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Relative performance evaluation of management

The effects on industrial competition and risk sharing*

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This paper shows that the use by firms in an oligopolistic industry of relative performance measures to evaluate their managers, may cause a conflict between the objectives of risk sharing and the implications for strategic competition derived from such performance measures, specially if firms compete in prices. We also show that if managers are risk neutral, the Stackelberg leader follower solution is among the multiple subgame perfect equilibria to the problem of strategic output competition; therefore relative performance evaluation of managers provides a mechanism to implement such solution.

1. Introduction

The strategic implications of evaluating managers' performance, taking into account the profits of the firms they manage and the profits of other firms in the industry, were recognized by Vickers (1985). The author illustrates that principals may want their agents to pursue objectives different from their own, when there is interdependency among the decisions of the agents. For example, it is shown that if one firm evaluates its managers as a function of the difference between the profits of the firm that he or she manages and the average profits of the other firms in the industry, while the others maximize absolute profits, then the firm with relative performance evaluation will end up with higher absolute profits than the others.

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Vickers considers this result to be an example of how firms may obtain strategic advantages using relative performance evaluation systems for their managers, which should be added to the informational advantages attributed to them by the literature on contract design.[1] However, the author does not prove that one firm using relative performance evaluation and the others not using it is the unique equilibrium for the industry, neither does he prove that at the equilibrium, if it exists, firms will end up with higher profits than those obtained under absolute profit maximization (and therefore it could be possible to talk about 'advantage' of the system). Finally, no consideration is given to the joint problem of information and risk sharing together with strategic competition. If relative performance evaluation has informational advantages that help to design more efficient risk sharing contracts then it is likely that, in order to achieve this objective, some restrictions will be imposed on the actual design of the contract. The question, therefore, is: Will the restrictions imposed by the optimal incentive contract from the risk sharing point of view, be compatible with those imposed by the nature of the strategic competition among managers?

This question is of particular interest in the context of relative performance evaluation because there are two potential reasons for using it and, we believe, it makes the problem of the strategic choice of managerial incentives more relevant than in the cases analysed by Fershtman and Judd (1987), Sklivas (1987) and, partially, by Vickers (1985). Here managers are motivated to maximize a linear combination of profits and sales, a function which does not have a prior justification, except for the potential benefit of strategic manipulation of the firm's reaction functions. Firms may end up achieving lower profits than they would have obtained if the usual Cournot outcomes had been contractually enforced. That is, as Fershtman and Judd (1987, p. 930) recognize, their analysis relies on the impossibility of enforcing this type of contract. Relative performance evaluation, if managers are risk averse, is of interest in its own right for a profit maximizing shareholder, since it may reduce the expected salary to be paid to the manager and, consequently, increase net profits. Independent of enforceability issues, the question is whether such an increase in net profit is compatible or not with the strategic implications of this type of performance evaluation.

Our paper departs from the previous literature on the strategic choice of managerial incentives in several respects. First, a model is formulated to take into account both strategic and risk sharing effects of relative performance evaluation when firms compete on quantities and when firms compete on prices. Secondly, as in Bonano and Vickers (1988), the results of the model take advantage of the properties of strategic substitutes and strategic

[1]See Holmstrom (1982), Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983). An empirical study on the use of relative performance by U.S.A. firms is presented in Antle and Smith (1986).

complements decision variables, to derive the subgame perfect equilibrium solutions to the shareholders problem. This problem is characterized as choosing the incentive function of their managers, subject to participation and incentive constraints, with two decision variables, namely quantities or prices and managerial effort. Thirdly, it shows that, under output competition, the Stackelberg equilibrium solution with risk-neutral managers is one of the subgame perfect equilibrium solutions to the shareholders problem, but not the only one. Fourthly, it analyses the 'advantages' of relative performance evaluation in terms of resulting net profits for the firms in the industry. Taken together, the above constitutes a generalization of Vickers' (1985) preliminary results. Finally, it differs from Fershtman and Judd (1987) and Sklivas (1987), in that it formulates a different problem and provides a complete characterization of the solution, without restricting it to linear demand and costs.

2. Description of the model

Let us consider an industry with n firms. The gross profits of the ith firm are determined as

$$P_i(q_i,\dots,q_n;e_i;\varepsilon)=\pi_i(q_i,\dots,q_n)+f(e_i,\varepsilon),\quad i=1,\dots,n,$$

where q_i, $i=1,\dots,n$, is the output of the ith firm, e_i is the manager's effort and ε is the realization of a random variable with known distribution. It is assumed that the effects on profits of the strategic variables q_i are separable from the joint effects of managerial effort and the random term. This would be the case, for example, if $f(e_i,\varepsilon)$ was interpreted as minus the fixed cost of the firm.

Additional assumptions on the profit function are the following:

(a) $\pi_i(q_1,\dots,q_n)$ is a continuous function, differentiable and concave in q_i;

(b) $\dfrac{\delta\pi_i}{\delta q_j}<0,\quad i\neq j;\quad \dfrac{\delta^2\pi_i}{\delta q_i\delta q_j},$

that is, the goods produced in the industry are strategic substitutes;[2]

(c) $\dfrac{\delta^2\pi_i}{\delta q_j^2}\geqq 0;$[3]

(d) $f(e_i,\varepsilon)$ has expected value and variance equal to $\mathrm{E}(e_i)$ and σ^2, respectively. The expected value $\mathrm{E}(e_i)$ is increasing and concave in e_i, while the

[2]See Bulow et al. (1985).

[3]This hypothesis is satisfied for convex demand functions.

variance σ^2 is assumed to be constant, independent of effort, and identical for each firm.

The manager of each firm is risk averse with a constant degree of risk aversion measured by the Arrow–Pratt coefficient of risk aversion Γ. The expected utility of an uncertain income y and a level of effort e_i is expressed in terms of its certainty equivalent, $\mathrm{CE}_i = \mathrm{E}[y] - c(e) - \Gamma/2\sigma^2 y$, where the cost function $c(e)$ is increasing and convex in the effort e. There is an outside opportunity with utility $\hat{u}$ for the manager.

Shareholders of firm i determine the income to be paid to the manager using the relative performance measure,

$$O_i = P_i(q_1, \ldots, q_n; e_i, \varepsilon) + \mu_i \left(\sum_{j \neq i} P_j(q_1, \ldots, q_n; e_j; \varepsilon) \right), \quad i = 1, \ldots, n,$$

where μ_i is a parameter to be determined. We assume that the compensation function is linear,

$$y_i = \alpha_i + \beta_i O_i, \quad i = 1, \ldots, n,$$

where α_i and β_i are parameters to be determined.[4]

The problem of the shareholders, assumed to be risk neutral, is characterized as[5]

$$\max_{\alpha_i, \beta_i, \mu_i} \pi_i(q_i, \ldots, q_n) + \mathrm{E}(e_i) - \alpha_i - \beta_i \left[\pi_i + \mathrm{E}(e_i) + \mu_i \left(\sum_{j \neq i} \pi_j + \mathrm{E}(e_j) \right) \right],$$

subject to

$$\alpha_i + \beta_i \left[\pi_i + \mathrm{E}(e_i) + \mu_i \left(\sum_{j \neq 1} \pi_j + \mathrm{E}(e_j) \right) \right] - c(e_i)$$

$$- \Gamma/2\beta_i^2(1 + \mu_i)^2 \sigma^2 \geqq u$$

[4]Holmstrom and Milgrom (1985) show that this characterization of uncertainty in the profit function, of managerial preferences and the use of linear compensation functions are justified under fairly general conditions.

[5]Assumption (d) implies that there is only common uncertainty, but not firm specific uncertainty. If both were present, profits would depend on the two random variables ε and ε_i. Therefore, assuming independence, $\sigma^2[f(e_i | \varepsilon, \varepsilon_i)] = \sigma^2 + \sigma_i^2]$ and $\sigma_y^2 = \beta_i^2[\mu_i^2(\sum_{j \neq 1} \sigma_j^2) + (1 + \mu_i)^2 \sigma^2]$. In general, it is assumed that $(\sum_{j \neq i} \sigma_j^2)/n - 1 < \sigma^2$ so that relative performance which substitutes common for specific uncertainty may be effective.

$$\max_{q_i, e_i} \alpha_i + \beta_i \left[\pi_i + \mathrm{E}(e_i) + \mu_i \left(\sum_{j \neq 1} \pi_j + \mathrm{E}(e_j) \right) \right] - c(e_i)$$

$$- \Gamma/2 \beta_i^2 (1+\mu_i)^2 \sigma^2, \quad i=1,\ldots,n.$$

The solution to this problem is obtained as the Nash perfect equilibrium of a two stage game in which managers first solve the n problems of deciding the values of q_i and e_i that maximize their compensation, for given values of α_i, β_i, and μ_i. We call $q_i^*(\mu_1,\ldots,\mu_n)$, $e_i^*(\beta_i)$, $i=1,\ldots,n$ the equilibrium solutions to the n manager's problems. It is assumed that these solutions satisfy: (e) the first order optimality conditions; (f) the second order conditions, and (g) the stability conditions. Secondly, shareholders decide the parameters of the compensation function of the managers, taking into account the participation constraint of these managers and how the values of α_i, β_i and μ_i influence the choice of the manager's decision variables q_i and e_i. Define by (μ_1^*, β_i^*), $i=1,\ldots,n$, the equilibrium solutions to the shareholder's problems.

3. Properties of the solution

This section characterizes, for $n=2$, the properties of the solution to the two stage game described above. The main results are summarized in a proposition which is proved in the appendix.

Proposition 1. Suppose there is a subgame perfect equilibrium of the game such that, at the second stage, competition is of the strategic substitute variety and equilibrium is stable. Then:

(a) The solution ($\mu_1^*=0, \mu_2^*=0$) is not an equilibrium solution.

(b) The equilibrium solution in which one firm uses relative performance evaluation and the other does not ($\mu_1^*=0, \mu_2^* \neq 0$) is part of the set of equilibria of the two stage game, when the manager is risk neutral, $\Gamma=0$. For this solution

$$\mu_1^* = -\frac{\delta^2 \pi_1 / \delta q_1 \delta q_2}{\delta^2 \pi_2 / \delta q_1 \delta q_2},$$

and the industry equilibria is of the Stackelberg leader–follower type, with the firm which uses relative performance evaluation acting as a leader and the other as a follower. If the profit functions are symmetric, $\mu_1^* = -1$.

(c) The equilibria solutions to the shareholders problem are non-positive, $\mu_i^* \leqq 0$, $i=1,2$. If the profit functions are symmetric $-1 \leqq \mu_i^* \leqq 0$, $i=1,2$.

(*d*) *If the output is homogeneous, the equilibrium of the two stage game implies a higher industry output and a lower price than in the Cournot profit maximizing equilibria* ($\mu_1=\mu_2=0$).
(*e*) *If the profit functions are symmetric and* $-1<\mu_i^*<0$, μ_i^*, *decreases with the degree of risk aversion of the managers.*
(*f*) *If managers are risk averse,* $\Gamma>0$, *in general the equilibrium will be unique and* $\mu_i^*<0$, $i=1,2$.

We shall now discuss the results set out in the proposition, together with the graphical illustration provided by the solution to an example of an industry with linear demand, $p=a-b(q_1+q_2)$ and linear costs, $c_1=c_2=c$. Initially it will be assumed that the managers are risk neutral, $\Gamma=0$. Under these assumptions, the output q_i for given values of μ_i is determined by

$$q_i=\frac{(1-\mu_i)(a-c)}{[4-(1+\mu_1)(1+\mu_2)]b}, \quad i=1,2,$$

which, in turn, implies a price and profit equal to

$$p=a-\frac{(a-c)[2-(\mu_1+\mu_2)]}{4-(1+\mu_1)(1+\mu_2)},$$

$$B_i(\mu_1,\mu_2)=\frac{(a-c)^2}{b}\,\frac{(1-\mu_1\mu_2)(1-\mu_i)}{[4-(1+\mu_1)(1+\mu_2)]^2}, \quad i=1,2,$$

The shareholders decide μ_1 and μ_2 maximizing the profit $B_i(\mu_1,\mu_2)$. The necessary conditions imply,

$$[4-(1+\mu_1)(1+\mu_2)](2\mu_1\mu_2-1-\mu_j)+2(1+\mu_j)(1-\mu_1\mu_2)(1-\mu_i)=0, \quad i\neq j=1,2.$$

These equations are satisfied for the value pair $\mu_1=\mu_2=1$; $\mu_1=0,\mu_2=-1$; $\mu_1=-1,\mu_2=0$; $\mu_1=\mu_2=-1/3$. Further, the second order conditions are satisfied for all of them except for $\mu_1=\mu_2=1$.

Fig. 1 shows the reaction functions and equilibria solutions for this problem. Point A represents the Cournot equilibrium with absolute profit maximization by the two firms. When relative performance evaluation is allowed, one firm, at least, has an incentive to shift its reaction function upwards [part (a) of Proposition 1]. The upward shift of the reaction function is explained by the fact that the decision variables are strategic

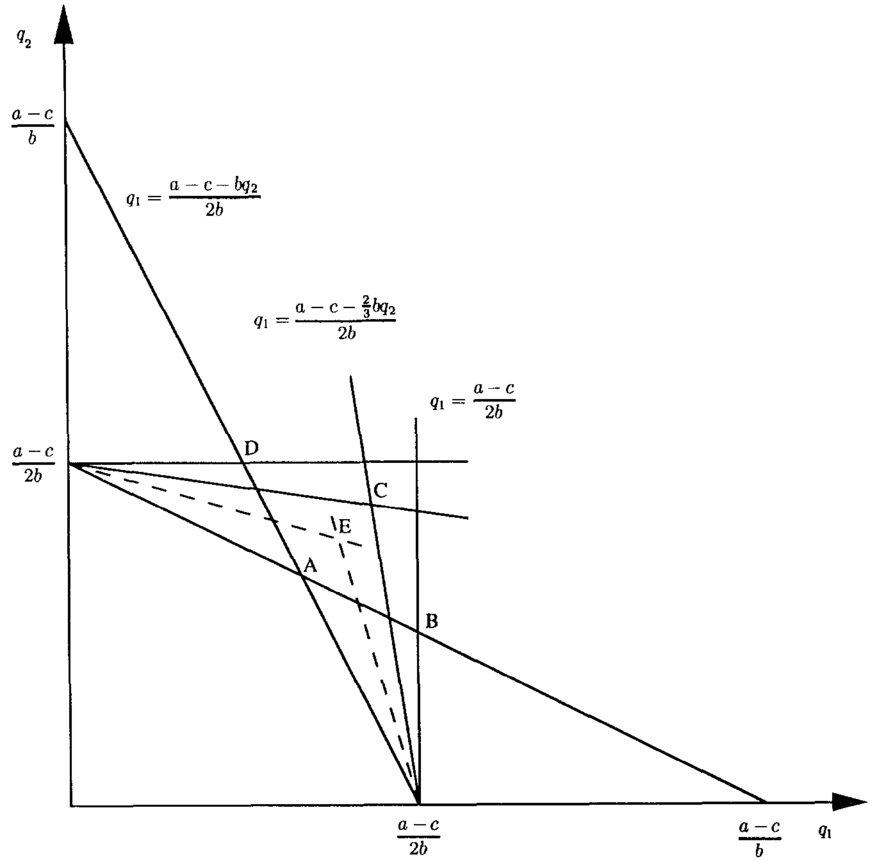

Fig. 1. Reaction functions and equilibria for the illustration with linear demand.

substitutes and by the stability conditions of the equilibrium solution: One firm expects that, by shifting its reaction function, it will increase its own output by a higher amount than it will lower the output of the other firm (stability condition) and this, in turn, will increase its profits. If both firms act in the same way, the two reaction functions will shift upwards and the new equilibrium point C will result. However, if only one firm uses relative performance evaluation, this firm chooses μ_i^* to take the maximum advantage of the situation and the equilibrium points are B or D, depending on whether it is firm 2 or firm 1 which uses relative performance evaluation. It is easy to show that D and B correspond to the Stackelberg leader–follower equilibria with $\mu^*_i = -1$, $\mu_j^* = 0$, as predicted by part (b) of the proposition, for symmetric cases like the one contemplated here. But it is important to note that, in general, multiple equilibria will exist and that the choice from amongst them will pose an additional problem.

Part (c) of Proposition 1 states that $\mu_i^* \leqq 0$; this is true because of the strategic substitutes nature of the decision variables, which in turn means that firms will tend to increase their output in order to simultaneously decrease the output of the other firms, although in a lower amount, given the stability conditions which determine the relative slope of the two reaction functions. The lower bound on μ_i^* for symmetric cases, $-1 \leqq \mu_i^*$, comes from the result which shows that

$$\mu_i^* = -\frac{\delta^2 \pi_i / \delta q_i \delta q_j}{\delta^2 \pi_j / \delta q_i \delta q_j},$$

when firm i uses relative performance evaluation and the other does not.[6] For the symmetric case $\mu_i^* = -1$ and since this is the Stackelberg equilibrium, we know that $\mu_i^* = -1$ will be the best choice of firm i when firm j chooses $\mu_j^* = 0$.

Comparing the equilibrium output with relative performance evaluation and with direct profit maximization, we can see that part (d) of Proposition 1 is also satisfied in our illustration. In general, the net increase in total industry output is explained by the stability conditions, which imply that the increase in the output of the firm shifting the reaction function is higher than the decrease in output induced in the others. Higher total output implies, in turn, lower output price.

We now move the the case where managers are risk averse and therefore $\Gamma > 0$. Part (e) of Proposition 1 establishes that an increase in Γ decreases the optimal value of μ_i^*; since $\mu_i^* \leqq 0$, this means that the absolute value of $\mu^*{}_i$ will increase and, consequently, the desired shift in the reaction function will be larger: Point E. When the managers are risk averse, net profits increase if managers share a lower amount of risk, since their expected salary will also be lower. The term $\beta_i^2(1+\mu_i)^2\sigma^2$, which determines the risk allocated to the manager, decreases as μ_i approaches -1; therefore, μ_i^* will depart from the equilibrium value under risk neutrality.

Point E gives a symmetric equilibria because we assume that the degree of risk aversion among the managers is the same. Asymmetric equilibria would occur if we were to allow for differences in risk aversion; in these equilibria, more risk averse managers would tend to produce more output than their less risk averse counterparts. For our special case, with only common uncertainty among firms but not specific uncertainty, point D (or B) could also be equilibrium solutions if the manager of the firm using relative

[6]For the asymmetric case, -1 will be a lower bound on μ_i^* as long as the firm using relative performance evaluation is also the firm with larger market share.

performance evaluation was risk averse, since for $\mu_i^* = -1$ the term $\beta_i^2(1+\mu_i)^2\sigma^2$ is equal to zero and therefore risk is totally eliminated from the expected utility of the manager. In general this will not be true; for example, if there is also specific uncertainty expressed by σ_i^2, the risk factor would be $\beta_i[\mu_i^2\sigma_j^2+(1+\mu_i)^2\sigma^2]$ and $\mu_i^* = -1$ would no longer be an equilibrium, neither would it reduce the risk factor to zero.

The uniqueness of the equilibrium with $\mu_i^* < 0$, $i=1,2$, when $\Gamma > 0$, is easy to see if we take into account that, in this case, it will not be optimal to have vertical reaction functions in order to optimize, in addition, the risk sharing effects. This is true for cases with common and specific risks affecting the distribution of profits.

The difference between the profits obtained with or without relative performance evaluation may be decomposed in two terms,

$$[\pi_i(q_i^*(\mu^*)) - \pi_i(q_i^*(0))]$$
$$+\{[\mathrm{E}(e_i^*(\mu_i^*)) - c(e_i^*(\mu_i^*)) - \Gamma/2\beta_i^{2*}(\mu_i^*)(1+\mu_i^*)^2\sigma^2]$$
$$-[\mathrm{E}(e_i^*(0) - c(e_i^*(0)) - \Gamma/2\beta_i^*(0)\sigma^2]\}.$$

The total industry gross profit with relative performance evaluation will be lower than with direct profit maximization because output is higher than Cournot output which, in turn, is already higher than collusive, monopoly, output. Therefore, in the symmetric case, gross profit per firm will also be lower which implies that the first bracket is negative. However, the second bracket is positive since $\mathrm{E}(e_i(\mu_i)) - c(e_i(\mu_i)) - \Gamma/2\beta_i^2(\mu_i)(1+\mu_i)^2\sigma^2$ is an increasing function of μ_i when $\mu_i \leq 0$.[7] The final conclusion to be drawn is that net profits may be higher or lower with relative performance evaluation than without it, depending on the importance of the strategic and the risk-sharing effects, since they move in opposite directions in determining the net profit of the firm.

4. Price competition

The profit function of firm i is now

$$P_i(p_1,p_2;e_i;\varepsilon) = \pi_i(p_1,p_2) + f(e_i,\varepsilon), \quad i=1,2.$$

The structure of the problem is identical to that obtained when firms

[7]Recall that β_i^* will be between zero and one. As μ_i^* approaches -1, $\beta_i^*(\mu_i)$ approaches 1, the last term approaches zero and effort converges to the first best level.

compete in quantities. All hypotheses are maintained except (b), (c) for which the inequality is reversed.

Proposition 2. Suppose there is a subgame perfect equilibrium of the game such that, at the second stage, competition is of the strategic substitute variety and equilibrium is stable. Then:

(a) The solution ($\mu_1^* = \mu_2^* = 0$) is not an equilibrium solution.
(b) If managers are risk neutral, $\Gamma = 0$, the equilibrium solutions μ_i^* are positive, $\mu_i^* > 0$, $i = 1, 2$.
(c) If managers are risk neutral, the equilibrium of the two stage game implies higher industry prices than in the profit maximizing case.
(d) If managers are risk averse, the sign of μ_i^* is undetermined as well as the level of industry prices in relation to the prices in the profit maximizing case.
(e) The equilibrium value μ_i^* decreases with the degree of risk aversion of the managers.

A partial demonstration of this proposition is presented in the appendix. We now turn to a discussion of its implications by way of an example of an industry with linear demand. Consider,

$$q_i = a - bp_i + dp_j, \quad i \neq j = 1, 2,$$

where a, b and d are positive parameters. Assuming that $c_1 = c_2 = c$ and $\Gamma = 0$, the reaction function of the ith firm is given by

$$P_i(\mu_i) = \frac{bc + a - \mu_i cd + dp_j(1 + \mu_i)}{2b} = p_i(p_j) + (p_j - c)\frac{\mu_i d}{2b},$$

where $p_i(p_j)$ is the reaction function when firms maximize absolute profits.

Firms are induced to manipulate their reaction functions to increase prices and profits ($\mu_i^* \neq 0$). Since goods are strategic complements, the reaction functions have positive slopes and therefore that objective is achieved by choosing positive values of μ_i^*. The net effect on equilibrium prices always implies that they will be higher than equilibrium prices under direct profit maximization. The linear-demand case makes this result clear, since for $\mu_i > 0$ the second term in the above expression is always positive, because negative profits are not allowed ($p_j > c$). Also note that under price competition the equilibrium solution will be unique, since the solution of one firm using relative performance evaluation cannot be an equilibrium; the reason is that for $\mu_i^* > 0$ it is not possible to transform the original reaction curve into a vertical, non-manipulable, reaction function.

Parts (d) and (e) of the proposition take into account that managers are risk averse. The final conclusions are now quite undetermined. The reason is that strategic competition considerations conflict with optimal risk allocation. The first ones imply that μ_i^* should be positive; but for $\mu_i^* > 0$ relative performance evaluation increases the risk allocated to the managers, the expected salary has to be higher to compensate the disutility created to the manager and net profits are lower. The final choice of μ_i has to weight these two conflicting objectives and depending on their relative importance the equilibrium value will be positive or negative. The equilibrium value of μ_i will be lower as we increase the degree of risk aversion of the managers, since this will mean higher weight to the risk-sharing considerations.

5. Conclusion

Will strategic competition and risk sharing considerations arrive at the same conclusion with respect to the opportunity of firms in oligopolistic industries using relative performance evaluation systems? According to the results of the model presented in this paper the answer to the question is yes, but only when firms compete on quantities (strategic substitutes), and not when they compete on prices (strategic complements). Therefore, under price competition the use of performance measures that weight the profit of other firms, has to be chosen taking into account the conflicting objectives of strategic competition and risk sharing.

When firms compete in quantities, our results indicate that strategic considerations make firms more aggressive than under direct profit maximization and that total industry output will increase (and prices decrease). Moreover we have shown that, if managers are risk neutral, then asymmetric equilibria cannot be ruled out, even in the case of symmetric firms, since the Stackelberg leader–follower equilibrium is part of the set of subgame perfect equilibria; this is achieved when one firm uses relative performance evaluation and the other does not.

If managers are risk averse, the more aggressive behaviour induced by relative performance evaluation does not necessarily imply lower profits for the firms, since this behaviour contributes towards the reduction in the expected salary of the managers, given that the risk shared by the managers is lower than the risk under no relative performance evaluation. Risk aversion also implies that the equilibrium will be unique and that all firms will use relative performance evaluation.

Appendix A

A.1. Demonstration of Proposition 1 and existence conditions

The manager i will decide e_i solving,

$$\max_{e_i} \beta_i \mathrm{E}(e_i) - c(e_i) - \Gamma/2\beta_i^2(1+\mu_i)^2\sigma^2 + \alpha_i + \beta_i\left(\sum_{j\neq i} \mathrm{E}(e_j)\right).$$

The necessary conditions, define the implicit function $e_i(\beta_i)$ increasing in β_i given the assumptions on $\mathrm{E}(e_i)$ and $c(e_i)$. The optimal value of β_i is obtained from

$$\max_{\beta_i} \mathrm{E}(e_i(\beta_i)) - c(e_i(\beta_i)) - \Gamma/2\beta_i^2(1+\mu_i)^2\sigma^2 - \bar{u}.$$

The first order conditions of optimality for this problem

$$(1-\beta_i)\frac{\mathrm{d}F}{\mathrm{d}e_i}\frac{\mathrm{d}e_i}{\mathrm{d}\beta_i} - \Gamma(1+\mu_i)^2\beta_i\sigma^2 = 0,$$

will define the implicit function $\beta_i(\mu_i)$, whose derivative with respect to μ_i is equal to

$$\frac{\mathrm{d}\beta_i}{\mathrm{d}\mu_i} = \frac{2\Gamma(1+\mu_i)\beta_i\sigma^2}{\dfrac{\mathrm{d}^2F}{\mathrm{d}e_i^2}\left(\dfrac{\mathrm{d}e_i}{\mathrm{d}\beta_i}\right)^2(1-\beta_i) - \dfrac{\mathrm{d}F}{\mathrm{d}e_i}\dfrac{\mathrm{d}e_i}{\mathrm{d}\beta_i} - \Gamma(1+\mu_i)^2\sigma^2} < 0 \text{ if } \mu_i \geqq -1.$$

Substituting $\beta_i(\mu_i)$ in the objective function of the problem, we define the function $h_i[e_i(\beta_i(\mu_i))] = H_i(\mu_i)$, whose derivative will have the same sign as $\mathrm{d}\beta_i/\mathrm{d}\mu_i$ since,

$$\frac{\mathrm{d}H_i}{\mathrm{d}\mu_i} = \underset{(+)}{\frac{\mathrm{d}h_i}{\mathrm{d}e_i}}\,\underset{(+)}{\frac{\mathrm{d}e_i}{\mathrm{d}\beta_i}}\,\frac{\mathrm{d}\beta_i}{\mathrm{d}\mu_i}.$$

The original problem of the shareholders is now formulated only in terms of $(\mu_1,\ldots,\mu_n)$.

$$V_i = B_i(\mu_1,\ldots,\mu_n) + H_i(\mu_i).$$

Demonstration of Proposition 1. (a) We will demonstrate the first point of the proposition, proving that if $\mu_i^* = 0$, the optimum choice for firm 1 is not $\mu_i^* = 0$. Given $(0,\mu_2)$ as the values of the factors of the evaluation functions which determine the equilibrium solution $q_i(\mu_2)$ and $\pi_i(q_1(0,\mu_2), q_2(0,\mu_2))$, we can prove that, for $\mu_2 = 0$,

$$\frac{\delta V_1}{\delta \mu_1}(0,\mu_2)=\frac{\delta \pi_1}{\delta q_1}\frac{\delta q_1}{\delta \mu_1}+\frac{\delta \pi_1}{\delta q_2}\frac{\delta q_2}{\delta \mu_1}+\frac{\mathrm{d}H_1}{\mathrm{d}\mu_1}=0. \tag{A.1}$$

The equilibrium outputs satisfy the necessary optimality conditions, which for $n=2$ are written as,

$$\frac{\delta \pi_i}{\delta q_i}+\mu_i\frac{\delta \pi_j}{\delta q_i}=0, \quad i\neq j=1,2. \qquad \text{(A.2),(A.3)}$$

Let us initially assume that $\mu_1=0$ and μ_2 takes whatever value. Eq. (A.2) implies $\delta\pi_1/\delta q_1=0$ and therefore, in order to prove (A.1), it is necessary to demonstrate that $\delta\pi_1/\delta q_2$ and $\delta q_2/\delta\mu_1$ are different from zero for the specific value of $\mu_2=0$. It is known that $\delta\pi_1/\delta q_2<0$ from hypothesis (b). In order to know $\delta q_2/\delta\mu_1$ we must totally differentiate (A.2) and (A.3), evaluate the result in $(0,\mu_2)$ and solve for $\delta q_2/\delta q_1$

$$\frac{\delta q_2}{\delta \mu_1}=\frac{\dfrac{\delta \pi_2}{\delta q_1}\left(\dfrac{\delta^2\pi_2}{\delta q_2\delta q_1}+\mu_2\dfrac{\delta^2\pi_1}{\delta q_2\delta q_1}\right)}{\mathrm{A}}, \tag{A.4}$$

$$\frac{\delta q_1}{\delta \mu_1}=\frac{-\dfrac{\delta \pi_2}{\delta q_1}\left(\dfrac{\delta^2\pi_2}{\delta q_2^2}+\mu_2\dfrac{\delta^2\pi_1}{\delta q_2^2}\right)}{\mathrm{A}}, \tag{A.5}$$

where

$$\mathrm{A}=\frac{\delta^2\pi_1}{\delta q_1^2}\left(\frac{\delta^2\pi_2}{\delta q_2^2}+\mu_2\frac{\delta^2\pi_1}{\delta q_2^2}\right)-\left(\frac{\delta^2\pi_1}{\delta q_1\delta q_2}\,\frac{\delta^2\pi_2}{\delta q_1\delta q_2}\right)-\mu_2\left(\frac{\delta^2\pi_1}{\delta q_1\delta q_2}\right)^2.$$

Consider first the case of the risk-neutral manager with $\Gamma=0$ and therefore $\mathrm{d}H_1/\mathrm{d}\mu_1=0$ since $\mathrm{d}\beta_1/\mathrm{d}\mu_1=0$. For $\delta q_2/\delta\mu_1$ to be equal to zero, which will make (A.1) equal to zero and therefore would imply that $\mu_1=0$ is an equilibrium solution, it is required that

$$\frac{\delta^2\pi_2}{\delta q_2\delta q_1}+\mu_2\frac{\delta^2\pi_1}{\delta q_2\delta q_1}=0, \tag{A.6}$$

taking into account that $\delta\pi_2/\delta q_1<0$. Eq. (A.6) cannot be satisfied for $\mu_2=0$ and therefore $\delta q_2/\delta\mu_1$ is different from zero when $\mu_2=0$.

If the manager is risk averse, $\Gamma>0$, then $\mathrm{d}H_1/\mathrm{d}\mu_1$ is always different from zero and therefore (A.1) would not be equal to zero even if (A.6) was satisfied.

(b) Recall that we now assume the managers to be risk neutral, $\Gamma=0$. We shall prove that if μ_j^* is chosen such that (A.6) is satisfied, then the equilibrium solution implies $\mu_i^*=0$ and that for this pair of parameters the industry will be in the Stackelberg leader–follower equilibrium.

Solving for μ_2^* in (A.6)

$$\mu_2^*=\frac{-\delta^2\pi_2/\delta q_2\delta q_1}{\delta^2\pi_1/\delta q_2\delta q_1}$$

Substituting in (A.4) we have $(\delta q_2/\delta q_1)=0$ and

$$\frac{\delta V_1}{\delta\mu_1}=\frac{\delta\pi_1}{\delta q_1}\frac{\delta q_1}{\delta\mu_1}+\frac{\delta\pi_1}{\delta q_2}\frac{\delta q_2}{\delta\mu_1}=0$$

requires $(\delta\pi_1/\delta q_1)=0$, since $(\delta q_1/\delta\mu_1)\neq 0$ [see (A.5)].

If (A.2) has to be satisfied and $(\delta\pi_1/\delta q_1)=0$, we have $\mu_1^*=0$, given that $\delta\pi_2/\delta q_1=0$. When $\mu_1^*=0$ eqs. (A.4) and (A.5) imply

$$\frac{\delta q_2}{\delta\mu_2}=\frac{-\dfrac{\delta\pi_1}{\delta q_2}\dfrac{\delta^2\pi_1}{\delta q_1^2}}{A},\quad \frac{\delta q_1}{\delta\mu_2}=\frac{\dfrac{\delta\pi_1}{\delta q_2}\dfrac{\delta^2\pi_1}{\delta q_1\delta q_2}}{A}.$$

Substituting in $(\delta V_2/\delta\mu_2)=0$ we obtain

$$\frac{\delta\pi_2}{\delta q_2}-\frac{\delta\pi_2}{\delta q_1}\frac{\delta^2\pi_1/\delta q_1\delta q_2}{\delta^2\pi_1/\delta q_1^2}=0,$$

which is identical to the Stackelberg leader–follower solution when firm 2 is acting as the leader.

As a final remark note that, if the profit functions are symmetric then $\mu_2^*=-1$, that is, the leader will evaluate the manager as a function of the absolute difference between the profits of the two firms. For $\mu_2^*=-1$, the derivative of $H_2(\mu_2)$ with respect to μ_2 is equal to zero if the manager is risk averse, $\Gamma>0$. Therefore, for this special case the Stackelberg equilibrium will also hold for risk-averse managers. The result may be explained by the fact that we assume there is only common uncertainty within the industry which is identical for all firms, but that there is no firm specific uncertainty. In this case, relative performance evaluation with $\mu_i=-1$ eliminates all the risk shared by the manager, the value of β_i^* can be made equal to one and the full efficient solution in terms of managerial effort e can be obtained.

(c) In order to prove $\mu_i^*\leqq 0$ we must prove that the alternative condition

$\mu_i^* > 0$ leads to a situation where $(\delta V_i/\delta\mu_i) = 0$, the equation which determines the optimal value of μ_i, cannot be satisfied.

If $\mu_i^* > 0$, then eqs. (A.2) and (A.3) together with hypothesis (b) $(\delta\pi_i/\delta q_j) < 0$, all imply that $(\delta\pi_i/\delta q_i) > 0$. Concavity of the profit function [hypothesis (a)], the second order condition and the stability condition all imply $A > 0$ and $(\delta q_i/\delta\mu_i) < 0$; therefore $(\delta q_j/\delta\mu_i) > 0$ as i and j are strategic substitutes. Moreover, we know that $dH_i/d\mu_i < 0$ if $\mu_i > 0$ since $d\beta_i/d\mu_i < 0$ in this case. All this implies

$$\frac{\delta B_i}{d\mu_i^*} = \underset{(+)}{\frac{\delta\pi_i}{\delta q_i}}\,\underset{(-)}{\frac{\delta q_i}{\delta\mu_i^*}} + \underset{(-)}{\frac{\delta\pi_i}{\delta q_j}}\,\underset{(+)}{\frac{\delta q_j}{\delta\mu_i^*}} + \underset{(-)}{\frac{dH_i}{d\mu_i^*}} < 0$$

Therefore $\mu_i^* > 0$ cannot be an optimum solution.

To prove that $\mu_i^* \gtreqq -1$ in the symmetric case, note that in (b) we proved that the best choice of firm i when firm j chooses $\mu_j^* = 0$ is $\mu_i^* = -1$.

(d) We now wish to prove that the inclusion of the profit made by the industry in the management evaluation function implies a higher output for that industry. In order to do this, we must take into account eqs. (A.4) and (A.5) as well as the condition by which $\mu_i^* = 0$.

In part (c) of the proposition it was proved that $(\delta q_i/\delta\mu_i^*) < 0$, $(\delta q_j/\delta\mu_i^*) > 0$ and $\mu_i^* \leqq 0$. Thus, when passing from a situation where the management only maximizes the profits of the firm, $\mu_i = 0$, $i = 1, 2$ to one where they maximize the function O_i with $\mu_i < 0$ for at least one firm, this will produce an increase of q_i for the firm with $\mu_i < 0$ and a decrease of q_j, $j \neq i$. The net effect on total output $q_i + q_j$, will be positive, that is to say, the total output of the industry will be higher, as the stability condition implies that $|\delta q_i/\delta\mu_i^*|$ is higher than $|\delta q_j/\delta\mu_i^*|$ [see eqs. (A.4) and (A.5)].

(e) In order to prove this result we will write $H_1(\mu_i, \Gamma)$, with $(\delta H_i/\delta\Gamma) < 0$, that is we shall take into account that the effort-risk sharing function $H(\cdot)$ is decreasing in the degree of risk aversion of the manager for β and μ given.

The first order conditions of the problem of determining μ_i are given by

$$\frac{\delta V_i}{\delta\mu_1} + \frac{\delta H_i}{\delta\mu_i} = 0, \quad i = 1, 2.$$

Total differentiation of the above equations with respect to Γ and solving for $(d\mu_i/d\Gamma)$ gives

$$\frac{d\mu_i}{d\Gamma} = \frac{\left(\dfrac{\delta^2 V_j}{\delta\mu_j^2} + \dfrac{\delta^2 H_j}{\delta\mu_j^2} - \dfrac{\delta^2 V_i}{\delta\mu_i\delta\mu_j}\right)\left(-\dfrac{\delta^2 H_i}{\delta\mu_i\delta\Gamma}\right)}{E} < 0$$

where E is the value of the determinant of the matrix defined by the system of equations, which has to be positive from second order conditions; the $\delta^2 H_i/\delta\mu_i\delta\Gamma$ is negative for $\mu_i > -1$ as can be checked from $\delta^2\beta_i/\delta\mu_i\delta\Gamma$, and the other term in the numerator is negative from stability conditions. Notice that we are making use of the assumption of symmetry in these conclusions.
(f) Under risk aversion, $(\mathrm{d}H_1/\mathrm{d}\mu_1) \neq 0$ and therefore even if $(\delta q_2/\delta\mu_1)=0$, $(\delta\pi_1/\delta q_1)$ will have to be different from zero. This, in turn, implies that μ_1 has to be different from zero in eq. (A.2). The uniqueness of the solution comes from the regularities imposed on the functions in order to guarantee the existence of such equilibrium.

Existence conditions. The conditions for the existence of a Nash equilibrium solution to the problem of the managers are given by the properties of the O_i function which assure the compliance of first and second order conditions, especially the latter, if we adopt the general assumption that the system of equations of the first order conditions is solvable. Knowing that μ_i^* will not be positive in the optimum, it is easy to prove that the concavity of π_i and the hypothesis (c) are sufficient for the second order conditions to be satisfied. Furthermore (a), (b) and (c) together with $\mu_i^* \leqq 0$ assure the stability condition and that $\mathrm{A}>0$.

A.2. Demonstration of Proposition 2

We will show only part (d) since the structure of the proof uses similar arguments to those used in Proposition 1.

The equilibrium value of μ_i^* satisfies

$$\frac{\delta V_i}{\delta\mu_i}(\mu_1^*, \mu_2^*) = \frac{\delta\pi_i}{\delta p_i}\frac{\delta p_i}{\delta\mu_i^*} + \frac{\delta\pi_i}{\delta p_j}\frac{\delta p_j}{\delta\mu_i^*} + \frac{\mathrm{d}H_i}{\mathrm{d}\mu_i} = 0, \tag{A.7}$$

where

$$\frac{\delta p_i}{\delta\mu_i^*} = \frac{-\left(\dfrac{\delta^2\pi_j}{\delta p_j^2} + \mu_j^*\dfrac{\delta^2\pi_i}{\delta p_j^2}\right)\dfrac{\delta\pi_j}{\delta p_i}}{\mathrm{A}}$$

$$\frac{\delta p_j}{\delta\mu_i^*} = \frac{-\left(\dfrac{\delta^2\pi_j}{\delta p_j\delta p_i} + \mu_j^*\dfrac{\delta^2\pi_i}{\delta p_j\delta p_i}\right)\dfrac{\delta\pi_j}{\delta p_j}}{\mathrm{A}},$$

and $\mathrm{d}H_i/\mathrm{d}\mu_i$ was defined above.

From the assumptions of the problem we know that $(\delta p_i/\delta\mu_i^*)>0$, $(\delta p_j/\delta\mu_i^*)>0$, $(\delta\pi_j/\delta p_i)>0$ and $(\mathrm{d}H_i/\mathrm{d}\mu_i) \leqq 0$ if $\mu_i \geqq -1$. Taking into account the necessary conditions from the problem of deciding p_i,

$$\frac{\delta \pi_i}{\delta p_i} + \mu_i \frac{\delta \pi_j}{\delta p_i} = 0 \tag{A.8}$$

it is easy to check that (A.7) and (A.8) can only be satisfied if $\mu_i > -1$. Therefore the sign of optimal μ_i is undetermined.

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