

# Obsolescence and modernization in the growth process

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## Abstract

In this paper, an endogenous growth model is built up incorporating Schumpeterian creative destruction and embodied technological progress. Under embodiment, long run growth is affected by two opposite effects: (i) obsolescence costs add to the user cost of capital, which have a negative effect on research efforts; and (ii) the modernization of capital increases the demand for investment goods, raising the incentives to undertake research activities. Applied to the understanding of the growth enhancing role of both capital and R&D subsidies, we conclude that the positive effect of modernization generally more than compensates the negative effect of obsolescence.

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## 1. Introduction

An important issue for growth and development theory is the role of subsidies to both capital accumulation and R&D activities. In the neoclassical growth framework,

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subsidizing capital has no permanent impact on the growth rate, since investment only matters in the short run while technological progress is the sole determinant of per capita growth in the long run. In contrast, facilitating and subsidizing R&D and technology adoption should foster long term growth. Moreover, in R&D based growth models, à la [Romer \(1990\)](#), subsidies to research activities, which precisely drive technological progress, are effective in boosting growth.

The dichotomy between capital accumulation and technological progress was at the heart of the *embodiment controversy* in the 1960s, as recently pointed out by [Hercowitz \(1998\)](#).<sup>1</sup> Supporters of the embodiment hypothesis argued that investment is the channel through which innovations are implemented. Since investment plays a *modernization* role under embodiment, it should be a decisive determinant of long run growth. In this paper, we claim that the growth enhancing role of capital subsidies needs to be analyzed in an endogenous growth model with embodied technical change, and we show that the main implications of the embodiment assumption for fiscal policy cannot be captured if the modernization role of investment is neglected, which gives a theoretical support to the *importance of the embodied question*.

[Howitt and Aghion \(1998\)](#) show that Romer's result is biased by the assumption that labor is the sole input in the production of research. Indeed, if the R&D sector employs capital as an input, subsidizing capital is growth enhancing. Paradoxically, Howitt and Aghion suggest that if new technologies are embodied in new machines, embodiment actually has the effect of weakening the result that a capital subsidy will affect long run growth. The reason is that replacement adds *obsolescence costs* to the user cost of capital, reducing the incentives to innovate.

Nonetheless, the empirical literature suggests that the modernization of capital is growth enhancing. [DeLong and Summers \(1991\)](#) find that countries with high growth rates are precisely those with both large investment rates and fast decline rates in the relative price of equipment. These observations capture the modernization role and the embodied nature of technological progress. [Wolff \(1991\)](#), for a sample of seven OECD countries, finds that catch-up in total factor productivity is highly correlated with capital accumulation. He also concludes that embodiment plays a central role in this relationship as productivity growth is highly sensitive to the age of the capital stock. [Bardhan and Pralle \(1996\)](#) notice the significant difference in the saving rate between Latin America and East Asia, and invoke the modernization role of investment to explain fast economic growth in East Asia and relative stagnation in Latin America.

In this paper, we introduce capital and embodied technical progress in a Schumpeterian growth model à la [Aghion and Howitt \(1992\)](#). The model we propose is an endogenous growth version of [Greenwood et al. \(1997\)](#). As in [Howitt and Aghion \(1998\)](#), the user cost of capital is increased by obsolescence costs. However, obsolescence costs are of a different nature: in Howitt and Aghion, the scrapping of machines due to the replacement of obsolete technologies increases the user cost of capital; in our framework, technologies are infinitely lived, but the investment-specific

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<sup>1</sup> See [Denison \(1964\)](#), [Phelps \(1962\)](#) and [Solow \(1960\)](#).

nature of technical progress permanently increases the productivity of the investment sector generating a stable decline of investment prices, which adds capital losses to the user cost of capital. More important, in our framework, research activities are exclusively addressed to the improvement of productivity in the investment sector. It is the simplest way to introduce embodied technical change. For this reason, profitability in the R&D sector depends crucially on the demand for investment goods. When technological progress is high, the demand for investment goods is high too, which raises the incentives to undertake research activities. This is the *modernization* role of embodiment. When applied to understanding subsidies to both capital and R&D activities, we show that the modernization effect more than compensates obsolescence costs, which contradicts the Howitt and Aghion claim that embodiment reduces the incentives to innovate and the efficacy of subsidies to capital.

The endogenous growth model with embodied technical change in this paper is close to the models by Krusell (1998) and Hsieh (2001), which use an explicit R&D sector à la Romer (1990), and the Arrowian learning-by-doing model designed by Bardhan and Priale (1996).<sup>2</sup> Firstly, our obsolescence mechanism is related to *investment-specific technical change*, as in Greenwood et al. (1997) and Krusell (1998). The occurrence of an innovation improves the productivity of the investment sector and lowers the relative price of investment goods. This reduction in the relative price of investment generates the same type of capital losses as in Solow (1960), reducing the incentives to invest and innovate. In Bardhan and Priale, the obsolescence mechanism is similar to the *inflationary wage scheme* first put forward by Solow et al. (1966). Technological progress pushes wages upward, which ends up by exhausting the quasi-rents extracted from the existing capital goods until they become obsolete and are scrapped. A similar mechanism is highlighted in the work of Solow (1960), as referred to by Hsieh. The equalization of labor marginal productivities across vintages reduces the allocation of labor to old sectors, reducing the value of old capital vintages. The associated capital losses, the so-called obsolescence costs, add to the user cost of capital. This kind of mechanism takes place under a vintage structure when some complementarity between capital and labor exists.

Secondly, for Bardhan and Priale, the modernization role of investment shows up in the negative relation between the growth rate and the lifetime of machines.<sup>3</sup> In our model, the modernization mechanism is different since the lifetime of capital goods is infinite. Actually, we identify a *modernization mechanism* based on the investment to capital ratio, an inverse function of the average age of capital. This mechanism requires investment being a determinant of the profitability of the R&D sector (which could be an adoption-imitation sector in the case of developing countries) to be effective. An increase in the rate of technical progress has multiplicative effects by raising the investment to capital ratio, which in turn affects the rate of technological progress. A similar mechanism is at work in Krusell and Hsieh.

<sup>2</sup> See also the LBD model with embodiment in Boucekine et al. (2003).

<sup>3</sup> A similar mechanism is at work in the second part of Hsieh (2001).

This paper is organized as follows. In Section 2, the proposed model is solved and its main properties are stressed. In particular, it is compared with the exogenous growth model with embodied technical change by Greenwood et al. (1997), and the R&D growth model with embodied technical change by Krusell (1998) and Hsieh (2001). Section 3 is devoted to the analysis of the effects on growth of capital and research subsidies. The results for the endogenous growth models with disembodied technical change are pointed out. The main economic mechanisms, obsolescence and modernization, are discussed. Sufficient conditions under which the modernization effect dominates the obsolescence effect are stated and interpreted. Section 4 concludes.

## 2. The model

The model in this paper is based on Aghion and Howitt (1992), and it introduces embodied technical change as in the work of Solow (1960). There are two final sectors, one producing a non-durable good, and another producing an investment good. Technology in the non-durable sector is Cobb-Douglas on capital and labor, and the non-durable good is allocated to consumption and as an input in both the production of intermediate goods and R&D activities. Technology in the investment goods sector is constant elasticity of substitution on a continuum of intermediate inputs. Both final sectors are competitive. In the intermediate sector, a continuum of differentiated goods is produced under monopolistic competition. Technology in this sector only employs non-durable goods as inputs, and benefits directly from innovations developed in the R&D sector. Finally, the R&D sector is competitive. For comparative reasons, we analyze two alternative technologies, one uses the non-durable good as an input and the other uses labor.

The reduced form of the economy is very similar to the two sector exogenous growth model of Greenwood et al. (1997), hereafter GHK, where one sector employs capital and labor to produce a non-durable good, and the other sector produces an investment good using the non-durable good as the sole input. In our model, the transformation of non-durable goods in investment goods requires an intermediate step, i.e. the production of intermediate inputs. In both models, technological progress benefits the investment sector, and requires new investments to propagate over the whole economy; the so-called embodied nature of technological progress.

Howitt and Aghion (1998), hereafter HA, combine the neoclassical growth model à la Solow (1956) and their 1992 creative destruction model, and share some key properties with our model, in particular that quality improvements are the engine of growth and profits in the intermediate sector are the main incentive to innovate. However and differently from our model, HA assume that technical progress is disembodied: A single final goods sector produces both consumption and investment goods, and benefits from quality improvements in the intermediate sector. As it is shown in the next, the different nature of technological progress is crucial to understand the different economic mechanisms at work in these two models.

## 2.1. Growth under embodiment

As in the standard optimal growth model, an infinitely lived *representative dynasty* endowed with one unit of labor maximizes intertemporal utility. The Euler equation related to the dynasty problem is

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\sigma}(r_t - \rho). \quad (1)$$

As usual, the growth rate of per capita consumption,  $C_t$ , depends on the difference between the interest rate  $r_t$  and the dynasty's discount rate  $\rho$ , multiplied by the intertemporal elasticity of substitution ( $1/\sigma$ ), which is supposed to be constant and strictly positive.

Technology in the *investment sector* displays constant returns to scale on a continuum of intermediate inputs in the interval  $[0,1]$ :

$$I_t = \left( \int_0^1 x_{jt}^\alpha dj \right)^{\frac{1}{\alpha}}, \quad (2)$$

where  $I_t$  is per capita investment,  $x_{jt}$  is the per capita amount of the intermediate good  $j$  used in the production of the investment good, and  $\alpha \in ]0,1[$ . The problem of the representative firm in the investment sector is purely static. She takes prices as given and maximizes current profits subject to the technological constraint (2). The optimal demand for the intermediate good  $j$  is:

$$x_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{\frac{1}{\alpha-1}} I_t, \quad (3)$$

where  $P_t$  is the price of the investment good and  $p_{jt}$  is the price of the  $j$ th intermediate good, both measured in units of the non-durable good (which is taken as the numeraire). The so-called 'true price index' associated with technology (2) is given by

$$P_t = \left( \int_0^1 p_{jt}^{\frac{\alpha}{\alpha-1}} dj \right)^{\frac{\alpha-1}{\alpha}}. \quad (4)$$

Technology in the *intermediate sector* is linear in the sole input, the non-durable good. The marginal productivity in the production of the  $j$ th intermediate good is  $q_{jt}$ . The sector is under monopolistic competition and each intermediate good is produced by one firm only. Given the demand function (3), the  $j$ th monopolist optimally sets the price

$$p_{jt} = \frac{1}{\alpha} \frac{1}{q_{jt}}. \quad (5)$$

The markup  $1/\alpha$  is constant and equal for all monopolists, and  $1/q_{jt}$  is the marginal cost. From Eqs. (4) and (5), we can compute the price of the investment good

$$P_t = \frac{1}{\alpha Q_t}, \quad (6)$$

where

$$Q_t = \left( \int_0^1 q_{jt}^{\frac{\alpha}{1-\alpha}} dj \right)^{\frac{1-\alpha}{\alpha}} \quad (7)$$

is a quality index of the inputs used in the production of the durable good. The price of the investment good is an inverse function of the average quality of intermediate inputs. In a stationary growth regime,  $Q_t$  must be growing at a positive constant rate, implying that the price of investment goods must be permanently declining. Notice also that the price of any intermediate good relative to the price of the investment good is  $p_{jt}/P_t = Q_t/q_{jt}$ . It depends on its relative quality only. More efficient intermediate goods are sold at smaller prices.

The per capita amount of non-durable goods employed in the production of intermediate goods is given by

$$X_t = \int_0^1 \frac{x_{jt}}{q_{jt}} dj = \frac{I_t}{Q_t}. \quad (8)$$

The equality on the left hand side of Eq. (8) can be easily obtained from Eq. (3),  $(p_{jt}/P_t) = (Q_t/q_{jt})$  and Eq. (7). It is important to notice that technological improvements in the intermediary sector lower the production costs of investment goods, since a reduced amount of nondurable goods is required to produce one unit of investment goods. Technological progress is embodied in new machines, implying that new investments are needed to profit from the advances in technology.

In the *non-durable goods sector*, technology is Cobb-Douglas,  $K_t^\alpha(1-l_t)^{1-\alpha}$ , where  $K_t$  is the per capita stock of capital and  $1-l_t \in [0,1]$  is the fraction of the labor endowment devoted to produce non-durable goods. The law of motion of the per capita capital stock is

$$\dot{K}_t = I_t - \delta K_t, \quad (9)$$

where  $\delta > 0$  is the depreciation rate.

In this sector, the representative firm takes prices as given and maximizes the discounted flow of profits subject to these technological constraints. Capital accumulation is subsidized at the rate  $\beta_K > 0$ , so that firms face the net of subsidies interest rate  $r_t - \beta_K$ . From the first order conditions of this problem, the marginal productivity of capital must be equal to the corresponding user cost and the marginal productivity of labor must be equal to the wage rate:

$$\alpha K_t^{\alpha-1} (1-l_t)^{1-\alpha} = P_t \left( r_t + \delta - \frac{\dot{P}_t}{P_t} - \beta_K \right), \quad (10)$$

$$(1-\alpha) K_t^\alpha (1-l_t)^{-\alpha} = W_t. \quad (11)$$

As expected, changes in the price of investment goods have a negative effect on the user cost of capital. From Eq. (6), the decline on investment prices is equal to the growth rate of the quality index, i.e.  $\dot{P}_t/P_t = -\dot{Q}_t/Q_t$ . Quality improvements in the intermediate sector move the technological frontier up, reducing the future price of investment goods

and acting as a brake on capital accumulation. This is the so-called *obsolescence cost* related to embodied technical change.

At equilibrium, per capita production of the non-durable good is allocated to consumption,  $C_t$ , and as inputs in the production of both the intermediate sector,  $X_t$ , and the R&D sector,  $N_t$ . All variables are in per capita terms. Formally,

$$C_t + X_t + N_t = K_t^\alpha (1 - l_t)^{1-\alpha}. \quad (12)$$

### 2.1.1. Embodied technical change

Combining Eqs. (1), (6), and (8) to (12), we get

$$\dot{K}_t = Q_t(K_t^\alpha(1 - l_t)^{1-\alpha} - C_t - N_t) - \delta K_t$$

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\sigma} (Q_t \alpha^2 K_t^{\alpha-1} (1 - l_t)^{1-\alpha} - (\rho + \delta + \gamma_t - \beta_K)),$$

where  $\gamma_t \equiv \dot{Q}_t/Q_t$  is the rate of embodied technical change.

This system is very close to the system representing the equilibrium of an optimal growth model with embodied technical change, as in GHK. Along a balanced growth path with positive growth, it can be easily shown that the growth rate of consumption is equal to  $(\alpha/1-\alpha)\gamma$ , and smaller than the growth rate of investment, equal to  $\gamma/1-\alpha$ . Consequently, our model reproduces the main empirical facts related to embodiment. Firstly, from Eq. (6) the relative price of investment permanently declines at the rate of embodied technical change  $\gamma$ . Secondly, the investment to output ratio permanently increases.<sup>4</sup>

The three main differences between our model and GHK are the following. Firstly, the production of the final good and the time endowment may also be allocated to R&D activities. Secondly, the obsolescence cost  $\gamma_t$  is endogenous. Its behavior is analyzed in the next subsection. Finally, monopolistic competition in the intermediate goods sector implies that the marginal productivity of capital is multiplied by  $\alpha$ , the inverse of the markup rate. Under exogenous growth, i.e.  $l_t = 0$ ,  $N_t = 0$ ,  $\gamma_t = \gamma > 0$ , and perfect competition in the intermediate goods sector, these two equations become equivalent to the dynamic system in GHK.

### 2.2. Schumpeterian R&D activities

The rate of embodied technical change is endogenized following Aghion and Howitt (1992). Let  $q_{jt} = q^{\kappa_{jt}}$ , where  $q^{\kappa_{jt}}$  represents the quality grad of the  $j$ th intermediate good at time  $t$ ,  $q > 1/\alpha$  being a constant.<sup>5</sup>  $\kappa_{jt}$  represents the number of quality improvements of the

<sup>4</sup> Based on the economic theory on index numbers, Licandro et al. (2002) find that under embodied technical progress the growth rate of output must be defined as in NIPA's methodology, i.e., a linear combination of the growth rate of both consumption and investment, the weights being the corresponding nominal shares. Consequently, the growth rate of output is smaller than the growth rate of investment.

<sup>5</sup> This condition states that the difference in quality between two successive innovations should be sufficiently large, that the current innovation displaces the previous one.

$j$ th intermediate good achieved up to date  $t$ . As usual in this literature, a researcher discovering a new quality grade is supposed to have the monopoly right to produce the good at the obtained quality. Consequently, when a new quality improvement occurs, incumbents lose automatically their monopoly rentals. This feature generates a Schumpeterian process of creative destruction.

How does the creative destruction process take place for the  $j$ th intermediate good? Let  $q^{\kappa_j}$  be the leading quality grade at time  $t$ . If a researcher successfully introduces an innovation at this time, the quality grade increases to  $q^{\kappa_j+1}$ . The innovation is assumed to come out according to a Poisson process, where  $\eta_{\kappa_j}$  denotes the Poisson arrival rate. We consider two alternative technologies in the R&D sector:

*Technology A*

$$\eta_{\kappa_j} = \lambda n_{\kappa_j} \phi(\kappa_j),$$

where  $n_{\kappa_j}$  is the amount of non-durable goods devoted to research and  $\lambda > 0$  measures the productivity of this sector.

*Technology B*

$$\eta_{\kappa_j} = \lambda Q_t^{\frac{\alpha}{1-\alpha}} l_{\kappa_j} \phi(\kappa_j),$$

where  $l_{\kappa_j}$  is the amount of labor services devoted to research and  $\lambda Q_t^{\frac{\alpha}{1-\alpha}} > 0$  measures the productivity of this sector.

In both cases, the Poisson arrival rate  $\eta$  is supposed to be a decreasing function of the research task, here captured by  $\kappa$ , representing the negative effect of research complexity. More precisely, we set  $\phi(\kappa) = q^{-(\kappa+1)\frac{\alpha}{1-\alpha}}$ , implying  $\phi'(\kappa) < 0$ . This assumption is consistent, as it is shown later, with an equilibrium Poisson probability ultimately independent of the complexity of the research task.

The expected value of an innovation discovered at time  $t$ ,  $V_{\kappa_j+1,t}$ , is equal to the expected flow of profits it generates. The instantaneous profits of the  $\kappa_j + 1$  innovator, for all time  $z \geq t$  until she will be displaced by the  $\kappa_j + 2$  innovator, are given by

$$\pi_{\kappa_j+1,z} = \left( p_{jz} - \frac{1}{q^{\kappa_j+1}} \right) x_{jz} = \frac{1-\alpha}{\alpha} \frac{I_z}{q^{\kappa_j+1}} \left( \frac{q^{\kappa_j+1}}{Q_z} \right)^{\frac{1}{1-\alpha}}.$$

The last equality comes after substitution of  $p_j$  from Eq. (5) and  $x_j$  from Eq. (3), using the condition  $p_j/P = Q/q_j$ . Given the embodied nature of technological progress, the benefits of R&D accrue to the investment sector. Consequently, the instantaneous profits of an innovator depend on the demand for investment goods only, in contrast to HA where technological progress is disembodied and benefits also the consumption sector. Under embodiment, the scope of R&D is restricted to the investment sector, enlarging the importance of investment in the growth process.

Then,

$$V_{\kappa_j+1,t} = \int_t^\infty \pi_{\kappa_j+1,z} e^{-\int_t^z (r_s + \eta_{\kappa_j+1,s}) ds} dz, \quad (13)$$



where the two exponential terms in the integrand represent respectively the discount factor and the probability of the quality grade  $q^{\kappa_j+1}$  still leading at time  $z > t$ .

Let us assume that the research sector is competitive and research is subsidized at the rate  $\beta_R$ . The arbitrage condition for a strictly positive amount of resources spent in R&D activities stipulates that the marginal cost of research should be equal to the expected present value of profits, that is:

$$\theta_t = \lambda \phi(\kappa_j) V_{\kappa_j+1,t}. \quad (14)$$

where

$$\theta_t = \begin{cases} 1 - \beta_R & \text{if technology A} \\ (1 - \beta_R) W_t Q_t^{\frac{\alpha}{1-\alpha}} & \text{if technology B.} \end{cases} \quad (15)$$

This arbitrage condition implies  $\frac{\dot{V}_{\kappa_j+1,t}}{V_{\kappa_j+1,t}} = \frac{\dot{\theta}_t}{\theta_t}$ , which yields by differentiation of Eq. (13)

$$V_{\kappa_j+1,t} = \frac{1-\alpha}{\alpha} \frac{q^{(\kappa_j+1)\frac{\alpha}{1-\alpha}} Q_t^{\frac{1}{1-\alpha}} I_t}{r_t + \eta_{\kappa_j+1} - \frac{\dot{\theta}_t}{\theta_t}}. \quad (16)$$

From Eqs. (14) and (16), it turns out that the Poisson arrival rate  $\eta_{\kappa_j+1}$  does not depend on the complexity of the research task, that is  $\eta_{\kappa_j+1} = \eta_{\forall j}$ . This means that quality improvements can occur for all types of intermediate goods with the same probability, whatever the quality grade is. This property of the model is entirely due to the specification of function  $\phi(\kappa)$ , as outlined by Barro and Sala-i-Martin (1995). The Poisson arrival rate  $\eta_{\kappa}$  is affected by  $\kappa$  in two opposite ways. First, the monopoly profits accruing to an innovator increase with  $\kappa$ , since its productivity depends directly on it. Secondly, by assumption, the probability of innovating decreases with the difficulty of the task, measured by  $\kappa$ . When the specification  $\phi(\kappa) = q^{-(\kappa+1)\frac{\alpha}{1-\alpha}}$  is adopted, the two effects exactly offset.

As set out above, at equilibrium Poisson arrival rates are equal for all intermediate goods. By the Law of Large Numbers, the average growth rate of  $Q(t)$  is

$$\gamma_t = \tilde{q} \eta_t, \quad (17)$$

where  $\tilde{q} = \frac{\alpha}{1-\alpha} (q^{\frac{\alpha}{1-\alpha}} - 1)$ . From Eqs. (16) and (17), the arbitrage condition (14) can be written as

$$\theta_t = \lambda \frac{1-\alpha}{\alpha} \frac{Q_t^{\frac{1}{1-\alpha}} I_t}{r_t + \frac{\gamma_t}{\tilde{q}} - \frac{\dot{\theta}_t}{\theta_t}}. \quad (18)$$

Under *technology A*, the equilibrium allocation of non-durable inputs to the production of R&D activities is given by  $N_t = \int_0^1 n_{\kappa_{jt}} dj$ . Since the equilibrium Poisson arrival rates are the same for all intermediate goods, using Eq. (17), we get after some trivial algebra:

$$N_t = \frac{\gamma_t}{\tilde{\lambda}} Q_t^{\frac{\alpha}{1-\alpha}}, \quad (19)$$

where  $\tilde{\lambda} = \frac{\lambda \tilde{q}}{q^{1-\alpha}}$ . The equilibrium allocation of labor to the production of R&D activities is  $l_t = 0$ .

Under *technology B*, the equilibrium allocation of labor to the production of R&D activities is given by  $l_t = \int_0^1 l_{kjt} dj$ . Since the equilibrium Poisson arrival rates are the same for all intermediate goods, using Eq. (17), we get after some trivial algebra:

$$l_t = \frac{\gamma_t}{\tilde{\lambda}}, \quad (20)$$

The equilibrium allocation of non-durable inputs to the production of R&D activities is  $N_t = 0$ .

### 2.3. Balanced growth path equilibrium

In order to characterize the equilibrium of this economy, some variable changes are introduced to eliminate trends. Concerning non-durable consumption and inputs,  $z_t = Z_t Q_t^{\frac{\alpha}{1-\alpha}}$  for  $Z \in \{C, X, N\}$ . Concerning capital and investment,  $z_t = Z_t Q_t^{\frac{1}{1-\alpha}}$  for  $Z \in \{I, K\}$ . Implicit in this transformation is that consumption grows at a smaller rate than both investment and capital, a direct implication of embodied nature of technical change. We define a balanced growth path (BGP) as an equilibrium path along which detrended variables are constant. It is characterized by the following equation system

$$\alpha k^{\alpha-1} (1-l)^{1-\alpha} = \frac{1}{\alpha} (r + \delta + \gamma - \beta_K) \quad (K)$$

$$\theta = \lambda \frac{1-\alpha}{\alpha} \frac{km}{r + \frac{\gamma}{q}} \quad (A)$$

$$r = \sigma \frac{\alpha}{1-\alpha} \gamma + \rho \quad (R)$$

$$m = \left( \frac{\gamma}{1-\alpha} + \delta \right). \quad (M)$$

$$\theta = \begin{cases} (1 - \beta_R) & \text{if technology A.} \\ (1 - \beta_R)(1 - \alpha)k^\alpha(1-l)^{-\alpha} & \text{if technology B.} \end{cases} \quad (W)$$

$$l = \begin{cases} 0 & \text{if technology A.} \\ \frac{\gamma}{\tilde{\lambda}} & \text{if technology B.} \end{cases} \quad (L)$$

Eq. (K) is the optimal condition for capital and it expresses the steady state value of capital intensity as a function of the user cost of capital multiplied by the markup rate. It is derived from Eqs. (10) and (6). Eq. (A) is the arbitrage condition in the R&D sector, Eq. (18), where  $m \equiv i/k$  is an index of the modernization of capital. The larger  $m$  is, the larger the weight of new machines in the stock of capital and the lower the average age of it. Eq. (R) is the standard Fisher equation showing how the interest rate depends on the growth

rate of consumption along the BGP. It comes from the Euler Eq. (1). We derive Eq. (M) from Eq. (9). It simply states that, at the balanced growth path, the investment to capital ratio must be equal to the depreciation rate plus the growth rate of capital. Eq. (W) defines the marginal cost of R&D activities and it is obtained from Eq. (15), after substitution of  $W_t$  from Eq. (11). Eq. (L) reports the amount of labor devoted to R&D activities.

Before pursuing the analysis, let us first establish a sufficient condition for existence and uniqueness of a balanced path with positive growth.

**Proposition 1.** *If  $\lambda > \underline{\lambda}$  there exists a unique solution to the system (K)–(L) with  $\gamma > 0$  and*

$$\underline{\lambda} = \begin{cases} \frac{(\delta + \rho - \beta_K)^{\frac{1}{1-\alpha}} \rho (1 - \beta_R)}{\Gamma \delta} & \text{if technology A} \\ \frac{(\rho + \delta - \beta_K) \rho (1 - \beta_R)}{\alpha \delta} & \text{if technology B} \end{cases}$$

where  $\Gamma = (1 - \alpha) \alpha^{\frac{1+\alpha}{1-\alpha}}$ .

**Proof.** To prove existence and uniqueness of strictly positive solutions for  $\gamma$ , some tedious algebra is needed.

*Technology A*

Indeed after successive substitutions from Eqs. (K), (R), (M), (W) and (L) into (A), we can write  $\gamma$  as an implicit function of the parameters of the problem:

$$1 - \beta_R = \frac{\Gamma \lambda \left( \frac{1}{1 - \alpha} \gamma + \delta \right)}{\left[ \left( \sigma \frac{\alpha}{1 - \alpha} + \tilde{q}^{-1} \right) \gamma + \rho \right] \left[ \left( \sigma \frac{\alpha}{1 - \alpha} + 1 \right) \gamma + \delta + \rho - \beta_K \right]^{\frac{1}{1-\alpha}}} \equiv \Lambda_A(\gamma). \quad (21)$$

It is easy to check that function  $\Lambda_A(\gamma)$  has the following properties: (i)  $\Lambda_A(0) = \frac{\Gamma \lambda \delta}{(\rho + \delta - \beta_K)^{\frac{1}{1-\alpha}} \rho}$ , (ii) the limit of  $\Lambda_A$  is zero when  $\gamma$  tends to infinity, (iii)  $\Lambda_A$  is continuous and strictly increasing in  $\lambda$  and (iv) there is at most one  $\gamma > 0$  such that  $\Lambda'_A(\gamma) = 0$ .

*Technology B*

After successive substitutions from Eqs. (K), (R), (M), (W) and (L) into (A), we can write  $\gamma$  as an implicit function of the sole parameters of the problem:

$$1 - \beta_R = \frac{\alpha \lambda \left( 1 - \frac{\gamma}{\lambda} \right) \left( \frac{1}{1 - \alpha} \gamma + \delta \right)}{\left[ \left( \sigma \frac{\alpha}{1 - \alpha} + \tilde{q}^{-1} \right) \gamma + \rho \right] \left[ \left( \sigma \frac{\alpha}{1 - \alpha} + 1 \right) \gamma + \delta + \rho - \beta_K \right]} \equiv \Lambda_B(\gamma) \quad (22)$$

It is easy to check that function  $\Lambda_B(\gamma)$  has the following properties: (i)  $\Lambda_B(0) = \frac{\alpha \lambda \delta}{(\rho + \delta - \beta_K) \rho}$ , (ii) the limit of  $\Lambda_B$  is zero when  $\gamma$  tends to  $\lambda \tilde{q} q^{-\frac{1}{1-\alpha}} > 0$ , (iii)  $\Lambda_B$  is continuous and strictly increasing in  $\lambda$  and (iv) there is at most one  $\gamma > 0$  such that  $\Lambda'_B(\gamma) = 0$ .

From properties (i)–(iv) follows that for all  $\lambda > \underline{\lambda}$ , there is only one strictly positive solution to (22).  $\square$

Proposition 1 states that productivity in the research sector should be large enough for a BGP with positive growth to be sustainable. Though this kind of conditions is very often required in endogenous growth models (even in the simplest ones, see [Romer, 1990](#)), it is absolutely needed in our framework to additionally rule out multiplicity.<sup>6</sup> Positivity of  $k$  can then be easily showed. Finally, as in the standard growth model the condition  $\rho > (1 - \sigma) \frac{\alpha}{1-\alpha} \gamma$  is required to get bounded utility, and it implies that  $c$  and  $i$  are strictly positive along the BGP.

#### 2.4. On Krusell and Hsieh

It is well-known in the endogenous growth literature that the different models of R&D have very similar reduced forms. Of course, this is also the case of endogenous growth models with embodied technical progress. For this reason, the reduced forms of [Krusell \(1998\)](#) and [Hsieh \(2001\)](#)'s models are very close to the system (K)–(L) in this section. Consequently, our main results can also be generated using any of these two models.

### 3. On the impact of subsidies

#### 3.1. Under disembodied technical progress

HA take a different view on technological progress, by imposing conditions that make it disembodied. The key assumption is that there is only one final good sector producing simultaneously the consumption and the investment good. Technology in the final sector is given by  $(1 - l_t)^{1-\alpha} \int_0^1 A_{jt} x_{jt}^\alpha dj$ , where as in the previous sections  $(1 - l_t)$  is the fraction of labor allocated to production,  $A_{jt}$  represents the state of technology in sector  $j$  and  $x_{jt}$  is the per capita amount of input  $j$  employed in the production of the final good. The production of one unit of the intermediary input  $j$  requires  $A_{jt} x_{jt}$  units of machines. Consistently, they define per capita aggregate capital as

$$K_t \equiv \int_0^1 A_{jt} x_{jt} dj = A_t x_t, \quad (23)$$

where  $A_t = \int_0^1 A_{jt} dj$ , and  $x_{jt} = x_t$  for all  $j$  is a direct implication of symmetry in sectorial technologies. From the previous assumptions, they get

$$C_t + I_t + N_t = A_t^{1-\alpha} K_t^\alpha (1 - l_t)^{1-\alpha}. \quad (24)$$

Given their definition of capital, technical progress is disembodied, the relative price of investment goods is unity and both consumption and investment grow at the same rate along the balanced growth path. Consequently, there are no obsolescence costs in the user

<sup>6</sup> Under  $\lambda < \lambda_c$ , multiple steady state equilibria are possible, as in [Hsieh \(2001\)](#). Strategic complementarities involved in the modernization mechanism are at the basis of the multiplicity of steady state equilibria in R&D based growth models with embodied technical change.

cost of capital and the benefits of innovation depend on total production instead of investment. More precisely, Eqs. (K) and (A) become

$$\alpha k^{\alpha-1}(1-l)^{1-\alpha} = \frac{1}{\alpha}(r + \delta - \beta_K) \quad (K_D)$$

$$\theta = \lambda(1-\alpha)\alpha \frac{k^{\alpha}(1-l)^{1-\alpha}}{r + \frac{\gamma}{\tilde{q}}}, \quad (A_D)$$

the other equations remaining identical.

Note that Romer (1990)'s model generates equations very similar to Eqs. (K<sub>D</sub>) and (A<sub>D</sub>), with the peculiarity that the term  $\gamma/\tilde{q}$  is zero, since Shumpeterian obsolescence costs are nil. Nevertheless, the main difference between HA and Romer is on the R&D technology. HA assume that R&D uses the final good as input instead of the labor endowment as in Romer. This assumption is critical, as stressed by HA, for the understanding of the role of capital subsidies on growth. When R&D uses labor as input, after substitution of Eq. (W) on Eq. (A<sub>D</sub>), we obtain

$$(1 - \beta_R) = \lambda\alpha \frac{1-l}{r + \frac{\gamma}{\tilde{q}}}.$$

From Eqs. (R) and (L),  $r$  and  $l$  are functions of  $\gamma$  implying that the growth rate does not depend on  $\beta_K$ . As said before, this result does not depend on the assumption that growth is Shumpeterian, but on the extreme assumption that capital is not required for R&D activities. As long as R&D uses capital as a production factor, a subsidy to the accumulation of capital would be good for technological progress and growth.

Subsidies to capital accumulation increase both capital per capita and output per capita, affecting the growth rate in two opposite ways. Firstly, an increase in final production causes the rise in profits to an innovator, right hand side of Eq. (A<sub>D</sub>), and secondly an increase in capital raises the marginal product of labor, raising R&D costs, Eq. (W). In the special case of disembodied technical change and labor as the sole input in R&D, as in Romer (1990), these two effects exactly cancel, because wages and profits are proportional to output per worker, as the previous equation shows. This implies that the growth rate does not depend on capital accumulation.

### 3.2. Modernization and obsolescence

Under embodiment, the growth rate is affected through two main economic mechanisms. The first is related to the effect of obsolescence on the user cost of capital, the so-called *obsolescence costs*, represented by the term  $\gamma$  on the right hand side of Eq. (K). It mitigates any positive effect on long run growth. The reason is straightforward. A rise in the rate of technical change increases the user cost of capital by raising the decline rate of investment prices, which generate capital losses to capital owners. This increase in the user cost of capital reduces the demand for capital, Eq. (K). Since the expected value of R&D is positively related to the demand for capital, Eq. (A), a decrease in the latter reduces the intensity of R&D activities and, consequently, the rate of technical progress.

The second feature is related to the crucial role of investment in the modernization of the capital stock. When R&D is stimulated by an exogenous parameter change that impinges on the research arbitrage condition (A), the resulting rise in productivity growth reinforces the stimulus by raising investment. This increase in investment comes about because a faster rate of innovation implies a modernization of capital, as can be seen in Eq. (M). This rise in investment reinforces the stimulus to R&D because it increases the size of the market that can be captured by a successful innovator. Consequently, when more resources are devoted to R&D, the associated increase in the rate of technical progress has multiplicative effects by raising the investment to capital ratio, which in turn affects the incentives to innovate and the rate of technological progress. This is the *modernization mechanism* that the embodiment assumption gives rise to.

The equation system (K)–(L) is very similar to the system governing the balanced growth path equilibrium in HA. There are however two main differences, both related to the diverse nature of technological progress, as it can be seen by comparing Eqs. (K)–(A) to (K<sub>D</sub>)–(A<sub>D</sub>). Firstly, under embodied technical change, the user cost of capital in the right hand side of Eq. (K) involves obsolescence costs  $\gamma$ , while it does not under disembodiment as it can be seen in (K<sub>D</sub>). Finally, under embodiment the expected value of R&D, on the right hand side of Eq. (A), depends on investment, while under disembodiment, on the right hand side of Eq. (A<sub>D</sub>), it depends on final production.

As said in the previous section, subsidies to capital accumulation affect the growth rate by increasing both profits to innovation and R&D costs. Under embodiment, even if labor is the sole input in R&D activities, the positive effect on profits more than compensates the negative effect on costs, and the net effect of the subsidy is positive. Therefore, under embodiment, subsidies to capital accumulation and innovation are growth enhancing. This result is established in Proposition 2.

**Proposition 2.** *Under technologies A and B, if  $\gamma > 0$  at steady state,  $\gamma$  increases when either  $\beta_K$  or  $\beta_R$  increases.*

**Proof.** Proposition 2 follows directly from Eqs. (21) and (22), since  $A_j(\gamma)$ ,  $j = A, B$ , are increasing functions of  $\beta_K$  and the left hand sides of Eqs. (21) and (22) are decreasing functions of  $\beta_R$ .  $\square$

### 3.3. Embodiment in Howitt and Aghion (1998)

In order to analyze the role of embodiment, Howitt and Aghion, Section 4.4, introduce the following assumption: When a new technology is discovered in sector  $j$ , all existing machines become obsolete.<sup>7</sup> It implies, in the particular case of technology A, that at each

<sup>7</sup> More precisely, they assume that machines associated to the old technology are instantaneously scrapped. HA never discuss the optimality of such a rule. However, it is well-known in vintage capital theory that the scrapping of old machines is not necessarily immediate. See Bardhan and Priale (1996) and Boucekine et al. (1997).

instant a fraction  $\lambda n$  of the stock of capital is destroyed. Consequently, the motion law of capital takes the following form

$$\dot{K}_t = X_t - (\delta + \lambda n_t)K_t, \quad (25)$$

where  $\delta > 0$  is the physical depreciation rate. At steady state, Eqs. (24) and (25) are very similar to the standard conditions in optimal growth models, with  $(1-\alpha)\dot{A}_t/A_t$  being the endogenous growth rate of *disembodied* technological progress and  $\delta + \lambda n$  the endogenous depreciation rate. This result is puzzling, because the authors start by assuming that technical progress is embodied, but their model behaves as if it were disembodied. In particular, it does not fit the empirical regularity that the price of equipment is permanently declining with respect to the price of non-durable consumption, neither the observation that the equipment to output ratio is permanently increasing in real terms. The only effect of embodiment is in the depreciation rate: it adds the obsolescence costs  $\lambda n$ , directly associated to the scrapping of machines embodying the replaced technology.

More important, HA's assumptions have strong implications for the incentives to undertake research activities. The zero profit condition in the research sector can be written as in Eq. (A<sub>D</sub>). As in Section 2.4, the main difference with respect to Eq. (A) is that in HA profits from R&D do not depend on investment, but on final production. Given that innovation is addressed to the production of final goods, profits depend on final goods demand not on investment demand. This kills the modernization effect. Additionally, if the R&D sector uses labor as sole input, the economy behaves as if technological progress were disembodied and subsidies to capital have no effect on innovation and the growth rate.

Let us propose the following variable change to better understand what is going on in the Howitt and Aghion model with embodied technical change.

We define

$$J_t = \left( \int_0^1 A_{it}^{1-\alpha} z_{it}^\alpha di \right)^{\frac{1}{\alpha}},$$

where  $z_{it} = A_{it}x_{it}$  represents the number of machines of type  $i$  operative at time  $t$  (measured in the same unit as  $X_t$ ). Given that this technology is constant elasticity of substitution, an improvement in the productivity of any of these technologies would improve the productivity of the others. It makes clear that there is no need of relying on any story about intermediary goods to understand the role of technical progress in this economy: The relevant assumption is that there is no perfect substitution between different vintages in terms of their ability to produce the nondurable good. Notice also that in HA,  $K_t$  measures (per capita) aggregate capital in terms of production costs, but  $J_t$  measures (per capita) aggregate capital in terms of their ability to produce the nondurable good.

It is easy to show that, at the symmetric equilibrium,  $J_t = Q_t K_t$ , where  $Q_t = A_t^{\frac{1-\alpha}{\alpha}}$ . It implies that Eq. (24) becomes

$$C_t + I_t + N_t = J_t^{\alpha} \quad (26)$$

From Eq. (25), we get

$$\dot{J}_t = \underbrace{Q_t X_t}_{I_t} - \left( \delta + \lambda n_t - \frac{\dot{Q}_t}{Q_t} \right) J_t \quad (27)$$

Under this definition of aggregate capital, the relative price of investment goods, given by  $1/Q_t$ , is permanently declining due to embodied technical progress, as in GHK. However, the law of motion of efficient capital includes two additional terms:  $\lambda n_t$ , representing scrapping, and  $\dot{Q}_t/Q_t$ , which represents the gains in productivity coming from the effects of time  $t$  innovations on the whole capital stock (this gain is due to the complementarity of different types of machine in the production of the nondurable good). The net depreciation rate of  $J_t$  is  $\delta + \lambda n_t - \dot{Q}_t/Q_t$ . Indeed, the negative term  $-\dot{Q}_t/Q_t$  cancels with the capital losses due to the permanent decline in investment prices, implying that the user cost of capital only depends on  $\delta + \lambda n_t$ .

Technological progress is driven by a common source, R&D in the investment sector, and it is embodied in new machines. However, given the nature of nondurable goods technology, this common source also generates disembodied technical progress as a result of the complementarity of different capital goods in the production of the nondurable good. It is represented by the term  $\dot{Q}_t/Q_t J_t$  in Eq. (27).

### 3.4. On the impact of subsidies under embodiment

The main concern of this section is to analyze the suggestion by HA that “incorporating embodiment actually has the paradoxical effect of weakening the result that capital subsidies affect long run growth.” HA, Section 4.4, provide an intuitive explanation of the role of obsolescence in their framework, by adding obsolescence costs to the user cost of capital, and found that capital intensity should be smaller at steady state. Consequently, they argue that an increase in capital subsidies has a lower positive effect on growth if obsolescence costs are to be considered. This paper shows that HA’s argument is incomplete, since it does not take into account the modernization effect of investment associated to the embodiment hypothesis. When taken into account, the rise in the rate of technical change due to capital subsidies yields indeed an increase in the investment to capital ratio, which again stimulates research and growth. This additional mechanism may well rule out the main conclusion of HA, namely the negative effect of embodiment on the efficiency of capital subsidies.

Proposition 3 establishes necessary and sufficient conditions for the positive effect of modernization more than compensate the negative effect of obsolescence.

**Proposition 3.** *Under technology A, the modernization effect is larger than the obsolescence effect if and only if  $r - \frac{\alpha}{1-\alpha} \gamma > \beta_K$ .*



**Proof.** The steady state of  $\gamma$  is implicitly defined by Eq. (21) as a function of  $\beta_K$  and  $\beta_R$ . Differentiating Eq. (21), after some algebra, we get

$$\frac{d\gamma}{d\beta_K} = \frac{\frac{1}{(1-\alpha)\tilde{r}}}{\frac{\sigma \frac{\alpha}{1-\alpha} + \tilde{q}^{-1}}{r + \gamma\tilde{q}^{-1}} + \frac{\sigma \frac{\alpha}{1-\alpha}}{(1-\alpha)\tilde{r}} + \underbrace{\frac{1}{(1-\alpha)\tilde{r}}}_{\text{obsolescence effect}} - \underbrace{\frac{1}{(1-\alpha)} \frac{1}{m}}_{\text{modernization effect}}} \quad (28)$$

$$\frac{d\gamma}{d\beta_R} = \frac{(\lambda\Gamma)^{-1}(r + \gamma\tilde{q}^{-1})\frac{1}{m}\tilde{r}^{\frac{1}{1-\alpha}}}{\frac{\sigma \frac{\alpha}{1-\alpha} + \tilde{q}^{-1}}{r + \gamma\tilde{q}^{-1}} + \frac{\sigma \frac{\alpha}{1-\alpha}}{(1-\alpha)\tilde{r}} + \underbrace{\frac{1}{(1-\alpha)\tilde{r}}}_{\text{obsolescence effect}} - \underbrace{\frac{1}{(1-\alpha)} \frac{1}{m}}_{\text{modernization effect}}} \quad (29)$$

where  $\tilde{r} = r + \delta + \gamma - \beta_K$  is the user cost of capital. The modernization effect is higher than the obsolescence effect if only if

$$\tilde{r} > m \Leftrightarrow r - \frac{\alpha}{1-\alpha} \gamma > \beta_K. \quad \square$$

A better understanding of Proposition 3 can be achieved by abstracting from the effects of  $\gamma$  others than those operating through the obsolescence and the modernization mechanisms. From Eq. (K), an increase in the obsolescence cost rises the user cost of capital and reduces the demand for capital. The elasticity of the demand for capital with respect to the obsolescence cost is proportional to the weight of obsolescence costs in the user cost of capital, i.e.  $\frac{dk}{k} = \frac{\gamma}{\tilde{r}} \frac{1}{\alpha-1} \frac{d\gamma}{\gamma}$ . In order for the modernization effect to fully compensate the obsolescence effect, the demand for investment must remain unchanged after the induced reduction in the demand for capital. If not, from Eq. (A) the reduced incentives for R&D should lower the rate of technical progress. Indeed, investment could be accommodated by an increase in the investment to capital ratio, such that  $(dk/k) = (dm/m)$ . From Eq. (M), such a reduction requires the following relation  $\frac{dm}{m} = \frac{\gamma}{1-\alpha} \frac{1}{m} \frac{d\gamma}{\gamma}$  to hold. Consequently, the positive effect of modernization fully compensates the negative effect of obsolescence costs if and only if the user cost of capital  $\tilde{r}$  is equal to the investment to capital ratio  $m$ . Nevertheless, if the user cost of capital is larger (smaller) than the investment to capital ratio, the modernization effect more (less) than compensate the obsolescence effect.

As stated at the end of Section 2.4,  $\rho > (1-\sigma) \frac{\alpha}{1-\alpha} \gamma$  is required to get bounded utility. This condition is equivalent to  $r - \frac{\alpha}{1-\alpha} \gamma > 0$ . Consequently, if capital subsidies are near zero, the modernization effect always dominates, and the positive effect of subsidies is larger under embodiment than under disembodied technical change.

#### 4. Conclusions

This paper introduces capital accumulation and embodied technical progress in a Schumpeterian creative destruction model, following a different strategy than [Howitt and Aghion \(1998\)](#). We show that the embodied nature of technical progress has two main implications. Firstly, the user cost of capital involves obsolescence costs, which affects negatively research incentives. Secondly, the modernization of capital through investment raises the incentives to undertake R&D activities. The modernization of capital is shown to offset the growth losses due to obsolescence costs, which in particular improves the growth enhancing role of subsidies.

The proposed model is consistent with the empirical evidence on embodied technical progress. Additionally, it stresses the modernization role of investment pointed out by the recent empirical literature on development economics. From a theoretical point of view, it reconciles the Schumpeterian approach with the literature on growth with embodied technical change, as in [Greenwood et al. \(1997\)](#), by stressing the importance of the investment-specific nature of technical progress.

Applied to the promotion of economic development, our results can serve to advocate the typical fiscal and trade policies ensuring the modernization of the capital stock. For example, capital subsidies are desirable under the strict condition that the purchased capital goods embody superior technologies. Though this policy is likely to generate relatively high obsolescence costs, the long term growth effects of the resulting modernization are likely to be high enough to compensate the latter costs. Naturally, such a modernization should take into account the necessarily limited capacity of technological absorption of the considered economies, of which the quality of the available skills is a key indicator. To properly address this issue, there is a need to construct less stylized models including a careful modelling of technological absorption capacity. This line of research is on top of our agenda.

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