THE JOURNAL OF FINANCE • VOL. XLVIII, NO. 4 • SEPTEMBER 1993

Brokerage Commission Schedules

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ABSTRACT

It is generally optimal for risk-sharing reasons to base a charge for information on the signal realization. When this is not possible, a charge based on the amount of trading, a brokerage commission, may be a good alternative. The optimal brokerage commission schedule is derived for a risk-neutral information seller faced with risk-averse purchasers who may differ in their risk aversion. Revenues from the brokerage commission are compared with those from a fixed charge for information and the optimal mutual fund management fee.

THE BROKERAGE INDUSTRY IS a significant source of information for investors about prospective security returns. One indication of this is that of the 11,817 members of the Association for Investment Management and Research who listed their occupations as investment management or research in 1991, 21 percent worked for brokers and investment dealers; another is that since the abolition of fixed commissions a distinction has arisen between full service brokers who provide investors with investment information, and discount brokers who charge lower commissions and provide only transaction services.[1] The information produced by broker research is typically provided free of charge to investors in the expectation that it will stimulate trade, rewarding the brokerage house with commissions.[2]

This paper compares the sale of investment information in return for a brokerage commission with direct sale for a fixed payment, and indirect sale through the provision of investment management services in return for a management fee,[3] and shows that under certain conditions it is efficient to sell information in return for a brokerage commission. In order for sale in

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[1] Full service brokers still account for 75 percent of all individual investor trading (*Wall Street Journal*, September 18, 1992).

[2] Payment for research by way of brokerage commissions is recognized in the Investment Company Act which allows fiduciaries to pay a higher commission if it is "reasonable in relation to the value of the brokerage and research services provided." (See Bloomenthal (1990), p. 73.) Investment analysts who work for brokerage houses are often remunerated on the basis of the amount of commissions generated by their recommendations. (See Brennan and Hughes (1991)).

[3] Following Admati and Pfleiderer (1990) we distinguish between a direct sale in which the purchaser receives the information, and an indirect sale in which the information is used to manage a portfolio on behalf of the purchaser, but without revealing the information.

return for a brokerage commission to be feasible it must be possible for brokers to monitor client trading without excessive cost,[4] and we assume that client trading is costlessly observable.

When information purchasers are homogeneous the advantage of a sale for a brokerage commission over a charge that is independent of the information signal, such as payment for a newsletter or an investment management fee, is that it allows the purchaser and the seller to share the risk about the signal realization. On the other hand, the brokerage fee distorts the investment decision of the purchaser and furthermore is unable to extract all of the purchaser's surplus, so that it is not obvious a priori which method of sale will raise the greatest revenue.[5]

When information purchasers are heterogeneous, both an investment management fee that depends on the funds under management and a brokerage commission have the advantage over a fixed fee of discriminating between investors with different risk tolerances. Although both create distortions in the investment decision, the brokerage commission has superior risk-sharing properties to the other two alternatives.

In order to concentrate on risk sharing and price discrimination considerations we abstract from other important considerations in the sale of information such as the reliability of the seller's information[6] and the externalities in the use of information that arise from competition between informed investors and the informativeness of security prices.[7] We also assume that purchasers of the information are not able to resell it. Given this setting, we find that when purchasers are homogeneous the brokerage commission is efficient relative to the other modes of sale if the information is precise, and when purchasers are heterogeneous it is dominated only by an indirect sale with a general (nonlinear) participation charge for imprecise signals.[8] Our conclusions must be tempered by the restrictive nature of the assumptions. For example, we do not consider the advantage of the mutual fund in overcoming problems of information resale and externality in use. Nevertheless, our results suggest that the provision of investment information by brokerage houses may be efficient if these problems are not too severe.

[4] There is an obvious incentive for investors to receive information from full service brokers and then to route the bulk of their trades through discounters. In practice brokers restrict the flow of information to investors who do not trade through them. Cf. Admati and Pfleiderer (1990), p. 906: "If the seller could observe the response of each trader to the information, he might wish to charge each trader as a function of his response."

[5] Kane and Marks (1990) point out that another advantage of a direct sale such as a newsletter is that it allows purchasers to combine different signals from competing sellers. An advantage of any direct sale over an indirect sale is that it allows the purchaser to combine the information of the seller with private information possessed by the purchaser.

[6] See Hirshleifer (1971), Leland and Pyle (1977), Campbell and Kracaw (1980), Bhattacharya and Pfleiderer (1985), and Allen (1990).

[7] See Admati and Pfleiderer (1986, 1990).

[8] Investment management services sold through mutual funds permit only fees that are proportional to the funds under management. (See Frankel (1978), p. 249.)

Section I analyses the case of homogeneous information purchasers. Section II extends this to heterogeneous purchasers and Section III concludes.

I. Charges for Information with Homogeneous Purchasers

Consider a setting in which there is a single risky asset with return, $\tilde{x}$, and initial price P, and let R be the gross riskless interest rate. An information seller faces a potential information purchaser (who behaves competitively), and after contracting with the purchaser observes a realization of a signal of asset payoff, $\tilde{y}$, where

$$\tilde{x} = \tilde{y} + \tilde{\varepsilon}, \tag{1}$$

and $\tilde{x}$, $\tilde{y}$, and $\tilde{\varepsilon}$ are distributed normally with variances σ_x^2, σ_y^2, and σ_ε^2. $\tilde{\varepsilon}$ is a mean zero error and the common mean of $\tilde{x}$ and $\tilde{y}$ is denoted μ. $\tilde{\varepsilon}$ is independent of $\tilde{y}$.

It can be shown that when both the seller and the purchaser of the information can invest on personal account, the Pareto-optimal schedule of charges for information will depend upon the signal, y, except in the special case in which the information purchaser and the seller have HARA utility with identical cautiousness. Thus, in general there are efficiency gains from sharing the risk associated with the signal realization between purchaser and seller.

For reasons of analytical tractability, we shall assume that the information seller is risk neutral, but that capital requirements imposed by regulators or by lenders restrict the ability of the seller to borrow cash or securities, so that the seller is able to take only a limited position in the security on personal account. It is assumed that the seller takes a portfolio position that is optimal given the signal and the constraints. The information purchaser is assumed to be risk averse with a utility function of the form:

$$U(W) = -\exp(-aW), \tag{2}$$

where a, the coefficient of absolute risk aversion, is common knowledge. This may be thought of as a situation in which the information seller is large relative to the purchaser, which would be representative, for example of financial intermediaries dealing with retail clients.

Under these assumptions $V_{Uy}(W_0)$, the expected utility of the information purchaser, with initial wealth W_0, conditional on the signal y, is given by:[9]

$$V_{Uy}(W_0) = -\exp\left\{-a\left[RW_0 + \frac{(y - RP)^2}{2a\sigma_\varepsilon^2}\right]\right\}, \tag{3}$$

[9] See Grossman and Stiglitz (1980).

and $V_U(W_0)$, the expected utility of the information purchaser, with initial wealth W_0, conditional on not receiving the signal, is:[10]

$$V_U(W_0) = -\exp\left\{-a\left[RW_0 + \frac{(\mu - RP)^2}{2a\sigma_x^2}\right]\right\}. \tag{4}$$

Since the information seller is risk neutral, the welfare gain of a signal-based charge for information over a fixed charge may be expressed as the difference between the expected revenue of an information monopolist under the two systems. The following theorem provides a basis for assessing this gain.

THEOREM 1: 1. (*Allen* (1990)) *The maximum noncontingent payment that the purchaser is willing to make for the information is given by*:

$$\frac{1}{aR}\left[\log\left(\frac{\sigma_x}{\sigma_\varepsilon}\right)\right]. \tag{5}$$

2. *The maximum signal-contingent payment schedule the information purchaser would be willing to agree to, $c(y)$, is given by*:

$$c(y) = \frac{1}{2aR}\left[\frac{(y - RP)^2}{\sigma_\varepsilon^2} - \frac{(\mu - RP)^2}{\sigma_x^2}\right]. \tag{6}$$

Proof: For (5) see Allen (1990) equation (13), and note that Allen implicitly assumes that the information is paid for at the end of the period. Under the revenue-maximizing schedule the information seller will extract the whole of the consumer surplus of the purchaser for each realization of the signal, y. Then substituting $[W_0 - c(y)]$ for W_0 in (3) and equating the result to (4) yields (6). Q.E.D.

The expected revenue under the signal-contingent payment schedule is obtained by taking expectations in (6),

$$E[c(y)] = \frac{1}{2aR}\left\{\frac{\sigma_y^2}{\sigma_\varepsilon^2} + (\mu - RP)^2\left[\frac{1}{\sigma_\varepsilon^2} - \frac{1}{\sigma_x^2}\right]\right\}. \tag{7}$$

Subtracting (5) from (7), the welfare gain from the signal-contingent schedule may be written as:

$$\frac{1}{2aR}\left\{\frac{\sigma_y^2}{\sigma_\varepsilon^2} - \log\left(\frac{\sigma_x^2}{\sigma_\varepsilon^2}\right) + (\mu - RP)^2\left[\frac{1}{\sigma_\varepsilon^2} - \frac{1}{\sigma_x^2}\right]\right\}. \tag{8}$$

The third term in (8) is positive so long as the expected return on the risky asset is not equal to the riskless interest return and $\sigma_y^2/\sigma_\varepsilon^2 \neq 0$; the differ-

[10] See Allen (1990), equation (9).

ence between the first two terms is also positive, since $\sigma_x^2/\sigma_\varepsilon^2 = 1 + \sigma_y^2/\sigma_\varepsilon^2$. Thus, the expected revenue of the information monopolist is higher under the signal-contingent payment schedule than under the fixed payment scheme.

However, it is unlikely in practice that it will be possible to make the payment for information depend on the signal, since information about the distribution of asset returns is typically of a qualitative nature that cannot be verified by an independent third party. In such cases the payment can at best depend upon a proxy for the signal. One such proxy is the payoff on the asset itself,[11] and, given the risk aversion of the purchaser, making the payment a function of the asset payoff will increase the discounted expected payment by allowing the risk-neutral seller to take an additional implicit position in the security. For example, the information purchaser is indifferent between paying a fixed charge K and giving the seller the payoff on K/P shares. The discounted expected payoff on these shares is $K\mu/RP$ which will exceed K if the shares have a positive risk premium. Unlimited further gains from trade are possible if the payment can be an arbitrary function of the asset payoff since such contracts allow the risk-neutral information seller to evade the limits on security positions mentioned above.[12] Therefore, in keeping with our assumption that the seller's ability to take positions in the security is limited, we assume that payoff-contingent payments for information are prohibited. A second proxy for the signal is the amount of trading done by the information purchaser, assuming that this is observable by the information seller. In the next section we consider a transaction-based charge for information under which the payment depends on the size of the trade by the information purchaser. As we shall see, a transaction-based charge imposes deadweight losses, since it affects the purchase's trade decision. Consequently, expression (8) provides an upper bound for the welfare gains possible from a transaction-based charge over a fixed fee.

A. Transaction-based Charges for Information

Let n_0 denote the information purchaser's initial position in the security, and $n \equiv n(y)$ denote the optimal position after the signal is received. $n(y)$ depends on the transaction charge $T(z)$ which is levied on the transaction $z \equiv n - n_0$ where positive and negative values of z correspond to purchases and sales respectively. $T(z)$ may be thought of as a brokerage commission. We have already remarked that there is a deadweight loss involved in a commission or transaction charge because of its effect on the optimal size transaction of the purchaser; since the information seller was able to extract all of the purchaser's consumer surplus under the signal-contingent payment scheme, this loss manifests itself in a reduction in the expected revenues of the information seller. In the following theorem we show that under a pure

[11] We are grateful to the referee for bringing this point to our attention.

[12] The information purchaser would be willing to trade the excess return on an unlimited amount of the security for an uninformative signal. With a positive risk premium the expected payoff on this contract would be unbounded.

commission charge structure, the information seller suffers loss by not being able to extract all of the purchaser's surplus.

THEOREM 2: *There does not exist a pure commission schedule that exhausts the purchaser's surplus for all values of the signal realization.*

Proof: It is readily shown, using the properties of the moment-generating function for normal random variables, that the decision problem of the information purchaser, given the signal y and an initial holding n_0, is of the form:

$$\max_n \left[(y - RP)n - T(n - n_0) - \frac{a\sigma_\varepsilon^2 n^2}{2} \right]. \tag{9}$$

Then $n(y)$, the optimal position of the information purchaser, conditional on the signal y, satisfies:

$$n(y) = \frac{y - RP - T'(n - n_0)}{a\sigma_\varepsilon^2}, \tag{10}$$

where $T'(n - n_0)$ is the first derivative of the commission schedule. Note that for sales of securities $n < n_0$ and $T'(n - n_0) < 0$. Rearranging (10), the first order condition for an optimum in the purchaser's decision problem can be written as:

$$y - RP - T'(n - n_0) - a\sigma_\varepsilon^2 n = 0. \tag{11}$$

Write the purchaser's reservation level of utility $V_U(W_0)$ as $-\exp(-aC)$. Then, noting that the purchaser's expected utility conditional on the signal y, when he purchases $n(y)$ shares, may be written as:

$$-\exp\left\{ -a\left[(y - RP)n(y) - T(n - n_0) - \frac{a\sigma_\varepsilon^2 [n(y)]^2}{2} \right] \right\}, \tag{12}$$

the condition for the purchaser to receive no surplus is that:

$$(y - RP)n(y) - T(n - n_0) - \frac{a\sigma_\varepsilon^2 [n(y)]^2}{2} = C, \qquad \forall y. \tag{13}$$

Differentiate (13) with respect to y:

$$n(y) + \left[y - RP - T'(n - n_0) - a\sigma_\varepsilon^2 n(y) \right] \frac{dn}{dy} = 0. \tag{14}$$

Combining (14) with (11), the condition for the purchaser to receive zero surplus for all signal levels is that $n(y) = 0$; but that is to say that the purchaser does not use the signal. Q.E.D.

The reason that a commission charge is unable to exhaust the purchaser's surplus is that the demand curve for shares depends upon the signal level, so that perfect price discrimination is not possible. This may be compared with the situation in Allen (1990) where the monopolist information seller is unable to charge a fee high enough to extract all the purchaser's surplus because of the need to satisfy a self-selection constraint arising from the information reliability problem which is the focus of that paper.

Since a pure commission schedule does not exhaust the purchaser's surplus, Theorem 2 implies that the optimal general commission structure consists of a fixed element, K, and a commission, $T(z)$, that depends upon the number of shares traded. However, in order to obtain a closed form expression for the commission schedule, we shall concentrate on the commonly observed pure commission structure which does not contain a fixed charge.[13]

The deadweight losses that result from a pure commission charge mean that, despite the risk-sharing advantages of such a payment scheme over a fixed fee, it is unclear whether a fixed fee or a commission charge will raise a higher level of expected revenue for the information seller.[14] As a preliminary to investigating this issue, we characterize in the following theorem the commission schedule that will maximize expected revenues. We shall assume that n_0, the number of shares initially held by the information purchaser, is given by

$$n_0 = \frac{\mu - RP}{a\sigma_\varepsilon^2}. \tag{15}$$

n_0 may be thought of as the number of shares the information purchaser holds over from the previous period when he received a null signal $y = \mu$. Then the commission is a function of the net trade $(n - n_0)$, and $T'(n - n_0) < 0$ for $n < n_0$.

THEOREM 3: *The pure commission schedule that maximizes the expected revenue of the information seller satisfies*

1. $$T'(n - n_0) \equiv t(n - n_0) = \frac{1 - F[y(n)]}{f[y(n)]} \quad \text{for } n > n_0$$
$$= -\frac{F[y(n)]}{f[y(n)]} \quad \text{for } n < n_0, \tag{16}$$

where $F[y]$ is the distribution function of the signal, $f(y)$ is its density function, and $y(n)$, the signal level that induces a final holding of n shares, is given from (11) *by*:

$$y(n) = RP + a\sigma_\varepsilon^2 n + t(n - n_0). \tag{17}$$

[13] A general commission structure might be difficult to implement in practice, since the explicit contract associated with the payment of a fixed fee would limit the ability of brokers to restrict the flow of information to investors who do not trade through them sufficiently. Investment advisors are required to treat all their clients fairly, in terms of timing and quality of information provided. (See Frankel (1978), pp. 390–405.)

[14] Of course, the general commission structure will always dominate both the fixed fee and the pure commission structure, which it includes as special cases.

2. *The marginal cost of transacting is decreasing in the absolute value of the size of the transaction,* $z \equiv n - n_0$:

$$\operatorname{sgn}[t'(z)] = -\operatorname{sgn}[z]. \tag{18}$$

Proof: M, the expected revenue of the information seller may be written as[15]

$$M = \int_{n_0}^{\infty} t(n - n_0)\{1 - F[y(n)]\}\, dn + \int_{n_0}^{-\infty} t(n - n_0)F[y(n)]\, dn, \tag{19}$$

where from (10), $y(n)$ is the signal which leads the purchaser to hold n shares, assuming that the second order condition for a maximum in (9) holds.

The first term in (19) corresponds to security purchases, and for each possible level of purchases, $n - n_0 \geq 0$, multiplies the marginal commission by the probability that the purchase will exceed that level; the second term corresponds to security sales. Pointwise maximization of (19) with respect to $t(n - n_0)$ yields (16).[16] The second-order conditions for a maximum in (19) for the seller and in (9) for the purchaser are verified in the Appendix along with part 2 of the theorem. □

The optimal commission schedule is thus determined by a comparison of the marginal benefit of increasing the transaction fee for a particular value of n, $t(n)$, with the marginal cost. The former is proportional to the probability that the nth share is inframarginal ($1 - F[y(n)]$ for $n > n_0$), while the latter depends upon the current level of the transaction fee, $t(n)$, and the marginal effect of the transaction fee on the probability that the nth share is not traded, which under our normal-exponential specification is proportional to the signal density.

The finding that the optimal marginal commission is declining in the absolute size of the transaction corresponds with what is observed in practice for the retail commission schedules of full service brokers, which decline with the size of trades.[17] This result is robust to changes in the specification of

[15] To see this, note that from the definition of $T(n - n_0)$

$$M = \int_{-\infty}^{\infty} T(n - n_0)\left\{\lim_{\Delta n \to 0}\left[\frac{F\left[y\left(n + \frac{\Delta n}{2}\right)\right] - F\left[y\left(n - \frac{\Delta n}{2}\right)\right]}{\Delta n}\right]\right\} dn$$

$$= \int_{-\infty}^{\infty} T(n - n_0) f[y(n)]y'(n)\, dn.$$

Integrating by parts, it can be shown that this is identical to (19).

[16] We are implicitly assuming that the commission schedule is differentiable. Brown and Sibley (1986) show that with diverse and continuously distributed consumers an optimal n-part tariff is Pareto dominated by an optimal $(n + 1)$-part tariff, which in the limit results in a continuous price schedule.

[17] See, for example, the evidence cited in Brennan and Hughes (1990).

preferences and beliefs so long as the hazard function of the signal is increasing. An interesting consequence of the declining commission structure is that there is a minimum level of the signal, y^*, required to induce a (positive) trade, where $y^* = \mu + h^{-1}[u^*]$ and $u^* = (y^* - \mu)/\sigma_y$.

Inspection of (42) reveals the following additional properties of the optimal commission schedule.

1. The marginal commission, interpreted as the marginal cost of buying a security for $z > 0$, and the marginal cost of selling a security for $z < 0$, is everywhere positive. This result is valid for arbitrary preferences and beliefs, given the additive signal structure (1).
2. The commissions on purchases and sales are identical:

$$t(z) = -t(-z). \tag{20}$$

This result is sensitive to the symmetry of the normal distribution and the assumption about the initial level of shareholdings, n_0. Given the symmetry in the commission structure, we report below only the comparative statics of the commission schedule for $z > 0$.

3. The optimal commission rate is decreasing in the conditional variance of the return:

$$\frac{\partial t(z)}{\partial \sigma_\varepsilon^2} = -\frac{\sigma_y h'(u)}{[h(u)]^2} \frac{\partial u}{\partial \sigma_\varepsilon^2} < 0. \tag{21}$$

This result, which is valid for all increasing signal hazard functions, accords with the intuition that it will be optimal for information sellers with less precise information to charge a lower commission rate.

4. It is optimal to charge lower commission rates for more risk-averse purchasers:

$$\frac{\partial t(z)}{\partial a} = -\frac{\sigma_y h'(u)}{[h(u)]^2} \frac{\partial u}{\partial a} < 0. \tag{22}$$

This result holds for any definition of risk aversion such that the more risk averse a purchaser the greater the signal value required to hold a given number of shares.

5. The optimal commission schedule does not depend on the signal variance or the risk of the stock:

$$\frac{\partial t(z)}{\partial \sigma_y^2} = \frac{\partial t(z)}{\partial \sigma_x^2} = 0. \tag{23}$$

This is true for arbitrary specifications of preferences and beliefs for the additive signal structure (1).

Substituting for $t(n - n_0)$ from (42) in (19) gives the expected revenue under the optimal commission schedule

$$M = \int_{n_0}^{\infty} h^{-1}[u(y(n))]\{1 - F[y(n)]\}\, dn + \int_{-\infty}^{n_0} h^{-1}[-u(y(n))]F[y(n)]\, dn, \tag{24}$$

where

$$y(n) = RP + a\sigma_\varepsilon^2 n + \sigma_y h^{-1}[u], \qquad n > n_0$$

$$y(n) = RP + a\sigma_\varepsilon^2 n - \sigma_y h^{-1}[-u], \qquad n < n_0$$

$$u = \frac{y(n) - \mu}{\sigma_y}.$$

In Section I.*C* we shall use (24) to compare the expected revenue of a monopoly information seller under direct sales with fixed and commission structures of charging for information with the expected revenue under an indirect sale. In the following section we consider the expected revenues attainable under an indirect sale.

B. Indirect Sale of Information

Under an indirect sale the information seller manages a mutual fund and charges purchasers either a fixed fee or a fee that depends on their participation in the fund. In contrast to the commission-based fee, the charge under an indirect sale is determined prior to the signal realization and thus does not allow sharing of the signal risk. Following Admati and Pfleiderer (1990), we define a unit participation in the fund as entitling the investor to the return on a purchase of θy shares of the risky asset when the signal is y. The information purchaser may also make a purchase of the risky asset on personal account; the amount of this purchase is denoted by $-q$. His end-of-period wealth is then given by (See Admati and Pfleiderer (1990) equation (A1))

$$\tilde{W} = RW_0 - RD(\theta) + (\theta\tilde{y} - q)(\tilde{x} - RP), \tag{25}$$

where $D(\theta)$ is the fund management charge for θ shares. The information purchaser chooses θ and q to maximize his expected utility. The following lemma, which is proven in the Appendix, characterizes the optimal decision of the information purchaser.

LEMMA 1: *The optimal holdings in the mutual fund and in the stock for an information purchaser with risk aversion a are*

$$q(a) = \frac{a\theta\left(\mu\sigma_\varepsilon^2 + RP\sigma_y^2\right) - (\mu - RP)}{a\sigma_x^2},$$

$$\theta(a) = \frac{2rR\sigma_y^2 + \sigma_y^2\sigma_\varepsilon^2 - \sqrt{4r^2R^2\sigma_y^2\sigma_x^2 + \sigma_y^4\sigma_\varepsilon^4}}{2arR\sigma_y^2\sigma_\varepsilon^2} \tag{26}$$

where $r \equiv r(\theta) \equiv D'(\theta)$ is the marginal cost of increasing participation in the fund.

The information seller's problem is to choose a charge structure to maximize $M = D(\theta)$. The optimal solution to this problem is to set the marginal participation charge r equal to zero to avoid distorting the purchaser's participation decision, and to impose a fixed charge which will extract all of the purchaser's surplus. This dominance of the fixed charge contrasts with the conclusion of Admati and Pfleiderer (1990) that a two-part charge is preferred, because in their setting it is optimal to restrict purchaser participation in the fund to take account of informational externalities.

The revenue from a pricing scheme with a proportional fee d such that $D(\theta) = \theta d$, is given by

$$M = \theta d = \frac{2dR\sigma_y^2 + \sigma_y^2\sigma_\varepsilon^2 - \sqrt{4d^2R^2\sigma_y^2\sigma_x^2 + \sigma_y^4\sigma_\varepsilon^4}}{2aR\sigma_y^2\sigma_\varepsilon^2}. \tag{27}$$

It follows that the revenue-maximizing proportional fee is given by

$$d = \frac{(\sigma_x^2 - \sigma_\varepsilon^2)\sigma_\varepsilon}{2R\sigma_x}. \tag{28}$$

Using the optimal fee from (28) in (27) the maximum revenue under a proportional fee is

$$M = \theta d = \frac{\sigma_x - \sigma_\varepsilon}{2aR\sigma_x}. \tag{29}$$

Unlike the optimal transaction charge which is independent of σ_x^2, the proportional fee is increasing in σ_x^2 because the value of the seller's signal is increasing in σ_x^2 for a given σ_ϵ^2. While the fee is independent of the investor's risk aversion, the total revenue is decreasing in the signal noise and the purchaser's risk aversion.

C. The Optimal Commission Schedule and a Comparison of Revenues from Sale of Information

In order to illustrate the results for a sale of information to homogeneous purchasers, the example described in Table I was considered for different values of the risk-aversion parameter (a) and the standard deviation of the signal noise (σ_ε). The parameter values were chosen so that n_0, the initial shareholding, was equal to zero, in order not to confound the results with different implicit assumptions about the level of the initial shareholding.

Table II reports the revenue-maximizing average commission rates for different size transactions under different assumptions about risk aversion

Table I

Parameters of Example

Unconditional expected return:	μ	\$11.00
Standard deviation of signal:	σ_y	\$ 1.00
Riskless interest rate:	R	1.10
Stock price:	P	\$10.00

Table II

Revenue-maximizing Commission Rates

Expected revenue-maximizing commission rates of information monopolist as a function of purchaser's risk aversion (a), signal error (σ_ε), and number of shares (z^*).

		σ_ε					
	z^*	0.1	0.3	0.5	0.7	0.9	1.0
	100	7.52%	7.46%	7.34%	7.16%	6.94%	6.82%
$a = 0.005$	200	7.51	7.39	7.16	6.83	6.44	6.22
	300	7.50	7.32	6.98	6.53	5.99	5.72
	25	7.52	7.44	7.28	7.06	6.79	6.63
$a = 0.025$	50	7.51	7.35	7.06	6.66	6.19	5.94
	75	7.50	7.27	6.85	6.30	5.68	5.37
	20	7.51	7.40	7.18	6.87	6.49	6.28
$a = 0.045$	40	7.50	7.28	6.87	6.33	5.72	5.41
	60	7.49	7.16	6.59	5.87	5.11	4.75
	15	7.51	7.38	7.14	6.81	6.40	6.18
$a = 0.065$	30	7.50	7.26	6.82	6.24	5.59	5.27
	45	7.48	7.13	6.52	5.75	4.97	4.60

and signal noise. For each level of risk aversion the maximum reported transaction size corresponds approximately to that induced by a signal which is three standard deviations from the mean when $\sigma_\varepsilon = 1$. As expected, for a given transaction size, the commission rates decline with increases in both the purchaser's risk aversion and the signal noise. They also decline with the size of the transaction, this effect being more pronounced the greater is the signal noise.

Table III reports the expected revenue of the information monopolist under the different information charge structures, for different assumptions about the purchaser's risk aversion and the signal noise. Note that the revenue from an indirect sale under the optimal two-part or unconstrained pricing scheme is the same as the revenue under a direct sale for a fixed charge, since it will be optimal for the mutual fund to charge a fixed amount for participation in order to avoid distortion of the investor's participation incentives; this fixed charge will extract all of the purchaser's surplus and the investment of the mutual fund will be optimal for the purchaser for each

Table III

Homogeneous Purchasers: Comparison of Revenues

Expected revenue of information monopolist as a function of purchaser's risk aversion (a) and signal error (σ_ε). BC denotes revenues under the brokerage commission structure; IP denotes revenues under indirect sale with a proportional participation fee; F denotes revenues under the fixed charge structure, and S denotes revenues under the signal-contingent structure. Bold figures denote the highest expected revenue excluding the signal-based charge.

		σ_ε					
		0.1	0.3	0.5	0.7	0.9	1.0
$a = 0.005$	BC	**$4100**	**$ 460**	**$170**	$ 84	$ 51	$ 41
	IP	82	65	50	40	30	27
	F	420	227	146	**101**	**73**	**63**
	S	9091	1010	364	186	112	91
$a = 0.025$	BC	**830**	**92**	**33**	17	10	8
	IP	16	13	10	8	6	5
	F	84	45	29	**20**	**15**	**13**
	S	1818	202	73	37	22	18
$a = 0.045$	BC	**460**	**51**	**18**	9	6	4
	IP	9	7	5	4	4	3
	F	47	25	16	**11**	**8**	**7**
	S	1010	112	40	21	12	10
$a = 0.065$	BC	**330**	**35**	**13**	6	4	3
	IP	6	5	4	3	3	2
	F	32	17	11	**8**	**6**	**5**
	S	699	78	28	14	9	7

signal realization. As the risk-sharing considerations discussed in Section I would suggest, the expected revenues under the signal-contingent structure are greatest, and the welfare losses from being unable to contract on the signal are large, particularly when the signal error is small.

Comparison of the fixed charge with the commission structure yields more ambiguous results. When the signal error is small relative to the variability of the signal itself, the expected revenues are larger under the commission structure, while the results are reversed when the signal error is large. This is because the advantage of the commission structure is that it shares the risk of the signal realization; the noisier is the signal, the less valuable it is for a given realization, and therefore the less risk is associated with the signal realization. As a result, the relative advantage of the commission structure is decreasing in the signal error.

For a given level of risk associated with the signal realization, the risk-sharing advantage of the commission is greater the more risk averse is the purchaser. However, this effect is offset by the fact that a more risk-averse purchaser takes a smaller position in the shares for any given signal realization, and this reduces the level of risk associated with the signal. The net effect is that the level of risk aversion does not significantly affect the relative revenues from the fixed charge and the commission in Table III.

The brokerage commission has superior risk-sharing properties compared to a mutual fund that charges a proportional fee for participation,[18] since the fee for mutual fund participation is levied ex ante. Both structures distort the investment decision—the mutual fund fee by a constant proportion across all signal realizations, the commission by an amount which is decreasing in the signal realization. The relative advantage of the commission is particularly pronounced when the signal error is low so that the risk associated with the signal realization is high. Overall, (direct or indirect) sale for a fixed charge tends to dominate when the information is less precise. A mutual fund with a proportional participation fee is never optimal.

Table IV compares the sizes of the transactions that are undertaken under the different structures. The distortionary effects of the commission are very large for low levels of risk aversion and for low signal noise. For higher levels of risk aversion and, more particularly, for high levels of signal noise, the effects are much less pronounced.

Table IV

Transaction Sizes in Absence of Transaction Costs

Number of shares purchased in the absence of transaction costs as a function of the number purchased under the expected revenue-maximizing commission structure (z^*) for different levels of purchaser's risk aversion (a), and signal error (σ_ε).

		σ_ε					
	z^*	0.1	0.3	0.5	0.7	0.9	1.0
	100	15060	1737	671	378	257	223
$a = 0.005$	200	15131	1808	743	451	335	302
	300	15202	1880	818	530	417	385
	25	3015	350	137	79	55	48
$a = 0.025$	50	3033	368	156	98	75	68
	75	3051	386	174	118	96	90
	20	1679	199	81	48	35	31
$a = 0.045$	40	1693	213	95	64	52	48
	60	1708	228	111	80	69	66
	15	1163	138	56	34	25	22
$a = 0.065$	30	1174	149	67	46	37	35
	45	1184	160	79	58	51	49

[18] This is the only structure that is possible under current institutional arrangements. Indirect sale does in principle allow the seller to discriminate between purchasers by offering quantity discounts on the management fees for units in a mutual fund. In practice, mutual funds are permitted to issue only one class of shares so that the burden of advisory fees is proportionally divided among the shareholders. (See Frankel (1978), p. 249.)

II. Heterogeneous Information Purchases

When information purchasers are homogeneous, a signal-based charge for information provides what Pigou (1932, chapter 7) defines as "third-degree" or perfect price discrimination, under which the purchaser is charged a nondistortionary fee that depends on the state of nature and extracts all of his surplus. The commission charge which, as we have seen, fails to extract all of the surplus, corresponds to "second-degree" or imperfect price discrimination across signal realizations; under this type of price discrimination individuals of different types or in different states face a single nonuniform price schedule (for trades) which serves to divert them towards price quantity combinations generating higher profits. A fixed charge for information is nondiscriminatory.

The situation is rather different when information purchasers are heterogeneous, for then, given our assumption that information cannot be resold, there is the possibility of discriminating across investor types as well as across signal realizations. When purchasers differ in their risk aversions a clear distinction emerges between direct sale for a fixed charge, and indirect sale with a general charge for participation in a mutual fund: these were equivalent when purchasers were homogeneous since the optimal general participation charge was in fact a fixed charge. With direct sale it is not possible to discriminate between purchasers with different risk aversions since only a single fixed charge is possible. But with indirect sale, investors may be charged according to the size of the position they take in the managed fund; this permits (imperfect) price discrimination across customers; however, since the mutual fund participation fee is levied ex ante it does not permit discrimination across signal realizations. A commission schedule, on the other hand, allows imperfect price discrimination across customer-signal realization pairs, based on the size of the trade.

In this section we consider the price discrimination properties of direct sales of information for a fixed charge and for a brokerage commission, and of indirect sales of information (Admati and Pfleiderer (1990)) with both general and proportional fee structures. Throughout, we assume that the risk aversion coefficient of information purchasers is distributed in some interval $[\underline{a}, \bar{a}]$ with a probability density function $g(a)$, and in order to obtain simple closed form expressions we shall sometimes impose the additional assumption that a is uniformly distributed.

A. Direct Sale of Information for a Fixed Charge

Under a direct sale, the information seller chooses a fixed charge, c, to maximize the expected revenues which are given by:

$$M = \max_{c} \int_{\underline{a}}^{\alpha(c)} c g(a)\, da. \tag{30}$$

In this expression $\alpha(c) = (1/cR)\log(\sigma_x/\sigma_\varepsilon)$, is the risk aversion of the marginal information purchaser who, as shown by expression (5), receives zero surplus from the information purchase. The first-order condition for a maximum in (30) may be expressed as:

$$cR\int_{\underline{a}}^{\alpha(c)} g(a)\,da = \log\left(\frac{\sigma_x}{\sigma_\varepsilon}\right) g\left(\frac{1}{cR}\log\left(\frac{\sigma_x}{\sigma_\varepsilon}\right)\right). \tag{31}$$

If a is uniformly distributed on $[\underline{a}, \underline{a} + b]$, then (30) can be written as

$$\begin{aligned} M &= \max_{\alpha} \int_{\underline{a}}^{\alpha} \frac{1}{\alpha bR}\log\left(\frac{\sigma_x}{\sigma_\varepsilon}\right) da, \\ &= \max_{\alpha} \frac{(\alpha - \underline{a})}{\alpha bR}\log\left(\frac{\sigma_x}{\sigma_\varepsilon}\right). \end{aligned} \tag{32}$$

It is easy to show that M is increasing in α, so that the information seller prefers to set the fixed charge, c, low enough to be able to sell to all potential purchasers. The following lemma summarizes our result.

LEMMA 2: *The information seller's maximum revenue under the fixed charge with uniformly distributed risk aversion is*

$$M = c = \frac{1}{(\underline{a} + b)R}\log\left(\frac{\sigma_x}{\sigma_\varepsilon}\right). \tag{33}$$

We note that for a given mean risk aversion $(\underline{a} + b/2)$, the revenue is decreasing in b, the dispersion parameter of the distribution of investor risk aversion. The intuition for this is that, if there is no dispersion the charge can be set to simultaneously exhaust the surplus of the average purchaser and sell to all potential purchasers; as the dispersion increases the seller must either reduce the charge or lose sales to the more risk-averse potential purchasers.

B. Brokerage Commissions

When investors are heterogeneous, the expected revenue of the information seller is obtained by integrating expression (19) with respect to the density of risk aversions:

$$\begin{aligned} M = \int_{\underline{a}}^{\bar{a}} \bigg[&\int_{n_0}^{\infty} t(n - n_0)\{1 - F[y_a(n)]\}\,dn \\ &+ \int_{n_0}^{-\infty} t(n - n_0)F[y_a(n)]\,dn \bigg] g(a)\,da, \end{aligned} \tag{34}$$

where $y_a(n)$ is the minimum signal level required to induce an investor with risk aversion a to purchase n shares. If the second-order condition for a maximum at n in problem (9) is satisfied then $y_a(n) = RP + a\sigma_\varepsilon^2 n + t(n - n_0)$ as in the homogeneous investor case. However, if the commission schedule slopes down sufficiently steeply at n, the second-order condition for the purchaser will not be satisfied at n for any signal realization. However, there will exist a minimum purchase size for any signal, n^*. Then $y_a(n) = y_a(n^*)$ for $n < n^*$.

As before, pointwise optimization of (34) yields the optimal marginal commission schedule:

$$\begin{aligned} t(n - n_0) &= \frac{\int_{\underline{a}}^{\bar{a}}(1 - F[y_a(n)])g(a)\,da}{\int_{\underline{a}}^{\bar{a}} f[y_a(n)]I(a,n)g(a)\,da} \qquad \text{for } n > n_0, \\ &= -\frac{\int_{\underline{a}}^{\bar{a}} F[y_a(n)]g(a)\,da}{\int_{\underline{a}}^{\bar{a}} f[y_a(n)]I(a,n)g(a)\,da} \qquad \text{for } n < n_0. \end{aligned} \tag{35}$$

where $I(a, n)$ is the indicator function for the second-order condition and is equal to one when the purchaser's second-order condition holds at n and is zero otherwise.

Substitution of the marginal commission schedule (35) in (34) gives the maximal expected commission revenues when the information purchasers are heterogeneous.

C. Indirect Sale of Information

Under an indirect sale in which the charge depends on the level of participation in the mutual fund the problem of the information seller may be represented as:

$$M = \max_{r(\theta)} \int_{\underline{a}}^{\bar{a}} D[\theta(a)]g(a)\,da, \tag{36}$$

where $D(\theta)$ is the aggregate fee for participating in θ units of the fund and $r \equiv r(\theta) \equiv D'(\theta)$. $\theta(a)$ is given in Lemma 1. The following theorem, which is proven in the Appendix, characterizes the optimal charge for participation in the mutual fund and the revenue of the information seller, when risk aversion is uniformly distributed.

THEOREM 4: *Assuming a uniform distribution for a on $[\underline{a}, \underline{a} + b]$, the optimal charge for information under an indirect sale is as follows.*

1. *The optimal proportional fee of the form $D(\theta) = \theta d$ is*

$$d = \frac{\sigma_y^2 \sigma_\varepsilon}{2R\sigma_x}. \tag{37}$$

The revenue of the information seller under the proportional fee is

$$M = \frac{d}{b}\int_{\underline{a}}^{\bar{a}} \theta(a)\, da = \frac{1}{b}\log\left(\frac{\bar{a}}{\underline{a}}\right)\left[\frac{2dR\sigma_y^2 + \sigma_y^2\sigma_\varepsilon^2 - \sqrt{4d^2R^2\sigma_y^2\sigma_x^2 + \sigma_y^4\sigma_\varepsilon^4}}{2R\sigma_y^2\sigma_\varepsilon^2}\right]. \tag{38}$$

2. *The optimal marginal participation charge, $r(\theta)$, is a solution to the following implicit equation*

$$\theta = \frac{\sqrt{4r^2R^2\sigma_y^2\sigma_x^2 + \sigma_y^4\sigma_\varepsilon^4} - 2rR\sigma_x^2}{\underline{a}\sigma_\varepsilon^2\sqrt{4r^2R^2\sigma_y^2\sigma_x^2 + \sigma_y^4\sigma_\varepsilon^4}}. \tag{39}$$

The revenue of the information seller under the optimal participation charge is

$$M = F + \int_{\theta(\bar{a})}^{\theta(\underline{a})} r(\theta)\int_{\underline{a}}^{a^*(\theta)} g(a)\, da\, d\theta. \tag{40}$$

where F is the fixed charge for participating in the fund; $a^*(\theta)$, the risk aversion of a purchaser who purchases θ shares in the fund when the marginal charge is $r(\theta)$; $\theta(\bar{a})$ and $\theta(\underline{a})$ are obtained from (26).

Note that under the assumption of uniformly distributed risk aversion the optimal proportional fee does not depend on the distribution of risk aversion of the information purchasers and so is the same as in the homogeneous case. (This property would not hold for arbitrary distributions of risk aversion.) The revenue of the information seller under the proportional fee decreases in the dispersion of the information purchasers' risk aversion. When nonproportional fees are possible the optimal marginal participation fee is decreasing in the level of participation. In the next section we compare the expected revenues under the above pricing structures.

D. Comparison of the Expected Revenues with Different Pricing Structures

Table V reports the expected revenues of the information seller under the different charge structures for different ranges of the uniformly distributed risk aversion parameter. The expected revenues under direct sale for a fixed charge (F), under direct sale for a brokerage commission (BC), under indirect sale (i.e., mutual fund) for a proportional fee (IP), under indirect sale for a general charge (IG) were computed using expressions (33), (34), (38), and (40) respectively.

As in the homogeneous investor case the brokerage commission provides the highest expected revenues for relatively precise signals, and always dominates the proportional mutual fund fee structure. However, for low-precision information the general mutual fund participation charge is preferred by the information seller. If an indirect seller is constrained to use a

Table V

Heterogeneous Purchasers: Comparison of Revenues

Expected revenue of information monopolist as a function of the signal error (σ_ε) and the distribution of the purchasers' risk aversion, which is assumed to be uniform in$[\underline{a}, \bar{a}]$. F denotes revenues under a direct or indirect sale for a fixed charge; IP denotes revenues for an indirect sale under the proportional participation charge; IG denotes revenues for an indirect sale under the general participation charge; BC denotes revenues under the brokerage commission structure. Bold figures denote the highest expected revenue.

		σ_ε					
		0.1	0.3	0.5	0.7	0.9	1.0
	F	$ 14	$ 8	$ 5	$ 3	$ 2	$ 2
$\bar{a} = 0.15$	IP	5	4	3	2	2	1
$\underline{a} = 0.05$	IG	16	9	6	4	3	**3**
	BC	**226**	**25**	**9**	**4**	**3**	2
	F	21	11	7	5	4	3
$\bar{a} = 0.1$	IP	6	4	3	2	2	2
$\underline{a} = 0.05$	IG	22	13	8	6	**4**	**4**
	BC	**285**	**32**	**11**	**6**	3	3
	F	8	11	5	3	2	2
$\bar{a} = 0.15$	IP	3	4	2	1	1	1
$\underline{a} = 0.1$	IG	8	5	5	**4**	**3**	**2**
	BC	**167**	**18**	**7**	3	2	2

proportional fee,[19] then a direct sale for a fixed charge or a brokerage commission is always preferred.

Of course, the superiority of the brokerage commission that is evident in Table V does depend upon the degree of investor heterogeneity, and as this is reduced the findings will approximate the homogeneous case of Table III in which the fixed charge for indirect or direct sale often dominates the brokerage commission.

III. Conclusion

In this paper we have considered three alternative structures for an information monopolist to use in the sale of information: a direct or indirect sale for a fixed charge, a direct sale for a brokerage commission, and an indirect sale for a mutual fund management fee. We have abstracted from three important considerations which have been addressed by others, the resale problem, the dilution in the value of information by leakage through security prices, and the need for the information seller to guarantee its

[19] See footnote 18.

reliability, in order to concentrate on two others, the optimal sharing of risk between the information seller and the investor, and the ability of the seller to discriminate between different types of buyer. We have argued that it would in general be efficient for the seller to charge on the basis of the signal realization. However, when the signal is not contractible, the only alternatives are a charge that is fixed in anticipation of the signal realization, a commission structure that charges the purchaser according to how much it is used, some combination of the two, or a charge that depends on the realized security return. In this paper we have analyzed two pure structures: a charge that is fixed in anticipation of the signal realization, and a brokerage commission. The advantage of the fixed charge structure when purchasers are homogeneous is that it does not distort the investment decision of the purchaser: the advantage of the commission structure is that it leads to improved risk sharing between seller and purchaser. For analytic tractability we considered a risk-neutral information monopolist and a risk-averse information purchaser. The assumption of monopoly and risk neutrality on the part of the information seller are restrictive. The risk-sharing advantage of brokerage commissions will be reduced if the information seller is also risk averse but it will not be eliminated except in the special case in which purchaser and seller have HARA utility with identical cautiousness. It would be interesting to extend the analysis to competing risk-averse information sellers.

When information purchasers are homogeneous, the expected revenue-maximizing commission schedule conforms with the type of commission structure generally observed in the United States in that the proportionate commission rate declines with the size of the transaction.[20] Our numerical example suggests that a direct or indirect sale for a fixed charge is likely to be optimal when the signal is relatively imprecise; otherwise, the brokerage commission provides higher expected revenues to the seller as well as leaving the purchaser with some surplus. For none of the parameter values considered is a mutual fund with a uniform participation fee optimal.

When purchasers are heterogeneous the relative advantage of the brokerage charge increases and it is dominated only by the general mutual fund participation charge for imprecise signals. The mutual fund with a proportional fee structure, the only structure that is feasible under current conditions, is always dominated. While our results depend on the precise assumptions we have made about the distribution of risk aversion across purchasers, they do suggest that the widespread use of the indirect mode of sale for a proportional fee is due to factors we have not considered such as the need to limit the use of information in order to preserve its value. Alternatively, it is possible that investment management services represent something more than simply the indirect sale of information that might otherwise be sold directly to investors. One indication that this may be the case is that mutual

[20] We should note that this is not general in other countries; nor is making the commission rate a function of the price of shares traded.

fund managers themselves frequently purchase information from brokers in return for brokerage commissions.

Appendix

Proof of Theorem 3: 1. In order to verify that the second-order condition for the seller in (19) is satisfied it is convenient to write the commission schedule in terms of the hazard function of the standard normal distribution. To this end, define the net trade $z \equiv n - n_0$, and the standard normal variate $u(z) \equiv (y(z + n_0) - \mu)/\sigma_y$. Then, using (15) and (17), and the definition of the net trade z, $u(z)$ may be written as:

$$u(z) = \frac{a\sigma_\varepsilon^2 z + t(z)}{\sigma_y}. \tag{41}$$

Then we have the following corollary:

COROLLARY 1: *The commission schedule that maximizes the expected revenue of the information seller may be written as*:

$$\begin{aligned} t(z) &= \sigma_y\{h[u(z)]\}^{-1} \qquad \text{for } z > 0 \\ &= -\sigma_y\{h[-u(z)]\}^{-1} \quad \text{for } z < 0. \end{aligned} \tag{42}$$

where $h(u) \equiv g(u)/(1 - G(u))$ is the hazard function for the standard normal density, $g(u)$.

Then, to verify the second-order conditions for a maximum of (19) note that:

a. $n > n_0$:

$$\frac{d^2M}{dt(z)^2} = -2f[y(n)] - t(z)f'[y(n)]. \tag{43}$$

Using the property of the standard normal density that $g'(u) = -ug(u)$, and noting that $u = (y - \mu)/\sigma_y$ so that $\sigma_y f(y) = g(u)$, (43) may be written as:

$$\frac{d^2M}{dt(z)^2} = \frac{1}{\sigma_y}\left[-2g(u) + \frac{tug(u)}{\sigma_y}\right]. \tag{44}$$

Finally, substituting for t from (42) and using the properties of the standard normal density

$$\begin{aligned} \frac{d^2M}{dt(z)^2} &= \frac{1}{\sigma_y}[-2g(u) + u[1 - G(u)]] \\ &= \frac{1}{\sigma_y}[1 - G(u)][-2h(u) + u] < 0. \end{aligned} \tag{45}$$

b. $n < n_0$:

$$\frac{d^2M}{dt(z)^2} = -2f[y(n)] - t(z)f'[y(n)]$$

$$= \frac{1}{\sigma_y}\left[-2g(u) + \frac{tug(u)}{\sigma_y}\right]. \tag{46}$$

Substituting for t from (42)

$$\frac{d^2M}{dt(z)^2} = \frac{1}{\sigma_y}[-2g(u) - uG(u)] < 0 \quad \text{i.f.f.} \ \frac{2g(u)}{G(u)} > -u. \tag{47}$$

But since $g(u) = g(-u)$, and $G(u) = 1 - G(-u)$, (47) may be written as:

$$\frac{2g(-u)}{1 - G(-u)} > -u. \tag{48}$$

But (48) is satisfied by the properties of the standard normal distribution.

2. The second-order condition for the purchaser in (9) is $-t'(n - n_0) - a\sigma_\varepsilon^2 < 0$. This condition is verified from (49) below.

3. To prove part 2 of the theorem for $z > 0$ differentiate (42) w.r.t. z and simplify to obtain:

$$t'(z) = -\frac{h'(u)a\sigma_\varepsilon^2}{[h(u)]^2 + h'(u)} < 0. \tag{49}$$

Since $h'(u) > 0$, it follows that $t'(z) > a\sigma_\varepsilon^2$. The proof for $z < 0$ is analogous.

Proof of Lemma 1: We now use the following result from Admati and Pfleiderer (1990) equation (A2). Let $\tilde{\eta}_1$ and $\tilde{\eta}_2$ be jointly normally distributed with means μ_1 and μ_2 and variance-covariance matrix Σ. Then $E[-\exp(-\tilde{\eta}_1\tilde{\eta}_2)]$ equals

$$-|\Sigma|^{-0.5}|A|^{-0.5}\exp\left[-\mu_1\mu_2 + \frac{1}{2}(\mu_1\,\mu_2)A^{-1}(\mu_1\,\mu_2)^T\right], \tag{50}$$

where T denotes the transpose and $A \equiv \Sigma^{-1} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. □

Let $\tilde{\eta}_1 \equiv (\theta\tilde{y} - q)$ and $\tilde{\eta}_2 \equiv a(\tilde{x} - RP)$. Then $\mu_1 = \theta\mu - q$, $\mu_2 = a(u - RP)$, $\Sigma_{11} = \theta^2\sigma_y^2$, $\Sigma_{22} = a^2\sigma_x^2$, and $\Sigma_{12} = \Sigma_{21} = a\theta\sigma_y^2$.

Optimizing (50) w.r.t. q gives the optimal q in (26). Then substituting for the optimal q from (26) in (50) the maximization problem can now be written as

$$\max_\theta f_1(\theta)\exp[f_2(\theta)]\exp[f_3(\theta)], \tag{51}$$

where

$$f_1(\theta) = -\frac{1}{\sqrt{1 + 2a\theta\sigma_y^2 - a^2\theta^2\sigma_y^2\sigma_\varepsilon^2}},$$

$$f_2(\theta) = -\mu_1\mu_2 + \frac{1}{2}(\mu_1\mu_2)A^{-1}(\mu_1\mu_2)^T = -\frac{(\mu - RP)^2}{2\sigma_x^2},$$

$$f_3(\theta) = -aR(W_0 - D(\theta)).$$

Since $f_2'(\theta) = 0$, the first-order condition for (51) becomes $f_1'(\theta) + f_1(\theta)f_3'(\theta) = 0$. This leads to a quadratic equation for θ. Imposing the second-order condition for a maximum under the constraint that $\theta \geq 0$ gives the optimal θ in (26).

Proof of Theorem 4: 1. Under the proportional fee structure the revenue of the mutual fund is

$$\max_d M(d) = \max_d \frac{d}{b}\int_{\underline{a}}^{\bar{a}} \theta(a)\, da. \tag{52}$$

where $\theta(a)$ is given by setting $r = d$ in (26). Then, substituting for $\theta(a)$ in (36) gives

$$\max_d M(d) = \max_d \left[\frac{2dR\sigma_y^2 + \sigma_y^2\sigma_\varepsilon^2 - \sqrt{4d^2R^2\sigma_y^2\sigma_x^2 + \sigma_y^4\sigma_\varepsilon^4}}{2R\sigma_y^2\sigma_\varepsilon^2}\right]\int_{\underline{a}}^{\bar{a}} \frac{1}{a} g(a)\, da. \tag{53}$$

Differentiating (53) w.r.t. d and rearranging the first-order condition gives the optimal d in (37). The second-order condition for the maximization problem in (53) holds strictly.

2. Under a general participation charge the revenue is

$$M = \max_{r(\theta)} \left[F\int_{\underline{a}}^{\alpha} g(a)\, da + \int_{\theta(\bar{a})}^{\theta(\underline{a})} r(\theta)m(\theta)\, d\theta\right], \tag{54}$$

where $m(\theta) \equiv \int_{\underline{a}}^{\min(\alpha,\, a^*(\theta))} g(a)\, da$, is the proportion of investors who buy at least θ shares and therefore incur the charge $r(\theta)$; $a^*(\theta)$, the risk aversion of a purchaser who purchases θ shares in the fund when the marginal charge is $r(\theta)$, is obtained by rearranging (26); α is the risk aversion of the investor who receives zero surplus from the purchase of information.

Since $\operatorname{sgn}[\partial M/\partial F] = \operatorname{sgn}[\alpha - \bar{a}]$, the optimal α is set equal to $\bar{a}$. In other words, the fixed portion of the charge is set so that all potential information purchasers invest in the mutual fund.

Pointwise maximization of (54) w.r.t. $r(\theta)$ gives an implicit equation for θ and $r(\theta)$, which is presented in (39). The revenue of the information seller under the optimal participation charge can be written as in (40).

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