

MONETARY POLICY AND UNCERTAINTY IN AN EMPIRICAL SMALL OPEN-ECONOMY MODEL[†]

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SUMMARY

This paper explores optimal policy design in an estimated model of three small open economies: Australia, Canada and New Zealand. Within a class of generalized Taylor rules, we show that to stabilize a weighted objective of output consumer price inflation and nominal interest variation optimal policy does not respond to the nominal exchange. This is despite the presence of local currency pricing and due, in large part, to observed exchange rate disconnect in these economies. Optimal policies that account for the uncertainty of model estimates, as captured by the parameters' posterior distribution, similarly exhibit a lack of exchange rate response. In contrast to Brainard (1967), the presence of parameter uncertainty can lead to more or less aggressive policy responses, depending on the model at hand. Copyright © 2010 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Recent theoretical analyses have emphasized the importance of pricing to market assumptions for optimal exchange rate policy, monetary policy and macroeconomic dynamics. Whether a country has producer currency pricing or local currency pricing can give rise to rather different policy recommendations, even when the sole objective of policy is to stabilize the aggregate inflation rate. For instance, Devereux and Engel (2003) show in a two-country model with local currency pricing that optimal monetary policy stipulates stabilization of the nominal exchange rate. Similarly, Monacelli (2005) shows that local currency pricing induces a trade-off in stabilizing aggregate price inflation and the output gap that is not present when the law of one price holds.

Despite these theoretical contributions there has been relatively little work on policy evaluation in empirical, small open-economy models. This paper seeks to fill this gap by exploring optimal policy design within an estimated structural model using data for Australia, Canada and New Zealand. Of particular interest is whether policies in a class of generalized Taylor rules optimally respond to exchange rate variations as predicted by theory. Moreover, we assess the consequence of various sources of model uncertainty for the design of optimal monetary policy. To our knowledge, this is the first such study in a fully estimated small open-economy model.¹

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¹ Levin *et al.* (2005) pursue a similar analysis for the closed-economy case.

The analysis is pursued using generalizations of the small open-economy framework proposed by Gali and Monacelli (2005) and Monacelli (2005), in which a small and large country each specialize in the production of a continuum of goods subject to imperfect competition and price rigidities.² Following the latter, imports are subject to local currency pricing (through what could be considered a retail sector providing distribution services) giving rise to deviations from the law of one price. We depart from their framework, by considering incomplete asset markets, the addition of other rigidities—such as indexation and habit formation—as well as a large set of disturbances which have been found crucial in taking closed-economy models to the data as documented by Christiano *et al.* (2005) and Smets and Wouters (2003), among others.

Using the empirical model, the optimal policy rule within a generalized class of Taylor rules is determined to minimize a weighted objective function in the variance of aggregate consumer price inflation, output and interest rates, subject to the constraints imposed by the estimated model. The Taylor rule posits that nominal interest rates are adjusted in response to output, output growth, inflation, nominal exchange rate growth, and past interest rates. Optimization occurs subject to two different assumptions about the central bank's knowledge of the economy. First, policy is determined assuming estimated model parameters are known with certainty to the policymaker. Second, we consider optimal policy that results from taking into account all uncertainty regarding model parameters by using the posterior distribution of our estimates. This is rendered feasible by adopting a Bayesian approach to inference.

The central insights from our analysis are as follows. First, we find that optimal policies do not respond to the nominal exchange rate. This is true regardless of whether parameter uncertainty is taken into account or not. Furthermore, this result is robust to a wide range of weight combinations for the components of the loss function, to the set of observables used to estimate the model, and the precise shocks included in estimation. This finding contrasts with Smets and Wouters (2002), which provides evidence that optimal policies stipulate a response to exchange rate variations.

The finding that it is not optimal to respond to exchange rate variation can be sourced to specific properties of the empirical model. There exists a 'disconnect' between nominal exchange rate movements and the evolution of domestic series. Indeed, cost-push and risk premium shocks account for between 69% and 84% of variation in the exchange rate, while accounting for a substantially lower share of the variation in output, interest rates and aggregate inflation across these three small open economies. Active stabilization of the nominal exchange rate exacerbates variability in output, inflation and nominal interest rates by connecting the evolution of these series more tightly to cost-push and risk premium shocks. And, even if this disconnect were not too strong, active stabilization of nominal exchange rates in the class of policies considered would still engender greater volatility in domestic variables.

Second, the implications of parameter uncertainty for monetary policy design are ambiguous. Depending on the country, the weight given to output stabilization, and the specific policy coefficient under consideration, policy can be more or less aggressive. The classic attenuation result of Brainard (1967) need not obtain, though our findings are consistent with multivariate generalizations of that analysis by Chow (1975). Similar results have also been documented for the closed-economy case in the robust control literature (see Giannoni, 2002). We conclude that

² The model is technically a semi-small open-economy model, as domestic goods producers have some market power. The model will nonetheless be referred to as a small open economy. Note also that our analysis appeals to an earlier interpretation of the Gali and Monacelli (2005) of a small–large country pair, rather than as an analysis of a continuum of small open economies.

the implications of parameter uncertainty for policy design need to be assessed on a case-by-case basis.

In exploring the robustness of our conclusions, we give particular attention to the modeling of the foreign block, which in the baseline model is fitted to observed US series. In an alternative model, the foreign block is treated as latent. By confronting the model with fewer observable series, greater flexibility exists to find mechanisms that could warrant an exchange rate channel for output and inflation stabilization. This is not the case and our characterization of optimal policy remains qualitatively unchanged.

An emergent issue in our robustness analysis concerns the impact of parameter identification for the design of optimal policy. For Australia, modeling the foreign block as latent gives rise to two modes with almost identical posterior densities. One is shown to favor a fairly high degree of nominal rigidities, while the other presents more persistent and volatile technology shocks. Although both modes confirm our conclusion that it is not optimal to respond to exchange rate variations, the policy coefficients on inflation and output growth are different, and each policy engenders rather different losses. We source these discrepancies to changes in the implied contribution of shocks and the transmission mechanisms of disturbances.

A number of recent papers have raised concerns about identification in dynamic stochastic general equilibrium (DSGE) models; see Lubik and Schorfheide (2005) and Justiniano and Preston (2006) for discussions in the context of open-economy models. More generally, Beyer and Farmer (2005), Fukac *et al.* (2006), Canova and Sala (2005), Cochrane (2007) and Iskrev (2007) explore sources of identification problems and their implications for inference and speculate on their consequences for policy evaluation. Our discussion provides a novel example of the problems that identification poses for policy design and underscores that care is warranted in the estimation of this class of models.

This paper most closely related to ours is Smets and Wouters (2002) and the references therein on policy evaluation in empirical small open-economy models. Lubik and Schorfheide (2003) also consider whether there is evidence that Australia, Canada, New Zealand and the UK have had monetary policies that depend on nominal exchange rate variations. However, they do not address the question of optimal policy or the consequences of model uncertainty. Our analysis also builds on the ever-growing literature on estimating small open-economy models using Bayesian methods (see Ambler *et al.*, 2004; Bergin, 2003, 2004; Del Negro, 2003; Dib, 2003; Ghironi, 2000; Justiniano and Preston, 2004, 2006; Lubik and Schorfheide, 2003, 2005; Lubik and Teo, 2005; Rabanal and Tuesta, 2005).

The paper proceeds as follows. Section 2 lays out the theoretical model. Section 3 discusses the data. Section 4 outlines the estimation methodology and adopted priors. Section 5 presents the baseline estimation results and properties of the model implied second-order moments. Section 6 presents the optimal policy exercises and assesses the implications of parameter uncertainty for policy design. Section 7 analyzes the robustness of our conclusions to the specification of the foreign block. Section 8 concludes.

2. A SIMPLE SMALL OPEN-ECONOMY MODEL

The following section sketches the derivation of key structural equations implied by the model proposed by Monacelli (2005) and its closely related precursor Gali and Monacelli (2005) when allowing for incomplete asset markets, habit formation and indexation of prices to past inflation.

These papers extend the microfoundations of the kind described by Clarida *et al.* (1999) and Woodford (2003) for analyzing monetary policy in a closed-economy setting to an open-economy context. For additional detail the reader is encouraged to consult Monacelli (2005).

2.1. Households

Households are assumed to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{\varepsilon}_{g,t} \left[\frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

where N_t is the labor input; $H_t \equiv hC_{t-1}$ is an external habit taken as exogenous by the household; $\sigma, \varphi > 0$ are the inverse elasticities of intertemporal substitution and labor supply respectively; and $\tilde{\varepsilon}_{g,t}$ is a preference shock. C_t is a composite consumption index:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where $C_{H,t}$ and $C_{F,t}$ are Dixit–Stiglitz aggregates of the available domestic and foreign produced goods given by

$$C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where α is the share of foreign goods in the domestic consumption bundle; $\eta > 0$ the elasticity of substitution between domestic and foreign goods; and $\varepsilon > 1$ is the elasticity of substitution between types of differentiated domestic or foreign goods.

Assuming the only available assets are one-period domestic and foreign bonds, optimization occurs subject to the flow budget constraint

$$P_t C_t + D_t + e_t B_t = D_{t-1}(1 + \tilde{r}_{t-1}) + e_t B_{t-1}(1 + \tilde{r}_{t-1}^*) \phi_t(A_t) + W_t N_t + \Pi_{H,t} + \Pi_{F,t} + T_t$$

for all $t > 0$, where D_t denotes the household's holding of one-period domestic bonds, and B_t holdings of one-period foreign bonds with corresponding interest rates \tilde{r}_t and \tilde{r}_t^* . The nominal exchange rate is \tilde{e}_t . P_t , $P_{H,t}$, $P_{F,t}$ and P_t^* correspond to the domestic CPI, domestic goods prices, the domestic currency price of imported goods and the foreign price, respectively, and are formally defined below. Wages W_t are earned on labor supplied and $\Pi_{H,t}$ and $\Pi_{F,t}$ denote profits from holding shares in domestic and imported goods firms. T_t denotes lump-sum taxes and transfers. Following Benigno (2001), Kollmann (2002) and Schmitt-Grohe and Uribe (2003), the function $\phi_t(\cdot)$ is interpretable as a debt elastic interest rate premium given by

$$\phi_t = \exp[-\chi(A_t + \tilde{\phi}_t)]$$

where

$$A_t \equiv \frac{\tilde{e}_{t-1} B_{t-1}}{\bar{Y} P_{t-1}}$$

is the real quantity of outstanding foreign debt expressed in terms of domestic currency as a fraction of steady-state output and ϕ_t a risk premium shock. The adopted functional form ensures stationarity of the foreign debt level in a log-linear approximation to the model.

Implicitly underwriting this expression for the budget constraint is the assumption that all households in the domestic economy receive an equal fraction of both domestic and retail firm profits. Hence, nominal income in each period is $W_t N_t + \Pi_{H,t} + \Pi_{F,t}$ which in equilibrium equals $P_{H,t} Y_{H,t} + (P_{F,t} - \tilde{e}_t P_t^*) C_{F,t}$ for all households. Absent this assumption, which imposes complete markets within the domestic economy, the analysis would require modeling the distribution of wealth across agents. That same assumption also ensures that households face identical decision problems and therefore choose identical state-contingent plans for consumption.

The household's optimization problem requires allocation of expenditures across all types of domestic and foreign goods, both intratemporally and intertemporally. This yields the following set of optimality conditions. The demand for each category of consumption good is

$$C_{H,t}(i) = (P_{H,t}(i)/P_{H,t})^{-\theta} C_{H,t} \quad \text{and} \quad C_{F,t}(i) = (P_{F,t}(i)/P_{F,t})^{-\theta} C_{F,t}$$

for all i with associated aggregate price indexes for the domestic and foreign consumption bundles given by $P_{H,t}$ and $P_{F,t}$. The optimal allocation of expenditure across domestic and foreign goods implies the demand functions

$$C_{H,t} = (1 - \alpha)(P_{H,t}/P_t)^{-\eta} C_t \quad \text{and} \quad C_{F,t} = \alpha(P_{F,t}/P_t)^{-\eta} C_t \quad (1)$$

where $P_t = [(1 - \alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$ is the consumer price index. Allocation of expenditures on the aggregate consumption bundle and optimal labor supply satisfy

$$\lambda_t = \tilde{e}_{g,t}(C_t - H_t)^{-1/\sigma} \quad (2)$$

$$\lambda_t = \tilde{e}_{g,t} P_t N_t^\varphi / W_t \quad (3)$$

and portfolio allocation is determined by the optimality conditions

$$\lambda_t \tilde{e}_t P_t = E_t[(1 + \tilde{v}_t^*) \beta \phi_{t+1} \lambda_{t+1} \tilde{e}_{t+1} P_{t+1}] \quad (4)$$

$$\lambda_t P_t = E_t[(1 + \tilde{v}_t) \beta \lambda_{t+1} P_{t+1}] \quad (5)$$

for Lagrange multiplier λ_t . The latter condition when combined with (2) gives the usual Euler equation.

2.2. Domestic Producers

There is a continuum of monopolistically competitive domestic firms producing differentiated goods. Calvo-style price setting is assumed, allowing for indexation to past domestic goods price inflation. Hence, in any period t , a fraction $1 - \theta_H$ of firms set prices optimally, while a fraction $0 < \theta_H < 1$ of goods prices are adjusted according to the indexation rule

$$\log P_{H,t}(i) = \log P_{H,t-1}(i) + \delta_H \pi_{H,t-1} \quad (6)$$

where $0 \leq \delta_H \leq 1$ measures the degree of indexation to the previous period's inflation rate and $\pi_{H,t} = \log(P_{H,t}/P_{H,t-1})$. Since all firms having the opportunity to reset their price in period t face the same decision problem they set a common price $P'_{H,t}$. The Dixit–Stiglitz aggregate price index therefore evolves according to the relation

$$P_{H,t} = \left[(1 - \theta_H) P'^{(1-\varepsilon)}_{H,t} + \theta_H \left(P_{H,t-1} \left(\frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\delta_H} \right)^{1-\varepsilon} \right]^{1/(1-\varepsilon)}. \quad (7)$$

Firms setting prices in period t face a demand curve

$$y_{H,T}(i) = \left(\frac{P_{H,t}(i)}{P_{H,T}} \cdot \left(\frac{P_{H,T-1}}{P_{H,t-1}} \right)^{\delta_H} \right)^{-\varepsilon} (C_{H,T} + C^*_{H,T}) \quad (8)$$

for all t and take aggregate prices and consumption bundles as parametric. Good i is produced using a single labor input $N_t(i)$ according to the relation $y_{H,t}(i) = \tilde{\varepsilon}_{a,t} N_t(i)$, where $\tilde{\varepsilon}_{a,t}$ is an exogenous technology shock.

The firm's price-setting problem in period t is to maximize the expected present discounted value of profits:

$$E_t \sum_{T=t}^{\infty} \theta_H^{T-t} Q_{t,T} y_{H,T}(i) \left[P_{H,t}(i) \left(\frac{P_{H,T-1}}{P_{H,t-1}} \right)^{\delta_H} - P_{H,T} MC_T \right]$$

where $MC_T = W_T/(P_{H,T} \tilde{\varepsilon}_{a,T})$ is the real marginal cost function for each firm, assuming homogeneous factor markets, subject to the demand curve, (8). The factor θ_H^{T-t} in the firm's objective function is the probability that the firm will not be able to adjust its price in the next $(T-t)$ periods. The firm's optimization problem implies the first-order condition

$$E_t \sum_{T=t}^{\infty} \theta_H^{T-t} Q_{t,T} y_{H,T}(i) \left[P_{H,t}(i) \left(\frac{P_{H,T-1}}{P_{H,t-1}} \right)^{\delta_H} - \frac{\theta_H}{\theta_H - 1} P_{H,T} MC_T \right] = 0. \quad (9)$$

2.3. Retail Firms

Retail firms import foreign differentiated goods for which the law of one price holds at the docks. In determining the domestic currency price of the imported good, firms are assumed to be monopolistically competitive. This small degree of pricing power leads to a violation of the law of one price in the short run.

Retail firms face a Calvo-style price-setting problem allowing for indexation to past inflation. Hence, in any period t , a fraction $1 - \theta_F$ of firms set prices optimally, while a fraction $0 < \theta_F < 1$ of goods prices are adjusted according to an indexation rule analogous to (6). The Dixit–Stiglitz aggregate price index consequently evolves according to the relation

$$P_{F,t} = \left[(1 - \theta_F) P'^{(1-\varepsilon)}_{F,t} + \theta_F \left(P_{F,t-1} \left(\frac{P_{F,t-1}}{P_{F,t-2}} \right)^{\delta_F} \right)^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \quad (10)$$

and firms setting prices in period t face a demand curve

$$C_{F,T}(i) = \left(\frac{P_{F,t}(i)}{P_{F,T}} \cdot \left(\frac{P_{F,T-1}}{P_{F,t-1}} \right)^{\delta_F} \right)^{-\varepsilon} C_{F,T} \quad (11)$$

for all t and take aggregate prices and consumption bundles as parametric. The firm's price-setting problem in period t is to maximize the expected present discounted value of profits:

$$E_t \sum_{T=t}^{\infty} \theta_H^{T-t} Q_{t,T} C_{F,T}(i) \left[P_{F,t}(i) \left(\frac{P_{F,T-1}}{P_{F,t-1}} \right)^{\delta_F} - \tilde{e}_T P_{F,T}^*(i) \right]$$

subject to the demand curve, (11). The factor θ_F^{T-t} in the firm's objective function is the probability that the firm will not be able to adjust its prices in the next $(T - t)$ periods. The firm's optimization problem implies the first-order condition

$$E_t \sum_{T=t}^{\infty} \theta_F^{T-t} Q_{t,T} \left[P_{F,t}(i) \left(\frac{P_{F,T-1}}{P_{F,t-1}} \right)^{\delta_F} - \frac{\theta_H}{\theta_H - 1} \tilde{e}_T P_{H,T}^*(i) \right] = 0.$$

2.4. International Risk Sharing

From the asset-pricing conditions that determine domestic and foreign bond holdings, the uncovered interest rate parity condition

$$E_t \lambda_{t+1} P_{t+1} [(1 + \tilde{v}_t) - (1 + \tilde{v}_t^*)(\tilde{e}_{t+1}/\tilde{e}_t)\phi_{t+1}] = 0 \quad (12)$$

follows, placing a restriction on the relative movements of the domestic and foreign interest rate, and changes in the nominal exchange rate.

The real exchange rate is defined as $\tilde{q}_t \equiv \tilde{e}_t P_t^*/P_t$. Since $P_t^* = P_{F,t}^*$, when the law of one price fails to hold, we have $\tilde{\Psi}_{F,t} \equiv \tilde{e}_t P_t^*/P_{F,t} \neq 1$, which defines what Monacelli (2005) calls the law of one price gap. The models of Gali and Monacelli (2005) and Monacelli (2005) are, respectively, characterized by whether or not $\tilde{\Psi}_{F,t} = 1$.

2.5. General Equilibrium

Goods market clearing requires

$$Y_{H,t} = C_{H,t} + C_{H,t}^* \quad (13)$$

in the domestic economy. The model is closed assuming foreign demand for the domestically produced good is specified as

$$C_{H,t}^* = \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\lambda} Y_t^*$$

where $\lambda > 0$. This demand function is standard in small open-economy models (see Kollmann, 2002; McCallum and Nelson, 2000) and nests the specification in Monacelli (2005) by allowing λ

to be different from η , the domestic elasticity of substitution across goods in the domestic economy, in order to give additional flexibility in the transmission mechanism of foreign disturbances to the domestic economy. However, our results are unaffected by the parametrization of this demand function.³ Domestic debt is assumed to be in zero net supply so that $D_t = 0$ for all t .⁴

The analysis considers a symmetric equilibrium in which all domestic producers setting prices in period t set a common price $P_{H,t}$. Similarly, all domestic retailers choose a common price $P_{F,t}$. Finally, households are assumed to have identical initial wealth, so that each faces the same period budget constraint and therefore makes identical consumption and portfolio decisions.

Finally, monetary policy is assumed to be conducted according to a Taylor-type rule discussed in the subsequent section. Fiscal policy is specified as a zero debt policy, with taxes equal to the subsidy required to eliminate the steady-state distortion induced by imperfect competition in the domestic and imported goods markets.

2.6. Log-Linear Approximation to the Model

The empirical analysis employs a log-linear approximation of the model's optimality conditions around a non-stochastic steady state. Here we discuss the key structural equations that emerge from this analysis. All variables are properly interpreted as log deviations from their respective steady-state values. For simplicity, these expressions assume $\bar{s} = 1$. Relations pertaining to the domestic economy are discussed, followed by those for the foreign economy.

A log-linear approximation to the domestic household's Euler equation (5) provides

$$c_t - hc_{t-1} = E_t(c_{t+1} - hc_t) - \sigma^{-1}(1-h)(i_t - E_t\pi_{t+1}) + \sigma^{-1}(1-h)(\varepsilon_{g,t} - E_t\varepsilon_{g,t+1}). \quad (14)$$

In the absence of habit formation, when $h = 0$, the usual Euler equation obtains. To derive a relationship in terms of domestic output, a log-linear approximation to the goods market clearing condition implies

$$(1-\alpha)c_t = y_t - \alpha\eta(2-\alpha)s_t - \alpha\eta\psi_{F,t} - \alpha y_t^* \quad (15)$$

where

$$\psi_{F,t} \equiv (e_t + p_t^*) - p_{F,t}$$

denotes the law of one price gap, the difference between the world currency price and the domestic currency price of imports, and $s_t = p_{F,t} - p_{H,t}$ gives the terms of trade. Time differencing the terms of trade definition implies

$$\Delta s_t = \pi_{F,t} - \pi_{H,t}. \quad (16)$$

Equilibrium domestic consumption depends on domestic output and three sources of foreign variation: the terms of trade, deviations from the law of one price and foreign output.

The terms of trade and the real exchange rate are related according to

$$q_t = e_t + p_t^* - p_t = \psi_{F,t} + (1-\alpha)s_t \quad (17)$$

³ Constraining λ to equal η results in identical insights from the estimation, and therefore we report results based on this more general specification.

⁴ A similar condition holds for the foreign economy once it is noted that domestic holdings of foreign debt, B_t , are negligible relative to the size of the foreign economy.

so that the real exchange rate varies with deviations from the law of one price and also differences in consumption bundles across the domestic and foreign economies.

A log-linear approximation to domestic firms' optimality conditions for price setting and the price index, (7), imply the relation

$$\pi_{H,t} - \delta\pi_{H,t-1} = \theta_H^{-1}(1 - \theta_H)(1 - \theta_H\beta)mc_t + \beta E_t(\pi_{H,t+1} - \delta\pi_{H,t}) \quad (18)$$

where

$$mc_t = \varphi y_t - (1 + \varphi)\varepsilon_{a,t} + \alpha s_t + \sigma(1 - h)^{-1}(c_t - hc_{t-1})$$

is the real marginal cost function of each firm. Thus domestic price inflation, $\pi_{H,t} = p_{H,t} - p_{H,t-1}$, is determined by current marginal costs, expectations about inflation in the next period and the most recent observed inflation rate. The latter appears as a result of price indexation. In the case of zero indexation to past inflation, $\delta = 0$, the usual forward-looking Phillips curve arises. In contrast to a closed-economy setting, domestic goods price inflation depends on three sources of foreign variation. There is a direct and an indirect effect of the terms of trade on firms' marginal costs, with the latter operating through the terms of trade implications for equilibrium consumption. There are also the effects of foreign output and deviations from the law of one price (recall relation (15)).

The optimality conditions for the retailers' pricing problem yields

$$\pi_{F,t} - \delta\pi_{F,t-1} = \theta_F^{-1}(1 - \theta_F)(1 - \theta_F\beta)\psi_{F,t} + \beta E_t(\pi_{F,t+1} - \delta\pi_{F,t}) + \varepsilon_{cp,t}. \quad (19)$$

Here, inflation in the domestic currency price of imports, $\pi_{F,t} = p_{F,t} - p_{F,t-1}$, is determined by current marginal cost conditions given by $\psi_{F,t}$ and expectations about next-period's inflation rate. A cost-push shock has also been added, capturing inefficient variations in mark-ups. Again, that prices are indexed to past inflation induces a history dependence on the most recent observed inflation rate. The domestic CPI and home goods prices are related according to

$$\pi_t = \pi_{H,t} + \alpha\Delta s_t. \quad (20)$$

The CPI and domestic goods price inflation differ insofar as imported goods prices deviate from domestic goods prices, with the difference weighted by the importance of those goods in the CPI—recall equation (16).

The uncovered interest rate parity condition gives

$$(i_t - E_t\pi_{t+1}) - (i_t^* - E_t\pi_{t+1}^*) = E_t\Delta q_{t+1} - \chi a_t - \phi_t \quad (21)$$

while the flow budget constraint implies

$$c_t + a_t = \beta^{-1}a_{t-1} - \alpha(s_t + \psi_{F,t}) + y_t \quad (22)$$

where $a_t = \log(e_t B_t / (P_t \bar{Y}))$ is the log real net foreign asset position as a fraction of steady-state output.⁵

⁵ In steady state, the foreign economy is assumed to have a zero debt-to-GDP ratio.

The model is closed by specifying monetary policy which is conducted according to the Taylor-type rule

$$\dot{i}_t = \rho_i \dot{i}_{t-1} + \psi_\pi \pi_t + \psi_y y_t + \psi_{\Delta y} \Delta y_t + \psi_e \Delta e_t + \varepsilon_{M,t}. \quad (23)$$

The nominal interest rate is determined by past interest rates and also responds to the current all-goods CPI inflation rate, output, output growth and the change in the nominal exchange rate. The final term, $\varepsilon_{M,t}$, is a monetary policy shock or implementation error in the conduct of policy.⁶

The domestic block of the economy is therefore given by equations (14)–(23) in the unknowns $\{c_t, y_t, \dot{i}_t, q_t, s_t, \pi_t, \pi_{H,t}, \pi_{F,t}, \psi_{F,t}, a_t\}$. Combined with the processes for the exogenous disturbances $\{\varepsilon_{a,t}, \varepsilon_{M,t}, \varepsilon_{g,t}, \varepsilon_{s,t}, \varepsilon_{cp,t}\}$ and $\{\pi_t^*, y_t^*, \dot{i}_t^*\}$, and the definitions $\Delta s_t = s_t - s_{t-1}$ and $\Delta q_t = q_t - q_{t-1}$, these relations constitute a linear rational expectations model which can be solved using standard methods. Together these relations also comprise the equations used to construct the likelihood for estimation. The disturbances $\{\varepsilon_{a,t}, \varepsilon_{g,t}, \varepsilon_{s,t}\}$ are assumed to be independent AR(1) processes and $\{\varepsilon_{M,t}\}$ an i.i.d. process. The determination of the foreign block $\{\pi_t^*, y_t^*, \dot{i}_t^*\}$ is discussed in the next section. In estimation, we only make use of observable series for $\{y_t, \dot{i}_t, \pi_t, q_t, s_t, \pi_t^*, y_t^*, \dot{i}_t^*\}$ and therefore exploit only a subset of cross-equation restrictions implied by the model.

2.7. The Foreign Economy

In Monacelli (2005) the foreign economy is specified as the closed-economy variant of the model described above. However, because the foreign economy is exogenous to the domestic economy, we have some flexibility in specifying the determination of foreign variables. Rather than take a literal interpretation of the Monacelli model, we instead assume that the paths of $\{\pi_t^*, y_t^*, \dot{i}_t^*\}$ are determined by a vector autoregressive processes of order two.

3. DATA

For all three countries, estimation uses quarterly data on output, inflation, interest rates, the real exchange rate and the terms of trade. GDP is per capita in log deviations from a linear trend. The inflation series corresponds to the annualized quarterly log-difference in the consumer price index (all goods), which includes both home and imported goods. For Australia, an adjustment is made to this series to take into account the effects of the introduction of the goods and services tax in 2000–2001. For Canada, we use an inflation measure excluding food and energy, given numerous references to this core series in the conduct of monetary policy by the Bank of Canada. Similar considerations to those in Australia dictate adjusting the large outlier in the first quarter of 1991 with the use—for that year only—of a measure that also excludes the effects of indirect taxes. Finally, we use the cash rate in Australia and, for Canada and New Zealand, averages of 3-month bank rates (all expressed in annualized percentages) for interest rates.

All Australian data were downloaded from the statistical tables published by the Reserve Bank of Australia. For Canada and New Zealand all data were obtained from Data Stream International.

⁶ Policy is assumed to respond to the linear detrended level of output and the change in this measure, as opposed to the model-theoretic measure of the output gap. This is motivated by recent research suggesting that model theoretic output gap measures do not accord with more traditional measures of economic slack used by actual policymakers (see Neiss and Nelson, 2005; Andreas *et al.*, 2005). This has relevance given our interest in assessing the historical stance of policy.

We constructed a model consistent real exchange rate using US price data discussed below, each country's CPI (as described above) and the bilateral nominal exchange rate. The real exchange rate is expressed in log-differences for the estimation. The terms of trade are measured as the price of imports to exports using the corresponding price deflators from the national accounts in each country. As with the real exchange rate, we use the log-difference of this series when taking the model to the data.

For specifications in which the foreign block is observable we assume it to be reasonably proxied by US data. The US series are the annualized quarterly log percentage change in the CPI, the log deviations of per capita GDP from a linear trend and the Fed Funds rate (annualized percentage), all taken from the Database at the Federal Reserve Bank of St Louis. Our samples run from 1984:I until 2007:I for Australia, and 1988:III–2007:I for New Zealand, following the move in each country towards a flexible exchange rate regime. For Canada, the sample covers the period 1982:I–2007:I, to coincide with the abandonment of monetary targeting with the Bank of Canada.⁷

In summary, for each country the model is taken to the data using eight observable series and the same number of disturbances. We demean the series before the estimation.

4. ESTIMATION

Our objective is not only to obtain point estimates for the parameters of the DSGE model specified in the previous section, but also to provide accurate measures of uncertainty surrounding these estimates. Therefore, using Bayesian methods, we aim to characterize the posterior distribution of the model parameters $\theta \in \Theta$. Given a prior, $\pi(\theta)$, the posterior density is proportional to the product of the likelihood and the prior. As described by Schorfheide (2000), posterior draws for this density can be generated using a random-walk metropolis algorithm and the state-space representation implied by the solution of the linear rational expectations model and the Kalman filter. Measures of location and scatter are obtained from the draws by computing, for instance, the median and standard deviations as well as posterior probability bands. Furthermore, given the draws, it is possible to characterize the posterior distribution of any functional of interest by computing the corresponding functional for each of the draws. This property will later be exploited to analyze the implication of model uncertainty on optimal policy.

An optimization algorithm is used to obtain an initial estimate of the mode. We start the maximization algorithm from a number of random starting values—before launching the Markov chain Monte Carlo (MCMC) chains—and check that the optimization routine always converges to the same value.⁸ This is a useful diagnostic for the presence of identification problems, conditional on a given set of priors. Indeed, our experience is that this is crucial to identifying local modes which may achieve almost identical values of the posterior with sometimes rather different configurations of coefficients. Of course, this procedure remains silent on the role of

⁷ We use four observations before the start of the sample dates listed above to deal with the initialization of the Kalman filter. These four initial data points are excluded from the computation of the likelihood and consequently from our estimates. Note that this does not represent the use of a training sample prior.

⁸ For the baseline model discussed, over 50 optimization runs were launched using random draws from the prior or an equally spaced grid covering the parameter space. All runs converged to the same mode. Note that obtaining different modes with substantially different values of the posterior/likelihood need not reflect identification issues but rather the properties of the optimization routine in place. In this respect, we differ from Canova and Sala (2005) in that we view the convergence to multiple modes with similar fit as problematic, not the convergence to multiple modes per se.

priors in achieving local identification, which may be discerned by looking at univariate or two-dimensional plots of the likelihood or the Hessian. The existence of multiple modes, related identification issues, and their implications for policy design are the focus of a later section.

Having ensured a unique mode for the baseline model, the Hessian from the optimization routine is used as a proposal density, properly scaled to yield a target acceptance rate of 25%. For the MCMC results, five chains of 100,000 draws each were initialized by randomly selecting starting values (using an over-dispersed normal density centered at the mode with a scaled-up Hessian as variance covariance matrix). For each chain, following a burn-in phase of 40,000 draws, convergence is monitored using CUMSUM plots and, for the overall chains, the potential scale reduction factors and confidence interval variants of Brooks and Gelman (1998).

The priors are described in the first three columns of Table I. The same priors are used for all countries except for the openness parameter, α , which we calibrate to the average share of exports and imports to GDP in each country using national accounts data. Over our sample period this results in values for α of 0.185, 0.28 and 0.29 for Australia, Canada and New Zealand. Attempts to estimate this parameter often led to implausibly low values.

We adopt fairly loose Gamma priors, with large tails, for the inverse Frisch elasticity of labor supply as well as the elasticity of substitution between domestic and foreign goods, considering the diverse estimates emerging from macro and micro studies. Similarly, our prior for the intertemporal elasticity of substitution easily accommodates values of 1 or 0.5 as used in the international business cycle literature, as well as substantially larger estimates that may result from the absence of capital and the consumption of durables in our model (see Rotemberg and Woodford, 1999). Priors for the Calvo price parameters assume the presence of nominal rigidities, centered at a compromise between traditionally large values obtained in macro studies and recent evidence of greater flexibility in prices using disaggregated data for the USA (see Bills and Klenow, 2007). For imported goods, it may be reasonable to assume a lower degree of stickiness. Nonetheless, estimated open-economy models tend to produce fairly large deviations from the law of one price. Therefore, just as in the case of domestic prices, we opt for a compromise in choosing our prior. We follow Lubik and Schorfheide (2003) in specifying the prior for the parameters of the Taylor rule, except for output growth, which is not considered in their analysis.

Habit and indexation have been found to be crucial for fitting closed-economy models, which suggests considering possibly large values for the parameters governing these intrinsic mechanisms of persistence. However, a priori it is possible that the dynamics in the foreign block may provide an alternative source of persistence in the model. To allow for this possibility, we specify very flat priors on habit as well as the indexation coefficients of both domestic and imported goods.

The exogenous stochastic disturbances (risk premium, technology, preference and import cost-push shocks) are assumed to be fairly persistent, reflected in a beta prior with a mean of 0.8 for the autoregressive coefficients. For the VAR(2) in the foreign block, we choose priors suggested by pre-sample individual autoregressions.⁹

Finally, the priors for the standard deviations of the shocks are the same for foreign and domestic shocks. To allow for a wide set of values a priori we specify Inverse-Gamma 1 densities, with infinite variance by fixing the degrees of freedom at 2. The scale parameters are chosen to obtain

⁹ For the first-order autoregressive coefficients, we specify a $N(0.59, 0.2^2)$ for inflation and $N(0.9, 0.1^2)$ for output and interest rates. Second-order own lags have a $N(0, 0.25^2)$ prior, while the off-diagonal elements of the first and second lag matrices are specified a priori as $N(0, 0.3^2)$ and $N(0, 0.15^2)$ respectively. Results using a prior centered at the pre-sample OLS estimates of a VAR(2) did not alter our results, although it induced some convergence problems in the MCMC chains in the case of Canada.

a mean of 0.5. We do not normalize the impact of any shocks as is sometimes done in closed-economy models.

5. RESULTS

The following section details a number of properties of the estimated models. The baseline estimates are presented for each country and the model's ability to fit particular second-order characteristics of the data discussed.

5.1. Estimates

Table I reports the estimation results for the baseline model in which the foreign block is observed. The intertemporal elasticity of substitution is a little below unity, taking values around 0.75 for Australia and New Zealand, and a larger value of 1.1 in Canada. The inverse elasticity of labor supply, a parameter notoriously poorly identified in DSGE models, takes values slightly above unity, although it has fairly wide posterior probability bands. Optimal price setting in the production of home goods displays some variation across countries. At the median of our parameter estimates, firms reoptimize prices approximately every 5, 3 and 3 quarters in Australia, Canada and New Zealand, respectively. The latter numbers accord well with survey evidence for the USA in Blinder *et al.* (1998) and values reported in Woodford (2003). Prices in the imported goods sector for these countries are adjusted more frequently than home goods prices, being reoptimized on average every 2.2, 1.7 and 1.4 quarters.

The elasticity of substitution between domestic and foreign goods is somewhat low, with median estimates between 0.6 and 0.76, despite a prior that allows for far larger values. These values have relevance for papers such as Obstfeld and Rogoff (2000), which proposes a model in which a fairly large elasticity of substitution between domestic and foreign goods—together with transaction costs—help explain a number of prominent puzzles in international macroeconomics. In estimated open-economy models, inference on this parameter has tended to produce either very small elasticities, particularly with complete markets, or seemingly implausibly large values; see Rabanal and Tuesta (2005) and Adolfson *et al.* (2005), respectively.

Habit formation appears to play a less prominent role than in other studies, having a maximum value of 0.33 in Australia. Even more surprisingly, price indexation presents a limited source of endogenous persistence in both domestic and imported goods sectors, with coefficient values of at most 0.11. These findings contrast with many closed-economy analyses (see, for example, Christiano *et al.*, 2005; Smets and Wouters, 2003) and the closely related open-economy analysis Justiniano and Preston (2006).¹⁰ The differences relative to closed-economy models are driven by the fact that the open-economy dimension of the model explains some domestic fluctuations. The differences relative to open-economy models is most likely due to the chosen set of shocks: in particular, to the presence of a cost-push shock in the imported good sector as opposed to the pricing of domestically produced goods. Because of this assumption, the persistence of home goods

¹⁰ An earlier version of this paper discussed this property of the estimates in great detail, using posterior odds ratios to examine relative fit across a range of models. Owing to the number of results now reported, this discussion is excluded, but such model comparison exercises would reveal that models that excluded price indexation provide a superior characterization of the data for these three economies. The inclusion or exclusion of these model features matters not for our policy conclusions.

inflation is in large part explained by real factors according to the assumed theory of marginal cost; that is, the autocorrelation of technology and preference shocks imparts inertia in domestic inflation rather than relying on a cost-push shock for the high-frequency variation for the change in home goods prices and a high degree of indexation for its persistence. Regardless of these modeling assumptions, the results on optimal policy are unaffected.¹¹

The policy parameters bear some resemblances across countries. Differences emerge in the responses to inflation, the nominal exchange rate and output growth. The response to inflation is largest in New Zealand and smallest in Australia. The reverse is true for the coefficient on output growth. The estimated responses to the level of output are small, consistent with substantial evidence from closed-economy models. Finally, the response to the nominal exchange rate is largest in Canada, with a coefficient of 0.29. This is consistent with the findings of Lubik and Schorfheide (2003).

As expected, the estimates of the foreign block (excluded due to space considerations) are remarkably similar across countries. Even though the foreign block is exogenous, in the sense that economic developments in each small country under consideration cannot feed back into the foreign series, it is not true that the foreign block is exogenous econometrically speaking. The cross-equation restrictions that result from uncovered interest parity tie the estimates of the foreign data-generating process to domestic parameters.

Finally, the cost-push, preference and risk premium disturbances are highly persistent, having autoregressive coefficients between 0.87 and 0.96 across all three countries. The estimated standard deviations are for the most part plausible, with the biggest differences across countries emerging for the cost-push shock, which is the most volatile disturbance for all economies. The standard deviations are 1.6, 2.01 and 7.27 for Australia, Canada and New Zealand, respectively. It is worth bearing in mind that the terms of trade and exchange rates (nominal and real) in these countries are quite volatile, particularly for New Zealand. Moreover, as we do not normalize these shocks—e.g. modify them such that they enter the corresponding equation with a unit coefficient—their scale is affected by other estimates and hence difficult to interpret at face value.

5.2. Second-Order Properties

Table II presents a set of second-order moments for the data and the corresponding statistics implied by the estimated model. We report medians as well as (5,95)% probability bands for the moments of each DSGE which account for both parameter and small-sample uncertainty.¹² Providing information on second-order properties provides a measure of absolute fit rather than posterior odds ratios, for example, that characterize relative fit.

Taking Australia first, the small open economy model matches the second-order properties of the data quite well. The median implied standard deviations for inflation, the real exchange rate and output are very close to their empirical counterparts. While this is not true for interest rates and the terms of trade, the empirical standard deviations are nonetheless contained in the 90% posterior bands generated by the model, albeit close to their edges. As for persistence, the model does very

¹¹ Had a cost-push shock been included in home goods pricing, as in Justiniano and Preston (2006), it would have explained a significant part of inflation variation and real factors would be less important. Moreover, an earlier version of this paper excluded the cost-push shock in imports, yielding higher estimates of indexation but with the same insights on policy design.

¹² For each parameter draw obtained with the MCMC chains we simulate 500 samples of length equal to the data after discarding the first 50 observations.

Table II. Data and model implied standard deviations and first order autocorrelations. Median and [5, 95] posterior band implied by the estimated baseline model^a

Australia			
	Data SD	Model SD	
Inflation	0.76	0.79	[0.63, 1.00]
Real exchange rate (fd) ^b	4.72	4.78	[4.04, 5.66]
Interest rate	1.09	0.70	[0.46, 1.10]
Output	1.98	1.82	[1.27, 2.79]
Terms of trade (fd)	1.98	2.43	[1.96, 3.01]
	Data autocorrelations	Model autocorrelation	
Inflation	0.63	0.59	[0.42, 0.74]
Real exchange rate (fd)	0.15	0.00	[−0.17, 0.18]
Interest rate	0.97	0.91	[0.82, 0.96]
Output	0.92	0.88	[0.78, 0.94]
Terms of trade (fd)	0.44	0.56	[0.40, 0.70]
Real exchange rate (level)	0.93	0.92	[0.81, 0.97]
Terms of trade (level)	0.93	0.96	[0.91, 0.98]
Canada			
	Data SD	Model SD	
Inflation	0.61	0.61	[0.48, 0.78]
Real exchange rate (fd)	2.25	2.42	[2.06, 2.84]
Interest rate	0.88	0.65	[0.42, 1.05]
Output	2.88	2.34	[1.53, 3.80]
Terms of trade (fd)	1.32	1.74	[1.45, 2.08]
	Data autocorrelations	Model autocorrelation	
Inflation	0.66	0.60	[0.42, 0.75]
Real exchange rate (fd)	0.27	0.07	[−0.10, 0.23]
Interest rate	0.92	0.91	[0.81, 0.96]
Output	0.97	0.93	[0.87, 0.97]
Terms of trade (fd)	0.18	0.49	[0.33, 0.63]
Real exchange rate (level)	0.98	0.93	[0.85, 0.97]
Terms of trade (level)	0.98	0.95	[0.90, 0.98]
New Zealand			
	Data SD	Model SD	
Inflation	0.56	0.77	[0.61, 0.99]
Real exchange rate (fd)	4.05	4.38	[3.68, 5.22]
Interest rate	0.73	0.76	[0.48, 1.23]
Output	2.34	2.80	[1.88, 4.48]
Terms of trade (fd)	2.09	2.55	[2.11, 3.08]
	Data autocorrelations	Model autocorrelation	
Inflation	0.40	0.55	[0.36, 0.72]
Real exchange rate (fd)	0.41	0.02	[−0.15, 0.19]

Table II. (*Continued*)

New Zealand			
	Data SD	Model SD	
Interest rate	0.92	0.93	[0.85, 0.97]
Output	0.88	0.89	[0.79, 0.95]
Terms of trade (fd)	−0.05	0.39	[0.21, 0.54]
Real exchange rate (level)	0.96	0.93	[0.83, 0.97]
Terms of trade (level)	0.87	0.94	[0.88, 0.97]

^a Model standard deviations and first-order autocorrelations are computed by generating, for each parameter draw, 100 replications of length equal to the sample size for each country, after discarding the first 50 observations. For each replication and parameter pair we compute the standard deviation and autocorrelations. We report medians and [5, 95] posterior bands of the implied statistics.

^b fd corresponds to the log first-difference.

well once again for inflation and output, with the 90% interval for the remaining observable series encompassing the autocorrelations in the data, except for the interest rate which is marginally outside its band. Although we use first differences in the real exchange rate and the terms of trade in the estimation, we wish to check that the model can account for the persistence in their levels, since this has been a challenge for open-economy models. In both cases the autocorrelation in the data fall comfortably within the estimated 90% bands for the same statistic in the model.

For Canada, the model provides a similarly reasonable characterization of the data. The model matches the volatility of inflation, output, real exchange rate and interest rate. The only exception is the terms of trade, which are somewhat over-predicted in the model. The serial correlation properties are for the most part well matched, except for the autocorrelation in the real exchange rate, which is outside the posterior bands, particularly for first differences. Finally, for New Zealand similar remarks to Australia apply for the standard deviations. For the autocorrelations, the model matches the corresponding sample moments, with the exception of real exchange rate and terms of trade growth. This is not surprising, given the random walk hypothesis of the exchange rate, and the associated difficulty that structural models have fitting the persistence and volatility of these series; see, for example, Chari *et al.* (2002) and Justiniano and Preston (2006) for calibration and estimation-based studies.

Overall, the model performs reasonably well for all three countries, perhaps with the exception of the serial correlation properties of the log difference in the real exchange rate, a feature shared by most structural and reduced-form open-economy models. Nonetheless, in all countries the level of the real exchange rate is correctly characterized as a very persistent process.

6. MONETARY POLICY DESIGN AND UNCERTAINTY

Recent theoretical analyses have emphasized the importance of pricing to market assumptions for optimal exchange rate and monetary policy. Whether a country has producer currency pricing or local currency pricing can give rise to different policy recommendations, even when the sole objective of policy is to stabilize the aggregate inflation rate. For instance, Devereux and Engel (2003) show in a two-country model with local currency pricing that optimal monetary policy stipulates stabilization of the nominal exchange rate. Similarly, Monacelli (2005), in a model

nested by the one estimated in this paper, shows that deviations from the law of one price lead to a trade-off in the stabilization of inflation and output in the absence of inefficient variations in markups. His analysis overturns the closed-economy result that stabilizing the inflation rate serves to simultaneously stabilize economic activity and introduces an explicit motive to respond to the exchange rate even when consumer prices are the sole objective of policy.

Despite these theoretical contributions there has been relatively little work on policy evaluation in empirical open-economy models. In the small open-economy literature, Smets and Wouters (2002) consider the implications of imperfect pass-through for optimal monetary policy, demonstrating that welfare-maximizing policies introduce a motive to stabilize the exchange rate (see also the references therein). Lubik and Schorfheide (2003), rather than explore the question of optimal policy, instead seek to identify whether in the small open economies considered here (as well as the UK) there is evidence that monetary authorities have responded to nominal exchange rate fluctuations. They find that only in the case of Canada does there exist strong evidence supporting such responses.

The following sections build on these analyses by considering optimal policy in our estimated model. Two exercises are pursued. First we look at the design of optimal monetary policies within the class of the Taylor-type rule adopted in the empirical model. Policy rule coefficients are chosen to minimize a quadratic loss function assuming that the remaining estimated model parameters take their median values. This elucidates whether optimal policy requires nominal interest rates to be adjusted in response to nominal exchange rate fluctuations or not.

Second, we determine the optimal policy rule that takes into account all parameter uncertainty implied by the estimated model. That is, we compute the policy rule that minimizes the expected loss, where expectations are also taken with respect to the posterior distribution of the remaining model parameters. As explained later, this analysis is facilitated by a Bayesian approach, which also allows taking into account the covariance between inferred parameters when quantifying the dispersion around our estimates. This permits addressing an old question of whether parameter uncertainty leads to more cautious policy prescriptions, as suggested by the seminal analysis of Brainard (1967).

6.1. The Optimal Policy Problem

The policymaker seeks to minimize the objective function

$$W_0 = E_0 \sum_{t=0}^{\infty} \beta^t L_t \quad (24)$$

where $0 < \beta < 1$ coincides with the household's discount factor and

$$L_t = \pi_t^2 + \lambda_y y_t^2 + \lambda_i i_t^2 \quad (25)$$

is the period loss at any date $t \geq 0$. The policymaker is therefore assumed to stabilize variation in aggregate consumer price inflation, output and nominal interest rates, where the weights λ_x , $\lambda_i > 0$ determine the relative priority given to each of these objectives. To simplify further, we consider the limiting case of this objective when β goes to unity. This transforms the analysis of the loss function (25) into the analysis of the objective

$$\overline{W}_0(\theta) = \text{var}(\pi_t) + \lambda_y \text{var}(y_t) + \lambda_i \text{var}(i_t)$$

a weighted sum of variances, and θ makes explicit the dependency of the variance calculation on model parameters.

The assumption of arbitrary weights (λ_y, λ_i) , and the assertion that consumer price inflation, output and nominal interest rate variation ought to be stabilized is questionable. To address these concerns the robustness of our conclusions is gauged by analyzing the above loss function as the weights (λ_y, λ_i) are varied over a fine grid on the unit square. Our conclusions are largely unaffected by the precise choice of weights in the objective function. For presentation purposes, we focus on how varying the relative weight on output stabilization affects outcomes, since this dimension has played a prominent role in the analysis of optimal policy (see Svensson, 1999, 2000).

Attention is restricted to optimal policies within a class of Taylor-type rules of the form

$$i_t = \rho_i i_{t-1} + \psi_\pi \pi_t + \psi_y y_t + \psi_{\Delta y} \Delta y_t + \psi_e \Delta e_t. \quad (26)$$

As in estimation, policy is assumed to adjust nominal interest rates in response to contemporaneous values of inflation, output, output growth, the nominal exchange rate growth and lagged observations of the nominal interest rate. Note that the response coefficients in equation (26) are not multiplied by $(1 - \rho_i)$ as in (23) since we wish to consider the possibility of rules having ρ_i very close to one. Care should be taken in comparing the optimal policy coefficients described in subsequent sections to the corresponding estimated policy parameters of Section 5.

To fix notation, partition the estimated parameters for a given model as $\theta = \{\theta_p, \theta_s\}$ where θ_s collects structural parameters other than those determining policy, denoted $\theta_p = \{\rho_i, \psi_\pi, \psi_y, \psi_{\Delta y}, \psi_e\}$. Conformably partition the associated parameter space as $\Theta = \{\Theta_p, \Theta_s\}$.¹³ Let $\bar{\theta}_s$ denote the estimated median value of the structural parameters.

6.2. Optimal Policy under Parameter Certainty

In our first policy experiment the optimal policy coefficients are chosen assuming the structural parameters are known and equal to $\bar{\theta}_s$, the median of the MCMC draws. Thus the effects of parameter uncertainty are ignored and optimal policy is determined as

$$\theta_p^* = \arg \min_{\theta_p \in \Theta_p} \bar{W}_0(\theta_p \mid \bar{\theta}_s)$$

where the minimization is subject to the constraints that policy is given by (26) and aggregate dynamics are as determined in Section 2. The final restriction placed on the policy design is that the coefficient on the lagged nominal interest rate must satisfy $0 \leq \rho_i \leq 1$. The study of super-inertial interest rate rules is left for future research.

Table III provides results for this optimal policy problem for three different objective functions, which differ according to the weight assigned to output stabilization.¹⁴ Consider the results for Australia when output is assigned a weight of $\lambda_y = 0$ in the objective function. Optimal policies are highly inertial, characterized by a unit coefficient on the lagged interest rate, prescribing the stance of policy in terms of the evolution of the first difference of nominal interest rates, rather

¹³ Note that the MCMC posterior simulator produces joint and marginal posterior densities which validate this approach.

¹⁴ As throughout the paper, the variances correspond to median volatilities from 100 samples of length equal to the data after discarding the first 50 observations. Qualitatively, results are unchanged when using the asymptotic variances instead. The approach pursued here has the advantage of incorporating small-sample uncertainty into the analysis.

than the level. The optimal response to inflation may seem smaller relative to typical estimates of this parameter and the estimated policy reaction function in Section 5. Recall, however, that these are not multiplied by one minus the coefficient on lagged interest rates, and that optimal policies exhibit a greater degree of inertia. In contrast, the response to output, output growth and the nominal exchange rate is zero to the second decimal place.

The second and third columns give results for an objective function that places greater weight on output stabilization. The response to output and output growth tends to rise with greater concern for output variability. For a unit weight on output in the objective function, both coefficients are roughly ten times that observed in the first column. Concomitantly, the variances of inflation and output under optimal policy increase and decline, respectively, as greater weight is placed on output stabilization. Hence a Taylor frontier is mapped out, delineating the inherent inflation–output stabilization trade-off present in this model. Regardless of the relative weights appearing in the objective function, it is never optimal to respond strongly to nominal exchange rate variations.

This last result is particularly surprising: despite the open-economy dimension of the model and the existence of deviations from the law of one price, optimal policy does not prescribe a direct response to exchange rate fluctuations to ensure that its objectives of stable output and inflation are met. This is at odds with the theoretical literature which underscores that models characterized by local currency pricing should give cause to respond to exchange rate fluctuations. Furthermore, it also suggests the finding of Lubik and Schorfheide (2003), of little evidence that the Reserve Bank of Australia has responded to exchange rate fluctuations, is part of an optimal policy framework, at least in this restricted family of Taylor-type rules.

The broad theme of these results is applicable to Canada and New Zealand. Policies are highly inertial, leading to a difference rule for the nominal interest rate. Both countries respond more aggressively to inflation than does Australia, and both show little response to the level of real economic activity regardless of the objective function. Higher preference for output stabilization is associated with stronger responses to output growth. Again there is little evidence supporting the desirability of policies responding to the nominal exchange rate.

These conclusions are valid regardless of the weight placed on output and interest rate stabilization. Figure 1 plots the optimal exchange rate coefficient as the weights (λ_y, λ_i) are varied on the unit square. This coefficients attains maximum values of 0.01, 0.05 and 0.02 for Australia, Canada and New Zealand.¹⁵

Further insights into the characteristics of these optimal rules emerge from comparing the standard deviations under optimal policy and the volatility observed in the data. For a zero weight on output stabilization, the standard deviations of inflation implied by these policy rules are 0.13, 0.03 and 0.06 for Australia, Canada and New Zealand (see Table III). Comparison to Table II makes clear that optimal policy implies very limited variation in inflation relative to historical data, roughly approximating inflation targeting. As output stabilization becomes relatively important, the case for strict inflation targeting weakens for obvious reasons, though the implied volatilities for inflation are close those observed in the data for the case of Australia and Canada. This is consistent with the notion of flexible inflation targeting (see Svensson, 1997, 1999).

¹⁵ Note that the coefficient magnitudes themselves are not sufficient to infer the relevance of the exchange rate—one also must consider the magnitude of exchange rate variations. Furthermore, a one standard deviation change in nominal exchange rate growth would imply—in partial equilibrium—at most a 10 basis point increase in nominal interest rates for the case of Canada. The response coefficients for Australia and New Zealand are considerably smaller.

Table III. Optimal policy and uncertainty for baseline

	Median of draws ^a			Over draws ^b		
	Relative weight on output			Relative weight on output		
	0	0.5	1	0	0.5	1
Panel A: Australia						
<i>Coefficients</i>						
Interest rate	1.00	1.00	1.00	1.00	1.00	1.00
Inflation	1.01	0.91	0.80	1.02	1.14	1.07
Output	0.00	0.06	0.10	0.00	0.06	0.08
Nominal exchange rate	0.00	0.00	0.00	0.00	0.00	0.01
Output growth	0.28	2.02	3.15	0.25	2.06	3.07
<i>Variance</i>						
Inflation	0.13	0.59	0.87	0.11	0.53	0.78
Interest rates	0.31	0.35	0.47	0.29	0.34	0.45
Output	4.38	1.44	0.89	5.18	2.39	1.80
Loss	0.44	1.66	2.22	0.40	2.06	3.04
Panel B: Canada						
<i>Coefficients</i>						
Interest rate	1.00	1.00	1.00	1.00	1.00	1.00
Inflation	2.18	2.27	1.57	2.09	2.38	1.85
Output	0.00	0.01	0.04	0.00	0.01	0.03
Nominal exchange rate	0.00	0.02	0.01	0.00	0.00	0.02
Output growth	0.25	2.57	3.10	0.24	2.72	3.62
<i>Variance</i>						
Inflation	0.03	0.39	0.85	0.03	0.38	0.79
Interest rates	0.20	0.20	0.30	0.22	0.22	0.32
Output	7.38	5.59	4.83	7.14	5.31	4.62
Loss	0.23	3.39	5.97	0.25	3.26	5.73
Panel C: New Zealand						
<i>Coefficients</i>						
Interest rate	1.00	1.00	1.00	1.0	1.0	1.0
Inflation	1.49	1.91	1.48	1.4	1.7	1.5
Output	0.00	0.03	0.07	0.0	0.0	0.1
Nominal exchange rate	0.00	0.02	0.01	0.0	0.0	0.0
Output growth	0.18	1.88	2.56	0.1	1.7	2.5
<i>Variance</i>						
Inflation	0.06	0.72	1.56	0.07	0.73	1.54
Interest rates	0.34	0.28	0.37	0.32	0.28	0.37
Output	10.53	7.81	6.53	11.59	8.63	7.40
Loss	0.40	4.90	8.45	0.39	5.33	9.31

^a Optimal coefficients are obtained by minimizing the weighted sum of variances for inflation, nominal interest rates and output, with equal weights on inflation and interest rates, but varying the relative weight on output. All parameters other than those in the Taylor-type rule are fixed at the median of the MCMC estimates. The variances are obtained by simulation with the same settings as reported in Table II.

^b In optimizing over the draws, we use a subset of 5000 draws, taken at equally spaced intervals, from the generated samples obtained with the MCMC simulator. For each candidate set of policy parameters we compute the loss over these draws and average the resulting loss. Once again, variances are obtained by simulation.

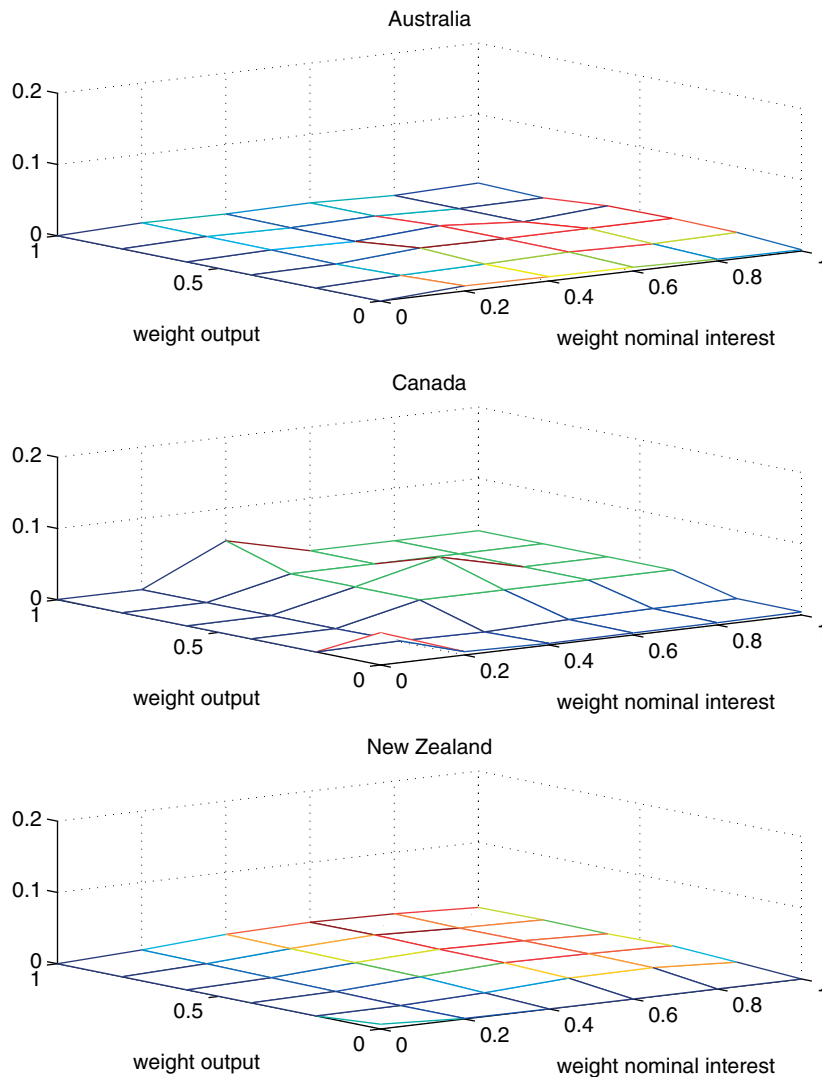


Figure 1. Optimal coefficient on exchange rate as weights vary. This figure is available in color online at www.interscience.wiley.com/journal/jae

6.3. Sourcing the Result

The striking result from these optimal policy exercises is the lack of response of nominal interest rates to exchange rate fluctuations. One interpretation of this finding is that the trade-off generated by deviations from the law of one price is not particularly important for imported goods price inflation dynamics and therefore CPI inflation dynamics. However, this is not generically true for the presented theoretical model. Consequently, it is worth considering further why the empirical model identifies a parameter configuration that engenders optimal policies without an active role for exchange rate stabilization.

The finding that it is not optimal to respond to the exchange rate can be sourced to two features of the empirical model. First, there exists a ‘disconnect’ of the real and nominal exchange rates from the remaining domestic series; see Obstfeld and Rogoff (2000), among others, for a detailed discussion. Indeed, variance decompositions reveal that cost-push shocks in the imported goods sector and risk premium shocks together account for 84%, 69% and 83% of the variation in nominal exchange rates in Australia, Canada and New Zealand, respectively, with almost identical shares for the real exchange rate. At the same time, both shocks play a substantially more muted role for inflation, output and domestic interest rates.¹⁶ A consequence of this disconnect is that by responding to the exchange rate monetary policy ties the evolution of the domestic economy to cost-push and risk premium shocks, and may give rise to increased variability.

Second, even if risk premium and cost-push shocks are not negligible for output, inflation and interest rate variations, policy responses to stabilize the exchange rate exacerbate variability in these series. Figures 2 and 3 shed light on these mechanisms, presenting impulse responses of various series for cost-push and risk premium shocks in the case of Australia. Similar insights hold for the other two countries. Three impulse responses are shown for each variable, each being associated with three different policy coefficients on the exchange rate. The baseline with $\phi_e = 0$ corresponds to the optimal policy coefficients when $\lambda_y = 0.5$ and remaining model parameters as shown in Table I.¹⁷ The second and third impulse responses are generated assuming, counterfactually, that $\phi_e = 0.2$ and $\phi = 0.4$, holding all other parameters fixed.

Consider the case of the cost-push shock when $\phi_e = 0$ (solid lines). An innovation to this disturbance causes an appreciation (i.e. decline) in the exchange rate (nominal and real) and a negative deviation in the law of one price gap. Because the latter is the marginal cost of imported goods, some of the direct effect of the cost-push shocks on imported goods price inflation is offset. Regardless, imported goods prices rise substantially, leading domestic demand to shift towards domestically produced goods, although price responses in the home goods sector are rather muted. Nominal and real interest rates fall slightly.

Increasingly strong responses to the exchange rate (dashed lines) tend to counteract the degree of exchange rate appreciation, which reduces the decline in the marginal costs of imported goods and leads to larger price pressures in this sector. Greater declines in real rates also exacerbate domestic price inflation: a given-sized cost-push shock is therefore more inflationary. Moreover, the stronger response to the exchange rate triggers larger variations in nominal interest rates and output. As a result, responding to exchange rates induces increased variability and larger losses in equation (25).

In the case of risk premium shocks, the depreciation (i.e. increase) in the exchange rate calls for an interest rate tightening. This has two counteracting effects on inflation. On the one hand, responding to exchange rate movements serves to stabilize imported goods price inflation. On the other hand, higher nominal and real interest rates tend to cause a contraction in domestic activity: output and domestic inflation fall. This might suggest that there is some scope to stabilize inflation through an exchange rate channel. However, two points should be made. First, responding more aggressively to exchange rate variations leads to larger movements in nominal rates and a

¹⁶ For Australia the combined variance share for risk premium and cost-push shocks in output, inflation and interest rates are 16%, 17% and 22%, respectively; for Canada and the same series order: 3%, 15% and 16%; while for New Zealand these shocks combined explain 1%, 21% and 19% respectively.

¹⁷ Similar insights result from using the estimated—as opposed to optimal—policy coefficients.

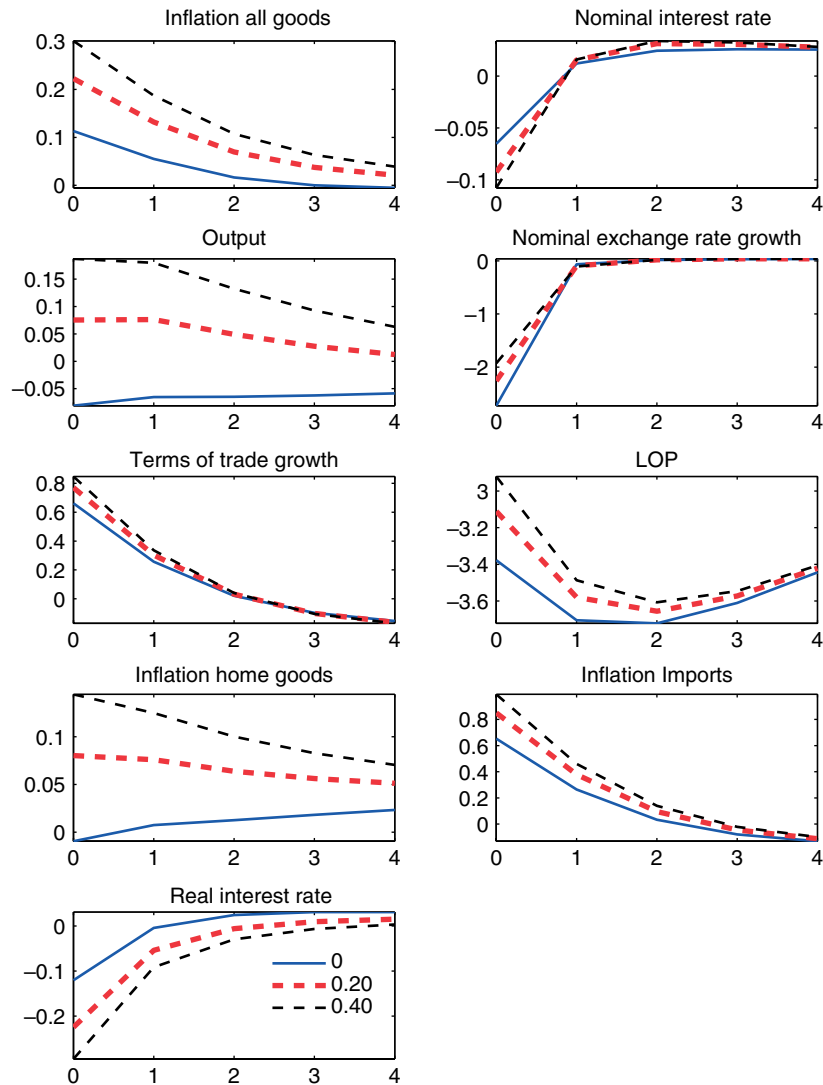


Figure 2. Impulse responses to import cost-push shock as coefficient on exchange rate varies. For Australia using optimal coefficients when weight on output is 0.5 (Table III). Optimal coefficient on exchange rate: solid line; counterfactually increased to 0.2: longer dashed line; further counterfactual increase to 0.4: shorter dashed line. This figure is available in color online at www.interscience.wiley.com/journal/jae

larger contraction in domestic activity—these effects outweigh the positive stabilizing influence on import goods price inflation, leading to larger losses.

Second, the optimal policy rules determined in the previous section are not conditional on a given shock. They are unconditional optimal policies. While our discussion of the effects of cost-push shocks and risk premium shocks can provide intuition for why more aggressive exchange rate policy is undesirable, it by no means rules out the possibility that, conditional on a single shock,

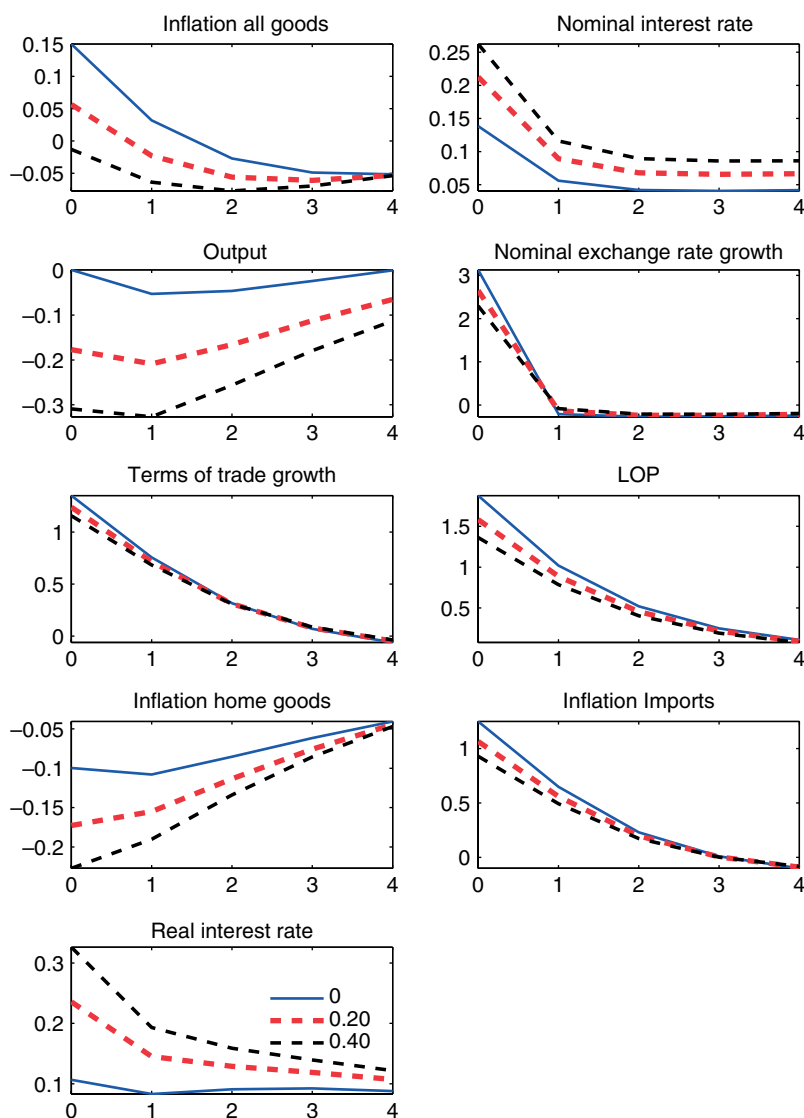


Figure 3. Impulse responses to risk premium shock as coefficient on exchange rate varies. For Australia using optimal coefficients when weight on output is 0.5 (Table III). Optimal coefficient on exchange rate: solid line; counterfactually increased to 0.2: longer dashed line; further counterfactual increase to 0.4: shorter dashed line. This figure is available in color online at www.interscience.wiley.com/journal/jae

there may be welfare improvements from managing exchange rate variations. However, taking into account all sources of variation and the associated property of exchange rate disconnect, our results suggest that stabilizing exchange rate fluctuations is undesirable.

These findings differ from Smets and Wouters (2002), who present evidence in an empirical small open-economy model with local currency pricing that optimal policy does respond to

exchange rate fluctuations.¹⁸ While the precise details of the underlying models differ, they do have the same basic elements. There are two sources of discrepancy in the two studies worth mentioning. First, Smets and Wouters use the theoretical-based output gap for their analysis, while we work with detrended output (see footnote 6). Second, our analyses differ in the estimation methodology. Smets and Wouters estimate a small subset of model parameters by matching impulse response functions. Our conjecture is that confronting the model with data on a greater number of dimensions, as done in the likelihood-based estimation procedure of this paper, engenders considerably different second-order moments, which in turn delivers different optimal policy prescriptions. Given these differences in policy implications, future research should attempt to sort out the effects of these alternative assumptions and estimation procedures on the characterization of optimal policy.

6.4. Optimal Policy under Parameter Uncertainty

We now determine the optimal policy that takes into account the effects of uncertainty regarding θ_s on the choice of optimal policy coefficients. The policy problem is

$$\hat{\theta}_p^* = \arg \min_{\theta_p \in \Theta_p} \int_{\Theta_s} \bar{W}_0(\theta_p | \theta_s) p(\theta_s | Y_t) d\theta_s$$

where minimization is subject to the same constraints as before and where $p(\theta_s | Y_t)$ is the estimated posterior distribution of the structural parameters. In determining the optimal policy coefficients the policymaker integrates out the uncertainty surrounding structural parameters by making use of the posterior distribution for these parameters obtained from model estimation.¹⁹ In contrast to Section 6.2, this problem accounts for the covariance across all estimated model parameters, including the standard deviations of the shocks, which are part of θ_s .

Columns 4–6 of Table III report results of this exercise based on 5000 draws for three objective functions.²⁰ For Australia, and a zero weight on output in the objective function, there is little evidence of attenuated policy responses once parameter uncertainty is taken into account—compare column 1. Indeed, optimal policies are virtually identical. As the preference for output stabilization increases, the response to both output growth and inflation rise, while other coefficients are roughly unchanged. Hence, optimal policy under parameter uncertainty demands more aggressive policy in response to inflation variations when output stabilization is relatively important. Concomitantly, the inflation and output variances attached to these policies are lower and higher than when model parameters are known with certainty to the policymaker. Brainard's seminal insight appears not to hold true in this estimated small open-economy model. Note that

¹⁸ This accords with the analysis of Batini and Pearlman (2007), which considers the role of balance sheet effects in a calibrated model.

¹⁹ It is important to note that this second approach to policy design, which entails discarding the draws of the policy parameters and retaining those of the non-policy block to represent $p(\theta_s | Y_t)$, is consistent with our estimation of the DSGE models. This is because Bayesian MCMC methods yield draws that correspond to the marginal densities of the model parameters. What we would not have been able to do, given our approach to inference, is to make any statements that required the conditional densities, say $p(\theta_s | Y_t, \theta_p)$, since we do not have samples from these ordinates in the estimation.

²⁰ As in Section 6.2, we also account for small-sample uncertainty. For each parameter draw we generate 100 artificial samples of length equal to the data, after discarding the initial 50 observations. Optimal policy hence minimizes the average loss over 250,000 samples. Computational capacity prevents using all parameter draws generated by the MCMC. However, the dispersion in a sample of 5000 is almost identical to that in the pooling of all draws, since the former are closer to an ideal independent sample.

uncertainty does effect outcomes, judging from the substantially larger losses in output in the last three columns, which is rationalized by the larger responses to inflation already mentioned.

For Canada, results are broadly similar. In the case of a low weight on output stabilization, $\lambda_y = 0$, there is little change in the optimal policy coefficients relative to the certain parameter case. As output stabilization becomes a greater priority, policy becomes more aggressive when compared to the certain parameter case not only for output but, as in Australia, for inflation as well. As before, uncertainty does not engender attenuated policy responses. And, in contrast to the Australian case, the variability of output need not increase once uncertainty is taken into account.

New Zealand reveals yet a different pattern of results. For objective functions giving less weight to output stabilization, policy response coefficients tend to be attenuated. This is true for both $\lambda_y = 0$ and $\lambda_y = 0.5$. When a unit weight is given to output stabilization, the optimal policy coefficients under uncertainty are roughly equal to those obtained ignoring the dispersion in the non-policy parameters. Despite this near equality on policy coefficients, taking into account uncertainty produces larger output losses.

Taken together, the results indicate that parameter uncertainty fails to have clear implications for the design and outcomes of simple optimal monetary rules. Depending on the country at hand, more or less aggressive policy responses might obtain. As Chow (1975) notes, in a multivariate setting the conclusions of Brainard (1967) for attenuation in policy need not hold, depending on the covariance properties of the uncertain model parameters. Similarly, the robust control literature on optimal policy design demonstrates that model uncertainty can lead to more aggressive policy settings (see Giannoni, 2002). In addition, the associated losses may be larger or smaller once we account for parameter uncertainty, with differences stemming mostly from the variability of output. It follows that resolution of the implications of uncertainty for policy design is largely an empirical matter.

What is clear from the present analysis is that, regardless of whether policymakers face parameter uncertainty or not, the optimal coefficients on the exchange rate are always small. This is because of the exchange rate disconnect property and the additional variability in output, inflation and interest rates engendered by stabilizing the exchange rate in the estimated model described earlier.

7. ROBUSTNESS AND IDENTIFICATION

This section turns to some robustness exercises and discussion of identification in our empirical model.

7.1. Unobserved Foreign Block

Rather than modeling the foreign block as being driven by a VAR in observed US inflation, output and nominal interest rates, we instead treat this component of the model as unobserved following the analysis of Lubik and Schorfheide (2003). Two observations motivate this alternative specification. First, while for Canada the use of US data as proxy for the foreign block may be plausible, it seems less appropriate in the case of Australia and New Zealand, where construction of trade-weighted indices of the relevant foreign variables—including, for instance, Japan—would be more desirable. Furthermore, this renders the model more agnostic about the precise nature of the foreign disturbances and allows evaluating the sensitivity of results to the choice of observables used in estimation. Second, and related to this last point, the estimated model of Section 2 is

prone to some of the difficulties detailed in Justiniano and Preston (2006). In particular, variance decompositions reveal a limited role for foreign-sourced disturbances in the evolution of domestic variables. The following investigates whether it is this feature of the model which engenders a negligible role for stabilizing exchange rate fluctuations in the design of optimal policy rules.

We assume that foreign output, inflation and interest rate shocks follow second-order autoregressive processes. The priors used in estimation coincide with those employed in Section 4, with appropriate adjustments arising from the different treatment of the foreign block.

Table IV presents the resulting estimates. For all three countries, while the intertemporal elasticity of substitution is very similar to the baseline model (observable foreign block), the inverse Frisch elasticity is slightly higher here. The Calvo parameters are quite stable as well, except for the degree of stickiness in home goods prices for Canada, which is substantially larger with an unobserved foreign block. Canada also exhibits a greater degree of habit persistence and more aggressive responses to inflation relative to the baseline model. Regarding the properties of shocks, risk premium disturbances are less persistent for all three countries, while disturbances to foreign interest rates are somewhat more volatile.

These parameter shifts largely take place to exploit the flexibility permitted by having an unobserved foreign block. Because the model is no longer constrained to fit the US time series it is free to exploit the variation inherent in these shocks to fit the domestic observable series. In particular, the restriction imposed by interest parity would seem to be substantially loosened here. Not surprisingly, foreign disturbances are now found to explain a greater fraction of the variation in domestic observables than in the model with an observable foreign block.

Given these estimates, we revisit the optimal policy exercises conducted earlier: the results are reported in Table V. Casual inspection reveals the optimal policy coefficients on the nominal exchange rate to be less than 0.02 when model parameters are known with certainty to policymakers, and less than 0.05 when model parameters are uncertain. As noted earlier, for these response coefficients a one standard deviation movement in the exchange rate implies a very small change in nominal interest rates. The intuition for this finding is similar to the baseline case: exchange rate disconnect divorces movements in the exchange rate from movements in other domestic series. With the foreign block unobserved this disconnect is less striking, particularly for output, than when the foreign block is observed. Nonetheless, having monetary policy respond to exchange rate movements forces inflation and interest rates to inherit the variability of risk premium and particularly cost-push shocks. This increase in their variance results in larger losses.²¹

As to the question of whether parameter uncertainty leads to cautious or aggressive policy, the results portray a mixed message once again. Depending on the country, the weight given to output stabilization, and the particular policy coefficient under consideration, policy can be more or less aggressive. This is consistent with the theory referenced earlier. As for outcomes, the resulting losses may differ, sometimes substantially, once parameter uncertainty is accounted for. This is mostly due to the variance of output and aligns well with the changes in optimal coefficients. We conclude that the policy implications of parameter uncertainty are model and data specific and must be examined carefully on a case-by-case basis.

²¹ In an earlier version of this paper we did not use the terms of trade and had also arbitrarily removed a few shocks, to force an even greater role for the unobserved foreign disturbances. Nonetheless, optimal policy was once again characterized by a lack of response to the exchange rate.

Table IV. Posterior estimates for unobserved foreign block^a

Coefficients		Posterior ^b								
		Australia			Canada			New Zealand		
		Median	SD	[5, 95] prob.	Median	SD	[5, 95] prob.	Median	SD	[5, 95] prob.
Inverse intertemporal elasticity of Substitution	σ	1.37	0.35	[0.88, 2.01]	0.91	0.30	[0.53, 1.50]	1.24	0.30	[0.80, 1.79]
Inverse Frisch	φ	1.15	0.60	[0.46, 2.37]	1.46	0.65	[0.64, 2.74]	1.29	0.69	[0.46, 2.71]
Calvo domestic prices	θ_H	0.80	0.08	[0.61, 0.87]	0.82	0.06	[0.69, 0.89]	0.65	0.07	[0.53, 0.77]
Calvo import prices	θ_F	0.52	0.07	[0.40, 0.62]	0.38	0.06	[0.28, 0.48]	0.29	0.05	[0.21, 0.38]
Elasticity H-F goods	η	0.63	0.07	[0.55, 0.77]	0.69	0.07	[0.61, 0.82]	0.73	0.08	[0.62, 0.87]
Habit	h	0.35	0.09	[0.19, 0.50]	0.54	0.09	[0.39, 0.68]	0.09	0.06	[0.02, 0.21]
Indexation domestic	δ_H	0.05	0.05	[0.01, 0.16]	0.05	0.05	[0.01, 0.15]	0.13	0.12	[0.03, 0.39]
Indexation foreign	δ_F	0.07	0.08	[0.01, 0.25]	0.11	0.10	[0.02, 0.33]	0.10	0.10	[0.02, 0.33]
Taylor rule, smoothing	ψ_i	0.84	0.03	[0.77, 0.88]	0.77	0.04	[0.70, 0.82]	0.81	0.03	[0.76, 0.86]
Taylor rule, inflation	ψ_π	1.82	0.22	[1.48, 2.19]	1.79	0.18	[1.53, 2.10]	2.26	0.25	[1.90, 2.71]
Taylor rule, output	ψ_y	0.09	0.06	[0.03, 0.22]	0.10	0.04	[0.04, 0.16]	0.05	0.03	[0.02, 0.11]
Taylor rule, exchange rate	$\psi_{\Delta e}$	0.13	0.04	[0.07, 0.20]	0.33	0.07	[0.23, 0.46]	0.06	0.03	[0.03, 0.12]
Taylor rule, output growth	$\psi_{\Delta y}$	0.71	0.23	[0.36, 1.14]	0.63	0.16	[0.39, 0.91]	0.42	0.13	[0.23, 0.65]
Technology	ρ_a	0.66	0.14	[0.46, 0.92]	0.78	0.10	[0.59, 0.91]	0.87	0.06	[0.76, 0.95]
Preferences	ρ_g	0.92	0.02	[0.87, 0.95]	0.90	0.03	[0.85, 0.94]	0.92	0.03	[0.87, 0.96]
Risk premium	ρ_{rp}	0.86	0.10	[0.64, 0.95]	0.81	0.12	[0.58, 0.94]	0.81	0.11	[0.58, 0.94]
Import cost-push shock	ρ_{cp}	0.96	0.03	[0.90, 0.98]	0.98	0.01	[0.95, 0.99]	0.98	0.01	[0.97, 0.99]
sd foreign inflation	sd_{π^*}	0.26	0.11	[0.15, 0.50]	0.30	0.27	[0.16, 0.84]	0.27	0.19	[0.15, 0.66]
sd foreign output	sd_{y^*}	0.34	0.38	[0.16, 1.47]	0.41	0.29	[0.17, 1.07]	0.58	0.81	[0.18, 2.60]
sd foreign interest rates	sd_{i^*}	0.28	0.09	[0.17, 0.47]	0.17	0.04	[0.12, 0.25]	0.21	0.10	[0.14, 0.37]
sd technology	sd_a	0.37	0.11	[0.28, 0.62]	0.27	0.08	[0.20, 0.45]	0.90	0.34	[0.53, 1.65]
sd Taylor rule	sd_{mp}	0.26	0.03	[0.22, 0.32]	0.25	0.03	[0.21, 0.31]	0.23	0.03	[0.19, 0.28]
sd preferences	sd_g	0.17	0.03	[0.13, 0.23]	0.20	0.03	[0.15, 0.26]	0.24	0.04	[0.17, 0.32]
sd risk premium	sd_{rp}	0.30	0.11	[0.16, 0.51]	0.18	0.04	[0.12, 0.26]	0.22	0.08	[0.14, 0.37]
sd import cost-push	sd_{cp}	1.86	0.73	[1.11, 3.45]	2.36	0.71	[1.50, 3.79]	7.10	2.13	[4.33, 10.97]

^a Priors and calibrated parameters are the same as in Table I, except for the foreign block, which is now driven by independent AR(2) processes for the latent foreign variables.

^b Corresponds to median and posterior percentiles from 5 MCMC chains of 100,000 draws each, in which 40,000 draws were used as an initial burn-in phase, and only one in every ten draws retained from the remaining 60,000, in each chain. Convergence diagnostics were assessed using trace plots and the potential scale reduction factors for the variance and 95% posterior intervals.

Table V. Optimal policy and uncertainty when foreign block is unobserved

	Median of draws ^a			Over draws ^b		
	Relative weight on output			Relative weight on output		
	0.0	0.5	1.0	0.0	0.5	1.0
Panel A: Australia						
<i>Coefficients</i>						
Interest rate	1.00	1.00	1.00	1.00	1.00	1.00
Inflation	0.92	0.81	0.71	0.73	0.82	0.79
Output	0.00	0.09	0.14	0.00	0.07	0.10
Nominal exchange rate	0.00	0.01	0.00	0.01	0.04	0.05
Output growth	0.24	1.88	3.01	0.14	1.58	2.58
<i>Variance</i>						
Inflation	0.13	0.58	0.82	0.15	0.56	0.87
Interest rates	0.30	0.38	0.50	0.42	0.51	0.64
Output	4.37	1.22	0.70	6.91	3.80	3.18
Loss	0.43	1.57	2.02	0.56	2.97	4.69
Panel B: Canada						
<i>Coefficients</i>						
Interest rate	1.00	1.00	1.00	1.00	1.00	1.00
Inflation	1.05	0.59	0.53	0.90	0.83	0.85
Output	0.00	0.05	0.06	0.00	0.04	0.01
Nominal exchange rate	0.00	0.01	0.00	0.02	0.02	0.02
Output growth	0.13	1.94	3.29	0.01	1.98	3.59
<i>Variance</i>						
Inflation	0.06	0.78	1.13	0.06	0.78	1.23
Interest rates	0.21	0.42	0.64	0.24	0.41	0.67
Output	8.95	1.96	1.14	11.04	3.43	2.46
Loss	0.27	2.17	2.91	0.31	2.91	4.35
Panel C: New Zealand						
<i>Coefficients</i>						
Interest rate	1.00	1.00	1.00	1.00	1.00	1.00
Inflation	1.52	1.86	1.56	1.38	2.01	1.54
Output	0.00	0.02	0.05	0.00	0.01	0.01
Nominal exchange rate	0.00	0.02	0.01	0.00	0.02	0.02
Output growth	0.12	1.75	2.58	0.10	1.62	2.13
<i>Variance</i>						
Inflation	0.04	0.63	1.44	0.06	0.58	1.25
Interest rates	0.27	0.24	0.33	0.27	0.24	0.29
Output	13.01	10.44	9.20	12.24	9.91	8.92
Loss	0.31	6.08	10.96	0.33	5.78	10.46

^a Optimal coefficients are obtained by minimizing the weighted sum of variances for inflation, nominal interest rates and output, with equal weights on inflation and interest rates, but varying the relative weight on output. All parameters other than those in the Taylor-type rule are fixed at the median of the MCMC estimates in Table IV. The variances are obtained by simulation with the same settings as reported in Table II.

^b In optimizing over the draws, we use a subset of 5000 draws, taken at equally spaced intervals, from the generated samples obtained with the MCMC simulator. For each candidate set of policy parameters we compute the loss over these draws and average the resulting loss. Once again, variances are obtained by simulation.

7.2. Matters of Identification

A number of recent papers have addressed identification problems and conditions for identification in medium-scale dynamic stochastic general equilibrium models. Lubik and Schorfheide (2005) and Justiniano and Preston (2006) discuss specific identification issues in open-economy models. More general discussions are provided by Beyer and Farmer (2005), Fukac *et al.* (2006), Canova and Sala (2005), Cochrane (2007) and Iskrev (2007). These papers explore a range of identification issues emerging from both the nature of estimation—method of moments and likelihood-based estimation—and economic structure. Adolfson and Linde (2007) perform a number of Monte Carlo exercises to examine local identification in a medium-scale small open-economy model. Collectively, these papers underscore that identification problems can plague estimation of models of the kind analyzed here.

While considerable care was taken to ensure estimation resulted in a unique mode for our baseline model, when the foreign block is treated as unobserved a non-trivial identification issue arises for Australia. Two modes are estimated that achieve almost identical posterior densities. Table VI reports parameters that exhibit differences across these two modes, together with the associated log posteriors. Most notable are the higher degree of nominal rigidity in home good prices, θ_H , for the first mode and the greater persistence and volatility of technology shocks for the second mode (ρ_a and sd_a).

The variance decompositions in Table VII evidence that these two parameter configurations imply rather different contributions of shocks for output and inflation. Preference shocks explain almost half of inflation variability in the second mode, compared to 34% of its variance in the first mode. The reverse pattern is true for technology shocks (20% versus 38%). In contrast, technological disturbances explain the bulk of output variations in the second mode (92% variance share), while they retain an important but more modest role in the first mode (38%).

It is difficult to isolate how individual parameters affect these results. Scrutiny of unreported impulse response functions suggests that for inflation the changing contribution of shocks is mostly attributable to differences in the estimated Calvo parameter for home goods. Indeed, the lower degree of price stickiness in the second mode rationalizes larger responses to preference shocks, all else equal, and a more muted response to technology disturbances. This is a salient difference of the impulse response functions across these two modes. As for output, the higher variance

Table VI. Selected coefficients from the two modes for Australia when foreign block is unobserved^a

Coefficients		First mode	Second mode
Inverse intertemporal ES	σ	1.43	1.30
Calvo domestic prices	θ_H	0.82	0.62
Elasticity H-F goods	η	0.59	0.71
Habit	h	0.38	0.23
Taylor rule, inflation	ψ_π	1.71	1.91
Taylor rule, output growth	$\psi_{\Delta y}$	0.75	0.52
Technology	ρ_a	0.65	0.93
sd technology	sd_a	0.31	0.51
sd import cost-push	sd_{cp}	1.88	2.22
Log posterior		−870.07	−870.54

^a Priors are as in Table IV.

Table VII. Variance decomposition for all-goods inflation and output in Australia for two modes when foreign block is unobserved^a

Series/shock	Foreign shocks	Neutral technology	Monetary policy	Preference	Risk premium	Import cost-push
Panel A. First mode						
Inflation	0.11	0.38	0.10	0.34	0.02	0.04
Output	0.08	0.30	0.08	0.25	0.00	0.29
Panel B. Second mode						
Inflation	0.10	0.20	0.19	0.46	0.03	0.02
Output	0.01	0.92	0.01	0.02	0.00	0.03

^a Stationary variance decomposition at each of the modes reported in Table VI for Australia.

Table VIII. Optimal Policy for Two Modes in Australia when Foreign Block is Unobserved¹

	First mode			Second mode		
	Relative weight on output			Relative weight on output		
	0.0	0.5	1.0	0.0	0.5	1.0
<i>Coefficients</i>						
Interest rate	1.00	1.00	1.00	1.00	1.00	1.00
Inflation	0.79	0.74	0.68	1.76	2.01	1.83
Output	0.00	0.07	0.09	0.00	0.00	0.01
Nominal exchange rate	0.00	0.00	0.00	0.00	0.00	0.02
Output growth	0.20	1.98	3.34	0.00	1.69	2.71
<i>Variance</i>						
Inflation	0.13	0.49	0.67	0.02	0.22	0.51
Interest rates	0.28	0.39	0.49	0.27	0.25	0.27
Output	4.24	1.14	0.71	6.13	5.18	4.75
Loss	0.41	1.45	1.87	0.29	3.06	5.53

¹ Optimal coefficients are obtained by minimizing the weighted sum of variances for inflation, nominal interest rates and output, with unit weights on inflation and interest rates, but varying the relative weight on output. As in Tables III and V, the variances are obtained by simulation. The two modes are reported in Table VI.

and autocorrelation of technology shocks accounts, at least in part, for the drastic increase in the contribution of these shocks for the second mode, despite the lower degree of nominal rigidities.

Table VIII characterizes optimal policy assuming policymakers treat as certain the parameters from each individual mode, as opposed to the median of the draws reported in Table V. While policy remains highly inertial—a difference rule for the nominal interest rate is optimal—the prescribed optimal policy coefficients are rather different for inflation and output growth. The first mode has much weaker response coefficients to inflation for all weights on output stabilization. In contrast, optimal policy tends to respond more strongly to output growth. Intuitively, it would be reasonable to conjecture—given the differences in estimated θ_H , ρ_a and sd_a —that optimal policy would prescribe strong responses to inflation in the first mode and a weaker response in the second. That this is not the case stems from the changing contribution of shocks adduced above, which calls for more activist monetary policy in response to preference shocks—and hence variations in inflation—in the second mode. Overall, as evidenced by the final row of Table VIII,

which reports the losses, the policy implications of these two parameter configurations are clearly different.

Comparing the optimal policy results of Table V to those in Table VIII permits an additional insight into how identification impacts policy design. The calculations in Table V were based on estimates from the MCMC Metropolis–Hastings algorithm using as starting values for the multiple chains draws around the first, highest mode in Table VII. Focusing on the inflation response coefficients in each of these tables, an interesting pattern emerges: for the cases in which $\lambda_y > 0$, the optimal coefficients of Table V lie between the policy coefficients associated with each of the two modes reported in Table VIII. The identification problem affects inference in the neighborhood of the first mode as the MCMC algorithm takes some draws from the posterior distribution of the second mode.²² Indeed, the posterior distribution of ρ_a , for instance, is clearly bimodal. Even though it may appear that local identification is achieved, a second local peak affects inference and policy design.

This example underscores that identification problems can have implications for policy design. Moreover, it emphasizes the importance of using additional data to mitigate identification issues. In our baseline, using observed series to fit the foreign block of this small open-economy model helps to better disentangle the effects of various latent variables and exogenous shocks. By dropping these observables, there is insufficient information in the domestic series, the terms of trade and the real exchange rate to pin down the effects of the various disturbances. This leads to the possibility of multiple modes.

As a final example, an earlier version of this paper estimated the unobserved foreign block model without using terms of trade data but reducing the number of domestic shocks. In this case identification problems appeared to be ameliorated. Nonetheless, the absence of a response to the nominal exchange rate in optimal policy was seen once again, for the reasons discussed earlier.

8. CONCLUSIONS

This paper analyzes optimal policy design in an estimated small open economy for Australia, Canada and New Zealand. Motivated by the theoretical literature on local currency pricing, the central question is whether optimal policy responds to nominal exchange rate variations in a class of generalized Taylor rules. The role of parameter uncertainty in policy design is also evaluated.

The central findings are twofold. First, within the class of rules that we consider, it is not optimal for policy to respond to nominal exchange rate variations. This is true regardless of country, whether policymakers face parameter uncertainty or not, the precise set of observables and shocks used to estimate the model, as well as the relative weight of the objectives in the loss function. This result is somewhat surprising given the presence of frictions in import goods markets that generate departures from the law of one price. Several recent papers have focused on this aspect of the specification to provide a rationale for managing exchange rate fluctuations in order to achieve inflation and output stabilization.

Second, parameter uncertainty may lead policymakers to respond more or less aggressively to variables that appear in their policy rule. Depending on the country, model and specific policy

²² We use a t distribution, rather than a normal, as a proposal for the MCMC and use very low degrees of freedom to allow for possibly large steps that may facilitate the transition across modes. We have also tried starting the MCMC sampler around the second, lower mode. In both cases the draws are still mostly drawn from the higher mode although for some parameters kernel estimates reveal the presence of a second peak.

weights under consideration, either outcome is possible. This suggests that generic empirical implications of parameter uncertainty for policy design are unlikely to be available, consistent with the theoretical predictions of Chow (1975).

Finally, we provide an example of how parameter identification may affect policy design and its associated outcomes. A more thorough and general analysis of this last issue is required given the growing role of DSGE models as inputs for the conduct of monetary policy in various central banks.

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