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Intermediated Search

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If buyers cannot observe product characteristics, they search available sellers to find better matches. In this situation, market go-betweens who 'manage' a variety of products emerge, offering consumers a variety of trading opportunities which reduce the uncertainty of search and improve the quality of consumer–producer trades. The distribution of the benefits from this intermediation depend critically on the number of products available. When variety is low, increases in capacity heighten competition, thereby lowering prices. On the other hand, with many products on offer, retail firms act monopolistically. In this case, increased capacity raises prices.

Introduction

Buyers often, perhaps typically, purchase goods and services through retailers and go-betweens rather than exchanging directly with producers. Department stores, supermarkets and other outlets offer a wide selection of goods from a number of different sources. Similarly, consultancies that offer professional and technical advice have a variety of specialists available to deal with customer inquiries. Individuals seeking legal advice, for example, generally contact a law firm rather than going directly to an individual lawyer. The firm then puts the client in touch with the appropriate expert.

This paper examines the way in which this type of intermediation influences trade. In particular, it focuses on the interaction between retailers who offer a variety of goods and services and consumers with differentiated tastes who have imperfect information. Acting as the middleman or manager for producers, the retailer sets a price for its goods or services. Buyers, who do not know the characteristics of these options, shop around to find the match that suits them best.[1] While buyers do not face a distribution of prices, they search over a variety of potential sellers who offer different trading opportunities or products.

The costs and benefits associated with this shopping around depend on the number of products available from the retailer. The extent of the product range determines two equilibrium regimes, each with fundamentally different welfare implications. If the costs of carrying a large product range are high, and hence retailers are offering only a few options, buyers are not likely to obtain a good match at a particular outlet and often decide to shop elsewhere. As retailers in the market expand their capacity (say, if the costs of carrying goods falls) this search option improves, and to retain customers each intermediary is forced to lower its price: competition and the possible loss of a sale puts pressure on the seller to lower its price and hence its payments to producers. With more options at a lower price, buyers search less widely and are better off. Producers, in contrast, become worse off and either receive less compensation or exit the market.

If, on the other hand, retailers offer a sufficiently diverse selection, the buyer is likely to obtain a good match at any given intermediary and does not shop around. In this situation, the buyer becomes a captive audience and the retailer, after a contact occurs, in effect has monopoly power. When the retailer expands from this position, it uses its market power to extract the benefits of better matches. Moreover, as sellers grow and offer a wider range of goods and services, perceived differences among them decline. This increased similarity lowers the buyer's value of search and each retailer is able to capitalize, to a greater extent, on each trade. This results in even higher prices, which, despite better matching opportunities, make the buyer worse off.

In both equilibrium regimes intermediation creates higher-quality matches, leading to greater specialization among producers. As intermediaries direct buyers to more suitable products, they serve clients more effectively and searchers become more selective. Greater specialization results as product-makers are asked to serve a more limited range of buyers. Since higher-quality matches occur as intermediaries offer a wider range of products, expansion of intermediation capacity generates an apparent rise in the division of labour.

Intermediation of this sort offers an alternative perspective on the role of middlemen in search models. Rubinstein and Wolinsky (1987) and Yavas (1994) analyse middlemen who trade only one good but are able to exist and influence the pattern of exchange because of their superior ability to locate traders. In a model with similarities to the one proposed here, Hänchen and von Ungern-Sternberg (1985) examine the effect of intermediaries such as test magazines who broker information gained from their own search. Here, intermediaries act as agencies that offer a stock of sellers to a flow of buyers, as in Coles and Smith (1998).[2] In this paper, however, if an agency-retailer does not satisfy a buyer, the buyer searches other intermediaries rather than waiting for an acceptable seller to arrive at a given 'marketplace'. Waiting at a particular firm is not productive, since the intermediary's stock of sellers, i.e. products, does not vary over time.

An outline of the paper is as follows. The next section describes the trading environment and the way in which intermediaries operate. Given this environment, buyers formulate search strategies while producers post prices. Section II characterizes the resulting equilibrium. Section III then establishes the welfare properties of intermediation. The final section summarizes the results.

I. Intermediaries and Trade

Trade takes place over an infinite number of discrete time periods. In each period a flow of new buyers with differentiated tastes enters the market looking to purchase a single product and then leave. Let preferences correspond to locations on the unit circle. Let e denote the size or measure of the in-flow of buyers and hence represent market demand.

The supply of differentiated products awaiting these buyers is produced by a mass of S agents organized within I retail establishments that intermediate trade. Each agent in a retail establishment receives the competitively determined per-period compensation w. Aggregate supply—market entry—of these agents is a positive and weakly increasing function of pay at the retail establishments: $S = S(w)$ where $S'(w) \geqslant 0$.

Retail establishments organize these producer agents as part of their selection of a product range—the number of goods or services on offer and the location of these products—prior to the beginning of trade.[3] To highlight the role of the intermediation between buyers and the producers, the production technology within the retail establishments is stylized. The only input is labour, and each product on offer in an outlet requires one worker to create, maintain or deliver it. Thus, a retailer's product range is equivalent to the number and location of its employees.

A capacity constraint, given by $N \in \mathbb{N}^+$, limits the maximum number of products that a retailer can offer or employees it can manage. To focus on the pricing decision of the intermediary, suppose that all retailers decide to operate at this capacity: all firms are the same size. Further, suppose that retailers equally spread these products or employees around the unit circle, with the location of the first chosen randomly. Despite the even spread of products, this allocation generates differences among retailers owing to the idiosyncratic location of the first product. The quality of a buyer–intermediary pairing will be match-specific and random.

Retailers such as supermarkets, law firms and so on are not, of course, capacity-constrained in this way. The constraint is adopted to provide a simple and convenient way of capturing the costs of managing a range of products. As this constraint plays a central role in what follows, the relationship between the selection and cost of a product range deserves elaboration. In a narrow interpretation, the capacity constraint is equivalent to a cost function in which the marginal cost of adding a product is constant (and equal to the employee's wage) up to N products and infinite thereafter. As such, the constraint technology represents a limiting case of increasing marginal costs of providing products. A richer and more satisfying interpretation is, however, possible. As detailed in the Appendix, the selection of a product range is directly linked with the costs of managing these products.

Rather than obscure the central analysis with these details, the approach adopted here is to remove a layer of complexity by allowing product range to effectively be exogenous while recognizing (as described in the Appendix) that costs determine the product range, which in turn determines price. In addition, adopting a binding capacity constraint avoids the following difficulty. In this framework the choice of the number of products in an establishment depends not only on the costs of operating an outlet of size N but also on the way in which the size of the establishment affects the arrival rate of buyers. With an exogenous symmetric allocation, this second concern is obscured as all retailers are the same size and are treated identically. With an endogenous product range or establishment size, formulating the way in which buyers select a retailer becomes problematic. At one extreme, buyers might be equally likely to select an outlet regardless of size, so that the number of employees has no effect on the arrival rate of buyers. Alternatively, one might reasonably expect the arrival rate of buyers to be positively correlated with the size (absolute or relative) of the outlet. More workers can devote a greater effort to drumming up business, or they may give the retailer a larger network of contacts. More employees may generate more customers so that 'balanced matching' occurs (see Burdett and Vishwanath 1988).

In return for organizing and operating the network of N employees, each intermediary sets a price for its goods, collects revenue and claims any profits. (Profits, however, are competed away in equilibrium.) Retailers do not actively search. Retailers initiate no advertising or contacts: they passively wait for searching buyers to drop in.

Before buyer search commences, the preferences or locations of the inflow of e new consumers are arbitrarily assigned and uniformly distributed at entry at the beginning of the period. A buyer then initiates trade by randomly selecting and contacting retailers, a time-consuming but otherwise costless activity. A buyer can make only one contact in a period, and before generating a contact cannot observe the characteristics—the price and location of opportunities—of an outlet. Retailers are *ex ante* identical to the buyer. It is as if a buyer enters the market, gets a list of the retail establishments in the market—say, from a telephone book or an internet search engine—and then arbitrarily calls retailers one by one until the buyer finds an acceptable product and concludes trade.

Location preferences of the buyer represent idiosyncratic features in the buyer–seller relationship that can arise even in highly specialized markets. Consider the example of legal services. One interpretation of idiosyncratic tastes is that different locations correspond to different legal fields such as real estate law, marital law and criminal law. Buyers enter the market unaware of the fields covered by different law firms and must select randomly. In some areas the legal profession may be more highly organized, and clients in need of legal advice will have some idea of the goods and services that more specialized retailers provide. In this case buyers can direct their search to retailers in a niche of the product space. None the less, idiosyncracies will continue in this reduced product space. For example, among divorce lawyers there are different methods of relating to clients and of approaching marital breakdown negotiations. Such information is not readily observed or advertised.

After a buyer contacts a retailer, the retailer's dispatcher or manager notes the locational preference of the buyer and then informs the buyer of both its price and its most appropriate product.[4] Put differently, once a buyer contacts a retailer, the buyer learns the intermediary's price and product range. The value of a trade between a buyer and any given product equals the buyer's utility from consumption U less the distance ε from the location of the buyer's most preferred product to the location of the product on the unit circle. The most appropriate product, therefore, is the one nearest the buyer's taste or position. Since there are no production costs, the retailer's share of the gains to trade (if consummated) is given by the price. Accordingly, the buyer's gain to trade is consumption utility less distance and price: $U - \varepsilon - p$.

Figure 1 illustrates a simple example of a buyer's preference over the product ranges of two retailers where both firms have three products available. Measuring locations by the conventional 12-hour position, retailer 1 has products for sale at 4, 8 and 12 o'clock while retailer 2 has products at 2, 6 and 10 o'clock. Provided that prices are equal at both outlets, a buyer located between 3 and 4 o'clock prefers buying retailer 1's 4 o'clock product over retailer 2's 2 o'clock product. However, since the buyer does not observe both intermediaries simultaneously and may find better matches while searching,

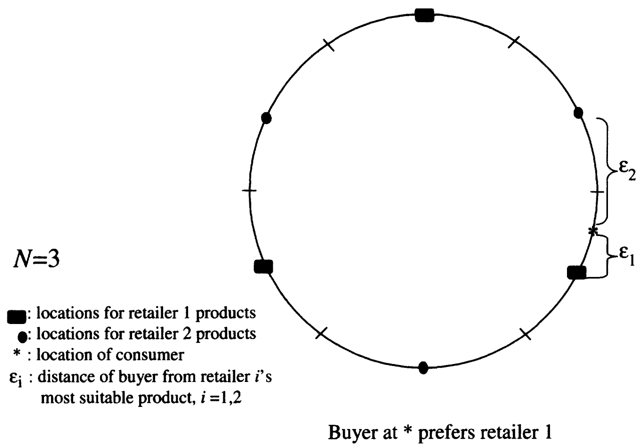

FIGURE 1. Two retailers and a buyer.

little as yet can be said regarding the decision to purchase at either of these firms.

In what follows, the analysis focuses on competitive steady-state behaviour. In a steady state agents face a stationary environment. The number and distribution of participating traders remains constant through time; likewise for the rate at which buyers contact a retailer and for the pricing behaviour of retailers. The objective is to characterize the decisions of buyers and sellers (given prices and wages) as they maximize expected discounted returns in a stationary economy and then find a steady state consistent with agents making these decisions.

(a) The retailer's problem

Given a distance and price, a buyer accepts a retailer's offer if doing so outweighs the option of continued search, denoted by V. Let $\Pr(U - \varepsilon - p \geqslant V)$ represent the probability that an arbitrary customer is willing to purchase from the selected intermediary at price p. The resulting expected per-period profit is then given by

$$\Pi(p; N) = \gamma \cdot p \cdot \Pr(U - \varepsilon - p \geqslant V) - w \cdot N,$$

where γ is the arrival or contact rate per period of buyers at a retailer. Notice that in a steady state γ, V and w are constant through time.[5] They are also exogenous to the retailer although endogenously determined in equilibrium.

In a steady state, retailers will not want to update their price or product range before trade in any period. Stationarity makes updating the choices before any period equivalent to selection at the start of time. The important feature is that the retailer cannot rearrange its fee or product variety to suit a particular taste once a contact has occurred.

Owing to the equal distribution of products within an outlet, the maximum distance a buyer can be from a product or employee is $1/2N$, and the distribution of distances between a buyer and the nearest employee will be uniform over this interval: $\varepsilon \sim U[0, 1/2N]$. Thus,

$$\text{(1)} \quad \Pr(U - V - p \geqslant \varepsilon) = \begin{cases} 0 & p > U - V \\ 2N(U - V - p) & U - V - \frac{1}{2N} < p \leqslant U - V \\ 1 & p \leqslant U - V - \frac{1}{2N} \end{cases}$$

This probability, which in effect corresponds to the retailer's conditional (on a contact occurring) demand curve, is kinked. As in monopolistically competitive models of location (Salop 1979), there exists a 'competitive' portion and a 'monopolistic' section of this curve and the optimal price may exist in either region—see Figure 2 below. Specifically, since this probability (or conditional demand curve) is constant and equal to one for $p \leqslant U - V - 1/2N$, profit is linear and increasing for these prices. At a minimum, the retailer sets price equal to this limit. For all other potentially accepted prices, the probability of acceptance is linear in price and profit is quadratic with a maximum at $p = (U - V)/2$. It follows that the larger of these two prices maximizes profit, i.e.

$$\text{(2)} \qquad p^* = \arg\max_p \Pi(p; N) = \max\left\{U - V - \tfrac{1}{2N}, (U - V)/2\right\}.$$

The optimal price is independent of both market demand, embodied in the contact rate γ, and the wage. Given the structure of the problem, this is not surprising. As wages are paid regardless of sales, they effectively are fixed costs associated with the product range. Similarly, as buyers do not *ex ante* observe the intermediary's characteristics, including price, there is no interaction between price and the arrival rate of buyers. Therefore, if a seller deviates from this pricing policy (plays out of equilibrium), consumers do not observe this alteration until after a contact has occurred. Without any change in consumer search behaviour, such deviations from the above pricing policy are unprofitable.

If the retailer sets price equal to $U - V - 1/2N$, all potential buyers are enticed to purchase so that the price 'spans' or covers all possible matches. This occurs when the intermediary has a sufficient number of employees, that is for a technology such that $N \geqslant 1/(U - V)$. Alternatively, when N is small, i.e. when $N < 1/(U - V)$, price equals $(U - V)/2$. In this case, retailers forgo poor or

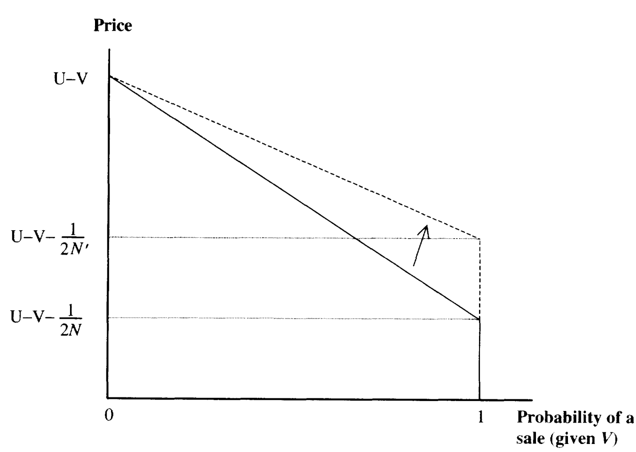

FIGURE 2. Increased intermediation capacity from N to N'.

distant matches in order to extract greater rents (using a higher price) from those buyers whose tastes are located near an available employee.

(b) The buyer's problem

The buyer faces a standard stationary search problem. Given the price and the location of the retailer's nearest employee, the buyer decides whether to accept the trade or seek out other opportunities. The utility of buying at price p and distance ε is simply $U - p - \varepsilon$. If a buyer declines this period's trading opportunity, in the next period he chooses between a new intermediary's offer or search yet again. Given that retailers randomly locate the initial employee and evenly space the remainder, the value of purchasing, $U - p - \varepsilon$, as well as the value of continued search, V, are independent of the buyer's location. Buyers do not have a preferred position.

Since the buyer chooses the best of these options, the value of continued search satisfies the equation

$$V = \frac{1}{1+r} E_\varepsilon \max\{V, U - p - \varepsilon\}$$

where r is the discount rate. There are no direct search costs. This Bellman equation can be rearranged as

$$rV = E_\varepsilon \max\{0, U - p - \varepsilon - V\}.$$

II. EQUILIBRIUM

Symmetric, competitive steady-state equilibria are now characterized.[6] Define the reservation distance ε^* as the distance at which a buyer is indifferent between buying from the current retailer and continuing the search, $U - p - \varepsilon^* = V$. With similarly sized outlets all charging the same equilibrium price p^*, the value of continued search satisfies

$$V = U - \varepsilon^* - p^*.$$

Subtracting this search value from the value of buying at a given distance $\varepsilon \leqslant \varepsilon^*$ gives

$$U - p^* - \varepsilon - V = \varepsilon^* - \varepsilon > 0.$$

Substituting this result into the above equation for rV, the reservation distance thus becomes

$$rV = \int_0^{\varepsilon^*} (\varepsilon^* - \varepsilon) \mathrm{d}G_N(\varepsilon) = N\varepsilon^{*2}, \tag{3}$$

where $\mathrm{d}G_N(\varepsilon) = 2N\,\mathrm{d}\varepsilon$ is the probability distribution for a uniformly distributed random variable over [0, 1/2N]: $\varepsilon \sim U[0, 1/2N]$. The uniformity of ε follows from the even distribution of employees within and across establishments.

From equation (2), two distinct equilibrium outcomes emerge, one each from the possible pricing strategies. When a retailer sets price $p^* = U - V - 1/2N$, all buyers are willing to purchase from this intermediary, with buyers located as far as possible from an employee (at the distance $\varepsilon = 1/2N$) indifferent between purchasing and continued search. With all retailers setting this price, the reservation distance equals the maximum possible distance:

$$\varepsilon^* = \frac{1}{2N}.$$

Plugging this ε^* into (3) yields

$$rV = \frac{1}{4N}.$$

The expected distance to the nearest product or employee is half the reservation distance.

Price equals $U - V - 1/2N$ in markets with large retailers, or more precisely in markets with retailers offering a large product range. The threshold size at which retailers charge this price occurs when[7]

$$N > N^c \equiv 1/(U - V).$$

Substituting in $V = 1/4rN^c$ into this equation implies that the critical size depends negatively on the interest rate and negatively on the buyer's benefit from consumption U:

$$N^c = \frac{1}{U}\left(1 + \frac{1}{4r}\right).$$

On the other hand, when retailing capacity is limited such that $N < N^c$, the alternative equilibrium outcome occurs and price equals $p^* = (U - V)/2$. At this price, buyers will on occasion prefer to search rather than accept the existing offer. From (1), the probability that a buyer decides not to buy at this price equals $1 - N(U - V)$.

Substitution of price $p^* = (U - V)/2$ into the value of search as given by $V = U - \varepsilon^* - p^*$ reveals that the price equals the reservation distance: $p^* = \varepsilon^*$. From (3), it follows that

$$rV = \int_0^{(U-V)/2} [(U - V)/2 - \varepsilon]2N\mathrm{d}\varepsilon = \frac{(U - V)^2}{4}N$$

Using the positive root from the quadratic formula it further follows that the flow value of search is

$$rV = rU + \frac{2r}{N}\left[r - (r^2 + rNU)^{1/2}\right].$$

This value of search then yields the price (and reservation distance)

$$p^* = (U - V)/2 = \frac{(r^2 + rNU)^{1/2} - r}{N} = \varepsilon^*.$$

Accounting for critical capacity as defined by N^c, these two outcomes can be combined to express the value of consumer search together as

$$(4) \qquad V = \begin{cases} \frac{\varepsilon^*}{2r} & N \geqslant N^c, \\ U - 2\varepsilon^* & N \leqslant N^c, \end{cases}$$

where the reservation distance is given by

$$\varepsilon^* = \begin{cases} \frac{1}{2N} & N \geqslant N^c, \\ \frac{[r^2+rNU]^{1/2}-r}{N} & N \leqslant N^c. \end{cases} \tag{5}$$

Now consider the entry decision of retail intermediaries and their producers as well as the determination of wages. The zero (per-period) profit condition for the retailer,

$$\Pi(p^*; N) = \gamma p^* \Pr(U - \varepsilon - p^* \geqslant V) - wN = 0,$$

depends upon (i) wages and (ii) the retailer's demand represented by the arrival rate of buyers, γ. Both are endogenous.

Since retailers are *ex ante* identical, and since buyers arbitrarily select retailers to call upon, each retailer has the same probability of being selected by a given buyer in the market. In a market where retailers have an extensive product range, $N \geqslant N^c$, the price is set in such a way that all buyers will purchase from the most suitable employee at the first intermediary contacted. In equilibrium, buyers search only once. In this case the arrival rate is proportional to the number of retailers:

$$\gamma = e/I, \qquad N \geqslant N^c.$$

Alternatively, when the market consists of retailers with a limited selection of goods, $N < N^c$, price is set such that $p^* = \varepsilon^* = (U - V)/2$. In this equilibrium outcome, buyers reject some retailers and search next period for better matches. From (1), at this price the buyer fails to meet an acceptable employee with probability $1 - 2N\varepsilon^*$, which in a steady state generates

$$\sum_{t=0}^{t=\infty} e(1 - 2N\varepsilon^*)^t = \frac{e}{2N\varepsilon^*}$$

buyers searching in the market. Given that these buyers call in on the retail establishments with equal probability, the arrival rate in a market with small retailers is

$$\gamma = \frac{e}{2N\varepsilon^* I}, \qquad N \leqslant N^c.$$

In the zero profit condition, optimal prices from (2) along with their associated arrival rates γ yield the number of operating retailers as a function of the wage: $I = ep^*/wN$, or

$$I(w) = \begin{cases} \frac{e}{wN}[U - V - 1/2N] & N \geqslant N^c, \\ \frac{e}{2wN}[U - V] & N \leqslant N^c. \end{cases}$$

For producer entry, setting labour demand equal to labour supply, $N \cdot I(w) = S(w)$ determines the aggregate payments to the producers within retail establishments as a function of market parameters:

$$wS(w) = \begin{cases} \frac{e(NU - 1/4r - 1/2)}{N} & N \geqslant N^c \\ \frac{e([r^2+rNU]^{1/2}-r)}{N} & N \leqslant N^c \end{cases} \tag{6}$$

For a given producer supply function $S(w)$, equations (2), (4), (5) and (6) characterize an equilibrium and highlight the distinction between markets with large and small retailers as determined by N^c. In a market with retailers at and above this critical size, intermediation provides a sufficiently diverse product range that buyers perceive only small (idiosyncratic) differences among sellers. In other words, the distance between the buyer and the nearest product or employee is small when there are many different workers, and hence products, available. In this case, retailers price so that the most distant potential buyer is just indifferent between continued search and buying. This price occurs at the kink in the conditional demand curve.

In contrast, the optimal price in markets with smaller retailers ($N<N^c$) lies on the downward-sloping portion of the demand curve. Intermediaries with fewer products will on occasion encounter poorly matched buyers. Lowering price to serve these distant customers entails too great a loss of revenue from the more appropriately matched buyers. Consequently, these small outlets set the price so that not all customers are satisfied and further search regularly occurs.

III. Capacity and Welfare

The distinction between markets with large and small intermediaries extends to welfare. As characterized in the following proposition, the two equilibrium regimes exhibit fundamentally different responses to changes in retail capacity.

Proposition 1. For $N<N^c$, an increase in retail capacity lowers prices. The value of search increases while wages and/or the entry of producers declines. For $N>N^c$, a rise in capacity generates higher prices. The expected payoff to buyers declines while wages and/or the entry of producers increases.

Proof. Differentiation of the equilibrium conditions (2), (4), (5) and (6) establishes that

$$\begin{array}{llll} \partial V/\partial N<0; & \partial p^*/\partial N>0; & \partial[wS(w)]/\partial N>0 & \text{for } N^c \\ \partial V/\partial N>0; & \partial p^*/\partial N<0; & \partial[wS(w)]/\partial N<0 & \text{for } N^c. \end{array} \quad \blacksquare$$

The welfare implications follow directly from the comparative statics of Proposition 1.[8] The value of search V represents the expected utility of buyers. Proposition 1 therefore illustrates that buyers initially benefit from rising capacity. However, improvements in capacity beyond the threshold N^c prove harmful to the buyer. The purchase price mirrors these movements around N^c. A rise (fall) in the value of search coincides with a fall (rise) in the purchase price.

Sellers experience reverse fortunes around the N^c threshold. The changes in price induced by capacity movements alter the retail revenue that gets passed along to the producers in these establishments, thereby creating pressure on wages. Ultimately, this pressure leads to adjustments in compensation and entry. If the supply of producers $S(w)$ is perfectly elastic, so that w is constant, workers enter or exit accordingly to restore balance. If $S(w)$ is perfectly inelastic, so that S is constant, wages alone absorb the price pressure. Elasticities between these two extremes lead to a combination of both responses. Since the wage represents the flow utility of sellers, sellers lose (gain) from falling (rising) prices unless $S(w)$ is perfectly elastic.

Although technological advances in capacity involve better buyer–seller matches, i.e. more total surplus, an overshooting of sorts occurs. The additional benefits from a rise in N are not shared, hence technological advancement is not Pareto-improving. The mechanics of this 'overshooting of welfare' can be decomposed into two effects on the probability a buyer accepts an offer. First, for a given value of V, an increase in N flattens the slope of the conditional demand curve in (1). As seen in Figure 2, the kink in demand occurs at a higher price, making the downward sloping region of this curve more elastic, that is, more competitive. Second, the value of search V adjusts to the greater availability of products as well as any change in prices charged at competing outlets. As V changes, the demand curve shifts vertically.

The outcome of these two effects depends on the firm's pricing policy and consequently on the intitial level of N. Suppose that at the existing capacity outlets are pricing to serve all buyers ($N > N^c$ and $p^* = U - V - 1/2N$). This price coincides with the kink in the demand curve. From the first effect, greater capacity implies a higher kink in the demand curve, allowing the retailer to raise its price without losing customers. Given a fixed V, a retailer can extract more of the gains from trade from all consumers with a more diverse product range.[9] The firm of course does so. In addition, with all intermediaries charging higher prices as well as extracting more benefits of higher-quality matches, the value of search falls and buyers become worse off. As V falls, the conditional demand curve shifts up (the second effect in the above decomposition), thereby reinforcing the initial price response. The higher retail price in general translates into higher wages and hence greater welfare for the producer employees.

On the other hand, in markets where intermediaries price such that some buyers prefer to search for better-quality matches (markets with $N < N^c$), the optimal price lies on the downward-sloping portion of the demand curve. For a constant V, a rise in capacity N increases the probability that a buyer finds an acceptable match but leaves the optimal price unaffected. With V and p^* constant but matching improved (i.e. with only the first effect of the decomposition), the benefits resulting from a larger product range would then be shared between buyers and sellers. Better-quality matches, however, raise the value (but not the actual amount) of search. From the second effect in the decomposition, the intercept of the demand curve falls as retailers become more competitive with each other and there is downward pressure on prices. Despite providing better-quality matches, prices and hence wages fall, making producer-employees worse off. In contrast, buyers experience improvements in match quality as well as lower prices. Both of these changes raise the welfare of the buyer.[10]

The characterization of equilibrium can be extended to specialization and the division of labour. Recall that each employee in an outlet serves buyers at a maximum distance of $1/2N$. This distance can be interpreted as a measure of the technical division of labour resulting from the capacity constraint. The observed degree of specialization, however, differs from this technological limit in markets with small retailers. Specifically, when $N < N^c$, producers within the outlet serve clients only up to the distance $2\varepsilon^*$, which is strictly less than their technical specialization $1/2N$, as defined above.

As capacity expands, both the observed and the technical measures of the division of labour imply greater specialization, regardless of the extent of intermediation. However, in markets with small retailers the observed measure of specialization increases faster than the technical limit ($2\varepsilon^*$ decreases faster than $1/2N$) until the two become equal at $N = N^c$. In markets where retailers exceed this value $N > N^c$, the technical and observed measures are equal. While improvements in capacity result in each employee serving a more narrow range of clients, and hence exhibiting greater specialization, the scope of the firm—the total number of clients served as given by $2N\varepsilon^*$—rises as capacity increases, provided that $N < N^c$.[11] As technology improves and N increases, the scope of the outlet increases until it spans all potential customers. This discussion leads to Proposition 2.

Proposition 2. For all N, a rise in retail capacity creates higher quality matches and more specialization. For $N < N^c$, increased capacity extends the scope of the firm.

Proof. This follows by applying $\partial\varepsilon^*/\partial N < 0$ from (5) according to the above discussion. ∎

IV. Discussion

If consumers cannot observe the characteristics of a particular product-maker's output, they must search available sellers in order to find a good match. For this reason, intermediaries such as department stores develop. Retail intermediaries exist by offering consumers a variety of trading opportunities which reduce the uncertainty of search and improve the quality of consumer–producer trades.

The resulting reduction in costs, however, is not necessarily shared between buyers and sellers. When intermediaries can offer only a limited product range they are vulnerable to competition, and the loss of trade as buyers may credibly go elsewhere. As a result, buyers benefit from searching and paying less while sellers lose out in the presence of expanded retailing. When retailers offer a sufficiently wide variety of products, intermediaries appear relatively homogeneous to buyers. At this point consumers have no cause to shop around; therefore, improvements in intermediation cannot lower the level of search activity. If buyers have little incentive to search over similar opportunities, sellers are able not only to capture the direct gains from improved intermediation, but also to extract further rents from buyers. In this case, sellers gain at the expense of buyers.

Regardless of its effect on price, the introduction of intermediation leads to greater specialization. Retailers improve match quality, and therefore buyers become more selective about who or what products they are paired with. Consumers who trade only with sellers or products having specific characteristics induce more specialization in their partners. In this case, specialization is driven by the search behaviour of buyers, not the mechanics of production.

In recent years, large superstores with a vast array of products have emerged, replacing smaller and more limited retailers. Likewise, improvements in management have enabled other retailers of goods and services to offer clients a wider range of products. Employees within a variety of organizations—medical care-givers for example—often appear to serve a more narrow range of clients. These trends have not been uniformly welcomed. This paper suggests that such progress does not impact equally on traders. Rather than benefiting all traders, these advancements favour one side and harm the other side of the market. These issues and observations, of course, require further examination, first to determine whether such effects have indeed occurred and then, if so, to determine the relative winners and losers.

APPENDIX

(a) The role of search

To highlight the role of search in this model, consider the situation in which a buyer searches only once and then retires from the market regardless of whether a purchase is made. In this situation, a consumer will purchase if and only if $U - \varepsilon - p \geqslant 0$. Following the derivations in the text, profit is now given by

$$\Pi(p; N) = \gamma \cdot p \cdot \Pr(U - \varepsilon - p \geqslant 0) - w \cdot N$$

so that $p^* = \max\{U/2,\ U - 1/2N\}$ and $N^c = 1/U$.

The value of consumer search becomes

$$V = E_\varepsilon \max\{U - p^* - \varepsilon, 0\} = \int_0^{\varepsilon^*} (U - p^* - \varepsilon) 2N \mathrm{d}\varepsilon$$

- For $p^* = U/2$, $\varepsilon^* = U/2$ and $V = NU^2/4$.
- For $p^* = U - 1/2N$, $\varepsilon^* = 1/2N$ and $V = 1/4N$.

In each period only the e entrants contact retailers, so $\gamma = e/I$ and the zero profit condition is

$$\Pi(p; N) = \frac{e}{I} \cdot p^* \cdot \Pr(U - \varepsilon - p \geqslant 0) - w \cdot N = 0.$$

- For $p^* = U/2$, $\Pi(p;\ N) = 0$ implies $I = eU^2/2w$.
- For $p^* = U - 1/2N$, $\Pi(p;\ N) = 0$ implies $I = e/wN$ (U $- 1/2N$).

Equating supply and demand ($S(w) = NI$) gives

- For $p^* = U/2$, $wS(w) = eNU^2/2$.
- For $p^* = U - 1/2N$, $wS(w) = e(U - 1/2N)$

Although the price is constant for $N < N^c = 1/U$, buyer welfare exhibits the same pattern since matches improve with N. The significant difference is that seller welfare (in the form of aggregate compensation) increases for all N. The opportunity for consumer search in subsequent periods fundamentally affects the distribution of the gains from trade, especially at low capacity levels.

(b) Product selection

The optimal product range depends on consumer behaviour and the actions of competitors. There may therefore be several interesting equilibrium outcomes. The aim here is not to characterize all possible equilibria, but rather to

1. demonstrate that the imposed allocation in which retailers operate at capacity with an even but random distribution of products is an equilibrium outcome;
2. establish that firms choose first product range and then price;
3. relate the optimal product range choice (as embedded by the capacity constraint) to the costs of operating an outlet.

Let us consider these objectives in order.

1. In the imposed solution, a retailer and a buyer both expect the competition to locate its first product randomly and then to evenly space the remaining ones. Thus, it needs to be shown that this behaviour is an optimal strategy, given this expectation and the uniform distribution of consumers. Since there are a large number of buyers and sellers, this candidate appears, in an informal sense, to be reasonable. It has the added advantage of capturing the concept of idiosyncratic preferences (the absence of a superior location) central to the paper.

Since the buyer expects intermediaries to distribute a fixed number of products randomly and evenly, location does not alter the value of continued search, V. When the buyer then contacts an outlet, let $\hat{\varepsilon}(p)$ represent its reservation distance at the price p. Given the reservation distance, the maximum number of total locations that the N employees can serve equals $\max\{2N\hat{\varepsilon}, 1\}$. Since evenly spreading the employees achieves this maximum, it optimizes the probability that a buyer accepts. It is an optimal allocation of the N employees although not a unique best assignment if $2N\hat{\varepsilon}<1$. Moreover, no position is preferred when the manager assigns a location to the first product. Expectations are rational.

2. Let $\bar{N}$ be the (common) choice of (an evenly distributed) product range for all but one of the retailers. Let $\bar{p}$ be the associated price. The remaining intermediary's own choice of price, given an own size of N, solves

$$\max_{p} \Pi(p; N, \bar{N}) = \gamma \cdot p \cdot \Pr(U - \varepsilon - p \geqslant V) - w \cdot N - c(N),$$

where $c(N)$ represents the direct costs of operating a product range of size N. Assume that for $N>0$ this cost function is continuous, twice differentiable, increasing and convex. Fixed costs, however, allow average cost to exceed marginal cost over some range. Wages and the value of search are again exogenous and are determined in equilibrium by $\bar{N}$ but not N and p.

It is straightforward to establish that, as with wages, operating costs $c(N)$ do not influence a firm's pricing decision: p (as well as $\bar{p}$) depends in the same way as before on N (and $\bar{N}$). Recall that consumers do not observe the price before a contact is made, leading to a pricing policy that is a function of N (and indirectly of $\bar{N}$ through V):

$$p^* = \begin{cases} (U-V)/2, & N<\bar{N}^c, \\ U - V - 1/2N, & N \geqslant \bar{N}^c, \end{cases}$$

where $\bar{N}^c = 1/(U-V)$. Accounting for this pricing policy, profit for a given N becomes

$$\pi(N; \bar{N}) = \begin{cases} 2\gamma[(U-V)/2]^2 N - wN - c(N), & N<\bar{N}^c, \\ \gamma[U - V - 1/2N] - wN - c(N), & N \geqslant \bar{N}^c. \end{cases}$$

This formulation illustrates that it is possible to treat price as a function of N without concern about feedback effects, as has been done in the paper using the capacity constraint. In other words, it is justified first to determine the product range and then to examine the price. The price response to movements in the determinants of product range (that is, the cost function $c(N)$ as shown below) are only a reaction to the alteration in the retailer's products.

3. As discussed in the text, the arrival rate of consumers at a firm may be linked to its size and the sizes of its competitors in a number of ways. To avoid taking a stand, let the arrival rate γ depend on the retailer's own size choice N and the (common) choice of the other intermediaries $\bar{N}$ subject to the constraint that total number of contacts equals the number of searching consumers: $\gamma = \gamma(N; \bar{N})$. Allowing for this dependence, the

derivative of the profit function is given by

$$\frac{\partial\pi(N;\bar{N})}{\partial N}=\begin{cases}2\gamma[(U-V)/2]^2+2\gamma_1[(U-V)/2]^2N-w-c'(N), & N<\bar{N}^c,\\ \gamma/2N^2+\gamma_1(U-V-1/2N)-w-c'(N), & N\geqslant\bar{N}^c,\end{cases}$$

where $\gamma_1\equiv\partial\gamma(N;\bar{N})/\partial N$. Similarly,

$$\frac{\partial^2\pi(N;\bar{N})}{\partial N^2}=\begin{cases}2[(U-V)/2]^2(2\gamma_1+\gamma_{11}N)-c''(N), & N<\bar{N}^c,\\ 1/N^3(\gamma_1N-\gamma)+\gamma_{11}(U-V-1/2N)-c''(N), & N\geqslant\bar{N}^c,\end{cases}$$

where $\gamma_{11}\equiv\partial^2\gamma(N;\bar{N})/\partial N^2$. Given decreasing returns to matching rates ($\gamma_{11}<0$), π is concave in N for $N>\bar{N}^c$. Decreasing returns in the contact function, however, are not sufficient for concavity of π for $N<\bar{N}^c$. Additional assumptions making γ sufficiently flat or curved ($2\gamma_1+\gamma_{11}N<0$) are more than adequate. Assume that γ and c are such that π is everywhere concave.

We now establish conditions under which an $N<N^c$ equilibrium exists and under which an $N\geqslant N^c$ equilibrium exists. We then examine when these conditions.

Case A: *$\bar{N}<N^c$ equilibrium.* $\bar{N}<N^c$ implies

$$\bar{p}=(U-V)/2=\frac{(r^2+r\bar{N}U)^{1/2}-r}{\bar{N}}.$$

Zero profit condition further implies

$$w+\frac{c(\bar{N})}{\bar{N}}=2\gamma(\bar{N};\bar{N})\left(\frac{(r^2+r\bar{N}U)^{1/2}-r}{\bar{N}}\right)^2,$$

while profit maximization gives

$$\text{(A1)}\qquad w+c'(\bar{N})=2[\gamma(\bar{N};\bar{N})+\gamma_1(\bar{N};\bar{N})\bar{N}]\left(\frac{(r^2+r\bar{N}U)^{1/2}-r}{\bar{N}}\right)^2$$

Combining gives

$$c'(\bar{N})-\frac{c(\bar{N})}{\bar{N}}=2\gamma_1(\bar{N};\bar{N})\bar{N}\left(\frac{(r^2+r\bar{N}U)^{1/2}-r}{\bar{N}}\right)^2\equiv\Psi_A(\bar{N}).$$

For a given $\bar{N}<N^c$ satisfying (7) to be an equilibrium, it is sufficient (given the piece-wise concavity of π) to establish that

$$\lim_{N\to N^c-}\partial\pi(N;\bar{N})/\partial N\leqslant 0.$$

Note that concavity, along with the first-order condition

$$\partial\pi(N;\bar{N})/\partial N_{|N=\bar{N}}=0,$$

implies that $\partial\pi(N;\bar{N})/\partial N<0$ for all $N\in(\bar{N},N^c)$. Further, note that

$$\lim_{N\to N^c+}\partial\pi/\partial N-\lim_{N\to N^c-}\partial\pi/\partial N=$$
$$2(\gamma+2\gamma_1)\left(\frac{(r^2+r\bar{N}U)^{1/2}-r}{\bar{N}}\right)^2-\frac{\gamma}{2(N^c)^2}-\gamma_1\left(U-V-\frac{1}{2}N^c\right)$$

where γ and γ_1 are evaluated at $(N^c;\bar{N})$. As

$$U-V-\frac{1}{2}N^c=\frac{(U-V)}{2}=\frac{(r^2+r\bar{N}U)^{1/2}-r}{\bar{N}},$$

this difference is non-negative (assuming $\gamma_1\geqslant 0$), which completes this case.

Case B: $\bar{N} \geqslant N^c$ equilibrium. $\bar{N} \geqslant N^c$ implies

$$\bar{p} = U - V - \frac{1}{2}\bar{N} = U - \frac{1}{4}r\bar{N} - \frac{1}{2}\bar{N}.$$

Likewise, the zero profit condition implies

$$w + \frac{c(\bar{N})}{\bar{N}} = \frac{\gamma(\bar{N};\bar{N})}{\bar{N}}\left(U - \frac{1}{4}r\bar{N} - \frac{1}{2}\bar{N}\right),$$

while profit maximization gives

$$\text{(A2)} \qquad w + c'(\bar{N}) = \frac{\gamma(\bar{N};\bar{N})}{2\bar{N}^2} + \gamma_1(\bar{N};\bar{N})\left(U - \frac{1}{4}r\bar{N} - \frac{1}{2}\bar{N}\right).$$

Combining gives

$$c'(\bar{N}) - \frac{c(\bar{N})}{\bar{N}} = \left(U - \frac{1}{4}\bar{N} - \frac{1}{2}\bar{N}\right)(\gamma_1\bar{N} - \gamma) + \frac{\gamma}{2\bar{N}^2} \equiv \Psi_B(\bar{N}).$$

For an $\bar{N} \geqslant N^c$ satisfying (A2) to be an equilibrium, it is sufficient (given the piecewise concavity of π) to establish that

$$\lim_{N \to N^c+} \partial\pi(N;\bar{N})/\partial N \geqslant 0.$$

Note that concavity, along with the first-order condition, implies

$$\lim_{N \to N^c-} \partial\pi(N;\bar{N})/\partial N > 0.$$

Moreover,

$$\lim_{N \to N^c+} \partial\pi/\partial N - \lim_{N \to N^c-} \partial\pi/\partial N =$$
$$2\gamma[(U-V)/2]^2 + 2\gamma_1[(U-V)/2]^2 N^c - \frac{\gamma}{2(N^c)^2} - \gamma_1(U - 1/4rN^c - 1/2N^c)$$

where γ and γ_1 are evaluated at $(N^c;\bar{N})$. As

$$U - 1/4rN^c - 1/2N^c = (U - 1/4rN^c)/2,$$

this difference equals zero and completes case B.

Discussion. The existence of a case A or case B solution—a positive solution to either (A1) or (A2)—clearly depends both on the cost structure and on the functions Ψ_A and Ψ_B. Assume for the purposes here that $c'(N) - c(N)/N$ is increasing in N, strictly negative at $N = 0$, and positive after some finite N. This is the specification often assumed in introductory economics textbooks. On the other hand, it can be shown that

(i) $\Psi_A(N^c) = \Psi_B(N^c)$ so that continuity holds;
(ii) $\lim_{N \to 0} \Psi_A(N) = \infty$ for $\gamma_1 > 0$;
(iii) $\lim_{N \to \infty} \Psi_B(N) = U \times \lim_{N \to \infty}[\gamma_1(N;N)N - \gamma(N;N)]$.

Assuming that

$$U \times \lim_{N \to \infty}[\gamma_1(N;N)N - \gamma(N;N)] < \lim_{N \to 0}[c'(N) - c(N)/N],$$

(i)–(iii), along with the basic assumptions on $c'(N) - c(N)/N$, are sufficient for the existence of a positive intercept of $c'(N) - c(N)/N$ and either $\Psi_B(N)$ or $\Psi_B(N)$, and hence an equilibrium. The point of intersection obviously depends on the position of $c'(N) - c(N)/N$. Appropriately specifying the cost function makes either type of equilibrium feasible.

Without further restrictions, little can be said regarding uniqueness. Note as well that, if there are constant returns to scale ($c'(N) = c(N)/N$ for all N), and if

$\lim_{N \to \infty}[\gamma_1(N; N)N - \gamma(N; N) = 0]$, then the likely equilibrium is one big retailer. On the other hand, if $c'(N) = c(N)/N$ for all N along with $\gamma_1 = 0$, then all $N \in [0, N^c]$ are equilibria.

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NOTES

1. This role for intermediation is related to search-theoretic explanations for the existence of money. Money and intermediaries both exist to overcome search frictions. Kiyotaki and Wright (1993) develop the search approach for money. Li (1998) extends this framework to allow for intermediation.
2. A general definition of intermediation incorporates several activities. For example, estate agents intermediate by introducing buyers and sellers, while middlemen such as used car dealers buy and then sell goods. The framework presented here is compatible with both interpretations. Retailers offer multiple alternatives to potential customers and pass along payments to the seller. It is not important whether the retail firm buys the product and then sells (as many retail outlets clearly do), or directs the customer to the relevant producers (as can happen in other situations).
3. Although the intermediary selects its product range before trade begins, actual production occurs at the time of purchase: there are no inventories.
4. Again, to keep things simple, there are no capital costs or opportunity costs for the manager.
5. As in much of the undirected search literature, it is also assumed that the contact rate γ is independent of the price charged, p. Moreover, two or more buyers cannot simultaneously contact the same firm. Allowing for this possiblity generates a Poisson distribution of customers. However, as the time interval becomes small, this outcome becomes inconsequential.
6. Other equilibria may exist but are not examined. For example, there may exist equilbria involving strategies linked to buyer locations so that some positions are preferred to others. These strategies undermine the idiosyncratic nature of matching and can require a high level of coordinated behaviour not found in the equilibrium analysed here.
7. The threshold N^c occurs when the two price possibilities are equal and therefore solves $U - V - 1/2N^c = (U - V)/2$.
8. Comparative statics with respect to other parameters follow similarly.
9. With a large range of products, idiosyncratic differences, captured by the expected distance of ε, among outlets are small. Larger retailers appear more homogeneous, which allows them to extract higher rents, a result related to Diamond's (1971) result on monopoly pricing by firms in a search economy. However, this model differs somewhat from the Diamond result since consumers receive some gains from trade for all finite N. Owing to *ex post* differences in matching payoffs, the monopolist does not fullly extract all rents. As in the literature, increased heterogeneity undermines seller's ability to post prices that capture all of the gains to trade. The equilibrium presented here, however, does not generate a distribution of prices. The law of one price holds. Several authors have demonstrated that dispersion can arise if there are *ex ante* differences among buyers.
10. Consumer search is critical for the existence of this overshifting of welfare gains. The vertical shift in demand—the second effect in the decomposition—relies on adjustments in the value of search. As such, search is similar to a public good for consumers. Less search, arising from greater variety, lowers the public good and allows retailers to raise prices. When outlets are large ($N > N^c$) this indirect effect outweighs the direct effect and consumers become worse off. The Appendix demonstrates that without consumer search sellers will always benefit from an expansion in capacity.
11. For $N > N^c$, the retailer serves all possible consumers so the scope is constant and equal to one as N changes. At a non-spanning price, i.e. when $N < N^c$, a retailer is indifferent between an evenly spread allocation of employees and some uneven distributions. Given this price, a retailer would earn the same expected revenue (and profit) if it could relocate products in such a way that all prospective buyers are never willing to purchase from two employees in the outlet (except in the case where the buyer is indifferent between buying and continued search). This occurs when there are potential clients unwilling to purchase from any

employee. As such, $2N\varepsilon^*$ reflects the probability that a contact is consummated, as well as how many consumer types a retailer covers at the chosen price, while $1 - 2N\varepsilon^*$ captures the amount of continued search in the economy.

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