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A business cycle model with variable capacity utilization and demand disturbances

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Abstract

We develop a dynamic general-equilibrium model with demand (preference) shocks, estimated using Hall's (J. Labour Economics 15 (1997) 223) residual, that replicates U.S. business cycles well, at least compared to the real business cycle models. The key factor is cyclical capital utilization, which is based on imperfect competition, slow adjustments in capital stock, and fixed requirement of labor input. We also demonstrate theoretically that a representative-agent economy with preference shocks could be viewed as the reduced form of a heterogeneous-agents economy with incomplete markets. Specifically, a heterogeneous-agents economy with incomplete markets is aggregated into a representative-agent economy with preference shocks. This result would provide a microeconomic foundation for preference shock models. It is also shown that a shock to marginal utility of consumption and a shock to marginal disutility of labor have different effects.

JEL classification: E22; E32

Keywords: Aggregation; Preference shocks; Capacity utilization; Business cycles

1. Introduction

The aim of this paper is to show that business cycles could be accounted for by "demand (preference) shocks," together with variation in capital utilization. There are two theoretical contributions. First, we show that a representative-agent economy with stochastic preferences can be interpreted as a reduced form of a heterogeneous-agents

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economy with incomplete markets. This result provides a microeconomic foundation for preference shock models. Second, we offer a model that generates variations in capital utilization. The simulation results show that the model can reproduce U.S. business cycles relatively well.

The real business cycle (RBC) theory has been among the most influential theories of business cycles. The main result of the RBC theory is that as long as one identifies the Solow–Prescott residual as exogenous technology shocks, business fluctuations are explained quite well by the simple optimal growth model. [1] Recent empirical evidence shows, however, that, after correcting for factor utilization, fluctuations in the productivity growth rate are not only small, but also have nearly zero correlation with fluctuations in output. [2] If the true productivity growth rate is acyclical, it would be very difficult to accept the RBC hypothesis. [3]

In this paper, we examine demand (preference) shocks, identified as the difference between the "observable" component of the marginal rate of substitution and marginal product of labor, which we call Hall's residual (Hall, 1997) following Galí et al. (2002). The importance of this residual in accounting for fluctuations in labor has been emphasized by Hall (1997) for the U.S. Holland and Scott (1998) obtain a similar finding for the U.K., using a one-sector growth model with technology and preference shocks. Here, we examine if it can account for other macro variables as well. However, there are (at least) two problems with the use of a preference-shock model.

The first problem is its interpretation. Taken literally, preference-shock models assume that everyone's marginal utility fluctuates similarly over time, which does not sound very plausible. Furthermore, as Galí et al. (2002) show, Hall's residual responds endogenously to a monetary policy shock, and hence, cannot be totally due to exogenous preference shifts. Based on the theoretical result in Section 2, we interpret a representative-agent economy with preference shocks as a reduced form of a heterogeneous-agents economy with incomplete markets. According to this interpretation, Hall's residual does not represent exogenous shocks. It is the result of more fundamental shocks (such as monetary shocks, oil-price shocks, etc.). We do not ask what the driving forces of Hall's residual are. Instead, our hypothesis is that, whatever the driving forces of business cycles are, they work through Hall's residual. To examine this, we take Hall's residual as given, feed it into the model, and see the extent to which the observed sequence of Hall's residual accounts for U.S. business cycles.

The second problem of preference shock models is that they tend to generate counterfactual behavior such as countercyclical wage rates, and countercyclical investment (Baxter and King, 1991). This problem remains as long as the standard CRS technology is used. Hence, for example, if we simulate the models of Hall (1997) or Holland and Scott (1998) only with preference shocks, we would obtain such counterfactual behavior. We avoid this problem by considering a model with cyclical capital

[1] For example, Hansen (1985) and Prescott (1986).

[2] For example, Basu and Fernald (1995), Burnside et al. (1995a, b) and Shapiro (1996).

[3] Note that introducing cyclical factor utilization into the RBC model, as done, for example, by Burnside and Eichenbaum (1996), does not solve the problem. Such a model could reproduce business cycles with smaller disturbances, but the problem of correlation remains.

utilization.[4] The key features of the model are: (i) imperfect competition, (ii) capital stock is adjusted less frequently than labor input, and (iii) there is a fixed requirement for labor input to operate a firm. Under these assumptions, in each period firms are active only a fraction of period, which varies cyclically. This feature of our model is consistent with the empirical research such as Bresnahan and Ramey (1994) and Shapiro (1996). Burnside and Eichenbaum (1996) consider a model of cyclical factor utilization which is different from ours. They assume that a higher utilization rate is costly because it depreciates capital faster and it requires higher effort of workers.[5] Closer to our model is the model of Cooley et al. (1995). They consider a continuum of firms, each of which is subject to idiosyncratic technology shocks every period after it has rented capital and before it hires labor. They also assume that there is a fixed labor requirement to operate a firm. It then follows that only a fraction of firms operate each period, and that the fraction varies according to the aggregate technology shock.

The numerical results show that our preference-shock model with cyclical capital utilization could account for U.S. business cycles relatively well, at least compared to the RBC model. In particular, it replicates fluctuations of the Solow–Prescott residual rather well. We also analyze theoretically the difference between the effects of a shock to marginal utility of consumption and those of a shock to marginal disutility of labor, and the difference between the effects of a shock to intertemporal marginal rate of substitution and those of a shock to intratemporal marginal rate of substitution. Given that business cycles could be due to fluctuations in Hall's residual, an interesting next question would be to investigate their cause(s). Galí et al. (2002) emphasize countercyclical wage markup. This is closely related to our incomplete-market interpretation in that both stories are based on labor-market frictions. In particular, uninsured employment risk could be due to wage rigidities.

The remainder of this paper is organized as follows. In Section 2 we give a theoretical foundation for preference shock models. In Section 3 we present the model economy. Section 4 describes simulation results. Section 5 concludes the paper.

2. Market incompleteness and Hall's residual

Hall (1997) considers a representative-agent economy in which the agent has stochastic period utility of the form

$$U(c_t, h_t, \psi_t) = \psi_t \ln c_t - a \frac{\chi}{1+\chi} h_t^{(1+\chi)/\chi},$$

where c is consumption, h is hours of work, ψ is a stochastic shock to the period utility function, and a and χ are parameters. Assuming a Walrasian equilibrium in the labor market, the (log) preference shock, $\ln \psi_t$, is identified as the residual:

$$\mathrm{mrs}_t - \mathrm{mpl}_t, \tag{1}$$

where $\mathrm{mrs}_t \equiv \ln a + \ln c_t + (1/\chi) \ln h_t$ is the "observable component" of the (log) marginal rate of substitution of the representative agent, and mpl_t is the (log) marginal

[4] Baxter and King (1991) consider externalities.

[5] Similar models are used by Greenwood et al. (1988), Bils and Cho (1994), and Imbs (1999).

product of labor. Following Galí et al. (2002), we call this Hall's residual.[6] In addition to preference shocks, Hall (1997) considers technology and government-purchase shocks, and finds that almost all employment fluctuations in the U.S. are attributed to preference shocks. Holland and Scott (1998) obtain a related finding for the U.K.

The results in Hall (1997) and Holland and Scott (1998), among others, suggest an important role of Hall's residual, at least, to account for employment fluctuations. Taken literally, however, a representative-agent model with preference shocks assumes that everyone in the economy experiences similar shifts in preferences. This does not appear a very plausible assumption. Moreover, Galí et al. (2002) provide VAR evidence that Hall's residual responds endogenously to an exogenous monetary policy shock, which suggests that it cannot simply reflect exogenous variations in preferences.

In response to such criticisms against preference-shock models, we illustrate that incomplete-market economies (without preference shocks) are generally aggregated into representative-agent economies with preference shocks. In this sense, a representative-agent economy with preference shocks can be viewed as a reduced form of a heterogeneous-agents economy with incomplete markets. Hence, if the asset market is incomplete, even though individuals do not have any preference shifts, there appear stochastic shifts in the "aggregate preferences." If we take this interpretation, it is natural to expect that Hall's residual is affected by, say, monetary shocks.[7]

The intuition behind this aggregation result is simple. Consider an economy with many agents. As is well known, if the asset market is complete, aggregation is possible in the following sense. Given any equilibrium in the original economy, construct an economy with a single agent whose utility function is the sum of the utility functions of the individuals in the original economy, using the inverses of their marginal utilities of wealth as Pareto weights. The single-agent economy constructed this way has the same equilibrium as the original economy. Even if the asset market is incomplete, similar aggregation is possible, except that the constructed preferences of the representative agent have stochastic components. This is because marginal rates of substitution of different agents are not equalized in equilibrium, and hence, marginal utilities of wealth of agents fluctuate stochastically over time.

To be specific, consider the following economy.[8] Individuals are infinitely lived and indexed by $i \in \mathscr{I} = \{1, \ldots, I\}$. The individual employment status in each period is the only uncertainty in the economy, which is represented by a stochastic process, $\{s_t\}$. At the beginning of each period t, a shock $s_t \in \mathscr{S} = \{1, \ldots, S\}$ realizes, and determines who are employed in that period. The function $\varepsilon^i : \mathscr{S} \to \{0, 1\}$ describes the employment status of individual i: if $\varepsilon^i(s) = 1$ individual i is employed; if $\varepsilon^i(s) = 0$ she is unemployed. That is, $\varepsilon^i(s_t), i \in \mathscr{I}$, are idiosyncratic employment shocks. Assume that $\sum_i \varepsilon^i(s) > 0$, each s. Let $s^t = (s_0, s_1, \ldots, s_t)$ denote the history of shocks up through period t. Given a t-period history s^t, let $s^{t+j}|s^t$ denote a $(t + j)$-period history which is a successor

[6] Earlier work that uses similar measures of preference shifts include, among others, Baxter and King (1991), Kennand (1988) and Parkin (1988). Holland and Scott (1998) use a similar measure for the U.K.

[7] In this paper, by "demand shocks" we mean stochastic shifts in *aggregate* preferences.

[8] Related models include, among others, Scheinkman and Weiss (1986), Aiyagari (1994), Krusell and Smith (1998), and Krebs (2002).

of s^t. Let $\pi(s^t)$ denote the probability of history s^t, and $\pi(s^{t+j}|s^t)$ the probability of s^{t+j} conditional on the occurrence of s^t. The initial shock, s_0, is given.

The preferences of agent i are given by

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\{u[c_t^i(s^t)]-v[h_t^i(s^t)]\}\pi(s^t),$$

where $c_t^i(s^t)$ is consumption at s^t, $h_t^i(s^t)$ is hours worked at s^t, and $\beta\in(0,1)$ is the discount factor. The period utility function, $u(c)-v(h)$, satisfies the standard assumptions of monotonicity, continuity, concavity, differentiability, as well as the boundary conditions: $\lim_{c\to 0}u'(c)=+\infty$; $v(0)=v'(0)=0$. Capital stock is the only store of value, and hence, the asset market is incomplete (individual labor shocks are not insurable). The flow budget constraint of agent i is

$$k_{t+1}^i(s^t)+c_t^i(s^t)=\left\{1+R_t(s^t)-\delta\right\}k_t^i(s^{t-1})+\varepsilon^i(s_t)w_t(s^t)h_t^i(s^t), \tag{2}$$

where $k_t^i(s^{t-1})\geqslant 0$ is the stock of capital held by agent i at the beginning of s^t; $R_t(s^t)$ is the rental rate of capital; $\delta\in(0,1)$ is the depreciation rate; $w_t(s^t)$ is the wage rate. The initial holdings of capital are $k_0^i>0$, $i\in\mathscr{I}$.

Let $\beta^t\lambda_t^i(s^t)\pi(s^t)$ be the multiplier on the flow budget constraint at s^t. The first-order conditions are:

$$u'[c_t^i(s^t)]=\lambda_t^i(s^t),\qquad v'[h_t^i(s^t)]=\lambda_t^i(s^t)\varepsilon^i(s_t)w_t(s^t), \tag{3}$$

and

$$1=\sum_{s^{t+1}|s^t}\frac{\beta\lambda_{t+1}^i(s^{t+1})\pi(s^{t+1}|s^t)}{\lambda_t^i(s^t)}\{R_{t+1}(s^{t+1})+1-\delta\}, \tag{4}$$

as well as the flow budget constraint (2). Note that, since $v'(0)=0$, $h^i(s^t)=0$ when $\varepsilon^i(s_t)=0$. The transversality condition is

$$\lim_{t\to\infty}\beta^t\sum_{s^t}\lambda_t^i(s^t)\{R_t(s^t)+1-\delta\}k_t^i(s^{t-1})\pi(s^t)=0. \tag{5}$$

Let $k_{t+1}(s^t)$, $h_t(s^t)$, and $c_t(s^t)$ denote aggregate capital, aggregate hours worked, and aggregate consumption, respectively. The aggregate resource constraint is

$$c_t(s^t)+k_{t+1}(s^t)-(1-\delta)k_t(s^{t-1})=F[k_t(s^{t-1}),h_t(s^t)], \tag{6}$$

where $F(k,h)$ is a standard CRS production function. We assume that the wage rate of an employed agent equals her marginal product of labor.

$$w_t(s^t)=F_h[k_t(s^{t-1}),h_t(s^t)].$$

The rental market equilibrium requires

$$R_t(s^t)=F_k[k_t(s^{t-1}),h_t(s^t)].$$

As is well known, if the asset market is complete, a multi-agent economy like ours is aggregated into a representative-agent economy. Suppose for a moment that there is a complete set of contingent claims. Let $\{q_t(s^t)\}$ be the price system of those contingent claims ($q_0(s_0)$ is normalized to unity). Then, the flow budget constraints reduce to a single, lifetime budget constraint:

$$\sum_{t=0}^{\infty}\sum_{s^t} q_t(s^t)c_t^i(s^t) \leqslant \sum_{t=0}^{\infty}\sum_{s^t} q_t(s^t)\varepsilon^i(s_t)w_t(s^t)h_t^i(s^t) + k_0^i. \tag{7}$$

Let Λ^i be the Lagrange multiplier for this lifetime budget constraint. Note that Λ^i is a constant and is related to the multipliers on the flow budget constraints (2), $\lambda^i(s^t)$, as

$$\lambda_t^i(s^t) = \frac{q_t(s^t)}{\beta^t \pi(s^t)}\Lambda^i \quad \text{all } t, s^t.$$

Then, the ratio of marginal utilities of consumption of two agents is constant over time and across histories:

$$\frac{u'[c_t^i(s^t)]}{u'[c_t^j(s^t)]} = \frac{\lambda_t^i(s^t)}{\lambda_t^j(s^t)} = \frac{\Lambda^i}{\Lambda^j} \quad \text{all } t, s^t, i, j.$$

It follows that individual consumption is written as a time-and-history-independent function, say f^i, of aggregate consumption:

$$c_t^i(s^t) = f^i[c_t(s^t); \Lambda^1, \ldots, \Lambda^I].$$

Given an equilibrium with complete asset market, the corresponding representative-agent economy is constructed using *constant* Pareto weights, $1/\Lambda^i$, $i \in \mathscr{I}$. A key for this result is that the constraint faced by each individual is given by a single, lifetime budget constraint (7), so that the ratios of marginal utilities of wealth are constant over time and across histories: $\lambda_t^i(s^t)/\lambda_t^j(s^t) = \Lambda^i/\Lambda^j$.

Now turn back to the incomplete-market case: no contingent claims are traded and physical capital is the only asset. The aggregation result is summarized in the following proposition.

Proposition. *Let an incomplete-market equilibrium,* $\{\hat{c}_t^i(s^t), \hat{h}_t^i(s^t), \hat{k}_t^i(s^t), \hat{\lambda}_t^i(s^t), \hat{R}_t(s^t), \hat{w}_t(s^t)\}$, *be given. A corresponding representative-agent economy is given by*

1. *the representative agent with preferences*:

$$\sum_{t=0}^{\infty}\sum_{s^t} \beta^t U(c_t(s^t), h_t(s^t), s_t \mid \hat{\lambda}_t^i(s^t), i \in \mathscr{I})\pi(s^t),$$

where the period utility is defined by

$$U(c, h, s | \lambda^i \in \mathscr{I}) = \max_{c^i, h^i, i \in \mathscr{I}} \left\{ \sum_{i \in \mathscr{I}} \frac{\lambda^1}{\lambda^i}(u(c^i) - v(h^i)) \right\}, \tag{8}$$

subject to

$$\sum_{i\in\mathscr{I}} c^i = c, \quad \text{and} \quad \sum_{i\in\mathscr{I}} \varepsilon^i(s)h^i = h.$$

2. *the aggregate resource constraint* (6).

The proof is in Appendix A.[9] The key difference from the complete-market case is that the ratios of marginal utilities of wealth, $\lambda_t^1(s^t)/\lambda_t^i(s^t)$, $i\in\mathscr{I}$, are not constant and fluctuate stochastically. As a result, the Pareto weights fluctuate, so that the period utility function of the representative agent (8) is stochastic, even if there are no preference shocks in the original incomplete-market economy.

An interesting special case is the one in which the period utility function of each individual takes the constant-elasticity form:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

$$v(h) = \frac{\chi}{1+\chi} h^{(1+\chi)/\chi}.$$

In this case, the aggregate period utility function (8) becomes

$$U(c,h,s|\lambda) = \psi(\lambda)\,\frac{1}{1-\gamma}\,c^{1-\gamma} - \phi(\lambda,s)\,\frac{\chi}{1+\chi}\,h^{(1+\chi)/\chi}, \tag{9}$$

where $\lambda \equiv (\lambda^1,\ldots,\lambda^I)$, and ψ and ϕ are "preference shocks" given by

$$\psi(\lambda) \equiv \lambda^1 \left[\sum_{i\in\mathscr{I}} (\lambda^i)^{-1/\gamma}\right]^{\gamma},$$

$$\phi(\lambda,s) \equiv \lambda^1 \left(\sum_{i\in\mathscr{I}} (\lambda^i\varepsilon^i)^{\chi}\right)^{-1/\chi}.$$

The aggregate preferences of form (9) is the one we use for the quantitative analysis in what follows.

We have illustrated that a representative-agent economy with preference shocks can be viewed as the reduced form of a heterogeneous-agents economy with incomplete markets. Of course, this is not the only interpretation of Hall's residual. Galí et al. (2002), for example, suggest that it reflects countercyclical markup variation, caused by price and wage rigidities. Their interpretation and ours are viewed as complementary. Both interpretations emphasize imperfections in the labor market. Uninsurable labor-income shocks are likely due to wage rigidities. Conversely, if the real wage is rigid, it is reasonable to expect uninsurable labor-income fluctuations.

In the next section, we use the reduced-form, representative-agent economy with preference shocks to account for U.S. business cycles. Note that under our incomplete-market interpretation preference shocks (Hall's residual) are not exogenous preference

[9] As discussed in Appendix A, the utility function of the representative agent is not unique. In (8) the Pareto weights are normalized using $\lambda_t^1(s^t)$, but we could have normalized them by $\lambda_t^j(s^t)$ for any j.

shifts. They are the result of more fundamental shocks such as monetary shocks, oil-price shocks, etc.[10] We do not ask what the driving forces of Hall's residual are. Instead, our hypothesis is that, whatever the driving forces of business cycles are, they work through Hall's residual. To examine this, we take Hall's residual as given, feed it to the model, and see the extent to which the observed sequence of Hall's residual accounts for U.S. business cycles. We will see that with variable capacity utilization Hall's residual can replicate them well.

3. Model economy

Time is discrete and indexed by $t=0,1,2,\ldots$. The length of each period is normalized to one. There are two types of stochastic disturbances to the economy: preference shocks, (ψ_t, ϕ_t), and government spending shocks, ξ_t (ξ_t is the share of government purchases in GDP). Let $s_t = (\psi_t, \phi_t, \xi_t)$, and $s^t = (s_0, \ldots, s_t)$. At the beginning of each period, s_t realizes and is observed. Hence, the decision of consumers and firms at period t depends on s^t. We assume that s_t follows a Markov process, which is specified in Section 3.6. As in the previous section, $\pi(s^t)$ denotes the probability of history s^t.

3.1. Differentiated products

As we shall see below, our formulation of variable capacity utilization makes the production technology non-convex, which leads us to assume imperfect competition in the goods market. Suppose that there is a continuum of firms, $j \in [0,1]$, that produce differentiated products. These goods are used for consumption, for investment, and by the government. Following the literature, we assume that all of these demanders are interested in a single composite good.[11] Specifically, consumption, c, investment, i, and government purchases, g, are given as

$$x = \left[\int_0^1 x(j)^{1/\mu} \, dj \right]^{\mu}, \quad x = c, i, g, \tag{10}$$

where $x(j)$ is the jth differentiated product used for purpose $x = c, i, g$, and $\mu \geqslant 1$. Let $p(j)$ be the price of differentiated product j. Then, the cost minimization yields the derived demand function:

$$x(j) = x \left[\frac{p(j)}{p} \right]^{\mu/(1-\mu)}, \quad x = c, i, g, \quad j \in [0,1], \tag{11}$$

[10] Note that Rotemberg and Woodford (1997) and Ireland (2000) consider a monetary model with nominal price rigidities, and find that monetary policy shocks explain very little of U.S. business cycles. As the evidence by Galí et al. (2002) suggests, however, it might be the case that such shocks work through Hall's residual.

[11] The literature on macroeconomic models of monopolistic competition includes, among others, Hornstein (1993), and Rotemberg and Woodford (1995).

where p is the price index defined by

$$p = \left[\int_0^1 p(j)^{1/(1-\mu)} \, \mathrm{d}j \right]^{1-\mu}.$$

In what follows, we take the composite good as the numeraire so that $p = 1$.

3.2. Households

There is a representative individual who consumes, whose preference are given by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t U[c_t(s^t), h_t(s^t), \psi_t, \phi_t] \pi(s^t),$$

where $c_t(s^t)$ is the composite good used for consumption as defined in (10), $h_t(s^t)$ is hours worked, and the period utility function, U, is

$$U(c, h, \psi, \phi) = \psi \ln(c) - \phi \frac{a\chi}{1+\chi} h^{(1+\chi)/\chi}, \tag{12}$$

where a and χ (wage-elasticity of Frisch labor supply) are constants.[12]

The flow budget constraint for the household is

$$c_t(s^t) + i_t(s^t) = w_t(s^t) h_t(s^t) + R_t(s^t) k_t(s^{t-1}) - \tau_t(s^t) + d_t(s^t), \tag{13}$$

where $i_t(s^t)$ is the composite good used for investment as defined in (10), $w_t(s^t)$ the wage rate, $R_t(s^t)$ the rental rate, δ the depreciation rate, $k_t(s^{t-1})$ the capital stock, $\tau_t(s^t)$ the lump-sum tax, and $d_t(s^t)$ is the dividends received from the firms. The capital stock evolves as

$$k_{t+1}(s^t) = i_t(s^t) + (1-\delta) k_t(s^{t-1}), \tag{14}$$

where the initial stock, $k_0 > 0$, is given.

Let $\beta \lambda_t(s^t) \pi(s^t)$ be the multiplier on the flow budget constraint. The first-order conditions are:

$$c_t(s^t) = \frac{\psi(s^t)}{\lambda_t(s^t)}, \tag{15}$$

$$h_t(s^t) = A \left[\frac{w_t(s^t) \lambda_t(s^t)}{\phi_t} \right]^{\chi}, \tag{16}$$

$$\lambda_t(s^t) = \beta \sum_{s^{t+1}|s^t} \lambda_{t+1}(s^{t+1}) [R_{t+1}(s^{t+1}) + 1 - \delta] \pi(s^{t+1}|s^t), \tag{17}$$

[12] This form of utility is assumed to make a balanced growth path exist.

where $A \equiv a^{-\chi}$. The transversality condition is

$$\lim_{t\to\infty} \beta^t \sum_{s^t} \lambda_t(s^t)\{R_t(s^t) + 1 - \delta\}k_t(s^{t-1})\pi(s^t) = 0. \tag{18}$$

3.3. Government

The government purchases the composite good, $g_t(s^t)$. Let $y_t(s^t)$ be total output:

$$y_t(s^t) = c_t(s^t) + k_{t+1}(s^t) - (1-\delta)k_t(s^{t-1}) + g_t(s^t). \tag{19}$$

Assume that the share of government purchases in GDP, ξ_t, fluctuates stochastically:

$$g_t(s^t) = \xi_t y_t(s^t). \tag{20}$$

The government finances its purchases through lump-sum taxes, $\tau_t(s^t)$. Assume balanced budget:

$$\tau_t(s^t) = g_t(s^t). \tag{21}$$

3.4. Firms

Production takes place in continuous time. At the beginning of each period, after the shocks have realized, each firm decides the amount of capital stock it rents, and the number of hours it operates during the period. The number of operation hours determines the amount of labor hired in the period. Notice that the amount of capital stock remains the same within each period, although the level of labor input can change. This assumption reflects the idea that capital stock is adjusted less frequently than labor input. The number of operation hours corresponds to the capital utilization rate. There are no aggregate nor idiosyncratic technology shocks.

Consider firm $j \in [0,1]$ in period t. To simplify notation, we omit the history-index s^t for a while. After observing the current shock, it rents capital, $k_t(j)$, and decides the length of time it operates, $u_t(j) \in [0,1]$. That is, firm j produces output ("active") over the time $[t, t+u_t(j)]$, and does not ("inactive") over $(t+u_t(j), t+1]$. To be active, a fixed amount of labor, $\bar{h}$, is required, and the firm produces a constant flow of output, $z_t k_t(j)^\theta$, where z_t is the economy-wide level of technology. Assume that using more labor does not increase the output flow. Also assume that even when the firm is inactive, $\underline{h}$ units of labor are required for maintenance.[13] Let $h_t(j,r)$ be the labor input of firm j at time $r \in [t, t+1)$. Then, given $u_t(j)$, we have

$$h_t(j,r) = \begin{cases} \bar{h} & \text{for } r \in [t, u_t(j)], \\ \underline{h} & \text{for } r \in (u_t(j), t+1). \end{cases}$$

The output flow, $y_t(j,r)$, at each point in time r is

$$y_t(j,r) = \begin{cases} z_t k_t(j)^\theta & \text{for } r \in [t, u_t(j)], \\ 0 & \text{for } r \in (u_t(j), t+1). \end{cases}$$

[13] As we shall see in Section 4 assuming $\underline{h} > 0$ makes output more volatile than labor.

Here, the key features of the production side of the economy are (i) imperfectly competitive firms; (ii) capital stock each firm rents is constant within each period, $k_t(j,r)=k_t(j)$, all $r\in[t,t+1)$; (iii) labor input at each instant can take only two values, $h_t(j,r)\in\{\underline{h},\bar{h}\}$, all $r\in[t,t+1)$. With imperfect competition, each firm takes into account the demand function (11) when deciding the level of output. In particular, aggregate demand directly affects its decision. The rental market equilibrium (Eq. (25) below) requires that the stock of capital each firm rents equals its aggregate supply, which is predetermined. Since the firm cannot change the level of capital within a period and the labor input takes only two values, the level of output is adjusted only through changing the utilization rate, $u_t(j)$. This is the mechanism of cyclical capital utilization in our model.

The total output that firm j produces in period $t, y_t(j)$, is

$$y_t(j)=\int_t^{t+1} y_t(j,r)\,\mathrm{d}r = z_t u_t(j)k_t(j)^\theta, \tag{22}$$

and the total labor input, $h_t(j)$, is

$$h_t(j)=\int_t^{t+1} h_t(j,r)\,\mathrm{d}r = u_t(j)\bar{h}+(1-u_t(j))\underline{h}.$$

This equation can be solved for the capital utilization rate, $u_t(j)$, as

$$u_t(j)=\frac{h_t(j)-\underline{h}}{\bar{h}-\underline{h}}. \tag{23}$$

Substituting for $u_t(j)$ from this equation into (22) yields

$$y_t(j)=z_t k_t(j)^\theta\,\frac{h_t(j)-\underline{h}}{\bar{h}-\underline{h}},$$

with $\underline{h}\leqslant h_t(j)\leqslant\bar{h}$.

Notice that the technology exhibits increasing returns in k and h as long as capacity is not fully utilized. Notice also that if $\underline{h}$ is equal to zero, the production function can be rewritten as

$$y_t(j)=\bar{h}^{\theta-1}z_t[u_t(j)k_t(j)]^\theta[h_t(j)]^{1-\theta},$$

which is homogeneous of degree one in uk and h. Hence as long as ξ is small, the technology here is consistent with the empirical findings by Basu and Fernald (1995) and Burnside et al. (1995a).

Eq. (11) implies that the total demand for product j is given by

$$y_t(j)=y_t\left[\frac{p_t(j)}{p_t}\right]^{\mu/(1-\mu)},$$

where $p_t=1$, and y_t is

$$y_t=\int_0^1 y_t(j)\,\mathrm{d}j.$$

We restrict our attention to the symmetric equilibrium in which $y_t(j)=y_t$, $p_t(j)=1$, $h_t(j)=h_t$, and $k_t(j)=k_t$. Also, we assume that $\underline{h}\leqslant h_t(s^t)\leqslant\bar{h}$ holds all s^t. In such an equilibrium, the factor market equilibrium conditions at s^t are given by

$$w_t(s^t) = \frac{z_t k_t(s^t)^\theta}{\mu(\bar{h} - \underline{h})}, \tag{24}$$

$$R_t(s^t) = \frac{\theta}{\mu} z_t k_t(s^t)^{\theta-1} \frac{h_t(s^t) - \underline{h}}{\bar{h} - \underline{h}}. \tag{25}$$

The aggregate production function is

$$y_t(s^t) = z_t k_t(s^t)^\theta \frac{h_t(s^t) - \underline{h}}{\bar{h} - \underline{h}}. \tag{26}$$

The dividends (profits) are

$$d_t(s^t) = y_t(s^t) - \frac{1}{\mu} z_t k_t(s^t)^\theta \frac{(1+\theta)h_t(s^t) - \theta\underline{h}}{\bar{h} - \underline{h}}. \tag{27}$$

It follows from (24)–(27) that the share of capital is constant, θ/μ, that the shares of labor fluctuate countercyclically:

$$\text{Share of labor} = \frac{1}{\mu} \frac{h_t(s^t)}{h_t(s^t) - \underline{h}},$$

and that the share of profit varies procyclically.

We assume that the level of technology, z_t, grows at a constant rate γ:

$$z_t = z_0 \gamma^t. \tag{28}$$

Eq. (24) shows that, even in the absence of technology shocks, our model generates (mildly) procyclical wage rates, because of variable capital utilization.

3.5. Equilibrium

Given $\{z_t\}$, initial condition k_0, and stochastic process of shocks $\pi(s^t)$, a symmetric equilibrium is an allocation, $\{c_t(s^t), h_t(s^t), k_{t+1}(s^t), y_t(s^t), d_t(s^t)\}$, a price system, $\{w_t(s^t), R_t(s^t)\}$, fiscal policy, $\{g_t(s^t), \tau_t(s^t)\}$, and multipliers, $\{\lambda_t(s^t)\}$, that satisfy the individual optimization conditions, (15)–(18), the aggregate resource constraint, (19), the government policy, (20)–(21), the factor market equilibrium conditions, (24)–(25), the production function, (26), and the dividend equation, (27).

3.6. Shocks

We assume that the share of government purchases, $\ln \xi_t$, follows an AR(1) process:

$$\ln(\xi_t) = (1 - \rho_g)\ln(\bar{\xi}) + \rho_g \ln(\xi_{t-1}) + \varepsilon_t, \tag{29}$$

where $\bar{\xi}$ is the steady-state share of government purchases, and ε_t is a serially uncorrelated process with mean 0 and standard deviation σ_ε.

The marginal rate of substitution between consumption and leisure is given by

$$\frac{a\phi_t}{\psi_t}\, c_t(s^t)h_t(s^t)^{1/\chi}.$$

It follows that Hall's residual (1) determines only $\{\phi_t/\psi_t\}$. We need additional restriction to identify $\{\phi_t\}$ and $\{\psi_t\}$. One approach is to use an additional equation (such as the Euler equation) to identify them. Here, we take a more indirect approach. That is, we consider the following two polar cases:

Model 1: $\phi_t = 1$, all t, and $U = \psi \ln(c) - \frac{a\chi}{1+\chi} h^{(1+\chi)/\chi}$, with

$$\ln(\psi_t) = (1-\rho_p^1)\ln(\bar{\psi}) + \rho_p^1 \ln(\psi_{t-1}) + v_t^1.$$

Model 2: $\psi_t = 1$, all t, and $U = \ln(c) - \phi \frac{a\chi}{1+\chi} h^{(1+\chi)/\chi}$, with

$$\ln(\phi_t) = (1-\rho_p^2)\ln(\bar{\phi}) + \rho_p^2 \ln(\phi_{t-1}) + v_t^2.$$

Here, $\bar{\psi}$ and $\bar{\phi}$ denote the steady-state values, and the innovation process, v_t^i, $i=1,2$, is a serially uncorrelated process with mean 0 and standard deviation σ_v^i. Then, we shall show that if the elasticity of labor supply is adequately chosen each model can replicate the U.S. business cycles relatively well. Since the true preference-shock process is a convex combination of these two polar cases, it follows that our model with identified ψ and ϕ would also be able to replicate the U.S. business cycles with an adequately chosen elasticity of labor supply.

In the first model of preference shocks, shocks are to marginal utility of consumption alone, and in the second model they are to marginal disutility of labor alone. In most of the existing work, the first model is used, although Hall's residual itself does not tell that is the right specification. Does it make a difference how to model the preference shock process? It may, in principle. In particular, the effect of Hall's residual on investment may differ between the two specifications. To see this, consider the following deterministic, partial-equilibrium example. Note that a higher value of Hall's residual corresponds to a higher ψ in Model 1, and a lower ϕ in Model 2. Suppose that the sequence of preference shifts in Model 1 is given by

$$\psi_0 = \exp(\eta), \quad \psi_t = 1 \quad \text{all } t \geqslant 1, \tag{30}$$

in Model 2 by

$$\phi_0 = \exp(-\eta), \quad \phi_t = 1 \quad \text{all } t \geqslant 1, \tag{31}$$

where $\eta > 0$. That is, there is a temporary positive shock to Hall's residual at date 0, and it takes a constant value from date 1 on. We consider how the shock to Hall's residual, η, affects the individual's choice of consumption, investment, and labor supply.

In Model 1, the individual's date-0 decision problem can be written as

$$\max_{c_0,h_0,k_1} \psi_0 \ln c_0 - \frac{a\chi}{1+\chi} h_0^{(1+\chi)/\chi} + \beta v(k_1), \tag{32}$$

subject to the flow budget constraint

$$c_0 + k_1 = w_0 h_0 + (R_0 + 1 - \delta)k_0, \tag{33}$$

where $v(k_1)$ is the value function for the problem from period 1 on (since $\psi_t = 1$, all $t \geqslant 1$, are given, the value after period 1 does not depend on ψ_0). Let λ_0 denote the Lagrange multiplier on the flow budget constraint. The first-order conditions are

$$\lambda_0 = \beta v'(k_1), \quad \frac{\psi_0}{c_0} = \lambda_0, \quad a h_0^{1/\chi} = \lambda_0 w_0, \tag{34}$$

as well as the flow budget constraint. Now let us see how an increase in η affects the choice of the agent (given prices). An increase in η obviously implies a higher ψ_0, which increases λ_0 (the proof is in the Appendix A). The first-order conditions, (33)–(34), immediately imply that (i) labor supply at date 0 is higher ($\uparrow h_0$); (ii) investment at date 0 is lower ($\downarrow k_1$, because of the concavity of $v(k_1)$); (iii) consumption at date 0 is higher ($\uparrow c_0$) (this follows from (33)). In Model 1, therefore, a temporary increase in Hall's residual has an effect that tends to make investment move *countercyclically*. Let us call this the *substitution effect* of ψ.

In Model 2, on the other hand, that is not true. The individual's problem at date 0 is

$$\max_{c_0,h_0,k_1} \ln c_0 - \phi_0 \frac{a\chi}{1+\chi} h_0^{(1+\chi)/\chi} + \beta v(k_1), \tag{35}$$

subject to the flow budget constraint (33). The first-order conditions are (33) and

$$\lambda_0 = \beta v'(k_1), \quad \frac{1}{c_0} = \lambda_0, \quad a\phi_0 h_0^{1/\chi} = \lambda_0 w_0. \tag{36}$$

A rise in Hall's residual, η, reduces ϕ_0, which, in turn, reduces λ_0 (see Appendix A). The first-order conditions, (33) and (36), imply that (i) labor supply at date 0 is higher ($\uparrow h_0$); (ii) investment at date 0 is higher ($\uparrow k_1$); (iii) consumption at date 0 is higher ($\uparrow c_0$). In particular, in Model 2, investment is procyclical. The intuition is very simple: a temporary reduction in ϕ makes the current period a good time to work, which implies higher income; due to consumption smoothing, higher income today yields higher investment.

We have seen how the specification of the preference shock process makes a difference.[14] Note that the above example considers only a temporary shock (ψ_0 and ϕ_0 do not affect their future realizations), and also does not take into account general-equilibrium effects through w and R. Whether or not investment moves countercyclically depends on the relative strengths of the substitution effect, the persistence effect, and the general-equilibrium effect. The numerical results in the next section show that (i) investment is less volatile in Model 1, and (ii) for some parameter values investment does move countercyclically in Model 1, because of the substitution effect of ψ.[15] We shall see, however, that, in spite of those differences, both models can replicate the

[14] Also see the discussion in Appendix A on the distinction between shocks to the intertemporal marginal rate of substitution and shocks to the intratemporal one.

[15] Note that, assuming Model 1 of preference shock process, Baxter and King (1991) provide a numerical result that with the standard Cobb–Douglas technology investment is countercyclical.

data fairly well as long as the elasticity of labor supply, χ, is adequately chosen for each model. As the discussion here suggests, the elasticity of labor supply required for Model 1 to replicate the U.S. business cycles is higher.

4. Numerical results

In this section, we examine numerical properties of our model, and compare them to those of Hansen's (1985) real business cycle model. The log-linear approximation method of King et al. (1988) is used to solve those models. [16]

4.1. Measuring preference shocks

Using Hall's residual, preference shocks in Model 1, $\{\psi_t\}$, are measured by

$$\ln(\psi_t) = \ln(c_t) + \frac{1}{\chi}\ln(h_t) + \ln(h_t - \underline{h}) = \ln(y_t) + \text{constant},$$

and in Model 2, by

$$\ln(\phi_t) = -\ln(c_t) - \frac{1}{\chi}\ln(h_t) - \ln(h_t - \underline{h}) + \ln(y_t) + \text{constant}.$$

If we assume the same parameter values in the two models, $\ln(\psi_t) = -\ln(\phi_t)$. But, note that we shall assume different values of χ for the two models, and that, even when $\ln(\psi_t) = -\ln(\phi_t)$, the two models generate different equilibria, as we have discussed in Section 3.6.

These measures are estimated using U.S. data on consumption, labor, and output, as well as the parameter values given in the next subsection. Table 1 shows the regression results for different values of the elasticity of labor supply, χ. The standard deviation of innovations monotonically decreases with the elasticity of labor supply. This is because as labor supply becomes less elastic, the larger shocks are needed to be compatible with the observed amount of labor fluctuations. In contrast, the AR coefficient of the preference shock process does not appear to be sensitive to the choice of labor supply elasticity.

Alternatively, as Baxter and King (1991) do, one could measure preference shocks using the data on wages, instead of marginal product of labor. The numerical results using their measure are essentially identical, so not reported here. [17]

[16] The deterministic steady state is defined by detrending relevant variables by $z_t^{1/(1-\theta)}$. It is unique, and is saddle-point. Hence, the equilibrium uniquely exists, at least around the steady state. Theoretically, this is because our economy does not satisfy the necessary condition for indeterminacy in Benhabib and Farmer (1994).

[17] They are available from the author upon request.

Table 1
Estimates of stochastic process for preference shocks

χ	0.2	0.5	1	3	5
ρ_p	0.9827	0.9845	0.9851	0.9840	0.9833
σ_v	0.0497	0.0249	0.0170	0.0123	0.0115

Notes: Output—real GDP; Consumption—real consumption of nondurables and services; Investment—real fixed investment plus consumer durables; Hours—total hours worked (Household Survey). Wages—real compensation per manhour. All data is taken from CITIBASE. Sample period 1959:III–1993:IX.

4.2. Calibrating parameters

The parameters are chosen so that along a deterministic balanced growth path (i) the quarterly growth rate is 1.0037, (ii) the share of private consumption expenditure is 0.59, (iii) the share of investment is 0.18, (iv) the quarterly rate of interest is 0.016, (v) the share of labor is 0.58, (vi) the share of monopoly profits is 0.02, (vii) the capacity utilization rate is 0.82, (viii) the hours worked is 22 percent of the available hours. These values are fairly standard.[18] The average capacity utilization rate is taken from Cooley et al. (1995). In addition, we have to specify the ratio of $\underline{h}$ to $\bar{h}$. Based on the evidence reported in Aizcorbe (1994), we set this value to 10 percent. The benchmark value of the elasticity of labor supply χ is set to 3 in Model 1 and 1 in Model 2. Finally the government expenditure process and the preference shock process are parameterized according to the regression results. The autocorrelation coefficients ρ_g is 0.99, and the standard deviations of innovation σ_ε is 0.013. (See Table 1 for ρ_p and σ_v.) These conditions determine the benchmark parameter values for our preference shock model. For the Hansen model, we use the technology shock process with autocorrelation coefficient 0.95 and standard deviation of innovation 0.007; the rest of the parameters are estimated using conditions (ii)–(v) and (viii) above.

4.3. Results

4.3.1. Benchmark parameter values

Assume the benchmark parameter values. Figs. 1–2 show the responses to a one-standard-deviation shock in Models 1 and 2, respectively (a positive shock to ψ in Model 1 and a negative shock to ϕ in Model 2). The vertical axis in each graph measures the percentage deviation of a relevant variable from its steady state value. The unit of the horizontal axis is a quarter. Those figures show: (i) all variables are very persistent; (ii) the immediate effect on all variables except for consumption is larger in Model 2; (iii) the dynamic paths of output are different; (iv) the wage rate is procyclical but varies less than other variables. All variables are very persistent because the shocks are persistent (i.e., both ρ_p and ρ_g are very close to one). Consumption is more volatile and other variables are less volatile in Model 1 because of the substitution

[18] See, for example, King et al. (1998), and Cooley and Prescott (1995).

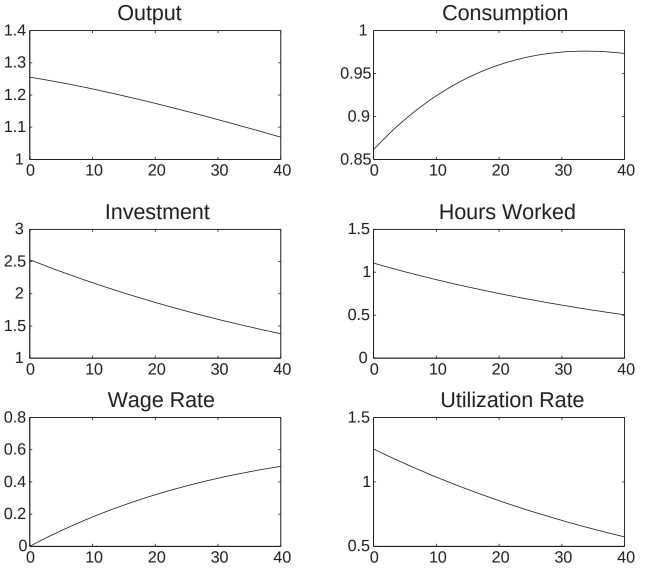

Fig. 1. Dynamic effects of preference shock 1.

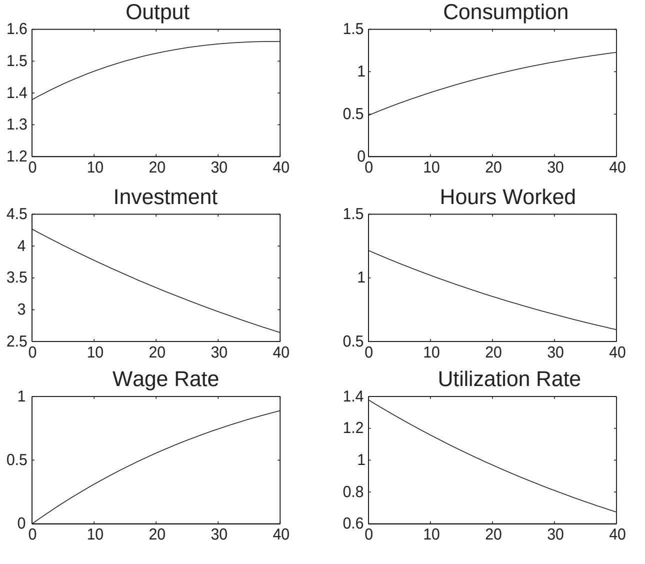

Fig. 2. Dynamic effects of preference shock 2.

effect of ψ. Because ψ_t is very persistent (persistence effect) and because higher labor supply increases the marginal product of capital (general-equilibrium effect), the overall effect of ψ on investment is positive, but the substitution effect makes investment (labor

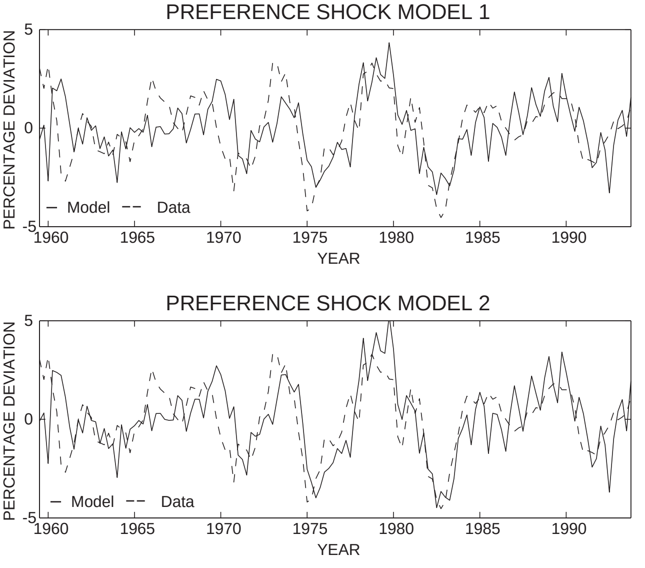

Fig. 3. Actual and predicted HP-filtered output.

and output as well) less volatile in Model 1.[19] Output in Model 2 increases at first because the increase in capital stock due to high investment overcomes the decrease in labor over time. The wage rate is procyclical (but varies very less than other variables) because, in our model of capital utilization, the wage rate depends only on capital stock, as shown in Eq. (24).

Figs. 3–7 compare the predictions of the two models to the U.S. data on output, consumption, investment, labor, and the Solow–Prescott residual. All variables are filtered using the Hodrick–Prescott filter. The solid lines represent the predictions of the models, and the dashed lines the U.S. quarterly data. Except for consumption, Model 2 appears to do a better job, but both models replicate the actual data relatively well. It would be of particular interest to see the models' prediction on the Solow–Prescott residual. Following the real business cycle literature, the Solow–Prescott residual $\ln z$ is measured as[20]

$$\ln(z_t) - \ln(z_{t-1}) = \ln(y_t) - \ln(y_{t-1}) + \text{Share of labor} \cdot [\ln(h_t) - \ln(h_{t-1})].$$

[19] Note that we have different values of χ in the two models (3 for Model 1 and 1 for Model 2). Thus, as Table 1 shows, σ_v for Model 2 is higher. But the higher immediate effects in Model 2 are by no means the result of this. If we use the same χ value (and hence the same σ_μ), the difference in the immediate effect between the two models becomes greater.

[20] See, for example, Prescott (1986) and Cooley and Prescott (1995). Note that the estimated residual is not TFP (thus, not the Solow residual), strictly speaking. But, we follow the convention to use this measure, which is typically justified by the fact that capital stock varies little over business cycles. This is why we call it the Solow–Prescott residual.

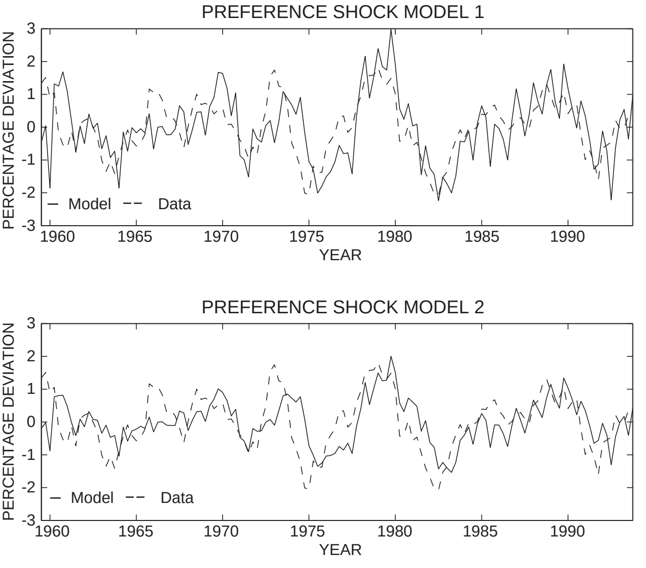

Fig. 4. Actual and predicted HP-filtered consumption.

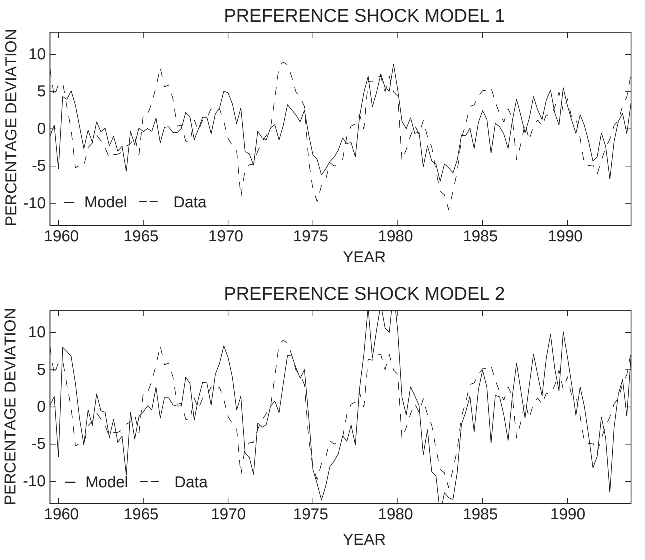

Fig. 5. Actual and predicted HP-filtered investment.

Fig. 7 shows that our models can replicate the time series of the Solow–Prescott residual quite well without technology shocks.

Tables 2–3 show second-moment properties of the two models and the Hansen model. Selected statistics for the U.S. economy are shown in the first

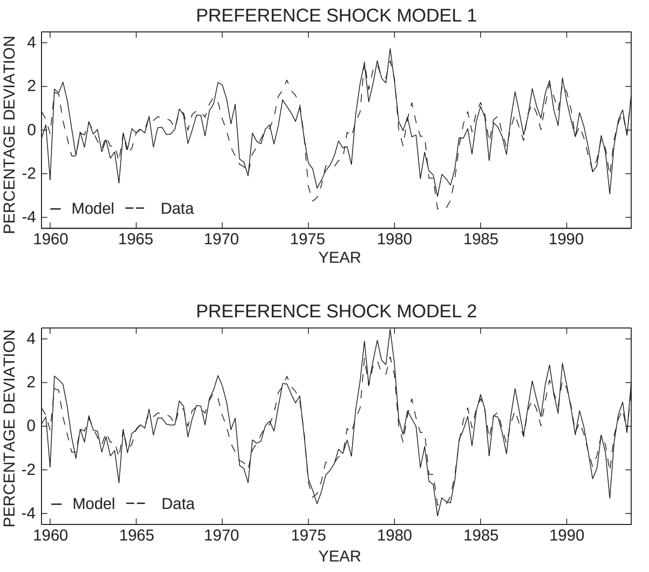

Fig. 6. Actual and predicted HP-filtered hours worked.

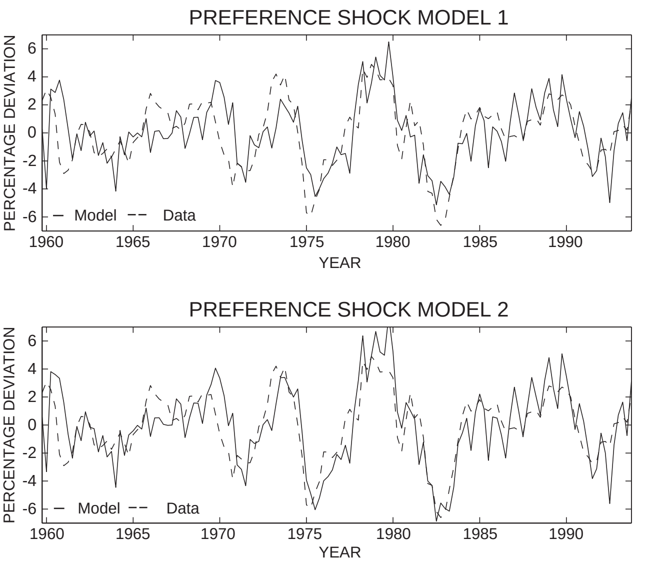

Fig. 7. Actual and predicted HP-filtered Solow residual.

column.[21] Table 2 shows the simulation result when the government spends a constant fraction of GDP, namely, when $\sigma_\varepsilon = 0$, and Table 3 the result with the government

[21] The number of quarters of the U.S. data is 138. We first generate 1000 pseudo-random samples, each one consisting of 188 periods, and then discard initial 50 periods in each sample. The statistics listed in Tables 2–5 are computed by taking averages over those 1000 samples.

Table 2
Without government spending shocks

	Data	Hansen	Pref shock 1	Pref shock 2
a. *Standard deviation*				
Output	1.71	1.62	1.58	1.73
Consumption	0.88	0.41	1.08	0.62
Investment	4.40	5.67	3.20	5.39
Hours	1.40	1.26	1.40	1.54
b. *Relative standard deviation*				
Consumption	0.51	0.26	0.69	0.36
Investment	2.57	3.50	2.02	3.11
Hours	0.82	0.78	0.88	0.89
c. *Contemporaneous correlation with output*				
Consumption	0.83	0.91	1.00	0.98
Investment	0.91	1.00	1.00	1.00
Hours	0.83	0.99	1.00	1.00
d. *AR*(1) *coefficient*				
Output	0.84	0.69	0.70	0.70
Consumption	0.86	0.77	0.70	0.70
Investment	0.86	0.69	0.70	0.69
Hours	0.83	0.68	0.70	0.69

Notes: (1) All series are HP filtered. (2) Hansen—Hansen (1985) model; Pref. shock j—the model presented in Section 3 with the jth specification of preference shocks. (3) For data, see Table 1.

spending shock. Not surprisingly, government spending shocks increase volatility of all variables, and reduce the correlation between consumption and output. In terms of the second moments, the predictions of the preference shock models here are as well as those of the Hansen model. Note that the requirement of labor for maintenance, $\underline{h}$, is the reason why output varies more than hours worked in our model (see the sensitivity analysis in Section 4.3.3).

4.3.2. Sensitivity to the elasticity of labor supply

Tables 4–5 report the second-moment properties of the two models for different values of the elasticity of labor supply, χ. It is a very important parameter in our models because, in the absence of technology shocks, short-run fluctuations in output reflect primarily those in hours worked. Qualitatively, how this parameter affects the variability of hours worked and output is similar in the two models: the higher the elasticity of labor supply is, the more variable hours worked and output will be. Tables 4–5 show that as the elasticity of labor supply increases from 0.2 to 5 the standard deviations of hours worked and output go up from 0.74 and 0.87 to 1.81 and 2.05 in Model 1; from 1.17 and 1.32 to 2.73 and 3.08 in Model 2. For each value of the elasticity of labor supply, the standard deviation of hours worked is lower in Model 1 because of the substitution effect of the shock to marginal utility of consumption, ψ.

Table 3
With government spending shocks

	Data	Hansen	Pref shock 1	Pref shock 2
a. *Standard deviation*				
Output	1.71	1.68	1.74	1.78
Consumption	0.88	0.55	1.11	0.70
Investment	4.40	5.76	3.59	5.45
Hours	1.40	1.51	1.54	1.58
b. *Relative standard deviation*				
Consumption	0.51	0.32	0.64	0.39
Investment	2.57	3.43	2.07	3.05
Hours	0.82	0.90	0.88	0.89
c. *Contemporaneous correlation with output*				
Consumption	0.83	0.45	0.82	0.74
Investment	0.91	0.99	1.00	0.99
Hours	0.83	0.95	1.00	1.00
d. *AR*(1) *coefficient*				
Output	0.84	0.69	0.70	0.70
Consumption	0.86	0.73	0.70	0.71
Investment	0.86	0.68	0.69	0.70
Hours	0.83	0.69	0.69	0.70

Notes: See Table 2.

How the elasticity of labor supply affects the second-moment properties of consumption and investment is very different between the two models. Let us start with the case where shocks are to marginal disutility of labor, ϕ (Model 2, Table 5). In this case, the relative standard deviations and the contemporaneous correlations with output are remarkably invariant for different values of the elasticity of labor supply. Indeed, in our model, a shock to ϕ works in a very similar fashion as a technology shock in the real business cycle model. There are two reasons for this. First, as we have seen in Section 3.6, the partial-equilibrium effect of a shock to ϕ tends to move consumption, investment, and hours in the same direction: a reduction in marginal disutility of labor, ϕ, makes the current period a good time to work, which increases income; due to consumption smoothing, higher income today leads to higher investment. Second, because of variable capital utilization, the general-equilibrium effect works in the same direction as the partial-equilibrium effect. In the absence of variations in capital utilization, the general-equilibrium effect would work against the partial-equilibrium effect (for example, through countercyclical wage rates).

Now turn to the case where shocks are to marginal utility of consumption, ψ (Model 1, Table 4). In contrast to Model 2, the second-moment properties of consumption and investment in Model 1 depend strongly on the elasticity of labor supply. This is due to the substitution effect of ψ, which tends to move consumption and investment in opposite directions. With our benchmark parameter value (Tables 1–3), the

Table 4
Sensitivity of preference shock model 1 to the elasticity of labor supply

χ	0.2	0.5	1	3	5
a. *Standard deviation*					
Output	0.87	1.01	1.20	1.73	2.05
Consumption	2.69	1.52	1.20	1.11	1.15
Investment	4.99	0.62	1.31	3.59	4.69
Hours	0.74	0.88	1.06	1.53	1.81
b. *Relative standard deviation*					
Consumption	3.11	1.51	0.99	0.64	0.56
Investment	5.77	0.61	1.09	2.07	2.29
Hours	0.85	0.88	0.88	0.88	0.88
c. *Contemporaneous correlation with output*					
Consumption	0.95	0.88	0.81	0.81	0.84
Investment	−0.98	−0.86	0.98	1.00	1.00
Hours	0.98	1.00	1.00	1.00	1.00
d. *AR*(1) *coefficient*					
Output	0.70	0.69	0.70	0.70	0.70
Consumption	0.70	0.70	0.70	0.70	0.70
Investment	0.70	0.70	0.70	0.69	0.70
Hours	0.70	0.69	0.70	0.69	0.70

Notes: For each value of χ, the corresponding values of ρ_p and σ_v are given in Table 1.

persistence and general-equilibrium effects outweigh the substitution effect, so that the model's predictions are fairly consistent with the data. However, when the elasticity of labor supply is small, the general-equilibrium effect becomes small and the substitution effect tends to be stronger than the other two effects. In other words, when the elasticity of labor supply is small, an increase in ψ (a reduction in Hall's residual) does not increase labor supply much. Thus output does not rise much, either. Due to the substitution effect of ψ, however, the consumer wants to increase current consumption, which is only possible by reducing investment given little increase in output.

4.3.3. Sensitivity to the labor requirements ratio

In the benchmark parameter values, we have chosen the labor requirements ratio, $\underline{h}/\bar{h}$, to be 10 percent. This is why the standard deviation of output is about 13.6 percent higher than that of hours worked in the two models with the benchmark parameters (Table 3). To see this analytically, log-linearize the aggregate production function (26) around the deterministic steady state:

$$\hat{y}_t = \theta\hat{k}_t + \frac{\tilde{h}}{\tilde{h} - \underline{h}}\hat{h}_t, \tag{37}$$

where a variable with a hat denotes the log-deviation from its detrended steady-state value, and $\tilde{h}$ is the steady state value of hours worked (0.22). Since the steady-state

Table 5
Sensitivity of preference shock model 2 to the elasticity of labor supply

χ	0.2	0.5	1	3	5
a. *Standard deviation*					
Output	1.32	1.50	1.78	2.59	3.08
Consumption	0.65	0.67	0.71	0.87	0.97
Investment	4.09	4.58	5.43	8.11	9.78
Hours	1.17	1.33	1.57	2.29	2.73
b. *Relative standard deviation*					
Consumption	0.49	0.45	0.40	0.33	0.32
Investment	3.10	3.06	3.06	3.14	3.18
Hours	0.89	0.89	0.89	0.89	0.89
c. *Contemporaneous correlation with output*					
Consumption	0.64	0.68	0.74	0.84	0.87
Investment	0.99	0.99	0.99	0.99	0.99
Hours	1.00	1.00	1.00	1.00	1.00
d. *AR*(1) *coefficient*					
Output	0.70	0.69	0.70	0.69	0.70
Consumption	0.71	0.71	0.71	0.72	0.72
Investment	0.69	0.69	0.69	0.69	0.69
Hours	0.69	0.69	0.69	0.69	0.69

Notes: For each value of χ, the corresponding values of ρ_p and σ_v are given in Table 1.

Table 6
Sensitivity of preference shock model 1 to the labor requirements ratio

$\underline{h}/\bar{h}$	0	0.1	0.2
a. *Standard deviation*			
Output	1.71	1.75	1.73
Consumption	1.06	1.12	1.16
Investment	3.67	3.63	3.35
Hours	1.71	1.55	1.33
b. *Relative standard deviation*			
Consumption	0.62	0.64	0.67
Investment	2.15	2.07	1.94
Hours	1.00	0.88	0.77
c. *Contemporaneous correlation with output*			
Consumption	0.79	0.81	0.84
Investment	1.00	1.00	1.00
Hours	1.00	1.00	1.00
d. *AR*(1) *coefficient*			
Output	0.70	0.70	0.70
Consumption	0.70	0.70	0.70
Investment	0.70	0.70	0.70
Hours	0.70	0.70	0.70

Notes: The parameters other than the labor requirement ratio are the same as in the benchmark case.

Table 7
Sensitivity of preference shock model 2 to the labor requirements ratio

$\underline{h}/\bar{h}$	0	0.1	0.2
a. *Standard deviation*			
Output	1.67	1.78	1.88
Consumption	0.63	0.71	0.80
Investment	5.30	5.44	5.44
Hours	1.68	1.57	1.45
b. *Relative standard deviation*			
Consumption	0.38	0.40	0.43
Investment	3.17	3.06	2.90
Hours	1.00	0.89	0.77
c. *Contemporaneous correlation with output*			
Consumption	0.67	0.74	0.80
Investment	0.99	0.99	0.99
Hours	1.00	1.00	0.99
d. *AR(1) coefficient*			
Output	0.70	0.70	0.70
Consumption	0.72	0.71	0.71
Investment	0.70	0.69	0.70
Hours	0.70	0.69	0.70

Notes: The parameters other than the labor requirement ratio are the same as in the benchmark case.

capacity utilization rate is 0.82, Eq. (23) implies that

$$\frac{\tilde{h}-\underline{h}}{\bar{h}-\underline{h}}=0.82.$$

Substituting this equation for $\tilde{h}$ into (37) yields

$$\hat{y}_t=\theta\hat{k}_t+\frac{0.82\bar{h}+0.18\underline{h}}{0.82\bar{h}-0.82\underline{h}}\,\hat{h}_t. \tag{38}$$

The coefficient on $\hat{h}_t$ is 1.136 when the labor requirements ratio, $\underline{h}/\bar{h}$, is 0.1.

Tables 6 and 7 show the second-moment properties of the two models with the labor requirements ratios of 0, 0.1, and 0.2. Eq. (38) predicts (assuming that capital stock fluctuates little over cycles) that the corresponding relative standard deviations of hours worked are 1, 0.88, and 0.77, respectively. This is confirmed in the tables. Except for the standard deviation of hours worked, however, the other statistics of the models are essentially unaffected by $\underline{h}/\bar{h}$.

5. Conclusion

The purpose of this paper is to show that there is a plausible dynamic model in which demand (preference) shocks alone can replicate the actual U.S. business cycles. To rationalize the use of preference shocks, we first show that incomplete-market economies can be aggregated into representative-agent economies with preference shocks. This gives a microeconomic foundation for the use of preference shocks. We then construct a dynamic general equilibrium model with variable capacity utilization and preference shocks. Two forms of preference shocks are considered, one is shocks to marginal utility of consumption and the other is shocks to marginal utility of leisure. We find that the model driven by each form of preference shocks can account for the U.S. business cycles rather well, at least compared to the standard RBC model. In particular, it can replicate the Solow–Prescott residual without assuming stochastic technology.

Without an ad-hoc assumption, Hall's residual alone cannot identify ψ and ϕ separately. As we have seen, however, ψ and ϕ affect the economy differently. Hence, it would be interesting if we identify them using additional equation, such as the Euler equation. Also, in our interpretation, Hall's residual is "endogenous." What the ultimate causes of Hall's residual are is an important question. These topics are left for future research.

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Appendix A

A.1. Proof of the proposition

In a complete-market equilibrium, the utility function of the representative agent is constructed uniquely (up to a constant). In an incomplete-market equilibrium, it is not. This is because a "state-price" process is not unique in an incomplete-market equilibrium. Let $\{\hat{q}_t(s^t)\}$ be an equilibrium state-price process in the original economy: that is, any strictly positive process that satisfies

$$\hat{q}_t(s^t) = \sum_{s^{t+1}|s^t} \hat{q}_{t+1}(s^{t+1})\{F_k[\hat{k}_{t+1}(s^{t+1}), \hat{h}_{t+1}(s^{t+1})] + 1 - \delta\}.$$

The existence of such a process is very standard (for example, Duffie, 2001). Pick any state-price process $\{\hat{q}_t(s^t)\}$.

Consider the following period utility function:

$$U[c_t(s^t), h_t(s^t), s^t \mid \hat{q}_t(s^t), \hat{\lambda}_t^i(s^t), i \in \mathscr{I}]$$
$$= \max_{c^i, h^i} \frac{\hat{q}_t(s^t)}{\beta^t \pi(s^t)} \left\{ \sum_{i \in \mathscr{I}} \frac{1}{\hat{\lambda}_t^i(s^t)} [u(c^i) - v(h^i)] \right\} \tag{A.1}$$

subject to

$$\sum_{i \in \mathscr{I}} c^i = c_t(s^t), \quad \text{and} \quad \sum_{i \in \mathscr{I}} \varepsilon^i(s_t) h^i = h_t(s^t).$$

In the main text the following particular state-price process is used:

$$\hat{q}_t(s^t) = \beta^t \hat{\lambda}_t^1(s^t) \pi(s^t),$$

but the aggregate utility function can be constructed using any equilibrium state-price process.

The maximization in (A.1) implies

$$\frac{\hat{q}_t(s^t)}{\hat{\lambda}_t^i(s^t)} u'[c_t^i(s^t)] = \beta^t U_c(s^t) \pi(s^t), \tag{A.2}$$

$$\frac{\hat{q}_t(s^t)}{\hat{\lambda}_t^i(s^t)} v'[h_t^i(s^t)] = \beta^t U_h(s^t) \pi(s^t) \varepsilon^i(s_t), \tag{A.3}$$

where $U_c(s^t)$, $U_h(s^t)$ are the partial derivatives of U with respect to c and h, respectively.

The representative agent maximizes the present-discounted value of flow utilities (A.1) subject to the sequence of resource constraints (6). Let $\beta^t \mu_t(s^t)\pi(s^t)$ be the multiplier on (6). The first-order conditions for the utility maximization of the representative agent are

$$U_c(s^t) = \mu_t(s^t), \qquad U_h(s^t) = \mu_t(s^t) F_h(s^t), \tag{A.4}$$

and

$$1 = \sum_{s^{t+1}|s^t} \frac{\beta \mu_{t+1}(s^{t+1}) \pi(s^{t+1}|s^t)}{\mu_t(s^t)} \left\{ F_k(s^{t+1}) + 1 - \delta \right\}. \tag{A.5}$$

The transversality condition is

$$\lim_{t \to \infty} \beta^t \sum_{s^t} \mu_t(s^t) \left\{ R_t(s^t) + 1 - \delta \right\} k_t(s^{t-1}) \pi(s^t) = 0. \tag{A.6}$$

It is straightforward to see the equivalence between the first-order conditions, (3)–(4), in the original economy and those, (A.2)–(A.5), in the representative-agent economy. Given non-negativity of capital, the transversality conditions for the two economies, (5) and (A.6), are equivalent as well.

A.2. Effects of ψ and ϕ on λ

Here, we determine the effect on λ_0 of ψ_0 and ϕ_0 in the example considered in Section 3.6. Let us start with Model 1. Suppose that $\{\psi_t\}$ is given by (30). It is straightforward to see that the individual problem (32) is rewritten as (without loss of generality, we assume $a = 1$)

$$\max \sum_{t=0}^{\infty} \beta^t \left(\psi_t \ln c_t - \frac{\chi}{1+\chi} h_t^{(1+\chi)/\chi} \right),$$

subject to the intertemporal budget constraint

$$\sum_{t=0}^{\infty} q_t c_t \leqslant k_0 + \sum_{t=0}^{\infty} q_t w_t h_t, \tag{A.7}$$

where q_t are intertemporal prices of consumption:

$$\frac{q_{t+1}}{q_t} = (R_{t+1} + 1 - \delta)^{-1}, \quad q_0 = 1.$$

Let λ_0 be the Lagrange multiplier on (A.7), which is, of course, equal to λ_0 used in problem (32). The first-order conditions are

$$\beta^t \frac{\psi_t}{c_t} = \lambda_0 q_t, \quad \beta^t h_t^{1/\chi} = \lambda_0 q_t w_t,$$

and the intertemporal budget constraint satisfied with equality. Substituting for c_t and h_t from these equations into the intertemporal budget constraint, we obtain

$$\psi_0 + \frac{\beta}{1-\beta} = \lambda_0 k_0 + \lambda_0^{1+\chi} \sum_{t=0}^{\infty} \beta^{-\chi t} (q_t w_t)^{1+\chi}.$$

This shows that a rise in ψ_0 increases λ_0.

Next, consider Model 2. The sequence $\{\phi_t\}$ is given by (31). The individual maximizes

$$\max \sum_{t=0}^{\infty} \beta^t \left(\ln c_t - \phi_t \frac{\chi}{1+\chi} h_t^{(1+\chi)/\chi} \right),$$

subject to the intertemporal budget constraint (A.7). The first-order conditions are

$$\beta^t \frac{1}{c_t} = \lambda_0 q_t, \quad \beta^t \phi_t h_t^{1/\chi} = \lambda_0 q_t w_t,$$

and the intertemporal budget constraint satisfied with equality. Substituting for c_t and h_t from these equations into the intertemporal budget constraint, we obtain

$$\lambda_0^{-\chi} \left(\frac{\lambda_0^{-1}}{1-\beta} - k_0 \right) = \phi_0^{-\chi} w_0^{1+\chi} + \sum_{t=1}^{\infty} \beta^{-\chi t} (q_t w_t)^{1+\chi},$$

which implies that ϕ_0 and λ_0 are positively related, since $1/(1-\beta) > \lambda_0 k_0$ (this, in turn, is because the right-hand side of the above equation is positive).

A.3. Shocks to intertemporal MRS and to intratemporal MRS

In the main text, we see the difference between the effects of a shock to marginal utility of consumption and those of a shock to marginal disutility of labor. Another useful distinction is between a shock to the "intertemporal" marginal rate of substitution (MRS) and a shock to the "intratemporal" MRS. Let us rewrite the period utility function as

$$u(c,h;\tilde{\psi},\tilde{\phi}) = \tilde{\psi}\left(\ln c - \tilde{\phi}\frac{\chi}{1+\chi}h^{(1+\chi)/\chi}\right).$$

Here, $\tilde{\psi}$ reflects a shock to the intertemporal MRS, and $\tilde{\phi}$ a shock to the intratemporal MRS. Hall's residual identifies only $\tilde{\phi}$. Model 1 corresponds to the case: $\tilde{\psi} = 1/\tilde{\phi}$, that is, shocks to the intertemporal and intratemporal MRS move in the opposite directions by the same amount. Model 2 assumes that $\tilde{\psi} = 1$, so that there are no shocks to the intertemporal MRS.

Obviously, holding $\tilde{\psi}$ constant, a shock to the intratemporal MRS, $\tilde{\phi}$, has the same effects as a shock to ϕ has in Model 2: it moves output, consumption, hours worked, and investment in the same direction as shown in Section 3.6. What about a shock to $\tilde{\psi}$? To see this, assume that $\tilde{\phi}_t \equiv 1$, let $\tilde{\psi}_t = 1$, all $t \geqslant 1$, and consider the individual's problem:

$$\max_{c_0,h_0,k_1} \tilde{\psi}_0\left(\ln c_0 - \frac{a\chi}{1+\chi}h_0^{(1+\chi)/\chi}\right) + \beta v(k_1).$$

The first-order conditions are the budget constraint (33) and

$$\lambda_0 = \beta v'(k_1), \qquad \frac{\tilde{\psi}_0}{c_0} = \lambda_0, \qquad a\tilde{\psi}_0 h_0^{1/\chi} = \lambda_0 w_0.$$

We can show that a rise in $\tilde{\psi}$ increases λ_0. Given w, the last two equations show that c_0 and h_0 should move in the opposite direction. It follows that a higher $\tilde{\psi}$ implies: (i) consumption rises ($\uparrow c_0$); (ii) investment falls ($\downarrow k_1$); (iii) hours worked falls ($\downarrow h_0$). Thus, consumption moves in the opposite direction to investment and labor. This is due to the substitution effect of a shock to the intertemporal MRS.

These theoretical consideration would support the view that it is shocks to the intratemporal MRS rather than to intertemporal MRS that are important to account for business cycles (i.e., comovement of macro variables).

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