# Private enforcement and social efficiency

## James E. Anderson[a], Oriana Bandiera[b],*

[a]*Boston College and NBER, United States*
[b]*London School of Economics and CEPR, UK*

## Abstract

This paper makes precise the distributional consequences and social efficiency of private enforcement of property rights. Properties of different values are subject to predation and owners choose between self-defense and private enforcement services. A distributional conflict of interest arises as private protection purchased by rich owners deflects predators on low value properties. The market structure of enforcement and development affect the distribution of property income through relative changes in the security of high and low values property. Moreover, because of the externality enforcers impose on poorer owners, the availability of private enforcement may constrain the policy of a benevolent State.
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## 1. Introduction

In most societies, both past and present, property rights enforcement differs enormously from the well-functioning and impartial system that lies at the foundation of economic theory. Property protection by private and State enforcers under various organizational and behavioral assumptions has been the subject of recent studies focused on economic

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\* Corresponding author. Department of Economics, LSE, Houghton St., WC2A 2AE London, UK.
  *E-mail address:* o.bandiera@lse.ac.uk (O. Bandiera).

efficiency.[1] Property rights enforcement also has important distributional consequences as the poor suffer more from insecurity than the rich (deSoto, 2000). This paper brings distribution to the fore.

We study property rights enforcement in a model where properties of different values are subject to predatory attacks and owners choose between self-defense and purchasing private enforcement services. The number of predators, the choice of self or specialized protection, the share of protected properties and the number of enforcers are all endogenous in the model. After characterizing the positive economics of the model, we consider the social efficiency of private enforcement.

The model has three distinctive features. First, properties have different values, implying a distributional conflict of interest between high and low value property owners as enforcement purchased by the former deflects predators on the latter. Second, we allow for both self-defense and specialized private enforcement, where the choice between the two is endogenous and left to the property owner. We show that in equilibrium the two can plausibly co-exist, with richer owners buying specialized enforcement and poor owners choosing self-defense. Taken together these two features allow us to analyze the distributional consequences of private enforcement, thus complementing the existing results on economic efficiency. The third key feature of the model is that we analyze two market structures, free entry and collusion. Enforcers offer geographically differentiated services and thus enjoy some market power in their regions. In the free entry case profits are driven to zero while collusion assumes that a coalition restricts entry to maximize profits.

Our main results are as follows. First when the market structure goes from free entry to collusion we find, quite reasonably, that security worsens and the price of private enforcement increases. As a consequence, enforcers are better off and owners are worse off. More interestingly, we find that the change in market structure has an important distributional consequence: the coalition increases inequality as the poorer properties become relatively more insecure. This follows from the endogenous allocation of predators between properties of different value.

Second, we consider the impact of development on insecurity. Under free entry, an 'equalizing' development path, increasing the wage relative to property values, makes all properties more secure and has an amplified equalizing effect as low value property

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[1] History provides many examples of private enforcement. In medieval Europe law merchants and of merchant guilds enforced property rights in the Champagne fairs and in long distance commerce, respectively (Milgrom et al., 1990; Greif et al., 1994). During the early nineteen century, the ranchers of the American West legendarily used self enforcement aided by banding together with neighbors in vigilante groups; claim associations protected and enforced settlers' property rights in the Midwest and miners' organizations enforced rights during the Gold Rush (de Soto, 2000). Monopoly enforcement was provided by the dominant Pinkerton's Detective Agency to U.S. mining and industrial interests across the country in the late 19th and early 20th centuries (Rowan, 1931). The Mafia originated as a coalition of specialized private property rights enforcers in rural western Sicily (Gambetta, 1993). Similar origins are described for the Japanese yakuza (Milhaupt and West, 2000). Both continue to operate today and, among other activities, they protect property rights that are not recognized by the State. Contemporary examples of private enforcement include the security services purchased by firms, universities and gated communities in developed countries, and the operation of gangs in their 'wilder' areas (Akerlof and Yellen, 1994; Jankowsky, 1991). Other examples of private enforcement and dispute settlement in specific sectors abound both in developed and developing countries (McMillan and Woodruff, 2000).

becomes relatively more secure. In contrast, a 'trickle down' development path which raises property values relative to the wage amplifies inequality by lowering the security of low value property. If the enforcers form a coalition, in contrast, development leads to higher profits while leaving security unchanged. Similarly, exogenous improvements in security, as when the technology of enforcement improves, increase the owners' welfare and have an equalizing effect when there is free entry whereas the gains from improved security are entirely captured by the enforcers in coalition.

Third we find that the market outcome generally differs from the utilitarian social optimum. Interestingly, private enforcers might offer too little but also too much enforcement. On the one hand, market power makes private enforcers protect too few properties. On the other hand, the negative externality imposed on the poor by the protection of the property of the rich leads to too much enforcement. Socially efficient enforcement trades off the monopoly distortion against the negative externality, the former afflicting the rich to the benefit of the poor and the latter afflicting the poor to the benefit of the rich. The utilitarian state (taken as a benchmark for states which care at least somewhat about the poor) includes in its objective function the welfare of the poor whose property is too low in value to efficiently be covered by specialized enforcement. The analysis shows that in an intermediate range of parameters the two distortions nearly offset, hence there is not much benefit to replacing private with public enforcement.

For sharper focus, the model abstracts from many details of enforcement and many elements of real economies. We assume that agents' choice between predation, ownership and enforcement is made at an earlier stage that we do not analyze here. To isolate distributional from efficiency considerations, we assume that the only productive activities in the model are the receipt of endowments by consumer/owners and the enforcement of rights to the endowments against predators. In this setup, enforcement has only a redistributive role.

Our paper is part of a literature on the implications of alternative enforcement institutions, recently reviewed by Dixit (2004). In a seminal contribution, deMeza and Gould (1992) allow owners to choose whether to self-defend by restricting access to their properties. They show that decentralized decisions generally do not yield the socially optimal level of enforcement because enclosure of any property has important effects on the rest of the economy which individual owners do not internalize. Like theirs, our model captures this important externality and brings the analysis one step further by examining its distributional consequences.

Grossman and Noh (1994), Grossman (2002), Moselle and Polak (2001) and Olson (1993) analyze enforcement provision by a "predatory" monopolistic ruler that maximizes revenues. Konrad and Skaperdas (1999) compare outcomes under monopolistic and under competition. The bottom line is that while private enforcers do not provide the first best level of enforcement, they lead to an increase in efficiency compared to anarchy. Our contribution highlights the distributional effects of private enforcement and is thus complementary to existing analyses of efficiency.

Finally, Grossman (1995) and Marcouiller and Young (1995) analyze the interdependence of state and private enforcement when both are revenue maximizers who operate in distinct markets. Here, in contrast, we compare private and state enforcers that provide

protection in the same market, but whereas private firms maximize profits the state maximizes social welfare.

The remainder of the paper is organized as follows. Section 2 sets out the basic elements of the model of private enforcement and derives the equilibrium when there is free entry and when the enforcers form a coalition. Section 3 analyses the distributional consequences of the two market structures and of the development process. Section 4 contrasts private enforcement with a welfare-maximizing state enforcement policy. Section 5 concludes.

## 2. A model of private enforcement

### 2.1. Elements of the model

#### 2.1.1. Set up

There are three types of agents in the economy: property owners, predators and specialized enforcers. We assume that occupational choices are made at an earlier stage which is suppressed here for simplicity.

*Property owners* of unit mass are uniformly located on a unit circle and at each location on the circle there is an identical distribution of property ranked from high to low value. The potential buyers' valuation of property at each location is thus distributed according to $V(\alpha)$, where $\alpha$ is the proportion of buyers on the radial section with valuation greater than or equal to $V$, and $V_\alpha < 0$.

All property is subject to attack by *predators* of mass $B$ (for Bandits). The number of predators is endogenously determined and is equal to the number agents whose alternative labor market option is worse than the expected payoff from predation. We normalize the maximum potential number of predators to one and assume that alternative options are uniformly distributed on $[0, w]$. Predators spread themselves across the region to equalize their rate of success across locations.

*Specialized enforcers* sell protection against predatory attack. We assume that enforcement services are geographically differentiated so that each enforcer is a monopolist in his region. This is the reduced form of a model where the capability of each enforcer decreases with distance or his costs increase with distance, both of which yield a spatial equilibrium where the assignment of buyers to sellers of enforcement is unique. The number of enforcers in equilibrium is endogenous and depends on whether there is free entry or collusion in the enforcement market.

We analyze the symmetric equilibrium of our spatial structure. With $n$ denoting the number of active enforcers, $B/n$ of the predators' mass is located in each enforcer's market area and $1/n$ of the property owners' mass is located in each enforcer's market area. Within each location, a fraction $\alpha*$ of properties are protected and a fraction $\lambda$ of predators chooses to prey on protected property.

#### 2.1.2. The probability of successful defense

Denote by $\beta$ and $\pi$ the objective probabilities that an owner enjoys his property when he chooses self-defense and when he buys protection, respectively. When owners choose

self defense they have some capacity to evade the predators through hiding their goods, moving them about or coordinating warnings. We assume plausibly that in this anonymous hide-and-seek interaction the success rate depends on the ratio of the numbers on each side. Specifically, the realized (objective) probability of successful ownership is equal to

$$\beta = \frac{1}{1 + \theta \dfrac{B(1-\lambda)/n}{(1-\alpha)/n}} = \frac{1}{1 + \theta B \dfrac{1-\lambda}{1-\alpha}}. \tag{1}$$

Here $B(1-\lambda)/n$ is the mass of predators who choose to attack unprotected property on each market segment. Similarly $(1-\alpha)/n$ is the mass of unprotected property owners in each market segment. Then $B\,(1-\lambda/1-\alpha)$ is the average intensity of predator to prey on unprotected property. $\beta$ is increasing in $\lambda$ and homogeneous of degree zero in $(1-\alpha, 1-\lambda)$. To interpret the interaction of predators and prey represented by Eq. (1), note that if $\theta$ is equal to 1 and the mass of predators and unprotected property owners is equal, the probability of successful evasion is equal to 1/2. Starting from this neutral benchmark, a rise in $\theta$ lowers the probability of successful evasion as predators become relatively more effective. Thus $\theta$ is a technological parameter reflecting the relative effectiveness of predators and prey interacting on unprotected property.

A key property of the model is the negative externality inflicted by those who purchase enforcement on those who do not: $\partial \beta / \partial \alpha < 0$. Intuitively, those who buy enforcement deflect thieves onto those who do not, lowering their probability of successful defense. The analytic derivative is special to the logistic functional form,[2] but its sign (the negative externality) would arise in any reasonable model of anonymous interaction.

Property protected by the specialized enforcer is plausibly assumed to be unconcealed and indeed clearly labelled to deter attack. The enforcer is assumed to be able to defeat some attacks and/or recover some stolen goods. Success is plausibly decreasing in the ratio of predators to enforcers. Again we adopt the logistic function: the realized probability of successful ownership when protected by the specialized enforcer is assumed at each location to be equal to

$$\pi = \frac{1}{1 + \theta \dfrac{B\lambda/n}{R}}. \tag{2}$$

Here, $B\lambda/n$ is the mass of predators who choose to attack protected property in each market, while $R$ represents the enforcement capability of the enforcer to deter attack or to recover the value taken by predators who choose to attack. $\theta/R$ gives the relative effectiveness of predators against the enforcer.

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[2] The logistic function has been widely used in the conflict literature (e.g., Grossman, 1995; Skaperdas and Syropoulos, 1995). There it is used to model interaction of one predator and one prey as a contest, with success determined by predetermined force levels which are optimally chosen at an earlier stage. The anonymous interaction model developed in this paper is distinct because (i) agents are probability takers and (ii) success depends on relative numbers due to interaction in which one side seeks and the other side hides.

### 2.1.3. The demand for protection

If the owner purchases specialized enforcement, his subjective probability of enjoying his property is equal to $\pi'$. If he does not buy enforcement his subjective probability of enjoying his property is equal to $\beta'$. The value of specialized enforcement to the marginal buyer is $(\pi' - \beta')V(\alpha)$, hence for specialized enforcement to be purchased at all, $\beta' < \pi'$. The value of each property is known only to its owner, but enforcers know the distribution of values. All buyers with valuation greater than or equal to $(\pi' - \beta')V(\alpha)$ will buy enforcement when the enforcer charges a price equal to $(\pi' - \beta')V(\alpha)$. Inframarginal property owners enjoy a surplus.

In rational expectations equilibrium, the subjective value of $\beta'$ must be equal to the realized value of $\beta$ in the interaction of predators and prey and subjective value of $\pi'$ must be equal to the objective performance of the enforcer $\pi$. The property owners are probability takers, as is plausible when they are in large numbers.

### 2.1.4. The predators' problem

Predators know whether property is protected or not, which gives them information on its expected value. The predators share the common beliefs, so those who choose to attack random pieces of unprotected property have a subjective probability of successful stealing equal to $1 - \beta'$, and those who choose to attack random pieces of protected property have a subjective probability of stealing equal to $1 - \pi'$.

The proportion of predators who attack protected property, $\lambda$, is determined by the equality of expected return in attacks on the two types of property.

$$[1 - \beta']V^{\mathrm{L}} = [1 - \pi']V^{\mathrm{H}}, \tag{3}$$

where the left hand side of the equation is the return from attacking unprotected (low value) property and the term on the right is the expected return from attacking protected (high value) property.[3] Here, $V^{\mathrm{H}}$ and $V^{\mathrm{L}}$ denote the average high and low value properties:

$$V^{\mathrm{H}} \equiv \mathop{E}_{x < \alpha^*}[V(x)], \quad V^{\mathrm{L}} \equiv \mathop{E}_{x \geq \alpha^*}[V(x)].$$

Finally, the mass of predators $B$ includes all agents whose alternative option drawn from $[0, w]$ is worse than the expected payoff from predation. With alternative options distributed uniformly, this implies a supply schedule

$$B = \frac{(1 - \beta')V^{\mathrm{L}}}{w}. \tag{4}$$

### 2.1.5. The supply of private protection

Enforcers sell protection against predatory attacks. The total number of enforcers is $n$, endogenously determined as explained in detail below. Each enforcer serves a market of size

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[3] We abstract from punishment of predators who are caught preying on protected property, without loss of generality. With a sufficiently harsh punishment, no predators will ever attack protected property. For less harsh punishments, the effect of the size of punishment is simply to raise the equilibrium $\pi$ and lower the equilibrium $\beta$ without changing any qualitative properties of the system.

$1/n$ and protection services are geographically differentiated, so that each enforcer has market power in his region. Such would be the equilibrium of a game where enforcers decide where to locate in the circle and either their capability (and thus the price they can charge) decreases with distance or their costs increase with distance. Enforcers are assumed to be honest, namely they do not to collude with bandits or sponsor predatory attacks. Implicitly we assume that honesty is sustained by repeated interactions that we do not model here.

Each enforcer incurs a cost $f$ of establishing capability $R$.[4] The capacity of the enforcer is for convenience effectively made exogenous by assuming that the capacity cost function steps upward in the relevant range such that an attempt by an aggressive firm to penetrate its neighbors markets, increasing the area to be defended from $1/n$ to $3/n$, would unprofitably raise its capacity costs.[5]

The enforcer maximizes profits under our assumptions by selecting the optimal proportion of customers in his area to serve, $\alpha$. Enforcers cannot price discriminate because they cannot observe the valuations of their customers and are assumed to be probability takers, just like the property owners and predators. Probability-taking implies that the enforcers naively fail to account for (i) the effect of $\alpha$ on $\beta$ and hence on $(\pi - \beta)V^H$, the willingness-to-pay of protected owners, and (ii) the effect of $\alpha$ on $\lambda$ in equilibrium; i.e. the share of predators who attack protected property.

Each enforcer takes the probability $\beta'$ as exogenous because it reflects the equilibrium interaction of anonymous predators and unprotected property owners across, in principle, the entire region.[6] The enforcer also takes the number of predators he faces, $\lambda B/n$; as given because again this reflects the interaction of predators and enforcers across the entire region. Moreover, since reputation $R$ is fixed, this implies that $\pi$ is exogenous. Our assumption is closely related to standard assumptions in the theory of monopolistic competition and the large trade theory literature on imperfect competition in general equilibrium whereby the monopolist takes prices in all markets other than his own as given (see, e.g., Helpman and Krugman, 1989).

The enforcer's choice problem is:

$$\max_{\alpha} \frac{\alpha}{n}(\pi' - \beta')V(\alpha) - f. \tag{5}$$

The enforcer has market power through acting on the knowledge that $V(\alpha)$ is declining in $\alpha$. Expectations are realized in rational expectations equilibrium, $\pi' = \pi$ and $\beta' = \beta$. We

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[4] We avoid complex questions of the origin of reputation in a dynamic setting by assuming expenditure of $f$ creates a capability $R$ which results in anticipated reputation for effectiveness.

[5] More general treatment of the capacity cost makes it an increasing and convex function $C(1/n)$ of the size of the market served, $1/n$. The enforcer has the ability to undercut prices at the edge of the market. It does not pay to do so with strictly convex 'plant' costs because profits fall with expansion beyond the optimum size. The linear case $C(1/n)=f/n$ implies that the size of the competitive, probability-taking firm is indeterminate until the free entry equilibrium condition is imposed. With linear plant costs there is no equilibrium with undercutting, so the plant size is the free entry equilibrium value of $1/n$. We use a step function for $C(1/n)$ in the text for simplicity, removing variable capacity cost as a force partially determining $n$.

[6] In equilibrium of course, as $B(1 - \lambda)/n$ predators attack the $(1 - \alpha)/n$ unprotected properties on his segment, the interaction on his own market segment is the same as that anywhere else. A sophisticated enforcer might understand the equilibrium and hence be able to optimize the effect of $\alpha$ on $\beta$ while assuming that all other enforcers would similarly optimize the effect of $\alpha$ on $\beta$ on their market segments.

assume that the property value function $V$ is such that revenue is concave in $\alpha$:$2V_\alpha + \alpha V_{\alpha\alpha} < 0$. Thus the profit-maximizing proportion of property owners served, $\alpha^*$, is unique. At an interior maximum $\alpha^*$ is determined by the first order condition for Eq. (5):

$$\frac{(\pi - \beta)}{n} [V(\alpha^*) + \alpha^* V_\alpha(\alpha^*)] = 0. \tag{6}$$

The equilibrium number of enforcers $n$ depends on whether there is free entry or collusion in the enforcement market as discussed below.

### 2.2. Case I: free entry

With free entry the number of enforcers $n$ adjusts to yield zero profits:

$$\frac{\alpha^*}{n} [\pi - \beta] V(\alpha) - f = 0. \tag{7}$$

We assume that $f$ is invariant to $n$; rationalized by infinitely elastic supply of enforcers from outside the region at a fixed opportunity cost.

The full equilibrium of the system is reached when the predators allocation condition (Eq. (3)), the predators' entry condition (Eq. (4)), the enforcer's choice of customers (Eq. (6)) and the free entry condition (Eq. (7)) are all satisfied with the anticipated probabilities being equal to the values implied by the logistic success functions (2) and (1). This 6 equation system determines $(B, \lambda, \alpha, \pi, \beta, n)$. Eq. (6) implies that $\alpha^*$ is independent of the other variables and parameters except for the parameters of $V(\alpha)$.

The system has three type of equilibria, an interior equilibrium and two corners at $\alpha^* = 0$ (no private enforcement is offered), and $\alpha^* = 1$ (all property is protected).[7] In the interior equilibrium, wealthy property owners purchase private protection while poor owners rely on self defense. We are mostly interested in the interior equilibrium because of its distributional properties.

In the Appendix we prove:

**Proposition 1.** *For $V(0) = f$ sufficiently large and $V(1) = f$ sufficiently small, and $w < (\theta V^L / (1 - \alpha^*))$; specialized enforcement is offered in symmetric equilibrium to a unique fraction of property $\alpha^*$.*

While the term $\theta V^L / (1 - \alpha^*)$ is generally endogenous, it gives the most economically meaningful characterization of equilibrium types. Interpreting the first two conditions, when $V(0) = f$ is too small, no specialized enforcer can break even; when $V(1) = f$ is too large, all property can be protected.[8]

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[7] Note that, in contrast, the model of deMeza and Gould (1992) typically yields corner solutions because, interpreted in our setting, the property value function assigns the same value to all property and only self-defense is available.

[8] The objective probability $\beta$ is undefined since $\alpha = \lambda = 1$. The equilibrium depends on the expectations of self-defense success $\beta'$. There are two cases: if property owners and predators are pessimistic about the effectiveness of their self-defense and $\beta' < (Rw - \theta f / Rw)$ there is an interior solution with $\pi = (Rw\beta' / Rw - \theta f)$, $B = ((Rw(1 - \beta') - \theta f) V(1) / Rw - \theta f)$, $n = (\beta' \theta V(1) / Rw - \theta f)$. This solution exists as long as $Rw - \theta f > 0$. Alternatively, if property owners and predators are optimistic about self-defense ($\beta' > (Rw\theta f / Rw)$), the equilibrium is at $\pi = 1$, $B = 0$, $n = V(1)(1 - \beta'/f)$.

When $w \geq (\theta V^L/(1 - \alpha^*))$, the solution is self protection equilibrium, with $\alpha^* = n = \lambda = B = 0$, $V^L = E[V]$ and $\pi$ undefined. That there is no demand for specialized enforcement whenever the outside option of the highest paid potential predators is too high is intuitive because with light predation it does not pay to purchase protection. More surprising, however, the self enforcement equilibrium is secure, i.e. predation does not occur, even though there are always potential predators with low (even zero) reservation price in the model. Secure equilibrium arises because as the proportion of predators rises, their opportunity cost $\theta B$ rises proportionately while their success rate $\theta B = (1 + \theta B)$ rises less than proportionately. Thus even the low option predators cannot collectively provide a sufficiently high probability of success $1 - \beta$ to make predation pay; and so potential predation unravels and the only equilibrium is secure.

## 2.3. Case II: collusion

Here we assume that enforcers agree on a division of territory and limit numbers to maximize profits per member, a plausible form of coalition. The natural alternative to the coalition is a centralized monopoly that also dictates pricing/service policies in each location. The exact nature of the coalition is however inessential to our main purpose of comparing collusion with free entry private enforcement on the one hand and with state enforcement on the other hand.

Our choice is dictated by two considerations. First, we focus on the "weakest" possible form of collusion as this enables us to isolate the effect of coordination from more sophisticated strategies available to a stronger monopoly. Second, the coalition is closer to the observed organizational structure of private enforcement firms (see Gambetta, 1993; Milhaupt and West, 2000 for evidence on the Sicilian mafia and the Japanese yakuza), possibly because is more likely to be stable. Indeed, although detailed price directives could potentially lead to higher profits, they would also offer more opportunities to cheat. This effect is absent in the coalition case as in our symmetric model all enforcers choose to protect the same share of properties and hence receive the same price in equilibrium.

### 2.3.1. The equilibrium

Formally, the coalition optimizes profits per member. In selecting the optimal $n$, the coalition takes into account that each enforcer will choose to protect a share $\alpha^*$ of the properties in his area, where $\alpha^*$ is chosen so as to achieve Eq. (6). The free selection of $n$ may be constrained from below by considerations of coalition formation and entry prevention which we do not model here. The exact form of the profit-maximizing $n$ plays no significant role in our main results. For simplicity we assume that the coalition can form at no cost. Assuming linear coordination costs do not change the basic results, as discussed below.

Considering the selection of optimal $n$ as part of the per member profit maximization problem we have:

$$\max_n \frac{\alpha^*}{n} \left( \frac{1}{1 + \theta \dfrac{B\lambda}{nR}} - \frac{1}{1 + \theta \dfrac{B(1-\lambda)}{1 - \alpha^*}} \right) V^* - f.$$

In selecting $n$ we assume, as discussed above, that the coalition takes $\lambda$ and $B$ as given, meaning that the number of predators who attack property under his protection is given.

Given the setup, the first order condition for $n$ is:

$$\frac{\alpha}{n^2}(\beta - \pi^2) \leq 0. \tag{8}$$

The second-order condition for a maximum is always satisfied.[9] An interior maximum, $\beta = \pi^2$. Note that the first-order condition is the same if we allow for linear coordination costs.

The equilibrium system for determining all endogenous variables with a coalition of enforcers is formed by replacing the zero profit condition of free entry (Eq. (7)) with the first-order condition in $n$: again, we are mostly interested in the interior equilibrium. Then:

**Proposition 2.** *For (i) $V(0)/f$ sufficiently large and $V(1)/f$ sufficiently small, (ii) $V^L/V^H > 1/2$ and (iii) $w < (\theta(V^L - V^H)^2/(1-\alpha^*)V^L)$ a unique interior coalition solution exists.*

**Proof.** See the Appendix. □

Conditions (i) and (iii) guarantee that the free entry interior solution exists. Condition (ii) is required for $\pi < 1$. The conditions guarantee that the members' profits, evaluated at the optimal membership size, are positive—a necessary condition for the coalition to form.

Interior equilibrium is not feasible for all property value distributions. When the conditions above cannot be met, the full equilibrium will occur where the coalition size $n$ will be the lowest feasible value, either 1 or some lower bound $\underline{n}$ based on considerations we do not model.[10] The essential property of the model, verified below in the next Proposition, is that the coalition reduces $n$.

## 3. Free entry vs. coalition and other comparative statics

The aim of this section is twofold. First we compare the interior equilibrium under free entry to the interior equilibrium under collusion. We show that the comparison yields interesting implications for the distribution of property income net of predation. Second, we discuss the effect of development and of changes in other exogenous parameters on inequality.

### 3.1. Free entry vs. coalition

Collusion restricts the level of enforcement, as we now verify. This intuitive property holds whether or not the level of $n$ reaches a lower bound. Formally:

**Proposition 3.** *The optimal $n$ chosen by the coalition, if it exists, $n^m$, is lower than the one that would result under free entry. Property is less secure and the price of private enforcement is higher than under free entry.*

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[9] The second derivative is $-2\pi^2(1 - \pi/n) < 0$.

[10] As an example of a corner solution $n$; an interior solution defined by Eq. (8) is impossible when the $V$ function is linear because $V^L/V^H = 1/2$. In contrast, if the property value schedule is a step function with $V^L/V^H > 1/2$, or if $V = V^0 - \alpha^\varepsilon$, $0 \leq V^0 - 1 < \varepsilon < 1$; then the interior equilibrium is feasible.

**Proof *(Sketch)*.** To compare the equilibria we solve the sub-system which constrains both forms of organization for given **n**:

$$\pi^* = \frac{1}{1 + \theta B \dfrac{\lambda/n}{R}}$$

$$\beta^* = \frac{1}{1 + \theta B \dfrac{1 - \lambda}{1 - \alpha}}$$

$$[1 - \beta]V^{\mathrm{L}} = [1 - \pi]V^{\mathrm{H}}$$

$$B = \frac{(1 - \beta)V^{\mathrm{L}}}{w}.$$

This yields $\boldsymbol{\pi^*}$ **(n)**, $\boldsymbol{\beta^*}$ **(n)**, $\boldsymbol{\lambda^*}$ **(n)**, $\boldsymbol{B^*}$ **(n)**. The Appendix shows that profits are monotonically decreasing in **n**. This implies that the equilibrium number of enforcers must be smaller under the coalition than under free entry.[11]                                           □

Security of property suffers from the organization of the coalition the price of enforcement will be higher, there will be more predators and the share of predators that choose to attack protected property will be higher.[12] The next corollary shows that enforcement market structure affects the distribution of property income as well.

**Corollary 1.** *When enforcers form a coalition the distribution of property income becomes more unequal as the reduction in security is larger for low value properties.*

**Proof.** Proposition 3 implies that both $\pi$ and $\beta$ fall when enforcers collude yet $\beta$ has to fall more than $\pi$ in order to keep the allocation constraint (Eq. (3)) satisfied. Since the share of protected property is the same under free entry and under the coalition this implies that the property income of the poorer owners (i.e. those with $\alpha > \alpha^*$) falls relative to the income of the richer owners ($\alpha \leq \alpha^*$).                                           □

The intuition behind the corollary is that since self-defense is the alternative option to purchasing specialized enforcement, owners are willing to pay a higher price for enforcement if and only if its relative value increases. For this to happen, the fall in security on unprotected properties must be larger than the corresponding reduction on protected properties.

Another key difference between free entry and collusion is that under the former changes in the exogenous parameters of the model affect property security whereas under the latter they do not. Collusive enforcers are able to charge higher prices by restricting

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[11] Note also that the result does not depend on the condition that determines $n$ under collusion, therefore the coalition restricts $n$ regardless of whether it internalizes $\lambda$ and $B$.

[12] That is, $(\partial\pi^*(n)/\partial n) > 0$, $(\partial\beta^*(n)/\partial n) > 0$, $(\partial(\pi^*/(n) - \beta^*(n))/\partial n) < 0$, $(\partial B^*(n)/\partial n) < 0$, $(\partial\lambda^*(n)/\partial n) > 0$. Offsetting this disadvantage of monopoly, if uncoordinated enforcers have higher costs of obtaining a reputation $R$, the coalition equilibrium can be more secure than the free entry equilibrium.

entry while under free entry higher profits attract more enforcers, which improves security. Formally:

**Corollary 2.** *In the free entry case, improvements in the outside option of predators, in the effectiveness and cost of the enforcement technology improve security on both protected and unprotected properties. In the collusion case, security is unaffected as the coalition adjusts its membership size to capture all surplus.*

In the Appendix we report the full comparative statics for both the free entry and the coalition equilibria. We note that, however, when $n$ is constrained at its lower bound, increases in $w$, $R$ and decreases in $\theta$ will raise security just as they do in the free entry case. The contrast is because the coalition cannot adjust membership to capture the change in surplus.

### 3.2. The effect of development on security and inequality

We interpret 'equalizing' economic development in the context of our model as a rise in $w$, the parameter which shifts upward the distribution of alternative options of potential predators. This interpretation corresponds to a rise in the outside opportunities of labor relative to the return on property where changes in property returns are interpreted as a multiplicative shift factor $v$ in the property value function. This approximates a standard idea of development in a neoclassical growth model, the return to labor rises relative to returns on forms of capital.[13] Development paths which decrease the wage relative to the return on property, trickle-down paths, have opposite implications for distribution.

Propositions 1 and 2 imply that, other things equal, asymptotically as the wage rises relative to the return on property, private enforcement disappears and a self enforced equilibrium with no predation emerges. At the interior equilibrium, the effect of development depends on the market structure for enforcement as specified below.

**Corollary 3.** *In the case of free entry development paths which raise the relative wage improve property security and decrease inequality among owners as poorer properties become relatively more secure. If enforcers form a coalition development increases the coalition's payoff and has no effect on security in the interior equilibrium.*

If there is free entry, an increase in $w$ implies that there will be fewer predators ($B$ decreases) but also fewer enforcers ($n$ decreases) in equilibrium. The intuition is that since there are fewer predators, both $\pi$ and $\beta$ increase, yet $\beta$ has to increase more than $\pi$ in order to maintain the allocation constraint (Eq. (3)). It follows that, given $\alpha^*$, the price of enforcement is lower, which makes some enforcers exit the market. The *Ceteris paribus* derivatives with respect to $w$ apply even when $v$, $f$ are increasing, since the interior equilibrium solution is homogeneous of degree zero in $f$, $v$, $w$.[14] Thus if development

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[13] Our assumption of a uniform distribution of returns to productive labor implies that the schedule of outside opportunities shifts upward while preserving uniformity.

[14] This follows immediately by inspecting the equilibrium system. The first-order condition for setting $\alpha$ is invariant to $v$, as is the predator allocation condition. The zero profit condition and the predator entry condition are respectively linear in $f$, $v$ and in $v$, $w$. The objective probabilities are invariant to $f$, $v$, $w$.

entails an increase in the relative wage, the demand for specialized enforcement falls and security improves as long as there is free entry.

If enforcers collude, an increase in $w$ reduces the number of predators as in the free entry case, but leaves security unchanged, unlike the free entry case, because the coalition adjusts its membership size to capture all surplus. In particular the number of enforcers falls and, as shown below, profits per member increase.

### 3.3. Related issues

While we compare the properties of the equilibrium under free entry and under collusion, we do not formally endogenize the transition from one market form to the other. Nevertheless some insights on the forces that might favor the formation of the coalition can be obtained by looking at its profits under the reasonable assumption that the coalition is more likely to form when it is more profitable to do so.

The profits of each coalition member are equal to:

$$P* = \frac{\alpha^* R w}{V^{\mathrm{L}}} \frac{(2V^{\mathrm{L}} - V^{\mathrm{H}})(V^{\mathrm{H}} - V^{\mathrm{L}})^2}{V^{\mathrm{H}}(\theta(V^{\mathrm{H}} - V^{\mathrm{L}})^2 - (1 - \alpha^*)wV^{\mathrm{L}})} - f$$

Profits per member are thus increasing in $R$ (the strength of the enforcers), decreasing in $\theta$ (the strength of the bandits) and increasing in $w$ (the outside option of bandits). The comparative statics with respect to $w$ provide a channel through which development can shift the organization of enforcement. If development entails an improvement in the outside opportunity of bandits, coalition profits increase with development. But for sufficiently high $w$, self enforcement takes over for all property and specialized enforcement withers away, suggesting that, overall, private enforcement coalitions are more likely to form at intermediate stages of development.

The existence of the coalition raises two further issues that we do not analyze here due to their marginality with respect to the central focus of the paper. First, it could be argued that the enforcers might engage in extortion in order to enhance their protection business, especially when they collude in a coalition. We address this issue in a working paper (Anderson and Bandiera, 2001) where we show that the coalition optimally sponsors predatory attacks on unprotected properties if it is able to do so. This however merely reinforces the results of distributional analysis as it gives the coalition one more instrument for extracting rent from the property owners.

Second, we do not analyze the process of coalition formation and its stability. It could be argued that the coalition can form only if all existing enforcers agree. In that case the relevant objective function is total profit because, in the formation of the coalition, some of the original number of enforcers must be retired and compensated with a share of the profits. Moreover, the coalition may face the need to increase $n$ sufficiently to prevent entry by ensuring that potential entrants cannot cover their costs. Both cases are analyzed in the working paper.

## 4. Socially efficient enforcement

Private enforcement equilibrium is usefully contrasted to the enforcement provided by a State that maximizes the utilitarian social welfare of its citizens. The most interesting result of this contrast is that in our model private enforcement may approximate optimal enforcement by the State closely enough to rationalize tolerance of private enforcement in intermediate parameter ranges.

Social efficiency trades off two distortions; the local monopoly distortion which pushes toward equilibrium where too few enforcers protect too small a share of property on the one hand and the uncorrected negative externality which pushes toward equilibrium where too many enforcers protect too large a share of property on the other hand. For low capability private enforcement, the analysis reveals that it is socially efficient to increase the proportion of protected property while for high capability private enforcement it is socially efficient to reduce the proportion. Despite the fact that high capability equilibrium has lower absolute value of the negative externality, it also has lower value of the monopoly distortion and the latter effect predominates. Free entry into private enforcement produces socially excessive intensity of enforcement while coalition may restrain entry excessively.

These results in combination suggest that coalitions of intermediate capability may approximate efficient enforcement fairly closely. The added effects of ignorance and political economy may nullify smallish gains from efficient state policy, while the rent extraction activity of public sector unions may make achieving those gains more costly than private enforcement resources. In contrast, very high or very low levels of capability, or competitive enforcement, may rationalize state replacement of private enforcement.

In what follows we use two specifications for the State's social welfare function. First we assume that the State cares about property owners only (both rich and poor), and then we allow the State to care about the welfare of predators as well.

### 4.1. Owners' welfare

We model a state with a utilitarian social welfare function that weights all owners' welfare equally as a clean special case which introduces considerations which are generally relevant.[15] Public enforcement in many states will presumably be at least somewhat responsive to the interests of low value property owners who cannot efficiently be afforded private protection; the utilitarian social welfare function is a convenient way to represent this.

To keep the comparison clean we maintain that state and private enforcers have equal capability and that, as private enforcers, the utilitarian state takes the number of predators as given. In reality, of course, private enforcement might be more effective if it can use illegal punishment methods, state enforcement might have better technology and states may powerful enough to acquire sophistication in playing the enforcement game. Incorporating such complications is straightforward and they have easily anticipated effects.

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[15] Kleptocratic states maximize profits, like private enforcers. While this extreme behavior apparently does characterize some states, many states appear more responsive to the welfare of their citizens. A convex combination of kleptocratic and utilitarian behavior should characterize most states.

We further simplify the analysis by assuming that the state can collect lump sum taxes from some unmodeled sector to pay for the balance of the cost of enforcement over property tax collection, or alternatively return to that sector the surplus collected from property protection. The purpose of this simplifying assumption is to isolate consideration of efficiency in enforcement from well-understood considerations of inefficient tax structure.

The welfare function of the state when it only takes into account property owners' welfare is the expected value of property less the cost of defense:

$$W(n, \alpha; B, \lambda) \equiv \pi(n, B, \lambda)S(\alpha) + \beta(\alpha, B, \lambda)D(\alpha) - fn.$$

$S(\alpha)$ and $D(\alpha)$ are the surpluses associated with protected and unprotected property respectively:[16]

$$S(\alpha) \equiv \int_0^\alpha V(x)\mathrm{d}x - \alpha V(\alpha), \;\; D(\alpha) \equiv \int_0^1 V(x)\mathrm{d}x - V(1) - S(\alpha).$$

The objective probability functions $\pi(\cdot)$ and $\beta(\cdot)$ are the same as in our earlier analysis.

We calculate the welfare improving direction of change in the share of protected property at the level chosen by the private enforcers, $\alpha^*$, the same in either free entry or coalition equilibrium: also, for given $\alpha$, we calculate the welfare improving direction of change in the number of enforcers $n$, a measure of the *intensity* of protection, evaluated at the number of enforcers in the free entry and coalition cases.[17]

Welfare changes with $\alpha$ and $n$ according to:

$$W_\alpha = (\pi - \beta)(-\alpha V_\alpha) + D\beta_\alpha$$

$$W_n = \pi_n S - f. \tag{9}$$

The private enforcement equilibrium value of $\alpha$ is implied by the first-order condition:

$$(\pi - \beta)(V + \alpha V_\alpha)/n = 0. \tag{10}$$

Equilibrium $\alpha$ is invariant to $n$. The number of enforcers $n$ under free entry is given by the zero profit condition

$$(\pi - \beta)\alpha V/n = f.$$

When there is a coalition interior solution, the private selection of $n$ is defined by

$$(\beta - \pi^2)\alpha V/n^2 = 0.$$

When the number of enforcers is bounded, $\pi^2 \geq \beta$ and $n^m$ is at its lower bound.

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[16] The welfare function is formed as follows. The total expected surplus enjoyed by the protected is $\pi \int_0^\alpha V(x)\mathrm{d}x$. Subtract from this their tax payment $(\pi - \beta)V\alpha$ to form their expected net surplus. The expected surplus enjoyed by the unprotected is $\beta[\int_\alpha^1 V(x)\mathrm{d}x - V(1)]$. Adding together all terms produces the expression below.

[17] The socially optimal level of enforcement may of course be very far from the local changes we analyze here. Since the welfare function set out here is not globally concave or convex, there may be no interior maximum. For example, at $\alpha = 0$; welfare is locally decreasing and convex in $\alpha$, and this may be the global maximum. A full analysis of optimal state enforcement requires a more realistic and complete treatment of the cost of enforcement, which should act to make the welfare function concave and produce well behaved interior maxima.

Regarding the social desirability of the privately protected proportion of property we find that:

**Proposition 4.** *(a) The utilitarian state has an incentive to defend a smaller proportion of property ($\alpha$) than would private enforcers (under either free entry or coalition) if the parameters are such that $\beta > 1/2$. (b) A necessary condition for the utilitarian state to defend a larger proportion of property than private enforcers (under either free entry or coalition) is $\beta \leq 1/2$.*

**Proof.** See Appendix.                                                                                    □

The economic intuition of Proposition 4 trades off the negative externality which enforcement inflicts on unprotected property and the monopoly restriction of enforcement. For parameters such that in equilibrium $\pi > \beta > 1/2$, the negative externality outweighs the monopoly distortion.[18] For parameters such that $\beta < 1/2$ it is possible that the monopoly distortion will be more important than the negative externality. In the Appendix we report definite results that obtain with a linear (uniform) distribution of property values $V(\alpha) = V^0 (1 - \alpha)$.

Next, we consider the utilitarian state's desired direction of change in the number of enforcers ($n$):

**Proposition 5.** *Other things equal, the utilitarian state has an incentive to: (a) hire less enforcers compared to free entry and (b) hire less (more) enforcers than would a weak (strong) coalition.*

**Proof.** See Appendix.                                                                                    □

Monopolistic competition theory has no general results on under or over-provision of service, but in this case strong results obtain in free entry equilibrium. Intuitively, an added enforcer at cost $f$ raises social welfare according to $\pi_n S$ while free entry equilibrium spends cost $f$ to secure private gain $(\pi - \beta)V\alpha^*$. The latter exceeds the former. Part (b) offsets the tendency of part (a) because coalition restrains entry.[19]

Finally, note that the free entry, the collusive and the socially optimal levels of efficient protection and predation are invariant to neutral economic development defined as equiproportionate rises in $f$, $w$, $v$, provided the change in $f$ does not shift collusive

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[18] If the coalition solution lies in the interior, this requires $[(V^H - V^L)/V^H]^2 > 1/2$, which requires appropriate restrictions on the shape of $V$, the distribution of property values. For the linear case, the solution must be at a constrained value of $n$. The solution value of $\beta$ for constant $n$ is increasing in $R$, so a sufficiently large protective capability when $V$ is linear (there is a uniform distribution of property values) satisfies the sufficient condition for $W_\alpha < 0$.

[19] If the state were "sophisticated", i.e. if it were able to internalize $B$ and $\lambda$, it would be more likely to protect a greater proportion of property more intensively (i.e. choose higher $\alpha$ and higher $n$) than either form of private enforcement. This result is because the general equilibrium effects of enforcement on driving out predators tend to dominate—greater enforcement effort lowers the return on predation even on unprotected property due to the reallocation of predators, while protecting a greater proportion of property can also reduce the level of predation. The result is however sensitive to the elasticity of predation with respect to enforcement. Simulations, available from the authors upon request, show that there are conditions under which a sophisticated state would also reduce $\alpha$ and $n$.

equilibrium between interior and boundary solutions. The incentive for the state to alter private levels of protection also remains constant. In contrast, a rise in $w$ relative to $f$ and $v$ will raise security (both $\pi$ and $\beta$) thus potentially changing the State's optimal policy relative to private enforcement.

### 4.2. Predators' welfare

The utilitarian state might care about predators as well as owners. On the margin, increases in predation transfer rents to a poor part of the population whose opportunity cost in outside activity is low. Predation can then be seen as a form of redistribution, which might be tolerated when more efficient alternatives are not available. We show that the utilitarian objective function of the state has similar qualitative properties regardless of whether the welfare of predators is included or not. Incorporating the welfare of predators into the evaluation of social efficiency generally reduces both the socially efficient level of protection offered and the proportion of property protected.

Predators with inframarginal opportunity cost receive rents in predation. The rent for any individual predator is the difference between the equilibrium return (equal to the alternative option of the *marginal* predator) and the alternative option of the individual. Under the uniform distribution (linear predator supply) assumption of the model, the total rent to predators, $\rho$, is given by 1/2 times the product of the equilibrium return and the mass of predators $B$, or $\rho = [(1 - \beta)V^L]B/2$.

A utilitarian state which cares as much about the predators as it does its property-owning agents will base considerations of optimal policy at the margin on $W_\alpha + \rho_\alpha$ and $W_n + \rho_n$.[20] In calculating $\rho_\alpha$, we assume that the state uses $(1 - \beta)V^L$ as the equilibrium return and exploits its forecast of $\beta_\alpha$ and $V_\alpha^L$. The state is assumed to take the mass of predators as given in making its welfare evaluations.[21]

Then from the state's point of view, the effect of changes in $\alpha$ and $n$ on predators' welfare is given by

$$\rho_\alpha = \left[ - V^L \beta_\alpha + (1 - \beta)V_\alpha^L \right]B/2$$

$$\rho_n = \left[ - V^L \beta_n \right]B/2.$$

The first equation shows that as the share of protected property increases, predators' welfare might increase or decrease. On the one hand, raising the proportion of property protected lowers the return to predation for given success rates; on the other, however,

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[20] The utilitarian state is plausibly assumed not to care about rents earned by a coalition of enforcers.

[21] The most plausible forecast of $\rho_\alpha$ is rather tricky to specify. A fully sophisticated state solves the full general equilibrium model, but this is rather implausible and is inconsistent with the treatment of the state above. The assumption in the text implies that in forming $\rho_\alpha = - \beta_\alpha V^L + (1 - \beta)V_\alpha^L$ the state makes use of its forecast of $\beta_\alpha$, and its knowledge of $V(\alpha)$, just as it does elsewhere. Since the state might be expected to know the equilibrium condition for allocating predators between protected and unprotected property, $(1 - \beta)V^L = (1 - \pi)V^H$, the assumptions imply that the state must be residually assigning a value to the change in $(1 - \pi)V^H$ and given its knowledge of $V(\alpha)$; a residual value to $d\pi = d\alpha$.

increasing $\alpha$ also makes unprotected property less safe and thereby raises the return to predation.[22] The second equation shows that predators' welfare is unambiguously decreasing in $n$, the number of enforcers. Intuitively, more enforcers make all properties safer and hence lower the return to predation.[23]

Overall, including predators' welfare makes the state more likely to want to reduce enforcement intensity by reducing the number of active enforcers. Generally, including predators' welfare also makes the state want to reduce the share of protected property $\alpha$. The Appendix reports the results for the linear case. The key message is that no significant qualitative difference emerges due to including the welfare of predators in the analysis.

## 5. Discussion and conclusions

Our analysis indicates that private protection of property rights has important consequences both for efficiency and the distribution of property income. Private protection purchased by richer owners deflects predators into low value unprotected properties, thus making the distribution of property income net of predation more unequal. This externality is key to understanding how the market structure of private enforcement and the development process affect the relative security of rich and poor properties.

The externality also drives the comparison between privately provided enforcement and its socially optimal level. Welfare maximizing state policy trades off the distortion arising from the private enforcers' monopoly power on the one hand and the distortion arising from the negative externality on the other hand. The results indicate that a utilitarian state facing highly capable private enforcement has an incentive to reduce the proportion of protected property while a state facing low capability private enforcement has an incentive to increase the proportion. For an intermediate range of parameter values, there is little net benefit to the state in regulating or replacing the private enforcement equilibrium proportion.

These incentives may offer insights into why states often tolerate private enforcement—the benefits of change are small when the capability of private enforcers is intermediate.

The welfare analysis also uncovers an potential difficulty with efficient state enforcement policy and casts a shadow on the effect of private governance on the development process. Private institutions that provide property enforcement may be beneficial when state protection is absent or ineffective. But as the state grows more capable, the potentially excessive private enforcement of high value property may make a shift to public enforcement desirable.

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[22] A clear result $\rho_\alpha(\alpha^c)<0$ obtains in the case of linear distribution of property values $= V^0(1-\alpha)$. Under linearity $\rho_\alpha = -(1-\beta)^2 V^0 B/4$ using $\beta_\alpha = -\beta(1-\beta)/(1-\alpha)$, $L=V^0(1-\alpha)/2$ and $\alpha^c=1/2$: For distributions of property values which raise $V^L(\alpha^c)$ relative to its linear case value, it is possible that increases in $\alpha$ actually increase rents to predators.

[23] Using the equilibrium allocation of predators between protected and unprotected property yields $\beta=1-(1-\pi)V^H/V^L$, hence $\beta_n=(V^H/V^L)\pi(1-\pi)/n>0$ using $\pi_n=\pi(1-\pi)/n$.

A utilitarian government in these circumstances may be unable to implement an efficient policy because the high value property owners prefer the greater enforcement potentially offered by the private sector. Of course, the state may be strong enough to prevent private enforcement from cream skimming its customers, but a welfarist state which defies the wishes of all its wealthier citizens by preventing private enforcement appears implausible.[24] The distributional conflict between high and low value property owners thus provide a useful insight into competition between the state and the private enforcers, suggesting elements of a theory of private vs. state enforcement.

## Acknowledgements

## Appendix A

### A.1. Solution and comparative statics-free entry

#### A.1.1. Solution

System :

$$\pi^* = \frac{1}{1 + \theta B \dfrac{\lambda/n}{R}}$$

$$\beta^* = \frac{1}{1 + \theta B \dfrac{1-\lambda}{1-\alpha}}$$

$$[1 - \beta]V^L = [1 - \pi]V^H$$

$$\pi - \beta = \frac{nf}{V^*\alpha^*}$$

$$B = \frac{(1-\beta)V^L}{w}$$

Solution :

$$\pi^* = \frac{V^L}{V^H}\beta^* + \frac{V^H - V^L}{V^L}$$

$$\beta^*$$

$$\lambda^* = 1 - \frac{w(1-\alpha^*)}{\theta V^L \beta^*}.$$

$$B^* = (1 - \beta^*)\frac{V^L}{w}$$

$$n^* = V^*\alpha^* \frac{V^H - V^L(1 - \beta^*)}{f V^H}$$

Where $\beta^*$ is a root of $\alpha\beta^{*2} + b\beta^* + c = 0$ where,

$a = (fV^H (V^L)^2\theta + RV^*\alpha^*wV^L (V^H - V^L));$
$b = (+fV^HV^L\theta (V^H - V^L) - fV^HV^Lw (1 - \alpha) - RV^*\alpha^*wV^L (V^H - V^L))$
    and $c = -fV^Hw (1 - \alpha) (V^H - V^L)$

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[24] As an example of tolerated private enforcement, think of the growth of gated communities containing people who shelter their incomes in offshore tax havens. The deflected predators batten onto the low value state protected or unprotected property with greater intensity, increasing the incentive to defect from state enforcement.

The polynomial has two roots: $\rho_1 = \frac{-b - \sqrt[2]{b^2 - 4ac}}{2a}$ and $\rho_2 = \frac{-b + \sqrt[2]{b^2 - 4ac}}{2a}$.

### A.1.2. Existence and uniqueness

An interior solution exist if $\rho_1, \rho_2$ are real and if at least one of them is between 0 and 1.

1. $(V^H - V^L) > 0 \Rightarrow c < 0 \Rightarrow b^2 - 4ac > 0 \rightarrow \rho_1, \rho_2$ are real
2. We show that the smallest root of the quadratic expression above is always negative, thus if a solution exists it is unique:

$$\frac{-b - \sqrt[2]{b^2 - 4ac}}{2a} < 0 \ \ if \ \ -b < \sqrt[2]{b^2 - 4ac}$$

   If $b > 0$ this is always true.
   If $b < 0 : 4ac < 0 \Rightarrow b^2 < b^2 - 4ac$ always.
3. An interior solution exists if the largest root lies between 0 and 1: $\frac{-b + \sqrt[2]{b^2 - 4ac}}{2a} > 0$ if $\sqrt{b^2 - 4ac} > b$.
   If $b < 0$ it is always true.
   If $b > 0$ then $4ac < 0 \Rightarrow b^2 - 4ac > b^2$ always. $\frac{-b + \sqrt[2]{b^2 - 4ac}}{2a} < 1$ if $\sqrt{b^2 - 4ac} < b + 2a$.
   Taking squares and rearranging we get: $a + b + c > 0$; which is satisfied iff $w < (\theta V^L / (1 - \alpha^*)).$[25]
   Thus $\beta^* = \frac{-b + \sqrt[2]{b^2 - 4ac}}{2a}$.
4. If $w > \theta V^L / (1 - \alpha^*)$, then the solution is $\alpha^* = 0 = \lambda$, which means $V^L = E[V]$.

Then the model reduces to the solution to $\beta = 1/(1 + \theta B)$ and $(1 - \beta) E[V] = wB$. Substituting the solution for $B$ into the objective probability equation yields the quadratic expression

$$-x\beta^2 + (1 + x)\beta - 1 = 0, x \equiv \theta E[V]/w.$$

The roots are $\beta = 1$ and $\beta \equiv 1/x > 1$ given the parameter values of this case; so the only relevant solution is a secure equilibrium.

### A.1.3. Comparative statics

Note that:

$$\frac{\partial \beta}{\partial X} = \frac{-\left(\frac{\partial a}{\partial X}\beta^2 + \frac{\partial b}{\partial X}\beta + \frac{\partial c}{\partial X}\right)}{2a\beta + b} \ \ \text{where } X = R, w, \theta, f$$

Note also that $2\alpha\beta + b = \sqrt[2]{b^2 - 4ac} > 0$

By straightforward (and tedious) computation we can show that:

$$\frac{\partial \beta}{\partial R} = \frac{\beta(1 - \beta)V^*\alpha^* wV^L(V^H - V^L)}{2a\beta + b} > 0;$$

$$\frac{\partial \beta}{\partial w} = \frac{\beta(1 - \beta)V^*\alpha^* RV^L(V^H - V^L) + \beta f V^H V^L(1 - \alpha) + f V^H(1 - \alpha)(V^H - V^L)}{2a\beta + b} > 0$$

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[25] $w < (\theta \overline{V}^L / (1 - \alpha^*))$ is also sufficient to guarantee that $b + 2a > 0$.

$$\frac{\partial}{\partial \theta} = \frac{-(\beta^2 f V^{\mathrm{H}}(V^{\mathrm{L}})^2 + \beta f V^{\mathrm{H}} V^{\mathrm{L}}(V^{\mathrm{H}} - V^{\mathrm{L}}))}{2a\beta + b} < 0$$

$$\frac{\partial \beta}{\partial f} = -\frac{1}{f} \frac{\beta(1-\beta)V^*\alpha^* R V^{\mathrm{L}}(V^{\mathrm{H}} - V^{\mathrm{L}})}{2a\beta + b} < 0$$

which implies that:

$$\frac{\partial \pi}{\partial R} = \frac{V^{\mathrm{L}}}{V^{\mathrm{H}}} \frac{\partial \beta}{\partial R} > 0; \quad \frac{\partial \pi}{\partial w} = \frac{V^{\mathrm{L}}}{V^{\mathrm{H}}} \frac{\partial \beta}{\partial w} > 0; \quad \frac{\partial \pi}{\partial \theta} = \frac{V^{\mathrm{L}}}{V^{\mathrm{H}}} \frac{\partial \beta}{\partial \theta} < 0; \quad \frac{\partial \pi}{\partial f} = \frac{V^{\mathrm{L}}}{V^{\mathrm{H}}} \frac{\partial \beta}{\partial f} < 0;$$

and:

$$\frac{\partial B}{\partial R} = -\frac{V^{\mathrm{L}}}{w} \frac{\partial \beta}{\partial R} < 0; \quad \frac{\partial B}{\partial \theta} = -\frac{V^{\mathrm{L}}}{w} \frac{\partial \beta}{\partial \theta} > 0; \quad \frac{\partial B}{\partial f} = -\frac{V^{\mathrm{L}}}{w} \frac{\partial \beta}{\partial f} > 0;$$

$$\frac{\partial B}{\partial w} = -\frac{V^{\mathrm{L}}}{w} \frac{\partial \beta}{\partial w} - (1 - \beta^*) \frac{V^{\mathrm{L}}}{w^2} < 0;$$

and:

$$\frac{\partial \lambda}{\partial R} = \frac{w(1-\alpha^*)}{\theta V^{\mathrm{L}} \beta^2} \frac{\partial \beta}{\partial R} > 0; \quad \frac{\partial \lambda}{\partial f} = \frac{w(1-\alpha^*)}{\theta V^{\mathrm{L}} \beta^2} \frac{\partial \beta}{\partial f} < 0$$

and finally that:

$$\frac{\partial n}{\partial R} = -\frac{V^*\alpha^*(V^{\mathrm{H}} - V^{\mathrm{L}})}{f V^{\mathrm{H}}} \frac{\partial \beta}{\partial R} < 0; \quad \frac{\partial n}{\partial \theta} = -\frac{V^*\alpha^*(V^{\mathrm{H}} - V^{\mathrm{L}})}{f V^{\mathrm{H}}} \frac{\partial \beta}{\partial \theta} > 0;$$

$$\frac{\partial n}{\partial w} = -\frac{V^*\alpha^*(V^{\mathrm{H}} - V^{\mathrm{L}})}{f V^{\mathrm{H}}} \frac{\partial \beta}{\partial w} > 0.$$

## A.2. Solution and comparative statics-coalition case

System :

$$\pi^* = \frac{1}{1 + \theta B \dfrac{\lambda/n}{R}}$$

$$\beta^* = \frac{1}{1 + \theta B \dfrac{1-\lambda}{1-\alpha}}$$

$$[1 - \beta]V^{\mathrm{L}} = [1 - \pi]V^{\mathrm{H}}$$

$$\beta^* - \pi^{*2} = 0$$

$$B = \frac{(1-\beta)V^{\mathrm{L}}}{w}$$

Solution :

$$\pi^* = \frac{(V^{\mathrm{H}} - V^{\mathrm{L}})}{V^{\mathrm{L}}}$$

$$\beta^* = \frac{(V^{\mathrm{H}} - V^{\mathrm{L}})^2}{(V^{\mathrm{L}})^2}$$

$$\lambda^* = \frac{\theta(V^{\mathrm{H}} - V^{\mathrm{L}})^2 - (1-\alpha^*)wV^{\mathrm{L}}}{\theta(V^{\mathrm{H}} - V^{\mathrm{L}})^2}$$

$$n^* = V^{\mathrm{H}} \frac{\theta(V^{\mathrm{H}} - V^{\mathrm{L}})^2 - (1-\alpha^*)wV^{\mathrm{L}}}{RwV^{\mathrm{L}}(V^{\mathrm{H}} - V^{\mathrm{L}})}$$

$$B^* = \frac{(1 - \beta^*)V^{\mathrm{L}}}{w}$$

### A.2.1. Existence and uniqueness

An interior equilibrium exists when $n>0$, $B>0$, $0<\lambda<1$. That is when:

(i) The range of property value is sufficiently narrow $V^L/V^H>1/2$;
(ii) the outside opportunity of predators is not too high i.e. $w<\frac{\theta(V^H-V^L)^2}{(1-\alpha^*)V^L}$ and
(iii) $V(0)/f$ is sufficiently large and $V(1)/f$ sufficiently small.

Condition (i) is required for $\pi<1$. When it is not met, as it necessarily cannot be in the linear case, then provided that conditions (ii)–(iii) are met, the solution is $n=1$, or more generally the optimal number of firms is driven to a minimum dictated by entry and coalition formation considerations which we do not model. The other conditions guarantee the existence of a unique interior free entry solution, by Proposition 1. In the interior equilibrium individual profits must be positive, that is:

$$\frac{\alpha^* R w}{V^L} \frac{(2V^L - V^H)(V^H - V^L)^2}{V^H(\theta(V^H - V^L)^2 - (1 - \alpha^*)wV^L)} - f > 0$$

Note that (i) and (ii) are sufficient for the first term to be positive.

Note also that, intuitively, profits are increasing in $R$ and decreasing in $\theta$. Profits are also increasing in $w$; indeed:

$$\frac{\partial}{\partial w}\left(\frac{\alpha^* R w}{V^L} \frac{(2V^L - V^H)(V^H - V^L)^2}{V^H(\theta(V^H - V^L)^2 - (1 - \alpha^*)wV^L)} - f\right)$$

$$= \frac{\alpha^* R \theta(2V^L - V^H)(V^H - V^L)^4}{V^L V^H(\theta(V^H - V^L)^2 - (1 - \alpha^*)wV^L)^2} > 0.$$

Thus an interior equilibrium with positive profits exists for $(f, \theta)$ low enough and $(w, R)$ high enough.

### A.2.2. Comparative statics

Note that $\pi^*$, $\beta^*$ only depend on property values. Other comparative statics as follows:

(1) $(\partial n^*/\partial w)<0$, $(\partial n^*/\partial \theta)>0$, $(\partial n^*/\partial R)<0$, in particular $(\partial n^*/\partial w)= -\theta V^H((V^H - V^L)/V^L w^2 R)$
(2) $(\partial B^*/\partial w)<0$, $(\partial B^*/\partial \theta)=0$, $(\partial B^*/\partial R)=0$;
(3) $(\partial \lambda^*/\partial w)<0$, $(\partial \lambda^*/\partial \theta)>0$, $(\partial \lambda^*/\partial R)=0$; in particular $(\partial \lambda^*/\partial \theta)=((1 - \alpha^*)wV^L/\theta^2(V^H - V^L)^2)$

### A.3. Coalition vs. free entry

This section compares the outcomes in the cases of free entry and coalition. Since the two cases differ only for the equation that determines $n$, instead of comparing

parameter values directly we solve the system as a function of $n$ and analyze how parameters change with $n$.

$$\pi = \frac{1}{1 + \theta B \dfrac{\lambda/n}{R}}$$

$$\beta = \frac{1}{1 + \theta B \dfrac{1 - \lambda}{1 - \alpha}}$$

$$[1 - \beta]V^{\mathrm{L}} = [1 - \pi]V^{\mathrm{H}}$$

$$B = \frac{(1 - \beta)V^{\mathrm{L}}}{w}$$

the interior solution is:

$$\beta^* = \frac{w(1 - \alpha)}{\theta V^{\mathrm{L}}(1 - \lambda^*)}; \ \pi^* = \frac{V^{\mathrm{L}}}{V^{\mathrm{H}}}\beta^* + \frac{V^{\mathrm{H}} - V^{\mathrm{L}}}{V^{\mathrm{H}}}; \ B = \frac{V^{\mathrm{L}}}{w} - \frac{(1 - \alpha)}{\theta(1 - \lambda^*)} \ \text{and}$$

$$\lambda^* = \frac{-b - \sqrt[2]{b^2 - 4ac}}{2a}$$

where:[26]

$a = \theta \ (V^{\mathrm{H}} - V^{\mathrm{L}})$
$b = - \ (\theta \ (V^{\mathrm{H}} - V^{\mathrm{L}}) + w \ (1 - \alpha) + Rnw)$
$c = Rnw$

Total profits are equal to: $P(n) = (V^{\mathrm{H}} - V^{\mathrm{L}}/V^{\mathrm{H}})(1 - \beta^* \ (n)) \ V^*\alpha^* - fn$.
We can show that:

1. If a coalition is formed there will be fewer enforcers. Note that $\frac{\partial \beta^*}{\partial \lambda^*} > 0$ and that $\frac{\partial \lambda^*}{\partial n} = \frac{Rw(1 - \lambda^*)}{\sqrt[2]{b^2 - 4ac}} > 0$, from which it follows that $\frac{\partial P}{\partial n} < 0$. Since profits must be positive if there is a coalition and zero under free entry it follows that, given $\frac{\partial P}{\partial n} < 0$ there must be more enforcers in the monopolistically competitive case.
2. If a coalition is formed every property will be less secure, there will be more bandits and the share of bandits attacking protected property will be higher.

Note that $\frac{\partial \beta^*}{\partial \lambda} > 0, \frac{\partial \pi^*}{\partial \lambda} > 0, \frac{\partial B^*}{\partial \lambda} < 0$. The result follows from $\frac{\partial \lambda^*}{\partial n} > 0$ and the fact that $n$ is smaller when a coalition is formed.

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[26] It is easy to show that the other solution is always larger than 1.

## A.4. Comparative statics with fixed n

The preceding subsection gives the solution for fixed $n$: the qualitative comparative static properties resemble the free entry case. Note that security rises with $w$: $\partial\beta^*/\partial w = (\partial\beta^*/\partial\lambda^*)\partial\lambda^*/\partial w > 0, \partial\pi^*/\partial w > 0, \partial B^*/w < 0$.

Moreover, $\partial\lambda^*/\partial\alpha^* > 0$, $\partial B^*/\partial\alpha^* = (V^L/w - B^*)[1/(1-\alpha) - \lambda^*_{\alpha*}/(1-\lambda^*)] \lesseqgtr 0$, and $\partial\beta^*/\partial\alpha^* = \beta^*[\lambda_{\alpha*}/(1-\lambda^*) - 1/(1-\alpha^*)] \lesseqgtr 0$. Finally, note that $\partial B^*/\partial n = -(V^L/w - B^*)\lambda^*_n/(1-\lambda^*) < 0$.

## A.5. Public enforcement

**Proof of Proposition 4.** The welfare-increasing direction of change in the proportion of protected property is inferred by examining $W_\alpha$ evaluated at the interior private solution using $\beta_\alpha = -\beta(1-\beta) = (1-*=\alpha)$. This yields

$$W_\alpha = (\pi - \beta)V - D\beta(1-\beta)/(1-\alpha).$$

In the coalition solution and in the free entry solution, $\beta \leq \pi^2$ by Proposition 3 and condition (8), hence $(1-\alpha)W_\alpha \geq \pi(1-\pi)V(1-\alpha) - \beta(1-\beta)D$: it can be shown that $V(1-\alpha) - D < 0$ for $\alpha > 0$[27]. Then a necessary condition for $W_\alpha > 0$ is $\beta \leq 1/2$ and a sufficient condition for $W_\alpha < 0$ is $\beta > 1/2$. Here we use $\pi > \beta$ and $d[x(1-x)]/dx = 1 - 2x \geq 0$ for $x \geq 1/2$ to establish the inequality.    □

Proposition 4 in the linear case.

With a linear (uniform) distribution of property values $V(\alpha) = V^0(1-\alpha)$, $\alpha^c = \alpha^m = 1/2$, $V^L = V^0/4$, $V^H = 3V^0/4$. From the equilibrium allocation of predators between protected and unprotected property, it is always true that $\beta = 1 - (1-\pi)V^H/V^L$, while in the linear distribution case $\beta = 3\pi - 2$, hence $\pi - \beta = 2(1-\pi)$ and $1 - \beta = 3(1-\pi)$. Using $\alpha^c = 1/2$ and evaluating $D(1/2) = 3V^0/8$; we obtain

$$\frac{W\alpha}{V^0(1-\pi)} = 1 - \frac{3}{4}[3(3\pi - 2)] = \frac{22}{4} - \frac{27}{4}\pi.$$

In the linear case $W_\alpha < (>)0$ as $\pi > (<)22/27$ or equivalently $\beta > (<)4/9$. While $\pi$ and $\beta$ are of course endogenous variables, the parameters which change their equilibrium values, such as $R$ and $\theta$, can always be manipulated to drive the solution into the relevant range.[28]

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[27] $y \equiv V(1-\alpha) - D = V(\alpha)(1-\alpha) - K + S(\alpha)$ by definition of $D$, where $K$ is a constant. Differentiating, $y_\alpha = (1-2\alpha)V_\alpha - V < 0$ for $\alpha \leq \alpha^*$, where $\alpha^*$ denotes the revenue-maximizing level of $\alpha$. Thus $y(\alpha) = \int_0^\alpha y_\alpha(x)dx < 0.||$ For example, in the linear case where $V(1) = 0$, $\alpha^m = 1/2$ and thus $V(1/2) = 2 - D < 0$; as is evident from examining the graph of $V$ plotted against $\alpha$ and using the definition of $D$.

[28] For instance consider the case where: $V = 10(1-\alpha) \Rightarrow \alpha^c = 1/2$, $V^H = 7.5$; $V^L = 2.5$. Set $\theta = 1 = w = R$; and set $f = 0.1$. Then the free entry solution is

$n^c = 4.9$

$\pi^c = 0.90$

$\beta^c = 0.71$.

We ignore the integer constraint on n for simplicity. At these free entry equilibrium values, the welfare derivative is $W_\alpha = -0.58$. For $\theta = 2$, $W_\alpha = 0.03$.

**Proof of proposition 5.** Substituting $\pi_n = \pi(1-\pi)/n$ into Eq. (9) yields

$$W_n = \pi(1-\pi)S/n - f.$$

For free entry enforcement, $f = (\pi - \beta)\alpha V/n$ while $\pi(1-\pi) \leq \pi - \beta$ by Proposition 3 and condition (8). Then at the free entry enforcement level $W_n \leq (\pi - \beta)[S - \alpha V]/n < 0$ since $S - \alpha V < 0$.[29] This proves part (a).[30] As for part (b), evaluating the welfare derivative at the profit maximizing solution, $W_n \leq (\pi - \beta)[S - \alpha V]/n + P$, where we use $f = (\pi - \beta)\alpha V/n - P$ by definition of per member profits $P$. With low profits, associated with weak coalition, the welfarist state desires less enforcement than does the coalition.     □

Joint owners' and predators' welfare in the linear case.

In the case of linear distribution of property values $V = V^0(1-\alpha)$, $\rho_\alpha = -(1-\beta)^2 V^0 B/4$ using $\beta_\alpha = -(1-\beta)/(1-\alpha)$, $V^L = V^0(1-\alpha)/2$ and $\alpha^* = 1/2$.

Thus combined welfare changes with the proportion of protected property according to:

$$\frac{W_\alpha + \rho_\alpha}{V^0(1-\pi)} = 1 - \frac{27}{4}\pi + \frac{18}{4} - \frac{3}{4}B(1-\pi)$$

where $B = 3(1-\pi)V^L/w \leq 1$. This expression is negative at $\pi = 1$ and positive at $\pi = 0$, and the expression is decreasing in $\pi$. Thus there is a unique crossover value of $\pi$ below which $W_\alpha + \rho_\alpha > 0$ and above which $W_\alpha + \rho_\alpha < 0$. Despite allowing for the preferences of the state to value the rents earned by predators as well as the deflection of predators onto unprotected property, in the linear case there always exists a range of values of low $\pi$ and $\beta$ for which a welfare maximizing state would choose to protect a higher proportion of property than would local monopoly enforcers.

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[29] Define $z \equiv S(\alpha) - \alpha V(\alpha)$. Then $z_\alpha(x) = -V(x) - 2x V_\alpha(x) < 0$ for $x \leq \alpha^*$ where $\alpha^*$ denotes the revenue maximizing level of $\alpha$. Then $z(\alpha) = \int_0^\alpha z_\alpha(x)dx < 0$.

[30] In the example above $W_n = -0.08$.

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