

# Polluting non-renewable resources, innovation and growth: welfare and environmental policy

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## Abstract

We analyze the impact of the pollution generated by the use of non-renewable resources on the standard results of growth models. In this context, we obtain a Hotelling rule which is not a pure efficiency condition any longer. Subsequently, we show that some of the optimal paths' standard properties change: in particular, an increase in the households' psychological discount rate leads to a slower extraction of the resource. Moreover, we present a simple endogenous growth model that allows us to study the effects of an environmental policy aimed at correcting the distortion introduced at the equilibrium. We show that the tax level does not matter, and that a decreasing tax on the resource use yields the optimum.

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## 1. Introduction

When considering the standard analysis of economic growth in the presence of non-renewable natural resources, several main questions arise: is positive long term growth possible despite the fact that one of the production process' inputs is only available in a

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finite quantity? What is the optimal growth path (namely, what is the optimal rate of resource extraction)? What is the equilibrium path? What are its properties, in particular, is it optimal?

The literature has answered these questions through two main types of models: the Ramsey growth model (where we can quote [Stiglitz, 1974; Garg and Sweeney, 1978] or Dasgupta and Heal, 1979) and endogenous growth models ([Schou (1996); Aghion and Howitt (1998); Scholz and Ziemes (1999); Barbier (1999)] or Grimaud and Rouge (2003)). Both kinds of models show that long-term positive growth is possible under certain technological conditions. In the former, the key optimality point is given by Hotelling's rule, an efficiency condition that characterizes the optimal resource extraction rate. It states that the marginal productivity of capital must equal the resource's marginal productivity growth rate. At market equilibrium, one of the conditions implies that the growth rate of the resource's price must equal the interest rate. As competition is assumed to be perfect in all markets, the unit prices of capital and of the resource must be respectively equal to their marginal productivities. Therefore, Hotelling's rule is verified at the equilibrium, and the equilibrium and optimal paths are identical.

In the second type of models, whether we consider those with horizontal (Romer) or vertical (Aghion–Howitt) differentiation, the optimal paths are in general not followed by the market equilibrium—a feature essentially due to intertemporal externalities that arise from the fact that knowledge is a public good, and to the mark-up imposed by monopolies that sell the intermediate goods.

A common feature of the papers cited above is that they neglect a key characteristic of nearly all non-renewable resources that are presently used (fossil fuels/mineral resources). Indeed, the combustion of petroleum, coal and, to a lesser extent, natural gas is responsible for an important part of CO<sub>2</sub> emissions – the main greenhouse gas – as well as emissions of precursors of acid rain (such as SO<sub>x</sub> and NO<sub>x</sub>) and other atmospheric pollutants. The consumption of hydrocarbons thus generates climatic change and deteriorates air and land quality. A part of the literature has already incorporated this aspect when considering the problem of non-renewable resources. Kolstad and Krautkraemer (1993) realise a survey of this literature. More recently, we can quote for example [Tahvonen's (1997, 2001)] or [Schou's (2000, 2002)] contributions. The latter uses Uzawa–Lucas and Romer-type growth models to consider the problem; we will refer to his results further below.

When polluting emissions are a by-product of the use of non-renewable resources, several questions arise. One can first ask whether the emissions thus generated constitute a stock or a flow. Several elements may lead to believe that pollution must be treated as a stock variable. Indeed, though carbon dioxide emissions are retained by the atmosphere, a part of these is also seized by oceans and forests and trapped for long periods of time. It therefore appears that a dynamic analysis of their effects is required. However, the fact that this can sensibly complicate calculations must also be taken into account. We will see further on that the growth model we develop initially incorporates two state variables (the stocks of knowledge and of the resource), and that the analysis of the stock of pollution requires us to incorporate a third one. Yet, as Kolstad and Krautkraemer (1993) put it: “in general, it is difficult or impossible to characterize the qualitative features of a model with three state variables without restrictive assumptions about the functional forms of important relationships”. For the preceding reason we will treat pollution uniquely as a

flow, as was done for example in Schou (2000), keeping in mind that it is only a first step in the analysis.

Another major question concerns the effects generated by this pollution. In fact, we can assume that it affects two targets in particular: the production process and households' utility. In the first case, for example, its effects could be an increased depreciation of capital due to air and land pollution. Schou (2000) focuses on this case. Here, we will only consider the effects on the second target (pollution reduces household's utility), because it appears to be more straightforward, and because it is the subject of a great part of the literature (see for example Gradus and Smulders, 1993 or Aghion and Howitt, 1998). Note, however, that the simultaneous analysis of both effects is a line of research to be considered in the future.

We therefore introduce an endogenous growth model with Romer horizontal differentiation in the presence of non-renewable resources whose use in the production process generates a flow of pollution that negatively affects households' utility. It is then clear how our problem differs from the problem posed by the standard growth models with non-renewable natural resources presented above. Indeed, the question before was: how should a finite stock of resources be distributed among an infinite set of generations? Now, a supplementary question is added to the problem: how to distribute the associated finite stock of pollution among an infinite set of generations? By introducing an externality linked to the use of the resource, we have added an additional dimension to the problem: the choice of an extraction path now implies the simultaneous choice of a pollution path. We study the problem in three steps: optimum, equilibrium and corrective environmental policies. For each step, we first present the general solutions, and we go on to specify standard functional forms that allow us to compute the complete characterization of the steady-state paths. At the optimum, we obtain a non-standard version of the Hotelling rule, which is no longer a pure efficiency condition. In the specified case, we show that higher household preferences for the present imply a slower resource extraction rate, a result contrary to that obtained by the standard literature (see Stiglitz, 1974 for example).

Next we adopt a particular type of economic decentralization. Given that our main objective is to concentrate on the consequences of introducing pollution into the standard theory of non-renewable resources, we eliminate all other distortions in order to isolate its effects. In so doing, we also wish to avoid some of the technical difficulties that are usually associated with this kind of models. Our endogenous growth model therefore has no intermediate goods, and we assume that innovations are indivisible public goods directly financed by patents. Naturally, such a characterization of decentralization cannot be considered as a reference equilibrium (we will come to this later), but we believe it is interesting for the reasons discussed above. We then show the differences between equilibrium and optimal conditions and we interpret this non-optimality. We go on to characterize the specified equilibrium steady state and show that resources are extracted too rapidly (with respect to the optimal rate) and that the equilibrium level of R&D is too high.

Finally, we implement an environmental policy aimed at correcting the model's sole distortion, which arises from the polluting emissions generated by the use of the resource. The policy is a unit tax on resource purchases. Contrary to Schou (2002), we show that such an instrument is necessary in order to correct equilibrium conditions and bring them closer

to optimality. Moreover, our analysis indicates that it is the tax's growth rate, and not its level, which allows the distortion to be corrected. We study the sign of the general expression of the optimal tax growth rate, which can be positive or negative. In the specified case, we analyze the effects of the environmental policy on the model's variables and we compute the tax's optimal growth rate (which, under our specification, turns out to be negative).

In the section that follows, we present the model and the welfare analysis. We define the economy's market equilibrium and present its properties in Section 3. Then, in Section 4, we study the effects of the environmental policy and we characterize the optimal policy. Finally, we give concluding remarks in Section 5.

## 2. Presentation of the model and welfare analysis

### 2.1. The model

We first present a non-specified version of the model in order to get the general characteristic conditions at the optimum (Section 2.2) and at the equilibrium (Section 3.1). Then we compare them and we define a general formulation for the optimal environmental policy (Section 4.1).

At each time  $t$ , a quantity  $Y_t$  of a homogeneous good is produced according to the following technology:

$$Y_t = F(L_{Yt}, A_t, R_t), \quad (1)$$

where  $L_{Yt}$  is the amount of labour devoted to production,  $A_t$  is the stock of knowledge and  $R_t$  is the flow of a non-renewable resource used within the production process. We will denote by  $F_L$ ,  $F_A$  and  $F_R$  the marginal productivities.

We consider that each innovation is a public, indivisible and infinitely durable good, which is simultaneously used by the homogeneous good sector and by the R&D sector.<sup>1</sup> Formally, it is a point on the segment  $[0, A_t]$ . At each time  $t$ , the stock of knowledge evolves as follows:

$$\dot{A}_t = q(A_t, L_{RDt}), \quad (2)$$

where  $L_{RDt}$  is the amount of labour devoted to R&D. We will denote by  $q_A$  and  $q_L$  the marginal productivities. Note that the flow of knowledge depends on the pre-existing stock.

**Remark.** In this model, contrarily to what is done in the standard endogenous growth literature (see for instance Aghion and Howitt, 1992, Grossman and Helpman, 1991, or Romer, 1990), knowledge is not embodied inside intermediate goods.

We suppose that the technologies  $F(\cdot)$  and  $q(\cdot)$  exhibit constant returns to scale in private inputs, that is,  $F(\lambda L_Y, A, \lambda R) = \lambda F(L_Y, A, R)$  and  $q(A, \lambda L_{RD}) = \lambda q(A, L_{RD})$ . This

<sup>1</sup> For instance, we think of a scientific report which embodies a new theory that can be used within the production process.

assumption is justified by the standard replication argument: see for instance [Romer \(1990\)](#) or [Jones \(2001\)](#).

Population is assumed constant, normalized to one, and each individual is endowed with one unit of labour. Thus, we have:

$$1 = L_{Yt} + L_{RDt}. \quad (3)$$

The resource is extracted from an initial finite stock  $S_0$ , and we have the standard resource stock law of motion:

$$\dot{S}_t = -R_t. \quad (4)$$

The entire production of the homogeneous good is consumed by the representative household:

$$Y_t = C_t. \quad (5)$$

The household's instantaneous utility function depends both on consumption  $C_t$  and on the flow of pollution  $P_t$ . We write the intertemporal utility function as:

$$U_0 = \int_0^{+\infty} U(C_t, P_t) e^{-\rho t} dt, \quad (6)$$

where the partial derivatives of the utility function with respect to consumption and pollution are denoted by  $U_C$  and  $U_P$  and are positive and negative, respectively.

Pollution is generated by the use of the non-renewable natural resource within the production process, and is given by

$$P_t = h(R_t), \quad \text{with } h' > 0. \quad (7)$$

## 2.2. Welfare analysis

The social planner maximizes  $\int_0^{+\infty} U(C_t, P_t) e^{-\rho t} dt$  subject to (1)–(5) and (7).

After eliminating the co-state variables, the first order conditions reduce to the two following characteristic conditions which hold at each time  $t$  (we drop time subscripts for notational convenience):

$$\rho - \frac{\dot{U}_C}{U_C} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A \quad (8)$$

and

$$\frac{\dot{F}_R}{F_R} - \frac{U_P h'}{U_C F_R} \left( \rho - \frac{\dot{U}_P h'}{U_P h'} \right) = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A. \quad (9)$$

**Proof.** See [Appendix A](#).  $\square$

Eqs. (8) and (9) are modified versions of the Ramsey–Keynes and Hotelling rules, respectively.<sup>2</sup> There are three differences with respect to their standard form. First, contrary to the standard neo-classical growth model, we do not have physical capital but a

<sup>2</sup> A complete economic interpretation of these two conditions is available from the authors upon request.

stock  $A_t$  of knowledge: this explains that the marginal productivity of capital is replaced by the RHS in (8) and (9). The relative complexity of this expression comes from the fact that a variation of  $C_t$  is compensated by a transfer of labor between the consumption good sector ( $L_Y$ ) and the research sector ( $L_A$ ) which modifies the whole trajectory of variable  $A_t$ . The second difference is that, in (8),  $\dot{U}_C$  is equal to  $U_{CC}\dot{C} + U_{CP}\dot{P}$  ( $U_{CP}$  is nil in the case of a separable utility function). Third, the Hotelling rule is no longer strictly a pure efficiency condition as in the standard model without pollution (see Withagen, 1999, p. 52). Indeed, extracting one unit of resource not only allows to produce more (as in the standard case), but it also entails an increase in pollution, which creates a disutility. This explains the term between brackets in (9).

### 2.3. Specification and steady-state analysis

#### 2.3.1. Characterization and existence

In order to further develop the analysis and to be able to later study the effects of a green tax, we consider the following standard specifications. As mentioned above, production and R&D technologies exhibit constant returns to scale in private inputs. We can thus set:

$$Y_t = L_{Yt}^\alpha R_t^{1-\alpha} A_t^\nu, \quad \text{with } 0 < \alpha < 1 \text{ and } \nu > 0, \quad (10)$$

$$\dot{A}_t = \delta L_{Rt} A_t, \quad \text{with } \delta > 0. \quad (11)$$

We assume that emissions are a linear function of extraction flows:

$$h(R_t) = \gamma R_t, \quad \text{with } \gamma > 0. \quad (12)$$

Note that this relation does not take into account certain technologies, such as carbon sequestration for instance.

Finally we take the separable instantaneous utility function used by Aghion and Howitt (Chapter 5, 1998) for example:

$$U(C_t, P_t) = \frac{C_t^{1-\varepsilon}}{1-\varepsilon} - \frac{P_t^{1+\omega}}{1+\omega}, \quad \text{with } \varepsilon > 0 \text{ and } \omega > 0. \quad (13)$$

*Characterization:* Using (10)–(13), conditions (8) and (9) become (we denote  $g_{zt}$  as the growth rate,  $\dot{z}_t/z_t$ , of any variable  $z_t$ ):

$$\rho + (\varepsilon - 1)g_{Yt} = -g_{L_Yt} + \frac{\delta\nu}{\alpha}L_{Yt}, \quad (14)$$

$$-g_{Rt} + \frac{(\gamma R_t)^{1+\omega}}{(1-\alpha)Y_t^{1-\varepsilon}}(\rho - \omega g_{Rt}) = -g_{L_Yt} + \frac{\delta\nu}{\alpha}L_{Yt}. \quad (15)$$

Moreover, differentiating (10) with respect to time yields the additional condition:

$$g_{Yt} = \alpha g_{L_Yt} + (1-\alpha)g_{Rt} + \nu\delta(1-L_{Yt}). \quad (16)$$

We focus now on the study of steady-state paths, i.e. on paths along which the growth rate of any variable is constant.

**Proposition 1.** *At the steady-state optimum, the quantities and growth rates take the following values (the superscript  $o$  indicates the optimum):*

$$L_{RD}^o = \frac{(\delta v - \rho\alpha)(\varepsilon(1 - \alpha) + \alpha + \omega)}{\delta v(\varepsilon + \omega(1 - \alpha + \varepsilon\alpha))}, \quad (17)$$

$$L_Y^o = 1 - L_{RD}^o, \quad (18)$$

$$g_A^o = \delta L_{RD}^o, \quad (19)$$

$$g_R^o = g_P^o = g_S^o = \frac{(\delta v - \rho\alpha)(1 - \varepsilon)}{\varepsilon + \omega(1 - \alpha + \varepsilon\alpha)}, \quad (20)$$

$$g_C^o = g_Y^o = \frac{(\delta v - \rho\alpha)(1 + \omega)}{\varepsilon + \omega(1 - \alpha + \varepsilon\alpha)}. \quad (21)$$

**Remark.** At the steady-state optimum, given  $S_0$  and  $A_0$ , the initial quantities are  $Y_0^o = C_0^o = (L_Y^o)^\alpha (-S_0 g_R^o)^{1-\alpha} (A_0)^\nu$ ,  $R_0^o = P_0^o/\gamma = -S_0 g_R^o$  (dividing (4) by  $S_t$  yields this result), and at each time  $t$  we have  $x_t^o = x_0^o e^{g_x^o t}$  for any variable  $x$ .

**Proof.** Using Eqs. (14)–(16) and considering that all rates of growth are constant, the results follow.  $\square$

*Existence of an interior optimum:* The two transversality conditions of the social planner's program are respectively  $\lim_{t \rightarrow +\infty} \mu_t A_t^o e^{-\rho t} = 0$  and  $\lim_{t \rightarrow +\infty} v_t S_t^o e^{-\rho t} = 0$ , and they respectively imply  $L_{RD}^o < 1$  and  $g_R^o < 0$  (see Appendix C for details). Moreover, we need  $L_{RD}^o > 0$  to have a solution. The three conditions are jointly fulfilled if  $\varepsilon > 1$  and  $\rho < \delta v/\alpha$  (see Eqs. (17), (18) and (20)).<sup>3</sup> Note that for this set of parameters' values, we have  $g_Y^o > 0$  (see (21)). These results are summarized in Fig. 1.

### 2.3.2. Properties of the optimal path

We now perform some comparative statics so as to describe the main properties of the path we just defined, and to compare it with what has been obtained in more standard growth models. Letters S and GR indicate that similar results were obtained by Stiglitz (1974) and Grimaud and Rouge (2003), respectively in a “à la Ramsey” and a “à la Aghion–Howitt” standard models with a non-renewable resource (which does not pollute) (Table 1).

- The effects of a change in  $\delta$  or  $\varepsilon$  on the variables of the model are quite standard. In our model,  $\delta$  is a parameter which characterizes the effectiveness of the R&D sector, and it can be seen as the exogenous part of technological progress: it corresponds to  $\eta$  in Stiglitz. A higher  $\delta$  means that devoting labour to research is better, hence labour in research ( $L_{RD}^o$ ) and the growth rate of knowledge ( $g_A^o$ ) grow. This will increase the output

<sup>3</sup> Similar conditions are obtained by Aghion and Howitt (1998) in a “Schumpeterian Approach to Pollution”, p. 161.

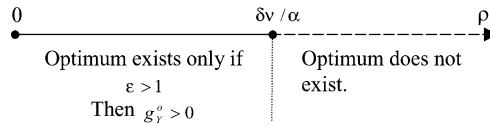


Fig. 1. Existence of interior optimum.

growth rate, despite a lower resource extraction growth as in Stiglitz and Grimaud–Rouge. On the reverse, an increase in the elasticity of marginal utility,  $\varepsilon$ , means that more utility is derived from uniform consumption paths. That is why a social planner will invest less in R&D ( $L_{RD}^0$  and  $g_A^0$  decrease). Moreover, this planner will lower consumption growth and therefore will choose a lower resource extraction growth rate  $g_R^0$  (we have the same result in Stiglitz and Grimaud–Rouge).

- Here, taking into account the pollution emitted by the use of the non-renewable resource modifies the impact of a change in  $\rho$  (the psychological discount rate) on  $g_R^0$ , with respect with the standard literature.

An increase in  $\rho$  means that utility derived from current times is more valued relative to utility derived from future times. In the literature mentioned above, this implies that the representative household values more current consumption relative to future consumption. Thus, the steady-state growth rate of the output will decrease, which corresponds to more output today and less tomorrow (relative to the situation before the increase in  $\rho$ ). In order to produce more today, the social planner will increase the amount  $L_Y^0$  of labour and the flow  $R^0$  of resource both used within the production process. An increase in the “production-labour” today means a decrease in the “R&D-labour” today, thus a decrease in the growth rate of knowledge (see (2)). An increase in the flow of resource used today implies a decrease in the flow used tomorrow:  $g_R^0$  decreases.

In our framework, the fundamental difference comes from the fact that valuing more utility derived from current times does not only mean valuing more current consumption. This also means valuing more the current state of the environment (see (6)): improving the first generations’ welfare means also improving their environment. Thus,  $P_0^0$  (i.e.  $R_0^0$ ) must decrease. So, as above, the planner will increase  $L_Y^0$  today, that is, decrease  $L_{RD}^0$  and thus decrease  $g_A^0$ ; but at the same time, the planner will decrease the level of pollution

Table 1  
Properties of the optimal path

	$\xi = \delta$	$\xi = \varepsilon$	$\xi = \rho$	$\xi = v$	$\xi = \omega$
$\frac{\partial L_{RD}^0}{\partial \xi}$	>0 GR	<0 GR	<0 GR	>0	<0
$\frac{\partial g_A^0}{\partial \xi}$	>0 GR	<0 GR	<0 GR	>0	<0
$\frac{\partial g_R^0}{\partial \xi}$	<0 S,GR	<0 S,GR	>0	<0	>0
$\frac{\partial g_Y^0}{\partial \xi}$	>0 S,GR	<0 S,GR	<0 S,GR	>0	>0



today, and will increase this level tomorrow (recall that, in our framework, the stock of pollution is finite): that is why, contrary to what is obtained in models without pollution,  $g_P^0$  (that is,  $g_R^0$ ) increases.

- A higher  $\omega$  corresponds to an increase in household's concern for a cleaner environment. One fundamental feature of our model, as we just recalled, is that pollution is created by the use of a non-renewable resource. That is why the stock of pollution is finite; for this reason, at the steady-state, polluting more (resp. less) today means polluting less (resp. more) in the future. Thus, for a given time preference, an increase in  $\omega$  implies less pollution today and more tomorrow. Hence, at the steady-state, the optimal growth rate of resource extraction will increase: less resource is consumed today (i.e. less pollution is emitted) and more tomorrow (i.e. more pollution tomorrow). To compensate the fall in  $R_0^0$ , the planner will have to increase  $L_Y^0$  today, and thus to decrease  $L_{RD}^0$ ; that is why the growth rate of knowledge decreases. From (10), we have  $g_Y^0 = (1 - \alpha)g_R^0 + \nu g_A^0$  at the steady-state. It turns out that the combination of both effects (a rise in  $g_R^0$  and a fall in  $g_A^0$ ) results in a domination of the former:  $g_Y^0$  increases.

So, the more households will value their environment, the slower the resource will be extracted, and the more rapidly output will grow.

**Remark.** With a non-separable utility function (as in Schou (2002)), one could not disentangle the effects of  $\omega$  and  $\rho$ . For instance, a higher  $\rho$  would lead to a slower extraction provided that households care enough for their environment.

### 3. Decentralized equilibrium

#### 3.1. Behaviour of agents and equilibrium conditions

Let us consider the non-specified version of the model presented in Section 2.1. The price of good  $Y$  is normalized to one, and  $w_t, p_t^R$  and  $r_t$  are, respectively, the wage, the resource price, and the interest rate on a perfect financial market. In order to eliminate the market failure arising from the fact that firms do not take into account the negative externality due to the use of the non-renewable resource within the production process, i.e. pollution, we implement a corrective policy: a tax on the demand for resource.

**Remark.** The tax is not levied on  $P_t$  but on  $R_t$ . Indeed, since  $P_t$  and  $R_t$  are linked by a functional relation, we can implement the optimum by only taxing the resource use. Moreover, it is generally more convenient to monitor the quantity of resource and thus to use such a policy instrument.

*Consumption good sector:* At each time  $t$ , the firm's profit is

$$\pi_t^Y = F(L_{Yt}, A_t, R_t) - w_t L_{Yt} - p_{Rt}(1 + \sigma_t)R_t \quad (22)$$

where  $\sigma_t$  is the unit tax on the resource use. Henceforth, we will denote  $\tau_t = 1 + \sigma_t$  for computational convenience.

Differentiating  $\pi_t^Y$  with respect to  $L_{Yt}$  and  $R_t$ , and equating to zero yields the two following first order conditions:

$$F_L = w_t, \quad (23)$$

and

$$F_R = \tau_t p_{Rt}. \quad (24)$$

*R&D sector:* As we mentioned above, knowledge is not embodied inside intermediate goods, implying that it cannot be financed by the sale of these goods. We suppose here that knowledge is directly financed. Nonetheless, as the sectors using knowledge exhibit constant returns to scale in private goods, their profits are nil; thus we assume that once an innovation has occurred, the government pays to the innovator a sum equal to the willingnesses to pay of both sectors.

**Remark.** This type of decentralization defines an equilibrium that is a benchmark: in the absence of any pollution, this equilibrium is a first best optimum (as we will show below). This is a formalization of ideas already expressed by [Arrow (1962); Dasgupta et al. (1996)] or [Scotchmer (1991, 1999)]. The main interest of designing such an equilibrium is that it allows us to focus on a single distortion: that caused by pollution. Hence we avoid other distortions (monopolies rents, intertemporal spillovers, etc.) inherent to the equilibria used in the standard endogenous growth models. Indeed, these distortions entail a great complexity of the model. Note that, as seen above, considering such an equilibrium leads to assume that the government entirely finances R&D activities. However, assuming that the two sectors using innovations (the final and R&D sectors) are imperfectly competitive, it is possible to construct an equilibrium in which R&D is entirely and privately financed by these sectors (we find a similar equilibrium path in this case).

At each time  $t$ , the value of an innovation is:

$$V_t = \int_0^{+\infty} v_s e^{-\int_t^s r_u du} ds, \quad (25)$$

where  $v_s$  is the sum of the willingnesses to pay of the homogeneous good sector ( $v_s^Y$ ) and the R&D sector ( $v_s^{RD}$ ):  $v_s = v_s^Y + v_s^{RD}$ .

The profit on innovations produced at  $t$  is:

$$\pi_t^{RD} = q(A_t, L_{RDt})V_t - w_t L_{RDt}. \quad (26)$$

The maximization of this profit function with respect to  $L_{Rt}$  leads to the following first order condition:

$$q_L V_t = w_t. \quad (27)$$

From (22) and (26), we have:

$$v_t^Y = \frac{\partial \pi_t^Y}{\partial A_t} = F_A$$

$$\text{and } v_t^{\text{RD}} = \frac{\partial \pi_t^{\text{RD}}}{\partial A_t} = q_A V_t.$$

Thus, we get

$$v_t = F_A + q_A V_t. \quad (28)$$

*Representative household:* At each time  $t$ , the representative household maximizes the utility function  $\int_0^{+\infty} U(C_t, P_t) e^{-\rho t} dt$  subject to  $\dot{B}_t = w_t + r_t B_t + p_{Rt} R_t - T_t - C_t$ , where  $B_t$  is the stock of bonds at  $t$ , and  $T_t$  is a lump-sum tax levied by the government to finance research. This maximization leads to the following condition:

$$\rho - \frac{U_{CC}\dot{C} + U_{CP}\dot{P}}{U_C} = r_t. \quad (29)$$

*Resource sector:* On the competitive natural resource market, the maximization of the profit function  $\int_t^{+\infty} p_{Rs} R_s e^{-\int_t^s r_u du} ds$ , subject to  $\dot{S}_s = -R_s$ ,  $S_s \geq 0$ ,  $R_s \geq 0$ ,  $s \geq t$ , yields the standard equilibrium “Hotelling rule”:

$$\frac{\dot{p}_{Rt}}{p_{Rt}} = r_t, \text{ for all } t. \quad (30)$$

The transversality condition of this program implies  $\lim_{t \rightarrow \infty} S_t = 0$ . Observe that, since  $p_{Rt} = F_R$  when no environmental policy is applied (see (24) above), the decentralized economy does not implement the optimum Hotelling rule given by (9) (we come back to this point later in the text).

*Government:* The government’s budget constraint is<sup>4</sup>

$$T_t + \sigma_t p_{Rt} R_t = (v_t^Y + v_t^{\text{RD}}) A_t, \text{ for all } t.$$

Our objective is now to present the two characteristic equilibrium conditions (which hold at each time  $t$ ) that can be compared to the optimal ones ((8) and (9)). We obtain:

$$\rho - \frac{U_{CC}\dot{C} + U_{CP}\dot{P}}{U_C} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A \quad (31)$$

and

$$\frac{\dot{F}_R}{F_R} - \frac{\dot{\tau}_t}{\tau_t} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A \quad (32)$$

**Proof.** Differentiating (25) and (27) with respect to time gives  $\frac{\dot{w}_t}{w_t} = r_t - \frac{v_t}{V_t} + \frac{\dot{q}_L}{q_L}$ . Using (23), (27) and (28), we get:  $r_t = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A$ . This equation, together with (29), gives (31).

The same equation, together with (24) and (30), leads to (32).  $\square$

<sup>4</sup> We assume here that the government possesses all the information and thus can perfectly determine  $v^Y$  and  $v^{\text{RD}}$ . Of course this is just a benchmark and informational asymmetries could be introduced.

First of all, we note that (8) and (31) are identical: in other words, the optimum Ramsey–Keynes rule is verified at the equilibrium.

However, we also note that, in absence of any environmental policy, the optimum Hotelling rule is not verified (compare (9) and (32)). This comes from the fact that the social cost of pollution is not reflected in the resource price. We analyse this point in Section 4 and we show that the optimum can be implemented with a unit tax and, in the specified case, that the resource is extracted too fast at the equilibrium. Nonetheless, if pollution has no impact on the utility of households ( $U_P = 0$ ), Eqs. (9) and (32) are identical and the first best optimum is reached, as mentioned above (refer to the remark after (24)).

### 3.2. Specification

#### 3.2.1. Characterization and existence

*Characterization:* Consider specifications (10)–(13) (presented in Section 2.3.1). As in the analysis of the optimum, we obtain conditions (14) and (16). Condition (32) becomes

$$-g_{Rt} - g_{\tau t} = -g_{LYt} + \frac{\delta v}{\alpha} L_{Yt}. \quad (33)$$

Using Eq. (14), and replacing  $-g_{LYt} + (\delta v/\alpha)L_{Yt}$  by  $\rho + (\varepsilon - 1)g_{Yt}$  in (16) and (33), we get  $g_{Yt}$  and  $g_{Rt}$  as functions of  $g_{\tau t}$ . Thus, if  $g_{\tau}$  is constant (i.e. independent of time), then  $g_Y$  and  $g_R$  are also constant. Moreover, since  $\lim_{t \rightarrow \infty} S_t = 0$  (see the transversality condition above), we have  $S_t = \int_t^\infty R_u du$ . Since  $g_R$  is constant, one gets  $g_S = g_R$ . We assume now that  $g_{\tau}$  is constant and we characterize the steady-state equilibrium through the following proposition.

**Proposition 2.** *At the steady-state equilibrium, the quantities and rates of growth take the following values (the superscript  $e$  is used for equilibrium):*

$$L_{RD}^e = \frac{\delta v(\varepsilon(1 - \alpha) + \alpha) - \alpha\rho + \alpha(\varepsilon - 1)(1 - \alpha)g_{\tau}}{\varepsilon\delta v}, \quad (34)$$

$$L_Y^e = \frac{\delta v\alpha(\varepsilon - 1) + \alpha\rho - \alpha(\varepsilon - 1)(1 - \alpha)g_{\tau}}{\varepsilon\delta v}, \quad (35)$$

$$g_A^e = \delta L_{RD}^e, \quad (36)$$

$$g_R^e = g_P^e = g_S^e = \frac{\delta v(1 - \varepsilon) - \rho}{\varepsilon} - \frac{\alpha\varepsilon + 1 - \alpha}{\varepsilon} g_{\tau}, \quad (37)$$

$$g_C^e = g_Y^e = \frac{\delta v - \rho}{\varepsilon} - \frac{1 - \alpha}{\varepsilon} g_{\tau}. \quad (38)$$

**Remark.** Given  $S_0$  and  $A_0$ , the initial quantities are  $Y_0^e = C_0^e = (L_Y^e)^\alpha (-S_0 g_R^e)^{1-\alpha} (A_0)^\nu$ ,  $R_0^e = P_0^e/\gamma = -S_0 g_R^e$ , and at each time  $t$  we have  $x_t^e = x_0^e e^{g_x^e t}$  for any variable  $x$ .

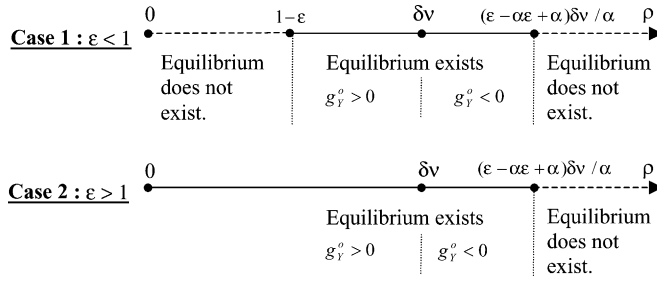


Fig. 2. Existence of interior equilibrium.

At the equilibrium, prices are:

$$w_t = \alpha(L_Y^c)^{\alpha-1}(R_t^c)^{1-\alpha}(A_t^c)^\nu, \quad (39)$$

$$p_{Rt} = (1 - \alpha)(L_Y^c)^\alpha(R_t^c)^{-\alpha}(A_t^c)^\nu(\tau_0 e^{g_R^c t})^{-1}, \quad (40)$$

and  $r = \delta v - (1 - \alpha)g_\tau$ ,  $v_t^Y = v(L_Y^c)^\alpha(R_t^c)^{1-\alpha}(A_t^c)^{\nu-1}$ ,  $v_t^{RD} = L_{RD}^c \alpha(L_Y^c)^{\alpha-1}(R_t^c)^{1-\alpha}(A_t^c)^{\nu-1}$ ,  $V_t = (\alpha/\delta)(L_Y^c)^{\alpha-1}(R_t^c)^{1-\alpha}(A_t^c)^{\nu-1}$ , where  $R_t^c = -S_0^c g_R^c e^{g_R^c t}$  and  $A_t^c = A_0^c e^{g_A^c t}$ .

*Existence of an interior equilibrium:* For the analysis that follows, we suppose that there is no environmental policy, that is,  $\tau_t = 1$  (or equivalently,  $\sigma_t = 0$ ).

As we did for the optimal path, we are looking for the set of parameter values in which  $0 < L_{RD}^c < 1$ , and  $g_R^c < 0$ . To do so, we use Eqs. (34) and (37). The set is described in Fig. 2.

### 3.2.2. Equilibrium properties

As it is done for the optimum, we now study the impact of different exogenous parameters variations on  $L_{RD}^c$ ,  $g_A^c$ ,  $g_R^c$  and  $g_Y^c$ . The results are depicted in Table 2.

Letters GR indicate that similar results have been obtained by Grimaud and Rouge (2003) in a model “à la Aghion–Howitt” with a non-polluting non-renewable resource.

The effects of the parameters’ variations presented in Table 2 result from market mechanisms (in Table 1, they resulted from the planner’s decisions). We can see that the

Table 2  
Properties of the equilibrium path

	$\xi = \delta$	$\xi = \varepsilon$	$\xi = \rho$	$\xi = v$	$\xi = \omega$
$\frac{\partial L_{RD}^c}{\partial \xi}$	>0 GR	<0 GR	<0 GR	>0	=0
$\frac{\partial g_A^c}{\partial \xi}$	>0 GR	<0 GR	<0 GR	>0	=0
$\frac{\partial g_R^c}{\partial \xi}$	<0 if $\varepsilon > 1$ GR	<0 if $g_Y^c > 0$ GR	<0 GR	<0 if $\varepsilon > 1$	=0
$\frac{\partial g_Y^c}{\partial \xi}$	>0 GR	<0 if $g_Y^c > 0$ GR	<0 GR	>0	=0

properties of the equilibrium path are very similar to the properties of the optimal one. There are only two differences. First,  $g_R^e$  is an decreasing function of  $\rho$ . Indeed, if  $\rho$  gets higher, households derive more utility from current consumption, thus they want to consume more today:  $g_C^e$  decreases. Hence firms will produce more today and less tomorrow. In order to increase output today, they will use more inputs: more labour and more resource. Indeed, contrarily to the social planner, firms do not take into account the effects of the resource use on the utility of consumers. As more resource is extracted today, the stock being finite, less will be extracted tomorrow:  $g_R^e$  decreases. The second difference with the properties of the optimal path is that  $\omega$  has no effect on the equilibrium variables. Indeed this parameter represents the taste of households for their environment (or equivalently their vulnerability towards pollution); however, in the absence of any environmental policy, pollution is not priced in this model, thus  $\omega$  affects no market.

#### 4. Impact of the environmental policy and implementation of the optimum

##### 4.1. General analysis

Let us first study the non specified version of the model. Comparing Eqs. (8) and (9) (optimum) and (31) and (32) (equilibrium), it becomes clear that, at each time  $t$ , the optimum path is implemented if the growth rate of the tax is:

$$g_{\tau t}^o = -\frac{U_P h'}{U_C F_R} \left( \frac{\dot{U_P h'}}{U_P h'} - \rho \right). \quad (41)$$

Note that it is the growth rate of the tax and not its level that matters here. We will further develop this issue in Section 4.2.

We now give some intuitions about the sign of  $g_{\tau t}^o$ . First, observe that the term  $-U_P h' / U_C F_R$ , which can be seen as the relative disutility of resource extraction, is unambiguously positive. Consequently,  $g_{\tau t}^o$  has the same sign as the term between brackets. For the sake of simplicity, we will henceforth assume that  $h'$  is constant, in accordance with the chosen specification (see (12)).

Assume  $\dot{U}_P = 0$ , which means that the disutility of pollution is constant. Then  $g_{\tau t}^o$  is negative. In this case, at each time  $t$ , the optimal environmental policy consists in taxing more today than tomorrow, which implies postponing part of the pollution flow. The bigger  $-U_P h' / U_C F_R$  and  $\rho$ , the lower is  $g_{\tau t}^o$ . This means that the more people dislike pollution and the more they prefer the present, the more present pollution will be taxed relative to future pollution.

Consider the case where  $\dot{U}_P / U_P < 0$  (i.e.  $\dot{U}_P > 0$ ), which means that people's dislike of pollution decreases across time: the lower this term, the stronger is the effect presented above. In the case of a positive  $\dot{U}_P / U_P$  (in particular if the revenue of household is increasing over time because of economic growth so that  $\dot{U}_P < 0$ ), this effect is weakened, and can even be reversed. Indeed, in this case, the disutility of pollution is growing with

time, and it can even be optimal to tax more tomorrow than today, in order to transfer future pollution to present times.

## 4.2. Specification

### 4.2.1. Optimum versus equilibrium

We now focus on the specified version of the model (presented in Sections 2.3 and 3.2) at the steady-state. First of all, Figs. 1 and 2 show that both the optimal path and the equilibrium path are defined together if  $\varepsilon > 1$  and  $\rho < \delta v/\alpha$ . Therefore, since our aim is to compare these paths and to analyse the effects of an environmental policy – whose main goal is the implementation of the optimum – these conditions will hold throughout the remainder of this section.

**Proposition 3.** *In absence of any environmental policy, we have:*

$$L_{RD}^e > L_{RD}^o, \quad (42)$$

$$g_R^e < g_R^o, \quad (43)$$

$$g_Y^e < g_Y^o. \quad (44)$$

**Proof.** In the set of parameter values described above, we compare  $L_{RD}^e$  (see (34)) and  $L_{RD}^o$  (see (17)),  $g_R^e$  (see (37)) and  $g_R^o$  (see (20)), and  $g_Y^e$  (see (38)) and  $g_Y^o$  (see (21)). The results follow.  $\square$

In this model, there is a single distortion: the pollution generated as a by-product of the resource's use. If there is no pollution (or, equivalently, if consumers do not value their environment), the decentralized equilibrium is a first best optimum (see above). If there is pollution, but no tax, the equilibrium remains unchanged because firms do not take into account this pollution: this equilibrium remains equivalent to the first best without pollution.

Hence, comparing the equilibrium (without any public intervention) and the optimal paths is exactly the same as comparing the optimal path in the absence of pollution and the optimal path when the resource use yields a flow of pollution that affects negatively consumers' utility.

In Section 2.3.2, we showed that the more consumers value their environment, the slower the resource is extracted:  $g_R^o$  increases in order to allow for less pollution today and more tomorrow (i.e. exhausting the finite stock of pollution less rapidly). Therefore, the limit case corresponding to an indifference towards environment ( $P_t$  does not enter the utility function) will be characterized by a lower  $g_R^o$  (i.e. a lower  $g_p^o$ ), because pollution is not taken into account by the planner. Market mechanisms yield the same result: at the decentralized equilibrium, the resource is extracted more rapidly because pollution is not priced (see Fig. 3). We also proved in Section 2.3.2 that a higher concern for the environment leads to a lower amount of labour devoted to R&D in order to compensate for the increase in  $g_R^o$ . For the same reasons,  $L_{RD}^e$  is over-optimal: there is too much R&D at the equilibrium.

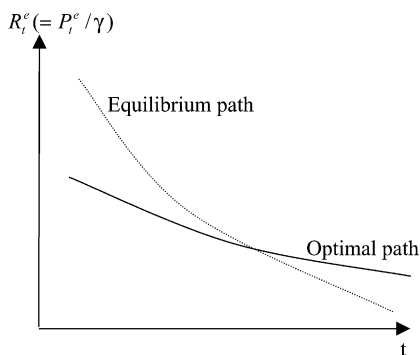


Fig. 3. Optimum vs. equilibrium without any public intervention ( $t$ : time;  $R_t^e$ : resource extraction;  $P_t^e$ : flow of pollution).

#### 4.2.2. Impact of the environmental policy

Here we want to study the effects of our environmental policy instrument (a tax on the use of the non-renewable resource) on the main relevant variables of the model. At the steady-state, the tax can be written:  $\tau_t = \tau_0 e^{g_\tau t}$ .

#### Proposition 4.

- A change in the tax level has no impact on the equilibrium except that it leads to a rent transfer from the resource sector to the government.
- A change in the tax's growth rate affects all prices, quantities and growth rates (as summarized in Table 3).

This proposition contrasts with most of the results obtained in the standard literature, where the tax level generally has an impact on economic variables. First of all, note that an increase (resp. a decrease) in  $\tau_0$  has a the following unique effect on the equilibrium defined in Section 3.2.1: it decreases (resp. increases)  $p_{Rt}$  (see (40)) in such a way that the total unit cost of the resource, i.e.  $\tau_t p_{Rt}$ , remains unchanged for the final sector firm. In other words, a change in the tax level (for a given growth rate  $g_\tau$ ) has a single effect on the economy: it causes the resource price level to move in a sense opposite to the tax level, implying a rent transfer from the resource sector to the government. The other prices and the quantities and rates of growth remain unchanged. We obtain in this framework a result similar to what is expressed in Nordhaus and Boyer (2000, Chapter 3) (this also corresponds to an idea already mentioned in [Schou, 2000, 2002]). In fact, this can be seen

Table 3  
Effect of the environmental policy

	$\xi = L_{RD}^e$	$\xi = g_A^e$	$\xi = g_R^e$	$\xi = g_Y^e$	$\xi = r$
$\frac{\partial \xi}{\partial g_\tau}$	$>0$ if $\varepsilon > 1$	$>0$ if $\varepsilon > 1$	$<0$	$<0$	$<0$



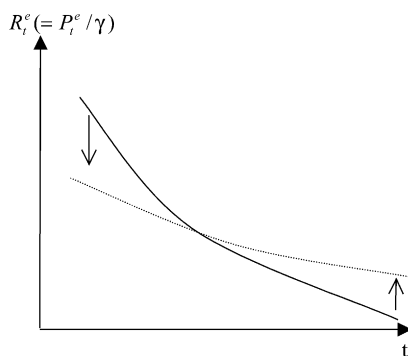


Fig. 4. Impact of the environmental policy consisting in a decrease in  $g_\tau$  ( $t$ : time;  $R_t^e$ : resource extraction;  $P_t^e$ : flow of pollution).

as a generalization in the dynamic case of the following elementary static example. Let us consider a market for which the demand is  $D = D(p\tau)$  ( $p$ : price,  $\tau = 1 + \sigma$ , where  $\sigma$  is a tax). If supply  $S$  is fixed (as the resource stock in our model), we have  $D(p\tau) = S$ , and thus  $p\tau = D^{-1}(S)$  which is constant: an increase in  $\tau$  only leads to a decrease in  $p$  and does not modify the quantities (obvious in the static case).

Conversely, any change in  $g_\tau$  modifies the entire path. Moreover, Proposition 5 below proves that the optimal  $g_\tau$  (the one that allows the implementation of the optimal path) is negative. Consider first a constant  $\tau$ . We then allow it to increase during the initial periods and to decrease during the latter periods:  $g_\tau$  becomes negative. Using Eqs. (34)–(38), we obtain the results summarized in Table 3.

First of all, note that the price ratio  $\tau_t p_{Rt} / w_t$  is a decreasing function of  $g_\tau$  (see (40) and (39)). Thus, at steady-state, the lower  $g_\tau$  is, the higher  $\tau_t p_{Rt}$  (the price paid by the firm for the resource) is relative to the wage. For this reason, the firm producing the consumption good will use more labour:  $L_Y$  increases. Hence, by a general equilibrium effect,  $L_{RD}$  decreases, and thus there is less technological progress (see Eq. (11)).

Moreover, at steady-state,  $g_{\tau_t p_{Rt} / w_t}$  is equal to  $g_\tau + g_{p_R} - g_w$ . Thus, Eqs. (30) and (39) allow us to write  $g_{\tau_t p_{Rt} / w_t} = g_\tau + r - g_Y^e$ , which is an increasing function of  $g_\tau$ . Therefore, whereas a lower  $g_\tau$  makes the resource price higher relative to the wage, this effect diminishes along time. The firm will thereby choose to buy less resource today and more tomorrow relative to the path it chose with the initial  $g_\tau$  (see Fig. 4):  $g_R^e$  increases and the resource is extracted at a lower path, as it is shown in Table 3.

#### 4.2.3. Implementation of the optimum

**Proposition 5.** *If  $g_\tau = -\frac{\delta v \omega (\varepsilon - 1) + \rho (\varepsilon + \omega)}{\varepsilon + \omega (1 - \alpha + \alpha \varepsilon)}$  at the steady-state, then the equilibrium path is optimal.*

Note that in this particular case (see a general discussion in Section 4.1), the optimal  $g_\tau$  is negative, which means that, along the equilibrium growth path, the tax level decreases with

the use of the resource ( $g_R^e$  is negative). Moreover, Eq. (24) implies that  $\tau_t$  is positive, implying that in the long-run  $\tau_t$  tends to zero, for any initial level  $\tau_0$ .

**Proof.** Simple computations allow us to find  $g_\tau$  such that  $L_{RD}^e$  (see (34)) and  $L_{RD}^0$  (see (17)) be equal.  $\square$

## 5. Conclusion

In this paper we considered the following problem: how are the standard results of non-renewable resources (growth) theory modified when we take into account the presence of polluting emissions arising from the use of those resources?

Firstly, we were able to characterize optimality conditions in a very general manner (that is, without specifying functional forms) since we avoided the technical complexity inherent to certain hypothesis of the standard endogenous growth models. We thus defined a Hotelling rule that was modified in two senses. On one hand, it is no longer strictly an efficiency condition; since pollution affects households' preferences, these are accounted for in the new Hotelling rule. On the other hand, it is now the accumulation of intellectual capital (knowledge), and not the accumulation of physical capital, which delays the depletion of the resource. Next, we presented a specified version of the model in order to conduct steady-state analysis and to give a complete characterization of the optimal path. We also described its properties; namely, we found once more that increases in households' impatience (or in the egoism of present generations), or in their preference for environmental quality, result in a slower extraction of the resource.

We went on to present a decentralization of the economy, such that pollution is the source of the model's only distortion. There, the violation of the Hotelling rule at the market equilibrium – when no environmental policy has been implemented – is clearly identified. Agents fail to incorporate the negative externality caused by the use of the resource since there is no market for pollution. We also characterized the steady-state's equilibrium path in the specified case and showed that the growth rate at which the resource is extracted is under-optimal: the resource is depleted too rapidly (or equivalently, there is too much pollution today). Moreover, contrary to that observed at the optimum, an increase in households' impatience makes the economy deplete the resource even faster.

Finally, the fact that there is only one distortion in the model allows us to implement a simple economic policy: a unit tax on the demand for the resource. In the general case, we showed that choosing the right growth rate for the tax allows us to implement the optimal path. Then we studied the sign of this growth rate and found that it can be positive or negative, depending on the strength of the psychological discount rate and the evolution of the disutility of pollution along time. In the specified case, we showed that, contrarily to what is found in a large part of the literature, a change in the tax level at the steady-state has no effect on the real variables of the economy and does not allow us to correct the equilibrium path (in order to make it optimal). Indeed, such a policy only entails a rent transfer from the firm that extracts the resource to the government (who levies the tax). Conversely, we studied the impact of a change in the tax's growth rate on the relevant

variables (prices, quantities and growth rates). We calculated the exact level of this growth rate (negative) which sends the appropriate signal to markets and for which the optimum is attained at equilibrium.

Certain elements – such as considering pollution as a stock and not a flow – could certainly add to our results. Moreover, allowing for the possibility of substituting the polluting non-renewable resource with a non-polluting renewable resource (such as solar energy or hydrogen fuel cells) remains a very interesting line of future research.

## Acknowledgment

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## Appendix A. Non specified optimality conditions

The current value Hamiltonian of the social planner's program is

$$H = U[F(1 - L_{RDt}, A_t, R_t), h(R_t)] + \mu_t q(A_t, L_{RDt}) - \nu_t R_t.$$

where  $\mu_t$  and  $\nu_t$  are the costate variables.

The first order conditions  $\partial H / \partial L_{RDt} = 0$  and  $\partial H / \partial R_t = 0$  yield

$$\mu_t = \frac{U_C F_L}{q_L} \quad (\text{A.1})$$

$$\nu_t = U_C F_R + U_P h'. \quad (\text{A.2})$$

Moreover,  $\partial H / \partial A_t = \rho \mu_t - \dot{\mu}_t$  and  $\partial H / \partial S_t = \rho \nu_t - \dot{\nu}_t$  yield

$$\frac{\dot{\mu}_t}{\mu_t} = \rho - \frac{F_A q_L}{F_L} - q_A \quad (\text{A.3})$$

$$\text{and } \frac{\dot{\nu}_t}{\nu_t} = \rho. \quad (\text{A.4})$$

Differentiating (A.1) with respect to time, we have

$$\frac{\dot{\mu}_t}{\mu_t} = \frac{U_{CC} \dot{C} + U_{CP} \dot{P}}{U_C} + \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L}. \quad (\text{A.5})$$

(A.5), together with (A.3) gives

$$\rho - \frac{U_{CC} \dot{C} + U_{CP} \dot{P}}{U_C} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A, \quad (\text{A.6})$$

that is, the Ramsey–Keynes condition.

Differentiating (A.2) with respect to time, we have

$$\frac{\dot{\nu}_t}{\nu_t} = \frac{U_C F_R ((U_{CC} \dot{C} + U_{CP} \dot{P} / U_C) + (\dot{F}_R / F_R)) + U_P h' ((U_{PC} \dot{C} + U_{PP} \dot{P} / U_P) + (\dot{h}' / h'))}{U_C F_R + U_P h'}. \quad (\text{A.7})$$

(A.7) together with (A.4) gives

$$\rho - \frac{U_{CC}\dot{C} + U_{CP}\dot{P}}{U_C} = \frac{\dot{F}_R}{F_R} + \frac{U_P h'}{U_C F_R} \left( \frac{U_{PC}\dot{C} + U_{PP}\dot{P}}{U_P} + \frac{\dot{h}'}{h'} - \rho \right). \quad (\text{A.8})$$

Plugging (A.6) into (A.8) yields:

$$\frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A = \frac{\dot{F}_R}{F_R} + \frac{U_P h'}{U_C F_R} \left( \frac{U_{PC}\dot{C} + U_{PP}\dot{P}}{U_P} + \frac{\dot{h}'}{h'} - \rho \right),$$

where

$$\frac{U_P h'}{U_C F_R} \left( \frac{U_{PC}\dot{C} + U_{PP}\dot{P}}{U_P} + \frac{\dot{h}'}{h'} - \rho \right) = \frac{U_P h'}{U_C F_R} \left( \frac{\dot{U}_P h' + U_P \dot{h}'}{U_P h'} - \rho \right) = \frac{U_P h'}{U_C F_R} \left( \frac{\dot{U}_P h'}{U_P h'} - \rho \right).$$

One gets finally

$$\frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A = \frac{\dot{F}_R}{F_R} + \left( \frac{\dot{U}_P h' - \rho U_P h'}{U_C F_R} \right), \quad (\text{A.9})$$

that is the Hotelling condition.

## Appendix B. Transversality conditions at the steady-state optimum

The first transversality condition is  $\lim_{t \rightarrow +\infty} \mu_t A_t^0 e^{-\rho t} = 0$ , which is equivalent to  $\lim_{t \rightarrow +\infty} \mu_0 e^{g_\mu t} A_0^0 e^{g_A^0 t} e^{-\rho t} = 0$  at the steady state. Moreover, with the specifications presented in Section 2.3.1. within in the main text, Eq. (A.3) in Appendix A gives us  $g_\mu = \rho - \frac{\delta v}{\alpha} + \delta L_{RD}^0 \left( \frac{v - \alpha}{\alpha} \right)$ . Thus, we can see that  $g_\mu + g_A^0 - \rho$  is negative if and only if  $L_{RD}^0$  is lower than one.

The second transversality condition is  $\lim_{t \rightarrow +\infty} v_t S_t^0 e^{-\rho t} = 0$ , which is equivalent to  $\lim_{t \rightarrow +\infty} v_0 e^{g_v t} S_0^0 e^{g_S^0 t} e^{-\rho t} = 0$  at the steady-state. Eq. (A.4) in Appendix A allows us to find that  $g_v + g_S^0 - \rho$  is negative if and only if  $g_S^0 (= g_R^0)$  is negative.

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