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MODELING GEOGRAPHIC FERROUS SCRAP MARKETS: REGIONAL PRICES AND INTERREGIONAL TRANSACTIONS IN THE UNITED STATES*

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ABSTRACT. The *U.S. Regional Ferrous Scrap Model* analyzes spatial variations in prices for two grades of ferrous scrap using a logistic model of choice under differentiated products. The model uses a computer-generated equilibrium framework to solve for prices that support the observed spatial distribution of supply and demand quantities. This paper presents the model's formal structure and its solution algorithm. The model specification is highly disaggregated with 1,212 supply and 240 demand regions. Characteristics of the equilibrium solution are described for prices and interregional flows. Sensitivity of equilibrium values to changes in model parameters is reported.

1. INTRODUCTION

When Joseph Schumpeter coined the phrase "Creative Destruction" he used U.S. Steel as an example of a company on the cutting edge of competition (Schumpeter, 1950, p. 83). At that time, U.S. Steel was revitalizing the market from within by beating old-line manufacturers. U.S. Steel remains a competitive firm in a competitive market, but now U.S. Steel is one of a number of firms being challenged by new competitors. Intra-industry competition in the steel industry today is a clash of two technologies, one based on the reduction of iron ore and its transformation into steel and the other based on the direct melting of ferrous scrap. U.S. Steel is still one of the leading ore-based producers; the domestic challengers depend on ferrous scrap.

The geography of ferrous scrap markets, as defined by interregional scrap flows and the spatial distribution of prices, is central to the geography of competition in steel markets. The economic model presented here characterizes regional markets for ferrous scrap in great detail: its equilibrium is defined by a set of prices that balances interregional flows of ferrous scrap among 1,212 supply regions and 240 demand regions, which encompass the continental United States. The model

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can be used to describe pricing patterns and interregional trade, and it also can serve as a basis for simulating changes in basic conditions related to supply and demand in narrowly defined geographic areas. Our objective in this paper is to present the structure of the *U.S. Regional Ferrous Scrap Model* and to characterize its equilibrium, and we do so by focusing on spatial price surfaces and interregional scrap flows. The primary use of this model is to provide an equilibrium solution of the spatial structure of prices and quantities that can be used as a basis for counterfactual analysis of changes in initial conditions. Thus, the model has been designed to analyze major market events such as the building or shut-down of a steel mill or the conversion of ore-based steelmaking capacity to scrap-based steelmaking capacity. Used in this way, the model can provide critical information on the spatial structure of prices and interregional shipments before and after the event. The model should not be used for price forecasting.

For parts of this paper, we draw on material published in Giarratani, Gruver, and Richmond (2002) to help describe market characteristics and the model's structure. The goal is to provide more extensive explanation of the formal model and to report and analyze model equilibrium spatial flows of scrap as well as prices and quantities. The earlier work is also extended by providing analysis of the model's sensitivity to parameter changes. Specifically, we provide evidence that the logistic function specification in the model, which is used to describe the spatial distribution of scrap supply, yields solutions that are relatively insensitive to moderate parameter variations.

2. CHARACTERISTICS OF THE MARKET FOR FERROUS SCRAP

In the past 25 years, there has been significant improvement and wide adoption of electric arc furnaces (EAFs) and continuous casters. As a result, the demand for ferrous scrap has dramatically increased. Electric arc furnace operators can, and often do, use 100 percent scrap for their metal input or furnace "charge." This compares to the basic oxygen furnace (BOF), the alternative technology for converting iron into steel, for which operators face a metallurgical maximum proportion of scrap to pig iron. The metallic charge in BOF furnaces averages 25 percent ferrous scrap (Houck, 1994, Table 4).

Electric furnace capacity has increased, not only in the United States, but also in many countries around the world. In many countries, the resulting demand for scrap exceeds domestic supply and the United States has been a significant exporter to satisfy that excess demand. In 1994, the year we pick as the baseline for this paper, scrap exports were 8.8 million metric tons compared to 1.7 for imports (Houck, 1994, Table 1). More recently, the Asian financial crisis, new export supply from the former Soviet Union (Marley, 2000), and a dramatic increase in demand from China have had a significant impact on the U.S. ferrous scrap market.

Ferrous scrap is demanded for export, foundries, and steelmaking. In addition to imports, ferrous scrap is supplied by three distinct domestic sources: home scrap, prompt scrap, and obsolete scrap. Scrap quality considerations and scrap price responsiveness are closely related to these scrap sources:

- Home scrap is the byproduct of foundry and steel mill operations in that normally product is lost during the manufacturing process due to spillage, cutting, and quality problems. Another term used to describe home scrap production is “yield loss.”[1]
- Prompt scrap results as a byproduct of manufacturing activities that use iron and steel as raw materials.
- Obsolete scrap comes from products and structures that are obsolete in the sense of having been used and discarded.

Because home and prompt scrap come from new products, the metallurgical characteristics of the resulting material are known. Typically, scrap from these sources has a low level of “residual” elements such as copper and other nonferrous metals, and residuals can affect the properties of steel in use. As a result, most purchased low-residual scrap comes from these sources. Unlike home scrap, which is produced at mills or foundries that use scrap, prompt scrap typically is produced at other locations and must be purchased and transported.

In contrast, obsolete scrap is a “postconsumer” product, and its metallurgical characteristics tend not to be uniform or well known. Most obsolete scrap has high-residuals content. There is a substantial stock of obsolete scrap, the spatial distribution of which is closely linked with the spatial distribution of population. This fact, as well as the collection and processing costs associated with this product, tends to make the supply of this source of scrap much more dependent on the price of scrap than is the case for home and prompt scrap, which result as the byproduct of other production activities.

Ferrous scrap is a heterogeneous input. Important factors contributing to this heterogeneity include distance between supplier and consumer, type and quality of scrap, and timing of demand over the year. Competitive small-scale suppliers and large-scale consumers have traditionally characterized the market structure; however, significant consolidation on the supply side has occurred in recent years.

3. RELATED LITERATURE

The roots of the spatial pricing literature go back, at least, to the well-known model of Hotelling (1929). As noted by d’Aspremont, Gabszewitz, and Thisse (1979), the existence of an equilibrium solution in formal spatial models is problematic. Discontinuities in the demand functions result when customers are overly sensitive to price or when there are discrete distributions of customers. These discontinuities often imply that solutions to spatial pricing problems do not exist. Yet, in applied modeling, it is often difficult to avoid discrete distributions of demand and discrete choices: to buy or not to

[1]The introduction of continuous casting as a replacement for ingot casting has decreased yield loss significantly. The decrease in prompt scrap resulting from lower yield loss is estimated to range from 10 to 20 percent of the raw steel cast.

buy. De Palma et al. (1985) argue that introducing heterogeneity into the model specification may solve many of the existing problems in spatial competition models like Hotelling's. De Palma and colleagues argue that actual markets are characterized by heterogeneity as a rule and that heterogeneity extends to consumers, suppliers, and products. Even if economic agents or goods were homogeneous, the data we use in modeling would inevitably mix "apples" and "oranges" and call the resulting composite "fruit." Thus, participants may be modeled as heterogeneous either on grounds of the empirical accuracy or because of technical tractability. In the case of the ferrous scrap industry, both arguments may be invoked. Consumers, suppliers, and the scrap itself are all heterogeneous, while the published data aggregates over time as well as products and consumers of various types. We consider only two grades of scrap whereas a dozen or more grades are widely traded. Each heterogeneous grade is defined by types and quantities of residual metals and will be further differentiated by being shredded, bailed, bundled, and so on. Furthermore, at a specific date during the one-year period of the model, a given supply location may not have sufficient inventory of the specific grade of scrap demanded. This is an example of heterogeneity over time. Each of these types of heterogeneity is accommodated by the specification of the logistic distribution function.

De Palma et al. introduce heterogeneity to the behavior of agents by making choice a probabilistic function of underlying characteristics, and the logistic function is the means by which this is accomplished. The reader is referred to de Palma et al. (1985), de Palma et al. (1994), Sheppard, Haining, and Plummer (1992), and Anderson and de Palma (1992a, b) for a discussion of the theoretical issues involved in this strategy. The *U.S. Regional Ferrous Scrap Model* builds on this tradition.

The logistic function has two primary advantages in our model. First, by using this function (which is presented in Appendix equations [4a] and [5a]) a wide range of spatial substitution behaviors may be replicated by varying the spatial allocation parameter (B) between infinity and zero. As this parameter approaches infinity, choice is highly substitutable and completely dependent on relative prices, which means that a merchant can capture the whole market by a small advantage over the competitors' offers. For instance, increasing the offer price by some very small amount may cause a supplier to switch from supplying none to supplying all of the quantity available at a given location. As B approaches zero, the allocation shares are much less responsive to price differentials. With this parameter value, the model form captures complete nonsubstitution behavior over regions. A second strength of this model is its concise first derivative with respect to the merchant's strategic variables. The chief advantage of the form of this derivative is that it facilitates the calculation of numerical solutions.

Sheppard, Haining, and Plummer (1992) and de Palma et al. (1994) have no economic behavior on the side of the market that evaluates the apportionment function (demand in their models) beyond the apportionment itself.

Total demand is fixed. Supply at each location accommodates demand under the assumption of fixed marginal costs. The *U.S. Regional Ferrous Scrap Model* differs from this construction in important ways. We model explicitly two grades of ferrous scrap, high-residual and low-residual, and account for substitution possibilities between these grades. For high-residual scrap, the regional supply functions are upward sloping and nonlinear, and the supply side of the market apportions the quantities as specified by the logistic function. In effect, each scrap supplier faces a set of customers and allocates supply to those customers in a probabilistic way. In our model, the demand side of the market has an input grade substitution function, but total demand is fixed. Switching the side of the market that evaluates the apportionment function is an important difference between the ferrous scrap model and previous models.

The model represents a single-period spatial equilibrium. Because low-residual scrap is primarily the byproduct of steel-consuming production (e.g., automobile production), the supply of low-residual scrap is assumed not to be price responsive in a given period, and it is specified as fixed for each region. Because high-residual scrap comes primarily from obsolete products and structures, stock is always available. To make these stocks useable at steel furnaces, costs must be incurred to separate components containing copper, lead, and other metals. Processing costs are involved for activities such as crushing, shredding, and bailing. In each period, scrap dealers must decide how much stock to hold for the future and how much processing cost to incur to make supplies of high-residual scrap available in the current period. Consistent with this characteristic, the model specifies per capita high-residual scrap supply as an increasing function of Free on Board (F.O.B.) prices. The per capita supply function is estimated from national data, and the estimated model includes a one-year price lag to account for the fact that the stock of obsolete scrap carried over from the previous year may affect the quantity of high-residual scrap supplied for a given price. Given that the U.S. Ferrous Scrap Model is based on a single-period equilibrium, lagged terms in the equation for scrap supply are constant.

From the demand side, the quantity demanded for both grades of ferrous scrap depends on the price of low-residual scrap relative to the price of high-residual scrap; however, the total scrap demanded in each region is fixed. We allow substitution between scrap grades, but we do not model the effect of scrap prices on steel demand. Our decision to model the market in this way is based primarily on the practical considerations involved in building a tractable model. Beyond this, however, we believe that the most significant price effects play out only in the longer run as capacity changes come to reflect plant start-ups, plant shut-downs, or the adoption of new technologies.

4. FORMAL STRUCTURE

Let the P_{ik}^L, P_{ik}^H identify the Cost, Insurance, and Freight (C.I.F.) prices and p_{ik}^L, p_{ik}^H the F.O.B. prices for scrap sold by region i of supply to region of demand k. The superscripts identify low-residual and high-residual scrap quality. Then the following equations hold for $i = 1 \ldots I$; $k = 1 \ldots K$:

(1)
$$p_{ik}^{L} = \max\left[P_{ik}^{L} - t_{ik}, 0\right]$$

$$p_{ik}^{H} = \max\left[P_{ik}^{H} - t_{ik}, 0\right]$$

The transportation and other cost of transfer t_{ik} over the distance d_{ik} from i to k is specified as,

$$t_{ik} = \alpha + \beta d_{ik}$$

where, $\alpha = \alpha_1$, $\beta = \beta_1$, if $d_{ik} < \bar{d}$, otherwise $\alpha = \alpha_2$, $\beta = \beta_2$.

Trucks are assumed to be the mode for shipments less than the fixed distance parameter $\bar{d}$ and rail; otherwise, each mode has different cost parameters. The quantities of low-residual and high-residual scrap sold by region of supply i to region of demand k are represented by q_{ik}^{L} and q_{ik}^{H}. These quantities are shares ρ_{ik}^{L} and ρ_{ik}^{H} of the total quantities Q_i^{LS} and Q_i^{HS} supplied by region i to all K demand regions. That is,

$$q_{ik}^{L} = \rho_{ik}^{L} Q_i^{LS}$$

$$q_{ik}^{H} = \rho_{ik}^{H} Q_i^{HS}$$

Total low-residual scrap Q_i^{LS} is estimated and fixed exogenous to the model. Total high-residual scrap Q_i^{HS} is assumed proportional to population (Pop_i) and the per capita supply $\left(f\left(\bar{p}_i^{H}\right)\right)$ is an endogenous increasing function of the weighted average F.O.B. price,

$$Q_i^{LS} = \hat{Q}_i^{LS}$$

$$Q_i^{HS} = f(\bar{p}_i^{H}) \cdot Pop_i, \text{where}$$

$$\bar{p}_i^{H} = \sum\nolimits_k \rho_{ik}^{H} p_{ik}^{H}, and\ \bar{p}_i^{L} = \sum\nolimits_k \rho_{ik}^{L}\, p_{ik}^{L}$$

The shares ρ_{ik}^{L} and ρ_{ik}^{H} supplied by region i to k demand region are specified as logistic functions of relative F.O.B. prices:

(2)
$$\rho_{ik}^{L} = \frac{k_{ik}^{L} e^{B\tilde{p}_{ik}^{L}}}{\sum_k k_{ik}^{L} e^{B\tilde{p}_{ik}^{L}}}$$

where $k_{ik}^{L} = 1$ if $p_{ik}^{L} > 0$, and $k_{ik}^{L} = 0$ otherwise.

$$\rho_{ik}^{H} = \frac{k_{ik}^{H} e^{B\tilde{p}_{ik}^{H}}}{\sum_k k_{ik}^{H} e^{B\tilde{p}_{ik}^{H}}}$$

where $k_{ik}^{H} = 1$ if $p_{ik}^{H} > 0$, and $K_{ik}^{H} = 0$ otherwise.

The larger the parameter B, the greater the sensitivity of the share to differentials in relative prices. Equilibrium solutions to the model have been obtained for values of B ranging from 30 to 60. A value for B of approximately 60 produces solutions consistent with the degree of regional price variation observed in published sources. The share is made a function of relative but not absolute prices by dividing each price by the average price $\bar{p}_i^L$ or $\bar{p}_i^H$ so the normalized prices are distributed around unity. This normalization allows the parameter value of B to be independent of absolute price levels. That is,

$$\tilde{p}_{ik}^L = \frac{p_{ik}^L}{\bar{p}_i^L} \tag{3}$$

and

$$\tilde{p}_{ik}^H = \frac{p_{ik}^H}{\bar{p}_i^H}$$

Combining the above specifications we can write,

$$q_{ik}^L = \frac{k_{ik}^H e^{B\tilde{p}_{ik}^H}}{\sum_k k_{ik}^H e^{B\tilde{p}_{ik}^H}} Q_i^{LS} \tag{4}$$

$$q_{ik}^H = \frac{k_{ik}^H e^{B\tilde{p}_{ik}^H}}{\sum_k k_{ik}^H e^{B\tilde{p}_{ik}^H}} Q_i^{HS}$$

The equilibrium conditions balancing supply and demand for each demanding region and each type of scrap are:

$$\sum\nolimits_i q_{ik}^L = Q_i^{LD}$$

and

$$\sum\nolimits_i q_{ik}^H = Q_i^{HD}$$

The proportion of low-residual scrap demanded is regulated by a technical minimum requirement of low-residual and the substitution of low-residual for high-residual as the price of low-residual approaches that of high-residual from above as follows,

$$\frac{Q_k^{LD}}{Q_k^{TD}} = min\left\{1, \frac{a}{\left(\frac{P_k^L}{P_k^H}\right)-1} + \frac{Q_k^{LMin}}{Q_k^{TD}}\right\} \tag{5}$$

where Q_k^{LMin} is the technical minimum quantity of low-residual scrap and,

$$Q_k^{TD} = Q_k^{LD} + Q_k^{HD}$$

The larger the value of parameter a, the greater the substitution of low-residual for high-residual scrap for any ratio of low-residual to high-residual prices up to the point at which 100 percent low-residual scrap is used. A value of .023 was used in the solution reported here. Issues related to choice of parameter values are discussed below.

The absolute level of prices is driven by the price of high-residual scrap. The input substitution function insures that the price for high-residual scrap represents a lower limit for the price of low-residual scrap. Adjustments in the high-residual price cause the changes in the quantity of high-residual scrap supplied, and these changes are required to bring total supply into equality with total demand.

The assumption that the buyers do not discriminate with respect to price implies that the C.I.F. price at each demand region k is independent of the region of supply i,

$$P_{ik}^L = P_k^L \text{ and } P_{ik}^H = P_k^H \text{ for all } i \text{ and } k$$

5. A DESCRIPTION OF THE SOLUTION ALGORITHM

The model is driven by the C.I.F. prices and the solution algorithm follows a process of adjusting these prices until the equilibrium conditions are satisfied. The process iterates through the following steps:

1. Pick initial values of $P_{ik}^L = P_k^L$ and $P_{ik}^H = P_k^H$ for all i and k.
2. Use P_k^L and P_k^H to calculate Q_k^{LD} and Q_k^{HD}.
3. Use $P_{ik}^L = P_k^L$ and $P_{ik}^H = P_k^H$ for all i and k to calculate p_{ik}^L and p_{ik}^H.
4. Use p_{ik}^L and p_{ik}^H to calculate ρ_{ik}^L and ρ_{ik}^H.
5. Use p_{ik}^L, p_{ik}^H, ρ_{ik}^L, and ρ_{ik}^H to calculate Q_i^{HS}.
6. Use p_{ik}^L, p_{ik}^H, ρ_{ik}^L, ρ_{ik}^H, and Q_i^{HS} to calculate q_{ik}^L and q_{ik}^H.
7. Use q_{ik}^L and q_{ik}^H to check for equilibrium with Q_k^{LD} and Q_k^{HD}.
   a. If equilibrium is satisfied, stop.
   b. If equilibrium is not satisfied, use excess demand values to adjust $P_{ik}^L = P_k^L$ and $P_{ik}^H = P_k^H$ for all i and k and return to step 2.

This process may be described in words as follows. For C.I.F. mill bid prices at each demand location, the algorithm works through each supply region. Transport costs are subtracted from 240 mill bid prices resulting in a matrix of 240 × 1,212 local F.O.B. bid prices. For each supply region, primary and secondary supply functions are calculated to determine first, how much total supply to bring to the market, and second, what portion of that supply will go to each customer. A running sum is made of the quantity supplied to each customer. When the algorithm has passed through every supplying location, the total supply going to each customer is then known. This total is compared to the customer's demand target, which is a price independent fixed quantity. The customer may have been supplied with too little or too much. If the quantity supplied is close enough to the target in every case, then the algorithm ends. Otherwise each customer updates her price, adapting it so as to acquire a quantity closer to her target. The algorithm begins again. Efficient convergence to a market solution is attained by adaptations of the Newtonian algorithm outlined in the Appendix.

6. DATA

The *U.S. Regional Ferrous Scrap Model* is data intensive. The basic units that constitute the supply and demand data sets are counties and plants, respectively. Some aggregating is done to both. This process reduces the number of consumers from more than 500 to 240 demand regions, and the number of supplying locations from more than 3,000 to 1,212.

Figure 1 provides an overview of the data structure. On the demand side, consumers represent steel mills, foundries, and exporters. An extensive independent database on U.S. steel plants has been developed for use with the model. This database is used in conjunction with data from *County Business Patterns* (various years) and export data from the Department of Commerce to determine levels of demand and input-grade substitution parameters for each individual consuming agent.

On the supply side, 1,212 supplying agents for high-residual scrap are parameterized by using population data at the county level from the U.S.

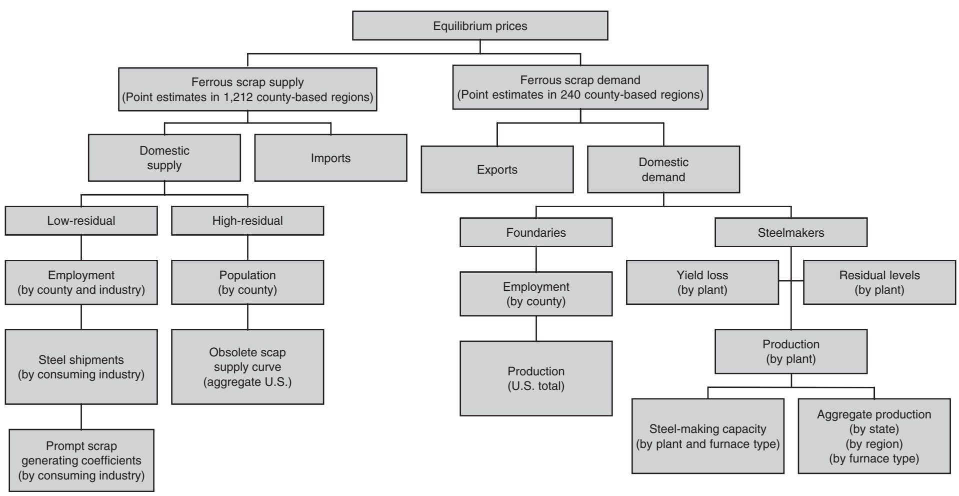

FIGURE 1: An Overview of Data Structure.

Department of Commerce, Bureau of the Census, and a per capita obsolete scrap supply function. The supply function is estimated using scrap prices from the *American Metal Market* and consumption data obtained electronically and from *Iron and Steel Scrap 1994* from the U. S. Geological Survey (Houck, 1994). Low-residual scrap supply is estimated as prompt scrap by applying industry-specific scrap-generating coefficients to steel shipment data from the *Annual Statistical Report* of the American Iron and Steel Institute (various years). Employment data related to steel-consuming manufacturing sectors from the Regional Economic Information System (various years) is used to allocate low-residual scrap supply to geographical regions. *Carload Waybill Statistics* are used to help estimate transport costs parameters.[2]

7. CHOOSING PARAMETER VALUES

A wide range of economic behaviors are captured in the possible values that can be assigned to the spatial allocation parameter in the logistic function, and there is little experience to guide parameterization, because use of the logistic function in an empirical spatial pricing model is relatively novel. De Palma et al. (1994) set the parameter corresponding to B by first determining the lowest levels where solutions still exist and then picking a higher value. Selection of the parameter levels investigated for the present model began with an intuitive determination of a reasonable range for the parameter.

For purposes of illustration, Table 1 presents a hypothetical example whereby a single seller confronts eight possible consumers. The offer price of the first consumer translates back to an F.O.B. price of 80 for the seller,

TABLE 1: Example of Allocation Shares Based on the Logistic Function

F.O.B. Prices	Prices Normalized by Mean = 104.9	Allocation Shares for Different Spatial Allocation Parameters	
		$B = 30$	$B = 60$
80	0.763	0.000	0.000
90	0.858	0.000	0.000
100	0.954	0.002	0.000
105	1.001	0.007	0.000
110	1.049	0.028	0.002
115	1.097	0.116	0.035
119	1.135	0.364	0.347
120	1.144	0.484	0.615

[2]Transport costs assume a minimum-size load and a fixed and variable cost per mile. The variable cost for shorter hauls is based on truck rates and the longer hauls assumes rail. Distances used in the solution reported here are minimum straight-line distances between a central point of the supply region and the demand region. For application of this model specific to a particular plant or geographical location, one could make specific adjustments to the structure of the transportation costs in a region around the central place of interest.

whereas the offer price of the eighth consumer translates back to an F.O.B. price of 120. The remaining consumers are distributed within this range. Dividing by the mean of 104.9 normalizes the prices. The two columns on the right are associated with two different values of the spatial allocation parameter B, and for each parameter value, the table shows the shares allocated to each consumer based on its offer price. At $B = 30$, a consumer that represents an F.O.B. price of 120 to the seller would be allocated 48.4 percent of the seller's supply. For $B = 60$, the same consumer would be allocated 61.5 percent of the seller's supply. Moreover, at the higher value for the spatial allocation parameter, 96.2 percent of the total supply would go to only two consumers, whereas significant supply amounts would be allocated to three or four consumers at a value of $B = 30$. In effect, the parameter B captures the differentiating effect of space. If it is large, all or nearly all of the scrap available from a given supply source will go to the customer (demand region) who offers the seller the highest F.O.B. price. If B is small, space has a differentiating effect and more of the supply is distributed to distant customers.

The speed with which the apportionment shifts depends both on the parameter in the logistic function and on how rapidly relative prices change as a function of variable transport costs. Given transport costs, the proximity of a supply region can be translated directly into a level of price advantage. Assuming a rail freight charge of 3.8 cents per mile, having a location that is 26 miles closer to a supplier is roughly equivalent to a $1 price advantage for a consumer.

For our analysis, a small number of scrap dealers were asked how long they would withhold supply for an expected increase in price of a given size. Dealers seemed only marginally sensitive to a $1 price difference, but very sensitive to a $10 difference. Translating dollars into miles, sharing should happen at a 26-mile proximity advantage but would be unlikely at a 260-mile advantage. In the simulations reported here, a value of 60 is used for the parameter B. This implies that for this example, a consumer with a $1 offer price disadvantage ($119 versus $120 and equivalent to 26 miles further distant) will still receive a 35 percent allocation of the supply. The consumer with a $5 disadvantage ($115 versus $120) will receive a 3.5 percent allocation.

The input-grade substitution parameter (a) also is important in determining the equilibrium solution. Specifically, the rate at which low-residual scrap is substituted for high-residual scrap as the price of the former approaches the price of the latter is determined by the value of this parameter. Figure 2 illustrates this point. For the model solutions reported in this paper, the value for the substitution parameter is $a = .023$. As shown in Figure 2, when a is set at this value, little substitution will occur until the price premium for low-residual scrap is less than 20 percent. For example, when the price premium for low-residual scrap is 20 percent, only about 12 percentage points more low-residual scrap will be used than the minimum assumed in the

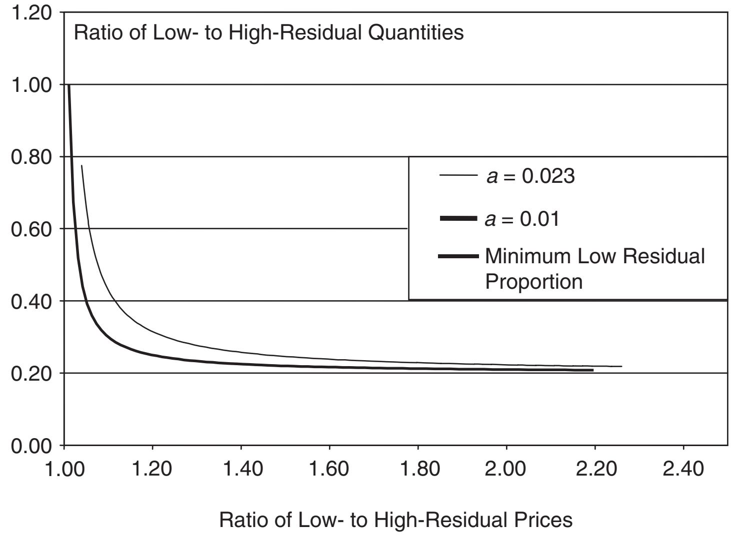

FIGURE 2: Substitution of Low-Residual for High-Residual Scrap.

example (i.e., 32 percent rather than 20 percent). If the substitution parameter had been set at a = .01, the rate of substitution would have been less rapid. At a 20 percent premium, the use of low-residual scrap would have been increased by only 5 percentage points relative to the minimum requirement. The choices of both parameters, *a* and *B*, have been chosen to help the model replicate known characteristics of markets. The primary effect of a larger value for parameter *a* is to increase the premium paid for low-residual over high-residual scrap in the equilibrium solution. The primary effect of an increase in either the variable transportation cost or the parameter *B* is a decrease in the geographic size of the supply region for each demand region.

8. NOTES ON MODEL VALIDATION

Ideally, validation of this model would be based on a close comparison of solution values for regional ferrous scrap prices and interregional ferrous scrap flows with published data. In reality, such comparisons are not simple, and in some critical respects, they also are not possible.

There are two sources of published ferrous scrap prices: *American Metal Market* and *Iron Age: New Steel*. Both industry magazines publish prices on nearly identical locations and collect their data in a similar manner. Prices from these sources are available for a number of grades of ferrous scrap and are reported for 12 to 15 city-regions, depending on the grade of the scrap of interest. In addition to numerous grades, three types of prices are cited: broker buying prices, consumer buying prices, and export yard buying prices. Broker buying prices are quoted F.O.B., on rail cars, at the seller's location.

The other two price types are both C.I.F. prices and include transport costs and the brokerage fee. The distinction between export yards and domestic consumers reflects a difference in quality standards.

The *U.S. Regional Ferrous Scrap Model* generates solution values of two "generic" grades of scrap (high-residual and low-residual) in 1,212 supply regions. Only 12–15 of these regions correspond to regions where published data would permit comparison for purposes of model validation; however, these same regions account for a significant proportion of total scrap consumed. Also, the model solution is based on supply and demand estimates taken for a given year. As with most commodities, however, the price of scrap varies significantly from month to month. For example, the composite price for shredded scrap began 1998 at nearly $150; it dropped to $90 at the beginning of 1999 and had risen back to over $130 by the beginning of 2000 (American Metal Market, 2000, February 4, p. 24). Hence, in any year, the average annual price for a given grade may represent the market poorly.

Comparisons between published prices and the prices generated as solution values in the model may be difficult, but validation of the model based on solution values for interregional flows is even more problematic. No published data are available on interregional flows of ferrous scrap.

When one compares published prices to solution values, the model performance is found to characterize observed patterns of pricing behavior very well. Giarratani, Gruver, and Richmond (2002) report this comparison. For prices of high-residual ferrous scrap in 15 city-regions, they find that model solutions track much more closely to No. 1 Heavy Melt than they do to No. 2 Heavy Melt, and this would be expected based on the metallurgical properties of these grades and the metallurgical assumptions made in estimating high-residual scrap demand and supply in the model. Model solutions for high-residual scrap are extremely close to published prices for 8 of the 15 city-regions reported (differences of $1 to $5 dollars per ton on a basis of $100 to $120). For the remaining city-regions where the gap between solutions values and published prices is larger ($10–$20 per ton on the same basis), one observes consistent ordering of prices such that if published prices between any two city-regions are ranked in a particular order, so too are the corresponding solution values. Price comparisons for low-residual ferrous scrap also are very good. There is close tracking in 5 of the 12 reporting city-regions for this grade, and there is good correlation where tracking is less precise. Importantly, for both grades, the model captures very well the outlying behavior of prices on the West Coast.

In assessing the validity of the model's solution values, one also may analyze the premium paid for low-residual scrap as compared with high-residual scrap. The model generates solution values for both grades in each of the 1,212 supply regions. Published data are available for both grades in 12 city-regions. Giarratani, Gruver, and Richmond (2002) also report this comparison. For 6 of the 12 city-regions where comparisons are possible, the difference in the price premium is equal to or less than 10 percentage points.

Differences in the low-residual premium of 14 to 23 percentage points are found in 4 of the remaining 6 regions. Two regions, Boston and Los Angeles, are international ports with anomalous characteristics. The published data for Boston show no price premium for low-residual scrap, and for Los Angeles, the published data show a negative premium (low-residual scrap costs less than high-residual scrap). The model predicts a 29 percent premium for low-residual scrap in Boston and a 1 percent premium in Los Angeles.

Because validation based on published data is problematic, and to better understand the characteristics of the model, we investigated the sensitivity of the solution to changes in the parameters a and B. Two simulations were conducted, one in which the parameter a was reduced from 0.23 to 0.20 and another in which the parameter B was reduced from 60 to 53. For each simulation, numerical elasticities were calculated in each region for price with respect to the parameter change and for quantity with respect to the parameter change.

Simulation results are summarized in Table 2, which reports the range of elasticity values for both parameters by type of region (i.e., supply region and

TABLE 2: Sensitivity Results: Elasticity of Equilibrium Solution Values with Respect to Parameter Change

	Range of Elasticities over Regions	
	B	a
Demand regions		
Low-residual		
Price	−0.05 to +0.06	+0.12 to +0.29
	[−0.02 to +0.03]*	[+0.14 to +0.21]
Quantity	0.00	0.00
	(constant in model)	(constant in model)
High-residual		
Price	−0.06 to +0.06	−0.02 to +0.10
	[−0.02 to +0.04]	[−0.02 to +0.05]
Quantity	−0.03 to +0.03	−0.01 to +0.05
	[−0.01 to +0.02]	[−0.01 to +0.02]
Supply regions		
Low-residual		
Price	−0.07 to +0.11	+0.13 to +0.26
	[−0.03 to +0.07]	[+0.13 to +0.19]
Quantity	−0.43 to +0.32	−0.66 to +0.27
	[−0.15 to +0.04]	[−0.22 to +0.11]
High-residual		
Price	−0.08 to +0.09	−0.02 to +0.08
	[−0.04 to +0.06]	[−0.02 to +0.06]
Quantity	−10.50 to +2.47	−2.41 to +0.99
	[−0.66 to +0.06]	[−0.47 to +0.06]

*Values in square brackets are 5 to 95 percentile, e.g., the low-residual price elasticities from supply regions range from −0.05 to +0.06; however, in only 5% of the regions are they less than −0.02 and in only 5% are they greater than +0.03.

demand region). The 5th and 95th percentile elasticity values are also reported in the square brackets. For parameter B, we find that sensitivity of the solution values for price is very low. Whether one considers demand regions or supply regions, and whether one considers the price of low-residual scrap or high-residual scrap, elasticity values never exceed an absolute value of 0.11, and they are typically much lower than this value. For parameter a, the solution values for price are only slightly more sensitive—the largest being +0.29 for price of low-residual scrap in supply regions.

Given the model structure, an increase in B implies that supply regions are more supply responsive to small premiums in F.O.B. offer prices, and as a result, demand regions with excess demand for scrap attract supply from a smaller geographical radius for a given vector of offer prices. The local spatial distribution of prices and quantities over regions is affected, yielding negative elasticities in some regions and positive elasticities in other regions for each price or quantity elasticity calculated. Alternatively, an increase in a makes it more difficult to substitute high-residual for low-residual scrap as the upper-limit percentage of high-residual is approached;[3] it follows that the low-residual price elasticities are positive in all regions for both demand and supply regions.

The sensitivity of some solution values related to quantity with respect to parameter changes are more variable than for prices. By assumption, the regional supply of low-residual scrap is fixed and the calculated elasticities must necessarily be zero for changes in either parameter. This assumption does not apply to high-residual scrap. Nevertheless, for this grade, the sensitivity of solution values for quantity is very low in supply regions. Whether one simulates a change in parameter B or parameter a, the absolute value of measured elasticity is always less than 0.05 for the high-residual grade.

In demand regions, quantity variables remain very insensitive to parameter changes for the low-residual grade and also for the high-residual grade with the exception of about 10 percent of the regions in the lower and upper tails of the distribution. Calculated elasticities for the quantity of high-residual scrap in the demand regions range from −10.50 to −0.66 in the 5 percent of regions with the smallest elasticities and from +0.06 to +2.47 in the 5 percent of regions with the largest elasticities for changes in parameter B. Calculated elasticities for the quantity of high-residual scrap in the demand regions range from −2.41 to +0.47 in the 5 percent of regions with the smallest elasticities and from +0.06 to +0.99 in the 5 percent of regions with the largest elasticities for changes in parameter a. It follows that, whether one simulates a change in parameter B or parameter a, the absolute value of measured elasticity for quantity variables in 90 percent of the demand regions is less than 0.66 in absolute value for the high-residual grade. All of the 10

[3]Alternatively, we can say that for a given delivered price ratio of high- to low-residual scrap, more low-residual scrap will be demanded in that region if parameter a is larger.

percent of regions in the tails of the distribution are regions with small scrap demand; the regions with the most negative elasticities are in Texas, Oklahoma, and the Dakotas. Most importantly, these results strongly support the use of our model for counterfactual simulations of market events in key steel-producing regions.

We also relied on maps to help assess the sensitivity of solution values to parameter changes. A comparison of maps before and after parameter changes clearly shows that changes in both price and quantity solution values are of a continuous nature. The new price gradients and supply pattern values change at the geographical margins, but they do not depict radical shifts in the nature of the solution pattern.

We do not report sensitivity of flows to changes in parameter values. Given the initial data, the logic of the model, and the insensitivity of price and quantity values to parameter variations, it follows that flow solution values will also be relatively insensitive to parameter variation.

9. MAPPING SOLUTION VALUES FOR EQUILIBRIUM FERROUS SCRAP PRICES

The solutions presented here are approximate representations for the 1994 ferrous scrap market in the United States in that the data upon which the model depends most critically have been compiled for that year. Mapping the prices to supply areas portrays the geographic structure of prices offered by the model very well in that the geographic price surface can be exposed. We present two maps: one for low-residual ferrous scrap prices and one for high-residual ferrous scrap prices. In both maps, we present the average F.O.B. price in each of 1,212 supply locations, by representing the absolute price level in each supply region as map shading. The maps show boundaries of the counties; however, the solution is obtained for the 1,212 regions each representing an aggregation of counties. The regions average three counties each, with more than three if counties are small, and fewer than three if counties are large.

Figure 3 shows the modeling solution for equilibrium low-residual ferrous scrap prices in 1994 in terms of U.S. dollars per gross ton. As indicated by the map, scrap pricing in the United States is focused on a geographic core surrounding the Great Lakes extending eastward to encompass the Pittsburgh region. Prices in the core region range from \$130 to \$150 per gross ton, and prices decrease monotonically from the core in radial regions. Similarly, Figure 4 shows the modeling solution for equilibrium high-residual ferrous scrap prices in 1994. Here, the pattern also is clearly that of core and periphery. The core is still focused on the Chicago region; however, the higher prices extend more to the west and less to the east. Notice also that the price gradient in high-residual scrap is less steep than that for low-residual scrap going west or south from Chicago, but the opposite is true going east. This is most evident if one focuses on the two highest-price bands in each map.

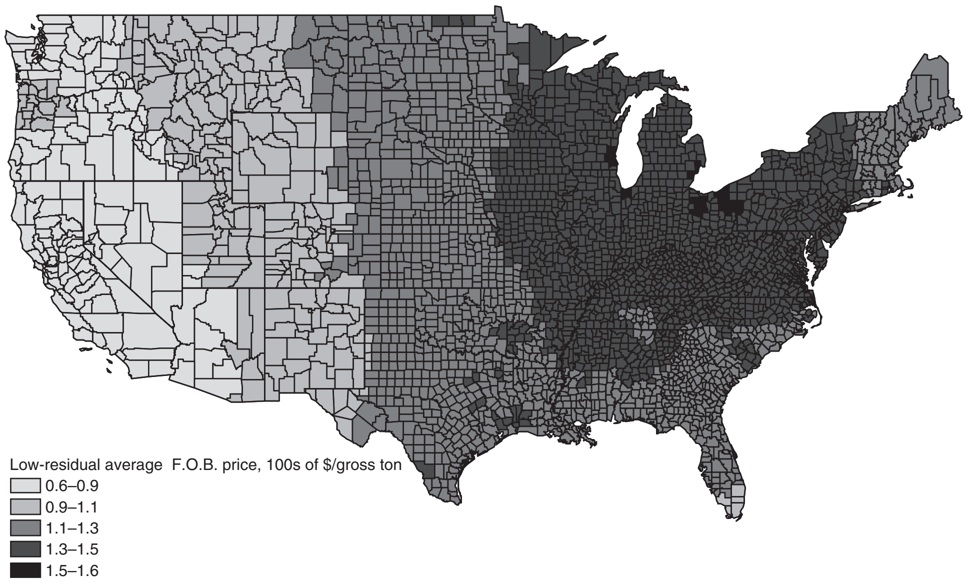

FIGURE 3: Spatial Distribution of Equilibrium Prices for Low-Residual Ferrous Scrap, 1994.

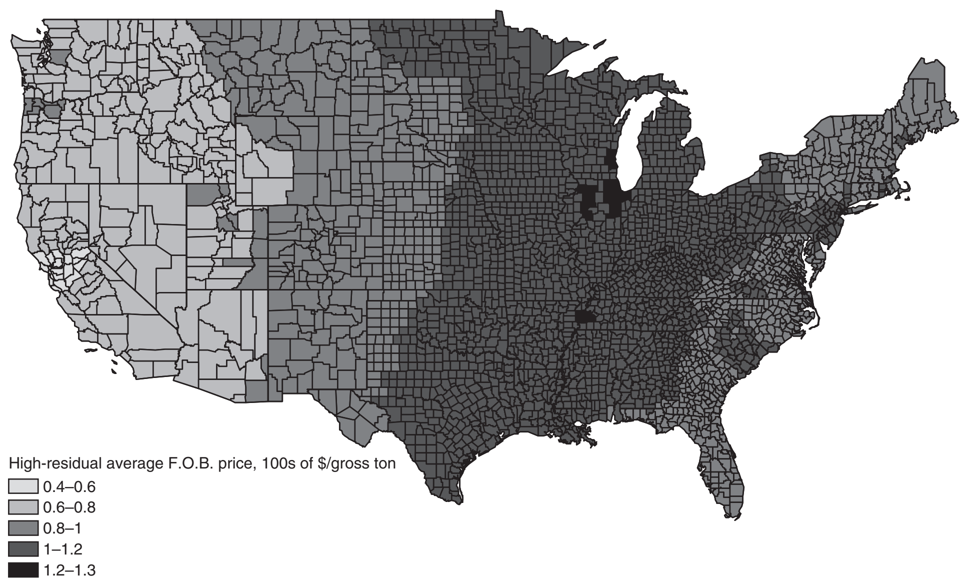

FIGURE 4: Spatial Distribution of Equilibrium Prices for High-Residual Ferrous Scrap, 1994.

For the high-residual solution (Figure 4), this combined core extends southward all the way to the Gulf of Mexico, westward into Texas and Nebraska. As compared to the same band for the low-residual solution (Figure 3), this combined band extends much less eastward toward coastal areas.

These results speak well for the validity of the model in that demand for low-residual scrap is highly concentrated in the Chicago–Pittsburgh core, where integrated steel makers have focused on the high-value-added sheet steel required in automobile production. By contrast, demand for high-residual scrap is much more diffused to the west and south. This grade of scrap is used extensively by minimills, which dominate the market for “long” products, and the locational pattern for minimills also is very diffused. These patterns also reflect the importance of the geographic distribution of the supply of ferrous scrap. A key source of low-residual scrap is steel using manufacturing activity as compared to the source of high-residual scrap, which is comprised primarily of postconsumer steel products. Because manufacturing activity is more highly concentrated in space than the consumer population, the supply of high-residual scrap is more diffused than that of low-residual scrap. The plant location patterns described here result from intense intra-industry competition between integrated steel makers and minimills, as explained by Ahlbrandt, Fruehan, and Giarratani (1996).

The importance of the Chicago–Pittsburgh core also is exposed in Figure 5, which shows the spatial distribution of demand for ferrous scrap in 1994. The geographic focus of demand clearly extends from the Great Lakes eastward to encompass the Pittsburgh region with other regions of lesser importance.

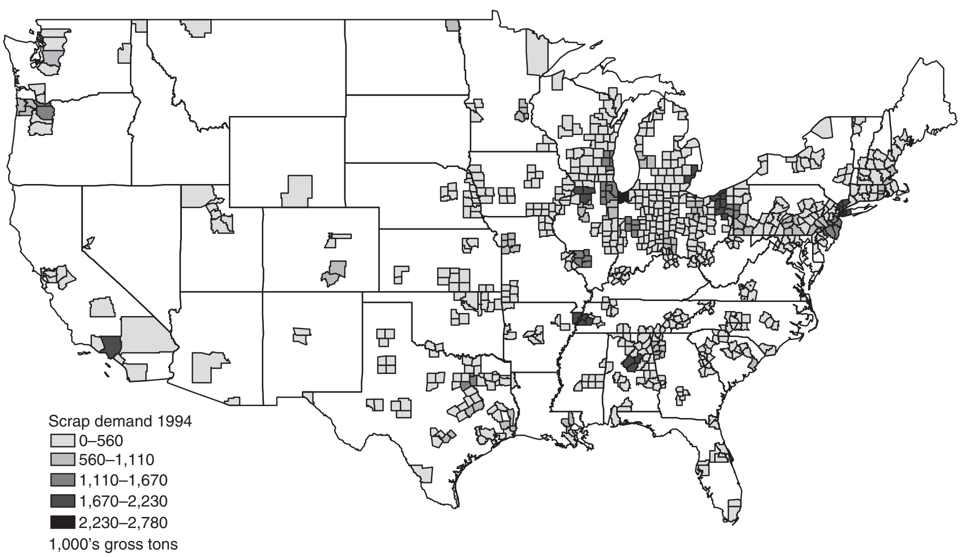

FIGURE 5: Spatial Distribution of Demand for Ferrous Scrap, 1994.

Table 3 describes the distribution of demand for both high-residual and low-residual scrap in the United States relative to St. Louis in 1994 and further exposes the dominance of the Chicago–Pittsburgh core. For this table, the continental United States is divided into quadrants using the latitude of 38 degrees to divide North from South and the longitude of −89 degrees (measured from east to west) to divide East from West. Evidence for the dominance of the Chicago–Pittsburgh core is found in that 11.8 million gross tons (MGT) of ferrous scrap originates from regions northeast of St. Louis, and this is out of a U.S. total of 20.0 MGT of low-residual demand. The Northeast also dominates the demand for high-residual scrap, but to a lesser degree. Out of a total high-residual demand of 38.4 MGT, 17.1 originate from regions in the Northeast.

10. MAPPING SOLUTION VALUES RELATED TO INTERREGIONAL FLOWS OF FERROUS SCRAP

Total scrap demand and low-residual supply by region are estimated exogenously in the model; however, the mix of low-residual and high-residual scrap demanded and the supply of high-residual by region are determined as part of the model's equilibrium solution. The solution values can help to further explain the geographic structure of this market by using these data to estimate geographic patterns of excess demand.

Figure 6 focuses on the market for low-residual ferrous scrap and shows excess demand prior to interregional scrap flows. The dark regions depicted in Figure 6 demand more low-residual scrap than they supply, and the light regions supply more than they demand. Not surprisingly, most of the regions with excess demand and regions with excess supply over 0.1 MGT are located in the industrial Northeast.

Figure 7 is similar to Figure 6, but it focuses on high-residual ferrous scrap. The dark regions depicted in Figure 7 demand more high-residual scrap than they supply, and the light supply more than they demand. Again, most of the regions with excess demand are located in the industrial Northeast.

TABLE 3: The Demand for Ferrous Scrap in the United States Relative to St. Louis, Missouri, 1994

	Low-residual (1,000's gross tons)		High-residual (1,000's gross tons)	
	West	East	West	East
North	2,911	11,845	6,714	17,133
South	3,762	1,456	8,869	5,663
Subtotal	6,673	13,301	15,583	22,796
Subtotal		19,974		38,380
Total				58,353

Source: Estimates from the *U.S. and Regional Ferrous Scrap Model*. Numbers in this table are based on model calculated equilibrium solutions.

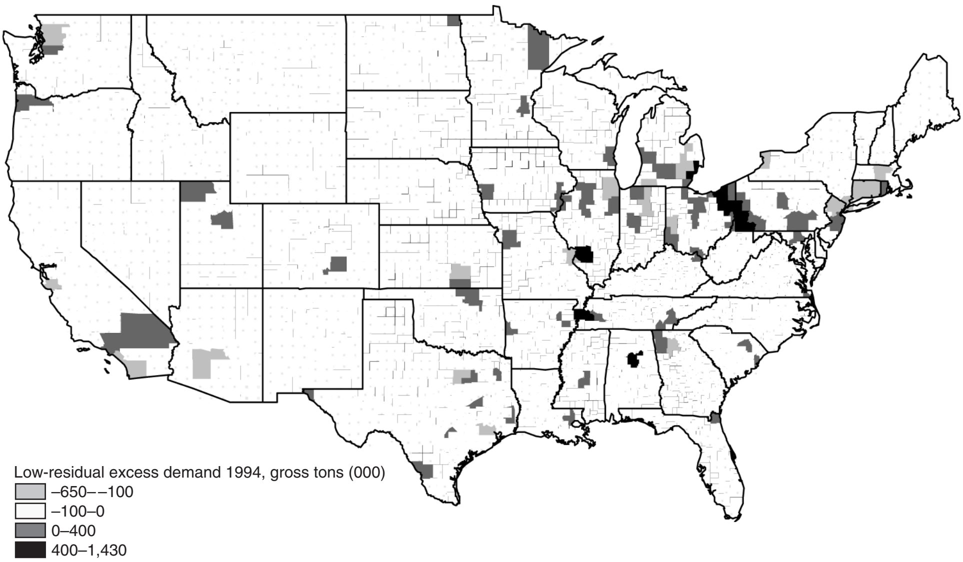

FIGURE 6: Excess Demand: Low-Residual Ferrous Scrap, 1994.

However, regions with large excess supply (over 0.1 MGT) appear to be more spatially dispersed.

Table 4 provides data on excess demand for low-residual and high-residual ferrous scrap. Once again in this table, the geographic distribution

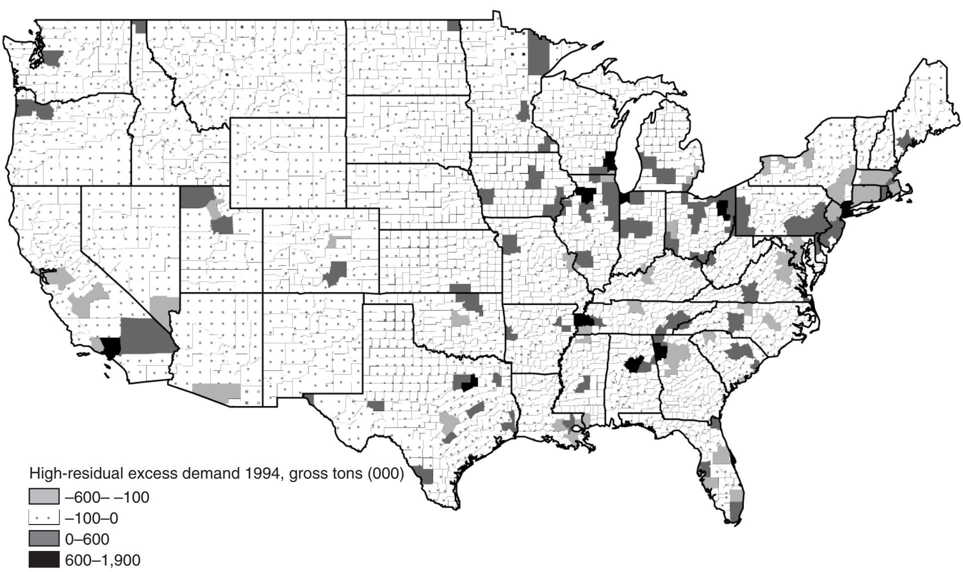

FIGURE 7: Excess Demand: High-Residual Ferrous Scrap, 1994.

TABLE 4: Excess Demand for Ferrous Scrap in the United States Relative to St. Louis, Missouri, 1994

	Low-residual (1,000's gross tons)		High-residual (1,000's gross tons)	
	West	East	West	East
North	−136	2,941	−126	1,492
South	−865	−1,939	1,094	−2,460
Subtotal	−1,001	1,001	968	−968
Subtotal		0		0
Total				0

Source: Estimates from the *U.S. and Regional Ferrous Scrap Model*.

is described relative to St. Louis. The data in Table 4 complement the information provided by Figures 6 and 7. Table 4 summarizes that there is a low-residual supply deficit of 2.9 MGT in the Northeast, which is largely balanced by the supply surplus of 1.9 MGT in the Southeast and 0.9 MGT in the Southwest. The table also shows that there is a high-residual supply deficit not only in the Northeast (2.9 MGT) but also in the Southwest (1.1 MGT). For high-residual scrap, this deficit is almost completely balanced by the supply surplus of 2.5 MGT in the Southeast.

11. TRACKING THE SOURCES AND DESTINATIONS OF INTERREGIONAL FERROUS SCRAP FLOWS

The elimination of excess demand across regions is achieved by interregional flows of ferrous scrap. The equilibrium solution of the model calculates supply flows for each region of scrap demand for both grades of scrap. The data provided in the solution offers a wealth of information about economic linkages for particular regions, and this information can be exploited to better understand the nature of regional dependencies.

To illustrate the model's ability to bring valuable information to bear on interregional linkages, we focus on the linkages for a single demand region, Gary, Indiana, which is one of the most important steel-producing areas in the United States. Figures 8 and 9 depict the supply sources for the Gary region for low- and high-residual scrap, respectively. We see that the low-residual sources of supply are much more spatially separated than the high-residual sources. This is not surprising given that low-residual scrap has its origin in steel-using manufacturing activities. In contrast, high-residual scrap is more closely distributed in proportion to population, because its primary source is obsolete, or postconsumer, steel products.

It is notable that the gradients of the supply maps for low- and high-residual scrap are each consistent with the price gradients noted in Figures 3 and 4. That is, price and quantity gradients are inversely related. The supply regions for low-residual scrap reach far to the west, including California but not east past Detroit and mid-Ohio; low-residual prices were high all the way

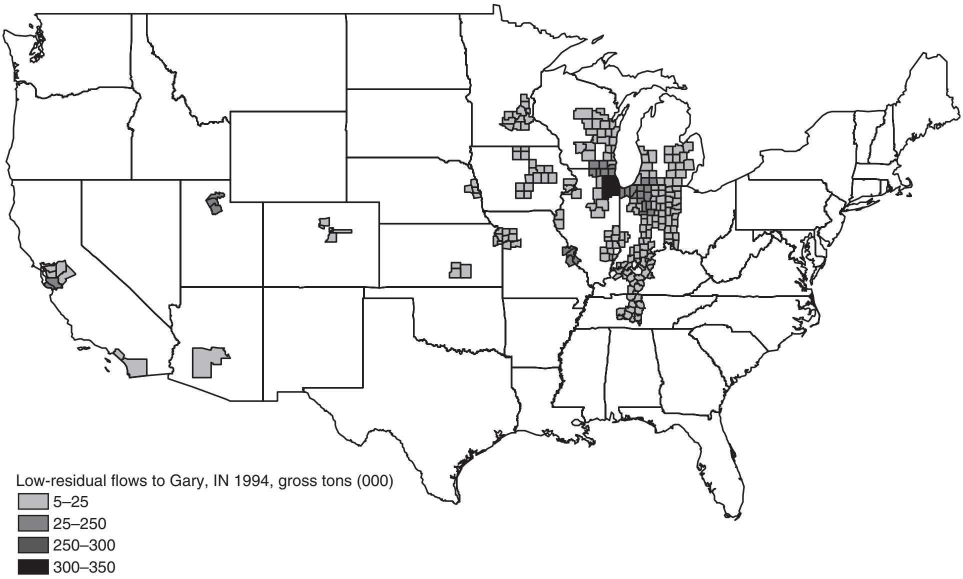

FIGURE 8: Supply Sources for Gary, Indiana: Low-Residual Ferrous Scrap, 1994.

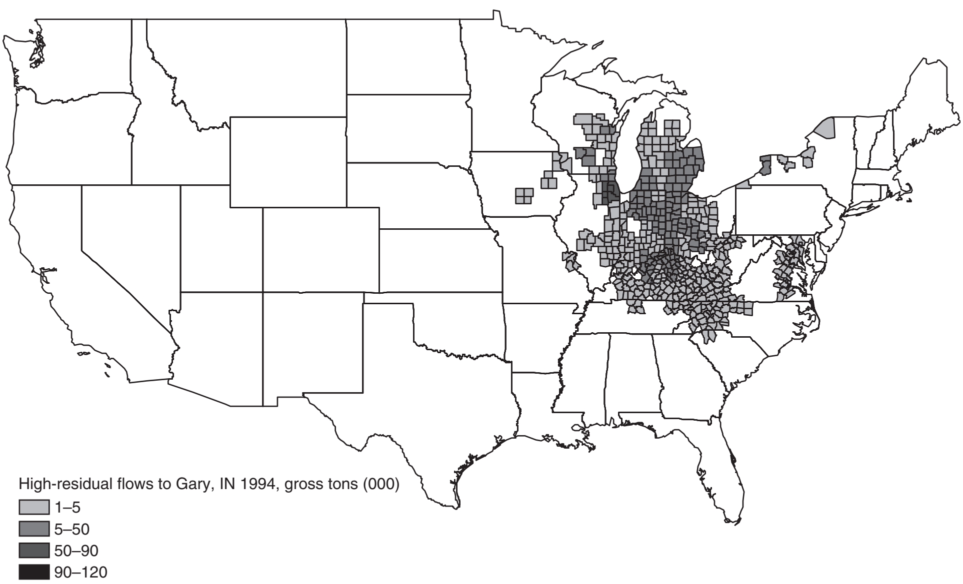

FIGURE 9: Supply Sources for Gary, Indiana: High-Residual Ferrous Scrap, 1994.

to the east coast but tapered off more quickly going west. By comparison, the supply regions for high-residual scrap reach to the east coast but not west much beyond the Mississippi river; high-residual prices were high to the west coast but tapered off more quickly going east.

Table 5 aggregates the supply flows around Gary by geographic quadrants. When the data are viewed in this way, the conclusions drawn from the maps can be reinforced. Table 5 summarizes that 63 percent of Gary's low-residual scrap comes from the West, while only 19 percent of its high-residual scrap comes from that direction.

12. CONCLUSIONS

Our objective in this modeling exercise is to represent the behavior of economic agents sufficiently well so as to capture essential characteristics of actual market outcomes. An additional objective is to generate evidence that a logistic function of spatial distribution provides a tractable specification for practical policy-modeling purposes. The tasks involved are immense in that data requirements on both sides of the market (supply and demand) are substantial. It is commonly acknowledged that the market for ferrous scrap in the United States is highly competitive, and this competition is furthered by the fact that interregional trade links the hundreds of individual scrap markets we model.

Nevertheless, in some of those markets, elements of imperfect competition may exist in that a dominant scrap supplier or a dominant steel mill could conceivably exercise some degree of market control because of its size vis-à-vis the local market or an advantage in terms of information about the local market. Transport networks also are complex and some regional scrap markets are better "connected" with other markets because of network differences.

For all of these reasons, and more, we acknowledge that the model we have developed departs from the real markets in which ferrous scrap is transacted. Nevertheless, whether one focuses on the major attributes of pricing behavior or interregional flows, the *U.S. Regional Ferrous Scrap*

TABLE 5: Geographic Origins of Ferrous Scrap Supply for Gary Indiana, 1994

	Low-residual West (1,000's gross tons)	%	East (1,000's gross tons)	%	High-residual West (1,000's gross tons)	%	East (1,000's gross tons)	%
North	699	38	394	21	147	16	430	47
South	481	26	289	16	29	3	316	34
Subtotal	1,180	63	683	37	176	19	746	81
Subtotal			1,863	100			922	100
Total							2,785	

Source: Estimates from the *U.S. and Regional Ferrous Scrap Model*.

Model seems to conform well to known market structures. Furthermore, based on parameter sensitivity analysis reported here, the equilibrium solutions change in a relatively continuous and insensitive manner as parameter values are varied from the values used for the baseline solutions.

The model's output is detailed, including information on quantities of supply and demand, on the direction and volume of material transport, and on the prices paid for each transaction. The output could be aggregated to represent average prices, market areas, or the degree of market sharing. The extraction and aggregation of predictions makes it possible to match observable data with less measurement and specification error. The model output may be combined to represent relative prices or comparative advantage of some locations over others. The output may be used as an input to construct a more encompassing model of optimal location. For instance, one might construct a profit surface by combining layers representing both costs and revenues. Finally, the output can be used as a descriptive tool to discuss the large geographic features of a market and how those features are likely to adapt to trends in supply and demand.

The *U.S. Regional Ferrous Scrap Model* is well suited to perform counterfactual simulations. It is based on a well-defined equation structure that captures fundamental properties of supply and demand in regional markets. By changing parameter values, initial supply, or initial demand conditions, one can use the model to examine the impact of a wide range of possible interventions on market outcomes. We expect that the structure of this model will apply equally well in academic and business analysis related to ferrous scrap markets and closely related markets for steel.

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APPENDIX

CALCULATING PRICE CHANGES

Consider a specific demand location $k = j$ and the effect of a change in P_j^L on ρ_{ij}^L. Using (1), (2), and (3) and differentiating with respect to P_j^L

$$\text{(1A)} \qquad \frac{\partial \rho_{ij}^L}{\partial P_j^L} = \frac{\partial \rho_{ij}^L}{\partial p_{ij}^L}\frac{\partial p_{ij}^L}{\partial P_j^L} = \frac{\partial \rho_{ij}^L}{\partial p_{ij}^L}$$

as it follows from (1) that $\left(\partial p_{ij}^L \middle/ \partial P_j^L\right) = 1$ for all i such that $p_{ij}^L > 0$. Then

$$\frac{\partial \rho_{ij}^L}{\partial p_{ij}^L} = \frac{\partial \rho_{ij}^L}{\partial \tilde{p}_{ij}^L}\frac{\partial \tilde{p}_{ij}^L}{\partial p_{ij}^L}$$

by differentiating (3) with respect to p_{ij}^L, the effect of including the normalization can be demonstrated.

$$\text{(2A)} \qquad \frac{\partial \tilde{p}_{ij}^L}{\partial p_{ij}^L} = \frac{1}{\bar{p}_i^L}\left[1 - \frac{p_{ij}^L}{\bar{p}_i^L}\frac{\partial \bar{p}_{ij}^L}{\partial p_{ij}^L}\right]$$

From (3), expression $\left(\partial \bar{p}_{ij}^L \middle/ \partial p_{ij}^L\right)$ in (2A) may be written as

$$\frac{\partial \bar{p}_{ij}^L}{\partial p_{ij}^L} = \rho_{ij}^L + p_{ij}^L\frac{\partial \rho_{ij}^L}{\partial p_{ij}^L} + \sum\nolimits_{k \neq j} p_{ik}^L \frac{\partial \rho_{ik}^L}{\partial p_{ij}^L}$$

For each iteration, $\left(\partial \rho_{ik}^L \middle/ \partial p_{ij}^L\right)$ will be small for $k \neq j$, and to simplify the price adjustment process, we assume it to be zero.

The simplified version of (2A) is

$$\text{(3A)} \qquad \frac{\partial \tilde{p}_{ij}^{L}}{\partial p_{ij}^{L}} = \left[\frac{1}{\bar{p}_i^L} - \frac{p^L}{\bar{p}_i^{L^2}} \left[\rho_{ij}^L + p_{ij}^L \frac{\partial \rho_{ij}^L}{\partial p_{ij}^L} \right] \right]$$

Next consider the effect of a change in the scaled F.O.B. price $\tilde{p}_{ij}^L$ on ρ_{ij}^L,

$$\text{(4A)} \qquad \frac{\partial \rho_{ij}^L}{\partial \tilde{p}_{ij}^L} = \frac{Bk_{ij}^L e^{B\tilde{p}_{ik}^L}}{\sum_k k_{ik}^L e^{B\tilde{p}_{ik}^L}} - \frac{B\left[k_{ij}^L e^{B\tilde{p}_{ij}^L}\right]^2}{\left[\sum_k k_{ik}^L e^{B\tilde{p}_{ik}^L}\right]^2} = B\left[\rho_{ij}^L - \rho_{ij}^{L^2}\right]$$

Substituting (4A), (2A), and (3A) into (1A) yields,

$$\text{(5A)} \qquad \frac{\partial \rho_{ij}^L}{\partial p_{ij}^L} = B\left[\rho_{ij}^L - \rho_{ij}^{L^2}\right] \left[\frac{1}{\bar{p}_i^L} - \frac{p_{ij}^L}{\bar{p}_i^{L^2}} \left[\rho_{ij}^L + p_{ij}^L \frac{\partial \rho_{ij}^L}{\partial p_{ij}^L} \right] \right]$$

Solving (5A) for $\left(\partial \rho_{ij}^L \big/ \partial p_{ij}^L\right)$, we can write (4A) as

$$\text{(6A)} \qquad \frac{\partial \rho_{ij}^L}{\partial P_j^L} = \frac{B\left[\rho_{ij}^L - \rho_{ij}^{L^2}\right]\left[\frac{\bar{p}_i^L - \rho_{ij}^L p_{ij}^L}{\bar{p}_i^{L2}}\right]}{1 + \frac{p_{ij}^{L2}}{\bar{p}_i^{L2}} B\left[\rho_{ij}^L - \rho_{ij}^{L^2}\right]}$$

Now use (4) and (6A) to consider the effect of a change in P_j^L on q_{ij}^L and on the sum, Q_j^{LS}, of all q_{ij}^L shipments to $k = j$.

$$\text{(7A)} \qquad \frac{\partial Q_j^{LS}}{\partial P_j^L} = \sum_i \frac{B\left[\rho_{ij}^L - \rho_{ij}^{L^2}\right]\left[\frac{\bar{p}_i^L - \rho_{ij}^L p_{ij}^L}{\bar{p}_i^{L2}}\right]}{1 + \frac{p_{ij}^{L2}}{\bar{p}_i^{L2}} B\left[\rho_{ij}^L - \rho_{ij}^{L^2}\right]} Q_i^{LS}$$

Next consider how a change in P_j^L affects the demand, Q_j^{LD}, at j for low-residual scrap. Assuming that $p_{ij}^L > p_{ij}^H$, the substitution relationship between low- and high-residual scrap in (5) can be rewritten as

$$\text{(8A)} \qquad Q_j^{LD} = \left[\frac{a}{\frac{P_j^L}{P_j^H} - 1} + \frac{Q_j^{LMin}}{Q_j^{TD}} \right] Q_j^{TD}$$

differentiating with respect to P_j^L yields (note that Q_j^{LMin} and Q_j^{TD} are constants).

$$\text{(9A)} \qquad \frac{\partial Q_j^{LD}}{\partial P_j^L} = \frac{-a}{\left[\frac{P_j^L}{P_j^H} - 1\right]^2 P_j^H} Q_j^{TD}$$

Now consider the effect of a change in P_k^L on the excess demand, Q_j^{EDL} at k. The first-order approximation is given by (10A)

$$\Delta Q_j^{EDL} = \frac{\partial}{\partial P_j^L}\left[Q_j^{LD} - Q_j^{LS}\right]\Delta P_j^L \tag{10A}$$

substituting (7A) and (9A) into (10A) yields (11A)

$$\Delta Q_j^{EDL} = \left\{ \frac{-a}{\left[\frac{P_j^L}{P_j^H} - 1\right]^2 P_j^H} Q_j^{TD} - \sum\nolimits_i \frac{B\left[\rho_{ij}^L - \rho_{ij}^{L^2}\right]\left[\frac{\bar{p}_i^L - \rho_{ij}^L p_{ij}^L}{\bar{p}_i^{L2}}\right]}{1 + \frac{p_{ij}^{L2}}{\bar{p}_i^{L2}} B\left[\rho_{ij}^L - \rho_{ij}^{L^2}\right]} Q_i^{LS} \right\} \Delta P_j^L \tag{11A}$$

Equation (11A) can be solved for the approximate ΔP_j^L required to reach $Q_j^{EDL} = 0$, that is,

$$\Delta P_j^L = \frac{\Delta Q_j^{EDL}}{\left\{ \frac{-a}{\left[\frac{P_j^L}{P_j^H} - 1\right]^2 P_j^H} Q_j^{TD} - \sum\nolimits_i \frac{B\left[\rho_{ij}^L - \rho_{ij}^{L^2}\right]\left[\frac{\bar{p}_i^L - \rho_{ij}^L p_{ij}^L}{\bar{p}_i^{L2}}\right]}{1 + \frac{p_{ij}^{L2}}{\bar{p}_i^{L2}} B\left[\rho_{ij}^L - \rho_{ij}^{L^2}\right]} Q_i^{LS} \right\}} = Q_j^{EDL}$$

Similar calculations can be made for the high-residual case. First substitute $Q_j^{LD} - Q_j^{HD}$ for Q_j^{TD} in Equation (8A) and solve for Q_j^{HD} to obtain

$$Q_j^{HD} = \left[\frac{a}{\frac{P_j^L}{P_j^H} - 1} + \frac{Q_j^{LMin}}{Q_j^{TD}}\right]^{-1} Q_j^{LD} - Q_j^{LD} \tag{12A}$$

Now differentiate (12A) to obtain

$$\frac{\partial Q_j^{HD}}{\partial P_j^H} = -\left[\frac{a}{\frac{P_j^L}{P_j^H} - 1} + \frac{Q_j^{LMin}}{Q_j^{TD}}\right]^{-2}$$

$$\left[-a\left(\frac{P_j^L}{P_j^H} - 1\right)^{-2}\left(-P_j^L P_j^{H^{-2}}\right)\right] Q_j^{LD}$$

The high-residual case has an additional term because Q_i^{LS} is fixed but Q_i^{HS} is a function of the price at supply location i. Thus, the equivalent of (11A) for the high-residual case is

$$\text{(13A)}\qquad \Delta Q_j^{EDH} = -\left[\frac{-a}{\left[\frac{P_j^L}{P_j^H}-1\right]} + \frac{Q_j^{LMin}}{Q_j^{TD}}\right]^{-2} a\left[\frac{P_j^L}{P_j^H}-1\right]^{-2}\left(\frac{P_j^L}{P_j^{H^2}}\right)Q_j^{LD}\Delta P_j^H$$
$$-\sum_i\left[\frac{B\left[\rho_{ij}^H-\rho_{ij}^{H^2}\right]\left[\frac{\bar{p}_i^H-\rho_{ij}^H p_{ij}^H}{\bar{p}_i^{H^2}}\right]}{1+\frac{p_{ij}^{H^2}}{\bar{p}_i^{H^2}}B\left[\rho_{ij}^H-\rho_{ij}^{H^2}\right]}Q_i^{HS}+\rho_{ij}^H\frac{\partial Q_i^{HS}}{\partial p_{ij}^H}\right]\Delta P_j^H$$

The equation for $\left(\partial Q_i^{HS}\big/\partial p_{ij}^H\right)$ is obtained by differentiating the product of the high-residual supply function.

$$\text{(14A)}\qquad \frac{\partial Q_i^{HS}}{\partial p_{ij}^H} = \frac{\partial}{\partial p_{ij}^H}\left[s_1+s_2\bar{p}_i^H+s_3(\bar{p}_i^H)^2\right]\frac{Pop_i}{1000}$$

Using the fact that

$$\text{(15A)}\quad \frac{\partial \mathrm{p}_i^H}{\partial \mathrm{p}_{ij}^H} = \frac{\partial}{\partial \mathrm{p}_{ij}^H}\sum\nolimits_k \mathrm{p}_{ik}^H\rho_{ik}^H = \frac{\partial \mathrm{p}_{ij}^H}{\partial \mathrm{p}_{ij}^H}\rho_{ij}^H + \frac{\partial \rho_{ij}^{HS}}{\partial \tilde{\mathrm{p}}_{ij}^H}\frac{\partial \tilde{\mathrm{p}}_{ij}^H}{\partial \mathrm{p}_{ij}^H}\mathrm{p}_{ij}^H + \sum\nolimits_{k\neq j}\left[\frac{\partial \rho_{ik}^{HS}\partial \tilde{\mathrm{p}}_{ij}^H}{\partial \tilde{\mathrm{p}}_{ij}^H\,\partial \mathrm{p}_{ij}^H}\mathrm{p}_{ik}^H\right]$$

Substituting the high-residual equivalent of Equation (6A) and assuming that the cross partial derivatives $\frac{\partial \rho_{ik}^{HS}}{\partial \tilde{p}_{ij}^H}$ are sufficiently small that they can be disregarded and still obtain an efficient price adjustment direction (15A) simplifies to

$$\rho_{ij}^H + \frac{B\left[\rho_{ij}^H-\rho_{ij}^{H^2}\right]\left[\frac{\bar{p}_i^H-\rho_{ij}^H p_{ij}^H}{\bar{p}_i^{H^2}}\right]}{1+\frac{p_{ij}^{H^2}}{\bar{p}_i^{H^2}}B\left[\rho_{ij}^H-\rho_{ij}^{H^2}\right]}p_{ij}^H$$

We can write (14A) as

$$\frac{\partial Q_i^{HS}}{\partial p_{ij}^H} = \frac{Pop_i}{1000}\left[s_2\frac{\partial \bar{p}_i^H}{\partial p_{ij}^H}+2s_3\frac{\partial \bar{p}_i^H}{\partial p_{ij}^H}\bar{p}_i^H\right]$$
$$= \frac{Pop_i}{1000}\left[s_2+2s_3\bar{p}_i^H\right]\left[\rho_{ij}^H + \frac{B\left[\rho_{ij}^H-\rho_{ij}^{H^2}\right]\left[\frac{\bar{p}_i^H-\rho_{ij}^H p_{ij}^H}{\bar{p}_i^{H^2}}\right]}{1+\frac{p_{ij}^{H^2}}{\bar{p}_i^{H^2}}B\left[\rho_{ij}^H-\rho_{ij}^{H^2}\right]}\right]p_{ij}^H$$

Substituting this version of (14A) into (13A) yields (16A) to use in the model.

$$
\begin{aligned}
{}_{\Delta}Q_j^{EDH} = &-\left[\frac{-a}{\left[\frac{P_j^L}{P_j^H}-1\right]}+\frac{Q_j^{LMin}}{Q_j^{TD}}\right]^{-2} a\left[\frac{P_j^L}{P_j^H}-1\right]^{-2}\left(\frac{P_j^L}{P_j^{H^2}}\right)Q_j^{LD}\Delta P_j^H \\
&-\left[\sum_i \frac{B\left[\rho_{ij}^H-\rho_{ij}^{H^2}\right]\left[\frac{\bar{p}_i^H-\rho_{ij}^H p_{ij}^H}{\bar{p}_i^{H^2}}\right]}{1+\frac{p_{ij}^{H^2}}{\bar{p}_i^{H^2}}B\left[\rho_{ij}^H-\rho_{ij}^{H^2}\right]}Q_i^{HS}\right]\Delta P_j^H \\
&-\left\{\sum_i \rho_{ij}^H\frac{Pop_i}{1000}\left[s_2+2s_3\bar{p}_i^H\right]\left[\rho_{ij}^H+\frac{B\left[\rho_{ij}^H-\rho_{ij}^{H^2}\right]\left[\frac{\bar{p}_i^H-\rho_{ij}^H p_{ij}^H}{\bar{p}_i^{H^2}}\right]}{1+\frac{p_{ij}^{H^2}}{\bar{p}_i^{H^2}}B\left[\rho_{ij}^H-\rho_{ij}^{H^2}\right]}\right]p_{ij}^H\right\}\Delta P_j^H
\end{aligned}
$$