The Sport League's Dilemma: Competitive Balance versus Incentives to Win*

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Abstract

We analyze a dynamic model of strategic interaction between a professional sport league that organizes a tournament, the teams competing to win it, and the broadcasters paying for the rights to televise it. Teams and broadcasters maximize expected profits, the league represents a cartel of teams and its objective is to maximize the teams' joint profits. Demand for sport depends positively on symmetry among teams (competitive balance) and how aggressively teams try to win (incentives to win). Revenue sharing increases competitive balance but decreases incentives to win. We show that if the difference in wealth between teams is large, full revenue sharing (used by many US leagues) is always optimal. Conversely, if the wealth difference is small, a performance-based reward scheme (as used by some European soccer leagues) may be optimal.

Keywords: Sport league, Revenue sharing, competitive balance, incentives to win.
JEL Classification: L19, L83

1 Introduction

Revenue sharing is a controversial topic in the organization of many professional sport leagues. In recent years, its importance has become even more evident given the large payments American and European leagues fetch from television broadcasters.[1] In this paper, we present a rigorous analysis of the opposing views in this controversy.

The argument in favor of revenue sharing in sports observes that there are large differences among revenues and wealth of teams. For example, Scully (1995) and Fort and Quirk (1995) provide evidence on large disparities of ticket sales and revenues from local TV deals among teams located in different cities. As a consequence, richer teams tend to be more successful since they can afford better players.[2] A mechanism which redistributes income from richer to poorer teams makes future competition more balanced, hence more enjoyable to the fans. A consequence of this argument is that revenue sharing increases future demand for the sport, hence increasing the revenues of the league. Furthermore, if teams are profit maximizers, revenue sharing also decreases the price teams pay for top players since their marginal value decreases. Hence, revenue sharing also has a positive impact on the profit of teams.

The case against revenue sharing is based on the idea that if there are no prizes for winning, teams' profits are independent of a competition's outcome. In other words, without a prize there are no monetary incentives for a team to win. In the end, this may have a negative effect on demand since the lack of incentives for team owners induces lack of incentives for players.[3] As noticed by Daly (1992) and Fort and Quirk (1995), if teams have nothing to compete for, fans may strongly doubt the integrity of the competition on the playing field with an obvious negative effect on demand. Hence, revenue sharing has a negative impact on current demand and team profits.

[1]The latest reported television deals for NFL and NBA, for example, are $17.6 billion over eight years and $2.4 billion over four years respectively (see Araton 1998).

[2]See Scully (1995) for detailed evidence.

[3]An example of this effect is given by the higher TV ratings for playoff matches when compared to regular season ones.

Unsurprisingly, professional economists have debated different revenue sharing arrangements, and their consequences for teams and players as well as for the demand for sports (see Fort and Quirk (1995) for a comprehensive review). In the end, the question is how to allocate revenues if the product is the result of a joint production of effort by several participating firms. Surprisingly, there is relatively little work on this question.

As seen above, revenue sharing has different consequences for current and future demand for the sport. Therefore, in order to derive the optimal level of revenue sharing in sport leagues, we study a dynamic model where professional teams compete to win a tournament and the league decides how to allocate revenues generated by the competition among winners and losers. In particular, each period has the following sequence of moves. First, the league announces a prize allocation scheme, i.e., prizes for the winner and the loser as a function of TV revenues. Then the leagues sells broadcasting rights to TV networks. Finally, the teams compete to win the tournament organized by the league. In this framework, we can properly address the following question: how should a professional sport league allocate revenues among participating teams?

The starting point is a description of aggregate demand for a sporting competition. This determines how much money the league may obtain for selling the rights to broadcast the event. Aggregate demand for a sport is ultimately determined by how much the fans enjoy watching the tournament in which the teams compete. Following the literature surveyed in Fort and Quirk (1995), we assume it depends on three factors. These are quality of the league, how hard teams try to prevail, and competitive balance in the tournament.

Quality of the league reflects its ability to attract talented athletes. It is measured by the combined wealth of the teams. A wealthier league (i.e., a league with a larger total wealth of teams) attracts better players. Therefore, teams' combined wealth has a positive effect on demand. The size of this effect is influenced by the environment in which the league operates. For example, US sport leagues are essentially monopsonists in the market for players in a given sport.[4] In this case, only intra-league trades are observed and league-wide talent is roughly constant. European sport leagues, on the other hand, compete with each other for top players.

[4]There are very few cases of athletes being able to play more than one sport at professional level.

In this case, inter-league trades of top players are observed frequently.

Willingness to win by teams is reflected by the salaries teams pay to their athletes. If the effort players produce is observable, a higher salary is the consequence of a higher effort. If the effort is not observable, higher prize when winning the competition generates a higher effort.

Competitive balance is measured by uncertainty of the outcome. Fans derive more enjoyment from sporting events whose winners are not easy to predict. In other words, the more symmetric the winning chances of the competitors the more exciting the tournament is to watch. Since a team's probability of winning ultimately depends on the athletes playing for it, competitive balance also depends on a team's wealth and how much it pays its athletes.

When the league chooses a monetary reward scheme conditional on the tournament's outcome, it knows this choice influences how teams compete in the event. Hence, the league knows that the level of revenue sharing influences aggregate demand for the sport. In a dynamic setting, revenue sharing has two effects on demand. The first effect we call *competitive balance.* Increased revenue sharing at time t increases demand at time $t+1$ by making the teams' future winning chances more equal. This effect has consequences for competitive balance at time $t+1$ even if teams are identical at time t; a large prize for today's winner introduces an asymmetry in tomorrow's winning probabilities. The second effect we call *incentives to win.* Increased revenue sharing decreases current demand by lowering the monetary value of winning and consequently diminishing teams' interest and effort toward winning. This effect lowers demand since fans enjoy more effort from players.

Considering the league as a cartel of profit maximizing firms and assuming it maximizes the teams' joint profit (as assumed by Atkinson, Stanley and Tschirhart (1988)), we derive the optimal level of revenue sharing in a repeated tournament by analyzing the trade-off between competitive balance and incentives to win. We show that in league with relatively homogenous teams, a performance based reward scheme may be optimal. Conversely, in a league with large wealth differences between teams, full revenue sharing is always optimal. The intuition for these results is the following. If the teams are relatively homogenous in terms of wealth then, even if the wealthier team wins the competition in the first period, the future wealth difference between the teams is small and the level of competitive balance does not fall too much. In this situation, a performance based reward scheme enhances effort in early periods without altering too much the level of competitive balance in latter periods. Conversely, if the difference in wealth between teams is large, the main objective of the league is to maintain the level of competitive balance. This is achieved by splitting TV revenues evenly between teams.

These results may explain the difference of TV revenue redistribution schemes chosen by some sports leagues. In European top soccer leagues, broadcasting revenues are distributed according to performance (see Tables 1 and 2).[5] Conversely, full revenue sharing of national TV deals is more common in US sport leagues (see Scully, 1995).

Our paper extends the existing literature in several ways. First, we consider a multi-period model. Therefore, we are able to capture the intertemporal trade-off generated by revenue sharing between demand and profits today and demand and profits tomorrow. Second, we consider the possibility that a league faces competition from other leagues and that they compete for top players as is the case in Europe. Therefore, we can study the influence of revenue sharing at time t on league-wide talent at time $t+1$. Related papers in a competitive environment are those of Hoen and Szymanski (1999), Palomino and Sakovics (2001). Hoen and Szymanski study the impact of the participation of top clubs in international competitions on the competitive balance of the domestic leagues. They do not address the issue of the optimal level of revenue sharing. Palomino and Sakovics (2000) consider a static model and compare the organization of markets for talent when a league is isolated and when it operates in a competitive environment. They show that full revenue sharing in US sports may result from monopsony power in the market for talent rather then from competitive balance considerations.

Other papers (El Hodiri and Quirk (1971), Atkinson, Stanley and Tschirhart (1988), Fort and Quirk (1995), Vrooman (1999), Szymanski (2001)) consider static models and focus on the case where leagues that do not face competition. Among these papers, Szymanski (2001) is the most related since he considers the case in which broadcast income is redistributed as a prize. He compares the impact of different types of redistribution schemes of broadcast income and

[5] See also Hamil, Michie and Oughton (1999) for more details about England.

competitive balance and investment in talent. He shows that a performance based redistribution leads to higher investment levels but a lower level of competitive balance.

The analysis carried out in this paper goes beyond the sports literature. Our model presents an example of a repeated moral-hazard problem between a principal and multiple agents in which the difference in output produced by the agents is detrimental to the principal.[6] An example of such a situation is the production and distribution of electricity.[7] In this setting, the principal faces a trade-off between "output balance" among agents and incentives to produce large quantities. The solution for the principal is to propose agent-specific contracts, the agent with the higher productivity rate receiving a lower marginal revenue in order to decrease his incentives to produce. The specific feature of a sport competition (or any other contest) is that the contract the principal proposes has to be the same for all agents. Rewards can only be based on relative performances. The reward for the winner cannot vary with his identity. (Any reward scheme based on identities would probably be considered as unfair by fans and lead to a low demand)

Our model is also related to Moldovanu and Sela (2001) who study the optimal allocation of prizes in contests. They consider a contest where the highest bidder (or agent making the highest effort) wins and the goal of the designer is to maximize the sum of all bids. In this framework, they show that if the contestants face linear or concave cost functions, then the allocation of the entire prize money to the winner is optimal. Conversely, if the contestants face convex cost functions, several prizes may be optimal. Our model differs from Moldovanu and Sela's in two crucial ways. First, we consider sequential contests and both the bids (the effort in our context) and the outcome of the contest at time t influence bids made at time $t+1$. Hence, when deciding on the allocation of prizes at time t, the contest designer (i.e., the league in our case) must take the impact of the reward scheme chosen at time t on future contests into account. Second, one of

[6] For example, consider a situation such that there are two agents 1 and 2, the income of the principal at time t is $\mathrm{Min}(q_{t,1}, q_{t,2})$, $q_{t,i}$ being the output of agent i at time t. Moreover, $q_{t,i}$ depends on agent i's (unobservable) effort, his productivity and some noise, and the productivity at time t depends on past income. (One can think of productivity as being the consequence of investment in more or less sophisticated machines.)

[7] We would like to thank Ines Macho-Stadler for suggesting this example.

the specificity of sport events is that competitive balance is valuable. This reduces incentives for the contest designer to award all the prize money to the winner at time t since it may decrease competitive balance at time $t+1$.

The organization of the paper is as follows. Section 2 introduces the basic model and Section 3 derives its equilibrium. Section 4 considers some extensions, Section 5 discusses the robustness of the results and Section 6 concludes.

2 The Model

In this section, we present a simple two-period model of strategic interactions between professional sport teams, a professional sport league, and television broadcasters. Two teams compete in the tournament organized by the league and shown on television by a broadcaster. More specifically, the following sequence of moves occurs in each period:

- The league announces a revenue sharing schedule: the league decides the amounts of money to be awarded to the loser and the winner of the tournament, as functions of the revenue from the sale of the TV rights.
- The broadcaster decides how much to pay for the exclusive right to televise the sporting event. The contract between the league and the broadcaster also states that if the reward scheme effectively chosen by the league deviates from the schedule announced previously, the league faces an infinite penalty.[8]
- The league announces the prizes for the winner and the loser.
- The teams simultaneously decide how much to spend on players' incentives.
- At the end of the period, the tournament is played, the winner is determined, and money is awarded accordingly.

[8] Such an assumption solves the commitment problem between the league and the broadcaster. In absence of such a penalty, the league could just choose the reward scheme that minimizes team's costs.

Before presenting the model in detail, two remarks are in order. First, we focus on the sale of rights to national TV networks and on the allocation of the corresponding revenues among teams. Of course, teams have other sources of revenues (for example, ticket sales, sponsoring, merchandising, local TV deals). In our model, these are captured by differences in initial wealth among teams; in other words, we assume these other sources of profits are constant over the two periods. However, the impact of other sources of revenues on the level of revenue sharing is studied in Section 4.3. Second, the model does not explicitly include a market for talent.[9,10] Although both these aspects deserve attention, our focus is on a model simple enough to capture the basic trade-off between competitive balance and incentives.

Demand

Demand for the sport depends on three sets of variables: the talent of the athletes playing the sport (league quality), their attempt to prevail in the competition (willingness to win), and competitive balance of the tournament. The league's quality is measured by the wealth of the participating teams; this reflects their ability to attract skilled athletes away from other sports. Willingness to win is measured by incentives to induce players' effort; it is important because fans enjoy athletes playing hard.[11] Competitive balance is measured by uncertainty of the tournament's outcome; fans enjoy sporting events more if the winner is not easy to predict. In other words, the more symmetric are the teams' winning chances, the more exciting is the tournament.

In each period t, D_t denotes demand in monetary terms, $e_{t,i}$ denotes team i's effort, $p_{t,i}$ its probability of winning, and $W_{t,i}$ its wealth; similarly for team j. Then, demand is given by

$$D_t = \gamma \left(e_{t,i} + e_{t,j}\right) + \delta[1 - (p_{t,i} - p_{t,j})^2] + \nu \left(W_{t,i} + W_{t,j}\right), \tag{1}$$

[9]See Palomino and Sakovics (2000) for a market for talent with competing leagues.

[10]Implicitly, we assume that talent is linear in price and that teams maximize their expected profit from talent under the constraint that they cannot borrow. In this case, if for a cost of talent equal to the total wealth of a team, the marginal profit is larger than the marginal cost, teams invest their entire wealth.

[11]A possible fourth set of variables may measure fans' attachment to a team. Since we model demand for the sport, we assume that these "individual team" effects wash out in the aggregate.

with γ, ν and δ positive.[12] Note that δ represents the monetary value of one unit of competitive balance. Equation (1) can loosely be interpreted as measuring fans welfare from watching the tournament. In this respect, the first term measures the importance of watching athletes 'give their best'. The second term measures the importance of watching balanced games, which outcome is uncertain. The third term measures the importance of watching good athletes. Roughly, this last effect allows for several competing leagues where each league talent level depends on the total wealth of its teams.[13]

Broadcasters

The market for TV rights is perfectly competitive. Therefore, a broadcaster expects zero profits in equilibrium. Since demand is expressed in monetary terms, we assume that broadcasting of the games generates income from advertising and this income increases with the audience that watches them. In particular, we let K_t denote the amount paid by the broadcaster and impose $K_t = D_t$ in each period. In other words, at this stage we only consider one-period deals between broadcasters and the league. Later in the paper (see Section 4.1), we extend the framework to include multi-period deals.

League

After receiving K_t from the broadcaster, the league decides how to allocate this sum between the two teams. We assume this allocation can only be contingent on the outcome of the tournament. In particular, this rules out allocations which explicitly depend on the teams' wealth.[14] We

[12]Many simplifying assumptions, like the functional form chosen for the demand for the sport, are made for simplicity and are not necessary for our qualitative results.

[13]This assumption corresponds to the case of European sport leagues who organize domestic competitions and sign TV deals with national broadcasters. Top players often switch from one league to another, hence changing league-wide talents. US sport leagues, on the other hand, are in an isolated environment where league-wide talent is given and only intra-league trades occur (with few exceptions at the draft level).

[14]This does not seem an unrealistic assumption since we do not see tournament prizes of the sort "since the richer team has won, amount x goes to the winner and amount y goes to the loser".

denote $K_{t,w}$ and $K_{t,l}$ the amounts allocated to period t winner and loser, respectively (obviously, $K_{t,w} + K_{t,l} = K_t$)

Following Atkinson, Stanley and Tschirhart (1988), we assume that a league is a cartel of the teams involved in the championship and its objective is to maximise the teams' joint profit.[15] Also, it is assumed that the league cares about teams' long run solvency, i.e., a losing team is able to compensate players.[16] Formally, we assume that the league chooses $K_{t,l}$ so that $K_{t,l} > \text{Max}_{i=1,2} e_{t,i}$.

Probability of winning the tournament

The tournament's outcome depends on the quality of each team and on their choices of incentives for players. The first aspect represents teams' initial ability; the second represents the effort spent towards winning. A richer team can buy better players, hence having an initial advantage. However, a poorer team can compensate this initial disadvantage by producing a higher effort level. In order to make players produce a higher effort level, teams must reward them. Here, the effort level is measured in monetary terms. We capture these ideas modeling the probability of winning.

The probability that team i wins period t tournament depends on its players' talent and how hard they play. Talent can be thought of as a team's ability to sign players at the beginning of the season; therefore, it is measured by the team's wealth $W_{t,i}$. How hard players try to win

[15] In practice, the governing body of a league is comprised of one voting representative from each member club and major issues must be approved by majority or supermajority vote. (See Flynn and Gilbert, 2001). Here, we implicitly assume that the maximization of the joint profits has been approved as the objective of the league and its implementation has been delegated to a commissioner.

[16] In our model, the assumption is made for the sake of tractability. The possibility of negative payoff requires to model also what happens if a team has a negative wealth at the end of the first period and cannot field a team in the second period.

The assumption that the league cares about teams' long run solvency is also rather realistic. For example, in France, a financial audit of 1st Division teams by the financial commission of the league (DNCG) takes place every year before the beginning of season. Teams can participate in the championship only if they receive the green light of the DNCG. If they do not, they are relegated to a lower division.

can be thought of as effort, and is measured by the incentives necessary for players to perform during the season $e_{t,i}$. Formally, the probability that team i wins in period t is

$$p_{t,i} = \begin{cases} \alpha\frac{e_{t,i}}{e_{t,i}+e_{t,j}} + \beta\frac{W_{t,i}}{W_{t,i}+W_{t,j}} & \text{if} \quad e_{t,i}+e_{t,j} > 0 \\ \frac{\alpha}{2} + \beta\frac{W_{t,i}}{W_{t,i}+W_{t,j}} & \text{if} \quad e_{t,i}+e_{t,j} = 0 \end{cases}$$

with $\alpha + \beta = 1$. Quite obviously, $p_{t,j} = (1 - p_{t,i})$ since there are only two teams. The probability of winning increases with the difference in effort and the difference in wealth. When the two teams are equally wealthy and produce the same effort level, their probability of winning is $\frac{1}{2}$. One can think of $\frac{\alpha}{\beta}$ as a measure of how winning depends on incentives relative to initial quality. If $\frac{\alpha}{\beta} > 1$ the marginal return to effort is higher than the marginal return to wealth; loosely speaking, trying hard is more important than being better.

Teams

Each team's objective is to maximize current expected profits with an appropriate choice of incentives for its players. This choice is made knowing the prizes the league will award to winner and loser of the current tournament. Formally, team i's profits are:

$$\Pi_{t,i} = p_{t,i}K_{t,w} + (1 - p_{t,i})K_{t,l} - e_{t,i}$$

Implicitly, here we assume that the effort produced by players is observable.

Team i has an initial wealth equal to $W_{1,i}$. If it wins the tournament, its wealth in the second period is

$$W_{2,i} = W_{1,i} + K_{1,w} - e_{1,i}.$$

If it looses the tournament, its wealth in the second period is

$$W_{2,i} = W_{1,i} + K_{1,l} - e_{1,i}.$$

Therefore, the prizes awarded to the winner and loser of the first period have an effect on competitive balance in the second period.[17] In other words, the probability of winning in period

[17]Hence, a fully rational team should consider the influence of its strategy in period 1 on the game that will be

2 depends on the outcome of period 1 because of the effect prizes have on the teams' wealth, hence on teams' talent. Here, talent should be interpreted as a durable good. Team i invests $W_{i,1}$ at the beginning of period 1 and $\pi_{i,1}$ at the beginning of period 2, and the amount of talent accumulated at the beginning of period 2 is $W_{i,2} = W_{i,1} + \pi_{i,1}$.

Finally, we assume that in each period, teams are financially constrained and cannot borrow. This assumption is made mainly for tractability. In Section A.3, we discuss how results still holds if this assumption is relaxed.

3 The Equilibrium

In this section, we characterize the equilibrium of the game described previously. We begin by analyzing the subgame starting at the beginning of period 2. The solution concept we use is subgame perfect Nash equilibrium. We start with period 2 subgame and look at three optimization problems. First, the teams' optimal effort choices given their wealth, the prizes decided by the league, and the TV rights; second, the league's optimal prize choice, given the TV rights, and the teams' equilibrium play that follows; finally, the broadcaster's optimal TV rights choice, given teams' and league equilibrium plays. Then, we repeat a similar procedure for period 1, considering equilibrium play in period 2.

3.1 The Effort Game

Period 2 ends with the teams playing a simultaneous move game in which they choose incentives for players. Each team i chooses the level of incentives to maximize period 2 profits. Formally, team i solves $\max_{e_{i,t}} \pi_{t,i}$ where

$$\pi_{t,i} = p_{t,i} K_{t,w} + (1 - p_{t,i}) K_{t,l} - e_{t,i}. \tag{2}$$

played in period 2. At this point, we consider the behavior of myopic teams. This seems realistic since a league is usually made of a relatively large number of teams and the strategic influence of a specific team on the revenue of the league in the following period is small. In any case, the behavior of farsighted fully rational teams is analyzed in Appendix A.2.

In other words, in every period t we have a simultaneous moves game in effort.[18] The equilibrium of this game is characterized by the following proposition.

Proposition 1 *In the equilibrium of the effort game, the optimal strategies are*

$$e_{t,i} = e_{t,j} = \frac{\alpha}{4}\Delta K_t \tag{3}$$

where $\Delta K_t \equiv K_{t,w} - K_{t,l}$.

Proof: See Appendix.

This proposition says that the effort produced by teams increases with the difference between the prize money going to the winner and the loser. For any given K_t, we can interpret ΔK_t as a measure of (lack of) revenue sharing; in particular, full revenue sharing corresponds to $\Delta K_t = 0$ (and therefore $K_{t,w} = K_{t,l} = \frac{1}{2}K_t$) while no revenue sharing corresponds to $\Delta K_t = K_t$. Then, Proposition 1 gives a relationship between the level of revenue sharing and the equilibrium choices of incentives by the teams. The larger the amount of revenue sharing (i.e., the smaller ΔK_t), the smaller the effort level produced by teams. Intuitively, players need incentives to produce effort; the size of the prize for winning the tournament determines the incentives teams are willing to pay to their players.

Before proceeding, it should be noted that the symmetry of the equilibrium (i.e., $e_{t,1} = e_{t,2}$) is not due to the assumption that the probability of winning is separable in relative effort and relative wealth. As shown in Section 5.3, symmetric equilibria are also obtained for probability of winning functions which do not have this separability property. Also, our results on revenue sharing hold qualitatively if the probability of winning function is such that teams do not choose the same effort level in equilibrium.

Using Proposition 1, we can derive the demand for sport at time t as a function of the level of revenue sharing at time t.

$$D_t = \gamma\frac{\alpha\Delta K_t}{2} + \delta\left[1 - \left(\beta\frac{W_{t,i} - W_{t,j}}{W_{t,i} + W_{t,j}}\right)^2\right] + \nu\left(W_{t,i} + W_{t,j}\right) \tag{4}$$

[18] As noticed previously, we assumer teams are myopic; hence, the game they play in period 1 is identical to the one in period 2.

Since the effort level is symmetric, the competitive balance part of the demand is unaffected by effort: it only depends on the wealth differential. The willingness to win part of the demand, instead, is determined by the prize difference between winning and losing. Equilibrium in the effort game implies that the strategically relevant variable from the league's point of view is ΔK_t.

3.2 Strategy of the league in the second period

Knowing the effort produced by teams and the demand for sport in the second period, we can turn to the problem of the league.

Given the assumption that the market for broadcasting rights is perfectly competitive broadcasters are willing to pay the full monetary value of demand. Therefore, $K_t = D_t$. It follows that teams' joint profit in Period 2 is

$$\Pi_2 = K_2 - e_{2,1} - e_{2,2} = \frac{\alpha}{2}(\gamma - 1)\Delta K_2 + \delta \left[1 - \left(\beta \frac{W_{2,i} - W_{2,j}}{W_{2,i} + W_{2,j}}\right)^2\right] + \nu \left(W_{2,i} + W_{2,j}\right) \qquad (5)$$

The league's optimal allocation for a given amount of funds available is described by the following proposition.

Proposition 2 *If $\gamma \leq 1$, then $\Delta K_2 = 0$. If $\gamma > 1$, then*

$$\Delta K_2 = \frac{2}{2+\alpha} K_2 \qquad (6)$$

Proof: see Appendix.

Given the equilibrium in the effort game, the competitive balance in the second period is independent of the level of revenue sharing chosen in that period. Therefore, when choosing the level of revenue sharing in the last period, the league compares the marginal revenue and the marginal cost of effort. If $\gamma < 1$, the leagues sets the level of revenue sharing so as to minimize the effort level. Conversely, if $\gamma > 1$ then the league sets the level of revenue sharing so as to maximize the effort level.

TV revenues in Period 2

Using Proposition 2, we can write the demand in period 2 as a function of K_2:

$$D_2 = \gamma \frac{\alpha K_2}{2+\alpha} + \delta \left[1 - \left(\beta \frac{W_{2,i} - W_{2,j}}{W_{2,i} + W_{2,j}} \right)^2 \right] + \nu \left(W_{2,i} + W_{2,j} \right). \tag{7}$$

Given that $K_2 = D_2$, we can derive the amount broadcasters pay in Period 2.

Corollary 3 *If* $\gamma \leq 1$ *then*

$$K_2 = \delta \left[1 - \beta^2 \frac{\left(W_{2,i} - W_{2,j}\right)^2}{\left(W_{2,i} + W_{2,j}\right)^2} \right] + \nu \left(W_{2,i} + W_{2,j} \right) \tag{8}$$

If $1 < \gamma < \frac{2}{\alpha} - 1$, *then*

$$K_2 = \frac{2+\alpha}{2+\alpha(1-\gamma)} \left\{ \delta \left[1 - \left(\beta \frac{W_{2,i} - W_{2,j}}{W_{2,i} + W_{2,j}} \right)^2 \right] + \nu \left(W_{2,i} + W_{2,j} \right) \right\}. \tag{9}$$

In the rest of the paper, we restrict our attention to sets of parameters such that $0 < \gamma < \frac{2}{\alpha} - 1$, i.e., sets of parameters such that an competitive equilibrium exists in the market for broadcasting rights.

3.3 Strategy of the league in the first period

The two periods are linked through the decisions players make in the first period and through the outcome of the tournament in the first period. In particular, when the winner and the loser in period 1 perceive different prizes, this reflects on their wealth in period 2. Therefore, broadcasters and league behavior in period 1 influences the equilibrium in period 2 through the teams' wealth. In particular, two period 2 variables are relevant: the aggregate wealth and the wealth difference.

Period 2 aggregate wealth does not depend on the outcome of the tournament which took place in the first period:

$$\begin{aligned} W_{2,i} + W_{2,j} &= W_{1,i} + W_{1,j} + K_1 - (e_{1,i} + e_{1,j}) \\ &= W_{1,i} + W_{1,j} + K_1 - \frac{\alpha \Delta K_1}{2}. \end{aligned}$$

Period 2 difference in wealth, instead, depends on the outcome of this tournament. If team i has won, the difference is given by:

$$\begin{aligned} W_{2,i} - W_{2,j} &= W_{1,i} - W_{1,j} + \Delta K_1 - e_{1,i} + e_{1,j} \\ &= W_{1,i} - W_{1,j} + \Delta K_1 \end{aligned}$$

while the same difference when team i has lost is

$$\begin{aligned} W_{2,i} - W_{2,j} &= W_{1,i} - W_{1,j} - \Delta K_1 - e_{1,i} + e_{1,j} \\ &= W_{1,i} - W_{1,j} - \Delta K_1 \end{aligned}$$

The TV revenue and the effort chosen by the teams both depend on the outcome of the tournament in period 1. Given the results of Corollary 3, we study the cases $\gamma \leq 1$ and $\gamma \in (1, \frac{2}{\alpha} - 1)$ separately.

3.3.1 Case 1: $\gamma < 1$

Let K_2^i denote the TV revenue in period 2 when team i has won the previous tournament. From Corollary 3, we deduce that

$$K_2^i = \delta \left[1 - \beta^2 \frac{(W_{1,i} - W_{1,j} + \Delta K_1)^2}{(W_{1,i} + W_{1,j} + K_1 - e_{1,1} - e_{1,2})^2} \right] + \nu \left(W_{1,i} + W_{1,j} + K_1 - e_{1,1} - e_{2,1} \right) \quad (10)$$

Since the game teams play in period 1 is identical to the one in period 2, Proposition 1 holds. Therefore, we can compute the actual aggregate profits of the two teams in period 2 if team i won in period in the first period.

$$\Pi_2^i = \begin{array}{l} \delta \left[1 - \beta^2 \frac{(W_{1,i} - W_{1,j} + \Delta K_1)^2}{\left[W_{1,i} + W_{1,j} + K_1 - \frac{\alpha}{2} \Delta K_1 \right]^2} \right] \\ + \nu \left[W_{1,i} + W_{1,j} + K_1 - \frac{\alpha}{2} \Delta K_1 \right] \end{array} \quad (11)$$

We can now study the league's optimization problem in the first period. In Period 1, teams' joint profit is

$$\Pi_1 = \pi_{1,i} + \pi_{1,j} = K_1 - \frac{\alpha}{2} \Delta K_1 \quad (12)$$

Therefore, the objective of the league in the first period is to maximize

$$\Pi = \Pi_1 + p_{1,1}\Pi_2^1 + (1 - p_{1,1})\,\Pi_2^2$$

under the constraint $\frac{\alpha}{4}\Delta K_1 \leq K_{1,l}$, with

$$p_{1,i} = \frac{\alpha}{2} + \beta\frac{W_{1,i}}{W_{1,1} + W_{1,2}}$$

We have the following results about the level of revenue sharing chosen by the league.

Proposition 4 *Assume $\gamma < 1$. Full revenue sharing is optimal in both periods, i.e., $\Delta K_t = 0$ and $K_{t,w} = K_{t,l} = \frac{1}{2}K_t$.*

Proof: see Appendix.

3.3.2 Case 2: $\gamma \in (1, \frac{2}{\alpha} - 1)$

We proceed as in Case 1. If $\gamma \in (1, \frac{2}{\alpha} - 1)$, we deduce from Corollary 3 that

$$K_2^i = \frac{2+\alpha}{2+\alpha\,(1-\gamma)}\left\{\begin{array}{l} \delta\left[1 - \left(\beta\frac{W_{1,i} - W_{1,j} + \Delta K_1}{W_{1,i} + W_{1,j} + K_1 - \frac{\alpha}{2}\Delta K_1}\right)^2\right] \\ +\nu\left[W_{1,i} + W_{1,j} + K_1 - \frac{\alpha}{2}\Delta K_1\right] \end{array}\right\} \tag{13}$$

and from Proposition 2, we deduce

$$\Pi_2^i = K_2^i\left(1 - \frac{\alpha}{2+\alpha}\right) \tag{14}$$

The joint profit in period 1 is still given by Equation (12) and the objective function of the league maximizes in the first period is

$$\Pi = \Pi_1 + p_{1,1}\Pi_2^1 + (1 - p_{1,1})\,\Pi_2^2$$

where Π_2^i $(i = 1, 2)$ is given by Equation (14) and

$$p_{1,i} = \frac{\alpha}{2} + \beta\frac{W_{1,i}}{W_{1,1} + W_{1,2}}$$

The level of revenue sharing chosen by the league in period 1 (i.e., ΔK_1) influences teams' joint profit in three ways. Suppose the level of revenue sharing increases (ΔK_1 decreases). The first effect is to decrease period 1 demand and cost since the equilibrium level of effort in period 1 decreases. The second effect is to increase period 2 demand through the larger total wealth of the teams after they receive the tournament prizes. The third effect is to increase period 2 demand because more revenue sharing implies the league is more balanced, i.e. $|p_{2,1} - p_{2,2}|$ decreases. Disregarding the role of league's wealth in attracting talent, revenue sharing is chosen on the basis of the trade off between *incentives to win*, represented by how sensitive demand is to effort, i.e., players trying hard to win (the first effect above), and *competitive balance*, represented by how sensitive demand is to uncertainty of the tournament's outcome (the third effect above). We have the following result.

Proposition 5 *Assume* $\gamma \in (1, \frac{2}{\alpha} - 1)$. *There exists* A *such that if* $|W_{1,1} - W_{1,2}| < A$, *then* $\Delta K_1 > 0$.

Proof: See Appendix.

The proposition states that if the teams are relatively homogenous (in terms of wealth), then a performance based scheme may be optimal from the league viewpoint. The reason is that even if the wealthier team win the competition in the first period, the difference in wealth between the two teams will not be too large in the second period and the level of competitive balance will not be too low. In such a case, a performance based reward scheme enhances effort in period 1 without altering too much the level of competitive balance in period 2.

4 Extensions

In this section, we propose three extensions of the basic model. First, we examine the situation in which the league and the broadcaster sign a long term contract. Second, we analyze the case in which players effort is unobservable and hence teams makes players' compensation contingent of the outcome of the tournament. Last, we consider the case in which teams have several sources of revenue.

These extensions contribute to the realism of the model without changing the basic conclusion. Full revenue sharing is not always the way to maximize teams' joint profit.

4.1 Multi-period TV Contracts

In the previous section, TV deals were negotiated for one period at a time. In reality, broadcasting contracts span many years. To account for this, we consider a modification of the model where at the beginning of period 1 the league sells the rights to broadcast games for both seasons. In this case, the league decides two things: the allocation of total prizes between periods and the division of the total prize between winner and loser in each period.

This change in assumptions has two main implications. First, at the beginning of period 1, teams know prizes to be awarded in the second period. This was not the case before since K_2 was a function of the team that won in the first period. Second, we do not have $D_t = K_t$, $(t = 1, 2)$. If we denote K the revenue of the league at the beginning of period 1, then perfect competition in the broadcasting industry yields $K = D_1 + E(D_2^i)$.

Note that the problem faced by teams in each period remains unchanged. Hence, Proposition 1 holds and the effort level of each team in period t equals $\frac{\alpha}{4}\Delta K_t$. Therefore, the joint profit in period 2 depends on the winner of period 1 tournament in the way described by Equation (5). These observations imply that the problem of the league in the first period is to chooses K_1, K_2, ΔK_1 and ΔK_2 as functions of K so as to maximize

$$\Pi = \Pi_1 + p_{1,1}\Pi_2^1 + p_{1,2}\Pi_2^2$$

We have the following result about revenue sharing.

Proposition 6 *The equilibrium when there are two-period TV contracts has the following properties.*

(i) *If* $\gamma \leq 1 + \nu$ *then* $\Delta K_1 = 0$ *and* $K_1 = K$.

(ii) *If* $\gamma \in \left(1 + \nu, 1 + \nu\frac{2(2+\alpha)}{\alpha}\right)$, *then* $K_1 = K$ *and there exists* A' *such that if* $|W_{1,1} - W_{1,2}| < A'$, *then* $\Delta K_1 > 0$.

Proof: see Appendix.

As in the case of 1-period contracts, if the sensitivity of demand to effort (i.e., γ) is not too small and the leagues is relatively homogenous in wealth, then the league chooses a performance-based reward scheme.

Also, the league prefers to concentrate incentives provided by the teams in the first period since they increase total wealth in the next period. Also, for any given prize structure, the larger the amount distributed in the first period the smaller the difference in relative wealth between the two teams in the second period; hence the more balanced the competition. For these two reasons, the league distributes all the prize money in the first period.

4.2 Unobservable Effort

So far, we have implicitly assumed that effort produced by team players was observable, hence teams could offer effort-based compensation to players. In this section, we relax this assumption. A direct consequence is that teams can only offer performance-based contracts to players. Denote $\mu_{t,i}(w)$ and $\mu_{t,i}(l)$ the fraction of the gain paid to players when team i earns $K_{t,w}$ and $K_{t,l}$, respectively. The objective of team i is to maximize

$$\Pi_{t,i} = p_{t,i}\left(1 - \mu_{t,i}(w)\right) K_{t,w} + \left(1 - p_{t,i}\right)\left(1 - \mu_{t,i}(l)\right) K_{t,l}$$

subject to $\mu_{t,i}(w) \geq 0$, $\mu_{t,i}(l) \geq 0$, and

$$e^*_{t,i} \in \arg\max \ p_{t,i}\mu_{t,i}(w)K_{t,w} + (1 - p_{t,i})\mu_{t,i}(l)K_{t,l} \tag{15}$$

This last equation represents the incentive compatibility constraint for the players.

Let $\Delta K_{t,i} = \mu_{t,i}(w)K_{t,w} - \mu_{t,i}(l)K_{t,l}$. Then, proceeding as in the proof of Proposition 1, one shows that the equilibrium of the effort game is such that

$$e^*_{t,i} = \sup\left\{0, \frac{\alpha(\Delta K_{t,i})^2 \Delta K_{t,j}}{(\Delta K_{t,1} + \Delta K_{t,2})^2}\right\} \tag{16}$$

with $i \neq j$. It follows that if $e^*_{t,i} > 0$, then

$$p_{t,i} = \alpha\frac{\Delta K_{t,i}}{\Delta K_{t,1} + \Delta K_{t,2}} + \beta\frac{W_{t,i}}{W_{t,1} + W_{t,2}} \tag{17}$$

From these results, we derive the following proposition about the compensation of players by teams.

Proposition 7 *Assume that* $\Delta K_t > 0$*. There exists an equilibrium such that*
(i) $\mu_{t,i}(l) = 0$ *(*$i = 1, 2$*)*
(ii) *If* $W_{t,i} > W_{t,j}$*, then* $0 < \mu_{t,i}(w) < \mu_{t,j}(w)$ *and* $p_{t,i} > p_{t,j}$*.*

We deduce that

$$e^*_{t,i} = \frac{\alpha \mu_{t,i}(w)^2 \mu_{t,j}(w) K_{t,w}}{(\mu_{t,1}(w) + \mu_{t,2}(w))^2}$$

and

$$p_{t,i} = \alpha \frac{\mu_{t,i}(w)}{\mu_{t,1}(w) + \mu_{t,2}(w)} + \beta \frac{W_{t,i}}{W_{t,1} + W_{t,2}}$$

The proposition says that players are only compensated in case of success and the incentives are more important for the team with the smaller wealth. It follows that players from the wealthier team exert a lower effort. However, in equilibrium, the wealthier team has a higher probability of winning the competition.

It should be noted that the level of revenue sharing influences the effort level produced by teams in two ways: directly through the difference of gains between the winner and the loser, and indirectly through the compensation scheme of the players ($\mu_{t,i}(w)$).

With respect to the case with observable effort, the problem of the league is modified in the following way: teams do not choose the same effort level. Therefore, the level of competitive balanced at time t depends on the level of revenue sharing at time t. From part *(ii)* of Proposition 7, we deduce that the league has an additional motive to choose a performance based reward scheme. The reason is that if $W_{t,i} > W_{t,j}$ and $\Delta K_t > 0$, then the level of competitive balance at time t is

$$|p_{t,i} - p_{t,j}| = \alpha \frac{\mu_{t,i} - \mu_{t,j}}{\mu_{t,i} + \mu_{t,j}} + \beta \frac{W_{t,i} - W_{t,j}}{W_{t,i} + W_{t,j}}$$

while if $\Delta K_t = 0$ then

$$|p_{t,i} - p_{t,j}| = \frac{\alpha}{2} + \beta \frac{W_{t,i} - W_{t,j}}{W_{t,i} + W_{t,j}}$$

From part *(ii)* of Proposition 7, we know that if $W_{t,i} > W_{t,j}$ then $\mu_{t,i} < \mu_{t,j}$ and $p_{t,i} > p_{t,j}$. Therefore, the level of competitive balance is higher with a performance based scheme than with full revenue sharing.

4.3 Other sources of revenue

We now assume that teams have revenues that are not submitted to possible revenue sharing by the league. For example, these revenues may come from local TV deal, ticket sales or merchandising. We consider two different cases: First revenues are independent of past performances, and second revenues are dependent of past performances. In the second case, the idea is that the better a team is performing, the more attractive it is, hence the higher its revenue.

4.3.1 Other sources of revenue are not performance dependent.

In order to have an impact of the level of revenue sharing chosen by the league, these revenues must depend on the effort level and/or on the investment in talent.

If this source of revenue only depends on investment in talent, then effort levels chosen by teams is unchanged as a function of the level of revenue sharing. It implies that the problem of the league is the same as in Section 3. So, Proposition 5 holds qualitatively.

If the additional source of revenue depends on the effort level chosen by teams, the equilibrium level is modified and so is the problem of the league. To see this, consider the following simple example. Assume that the additional source of revenue of team i in period t is $S_i(e_{t,i}) = x_i Log(e_{t,i})$. In such a case, the first order condition of profit maximization for team i in period t is

$$\frac{e_{t,j}}{(e_{t,i}+e_{t,j})^2}\alpha\Delta K_t + \frac{x_i}{e_{t,i}} - 1 = 0$$

with $j \neq i$.

It implies that in equilibrium $e_{t,1} - x_1 = e_{t,2} - x_2$. The equilibrium effort level of team i is then the unique positive solution of

$$\frac{e_{t,i}(e_{t,i}+x_j-x_i)}{(2e_{t,i}+x_j-x_i)^2}\alpha\Delta K_t = e_{t,i} - x_i$$

From this last equation, it is straightforward that the effort level is increasing in ΔK_t.

Using the equilibrium condition $e_{t,1} - x_1 = e_{t,2} - x_2$ we deduce that the level of competitive balance is

$$1 - (p_{t,i} - p_{t,j})^2 = 1 - \left(\frac{\alpha(x_1-x_2)}{2e_{t,2}+x_1-x_2} + \frac{\beta(W_{t,1}-W_{t,2})}{W_{t,1}+W_{t,2}}\right)^2$$

Then, one can show that if $W_{t,i} > W_{t,j}$ and $x_i > x_j$, $(i = 1, 2,\ j \neq i)$ then the level of competitive balance is increasing in the effort level produced by teams. Since effort is increasing in ΔK_t, it implies that the additional source of revenues provides incentives for the league to choose a performance-based reward scheme.

If $W_{t,i} > W_{t,j}$ and $x_j > x_i$, $(i = 1, 2,\ j \neq i)$ then the level of competitive balance is increasing in the effort level produced by teams if

$$e_{t,1} + e_{t,2} < \frac{(x_i - x_j)(W_{t,i} + W_{t,j})}{W_{t,j} - W_{t,i}}$$

4.3.2 Other sources of revenue are performance dependent

Assume now that additional revenues are past-performance dependent. Here, the idea is that the better a team is performing, the more attractive it is, hence the higher its revenue. Formally, we assume that the winner of the competition in period t receives $K_{t,w} + P$ with P strictly positive and independent of the degree of revenue sharing chosen by the league. As before, the loser receives $K_{t,l}$.

Proceeding as in Section 3, one shows that the equilibriun effort level is

$$e_{t,1} = e_{t,2} = \frac{\alpha}{4}(\Delta K_t + P)$$

It implies that if $\gamma < 1$ then $\Delta K_2 = 0$ and if $\gamma > 1$, then

$$\Delta K_2 = \text{Max}\left(\frac{2K_2 - \alpha P}{2 + \alpha}, 0\right)$$

Hence, if P is not too large, the league chooses a performance reward scheme in the second period.[19]

In the first period, if P is small enough (so that the losing teams does not have a negative wealth in Period 2), the problem of the league is the same as in Section 3. As a consequence, Proposition 5 holds.

[19]We need

$$P < \frac{2(2+\alpha)}{\alpha[2(1+\gamma) + \alpha(1-\gamma)]}\left\{\delta\left[1 - \left(\beta\frac{W_{2,i} - W_{2,j}}{W_{2,i} + W_{2,j}}\right)^2\right] + \nu\left[W_{1,i} + W_{1,j} + K_1 - \frac{\alpha}{2}\Delta K_1\right]\right\}$$

5 Robustness of the results

The model we have presented is highly stylized. Here, we discuss how our results are robust to some generalization.

5.1 Teams are not financially constrained

Our model has assumed that teams are financially constrained and invest all their wealth to buy players. Therefore, wealth heterogeneity reduces competitive balance. Relaxing the assumption that teams are financially constrained does not affect our results qualitatively as long as rich teams have lower borrowing costs than poor teams. A possible reason for this difference in borrowing costs is that rich teams have more assets which can be used as collateral.[20] In such a case, rich teams invest more in talent than poor team and the league is not fully balanced either. As a consequence, the league faces the same trade-off between providing teams with incentives to produce effort in the first period and ensuring some degree of competitive balance in the second period. An example is provided in Appendix.

5.2 The shape of the demand for sport.

The demand function considered in our model is linear in effort and in level of competitive balance ($(p_{t,i} - p_{t,j})^2$). These assumptions were made for tractability: we are able to compute explicitly the joint profit in Period 2 which makes the analysis of the problem of the league in the first period easier.

Our results hold if the demand for effort is concave. In such a case, if the marginal demand for effort at $e = 0$ is smaller than one, then we are in a situation similar to the case $\gamma < 1$ in the model. Hence, the results of proposition 4 hold. Conversely, if the marginal demand for effort at $e = 0$ is greater than one, then we are in a situation similar to the case $\gamma > 1$. It follows that in the first period, the league chooses a performance based scheme if the difference on wealth

[20]For example, see Holmstrom and Tirole (1997), for a model in which the amount of asset used as collateral increases the level of investment.

between teams is small as stated by Proposition 5.

Concerning the demand for competitive balance, our results hold qualitatively for any function decreasing and concave in $(p_{t,i} - p_{t,j})^2$. A non linear demand for competitive balance only affects the threshold beyond which the league chooses full revenue sharing.

5.3 The shape of the probability-of-winning function

The probability-of-winning-the-tournament we used in the model is such that the effect of relative wealth and relative effort are additive. This implies that in equilibrium, effort levels chosen by the teams are identical and independent of their wealth. If the effort level chosen by teams is wealth dependent then results are modified in the following way.

First, if the poorer team chooses a higher effort level (as is the case when effort is unobservable) the league has additional motives to choose a performance base reward. The reason is that revenue sharing does not provide any incentives to produce effort while a performance-based reward scheme generate more effort from the poorer team than from the richer, hence a performance based reward scheme increases competitive balance.

Second, consider a probability-of-winning function of the following type

$$p_{t,i} = \begin{cases} \frac{e_{t,i}^{\alpha} W_{t,i}^{\beta}}{e_{t,i}^{\alpha} W_{t,i}^{\beta} + e_{t,j}^{\alpha} W_{t,j}^{\beta}} & \text{if} \quad e_{t,i} + e_{t,j} > 0 \\ \frac{W_{t,i}^{\beta}}{W_{t,i}^{\beta} + W_{t,j}^{\beta}} & \text{if} \quad e_{t,i} + e_{t,j} = 0 \end{cases}$$

As in the model developed in Sections 2 and 3, teams choose the same effort level in equilibrium. Here, we have

$$e_{t,1} = e_{t,2} = \text{Min}\left(K_{t,l}, \frac{\alpha W_{t,1}^{\beta} W_{t,2}^{\beta}}{\left(W_{t,1}^{\beta} + W_{t,2}^{\beta}\right)^2} \Delta K_t \right)$$

This implies that the level of competitive balance at time t depends only on the difference in wealth at time t:

$$(p_{t,i} - p_{t,j})^2 = \left(\frac{W_{t,1}^{\beta} - W_{t,2}^{\beta}}{W_{t,1}^{\beta} + W_{t,2}^{\beta}} \right)^2$$

It follows that the problem of the league in Period 1 is the same as in the model of Sections 2 and 3. The larger ΔK_1, the larger the effort exerted in period 1 but the smaller the expected level of competitive balance in period 2.

6 Conclusions

We presented a theoretical model of revenue sharing in sport leagues. Our main results derive conditions under which a performance-based reward scheme is optimal for a league. These can be summarized by looking at the relative importance of (current) incentives to win relative to (future) competitive balance. Higher revenues sharing increases future demand through a better competitive balance, but decreases current demand through a lower effort to win from teams. We have show that in league with relatively homogenous teams, a performance based reward scheme may be optimal while in a league with large wealth differences between teams, full revenue sharing is optimal. The reason is that if teams are relatively homogenous in terms of wealth (thus in investment in talent), then even if the wealthier team wins the competition in the first period, the difference in wealth between the two teams will not be too large in the latter periods and the level of competitive balance will not be too low. In such a case, a performance based reward scheme enhances effort in early periods without altering too much the level of competitive balance in latter periods. Conversely, if the difference in wealth between teams is large, the main objective of the league is to maintain the level of competitive balance. This is achieved by splitting TV revenues evenly between teams.

Our results contribute to the moral-hazard and contest design literatures. In a moral hazard context, our model is an example of a repeated agency problem between a principal and multiple agents in which the difference in output produced by the agents is detrimental to the principal. In this setting, the principal faces a trade-off between 'output balance' among agents and incentives to produce large quantities. Our results show that the principal may have incentive to 'invest' in output balance; that is, lower the output today in order to get a lower difference in outputs tomorrow.

For the contest design literature, our model describes a situation in which a winner-takes-all prize allocation may be optimal for the one-contest case but not optimal in the case of repeated contests. Multiple-prize allocations may be optimal for repeated contests if the outcome of the contest at time t influences bids (or effort) in the following contests.

A Appendix

A.1 Proofs

Proof of Proposition 1.

Firm i expected profits are:

$$\pi_{t,i}\left(e_{t,i},e_{t,j}\right)=\begin{cases}\left(\alpha\frac{e_{t,i}}{e_{t,i}+e_{t,j}}+\beta\frac{W_{t,i}}{W_{t,i}+W_{t,j}}\right)\Delta K_t+K_{t,l}-e_{t,i} & \text{if} \quad e_{t,i}+e_{t,j}>0\\ \left(\frac{\alpha}{2}+\beta\frac{W_{t,i}}{W_{t,i}+W_{t,j}}\right)\Delta K_t+K_{t,l} & \text{if} \quad e_{t,i}+e_{t,j}=0\end{cases}$$

Assume that $\Delta K_t>0$. Notice that

$$\pi_{t,i}\left(0,0\right)=\frac{W_{t,i}}{W_{t,i}+W_{t,j}}\Delta K_t+K_{t,l}$$

and

$$\pi_{t,i}\left(e,0\right)=\left(\alpha+\beta\frac{W_{t,i}}{W_{t,i}+W_{t,j}}\right)\Delta K_t+K_{t,l}-e.$$

Hence

$$\pi_{t,i}\left(e,0\right)-\pi_{t,i}\left(0,0\right)=\frac{\alpha}{2}\Delta K_t-e$$

Therefore, $e_{t,i}=0$ is not a best reply to $e_{t,j}=0$; this means aggregate effort equal to zero cannot be an equilibrium. Setting the derivative of $\pi_{t,i}$ with respect to effort equal to zero one gets:

$$\frac{e_{t,j}}{\left(e_{t,i}+e_{t,j}\right)^2}\alpha\Delta K_t-1=0$$

Therefore, team i's best response is:

$$e_{t,i}=\max\left\{0,\sqrt{\alpha e_{t,j}\Delta K_t}-e_{t,j}\right\}$$

and similarly for team j. Therefore, the Nash equilibrium of the game is

$$e_{t,i} = e_{t,j} = \frac{\alpha}{4}\Delta K_t.$$

Assume $\Delta K_2 = 0$. In this case, effort is costly and does not increase expected revenues. Hence, they choose $e_{t,i} = e_{t,j} = 0$.

□

Proof of Proposition 2.

The joint profit in period 2 is given by:

$$\Pi_2 = \frac{\alpha}{2}(\gamma - 1)\Delta K_2 + \delta \left[1 - \left(\beta \frac{W_{2,i} - W_{2,j}}{W_{2,i} + W_{2,j}}\right)^2\right] + \nu \left(W_{2,i} + W_{2,j}\right).$$

If $\gamma < 1$, Π_2 is decreasing in ΔK_2. Therefore, the league chooses $\Delta K_2 = 0$. If $\gamma > 1$, Π_2 is decreasing in ΔK_2. Therefore, the joint profit is maximized by choosing ΔK_2 as large as possible. The only limit to the size of ΔK_2 is the constraint that teams make no losses. This constraint implies

$$K_{2,l} = e_{2,i} = \frac{\alpha \left(K_{2,w} - K_{2,l}\right)}{4}$$

or

$$\frac{\alpha}{2}\left(K_{2,w} - K_{2,l}\right) = 2K_{2,l}.$$

This can be rewritten as

$$\frac{\alpha}{2}\left(K_{2,w} - K_{2,l}\right) = K_{2,l} + K_{2,w} + K_{2,l} - K_{2,w};$$

since $K_{2,w} + K_{2,l} = K_2$, we can write

$$\left(\frac{\alpha}{2} + 1\right)\left(K_{2,w} - K_{2,l}\right) = K_2$$

or

$$\Delta K_2 = \frac{2}{2 + \alpha} K_2$$

yielding the result.

□

Proof of Proposition 4.

Let $C = \nu(W_{1,1} + W_{1,2}) + \delta\left[1 - \beta^2\left(\frac{W_{1,1}-W_{1,2}}{W_{1,1}+W_{1,2}}\right)^2\right]$. Then, $K_1 = C + (\gamma-1)\frac{\alpha}{2}\Delta K_1$. After some substitution, we have

$$\Pi = \begin{array}{l} C + (\gamma-1)\frac{\alpha}{2}\Delta K_1 + \nu\left(W_{1,i} + W_{1,j} + C + (\gamma-1)\frac{\alpha}{2}\Delta K_1\right) \\ +\delta\left\{1 - \beta^2\frac{(W_{1,i}-W_{1,j})^2+(\Delta K_1)^2+2\beta\left(\frac{(W_{1,i}-W_{1,j})^2}{W_{1,i}+W_{1,j}}\right)\Delta K_1}{\left(W_{1,i}+W_{1,j}+C+(\gamma-1)\frac{\alpha}{2}\Delta K_1\right)^2}\right\} \end{array}$$

Differentiating this expression with respect to ΔK_1 we have:

$$\begin{array}{l} \frac{\partial\Pi}{\partial\Delta K_1} = \frac{\alpha}{2}(1+\nu)(\gamma-1) - \frac{\delta\beta^2}{D^2}\left[2\Delta K_1 + 2\beta\frac{(W_{1,i}-W_{1,j})^2}{W_{1,i}+W_{1,j}}\right] \\ +\frac{(\gamma-1)\alpha\delta\beta^2\Delta K_1}{D^3}\left[(W_{1,i}-W_{1,j})^2 + (\Delta K_1)^2 + 2\beta\left(\frac{(W_{1,i}-W_{1,j})^2}{W_{1,i}+W_{1,j}}\right)\Delta K_1\right] \end{array}$$

where

$$D = W_{1,i} + W_{1,j} + C + (\gamma-1)\frac{\alpha}{2}\Delta K_1$$

$\partial\Pi/\partial\Delta K_1$ is negative for any value of ΔK_1, hence $\Delta K_1 = 0$ at the optimum.

□

Proof of Proposition 5

Assume that $\alpha\Delta K_1/4 < K_{1,l}$. Then, the league's objective function in period 1 can be rewritten as

$$\begin{array}{rcl} \Pi & = & (\gamma-1)\frac{\alpha\Delta K_1}{2} + \delta\left[1 - \left(\beta\frac{W_{1,i}-W_{1,j}}{W_{1,i}+W_{1,j}}\right)^2\right] + \frac{3+\alpha(1-\gamma)}{2+\alpha(1-\gamma)}\nu\left(W_{1,i}+W_{1,j}\right) \\ & & +\frac{\nu}{2+\alpha(1-\gamma)}\left(C + (\gamma-1)\frac{\alpha}{2}\Delta K_1\right) + \frac{\delta}{2+\alpha(1-\gamma)} \\ & & -\frac{\delta\beta^2}{2+\alpha(1-\gamma)}\frac{\left[(W_{1,i}-W_{1,j})^2+(\Delta K_1)^2+2\beta\frac{\left(W_{1,i}-W_{1,j}\right)^2}{W_{1,i}+W_{1,j}}\Delta K_1\right]}{\left(W_{1,i}+W_{1,j}+C+(\gamma-1)\frac{\alpha}{2}\Delta K_1\right)^2} \end{array}$$

Let $D = W_{1,i} + W_{1,j} + C + (\gamma-1)\frac{\alpha}{2}\Delta K_1$ and

$$H = \frac{D\left(2\Delta K_1 + 2\beta^2\left(\frac{W_{1,1}-W_{1,2}}{W_{1,1}-W_{1,2}}\right)^2\right) - \frac{\alpha}{2}(\gamma-1)\left[(W_{1,i}-W_{1,j})^2 + (\Delta K_1)^2 + 2\beta\frac{(W_{1,i}-W_{1,j})^2}{W_{1,i}+W_{1,j}}\Delta K_1\right]}{D^2}$$

Then

$$\frac{\partial\Pi}{\partial\Delta K_1} = -\frac{\beta^2\delta}{2+\alpha(1-\gamma)}H + (\gamma-1)\frac{\alpha}{2}\left(1 + \frac{\nu}{2+\alpha(1-\gamma)}\right)$$

and

$$\text{Lim}_{\Delta K_1 \to 0} \frac{\partial \Pi}{\partial \Delta K_1} = (\gamma - 1)\frac{\alpha}{2}\left(1 + \frac{\nu}{2 + \alpha(1-\gamma)}\right) - \frac{\beta^2 \delta \left(W_{1,i} - W_{1,j}\right)^2}{2 + \alpha(1-\gamma)}\left[\frac{W_{1,i} + W_{1,j} + C}{\left(W_{1,i} + W_{1,j}\right)^2} - \frac{\alpha}{2}(\gamma - 1)\right]$$

Therefore, that there exists A such that if $(W_{1,1} - W_{1,2})^2 < A$, then $\text{Lim}_{\Delta K_1 \to 0} \frac{\partial \Pi}{\partial \Delta K_1} > 0$. So, if $(W_{1,1} - W_{1,2})^2 < A$ the league chooses $\Delta K_1 > 0$.

□

Proof of Proposition 6.

The league chooses K_1, K_2, ΔK_1 and ΔK_2 so as to maximize

$$\Pi = \Pi_1 + p_{1,1}\Pi_2^1 + p_{1,2}\Pi_2^2$$

This last equation can be rewritten as

$$\begin{aligned} \Pi = {} & C + (\gamma - 1)\tfrac{\alpha}{2}(\Delta K_1 + \Delta K_2) + \nu\left(W_{1,i} + W_{1,j} + K_1 + \tfrac{\alpha}{2}\Delta K_1\right) \\ & + \delta\left\{1 - \beta^2 \frac{\left[(W_{1,i} - W_{1,j})^2 + (\Delta K_1)^2 + 2\beta\frac{\left(W_{1,i} - W_{1,j}\right)^2}{W_{1,i} + W_{1,j}}\Delta K_1\right]}{\left[W_{1,i} + W_{1,j} + K_1 - \frac{\alpha}{2}\Delta K_1\right]^2}\right\} \end{aligned} \tag{18}$$

where

$$C = \nu(W_{1,1} + W_{1,2}) + \delta\left[1 - \beta^2\left(\frac{W_{1,1} - W_{1,2}}{W_{1,1} + W_{1,2}}\right)^2\right].$$

Proof of (i). Assume that $\gamma < 1$. It is straightforward that $\partial\Pi/\partial\Delta K_2 < 0$ and $\partial\Pi/\partial K_1 > 0$. Therefore, the league chooses $K_2 = 0$ and

$$\begin{aligned} \frac{\partial \Pi}{\partial \Delta K_1} = {} & \frac{\alpha}{2}(\gamma - \nu - 1) \\ & - \delta\beta^2 \frac{\left[2\Delta K_1 + 2\beta\frac{\left(W_{1,i} - W_{1,j}\right)^2}{W_{1,i} + W_{1,j}}\right](W_{1,i} + W_{1,j} + K_1) + \alpha(W_{1,i} - W_{1,j})^2\left(1 + \beta\frac{\Delta K_1}{W_{1,i} + W_{1,j}}\right)}{\left(W_{1,i} + W_{1,j} + K_1 - \frac{\alpha\Delta K_1}{2}\right)^3} \end{aligned} \tag{19}$$

Therefore, $\frac{\partial \Pi}{\partial \Delta K_1} < 0$. We deduce that if $\gamma \leq 1$ then $\Delta K_1 = 0$.

Now, assume that $\gamma \in (1, 1+\nu)$. In such a case, $\partial\Pi/\partial\Delta K_2 > 0$. Proceeding as in the case of one-period TV deals, one shows that the league chooses $\Delta K_2 = 2K_2/(2+\alpha)$. It follows that

$$\begin{aligned}\Pi = (\gamma-\nu-1)\tfrac{\alpha}{2}\Delta K_1 + \delta\left[1-\left(\beta\tfrac{W_{1,i}-W_{1,j}}{W_{1,i}+W_{1,j}}\right)^2\right] + 2\nu\left(W_{1,i}+W_{1,j}\right) \\ +\nu K_1 + (\gamma-1)\tfrac{\alpha}{2}\tfrac{K-K_1}{2+\alpha} + \delta\left\{1-\beta^2\frac{\left[(W_{1,i}-W_{1,j})^2+(\Delta K_1)^2+2\beta\frac{\left(W_{1,i}-W_{1,j}\right)^2}{W_{1,i}+W_{1,j}}\Delta K_1\right]}{\left[W_{1,i}+W_{1,j}+K_1-\frac{\alpha}{2}\Delta K_1\right]^2}\right\}\end{aligned} \tag{20}$$

Since $\nu > \gamma - 1$, $\frac{\partial\Pi}{\partial K_1} > 0$. This implies that $K_1 = K$ and $K_2 = 0$. Also, $\frac{\partial\Pi}{\partial\Delta K_1}$ is still given by (19). Therefore, if $\gamma < 1+\nu$, then $\frac{\partial\Pi}{\partial\Delta K_1} < 0$ and $\Delta K_1 = 0$.

Proof of (ii). The objective function of the league in the first period is given by (18). Therefore, as in the previous case, $\frac{\partial\Pi}{\partial\Delta K_2} > 0$ and $\frac{\partial\Pi}{\partial K_1} > 0$. Hence, $K_1 = K$ and $K_2 = 0$. Now, assume that $\alpha\Delta K_1/4 < K_{1,l}$. Then, $\frac{\partial\Pi}{\partial\Delta K_1}$ is still given by (19). Let

$$H = \frac{\left[2\Delta K_1 + 2\beta\frac{(W_{1,i}-W_{1,j})^2}{W_{1,i}+W_{1,j}}\right]\left(W_{1,i}+W_{1,j}+K_1\right) + \alpha\left(W_{1,i}-W_{1,j}\right)^2\left(1+\beta\frac{\Delta K_1}{W_{1,i}+W_{1,j}}\right)}{\left(W_{1,i}+W_{1,j}+K_1-\frac{\alpha\Delta K_1}{2}\right)^3}$$

$$\mathrm{Lim}_{\Delta K_1\to 0}H = \frac{\left[2\beta\frac{(W_{1,i}-W_{1,j})^2}{W_{1,i}+W_{1,j}}\right]\left(W_{1,i}+W_{1,j}+K_1\right) + \alpha\left(W_{1,i}-W_{1,j}\right)^2}{\left(W_{1,i}+W_{1,j}+K_1\right)^3}$$

Therefore, there exists A_2 such that if $|W_{1,1}-W_{1,2}| < A_2$ then $\mathrm{Lim}_{\Delta K_1\to 0}H > 0$. This implies that $\Delta K_1 > 0$.

□

Proof of Proposition 7.

Proof of part *(i)*. From equations (16) and (17), we derive that

$$\frac{\partial\pi_{t,i}}{\partial\mu_{t,i}(w)} = \frac{\alpha\Delta K_{t,j}K_{t,w}}{(\Delta K_{t,1}+\Delta K_{t,2})^2}\left[(1-\mu_{t,i}(w))K_{t,w} - (1-\mu_{t,i}(l))K_{t,l}\right] - p_{t,i}K_{t,w} \tag{21}$$

$$\frac{\partial\pi_{t,i}}{\partial\mu_{t,i}(l)} = -\frac{\alpha\Delta K_{t,j}K_{t,l}}{(\Delta K_{t,1}+\Delta K_{t,2})^2}\left[(1-\mu_{t,i}(w))K_{t,w} - (1-\mu_{t,i}(l))K_{t,l}\right] + (p_{t,i}-1)K_{t,l} \tag{22}$$

Assume that there exists an equilibrium with $\mu_{t,i}(w) > 0$. This implies that

$$(1-\mu_{t,i}(w))K_{t,w} - (1-\mu_{t,i}(l))K_{t,l} > 0$$

In turn, this implies that $\partial\pi/\partial\mu_{t,i}(l) < 0$ in equilibrium. Hence, $\mu_{t,i}(l) = 0$. Now, we need to show that the system of equations

$$\frac{\alpha\mu_{t,j}(w)}{(\mu_{t,1}(w) + \mu_{t,2}(w))^2}\left[(1 - \mu_{t,i}(w))K_{t,w} - K_{t,l}\right] - p_{t,i}K_{t,w} = 0 \qquad i = 1, 2 \qquad i \neq j \tag{23}$$

has a solution in $(0, 1) \times (0, 1)$ which satisfies the second order conditions of profit maximization.

From equation (21), it is straightforward that if $\mu_{t,i}(l) = 0$ then $\partial^2\pi_{t,i}/(\partial\mu_{t,i}(w))^2 < 0$. Now, when $\mu_{t,1}(w)$ and $\mu_{t,2}(w)$ converge to 0 at the same speed (so that there exists $H > 0$ such that $H < \mu_{t,i}(w)/\mu_{t,j(w)}$ ($i = 1, 2$ and $i \neq j$) as when $\mu_{t,1}(w)$ and $\mu_{t,2}(w)$ converge to 0), then the LHS of (23) goes to infinity. Furthermore, for any given $\mu_{t,i}(w) > 0$, $\alpha\mu_{t,j}(w)/(\mu_{t,1}(w) + \mu_{t,2}(w))^2$ converges to 0 as $\mu_{t,j}(w)$ converges to zero. Hence, we deduce that by continuity, there exist $\mu_{t,1}(w)$ and $\mu_{t,2}(w)$ such that the system of equations (23) has a solution in $(0, 1) \times (0, 1)$.

Proof of part *(ii)*: We use a contradiction argument. Assume that $W_{t,i} > W_{t,j}$ and $\mu_{t,i}(w) \geq \mu_{t,j}(w)$. This implies that $p_{t,i} > p_{t,j}$. From (23), it follows that

$$\frac{\mu_{t,j}(w)}{\mu_{t,i}(w)} > \frac{(1 - \mu_{t,j}(w))K_{t,w} - K_{t,l}}{(1 - \mu_{t,i}(w))K_{t,w} - K_{t,l}}$$

The LHS of this inequality is smaller than 1 while the RHS is larger than 1. Hence, the inequality does not hold and if $W_{t,i} > W_{t,j}$ then $\mu_{t,i}(w) < \mu_{t,j}(w)$. Now, $\mu_{t,i}(w) > \mu_{t,j}(w)$ implies $\pi_{t,i} > \pi_{t,j}$ follows directly from (23).

□

A.2 Fully Rational Teams

Fully rational teams take into account the impact of their action at time 1 on their wealth in period 2. Since probabilities of winning in period 2 and the revenue of the league in period 2 depend on teams' wealth, it follows that they take into the influence of their action in period 1 on their probability of winning in period 2 and on K_2. In period 2, the problem of the fully rational team is identical to that of a myopic team.

Formally, in period 1, fully rational team i solves the following problem

$$\begin{aligned} Max_{e_{1,i}}\Pi_{1,i} = Max_{e_{1,i}}p_{1,i}&\left[K_{1,w} + p_{2,i}(i)K^*_{2,w}(i) + (1 - p_{2,i}(i))K^*_{2,l}(i) - e_{2,i}(i)\right] \\ +(1 - p_{1,i})&\left[K_{1,l} + p_{2,i}(j)K^*_{2,w}(j) + (1 - p_{2,i}(j))K^*_{2,l}(j) - e_{2,i}(j)\right] - e_{1,i} \end{aligned} \tag{24}$$

with $i \neq j$and where $e_{2,i}(m)$, $K^*_{2,w}(m)$ and $K^*_{2,l}(m)$ represent the effort produced by team i in period 2, the amount awarded to the winner in period 2, the amount awarded to the loser in period 2, respectively, if team m wins in period 1 ($m = i, j$).

Proposition 8 *Assume that $\gamma \leq 1$. Then, there exists A_3 such that if $|W_{1,1} - W_{1,2}| < A_3$, then $\Delta K_1 = 0$.*

Proof: If $\gamma \leq 1$, then $e_{2,i} = 0$ and

$$K^*_2(i) = \delta \left[1 - \beta^2 \left(\frac{W_{1,i} - W_{1,j} + \Delta K_1}{W_{1,i} + W_{1,j} + K_1 - e1, 1 - e_{1,2}}\right)^2\right] + \nu(W_{1,1} + W_{1,2} + K_1 - e1, 1 - e_{1,2})$$

and

$$p_{2,i}(i) = \frac{\alpha}{2} + \beta \frac{W_{1,i} - W_{1,j} + \Delta K_1}{W_{1,i} + W_{1,j} + K_1 - e1, 1 - e_{1,2}}$$

and

$$p_{2,i}(j) = \frac{\alpha}{2} + \beta \frac{W_{1,i} - W_{1,j} - \Delta K_1}{W_{1,i} + W_{1,j} + K_1 - e1, 1 - e_{1,2}}$$

It is straighforward that $\lim_{W_{1,1} \to W_{1,2}} e_{1,1} = e_{1,2} = e$ since in the limit case the two teams are identical.

Also differentiating $\Pi_{1,i}$ with respect to $e_{1,i}$, we obtain

$$\lim_{W_{1,1} \to W_{1,2}} \frac{\partial \Pi}{\partial e_{1,i}} = \frac{\alpha}{4e} \Delta K_1 + \frac{2\delta\beta^2 \Delta K_1}{(W_{1,1} + W_{1,2} + K_1 - 2e)^2} - 1$$

Denote $F(e, \Delta K_1)$ the RHS of this last equation. The First order condition of profit maximization requires $F(e, \Delta K_1) = 0$, the Second order condition requires $\partial F / \partial E < 0$. Given that $\partial F / \partial \Delta K_1 > 0$, we deduce that

$$\lim_{W_{1,1} \to W_{1,2}} \frac{de}{d\Delta K_1} > 0$$

at the optimum.

Let $C = \delta + \nu(W_{1,1} + W_{1,2})$. When $W_{1,1}$ converges to $W_{1,2}$, the objective function of the league in period 1 converges to

$$\Pi = 2(\gamma - 1)e + C + \delta \left[1 - \beta^2 \left(\frac{(\Delta K_1)^2}{W_{1,i} + W_{1,j} + C + 2(\gamma - 1)e}\right)^2\right] + \nu(W_{1,1} + W_{1,2} + C + 2(\gamma - 1)e)$$

We deduce that

$$\frac{d\Pi}{d\Delta K_1}=2(1+\nu)(\gamma-1)\frac{de}{d\Delta K_1}-\frac{2\delta\beta^2\Delta K_1}{(W_{1,i}+W_{1,j}+C+2(\gamma-1)e)^2}+\frac{2\delta\beta^2(\delta K_1)^2(\gamma-1)\frac{de}{d\Delta K_1}}{(W_{1,i}+W_{1,j}+C+2(\gamma-1)e)^3}<0$$

Hence, by continuity, we derive that there exists A_3 such that if $|W_{1,1}-W_{1,2}|<A_3$, then $\Delta K_1=0$.

□

Assume now that $\gamma>1$. As already mentioned, teams' problem in period 2 is identical to the case of myopic teams. Therefore, we have

$$\begin{aligned} K_2^*(i) = {} & \tfrac{(2+\alpha)\delta}{2+\alpha(1-\gamma)}\left[1-\beta^2\left(\tfrac{W_{1,i}-W_{1,j}+\Delta K_1}{W_{1,i}+W_{1,j}+K_1-e_{1,1}-e_{1,2}}\right)^2\right] \\ & +\tfrac{(2+\alpha)\nu}{2+\alpha(1-\gamma)}\left(W_{1,1}+W_{1,2}+K_1-e_{1,1}-e_{1,2}\right)) \end{aligned}$$

The objective of team i in period 1 is

$$\begin{aligned} Max_{e_{1,i}}\Pi_{1,i} = {} & Max_{e_{1,i}}p_{1,i}\left[K_{1,w}+p_{2,i}(i)K_{2,w}^*(i)+(1-p_{2,i}(i))K_{2,l}^*(i)-e_{2,i}(i)\right] \\ & +(1-p_{1,i})\left[K_{1,l}+p_{2,i}(j)K_{2,w}^*(j)+(1-p_{2,i}(j))K_{2,l}^*(j)-e_{2,i}(j)\right]-e_{1,i} \end{aligned} \tag{25}$$

We have the following proposition:

Proposition 9 *Assume that $\gamma\in(1,\frac{2}{\alpha}-1)$. Then, there exists A_4 such that if $|W_{1,1}-W_{1,2}|<A_4$, then $\Delta K_1>0$.*

Proof: Proceeding as in the proof of Proposition 8, we obtain that $\lim_{W_{1,1}\to W_{1,2}}e_{1,1}=\lim_{W_{1,1}\to W_{1,2}}e_{1,2}=e$ and $\lim_{W_{1,1}\to W_{1,2}}\frac{de}{d\Delta K_1}>0$. Now, we know that, as in the case of myopic teams, teams'joint profit in period 2 is

$$\begin{aligned} \Pi_2^*(i) = \tfrac{2}{2+\alpha}K_2^*(i) = {} & \tfrac{2\delta}{2+\alpha(1-\gamma)}\left[1-\beta^2\left(\tfrac{W_{1,i}-W_{1,j}+\Delta K_1-e_{1,1}+e_{1,2}}{W_{1,i}+W_{1,j}+C+(\gamma-1)(e_{1,1}+e_{1,2})}\right)^2\right] \\ & +\tfrac{2\nu}{2+\alpha(1-\gamma)}\left(W_{1,i}+W_{1,j}+C+(\gamma-1)(e_{1,1}+e_{1,2}\right) \end{aligned}$$

Therefore, the objective of the league in period 1 is to maximize

$$\Pi=(\gamma-1)(e_{1,1}+e_{1,2})+C+p_{1,1}\Pi_2^*(1)+p_{1,2}\Pi_2^*(2)$$

We deduce that

$$\lim_{W_{1,1}\to W_{1,2}}\frac{d\Pi}{d\Delta K_1}=2(\gamma-1)\left(1+\frac{2}{2+\alpha(1-\gamma)}\right)\frac{de}{d\Delta K_1}-\frac{2\delta\beta^2}{2+\alpha(1-\gamma)}\left[\frac{2D(\Delta K_1)-4(\gamma-1)\frac{de}{d\Delta K_1}}{D^3}\right]$$

where

$$D = W_{1,1} + W_{1,1} + C + 2(\gamma - 1)e$$

We deduce that

$$\lim_{W_{1,1} \to W_{1,2}} \frac{d\Pi}{d\Delta K_1}\bigg|_{\Delta K_1 = 0} > 0$$

Therefore, by continuity, there exists A_4 such that if $|W_{1,1} - W_{1,2}| < A_4$, then $\Delta K_1 > 0$. □

A.3 Teams are not financially constrained: an example

Assume that teams' initial wealth takes the form of illiquid assets which cannot be sold in the short-run to finance investment in players. We consider two different cases:

- *Case 1:* At the end of each period, players can be sold at the same price as which they were bought. This implies that teams face no risk of bankruptcy.
- *Case 2:* With some probability $q > 0$, the investment in talent I_t is worth 0 at the end of the period, and with probability $1 - q$, players can be sold at the same price as which they were bough. In such a situation, teams face a risk of bankruptcy.

Case 1: In period 2, the only liquid asset teams have is their profit in the first period $\pi_{1,i}$ $(i = 1, 2)$. Therefore, team i maximizes

$$\pi_{2,i} = K_{2,l} + p_{2,i}\Delta K_2 - \max(I_{2,i} - \pi_{1,i})(1 + r_f)$$

where r_f is the risk-free interest rate.

There are three possible equilibria.

- Case 1a: If $\beta\Delta K_2/4 < \min_i \pi_{1,i}$, then $I_1 = I_2 = \beta\Delta K_2/4$
- Case 1b: If $\beta\Delta K_2/4 > \min_i \pi_{1,i}$ and $\max_i \pi_{1,i} = \frac{\beta\Delta K_2(1+r_f)}{(2+r_f)^2}$, then if $\pi_{1,i} > \pi_{1,j}$ $(i, j = 1, 2,$ $i \neq j)$

  $$I_i = \frac{\beta\Delta K_2(1 + r_f)}{(2 + r_f)^2}$$

  and

  $$I_j = \frac{\beta\Delta K_2}{(2 + r_f)^2}$$

- Case 1c: If $max_i \pi_{1,i} < \beta \Delta K_2/(2+r_f)^2$ then

$$I_1 = I_2 = \frac{\beta \Delta K_2}{(2+r_f)^2}$$

Case 2: If teams face the risk that bankruptcy, then illiquid assets will be used as collateral by lenders. This implies that the richer teams fill have a lower borrowing costs (as in Case 1b). As a consequence, the rich team invests more than the poor team.

A.4 Tables

Table 1. Revenue allocation in the LNF (season 1999-2000, in Million FF)

Ranking	Fixed Amount	Variable Amount	Total
1	54.5	45.5	100
2	54.5	40.25	94.75
3	54.5	36.75	91.25
4	54.5	31.5	86
5	54.5	29.75	84.25
6	54.5	28	82.5
7	54.5	24.5	79
8	54.5	21	75.5
9	54.5	19.25	73.75
10	54.5	17.5	72
11	54.5	14	68.5
12	54.5	10.5	65
13	54.5	8.75	63.25
14	54.5	7	61.5
15	54.5	5.25	59.75
16	54.5	2	56.5
17	54.5	2	56.5
18	54.5	2	56.5

Table 2. Ratio of revenues for the season 1999-2000 is some top European soccer leagues. Source: L'Equipe.

Country	Best/Worst
England	2.2
France	1.8
Germany	1.7

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