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Is an Exhaustible Resource Economy Sustainable?

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Abstract

The paper focuses on two alternative concepts of sustainability dominating the literature: (i) maximum permanently maintainable consumption level (Fisherian income) and (ii) the amount of consumption that leaves total value of wealth intact (Hicksian income). In the context of a pure exhaustible resource economy, the author derives an explicit relationship between the two sustainability criteria and shows that while such an economy is not sustainable in the former sense, it *is* in the latter sense provided social preferences are represented by a logarithmic utility function. The implications of the two concepts for greening of national income are derived. Finally, the paper shows the range of values of the parameters of the model for which the utilitarian optimal path can be close to paths satisfying the alternative sustainability criteria, suggesting that such outcomes are less likely for very poor resource-dependent countries than for the rich ones.

1. Introduction

Concerns about intergenerational fairness in allocation of natural resources have been rising over the past quarter of a century. Indeed, they have given rise to the concept of "sustainability," which was vigorously highlighted by the publication of *Our Common Future*, or Brundtland Report, by the World Commission on Environment and Development (WCED, 1987). Ever since, the concept has been powerfully influencing the thinking about long-run natural resource use and economic development policy and has resulted in an enormous and rapidly growing literature.[1] Nonetheless, sustainability has remained more or less a vague theoretical concept, and, perhaps because of this, has thus far provided little operational guidance (see, e.g., Pezzey, 1989; World Bank, 1997).

In the theoretical literature, two definitions of sustainability seem to have gained prominence. One notion, influenced by the Rawls' (1972) maximin criterion of intergenerational fairness, requires the aggregate consumption level, or the corresponding social utility level, to be permanently maintained at a constant level. This may be termed the "consumption/utility-constant" criterion. This definition was the cornerstone of the pioneering works by Solow (1974) and Hartwick (1977), which led to the well-known Solow–Hartwick sustainability rule. In the context of a closed economy that uses an exhaustible resource input and a reproducible capital with constant technology and population to produce a consumption good, the rule states that reinvesting resource rents to accumulate reproducible capital is a sufficient condition for enjoying a constant consumption path forever. Dixit et al. (1980) subsequently generalized the rule in the form of "zero net aggregate investment" for an economy endowed with multiple diverse capital goods (see also Solow, 1986, 1992). The other notion of

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sustainability, favored mostly by ecological economists, rests on a Hicksian definition of income (Hicks, 1946, ch. XIV): the amount that can be consumed while keeping the value of national wealth, including that of natural capitals, permanently constant (this we may term the "wealth-constant" criterion).[2]

These proposed concepts of sustainability pose a number of important questions that need to be closely examined. What are the implications of the two concepts for the paths of resource use and aggregate consumption/welfare? Perhaps more interestingly, what are their implications for greening of national income accounts? Can a dynamically optimizing economy be sustainable according to one of these concepts but not by the other? Furthermore, one is interested to know how significantly the optimal path of the economy may deviate from the sustainable paths implied by either of the two concepts, and whether such deviations are likely to be more accentuated in the case of poor developing countries than of rich industrial ones.

An examination of these important questions is the main objective of this paper. To bring out the distinction between the two concepts of sustainability in the sharpest and simplest fashion, I focus on the special case of a pure exhaustible resource (Hotelling) economy. Analyzing sustainability in the context of such a specialized economy can also be particularly instructive because it is widely held in the literature that such an economy is intrinsically unsustainable. For example, in an insightful paper analyzing the question "Are optimal paths sustainable?", Heal (2001, pp. 21–2) concludes that "[In] fact it is only the simple Hotelling model that does not produce sustainable paths. In the case of renewable resources, most possible optimal paths are sustainable in the sense of maintaining the resource base."

Section 2 briefly reviews the characteristics of the optimal consumption policy for the simple Hotelling economy and highlights the condition for sustainability of a constant consumption/utility path. Section 3 considers the "wealth-constant" criterion of sustainability and derives an explicit relationship between the conditions for the alternative sustainability concepts. In particular, I show that the condition for sustainability of a constant consumption path entails that the value of the national wealth rises over time at the constant market interest rate—a condition that is, however, never satisfied in a competitive exhaustible resource economy. In that section, I also show that the two definitions of sustainability imply different methods for greening of national income and hence lead to different measures of green NNP. Perhaps strikingly, in section 4 it is shown that an exhaustible resource economy *can* be sustainable in the sense of keeping the value of its resource asset constant, provided its representative citizen has a *logarithmic* utility function. This result sharply distinguishes the concepts of sustainability based on physical units (such as constant consumption flows or constant capital stocks), which are often invoked by ecologists or ecological economists, from its economic value-based concepts (such as constant wealth value). Further, I show that for a certain range of magnitudes of the social discount rate and social aversion to intergenerational inequality, the utilitarian optimal policy may not be too far from the paths implied by either of the two sustainability criteria. However, such possibilities seem less likely for very poor resource-dependent developing nations than for rich ones. Section 5 provides concluding remarks.

2. The Exhaustible Resource Economy Revisited

Consider a purely exhaustible resource economy and, following Hotelling (1931), assume: (i) it has a fully known and fixed initial stock of the resource of size $S_0 > 0$; (ii) the resource can be extracted costlessly; (iii) there is no technological change; (iv) the population size remains constant; and (v) citizens' preferences are identical and presented by the representative consumer's utility function, $u(c)$, which is a twice differentiable, increasing, and strictly concave function of the resource consumption rate (i.e., $u'(c) > 0$, $u''(c) < 0$ for all $c \geq 0$), with $\lim_{c\to 0} u'(c) = +\infty$ and $\lim_{c\to\infty} u'(c) = 0$. The utilitarian social planner uses a social welfare function defined as the discounted sum of the representative consumer's utility flow and her objective is to plan a path of resource extraction and consumption that maximizes this social welfare function given the resource stock constraint. Formally, she plans to

$$\max_{c(t)} \int_0^\infty e^{-\rho t} u(c(t))dt, \tag{1a}$$

$$\text{s.t.} \quad \dot{S}(t) = -c(t) \geq 0, \quad S(t) \geq 0, \quad S_0 \text{ (given)} \tag{1b}$$

where $\rho > 0$ is the social time preference rate, assumed constant. Notice that the social welfare function (1a) is in contrast to Rawls' maximin criterion that implies a different objective function and underlies his constant consumption path result.

Assuming the constraint $S(t) > 0$ holds, the current-value Hamiltonian of this problem, which does not depend directly on S, is

$$H(c(t), \lambda(t)) = u(c(t)) - \lambda(t)c(t), \tag{2}$$

where $\lambda(t)$ is the shadow price of the resource stock in utility units. The first-order conditions for an interior optimal path, ensured by the Inada conditions on the utility function, are

$$\frac{\partial H}{\partial c} = u'(c(t)) - \lambda(t) = 0, \tag{3}$$

$$-\frac{\partial H}{\partial S} = 0 = \dot{\lambda}(t) - \rho\lambda(t), \tag{4}$$

and the transversality condition is

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)S(t) = 0. \tag{5}$$

Differentiating (3) with respect to time, using (4), and denoting the elasticity of marginal utility of consumption by $\eta(c) = -cu''(c)/u'(c)$, the optimal consumption path is characterized by the familiar condition

$$\frac{\dot{c}(t)}{c(t)} = -\frac{\rho}{\eta(c)}. \tag{6}$$

It is immediate from (6) that, in general, the optimal policy for an exhaustible resource economy does *not* sustain a positive constant flow of consumption and hence utility. In fact, for the class of isoelastic utility function, $u(c) = c^{1-\eta}/1 - \eta$, $0 < \eta < \infty$, along the optimal path, the consumption level declines exponentially over time at the constant rate of ρ/η. That is:

$$c(t) = c(0)e^{-\frac{\rho}{\eta}t}, \tag{7}$$

where from the resource stock constraint $\int_0^\infty c(t)dt = S_0$ and from (7) one obtains $c(0) = (\rho/\eta)S_0$, so that (7) can be rewritten as

$$c(t) = \frac{\rho}{\eta} S_0 e^{-\frac{\rho}{\eta}t}, \quad \forall t \in [0, \infty). \tag{8}$$

It is important to note that for an optimal policy to exist it is necessary that $\rho > 0$. In particular, in the limiting cases of no utility discounting, $\rho = 0$, or a pure egalitarian social welfare function where $\eta \to \infty$, a positive constant consumption path ($c(t) = \bar{c} > 0, \forall t \geq 0$), as implied by (6) for a general utility function, $u(c)$, cannot be sustained permanently by an exhaustible resource economy. On the other hand, the constant zero consumption path ($c(t) = 0, \forall t \geq 0$) implied by (8) for these limiting cases and isoelastic utility function is evidently not optimal.

It is worth in passing to note how changes in the two ethical parameters ρ and η affect the optimal consumption path. As should be expected, all else equal, the optimal initial (or the current generation's) consumption level will be higher, and the rate of consumption decline will be faster, the larger is the social rate of time preference, ρ. Given the resource stock constraint, $\int_0^\infty c(t)dt = S_0$, this implies that a higher social rate of time preference would make the earlier generations (those indexed by $0 \leq t < T = \eta/\rho$) better off at the expense of later generations (those indexed by $t > T$). Thus, for instance, for values of $\rho = 0.01$ and $\eta = 2.5$,[3] a small increase in ρ would benefit the generations living in the first two and half centuries at the expense of all those living later on. Just the reverse of these effects holds for an increase in η, which reflects the degree of social aversion to intergenerational inequality or, equivalently, a greater degree of intergenerational egalitarianism.

It will prove useful if we express the optimal level at any time t as a function of the remaining resource stock, $S(t) = \int_t^\infty c(\tau)d\tau$, at that time. Using (7), it is easy to verify that

$$c(t) = \frac{\rho}{\eta} S(t). \tag{8a}$$

That, by the "constant-consumption path" criterion, an exhaustible resource economy is unsustainable is obvious, since a constant (positive) consumption path is simply not feasible in such an economy with infinite horizon. Nonetheless, to facilitate subsequent analysis, it will be useful to formally confirm this by invoking a general sustainability condition established in Farzin (2002, Propositions 1 and 2): namely, a necessary and sufficient condition for a dynamically optimizing economy to sustain a constant consumption/utility path is the *stationarity of the current-value Hamiltonian* along the optimal path; i.e., $dH/dt = 0, \forall t \geq 0$.[4] The intuition behind this general result derives from a basic insight from the classic paper by Weitzman (1976). That is, in a dynamically optimizing economy, along the optimal path, the current-value Hamiltonian *at time t*, $H(t)$, is related to the optimal utilitarian welfare/consumption *path*, $u(c^*(\tau))$, $\tau \in (t, \infty)$, according to the following relationship[5]:

$$\int_t^\infty e^{-\rho(\tau-t)} H(t) d\tau = \frac{H(t)}{\rho} = \int_t^\infty e^{-\rho(\tau-t)} u(c^*(\tau)) d\tau,$$

where $H(t)$ may be interpreted as the imputed social return on wealth at time t, measured in utils. Differentiating (2) with respect to t and using (3) and (4), we have

$$\frac{dH}{dt} = -\rho\lambda(t)c(t) < 0, \quad \forall t \geq 0. \tag{9}$$

Thus, for a purely exhaustible resource economy, the stationarity condition for sustainability is never met. This confirms and generalizes the nonexistence of a sustainable (positive) constant consumption path, as reflected in (7) (or (8)) for the special case of an isoelastic utility function.

3. Sustainability and Resource Asset Value

As is well known, the Hicksian definition of income requires that the *value of wealth* be kept intact.[6] Accordingly, in the green accounting literature, "keeping the value of wealth intact" has often been interpreted as a sustainability criterion.[7] Perhaps strikingly, in this section I show that, although unsustainable in the sense of maintaining a constant positive consumption flow, an exhaustible resource economy can under certain conditions be sustainable according to the criterion of maintaining a constant asset value of the resource stock. To see this, we need to consider the competitive market valuation of the resource stock. As is familiar, the socially optimal consumption policy characterized in section 2 can, under some idealized conditions, be decentralized through perfectly competitive resource markets. Since such a decentralization proves helpful in bringing out the main point to be made in this section, we may as well undertake it.

Formally, under the assumptions that (i) the resource is owned by identical competitive firms, and (ii) the representative firm has perfect foresight (or, equivalently, there is a complete set of forward markets), the representative firm chooses a path of resource extraction, $E(t) \geq 0$, so as to maximize the present value of its cash flow; i.e.,

$$\max_{E(t)} \int_0^\infty e^{-rt} p(t)E(t)dt, \tag{10}$$

$$\text{s.t.} \quad \dot{S} = -E(t) \geq 0, \quad S(t) \geq 0, \quad S(0) = S_0 > 0 \text{ (given)}$$

where $r > 0$ is the resource owners' discount rate (equal to the competitive interest rate), assumed to be constant,[8] and $p(t)$ is the competitive resource price at time t. The current-value Hamiltonian of this problem, denoted by, H_p, is

$$H_p(E(t), S(t), \mu(t)) = p(t)E(t) - \mu(t)E(t) = [p(t) - \mu(t)]E(t) \tag{11}$$

This yields the following first-order conditions for an interior optimum:

$$\frac{\partial H_p}{\partial E} = p(t) - \mu(t) = 0, \tag{11a}$$

$$\frac{\partial H_p}{\partial S} = \dot{\mu}(t) - r\mu(t) = 0, \tag{11b}$$

and the transversality condition

$$\lim_{t\to\infty} e^{-rt}\mu(t)S(t) = 0. \tag{11c}$$

To decentralize the socially optimal consumption path by the competitive equilibrium prices, $p(t)$, we note that the market clearing condition $c(t) = E(t)$ must be met at any time t. Further, using utility units (*utils*) as a *numéraire* in the decentralized economy, the competitive equilibrium price $p(t)$ should equal the shadow price of the resource, $\lambda(t)$, in utils terms, i.e., $\mu(t) = \lambda(t)$, as should the rate of interest (in utils terms) equal the utility discount rate,[9] i.e., $r = \rho$. From (3), (4), (11a), and (11b) one then has

$$u'(c(t)) = p(t) = \mu(t), \tag{12}$$

$$p(t) = p(0)e^{rt}. \tag{13}$$

Equation (13) is, of course, the familiar Hotelling rule that along an intertemporally efficient extraction path the resource's flow price (equal to resource stock price) should rise at the interest rate. The competitive equilibrium extraction/consumption path is determined by (12) and (13) according to[10]

$$c(t) = \overset{-1}{u'}(p(t)) = D(p(0)e^{rt}) \tag{14}$$

where $D(.) \equiv \overset{-1}{u'}(.)$ is the resource demand function. Thus, the determination of the efficient extraction/consumption path hinges on determining the initial resource price $p(0)$ optimally. Given the assumption of perfect foresight and the preferences of the representative consumer, $u(c)$, the initial price is determined efficiently and uniquely from the resource stock constraint

$$\int_0^\infty c(t)dt = \int_0^\infty D(p(0)e^{rt})dt = S_0. \tag{15}$$

It is worth noting in passing that the representative firm's value of Hamiltonian, which also measures the NNP,[11] *vanishes*; i.e., $H_p(E(t), \mu(t)) = p(t)E(t) - \mu(t)E(t) = 0$. This simply states that once the resource stock depreciation (which is the value of resources extracted and consumed at any time, $\mu(t)E(t)$) is deducted from the gross national product (which is the value of resource sales, $p(t)E(t)$), then the NNP, and hence the only sustainable level of consumption, is *zero*; again confirming that there is no *positive* sustainable consumption rate for a purely exhaustible resource economy.

Next, note that the competitive market value of the resource stock (or the economy's wealth) is the capitalized value of the cash flow from the resource sales, which using (13) can be written as

$$V(t) = \int_t^\infty e^{-r(\tau - t)} p(\tau)E(\tau)d\tau = p(t)S(t). \tag{16}$$

Differentiating (16) with respect to time and using (10), (13), and (16) gives the change in the value of the resource asset at any time as

$$\dot{V}(t) = rV(t) - p(t)E(t), \tag{17a}$$

$$= p(t)[rS(t) - E(t)]. \tag{17b}$$

To decentralize the socially optimal policy on the consumption side, we note that the representative household's wealth is $V(t)$, so that its budget constraint is given by (17a), with $E(t)$ replaced by $c(t)$. Thus, the household solves the problem of

$$\max_{c(t)} \int_0^\infty u(c(t))e^{-\rho t}dt \quad \text{s.t.} \quad V(t) = rV(t) - p(t)c(t), \quad V(0) = p(0)S_0.$$

However, by differentiating (16) with respect to time, one has $\dot{V}(t) = \dot{p}(t)S(t) + p(t)\dot{S}(t)$, which upon substituting this in the household's budget constraint and recalling from (13) that $\dot{p} = rp(t)$, so that $\dot{p}S(t) = rp(t)S(t) = rV(t)$, we obtain $\dot{S}(t) = -c(t)$. That is, the representative household solves the very same problem as the social planner does (namely the problem given by (1a) and (1b)). This completes the characterization of the competitive (efficient) equilibrium supporting the optimal consumption path obtained in section 2.

Equation (17a) simply states that the net change in the asset value of the resource stock is equal to the imputed interest income from the asset minus the asset value depreciation. Equation (17b) relates the change in the asset value to behavior of the resource extraction path and shows that at any time the asset value appreciates or depreciates according to whether the extraction rate as a percentage of the remaining resource stock is lower or higher than the interest rate.

Two interesting points emerge from (17a) and (17b). First, we have

$$\dot{V}(t)=0 \quad \Leftrightarrow \quad rV(t)=p(t)E(t), \tag{18a}$$

which provides what we may call the asset-value sustainability rule; namely, in order to keep the asset value of the resource stock constant over time the value of resources extracted and *consumed* (i.e., the economy's GNP as measured conventionally) should always equal the imputed interest on the resource asset. Note that while this rule is precisely in keeping with the Hicksian definition of income, it may be interpreted as dual to Solow–Hartwick's sustainability rule, which states that in order to sustain a constant consumption flow indefinitely the rents from resource extraction ($p(t)E(t)$) should always be *reinvested*.

Second, and equivalently, the asset-value sustainability rule holds if and only if the extraction rate as a percentage of the remaining resource stock (the extraction-reserves ratio) always equals the interest rate:

$$\dot{V}(t)=0 \quad \Leftrightarrow \quad E(t)=rS(t), \tag{18b}$$

which in turn implies, from (10), that the extraction rate should follow the path $E(t) = E(0)e^{-rt}$, $\forall t \geq 0$ with $E(0) = D(p(0))$ and $p(0)$ determined from (15).

So, one may express the asset-value sustainability rule also in the form of an extraction rule stating that: in order to keep the asset value of the resource stock intact, the extraction rate should decline over time at a rate equal to the interest rate. This "r-percent" *extraction* rule may be interpreted as dual to the Hotelling's "r-percent" *price* rule.

We can now explicitly see the relationship between the two sustainability concepts by relating the change over time in the optimal current-value Hamiltonian to the change in the asset value of the resource stock. This relationship can be stated in the form of the following proposition.

PROPOSITION 1. *For an exhaustible resource economy to be sustainable in the sense of maintaining a constant consumption (utility) flow indefinitely, the competitive asset value of the resource stock should* grow *over time at the market rate of interest.*

PROOF. Recalling that in the decentralized competitive market equilibrium, $\lambda(t) = p(t)$, $\rho = r$ (in utils terms), $c(t) = E(t) = -\dot{S}(t)$, and using (17a) and (13) yields

$$\frac{dH(t)}{dt}=-rp(t)E(t)=r[\dot{V}(t)-rV(t)]. \tag{19}$$

From (19) it follows that

$$\frac{dH(t)}{dt}=0 \quad \Leftrightarrow \quad \dot{V}(t)=rV(t), \quad \forall (t)\geq 0. \tag{20}$$

□

What is appealing about relation (20) is that it translates the stationarity of the current-value Hamiltonian as a necessary and sufficient condition for sustainability into a simple testable condition on the competitive market value of the resource asset.

However, noting that along the optimal extraction path $E(t) \geq 0$, it is obvious from (19) that $dH(t)/dt < 0$, $\forall t \geq 0$, implying that for such an economy there is *no positive* permanently sustainable level of consumption.[12]

Proposition 1 is interesting in two respects. First, rather strikingly, it states that intergenerational equity in terms of consumption allocation is not only compatible with future generations being successively and increasingly favored over earlier generations in terms of the asset value allocation, but also requires this. Second, it sharpens the difference between the two concepts of sustainability and their implications for greening of national income accounts. For it cautions us that it is the zero *value of the change* in the resource stock (here $p(t)S(t) = -p(t)E(t)$, or, more generally, zero net aggregate investment along the optimal path) that is the necessary and sufficient condition for *consumption sustainability* and *not*, as is sometimes mistakenly held, the zero *change in the value* of the resource stock (here $\dot{V}(t) = 0\ \forall t$, or, generally, the condition of keeping the value of aggregate capital intact). The importance of this distinction is better appreciated once it is noted that *in general* for an economy with many diverse (exhaustible and reproducible) capital stocks, the *change in the value* of aggregate capital is

$$\frac{d}{dt}\sum_j \lambda_j(t)S_j(t) = \sum_j \dot{\lambda}_j(t)S_j(t) + \sum_j \lambda_j(t)\dot{S}_j(t),$$

where $S_j(t)$, $j = 1, 2, \ldots, m$, denotes the stock quantity of the jth capital good at time t and $\lambda_j(t)$ its shadow price. It is then clear that the two measures of change in stock value will differ except in the special case of the steady state where the λ_j are all constants, implying that the net capital gain (loss), $\Sigma_j \dot{\lambda}_j(t)S_j(t)$, will be zero and hence that the change in the value of a stock will consist only of the value of the net change in the physical stock; i.e.,

$$\frac{d}{dt}\sum_j \lambda_j(t)S_j(t) = \sum_j \lambda_j(t)\dot{S}_j(t) \quad \forall t.$$

Accordingly, in attempting to correct the conventional measures of NNP to reflect sustainability, it is important to be clear about the specific concept of sustainability that one has in mind. Thus, to reflect *consumption sustainability*, it will be incorrect, outside of the steady state, to include the *change in the value* of aggregate capital

$$\frac{d}{dt}\sum_j \lambda_j(t)S_j(t) = \sum_j \dot{\lambda}_j(t)S_j(t) + \sum_j \lambda_j(t)\dot{S}_j(t)$$

as, for example, Repetto et al. (1989) do, since only the *value of the change* in capital stocks, $\Sigma_j \lambda_j(t)\dot{S}_j(t)$, should be included. On the other hand, to reflect sustainability in the sense of Hicksian income, it will be incorrect, outside of the steady state, not to account for the net capital gain (loss), $\Sigma_j \dot{\lambda}_j(t)S_j(t)$. For the special case of a purely exhaustible resource economy, the greening of NNP according to the former definition of sustainability implies $\text{NNP}(t) = p(t)E(t) + p(t)\dot{S}(t) = 0, \forall t \geq 0$, whereas greening it according to the latter definition implies $\hat{\text{NNP}} = p(t)E(t) + \dot{V}(t) = p(t)E(t) + p(t)\dot{S}(t) + \dot{p}(t)S(t) = \dot{p}(t)S(t) = rV(t) > 0$, which obviously always exceeds the former measure of green NNP.

4. Sustainability and Preferences

Although Rawls' maximin criterion may be viewed as an ethical justification for relying on a constant-consumption path as a concept of sustainability, the ethical justification for keeping the value of wealth constant is much less clear. Despite this, one would still be interested in an answer to the question: If instead of the stationarity of the optimal current-value Hamiltonian ($dH(t)/dt = 0, \forall t \geq 0$), implying a constant maximum consumption (utility) level, one were to adopt the constancy of the asset value of the resource ($\dot{V}(t) = 0, \forall t \geq 0$) as an alternative criterion of sustainability, what would be its implications? After all, as Koopmans (1967) originally argued, ethical values should be judged in terms of sensibleness of their implications. One would then be interested in comparing the implications of the criterion of keeping the asset value of the resource intact with those of the pure utilitarian principle explored earlier.

Now, by (18a), an immediate implication of this alternative criterion is that every generation would enjoy an income and consumption flow of the *same market value*, equal to the imputed (*constant*) flow of the interest income from the resource asset; that is

$$p(t)E(t) = p(0)E(0) = rV(0) = rV(t), \quad \forall t \geq 0. \tag{21}$$

Together with the consideration that the asset value of the resource stock would also be maintained intact at its initial level, this criterion may not seem intergenerationally too unfair. Interestingly, however, the possibility of sustainability in this sense turns out to depend critically on society's preferences and, in particular on the value of the parameter η, which reflects the society's degree of preferences for intergenerational egalitarianism when the preferences are presented by the isoelastic utility function. More specifically, one can state the following proposition.

PROPOSITION 2. *In order for an exhaustible resource economy to be sustainable in the sense of maintaining the asset value of the resource stock intact (i.e., $\dot{V}(t) = 0, \forall t \geq 0$), its representative citizen must have a logarithmic utility function, $u(c) = \ln c$, or, equivalently, extract and consume the resource at rates that decline over time at the interest rate (i.e., according to $c(t) = E(t) = rS_0e^{-rt}$).*[13]

PROOF. Recall that in the competitive equilibrium $c(t) = E(t)$ and $\rho = r$, so that, by substituting for these in (8a), the optimal extraction (consumption) path is governed by $E(t) = (r/\eta)S(t)$. Using this in (17b) yields

$$\dot{V}(t) = (\eta - 1)p(t)E(t), \tag{22}$$

so that in general along the optimal path

$$\dot{V}(t) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0, \quad \forall t \geq 0 \quad \text{as} \quad \eta \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 1. \tag{23}$$

For $\eta = 1$ the utility function $u(c) = c^{1-\eta}/(1 - \eta)$, $\eta > 0$, takes the logarithmic form of $u(c) = \ln c$, so the optimal (or, equivalently, the efficient market) extraction/consumption path takes the form given in (18b). □

Thus, the logarithmic utility function acts like a benchmark case,[14] with which we can compare the implications of optimal policies in other cases. For any utility function specified by a value of $0 < \eta < 1$, the optimal policy implies an exacerbated intergenerational inequity in that it favors earlier generations both in terms of consumption allocation and the value of inherited (resource) wealth more strongly that would be the case for $\eta = 1$. Such allocations may arguably be deemed less desirable than that resulting from the logarithmic utility function, which maintains the value of wealth intact and results in a relatively less inequitable consumption path. On the other hand, for any utility function specified by a value of $\eta > 1$, while the optimal policy is still biased in favor of earlier generations in terms of distribution of consumption flow (albeit to a lesser extent than when $\eta = 1$), it favors later generations in terms of the value of resource wealth. If one believes that, from an intergenerational equity perspective, the ideal allocation is one, that sustains a constant maximum flow of consumption, then, all else equal, the optimal paths associated with larger values of $\eta > 1$ should be judged preferable to that arising from the logarithmic utility. If, however, intergenerational fairness is judged based on keeping the value of wealth intact, then the allocation associated with the logarithmic utility function may be considered most equitable and hence preferred.

How significantly the optimal paths arising from the two sustainability criteria differ from one another depends crucially on the magnitude of the social discount rate and the extent of social aversion to intergenerational inequality. To see this more clearly, use (23a), (16), and $E(t) = (r/\eta)S(t)$ to obtain

$$\frac{\dot{V}(t)}{V(t)} = r\left(1 - \frac{1}{\eta}\right). \tag{24}$$

Accordingly, for any given discount rate $r > 0$, as $\eta \rightarrow 1$ the optimal path moves toward the policy that satisfies the "keep the value of wealth intact, $\dot{V}(t) = 0$" criterion, whereas as $\eta \rightarrow \infty$ it moves toward the policy that fulfills the "stationarity, $\dot{H}(t) = 0$" criterion of sustainability as given in (20). Of course, for any value of $0 < \eta < \infty$, the smaller the social discount rate, $r = \rho > 0$, the smaller will be the difference between the policies that satisfy the alternative sustainability conditions, so that for sufficiently small values of $r > 0$ and sufficiently large values of η the optimal policy becomes almost compatible with *both* concepts of sustainability.

Given the isoelastic utility function, one may then be curious to know whether individuals' preferences are actually represented by anything close to a logarithmic utility function. The available empirical evidence suggests values of η in the range of 1.5 to 2.5; implying that the optimal policy tends to be biased in favor of earlier generations in terms of consumption flow but biased in favor of later generations in terms of resource asset value. However, since a purely exhaustible resource economy is likely to be relatively poor, it may be argued that these estimated values of η, which are derived from models of household saving behavior in rich industrial countries, overestimate those applicable to the former economies. In that case, the utility function may be closer to a logarithmic one and the optimal path not far from the path which keeps the value of the resource wealth constant, although one should also bear in mind that the social time preference is likely to be higher for poor resource-based economies than for mature industrial nations. For example, for $\eta = 1.125$ and a conservatively large value of $r = 0.05$, the asset value of the resource stock will increase steadily over time at the rather insignificant rate of half a percent ($\dot{V}(t)/V(t) = 0.005$) along the optimal path. The optimal policy would then tend to be close to the path that keeps the value of capital intact, although in allocating the consumption flow it would strongly favor earlier generations. Obviously, if the relevant value of η for poor resource-dependent countries turns out to be less than unity, say, for instance $\eta = 0.75$, which is by no means an implausible figure, then the implied optimal path would fail to satisfy either of the sustainability criteria as it would allocate both the consumption flow and the wealth in favor of earlier generations, with the former declining at the constant rate of 6.67% and the latter at the rate of 1.67%. On the other hand, if we take $\eta = 2.5$ and $r = 0.01$ as plausible values for a rich resource-based country, then the implied policy will be somewhere in between the paths satisfying the alternative sustainability criteria; although probably tending to be closer to the path satisfying the stationarity condition, for then $\dot{V}(t)/V(t) = 0.006$ and $\dot{c}(t)/c(t) = -0.004$. These numerical stipulations may be taken to suggest that for very poor resource-dependent developing nations the optimal utilitarian policy is more likely to deviate from the policies defined by either of the two sustainability concepts that would be the case for rich nations.

5. Conclusion

I have argued that whether an economy is sustainable or not depends crucially on the specific concept of sustainability adopted. To sharpen this argument, I have focused on a purely exhaustible resource economy and examined the possibility of its sustainability according to two alternative concepts: (a) permanently maintaining a constant-consumption (utility) path, and (b) keeping the value of national wealth intact. I have shown that sustainability in the latter sense requires that the value of resources extracted and consumed should always be equal to the imputed interest income from the resource asset, or, equivalently, the extraction rate should decline over time at a rate equal to the market rate of interest. What seems appealing about these equivalent conditions are that: (i) they are empirically easily testable, and (ii) the former may be interpreted as dual to the Solow–Hartwick rule of reinvesting resource rents to sustain a constant consumption level, and the latter as dual to the Hotelling "*r*-percent" price rule.

Further, I have explicitly shown the relationship between the two sustainability criteria, and particularly that the sustainability of consumption flow requires that the resource asset value always appreciates at the market interest rate. Accordingly, while sustainability in the sense of constant consumption flow is not possible for an exhaustible resource economy, sustainability in the sense of keeping the value of national wealth intact *is*, provided preferences are presented by a *logarithmic* utility function. More generally, the relationship between the two sustainability criteria turns out to depend crucially on the magnitude of the social discount rate and the degree of social aversion to intergenerational inequality. Interestingly, for plausibly small values of the former and reasonably large values of the latter, the implied optimal path can be quite close to paths implied by alternative concepts of sustainability. Much in the spirit of Heal's (2001) conclusion, this finding may lessen to some degree concerns about alternative concepts of sustainability and about sustainability versus optimality. However, and perhaps ironically, such an outcome is more likely for the rich resource-based economies than for the very poor ones.

It is important to be cautious in interpreting the conclusions reached here based on a simplified model of a purely exhaustible resource economy. For one thing, augmenting such an economy with services of renewable natural resources (e.g., fisheries, forests, land and water sources, and renewable energy resources) and human-made capitals (e.g., manufactured capital, human capital, and social capital), provided their utilization rates remain within their respective regenerative capacities or reproduction limits, not only can ease the constraint of resource exhaustibility on sustaining reasonably high living standards, it can also narrow down the gap between the utilitarian optimal and sustainable development paths. On the other hand, steady and high population growth rates, as have been experienced by many poor developing countries, can act in the opposite direction, unless technological advances continue at sufficiently high rates to increase the productivities of natural and human-made capitals and enhance the possibilities of substituting the latter types of capitals for the former ones. What seems fundamental in this process of technological advancement to offset the effect of population growth, and poses a principal challenge for development policy, is the role of investment to transform a given population from its primitive form of *raw labor* with low productivity to its higher form of *human capital* (knowledge stock) with fantastically higher productivity.

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Notes

1. For a survey of the literature, see, for example, Aronsson et al. (1997). On the concept of sustainable development and an axiomatic approach to it, see Chichilnisky (1993, 1996). For a review of the theory of sustainability and green national accounting, see the special issue of *Environment and Development Economics* (February and May 2000).
2. For a theoretical justification of the concepts of sustainability, see Asheim et al. (2001).
3. See Mirrlees and Stern (1972), Stern (1977), and Arrow et al. (1996) for an excellent account of plausible values for these parameters.
4. As shown in Farzin (2002), the stationarity condition holds generally for economies characterized by an autonomous dynamic optimal control problem as well as for cases where the discount rate is time-dependent provided it does not decline too fast with time. A hyperbolic discount rate function is an example of the latter.
5. Since $u(c)$ is a single-valued, monotonic function of c, sustainability can be equivalently defined in terms of a constant utility or consumption flow. In fact, Weitzman assumed a linear utility function of the form $u(c(t)) = c(t)$.
6. Note that provided the real rate of return on wealth remains constant over time, this would imply a permanently constant consumption flow, equal to return on wealth. However, in general, this need not be the case.
7. Note that ecological economists have often favored this definition of sustainability by misinterpreting it to mean that it requires keeping the *physical* (as opposed to economic value of) stocks of natural capital intact.
8. Without loss of generality, I have normalized the number of resource owners to unity so that the representative resource owner has the same stock, s_0, as in the socially optimal case.
9. Note that using the consumption good (which is measured in the same units as the exhaustible resource) as a *numéraire*, the spot price of the resource is 1 in this one-good frictionless economy, and the real interest rate is zero. Denoting by $v(t)$ the marginal utility of income (wealth), we must then have $\lambda(t) = v(t)\mu(t)$, where $\mu(t)$ is constant in this economy and $v(t)$ therefore grows at the rate ρ.
10. That in utility units the interest rate equals the social discount rate can be also seen more formally by noting from (12) and (13) that $r = \dot{p}/p = -\eta(c)\dot{c}/c = \rho$. Also, notice that $E(\tau)$ cannot be determined from the optimization problem (11) of the competitive firm. It is entirely determined by the demand at each point in time.
11. Note that, by normalization, the representative firm's maximized profit at any time, $H_p(t)$, represents the economy's NNP.
12. Note that while the stationarity of the current-value Hamiltonian is a necessary and sufficient condition for sustainability in the sense of a constant consumption (utility) path, sustainability in the sense of keeping the value of wealth intact implies that the current-value Hamiltonian monotonically declines over time according to $dH(t)/dt = -r^2V(t)$ (see (19)).
13. Interestingly, Heal (2001) shows that a pure exhaustible resource economy can be sustainable in the ecological sense of maintaining the *physical* resource stock base, or a part of it, intact provided the utility function depends also on the remaining stock of the resource, $u(c(t), S(t))$; i.e., when the resource stock itself is also a source of value.
14. Notice further that, for the case of an isoelastic utility function, the market value of consumption, using (14), is $y(t) \equiv p(t)c(t) = [p(t)]^{1-1/\eta}$. Differentiating with respect to time and using (13) gives $\dot{y}(t) = r(1 - 1/\eta)[p(t)]^{1-1/\eta}$; so that

$$\dot{y}(t)\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}0 \quad \text{as} \quad \eta\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}1.$$

Accordingly, for a logarithmic utility function, the optimal policy will maintain both the values of consumption and the resource asset intact, respectively at $y(t) = y(0) = p(0)E(0) = 1$ and $V(t) = V(0) = p(0)S_0 = 1/r$. Notice, however, that for $\eta \neq 1$ the bias of intergenerational inequality is in the same direction both for asset value and consumption value allocations.