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An analysis of the multi-product newsboy problem with a budget constraint

Layek L. Abdel-Malek[a,*], Roberto Montanari[b]

[a]Department of Industrial and Manufacturing Engineering, New Jersey Institute of Technology, Newark, NJ 07102, USA
[b]Dipartimento di Ingegneria Industriale, Università degli Studi di Parma, Viale delle Scienze, 43100 Parma, Italy

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Abstract

This is a sequel of an earlier paper entitled ‘‘Exact, approximate, and generic iterative models for the multi-product newsboy problem with budget constraint’’ (Abdel-Malek et al., 2004) that appeared in this journal. Motivated by Lau and Lau's (Eur. J. Oper. Res., 94 (1996) 29) observation where infeasible ordering quantities (negative) were obtained when applying existing methods, the extension here examines the solution space of the problem in order to provide the necessary insight into this phenomenon. The resulting insight shows that the solution space can be divided into three regions that are marked by two distinct thresholds. The first region is where the budget is large and the solution is the same as the unconstrained problem. The second region is where the budget is medium and the constraint is binding, however the newsboy can order all the products on the list. The third region is where the budget is very tight and if the non-negativity constraints are relaxed negative order quantities may be obtained, and therefore some products have to be deleted from the original list. We show how the values of the thresholds that divide the regions are computed and extend the previous methods, when necessary, to cover each of the three-solution's domains in order to determine the optimum order quantity for the various products. Numerical examples are given to illustrate the application of the developed procedures.

Keywords: Inventory control; Newsboy problem; Nonlinear optimization; Budget constraint

1. Introduction

The newsboy problem, also known as the single period stochastic inventory model, is found to be a suitable tool for decision-making regarding stocking issues in today's supply chains. Motivated by the interest of the community, in an earlier publication entitled ‘‘Exact, approximate, and generic iterative models for the multi-product newsboy problem with budget constraint’’ (Abdel-Malek et al., 2004), we developed models to determine the optimum lot size for the

*Corresponding author. Tel.: +1 973 596 3648;
fax: +1 973 596 3652.
E-mail address: malek@njit.edu (L.L. Abdel-Malek).

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capacitated situation. The model formulation used is as follows:

$$\text{Minimize } E = \sum_{\tau=1}^{N} \Big[c_\tau x_\tau + h_\tau \int_0^{x_\tau} (x_\tau - D_\tau) f_\tau(D_\tau) \times \mathrm{d}D_\tau + v_\tau \int_{x_\tau}^{\infty} (D_\tau - x_\tau) \times f_\tau(D_\tau)\,\mathrm{d}D_\tau \Big], \tag{1}$$

subject to

$$\sum_{\tau=1}^{N} c_\tau x_\tau \leqslant B_G, \tag{2}$$

where E is the expected cost; N the total number of items; τ the item index; c_τ the cost per unit of product τ; x_τ the amount to be ordered of item τ which is a decision variable; h_τ the cost incurred per item τ for leftovers at the end of the specified period; D_τ the random variable of item's τ demand; $f_\tau(D_\tau)$ probability density function of demand for item τ; v_τ the cost of revenue loss per unit of product τ, and B_G the available budget.

This model is based on the classical one that was developed originally by Hadley and Whitin (1963). The solution is obtained using the Lagrangian method and it is as follows:

$$F(x_\tau^{**}) = \frac{v_\tau - (1+\lambda)c_\tau}{v_\tau + h_\tau}, \tag{3}$$

where x_τ^{**} is the optimal solution under budget constraint; $F(\cdot)$ the cumulative distribution function (CDF), and λ the Lagrangian multiplier.

As can be seen, this approach first ignores the budget constraint and finds the optimum values of the lot size for each product. Then, these values are plugged into the budget to see if they satisfy its constraint, otherwise the inequality constraint is set to equality and the Lagrangian approach is used. It should be also noted that Hadley and Whitin's model relaxes the non-negativity constraints of the order quantities. In fact most of the existing Lagrangian based models regarding the capacitated newsboy problem do not pay too much attention to the lower bounds of the order quantities (non-negativity constraints), see for example, Abdel-Malek et al. (2004), Ben-Daya and Abdul (1993), Erlebacher (2000), Gallego and Moon (1993), Khouja (1999), Moon and Silver (2000), and Vairaktarakis (2000). One should mention that if, when tackling a three-product problem, the non-negativity constraints are not relaxed and Kuhn–Tucker conditions are applied, the number of nonlinear equations to be solved simultaneously is more than 20. This could be one of the reasons that most existing models relax the lower bounds to make the problem tractable. Nevertheless, in doing so and as Lau and Lau (1995, 1996) were among the first to observe, this could lead to infeasible order quantities (negative) for some of the considered products. While modeling a case study for a large bakery, Lau and Lau experienced this phenomenon and offered a conceptual explanation for its occurrence.

To address the aforementioned situation, we extend the previous paper by Abdel-Malek et al. to efficiently solve the capacitated multi-product newsboy problem (CMPNP) and to help the decision-maker in recognizing the implications of the available budget. Hence, the decision-maker can avoid infeasible (negative) order quantities by deleting products from further consideration when the constraint is too tight. Additionally, this extension provides a means to conduct sensitivity analysis, when necessary, for increasing the budget to include desired products if the initial amount does not allow for their consideration.

Our taxonomy in this article is as follows. After this introduction, in Section 2, we present the problem and give some insights into its solution space. Section 3 follows where a general approach for solving the CMPNP and the necessary proofs are shown. Afterwards, Section 4 shows numerical examples to illustrate the application of these methods. Finally, Section 5 presents the conclusion of this paper.

2. The problem

As mentioned in the introduction many of the existing methodologies for solving the newsboy problem with budget constraint have used the Lagrangian approach that was originally implemented by Hadley and Whitin (1963) as the underpinning for further developments. The problem with these approaches is that they do not consider the lower bound of the order quantities. Unlike in situations where the demand is deterministic, relaxing the lower bounds of items' order quantities in stochastic environments can lead to infeasible (negative) order quantities as Lau and Lau (1996) experienced in one of their numerical examples. (One can prove that for the multi-product constrained problem with deterministic demand that relaxing the non-negativity constraints is of no consequence.) Therefore, to obtain the optimum order quantities for each of the considered items, their lower bounds should be imposed (non-negativity constraints) and the Kuhn–Tucker conditions must be observed. However, as stated before, adding the lower bound constraints makes the problem intractable. Addressing this issue is the crux of the paper.

To illustrate the solution space for a two-item problem, we refer to Fig. 1. Fig. 1a is a pictorial of a case where the available budget allows the ordering of the two items, and using the Lagrangian approach while relaxing the non-negativity constraints will yield the optimum solution. On the other hand, Fig. 1b exhibits a situation where the budget is too tight. In such a situation relaxing the non-negativity constraint and only imposing the Lagrangian multiplier for the budget constraint could lead to negative values for one of the decision variables (see Example 4.1 for further numerical illustration).

As stated previously, the main contribution of this extension is in providing an efficient taxonomy for solving the multi-product newsboy problem that is able to discern the different degrees of tightness of the budget constraint and accordingly select the proper approach to find the order quantities for each of the considered products. The following section shows our approach and its rationale.

3. Approach

Solving the CMPNP more efficiently requires first an insight into its solution space, that is how tight the budget constraint is. The developed approach here divides the solution space into three categories depending on the available budget, number of products, their unit costs, and their demand probability distribution functions. As shown in Fig. 2, there are three degrees of budget tightness that are separated by two thresholds.

The first range is defined by a budget which is large enough that if the constraint is relaxed, the resulting order quantities from obtaining the critical points of Eq. (1) (global optimum) do not violate the available amount of money, that is the budget constraint is unbinding. The second range occurs when the budget is tight but if the non-negativity constraints are relaxed all the optimum quantities of the products yielded by the Lagrangian approach will be positive. The last range is when the budget is too tight that the non-negativity constraints cannot be relaxed (consequently the Lagrangian approaches become intractable).

Before embarking on solving the CMPNP, one has to define the aforementioned two thresholds that divide these three ranges of the available budget. The following equations determine each of these two thresholds depending on the available budget and demand patterns (see Fig. 2).

Threshold 1 (*Lower limit of unbinding budget constraint***).** This threshold is given by

$$B_G^{(1)} = \sum_{\tau=1}^{N} c_\tau F_\tau^{-1}\left(\frac{v_\tau - c_\tau}{v_\tau + h_\tau}\right) = \sum_{\tau=1}^{N} c_\tau x_\tau^*. \tag{4}$$

This is the lower limit of the budget in order to achieve the global minimum of Eq. (1).

Threshold 2 (*Lower budget limit for relaxing the non-negativity constraints***).** Letting

$\lambda^- = \min_{j=1,\dots,N}(\lambda_j)\,\forall j = 1,\dots,N$

and noting that λ_j is the marginal utility at the lower limit of the feasible amount of the product to be ordered, and $F_j(0)$ is the value of the CDF at this point, where

$$\lambda_j = \frac{v_j - (v_j + h_j)F_j(0)}{c_j} - 1. \tag{5}$$

Consequently, one can express the second threshold by

$$B_G^{(2)} = \sum_{\tau=1}^{N} c_\tau F_\tau^{-1}\left(\frac{v_\tau - (1+\lambda^-)c_\tau}{v_\tau + h_\tau}\right). \tag{6}$$

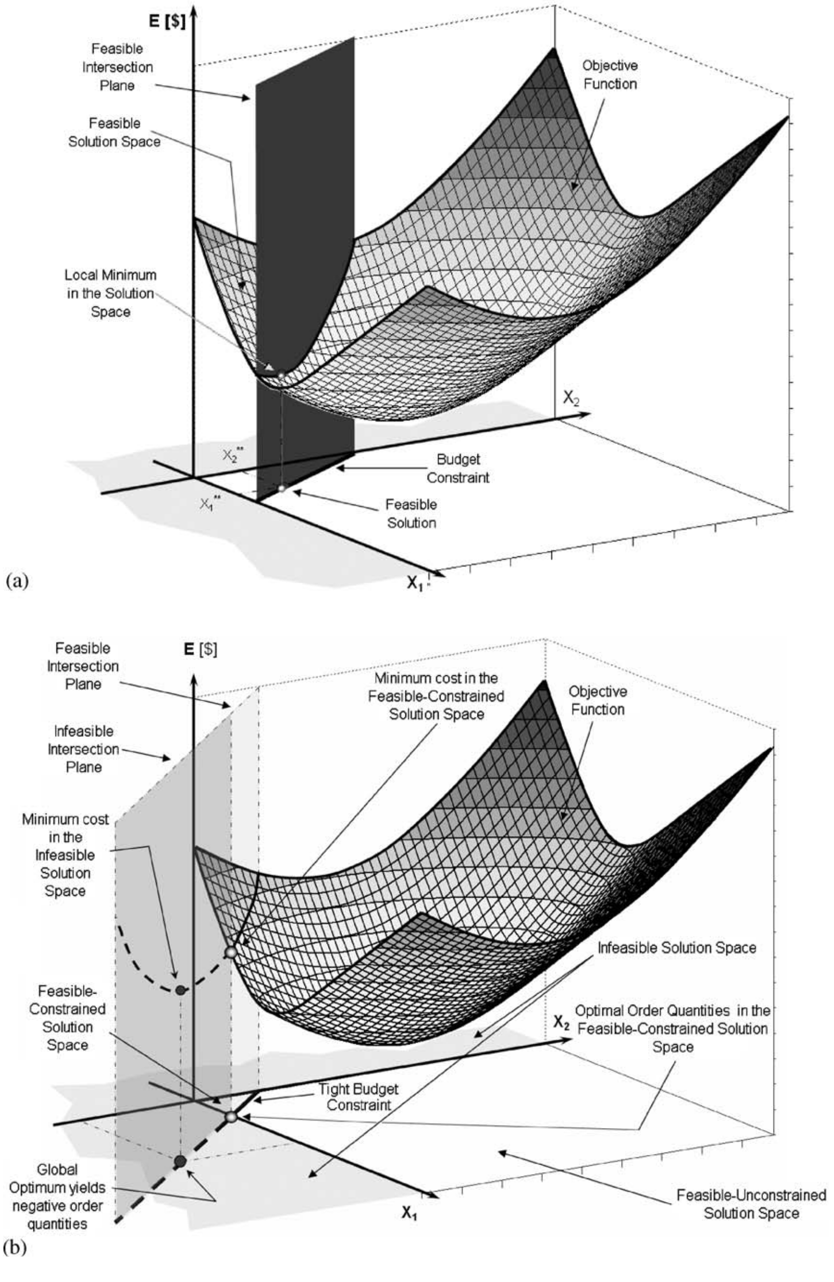

Fig. 1. (a) A solution space where non-negativity constraint can be relaxed. (b) A solution space where non-negativity constraints must be considered (relaxing the non-negativity constraints may lead to negative order quantities).

Once the two thresholds are defined, the solution methodology for each of the resulting three ranges can be implemented as shown in the following.

Range 1 $B_G^{(1)} \leqslant B_G$**.** In this case, one can obtain the optimum order quantities in a straightforward manner by applying the following

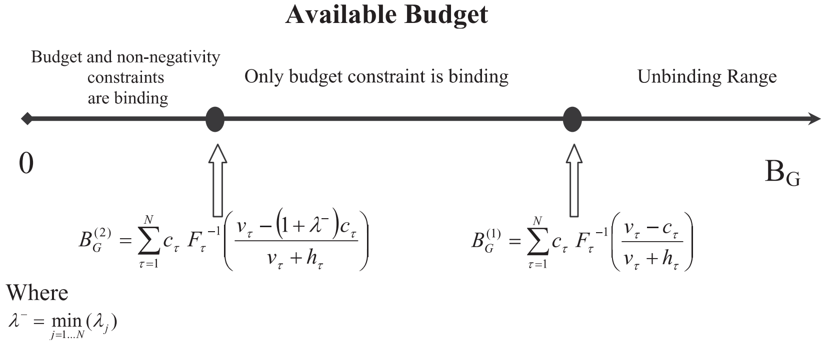

Fig. 2. Thresholds for the budget ranges.

equation:

$$x_\tau^* = F_\tau^{-1}\left(\frac{v_\tau - c_\tau}{v_\tau + h_\tau}\right) \quad \forall \tau = 1, \ldots, N, \tag{7}$$

where, x_τ^* is the optimal solution without budget constraint.

Range 2 $B_G^{(2)} \leqslant B_G < B_G^{(1)}$**.** In this range, one can relax the non-negativity constraint and the optimal amount is given by (as reported in Hadley and Whitin (1963)):

$$x_\tau^{**} = F^{-1}\left(\frac{v_\tau - (1+\lambda)c_\tau}{v_\tau + h_\tau}\right). \tag{8}$$

Depending on the demand probability density function, one can use one of the solution methods presented in the previous paper by Abdel-Malek et al. (2004).

Range 3 $B_G < B_G^{(2)}$**.** As mentioned before, the problem in this region becomes difficult particularly for large number of products since the non-negativity constraints have to be added to the original formulation given in Eqs. (1)–(2), that is:

$$x_\tau \geqslant 0, \quad \tau = 1, \ldots, N. \tag{9}$$

Adding this set of constraints strictly defines the solution space of the problem. And in this region it can be seen, as in Fig. 1b, that one or more products will have an order amount of "zero". However, in doing so, as we stated before, this complicates the problem and makes the Lagrangian based approaches hard to apply. Addressing this difficulty is the main contribution of our model.

The rationale of our approach is in deleting products, in ascending order, that have low marginal utility at the threshold of their lower bounds from further consideration (a similar rationale has been applied by Silver and Moon (2001), but for a non-capacitated model). This process keeps on lowering the cost function (since the loss of opportunity cost is not included for the eliminated products, see Figs. 1 and 2) until it intersects with budget constraint, thus avoiding making any of the order quantities negative. At this point, and for the remaining products, one can relax the non-negativity constraints and could apply one of the aforementioned procedures given in Abdel-Malek et al. (2004), GIM for example (see appendix for more details).

3.1. Explanation and proof

For the order quantities to be feasible the set of inequalities given in (9) has to be observed. More specifically, as mentioned before, we have to satisfy the Kuhn–Tucker conditions. The idea is to first relax the non-negativity constraints given in (9) and solve for λ, the marginal utility, which is shown in Eq. (10).

$$\lambda = \frac{v_j - (v_j + h_j)F_j(x_j)}{c_j} - 1. \tag{10}$$

Then we compute the marginal utilities of each product at $x_\tau = 0$, i.e. $F_\tau(0)$, as shown in Eq. (5). Afterwards we determine the effect of excluding products, starting with those which have the lowest marginal utility at $F_\tau(0)$ and proceeding in ascending order, on the expected cost function and continue the exclusion process until the total expected cost reduction yields

$$B_{G,j}^{(R)} \leqslant B_G, \tag{11}$$

where R is the number of products to be included in the new list $R<N$; $B_{G,j}^{(R)}$ the lower bound of budget required for including item j in the list, which is expressed by

$$B_{G,j}^{(R)} = \sum_{\tau=1}^{N} c_\tau F_\tau^{-1}\left(\frac{v_\tau - (1+\lambda_j)c_\tau}{v_\tau + h_\tau}\right). \tag{12}$$

Once this point is reached, we know the products to be considered for further analysis. Subsequently, one can relax the non-negativity constraints and the problem becomes tractable again. Based on duality theory, the method here excludes products with low marginal utility first. And since the marginal utility function is monotonic in nature so the optimum should be obtained on its boundary, therefore one can relax the non-negativity constraints. The following shows the proof that this function is monotonic.

Proof. Taking the partial derivative of Eq. (10) yields

$$\frac{\partial \lambda}{\partial x_j} = -\frac{v_j + h_j}{c_j}\frac{\partial F_j(x_j)}{\partial x_j}. \tag{13}$$

And since $\partial F_j(x_j)/\partial x_j > 0$, F_j is a cumulative distribution function, therefore $\partial\lambda/\partial x_j < 0$. Consequently for any

$$\lambda > \lambda_j \Rightarrow x_j < 0. \tag{14}$$

Now, consider the general budget equations, which can be derived using Eqs. (2) and (3):

$$B_G = \sum_{\tau=1}^{n} c_\tau F_\tau^{-1}\left(\frac{v_\tau - (1+\lambda)c_\tau}{v_\tau + h_\tau}\right). \tag{15}$$

Its derivative is

$$\frac{\partial B_G}{\partial \lambda} = -\sum_{\tau=1}^{N} \frac{c_\tau^2}{v_\tau + h_\tau}\frac{\partial F_\tau^{-1}(Y_\tau)}{\partial Y_\tau}, \tag{16}$$

where

$$Y_\tau = \frac{v_\tau - (1+\lambda)c_\tau}{v_\tau + h_\tau}. \tag{17}$$

Since $\partial F_\tau^{-1}(Y_\tau)/\partial Y_\tau > 0$ therefore $\partial B_G/\partial\lambda < 0$. Hence B_G is a decreasing monotonic function of λ and it follows that for any

$$B_G < B_{G,j} \Rightarrow \lambda > \lambda_j. \tag{18}$$

Consequently, using Eqs. (14) and (18), for any

$$B_G < B_{G,j} \Rightarrow x_j < 0. \tag{19}$$

This demonstrates that when a product j is deleted from the list, it is not optimum any more to reconsider it further for budgets that are less than $B_{G,j}$.

It should be emphasized that the proof holds only if the integrality constraints on the number of ordered items are relaxed. Nevertheless, the impact of relaxing the integrality constraints on the optimality of the solution diminishes when the number of items to be ordered is large (more than 10). If greater accuracy is sought, one can apply branch and bound techniques after obtaining the continuous solution yielded by the aforementioned approach.

Fig. 3 shows the first phase of the proposed solution method. In this phase (N items are still on the list), we start excluding the product with the lowest marginal utility $\lambda_j^{(N)}$ at $F_\tau(0)$ (that is we delete the product with largest contribution to the cost). Proceeding in ascending order of $\lambda_j^{(N)}$, Eq. (5), and letting $\lambda_j^{(N)} = \lambda_j^{(N-1)}$ we continue to compute $B_{G,j}^{(N-l)}$, Eq. (12), until it becomes equal to or less than the available budget. At this point the non-negativity constraints can be relaxed and one could apply GIM, to obtain the order quantities for the remaining products.

3.2. General approach

In Fig. 4, a flowchart of the general approach for obtaining the optimum order quantities of the products in the three ranges of the budget is described. The examples of Section 4 give more details of the solution methodologies.

4. Numerical examples

In this section, two numerical examples are presented. The first example is to show that

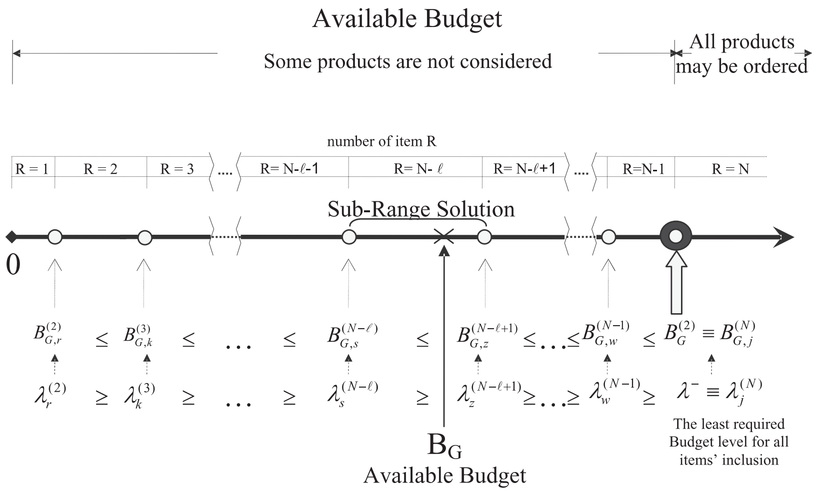

Fig. 3. Sub-range solution.

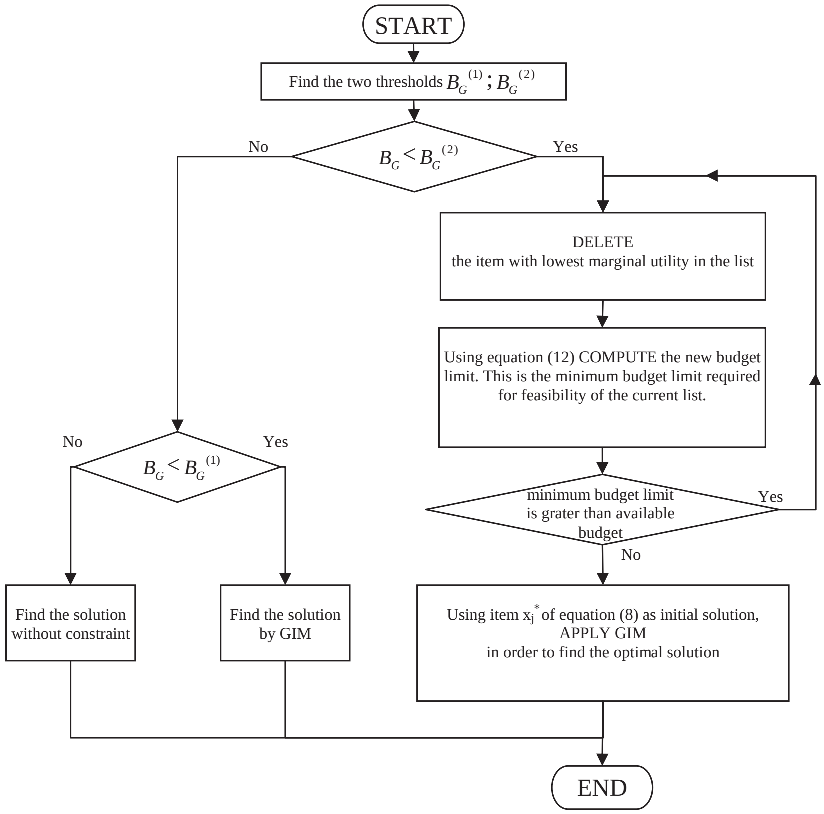

Fig. 4. Flowchart of the general approach procedures.

relaxing the non-negativity constraints could lead to infeasible order quantities which is an inherent problem in most of the Lagrangian based solution methods as observed by Lau and Lau (1996). The second example illustrates the application of the general approach developed here for large number of products (17).

4.1. Example 1

Table 1 shows the parameters of a three-product newsboy problem with \$300 budget constraint. Also, the table shows the solution results when relaxing and imposing the non-negativity constraints.

As can be seen, the optimum number of items varies for the products in this case if the non-negativity constraints are relaxed. One can see that the first item is negative, which is infeasible, when the non-negativity constraints are relaxed.

4.2. Example 2

Consider a 17-item situation with a budget constraint $B_G = \$2500$. Table 2 shows the pertinent data of the problem (note that the demand is normally distributed). Before embarking on solving the CMPNP and as shown in Fig. 4, one has to define the two thresholds; $B_G^{(1)}$ and if necessary $B_G^{(2)}$. Using Eq. (4), one can obtain $B_G^{(1)} > B_G$ (\$21998 > \$2500), then it is necessary to evaluate the second threshold $B_G^{(2)}$.

Using Eqs. (5)–(6), λ_j values $\forall_j = 1, \ldots, N$ and $B_G^{(2)}$ are determined as shown in Table 3. We find that $B_G^{(2)} > B_G$ (\$18 807 > \$2500), therefore the procedure of Range 3 has to be applied.

The value of $B_G^{(2)}$ has resulted from the inclusion of all items. So in order to reduce it, we begin by excluding item 9 (item with the lowest marginal utility at $F_j(0)$). Then as described in Fig. 4, we proceed in ascending order of λ_j. In this case $\lambda_{15}^{(16)} = 0.4242$. Substituting its value in Eq. (12) we compute $B_{G,15}^{(16)} = \$13546$ which is still larger than the available budget (at this juncture items 9 and 15 are excluded), in the same fashion we repeat this procedure until $B_{G,i}^{(N-l)} \leqslant B_G$. In this example it occurs at $\lambda_{11}^{(6)} = 0.9969$ when $B_{G,11}^{(6)} \leqslant B_G$ (\$2163 $\leqslant$ \$2500).

Therefore, products 9, 15, 2, 7, 3, 14, 16, 10, 1, 5, and 4 are excluded from the list. Then, in order to compute the initial solution required for GIM, which is to be engaged at this juncture (see flowchart in Fig. 4), we use Eq. (8) and noting

Table 2
Example's 2 parameters

Item	v_τ	h_τ	c_τ	μ_τ	σ_τ
1	7	1	4	102	51
2	12	2	8	73	18.3
3	30	4	19	123	30.8
4	30	4	17	95	23.8
5	40	2	23	62	15.5
6	45	5	15	129	43
7	16	1	10	69	34.5
8	21	2	10	83	41.5
9	42	3	40	120	30
10	34	5	20	89	22.3
11	20	3	10	115	38.3
12	15	5	7	91	30.3
13	10	3	4	52	17.3
14	20	3	12	76	38
15	47	2	33	66	16.5
16	35	4	21	147	36.8
17	22	1	11	104	34.7

Table 1
Three products problem solution with/without non-negativity constraints

Item_τ	v_τ	h_τ	c_τ	Normal distribution function		Solution	
						Without non-negativity constraints	With non-negativity constraints
				μ_τ	σ_τ	x_τ	x_τ
1	4	1	2	100	30	−32.488	0
2	3	2	1	180	60	129.503	129.503
3	6	2	3	300	50	78.491	56.832

Table 3
Example 2 threshold evaluation

Product	λ_j	$c_\tau F_\tau^{-1}\left(\frac{v_\tau-(1+\lambda^-)c_\tau}{v_\tau+h_\tau}\right)$
1	0.704499876	329.40441
2	0.499944549	488.79112
3	0.578890667	2023.2144
4	0.76464251	1467.4311
5	0.739072573	1314.6595
6	1.995500109	2073.5003
7	0.561324895	532.0612
8	1.047674857	784.69807
9	($\lambda^- = 0.049964353$)	0
10	0.699938212	1588.3487
11	0.996895075	1065.7881
12	1.139000094	573.53593
13	1.495612607	198.61525
14	0.623062381	700.9681
15	0.424195375	1814.2708
16	0.666607821	2751.9037
17	0.997177341	1100.2836
		$B_G^{(2)} = 18807$

Table 4
Initial solution for GIM (as obtained by Eq. (8))

Item	Item 11	Item 17	Item 8	Item 12	Item 13	Item 6
x_τ	0	1	12.6	41.4	34.4	106.6

Table 5
Optimum quantities of products to be ordered

Item	Item 11	Item 17	Item 8	Item 12	Item 13	Item 6
x_τ	15.58	15.23	14.02	42.20	34.56	106.86

that $\lambda = \lambda_{11}^{(6)} = 0.9969$. Table 4 shows the results for these initial values while; Table 5 exhibits the final results, which are obtained by applying GIM.

5. Conclusion

The extension in this paper has been propelled by the earlier observation of Lau and Lau (1996). In their article, they noted that if the budget is tight, the existing solutions methods, which are mostly Lagrangian based, could lead to negative optimum order quantities. Our study shows that the negativity of order quantities phenomenon stems from the fact that most of the existing models relax the lower bounds of the order quantities.

To address this problem, the solution space for this model is analyzed. Our study reveals insights leading to efficient solution methods that depend on the value of the available budget and the parameters governing the problem. It is found that the solution space can be divided into three distinct regions by two thresholds that have been defined in the paper. Formulae that are based on the available budget, products cost, and demand density functions are developed to determine the values of these thresholds in order to define these regions. The first region is where the budget is large enough to order the optimum quantity of each item without exceeding the allocated amount of money. In this case the classical model by Hadley and Whitin (1963) can be directly applied to obtain the optimum order quantities of each item while relaxing the budget constraint. In the second region, the budget constraint is binding. Therefore, the Lagrangian approach while relaxing the lower bound can be followed as basis to solve the problem. Depending on the type of probability density function of each product demand, one can choose from the solution models, exact, approximate or GIM (Generic Iterative Method) that are developed in Abdel-Malek et al. (2004) to obtain the lot size of each product. The third region, where the main contribution of this work lies, addresses the tight budget issue. That is when the budget is not enough to order all the products. Based on duality theory, our approach starts by deleting, in ascending order, products with lower marginal utilities at their lower bounds, until the remaining ones can fit within the available budget. Then, the non-negativity constraints are relaxed and one can apply one of the solution methods followed in the second region.

It should be mentioned that the approach proposed here relaxes the integrality constraints on the number of items ordered from each product. In practical sense, this is almost optimal when the number of items from each product is large, more than 10 (numerical experiments showed that the total costs are almost identical in most cases when rounding off the resulting solution). Nevertheless, if the number of items is small, the yielded solution can be an excellent starting point for Branch and Bound Approach since the objective function is convex.

In addition to its applicability to large number of products that could have different probability density functions, the proposed methodology can be used for post optimality analysis, where the decision-maker can decide on increasing the available budget for more customer satisfaction or even tighten it more depending on the marginal utilities of each product. This should be helpful in the analysis of a firm's inventory policies.

Finally, the model introduced in this paper complements existing Lagrangian based models. After defining the degree of tightness of the budget and deleting those products with low marginal utility, if any, one can apply other models that are already available in the literature. Also, the models here can be helpful as a first cut solution for more complex structures of the newsboy problem such as those developed in Moon and Silver (2000) and Silver and Moon (2001).

Acknowledgements

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Appendix

GIM is an iterative tool for solving the multi-product newsboy problem. It utilizes Taylor's expansion to approximate the cumulative-distribution-demand function by a straight line, see Fig. 5. This is expressed as follows:

$$f(x_\tau^{*(i-1)})(x_\tau^{*(i)} - x_\tau^{*(i-1)}) + F_\tau(x_\tau^{*(i-1)}) = F_\tau(x_\tau^{*(i)}). \tag{A.1}$$

Substituting Eq. (A.1) in Eq. (3), we get

$$x_\tau^{*(i)} = x_\tau^{*(i-1)} + \frac{1}{f(x_\tau^{*(i-1)})} \times \left[\frac{v_\tau - c_\tau(1+\lambda_G^{(i)})}{v_\tau + h_\tau} - F_\tau(x_\tau^{*(i-1)})\right]. \tag{A.2}$$

Then letting

$$B_G^{(i)} = \sum_{\tau=1}^{N} (c_\tau x_\tau^{*(i)}), \tag{A.3}$$

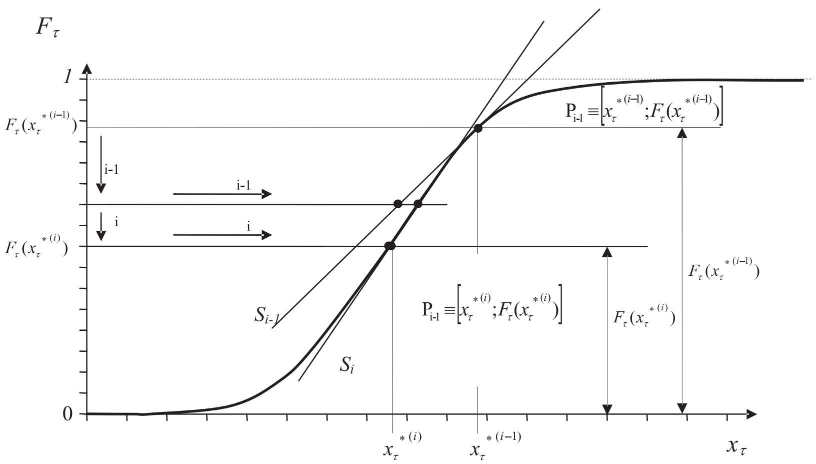

Fig. 5. Schematic diagram for GIM concept.

$$\Delta C^{(i)} = \sum_{\tau=1}^{N} (c_\tau x_\tau^{*(i-1)}) - B_G \tag{A.4}$$

and substituting Eq. (A.3) in Eq. (A.4), we get

$$\begin{aligned}\Delta C^{(i)} &= \sum_{\tau=1}^{N} c_\tau (x_\tau^{*(i-1)} - x_\tau^{*(i)}) \\ &= \sum_{\tau=1}^{N} c_\tau \left(x_\tau^{*(i-1)} - x_\tau^{*(i-1)} - \frac{1}{f(x_\tau^{*(i-1)})} \right. \\ &\quad \left. \times \left[\frac{v_\tau - c_\tau(1+\lambda_G^{(i)})}{v_\tau + h_\tau} - F_\tau(x_\tau^{*(i-1)}) \right] \right).\end{aligned}$$

After defining $\lambda_G^{(i)}$ as follows:

$$\lambda_G^{(i)} = \frac{\Delta C^{(i)} - (A^{(i)} + B^{(i)})}{A^{(i)}},$$

where

$$A^{(i)} = \sum_{\tau=1}^{N} \frac{c_\tau^2}{(v_\tau + h_\tau) f(x_\tau^{*(i-1)})},$$

$$B^{(i)} = \sum_{\tau=1}^{N} \left(\frac{c_\tau F_\tau(x_\tau^{*(i-1)})}{f_\tau(x_\tau^{*(i-1)})} - \frac{c_\tau v_\tau}{(v_\tau + h_\tau) f_\tau(x_\tau^{*(i-1)})} \right).$$

One can proceed iteratively at this point and at each iteration, i, the error is evaluated by

$$\begin{aligned}\varepsilon_G^{(i)} &= \frac{\sum_{\tau=1}^{N} c_\tau x_\tau^{*(i)} - B_G}{B_G} \\ &= \left[\frac{1}{B_G} \sum_{\tau=1}^{N} c_\tau F_\tau^{-1} \left(\theta_\tau - \frac{c_\tau \lambda_G^{(i)}}{v_\tau + h_\tau} \right) \right] - 1.\end{aligned}$$

We proceed in the same fashion until an acceptable level of error is reached.

As one can see from Fig. 5, at each point representing $x_\tau^{*(i)}$ values, a closer solution to the optimum is obtained. More clearly for $x_\tau^{*(i)}$'s values are closer than those of $x_\tau^{*(i-1)}$'s to the optimal solution.

Fig. 6 shows the flowchart of the procedure.

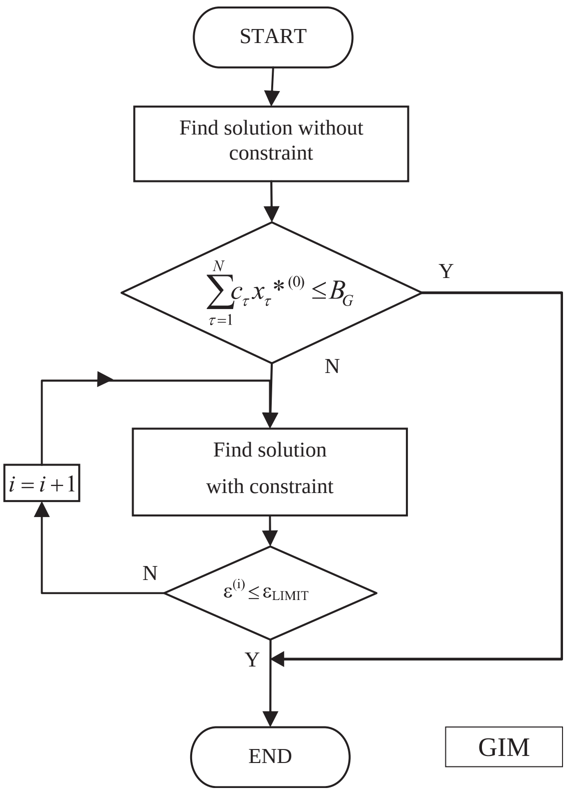

Fig. 6. GIM flowchart.

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