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Inflation, investment and economic performance: The role of internal financing

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Abstract

This paper studies a link between inflation and economic activity that is built on two hypotheses. First, firms mitigate informational frictions in financial markets by accumulating retained earnings over a period of time. Second, firms allocate earnings among three competing uses – dividends, current investment, and the accumulation of internal funds – and inflation directly distorts this allocation decision as well as the real value of accumulated internal funds. The model predicts that the level of inflation – both unanticipated and expected inflation – as well as the variability of inflation distort firms' internal financing decisions, increases frictions in financial markets, reduces the level and efficiency of investment, and reduces aggregate output. The marginal effects of inflation are increasing in the inflation rate.

JEL classification: E31; E44; G35

Keywords: Inflation; Credit markets; Internal financing

1. Introduction

There is considerable uncertainty as well as debate about the mechanism by which inflation affects economic activity. This paper adds to our understanding of this relationship by synthesizing existing ideas and models in specific areas. In particular, we incorporate into a model two important hypotheses that typically occur separately in the literature.

The first hypothesis posits that firms allocate earnings among three competing uses: dividends, current investment, and the accumulation of internal funds. This decision problem will be non-trivial if, as in our model, there are agency costs in financial

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markets. In these circumstances, firms can mitigate informational frictions in financial markets by using internal funds (retained earnings) to partly finance investment. The second hypothesis focuses on how inflation impacts on the decision problem in hypothesis one. It posits that inflation *directly* distorts the earnings allocation decision of the firm as well as the real value of any accumulated internal funds.

The idea that inflation directly reduces a firm's internal funds is the thesis of Lintner's (1975) presidential address to the American Finance Association in which he argues that inflation is *the most important determinant* of firms' internal funds. Lintner's interest in this relationship is for explaining the inverse relationship between inflation and stock returns, but his argument is relevant to understanding investment, financial market activity, and macroeconomic activity. Specifically, Lintner (1975, p. 265) states: "My analysis will show that...a company's relative dependence upon outside financing will *necessarily be higher* the higher the rate of inflation, whether expected or unanticipated..." (italics original). Lintner's hypothesis is that inflation reduces the real value of internal funds, and thus for any given scale of investment project, a firm will have greater external financing needs.

Our model includes the effect of inflation emphasized by Lintner. It also includes the effect of inflation on the earnings allocation decision. The main predictions of the model are that the level of inflation – both unanticipated and expected inflation – as well as the variability of inflation distort a firm's internal financing decisions, increase informational frictions in financial markets, reduce the level and efficiency of investment, and consequently reduce aggregate output. Moreover, credit market frictions increase more than proportionately with the inflation rate. This is broadly consistent with the view (McKinnon, 1973; Shaw, 1973) that capital market frictions in higher inflation (developing) countries are much greater than in lower inflation (developed) countries.

The remainder of the paper is organized as follows. Section 2 discusses related literature. Section 3 presents the model, Section 4 briefly outlines the equilibrium when there are no informational asymmetries, and Section 5 studies the equilibrium with informational asymmetries. The final section contains conclusions. Technical issues are collected in the Appendix.

2. Related literature

Two broad literatures are relevant for placing in context the contribution of the paper. The first literature emphasizes the role of internal funds for explaining investment as well as business cycles. Regarding investment, empirically a strong positive association between internal funds and investment has been documented at least as early as Greenberg (1964) and Hochman (1966). Many more recent studies invariably corroborate this finding (e.g. Chirinko and Schaller, 1995; Fazzari et al., 1988; Hubbard et al., 1995; Bernanke et al., 1996; Stevens, 1994). Theoretical models explain this empirical fact by showing that internal funds mitigate the consequences of informational asymmetries in financial markets (see the surveys by Bhattacharya and Thakor, 1993; Bernanke and Gertler, 1995; Hubbard, 1995; Cecchetti, 1995). These are real models in which money has no role.

Regarding business cycles, Bernanke and Gertler's (1989, 1990) seminal work showed that if internal funds are procyclical, then credit market frictions will move countercyclically, thus amplifying real business cycle (RBC) productivity shocks – the "financial accelerator". A number of papers have incorporated money into these modified RBC models by imposing a transactions demand for money (Carlstrom and Fuerst, 2001; Cooley and Nam, 1998; Fisher, 1999; Feurst, 1995; Bernanke et al., 2000). The focus of these studies is whether the financial accelerator amplifies or dampens a monetary shock compared to a monetary RBC model without a financial accelerator. The answer to this question appears to depend on certain characteristics of models (Carlstrom and Fuerst, 2001).

Two features of these modified monetary RBC models are noteworthy for understanding the contribution of this paper. First, in many of these models internal funds are not a choice variable (exceptions are Carlstrom and Fuerst, 2001; Bernanke et al., 2000). Second, money serves only a transactions role and firms' internal funds are fully indexed to nominal price movements. This stands in stark contrast to Lintner's argument and also to the empirical fact that real returns on financial assets are adversely affected by inflation (e.g. Choi et al., 1996). This might be important since Chari et al. (1996) argue that in order to generate empirically plausible effects of monetary policy on economic activity, money must have a role beyond a purely transactions role. They argue, in particular, for models in which monetary shocks *directly* impact the financial system. In the model used in the present paper, monetary policy has the capacity to directly impact firms' internal funds and thus financial market activity.

The second broad literature that is relevant to this paper is concerned with the link between inflation and economic activity. In the empirical growth literature, high inflation has a strong negative effect on growth (Barro, 1996; Bruno, 1995; Chari et al., 1995; Fischer, 1993; Ghosh and Phillips, 1998; Khan and Senhadji, 2001; Sarel, 1996). The variability of the inflation rate also negatively affects economic activity (Fischer, 1993; Zarnowitz and Lambros, 1987).

Because investment is central to growth, it has been argued that this is the most likely channel through which inflation reduces growth (Barro (1996)). In addition to reducing the level of investment, it has been argued that inflation reduces the efficiency of investment (Goldsmith, 1969). Since investment is generally highly correlated with financial market activity, various studies focus on the relationship between inflation and financial market activity.[1] Most of these studies rely on an inverse relationship between inflation and money as a store of value. Our theoretical argument also relies on an inverse relationship between inflation and the real returns on financial assets, but it differs from existing arguments in two key ways.

First, although internal funds play a role in some existing models (Boyd and Smith, 1998; Huybens and Smith, 1998, 1999), they are not a choice variable of firms. This

[1] For empirical evidence see Boyd et al. (2001), and for theoretical analyses see Azariadis and Smith (1996), Choi et al., (1996), Boyd and Smith (1998), De Gregorio and Sturzenegger (1997), and Huybens and Smith (1998, 1999). Levine et al. (2000) argue convincingly that financial system development *causes* growth.

may be important as firms do have leeway to allocate earnings among the three competing uses mentioned above.

The second main difference between our argument and existing models is that internal funds are not assumed to be fully indexed to the inflation rate. That is, in Boyd and Smith (1998), Huybens and Smith (1998, 1999), and also in monetary RBC models with a financial accelerator, internal funds are denominated in real terms and thus are not *directly* affected by monetary shocks. In these models, inflation affects internal funds and thus credit market distortions only indirectly through variations in the capital stock. In contrast, we propose that inflation *directly* affects the real value of firms' internal funds – this is Lintner's (1975) argument – and the decision as to how to allocate current earnings among competing uses. This difference might be important for the consequences of inflation, because in our model inflation has a *first-order effect* on internal funds, credit market distortions, and thus investment. This difference is plausible because internal funds are generally accumulated over a period of time – because of investment indivisibilities – and the cost of doing so will be positively related to the inflation rate *provided* real asset returns are negatively related to inflation (as documented by Choi et al. (1996)). Note that this proviso is also the critical assumption in the existing models discussed above.

3. The model

3.1. Overview

The main features of the model and sequence of events each period are as follows (see Fig. 1). First, there are two types of two-period lived agents, savers (S) and entrepreneurs (E). Endowments accrue only to savers in their first period of life. Second,

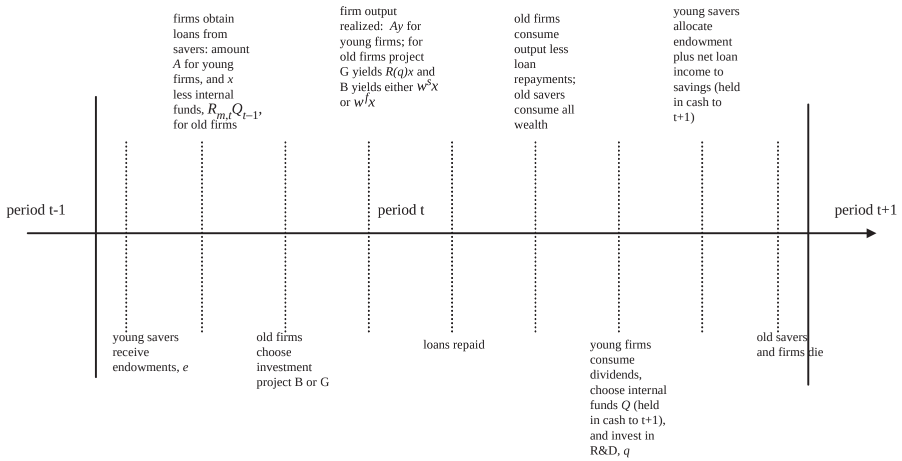

Fig. 1. Sequence of events each time period.

during a time period t savers provide intra-period loans to firms, which are owned by entrepreneurs, in order to produce the single consumption/capital good during the same period. Thus, an entrepreneur requires a loan in each of the two periods of his lifetime. Loans are repaid at the end of each period after production output is realized. Third, entrepreneurs have private information about the characteristics of their firms' production technologies, which leads to agency costs in financial markets and thus an incentive for firms to self-finance (internal funds). Fourth, after production takes place and loans are repaid (if possible), remaining earnings of a firm are allocated among: dividends (equals current consumption of entrepreneurs); investment to improve production efficiency in the next period, $t+1$; and retained earnings, or "internal funds", which are held until period $t+1$ in order to reduce the amount of external financing required to undertake production in period $t+1$. The assumption that internal funds must be held across periods captures the idea that firms must accumulate internal funds over a period of time to minimize agency costs in a world with lumpy capital needs. Fifth, fiat money is the only store of value across time periods. Thus, internal funds of firms and the savings of savers are held in cash balances.

A key assumption is that internal funds are in the form of cash balances. There are three reasons for making this assumption. First, it is arguably a close approximation, especially in developing countries in which financial stores of value are not immune to inflation (Choi et al., 1996). Also, Lintner (1975, p. 275) notes: "In keeping with observed practice, we also assume there is no interest income on cash, and that the terms on receivables are not adjusted for changes in the rate of inflation". The environment that Lintner describes evidently has strong similarities with cash-in-advance models, in which payment lags ensure that cash must be held for at least one period before it can be spent on capital investment. Second, this assumption makes transparent the effect of inflation on internal financing decisions of firms. An alternative environment in which there exists an interest-bearing store of value and no payment lags would produce the same qualitative results so long as the nominal return on internal funds rises less than one-for-one with the inflation rate. Note also that firms are not forced to carry internal funds across periods – a firm can avoid the inflation tax by paying dividends or by increasing expenditures on research and development. Finally, the idea that internal funds are associated with financial assets rather than real assets (such as a firm's capital stock) is consistent with all the empirical work on investment and internal funds discussed in Section 2.

At the start of period t, therefore, the stock of money is in the hands of old savers (from endowment sales when young) and old entrepreneurs' firms (internal funds). During a period money flows from old agents to young agents as payment for the single good. We embed this basic two-period model in an overlapping generations (OG) framework for two reasons: so that there exists an equilibrium with valued fiat money and to facilitate analysis of intertemporal variations in monetary policies.

3.2. Formal description

Consider a two-period OG model with a continuum of agents, of which fraction η are entrepreneurs. Savers receive an endowment of $e > 0$ units of the single, perishable consumption good when young and nothing when old. Preferences of savers are

$$V_t^{\mathrm{S}} = C_{t+1}^{\mathrm{S}}. \tag{1}$$

Thus, savers will wish to save all of their endowment until they are old. Within a period they will use their wealth to make intra-period loans to entrepreneurs.

Entrepreneurs have no endowment in either period of life, but they each own a technology for producing the consumption good. Entrepreneurs' preferences are

$$V_t^{\mathrm{E}} = C_t^{\mathrm{E}} + \beta C_{t+1}^{\mathrm{E}}, \tag{2}$$

where $\beta > 0$. We use the terms "entrepreneurs" and "firms" interchangeably. Firms are corporate entities with limited liability. Thus, entrepreneurs' consumption cannot be negative:

$$C_t^{\mathrm{E}} \geqslant 0, \quad \forall t. \tag{3}$$

This is an important restriction because it implies that loans made to firms could be risky (and will be if (3) binds in some state of the world). Entrepreneurs' consumption will derive from dividends paid by their firms to themselves, after repaying loans.

A firm requires capital to produce the consumption good. Capital can be created one-for-one instantaneously from the consumption good. In the first period of an entrepreneur's life, his firm requires $y > 0$ units of capital at the start of the period to produce $A \cdot y$ units of the consumption good at the end of the current period, where $A > 1$. Recall that entrepreneurs have no endowment and thus their firms must borrow from savers the full amount y. Clearly this loan is riskless since A is constant. The role of this first period investment is solely to generate income for firms so that we may study the earnings retention and investment decisions. Specifically, after this loan has been repaid, a firm has three possible uses for its earnings: distribute them to the entrepreneur (dividends); retain them within the firm to internally finance second-period capital needs of the firm; and/or use them for research and development to improve the efficiency of investment in the second period. Investment in this model should be viewed as physical capital investment plus expenditures on research and development.[2]

In an entrepreneur's second period of life, his firm can choose between two possible projects. Both projects require $\tilde{x} > 0$ units of capital, where $\tilde{x}$ is distributed across the population of entrepreneurs according to the distribution function $H(\tilde{x})$, $\tilde{x} \in [\underline{x}, \bar{x}]$. The variable $\tilde{x}$, which is public information and known when an entrepreneur is born, will be an important characteristic of an entrepreneur in financial markets. The only purpose of having $\tilde{x}$ vary across entrepreneurs is that it ensures a continuous relationship between the number of entrepreneurs that invest and the inflation rate; otherwise, either all entrepreneurs would produce or all would not produce. A sufficient condition for it to be physically possible to fund all firms is that the per capita endowment exceeds the

[2] This accords with the System of National Accounts: "Research and development are undertaken with the objective of improving efficiency or productivity or deriving other future benefits so that they are inherently investment – rather than consumption – type activities". (European Community, OECD, United Nations, and IMF/World Bank, 1993, p. 145.)

maximum possible per capita capital need of firms:

$$\eta\left(\int_{\underline{x}}^{\bar{x}} \tilde{x}\,\mathrm{d}H(\tilde{x}) + y\right) < e(1-\eta). \tag{4}$$

The two possible project choices available to a firm in the second period are denoted B and G. The "good project," project G returns $R(q_t)$ per unit invested with certainty, where q_t denotes expenditures on research and development, which are assumed to be observable. Project B is risky. It returns $w^{\mathrm{s}}(q_t)$ per unit invested with probability p and w^{f} per unit invested with probability $1-p$.

Assumption 1. *The function $R(q)$ satisfies $R(q) > 1$, $R'(q) > 0$, and $R''(q) < 0$ for all $q \in (0, \bar{q})$, where $\bar{q} \leqslant (A-1)y$. Furthermore $R'(0) = \infty$ and $R'(\bar{q}) = 0$.*

Assumption 2. *The function $w^{\mathrm{s}}(q)$ takes the form*

$$w^{\mathrm{s}}(q) = \frac{1}{p}[R(q) - \alpha], \tag{5}$$

with $\alpha < 1-p$ and $R(\bar{q}) - \alpha + (1-p)w^{\mathrm{f}} < 1$.

Assumption 2 has the implication that the marginal benefit of q is the same for both projects and thus q_t cannot be used to signal intentions about project choice in credit markets. We make this assumption to focus on the role of internal funds in this regard, as that has been the main focus of the literature concerned with credit market imperfections, as discussed in Section 2. It will become clear below that the assumption that $\alpha < 1-p$ is necessary for informational frictions in credit markets to matter.

It is easy to verify that the above two assumptions imply

$$w^{\mathrm{s}}(q) > R(q) > 1 > pw^{\mathrm{s}}(q) + (1-p)w^{\mathrm{f}}, \quad \forall q \in [0, \bar{q}]. \tag{6}$$

Thus, project G dominates project B – in the sense of first- and second-order stochastic dominance – and project B is socially wasteful. We also assume that external financing is required by firms to initiate second-period investment projects, and that loans to finance project B are risky. The following condition ensures this:

$$\underline{x} - (A-1)y\beta^{-1} > w^{\mathrm{f}}\underline{x}. \tag{7}$$

Informational frictions in financial markets arise because project selection occurs *after arranging financing* and is private information. This introduces an asset-substitution problem in external financing, as in Stiglitz and Weiss (1981) and others. This informational friction will matter if financial contracts are restricted to standard debt contracts (with contractual interest rate r).[3,4]

[3] A standard debt contract specifies a gross interest rate r on loan amount B, then the borrower repays B if he has sufficient funds, otherwise the borrower defaults on the loan and the lender recovers whatever he is able to.

[4] The optimality of standard debt contracts (and financial intermediation) can be incorporated if we were to assume costly state verification (Townsend, 1979; Gale and Hellwig, 1985; Williamson, 1986; Diamond, 1984). This is discussed in the Appendix.

Fiat money serves as a store of value. Denote the return on money from t to $t+1$ as $\tilde{R}_{m,t+1} \equiv (P_{t+1}/P_t)^{-1}$, where P_t is the nominal price level. The government prints money and distributes it in a lump-sum fashion to old savers. This assumption is not critical; what is critical is that the pattern of transfers is such that firms are affected by inflation. A “monetary policy” is a conditional cumulative distribution function $F_t(\tilde{R}_{m,t+1})$. It is assumed that the expected inflation rate is not so low that all agents will want to defer consumption:

$$\mathrm{E}_t\tilde{R}_{m,t+1} = \int_0^{\infty} \tilde{R}_{m,t+1}\,\mathrm{d}F_t(\tilde{R}_{m,t+1}) \leqslant \beta^{-1}, \quad \forall t. \tag{8}$$

The analysis concentrates on the stationary monetary equilibrium in which expectations of inflation coincide with the central bank's monetary rule, specified as a distribution $F_t(\tilde{R}_{m,t+1})$ with mean gross inflation rate $\mathrm{E}_t R^{-1}_{m,t+1}$. The market clearing condition for the money market is as follows (the goods market clears by Walras' Law):

$$(1-\eta)P_tS_t + \eta \int_{\underline{x}}^{\bar{x}} Q_t(\tilde{x})\,\mathrm{d}H(\tilde{x}) = \frac{1}{2}M_t, \tag{9}$$

where S is real savings of young savers, Q_t is nominal retained earnings (internal funds) of a young entrepreneur, and M_t is per capita money supply. Existence of a monetary equilibrium is straightforward as the only store of value is money, as in the prototype OG model. A sketch of existence given our monetary rule is presented in the Appendix.

4. A benchmark

As a benchmark, consider briefly the equilibrium in the absence of informational frictions. From one time period to the next, the return on savings and internal funds is $\tilde{R}_{m,t+1}$. Within a time period, loans have a contractual interest rate of $r(i)$, where i indexes project choice (since, at present, this choice is observable). Competition among lenders within a period will ensure that the expected return on lending is equal to its opportunity cost, unity. As we explain below, since this is impossible for project B (as its expected return is less than unity), lenders will never finance this project when its choice is contractible.

The decision problem of savers is to choose the set $\{S_t, L_t, L_{t+1}, C^{\mathrm{s}}_{t+1}\}$ to solve

$$\max \quad E_tC^{\mathrm{S}}_{t+1} \quad \text{s.t.} \tag{10}$$

$$S_t = e - L_t + rL_t, \tag{11}$$

$$C^{\mathrm{S}}_{t+1} = \tilde{R}_{m,t+1}S_t - L_{t+1} + rL_{t+1} + T_{t+1}, \tag{12}$$

where S_t is real savings when young, L_t is loans, and T_{t+1} is the real value of monetary transfers. Since $r=1$, the solution to (10)–(12) is obvious: $S^*_t = e$.

To solve the choice problem of an entrepreneur, begin with the second-period decision problem. If an entrepreneur retained nominal earnings equal to Q_t in period t, then real internal funds in period $t+1$ are $\tilde{R}_{m,t+1}Q_t$. Assuming the entrepreneur selects project G, second-period consumption therefore satisfies

$$C^{\mathrm{E}}_{t+1} = R(q_t)\cdot\tilde{x} - r(\tilde{x} - R_{m,t+1}Q_t), \tag{13}$$

where r is the interest rate on a loan of amount $(\tilde{x} - R_{m,t+1}Q_t)$. We have assumed an entrepreneur commits all available internal funds to the project, $R_{m,t+1}Q_t$, which is valid because with symmetric information the cost of internal funds could not be less than the cost of external funds (i.e. a gross interest rate of unity). As noted above $r=1$ for project G.

For project B, note from (6) that loans would necessarily earn a negative return. Thus, savers would not finance project B. Equivalently, any loan for project B with a contractual interest rate $r \geqslant (w^{\mathrm{s}}(q_t)\tilde{x})/(\tilde{x} - \tilde{R}_{m,t+1}Q_t)$ would lead to non-positive consumption of entrepreneurs that selected project B, and thus there would be zero demand for loans for project B.

In their first period of life, entrepreneurs are financed with loans of amount y. Thus, first-period earnings of all firms after loan repayments are $(A-1)y$. Young entrepreneurs therefore choose $\{q_t, Q_t\}$ to maximize

$$\mathrm{E}_t V^{\mathrm{E}}_t = [(A-1)y - q_t - Q_t] + \beta[\tilde{x}R(q_t) - (\tilde{x} - \mathrm{E}_t\tilde{R}_{m,t+1}Q_t)]. \tag{14}$$

The optimal plan for all entrepreneurs is $Q_t = Q^* = 0$, and $q_t(\tilde{x}) = q^*(\tilde{x})$, where $q^*(\tilde{x})$ solves $R'(q^*(\tilde{x})) = 1/\beta\tilde{x}$. The reason firms retain no internal funds is that inflation makes them costly to acquire and with symmetric information the net interest rate on external funds is zero.

Collecting results, the equilibrium with symmetric information has firms never retaining earnings and per-capita GDP is constant and given by

$$Y_t = \frac{1}{2}\left[(1-\eta)e + \eta(A-1)y + \eta\int_{\underline{x}}^{\bar{x}}(R(q^*_{t-1}(\tilde{x})) - 1)\tilde{x}\,\mathrm{d}H(\tilde{x})\right].$$

Thus, the real economy is completely unaffected by the level of the inflation rate and its variability.

5. Asymmetric information

Suppose now that project selection is unobservable but that lenders believe that a firm will select project G. If financing is raised at an interest rate appropriate for project G (i.e., a gross interest rate of unity) but the firm actually undertakes project B, then he will be better off provided $p(w^{\mathrm{s}}(q)-1) > R(q)-1$. By Assumption 2, this inequality is satisfied since $\alpha < 1-p$. This asset-substitution problem destroys the full-information equilibrium because firms would not in fact choose project G. The lender, believing mistakenly that the firm will choose project G, would earn a per unit expected return on a loan equal to $p + (1-p)((w^{\mathrm{f}})/\tilde{x}) < 1$, or a negative net rate of return on the loan. The only possible equilibrium has entrepreneurs signaling to lenders that they will in fact select project G – i.e. their project choice is incentive compatible.

If an entrepreneur simply ceased production operations in the first period, then they have no incentive to defer consumption and their expected lifetime utility is

$$V_t^{\mathrm{E}} = (A-1)y. \tag{15}$$

The alternative is to commit internal funds of I_{t+1} toward initiating a project in the second period in order to signal project selection. Project G is an incentive compatible choice for the firm if

$$R(q_t)\tilde{x} - (\tilde{x} - I_{t+1}) > p[w^{\mathrm{s}}(q_t)\tilde{x} - (\tilde{x} - I_{t+1})] \tag{16}$$

or

$$I_{t+1}(\tilde{x}) > \hat{I}(\tilde{x}) \equiv \tilde{x}\left(\frac{(1-p)-\alpha}{1-p}\right). \tag{17}$$

If (18) is not satisfied, then it is easily shown that there does not exist an interest rate such that a lender would be willing to finance the firm. Thus, a firm will be able to obtain external financing in the second period only if it has internal funds of at least $\hat{I}(\tilde{x})$. The only means of signaling commitment to undertaking project G is accumulating internal funds in the previous period. This is because (17) holds for any q_t and the asset substitution problem is not lessened by q_t – the right side of (17) is not affected by q_t. Even though 'capitalized R& D' is increasing in q_t, an entrepreneur that chooses a higher value of q_t would not have any advantage in credit markets over an entrepreneur with a lower value of q_t since the incentive compatibility constraints for the two entrepreneurs would be identical (assuming they have the same $\tilde{x}$). While it may be of interest to study signalling in financial markets through q_t, we focus on internal funds as that has been the focus of the literature (see Section 2), and it is also arguably the main means of signalling in practice.

Two factors can make it difficult for a firm to accumulate this required amount of internal funds. First, $\hat{I}(\tilde{x})$ is increasing in $\tilde{x}$ and thus a firm may be unable to accumulate the required amount of internal funds. In other words, firms with large external financing needs are more likely to be affected by the incentive constraint than firms with lower external financing needs. One could interpret this in terms of firm size, although our preferred interpretation is in terms of relative sizes of investment projects and external financing needs, for a given firm size. Second, the return on internal funds is stochastic and thus the ex post value of internal funds may be less than anticipated. In light of these two factors, second-period consumption of entrepreneurs equals $[R(q_t)\tilde{x} - (\tilde{x} - \tilde{R}_{m,t+1}Q_t)]$ if $\tilde{R}_{m,t+1}Q_t \geqslant \hat{I}(\tilde{x})$, and is equal to $\tilde{R}_{m,t+1}Q_t$ if $\tilde{R}_{m,t+1}Q_t < \hat{I}(\tilde{x})$. Expected second-period consumption is therefore

$$\begin{aligned} \mathrm{E}_t C_{t+1}^{\mathrm{E}} = {} & \int_{\hat{I}(\tilde{x})/Q_t}^{\infty} [R(q_t)\tilde{x} - (\tilde{x} - \tilde{R}_{m,t+1}Q_t)]\,\mathrm{d}F_t(\tilde{R}_{m,t+1}) \\ & + \int_0^{\hat{I}(\tilde{x})/Q_t} \tilde{R}_{m,t+1}Q_t\,\mathrm{d}F_t(\tilde{R}_{m,t+1}). \end{aligned} \tag{18}$$

Simplifying, and noting that first-period consumption is $C_t^{\mathrm{E}} = (A-1)y - q_t - Q_t$, gives expected lifetime utility

$$\begin{aligned} \mathrm{E}_t V_t^{\mathrm{E}} = {} & [(A-1)y - q_t - Q_t] + \beta[(R(q_t) - 1) \\ & \times \tilde{x}(1 - F_t(\hat{I}(\tilde{x})/Q_t)) + Q_t \mathrm{E}_t \tilde{R}_{m,t+1}]. \end{aligned} \tag{19}$$

The objective of entrepreneur $\tilde{x}$ is to maximize (19) by choosing $\{q_t, Q_t\}$. The first-order conditions for Q_t and q_t respectively are

$$\beta \mathrm{E}_t \tilde{R}_{m,t+1} + \left(\frac{\beta(R(q_t) - 1)\tilde{x} f_t(\hat{I}(\tilde{x})/Q_t)\hat{I}(\tilde{x})}{Q_t^2} \right) = 1, \tag{20}$$

$$\beta R'(q_t)\tilde{x}(1 - F_t(\hat{I}(\tilde{x})/Q_t)) = 1. \tag{21}$$

Eq. (20) equates the marginal benefit of retaining earnings with the marginal cost. The marginal benefit is the discounted expected real value of that wealth plus the benefit of an increased probability of mitigating incentive problems in financial markets and thus of being able to initiate production in the future. The left side of (21) is the marginal benefit of increasing the efficiency of investment. [5]

It is of interest to analyze the effects of both the mean and variability of $\tilde{R}_{m,t+1}$ on the equilibrium. It is assumed that $F_t(\tilde{R}_{m,t+1})$ is uniform with support $[L_t, U_t]$. Eq. (20) in this case yields

$$Q_t = \left(\frac{\beta(R(q_t) - 1)\tilde{x}\hat{I}(\tilde{x})}{(U_t - L_t)(1 - \beta \mathrm{E}_t \tilde{R}_{m,t+1})} \right)^{1/2}. \tag{22}$$

Note that (22) includes as distinct arguments the conditional mean $\mathrm{E}_t \tilde{R}_{m,t+1}$, and the spread of the distribution, $(U_t - L_t)$. The other first-order condition, Eq. (21), can be written as

$$\beta R'(q_t)\tilde{x} \left(\frac{U_t - \hat{I}(\tilde{x})/Q_t}{U_t - L_t} \right) = 1. \tag{23}$$

Finally, expected output in the second period (net of loan repayments) for entrepreneur $\tilde{x}$ is $[1 - F(\hat{I}(\tilde{x})/Q_t(\tilde{x}))][R(q_t(\tilde{x})) - 1]\tilde{x}$, and thus mean GDP in the economy is

$$\begin{aligned} \mathrm{E}_t(Y_{t+1}) = \frac{1}{2} \Bigg(& (1-\eta)e + \eta(A-1)y + \eta \int_{\underline{x}}^{\bar{x}} [1 - F(\hat{I}(\tilde{x})/Q_t(\tilde{x}))] \\ & \times [R(q_t(\tilde{x})) - 1]\tilde{x}\,\mathrm{d}H(\tilde{x}) \Bigg). \end{aligned} \tag{24}$$

[5] In writing (20) and (21) it has been assumed that the limited liability constraints (3) do not bind for entrepreneur $\tilde{x}$. For some parameterizations there could be entrepreneurs with high capital requirements in the second period, $\tilde{x}$, for which this assumption is not valid. Specifically, this will be the case for entrepreneur $\tilde{x}$ if $(A-1)y < Q_t^*(\tilde{x}) + q_t^*(\tilde{x})$, where $\{Q_t^*(\tilde{x}), q_t^*(\tilde{x})\}$ solve (20) and (21). These agents will therefore be at a corner solution, and will need to scale back q_t and Q_t relative to the desired levels or, if $\tilde{x}$ is sufficiently large, they will simply pay a liquidating dividend in the first period and not produce in the future. To simplify the exposition, the discussion focuses mainly on parameterizations for which corner solutions do not arise. Formally, this restriction is: $(A-1)y > Q_t^*(\bar{x}) + q_t^*(\bar{x})$. and the discussion at the end of the present section consider the effects of binding limited liability constraints.

Proposition 1. *If $F_t(\tilde{R}_{m,t+1})$ is uniform, then*

(a) *unanticipated inflation weakly decreases the likelihood that all firms in their second period of life produce, and thus weakly decreases GDP;*
(b) *higher expected inflation strictly decreases firms' internal funds and decreases the efficiency of investment, strictly decreases the probability of firms producing in their second period as well as the level of output of firms that do produce, and strictly decreases mean GDP;*
(c) *higher variability of inflation strictly decreases firms' internal funds and the efficiency of investment, strictly decreases the probability of firms producing in their second period as well as the level of output of firms that do produce, and strictly decreases mean GDP.*

In comparison, with symmetric information, expected GDP is (24) with $F(\hat{I}(\tilde{x})/Q_t(\tilde{x}))=0$, and $q_t(\tilde{x})=q(\tilde{x})$, for all t. Thus, with asymmetric information, expected GDP is unambiguously lower, and it is also time varying.

There are three links between inflation and economic activity in this model. First, both *unanticipated and anticipated inflation* is a tax on the real value of accumulated internal funds and thus increases reliance on external financing, exacerbates frictions in credit markets, and reduces investment. [6] This relationship between unanticipated or anticipated inflation and the need for external financing is Lintner's (1975) argument.

The second and closely related link is that higher *expected inflation*, or higher *variability of inflation*, increases the cost of signaling in credit markets, decreases firms' desired internal funds, and reduces investment. In other words, anticipated inflation and its variability distorts a firm's decision as to how to allocate earnings among competing uses – dividends, current investment, and the accumulation of internal funds. Whereas the link between expected inflation and the decision about the level of internal funds is fairly straightforward, the link between inflation variability and internal funds is more subtle. Specifically, inflation variability weakens the signalling value of internal funds. To see this, note that $\partial^2[1-F_t(\hat{I}(\tilde{x})/Q_t)]/\partial Q\partial U_t=-\hat{I}(\tilde{x})Q_t^{-2}/2(U_t-L_t)<0$.[7] In words, with more variable inflation, internal funds are not as potent in affecting the probability of being able to obtain external financing. As clearly reflected in the first-order condition (20) or (22), this reduces optimal internal funds because the objective function is concave in Q_t.

The third link between inflation and economic activity is that it decreases the efficiency of investment. The reason is that inflation makes it less certain that external financing will be forthcoming and thus reduces incentives to enhance the efficiency of investment. Intuitively, q_t and Q_t are complements, in that they respond in qualitatively

[6] By 'anticipated' inflation we are referring to the mean of the inflation distribution and by 'unanticipated' inflation we are referring to the difference between actual and mean inflation.

[7] This calculation incorporates the fact that $L_t = 2\mathrm{E}_t R_{m,t+1} - U_t$, where $\mathrm{E}_t R_{m,t+1}$ is constant for a mean preserving spread.

similar fashion to the likelihood external financing will be forthcoming – using (21), we have $\partial q_t/\partial Q_t > 0$.[8]

Next, we inquire into the form of non-linearities between the inflation rate and the key endogenous variables in the model. Our main finding is presented in the following proposition.

Proposition 2. *Internal funds* (Q_t) *and the efficiency of investment* (q_t) *are each increasing, concave functions of* $\mathrm{E}_t R_{m,t+1}$. *Actual firm output as well as the probability that any entrepreneur produces in his second period of life are also increasing concave functions of* $\mathrm{E}_t R_{m,t+1}$.

This result has the prediction that higher inflation not only harms economic activity and exacerbates frictions in financial markets, but also that the effects are more than proportionate to the inflation rate. This finding is broadly consistent with the view (McKinnon, 1973; Shaw, 1973) that capital market frictions are much more acute in (high-inflation) developing countries than in developed countries.

The model predicts a monotone relationship between $\mathrm{E}_t R_{m,t+1}$ and economic activity. However, it is an increasing concave function, implying that the marginal consequences of inflation are small at low rates of inflation. Indeed, it is easily shown that when the (gross) rate of inflation converges to entrepreneurs' discount factor (β), the marginal effect of inflation is zero. Recently, some empirical evidence suggests there may be threshold effects in the inflation–growth relationship, in that growth is negatively related to inflation only for inflation rates above a threshold level; below the threshold, some authors have found no statistically significant relationship between inflation and growth. The level of this inflation threshold has been estimated to be as low as 1% and upwards of 10% (Khan and Senhadji, 2001; Ghosh and Phillips, 1998). One difference between developing countries and industrial countries is that the latter typically have more developed financial markets including financial instruments that better insulate internal funds from inflation. For instance, developing countries often have significant financial restrictions that raise the effective inflation tax (e.g. ceilings on deposit rates), and underdeveloped markets for fixed-income investments which are the main financial store of value for firms. While the concavity results of Proposition 2 may shed some light on the threshold phenomenon, we also note that, in the context of our model, if there were better inflation hedging options at lower inflation rates, then this would help explain the empirical facts.

Some of the predictions in Proposition 2 hinge on having endogenous efficiency of investment, $R(q)$. Indeed, if efficiency of investment is exogenous, $R(q) = R$, then

[8] Relaxing the assumption that all entrepreneurs have sufficient cash flow in the first period to choose the optimal (interior) level of internal funds would mean that there is a group of entrepreneurs at a corner solution in their choice of internal funds: internal funds are set equal to their maximum feasible amount, $(A-1)y$. In addition, there may be another group of entrepreneurs that choose to cease production in the first period because their financing requirements in the second period are so high relative to their earnings in the first period that they would not be able to accumulate sufficient internal funds to mitigate the asset-substitution problem in capital markets. These generalizations lead to somewhat richer interaction between output and inflation, but do not alter the qualitative implications.

Table 1
Effects of inflation

$E_t R_{mt+1}$	Q_t	$\hat{I}$	Prob. produce	GDP
a. The mean inflation rate				
0.90	8.33	4.68	0.92	4.78
0.85	7.49	4.68	0.78	4.43
0.80	6.86	4.68	0.64	4.09
0.75	6.37	4.68	0.52	3.77
0.70	5.97	4.68	0.39	3.46
Benchmark	0	0	1	4.98
$U - L$	Q_t	$\hat{I}$	Prob. produce	GDP
b. The variability of the inflation rate				
0.6	7.92	4.68	0.85	4.59
0.7	7.33	4.68	0.73	4.30
0.8	6.86	4.68	0.65	4.09
0.9	6.47	4.68	0.58	3.94
1.0	6.13	4.68	0.54	3.82
Benchmark	0	0	1	4.98

there is convexity, rather than concavity, of the Q_t function – this is readily verified using (22).

To illustrate some key implications of the model (and to illustrate that these implications are present for a non-trivial parameter space) we consider briefly some numerical examples. To simplify, we focus on the special case in which $R(q) = R$; this focuses the discussion on internal funds.[9] We choose the following parameter values: $\alpha = 0.25$, $p = 0.6$, $\eta = 0.2$, $e = 4$, $A = 5.4$, $y = 2$, $R = 1.2$, $\underline{x} = 10$, $\bar{x} = 15$, $w^{\mathrm{f}} = 0.01$. We use a uniform distribution for $H(\tilde{x})$. There is nothing special about these parameter values – alternative values yield similar findings – other than they can be shown to satisfy all the parameter restrictions discussed in Section 2.

We consider the effects of the mean inflation rate and the variability of inflation. The first panel of Table 1 considers the effects of the mean $E_t R_{m,t+1}$ holding the spread of the distribution constant at the value $U - L = 0.8$ The second panel of the table shows the effects of mean-preserving spreads of the distribution, where the mean is $E_t R_{m,t+1} = 0.8$. The bottom row of each panel shows the values in the benchmark economy where there is symmetric information. The entrepreneur's Q_t value shown in the table corresponds to the mean entrepreneur. Also shown in the table is the probability that the mean entrepreneur is able to signal in financial markets, and thus operate his production technology.

Clearly, internal funds are decreasing in the inflation rate, as predicted by the model. Shown in the rightmost column of the table is per capita GDP. The consequences of

[9] We choose this case partly because in the general case solving (24) requires integrating over an unknown function $q(\tilde{x})$.

inflation are large the level of per capita GDP is 38% larger when $\mathrm{E}_t R_{m,t+1} = 0.9$ than when $\mathrm{E}_t R_{m,t+1} = 0.7$. The magnitude of non-linearities in this example is economically important. For example, non-linearity accounts for about 10–20% of the variation in Q_t in the table. The second panel in the table illustrates the consequences of the variability of inflation for Q_t and GDP. Again, as predicted by the model, the variability of inflation has negative consequences for internal funds and output.

5.1. Longer-lived entrepreneurs

If entrepreneurs live for more than two periods this endogenizes earnings at all dates beyond the first period and this has a noteworthy consequence because the level of earnings is important for firms' investment and internal financing decisions. We next briefly outline this consequence.

When firms are active for only two periods, inflation has persistent effects on output only if inflation is itself persistent. If, instead, firms last for three periods (or longer), then even a temporary inflation shock will reduce firms' earnings in the second period, and that will be propagated into the third period if the limited liability constraints bind for some entrepreneurs. The reason is that firms with binding limited liability constraints would have to scale back total expenditures on internal funds and on improving the efficiency of production, $Q_t + q_t$, to be no greater than current earnings.[10]

To illustrate this, assume that $R(q) = R$ and $w^{\mathrm{s}}(q) = (R - \alpha)/p$, so that the only decision of firms is their choice of internal funds. In the second period of life, period $t + 1$, the decision problem of entrepreneurs is the same as in the two-period model when entrepreneurs are in their first period of life. The only difference is that mean income is equal to $y_{t+1} = [1 - F(\hat{I}(\tilde{x})/Q_t)](R-1)\tilde{x} + \tilde{R}_{m,t+1}Q_t$, rather than the exogenous amount $(A - 1)y$. If we assume for the moment that the limited liability constraints do not bind, then it is easy to show that the optimal choices of Q_t and Q_{t+1} are independent of one another and, moreover, are given by expression (22) (with the appropriate time indexes). Proposition 1 therefore applies also to the additional period of an entrepreneur's life in a three-period version of the model. But mean firm earnings in the second period of life are decreasing in both the level and variability of inflation, and thus a temporary shock to the inflation rate or the distribution of the inflation rate that results in lower firm earnings in one period will be propagated forward if the limited liability constraints prevent firms from accumulating the required amount of internal funds to mitigate informational frictions in credit markets.

6. Conclusion

Mankiw (1994) states that the costs of inflation and the costs of reducing it are "one of the four most important unresolved questions in macroeconomics". The main objective of this paper is to study a channel through which inflation is harmful to investment and economic activity. The channel proposed in the paper rests on two

[10] Footnotes 5 and 8 discuss how the level of earnings can constrain optimal decisions.

hypotheses. These two hypotheses are, first, that firms' internal funds are important for mitigating informational frictions in financial markets and, second, that inflation distorts firms' incentives to accumulate internal funds as well as the real value of accumulated internal funds. The model predicts that when these two relationships co-exist, higher and more volatile inflation rates increase informational frictions in credit markets, reduce the level and efficiency of investment, and reduce GDP.

Standard arguments for why inflation variability is harmful to economic activity are that it makes relative prices less informative due to costs of changing prices and staggered price setting, and that greater uncertainty is harmful due to risk aversion. The model studied here complements these arguments by showing that increased variability of inflation is harmful even when all agents are risk neutral and relative prices are not distorted. Instead, the costs of inflation variability derive from the fact that it distorts firms' internal financing decisions, exacerbates frictions in credit markets, and thereby reduces the level and efficiency of investment. The costs of inflation also rise more than proportionately with the inflation rate.

In future work it would be interesting to adapt some aspects of our model to standard models of business cycles and growth. For example, in monetary RBC models with a financial accelerator mechanism it would be interesting to allow monetary shocks to directly affect the real value of accumulated internal funds as well as firms' decisions as to how to allocate current earnings among dividends, current investment, and accumulated internal funds. This may significantly alter the propagation of monetary shocks, compared to either the model without the financial accelerator or the model with the financial accelerator but when money plays only a transactions role.

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Appendix A

A.1. Standard debt contracts

Conditional on project G being chosen, Williamson (1986), Proposition 1, establishes that the optimal contract is a standard debt contract for the general case where project returns are stochastic. With a certain return on the project, monitoring does not occur in equilibrium, but serves to enforce debt repayment. Williamson's proof carries over to the case of unobservable project selection, for (as shown below) this simply introduces an additional incentive constraint which states that financing is only forthcoming if a firm can credibly signal that it will choose project G. This incentive constraint is that the firm must have a minimum level of internal funds (see Eq. (17)).

To see why debt contracts are optimal with costly state verification, consider the model in which project choice is observable. Assume there is at most one entrepreneur per lender for simplicity (having multiple lenders would simply introduce financial intermediaries as in Diamond (1984) and Williamson (1986)) and the loan amount is unity. Let the cost of verification and recovery of assets be the (pecuniary equivalent) of non-pecuniary costs $c > 0$. These costs are assumed to be borne by the entrepreneur (with competitive lenders the borrower will in effect always bear these costs, e.g. Gale and Hellwig (1985)). Since there is no uncertainty of project return, the payment from the borrower to lender is a constant amount, b. In addition, if $b \geqslant 1$ (the opportunity cost of lending) then the lender will clearly not proceed with verification and recovery of assets. If $b < 1$, the lender would need to verify and recover assets in order to maintain a return of no less than unity. Let the recovery value in such an event be μ, such that $b + \mu \geqslant 1$. It is obvious that, since the objective function of entrepreneurs is $R - b - I(c + \mu)$, where $I = 1$ if $b < 1$, the solution must be a contract that specifies $b = 1$ and that verification of assets does not occur in equilibrium.

With unobservable project selection, costly state verification ensures that the contractual payment from borrowers to lenders cannot be contingent on project return. Moreover, since there does not exist a contract with positive net return to lenders when project B is selected, lenders must ensure that project B will not be selected. This introduces the incentive compatibility constraint (17). With this satisfied, the determination of the optimal contract proceeds as above.

A.2. Sketch of existence of equilibrium

Using the market clearing condition we can write

$$P_t = \frac{\frac{1}{2} M_t - \eta \int_{\underline{x}}^{\bar{x}} Q_t(\tilde{x})\,\mathrm{d}H(\tilde{x})}{(1-\eta)S_t}. \tag{A.1}$$

For the imperfect information case (the perfect information case is the standard OG model which savers are the only users of money), imposing the optimality condition $S_t = e$, the price level must satisfy

$$P_t = \frac{\frac{1}{2} M_t - \eta \int_{\underline{x}}^{\bar{x}} Q_t(\tilde{x})\,\mathrm{d}H(\tilde{x})}{(1-\eta)e}. \tag{A.2}$$

Note that Q_t is, for given $\mathrm{E}_t R_{m,t+1}$, invariant to the price level. Consequently, given expectations of inflation, the equilibrium price is determined by (A.2). Finally, transfers, or equivalently money supply growth, determine the distribution of $R_{m,t+1}$. That is, the link between money supply growth and state-dependent inflation is

$$R_{m,t+1} = \frac{\frac{1}{2} M_t - \eta \int_{\underline{x}}^{\bar{x}} Q_t(\tilde{x})\,\mathrm{d}H(\tilde{x})}{\frac{1}{2} M_{t+1} - \eta \int_{\underline{x}}^{\bar{x}} Q_{t+1}(\tilde{x})\,\mathrm{d}H(\tilde{x})}. \tag{A.3}$$

The distribution of M_t (and thus transfers) must be such that the implied distribution of $R_{m,t+1}$ is the announced policy $F_t(R_{m,t+1})$.

Proof of Proposition 2. The proof proceeds by establishing each of the three parts in the proposition for an arbitrary entrepreneur $\tilde{x}$. Because the claims hold for all entrepreneurs, the consequences for GDP follow.

Part (a): Entrepreneur $\tilde{x}$ produces in the second period if, and only if $\tilde{R}_{m,t+1}Q_t(\tilde{x}) \geqslant \hat{I}(\tilde{x})$. As $\{\hat{I}(\tilde{x}), Q_t(\tilde{x})\}$ is predetermined in $t+1$, it follows that production activity is weakly decreasing in $\tilde{R}_{m,t+1}$.

Part (b): Let the implicit functions defined by the first-order conditions (20)–(21) be denoted by $F^1(Q_t, q_t, z)$ and $F^2(Q_t, q_t, z)$, respectively, where z denotes the exogenous variable of interest in the comparative statics. Let $F^1_{Q_t}(Q_t, q_t, z)$ denote the partial derivative with respect to Q_t, and similarly for the other variables. It is straightforward to show that $F^1_{Q_t} < 0$, $F^1_{q_t} > 0$, and $F^1_{\mathrm{E}_t\tilde{R}_{m,t+1}} > 0$, holding constant the spread of the distribution $(U_t - L_t)$. Similarly, $F^2_{Q_t} > 0$, $F^2_{q_t} < 0$. Finally, note that $\mathrm{E}_t\tilde{R}_{m,t+1} = (U_t + L_t)/2$, thus for fixed $U_t - L_t$, an increase in expected inflation implies that both U_t and L_t must increase. It follows that $F^2_{\mathrm{E}_t\tilde{R}_{m,t+1}} > 0$. The matrix version of the implicit function rule states that

$$\frac{\mathrm{d}Q_t}{\mathrm{dE}_t\tilde{R}_{m,t+1}} = \frac{-F^1_{\mathrm{E}_t\tilde{R}_{m,t+1}}F^2_{q_t} + F^2_{\mathrm{E}_t\tilde{R}_{m,t+1}}F^1_{q_t}}{|H|}, \tag{A.4}$$

where H is the Hessian matrix. By the second-order conditions $|H| > 0$, and thus it follows that $(\mathrm{d}Q_t/\mathrm{dE}_t\tilde{R}_{m,t+1}) > 0$. Next, note that

$$\frac{\mathrm{d}q_t}{\mathrm{dE}_t\tilde{R}_{m,t+1}} = \frac{-F^2_{\mathrm{E}_t\tilde{R}_{m,t+1}}F^1_{Q_t} + F^1_{\mathrm{E}_t\tilde{R}_{m,t+1}}F^2_{Q_t}}{|H|} > 0. \tag{A.5}$$

Thus, the probability of production in $t+1$, $1 - F(\hat{I}(\tilde{x})/Q_t(\tilde{x}))$, and actual output $R(q_t(\tilde{x}))\tilde{x}$, are both strictly increasing in $\mathrm{E}_t\tilde{R}_{m,t+1}$.

Part (c): As $\mathrm{E}_t\tilde{R}_{m,t+1} = (U_t + L_t)/2$, we have that $L_t = 2\mathrm{E}_t\tilde{R}_{m,t+1} - U_t$. Using this to eliminate L_t from (20) and (21), a mean-preserving spread in the distribution of $\tilde{R}_{m,t+1}$ therefore corresponds to an increase in U_t holding $\mathrm{E}_t\tilde{R}_{m,t+1}$ constant. Following the same approach as in part (b), we have $F^1_{U_t} < 0$ and $F^2_{U_t} = [2\beta R'(q_t)\tilde{x}((\hat{I}(\tilde{x})/Q_t) - (U_t + L_t)/2)]/(U_t - L_t)^2$, which may be of either sign. By the implicit function theorem

$$\frac{\mathrm{d}q_t}{\mathrm{d}U_t} = \frac{-F^2_{U_t}F^1_{Q_t} + F^1_{U_t}F^2_{Q_t}}{|H|}. \tag{A.6}$$

Using the actual expressions for the partial derivatives in the numerator of (A.6), it is easily shown that $\mathrm{sign}(\mathrm{d}q_t/\mathrm{d}U_t) = \mathrm{sign}[(\hat{I}(\tilde{x})/Q_t) - (U_t + L_t)]$, which is necessarily negative since $\hat{I}(\tilde{x})/Q_t < U_t$. Finally, taking the total differential of (20), we have

$$\frac{\mathrm{d}Q_t}{\mathrm{d}U_t} = \frac{-F^1_{q_t}}{F^1_{Q_t}}\frac{\mathrm{d}q_t}{\mathrm{d}U_t} - \frac{F^1_{U_t}}{F^1_{Q_t}}. \tag{A.7}$$

Collecting results from above, it follows that $\mathrm{d}Q_t/\mathrm{d}U_t < 0$. Thus, the probability of production in $t+1$, $1 - F(\hat{I}(\tilde{x})/Q_t(\tilde{x}))$, and actual output $R(q_t(\tilde{x}))\tilde{x}$, are both strictly increasing in mean-preserving spreads of the distribution of $\tilde{R}_{m,t+1}$.

Proof of Proposition 2. The implicit function rule can be applied to (20) and (21) to obtain second derivatives of the endogenous variables with respect to the expected inflation rate. Though tedious, this yields $\mathrm{d}^2Q/\mathrm{d}ER_m^2 = |H_1|/|H|$, where $|H| > 0$ by the second-order conditions. Noting that $F^1_{\mathrm{E}R_m\mathrm{E}R_m} = 0$, $F^1_{\mathrm{E}R_mQ} = 0$, $F^1_{\mathrm{E}R_mq} = 0$, $F^2_{\mathrm{E}R_m\mathrm{E}R_m} = 0$, and $F^2_{\mathrm{E}R_mQ} = 0$, $|H_1|$ is given as follows:

$$|H_1| = -F_q^2[F^1_{Qq}(Q')^2 + F^1_{Qq}(q')^2 + 2F^1_{qQ}Q'q'] \\ +F_q^1[2F^2_{\mathrm{E}R_mq}q' + F^2_{qQ}(Q')^2 + F^2_{qQ}(q')^2 + 2F^2_{qQ}q'Q']. \tag{A.8}$$

We have $F^1_{\mathrm{E}R_m}=\beta$, $F^1_{qQ}=-2\beta R'(q)\tilde{x}\hat{I}/((U-L)Q^3) < 0$, $F^2_{\mathrm{E}R_m}=\beta R'(q)\tilde{x}(\mathrm{d}U/\mathrm{d}\,\mathrm{E}R_m)/(U-L)$ (since $\mathrm{d}U/\mathrm{dE}R_m=1$), $F^2_{qQ}=\beta R''(q)\tilde{x}/(U-L) < 0$, and $F^2_{qQ}=\beta R''(q)\tilde{x}\hat{I}(U-L)/[(U-L)^2Q^2] < 0$. It follows that $\mathrm{d}^2Q/\mathrm{d}\,\mathrm{E}R_m^2 < 0$. Thus, Q is an increasing concave function of $\mathrm{E}R_m$.

We have $\mathrm{d}^2q/\mathrm{d}\,\mathrm{E}R_m^2 = |H_2|/|H|$. Thus, proceeding similarly,

$$|H_2| = F_Q^2[F^1_{Qq}(Q')^2 + F^1_{Qq}(q')^2 + 2F^1_{qQ}Q'q'] \\ -F_Q^1[2F^2_{\mathrm{E}R_mq}q' + F^2_{qQ}(Q')^2 + F^2_{qQ}(q')^2 + 2F^2_{qQ}q'Q']. \tag{A.9}$$

Using the results above we have $|H_2| < 0$, and thus $\mathrm{d}^2q/\mathrm{d}\,\mathrm{E}R_m^2 < 0$.

Actual output for entrepreneur $\tilde{x}$ is $y(\tilde{x}) \equiv R(q)\tilde{x}$. Thus, $\mathrm{d}^2y(\tilde{x})/d\mathrm{E}R_m = R''(q)\tilde{x}(\mathrm{d}q/\mathrm{d}\,\mathrm{E}R_m) + R'(q)\tilde{x}(\mathrm{d}^2q/\mathrm{dE}R_m^2) < 0$. Thus actual output is concave in $\mathrm{E}R_m$. Finally, the probability of obtaining external financing and therefore of production is $z(\tilde{x}) \equiv 1 - F(\hat{I}/Q)=[U-\hat{I}/Q]/(U-L)$. We have $\mathrm{d}^2z(\tilde{x})/\mathrm{d}\,\mathrm{E}R_m^2=(1/Q^2)(\mathrm{d}^2Q/\mathrm{d}\,\mathrm{E}R_m^2)-(\mathrm{d}Q/\mathrm{d}\,\mathrm{E}R_m)(2\hat{I}/Q^3)$. Since this is unambiguously negative, the final claim is established.

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