Economic Theory **26**, 167–175 (2005)
DOI: 10.1007/s00199-004-0526-0

Economic Theory

Consistent firm choice and the theory of supply*

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Received: April 11, 2003, revised version: April 26, 2004

Summary. This paper analyzes the problem of deriving predictions, regarding supply behavior of a competitive firm, from prior consistency postulates about input-output choices made by such a firm. It extends the literature by introducing a consistency postulate for firm choice, which is weaker than profit-maximization. This consistency postulate is nevertheless both necessary and sufficient for supply responses predicted by the standard theory of firm choice based on the postulate of profit-maximization. Furthermore, our rationality postulate, in conjunction with another condition, is shown to be equivalent to firm choice behavior that can be rationalized in terms of profit maximization.

Keywords and Phrases: Weak axiom of profit maximization, Supply inequality, Non-reversibility.

JEL Classification Numbers: D21.

1 Introduction

This paper analyzes the problem of deriving predictions, regarding supply behavior of a competitive firm, from prior consistency postulates about choices made by such a firm. It extends the literature by introducing a consistency postulate for firm choice, which does not entail profit-maximization. This consistency postulate, while weaker than the standard Weak Axiom of Profit Maximization, is nevertheless both necessary and sufficient for all supply responses predicted by the standard theory of firm choice based on the postulate of profit-maximization. Furthermore, our rationality postulate, in conjunction with another condition, is shown to be equivalent to firm choice behavior that can be rationalized in terms of profit maximization.

* I thank Bhaskar Dutta and two anonymous referees for helpful comments on earlier versions.

The standard choice-based theory of firm behavior posits that a competitive firm chooses its output-input combinations in such a way as to satisfy the so-called Weak Axiom of Profit Maximization (WAPM). Firm choice in accordance with WAPM implies the well-known Supply Inequality (SI), which in turn yields, as a special case, the familiar Law of Supply (LS): the supply of any output by a competitive firm must be non-decreasing in its own price, and the use of any input non-increasing in its own price. Furthermore, for a firm's choice behavior to be rationalizable in terms of profit-maximization with respect to some collection of feasible input-output combinations, it is both necessary and sufficient that such behavior satisfy WAPM.[1]

Apparent violations of WAPM, however, are frequently encountered in empirical studies. A standard practice in econometric analysis is to attribute "small" violations of this kind to measurement error, rather than to violation of the profit-maximization hypothesis itself.[2] For such small violations, deriving predictions about the firm's responses to price changes on the basis of SI may seem reasonable. If, however, violations are "large", then the theoretical justification for such predictions would appear unpersuasive. It has been argued that large departures from WAPM, and, indeed, from maximizing behavior per se, are in fact routine in reality, and that this seriously reduces the empirical scope and usefulness of the standard theory of producers' behavior.[3]

Even if firms actually satisfy WAPM, one could observe *seemingly* contrary behavior, if there are factors that are not taken into account by an external observer, but nevertheless influence firms' profitability. The external observer may typically be expected to have incomplete information about the firm's input output decisions, the nature of the constraints facing the firm and its cost structure, and may fail to take into account some aspects in his calculations.

The problem of predicting a competitive firm's supply responses to price changes, when it appears to violate the profit-maximization hypothesis, and thereby WAPM, seems to have generated two different types of responses at the theoretical level. As mentioned earlier, it is possible to hypothesize that such behavior is reconcilable with the profit maximization postulate by ascribing it to a perception failure on part of the observer.[4] Thus, it has been hypothesized that such observations may be largely due to mis-specification of the firm's input output decisions, cost or return structure. The proper procedure, in this view, is to identify some alternative specification that would make the firm's observed behavior consistent with profit maximization.[5] Once such a specification has been identified, a version of WAPM

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[1] For formal definitions, see Section 2. Samuelson (1947, chapter 4) introduced both WAPM and SI, showed that the former implies the latter (and thereby, LS), and noted that WAPM is necessary for profit maximization. See also Debreu (1959, p. 47). Hanoch and Rothschild (1972) pointed out that WAPM is also sufficient for a firm's choice behavior to be rationalizable in terms of profit-maximization with reference to some technology set. The name, WAPM, is due to Varian (1984).

[2] Varian (1985) provides a formal treatment of the notion of a "small" violation in this context.

[3] See, for example, Leibenstein (1976, 1979) and Simon (1979).

[4] Such arguments are advanced, for example, by Stigler (1976) in his critique of the notion of X-efficiency proposed by Leibenstein.

[5] This understanding, in large measure, seems to motivate the transactions cost approach to firm behavior. For a survey, see de Alessi (1983a,b). Leibenstein (1983) provides a response.

which conforms to that specification can be used to generate testable predictions in the standard way, via the corresponding SI. A second approach has been to argue that, even if individual firms do not necessarily maximize profits, the law of supply may be expected to hold in the aggregate as a market-wide phenomenon.[6]

Neither of the two approaches addresses the following questions. Is it possible to formulate, without relying on profit-maximization, an independent, alternative a priori justification for SI, in terms of it being a requirement of consistent choice, or a rationality postulate? If so, can one then perform the relevant factorization exercise with respect to WAPM, i.e., show that WAPM is in fact equivalent to a conjunction of SI, and other, conditions?

Providing answers to such questions would seem to be of considerable interest. First, by independently justifying SI in terms of an a priori consistency condition for firm choice, which does not necessarily imply profit maximization, one would be able to provide a more general and robust theoretical justification for deriving predictions regarding supply responses of a competitive firm from SI. Secondly, by providing a factorization result for WAPM, one would be able to better identify exactly what kind of consistent firm choice is entailed by WAPM, and whether such consistency carries independent intuitive justification.

The purpose of the present paper is to address these issues. We offer an independent justification for SI as a consistency restriction on the firm's choice of net output vectors, which we interpret as a rationality postulate for the firm's choice behavior. This restriction is weaker than WAPM, and thus can be satisfied even by firms whose choice behavior cannot be rationalized in terms of profit maximization. We also identify a second restriction on the firm's choice behavior, which we term Non-Reversibility. We show that Supply Inequality and Non-Reversibility, taken together, are in fact equivalent to the Weak Axiom of Profit Maximization.

The paper is organized as follows. Section 2 introduces the basic notation and definitions. Section 3 discusses the intuitive rationale for our re-interpretation of SI as an independent consistency restriction. Section 4 addresses the relationship between the Weak Axiom of Profit Maximization and SI. We also specify the condition of Non-Reversibility and present our factorization result. Section 5 concludes.

2 Notation and preliminaries

Let n be the number of commodities, $n \geq 2$, and let $N = \{1, 2, ...n\}$. Let $\Re$ and $\Re_{++}$ denote, respectively, the set of real numbers and that of positive real numbers. A competitive firm faces n-dimensional vectors of commodity prices and produces n-tuples of net outputs (inputs being interpreted, in standard fashion, as commodities for which the firm's net output is negative). We shall denote price vectors by p, p' etc. and net output vectors by v, v' etc. We assume that all prices are finite and positive, so that $\Re^n_{++}$ constitutes the set of all possible price vectors. The collection of all technologically feasible net output vectors that are available to the firm constitutes the non-empty *technology set,* T, $T \subset \Re^n$.

[6] See, for example, Becker (1962).

Definition 2.1. *A supply function* (SF) is a rule S which specifies, for every price vector p, exactly one element of T; i.e., $S : \Re^n_{++} \to T$.

Thus, given an SF, S, the competitive firm chooses the net output vector $S(p)$, from its technology set T, when faced with the price vector p. The firm's choice behavior is thus completely described by its supply function. The standard a priori restriction that a competitive firm's supply function is assumed to satisfy in the literature is the so-called Weak Axiom of Profit Maximization.

Definition 2.2. An SF, S, satisfies the *Weak Axiom of Profit Maximization* (WAPM) if, and only if, for every ordered pair of price vectors $\langle p, p' \rangle$, $[p(v - v') \geq 0]$, where $v = S(p), v' = S(p')$.

WAPM requires that, if the firm happens to choose some net output vector v when faced with the price vector p, then it cannot choose any net output vector under another price situation which would give it a higher profit under p.

An outside observer cannot directly observe the firm's technology set, but can only observe the firm's choices of net output vectors under different price situations. The firm's choice behavior, represented by its supply function, S, can be interpreted as being driven by the goal of profit-maximization, if one can construct a set of net output vectors, say Γ, such that, if the firm's technology set was indeed Γ, and it wished to maximize its profit, then it would be able to do so by choosing according to S. More formally, we have the following definition.

Definition 2.3. An SF, S, is *rationalizable in terms of profit maximization with respect to a technology set* (RN) if, and only if there exists some $\Gamma \subseteq \Re^n$ such that, for every price vector p, (i) $S(p) \in \Gamma$, and (ii) $[[pS(p) \geq pv']$ for all $v' \in \Gamma]$. The set Γ will be said to *rationalize* S in terms of profit maximization.

Remark 2.4. An SF, S, satisfies WAPM if, and only if, it also satisfies RN. A *closed and convex* set $\Gamma' \subseteq \Re^n$ exists which rationalizes S in terms of profit maximization if, and only if, S satisfies RN.[7]

The primary testable empirical implication of the behavioral restriction imposed by WAPM (or, equivalently, RN) is the so-called Supply Inequality.

Definition 2.5. An SF, S, satisfies *supply inequality* (SI) if, and only if, for every pair of price vectors $\langle p, p' \rangle$:

$$(p - p')(v - v') \geq 0,$$

where $v = S(p), v' = S(p')$.

The condition of supply inequality (SI) yields the familiar Law of Supply (the supply of any output by a competitive firm must be non-decreasing in its own price, and the use of any input non-increasing in its own price), along with positive semi-definiteness of the substitution matrix. The empirical content of the standard choice-based theory of *price-taking* firm behavior is entirely specified by SI, which

[7] See Varian (1984).

in turn is derived as the implication of WAPM.[8] Thus, in the standard framework, SI is not endowed with any independent intuitive justification, but is merely seen as a consequence of WAPM. We now turn to the problem of providing an independent justification for SI, by re-interpreting it as an alternative, plausible and weaker behavioral postulate.

3 Consistent firm choice

Suppose, faced with price vectors p, p', a firm is observed to choose the net output vectors v, v', respectively. Then, by its actions, the firm reveals that both these net output vectors are technologically feasible options for it, i.e., both belong to its technology set. What kind of minimal consistency requirement, carrying the flavor of a rationality condition, can one then intuitively expect the firm to satisfy, without making any a priori assumption regarding profit-maximization?

First consider the case where v yields more profit than v', say \$10 more, under the price vector p. Then, under p', if by choosing v the profit the firm can make is *greater* than \$10 *plus* the profit the firm would make by choosing v' instead, then there does not seem to be any reason for the firm to choose the latter, since the firm had chosen v earlier even with a lower profit gain. Hence, if the firm does switch to v' under the price vector p', it must be that the profit gain from choosing v rather than v' under p', is, at the most, \$10.

Now consider the case where v yields less profit than v', say \$10 less, under the price vector p, yet the firm nevertheless decides to choose v rather than v', for some reason. Then, by its choice under the price situation p, the firm reveals that it is willing to forego a profit of \$10 in order to choose v rather than v'. Consequently, it appears reasonable to expect the firm to choose v rather than v' under p' as well so long as, by doing so, it foregoes a profit of *less* than \$10. Thus, the firm should choose v' under p' *only if*, by continuing with v, it stands to forego a profit of \$10 or more.

The argument outlined above provides an a priori intuitive justification for a minimal rationality condition for the firm's choice behavior,[9] which can be specified formally as follows:

$$\text{for every pair of price vectors } \langle p, p' \rangle, p\,(v - v') \geq p'\,(v - v'), \qquad (1)$$
$$\text{where } v = S\,(p)\,, v' = S\,(p')\,.$$

From Definition 2.5, it is clear that the consistency condition for the competitive firm's choice behavior specified by (1) is in fact equivalent to SI. Thus, instead of deriving its justification from WAPM, one may directly interpret, and justify, SI as an a priori consistency condition for the firm's choice behavior.

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[8] Thus, when one tests to see whether SI is satisfied, one is actually testing the joint hypothesis of profit maximization and price-taking behavior.

[9] Note that our intuitive argument takes profitability into account as a factor in thinking about the rationality of the firm's choice, but not necessarily as the only factor.

4 Consistent choice and profit maximization

The intuitive justification provided in support of SI made no explicit presumption of profit-maximizing behavior. The next question therefore is, what exactly is the relationship between SI and WAPM, i.e., price-taking firm behavior that can be rationalized in terms of profit-maximization? We now proceed to address this issue.

Proposition 4.1. *WAPM implies, but is not implied by, SI.*

Proof of Proposition 4.1. Definitions 2.2 and 2.5 yield the first part of Proposition 4.1. Thus, to establish Proposition 4.1, we only need to show that SI does not imply WAPM.

The proof consists of a counter-example. Let the price of the i-th commodity be denoted by p_i, and let the quantity of the i-th commodity supplied be denoted by v_i, $i \in N$. Let the supply function $S(p)$ be given by: $v = (1, -\frac{1}{p_2}, 0, ..., 0)$.

Clearly, this SF violates WAPM. We need to show that it satisfies SI. Let p^*, p' be any arbitrary pair of price vectors, and let v^*, v' denote the corresponding net output vectors. We shall show that: $p^*(v^* - v') \geq p'(v^* - v')$.

First note that: $(v^* - v') = \langle 0, \frac{1}{p_2'} - \frac{1}{p_2^*}, 0, ..., 0 \rangle$. Hence, $p^*(v^* - v') = \frac{p_2^*}{p_2'} - 1$, while $p'(v^* - v') = 1 - \frac{p_2'}{p_2^*}$. Now let $k = p_2^* - p_2'$. Then:

$$\frac{p_2^*}{p_2'} + \frac{p_2'}{p_2^*} = 2 + \left(\frac{k^2}{p_2' p_2^*} \right) \geq 2.$$

It follows that $p^*(v^* - v') \geq p'(v^* - v')$. □

By Proposition 4.1, SI is a weaker restriction on firm behavior than WAPM. Thus, even price-taking firm behavior that cannot be rationalized in terms of profit maximization may satisfy SI, and, thereby, satisfy all the testable predictions of standard theory specified by that condition. Note that the supply function specified in the proof of Proposition 4.1 also shows that SI does not imply *cost minimization*.

Since SI is weaker than WAPM, the natural question to ask now is: what kind of additional restriction does WAPM entail for firm choice? We proceed to identify this restriction, which we call 'non-reversibility'.

Definition 4.2. An SF, S, satisfies *non-reversibility* (NR) if, and only if, for every ordered pair of price vectors, $\langle p, p' \rangle$, [if $p(v - v') > 0$, then $p'(v' - v) \geq 0$], where $v = S(p), v' = S(p')$.

The condition of non-reversibility requires the following. If the firm can make a higher profit by choosing v instead of v' under the price vector p, and does choose v, then it should not choose v' under the price vector p' unless, by doing so, it receives at least as much profit as from v (under p'). In other words, when, by its choices, the firm reveals both v and v' to be technologically feasible alternatives for it, if we see the firm choosing that net output vector which yields a higher profit under p, then the firm's choice under p', between this pair of alternatives, cannot involve foregoing additional profit. Unlike SI, NR does not appear to possess much independent intuitive appeal as a consistency (or rationality) requirement.

It remains to establish that supply inequality and non-reversibility in fact constitute independent restrictions on price-taking firm behavior.

Proposition 4.3. *SI neither implies, nor is implied by, NR.*

Proof of Proposition 4.3. Consider first the example introduced in the proof of Proposition 4.1. As established there, this SF satisfies SI. It however violates NR. To see this, consider any pair of price vectors p^*, p' such that $[p_2^* - p_2' > 0]$, and let v^*, v' denote the corresponding net output vectors. Then:

$$p^* \left(v^* - v'\right) = \frac{p_2^* - p_2'}{p_2'} > 0,$$

and

$$p' \left(v' - v^*\right) = \frac{-\left(p_2^* - p_2'\right)}{p_2^*} < 0.$$

This however violates NR.

The following example shows that an SF may satisfy NR, yet violate SI. Let the price of the i-th commodity be denoted by p_i, and let the quantity of the i-th commodity supplied be denoted by v_i, $i \in N$. Consider the supply function $S(p)$:

$$v = (1, -1, 0, ..., 0) \text{ if } \left[p_1 - p_2 \leq 0\right],$$

and

$$v = (0, 0, ..., 0) \text{ otherwise.}$$

Let p^*, p' be any arbitrary pair of price vectors. It is obvious that, if either $[[p_1^* - p_2^* \leq 0]$ and $[p_1' - p_2' \leq 0]]$, or $[[p_1^* - p_2^* > 0]$ and $[p_1' - p_2' > 0]]$, then $p^*(v^* - v') = p'(v' - v^*) = 0$. Consider now the case where $[[p_1^* - p_2^* \leq 0]$ and $[p_1' - p_2' > 0]]$. In this case, we have $[p^*(v^* - v') \leq 0$ and $p'(v' - v^*) < 0]$. Hence, the SF (trivially) satisfies NR. Since, however, in the last case $[p^*(v^* - v') < p'(v^* - v')]$, our SF violates SI. □

Note that, since WAPM implies SI (Proposition 4.1), the example used in the proof of Proposition 4.3 above, which shows that an SF may violate SI while satisfying NR, also establishes that NR does not imply WAPM.

Remark 4.4. A transparent way of contrasting the three conditions, WAPM, SI and NR, is the following. Consider any arbitrary pair of price vectors $\langle p, p' \rangle$, and let $v = S(p), v' = S(p')$. WAPM requires $[pv \geq pv']$ (Definition 2.2). However, NR only requires this weak inequality to hold *conditionally*: [*if* $p'v' > p'v$, *then* $pv \geq pv'$] (Definition 4.2). Now note that, while WAPM requires that the weak inequality be satisfied individually, i.e., [$pv \geq pv'$ *and* $p'v' \geq p'v$], SI merely requires that it hold *jointly*, i.e. $[pv + p'v' \geq pv' + p'v]$ (Definition 2.5).

The last step in our analysis consists in showing that SI and NR, together, turn out to be equivalent to WAPM, and, therefore, to rationalizability in terms of profit maximization as well.

Proposition 4.5. *An SF, S, satisfies WAPM if, and only if, it satisfies both SI and NR.*

Proof of Proposition 4.5. First note that WAPM is equivalent to the following condition:

$$\text{for every pair of price vectors } \langle p, p' \rangle, \ [p(v - v') \geq 0 \geq p'(v - v')], \tag{2}$$

where $v = S(p), v' = S(p')$.

That (2) implies both SI and NR is obvious. Now suppose that SI is satisfied. Consider any arbitrary pair of price vectors $\langle p, p' \rangle$. Given SI, if (2) is violated, it must be that either $0 > p(v - v') \geq p'(v - v')$, or $p(v - v') \geq p'(v - v') > 0$. In either case, NR would be violated. Hence, SI and NR together imply WAPM. □

In light of Definition 2.3 and Remark 2.4, Proposition 4.5 yields the following.

Corollary 4.6. *An SF, S, is rationalizable in terms of profit maximization with respect to a technology set if, and only if, it satisfies both SI and NR.*

5 Conclusion

In this paper, we have extended the choice-based theory of firm behavior in two main directions.

First, we have introduced an independent intuitive justification for the condition of Supply Inequality by re-interpreting it as an a priori consistency (or rationality) restriction for a competitive firm's choice of net output vectors. We have shown that this restriction is weaker than the Weak Axiom of Profit Maximization, and thus can be satisfied even by price-taking firms whose choice behavior cannot be rationalized in terms of profit maximization. We have thereby shown that all the standard predictions about supply responses of a competitive firm that are generated by the Supply Inequality actually flow from our consistency postulate, and do not require the satisfaction of WAPM. Thus, the empirical contents of the standard theory have been shown to be more robust than the premise of rationalizability of a firm's choice behavior in terms of profit maximization.

Second, we have shown that, given price-taking behavior, the Weak Axiom of Profit Maximization is equivalent to the combination of Supply Inequality and another condition, which we have termed Non-Reversibility.

In a series of recent papers, Bandyopadhyay, Dasgupta and Pattanaik (2004, 2002, 1999) have extended the standard choice-based theory of consumer behavior, founded on Samuelson's Weak Axiom of Revealed Preference, to allow for random choices by a competitive consumer. An analogous generalization, along stochastic lines, of the deterministic analysis carried out in this paper in the context of producer behavior, remains the natural subject for future investigation.

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