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Extending the Input-Output Model with Assets

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ABSTRACT In this paper, the input-output model is extended with assets. It allows us to examine the various assets that are held and used in production. The requirements of assets that must be held by each sector can thus be specified. Extending the input-output model with assets provides a better alternative to the capital stock matrix in the standard Systems of National Accounts. The input-output model is extended by taking the depreciation of fixed assets into full account. This extension allows for the calculation of total holding coefficients that express the amount of assets that are required to be held in each sector in order to satisfy a unit of final demand. In addition, a dynamic version of the extended model is presented. The extended input-output model has been widely applied in China for various purposes.

KEY WORDS: Assets, extended input-output models, total labor consumption coefficients, total holding coefficients, dynamic input-output model

Introduction

Since the 1970s, input-output tables have been widely issued in China (see Table 1, see also Chen, 1989, and Polenske and Chen, 1991). The first national input-output table of China was constructed for 1973 and was an experimental table compiled in physical units (Chen and Xue, 1984). In 1987, the State Council of China decided that every five years China would conduct a special input-output survey to construct national and regional input-output tables. Since 1987, the National Bureau of Statistics has constructed input-output tables regularly (1987, 1992, 1997, 2002). In 1987, 1992, and 1997, all provinces, autonomous regions, and municipalities - except Tibet and Hainan - constructed regional input-output tables (Liu and Wu, 1991). The sector classification, table scale, and table form of regional tables are similar to those in national input-output tables.

In addition to 'general' national and regional input-output tables for all sectors, many 'special' input-output tables have been compiled for sectors, such as agriculture;

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		Number of	· · · · · · · · · · · · · · · · · · ·		
Year	Type of table	sectors/products	Comments		
1973	Physical	61	First national table (experimental)		
1979	Physical	61	1973 table updated by RAS method		
	Value	21	MBS table		
1981	Physical	146	MBS table		
	Value	26	MBS table		
1983	Physical	146	1981 updated physical table by RAS method		
	Value	22	1981 updated value table by RAS method		
1987	Value	117	First table based on special input-output surveys		
1990	Value	33	1987 updated table by RAS method		
1992	Physical	151			
	Value	118	Tables based on special input-output surveys		
1995	Value	33	1992 updated table by RAS method		
1997	Value	124	Table based on special input-output surveys		
2000	Value	40	1997 updated table by RAS method		

Table 1. Chinese national input – output tables, 1973–2000

Note: MBS denotes Material Balance System.

mechanical and electronic industries, chemicals; color televisions; weapons; information industry; and copper (Polenske and Chen, 1991). Environmental protection-economy-input—output tables have been constructed for the city of Tianjin, for Liaoning province, and for China; for the urban water economy in Beijing; and for energy consumption and greenhouse-gas emissions in China. One of the important features of the applications of input—output techniques in China is the construction of enterprise level tables and their use in enterprise planning and cost calculation (Li, 1991; Tong and Wang, 1992).

Many Chinese economists find that input—output models are very useful for economic analysis, planning, management, and forecasting. In the process of using input—output analysis, we began in the 1980s to study an extended input—output model with assets. The main idea of the model is that assets, including fixed assets, inventories, labor, financial assets, and natural resources, play a very important role in production. The common input—output table, however, does not include assets. Therefore, assets have to be incorporated into the input—output table and a set of corresponding models has to be developed. Since 1980, Chinese input—output researchers have used the extended input—output model in grain output prediction, water conservancy, economic-environmental analysis of township and village enterprises, foreign trade, finance, and other areas.

Many scholars have done work on the issue of assets. In particular, Leontief *et al.* (1953) proposed a dynamic input—output model using a capital coefficient matrix. At the Harvard Economic Research Project, they not only studied dynamic models theoretically, but also led empirical work on the construction of matrices of capital coefficients, including fixed capital stocks and inventories by sector. Following Leontief, many scholars — for example, Ghosh (1964), Almon (1970), Carter (1970), Green (1971), Gossling (1975), Peterson and Schott (1979) — studied issues concerning capital coefficients and labor. In Japan there has been much meaningful research on capital measurement, including an estimation of the capital formation matrix every five years since 1970 and an estimation of the capital stock matrix in 1955, 1975, and other years (Kuroda and Nomura, 2004).

Concerning natural resources, Leontief *et al.* (1977) calculated the production and consumption of six metallic and three energy resources for 2000 and divided developing countries into two groups: developing countries with major mineral resource endowments and other developing countries. The following work is seen as an extension of this research agenda. The common input—output table, however, has up to now not regularly included assets held and used. In this paper we will introduce an extended input—output model with assets that has been applied in China.

2. Holding and Using Assets

In 1984, when we constructed the input—output table for agriculture, to study grain issues in China, we had to face the fact that both cultivated land and capital play a critical role in agricultural production. The common input—output table (and model), however, did not include land, labor, or any form of capital assets used in agricultural production. It was therefore necessary for us to extend the normal input—output models to include fixed assets, inventories, labor, land, water, and other assets.

Holding and using assets in production is a prerequisite for production. No production process can proceed without the presence of required quantities of assets, including capital, skilled or unskilled labor, and natural resources. Production scale and economic benefits also depend on the quantity and quality of the available assets. Natural resources play a critical role in some sectors, such as mining and agriculture.

In normal input-output models, the term 'input' represents the use of various production factors in the process of economic activities. It may be seen from the meaning of 'total input' in the input-output table. The total input is equal to the sum of intermediate inputs and primary inputs, and gives the 'consumption' of various materials, energy, services and primary factors. With assets, we refer to holding and using various assets by each sector at a point in time. Assets consist not only of fixed assets (such as machinery and construction), but also of inventories, financial assets, labor force (educated or not, skilled or not), natural resources, intangible assets, and others.

There exists close interrelationships between factor inputs, assets held and used, and output. First, assets are a *prerequisite* for input and output. Without assets there will be neither input nor output. The quantity and quality of output are directly dependent on the quantity and quality of assets held and used. In particular, modern production requires higher-quality assets. Second, assets are *related* with output and input. For example, some parts of output are used as fixed capital formation and are thus as an increase in stocks in order to improve the quality and quantity of fixed assets. For example, in order to get a more skilled or a highly skilled labor force, it is necessary to have more output in the education sector. Finally, input is *dependent* on the assets used. For example, because modern agriculture uses advanced agricultural machinery and highly skilled managers and workers as assets, its input coefficients, such as for oil, electricity, chemical fertilizer, and chemical pesticides, are different from the input coefficients of traditional agriculture.

The input-output model reflects the interdependence between factor inputs and outputs in production. However, the normal input-output model does not include an asset section; nor does it reflect the interrelationships between assets and outputs. Because there is no asset section, especially fixed assets and financial assets (other than machinery and construction in the capital stock matrix), the current input-output tables may lead to a

misconception. Consider the usual input—output equations to determine the vector of total sectoral outputs for a given final demand vector. That is,

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{f} \tag{1}$$

where \mathbf{x} denotes gross output vector, \mathbf{f} is the final demand vector, \mathbf{I} is the identity matrix, and \mathbf{A} represents the matrix of direct input coefficient matrix. In fact, even if \mathbf{f} has been determined, \mathbf{x} cannot be obtained if the availability of certain quantities of assets held and used (such as fixed assets, natural resources, and labor) is not assured (Chen, 1990).

The common input—output model shows interrelationships between inputs and outputs. The extended input—output model with assets shows not only relationships between factor inputs and output, but also relationships between assets and output, and linkages between assets and factor inputs.

From the point of view of the system of national accounts and statistics, factor inputs and assets held and used are different types of statistical indicators. Factor inputs fall in the category of flows, while assets belong to the category of stocks. In common input—output tables, all entries (including intermediate and primary inputs, final demands, and outputs) are flows. All assets held—for example, natural resources, labor, fixed assets, inventories, and financial assets—are stocks. As Hicks (1965) wrote, current activities of production and consumption are called 'flow activities', while the holding of assets is called 'stock activity'.

In an economic activity, flows reflect the creation, transformation, transaction, or exchange within a period of time. Their quantity depends directly on the length of the process. Steel output per year, for instance, is equal to roughly 12 times the output per month. However, stocks are in position or in 'holding' at a point in time. The quantity has no direct relation with the length of the production period. For example, the number of workers in the steel sector at the beginning of the year is roughly equal to that at the end of the year. Generally, stocks are *preconditions* for flows and the intensity and quantity of flows are determined by the status of stocks. Increases in stocks and reproduction of stocks are *dependent* on the flows. There is a close and reciprocal relationship between the two. In summary, the common input—output model shows relationships only between flows; the extended input—output model with assets shows not only linkages between flows, but also between stocks and flows and between stocks themselves.

3. The Extended Input-Output Model with Assets and its Main Characteristics 1

Table 2 describes the extended input—output table. Note that the actual compilation of such a table requires a large amount of data. Because, for most applications, data are scarce in China, we often had to use a more simplified form of the table. Whereas the normal input—output table includes only factor inputs in the column for a sector, the extended input—output table with assets includes factor inputs and the holding of various assets (Chen, 1990, 1998).

The upper part of Table 2 reflects the usual input—output table. The $n \times n$ matrix **W** gives the intermediate input deliveries w_{ij} from sector i to sector j. The matrix **F** with final demands may be huge and may comprise the following vectors and submatrices. (i) an $n \times n_c$ matrix with deliveries from sector i to each of the n_c consumption categories; (ii) an $n \times n$ matrix with deliveries from sector i for fixed capital formation in sector j; (iii)

Factor inputs	Intermediate demand	Final demand	Gross outputs	
Intermediate inputs	W	F	X	
Primary inputs Total inputs	$egin{array}{c} \mathbf{V} \ \mathbf{x}' \end{array}$			
Asset holding	Intermediate assets	Final assets	Gross assets	
Fixed assets	$\mathbf{W}_{[1]}^{\mathrm{O}}$	$\mathbf{F}_{[1]}^{\mathrm{O}}$	x ^O _[1]	
Inventories	$\mathbf{W}^{ ext{O}}_{[2]}$	$\mathbf{F}^{ ext{O}}_{[2]}$	$\mathbf{x}_{[2]}^{\mathrm{O}}$	
Financial assets	$\mathbf{W}_{[3]}^{\mathrm{O}}$	$\mathbf{F}_{[3]}^{\mathrm{O}}$	$\mathbf{x}_{[3]}^{\mathrm{O}}$	
Labor	$\mathbf{W}^{ ext{O}}_{[4]}$	$\mathbf{F}^{ ext{O}}_{[4]}$	$\mathbf{x}_{[4]}^{\mathrm{O}}$	
Natural resources	$\mathbf{W}^{\mathrm{O}}_{[5]}$	$\mathbf{F}_{[5]}^{ ext{O}}$	$\mathbf{x}_{[5]}^{\mathrm{O}}$	
Intangible assets $\mathbf{W}_{[6]}^{\mathbf{O}}$		$\mathbf{F}^{\mathrm{O}}_{[6]}$	$\mathbf{x}_{[6]}^{\mathrm{O}}$	

Table 2. The extended input-output table with assets

an $n \times n$ matrix with the changes in the inventories that sector j holds of products from sector i; (iv) a vector with exports; and (v) a vector with imports (all with a minus sign). The column vector with the rowsums of \mathbf{F} gives the final demand vector \mathbf{f} that was used in (1). The $k \times n$ matrix \mathbf{V} gives the primary inputs, where k categories are distinguished (such as depreciation of fixed capital, compensation of employees, net indirect taxes, and operating surplus).

The lower part of Table 2 describes the asset holdings and the use of assets in production. The matrix \mathbf{W}^{O} (here the superscript 'O' indicates asset) may be very large and could consist of the following submatrices. (i) An $n \times n$ matrix $\mathbf{W}_{[1]}^{0}$ that specifies the use of assets by each production sector and the physical contents of the fixed assets (e.g. construction, machinery, transport vehicles, etc). Its elements give the fixed assets originating from sector i that are held by sector j. (ii) An $n \times n$ matrix $\mathbf{W}_{[2]}^{O}$ that gives the inventories originating from sector i that are held by sector j. (iii) An $n_f \times n_f$ matrix $W_{[3]}^{0}$ with financial assets held by sector j for each of the n_f categories (such as currencies; deposits; loans; shares; securities other than shares; advances and trade credits; and other financial assets). (iv) An $n_l \times n$ matrix $\mathbf{W}_{[4]}^{O}$ with labor (typically expressed in physical units) used by sector j for each of the n_l schooling categories (such as illiterate; primary school; junior secondary school; senior secondary school; college; and higher level). (v) An $n_r \times n$ matrix $\mathbf{W}_{[5]}^{O}$ with the natural resources (typically also expressed in physical units) used by sector j for each of the n_r categories (such as land; water resources; subsoil assets; forest; and other natural resources). (vi) An $n_i \times n$ matrix $\mathbf{W}_{[6]}^{O}$ with intangible assets held by sector j for each of the n_i categories (such as patents; trade marks; and other intangible assets). The matrices $\mathbf{F}_{[i]}^{O}$ can be interpreted in a similar fashion. The rowsums of the intermediate and the final demands in the upper part of Table 2 yield the vector with gross outputs (x). The rowsums of intermediate and final assets in the lower part of the table yield the corresponding vectors $\mathbf{x}_{\text{fil}}^{\text{O}}$ of gross assets.

In the current system of national accounts (SNA) there are capital accounts, financial accounts, and so on. However, only the total amounts of fixed assets, inventories, financial assets, labor, and natural resources are usually shown in the related accounts of SNA. The current SNA does not include detailed capital data in an input—output format, i.e. by sector of use and origin. In the normal input—output table, fixed capital formation

and changes in inventories are column vectors, providing the total amounts summed over all sectors in the economy. The figures for fixed capital formation or changes in inventories for every sector of use are typically not included. This means that the current SNA and the normal input—output table cannot reflect the relationships between output and the assets held and used by each sector.

The advantage of the extended input—output model over the SNA model is that it specifies the quantity of asset elements held and used by every production sector and reflects the relationships between output and assets in every sector. This not only provides more information but also creates a very important analytical foundation, for example, to measure the growth of multi-factor or total factor productivity in each sector. In the extended input—output table, fixed capital formation and changes in inventories are two matrices of order n. There are three advantages we would like to point out.

First, the data aspect. Information, with respect to the amounts of fixed capital formation and changes in inventories in every sector, is an important element for economic analysis and forecasts. Second, the extension of the input-output table allows us to examine the relations between factor inputs and assets. There are two relationships: (i) between fixed capital formation (as a flow) in the final demands part and (the stock of) fixed assets held, and (ii) between changes in inventories (as a flow) in the final demands part and inventories of assets (as a stock). These two linkages can be expressed as follows. The amount of fixed assets at the beginning of the next year equals the amount of fixed assets at the beginning of the current year plus fixed capital formation in the current year minus consumption of fixed capital in the current year. Similarly, the amount of inventories at the beginning of the next year equals the amount of inventories at the beginning of the current year plus the net changes in inventories during the current year. Third, capital coefficients play a very important role in dynamic economic analysis. They are the basis of the dynamic input-output model. It is well known, however, that under the present SNA system it is difficult to obtain a capital coefficient matrix. The extended input-output table assists us in data collection, thus allowing us to calculate a capital coefficient matrix. To this end, we may use the $n \times n$ matrices for fixed assets and for inventories. If we then compile the extended input-output tables for two years (using the same sector classification), we can calculate not only the matrix with average capital coefficients, but also the matrix with marginal capital coefficients.

The estimation of assets is a very important issue. In order to estimate the assets in an appropriate way, it is necessary to employ special surveys. We used simple methods, however, to roughly estimate the assets. In current Chinese economic practice, the depreciation of fixed assets is calculated not according to the fixed asset classification, but according to the specific sector that uses the asset – a practice that makes general economic sense. Using data for the depreciation of fixed assets and data for the depreciation rate by sector, we estimated the approximate value of total fixed assets in each sector (which yields a vector of fixed asset values) on the basis of the total value of fixed assets of all sectors as estimated by the National Bureau of Statistics of China in the national accounts. In order to obtain a fixed asset stock matrix we have to disaggregate the value of fixed assets by sector of origin. For this purpose we used data from the fixed asset formation matrices from previous years to estimate the structure of fixed assets by sectors of origin.

For specific studies, we constructed specific extended input—output tables. For example, for the extended table with assets in agriculture (see Chen *et al.*, 1991), we had to disaggregate agricultural fixed assets, labor, irrigated land, and other factors by agricultural

sub-sectors (such as grain, cotton, and others). We used data from surveys on the costs and benefits of major agricultural products of China, organized by the National Development and Planning Commission, the National Economic and Trade Commission, the Ministry of Agriculture, the National Bureau of Forestry, the National Bureau of Light Industry, the National Bureau of Tobacco, the General Cooperative of Supply and Marketing of China, and other agencies. The surveys had been conducted in the 1300 counties of China, covering more than 60,000 rural households every year since 1980. These surveys provided a wealth of precious information, such as data on the costs of: depreciation of fixed assets, one day of labor, farm machinery and other materials, irrigation, taxes and benefits per *mu* (0.067 hectare) of sown area by major crops, various other factors for livestock products, aquatic products, and forest products.

From the upper part of Table 2, we obtain the usual accounting equation that states that the gross output (in sector i = 1, ..., n) is the sum of intermediate and final demand.

$$x_i = \sum_{j=1}^n w_{ij} + f_i \tag{2}$$

Note that f_i gives the total final demand in sector i, i.e. the sum of row i of the matrix \mathbf{F} in Table 2. Defining the direct input coefficients as $a_{ij} = w_{ij}/x_j$ implies that equation (2) can be rewritten as

$$x_i = \sum_{i=1}^n a_{ij} x_j + f_i \tag{3}$$

The lower part of Table 2 lists six types of assets. For the ease of exposition we stack the submatrices $\mathbf{W}_{[k]}^O$ into a single large matrix \mathbf{W}^O , and in the same way we construct \mathbf{F}^O and \mathbf{x}^O . The second accounting equation yields that the gross asset is the sum of intermediate and the final assets. We have termed the assets held and used by production sectors as the *intermediate* assets; and the assets held and used in the final demand categories as the *final* assets. Hence

$$\mathbf{x}_{i}^{\mathrm{O}} = \sum_{i=1}^{n} \mathbf{w}_{ii}^{\mathrm{O}} + \mathbf{f}_{i}^{\mathrm{O}} \tag{4}$$

Define the direct holding coefficient of the ith asset by the jth sector as

$$a_{ii}^{\mathcal{O}} = w_{ii}^{\mathcal{O}}/x_i \tag{5}$$

Note that this gives the amount of the *i*th asset held and used by sector *j* per unit of its gross output. Equation (4) can now be rewritten as

$$x_i^{O} = \sum_{j=1}^n a_{ij}^{O} x_j + f_i^{O}$$
 (6)

In matrix notation we have

$$\mathbf{x}^{\mathrm{O}} = \mathbf{A}^{\mathrm{O}}\mathbf{x} + \mathbf{f}^{\mathrm{O}} \tag{7}$$

On the basis of this extended input—output model we will introduce some new concepts,

models, and calculation formulae. For example, we will develop the total input coefficient that incorporates inputs of fixed assets, the total labor consumption coefficient, the coefficient for the total holding of fixed assets, financial assets, natural resources, and labor.

4. The Total Input Coefficients with Indirect Inputs of Fixed Assets

In input–output analysis, the standard static model is expressed as $\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{f}$. Its solution is given by

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{f} = \mathbf{\bar{B}} \mathbf{f} = (\mathbf{B} + \mathbf{I}) \mathbf{f}$$
(8)

where $\bar{\mathbf{B}} = (\mathbf{I} - \mathbf{A})^{-1}$ is the matrix of total requirement coefficients (or the Leontief inverse), and

$$\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1} - \mathbf{I} \tag{9}$$

is the matrix of total input coefficients (or matrix of total consumption coefficients). Element (i, j) of the Leontief inverse $(\mathbf{I} - \mathbf{A})^{-1}$ gives the output in sector i that is required to satisfy one unit of final demand in sector j. The intermediate inputs from sector i that are required (directly and indirectly) in satisfying this one unit final demand in sector j are given by element (i, j) of the matrix $\mathbf{B} = \mathbf{A}(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \cdots = (\mathbf{I} - \mathbf{A})^{-1} - \mathbf{I}$. So, the matrix \mathbf{B} is equal to the sum of the direct input coefficients and the total of all indirect input coefficients. The difference between the Leontief inverse and the matrix of total input coefficients is the identity matrix.

The total input coefficients matrix is widely used in China, for example, to calculate the total electricity consumption due to the final demand for steel, to calculate the total labor use per ton of final demand for grain, etc. In equation (8), however, a question arises concerning the calculation of total input coefficients. The expression for **B** in equation (9) does not include the indirect 'consumption' of fixed assets. For example, in the production of steel, some parts of this sector's factory buildings and metallurgical equipment are 'consumed'. The production of these buildings and equipment also requires electricity, which is obviously an indirect input of electricity to steel. It is not contained, however, in the total input (total consumption) of electricity to steel in the normal input—output model. Consequently, the total input coefficients defined by equation (9) are incomplete.

It was the acknowledgment of this incomplete character of the original input-output models that led us, among other problems referred to above, to develop an extended input-output model with assets. In order to completely include the indirect input of electricity via the fixed assets, a new equation for computing the total input coefficients is developed by extending the input-output model with assets as follows (Chen, 1990).

$$b_{ii}^* = a_{ij} + \sum_{k=1}^n b_{ik}^* a_{ki} + \alpha_i d_{ij} + \sum_{s=1}^n b_{is}^* \alpha_s d_{sj}$$
 (10)

In equation (10), the first term on the right-hand side denotes the direct consumption coefficient of electricity by steel, the second term gives the indirect electricity

consumption via intermediate inputs, the third term represents the direct electricity consumption via fixed assets (which is equal to the product of the depreciation rate α_i and the fixed asset holding coefficient d_{ij}), and the last term represents the indirect electricity consumption via fixed assets. In matrix notation, we have

$$\mathbf{B}^* = \mathbf{A} + \mathbf{B}^* \mathbf{A} + \hat{\alpha} \mathbf{D} + \mathbf{B}^* \hat{\alpha} \mathbf{D}$$
 (11)

Where \mathbf{B}^* represents a matrix of total input coefficients with the input of fixed assets, \mathbf{D} is a matrix of direct holding coefficients of fixed assets, and $\hat{\alpha}$ is the diagonal matrix of depreciation rates α_i of fixed assets. Rearranging terms yields

$$\mathbf{B}^*(\mathbf{I} - \mathbf{A} - \hat{\alpha}\mathbf{D}) = \mathbf{A} + \hat{\alpha}\mathbf{D}$$
 (12)

or

$$\mathbf{B}^* = (\mathbf{I} - \mathbf{A} - \hat{\alpha}\mathbf{D})^{-1} - \mathbf{I}$$
 (13)

Consider the matrix $\mathbf{W}_{[1]}^{O}$ in Table 2. Its typical element $w_{[1]ij}^{O}$ gives the amount of fixed assets produced in sector i that are held and used by sector j. Per unit of its gross output, sector j holds and uses an amount $d_{ij} = w_{[1]ij}^{O}/x_j$ of fixed assets. In the production process, however, a fraction α_i of these fixed assets are 'consumed'. So, a production of one unit of gross output in sector j uses not only a_{ij} of intermediate inputs, but also 'consumes' $\alpha_i d_{ij}$ of fixed assets. Note that both items are produced in sector i. Taking the 'consumption' of fixed assets into full account would imply that the matrix \mathbf{A} of direct input coefficients is replaced by the matrix $\mathbf{A} + \hat{\alpha} \mathbf{D}$. Because data on the actual 'consumption' rate of fixed assets are lacking, it is approximated by the depreciation rate of the fixed assets. Note that the matrix \mathbf{D} with coefficients for the direct holding of fixed assets is the first submatrix (corresponding to $\mathbf{W}_{[1]}^{O}$) of the matrix \mathbf{A}^{O} in equation (5). The new matrix of total input coefficients (which incorporates the input of fixed assets) is thus given by equation (13).

Note that the $n \times n$ matrix $(\mathbf{A} + \hat{\alpha}\mathbf{D})$ has only non-negative elements. Also note that the matrix $\hat{\alpha}\mathbf{D}\hat{\mathbf{x}}$ gives the total 'consumption' or depreciation of the *i*th fixed asset held and used by sector *j*. Its columnsums thus give the total depreciation of fixed assets held by sector *j*. But these are listed as part of the primary input matrix \mathbf{V} . Given that the columnsums of the matrices \mathbf{W} and \mathbf{V} in Table 2 yield the gross outputs, it readily follows that the columnsums of $(\mathbf{A} + \hat{\alpha}\mathbf{D})$ are all smaller than one. These two properties are sufficient to guarantee that $(\mathbf{A} + \hat{\alpha}\mathbf{D})$ is non-singular so that its inverse exists.

The elements of the total input coefficients \mathbf{B}^* as computed by equation (13) are obviously larger than those of \mathbf{B} . For example, based on the extended input-output table for the urban and rural economies of China in 1987, the total input coefficients of electricity to rice, coal, and metallurgical industries were, respectively, 0.01810, 0.08019, and 0.08626 when \mathbf{B} was used. They increased to 0.02045, 0.09334, and 0.09475, respectively, when \mathbf{B}^* was used.

As we indicated earlier, in current Chinese economic practice, the depreciation of fixed assets is not calculated by fixed assets classification, but according to the specific sector that uses the fixed assets – which makes general economic sense. Let $\hat{\beta}$ denote the diagonal matrix of fixed asset depreciation rates of the sectors that use the fixed assets.

We have

$$\mathbf{B}^* = (\mathbf{I} - \mathbf{A} - \mathbf{D}\hat{\mathbf{\beta}})^{-1} - \mathbf{I}$$
 (14)

Equations (13) and (14) give the two new formulations for the matrix of total input (or consumption) coefficients that take full account of the input of fixed assets. The corresponding matrices of total requirement coefficients are given as $\bar{\bf B}^* = ({\bf I} - {\bf A} - \hat{\alpha}\,{\bf D})^{-1}$ and $\bar{\bf B}^* = ({\bf I} - {\bf A} - {\bf D}\hat{\beta})^{-1}$.

What does this imply for the basic accounting equations of an input-output table?² Taking account of equation (1), or equivalently $\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{f}$, and replacing \mathbf{A} by $(\mathbf{A} + \bar{\alpha}\mathbf{D})$ implies that

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \hat{\mathbf{\alpha}}\mathbf{D}\mathbf{x} + \mathbf{f}^* \tag{15}$$

or $\mathbf{x} = (\mathbf{I} - \mathbf{A} - \bar{\boldsymbol{\alpha}} \mathbf{D})^{-1} \mathbf{f}^*$, where

$$\mathbf{f}^* = \mathbf{f} - \hat{\mathbf{\alpha}} \mathbf{D} \mathbf{x} \tag{16}$$

represents the vector of net final demands. These are equal to the final demands (including fixed capital formation) minus the depreciation of fixed assets. If the depreciation of fixed assets, as given by the vector $\hat{\boldsymbol{\alpha}} \mathbf{D} \mathbf{x}$, reflects or approximately equals the replacement of fixed assets, then \mathbf{f}^* reflects the vector of net final demands, excluding the replacement investments of fixed assets. In this case, \mathbf{f}^* would include the net investments (gross investments minus the replacement investments for fixed assets). Of course, if data are available with respect to such replacement investments, then we could define α as the vector with the replacement rates of fixed assets. In that case, the appropriate assumption would be that the consumption of fixed assets reflects, or approximately equals, the replacement of fixed assets.

5. Total Labor Consumption Coefficients

The total labor consumption coefficient is the sum of all direct and indirect labor consumption per unit product. It is a very important indicator to express social productivity for each sector. As we know from common input—output analysis, if the direct labor input coefficients are given by the row vector \mathbf{l}' , the *i*th element of the row vector $\mathbf{\mu}' = \mathbf{l}' (\mathbf{I} - \mathbf{A})^{-1}$ gives the total amount of labor that is (directly and indirectly) required to satisfy one unit of final demand in sector *i*.

This expression for the vector of total labor consumption coefficients does not include labor consumed in the production of the fixed assets. To overcome this problem, the new expression for the extended model with assets becomes $\mathbf{I}'(\mathbf{I} - \mathbf{A} - \hat{\alpha}\mathbf{D})^{-1}$ or $\mathbf{I}'(\mathbf{I} - \mathbf{A} - \mathbf{D}\hat{\boldsymbol{\beta}})^{-1}$. Its *i*th element gives the total amount of labor that is (directly and indirectly) required to satisfy one unit of net final demand – as defined in equation (16) – in sector *i*.

Similar results are obtained if the row vector with direct labor input coefficients is replaced by a vector with direct energy consumption coefficients, direct service input coefficients (given by the row vector s'), direct water input coefficients, direct input coefficients

for employees' compensations, direct input coefficient of net indirect taxes, and others. This implies that the methodology above can be applied to many issues. For example, the traditional method of calculating total service intensities of different industries is $\mathbf{s}'(\mathbf{I} - \mathbf{A})^{-1}$, see Bhowmik (2003). The new formula to calculate total service intensities is $\mathbf{s}'(\mathbf{I} - \mathbf{A} - \hat{\alpha}\mathbf{D})^{-1}$ or $\mathbf{s}'(\mathbf{I} - \mathbf{A} - \mathbf{D}\hat{\beta})^{-1}$. The difference between the two approaches is that the second includes the indirect service input, which is contained in the production of the fixed assets that have been consumed.

6. Total Holding Coefficients of Assets

The direct holding coefficient of an asset a_{ij}^{O} was defined in equation (5) as the ratio of the *i*th asset and the output of sector *j*. It indicates the holding intensity of the *i*th asset by production sector *j*. The matrix with direct holding coefficients of assets is thus given by

$$\mathbf{A}^{\mathrm{O}} = \mathbf{W}^{\mathrm{O}} \hat{\mathbf{x}}^{-1} \tag{17}$$

The matrix with total holding coefficients of assets ${\bf B}^{\rm O}$ can now be calculated by two methods. First, as ${\bf B}^{\rm O}={\bf A}^{\rm O}({\bf I}-{\bf A})^{-1}$ and, second, by using the extended model, which gives ${\bf B}^{\rm O*}={\bf A}^{\rm O}({\bf I}-{\bf A}-\hat{\boldsymbol{\alpha}}{\bf D})^{-1}$ or ${\bf B}^{\rm O*}={\bf A}^{\rm O}({\bf I}-{\bf A}-{\bf D}\hat{\boldsymbol{\beta}})^{-1}$. The total holding coefficients matrix could be used to calculate the total holding intensity of fixed assets, inventories, financial assets, employment, natural assets and others, per unit of final demands. For example, we have used the total holding coefficients matrix of employment to estimate the increase in employment in the People's Republic of China in response to an increase in exports from China to the United States.

7. The Dynamic Extended Input-Output Model with Assets

The Leontief dynamic input–output model can be written as follows (see Leontief *et al.*, 1953).

$$\mathbf{x} - \mathbf{A}\mathbf{x} - \mathbf{C}\dot{\mathbf{x}} = \tilde{\mathbf{f}}$$
 (in continuous form) (18)

$$\mathbf{x}(t) - \mathbf{A}\mathbf{x}(t) - \mathbf{C}[\mathbf{x}(t+1) - \mathbf{x}(t)] = \tilde{\mathbf{f}}(t)$$
 (in discrete form) (19)

where \mathbf{C} is the matrix with capital coefficients, $\tilde{\mathbf{f}}$ is the column vector of net final demands excluding capital formation, and $\dot{\mathbf{x}} = d\mathbf{x}/dt$. The time lag in these models is one year. They reflect the relationships between capital demand for increase in outputs in a later period (year t+1) and product use in the current period (year t). Two issues arise with respect to these models.

First, in order to expand production, not only more fixed assets and inventories are required, but also more labor, particularly skilled labor, is required. As Schultz (1961) has indicated, humans are the determinant factor of production, and in order to develop production it is first required to raise people's cultural, scientific, and technological knowledge. He put forward an important conception of human capital, investment in increasing human capability, and also pointed out that knowledge is the most powerful engine of

production. Based on US data, Schultz established that the contribution of human capital to profits is greater than that of physical capital.

In the dynamic extended input—output model with assets, we must take into account the fact that it takes a longer time to train skilled labor than to prepare and produce fixed assets and inventories. Therefore, future demand for labor for an expanding production should be linked with current human capital investments. Then, we get a new dynamic extended input—output model reflecting the relationship between human capital demand for an increase in outputs in later periods and inputs for training in the current period. We arrive at the following model.

$$\mathbf{x} - \mathbf{A}\mathbf{x} - (\mathbf{C} + \mathbf{H})\dot{\mathbf{x}} = \tilde{\mathbf{f}}^*$$
 (in continuous form) (20)

$$\mathbf{x}(t) - \mathbf{A}\mathbf{x}(t) - (\mathbf{C} + \mathbf{H})[\mathbf{x}(t+1) - \mathbf{x}(t)] = \tilde{\mathbf{f}}^*(t) \quad \text{(in discrete form)}$$

where ${\bf H}$ is the matrix of human capital coefficients, which gives the requirements for increasing output by one unit in the next period. The matrix is defined as ${\bf H}={\bf M}_L{\bf L}$, where ${\bf L}$ is marginal labor force coefficient matrix and ${\bf M}_L$ is the input matrix for training the labor force. Note that the final demands are adapted; for example, we have ${\bf \tilde f}^*={\bf \tilde f}+{\bf H}{\dot {\bf x}}$. This dynamic model shows the functional relationship between the use of products in the current period and the required skilled labor and capital for an increase in output in the next period. For simplicity's sake, we have assumed also here that the time lag is one year. If the time lag is more than one year, we can obtain a system of higher-order-differential equations (in continuous form) or a system of higher-order-difference equations (in discrete form). Because most asset elements have time-lag problems similar to that of human capital formation, we use alternative formulations where ${\bf C}^*$ replaces ${\bf C}+{\bf H}$. The matrix ${\bf C}^*$ is then the sum of the matrix ${\bf C}$ of marginal capital coefficients, the matrix ${\bf H}$ of human capital coefficients, and other matrices.

The second issue is that Leontief's dynamic model does not include the requirements to cover the consumption of fixed assets. From Leontief's dynamic model, it can be seen that if production is not expanded – that is, if $\dot{\mathbf{x}} = 0$ or $\mathbf{x}(t+1) - \mathbf{x}(t) = 0$ – then investments are not required. In the real world, however, gross capital formation is used not only to expand production, but also for maintenance, i.e. to cover the consumption of existing assets. From the figures in Table 3 we can conclude that in China, Japan, and the United States the ratios of depreciation of fixed assets and gross capital formation are very high: 36.2% in China (1997), 72.4% in Japan (2000), and 70.3% in the United States (1990). In the last two countries, most gross capital formation is to cover the consumption of fixed assets. This ratio tends to increase over time. In Japan, it was 33.0% in 1960, 40.6% in 1980, 52.2% in 1990, and 72.4% in 2000. This should not be ignored. The findings in Table 3 are thus an additional motivation for extending the Leontief dynamic model.

Just like we did earlier, the matrix **A** is now replaced by $(\mathbf{A} + \hat{\boldsymbol{\alpha}}\mathbf{D})$ and, similar to what was done in equation (17), $\tilde{\mathbf{f}}^*$ is replaced by $\tilde{\mathbf{f}}^{**} = \tilde{\mathbf{f}}^* - \hat{\boldsymbol{\alpha}}\mathbf{D}\mathbf{x}$. Note that **D** gives the average capital coefficients and if we assume that these are equal to the marginal

Table 3. The proportion of depreciation of fixed assets to gross capital formation in China, Japan
and the United States (in billions of the respective currency at current prices)

	Gross capital formation			Ratio of	Ratio of
	Total	Gross fixed asset formation	Depreciation of fixed assets	depreciation on gross fixed asset formation (%)	depreciation on gross capital formation (%)
China					
1981	151.7	118.9	41.1	34.6	27.1
1987	437.3	380.3	120.2	31.6	27.5
1992	963.8	831.7	353.7	42.5	36.7
1997	2,845.7	2,515.4	1,031.2	41.0	36.2
Japan					
1960	5,292.7	4,658.9	1,746.7	37.5	33.0
1970	28,617.5	26,257.9	9,531.2	36.3	33.3
1980	77,846.5	73,943.5	31,640.9	42.8	40.6
1985	88,049.9	86,558.2	44,330.5	51.2	50.3
1990	139,054.0	138,727.0	72,654.0	52.4	52.2
1995	141,782.7	139,721.7	80,800.7	57.8	57.0
2000	130,076.4	129,807.4	94,179.0	72.6	72.4
US					
1970	180.0		106.0		58.9
1980	540.0		365.0		67.6
1990	943.0		663.0		70.3

Sources: National Bureau of Statistics and State Planning Commission (1984); Uno (1989); National Bureau of Statistics (1991, 1996, 1999); Japan Statistical Bureau (2003); and OECD (2003).

capital coefficients in C, we can rewrite equations (20) and (21) as

$$\mathbf{x} - (\mathbf{A} + \hat{\boldsymbol{\alpha}}\mathbf{C})\mathbf{x} - (\mathbf{C} + \mathbf{H})\dot{\mathbf{x}} = \tilde{\mathbf{f}}^{**}$$
 (in continuous form) (22)

$$\mathbf{x}(t) - (\mathbf{A} + \hat{\mathbf{\alpha}}\mathbf{C})\mathbf{x}(t) - (\mathbf{C} + \mathbf{H})[\mathbf{x}(t+1) - \mathbf{x}(t)] = \tilde{\mathbf{f}}^{**}(t) \quad \text{(in discrete form)}$$
(23)

Taking account of the fact that labor eventually retires leads to a further extension. If $\hat{\alpha}_H$ denotes the diagonal matrix of labor retirement coefficients, the aspect of retirements would be included into the model if $(\mathbf{A} + \hat{\alpha}\mathbf{C})$ is replaced by $(\mathbf{A} + \hat{\alpha}\mathbf{C} + \hat{\alpha}_H\mathbf{H})$. Note that $\tilde{\mathbf{f}}^{***}$ must then be replaced by $\tilde{\mathbf{f}}^{****} = \tilde{\mathbf{f}}^{**} - \hat{\alpha}_H\mathbf{H}\mathbf{x}$.

8. Concluding Remarks

The holding and using of assets is a prerequisite for production. Production in modern society cannot occur without appropriate quantities of fixed assets, natural resources, labor, and financial assets. The scale of production and the economic benefits derived from the output also partially depend on the quality and quantity of production factors and assets. The output is obtained not only by factor inputs, but also by assets held and used. The input—output model extended with assets is a very important tool for analyzing

the relationships between sectors. Our contribution has been to formally incorporate assets into the input—output table and establish a corresponding model system to calculate total input coefficients and related indexes, for example, total holding coefficients of fixed assets.

In the 1980s, when we wanted to predict annual grain output in China, we discovered it was necessary to include assets in the input—output model. Since 1987, the model has been used in China to analyze many issues. Most of the published papers and books are in Chinese. Therefore, most input—output analysts in the world are still unaware of the extended input—output model with assets, or its applications. Up to the present, the extended input—output model with assets has been used in China for grain output prediction (Chen *et al.*, 2001); to study water conservancy issues of China and it's nine major river basins (Chen *et al.*, 2002); to study the key sectors of Chinese economic development in urban and rural economies and to calculate the amount of surplus labor (unemployment) in rural areas (Chen *et al.*, 1992); to predict, from a 1987 base, economic development indicators in Xinjiang in 1990, 1995, and 2000 and to study relations between Xinjiang and other regions of China; to study Shanxi Water Resource (Chen, 2000); to study Township and Village Enterprises (TVE) in China, in particular their coal and energy utilization and environmental pollution (Yang, 2001).

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Notes

¹In some previous books and many papers published in China and a few abroad, the model was called an input-'occupancy'-output model. The meaning of the term 'occupancy' – a literal translation of the Chinese 占用, in pinyin: zhan (4) yong (4) – was meant to stand for, in English, 'holding and using assets' at a point of time by a sector, where assets include fixed assets, inventory, financial assets, labor, natural resources, and so on.

²The following part is based on the comments by Erik Dietzenbacher.

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