, may be allocated to employment in a formal or modern sector, with the remainder, l , being spent in an informally organized sector. Output in the formal sector is produced by the CES production function:

$$\begin{aligned} dY &= B[\alpha((1-l)\bar{K})^{-\rho} + (1-\alpha)K^{-\rho}]^{-1/\rho}(dt + du) \\ &\equiv Z(dt + du) \quad 0 < \alpha < 1, \quad -1 < \rho < \infty \end{aligned} \quad (1a)$$

where K denotes the individual firm's stock of capital, \bar{K} is the average economywide stock of capital, so that $(1-l)\bar{K}$ measures individual labor in efficiency units. This is a generalization of the stochastic Cobb–Douglas production function employed by Corsetti (1997) and Turnovsky (2000), with the elasticity of substitution defined by $\varepsilon \equiv 1/(1+\rho)$. The stochastic variable, du , is temporally independent and normally distributed with mean zero and variance $\sigma_u^2 dt$ over the instant dt . This stochastic production function exhibits constant returns to scale in the private decisions, the fraction of time devoted to employment in the formal sector, and the private capital stock. In addition, the average stock of capital yields an externality such that in equilibrium, when $K=\bar{K}$, the production function is linear in the accumulating stock of capital, as in Romer (1986). Letting

$$\Omega(\bar{l}) \equiv [\alpha(1-\bar{l})^{-\rho} + (1-\alpha)]^{-1/\rho}$$

Aggregate (average) output, $d\bar{Y}$, is thus represented by:

$$d\bar{Y} = B[\alpha(1-\bar{l})^{-\rho} + (1-\alpha)]^{-1/\rho} \bar{K}(dt + du) \equiv B\Omega(\bar{l})\bar{K}(dt + du) \quad (1b)$$

Factor returns over the period $(t, t+dt)$ are generated as follows. The wage rate (return to labor) is described by the stochastic process

$$dA = a(dt + du) \quad (2a)$$

where

$$a \equiv (\partial Z / \partial (1-l))_{l=\bar{l}, K=\bar{K}} = B\alpha\Omega(\bar{l})^{1+\rho}(1-\bar{l})^{-(1+\rho)}\bar{K} \equiv \delta(\bar{l})\bar{K}$$

Likewise, the private rate of return on capital, dR_K , over the period $(t, t+dt)$ is specified by:

$$dR_K = r_K(dt + du) \quad (2b)$$

where

$$r_K \equiv (\partial Z / \partial K)_{l=\bar{l}, K=\bar{K}} = B(1-\alpha) \left[\alpha(1-\bar{l})^{-\rho} + (1-\alpha) \right]^{-\frac{1+\rho}{\rho}} \equiv B(1-\alpha)\Omega(\bar{l})^{1+\rho}$$

Eqs. (2a) and (2b) assume that the returns to capital and labor are represented by their respective aggregate stochastic marginal physical products. Eqs. (2a) and (2b) imply that the rate of return to capital is stationary, while over time, the wage rate grows with the aggregate capital stock.⁷ The stochastic shock in the formal sector is reflected proportionately in both factor returns.

Output in the informal sector depends upon labor in accordance with the production function

$$dQ = q\bar{l}\bar{K}(dt + dv) \quad (3a)$$

where the aggregate capital stock serves as a proxy for knowledge that conditions the productivity of individual labor. For simplicity, we assume that labor has constant productivity, parameterized by q .⁸ The production function also includes the assumption that unlike labor, individual capital cannot move between the two sectors. This is a reasonable first approximation for a developing economy in which banks are unlikely to lend to finance investment in the informal sector, as well as because the “types” or “vintages” of capital are different. The stochastic disturbance in the informal sector, dv , is temporally independent and normally distributed with mean zero and variance $\sigma_v^2 dt$ over the instant dt . The correlation between the two shocks is $\sigma_{uv} dt$. Aggregate output in the informal sector is

$$d\bar{Q} = q\bar{l}\bar{K}(dt + dv) \quad (3b)$$

In addition to holding capital, the agent may hold government bonds, b , the before-tax real rate of return on which is postulated to be

$$dR_B = r_B dt + du_B \quad (4)$$

where r_B and du_B will be determined endogenously in macroeconomic equilibrium. The bonds we shall consider have an endogenously determined variable price, P , but beyond

⁷ Together, Eqs. (2a) and (2b) imply $(1-\bar{l})dA + \bar{K}dR_K = B\Omega(\bar{l})\bar{K}(dt + du) = d\bar{Y}$.

⁸ It is straightforward and changes little if we assume that labor interacts with a fixed factor land, T , say, in accordance with the production function $dQ = q\bar{l}^\theta T^{1-\theta} \bar{K}(dt + dv)$.

that, their precise nature is unimportant. Equilibrium asset-pricing considerations will determine r_B and du_B in terms of the real shocks, du , dv to the economy, with P adjusting to support this equilibrium.

2.2. Consumer optimization

The representative consumer's asset holdings are subject to the wealth constraint

$$W = Pb + K \quad (5)$$

where W denotes real wealth. In addition, the agent is assumed to purchase output over the instant dt at the non-stochastic rate $C(t)dt$ out of income generated by these asset holdings. His objective is to select his portfolio of assets and the rate of consumption to maximize expected lifetime utility, taken to depend upon consumption, $C(t)$, as represented by the isoelastic utility function

$$E_0 \int_0^{\infty} \frac{1}{\gamma} C^\gamma e^{-\beta t} dt \quad -\infty < \gamma < 1 \quad (6)$$

subject to the wealth constraint, Eq. (5), and the stochastic wealth accumulation equation:

$$dW = W[n_B dR_B + n_K dR_K] + (1-l)dA - Cdt - dT \quad (7)$$

where $n_B \equiv Pb/W$ is the share of portfolio held in government bonds, $n_K \equiv K/W$ the share of portfolio held in capital, and dT taxes paid.

The government is assumed to tax income from capital and labor generating the aggregate flow of tax revenues

$$dT = r_K \bar{K} (\tau_K dt + \tau'_K du) + \delta(1-l)\bar{K} (\tau_W dt + \tau'_W du) \quad (8)$$

where we assume that only the formal sector is taxed. This specification allows for different tax rates on capital and wage income, as well as on the deterministic and stochastic components of each. Different values for τ_K , τ'_K and τ_W , τ'_W reflect the possibility that taxes might include offset provisions having the effect of reducing the degree of after-tax randomness of real returns. Without loss of generality, interest income is untaxed, the before-tax return adjusting to satisfy the equilibrium arbitrage conditions.

Substituting for n_i into Eq. (5) and for Eqs. (2a), (2b), (4) and (8) into Eq. (7), the stochastic optimization problem can be expressed as choosing the consumption–wealth ratio, C/W , and the portfolio shares, n_i to maximize expected intertemporal utility Eq. (6) subject to

$$dW = \left\{ \left[n_B r_B + n_K (1 - \tau_K) r_K - \frac{C}{W} \right] W + [(1 - \tau_W)(1 - l)\delta + ql]\bar{n}_K \bar{W} \right\} dt + W d\omega \quad (9a)$$

$$n_B + n_K = 1 \quad (9b)$$

where, for convenience, we denote the stochastic component of dW/W by

$$d\bar{w} \equiv \left[(1 - \tau'_K) n_K r_K + (1 - \tau'_W) \delta (1 - l) \bar{n}_K \frac{\bar{W}}{W} \right] du + n_B du_B + q l n_K \frac{\bar{W}}{W} dv \quad (9c)$$

In performing the optimization, the agent takes the rates of return of the assets and the relevant variances and covariances as given, although these will ultimately be determined in equilibrium.

Through the equilibrium wage rate, the individual's rate of wealth accumulation depends upon aggregate wealth, which accumulates as follows:

$$d\bar{W} = \bar{W} \left[\bar{n}_B r_B + \bar{n}_K (1 - \tau_K) r_K - \left(\frac{\bar{C}}{\bar{W}} \right) + [(1 - \tau_w) \delta (1 - \bar{l}) + q \bar{l}] \bar{n}_K \right] \times \bar{W} dt + \bar{W} d\bar{w} \quad (9a')$$

$$d\bar{w} \equiv [(1 - \tau'_K) r_K + (1 - \tau'_W) \delta (1 - \bar{l})] \bar{n}_K du + \bar{n}_B du_B + q l \bar{n}_K dv \quad (9c')$$

This renders the agent's optimization a two-state variable problem, the two states being the agent's individual wealth, W , which is under his direct control, and the aggregate stock of wealth, \bar{W} , the evolution of which follows Eqs. (9a') and (9c'), and which the individual takes as exogenous. However, although from the individual's point of view, there are two state variables since all agents are identical, with aggregate and individual shocks being identical and perfectly correlated, in the macroeconomic equilibrium, the two state variables evolve proportionately [$\bar{W}=W$]. Thus, along the equilibrium growth path, the dynamic evolution of the economy can be represented by a single-state variable.⁹

2.3. Government policy

Government policy is restrictive, its sole purpose being to respond to changes in the sectoral volatilities. For this purpose, it levies taxes and issues debt subject to its flow budget constraint:

$$d(Pb) = (Pb) dR_B - dT \quad (10)$$

It is straightforward to introduce stochastic government expenditure, which may or may not be productive, as in Turnovsky (1999), but this is unnecessary for present purposes.

2.4. Goods market equilibrium

Finally, the flow of physical goods in the economy to consumption, investment and government expenditure must satisfy the resource constraint

$$dY + dQ = dC + dK \quad (11)$$

⁹ As we will discuss in Section 3.1 and further in the Appendix A, the equilibrium is the continuous time analogue to the "recursive competitive equilibrium" concept defined by Stokey and Lucas (1989).

which using Eqs. (1a) (1b), (11) and $dC=C(t)dt$ implies that the equilibrium rate of capital accumulation (rate of growth) in the economy is

$$\frac{dK}{K} = \left[B\Omega(\bar{l}) + q\bar{l} - \frac{C}{n_K W} \right] dt + B\Omega(\bar{l}) du + ql dv \equiv \psi dt + dw. \quad (12)$$

3. Macroeconomic equilibrium

The solution of the model is derived in the Appendix A. This is based on the assumption that the equilibrium is a recurring one, in which risks and returns on assets are unchanging through time. This implies that the agent chooses the same allocation of portfolio wealth at each instant of time. Since all agents are identical, we drop the distinction between individuals and the aggregate by dropping the bars. The equilibrium is summarized by the stochastic growth path defined below.

3.1. Equilibrium growth path

Definitions of δ , Ω , r_K , ϕ

$$\delta = B\alpha\Omega^{1+\rho}(1-l)^{-(1+\rho)} \quad (13a)$$

$$\Omega = [\alpha(1-l)^{-\rho} + (1-\alpha)]^{-\frac{1}{\rho}} \quad (13b)$$

$$r_K = B(1-\alpha)\Omega^{1+\rho} \quad (13c)$$

$$\phi = r_B(1-n_K) + r_K(1-\tau_K)n_K \quad (13d)$$

Equilibrium labor allocation

$$\delta(1-\tau_W) - q = (1-\gamma)[B\Omega\delta(1-\tau'_W)\sigma_u^2 - (B\Omega - \delta l(1-\tau'_W)q\sigma_{uv} - q^2 l\sigma_v^2)] \quad (13e)$$

Equilibrium portfolio allocation

$$\begin{aligned} r_B = r_K(1-\tau_K) + (1-\gamma) & \left\{ \sigma_w^2 + (B\Omega\sigma_u^2 + ql\sigma_{uv}) \right. \\ & \left. \times \left[\frac{n_K}{1-n_K} (\tau'_K r_K + \tau'_W \delta(1-l)) - r_K(1-\tau'_K) \right] \right\} \end{aligned} \quad (13f)$$

Consumer budget constraint

$$\psi = \phi + [(1-\tau_w)\delta(1-l) + ql]n_K - \frac{C}{W} \quad (13g)$$

Goods market equilibrium

$$\psi = B\Omega + ql - \left(\frac{C}{W} \right) \frac{1}{n_K} \quad (13h)$$

Equilibrium volatility

$$\sigma_w = (B^2 \Omega^2 \sigma_u^2 + 2B\Omega q l \sigma_{uv} + q^2 l^2 \sigma_v^2)^{0.5} \quad (13i)$$

Equilibrium growth rate

$$\begin{aligned} \psi = & \frac{\phi - \beta}{1 - \gamma} - B^2 \Omega^2 \left[\frac{\gamma}{2} - (1 - \tau'_w) [1 - (1 - \alpha) \Omega^p] n_K \right] \sigma_u^2 \\ & - B\Omega [\gamma - n_K (2 - \tau'_w) + (1 - \tau'_w) (1 - \alpha) \Omega^p n_K] q l \sigma_{uv} \\ & + \left(n_K - \frac{\gamma}{2} \right) q^2 l^2 \sigma_v^2 \end{aligned} \quad (13j)$$

This system characterizes a recursive competitive equilibrium that holds along a balanced growth path. In this respect, the solution procedure is the continuous time analogue to the recursive competitive equilibrium concept defined by [Stokey and Lucas \(1989\)](#) and others. It is well known that for the constant elasticity utility function and stochastic labor income, it is, in general, impossible to derive an explicit closed-form expression for the consumption function.¹⁰ Nevertheless, despite this, using the equilibrium (13a) conditions (13b) conditions (13c) conditions (13d) conditions (13e) conditions (13f) conditions (13g) conditions (13h) conditions (13i) conditions (13j), one can determine an *equilibrium* relationship between consumption and wealth, one that holds along the balanced growth path.

The equilibrium has the following recursive structure. The first four equations repeat the definitions of the equilibrium output–capital ratio, the wage rate, the return to capital, which are all functions of the labor allocation decision, l , and the average return on asset income, l . The first critical equilibrium equation is the labor allocation condition (13e). This asserts that labor is allocated such that the risk-adjusted after-tax returns to labor in the two sectors are equal. Having thus determined l yields δ , Ω , r_K and, in turn, the volatility of the growth rate, σ_w , along the equilibrium growth path. The consumer budget constraint, goods market equilibrium, the equilibrium portfolio allocation condition and the equilibrium growth condition then jointly determine n_K , ψ , C/W , r_B . Eq. (13f) further reveals how the before-tax return on bonds adjusts to yield the equilibrium after-tax rate of return so that the real growth equilibrium is independent of the tax rates on interest income.

3.2. Initial prices and wealth effects

The equilibrium growth path (Eqs. (13a)–(13j)) describes a stable rational expectations equilibrium. As in any such equilibrium, its attainment, or the shift from one equilibrium to another resulting from a structural change, is brought about by an appropriate initial jump in the price of bonds, $P(0)$. To the extent that the representative agent holds bonds in his equilibrium portfolio, these jumps impose initial capital gains or losses, thereby

¹⁰ See, e.g., [Blanchard and Mankiw \(1988\)](#).

affecting initial wealth. With K evolving continuously in accordance with the stochastic process, Eq. (12), the initial stock at time 0, K_0 , is predetermined. Given constant portfolio shares, the initial dollar value of government bonds outstanding is determined by¹¹

$$P(0)b_0 = (n_B/n_K)K_0;$$

Thus, given b_0 and K_0 , any policy that generates a change in the relative portfolio shares, n_B/n_K , will lead to a jump in the initial market value of bonds, $P(0)b_0$. The corresponding initial wealth, $W(0)$, of the agent is thus

$$W(0) = (1/n_K)K_0. \quad (14)$$

3.3. Feasibility of equilibrium

Finally, the equilibrium must satisfy certain feasibility conditions. First is the transversality condition, which for the constant elasticity utility function is of the form:¹²

$$\lim_{t \rightarrow \infty} E[W^\gamma e^{-\beta t}] = 0. \quad (15)$$

Using Eqs. (13a)–(13j), condition (15) can be shown to be a generalization of the condition $C/W > 0$, originally due to Merton (1969), to which it reduces in the absence of labor income.¹³ With the equilibrium being one of balanced real growth, in which all real assets grow at the same rate, Eq. (15) also implies that the intertemporal government budget constraint is met. One can show that Eq. (15) automatically holds for the logarithmic utility function ($\gamma=0$). In other cases, this condition may impose restrictions in order for the tax rate to remain feasible. But provided Eq. (15) holds, the equilibrium is viable in the sense of being consistent with the intertemporal solvency of the government.

Second, with nonnegative stock of capital in existence, the equilibrium portfolio shares $n_K \geq 0$. This inequality impose further restrictions on government policy. If the government is permitted to borrow and lend, then no restriction on n_B is imposed. However, if such lending to the private sector is ruled out, the additional restriction $n_B \geq 0$ or $1 \geq n_K$ is required to be met.¹⁴

The third condition is associated with the endogeneity of the fraction of time allocated to the two forms of labor. This requires that the solution for l from the labor allocation condition (13e) lies in the range $0 \leq l \leq 1$. For the CES production function, the restrictions involved are hard to establish analytically although our numerical simulations always yield an interior solution.

¹¹ These equations hold at all points of time, including 0. Given the constancy of portfolio shares, they are the source of the proportionality of the stochastic growth rates summarized in Eq. (A.20).

¹² The utility function (as a function of wealth) can be shown to be of the form ϕW^γ .

¹³ This is derived for a simpler (Cobb–Douglas, one-sector) version of the present model by Turnovsky (2000).

¹⁴ The equilibrium in which the portfolio share of government bonds $n_B=0$ is one in which there are no outside bonds. Such an equilibrium still allows for the presence of inside bonds that are perfect substitutes for government bonds.

3.4. Distributional measures

The focus of our study is on the tradeoff between the mean growth rate, ψ , its volatility, σ_w , and the behavior of relative factor shares, a key determinant of which is the relative size of the formal to the informal sector. In general, this is summarized by the stochastic quantity

$$\frac{d\bar{Y}}{d\bar{Y} + d\bar{Q}} = \frac{B\Omega(dt + du)}{B\Omega(dt + du) + ql(dt + dv)} \quad (16)$$

For convenience, we shall consider the deterministic quantity

$$\theta \equiv \frac{E(d\bar{Y})}{E(d\bar{Y} + d\bar{Q})} = \frac{B\Omega}{B\Omega + ql} \quad (16')$$

which can be easily seen to be an increasing function of the labor allocated to the formal sector.¹⁵

In looking at factor shares, we want to make a distinction between what we will call the “wage” share and the “labor” share. By the former, we denote the share of formal-sector output earned by labor in that sector. We define it as

$$S_W \equiv \frac{(1-l)dA}{dY} = \frac{\alpha\Omega^\rho}{(1-l)^\rho} \quad (17a)$$

and is non-stochastic. The labor share is defined as the overall share of labor income in total GNP,

$$\frac{(1-l)dA + d\bar{Q}}{d\bar{Y} + d\bar{Q}}$$

and, in general, is stochastic. For convenience, we shall focus on the non-stochastic quantity

$$S_L \equiv \frac{(1-l)E(dA) + E(d\bar{Q})}{E(d\bar{Y} + d\bar{Q})} = \theta S_W + (1-\theta) \quad (17b)$$

which is a weighted average of the share of labor in the formal sector (S_W) and in the informal sector (l). The distinction between these two measures of distribution is important given that one of the sectors in our economy is informal. By definition, labor incomes generated by this sector will not appear in the national accounts, and as a result, any

¹⁵ The quantity in Eq. (16') can be seen to be the “zero-order” component in a Taylor expansion of $d\bar{Y}/(d\bar{Y} + d\bar{Q})$ about its mean. Performing the expansion while recalling the assumption that $d\bar{Y}$ and $d\bar{Q}$ are uncorrelated, the expression for θ given in Eq. (16') will be a reasonably accurate estimate of the mean of Eq. (16) as long as the difference between the relative volatilities of the two outputs, expressed as $\text{var}(d\bar{Y})/E(d\bar{Y})$, $\text{var}(d\bar{Q})/E(d\bar{Q})$, respectively, is not too great.

attempt to confront the predictions of the model with the data would need to consider the “measured” share of labor, i.e., S_W .

In general, S_W increases with labor in the formal sector if and only if $\varepsilon > 1$ and, in the case of the Cobb–Douglas production function, $S_W = \alpha$. The impact of labor allocation on the labor share is

$$\frac{dS_L}{dl} = \theta \frac{dS_W}{dl} + (S_W - 1) \frac{d\theta}{dl}$$

The first component reflects the impact in the formal sector and depends upon ε , while the latter reflects the fact that with labor being the only (direct) factor of production in the informal sector, a shift in labor from the formal sector raises the overall share of labor income. Hence, S_L increases in l even if the production function in the formal sector is Cobb–Douglas.

Given the stochastic specifications (Eqs. (2a) and (2b)), the relative volatility of labor income, capital income, and output of the formal sector all equal σ_u .¹⁶ The volatility of overall output is

$$\sigma_Y \equiv \frac{\sigma_W}{B\Omega + ql} = \left(\theta^2 \sigma_u^2 + 2\theta(1 - \theta)\sigma_{uv} + (1 - \theta)^2 \sigma_v^2 \right)^{0.5} \quad (18a)$$

while the volatility of the overall labor share of output is

$$\sigma_L \equiv \frac{\left(S_W^2 \theta^2 \sigma_u^2 + 2\theta(1 - \theta)S_W \sigma_{uv} + (1 - \theta)^2 \sigma_v^2 \right)^{0.5}}{\theta S_W + (1 - \theta)} \quad (18b)$$

3.5. Welfare

To assess the consequences of policy on economic welfare, we consider the welfare of the representative agent as specified by the intertemporal utility function (6) evaluated along the equilibrium path. By definition, this equals the value function used to solve the intertemporal optimization problem.

It can be shown that for the constant elasticity utility function the optimized level of utility, starting from an initial stock of wealth, $W(0)$, and computed in this way is given by

$$X(W(0)) = \frac{(1/\gamma)(C/W)^\gamma W(0)^\gamma}{\beta - \gamma[\psi + (1/2)(\gamma - 1)\sigma_w^2]} \quad (19)$$

Using the relationship (Eq. (14)), the welfare criterion (Eq. (19)) can be expressed as

$$X(K_0) = \frac{(1/\gamma)(C/W)^\gamma n_K^{-\gamma} K_0^\gamma}{\beta - \gamma[\psi + (1/2)(\gamma - 1)\sigma_w^2]} \quad (19')$$

where C/W , n_K are obtained from Eqs. (13a)–(13j). Assuming that these solutions are all positive and that the transversality condition is met so that the denominator is positive implies that $\gamma X(K_0) > 0$.

¹⁶ Since income from capital is derived from only the formal sector, its volatility is independent of the informal sector.

4. Some analytical properties

In general, formal analysis of the equilibrium (Eqs. (13a)–(13j)) is intractable, and we shall resort to numerical simulations. Despite that, a number of observations can be drawn from the structure.

$$(i) \quad \frac{\partial l}{\partial \sigma_u} > 0; \quad \frac{\partial l}{\partial \sigma_v} < 0; \quad (20a)$$

$$(ii) \quad \frac{\partial l}{\partial \tau_w} > 0; \quad \frac{\partial l}{\partial \tau'_w} < 0; \quad \frac{\partial l}{\partial \tau_K} = \frac{\partial l}{\partial \tau'_K} = 0; \quad (20b)$$

An increase in the production risk in the formal sector, σ_u , causes labor to move toward the informal sector, while greater production risk in the informal sector, σ_v , has the opposite effect. An increase in the deterministic component of the wage tax (levied only in the formal sector) causes labor to shift away from that sector. But an increase in the stochastic component of the wage tax reduces the volatility of net labor income and has the reverse effect. In contrast, labor allocation is invariant with respect to both the deterministic and stochastic components of the tax on capital income.

The response of the labor allocation is a critical element of the impact of risk on volatility. The effects of an increase in the volatility of production in the formal and informal sectors, given the covariances between the two shocks, are respectively:

$$\frac{\partial \sigma_w}{\partial \sigma_u} = \frac{(B\Omega)^2}{\sigma_w} \left\{ \sigma_u + \left[\left(\frac{ql}{B\Omega} \right)^2 \frac{\sigma_v^2}{l} + \frac{q}{B\Omega} \left(1 - \frac{S_W l}{1-l} \right) \sigma_{uv} - \frac{S_W \sigma_u^2}{1-l} \right] \frac{\partial l}{\partial \sigma_u} \right\} \quad (21a)$$

$$\frac{\partial \sigma_w}{\partial \sigma_v} = \frac{(ql)^2}{\sigma_w} \left\{ \sigma_v + \left[\frac{\sigma_v^2}{l} + \frac{B\Omega}{ql^2} \left(1 - \frac{S_W l}{1-l} \right) \sigma_{uv} - \left(\frac{B\Omega}{ql} \right)^2 \frac{S_W \sigma_u^2}{1-l} \right] \frac{\partial l}{\partial \sigma_v} \right\} \quad (21b)$$

Consider an increase in σ_u . The direct effect is to raise volatility by an amount that depends upon the output–capital ratio, $B\Omega$, in the formal sector. In addition, by causing labor to shift away from that sector, it has a secondary effect, which will reinforce or offset the first effect depending upon (i) the relative volatility and (ii) the relative output–capital ratios in the two sectors.

One issue that has received substantial attention in the literature concerns the impact of volatility on growth. Empirical evidence on this issue is mixed. Ramey and Ramey (1995) present evidence to suggest that volatility has a negative impact on the mean growth rate, although this contrasts with the earlier findings of Kormendi and Meguire (1985), for example. The simplest one-sector stochastic growth model implies a positive relationship if and only if the coefficient of relative risk aversion exceeds unity, as the empirical evidence strongly suggests. The present two-sector portfolio model is somewhat more ambiguous in this regard. The direct effect of an increase in volatility in either sector, given by the coefficient of σ_u^2 or σ_v^2 in Eq. (13j), is positive, as long as $\gamma < 0$. An increase in

σ_u^2 will tend to move the resources employed in the formal sector to the informal sector, and the net of effect of this on the overall volatility and growth will depend upon the relative volatility of the two sectors. In addition, the higher volatility σ_u^2 will affect the mean return and thus the mean growth rate. The relationship between volatility and growth is thus a complex one and further insight into this issue will be provided by the numerical analysis in Section 5.

A key issue upon which we wish to focus concerns the impact of volatility on the factor shares. Recalling the definitions of S_W, S_L in Eqs. (17a) and (17b), we see that these depend entirely upon the response of sectoral employment. Specifically, we can show that

$$\frac{dS_W}{S_W} = \left(\frac{1-\varepsilon}{\varepsilon} \right) (1-S_W) \left(\frac{l}{1-l} \right) \frac{dl}{l} \quad (22a)$$

$$\frac{dS_L}{S_L} = \frac{(1-S_L)}{S_L} \frac{l}{1-l} \left[S_W \left(\frac{1-\varepsilon}{\varepsilon} \right) + \frac{(1-\theta)[(1-l)(1-S_W)]}{l} \right] \frac{dl}{l} \quad (22b)$$

Eq. (22a) is conventional. An increase in employment in the formal sector (decrease in l), brought about by a reduction in σ_u , say, will raise the share of labor in the income of the formal sector if and only if the elasticity of substitution in that sector is greater than one. With the absence of private capital in the informal sector, any increase in employment in that sector will raise the overall share of labor income. Thus, if $\varepsilon > 1$, it is possible for S_W and S_L to respond in opposite ways to an increase in volatility, and an example of this is provided in our numerical simulations.

4.1. Some policy implications

The structure of the equilibrium (Eqs. (13a)–(13j)) has important implications for the options available to the policymaker to respond to an increase in the sectoral production volatilities, σ_u, σ_v . From these conditions, we may solve for the following key variables in the form:

$$l = l(\sigma_u, \sigma_v, \tau_W, \tau'_W) \quad (23a)$$

$$\sigma_W = \sigma_W[\sigma_u, \sigma_v, l(\sigma_u, \sigma_v, \tau_W, \tau'_W)] \quad (23b)$$

$$S_W = S_W[l(\sigma_u, \sigma_v, \tau_W, \tau'_W)] \quad (23c)$$

$$S_L = S_L[l(\sigma_u, \sigma_v, \tau_W, \tau'_W)] \quad (23d)$$

$$\psi = \psi[\sigma_u, \sigma_v, \tau_K, \tau'_K, \tau_W, \tau'_W, l(\sigma_u, \sigma_v, \tau_W, \tau'_W)] \quad (23e)$$

The first observation is that the labor shares, S_W, S_L , and the aggregate volatility, σ_w , are controlled by choosing the sectoral labor allocation, l . This, in turn, can be accomplished

by setting a linear combination of the tax rates on the deterministic and stochastic components of wage income. Indeed, this can be achieved by setting the tax rate on all labor income uniformly.¹⁷ Note further that the tax on capital income is irrelevant insofar as stabilizing these quantities is concerned. Taxes on capital may, however, play an important role in controlling the mean growth rate.

Consider an increase in the volatility, $d\sigma_u$, in the formal sector. This will cause labor to move from the formal to the informal sector. From Eqs. (23a) and (23c), we see that the sectoral labor allocation and thus labor shares will be stabilized if the tax on the deterministic component of labor income is reduced, or the tax on the stochastic component is increased, by the respective amounts:

$$d\tau_W = -\frac{\partial I/\partial \sigma_u}{\partial I/\partial \tau_W} d\sigma_u < 0; \quad d\tau'_W = -\frac{\partial I/\partial \sigma_u}{\partial I/\partial \tau'_W} d\sigma_u > 0 \quad (24)$$

In either case, the overall volatility of the growth rate increases by the direct amount $(B\Omega)(\sigma_u/\sigma_w)$.

Alternatively, the policymaker may choose to set the tax rate on labor income to stabilize the aggregate volatility. This can be achieved by setting

$$d\tau_W = -\frac{[\partial \sigma_w/\partial \sigma_u + \partial \sigma_w/\partial I \partial I/\partial \sigma_u]}{\partial \sigma_w/\partial I \partial I/\partial \tau_W} d\sigma_u \quad (25)$$

which is almost certainly positive (certainly in the simulations). This implies that the labor share will adjust by the (almost certainly positive) amount

$$dS_L = -\frac{\partial \sigma_w/\partial \sigma_u}{\partial \sigma_w/\partial I} d\sigma_u \quad (26)$$

There is thus a tradeoff between the amount of the adjustment due to a change in $d\sigma_u$ that is borne by the aggregate volatility and the amount that is borne by the factor shares.

A similar type of tradeoff applies in response to a change in the volatility, $d\sigma_v$, in the informal sector. In fact, our simulations show that seeking to stabilize the aggregate volatility, $d\sigma_w$, would be a disastrous policy, since it implies an astronomical increase in the wage tax, leading to a catastrophic decline in welfare. Indeed, for our chosen parameter set, it is infeasible, requiring a tax rate well in excess of 100%!

Lastly, we may note that the impact of an increase in volatility $d\sigma_u$ on the mean growth rate can be offset in several ways. It can either be accommodated by an adjustment in the tax on labor income or if that has been committed to some other objective, by adjusting the tax on capital income. Again, our simulations show that seeking to stabilize the mean growth rate is very undesirable from a welfare point of view.

¹⁷ Note that it is impossible to set S_W or S_L and σ_w simultaneously.

5. Calibration

To obtain further insight we resort to numerical simulations. These are based on the following parameter values that we take to be representative of a range of developing economies:

Preference parameters	$\gamma=-1.5, \beta=0.04$
Production parameters	$\alpha=0.6, B=0.4, q=0.275$
Elasticity of substitution	$\varepsilon=0.5, 1, 2$
Sectoral risk parameters	$\sigma_u=0, 0.125, 0.25$
	$\sigma_v=0, 0.125, 0.25$
Tax rates	$\tau_K=0.20, \tau_K'=0.20, \tau_W=0.20, \tau_W'=0.20$

The preference parameters are standard. They correspond to a coefficient of relative risk aversion of 2.5 and a rate of time preference of 4%. The production elasticity $\alpha=0.6$ implies that the share of labor in the Cobb–Douglas production function is 60%. The parameters B, q are arbitrary scale parameters, which play a big role in determining the equilibrium allocation of labor. The values of ε correspond to low, medium, and high elasticity of substitution, while the values for the σ_u, σ_v correspond to zero, medium (12.5%) and high (25%) sectoral risk (see Gavin and Hausmann, 1995). The percentage of household income paid in taxes varies widely across countries and across income groups within a country, most of the time falling between 15% and 25% for developing countries (see Jimenez, 1986).

5.1. Equilibrium

The equilibria are summarized in Tables 1a and 1b, which correspond to a low elasticity of substitution, the Cobb–Douglas, and a high elasticity of substitution. Table 1a

Table 1a
Effects of risk on equilibrium: distributional aspects

	$\sigma_v=0$					$\sigma_v=0.125$					$\sigma_v=0.25$				
	l	θ	S_W	S_L	Y/K	l	θ	S_W	S_L	Y/K	l	θ	S_W	S_L	Y/K
<i>Low elasticity of substitution: $\rho=1$ ($\varepsilon=0.5$)</i>															
$\sigma_u=0$	28.4	82.3	67.7	73.4	0.313	28.1	82.5	67.6	73.3	0.314	27.1	83.1	67.3	72.8	0.318
$\sigma_u=0.125$	30.0	81.3	68.2	74.1	0.306	29.6	81.5	68.1	74.0	0.308	28.6	82.2	67.8	73.5	0.312
$\sigma_u=0.25$	34.4	78.3	69.6	76.2	0.289	34.1	78.6	69.5	76.0	0.290	33.0	79.3	69.1	75.5	0.294
<i>Cobb–Douglas: $\rho=0$ ($\varepsilon=1$)</i>															
$\sigma_u=0$	45.3	71.6	60.0	71.4	0.260	44.7	72.0	60.0	71.2	0.262	42.7	73.3	60.0	70.7	0.267
$\sigma_u=0.125$	46.9	70.4	60.0	71.8	0.255	46.3	70.9	60.0	71.6	0.257	44.4	72.2	60.0	71.1	0.262
$\sigma_u=0.25$	51.3	67.4	60.0	73.0	0.242	50.7	67.8	60.0	72.9	0.245	48.9	69.1	60.0	72.4	0.249
<i>High elasticity of substitution: $\rho=-0.5$ ($\varepsilon=2$)</i>															
$\sigma_u=0$	64.7	59.1	47.1	68.7	0.225	63.8	59.8	47.5	68.6	0.226	60.9	61.7	48.4	68.1	0.231
$\sigma_u=0.125$	66.0	58.2	46.6	68.9	0.223	65.1	58.9	47.0	68.8	0.224	62.4	60.7	47.9	68.4	0.229
$\sigma_u=0.25$	69.4	55.8	45.4	69.5	0.219	68.6	56.4	45.7	69.4	0.220	66.2	58.1	46.6	69.0	0.223

All equilibrium quantities, except Y/K ratio, are in percentages.

summarizes the equilibrium values pertaining to income distribution; sectoral labor allocation, the relative size of the formal sector, labor shares in the formal sector and in aggregate, and the output–capital ratio. Table 1b summarizes the growth rate, volatility measures and the welfare effects. In bold type, we identify the benchmark case. This corresponds to the Cobb–Douglas production function, with medium degrees of sectoral volatilities $\sigma_u=0.125$, $\sigma_v=0.125$.

Welfare is measured as percentage “equivalent variations” in the initial capital stock, relative to this benchmark economy. Thus, for example, increasing σ_u and σ_v from 0.125 and 0.125 to 0.250 and 0.250, respectively, while leaving ε unchanged leads to an overall welfare loss equivalent to a 1.97% reduction in capital relative to the benchmark. Subtracting the values of ΔX also gives approximate losses for other economies. Thus, for example, increasing σ_u from 0.125 to 0.25, while holding σ_v fixed at 0 and 0.5, respectively, will reduce welfare by approximately $(7.07-2.73\%)=4.34\%$.

Tables 1a and 1b span the range of plausible values and the following can be noted. In the benchmark economy of Cobb–Douglas and medium risk, 53.7% of labor is allocated to the formal sector, which produces about 70.9% of GDP. Labor’s share of output is 60% in the formal sector and 71.6% overall. The capital–output ratio is around 3.9. The mean growth rate is 2.63%, with volatility 4.19% and aggregate output volatility of around 9.6%. These numbers are consistent with an “average” economy in the Gavin and Hausmann (1995) sample.

The percentage of labor allocated to the informal sector varies between 27.1% and 69.4%. Correspondingly, the wage share and the labor share vary between 47.1% and 69.6% and between 68.1% and 76.2%, respectively. For the Cobb–Douglas function, S_W is constant (0.6) but S_L still varies in response to the sectoral labor allocation. The allocation of labor to the informal sector increases with the elasticity of substitution in the formal

Table 1b
Effects of risk on equilibrium: growth, volatility, and welfare

	$\sigma_v=0$					$\sigma_v=0.125$					$\sigma_v=0.25$				
	ψ	σ_w	σ_Y	σ_L	ΔX	ψ	σ_w	σ_Y	σ_L	ΔX	ψ	σ_w	σ_Y	σ_L	ΔX
<i>Low elasticity of substitution: $\rho=1$ ($\varepsilon=0.5$)</i>															
$\sigma_u=0$	2.16	0	0	0	-1.19	2.20	0.97	2.20	1.81	-0.84	2.31	1.87	4.22	5.79	0.15
$\sigma_u=0.125$	2.34	4.47	10.2	9.34	-2.73	2.39	4.60	10.4	9.88	-2.36	2.52	4.94	11.2	11.2	-1.33
$\sigma_u=0.25$	2.83	8.55	19.6	17.9	-7.07	2.89	8.67	19.8	18.3	-6.68	3.06	8.98	20.5	19.4	-5.54
<i>Cobb–Douglas: $\rho=0$ ($\varepsilon=1$)</i>															
$\sigma_u=0$	2.40	0	0	0	0.74	2.48	1.54	3.50	4.92	1.21	2.67	2.94	6.68	9.45	2.56
$\sigma_u=0.125$	2.55	3.85	8.80	7.36	-0.48	2.63	4.19	9.58	8.99	–	2.84	5.00	11.4	12.4	1.36
$\sigma_u=0.25$	2.90	7.30	16.9	13.8	-3.87	2.99	7.56	17.4	15.0	-3.37	3.26	8.24	18.9	17.9	-1.97
<i>High elasticity of substitution: $\rho=-0.5$ ($\varepsilon=2$)</i>															
$\sigma_u=0$	2.76	0	0	0	3.78	2.87	2.19	5.03	7.33	4.31	3.17	4.19	9.56	14.0	5.80
$\sigma_u=0.125$	2.85	3.16	7.28	4.93	2.92	2.97	3.91	8.98	9.01	3.44	3.29	5.42	12.4	15.3	4.92
$\sigma_u=0.25$	3.09	6.03	14.0	9.11	0.53	3.23	6.54	15.1	12.2	1.04	3.60	7.78	17.9	18.1	2.47

All equilibrium quantities are in percentages. Welfare Changes are measured relative to the benchmark $\sigma_u=\sigma_v=0.125$, $\varepsilon=1$.

sector. This is because an increase in ε reduces the marginal physical product of labor and therefore the wage rate paid in the formal sector, inducing labor to move away from it. As a consequence, S_W and S_L both decrease with ε .

The mean growth rate ψ varies between 2.16% and 3.60%; the standard deviation (σ_w) of the growth rate varies between 0% and 9.0%. The latter figure is consistent with the figures of [Gavin and Hausmann \(1995\)](#) for developing economies. Overall volatility of GDP varies between 0 to around 20.5%, which is a plausible range for developing countries. The elasticity of substitution plays an important role. The mean growth rate increases with the elasticity of substitution, while all volatility measures decline with the elasticity of substitution. Greater flexibility in production allows for a more efficient use of resources, which increases the growth rate and has a stabilizing influence in the presence of risk. Lastly, welfare decreases with the elasticity of substitution if the degree of risk is small, and it increases with the elasticity of substitution if the degree of risk is high. Intuitively, flexibility in production is more desirable in a riskier environment.

5.2. Effects of risk

As we already saw analytically, the allocation of labor to the informal sector (l) increases with risk in the formal sector, σ_u , and decreases with risk in the informal sector, σ_v . Its sensitivity to σ_v increases with the elasticity of substitution in the formal sector since for high values of ε , the fraction of labor allocated to the informal sector is relatively high. The relative size of the formal sector, θ , being a decreasing function of l , thus decreases with σ_u and increases with σ_v .

The mean growth rate increases with both forms of risk. It is more sensitive to risk in the formal sector than in the informal sector, as under our choice of parameters the former is larger. Volatility increases with both sources of sectoral risk. As we already saw, high values of ε result in a large informal sector. Consequently, the sensitivity of the mean and the volatility of growth to the risk in the formal sector decrease with the elasticity of substitution in the formal sector, while their sensitivity to the risk in the informal sector increases with ε .

The relationship between risk and factor shares is complex. The wage share, S_W , increases with σ_u if $\varepsilon=0.5$ and increases with σ_u if $\varepsilon=2$. (It is unchanged for the Cobb–Douglas case.) It decreases with σ_v if $\varepsilon=0.5$ and increases with σ_v if $\varepsilon=2$. Meanwhile, the overall share of labor, S_L , increases with σ_u and decreases with σ_v for all values of the elasticity of substitution. These results have two implications. First, that the source of risk is a crucial determinant of the relationship between volatility and factor shares. Greater growth volatility is associated with a lower labor share only if it is due to greater risk in the informal sector. Second, note that in the case of a high elasticity of substitution, $\varepsilon=2$, an increase in σ_u or in σ_v has opposite effects on S_W and S_L . This means that if only incomes in the formal sector are measured, the observed changes in factor shares following an increase in volatility will not reflect the true impact on the labor share.

An increase in the risk in the formal sector reduces welfare, the reduction varying inversely with the elasticity of substitution. An increase in the risk in the informal sector is

welfare-improving. The reason for this paradoxical result is that taxing the formal sector only introduces a distortion into the economy, making the informal sector too large relative to the formal sector. Increasing the risk in the informal sector reduces the labor allocated to the informal sector, thereby correcting for this distortion. In the absence of taxes in the formal sector, increasing σ_v is also welfare-deteriorating, but less so than a corresponding increase in σ_u .

5.3. Policy responses

Table 2a and 2b summarize possible policy responses to increases in the two sources of risk from 12.5% to 25%, respectively, with the three panels corresponding to low, medium, and high values of the elasticity of substitution. Since, as we have already seen, there is always a degree of freedom in the choice of tax rates, we focus on the case where the deterministic and stochastic components of income are taxed at a uniform rate. We first consider Table 2a and shall focus on the panel characterized $\varepsilon=0.5$, since all other cases are parallel.

Row 1 describes as a benchmark no policy response. An increase in σ_u leads to a substantial reallocation of labor to the informal sector, raising the share of labor by

Table 2a
Policy responses to increase in σ_u from 0.125 to 0.25

	$\Delta\tau_W$	$\Delta\tau_K$	ΔS_L	$\Delta\psi$	$\Delta\sigma_w$	ΔX
<i>Low elasticity of substitution: $\rho=1$ ($\varepsilon=0.5$)</i>						
(1) Passive response	0	0	2.03	0.50	4.06	-4.42
(2a) Maintain fixed growth rate (adjusting τ_W)	4.36	0	5.56	0	3.58	-10.1
(2b) Maintain fixed growth rate (adjusting τ_K)	0	12.6	2.03	0	4.06	-10.4
(3) Maintain fixed shares (adjusting τ_W)	-3.57	0	0	0.90	4.43	-0.17
(4) Maintain fixed welfare (adjusting τ_W)	-3.71	0	-0.08	0.92	4.45	0
(5a) Maintain fixed welfare (adjusting τ_K)	0	-10.0	2.03	0.90	4.06	0
(5b) Maintain fixed shares and welfare	-3.57	-0.35	0	0.92	4.43	0
<i>Cobb–Douglas: $\rho=0$ ($\varepsilon=1$)</i>						
(1) Passive response	0	0	1.04	0.37	3.37	-3.37
(2a) Maintain fixed growth rate (adjusting τ_W)	4.23	0	3.48	0	2.87	-7.48
(2b) Maintain fixed growth rate (adjusting τ_K)	0	8.21	1.21	0	3.37	-7.40
(3) Maintain fixed shares (adjusting τ_W)	-3.05	0	0	0.64	3.72	-0.52
(4) Maintain fixed welfare (adjusting τ_W)	-3.62	0	-0.23	0.70	3.79	0
(5a) Maintain fixed welfare (adjusting τ_K)	0	-7.22	1.21	0.69	3.37	0
(5b) Maintain fixed shares and welfare	-3.05	-1.06	0	0.69	3.72	0
<i>High elasticity of substitution: $\rho=-0.5$ ($\varepsilon=2$)</i>						
(1) Passive response	0	0	0.57	0.26	2.63	-2.32
(2a) Maintain fixed growth rate (adjusting τ_W)	4.71	0	1.57	0	2.22	-5.30
(2b) Maintain fixed growth rate (adjusting τ_K)	0	5.05	0.57	0	2.63	-4.87
(3) Maintain fixed shares (adjusting τ_W)	-2.50	0	0	0.41	2.88	-0.72
(4) Maintain fixed welfare (adjusting τ_W)	-3.63	0	-0.27	0.48	3.00	0
(5a) Maintain fixed welfare (adjusting τ_K)	0	-4.77	0.57	0.50	2.63	0
(5b) Maintain fixed shares and welfare	-2.50	-1.45	0	0.48	2.88	0

2.03%. The increase in risk raises the mean growth rate by 0.50% and also its volatility by 4.06%, leading to a reduction in welfare of 4.42%. We have already commented that to stabilize the volatility at its original level is infeasible, requiring a tax on labor income of 3000%! Stabilization of the mean growth rate is feasible and can be achieved in one of two ways. First, by raising the tax on labor income by 4.36%, this leads to a substantial migration of labor to the informal sector, raising the overall share of labor by 5.56%. The shift in labor has a modest effect on reducing volatility and while this may be welfare improving, it is overwhelmed by the adverse effects of the decline in the mean growth rate, and the welfare loss is exacerbated to 10.1%. Alternatively, the growth rate may be stabilized by raising the tax on capital income by 12.6%, but since this has no impact on mitigating volatility, the impact on welfare is even more adverse.

But there are more promising policy responses. Row 3 summarizes the case where the government stabilizes the (gross) share of labor income by reducing the wage tax by 3.57%. This increases the mean growth rate by 0.90% while increasing the volatility only marginally more than in the case of the passive policy. The net effect is that the welfare loss resulting from the higher production risk is reduced to 0.17%.

Table 2b
Policy responses to increase in σ_v from 0.125 to 0.25

	$\Delta\tau_W$	$\Delta\tau_K$	ΔS_L	$\Delta\psi$	$\Delta\sigma_w$	ΔX
<i>Low elasticity of substitution: $\rho=1$ ($\varepsilon=0.5$)</i>						
(1) Passive response	0	0	-0.461	0.126	0.338	1.055
(2a) Maintain fixed growth rate (adjusting τ_W)	1.350	0	0.352	0	0.318	-0.663
(2b) Maintain fixed growth rate (adjusting τ_K)	0	2.735	-0.461	0	0.338	-0.393
(3) Maintain fixed shares (adjusting τ_W)	0.767	0	0	0.055	0.327	0.086
(4) Maintain fixed welfare (adjusting τ_W)	0.834	0	0.040	0.048	0.326	0
(5a) Maintain fixed welfare (adjusting τ_K)	0	1.997	-0.461	0.034	0.338	0
(5b) Maintain fixed shares and welfare	0.767	0.165	0	0.047	0.327	0
<i>Cobb–Douglas: $\rho=0$ ($\varepsilon=1$)</i>						
(1) Passive response	0	0	-0.515	0.212	0.808	1.363
(2a) Maintain fixed growth rate (adjusting τ_W)	3.247	0	0.871	0	0.864	-1.893
(2b) Maintain fixed growth rate (adjusting τ_K)	0	4.240	-0.515	0	0.808	-0.906
(3) Maintain fixed shares (adjusting τ_W)	1.200	0	0	0.133	0.826	0.173
(4) Maintain fixed welfare (adjusting τ_W)	1.373	0	0.074	0.122	0.829	0
(5a) Maintain fixed welfare (adjusting τ_K)	0	2.256	-0.515	0.084	0.808	0
(5b) Maintain fixed shares and welfare	1.200	0.333	0	0.117	0.826	0
<i>High elasticity of substitution: $\rho=-0.5$ ($\varepsilon=2$)</i>						
(1) Passive response	0	0	-0.431	0.323	1.515	1.430
(2a) Maintain fixed growth rate (adjusting τ_W)	9.849	0	1.866	0	1.979	-5.254
(2b) Maintain fixed growth rate (adjusting τ_K)	0	5.907	-0.431	0	1.515	-1.738
(3) Maintain fixed shares (adjusting τ_W)	1.690	0	0	0.263	1.596	0.263
(4) Maintain fixed welfare (adjusting τ_W)	2.207	0	0.095	0.250	1.615	0
(5a) Maintain fixed welfare (adjusting τ_K)	0	2.270	-0.430	0.176	1.515	0
(5b) Maintain fixed shares and welfare	1.690	0.508	0	0.236	1.596	0

Indeed, it is possible to eliminate the welfare loss entirely. By reducing the wage tax even further—by 3.71%—the policymaker can induce a larger modest increase in the mean growth rate to 0.92%, with a slightly larger volatility (4.45% rather than 4.43%). While this results in a slight reduction in the overall share of income being earned by labor (−0.08%), nevertheless, overall welfare is preserved. Welfare can also be preserved if instead, the government reduces the tax on capital by 10%. In this case, the mean growth rate will be increased by 0.90%, with volatility remaining at 4.06%, but with the share of labor increasing by 2.03%. Although these two tax policies achieve the same overall welfare, the former is more favorable to capital and the latter to labor. Finally by combining the tax cut on labor income of 3.57% [in row 3] with a 0.35% cut in the tax on capital income, the policymaker can neutralize the effects of the higher volatility on both the distribution of income and the overall welfare level.

Moving down the panels of Table 2a as the elasticity of substitution increases, the same pattern describing the policy responses is observed. The case of an increase in risk in the informal sector, discussed in Table 2b, is analogous. The one difference is that most of the policy responses will be reversed due to the fact that this form of risk induces a decrease, rather than an increase, in l , which needs to be offset in designing the optimal policy response.

6. Second best optimal policy responses

Thus far, the tax rates and the policy responses have been arbitrary. We now consider the consequences of redistributing the tax burdens from labor to capital on the equilibrium growth rate and welfare. In equilibrium, the existing (constant) tax rates on labor and capital income generate tax revenues equal to $(\tau_W \delta(l)(1-l) + r_K(l)\tau_K)K \equiv \bar{T}K$. This grows with the capital stock of capital and enables the policymaker to finance the constant fraction, $\bar{T}/B\Omega(l)$, of output. We abstract from the use of this revenue.¹⁸ We shall assume that the policymaker in setting the tax rates, τ_W, τ_K , maintains

$$\frac{(\tau_W \delta(l)(1-l) + r_K(l)\tau_K)}{B\Omega(l)} = \frac{\bar{T}}{B\Omega(l)} = \text{constant} \quad (27)$$

It is straightforward to show that if the policymaker wishes finance the constant share of output in Eq. (27) through taxes, then in the absence of risk, equalizing the tax rates $\tau_W = \tau_K$ will maximize the growth rate. Thus, the assumption we have made in our benchmark simulations ($\tau_W = \tau_K = 0.20$) corresponds to growth maximization in a deterministic

¹⁸ We are implicitly assuming that the revenue raised is rebated to all individuals as a lump-sum transfer. In our current setup, with homogeneous agents, all individuals pay and receive the same amount. In García-Peñalosa and Turnovsky (2004), we justify the use of this type of policy to attain distributional objectives when individuals differ in their capital endowments, in the context of a (non-stochastic) two-sector growth model. Our results in that paper and, in particular, the fact that the distribution of endowments does not affect aggregate outcomes implies that our present results are robust to the introduction of endowment heterogeneity.

economy. But it can also be shown that equalizing the tax rates does not maximize welfare in such an economy; this requires the tax rate on labor to be reduced below that on capital.¹⁹

In the absence of policy constraints, i.e., if both sectors could be taxed, it would be optimal to raise all revenue through a labor income tax since such a tax would not distort the allocation of labor across sectors. But the inability to tax the informal sector means that the wage tax reduces the labor supplied to the formal sector and, consequently, the marginal product of capital. Under these circumstances, shifting the tax burden from labor to capital has two effects: the higher tax on interest income reduces the net interest rate, while the lower wage tax increases employment in the formal sector and thus increases the marginal product of capital. With no risk, the distortion in the allocation of labor due to the wage tax is large, implying that the former effect always dominates.

Tables 3a and 3b present numerical solutions for varying degrees of risk. In the absence of risk ($\sigma_u = \sigma_v = 0$), we see that maintaining the tax rates at their equal initial levels ($\tau_W = \tau_K = 20\%$) is growth-maximizing, confirming the result just noted. The changes in these tables are taken about the corresponding economy, in Tables 1a and 1b, in which the tax rates are set at the baseline levels $\tau_W = \tau_K = 20\%$. Thus, for example, for the economy characterized by $\varepsilon = 0.5$, $\sigma_u = 0.125$, $\sigma_v = 0.25$, setting the tax rates at their respective second best optima, $\hat{\tau}_W = 15.4\%$, $\hat{\tau}_K = 28.9\%$, will raise welfare by 0.336% and reduce the growth rate by 0.03 percentage points to 2.49%.

A number of results emerge. The second best welfare maximizing tax policy is to reduce the tax on the wage rate from its initial benchmark level of 20% to around 12–15%, depending upon the relative importance of the two shocks, and increase the tax on capital income correspondingly. In all cases, this reduces the fraction of labor employed in the informal sector. However, the impact on the growth rate is mild, reflecting two offsetting effects. Whereas the increase in employment in the formal sector tends to raise the growth rate, the higher capital tax tends to be growth-inhibiting. In the cases where the formal sector is more volatile than the informal sector, this shift toward a capital tax will generate an increase in aggregate volatility, although if the informal sector is more volatile, aggregate volatility will be reduced. Higher volatility in the formal sector is associated with a reduction in the optimal tax on labor income. This is because the higher volatility decreases the desirability of working in that sector. Higher volatility in the informal sector decreases the return to employment in that sector and this requires a compensating higher tax on wages in the formal sector.

The welfare gains from moving toward the second best optimal tax policy are remarkably stable across the various configurations of risk and productivity parameters, ranging between 0.3% and 1.2%. There are patterns, however. The welfare gains increase with risk in the formal sector as long as $\varepsilon \leq 1$ and decrease otherwise. They decrease with risk in the informal sector and are more sensitive to σ_v than to σ_u . Finally, the welfare gains increase with the elasticity of substitution because of the opportunity this presents for the more efficient use of productive factors.

¹⁹ We shall discuss only second best tax policies. The first best tax policy is uninteresting in this model. Without distortions in the labor market, it is simply to set $\tau_W = 0$, while subsidizing capital so as to induce the agent to take account of the externality in production.

Table 3a
Second best optimal policy: welfare maximization

	$\sigma_v=0$						$\sigma_v=0.125$						$\sigma_v=0.25$					
	\hat{l}	$\hat{\tau}_W$	$\hat{\tau}_K$	$d\hat{\psi}$	$d\hat{\sigma}$	$d\hat{X}$	\hat{l}	$\hat{\tau}_W$	$\hat{\tau}_K$	$d\hat{\psi}$	$d\hat{\sigma}$	$d\hat{X}$	\hat{l}	$\hat{\tau}_W$	$\hat{\tau}_K$	$d\hat{\psi}$	$d\hat{\sigma}$	$d\hat{X}$
<i>Low elasticity of substitution: $\rho=1$ ($\varepsilon=0.5$)</i>																		
$\sigma_u=0$	19.3	13.3	32.5	−0.039	0.000	0.736	19.8	13.8	31.6	−0.049	−0.287	0.611	21.2	15.5	28.6	−0.062	−0.405	0.317
$\sigma_u=0.125$	20.8	13.1	33.1	0.007	0.383	0.764	21.4	13.7	32.0	−0.006	0.283	0.637	22.7	15.4	28.9	−0.032	0.087	0.336
$\sigma_u=0.25$	25.3	12.6	34.9	0.149	0.797	0.847	25.6	13.1	33.9	0.123	0.699	0.714	26.9	14.9	30.5	0.058	0.425	0.395
<i>Cobb–Douglas: $\rho=0$ ($\varepsilon=1$)</i>																		
$\sigma_u=0$	32.6	13.0	30.5	−0.059	0.000	1.022	32.9	13.5	29.8	−0.078	−0.403	0.862	34.4	15.2	27.2	−0.104	−0.576	0.477
$\sigma_u=0.125$	34.5	12.8	30.8	−0.002	0.520	1.023	34.8	13.3	30.1	−0.025	0.326	0.866	36.0	15.0	27.5	−0.068	−0.032	0.488
$\sigma_u=0.25$	39.8	12.3	31.6	0.151	0.997	1.025	40.0	12.8	30.8	0.117	0.834	0.877	40.8	14.5	28.3	0.037	0.439	0.515
<i>High elasticity of substitution: $\rho=-0.5$ ($\varepsilon=2$)</i>																		
$\sigma_u=0$	49.7	12.5	28.0	−0.077	0.000	1.158	50.0	13.1	27.3	−0.104	−0.472	1.005	50.2	14.6	25.7	−0.159	−0.736	0.614
$\sigma_u=0.125$	51.9	12.4	27.9	−0.019	0.585	1.126	51.9	12.9	27.4	−0.051	0.244	0.980	52.0	14.4	25.8	−0.118	−0.247	0.608
$\sigma_u=0.25$	57.3	12.1	27.7	0.117	1.030	1.043	57.1	12.5	27.4	0.081	0.807	0.917	56.9	14.0	25.9	−0.007	0.321	0.588

The optimal tax rates, $\hat{\tau}_W, \hat{\tau}_K$, and labor allocation, \hat{l} , are in percentages; $d\hat{\psi}$, $d\hat{\sigma}$ are in percentage point changes; $d\hat{X}$ is in percentage changes.

Table 3b

Second best optimal policy: growth maximization

	$\sigma_v=0$						$\sigma_v=0.125$						$\sigma_v=0.25$					
	\hat{l}	$\hat{\tau}_W$	$\hat{\tau}_K$	$d\hat{\psi}$	$d\hat{\sigma}$	$d\hat{X}$	\hat{l}	$\hat{\tau}_W$	$\hat{\tau}_K$	$d\hat{\psi}$	$d\hat{\sigma}$	$d\hat{X}$	\hat{l}	$\hat{\tau}_W$	$\hat{\tau}_K$	$d\hat{\psi}$	$d\hat{\sigma}$	$d\hat{X}$
<i>Low elasticity of substitution: $\rho=1$ ($\varepsilon=0.5$)</i>																		
$\sigma_u=0$	28.4	20.0	20.0	0	0	0	30.2	21.5	16.8	0.002	0.071	−0.350	35.4	26.1	5.83	0.033	0.569	−1.579
$\sigma_u=0.125$	24.6	16.0	28.0	0.014	0.228	0.634	26.3	17.5	25.1	0.005	0.116	0.410	31.7	22.3	15.0	0.004	−0.032	−0.435
$\sigma_u=0.25$	11.9	1.2	52.0	0.226	1.852	−0.783	13.7	2.9	49.7	0.184	1.616	−0.626	19.2	8.3	41.7	0.084	0.955	−0.172
<i>Cobb–Douglas: $\rho=0$ ($\varepsilon=1$)</i>																		
$\sigma_u=0$	45.3	20.0	20.0	0	0	0	47.7	21.8	17.3	0.004	0.104	−0.530	54.8	27.6	8.60	0.062	0.829	−2.527
$\sigma_u=0.125$	41.1	16.5	25.3	0.013	0.250	0.747	43.8	18.5	22.3	0.003	0.067	0.342	51.2	24.4	13.4	0.019	0.081	−1.189
$\sigma_u=0.25$	23.9	2.9	45.7	0.218	2.247	−0.577	27.3	5.1	42.4	0.160	1.777	−0.195	37.5	12.3	31.6	0.040	0.632	0.435
<i>High elasticity of substitution: $\rho=-0.5$ ($\varepsilon=2$)</i>																		
$\sigma_u=0$	64.7	20.0	20.0	0	0	0	67.5	22.3	18.0	0.006	0.128	−0.681	74.3	29.0	13.2	0.089	0.919	−2.986
$\sigma_u=0.125$	61.1	17.1	22.7	0.009	0.206	0.663	64.3	19.5	20.4	0.000	0.013	0.124	72.1	26.6	14.8	0.044	0.316	−1.939
$\sigma_u=0.25$	44.6	5.7	36.0	0.151	2.036	0.229	49.9	8.7	32.0	0.093	1.301	0.638	63.0	17.8	22.0	0.004	0.102	0.344

The optimal tax rates, $\hat{\tau}_W, \hat{\tau}_K$, and labor allocation, \hat{l} , are in percentages; $d\hat{\psi}, d\hat{\sigma}$ are in percentage point changes; $d\hat{X}$ is in percentage changes.

Comparing Tables 3a and 3b highlights the tradeoffs between growth maximization and welfare maximization and the contrasts between them. In general, maximizing the expected growth rate does not coincide with welfare maximization. The welfare gains from growth-maximizing tax policy range between 0.75% and –3.0% and thus, in some cases, leads to welfare losses relative to the corresponding benchmark economy. Maximizing the growth rate requires increasing volatility and thus imposes too much risk on the risk-averse representative agent. The relative size of the informal sector to the formal sector in the growth-maximizing economy relative to that in the welfare-maximizing economy thus depends upon the relative riskiness of the two sectors. Finally, whereas the optimal tax rates in a welfare-maximizing economy are rather insensitive to varying degrees of risk, in a growth-maximizing economy, they are highly sensitive to risk, particularly to σ_u .

7. Conclusions

Developing economies are subject to large fluctuations in the level of output and the rate of growth, yet little work has been done to try to understand the effect of volatility on distribution. In this paper, we have used an endogenous, stochastic growth model to examine the relationship between the volatility of growth and the factor distribution of income. Our framework incorporates two important features of developing economies: the co-existence of a modern and a traditional sector and the fact that traditional sector employment—or at least a large part of it—takes place outside the formal labor market mechanism. This implies that governments cannot tax incomes generated in the traditional sector, and this imposes constraints on policy responses and decisions.

Our analysis shows that the relationship between volatility and the labor share is a complex one. First, the source of risk matters, as an increase in volatility will be associated with a lower labor share only if it stems from greater risk in the traditional (or informal) sector. Increased risk in the formal sector, on the other hand, tends to shift labor away from this sector, increasing its marginal product and hence its share in income. A second difficulty in characterizing this relationship is due to the fact that for high values of the elasticity of substitution between capital and labor in the formal sector, the shares of labor in the formal sector and in the economy as a whole move in opposite directions in response to greater risk. This means that if only formal sector incomes can be measured, the observed changes in factor shares will not necessarily correspond to the true ones.

The government's policy constraint provides an advantage in terms of the possible responses to an increase in risk. Because only labor incomes in the formal sector can be taxed, changes in the wage tax affect the allocation of labor across sectors. This means that the government has two policy instruments: the capital income tax which affects savings and the wage tax which affects sectoral employment. This allows the policymaker to have two targets. By a suitable adjustment of the two tax rates following an increase in risk, it is possible to maintain both factor shares and welfare at their original levels.

An important question concerns the optimal tax structure in developing countries. If all labor incomes could be taxed, a wage tax would not distort the allocation of

employment across sectors and it would be optimal to raise all necessary revenue through it. However, the policy constraint implies that the wage tax results in a level of employment in the formal sector below the social optimum. This distortion has a stronger impact on the rate of return than the capital income tax, implying that the second best optimal policy response requires a wage tax below the capital income tax. In the presence of risk, we find that optimal policy exhibits a tradeoff between growth and welfare. Increasing the growth rate requires increasing volatility, which imposes too much risk on the risk-averse representative agent. Maximizing growth thus entails a welfare loss that increases with the risk in the economy. This suggests that economic policy should encompass a wider range of objectives than simply the rate of growth of GDP.

We conclude by noting some caveats. Our analysis has been based on a closed economy, in which agents have no access to world financial markets. However, to the extent that agents do have the opportunity to invest abroad they may have an avenue for avoiding the tax on capital income as well as that on labor income. Two aspects limit the strength of this effect. First, as [Rodrik and van Ypersele \(2001\)](#) have discussed, the higher the degree of international capital mobility, the more the after-tax rate of return on domestic capital is tied to the world rate of interest, and the less the ability of the economy to set an independent tax on capital. The access to world capital markets of a developing economy is likely to be low, implying that this factor would not be too large. The essence of our results will continue to hold, with the optimal tax policy involving a tradeoff between the ability to avoid the two types of income taxes. Second, a government has the power to tax the income of its residents, wherever it is earned. If the returns to capital invested at home and abroad are taxed at the same rate, having access to international capital markets will not change our conclusions.

Another important issue concerns the nature of the formal sector and the sources of tax revenues. In our analysis, the formal sector is something like the manufacturing sector. Yet, in some developing economies, a major source of tax revenues is derived from the export of natural resources. This raises the question of how the policy implications derived in this model would extend to a three-sector economy consisting of say a formal sector, informal sector and a resource-exporting sector. We can conjecture that to the extent that export revenues provide a source of taxation, one would expect our results to generally hold, with the tax on exports substituting for the tax on labor. Both these aspects merit careful analysis and illustrate important directions in which our analysis can be usefully extended.

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Appendix A

This appendix provides some of the technical details underlying the derivation of the equilibrium (13a) conditions (13b) conditions (13c) conditions (13d) conditions (13e) conditions (13f) conditions (13g) conditions (13h) conditions (13i) conditions (13j), which is based on the concept of a “recursive competitive equilibrium” due to [Stokey and Lucas \(1989, pp. 479–480\)](#). The critical feature of this equilibrium is that the value function, as well as the optimal policies, is part of the equilibrium. This has the desirable feature that the equilibrium conditions—which with identical agents includes the equality of individual and aggregate quantities—can be substituted into the value function. This determines an equilibrium value function that then depends upon only one state variable, and can be readily solved, in contrast to the intractability usually encountered when two state variables are present.

The representative agent’s stochastic optimization problem is to choose his individual consumption–wealth ratio, the fraction of time devoted to formal sector and to the informal sector, and portfolio shares to maximize:

$$E \int_0^{\infty} \frac{1}{\gamma} C(t)^{\gamma} e^{-\beta t} dt \quad -\infty < \gamma < 1, \quad 0 \leq \theta \leq 1, \quad \theta \gamma < 1, \quad \gamma(1 + \eta) < 1 \quad (\text{A.1a})$$

subject to his individual constraints:

$$dW = \left\{ \left[n_B r_B + n_K (1 - \tau_K) r_K - \frac{C}{W} \right] W [(1 - \tau_W)(1 - l)\bar{\delta} + ql] \bar{n}_K \bar{W} \right\} dt + W dw \quad (\text{A.1b})$$

$$n_B + n_K = 1 \quad (\text{A.1c})$$

$$dw \equiv \left[(1 - \tau'_K) n_K r_K + (1 - \tau'_W) \delta (1 - l) \bar{n}_K \frac{\bar{W}}{W} \right] du + n_B du_B + q \ln_K \frac{\bar{W}}{W} dv \quad (\text{A.1d})$$

where $\bar{\delta} \equiv B\alpha(1 - \bar{l})^{-(1+\rho)} \Omega^{1+\rho}$, and $r_K \equiv B(1 - \alpha) \Omega^{1+\rho}$. The aggregate constraints are

$$d\bar{W} = \left\{ (1 - \tau_K) \bar{n}_K r_K + \bar{n}_B r_B - \left(\frac{C}{W} \right) + [(1 - \tau_W) \delta (1 - \bar{l}) + ql] \bar{n}_K \right\} \bar{W} dt + \bar{W} d\bar{w} \quad (\text{A.1b}')$$

$$\bar{n}_B + \bar{n}_K = 1 \quad (\text{A.1c}')$$

$$d\bar{w} \equiv [(1 - \tau'_K) r_K + (1 - \tau'_W) \delta (1 - \bar{l})] \bar{n}_K du + \bar{n}_B du_B + ql \bar{n}_K dv \quad (\text{A.1d}')$$

Since the individual perceives two state variables, W, \bar{W} , we consider a value function of the form

$$V(W, \bar{W}, t) = e^{-\beta t} X(W, \bar{W})$$

The differential generator of the value function $V(W, \bar{W}, t)$ is

$$\begin{aligned} \Psi[V(W, \bar{W}, t)] = & \frac{\partial V}{\partial t} + \left\{ \left[n_B r_B + n_K (1 - \tau_K) r_K - \frac{C}{W} \right] W \right. \\ & + \left[(1 - \tau_W)(1 - l)\bar{\delta} + ql \right] \bar{n}_K \bar{W} \left. \right\} V_W + \left\{ (1 - \tau_K) \bar{n}_K r_K + \bar{n}_B r_B \right. \\ & - \left(\frac{C}{W} \right) + \left[(1 - \tau_W)\delta(1 - \bar{l}) + ql \right] n_K \left. \right\} \bar{W} V_{\bar{W}} \\ & + \frac{1}{2} \sigma_w^2 W^2 V_{WW} + \sigma_{w\bar{w}} W \bar{W} V_{W\bar{W}} + \frac{1}{2} \sigma_{\bar{w}}^2 \bar{W}^2 V_{\bar{W}\bar{W}} \end{aligned} \quad (\text{A.2})$$

where we shall assume that with all agents being identical, the aggregate and individual proportional shocks are identical and perfectly correlated.

The individual's formal optimization problem is to choose C, l, n_B, n_K to maximize the Lagrangian expression

$$e^{-\beta t} \frac{1}{\gamma} C^\gamma + \Psi[e^{-\beta t} X(W, \bar{W})] + \frac{\lambda}{\beta} (1 - n_B - n_K) \quad (\text{A.3})$$

In doing this, he takes the evolution of the aggregate variables and the externality they generate as given. Taking the partial derivative with respect to C, l, n_B, n_K and canceling $e^{-\beta t}$ yields

$$C^{\gamma-1} = X_W \quad (\text{A.4a})$$

$$[q - \delta(1 - \tau_w)] \bar{n}_K \bar{W} X_W + \text{cov}[dw, du_l] \frac{W^2 X_{WW}}{dt} + \text{cov}[d\bar{w}, du_l] \frac{W \bar{W} X_{W\bar{W}}}{dt} = 0 \quad (\text{A.4b})$$

$$r_B W X_W + \text{cov}[dw, du_B] \frac{W^2 X_{WW}}{dt} + \text{cov}[d\bar{w}, du_B] \frac{W \bar{W} X_{W\bar{W}}}{dt} = \frac{\lambda}{\beta} \quad (\text{A.4c})$$

$$\begin{aligned} (1 - \tau_K) r_K W X_W + \text{cov}[dw, (1 - \tau'_K) du_K] \frac{W^2 X_{WW}}{dt} \\ + \text{cov}[d\bar{w}, (1 - \tau'_K) du_K] \frac{W \bar{W} X_{W\bar{W}}}{dt} = \frac{\lambda}{\beta} \end{aligned} \quad (\text{A.4d})$$

where

$$du_l = \left\{ -\delta(1 - \tau'_w) du + q dv \right\} \bar{n}_K \frac{\bar{W}}{W} \quad (\text{A.4e})$$

In addition, the value function must satisfy the Bellman equation

$$\max \left\{ e^{-\beta t} \frac{1}{\gamma} C^\gamma + \Psi[e^{-\beta t} X(W, \bar{W})] \right\} = 0 \quad (\text{A.3'})$$

Being a function of two state variables, the Bellman equation is a partial differential equation in the individual and aggregate wealth, W and \bar{W} , which recalling ψ , Eqs. (A.1b) and (A.1b') can be written:

$$\begin{aligned} \frac{1}{\gamma} C^\gamma - \beta X(W, \bar{W}) + \frac{E(dW)}{dt} X_W + \frac{E(d\bar{W})}{dt} X_{\bar{W}} + \frac{1}{2} \frac{E(dW)^2}{dt} X_{WW} \\ + \frac{E(dW d\bar{W})}{dt} X_{W\bar{W}} + \frac{1}{2} \frac{E(d\bar{W})^2}{dt} X_{\bar{W}\bar{W}} \end{aligned} \quad (\text{A.6})$$

The next step is to take the partial derivative with respect to W of the Bellman Eq. (A.6), noting that n_M, I are independent of W , while through the optimality condition (A.4a), C is a function of W . This yields the following condition:

$$\begin{aligned} C^{\gamma-1} C_W - \beta X_W + \frac{E(dW) X_W}{dt} + [n_B r_B + (1 - \tau_K) n_K r_K - C_W] X_W \\ + \frac{E(d\bar{W})}{dt} X_{\bar{W}W} + W X_{WW} \sigma_W^2 + \frac{1}{2} W^2 X_{WW} \frac{\partial \sigma_W^2}{\partial W} + \frac{1}{2} \frac{E(dW)^2}{dt} X_{WWW} \\ + \bar{W} X_{W\bar{W}} \sigma_W^2 + W \bar{W} X_{W\bar{W}} \frac{\partial \sigma_{W\bar{W}}}{\partial W} + \frac{1}{2} \frac{E(d\bar{W})^2}{dt} X_{\bar{W}\bar{W}W} \\ + \frac{E(dW d\bar{W})}{dt} X_{W\bar{W}W} = 0 \end{aligned} \quad (\text{A.7})$$

Consider now $X_W = X_W(W, \bar{W})$. Taking the stochastic differential of this quantity yields:

$$\begin{aligned} dX_W = X_{WW} dW + X_{\bar{W}W} d\bar{W} + \frac{1}{2} X_{WWW} (dW)^2 + X_{W\bar{W}W} (dW)(d\bar{W}) \\ + \frac{1}{2} X_{W\bar{W}\bar{W}} (d\bar{W})^2 \end{aligned} \quad (\text{A.8})$$

Taking expected values of Eq. (A.8), dividing by dt and substituting the resulting equation along with Eq. (A.4a) into Eq. (A.7) leads to:

$$\begin{aligned} (\phi - \beta) X_W + [W X_{WW} \sigma_W^2 + \bar{W} X_{W\bar{W}} \sigma_{W\bar{W}}] + \left[\frac{1}{2} W^2 X_{WW} \frac{\partial \sigma_W^2}{\partial W} + W \bar{W} X_{W\bar{W}} \frac{\partial \sigma_{W\bar{W}}}{\partial W} \right] \\ + \frac{E(dX_W)}{dt} = 0 \end{aligned} \quad (\text{A.9})$$

where $\phi \equiv n_B r_B + n_K (1 - \tau_K) r_K$.

The solution to this equation is by trial and error. Given the form of the objective function, we propose a value function of the form:

$$X(W, \bar{W}) = c W^{\gamma-\gamma_2} \bar{W}^{\gamma_2} \quad (\text{A.10})$$

where the parameters γ, γ_2 are to be determined. From Eq. (A.10), we obtain:

$$\begin{aligned} X_W &= (\gamma - \gamma_2)X/W; & X_{\bar{W}} &= \gamma_2 X/\bar{W}; & X_{WW} &= (\gamma - \beta)(\gamma - \beta - 1)X/W^2; \\ X_{\bar{W}\bar{W}} &= \gamma_2(\gamma_2 - 1)X/\bar{W}^2; & X_{W\bar{W}} &= \gamma_2(\gamma - \gamma_2)X/W\bar{W} \end{aligned} \quad (\text{A.11})$$

Since our concern is solving for the macroeconomic equilibrium, we shall impose the following equilibrium conditions:

$$\bar{n}_i = n_i, \quad i = B, K; \quad \bar{W} = W; \quad \text{and hence } \sigma_w^2 = \sigma_{w\bar{w}} \quad (\text{A.12})$$

which by the nature of the recursive competitive equilibrium can eventually be substituted into the equilibrium value function.

We now consider $\partial \sigma_w^2 / \partial W$, $\partial \sigma_{w\bar{w}} / \partial W$. From Eq. (A.1d), we obtain

$$\begin{aligned} \sigma_w^2 &= \left[(1 - \tau'_K) n_K r_K + (1 - \tau'_W) \delta (1 - l) \bar{n}_K \frac{\bar{W}}{W} \right]^2 \sigma_u^2 + n_B^2 \sigma_B^2 + q^2 l^2 \bar{n}_K^2 \left(\frac{\bar{W}}{W} \right)^2 \sigma_v^2 \\ &+ 2 \left[(1 - \tau'_K) n_K r_K + (1 - \tau'_W) \delta (1 - l) \bar{n}_K \frac{\bar{W}}{W} \right] n_B \sigma_{uB} \\ &+ 2 \left[(1 - \tau'_K) n_K r_K + (1 - \tau'_W) \delta (1 - l) \bar{n}_K \frac{\bar{W}}{W} \right] q l \bar{n}_K \frac{\bar{W}}{W} \sigma_{uv} \\ &+ 2 n_B q l \bar{n}_K \frac{\bar{W}}{W} \sigma_{Bv} \end{aligned} \quad (\text{A.13})$$

Differentiating Eq. (A.13) with respect to W , while noting the stochastic component of the government budget constraint

$$n_B du_B = [n_B(r_K + \delta(1 - l)) + n_K r_K \tau'_K + n_K \delta(1 - l) \tau'_W] du + n_B q l dv \quad (\text{A.14})$$

we find that in equilibrium

$$\begin{aligned} \frac{\partial \sigma_w^2}{\partial W} &= -2(1 - \tau'_W) \delta (1 - l) \frac{n_K}{W} [(r_K + \delta(1 - l)) \sigma_u^2] \\ &- 2q l \frac{n_K}{W} [q l \sigma_v^2 + (r_K + \delta(1 - l)) \sigma_{uv}] \end{aligned} \quad (\text{A.15a})$$

Likewise, we find

$$\frac{\partial \sigma_{w\bar{w}}}{\partial W} = \frac{1}{2} \frac{\partial \sigma_w^2}{\partial W} \quad (\text{A.15b})$$

Substituting Eqs. (A.15a) and (A.15b) into Eq. (A.9) leads to:

$$\frac{E(dX_W)}{X_W dt} = (\beta - \phi) + (1 - \gamma) \left[\sigma_w^2 + \frac{W}{2} \frac{\partial \sigma_w^2}{\partial W} \right] \quad (\text{A.16})$$

Substituting for Eqs. (A.13) and (A.15a) and recalling the definition of r_K , this can be expressed in terms of the underlying stochastic shocks

$$\begin{aligned} \frac{E(dX_W)}{X_W dt} = & (\beta - \phi) + (1 - \gamma) \{ B^2 \Omega^2 [1 - (1 - \tau'_W)(1 - (1 - \alpha)\Omega^\rho)n_K] \sigma_u^2 \\ & + B\Omega[(2 - n_K) - (1 - \tau'_W)(1 - (1 - \alpha)\Omega^\rho)n_K] ql\sigma_{uv} + q^2 l^2 (1 - n_K) \sigma_v^2 \} \end{aligned} \quad (A.17)$$

Now returning to Eq. (A.4a) and computing the stochastic differential of this relationship and taking expected values yields:

$$\frac{E(dX_W)}{X_W} = (\gamma - 1) \frac{E(dC)}{C} + \frac{1}{2}(\gamma - 1)(\gamma - 2)E\left(\frac{dC}{C}\right)^2 \quad (A.18)$$

Focusing on a balanced growth path along which C/W is constant,

$$\frac{dC}{C} = \frac{dW}{W} \equiv \psi dt + dw$$

we may write:

$$\frac{E(dX_W)}{X_W dt} = (\gamma - 1)\psi + \frac{1}{2}(\gamma - 1)(\gamma - 2)\sigma_w^2 \quad (A.19)$$

Equating Eqs. (A.17) and (A.19), we may express the mean growth rate as:

$$\begin{aligned} \psi = & \frac{\phi - \beta}{1 - \gamma} - B^2 \Omega^2 \left\{ \left[\frac{\gamma}{2} - (1 - \tau'_W)(1 - (1 - \alpha)\Omega^\rho)n_K \right] \sigma_u^2 - B\Omega[\gamma - n_K(2 - \tau'_W) \right. \\ & \left. + (1 - \tau'_W)(1 - \alpha)\Omega^\rho n_K] ql\sigma_{uv} + q^2 l^2 \left(n_K - \frac{\gamma}{2} \right) \sigma_v^2 \right\} \end{aligned} \quad (A.20)$$

This is the equilibrium growth rate relationship given by Eq. (13j) in the text.

Combining Eqs. (A.20) with (A.1b), we can express the equilibrium consumption–wealth ratio as

$$\frac{C}{W} = \phi + [(1 - \tau_W)\delta(1 - l) + ql]n_K - \psi \quad (A.21)$$

Substituting Eq. (A.14) into Eq. (A.1d), and using the equilibrium condition the equilibrium stochastic component of wealth is

$$dw = B\Omega(l)du + ql dv \quad (A.1d')$$

implying that the variance is:

$$\sigma_w^2 = B^2 \Omega(l)^2 \sigma_u^2 + 2B\Omega(l)ql\sigma_{uv} + q^2 l^2 \sigma_v^2 \quad (A.22)$$

This is Eq. (13i) of the text.

Substituting the equilibrium conditions into Eq. (A.4e) the stochastic shock to wages is

$$du_l \equiv \{ -\delta(1 - \tau'_w)du + qdv \} n_K \quad (\text{A.4e}')$$

Combining this with Eq. (A.1d') in Eq. (A.4c) yields the equilibrium labor allocation condition (13e):

$$\delta(1 - \tau_w) - q = (1 - \gamma) [B\Omega\delta(1 - \tau'_w)\sigma_u^2 - (B\Omega - \delta l(1 - \tau'_w))q\sigma_{uv} - q^2 l\sigma_v^2] \quad (\text{A.23a})$$

In addition, substituting Eqs. (A.1d') and (A.14) into Eqs. (A.4c) and (A.4d), and subtracting, leads to the equilibrium portfolio allocation condition (13f):

$$r_B = r_K(1 - \tau_K) + (1 - \gamma) \left\{ \sigma_w^2 + (B\Omega\sigma_u^2 + ql\sigma_{uv}) \left[\frac{n_K}{1 - n_K} (\tau'_K r_K + \tau'_w \delta(1 - l)) - r_K(1 - \tau'_K) \right] \right\} \quad (\text{A.23b})$$

Finally, the consumer budget constraint (13g) is obtained directly from the deterministic component of Eq. (9a), while the goods market equilibrium condition (13h) follows directly from the deterministic component of Eq. (12), thus completing the derivation of the stochastic equilibrium.

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