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Sticky Prices versus Monetary Frictions: An Estimation of Policy Trade-offs[†]

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We develop a two-sector monetary model with a centralized and decentralized market. Activities in the centralized market resemble those in a standard New Keynesian economy with price rigidities. In the decentralized market agents engage in bilateral exchanges for which money is essential. This paper is the first to formally estimate such a model, evaluate its fit based on postwar US data, and assess its money demand properties. Steady-state welfare calculations reveal that the distortions created by the monetary friction may be of similar magnitude as the distortions created by the New Keynesian friction. (*JEL* C54, E12, E31, E41, E52)

This paper develops a dynamic stochastic general equilibrium (DSGE) model that bridges the gap between the search-based monetary theory initiated by Nobuhiro Kiyotaki and Randall Wright (1989) and the literature on estimable New Keynesian DSGE models laid out in the textbook by Michael Woodford (2003). Contrary to popular belief, it is fairly straightforward to combine interesting elements of the monetary micro-foundations literature with New Keynesian models and create empirical models that can be confronted with the data and used to study important substantive questions. In our model, following the basic structure of Ricardo Lagos and Wright (2005), henceforth LW, and Aruoba, Christopher J. Waller, and Wright (forthcoming), henceforth AWW, in every period economic activity takes place in two markets. In a decentralized market (DM), households engage in bilateral trade, with a fraction of households producing and a fraction of households consuming. The centralized market (CM) resembles a standard DSGE model with admittedly reduced-form nominal rigidities, where production is carried out by firms. Demand for money arises because the particular frictions in the decentralized market necessitate the facilitation of transactions by a medium of exchange. We represent monetary policy by an interest rate feedback rule, and introduce stochastic

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disturbances to technology, preferences, government spending, and monetary policy to make the model amenable to econometric estimation methods. While the structure of our model to a large extent resembles that of a canonical New Keynesian model with capital, the presence of the decentralized market provides a micro-founded motive for holding money and creates a nonseparability between consumption and the value of real money balances.

The proposed model is estimated and evaluated using postwar US data on output, inflation, interest rates, and inverse M1 velocity. While most of the work on search-based monetary models has been theoretical, we use the Bayesian techniques surveyed by Sungbae An and Frank Schorfheide (2007) to conduct a full-fledged econometric analysis. A novel feature of our estimation is that we construct a measure of the target inflation rate from low-frequency dynamics of inflation as well as inflation expectations and then use this series along with output, inflation, interest rates, and velocity to estimate the DSGE model. To assess the fit of the search-based DSGE model, we also estimate a vector autoregression (VAR) and a standard New Keynesian model in which real money balances enter the households' utility function model in a separable fashion and conduct a detailed comparison.

Overall time series fit in a Bayesian framework is summarized by the so-called marginal likelihood, which approximately takes the form of an in-sample goodness-of-fit measure penalized for the number of estimated parameters. In terms of marginal likelihoods, both the search-based DSGE model and the money in the utility function (MIU) model are clearly dominated by the VAR. This outcome is not uncommon in the comparison of DSGE models and VARs. The MIU model fares somewhat better than the search-based DSGE model. The separable utility specification of the MIU model implies that, to the extent that money balances are mostly driven by money demand shocks, money can evolve largely independent of output, inflation, and interest rates over the business cycle, which happens to be a feature of our data. In contrast, the search-based DSGE model that we build features an inherently nonseparable structure and creates a tighter link between the dynamics of money balances and the other macroeconomic aggregates. As such, while this tighter link is conceptually appealing, the joint dynamics predicted by the particular model considered in this paper are not fully consistent with postwar US data.

As explained in Schorfheide (2000), in situations where the structural models under consideration are dominated by a more richly parameterized VAR, in the sense that the posterior probabilities of the former are essentially zero, it is sensible to evaluate the DSGE models by comparing some of their specific implications, e.g., population moments or impulse responses, to those derived from the less restrictive VAR. Under the assumption that agents forecast the target inflation rate with a random walk model, we are able to identify impulse responses to a target inflation rate shock in the VAR. Matching the responses to this shock is desirable since we use the DSGE model subsequently to examine the effect of target inflation changes on welfare. Note that the VAR itself, while being the better empirical model from a Bayesian perspective, is not suitable to conduct such a welfare analysis. A comparison of DSGE and VAR impulse responses shows that the Bayesian estimates of different versions of the search-based DSGE model, as well as the MIU model, are successful in matching the responses of output, interest rates, and inflation. However, all of the models can capture the (small) short-run response but not the (large) long-run response of velocity. This finding also translates into the failure of the models to match the short- and long-run elasticities of velocity with respect to interest rates simultaneously.

Finally, we study the steady-state welfare implications of the estimated DSGE model to determine whether the monetary friction is quantitatively important for policy analysis. Specifically, we compute measures of welfare gain by changing the target inflation from our end-of-sample value of 2.5 percent to a new value $\overline{\pi}_*$. Our model incorporates two key channels through which inflation can affect welfare. First, nonzero inflation rates lead to relative price distortions and inefficient use of intermediate goods because it is costly for firms to adjust nominal prices. We label this channel the New Keynesian channel. Its strength is determined by the probability with which firms are unable to reoptimize their prices—the nominal rigidity in the centralized market is based on the mechanism proposed by Guillermo A. Calvo (1983)—and the degree to which nonoptimizing firms index their past price to lagged inflation. Second, nonzero nominal interest rates constitute a tax on money holdings and hence depress activity in the decentralized market. We label this channel the Friedman channel. It is to a large extent controlled by the probability with which households engage in bilateral exchange in the decentralized market, which in turn determines the interest elasticity of money demand. Since both the New Keynesian and the Friedman channels "agree" that positive target inflation rates are undesirable, we focus on the range of $\overline{\pi}_* \in [-2.5\%, 0\%]$, where there is a trade-off between the two channels. According to our parametrization, at $\overline{\pi}_* = -2.5$ percent the nominal interest rate is zero, which is the celebrated "Friedman rule." Unlike a cashless New Keynesian model, which favors a target inflation rate of zero, our estimated search-based models imply that the optimal inflation rate lies in the range of -2.5 percent to -1 percent. We interpret this finding as evidence that the distortions created by monetary frictions may be of similar magnitude as the distortions created by the New Keynesian friction.

Our paper is related to several strands of the literature in monetary economics and the estimation of DSGE models. While the literature on estimated DSGE models with New Keynesian features is large—see Schorfheide (2008) for an extensive survey—only very few papers use a measure of money as observable, and hence implicitly or explicitly estimate a DSGE model-implied money demand function. The search-theoretic literature for the most part has only recently started to conduct quantitative exercises. Our paper is the first in applying formal estimation methods to a model based on the LW framework and examining its money demand properties.

Since we are also analyzing the steady-state welfare properties of our search-based DSGE model, a few remarks about the literature on optimal monetary policy are in order. First, most of the policy analysis in the search-theoretic literature focuses on the optimal long-run monetary policy. In general the Friedman rule is found to be the optimal policy unless the model features some other frictions, e.g., endogenous participation in Guillaume Rocheteau and Wright (2005), credit rationing by banks in Aleksander Berentsen, Gabriele Camera, and Waller (2007), or government-financing in Aruoba and Sanjay K. Chugh (2010). The New Keynesian channel and its potential influence on monetary policy has not been analyzed in this class of models before.

Second, there is a large literature on monetary policy analysis in New Keynesian models. Much of it, as summarized in Woodford (2003), focuses on stabilization policies, assuming the absence of steady-state distortions. Among the few papers that study the optimal long-run target inflation and consider both New Keynesian and monetary frictions, the following three articles are most closely related to our work. Robert G. King and Alexander L. Wolman (1996) find that once monetary frictions that generate money demand, shopping time in their case, are added to the most stripped-down New Keynesian model, the Friedman rule is approximately optimal. Aubhik Khan, King, and Wolman (2003) use a framework where the probability of a price change for a firm depends on the time since last change. The optimal long-run inflation target in their benchmark calibration is -0.75 percent. Stephanie Schmitt-Grohe and Martin Uribe (2007) show that in a medium-scale New Keynesian model, one with more frictions than ours, and with a transaction cost of consumption to motivate money demand, the optimal policy is a long-run inflation target of -0.5 percent, but it is very sensitive to changing the degree of price stickiness. By and large, normative prescriptions derived from our estimated search-based DSGE model, while obtained under very different assumptions about the demand for money, are consistent with this earlier work, which gets a range of results between the Friedman rule and full price stability.

The remainder of the paper is organized as follows. We provide a detailed derivation and discussion of the search-based DSGE model in Section I. The Bayesian estimation results are presented in Section II and the welfare analysis is summarized in Section III. Finally, Section IV concludes. Detailed derivations as well as additional estimation results are provided in the Web Appendix.

I. The Model

The model is an extension of the two-sector model developed in AWW, consisting of a decentralized and centralized market. To generate price stickiness we replace the perfectly competitive CM firms by monopolistically competitive firms that are constrained in their ability to change nominal prices. The centralized market is essentially identical to the goods market in a standard New Keynesian DSGE model described, for instance, in Woodford (2003) with a nominal rigidity in the style of Calvo (1983).

A. *Households*

There is a continuum of ex ante identical households in the economy. In every period, households first trade in the DM. According to an idiosyncratic taste shock that is realized at the beginning of the period, households become buyers with probability σ, sellers with probability σ, or nonparticipants with probability $1 - 2\sigma$. These shocks are independent across time and across households. Given that there are equal measures of buyers and sellers, we assume there is an efficient matching technology that matches exactly one buyer with one seller. The taste shocks create a double-coincidence problem where frictionless barter cannot occur. AWW show that a search-based setup in which households meet at random leads to the same mathematical construct. All households are anonymous in this market, which means IOUs will not be accepted in trade. Kiyotaki and Wright (1989) show that a double-coincidence problem, and the anonymity of households, will make money "essential" in the decentralized market, since trade can happen only with a quid pro quo. The terms of trade in such a match are determined via one of two alternative schemes: generalized Nash bargaining (B) or price-taking (PT).[1] Our model features two other durable assets, bonds and capital claims, which in principle can serve as a medium of exchange. In our benchmark model, we follow AWW and assume this possibility away. In Section IIG we relax this restriction and allow that, in addition to money, a fraction of the claims to the capital stock are liquid and may be used to purchase DM goods.

Once the households leave the DM, they proceed to the CM where neither of the two frictions that create a role for money in the DM is present: the households are identical in their preferences and abilities, and they are not anonymous. Using labor and capital income, the households acquire the final goods produced in the CM and use them for consumption and to accumulate capital. Households also adjust their asset holdings. We assume that households have access to a set of claims contingent on all possible realizations of the aggregate states. To characterize the households' behavior in this economy, we start from the households' CM problem.

Household Activity in the Centralized Market.—The households take as given the aggregate price level P_t in the CM, the gross nominal interest rate R_t on one-period bonds, the wage W_t, the rental rate of capital R_t^k, and the set of aggregate shocks $\boldsymbol{\mathcal{S}}_t$, along with their laws of motion. We use $V_t^{CM}(\hat{m}_t, k_t, i_{t-1}, b_t, \boldsymbol{\mathcal{S}}_t)$ and $V_t^{DM}(m_t, k_t, i_{t-1}, b_t, \boldsymbol{\mathcal{S}}_t)$ to denote the period t value functions in the CM and DM, where $\hat{m}_t$ (m_t) is the money balance of a household entering the CM (the DM), k_t is its capital stock, i_{t-1} is lagged investment, and b_t denotes its bond holdings. As it is clear from the notation, $\boldsymbol{\mathcal{S}}_t$, the only source of uncertainty for the CM, is realized at the beginning of the period and the households are able to compute the outcomes in the CM when they are in the DM. The CM problem takes the form

$$(1)\quad V_t^{CM}(\hat{m}_t, k_t, i_{t-1}, b_t, \boldsymbol{\mathcal{S}}_t)$$

$$= \max_{x_t, h_t, m_{t+1}, i_t, k_{t+1}, b_{t+1}} \left\{U(x_t) - Ah_t + \beta E_t\left[V_{t+1}^{DM}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, \boldsymbol{\mathcal{S}}_{t+1})\right]\right\},$$

subject to the constraints

$$(2)\quad P_t x_t + P_t i_t + b_{t+1} + m_{t+1} \leq P_t W_t h_t + P_t R_t^k k_t + \Pi_t$$

$$+ R_{t-1} b_t + \hat{m}_t - T_t + \Omega_t,$$

[1] From an a priori sense, there is no reason to prefer one pricing mechanism over the other. Historically, Nash bargaining has been used in the search-based literature, and only recently has competitive pricing (price-taking) been introduced (by Rocheteau and Wright 2005). To add to the accumulating literature on the positive properties of these models, and because their normative implications are quite different, we chose to include both pricing mechanisms.

$$k_{t+1} = (1 - \delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right]i_t. \tag{3}$$

Here, $U(x_t)$ is the instantaneous utility from consuming x_t units of the final good, A is the disutility of one unit of labor, h_t is hours worked, T_t is a nominal lump-sum tax, Π_t denotes the total profits the household receives from intermediate good producers, and Ω_t is the household's net cash-in-flow from trading state-contingent securities. The assumption of quasi-linear preferences is crucial and leads to a degenerate distribution of asset holdings at the end of each period. This assumption can be motivated by the indivisible labor setup of Richard Rogerson (1988), and it is used in the monetary model of Thomas F. Cooley and Gary D. Hansen (1995), as well as in many of the New Keynesian models discussed in Woodford (2003).

Equation (3) determines the capital accumulation. The adjustment cost function $S(\cdot)$ satisfies properties $S(1) = 0$, $S'(1) = 0$ and $S''(1) > 0$. We adopt the timing convention that k_{t+1} and m_{t+1} denote capital and money holdings at the end of period t and do not depend on period $t + 1$ shocks. The individual state variables $(\hat{m}_t, k_t, b_t)$ do not appear in the household's optimality conditions, and thus for any distribution of assets $(\hat{m}_t, k_t, b_t)$ across agents entering the CM, the distribution of $(m_{t+1}, k_{t+1}, b_{t+1})$ is degenerate.[2] It can also be shown that $V_t^{CM}(\cdot)$ is linear in $\hat{m}_t$, which will be important in the DM problem below.

Household Activity in the Decentralized Market.—The value of starting the DM for a household whose taste shock has not been realized yet is given by

$$\begin{aligned} V_t^{DM}(m_t, k_t, i_{t-1}, b_t, \boldsymbol{\mathcal{S}}_t) = {} & \sigma V_t^b(m_t, k_t, i_{t-1}, b_t, \boldsymbol{\mathcal{S}}_t) + \sigma V_t^s(m_t, k_t, i_{t-1}, b_t, \boldsymbol{\mathcal{S}}_t) \\ & + (1 - 2\sigma) V_t^{CM}(m_t, k_t, i_{t-1}, b_t, \boldsymbol{\mathcal{S}}_t), \end{aligned} \tag{4}$$

where the values of being a buyer and a seller are

$$V_t^b(m_t, k_t, i_{t-1}, b_t, \boldsymbol{\mathcal{S}}_t) = \chi_t u(q_t) + V_t^{CM}(m_t - d_t^m, k_t, i_{t-1}, b_t, \boldsymbol{\mathcal{S}}_t), \tag{5}$$

$$V_t^s(m_t, k_t, i_{t-1}, b_t, \boldsymbol{\mathcal{S}}_t) = -c(q_t, k_t, Z_t) + V_t^{CM}(m_t + d_t^m, k_t, i_{t-1}, b_t, \boldsymbol{\mathcal{S}}_t). \tag{6}$$

In this transaction, q_t units of the consumption good are exchanged against d_t^m units of currency. A household that consumes q_t receives utility $\chi_t u(q_t)$. The disturbance χ_t is a preference shock for goods produced in the DM. Since money is essential to purchase DM goods, χ_t can also be interpreted as a money demand shock. A seller household in the DM experiences disutility $-e_t$, where e_t denotes the effort the household exerts to produce q_t units of the DM good according to the technology $q_t = Z_t f(e_t, k_t)$. The total factor productivity process Z_t is assumed to be exogenous and common across DM and CM. We invert the production function to express

[2] In the absence of investment adjustment costs, this statement is exactly correct. In the presence of investment adjustment costs, since i_{t-1} appears in the optimality conditions, we also need i_{t-1} to be identical across households. This can be achieved, for example, by assuming i_0 is identical across households.

the level of effort as $e = c(q, k, Z)$, which appears in (6). The terms of trade are determined via bilateral generalized Nash bargaining, which is one of the most common schemes in the search literature, or price taking, which was first considered by Rocheteau and Wright (2005).

Bargaining. Exploiting the linearity of the CM value function, and using threat points that have the agents continuing to the CM, our bargaining problem is

$$\max_{q,d^m} \left[\chi u(q) - U'(X)\frac{d^m}{P}\right]^{\theta}\left[U'(X)\frac{d^m}{P} - c(q, k^s, Z)\right]^{1-\theta} \quad \text{s.t. } d^m \leq m^b,$$

where $U'(X)$ is the marginal utility of CM consumption and θ is the bargaining power of the buyer. The first term captures the buyer's surplus and the second term is the seller's surplus. We dropped the time subscripts since the bargaining problem is static. Using the insights of LW and AWW, in any monetary equilibrium, $d^m = m^b$, that is, the buyer spends all his money in exchange for some q that the seller produces using his capital and effort.

Price-Taking. Let $\tilde{p}$ be the DM price level that is taken as given by buyers and sellers. To ensure the quid pro quo nature of the trade, the value of the goods purchased has to equal the value of the money transferred from buyer to seller: $\tilde{p}q = d^m$. Using this condition, we write the value functions as

$$(7)\quad V_t^b(m_t, k_t, i_{t-1}, b_t, \boldsymbol{\mathcal{S}}_t) = \max_{q_t} \{\chi_t u(q_t) + V_t^{CM}(m_t - \tilde{p}_t q_t, k_t, i_{t-1}, b_t, \boldsymbol{\mathcal{S}}_t)\}$$

$$\text{s.t. } \tilde{p}_t q_t \leq m_t,$$

$$(8)\quad V_t^s(m_t, k_t, i_{t-1}, b_t, \boldsymbol{\mathcal{S}}_t) = \max_{q_t} \{-c(q_t, k_t, Z_t) + V_t^{CM}(m_t - \tilde{p}_t q_t, k_t, i_{t-1}, b_t, \boldsymbol{\mathcal{S}}_t)\}.$$

It can be shown that in any monetary equilibrium buyers spend all of their money so that $q = m^b/\tilde{p}$ holds.

B. *Firms in the Centralized Market*

The setup of the centralized market resembles that of a New Keynesian DSGE model. Production is carried out by two types of firms in the CM: final good producers combine differentiated intermediate goods. Intermediate goods producing firms are subject to a Calvo-style friction (1983). They hire labor and capital services from the households to produce the inputs for the final good producers.

Final and Intermediate Goods Producers.—The final good Y_t in the CM is a composite made of a continuum of intermediate goods $Y_t(i)$:

$$(9)\quad Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda}}\, di\right]^{1+\lambda},$$

with elasticity of substitution $(1 + \lambda)/\lambda$. We constrain $\lambda \in [0, \infty)$. The final good producers buy the intermediate goods on the market, package them into Y_t units of the composite good, and resell them to consumers. These firms maximize profits in a perfectly competitive environment taking $P_t(i)$ as given, which yields the demand for good i:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} Y_t. \tag{10}$$

Combining this demand function with the zero profit condition, one obtains the following expression for the price of the composite good:

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda}}\, di\right]^{-\lambda}. \tag{11}$$

Inflation in the CM is defined as $\pi_t = P_t/P_{t-1}$.

Intermediate goods producers, indexed by i, face the demand function (10) and use a Cobb-Douglas technology with fixed costs $\mathcal{F}$:

$$Y_t(i) = \max\left\{Z_t K_t(i)^{\alpha} H_t(i)^{1-\alpha} - \mathcal{F}, 0\right\}. \tag{12}$$

The technology shock Z_t is identical to the one that appears in the DM production function. Following Calvo (1983), we assume that in the current period firms are only able to reoptimize their price with probability $1 - \zeta$. A random fraction ι of the firms that are not allowed to choose $P_t(i)$ optimally update their price $P_{t-1}(i)$ according to last period's inflation rate π_{t-1}, whereas the remaining $1 - \iota$ firms keep their price constant.[3] We treat ι, the degree of dynamic indexation, as a parameter to be estimated.

For a firm that is allowed to reoptimize its price, the problem is to choose a price level $P_t^o(i)$ that maximizes the expected present discounted value of profits in all future states in which the firm is unable to reoptimize its price. This firm uses the time t value of a dollar in period $t + s$ for the consumers, to discount future profits. Here we are considering only the symmetric equilibrium in which all firms that can readjust prices will choose the same $P_t^o(i)$. The solution of this problem leads to a dynamic relationship between the optimal price $p_t^o = P_t^o/P_t$ and marginal costs MC_t (New Keynesian Phillips Curve).

[3] In most estimated DSGE models, it is assumed that the fraction of firms $1 - \iota$ index their past price by the steady-state inflation rate. However, in order to preserve the steady-state effects of the New Keynesian distortion if $\iota < 1$, we do not make such an assumption. The sensitivity of policy analysis to this assumption has recently been emphasized by Guido Ascari and Tiziano Ropele (2007).

C. *Government Spending and National Accounts*

In period t, the government collects a nominal lump-sum tax T_t, spends G_t on goods from the centralized market, issues one-period nominal bonds B_{t+1} that pay gross interest R_t tomorrow, and supplies the money to maintain the interest rate rule. The government satisfies the following budget constraint every period:

$$P_t G_t + R_{t-1} B_t + M_t = T_t + B_{t+1} + M_{t+1}. \tag{13}$$

We assume that government spending G_t evolves exogenously as specified below.

Adding the households' CM budget constraints, the government budget constraint, and the profits of intermediate goods producers, we obtain

$$X_t + I_t + G_t = Y_t, \tag{14}$$

which is the resource constraint in the CM. Since there is no savings in the DM, there is a trivial resource constraint that sets consumption equal to output. The quantity of final goods in the CM is related to the total output of the intermediate goods firms according to

$$Y_t = \frac{1}{D_t}\left[Z_t K_t^{\alpha} H_t^{1-\alpha} - \mathcal{F}\right], \quad D_t = \int \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} di, \tag{15}$$

where D_t measures the extent of price dispersion. Unless $P_t(i) = P_t$ for all firms, D_t will be greater than unity, which in turn implies the economy will produce inside its production-possibilities frontier. Since we have a model with two sectors, we aggregate DM and CM output and inflation using a Fisher index to obtain a measure of GDP, $\mathcal{Y}_t^{GDP}$, as well as a GDP deflator inflation, π_t^{GDP}. Moreover, we use $\mathcal{Y}_t$ to denote total output across the two sectors measured in terms of the CM good.

D. *Monetary Policy*

Following authors like Thomas J. Sargent (1999) and Robert E. Lucas, Jr. (2000), we assume that low-frequency movements of inflation, such as the rise of inflation in the 1970s and the subsequent disinflation episode in the early 1980s, can be attributed to monetary policy changes. Unlike in the learning models considered by Sargent, Noah Williams, and Tao Zha (2006) or Giorgio E. Primiceri (2006), our DSGE models offer no explanation why monetary policy shifts occur over time and simply assumes a time-varying target inflation rate $\pi_{*,t}$. The central bank supplies money to control the nominal interest rate. Following the setup in Schorfheide (2005), we assume that it systematically reacts to inflation and output growth according to the rule

$$R_t = R_{*,t}^{1-\rho_R} R_{t-1}^{\rho_R} \exp\{\sigma_R \epsilon_{R,t}\}, \quad R_{*,t} = (r_* \pi_{*,t})\left(\frac{\pi_t^{GDP}}{\pi_{*,t}}\right)^{\psi_1}\left(\frac{\mathcal{Y}_t^{GDP}}{\gamma \mathcal{Y}_{t-1}^{GDP}}\right)^{\psi_2}, \tag{16}$$

where r_* is the steady-state real interest rate, γ is the gross steady-state growth rate of the economy, and $\epsilon_{R,t}$ is a monetary policy shock. With the exception of the time-varying inflation target $\pi_{*,t}$, the specification (16) is widely used in the literature on estimable monetary DSGE models. The parameter ρ_R captures interest rate smoothing, that is, within the period the central bank does not fully adjust the nominal rate to the desired level $R_{*,t}$. The coefficients ψ_1 and ψ_2 determine how strongly the central bank reacts to deviations of inflation and output growth from their respective target values. Finally, the monetary policy shock $\epsilon_{R,t}$ reflects short-run deviations from the systematic part of the interest rate feedback rule that are unanticipated from the perspective of the public.

E. *Closing the Model*

We consider five aggregate disturbances in our model economy. The random productivity term that affects production in both CM and DM is represented by Z_t, and g_t is a shock that shifts government spending according to

$$G_t = (1 - 1/g_t)\mathcal{Y}_t. \tag{17}$$

Government consumption goods are purchased in the centralized market. The money demand shock χ_t shifts preferences for goods produced in the DM. Finally, our model has two monetary policy shocks: $\epsilon_{R,t}$ is assumed to be serially uncorrelated and captures short-run shifts in monetary policy, whereas the time-varying inflation target $\pi_{*,t}$ captures long-run policy changes. We define $\tilde{Z}_t = \ln(Z_t/Z_*)$, $\tilde{\chi}_t = \ln(\chi_t/\chi_*)$, and $\tilde{g}_t = \ln(g_t/g_*)$, where Z_*, χ_*, and g_* are steady-state values of the respective exogenous disturbances. We assume that these exogenous disturbances evolve according to stationary AR(1) processes $\tilde{Z}_t = \rho_z \tilde{Z}_{t-1} + \sigma_z \epsilon_{z,t}$, $\tilde{\chi}_t = \rho_\chi \tilde{\chi}_{t-1} + \sigma_\chi \epsilon^\chi_{\chi,t}$, and $\tilde{g}_t = \rho_g \tilde{g}_{t-1} + \sigma_g \epsilon_{g,t}$. We also define $\tilde{\pi}_{*,t} = \ln(\pi_{*,t}/\pi_*)$, where π_* is a constant discussed below and $\tilde{\pi}_{*,t}$ evolves as a random walk $\tilde{\pi}_{*,t} = \tilde{\pi}_{*,t-1} + \sigma_\pi \epsilon^\pi_{\pi,t}$. The innovations are stacked in the vector $\boldsymbol{\epsilon}_t = [\epsilon_{z,t}, \epsilon_{\chi,t}, \epsilon_{g,t}, \epsilon_{\pi,t}, \epsilon_{R,t}]$ and are assumed to be independently and identically distributed according to a vector of standard normal random variables.

The law of motion for the exogenous processes completes the specification of our DSGE model. The equilibrium conditions are summarized in the Web Appendix. To solve the model, we compute the steady-state conditional on $\tilde{\pi}_{*,t} = 0$ and $\pi_* = 1.01$, corresponding to an annual inflation rate of 4 percent, which is the mean in our sample. Next, we use a log-linear approximation around this steady state to form a state-space representation that is used for the Bayesian estimation. While our model implies that the inflation target can move arbitrarily far away from π_*, in our sample, $\tilde{\pi}_{*,t}$ and $\tilde{\pi}_t$ are never greater than 10 percent in absolute value. It should be noted that these deviations are commensurable to the deviations in a model with a fixed target inflation rate that is equal to the sample mean.[4]

[4] AWW use a nonlinear solution scheme (projection method with Chebyshev polynomials) with no shocks and find that around a reasonable neighborhood of the steady state, the decision rules are well approximated linearly.

II. Empirical Analysis

We now turn to the DSGE model estimation. We use a Bayesian approach discussed in detail in An and Schorfheide (2007). Our dataset and the construction of the target inflation series is described in Section A. Functional forms are specified in Section B and a description of the prior distribution is provided. Parameter estimates, as well as implied steady states, are presented in Section C, and the implied model dynamics are analyzed via variance decompositions and impulse response functions in Section D. We assess the fit of the search-based DSGE model in Section E and discuss the properties of money demand in Section F. Finally, in Section G we study the sensitivity of key parameter estimates to some of our modeling choices.

A. *Data*

Our empirical analysis is based on quarterly US postwar data on aggregate output, inflation, inflation expectations, interest rates, and (inverse) velocity of money.[5] Our estimation sample ranges from 1965:I to 2005:I and we use likelihood functions conditional on data from 1964:I to 1964:IV to estimate our DSGE model and VARs. As explained in Section I, we assume that the target inflation rate $\pi_{*,t}$ is time varying. One could simply treat $\pi_{*,t}$ as a latent variable in the likelihood-based estimation of the DSGE model and use the Kalman smoother to obtain ex post estimates of $\pi_{*,t}$ based on the observations that are included in the construction of the likelihood function. We shall deviate from this commonly used approach for two reasons. First, we will assess the time series fit of the DSGE model and the propagation of unanticipated changes in the target inflation rate through a comparison with a VAR. To facilitate this comparison, it is helpful to treat the target inflation rate as observable. Second, from the perspective of the agents, $\pi_{*,t}$ can be interpreted as a long-run inflation expectation. Hence, we will incorporate survey expectations in the construction of the $\pi_{*,t}$ series.

In order to obtain a measure of the inflation target, we combine three inflation expectation measures which are plotted in the top panel of Figure 1: GDP deflator filtered through a one-sided band-pass filter, one-year- and ten-year-ahead inflation expectations.[6] Prior to 1986 these three measures of target inflation move together very closely. Between 1987 and 1992 the bandpass-filtered inflation series is about 1 percent lower than the inflation expectations. After 1992 the bandpass-filtered

Accumulated evidence from estimating New Keynesian DSGE models (see, for example, An 2007), also suggests that log-linear solution techniques work well for the approximation of equilibrium dynamics.

[5] Unless otherwise noted, the data are obtained from the FRED2 database maintained by the Federal Reserve Bank of St. Louis. Per capita output is defined as real GDP (GDPC96) divided by civilian noninstitutionalized population (CNP16OV). We take the natural log of this measure, extract a linear trend, and link the deviations from this trend to the stationary fluctuations around the deterministic steady state that our model produces. Inflation is defined as the log difference of the GDP deflator (GDPDEF) and our measure of nominal interest rates corresponds to the federal funds rate (FEDFUNDS). Money is incorporated as an observable by using inverse M1 velocity. We use the sweep-adjusted M1S series provided by Barry Z. Cynamon, Donald H. Dutkowsky, and Barry E. Jones (2006). The M1S series is divided by quarterly nominal output to obtain inverse velocity, and we relate the natural logarithm of the resulting series to the log deviations from $100 \times \ln(\mathcal{M}^*/\mathcal{Y}^*)$.

[6] Inflation expectations are from the Survey of Professional Forecasters with the following exception: the ten-year inflation expectations for the period 1979–1991 are from the Livingston Survey and the Blue Chip Economic Indicators. All series are provided by the Federal Reserve Bank of Philadelphia.

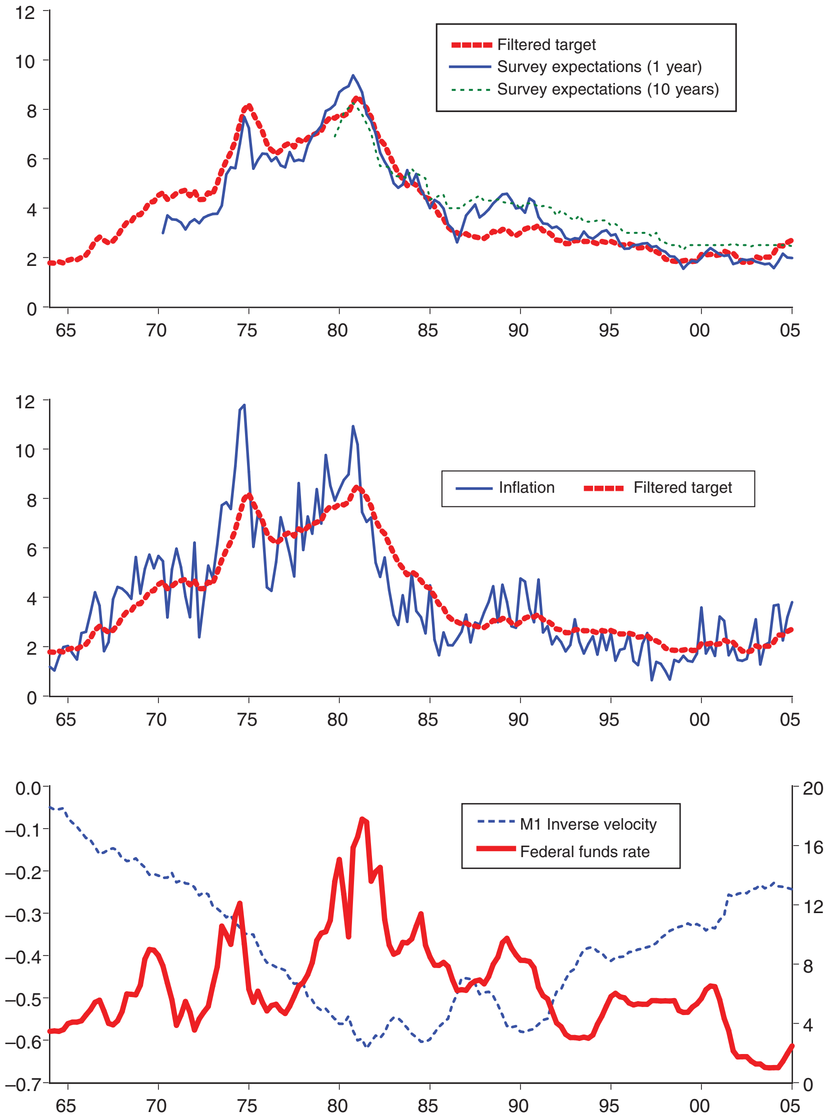

Figure 1. Inflation, Target Inflation, M1-Velocity, and Interest Rates

Notes: Inflation rates in the top two panels and Fed funds rate (right scale) in the bottom panel are annualized and measured in percentages. In the bottom panel inverse velocity (left scale) is depicted in natural logarithms.

inflation essentially tracks one-year-ahead survey forecasts, which tend to be slightly lower than the ten-year expectations. To combine the three series, we use a small state-space model and extract the common factor using the Kalman filter. The filtered target inflation series $\tilde{\pi}_{*,t}$ is displayed in the second panel of Figure 1, together with the GDP deflator inflation.[7] The dynamics of $\tilde{\pi}_{*,t}$ are well approximated by the

[7] If one regresses the filtered series $\tilde{\pi}_{*,t}$ on the three observed measures, the coefficients are 0.57 ($\tilde{\pi}_t^{BP}$), 0.22 ($\tilde{\pi}_t^{1y}$), and 0.23 ($\tilde{\pi}_t^{10y}$).

random walk that the DSGE model agents use to forecast the target inflation rate. Finally, the bottom panel of Figure 1 overlays the federal funds rate and M1 inverse velocity. According to our theoretical framework, the rise and fall of the nominal interest rate is to a large extent generated by exogenously changing preferences of monetary policymakers, as reflected in $\pi_{*,t}$. The postwar US data exhibit a strong negative correlation between inverse velocity and nominal interest rates that at least qualitatively resembles a money-demand relationship.

B. *Functional Forms, Restricted Parameters, and Priors*

We use the following functional forms in our estimation:

$$u(q) = \ln(q + \kappa) - \ln(\kappa), \quad U(x) = B\ln(x), \quad f(e,k) = e^{1-\alpha}k^{\alpha},$$

where κ is set equal to 1E-4 to make sure the threat point of a buyer in the DM, which involves $q_t = 0$, is well defined. The parameter B determines the relative weight of the utility from consuming the CM and DM goods. We use a natural logarithm for both utility functions and use the same Cobb-Douglas production function as the function used by the intermediate good producers in the CM. As Waller (2009) shows, these are necessary conditions for balanced-growth in this model.

One goal of our empirical analysis is to compare the propagation of shocks and the steady-state welfare implications for various specifications of our model. Hence, it is desirable to normalize and restrict a subset of the model parameters prior to estimation. The steady states of real GDP, $\mathcal{Y}_*$, and the DM preference shock process, χ_*, are normalized to one. The steady-state log inverse velocity is fixed at the sample mean -0.38. We fix H_*/Y_* at 0.03. To obtain this value we compute the sample average of quarterly hours worked per capita divided by quarterly real per capita GDP (in 1996 dollars). To a good approximation, the preference parameters A and B and the steady-state level of technology Z_* are determined by the steady-state hours, velocity, and labor productivity, respectively.

The DSGE model is log-linearized around the average inflation rate in our sample, which is approximately 4 percent. We let r_A be equal to the difference of the average federal funds rate and the average inflation rate between 1965 and 2005, and set $\beta = 1/(1 + r_A/400)$. We set $g_* = 1.2$, which is computed from the average ratio of government consumption plus investment and GDP. We fix the depreciation rate δ at 0.014. This value is obtained as the average ratio of fixed asset depreciation and the stock of fixed assets between 1959 and 2005.[8]

It is well known that the central bank's reaction to inflation deviations, ψ_1, is difficult to identify. Since the primary focus of the paper is not to estimate monetary policy rules, we set $\psi_1 = 1.7$. This value is taken from Schorfheide (2005), who estimated a model with a regime-switching target inflation rate over a similar time period. The steady-state gross growth rate of GDP, which is parameterized by γ in

[8] We use NIPA-FAT11 (current cost net stock) and NIPA-FAT13 (current cost depreciation) for fixed assets and consumer durables.

Table 1—Prior and Posterior Distributions

	Prior distributions				Posterior distributions			
					SBM(B)		SBM(PT)	
Name	Domain	Density	Para (1)	Para (2)	Mean	90% Interval	Mean	90% Interval
Households								
θ	[0,1)	Uniform	0.00	1.00	0.95	[0.95, 0.96]		
2σ	[0,1)	Beta	0.40	0.20	0.63	[0.56, 0.70]	0.59	[0.52, 0.66]
Firms								
α	[0,1)	Beta	0.30	0.025	0.32	[0.31, 0.34]	0.27	[0.26, 0.28]
λ	$\mathbb{R}^+$	Gamma	0.15	0.05	0.14	[0.12, 0.16]	0.19	[0.18, 0.21]
ζ	[0,1)	Beta	0.60	0.15	0.83	[0.79, 0.87]	0.84	[0.80, 0.88]
ι	[0,1)	Beta	0.50	0.25	0.72	[0.54, 0.91]	0.57	[0.31, 0.82]
S''	$\mathbb{R}^+$	Gamma	5.00	2.50	4.89	[2.50, 7.36]	5.08	[2.42, 7.71]
Central bank								
ψ_2	$\mathbb{R}^+$	Gamma	0.20	0.10	0.86	[0.64, 1.06]	0.83	[0.64, 1.02]
ρ_R	[0,1)	Beta	0.50	0.20	0.61	[0.56, 0.66]	0.60	[0.55, 0.65]
σ_R	$\mathbb{R}^+$	InvGamma	0.50	4.00	0.36	[0.31, 0.41]	0.37	[0.31, 0.42]
$\sigma_{R,2}$	$\mathbb{R}^+$	InvGamma	1.00	4.00	0.85	[0.63, 1.07]	0.85	[0.62, 1.08]
$\tilde{\pi}^*_{A,0}$	$\mathbb{R}$	Normal	0.00	2.00	0.05	[−3.21, 3.26]	0.02	[−3.22, 3.28]
σ_π	$\mathbb{R}^+$	InvGamma	0.05	4.00	0.05	[0.04, 0.05]	0.05	[0.04, 0.05]
Shocks								
ρ_g	[0,1)	Beta	0.80	0.10	0.84	[0.81, 0.88]	0.87	[0.83, 0.90]
σ_g	$\mathbb{R}^+$	InvGamma	1.00	4.00	1.01	[0.90, 1.11]	1.06	[0.94, 1.16]
ρ_χ	[0,1)	Beta	0.80	0.10	0.97	[0.97, 0.98]	0.96	[0.95, 0.97]
σ_χ	$\mathbb{R}^+$	InvGamma	1.00	4.00	1.80	[1.63, 1.97]	1.88	[1.70, 2.05]
ρ_Z	[0,1)	Beta	0.80	0.10	0.83	[0.76, 0.90]	0.83	[0.77, 0.89]
σ_Z	$\mathbb{R}^+$	InvGamma	1.00	4.00	1.04	[0.90, 1.17]	1.06	[0.91, 1.21]

Note: Para (1) and Para (2) list the means and the standard deviations for beta, gamma, and normal distributions; the upper and lower bound of the support for the Uniform distribution; and s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma \mid \nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$.

the interest rate feedback rule, is set equal to one since we model deviations from the steady state. Finally, we let $\mathcal{F} = 0$ (no fixed costs).

Suppose we stack the remaining DSGE model parameters in the vector $\boldsymbol{\vartheta}$ with elements ϑ_i, $i = 1, \ldots, k$. Our prior distribution for $\boldsymbol{\vartheta}$ takes the form $p(\boldsymbol{\vartheta}) \propto f(\boldsymbol{\vartheta}) \prod_{i=1}^{k} p_i(\vartheta_i)$. The marginal densities $p_i(\vartheta_i)$ capture prior information for individual parameters and are summarized in the first four columns of Table 1. Following Marco Del Negro and Schorfheide (2008), the function $f(\boldsymbol{\vartheta})$ is used to incorporate beliefs about the steady state that are functions of multiple parameters. In particular,

$$f(\boldsymbol{\vartheta}) = \exp\left\{-\frac{1}{2}\left[\frac{(I_*(\boldsymbol{\vartheta})\,\mathcal{Y}_*(\boldsymbol{\vartheta}) - 0.16)^2}{0.005^2} + \frac{(lsh(\boldsymbol{\vartheta}) - 0.060)^2}{0.01^2} + \frac{(mu_{DM}(\boldsymbol{\vartheta}) - 0.15)^2}{0.01^2} + \frac{(mu(\boldsymbol{\vartheta}) - 0.15)^2}{0.01^2}\right]\right\}.$$

Thus, $f(\boldsymbol{\vartheta})$ down-weighs the overall prior density at parameter combinations for which the investment output ratio, the labor share, and the markups in the DM and the overall economy deviate from 0.16, 0.60, and 0.15, respectively. For the price-taking version of the search-based DSGE model, the markup in the decentralized market is zero and we drop the corresponding term from the function $f(\vartheta)$.

The two remaining preference parameters are related to the search and matching frictions that generate a role for money demand. The probability of a single coincidence in the DM, σ is bounded between zero and 0.5 and we use an almost uniform prior on this interval. As we demonstrate below, this parameter affects the steady-state velocity and the responsiveness of money demand to changes in the interest rate. In the bargaining version of our model, the parameter θ measures the bargaining power of the buyer and affects the markup in the decentralized market. Our prior for θ is indirectly determined by $f(\vartheta)$. Turning to the firms, we use a uniform prior on the indexation parameter ι. Our prior for ζ is chosen to be broadly consistent with micro-evidence on the frequency of price changes. The parameter λ corresponds to the markup in the centralized market and is centered at 15 percent. The prior distributions for ρ_g, ρ_z, and ρ_χ reflect the belief that the government spending (demand) disturbance, the technology shock, and the DM preference shock are fairly persistent. The priors for the shock standard deviations were loosely chosen such that the implied distribution of the variability of the endogenous variables is broadly in line with the variability of the observed series over a presample from 1959 to 1964.

C. *Parameter and Steady-State Estimates*

Posterior means and 90 percent credible intervals for the estimated DSGE model parameters are reported in Table 1. The bargaining model is abbreviated as SBM(B) and the price-taking model as SBM(PT). The estimated single-coincidence probability is around 0.3. We will document in Section E that this estimate captures the fairly low short-run elasticity of money demand with respect to interest rates in the data. The estimate of $\theta = 0.95$ in SBM(B) is strongly influenced by the prior distribution that favors parameter values consistent with a markup of about 15 percent throughout the sectors of the economy. This leads to a DM markup of 17 percent, and along with the 14 percent markup in the CM and the DM share of 20 percent matches our target of aggregate markup. To provide a comparison, in AWW, θ was calibrated to be around 0.90 using a DM markup of 30 percent as the target.

Turning to the firms, we observe a number of departures from standard parameter estimates due to both the two-sector structure of our model and the differences in pricing mechanisms in the two sectors. The posterior mean of the CM markup is higher in the price-taking model, because the DM markup is zero and we are using a fairly tight prior that implies an economy-wide markup of about 15 percent. Much of the information about α stems from the prior distribution, which utilizes information about long-run averages not included in the likelihood function. The capital share α is significantly larger in SBM(B) than in SBM(PT). This is due to the holdup problems in SBM(B) which, everything equal, reduce the steady-state capital stock. Since we are using priors that restrict the investment-output ratio to be approximately 16 percent in both models, the holdup problem present in the bargaining model requires a larger capital share parameter in the production function.

The estimates of the price-stickiness parameter ζ and the degree of indexation ι are relatively high in both models, implying an average duration between price reoptimizations in the CM of about six quarters and a dynamic indexation of 60–70 percent. Our coefficient estimates would roughly translate into a Phillips curve slope of 0.02 (with respect to marginal costs) and the coefficient on lagged inflation would be about 0.42. Compared to the slope estimates surveyed in Schorfheide (2008), which range from 1E-3 to about 4, our estimate is fairly small but not unreasonable. Since the degree of indexation is inherently difficult to identify, the estimates of the coefficient on lagged inflation reported in the literature are essentially uniformly distributed over the range 0 to 0.5 and are very sensitive to auxiliary assumptions about the law of motion of exogenous shocks. Note that ζ and ι in our model affect only CM inflation dynamics, whereas aggregate inflation is a weighted average of CM and DM inflation. We will document subsequently that inflation in the DM lacks persistence, and prices in the DM are essentially flexible. Thus, in order to match observed inflation dynamics, CM prices need to be more rigid than in the one-sector model.[9] Since CM firms generate 80 percent of total production, the probability that a given price cannot be changed is $0.8 \times 0.83 = 0.66$. This implies an average duration of 8.8 months between price changes in the aggregate, perfectly in line with other empirical studies.

The estimates of the parameters that describe the central bank behavior and the evolution of the exogenous shocks are very similar across SBM(B) and SBM(PT). The estimated reaction coefficient to output growth is about 0.85 and the interest rate smoothing parameter is 0.6. The preference shock for DM goods is the most persistent among the shocks, with an autocorrelation of about 0.97. We treat the initial value of the target inflation rate as a parameter that appears as $\tilde{\pi}^*_{0,A}$ in Table 1. Since it is well known that interest rate feedback rules tend to fit poorly over the period 1979:I to 1982:IV, we allow the standard deviation of the monetary policy shock over this period $(\sigma_{R,2})$ to differ from the standard deviation for the remainder of the sample (σ_R). From an econometric perspective, this parametrization generates a heteroskedasticity correction for the monetary policy rule.

D. *Dynamics*

Variance decompositions for output, inflation, and interest rates are reported in Table 2. The decompositions are computed for business cycle frequencies ranging from 6 to 32 quarters per cycle. Since the decompositions for SBM(B) and SBM(PT) are very similar, we will focus on the bargaining version. Our model was built upon the assumption that the target inflation shock affects only low-frequency movements and we find, indeed, that its contribution to business cycle fluctuations is essentially zero. Technology shocks cause about 30 percent of the output fluctuations and the demand or government spending shocks explain roughly 50 percent. Technology shocks are also the most important source of inflation dynamics and generate 50 percent of its business cycle movements through marginal cost fluctuations. A key

[9] Our CM specification abstracts from real rigidities that are often introduced in New Keynesian models to generate inflation persistence.

Table 2—Posterior Variance Decomposition (Business Cycle Frequencies)

	SBM(B)		SBM(PT)	
Shock	Mean	90% Interval	Mean	90% Interval
Output				
Government spending	0.51	[0.43, 0.61]	0.53	[0.42, 0.60]
Money demand	0.05	[0.03, 0.07]	0.06	[0.03, 0.09]
Monetary policy	0.12	[0.07, 0.17]	0.12	[0.06, 0.18]
Technology	0.32	[0.23, 0.40]	0.29	[0.21, 0.38]
Target inflation	0.01	[0.00, 0.01]	0.01	[0.00, 0.01]
Inflation				
Government spending	0.18	[0.14, 0.23]	0.17	[0.13, 0.21]
Money demand	0.01	[0.00, 0.01]	0.01	[0.00, 0.02]
Monetary policy	0.23	[0.17, 0.28]	0.21	[0.15, 0.25]
Technology	0.50	[0.45, 0.58]	0.51	[0.45, 0.58]
Target inflation	0.08	[0.05, 0.12]	0.10	[0.06, 0.13]
Inverse velocity				
Government spending	0.44	[0.38, 0.49]	0.46	[0.40, 0.52]
Money demand	0.52	[0.46, 0.57]	0.50	[0.44, 0.55]
Monetary policy	0.02	[0.02, 0.03]	0.03	[0.02, 0.03]
Technology	0.02	[0.01, 0.03]	0.01	[0.00, 0.02]
Target inflation	0.00	[0.00, 0.00]	0.00	[0.00, 0.00]
Real money balances				
Government spending	0.11	[0.07, 0.14]	0.12	[0.08, 0.16]
Money demand	0.70	[0.65, 0.74]	0.69	[0.63, 0.73]
Monetary policy	0.13	[0.09, 0.17]	0.13	[0.09, 0.16]
Technology	0.07	[0.05, 0.11]	0.06	[0.03, 0.10]
Target inflation	0.00	[0.00, 0.00]	0.00	[0.00, 0.00]

Note: Real money balances are measured in terms of the CM good.

feature of the search-based models is their nonseparable structure, meaning that even under an interest-rate feedback rule, the economy is not insulated from money demand shocks. These shocks arise from time-varying taste for the goods produced in the decentralized market and explain around 5 percent of output fluctuations and about 70 percent of the cyclical fluctuations of real money balances.

Impulse response functions to a technology shock[10] for SBM(B) are depicted in Figure 2. A positive technology shock decreases current and future expected marginal costs. As a result, the increase in technology on impact creates an immediate decrease in prices in the DM, which is reflected in the response of DM/CM relative price and DM inflation. Due to the rise in productivity, CM and DM production increase on impact. According to the estimated monetary policy rule, the central bank responds to negative inflation and positive GDP growth by lowering the nominal interest rate. The drop in interest rates reduces the opportunity costs of holding money and raises the demand for DM goods and, hence, real money balances. Recall that according to our timing convention, time t real money balances reflect end-of-period holdings. As a result of this increased demand for DM goods, after period 1, DM inflation increases as does CM inflation. CM inflation (not shown) has a typical negative hump-shaped response since price adjustments in the CM are

[10] Impulse response functions to other shocks are available in our NBER Working Paper 14870.

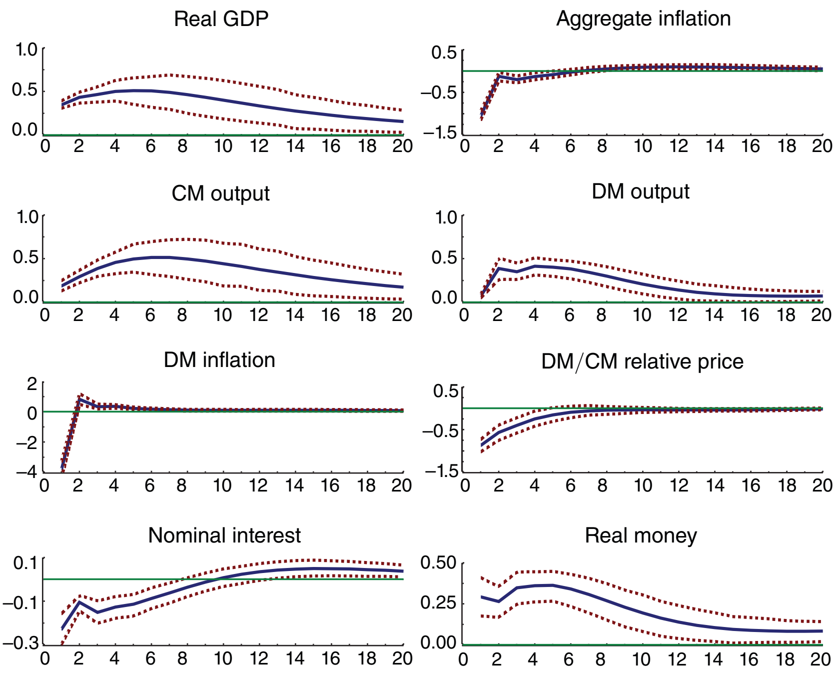

Figure 2. Impulse Responses to Technology Shock

Notes: The figure depicts pointwise posterior means and 90 percent credible intervals of impulse responses for the SBM(B) model. Responses of inflation and the federal funds rate are measured in percentage points; responses of real output, real money balances, and relative prices are measured in percentage deviations from the steady state.

subject to the Calvo friction. The DM inflation, on the other hand, reacts instantly to shocks and mimics very closely the changes in the interest rate. We consider this to be evidence that DM prices are less sticky than CM prices.[11] Output and consumption in both markets show a hump-shaped response after the shock prolonged by the expansionary policy of the central bank. Since the technology shock is transitory, CM and DM output eventually return to their steady-state levels.

E. *Model Fit*

In order to assess the fit of the estimated search-based DSGE model, we will consider two reference models. The first is a standard New Keynesian DSGE model in which real money balances directly enter the utility function (MIU) in an additively separable manner. The second reference model is a restricted vector autoregression, in which the target inflation rate evolves exogenously. We consider various measures of relative fit, including marginal log likelihood values, in-sample root mean

[11] Using a simulation of our model, we find that the aggregate inflation has an autocorrelation of between 0.34 and 0.53, which is broadly in line with the data, as it should be. This can be decomposed into CM inflation persistence of between 0.74 and 0.91 and DM inflation persistence of around −0.10.

squared errors (RMSE), and discrepancies between the DSGE model and the VAR impulse response functions.

MIU Model.—We construct the MIU model by shutting down the decentralized market ($\sigma = 0$) in the search-theoretic models described in Section I and adding a real-money balance term to the households' instantaneous utility function:

$$\mathcal{U}_t = U(x_t) - Ah_t + \frac{\chi_t}{1-\nu}\left(\frac{m_t}{P_t}\frac{A}{Z_*^{1/1-\alpha}}\right)^{1-\nu}. \tag{18}$$

The shock χ_t captures time-varying preferences for money, and the parameter ν controls the interest-rate elasticity of money demand. The scaling by $A/Z_*^{1/(1-\alpha)}$ can be interpreted as a reparameterization of χ_t, which has the effect that steady-state velocity stays constant as we change A and Z. To mimic the timing conventions in the search-based models, we assume that m_t is the (predetermined) money stock at the beginning of the period, and P_t is the price at which the final good is sold in period t. A detailed description of the model and its approximate solution can be found in the Web Appendix. For the common parameters, we impose the same restrictions and use the same prior distributions as in the estimation of SBM(B) and SBM(PT). In addition, we assume that the parameter ν is a priori distributed according to a gamma distribution with mean 20 and variance 5. The posterior estimate of ν is 31.75.

VAR.—We collect output, inflation, interest rates, and inverse velocity in the 4×1 vector $\mathbf{y}_{1,t}$ and the target inflation rate in the scalar $y_{2,t}$. Moreover, we let $\mathbf{y}_t = [\mathbf{y}_{1,t}', y_{2,t}]'$. We assume that y_t follows a Gaussian vector autoregressive law of motion subject to the restrictions that the target inflation rate evolves according to a random walk process and that the innovations to the target inflation rate are orthogonal to the remaining shocks. These restrictions are consistent with the assumptions that underlie our DSGE model and identify the propagation of unanticipated changes in the target inflation. The VAR takes the form

$$\mathbf{y}_{1,t} = \mathbf{\Phi}_0 + \mathbf{\Phi}_1 \mathbf{y}_{t-1} + \cdots + \mathbf{\Phi}_p \mathbf{y}_{t-p} + \mathbf{\Psi}\Delta y_{2,t} + \mathbf{u}_{1,t}, \tag{19}$$

$$y_{2,t} = y_{2,t-1} + \sigma_{\pi*}\epsilon_{\pi*,t}, \tag{20}$$

where $\mathbf{u}_{1,t} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_{11})$ and is independent of $\epsilon_{\pi*,t}$. We estimate the VAR composed of (19) and (20) with $p = 4$ using the version of the "Minnesota" prior described in Thomas Lubik and Schorfheide (2006).[12]

According to the log marginal likelihoods reported in Table 3, the bargaining version of the SBM is slightly preferred over the price taking version. A comparison of the RMSEs suggests that the ranking is mainly due to differences in the RMSE for

[12]The Minnesota prior tilts the estimates of the VAR coefficients toward univariate unit root representations. The hyperparameters are $\tau = 0.1$, $d = 3.1$, $w = 5$, $\lambda = 1$, $\mu = 1$. Our prior assumes that the elements of $\mathbf{\Psi}$ are independently distributed according to $\mathcal{N}(0, \lambda^{-2})$.

Table 3—Marginal Data Densities and RMSEs

			In-Sample RMSE			
Model		$\ln p(\mathbf{Y}^T)$	Output	Inflation	Interest	Inverse velocity
SBM(B)	$\hat{\sigma} = 0.32$	−998.43	0.81	1.18	1.41	2.17
SBM(PT)	$\hat{\sigma} = 0.30$	−1,007.26	0.83	1.18	1.42	2.32
MIU	$\hat{\nu} = 31.8$	−949.14	0.86	1.08	1.06	1.43
VAR(4)		−924.14	0.85	0.96	0.87	1.31
SBM(B)	$\sigma = 0.06$	−1,126.00	0.83	1.08	1.15	3.22
SBM(PT)	$\sigma = 0.06$	−1,126.59	0.83	1.08	1.15	3.20
MIU	$\nu = 5.15$	−1,092.52	0.86	1.09	1.07	2.39

Notes: The marginal data densities for all models are computed conditional on the four observations from 1964:I to 1964:IV that are used to initialize the lags of the VAR. The RMSEs are computed at the posterior mode and measured as follows: output is in percentage deviations from the linear trend, inverse velocity is in percentage deviations from the sample mean, inflation and interest rates are in annualized percentages.

inverse velocity. However, by and large the estimated models produce very similar impulse-response dynamics which makes it difficult to identify the pricing mechanism for the bilateral exchange from the aggregate data. The MIU model attains an even larger marginal likelihood value than SBM(B). While the MIU's in-sample output predictions are slightly less precise, the inflation, interest rate, and velocity forecasts are more accurate than those of the search-based models.

The two main differences between the MIU model and the SBMs are that, first, the MIU model has only one sticky-price sector, whereas the SBMs are composed of a sticky price and a flexible price sector that are aggregated into GDP. Second, the MIU model has a separable structure that insulates the economy from money demand shocks. We will focus on the latter aspect. The estimated value of ρ_χ in the MIU model is 0.98, and most of the variation in real money is explained by the highly persistent money demand shock $\tilde{\chi}_t$, which has no effect on output, inflation, and interest rates. Thus, the weak correlation between real money and the other variables in the data allows the money demand shock in the MIU model to capture real money balance fluctuations without compromising the fit for any other variable. In contrast, aggregate output and inflation in the search-based models are not insulated from money demand shocks.[13]

Peter N. Ireland (2004), using likelihood-based methods, finds that when money is included as observable, US data tend to prefer a separable MIU model to a more general MIU model where real money balances and consumption enter utility in a nonseparable way. The relative ranking of the MIU model and the search-based models we present in this paper is consistent with Ireland's (2004) finding. On the other hand, Giovanni Favara and Paolo Giordani (2009) provide VAR evidence that money demand shocks generate output and inflation fluctuations. This evidence cannot be reconciled with the separable MIU model, in which the economy is insulated from money demand shocks. Our interpretation of these findings is that nonseparable

[13] When we produce the filtered signals shutting down all shocks except for the money demand shocks (not shown), in the MIU model output, inflation and the interest rate are completely flat while for our model there are small but nontrivial fluctuations.

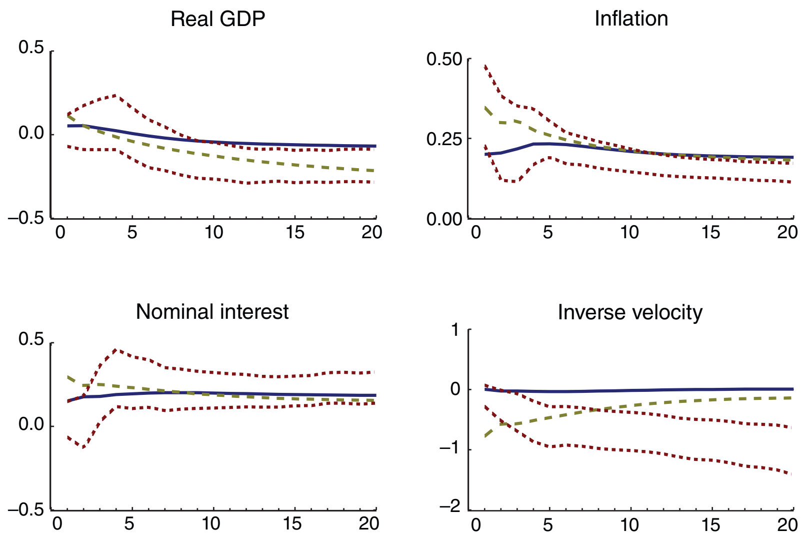

Figure 3. Impulse Responses to Inflation Target ($\epsilon_{\pi,t}$) Shock

Notes: Figure depicts pointwise posterior 90 percent credible intervals of impulse responses for VAR (short dashes) and posterior mean responses for SBM(B): σ estimated (solid); $\sigma = 0.06$ (long dashes). Responses of inflation and fed funds rate are measured in annualized percentages and responses of real output and inverse velocity are measured in percentage deviations from the steady state.

structure in the SBM could provide a useful starting point for understanding monetary aggregates, but it does suffer from some misspecification. This interpretation is consistent with the marginal likelihood differentials of the estimated DSGE models relative to the VAR. The VAR relaxes the restrictions that the DSGE models place on the joint dynamics of output, inflation, interest rates, and velocity, and thereby attains a substantially higher marginal likelihood than all three estimated DSGE models.

F. *Money Demand*

As discussed in detail in Schorfheide (2000), since the VAR attains a better time series fit than the DSGE models, we can use its implications with respect to the propagation of shocks as a benchmark to assess the search-based models. In a typical VAR, identification of structural shocks requires restrictions that may not necessarily be in line with the restrictions imposed by the DSGE model. In our setup, since we use the same law of motion for the inflation target in both the VAR and the DSGE model and treat it as observable in our DSGE model, the VAR identification scheme fits squarely with our assumptions. Figure 3 depicts impulse responses to a target inflation shock that raises inflation by about 20 basis points in the long run computed from the VAR (short-dashed bands show 90 percent credible sets) and SBM(B)(solid lines show the posterior mean). The DSGE model restrictions imply that the long-run responses of inflation, nominal interest rates, and target inflation are identical. While the long-run responses of the VAR are unconstrained, the impulse response bands for inflation and interest rates are approximately consistent with a 20-basis-point increase over long horizons.

The DSGE model is able to reproduce the VAR estimates of the real GDP, inflation, and interest rate response functions. The only striking discrepancy between DSGE model and VAR responses arises from inverse velocity. According to the VAR, the initial response of inverse velocity is sluggish, but after 20 periods it has fallen 100 basis points below its mean value, whereas the model-based responses are an order of magnitude smaller. The VAR estimates indicate that the interest elasticity of money demand is much lower in the short run than in the long run. According to the posterior mean VAR responses, interest rates increase 5 basis points in the first period and velocity drops roughly 10 basis points below its steady-state value. After five periods, the interest rate is up 26 basis points and velocity is down 58 basis points. In the long run, after 200 periods (not shown), the interest rate has increased by 29 basis points, whereas velocity has fallen about 210 basis points.

The discrepancy between the DSGE model and the VAR can be explained by the inability of our DSGE model to match both short- and long-run interest elasticities of money demand. In our model the nominal return on a bond is R_t, whereas the expected nominal return on holding money is

$$E_t\left[\sigma \frac{\chi_{t+1}\, u'(q_{t+1})}{c_q(q_{t+1}, K_{t+1}, Z_{t+1})} + (1 - \sigma)\right],$$

which depends on the realization of the idiosyncratic taste shock as well as the money demand shock. The term $\chi_{t+1}u'(q_{t+1})$ captures the marginal utility of consuming q_{t+1} units of the DM good, and $c_q(\cdot)$ captures the marginal disutility of producing it. Thus, the smaller the probability of participating in the DM, the more interest rate sensitive the return to holding money conditional on participating in the DM has to be to equate the expected returns on bond and money holdings. Since in equilibrium the return to holding money is (inversely) proportional to money balances, the interest rate elasticity of money demand has to be decreasing in σ. It can be shown through a log-linear approximation that in SBM(PT) both the short- and long-run interest semi-elasticity is given by $R_*/(R_* - 1 + \sigma)$.[14]

According to our posterior estimates, the interest semi-elasticity is about 3 in SBM(PT). Stephen M. Goldfeld and Daniel E. Sichel (1990) estimate the short-run interest semi-elasticity to be around one. Estimates of the long-run semi-elasticity reported in Lucas (2000), James H. Stock and Mark W. Watson (1993), and Laurence Ball (2001) range from 5 to 11. Thus, the likelihood-based estimation picks up the low short-run elasticity. In Figure 3, this is reflected in DSGE model–based velocity responses that are small at all horizons. Since the interest rate elasticity is important for the strength of the Friedman channel in our subsequent welfare calculation, we

[14] Note that for small interest rates, the elasticity with respect to the gross interest rate is equal to the semi-elasticity with respect to the net interest rate.

consider a second set of DSGE model parameter estimates in which we constrain σ to be approximately 0.06, which raises the interest semi-elasticity from 3 to 13 in SBM(PT).

Only the estimates of parameters that govern the dynamics of the money demand shock and the price rigidity in the CM are significantly affected by restricting σ. The estimated persistence of χ drops slightly and the standard deviation σ_χ increases dramatically because the velocity forecasts are deteriorating. The implied size of the decentralized market shrinks from 20 percent to 4 percent of GDP, which yields smaller estimates for ζ and ι. Less price rigidity in the CM is needed to capture the same aggregate inflation dynamics. Due to the parameter restriction, the log marginal data density for the two search-based models drops by more than 100 points (see Table 3) and the in-sample RMSE of inverse velocity rises from 2.17 to 3.22 for SBM(B) and from 2.32 to 3.20 for SBM(PT). The RMSEs for output, inflation, and interest rates do not change by the same order of magnitude. Thus, imposing a low value of σ in the search-based models leads to an unambiguous deterioration of time series fit.

Our estimated MIU model suffers from the same problem. Just as the search-based models, it is unable to match the long-run elasticity of money demand and capture the VAR-implied long-run response of inverse velocity to a target inflation rate shock (MIU responses are not shown in Figure 3). When we reestimated the MIU model subject to the restriction $\nu = 5.15$, which implies an increase of the interest semi-elasticity from 2 to 12, we observe a similar deterioration in fit as for SBM(B) and SBM(PT).[15]

Figure 3 also depicts the posterior mean impulse response to an inflation target shock from the restricted version of SBM(B) using long-dashed lines. For $\sigma = 0.06$ the initial response of inverse velocity is almost 100 basis points, which lies outside the VAR credible interval, while the unrestricted model captures the small short-run response of inverse velocity. After 20 periods, inverse velocity is about 10 basis points below its steady-state level in the restricted model, which is still small but closer to the VAR credible interval. The long-run response (not shown in the figure) is about -50 basis points, whereas the 90 percent VAR credible interval ranges from -60 to -390 basis points. Thus, given the restrictions generated by the search-based DSGE models, we can match either the short-run or the long-run interest rate elasticity of money demand, but not both.

G. *Sensitivity Analysis*

In order to examine the robustness of our key parameter estimates that control the magnitudes of the New Keynesian and the monetary distortion, we reestimated the SBM(PT) based on various assumptions about the target inflation rate as well as for different subsamples. Moreover, we consider a version of the price-taking model in which a fraction of the capital holdings is liquid and can be used for DM purchases.

[15] Pablo A. Guerron-Quintana (2009) points out the inability of a standard monetary model to match both elasticities, and considers a model where in every period only a fraction of households are able to reoptimize their money balances to successfully match them.

Table 4—Sensitivity Analysis for Key Parameters, SBM(PT)

		ζ		ι		2σ	
Specification	Sample	Mean	90% Interval	Mean	90% Interval	Mean	90% Interval
Benchmark	1965:I to 2005:I	0.84	[0.80, 0.88]	0.57	[0.31, 0.82]	0.59	[0.52, 0.66]
Estimated ψ_1	1965:I to 2005:I	0.86	[0.82, 0.90]	0.83	[0.71, 0.98]	0.69	[0.61, 0.78]
Latent $\pi_{*,t}$	1965:I to 2005:I	0.89	[0.87, 0.92]	0.84	[0.71, 0.97]	0.68	[0.60, 0.76]
Constant π_*	1965:I to 2005:I	0.91	[0.87, 0.95]	0.72	[0.46, 0.97]	0.35	[0.29, 0.43]
Constant π_*	1965:I to 1979:IV	0.89	[0.87, 0.91]	0.85	[0.71, 0.99]	0.57	[0.50, 0.63]
Constant π_*	1984:I to 2005:I	0.84	[0.78, 0.90]	0.61	[0.17, 0.99]	0.64	[0.52, 0.79]
Liquid capital	1965:I to 2005:I	0.83	[0.79, 0.88]	0.54	[0.28, 0.77]	0.59	[0.52, 0.66]

Note: For convenience, we reproduce the key parameter estimates for the specification reported in Table 1.

Since ζ, ι, and σ are the most important parameters for the magnitude of the two distortions of interest, we summarize their estimates in Table 4.

As discussed previously, we decided to fix the central bank's response to inflation deviations at $\psi_1 = 1.7$. If we estimate this parameter instead, the Markov chain seems to become less stable and the parameter drifts to a value close to one. Given the accumulated evidence about monetary policy rule coefficients, we decided to fix the coefficient at 1.7 for our benchmark empirical analysis, which spans the period from 1965 to 2005. The second row of Table 4 indicates that if ψ_1 is estimated despite the aforementioned problems, $\hat{\zeta} = 0.86$ stays roughly the same, and both $\hat{\iota} = 0.83$ and $\hat{\sigma} = 0.35$ increase compared to the benchmark estimation.

Similar results emerge if we treat the target inflation rate as a latent variable rather than an observable. If we assume that the target inflation rate had been constant between 1965 and 2005, the estimate of ζ increases to about 0.91 and $\hat{\sigma} = 0.18$ drops, implying a slightly larger interest elasticity of money demand. Subsample estimates under the assumption that the target inflation rate is constant are similar to the full sample estimates obtained if the target inflation rate is treated as a latent variable.

As we will discuss in more detail in Section III, larger values of ι weaken the New Keynesian distortion and create a greater incentive for the policymaker to choose a target inflation rate that keeps the nominal interest rate strictly below zero. On the other hand, large values for σ and the implied lower interest elasticity of money demand tend to reduce the monetary distortion and hence the welfare costs associated with positive nominal interest rates. We verified that, on balance, the parameter estimates obtained from this sensitivity analysis tend to push the optimal target inflation rate closer to the lower bound of -2.5 percent. Thus, the results reported in Section III, to the extent that they are sensitive to the assumptions made in the benchmark estimation, tend to overestimate the optimal inflation rate.

Finally, since the time period in the model is a quarter, it is likely that agents can liquidate some of their assets to make purchases in the DM during a period. To investigate this possibility, we consider an extension of SBM(PT) in which buyers can use a fraction a of their capital stock holdings k_t^b to acquire goods in the decentralized market.[16] The extended model is similar to the one studied by Lagos and Rocheteau (2008), with the main exception that in our version capital is used as a factor of production in the DM. Let d^m (d^k) denote the amount of money (capital) transferred from buyer to seller. The constraint of the buyer is now given by

$$\tilde{p}q = d^m + P\xi d^k, \qquad d^m \leq m^b, \qquad d^k \leq ak^b \tag{21}$$

and the value functions (7) and (8) have to be modified accordingly. The equality in (21) implies that the value of the purchased goods has to equal the value of the transferred assets. Here P is the price of the CM good in terms of the currency, and ξ is the price of a unit of capital in this transaction. In equilibrium, ξ is set such that the seller is indifferent between accepting money or capital. This price reflects that the seller can only rebalance her asset portfolio at the end of the decentralized market, which implies that she will earn the rental rate of capital for d^k while the CM is open. The two inequalities in (21) imply that the money and capital used in the transactions cannot exceed the buyer's holdings m^b and ak^b, where ak^b is the fraction of capital that is liquid. The remainder of the model is identical to the price-taking model described in Section I. A detailed description of the equilibrium conditions is provided in Aruoba and Schorfheide (2010).

If the fraction a of liquid capital is small, there exists an equilibrium in which money and liquid capital coexist as a medium of exchange in the DM. In this equilibrium, the buyer spends all her money and liquid capital in the bilateral meeting. In addition, there always exists an equilibrium in which money is not valued. However, since we are using the model to explain observations from an economy in which money is valued, we restrict our attention to the monetary equilibrium. Bayesian inference for the liquid capital model is based on the same prior distribution that we used for SBM(PT). In addition we have to specify a prior distribution for a. We use a Gamma distribution centered at 0.05 with standard deviation 0.03. The right tail of this prior distribution contains values for which the monetary equilibrium does not exist. Hence, we truncate the joint prior for all model parameters to ensure the existence of a unique rational expectations equilibrium in which money is valued. The resulting (truncated) marginal prior for a has a mean of 0.033 and a standard deviation of 0.02.

The liquid capital model leads to a more general money demand function that also includes the capital stock and the return of holding capital while the DM is open. While this generalized money demand function can in principle improve the fit of the search-based model, it turned out that our liquid capital specification was empirically not successful. The posterior distribution of a concentrates near zero and its marginal likelihood is lower than that of the SBM(PT) specification. The remaining parameter estimates are essentially identical to the ones reported in the last two columns of Table 1.

[16] We thank the coeditor and Ricardo Lagos for suggesting this extension.

III. Steady-State Welfare Implications

To illustrate the relative magnitude of the monetary distortion and the New Keynesian distortion in the estimated search-based DSGE model, we compute the steady-state welfare effects of changes in the long-run inflation target π_*. The only sources of uncertainty are the realization of the Calvo shock on the firm side and households' opportunity to engage in a bilateral exchange in the DM. All aggregate shocks are set to zero and hence aggregate outcomes are nonstochastic. We hereby expect to capture the most important first-order effects. The social welfare function, which places equal weights on all households up to a constant, is given by

$$V(\pi_*) = \sigma[u(q_*) - c(q_*, k_*, Z_*)] + U(x_*) - Ah_*. \tag{22}$$

We solve for the percentage change required in x_* and consumption in the DM (the q_* in $u(q_*)$) to make the households indifferent between two economies with different steady-state inflation rates. We use an annual inflation rate of 2.5 percent as a benchmark, which is the average inflation rate at the end of our sample.[17]

The monetary distortion and the New Keynesian distortion constitute opposing channels through which changes in the long-run inflation target affect welfare in our search-based DSGE model. First, an increase in inflation raises the opportunity cost of holding money, reduces real money balances, and reduces the equilibrium consumption in the DM, which will directly reduce welfare. Since capital is used as an input to DM production, the return to holding capital falls, due to the drop in DM consumption, leading to reduced investment in the CM. This will further depress real activity in the CM, including consumption. A version of this channel, which we label the Friedman channel, is present in virtually all monetary models and it underlies Friedman's prescription of a 0 percent net nominal interest rate. In traditional models of money demand, the opportunity cost of holding money can be measured by the area under the money demand curve, as first discussed by Martin J. Bailey (1956) and subsequently, for instance, by Lucas (2000). Ben Craig and Rocheteau (2008) show that this result extends to search-based models under certain conditions: in the basic LW model without holdup problems, the area under the money demand curve very closely approximates the consumption-equivalent welfare measure. Thus, the strength of the Friedman channel in our model is very sensitive to the interest rate elasticity of money demand, which in turn depends on the parameter σ.

Second, our model has a nominal rigidity that prevents a fraction of firms in each period from choosing their prices optimally. This relative price distortion is captured by the deviation of D_t in (15) from unity and moves the economy inside the production possibility frontier. The welfare loss associated with this distortion becomes more severe as the steady-state inflation rate moves away from 0 percent (in both directions) because it becomes more costly for firms not to adjust their prices. The magnitude of the distortion is an increasing function of the probability ζ that firms

[17] We replace $(1 - g_*^{-1})\mathcal{Y}_*$ with simply a constant G_* obtained from the estimations to prevent any welfare effects coming through this term.

are unable to reoptimize their price and it is decreasing in the degree of dynamic indexation ι. Moreover, monopolistic competition among intermediate good producers leads to a positive markup in the CM, given by λ, and generates an additional distortion by moving the real wage rate away from the marginal product of labor.[18] We label this link between long-run inflation and welfare the New Keynesian channel, discussed more extensively in Wolman (2001).

Our discussion makes it clear that the Friedman channel and the New Keynesian channel have opposing implications for welfare. The welfare loss of inflation from the Friedman channel is eliminated in the steady state if the central bank's inflation target equals minus the real interest rate, which is determined in our model by the rate of time preference. On the other hand, the loss due to the New Keynesian channel is minimized around a 0 percent inflation target. When both channels are present, the inflation rate that minimizes the overall distortions may be at either of the two extremes, or somewhere in between. Much of the recent literature that uses cashless New Keynesian models as tools for policy analysis operates under the premise that the distortion resulting from the Friedman channel is negligible. We will subsequently show that this is not the case for our estimated search-based DSGE model.

Figure 4 plots the welfare cost of deviating from the benchmark target of 2.5 percent inflation for four versions of the model using the posterior mean parameter estimates discussed in the previous section. In particular, we distinguish between the bargaining (B) and price-taking (PT) specifications and consider versions in which σ was either estimated to capture the short-run interest elasticity of money demand (SR) or fixed to capture the long-run interest elasticity (LR). All versions show that some target in the interval between the Friedman rule and price stability is strictly better than the benchmark target, with a gain between 0.2 percent and 0.6 percent of consumption. This is not surprising, since all the channels we identified above agree that positive inflation is not desirable. While welfare costs are fairly steep to the right of 0 percent, they are quite flat to the left of 0 percent. For some versions, the welfare difference between the Friedman rule and price stability is less than 0.05 percent. Thus, in this target inflation region, the Friedman channel and the New Keynesian channel work in opposite directions and their strengths are similar.

As we compare the four DSGE model versions, several simultaneous changes need to be accounted for. First, the Friedman channel is amplified in the bargaining version of the search-based model through two hold-up problems that are explained in detail in AWW: a buyer tries to take advantage of the fact that the seller's capital yields a return only if it is used for DM production. On the other hand, a seller tries to exploit that the buyer's money yields utility only if it is used to purchase DM goods. As a consequence, buyers (sellers) bring too little money (capital) to the DM relative to what is socially optimal. These holdup problems, especially the money holdup problem, become more severe as inflation increases, since higher inflation further reduces the benefit of holding money and accumulating capital. Thus, as we replace bargaining by price-taking, the holdup problems disappear, which reduces

[18] It is common to use a labor income subsidy to offset the effect of the positive markup. As Schmitt-Grohe and Uribe (2007) also note, we find this arbitrary and refrain from doing so, especially given our objective of finding the net welfare effect of all the distortions in our model.

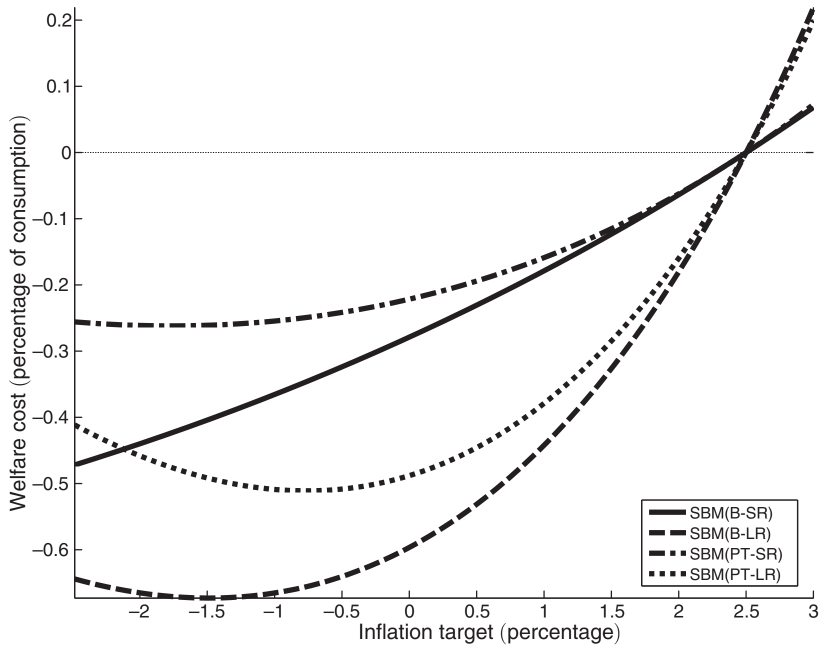

Figure 4. Steady-State Welfare Costs

Notes: Welfare costs of deviating from a 2.5 percent target in terms of consumption. They are calculated at the posterior mean parameter estimates of the four models. Negative numbers correspond to welfare gains.

the welfare gain of moving to the Friedman rule. Second, as we move from SR to LR versions, two effects are present. The money demand curve becomes steeper, which increases the area underneath the curve in the region to the left of 0 percent inflation. This rotation strengthens the Friedman channel. However, after the model is reestimated, the estimates of ζ are only slightly lower, while the estimates of ι drop significantly. The reduced dynamic indexation strengthens the New Keynesian channel substantially. As a result, the welfare cost curve for the LR versions are lower (reflecting the increased Friedman channel) and more convex (reflecting the increased New Keynesian channel).

To isolate the effects of the holdup channel and the slope of the money demand curve from that of the New Keynesian channel, Figure 5 plots welfare costs, holding the New Keynesian channel fixed at $\zeta = 0.81$ and $\iota = 0.09$. We choose these parameters because among the four sets of estimates reported in Section IIC, these values maximize the strength of the New Keynesian channel. Figure 5 highlights that the Friedman channel is larger in the bargaining version than in the price-taking version, and it becomes stronger as one increases the interest elasticity of money demand. Therefore, returning to Figure 4, it seems that going from SR to LR, the New Keynesian channel increases in strength relative to the Friedman channel. We deduce that except for the bargaining (SR) version, which has a very sharp prediction about welfare in favor of the Friedman rule, the welfare gains of reducing the

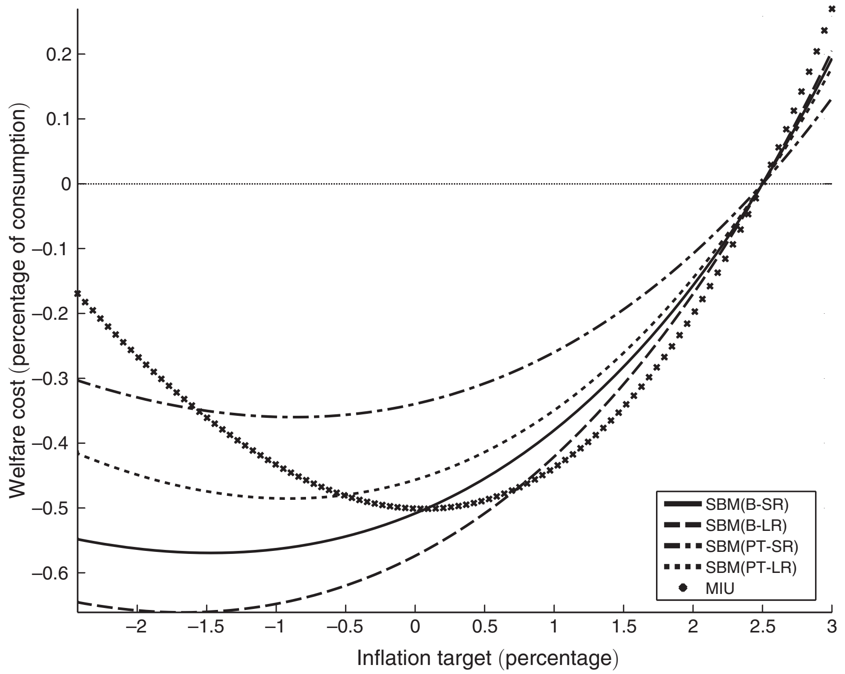

FIGURE 5. STEADY-STATE WELFARE WITH FIXED NEW KEYNESIAN CHANNEL

Notes: Figure depicts welfare costs fixing the parameters of New Keynesian channel at $\zeta = 0.81$ and $\iota = 0.09$.

target inflation rate below 0 percent are fairly insensitive to the actual value that is chosen.

We also plot the welfare costs implied by our estimated MIU model in Figure 5 for $\zeta = 0.81$ and $\iota = 0.09$. For the MIU model the monetary distortion is clearly dominated by the New Keynesian distortion, and welfare costs are almost symmetric around 0 percent inflation. Thus, we conclude that it matters how the monetary friction is modelled: based on our estimated search-based DSGE models, there is evidence that for inflation rates between -2.5 percent and 0 percent the distortion caused by the Friedman channel is not negligible relative to the New Keynesian distortion.

IV. Conclusion

As an alternative to the commonly used cashless New Keynesian model, or its "cash-filled" MIU counterpart, we have developed an estimable DSGE model in which the presence of a decentralized market creates an incentive for households to hold money, and money's role as a medium of exchange emerges endogenously. Using postwar US data on output, inflation, interest rates, and (inverse) velocity, we estimate several versions of our search-based DSGE model and document its empirical fit. While the money demand relationships derived from our particular model specifications had some difficulties capturing the relationship between monetary aggregates on the one hand and output, inflation, and interest rates on the other hand, we view this paper as a promising first step in bridging the gap between the theoretical literature on micro-founded models of money and the empirical literature on estimable DSGE models, and in providing empirical insights that help improve the theoretical models. A steady-state analysis of the welfare effects of inflation suggests that empirical versions of micro-founded models of money demand may pose a challenge to proponents of cashless New Keynesian models: in the vicinity of a zero-inflation steady state, the distortions from monetary frictions may be of similar magnitude as the distortions created by the New Keynesian friction, and hence may not be negligible after all.

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