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Economics Letters 88 (2005) 272–277

economics letters

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Measuring profit efficiency with McFadden's gauge function

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Received 12 June 2004; received in revised form 8 July 2004; accepted 22 February 2005
Available online 19 May 2005

Abstract

Traditional input or output-based technical efficiency measures render composed error stochastic profit frontier estimation intractable for cross-sectional data. An alternative measure based on the gauge function simplifies estimation, permitting econometric techniques commonly used for stochastic cost frontiers.

Keywords: Efficiency; Gauge function; Stochastic frontier analysis; Translog profit function

JEL classification: C13; C31; D24

1. Introduction

A common issue in applied analysis is measurement of relative efficiency across a sample of firms. The stochastic frontier analysis literature has developed techniques for measuring production, revenue and cost efficiency that are both theoretically consistent with a well-defined primal production technology and easily estimated from a cross-sectional data set (for a textbook treatment see Kumbhakar and Lovell, 2000). This approach estimates efficient production, revenue, or cost frontiers with a two-part additive composed error term. Following the work of Aigner et al. (1977) and Meeusen and van den Broeck (1978) for production frontiers and Schmidt and Lovell (1979) for cost frontiers, the first part of the error is stochastic noise, and the second is a random firm-specific efficiency parameter.

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doi:10.1016/j.econlet.2005.02.014

Theoretical complications arise in applying this technique to profit frontiers. The key issue is how to specify the source of inefficiency. Conceptually, a firm can earn less than maximum possible profit for two reasons. First, it may be technically inefficient, choosing a production plan below the efficient frontier of the technology set. Second, it may be allocatively inefficient, choosing the wrong mix of inputs and outputs, given prices.

Ali and Flinn (1989) estimated a simple composed-error profit frontier without attributing inefficiency to either of these causes. Later, Kumbhakar (1996) argued that this specification was not consistent with an underlying source of inefficiency. He claimed that profit inefficiency is a function of prices, even if only technical inefficiency is present. As a result, specifying technical efficiency as part of the error results in biased and inconsistent parameter estimates. This claim is discouraging since the composed error model is easy to estimate. On the contrary, the alternative proposed in Kumbhakar (2001) is quite cumbersome for cross-sectional data, and precludes simultaneous estimation of a profit function and derived share equations.[1]

Here, I show that these results are an artifact of a particular specification of technical efficiency rather than an intrinsic characteristic of profit frontiers in general. Typically, stochastic frontier analysis uses technical efficiency measures based on Shephard's (1970) output or input distance functions. A measure based on the output distance function allows use of the composed-error technique with production and revenue frontiers. An input distance function-based approach permits composed-error cost frontier estimation. A further advantage of these two approaches is that technical efficiency does not appear in derived revenue or cost share equations, thus facilitating estimation of a complete system. As shown by Kumbhakar, however, neither specification keeps these nice properties when applied to profit frontier estimation.

Basing the technical efficiency measure on McFadden's (1978) gauge function, however, permits straightforward application of composed-error techniques to profit frontiers. Unlike the other specifications, the gauge measure provides a direct link between technical inefficiency and profit inefficiency. The gauge measure enters the estimating equations independently of prices and other exogenous variables and does not enter the derived profit-share equations. As a result, the system of a profit frontier with associated netput share equations can be simultaneously estimated by adapting techniques developed for cost frontier estimation.

2. Production model

Firms are assumed to be allocatively efficient.[2] Differences in technical efficiency are modelled as differences in production technologies. A technically inefficient firm has a production possibility set contained by the production possibility sets of more efficient firms. Firms labelled technically inefficient operate below the frontier of the most efficient technology, but on the frontier of their own smaller technology set. Heterogeneity in production possibility sets is completely described by an unobserved scalar parameter $\gamma \in (0,1]$. These assumptions are standard in stochastic frontier analysis (Kumbhakar and Lovell, 2000).

[1] With panel data one can more easily model inefficiency as a firm-specific effect as in Schmidt and Sickles (1984).

[2] It is straightforward to adapt techniques developed by Kumbhakar (1996) to model allocative inefficiency.

2.1. Technology

In addition to γ, firms are characterized by observable fixed factors $\mathbf{z}\in\Re^m$. Production plans are denoted by the variable netput vector $\mathbf{x}\in\Re^n$. Let the frontier technology $T(\mathbf{z})\subset\Re^n$ denote the set of feasible production plans for an efficient firm. This set is assumed to have the following properties:

(T1) $T(\mathbf{z})$ is convex and continuous in $\mathbf{z}$;
(T2) For each $\mathbf{z}$: $T(\mathbf{z})$ is closed;
(T3) For each $\mathbf{z}$: $0\in T(\mathbf{z})$ (*inaction is feasible*);
(T4) For each $\mathbf{z}$: $T(\mathbf{z})$ is convex;
(T5) For each $\mathbf{z}$ and for each $\mathbf{x}\neq 0$: if $\mathbf{x}\in T(\mathbf{z})$ then $-\mathbf{x}\notin T(\mathbf{z})$ (*irreversibility*);

These properties are standard, and together ensure the existence of a well-behaved dual profit frontier for a non-degenerate set of netput prices $P(\mathbf{z})\subset\Re^n$ (McFadden, 1978).

Define the gauge function as:

$$g(\mathbf{x};\mathbf{z})\equiv\inf\{\lambda>0:\ \mathbf{x}\in\lambda T(\mathbf{z})\},\ \text{where}\ \inf\{\emptyset\}=\infty. \tag{1}$$

Roughly speaking, $g(\mathbf{x};\mathbf{z})$ measures the smallest amount by which the $T(\mathbf{z})$ can be radially expanded and contain $\mathbf{x}$. Fig. 1 illustrates the gauge function in netput space. For netput bundle $\mathbf{x}^{\mathrm{A}}=(x_1^{\mathrm{A}},x_2^{\mathrm{A}})$, $g(\mathbf{x}^{\mathrm{A}};\mathbf{z})=\mathrm{OA/OB}$. The gauge function provides a complete functional representation of $T(\mathbf{z})$ (McFadden, 1978), such that

$$\mathbf{x}\in T(\mathbf{z})\Leftrightarrow g(\mathbf{x};\mathbf{z})\leq 1. \tag{2}$$

Let $t(\mathbf{z},\gamma)\subseteq T(\mathbf{z})$ denote the set of production possibilities for a firm with technical efficiency parameter γ. Assume $t(\mathbf{z},\gamma)$ satisfies (T1)–(T5) for each γ. The following assumption formalizes the notion of technical efficiency.

$$\text{(T6)}\ \mathbf{x}\in t(\mathbf{z},\gamma)\Leftrightarrow g(\mathbf{x};\mathbf{z})\leq\gamma.$$

Given $\mathbf{z}$, the parameter γ indicates a radial contraction of the set of production possibilities from the frontier set $T(\mathbf{z})$.In other words, if $\mathbf{x}$ can be produced by a firm with characteristics $\mathbf{z}$ and γ, then $\mathbf{x}/\gamma$ can

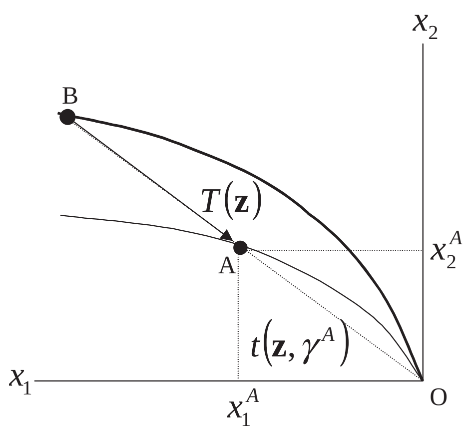

Fig. 1. Gauge function.

be produced by an efficient firm with $\mathbf{z}$. Again referring to Fig. 1, for a firm with technical efficiency γ^{A}=OA/OB, the set of production possibilities is $t(\mathbf{z},\gamma^{A})$=(OA/OB)$T(\mathbf{z})$.

2.2. Profit function

We now examine the implications of (T1)–(T6) for profit maximizing behavior. Define the restricted profit function $\pi(\mathbf{p},\mathbf{z},\gamma)$, as the maximum profit attainable by a firm with characteristics $\mathbf{z}$ and γ, facing prices $\mathbf{p}\in P(\mathbf{z})$:

$$\pi(\mathbf{p},\mathbf{z},\gamma)\equiv \sup_{x}\{\mathbf{p}'\mathbf{x} : g(\mathbf{x};\mathbf{z})\leq\gamma\}. \tag{3}$$

Note from Eq. (1) that $g(\mathbf{x};\mathbf{z})$ is positively linearly homogeneous in $\mathbf{x}$. Consequently, $\pi(\mathbf{p},\mathbf{z},\gamma)$ is positively linearly homogeneous in γ:

$$\pi(\mathbf{p},\mathbf{z},\gamma) = \gamma\pi(\mathbf{p},\mathbf{z},1). \tag{4}$$

Define profit efficiency as the ratio of actual to maximum possible profit:

$$\text{Profit efficiency}\equiv\frac{\pi(\mathbf{p},\mathbf{z},\gamma)}{\pi(\mathbf{p},\mathbf{z},1)} = \gamma. \tag{5}$$

Hence, if maximum profit is strictly positive, the gauge technical efficiency measure γ is a direct measure of profit efficiency.

Finally, consider cases when $\pi(\mathbf{p},\mathbf{z},\gamma)$ is differentiable in $\mathbf{p}$ and $\mathbf{z}$. Let $\mathbf{x}(\mathbf{p},\mathbf{z},\gamma)$ denote the optimal netput supply vector. Hotelling's Lemma indicates that $\mathbf{x}(\mathbf{p},\mathbf{z},\gamma)$ is simply a radial contraction of the netput supply vector for $\mathbf{x}(\mathbf{p},\mathbf{z},1)$:

$$\mathbf{x}(\mathbf{p},\mathbf{z},\gamma) = \frac{\partial\pi(\mathbf{p},\mathbf{z},\gamma)}{\partial p_i} = \frac{\partial\gamma\pi(\mathbf{p},\mathbf{z},1)}{\partial p_i} = \gamma x_i(\mathbf{p},\mathbf{z},1),\ i=1,\ldots,n. \tag{6}$$

Hence netput profit shares are independent of the level of technical efficiency:

$$\frac{\partial\pi(\mathbf{p},\mathbf{z},\gamma)}{\partial p_i}\cdot\frac{p_i}{\pi(\mathbf{p},\mathbf{z},\gamma)} = \frac{p_i x_i(\mathbf{p},\mathbf{z},1)}{\pi(\mathbf{p},\mathbf{z},1)},\ i=1,\ldots,n. \tag{7}$$

Kumbhakar (2001) showed that profit functions derived using either output or input-based technical efficiency measures are *not* generally homogeneous in the corresponding technical efficiency parameter. Therefore, using one of these measures one cannot generally separate technical efficiency parameters from the efficient frontier as in Eq. (4). Nor in these cases are profit shares independent of technical efficiency as in Eq. (7). These results are the source of misgivings about the appropriateness of the composed error model for measuring profit efficiency. Note that the gauge measure eliminates these problems. In the next section I briefly describe how one may use the gauge measure to estimate a translog profit frontier.

3. Estimation

If the efficient technology set $T(\mathbf{z})$ were known with certainty, a researcher with data on $\mathbf{x}$ and $\mathbf{z}$ could calculate the value of the gauge function directly for each firm. It follows from Eq. (5) that if firms behave as restricted profit maximizers, this value would indicate each firm's level of profit efficiency. In most applications, $T(\mathbf{z})$ is not known with certainty, so γ must be inferred from the data.

For the translog specification, after taking logs and appending a stochastic noise term v_0, Eq. (4) becomes:

$$\ln\pi(\mathbf{p},\mathbf{z},\gamma) = \alpha_0 + \sum_{i=1}^{m}\alpha_i \ln z_i + \sum_{i=1}^{n}\beta_i \ln p_i + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\alpha_{ij}\ln z_i \ln z_j + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\ln p_i \ln p_j + \sum_{i=1}^{n}\sum_{j=1}^{m}\delta_{ij}\ln p_i \ln z_j + \ln\gamma + v_0. \tag{8}$$

Symmetry conditions and positive linear homogeneity in prices can be imposed in the standard manner. After dropping one linearly dependent share equation, the corresponding profit share equations are:

$$\frac{p_i x_i(\mathbf{p},\mathbf{z},\gamma)}{\pi(\mathbf{p},\mathbf{z},\gamma)} = \beta_i + \sum_{j=1}^{n}\beta_{ij}\ln p_j + \sum_{j=1}^{m}\delta_{ij}\ln z_j + v_i, \; i = 1,\ldots,n-1. \tag{9}$$

Here, $v_1,\ldots, v_{n-1}$ represent stochastic noise.

Eqs. (8) and (9) can be estimated using an extension of the two-part additive error model. Let $\mathbf{v} \equiv (v_0, v_1,\ldots, v_{n-1})'$. I make the following standard assumptions regarding the stochastic elements of the model (Kumbhakar and Lovell, 2000):

(S1) $E[\mathbf{v}|\mathbf{p},\mathbf{z}]=\mathbf{0}$;

(S2) $E[v_0^3|\mathbf{p},\mathbf{z}]=0$;

(S3) ln γ has probability density function:

$$f(\ln\gamma;\sigma) = \begin{cases} \frac{2}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(\ln\gamma)^2}{2\sigma^2}\right], & \ln\gamma\leq 0 \\ 0, & \text{otherwise} \end{cases};$$

(S4) $E[\gamma|\mathbf{p},\mathbf{z}]=E[\gamma]$;

(S5) $E[\gamma\mathbf{v}]=0$.

Under (S1)–(S6), seemingly unrelated regression (SUR) consistently estimates all parameters except the intercept α_0, which is biased by E [ln γ] (Greene, 1980). Since v_0 is symmetric, the skewness of the SUR residuals from Eq. (8) can be manipulated to calculate consistent estimates of σ (for details see Olson et al., 1980). One can then calculate the mean of ln γ to correct the bias of α_0.

If interested in assigning an efficiency measure to each producer, it is necessary to impose additional structure on the noise terms. Specifically, if one is willing to assume v_0 is normally distributed and independent of $v_1,\ldots, v_{n-1}$, one can estimate the expected value of γ for each firm conditional on the residual from Eq. (8) by applying Kumbhakar's (1987) generalization of the methodology developed by Jondrow et al. (1982).

4. Conclusion

Composed error models have proven useful for estimation of production, revenue, and cost functions that allow for technical inefficiency. These models traditionally use technical efficiency specifications based on output or input distance functions. However, a reliance upon these two specifications has thwarted attempts to apply two-part additive error models to profit efficiency. This paper shows that an alternative technical efficiency measure based upon McFadden's gauge function enables straightforward application of the two-part additive error model. Consequently, in the presence of technical inefficiency estimation of a complete profit system of equations is no more complicated than cost frontier estimation.

Acknowledgements

The author is grateful to Robert G. Chambers, Valerie A. Mueller and Arthur A. Small III for valuable suggestions.

References

Aigner, D., Lovell, C., Schmidt, P., 1977. Formulation and estimation of stochastic frontier production function models. Journal of Econometrics 6, 21–37.

Ali, M., Flinn, J., 1989. Profit efficiency among basmati rice producers in Pakistan Punjab. American Journal of Agricultural Economics 71 (2), 303–310.

Greene, W.H., 1980. Maximum likelihood estimation of econometric frontier functions. Journal of Econometrics 13, 27–56.

Jondrow, J., Lovell, C., Materov, I., Schmidt, P., 1982. On the estimation of technical inefficiency in the stochastic frontier production function model. Journal of Econometrics 19, 233–238.

Kumbhakar, S., 1987. The specification of technical and allocative inefficiency in stochastic production and profit frontiers. Journal of Econometrics 34, 335–348.

Kumbhakar, S., 1996. Efficiency measurement with multiple outputs and multiple inputs. Journal of Productivity Analysis 7, 225–255.

Kumbhakar, S., 2001. Estimation of profit functions when profit is not maximum. American Journal of Agricultural Economics 83 (1), 1–19.

Kumbhakar, S., Lovell, C., 2000. Stochastic Frontier Analysis. Cambridge University Press.

McFadden, D., 1978. Cost, revenue and profit functions. In: Fuss, M., McFadden, D. (Eds.), Production Economics: A Dual Approach. North-Holland.

Meeusen, W., van den Broeck, J., 1977. Efficiency estimation from Cobb–Douglas production functions with composed error. International Economic Review 18, 435–444.

Olson, J., Schmidt, P., Waldman, D., 1980. A Monte Carlo study of estimators of stochastic frontier production functions. Journal of Econometrics 13, 67–82.

Schmidt, P., Lovell, C., 1979. Estimating technical and allocative inefficiency relative to stochastic production and cost frontiers. Journal of Econometrics 9, 343–366.

Schmidt, P., Sickles, R., 1984. Production frontiers and panel data. Journal of Business and Economic Statistics 2 (4), 367–374.

Shephard, R., 1970. The Theory of Cost and Production Functions. Princeton University Press.