Investing exhaustible resource rents and the path of consumption

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Abstract. We set out dollar-valued net national product for an economy with a wasting essential stock (oil deposit). We take up 'maintaining capital intact' and locally unchanging consumption. The percentage change in 'net investment' or 'genuine savings,' relative to the market rate of interest, denotes whether current consumption is rising, constant, or declining. JEL classification: O13, Q28, F0

Investir les rentes dérivées de l'exploitation d'une ressource épuisable et le sentier de consommation. On définit en dollars le produit national net d'une économie qui a un stock de ressources (un dépôt de pétrole) en train de s'épuiser. On envisage de maintenir le capital intact et la consommation locale inchangée. Le changement en pourcentage dans l'investissement net et dans les véritables épargnes, en relation avec le taux d'intérêt sur le marché, définit si la consommation courante croît, demeure constante, ou décline.

1. Introduction

Investing exhaustible resource rents in produced capital and its extension, 'zero net investment,' is an intrinsic part of Solow's (1974) original and generalized constant consumption model Hartwick (1977); Dixit, Hammond, and Hoel (1980); Hamilton (1995); and Sato and Kim (2002). And the magnitude of 'net investment' or 'genuine savings' has become a central focus in the measurement

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of the sustainability of an economy.[1] Here we take the theory beyond constant consumption and link investing exhaustible resource rents to growth in a model with consumption varying through time as in a model of optimal savings. We observe that, at a point of locally unchanging consumption, the percentage change in 'net investment' or genuine savings equals the prevailing market rate of interest (the utility discount rate at this point) and the level of net investment is negative.[2] Consumption is then increasing (decreasing) when the percentage change in net investment is below (above) the market rate of interest. Though we clarify the connection between zero 'net investment' and constant consumption, our results suggest that the sign of current 'net investment' is imperfectly linked to the sign of current change in consumption. Thus the usefulness of the sign of 'net investment' as an indicator of the sustainability of current consumption is circumscribed by our results. However, the sign of 'net investment' is a good indicator of the direction of movement of the current welfare integral (Hamilton and Clemens 1999), and we report on this.

We open with a brief analysis of 'maintaining capital intact' in an economy with two capital goods, one produced and one an exhaustible oil stock. The latter supplies an essential input flow to the economy at each date. We characterize dollar-valued national wealth and income. We link our local constant consumption result to the constant consumption result over an interval of Hamilton (1995) and Sato and Kim (2002).

2. The analysis with a wasting natural stock

Our economy needs current oil flow $R(t)$ to operate as in $Q(t) = F(K(t), N, R(t))$, where $R(t) = -\dot{S}(t)$ for $S(t)$ the current stock of oil remaining. $F(\cdot)$ is homogeneous of degree unity. Now two Euler equations[3] govern the accumulation of and the draw-down of $S(t)$, namely,

$$-\frac{dU_C(t)/dt}{U_C(t)} + \rho = F_K(t) - \delta,$$

$$\dot{F}_R(t) = r(t)F_R(t),$$

and the market rate, $r(t) = F_K(t) - \delta$. Corresponding to stock $K(t)$ with dollar value $K(t)$, we have stock value

1 Pearce and Atkinson [1993] is a well-known, empirical study which uses the sign of current net investment to measure the current sustainability of an economy. Asheim [1994] questioned the use of a measure associated with a globally constant consumption program in an economy exhibiting inherent non-constant consumption. Arrow, Dasgupta, and Maler [2003] have taken up the question of the usefulness of genuine savings as an indicator of sustainability in second best economies.

2 Hartwick, Long, Tian [2003] observed that net investment would be negative at this point and here we are able to be precise about the size of the negative wedge.

3 These equations emerge from the problem of maximizing $\int_0^\infty U(C(t))e^{-\rho t}dt$ subject to $\dot{K}(t) = F(K(t), N, R(t)) - C(t) - \delta K(t)$, $\dot{S}(t) = -R(t)$, $K(0) = K_0$ and $S(0) = S_0 = \int_0^\infty R(t)dt$.

$$V(S(t)) = \int_t^\infty \hat{R}(z)\hat{F}_R(z)\exp\left(-\int_t^z \hat{r}(y)dy\right)dz$$
$$= S(t)F_R(t)$$

as the current market value, in dollars, of remaining stock $S(t)$. Recall that $F_R(t)$ grows at rate $r(t)$.[4] Hats indicate values along the optimal path. It follows that[5]

$$\dot{V}(S(t)) = -\hat{R}(t)\hat{F}_R(t) + S(t)\hat{F}_R(t)r(t),$$

since $-\dot{S}(t) = R(t)$ and $\dot{F}_R(t) = F_R(t)r(t)$. We have

$$\begin{aligned}\dot{K}(t) &= F(K(t),N,R(t)) - \delta K(t) - C(t)\\ &= K(t)F_K(t) - \delta K(t) + NF_N(t) + R(t)F_R(t) - C(t)\end{aligned} \tag{1}$$

$$= r(t)K(t) + r(t)V(t) - \dot{V}(t) - [C(t) - NF_N(t)]. \tag{2}$$

We used constant returns to scale here. We can express the capital value of the future labour services as $Z(t) = Nv(t)$ for $v(t) = \int_t^\infty F_N(z)\ \exp\left(-\int_t^z r(x)dx\right)dz$.[6] Now we rewrite (2) as

$$C(t) + \dot{K}(t) + \dot{V}(t) + \dot{Z}(t) = r(t)K(t) + r(t)V(t) + NF_N(t) + \dot{Z}(t),$$

and the right-hand side reduces to $r(t)K(t) + r(t)V(t) + r(t)Z(t)$ because $\dot{Z}(t) = Z(t)r(t) - NF_N(t)$. Hence, we obtain the basic relation,

$$C(t) + \dot{W}(t) = W(t)r(t),$$

for $W(t) = K(t) + Z(t) + V(S(t))$.[7] $W(t)$ is total current capital value, in dollars or numeraire units, and $\dot{W}(t) = \dot{K}(t) + \dot{Z}(t) + \dot{V}(S(t))$. The steps followed are (1) take the accounting relation, $C(t) + \dot{K}(t) = F(\cdot) - \delta K(t)$, (2) 'expand,' using constant returns to scale, to get $C(t) + \dot{K}(t) = K(t)F_K(t) + NF_N(t) + R(t)F_R(t) - \delta K(t)$, (3) invoke a dynamic efficiency condition to get $C(t) + \dot{K}(t) = K(t)r(t) + NF_N(t) + R(t)F_R(t)$, (4) insert $\dot{V}(S(t))$ and $\dot{Z}(t)$ on the left-hand side and the appropriate corresponding 'expansions' on the right-hand side to get

$$\begin{aligned}C(t) + \dot{K}(t) + \dot{V}(t) + \dot{Z}(t) = {}& r(t)K(t) + NF_N(t) + R(t)F_R(t) + r(t)V(t)\\ &- R(t)F_R(t) + r(t)Z(t) - NF_N(t)\end{aligned}$$

4 $V(S(t)) = S(t)F_R(t)$ is a somewhat unusual stock valuation relation because $F_R(t)$ is essentially a flow price or rental rather than a stock or capital price. Hence, $\dot{F}_R(t) = r(t)F_R(t) + \zeta(t)$, with $\zeta(t) = 0$. If $F_R(t)$ were a standard durable good price, $\zeta(t)$ would not be zero.

5 $RF_R(t)$ is generally referred to as 'user cost' and $\dot{V}(S(t))$ as 'economic depreciation.'

6 $\int_t^\infty F_N(z)\ \exp\left(-\int_t^z r(x)dx\right)dz$ is a standard capital value or asset price, say, $v(t)$, here for labour. Its time derivative is $v(t)r(t) - F_N(t)$.

7 Recall that $K(t)$ is also the value of discounted future services in $K(t) = \int_t^\infty F_K(z)K(z)\ \exp\left(-\int_t^z [r(x) + \delta]dx\right)dz$.

(one invokes dynamic efficiency for these 'expansions'), and (5) collect terms.

One integrates $C(t) + \dot{W}(t) = W(t)r(t)$ to get the basic condition

$$W(t) = \int_t^\infty C(z)\ \exp\ (-\int_t^z r(x)dx)dz.$$

This resembles an Irving Fisher intertemporal budget constraint: the current value of capital goods can support a particular future consumption path. Here, the endowment of natural capital allows for a larger value of discounted future consumption.

If one starts with this capital value relation and differentiates, one gets, by the chain rule, $\dot{W}(t) = \dot{K}(t) + \dot{S}(t)F_R(t) + S(t)\dot{F}_R(t) + N\dot{v}(t)$ and, of course, $C(t) + \dot{W}(t) = W(t)r(t)$. Now, $\dot{K}(t) + \dot{S}(t)F_R(t)$ is $G(t)$ or genuine savings or 'net investment,' and $S(t)\dot{F}_R(t) + N\dot{v}(t)$ are 'anticipated capital gains.' $C(t) + G(t)$ is identified by many as green net national product or 'Hicksian income.' Hence $C(t) + G(t)$ is equal to $r(t)W(t)$ minus the capital gains terms. A reasonable 'expansion' of this right-hand side is as $Q(t) - \delta K(t) - R(t)F_R(t)$, namely, as current flow output minus 'depreciation' terms. This is, in a sense, a dual expression for green net national product. Of interest here is that $\delta K(t)$, which we are familiar with, and $R(t)F_R(t)$ are being treated symmetrically. So-called 'user cost' of natural capital, namely, $R(t)F_R(t)$, is a 'depreciation' term here in the same sense as $\delta K(t)$.

In the following we assume that labour is fixed, so that $Q(t) = F(K(t), R(t))$ and $W(t) = K(t) + V(t)$, and that consumption peaks away from time zero (cases with 'low' values of the utility discount rate[8]).

Now, $K(t)$ is net investment. Some $Q(t)$ (namely, $\delta K(t)$) has gone to replacing evaporated $K(t)$. $K(t)$ has been maintained intact. What about maintaining $V(S(t)$ intact? To be more precise, we ask: if the economy were moving along its efficient path under Ramsey savings and it was observed that $\dot{K}(t) + \dot{V}(t) = 0$, is current $C(t)$ locally unchanging? The answer is 'no'. When $\dot{W}(t) = 0$, $C(t) = r(t)W(t)$, implying that $\dot{C}(t) = \dot{r}(t)W(t) + r(t)\dot{W}(t)$. The latter term is zero, but the preceding term is not in general.

Next, we examine the link between genuine savings and the peaking of consumption. Dispensing with the assumption of constant returns to scale, we are able to express genuine savings in terms of discounted consumption-change quite generally along the growth path for the economy.

PROPOSITION 1. *Current consumption change takes the form:* $\dot{C} = (r - \dot{G}/G)G$.

Proof. We take the time derivative of the basic accounting relation

8 The 'peak' can occur at time zero for ρ relatively large. See Pezzey and Withagen (1998) and Hartwick, Long, and Tian (2003).

$$\dot{K}(t) = F(K(t),R(t)) - \delta K(t) - C(t)$$

to yield

$$\begin{aligned}\dot{C} &= \dot{K}F_K + \dot{R}F_R - \delta\dot{K} - \frac{d\dot{K}}{dt}\\ &= \dot{K}F_K + \dot{R}F_R - \delta\dot{K} - \frac{d\dot{K}}{dt} + R\dot{F}_R - R\dot{F}_R\\ &= (F_K - \delta)\dot{K} - (F_K - \delta)F_R R - \dot{G}, \text{ using } \dot{F}_R = (F_K - \delta)F_R\\ &= rG - \dot{G}.\end{aligned}$$

In summary, we have the RULE:

For $G > 0$, $C(t)$ is increasing (decreasing) (constant) as $\frac{\dot{G}}{G} < (>)(=) r(t)$.

If $G < 0$, the directions of the inequalities are reversed. □

It follows quite directly that at the peak of consumption, G takes the particular form $G = -\gamma F_R$, because $r(t)$ equals both $\dot{G}/G$ and $\dot{F}_R/F_R$. We read this as a local in time result in the neighbourhood of $C(t)$ at its peak. However, the result that $\dot{C} = 0$ over an interval implies $G = -\gamma F_R$ goes back to Dixit, Hammond, and Hoel (1980) and Hamilton (1995). Recently, Sato and Kim (2002) solved a closed-form example with a Cobb-Douglas production function. The presumption is that, though C remains constant over an interval, it cannot remain constant forever as in the Solow case, because, roughly speaking, $K(t)$ is not being accumulated fast enough relative to the decline in $S(t)$.

We have, in turn, the following characterization of genuine savings along the optimal path.

PROPOSITION 2. $G = \int_t^\infty \dot{C}(s)\, exp\left(-\int_t^s r(x)dx\right)ds$.

Proof. This is an integral of the basic condition, $\dot{C} = rG - \dot{G}$ derived above.

Hence, we have genuine savings at each instant equal to the present value of the change in consumption along the optimal path. Clearly, at the peak of consumption it must be that genuine savings, G, is negative. Hence, if genuine savings starts out positive, as it would in a growing economy, it must turn negative before the peak in consumption. That is,

COROLLARY 1. *If the consumption path has a peak away from time zero, then* $G < 0$ *before* $\dot{C} = 0$, *that is, before the peak is reached.*

This result suggests that the negative turn in genuine savings is a signal of impending 'collapse' in consumption. This conforms with intuition. When an intuitive measure of capacity expansion is shrinking, then the economy can be expected to turn down in the future. As C rises to its peak (G is negative), $\dot{\mathrm{G}}$ must be negative and be 'large' in absolute value, and as C passes its peak, $\dot{\mathrm{G}}$ (negative) becomes small in absolute value, so that $\dot{\mathrm{C}}$ becomes negative.

Sato and Kim (2002) invoke the energy conservation law of classical mechanics in their analysis of constant consumption paths. In classical mechanics, energy conservation takes the form of so-called conservative systems (no friction) of the Hamiltonian of the system remaining unchanging over time, and the value of this Hamiltonian is the sum of potential and kinetic energy. Withagen and Asheim (1998) emphasize that the Hamiltonian is unchanging in our economic models along constant consumption paths. Hence, we can identify the constant consumption level C with the unchanging energy sum in classical mechanics. That is, we have $H(t) = C + \dot{K} + \dot{S}F_R = C$. Presumably, one then associates $C + \dot{K}$ with 'output' or kinetic energy at an instant and $\dot{S}F_R = -RF_R$ with energy inflow or potential energy at an instant in this analogy. A more complete exploration of the capital theoretic nature of energy accounting in classical mechanics is taken up in Hartwick (2003).

3. Concluding remarks

Here, we have continued the investigation of the meaning or usefulness of zero 'net investment' or 'genuine savings' outside the realm of models with constant consumption. Previous work was reported in Asheim (1994), Long and Yang (1999), and Hartwick, Long, and Tian (2003). In a sense, this work represents a generalization of models constructed around constant consumption paths. We have presented some rules or characterizations of the relationship between consumption motion and the motion of net investment near points of consumption peaking or locally constant consumption. A key rule involved the percentage change in net investment relative to the market rate of interest. This and related rules may shed light on the question of sustainability of economies in new contexts.

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