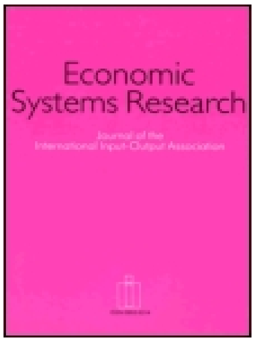

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A World Trade Model Based on Comparative Advantage with *m* Regions, *n* Goods, and *k* Factors

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ABSTRACT *This paper describes the World Trade Model, a linear program that determines world prices, scarcity rents, and international trade flows based on comparative advantage in a world economy with m regions, n goods, and k factors. The new model generalizes the World Model of Leontief et al. (1977) in ways that make it particularly useful for analyzing scenarios about sustainable development. Major properties of the model are demonstrated, and sources of the gains from trade are identified for the world as a whole and for individual regions. Illustrative results are reported for a 10-region, 8-good, 3-factor model of the world economy.*

KEY WORDS: International trade, world model, comparative advantage, linear programming

1. Introduction

A number of large-scale models and databases of the world economy were built in the 1970s, the most prominent initiated by Meadows and Meadows (1972) to identify and describe limitations to economic growth posed by environmental constraints, notably resource availability; by Leontief (1974) and Leontief *et al.* (1977) to evaluate ways of closing the gap in material well-being between rich and poor countries; and by, for example, Klein *et al.* (1976) to analyze the international transmission of business fluctuations. Others have been developed over the years, differing in questions addressed, level of sectoral detail, time horizon, and extent of reliance on economic theory for the representation of interdependence among regions and economic dynamics. In recent years equilibrium models have become the most widely used models of the world economy; namely, computable general equilibrium (CGE) and, to a lesser extent, spatial equilibrium models. The World Trade Model (WTM) presented in this paper is intended as an alternative to these models, and the characteristics that distinguish it are described.

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The Intergovernmental Panel on Climate Change (IPCC) has been extremely successful since its creation within the United Nations system in 1988 in mobilizing cooperation among various communities of natural scientists. Unprecedented numbers of scientists have collaborated on several co-authored assessment reports. These reports are based on a large and expanding body of research in which spatially-disaggregated models of the climate system, joined more recently by models of the biosphere, are used to project future changes in temperature, precipitation, and vegetation associated with increased carbon emissions to the atmosphere. Until now, the main question addressed by economic models within the IPCC framework has been to estimate the money cost of adjusting to these changes. However, the interest of many researchers is now shifting to a different question: what actions could be taken starting now that would have a chance of substantially diminishing human impact on climate, and on the environment more generally? A new generation of world model will be developed to address this question, and I believe that only an input–output model, such as the WTM, has many of the features that will be needed for this inquiry.

In this paper I describe the features of a new model of the world economy intended to interface with representations of the physical environment, such as life-cycle engineering analyses, detailed material flows, or a model of the biosphere. I propose a formulation that generalizes the model of Leontief *et al.* (1977) in several ways, in particular replacing their highly parameterized representation of international trade by a direct comparison of cost structures in different economies. I call the new model the World Trade Model to indicate its relation to their work, which came to be known as the World Model.

The WTM takes the form of a linear programming model of trade in a world with m regions, n goods, and k factors of production. It is a closure of a one-region input–output model for international trade. The values of endogenous variables – output, exports, imports, factor scarcity rents for each region, and world prices for traded goods – are determined through production assignments for all goods that are made according to comparative advantage.

The World Trade Model is intended for analyzing scenarios about actions that could be taken to achieve the environmental and social objectives associated with sustainable development. The availability and use of factor inputs, such as land and fresh water, are measured in physical units. The model minimizes factor use rather than adopting the more typical approach of maximizing consumption or growth. Scenarios describe substantial, not marginal, departures from current practices that are motivated mainly by considerations other than changes in prices and incomes. The model can assess if a scenario is cost-reducing or not relative to a baseline, but it does not require that to be the case. This model would be well-suited to evaluating the likely future impact of rapid economic growth in China and India on demand for resources, world trade in resources, resource prices, and impacts on the environment. The model could be used to investigate the potential for the substitution of a plant-based diet for an animal-based diet for reducing environmental pressures and to assess the extent to which such dietary changes in developed economies could offset the environmental impact of improving diets in the developing world. Another scenario that could be evaluated in this framework is the adoption of biomass-based fuels, even if they are more expensive than fossil fuels, based on environmental or political motivations. Formulating such scenarios requires collaboration with experts in disciplines other than economics, and such collaboration requires that common variables be measured in common units. The World Trade Model manipulates both physical and monetary measures of both factors and goods and can readily handle unpriced factors, such as fresh water, as well.

The World Trade Model can be applied for two distinct purposes. Using data for the past, trade flows and world prices computed by the model can be compared with actual values to assess how closely past trade patterns conform to values computed on the assumption of comparative advantage as embodied in the model and, in turn, to improve and further elaborate the model. Alternatively, the model can be used to analyze the impact of scenarios about substantial changes in the future while assuring consistency among projections of trade patterns, endowments, production, consumption, and prices in all economies. Before the model is used for these purposes, it is vital to document its properties, and that is the objective of this paper.

The remainder of this section describes the historical background of this effort and summarizes the main results of the paper. The next section presents the key assumptions distinguishing the WTM from other models of the world economy. It then describes a no-trade world model and a one-region world model, which are introduced to highlight, by contrast, central features of the World Trade Model. The major properties of the World Trade Model are stated and proved in the third section. Illustrative empirical results obtained for a 10-region, 8-good, 3-factor implementation are reported in the fourth section, and the final section indicates the directions to be pursued in ongoing research.

1.1. Background

One of the earliest models of the world economy was that developed by Leontief *et al.* (1977) and first described in Leontief's (1974) Nobel Lecture. While his earlier contributions on trade were, and remain to this day, enormously influential, by contrast economists have taken remarkably little note of his model of the world economy. (Leontief's work on trade is examined in Duchin, 2004.) One reason for this neglect is that trade flows and prices in the model of Leontief and his colleagues were not determined on the basis of comparative cost structures. My objective in this paper is to propose a linear program that maintains the many desirable features of this model but also represents a substantial improvement in the determination of trade. The new model achieves this objective by incorporating a representation of trade that extends notions of comparative advantage from the $2 \times 2 \times 2$ case to the general $m \times n \times k$ case.

Over half a century ago, Leontief (1953) analyzed the relationship between factor endowments and the factor contents of trade for the United States. His counter-intuitive findings stimulated many attempts to improve the explanatory power of standard interpretations of comparative advantage. Several relatively recent papers in that tradition have demonstrated the need for region-specific factor prices and technologies, in the form of input–output tables, to explain observed trade patterns (Trefler, 1993, 1995; Davis and Weinstein, 2001; Hakura, 2001). Other research has shown that a region's trade potential needs to be assessed relative to all potential trade partners, not one partner at a time (Davis and Weinstein, 2001; He and Polenske, 2001).

Also in the 1950s, Dorfman *et al.* (1958) described the surprisingly deep connection between linear programming and economic theory. For every linear program where the decision variables have a physical interpretation, there is a dual program whose decision variables, one for each constraint in the primal, have a price interpretation. One can solve these programs for optima to linearly constrained problems, in particular identifying the lowest-cost allocation of resources among competing uses. They illustrated the point with a simple linear program that could be solved for international trade flows and the benefits to participating countries.[1] Other chapters in the same volume described the relationship between input–output models and linear programs. More recently, Carter (1970), Leontief (1986), and Duchin and Lange (1995) used linear programming to identify the low-cost choice among alternative technological options in a single country, an approach that is generalized to the many-country case in this paper. Linear programming has been used in a small number of papers to recast existing theorems about international trade (Maiti, 1973; Minabe, 1977). In three articles appearing in this journal, ten Raa and his colleagues described a linear programming framework with endogenous determination of trade based on comparative advantage by integrating an inter-industry representation with standard trade theoretic assumptions. They used this framework in applications with two or three regions, two factors, and differing choices of objective functions and exogenous variables. In a two-country, two-factor model maximizing foreign earnings, they located India's comparative advantage relative to Europe and estimated its efficiency gains (ten Raa and Chakraborty, 1991). A subsequent study repeated Leontief's test of the Heckscher–Ohlin theory for two countries and two factors, with and without similar preferences and technologies, but (unlike Leontief) with trade flows determined endogenously on the basis of relative comparative advantage and utility maximization (ten Raa and Mohnen, 2001). Shestalova (2001) introduced a third region, made relative world prices (but not price levels) endogenous, and then used this framework to refine the measurement of total factor productivity. This body of work has different objectives and characteristics, but it shares with the model described in this paper the feature of direct comparison of alternative cost structures.

After a lapse of a quarter of a century, models of the world economy are once again in demand in connection with prospects for improving the international distribution of income and for reducing global pressures on the environment. While virtually all empirical models of the world economy make use of input–output matrices to achieve consistent sector-level disaggregation, only input–output models make full use of sectoral independence to determine production levels and prices. CGE models rely on the Armington assumption to distinguish goods by their place of origin. This representation allows a region to both import and export a given good but cannot evaluate comparative advantages. Spatial equilibrium models, by contrast, allow for competition among comparable goods, but solutions are not computed on the basis of a general comparison of cost structures. The World Trade Model developed in this paper is an input–output model that is intended to stand on its own in determining the global division of labor and prices according to alternative scenarios. In addition, the basic algorithm could be readily integrated, conceptually and operationally, into other kinds of models, or desirable features of other models could be incorporated into it. For example, selected goods could readily be disaggregated by place of origin in the World Trade Model in cases where the preference for certain origins is assumed more important than relative cost advantages.

1.2. Summary of Main Results

The properties of the World Trade Model are demonstrated in two theorems and four propositions. They are summarized here and presented with proofs in the third section. It will be shown that under the World Trade Model, the world as a whole benefits from trade because the same regional consumption vectors are satisfied as in the absence of trade, but with lower factor use.

World prices are set by the intermediate and factor costs of the producing regions that are designated by a solution to the model. Theorem 1(i) shows that these prices are lower than any region's no-trade prices if there are no scarcity rents.

However, producing regions attempting to satisfy world demand can be expected to exhaust one or more factors, and owners of fully utilized factors earn scarcity rents. In addition, a benefit-of-trade rent may be required to induce some regions to export, even though their production for export is beneficial for the world as a whole. With the emergence of both kinds of rents, prices of goods will rise to values that may exceed no-trade prices, at least the no-trade prices of the producing regions. This case is treated in Theorem 1(ii). Nonetheless, while there is a redistribution of income among factors and among regions, all regions can be said to benefit from trade.

A region is not required to have balanced trade. A net exporter benefits from trade because its factor earnings with trade exceed the cost of consumption at world prices. This is true even though the region uses no more factors than in the absence of trade.

Regions that are net importers also benefit from trade. While they experience a trade deficit, thus incurring debt in order to maintain their consumption, the debt at world prices is lower than the value of the factors saved, as shown in Theorem 2.

A number of illustrative results are reported to provide insights into characteristics of global comparative advantage that could not be examined in any other way. Some of these results include: the superiority of labor-intensive over modern technologies under certain assumptions, the redistribution of factor income toward land associated with an optimal solution, and the prospects for identifying alternative near-optimal solutions that have substantially different implications for the international division of labor and distribution of income.

2. The No-Trade, One-Region, and World Trade Models

Three models are described in this section, starting with a model of a world with no trade. Then a linear program is introduced to solve for trade flows and prices that are optimal for the world as a whole. In the latter model, the world is treated as a single compound region in that all regions' factors are pooled; that is, production takes place in the regions whose technologies are chosen and they are able to use, at their factor prices, factors that may originate in other regions. Finally, I describe the World Trade Model, which differs from that of the one-region world by requiring regions to use only their own factor endowments and by adding a constraint to assure that each region benefits from trade. The different formulations provide a basis for isolating the implications of specific assumptions; precise statements about the logical relationships among the solutions in different models can be found in the third section.

2.1. Key Assumptions

The major strengths of the WTM are the ability to represent scenarios about substantial departures from current practices, trade based on direct cost comparisons (i.e. a direct calculation of comparative advantage in the general $m \times n \times k$ case), the determination of scarcity rents on fully-utilized factors, and tracking of physical quantities in physical units as well as monetary ones.

Demand is exogenous and region-specific, and an increase in demand would be met by higher production – subject to factor constraints. Factor prices are exogenous and region-specific; but factors can also earn scarcity rents, which are endogenous. Factors that are fully utilized in a region earn a scarcity rent; a non-zero rent indicates that the factor is scarce in that region. (An unpriced factor, such as fresh water, may thus have a non-zero scarcity rent in water-stressed regions.) If total world demand cannot be met with the factors available globally, the linear program will be physically infeasible and identified as such by the solution algorithm.

The World Trade Model minimizes factor inputs; and since the factors are measured in physical units, a set of prices is needed to weight the quantities of different factors. The exogenous factor prices, not inclusive of the factor rents, are used for this purpose. These are the same region-specific factor prices that are used to determine the cost of production in the regions that produce for export.

The World Trade Model imposes a constraint to assure that no region will enter into trade unless it benefits economically. There is no balance of trade constraint, but some limitations on trade deficits would be called for in a multi-period or dynamic formulation.

All three model variants are expressed in terms of the following variables and parameters representing n goods and k factors in m regions:

$\mathbf{A}_i$	$n \times n$ matrix of inter-industry production coefficients in region i (exogenous)
$\mathbf{F}_i$	$k \times n$ matrix of factor inputs per unit of output in region i (exogenous)
$\mathbf{x}_i$	$n \times 1$ vector of output in region i (endogenous)
$\mathbf{y}_i$	$n \times 1$ vector of domestic consumption in region i (exogenous)
$\mathbf{f}_{nt,i}$	$k \times 1$ vector of factor use in absence of trade in region i (exogenous)
$\mathbf{f}_i$	$k \times 1$ vector of factor endowments in region i, where $\mathbf{f}_i \geq \mathbf{f}_{nt,i}$ (exogenous)
π_i	$k \times 1$ vector of factor prices in region i (exogenous)
$\mathbf{p}_{nt,i}$	$n \times 1$ vector of commodity prices in absence of trade in region i (endogenous)
$\mathbf{p}_0$	$n \times 1$ vector of world commodity prices (endogenous)
$\mathbf{r}_i$	$k \times 1$ vector of factor scarcity rents in region i (endogenous, simply $\mathbf{r}$ in the one-region world model)
α_i	scalar, benefit-to-trade shadow price (see text) in region i (endogenous)

Note that $\{\mathbf{x}_i\}_j$ denotes the jth entry of the vector $\mathbf{x}_i$.

2.2. No-Trade Model: Production and Consumption in m Closed Economies

A world economy with production and consumption involving n goods and k factors in m closed economies is represented by an input–output quantity model (1) and price model (2) and income equations (3) as follows:

$$\begin{bmatrix} (\mathbf{I}-\mathbf{A}_1) & & 0 \\ & \ddots & \\ 0 & & (\mathbf{I}-\mathbf{A}_m) \\ -\mathbf{F}_1 & & 0 \\ & \ddots & \\ 0 & & -\mathbf{F}_m \end{bmatrix} \begin{bmatrix} \mathbf{x}_{nt,1} \\ \vdots \\ \mathbf{x}_{nt,m} \end{bmatrix} = \begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_m \\ -\mathbf{f}_{nt,1} \\ \vdots \\ -\mathbf{f}_{nt,m} \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} (\mathbf{I}-\mathbf{A}_1') & & 0 & -\mathbf{F}_1' & & 0 \\ & \ddots & & & \ddots & \\ 0 & & (\mathbf{I}-\mathbf{A}_m') & 0 & & -\mathbf{F}_m' \end{bmatrix} \begin{bmatrix} \mathbf{p}_{nt,1} \\ \vdots \\ \mathbf{p}_{nt,m} \\ \\ \pi_\mathbf{1} \\ \vdots \\ \pi_\mathbf{m} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \tag{2}$$

$$\mathbf{p}_{nt,i}'\mathbf{y}_i = \pi_\mathbf{i}'\mathbf{F}_i\mathbf{x}_{nt,i} = \pi_\mathbf{i}'\mathbf{f}_{nt,i}, \quad \forall i \tag{3}$$

Given technologies (represented by the columns of coefficients in $\mathbf{A}_i$ and $\mathbf{F}_i$), consumption requirements ($\mathbf{y}_i$), and factor prices ($\pi_\mathbf{i}$), one can compute output ($\mathbf{x}_i$), factor use ($\mathbf{f}_{nt,i}$),[2] and commodity prices ($\mathbf{p}_{nt,i}$). The equality (equation (3)) of factor payments with the value of final deliveries for region i is derived by transposing the ith matrix equation in equation (2), multiplying through by the vector of output, $\mathbf{x}_i$, and substituting from equation (1). (Note that $-\mathbf{F}$ appears in equations (1) and (2) rather than $\mathbf{F}$ in both places for closer correspondence with the matrix notation in later models. In those models, the sign will control the direction of an inequality: $a \le b \Leftrightarrow -a \ge -b$.)

This is an adaptation of the static, one-region input–output model to the case of m closed regions; it will be used as a point of comparison to quantify a region's benefits from trade in later formulations. It determines for each region the output, factor use, and prices that satisfy given consumption requirements at given factor prices in the absence of trade.

2.3. One-Region World

If the world is treated as one region, determining the optimal division of labor is equivalent to the problem of selecting the most factor-saving technologies in a country that has m choices for producing each good. The latter problem was addressed in (Duchin and Lange, 1995), using an approach that is adapted here to a world model by the pooling of factors. The linear program is set up as follows.

In the primal program:

$$\text{Minimize } Z = \sum_i \pi_i{}'\mathbf{F}_i\mathbf{x}_i$$

subject to

$$\begin{bmatrix} (\mathbf{I}-\mathbf{A}_1) & (\mathbf{I}-\mathbf{A}_2) & \cdots & (\mathbf{I}-\mathbf{A}_m) \\ -\mathbf{F}_1 & -\mathbf{F}_2 & \ddots & -\mathbf{F}_m \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_m \end{bmatrix} \ge \begin{bmatrix} \sum_i \mathbf{y}_i \\ -\sum_i \mathbf{f}_i \end{bmatrix} \tag{4}$$

with

$$\mathbf{x}_i \geq 0, \quad \forall i.$$

Or, in the dual program:

$$\text{Maximize } Z = \mathbf{p}_0' \sum_i \mathbf{y}_i - \mathbf{r}' \sum_i \mathbf{f}_i$$

subject to

$$\begin{bmatrix} (\mathbf{I} - \mathbf{A}_1') & -\mathbf{F}_1' \\ (\mathbf{I} - \mathbf{A}_2') & -\mathbf{F}_2' \\ \vdots & \vdots \\ (\mathbf{I} - \mathbf{A}_m') & -\mathbf{F}_m' \end{bmatrix} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{r} \end{bmatrix} \leq \begin{bmatrix} \mathbf{F}_1' \boldsymbol{\pi}_1 \\ \mathbf{F}_2' \boldsymbol{\pi}_2 \\ \vdots \\ \mathbf{F}_m' \boldsymbol{\pi}_\mathbf{m} \end{bmatrix} \tag{5}$$

with

$$\mathbf{p}_0, \mathbf{r} \geq 0.$$

So, since the programs have a common optimum value:

$$\mathbf{p}_0' \left(\sum_i \mathbf{y}_i \right) - r' \left(\sum_i \mathbf{f}_i \right) = \sum_i \pi_i' \mathbf{F}_i \mathbf{x}_i. \tag{6}$$

The solution to the primal equation (4) is the vector of sub-regional outputs; the program determines the choice of technologies (and thereby the sub-regions where production takes place) that satisfies consumption requirements while minimizing total factor costs evaluated at the relevant factor prices. The solution to the dual equation (5) determines the vector $\mathbf{p}_0$ of world prices and $\mathbf{r}$, a k-vector of scarcity rents on the k factors. The ith matrix inequality in the dual specifies for each of the n goods in the ith sub-region the relationship between world prices, world scarcity rents, and factor costs.

For the optimal solution, the values of the primal and dual objective functions are equal by the Duality Theorem of linear programming (Luenberger, 1989). This identity, shown as equation (6), is analogous to the income equation in the no-trade model (equation (3)) and assures that worldwide final demand, evaluated at world prices, is equal to the payments for utilized factors plus the rents on scarce factors.

2.4. The World Trade Model

The one-region world model makes an optimal selection of technologies from the point of view of the world as a whole. However, in reality production cannot follow the logic it imposes because factors of production have limited mobility and, when they do move, it is generally to regions where they earn the highest return, which may not be where they would satisfy a global cost-minimizing criterion. Some results obtained with this model are reported in the fourth section. The World Trade Model, described below, solves for the optimal international division of labor and world prices based on each region's comparative advantage as reflected in exogenously fixed, region-specific technologies, consumption, factor endowments, and factor prices. A benefit-of-trade constraint assures that a region will enter into trade only to the extent that its imports at no-trade prices are worth at least as much as its exports.

In the primal program:

$$\text{Minimize } Z = \sum_i \pi_i{}'\mathbf{F}_i\mathbf{x}_i$$

subject to

$$\begin{bmatrix} (\mathbf{I}-\mathbf{A}_1) & \cdots & \mathbf{I}-\mathbf{A}_m) \\ -\mathbf{F}_1 & & 0 \\ & \ddots & \\ 0 & & -\mathbf{F}_m \\ -\mathbf{p}'_{nt,1}(\mathbf{I}-\mathbf{A}_1) & & 0 \\ & \ddots & \\ 0 & & -\mathbf{p}'_{nt,m}(\mathbf{I}-\mathbf{A}_m) \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_m \end{bmatrix} \geq \begin{bmatrix} \sum_i \mathbf{y}_i \\ -\mathbf{f}_1 \\ \vdots \\ -\mathbf{f}_m \\ -\mathbf{p}'_{nt,1}\mathbf{y}_1 \\ \vdots \\ -\mathbf{p}'_{nt,m}\mathbf{y}_m \end{bmatrix} \tag{7}$$

with

$$\mathbf{x}_i \geq 0, \quad \forall i.$$

Or, in the dual program:

$$\text{Maximize } Z = \mathbf{p}'_0 \sum_i \mathbf{y}_i - \sum_i \mathbf{r}'_i\mathbf{f}_i - \sum_i \alpha_i(\mathbf{p}'_{nt,i}\mathbf{y}_i)$$

subject to

$$\begin{bmatrix} (\mathbf{I}-\mathbf{A}'_1) & -\mathbf{F}'_1 & & 0 & -(\mathbf{I}-\mathbf{A}'_1)\mathbf{p}_{nt,1} & & 0 \\ \vdots & & \ddots & & & \ddots & \\ (\mathbf{I}-\mathbf{A}'_m) & 0 & & -\mathbf{F}'_m & 0 & & -(\mathbf{I}-\mathbf{A}'_m)\mathbf{p}_{nt,m} \end{bmatrix} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{r}_1 \\ \vdots \\ \mathbf{r}_m \\ \alpha_1 \\ \vdots \\ \alpha_m \end{bmatrix}$$

$$\leq \begin{bmatrix} \mathbf{F}'_1\pi_1 \\ \mathbf{F}'_2\pi_2 \\ \vdots \\ \mathbf{F}'_m\pi_m \end{bmatrix} \tag{8}$$

with

$$\mathbf{p}_0,\, \mathbf{r}_i,\, \alpha_i \geq 0, \quad \forall i.$$

So, since the programs have a common optimum value:

$$\mathbf{p}_0'\left(\sum_i \mathbf{y}_i\right) - \sum_i \mathbf{r}_i'\mathbf{f}_i - \sum_i \alpha_i(\mathbf{p}_{nt,i}'\mathbf{y}_i) = \sum_i \pi_i'\mathbf{F}_i\mathbf{x}_i. \tag{9}$$

The solution to the primal equation (7) is the vector of regional outputs. The last m inequalities in the primal assure for all regions that the value of imports at no-trade prices exceeds that of exports. The dual (8) solution includes (besides world prices and region-specific factor scarcity rents) a vector of shadow prices, α_i, corresponding to the last m inequalities in the primal, that determine a benefit-of-trade rent and assure that world prices are high enough to accommodate this rent. Equality of the primal and dual objective functions (9) assures that the value of world final demand equals the cost of utilized factors plus rents on scarce factors and benefit-of-trade rents.

3. Properties of the World Trade Model

In the World Trade Model, technologies, factor endowments, factor prices, and the level and composition of final deliveries are exogenous and region-specific. There are a common unit of currency and no barriers to trade or transportation costs. Inter-industry production requirements are accounted for. Factor endowments need not be fully utilized, and there is no constraint on the balance of trade (in nominal, or post-trade, prices). Every region benefits from trade in that its imports are worth no less than its exports at no-trade prices. A solution for output, world prices, and scarcity rents is optimal in that it requires the fewest factor inputs (valued in the factor prices of the regions employing the factors) for the world as a whole among all feasible scenarios, including the no-trade case.

Basic Attributes

Proposition 1

The unique solution in the no-trade model is a point inside the feasible polyhedron for the World Trade Model, which is itself a subset of the feasible polyhedron for the one-region world model.

Proof

All three models use the exogenous variables $\mathbf{y}_i$, $\pi_\mathbf{i}$, $\mathbf{A_i}$, and $\mathbf{F}_i$; let these inputs be common and fixed for all the models. The no-trade model determines $\mathbf{f}_{nt,i}$, and for the other two, the variables $\mathbf{f}_i$ are exogenously specified in such a way that $\mathbf{f}_i \geq \mathbf{f}_{nt,i}$ for all i. Although other variables are involved in the solutions to the models, we can regard the n-vectors $\mathbf{x}_i$, designating production levels for goods in region $i = 1, \ldots, m$, as the decision variables for the linear programs because all others can be deduced from $\mathbf{x}_i$ and the exogenous variables. Therefore, consider each linear program to designate a feasible polyhedron in the real (mn)-space where each axis measures the value of some $\{\mathbf{x}_i\}_j$.

The amount of factor use is the scalar $\sum_i \pi_i'\mathbf{F}_i\, \mathbf{x}_i$, where the quantities of factors used in region i, the k-vector $\mathbf{F}_i\, \mathbf{x}_i$, are weighted by the region's factor prices, the k-vector π_i.

The expression $\sum_i \pi_i{}'\mathbf{F}_i$ is precisely the objective function of the primal, which is minimized in the linear programs.

The unique solution to the no-trade system of equations is a point in this (mn)-space, which is clearly feasible for the two linear programs because it meets their constraints while adding the requirement that regions produce all the goods necessary for their own consumption.

As we pass from the one-region world model to the World Trade Model to no-trade, each successive model adds additional constraints to the preceding one, thus reducing the space of feasible solutions, with the no-trade solution the most constrained of all. This completes the proof.

The significance of this nesting is that the benefit from trade for the world as a whole consists of the conservation of factors for given consumption, and the primal objective function measures the total amount of factor use (where the quantity of each factor is weighted by the factor price). Thus the proposition states that factor use is lowest in the one-region world model, is greater with region-specific technologies and factors and the benefit-to-trade constraint of the World Trade Model, and is greater still in the absence of trade. This should be thought of in the context that the one-region world model is underconstrained for accurate descriptions of trade patterns: regional self-interest is not adequately represented. On the other hand, requiring regions to rely on domestic production only is obviously overconstrained.

Proposition 2

For each region,

(i) The amount of production of the jth good and the slack in the price equation for that good are not both non-zero:

$$\mathbf{x}_i'[(\mathbf{I}-\mathbf{A}_i)'\mathbf{p}_0 - \mathbf{F}_i' r_i - (\mathbf{I}-\mathbf{A}_i)'\mathbf{p}_{nt,i}\alpha_i - \mathbf{F}_i'\pi_\mathbf{i}] = 0 \quad \forall i \tag{10}$$

(ii) The world price of a traded good and excess production of the good are not both non-zero.

$$\mathbf{p}_0'[\Sigma_i(\mathbf{I}-\mathbf{A}_i)\mathbf{x}_i - \Sigma_i(y_i)] = 0 \quad \forall i \tag{11}$$

(iii) The scarcity rent on a factor and the amount of excess capacity of that factor are not both non-zero:

$$\mathbf{r}_i'(\mathbf{F}_i\mathbf{x}_i - \mathbf{f}_i) = 0 \quad \forall i \tag{12}$$

(iv) The benefit-of-trade rent and the value of net exports at no-trade prices are not both non-zero:

$$\alpha_i'[\mathbf{p}_{nt,i}'((\mathbf{I}-\mathbf{A}_i)\mathbf{x}_i - \mathbf{y}_i)] = 0 \quad \forall i \tag{13}$$

Proof

Each part of this proposition is an application of the Complementarity Slackness Theorem of linear programming (Luenberger, 1989), and these four parts are the only complementarity results for the World Trade Model. This completes the proof.

In the proposition above, the inner product in equation (10) involves two vectors: a variable from the primal program and a slack from the dual. In equations (11)–(13), the variable is from the dual and the slack from the primal. Since the inner product of two vectors is the sum of their entrywise products, this means that at least one (and possibly both) of the vectors has a zero in each position. If the slacks and variables in equation (10) are both zero in any position, this is called primal degeneracy; in equations (11)–(13) it is called dual degeneracy (Paris, 1991).

In equation (10), the inequalities determine prices and the slack represents cost savings for regions that do not produce the good in question but instead import it. For all regions that do produce the good, any rents adjust so that the world price is just equal to the region's cost, including rents. In equation (11), the slacks represent the excess of world production over world demand. In equation (12), the inequalities determine factor use and the slack represents the amount of unutilized capacity. (In the single-region world model, there is one set of factor rents ($\mathbf{r}$) for fully utilized factors while the rents are region-specific ($\mathbf{r}_i$) in the World Trade Model.) In the benefit-of-trade inequalities (13), the slacks represent the excess value of imports over exports at no-trade prices.

Commodity Prices

What follows is one of the main results of this paper, demonstrating the relationship between world prices and no-trade prices in the absence and presence of scarcity rents.

Theorem 1

(i) If no scarcity rents are earned, then:
   (a) The price of every good will be no greater than the lowest no-trade price for that good. That is,
   $\{\mathbf{p}_0\}_j \leq \{\mathbf{p}_{nt,i}\}_j \quad \forall i, j.$
   (b) If the matrix $(\mathbf{I}-\mathbf{A}')^{-1}$ is strictly positive, then if the world price for any one good is less than its lowest no-trade price, this implies that all goods have this property: $\{\mathbf{p}_0\}_j < \{\mathbf{p}_{nt,i}\}_j \quad \forall i, j.$
   (c) For non-degenerate choices of values of exogenous variables, each good will be produced in one region only.

(ii) When any scarcity rent is non-zero, a corresponding good may be produced in more than one region. The world price of the good is lower than the cost of production would be in any region that does not in fact produce it but may be higher than no-trade prices in producing regions.

Proof

(i) (a) If scarcity rents $\mathbf{r}_i$ and α_i are zero, the dual (8) reduces to

$$\begin{bmatrix} (\mathbf{I}-\mathbf{A}_1)' \\ \vdots \\ (\mathbf{I}-\mathbf{A}_m)' \end{bmatrix} [\mathbf{p}_0] \leq \begin{bmatrix} \mathbf{F}_1'\pi_1 \\ \vdots \\ \mathbf{F}_m'\pi_m \end{bmatrix},$$

or $\mathbf{p}_0 \leq (\mathbf{I} - \mathbf{A}_i')^{-1}\mathbf{F}_i'\pi_i$ for $i = 1,\ldots, m$. But $\mathbf{p}_{nt,i} = (\mathbf{I} - \mathbf{A}_i')^{-1}\mathbf{F}_i'\pi_i$ from (2). Therefore, $\mathbf{p}_0 \leq \mathbf{p}_{nt,i}$ for $i = 1,\ldots, m$, so that $\{\mathbf{p}_0\}_j \leq \min_i \{\mathbf{p}_{nt,i}\}_j$.

(i) (b), we begin with the assumption that $\{\mathbf{p}_0\}_j < \min_i \{\mathbf{p}_{nt,i}\}_j$ for some j. In the general case where $\mathbf{A}$ is indecomposable, the matrix $(\mathbf{I}-\mathbf{A}')^{-1}$ is strictly positive (Takayama, 1985), meaning that all goods depend, however indirectly, on inputs of all other goods. This being the case, there is a multiplier effect, which ensures that world prices drop as a result of that one diminished price: the price of every other good drops because of the lowered price of good j, which is an input. Therefore, $\{\mathbf{p}_0\}_j < \{\mathbf{p}_{nt,i}\}_j$ for all i and j.

Thus the world price for each good is lower than the no-trade price in any region, in particular in producing regions, and the difference between the world price and the no-trade price of the lowest-cost producing region is due precisely to the inter-industry multiplier effect.

(i) (c) If there are no binding factor constraints, the low-cost region will produce until all demand is satisfied. The only exception can occur when there is the coincidence (a degeneracy in the exogenous variables from the point of view of the linear program) that two regions have the same cost of production for a good.

(ii) This follows from the argument in the proof of (i).

This completes the proof.

Proposition 3

The following income equation holds for each region:

$$(\pi_i + \mathbf{r}_i)'\mathbf{F}_i\mathbf{x}_i = (\mathbf{p}_0 - \mathbf{p}_{nt,i}\alpha_i)'\mathbf{y}_i + \mathbf{p}_0'[(\mathbf{I} - \mathbf{A}_i)\mathbf{x}_i - \mathbf{y}_i] \tag{14}$$

Proof

Combine equations (10) and (13). This completes the proof.

It shows that earnings and rents on factors of production are equal to domestic outlays for consumption plus payments received for exports, with the benefit-of-trade rent permitting a reduction in domestic prices.

For the world as a whole, the income equations (3), (6) and (9) reflect that factor earnings cover the cost of final deliveries: all trade flows cancel out because one region's imports are other regions' exports. In contrast, trade flows need to be represented in the income equation for an individual region. The benefit-of-trade rent in region i, namely α_i, is paid by consumers in importing regions, who are charged $\{\mathbf{p}_0\}_j$ for imports of good j. The domestic price of good j may be as low as $\{\mathbf{p}_0 - \alpha_i\mathbf{p}_{nt,i}\}_j$, although the rent could be distributed in other ways instead of allowing the full benefit to consumers.

Gains from Trade

While for the world as a whole the optimal solution to the World Trade Model supports given consumption requirements with lower factor use than the no-trade solution, there is no assurance that this is true for individual regions, which may experience positive or negative trade balances in post-trade prices. The next proposition describes the consequences of trade for factor use in a region. The final theorem describes the gains from trade.

Proposition 4

The change in factor use that a region experiences in going from no trade to trade based on comparative advantage is equal to the balance of trade valued in no-trade prices. For a net importer, the region's saving in factor use is equal to its balance-of-trade deficit (in no-trade prices). For a net exporter, factor use is the same as in the absence of trade. That is,

$$\mathbf{p}'_{nt,i}[\mathbf{y}_i - (\mathbf{I} - \mathbf{A}_i)\mathbf{x}_i] = \pi'_i\mathbf{F}_i\mathbf{x}_{nt,i} - \pi'_i\mathbf{F}_i\mathbf{x}_i. \tag{15}$$

Proof

Combine equations (1) and (2). This completes the proof.

The left-hand side of equation (15) is the vector of net imports (exports if negative) valued in no-trade prices, while the right-hand side is the value of extra factors saved (utilized, if negative). The World Trade Model, by constraining the left-hand side to be positive, assures that in addition to the excess value of imports over exports, aggregate factor use is no greater than in the absence of trade.

The following is the second main result of this paper, describing the gains from trade for every region.

Theorem 2

(i) Trade alters the relative prices of goods such that the terms of trade improve for every region. That is,

$$\mathbf{p}'_0[(\mathbf{I} - \mathbf{A}_i)\mathbf{x}_i - \mathbf{y}_i] \geq \mathbf{p}'_{nt,i}[(\mathbf{I} - \mathbf{A}_i)\mathbf{x}_i - \mathbf{y}_i] \quad \forall i, \tag{16}$$

where $(\mathbf{I} - \mathbf{A}_i)\mathbf{x}_i - \mathbf{y}_i$ is the vector of net exports for region i under the World Trade Model.

(ii) If there is at least one factor in region i that is not fully utilized in the no-trade case but is fully utilized in the World Trade Model, then the inequality is strict. That is, for that region

$$\mathbf{p}'_0[(\mathbf{I} - \mathbf{A}_i)\mathbf{x}_i - \mathbf{y}_i] > \mathbf{p}'_{nt,i}[(\mathbf{I} - \mathbf{A}_i)\mathbf{x}_i - \mathbf{y}_i] \tag{17}$$

Proof

For (i), assume the negation of the inequality to be proven: $\mathbf{p}'_0[(\mathbf{I} - \mathbf{A}_i)\mathbf{x}_i - \mathbf{y}_i] < \mathbf{p}'_{nt,i}$ $[(\mathbf{I} - \mathbf{A}_i)\mathbf{x}_i - \mathbf{y}_i]$. Now use equation (14) to substitute for the left-hand side and equation (15) to substitute for the right-hand side: $(\pi_i + \mathbf{r}_i)'\mathbf{F}_i\mathbf{x}_i - (\mathbf{p}_0 - \mathbf{p}_{nt,i}\alpha_i)'\mathbf{y}_i < -\pi'_i\mathbf{F}_i\mathbf{x}_{nt,i} +$ $\pi_i{}'\mathbf{F}_i\mathbf{x}_i$.

After cancellation and rearrangement, a lower bound for $\mathbf{p}_0{}'\mathbf{y}_i$ is obtained: $\mathbf{r}'_i\mathbf{F}_i\mathbf{x}_i + \pi'_i\mathbf{F}_i\mathbf{x}_{nt,i} + \alpha_i\mathbf{p}_{nt,i}\mathbf{y}_i < \mathbf{p}'_0\mathbf{y}_i$.

On the other hand, according to equation (8): $(\mathbf{I} - \mathbf{A}'_i)\mathbf{p}_0 - \mathbf{F}'_i\mathbf{r}_i - (\mathbf{I} - \mathbf{A}'_i)\mathbf{p}_{nt,i}\alpha_i \leq \mathbf{F}'_i\pi_i$.

Transposing, rearranging, multiplying by $\mathbf{x}_{nt,i}$, and substituting $\mathbf{y}_i = (\mathbf{I}-\mathbf{A}_i)\mathbf{x}_{nt,i}$ from equation (1), this yields an upper bound for $\mathbf{p}_0'\mathbf{y}_i$: $\mathbf{p}_0'\mathbf{y}_i \leq (\mathbf{r}_i + \pi_i)'\mathbf{F}_i\mathbf{x}_{nt,i} + \alpha_i\mathbf{p}_{nt,i}'\mathbf{y}_i$.

Together, these upper and lower bounds for $\mathbf{p}_0'\mathbf{y}_i$: give the inequality $\mathbf{r}_i'\mathbf{F}_i\mathbf{x}_i < \mathbf{r}_i'\mathbf{F}_i\mathbf{x}_{nt,i}$. By equation (12), $\mathbf{r}_i'(\mathbf{F}_i\mathbf{x}_i - \mathbf{f}_i) = 0$, allowing us to replace $\mathbf{r}_i'\mathbf{F}_i\mathbf{x}_i$ with $\mathbf{r}_i'\mathbf{f}_i$. Also recall that $\mathbf{f}_{nt,i} = \mathbf{F}_i\mathbf{x}_{nt,i}$ by definition. Thus $\mathbf{r}_i'(\mathbf{f}_i - \mathbf{f}_{nt,i}) < 0$. But this is impossible, since each entry of $\mathbf{r}_i$ is non-negative, and factor endowments are always at least as great as no-trade factor use. This is the desired contradiction.

(ii) Following the same reasoning as the last proof leads to the inequality $\mathbf{r}_i'\mathbf{f}_i - \mathbf{f}_{nt,i}) \leq 0$. Repeating the previous argument, it must be the case that $\mathbf{r}_i'\mathbf{f}_i - \mathbf{f}_{nt,i}) = 0$, which implies that every summand (since all are non-negative) is zero. But if, as in the hypothesis, some factor is not fully utilized in the no-trade model, then there is some j such that $\{\mathbf{f}_i\}_j - \{\mathbf{f}_{nt,i}\}_j > 0$, which forces that rent $\{\mathbf{r}_i\}_j$ to be zero. The hypothesis also states that factor j is fully utilized in the World Trade Model, which means that the corresponding slack is zero. Only in the case of dual degeneracy can both slack and rent be zero (see discussion after Proposition 2). Thus, for non-degenerate programs, the jth summand is positive, a contradiction. This completes the proof.

The theorem states that world prices provide no less favorable terms of trade for each region than would its own no-trade prices; furthermore, the terms of trade are strictly more favorable under a hypothesis that is commonly satisfied. Taking this result together with Proposition 4 means that a region may have a trade surplus in world prices even though no more factors are utilized than in the absence of trade; for a region experiencing a trade deficit, the deficit is smaller than the value of the factors saved. This improvement in the terms of trade, with no additional factor use, is the benefit to the region from entering into trade.

4. Illustrative Empirical Results

The World Trade Model presented in this paper was applied to a database for 1990 for ten regions, eight goods, and three factors of production (see the Appendix for data classification and sources). The factors are land, labor, and capital, where land is specific to agriculture and the other two factors are required for all goods in all regions. Where no data were available, or where data inconsistencies became apparent, I introduced my own estimates into the database. While the data are not of sufficient detail or quality to support a close interpretation of the numerical results, the following preliminary results are suggestive of the capabilities of the model and the insights into a general conception of comparative advantage that it offers.

4.1. One-Region World: Labor-Intensive Technologies

The value of world factor use is lowest for the one-region world model (as assured by Proposition 1). In this solution, most production takes place using labor-intensive technologies at low labor costs with the greatest export surpluses earned by Eastern Europe and low-income Asia. The quantities of land and labor used are far larger than in the absence of trade or under the assumptions of the World Trade Model and exhaust the world's supply of them. Consequently, scarcity rents are earned on labor and land, while the volume of capital used is only slightly higher than in the absence of trade. The rents, and therefore the earnings, of land increase commensurately with increased usage, but the total earnings of labor are by far the lowest of the three models.

The model formulation allows a choice between what could be characterized as labor-intensive technologies coupled with low wages and capital-intensive technologies coupled with high-cost labor. The results described in the previous paragraph suggest that the former can be competitive when labor and land are abundant. But modern technologies that are less land- and labor-intensive are needed in order to expand output and raise the material standard of living, given a fixed world endowment of limiting resources. This result lends support to Boserup's theory that the imperative for change due to the exhaustion of limited factors when faced with increasing demand – and not simply presumed advantages of greater factor productivity – is generally responsible for the adoption of new technologies (Boserup, 1981). The values of the objective functions and quantities of factor use are shown in Table 1 for the three models.

4.2. Specialization

With trade, the worldwide volume of output of most goods changes very little from the no-trade situation. However, total energy use falls and the fuel mix shifts: under the World Trade Model substantially more coal and less gas are used than in the absence of trade. While there is overall less energy use with trade, a supplementary computation shows that total carbon emissions (using carbon coefficients from Duchin and Lange, 1994) hardly change because the reduction in fuel use is compensated by the shift toward coal. This result suggests the interaction of economic and environmental consequences that can be anticipated as manufacturing production continues to migrate in the coming decades to the developing world.

A solution to the World Trade Model does not generally involve complete specialization because of constraints on factor availability. In this implementation, however, demand for each energy product is satisfied by only one region. The ability of a single region to satisfy total world demand reflects both the high degree of geographic and sectoral aggregation and the fact that no constraints have been imposed on resource endowments other than land. Unlike the case of energy products, the optimal solution does not involve complete regional specialization in services, manufacturing, and especially agricultural goods. Seven regions produce enough agricultural goods to satisfy all (or nearly all) their domestic demand while jointly satisfying the demand of the other regions as well.

Table 1. Factor use in price and physical units in 1990 (Z: 10^9 US\$, land: 10^9 hectares, labor: 10^6 workers, captial: 10^9 US\$)

	$Z = \sum \pi' \mathbf{Fx}$ Total Factors	**Fx** Land	**Fx** Labor	**Fx** Capital
No Trade	17,873	1,380	1,681	6,724
World Trade Model	14,990	1,649	1,546	5,766
One-Region World	10,511	2,425*	2,247*	7,886

*world capacity fully utilized.

Note: In moving from No-Trade to the World Trade Model, the mobility of goods is introduced. The move from there to the One-Region World allows also for factor mobility (at factor prices of the destination region).

Naturally, all but one of the producing regions runs out of some factor of production. In all of these cases, that factor turns out to be land.

In the current model formulation that minimizes factor use, there may be regions where all costs of production are sufficiently high relative to the rest of the world that the optimal outcome is for them to produce nothing even though they consume. This is the outcome for the rest-of-world region comprised of Africa and the poor Latin American countries, a region that is entirely reliant on imports in these computations. In a dynamic framework, sustained balance of trade deficits would not be tolerated, and the situation could lead to emigration out of the region. When the model is used to analyze scenarios about the feedback from climate change entailing great geographic disparities, this kind of outcome could be encountered.

4.3. Absolute versus Comparative Advantage

In the application of the World Trade Model to this database, the producers of most goods are limited to, or at least include, the regions with the lowest absolute no-trade cost of production for that good. But there are several cases where the region with the lowest absolute cost does not produce the good. Each of these regions necessarily encounters at least one factor constraint reflecting the fact that its scarce factors are better used where it has greater comparative advantage. (It is precisely the distinction between absolute and comparative advantage that limits the usefulness of "chains" of comparative advantage.)

4.4. Increased World Prices

World prices for individual goods are higher in some regions than the no-trade prices, especially for agriculture, where the intermediate and factor costs of the least efficient of the producing regions need to be met. However, for nine of the ten regions, the cost of the entire consumption bundle is lower under the assumptions of the World Trade Model than in the absence of trade; for the tenth region, namely China, the increased factor earnings in the form of rents are sufficient to cover the cost of the higher-priced bill of goods. Production according to comparative advantage does not necessarily lower the nominal prices of all goods when new scarcity rents are taken into account, a result that could be anticipated on the basis of Theorem 1(ii).

4.5. Distribution of Benefits from Trade

The world as a whole benefits from trade based on comparative advantage in that total factor use falls by over 15% (see Table 1). It was demonstrated that the benefits necessarily improved terms of trade for every region, meaning that the value of net exports is greater when evaluated in world prices than in no-trade prices. But the distribution of benefits is uneven, and computations show that the greatest terms-of-trade benefit at about \$550 billion (1990 US dollars) accrues to the middle-income Latin American region, with large improvements in the terms of trade also for Western Europe, North America, and China. The greatest benefit is achieved when a region is able to import cheaply (because of other regions' cost structures) goods that would be particularly expensive to produce domestically.

4.6. Redistribution of Income

The World Trade Model satisfies a given level and composition of world demand through an allocation of resources that effectively achieves a redistribution of income among factors of production and among regions, as shown in Table 2. In moving from a world with no trade to one with optimal patterns of trade, there is a significant increase in the use of land, accompanied by moderate declines in use of the other factors. The major change in the earnings of factors is likewise the increased earnings of land. In the absence of trade, a fraction of a percent of total factor payments is allocated to land. Under the World Trade Model, land earns nearly 5% of total world income. While the percentage gain to income from land is great, the relative loss to other factors is small because their earnings are much larger.

In the absence of trade, the distribution of world factor earnings is nearly 62% to labor and 38% to capital. In the World Trade Model, the shares of labor and capital fall to just under 60% and 36%, respectively. The cost of the world consumption bundle falls nearly 10% since most prices have fallen. The notable exception is agricultural products, for which the price is higher than the no-trade price for most regions because of scarcity rents on land.

In some regions the redistribution of income is more dramatic: notably in middle-income Latin America, where land's share rises from under 1% to nearly a third of regional income. The distribution among regions also shifts, with the greatest increase in factor income experienced by low-income Asia, and especially China, where it grows by almost 75%. It would be possible to distinguish the changes in real income of owners of land, labor, and capital, respectively, once social accounting matrices are incorporated into the representation of each economy.

Table 2. Income and consumption by region and income by factor (10^9 US$, factor incomes include rents)

	No Trade Income = Consumption	WTM Factor Income	WTM Cost of Consumption
North America	4,967	5,182	4,881
Western Europe	4,083	4,083	3,633
Former Soviet Union	1,927	1,888	1,865
Low-Income Asia	406	490	279
China	735	1,279	961
Japan	1,458	1,396	1,374
Oil-Rich Middle East	484	134*	374*
Eastern Europe	809	847	590
Medium-Income Latin America	986	962	741
Rest-of-World	2,018	0*	1,563*
Total	17,873	16,262	16,262
	No Trade	WTM	
Land	62	720	
Labor	11,088	9,688	
Capital	6,724	5,853	
Total	17,873	16,262	

*Net importing regions.

4.7. Near-Optimal Solutions

Even with a problem involving only ten regions, eight goods, and three factors, the linear program for the World Trade Model (with a matrix of dimensions $(m + n + mk) \times (mn)$, or 48×80 for this database) generates a huge number of feasible solutions (so many that they could not be enumerated exhaustively but had to be sampled), and thousands of them (a substantial number, although a small proportion) have values of their objective functions within 1% of the optimal one. An effort was made to classify and characterize the near-optimal feasible solutions with respect to common patterns of specialization and world prices. The preliminary results are highly suggestive. A solution that is nearly equivalent to the optimal one in terms of total factor use (the value of the primal objective function) may be substantially different in terms of the international division of labor or the distribution of income. Linear geometry in high dimensions (even as few as $mn = 80$) can produce results of this type, as counterintuitive as they may appear (Duchin, 2003). It follows, for example, that solutions that are practically indistinguishable in global factor use could correspond to different development strategies or have vastly different implications for the environment. The model could be used to identify a variety of near-optimal arrangements for sustainable development.

5. Research Program

The determination of trade flows and world prices in the model described in this paper is based on a comparison of input cost structures and factor endowments of potential trading partners. It is a generalization of the World Model of Leontief *et al.* (1977) achieved, in turn, through the generalization of a two-country, two-good, two-factor model of international trade to the $m \times n \times k$ case. The model is intended for empirical analysis of the global economy.

A key requirement for the implementation of a full-scale model is the development of a new database. The input–output tables that provide the bulk of the required data are regularly compiled for many countries, but integrating them into a common database remains a substantial challenge. Since input–output tables are used in all world models, this is a challenge that should be undertaken with the collaboration of national statistical offices.

The World Trade Model especially requires a systematically defined body of data about factor endowments and factor use not only for labor and capital but also for land, water, oil and other resources such as copper or chromium. The factors of production need to be measured in physical units. A number of statistical offices have begun to collect such data on material flows, and this effort will surely be accelerated as the ability to incorporate the data into analytic frameworks improves. Data on factor prices are particularly crucial and hard to come by.

New research directions based on the World Trade Model are being established. It has been used to analyze the extent to which new patterns of production and trade in agricultural products can provide a mechanism for adapting to climatic change. This work involved the disaggregation of both geographic regions and agricultural sectors and a detailed treatment of land and changing crop yields attributable to climate change (Julia, 2004). In other work, my colleagues and I have extended the model to take distance-based transportation costs into account in determining bilateral trade flows

(Strømman and Duchin, 2005) and are using it to investigate the prospects for a changing international division of labor in the processing of energy-intensive resources (Strømman, Hertwich and Duchin, 2005). The formulation in this paper of a one-region world will be useful for analyzing the potential advantages of regional integration.

A solution to the World Trade Model corresponds to an optimal static allocation of resources. An adequate dynamic framework needs to reflect shifts in comparative advantage and their causes and consequences. One cause is changing factor endowments: stocks and flows of not only capital but also resources need to be tracked. Other dynamic phenomena include changes in technologies and lifestyles: technological innovation and the international transfer of technologies, which affect production capabilities; and innovations in lifestyles and the international emulation of lifestyles, which affect consumption patterns. Rents on scarce factors in a particular region need to have an impact on factor prices in that region in succeeding periods; in a dynamic framework, factor prices would be endogenous after the initial period. The World Trade Model also needs to determine exchange rates endogenously, taking current and accumulated trade deficits or surpluses into account. Progress with the database, with the theoretical extension of the model, and with scenario analysis can be carried out simultaneously, making use of a relatively well-defined division of labor.

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Notes

[1]They built on Graham's work dating back to the 1920 s. As Whitin (1953: 521) has observed, "Graham's models [of international trade among many countries in many commodities] possess the basic characteristics of 'linear programming'".

[2]Endowments, $\mathbf{f}_i$, are not explicitly represented in the no-trade model, and it is assumed that $\mathbf{f}_{nt,\ i} \leq \mathbf{f}_i$.

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Appendix

Data Classifications and Units

Regions (10): North America, Western Europe, Former Soviet Union, Low-Income Asia, China, Japan, Oil-Rich Middle East, Eastern Europe, Middle-Income Latin America, and Rest-of-World (Africa and Low-Income Latin America).
Goods (8): Coal, Oil, Gas (each in 10^6 tons of coal equivalent), Electricity, Mining, Agriculture, Manufacturing, and Services (each in 109 of 1990 US dollars).
Factors (3): Land (in 10^6 hectares), Labor (in 10^6 workers), and Capital (in 10^9 of 1990 US dollars).

Data Sources

Input–output tables and consumption vectors: Duchin and Lange (1994).
Actual and potential available arable land: Food and Agricultural Organization (2000).
Labor force, wage rates, employment by sector and population: Summers and Heston (1991); International Labour Office (various years, Tables 2A, 3B, 16 and 21); United Nations (1992, Table 5); and Bureau of Labor Statistics (1997).
Capital expenditures and rates of return: United Nations (1993); and Bureau of the Census (1994: Table 2).