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WELFARE-MAXIMIZING MONETARY POLICY UNDER PARAMETER UNCERTAINTY

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SUMMARY

This paper examines welfare-maximizing monetary policy in an estimated micro-founded general equilibrium model of the US economy where the policymaker faces uncertainty about model parameters. Uncertainty about parameters describing preferences and technology implies uncertainty about the model's dynamics, utility-based welfare criterion and the 'natural' rates of output and interest that would prevail absent nominal rigidities. We estimate the degree of uncertainty regarding natural rates due to parameter uncertainty. We find that optimal Taylor rules under parameter uncertainty respond less to the output gap and more to price inflation than would be optimal absent parameter uncertainty. We also show that policy rules that focus solely on stabilizing wages and prices yield welfare outcomes very close to the first-best. Copyright © 2009 John Wiley & Sons, Ltd.

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1. INTRODUCTION

This paper examines welfare-maximizing monetary policy in an estimated dynamic stochastic general equilibrium (DSGE) model of the US economy where the central bank faces uncertainty about the values of model parameters. The design of optimal monetary policy depends on the nature of the dynamics of the economy, the natural rates of output and interest, and the central bank objective function. Traditional analysis of monetary policy under uncertainty has treated these three factors as independent and studied them in isolation (see, for example, Brainard, 1967; Rudebusch, 2001), But modern micro-founded models imply that the structural parameters describing preferences and technology jointly determine all three factors. Therefore, an analysis of monetary policy under parameter uncertainty requires that these consequences of parameter uncertainty be analyzed in unison.

Recent papers by Giannoni (2002), Levin and Williams (2005), and Levin *et al.* (2005; henceforth LOWW), have studied monetary policy under parameter uncertainty in micro-founded models. The latter two papers imposed the linkage between parameter uncertainty and uncertainty about the welfare costs of fluctuations, but neither examined the role of natural rate uncertainty in the design of optimal policy. Aoki and Nikolov (2004) highlighted the connection between parameter uncertainty and uncertainty about the natural rate of interest—defined to be the real interest rate that would prevail absent nominal rigidities—but did not explore further the role of natural rates in the design of optimal policy under uncertainty.

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We use a small estimated micro-founded model as a laboratory to explore how parameter uncertainty and the associated uncertainty about natural rates and welfare costs of fluctuations affects the design of optimal monetary policy. We analyze the implications of parameter uncertainty from the perspective of a Bayesian policymaker who aims to maximize expected household welfare. We first show that parameter uncertainty implies a non-trivial degree of uncertainty about the natural rates of output and interest. We then show that optimal Taylor rules under parameter uncertainty respond less to the output gap and more to price inflation than would be optimal absent parameter uncertainty. This conclusion is consistent with that found in the literature on optimal policy rules using traditional models, despite the very different analytical frameworks (see Orphanides and Williams, 2002, and references therein). Finally, we show that policy rules that respond solely to wage and price inflation yield welfare outcomes very close to the first-best.

The paper is organized as follows. Section 2 describes the model. Section 3 describes its estimation. Section 4 examines optimal monetary policy when model parameters are known. Section 5 considers optimal policy under parameter uncertainty. Section 6 concludes.

2. THE MODEL

Our analysis uses a small micro-founded model with various frictions that interfere with instantaneous full adjustment of quantities and prices to shocks. To make the analysis tractable, we abstract from many features present in recently developed larger DSGE models (see, for example, Christiano *et al*., 2005; Smets and Wouters, 2003). We first present the model's preferences and technology and then describe the firms' and households' optimization problems. Mathematical descriptions of these problems are given in Appendix A of Edge *et al*. (2008; henceforth ELW) along with the model's nonlinear and linearized first-order conditions and steady-state solution. Throughout, we denote the log of variables by lower-case letters.

2.1. Production Technology and Preferences

The economy's final good, $Y_{f,t}$, is produced according to the Dixit–Stiglitz technology:

$$Y_{f,t} = \left(\int_0^1 Y_{f,t}(x)^{\frac{\Theta_{p,t}-1}{\Theta_{p,t}}} \mathrm{d}x \right)^{\frac{\Theta_{p,t}}{\Theta_{p,t}-1}} \tag{1}$$

where $Y_{f,t}(x)$ denotes the quantity of the xth differentiated goods used in production and $\Theta_{p,t}$ is the time-varying elasticity of substitution between the production inputs. Final goods producers obtain their inputs from the economy's differentiated intermediate goods producers who supply $Y_{m,t}(x)$. Not all of the intermediate goods producers' differentiated output is realized as final goods' inputs; some is absorbed in price formulation, following the adjustment cost model of Rotemberg (1982). We modify the Rotemberg model so that the cost of adjusting prices is relative to a rule-of-thumb price adjustment equal to a weighted average of steady-state inflation and last period's inflation rate. This allows for intrinsic inertia in inflation. Specifically, the relationship between $Y_{f,t}(j)$ and $Y_{m,t}(j)$ is given by

$$Y_{f,t}(j) = Y_{m,t}(j) - \frac{\chi_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - (1-\gamma_p)\Pi_{p,*} - \gamma_p \Pi_{p,t-1} \right)^2 Y_{m,t} \tag{2}$$

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where the second term in (2) denotes the cost of setting prices, $P_t(j)$ is the price charged by firms j for a unit of its output, $\Pi_{p,*}$ is the steady-state price inflation rate, and $\Pi_{p,t-1}$ is the lagged price inflation rate. Our choice of quadratic adjustment costs for modeling nominal rigidities contrasts with that of many other recent studies, which rely instead on staggered contracts as in Calvo (1983). ELW extends the analysis in this paper to a Calvo-based model of wages and prices. The results regarding the effects of uncertainty on optimal policy are qualitatively the same with Calvo contracts as with quadratic adjustment costs.

The differentiated intermediate goods, $Y_{m,t}(j)$ for $j \in [0, 1]$, are produced by combining each variety of the economy's differentiated labor inputs that are supplied to market activities (that is, $\{L_{y,t}(z)\}$ for $z \in [0, 1]$). The composite bundle of labor, denoted $L_{y,t}$, that obtains from this aggregation implies, given the current level of technology A_t, the output of the differentiated goods, $Y_{m,t}$. Specifically, production is given by

$$Y_{m,t}(j) = A_t L_{y,t}(j) \text{ where } L_{y,t}(j) = \left(\int_0^1 L_{y,t}(x, j)^{\frac{\Theta_{w,t}-1}{\Theta_{w,t}}} \mathrm{d}x \right)^{\frac{\Theta_{w,t}}{\Theta_{w,t}-1}} \tag{3}$$

where $\Theta_{w,t}$ is the time-varying elasticity of substitution between the differentiated labor inputs. The log-level of technology, A_t, is modeled as a random walk:

$$\ln A_t = \ln A_{t-1} + \varepsilon_{A,t} \tag{4}$$

where $\varepsilon_{A,t}$ is an i.i.d. innovation. We abstract from trend growth in productivity.

Households own shares in the firms in the economy. They derive utility from the consumption good C_t and from leisure time, equal to their time endowment $\overline{L}$ less the $0 \leq L_{u,t}(i) \leq \overline{L}$ hours allocated to non-leisure activities. The household members live forever and there is no population growth. Preferences exhibit an additive habit (equal to a fraction $\eta \in [0, 1]$ of its consumption last period) and are non-separable between consumption and leisure. Specifically, preferences of household i are given by

$$E_0 \frac{1}{1-\sigma} \sum_{t=0}^{\infty} \beta^t \Xi_{c,t} [(C_t(i) - \eta C_{t-1}(i))(\overline{L} - L_{u,t}(i))^{\zeta}]^{1-\sigma} \tag{5}$$

where β is the household's discount factor and $\Xi_{c,t}$ is a stochastic preference shifter that is assumed to follow an AR(1) process in logs. The economy's resource constraint implies that $\int_0^1 C_t(x)\mathrm{d}x \leq Y_{f,t}$, where $Y_{f,t}$ denotes the output of the economy's final good.

Household i supplies $L_{y,t}(i)$ hours to the labor market and devotes time to setting wages. Consequently, the time allocated to non-leisure activities, $L_{u,t}(i)$, is given by

$$L_{u,t}(i) = L_{y,t}(i) + \frac{\chi_w}{2} \left(\frac{W_t(i)}{W_{t-1}(i)} - (1-\gamma_w)\Pi_{w,*} - \gamma_w \Pi_{w,t-1} \right)^2 L_{u,t} \tag{6}$$

where (analogous to the cost of setting prices) the second term in (6) is the cost of setting wages in terms of labor time. $W_t(i)$ is household i's wage for a unit of its time, $\Pi_{w,*}$ is the steady-state wage inflation rate, and $\Pi_{w,t-1}$ is the lagged wage inflation rate.

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2.2. Firms' and Households' Optimization Problems

The final goods producing firm takes as given the prices set by each intermediate-goods producer for their differentiated output, $\{P_t(j)\}_{j=0}^1$, and chooses inputs, $\{Y_{f,t}(j)\}_{j=0}^1$, to minimize the cost of producing its final output $Y_{f,t}$, subject to its production technology, given by equation (1). Each intermediate-goods producing firm chooses the quantities of labor that it employs in production and the price that it will set for its output. In deciding the quantities of labor to employ, firm j takes as given the wage $\{W_t(i)\}_{i=0}^1$ set by each household for its variety of labor and chooses $\{L_{y,t}(i,j)\}_{i=0}^1$ to minimize the cost of attaining the aggregate labor bundle $L_{y,t}(j)$ that it needs for production.

In setting its price, $P_t(j)$, the intermediate-goods producing firm takes into account the demand schedule for its output that it faces from the final goods sectors and the fact—as summarized in equation (2)—that its price affects the amount of its output that it can sell to final goods producers. The intermediate-goods producing firm j takes as given the marginal cost MC_t for producing $Y_{m,t}(j)$, the aggregate price level P_t, and aggregate final-goods demand $Y_{f,t}$, and chooses its price $P_t(j)$ to maximize its profits subject to the cost of resetting its price and the demand curve it faces for its differentiated output. We assume a subsidy on production (which we acknowledge does not in practice exist), equal to $(\Theta_{p,*}-1)^{-1}$, which offsets the distortionary effects of monopolistic competition.

The household, taking as given the expected path of the gross nominal interest rate R_t, the price level P_t, the aggregate wage rate W_t, its profits income, and its initial stock of bonds, chooses its consumption $C_t(i)$ and its wage $W_t(i)$ to maximize its utility subject to its budget constraint, the cost of resetting its wage, and the demand curve it faces for its differentiated labor. As with production we assume a subsidy on labor supply, equal to $(\Theta_{w,*}-1)^{-1}$, which ensures Pareto optimality of the flexible wage and price equilibrium. These subsidies are funded by lump sum taxes.

2.3. Natural Rate Variables

The model has a counterpart in which all nominal rigidities are absent, that is, prices and wages are fully flexible ($\chi_p = \chi_w = 0$). We refer to the levels of output, hours, and the real one-period interest rate in this equilibrium as the natural rates of output, $\widetilde{Y}_t$, hours, $\widetilde{L}_t$, and interest, $\widetilde{R}_t$, respectively. We also define log deviations of these variables from their steady-state values, $\tilde{y}_t \equiv \log\ \widetilde{Y}_t - \log\ Y_*$ and $\tilde{r}_t \equiv \log\ \widetilde{R}_t - \log\ R_*$. These natural rates are functions of our model's structural shocks and are derived in Appendix A of ELW.

2.4. Monetary Authority

We assume that the central bank uses the short-term interest rate as its instrument. For estimation purposes, we assume that the short-term interest rate responds to deviations of price inflation from its steady-state level, $\pi_{p,t} \equiv \log\ \Pi_{p,t} - \log\ \Pi_{p,*}$, and to the output gap, $x_t = \log\ Y_t - \log\ \widetilde{Y}_t$. We also allow for policy inertia by including the lagged short-term interest rate in the feedback equation. Monetary policy is described by

$$r_t = \phi_r r_{t-1} + (1-\phi_r)\{\phi_p \pi_{p,t} + \phi_x x_t\} + \varepsilon_{r,t} \tag{7}$$

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where $r_t \equiv \log R_t - \log R_*$, $y_t \equiv \log Y_t - \log Y_*$, and $\varepsilon_{r,t}$ is an i.i.d. policy shock. Note that we have suppressed the constant that incorporates the steady-state levels of the interest and inflation rate. In the analysis of optimal monetary policy, we specify a generalized version of this policy rule, as described in Section 4.

3. ESTIMATION

In order to analyze optimal Bayesian monetary policy under parameter uncertainty, we need a posterior distribution of the model parameters, which we obtain by following a limited-information approach. Specifically, we estimate several of the structural parameters of our model by minimum distance estimation based on impulse responses to monetary policy and technology shocks from a VAR estimated on quarterly US data.[1]

3.1. VAR Specification and Identification

Our VAR is determined by the model developed in the previous section and our identification strategy for the structural shocks. For the latter, we follow Galí (1999) and assume that the technology shock is the only shock that has a permanent effect on the level of output per hour. The monetary shock is identified by the restriction that the last variable in the VAR (the funds rate) is Wold-causal for the preceding variables. Our model and identifying assumptions suggest the inclusion of five variables in the VAR: the first difference of log output per hour, price inflation (the first difference of the log of the GDP deflator), the log labor share, the first difference of log hours per person, and the nominal funds rate. Output per hour, the labor share, and hours are the Bureau of Labor Statistics' (BLS) measures for the non-farm business sector. Population is the civilian population age 16 and over. Letting Y_t denote the vector of variables in the VAR, we view the data in the VAR as corresponding, up to constants, to the model variables $Y_t = [\Delta(y_t - l_t),\ \pi_t,\ y_t - l_t - w_t,\ \Delta l_t,\ r_t]'$, where lower-case letters denote logs of the model variables. We estimate the VAR over the sample 1966q2 to 2006q2, with four lags of each variable.

The structural form of the VAR is given by $A_0 Y_t = \text{constant} + A(L)Y_{t-1} + \varepsilon_t$, where Y_t is as defined above. The short-run assumption implies that the last column of the contemporaneous multiplier matrix A_0 has all zeros above the main diagonal. The fifth element of ε_t is identified as the funds rate shock $\varepsilon_{r,t}$ in (7). The long-run identifying restriction of Galí (1999) is that permanent shocks to technology are the only shocks to have a permanent effect on labor productivity. Using this assumption, we identify the first element of ε_t as the technology shock $\varepsilon_{a,t}$ in (4). This implies that the first row of the matrix of long-run (cumulative) effects of ε_t on Y_t, $(I - A(1))^{-1}A_0^{-1}$, consists of zeros except for the first element. Appendix B of ELW provides further details.

The dashed lines in the first column of Figure 1 show the impulse responses to a permanent 1% increase in the level of technology. The dashed-dotted lines are one standard deviation bands around the responses, computed by bootstrap methods.[2] The second column shows the impulse

[1] Rotemberg and Woodford (1997), Amato and Laubach (2003), and Christiano *et al*. (2005) use this estimation strategy. See Justiniano and Preston (forthcoming) and LOWW for examples of the Bayesian approach and Karagedikli *et al*. (forthcoming) for a discussion of alternative estimation methods.

[2] To prevent the standard error bands from diverging over time, we discard draws for which the implied reduced-form VAR is unstable, such as draws for which the largest eigenvalue of the coefficient matrix in the reduced form, written in companion form, exceeds 0.99. In total, about 14% of draws are rejected.

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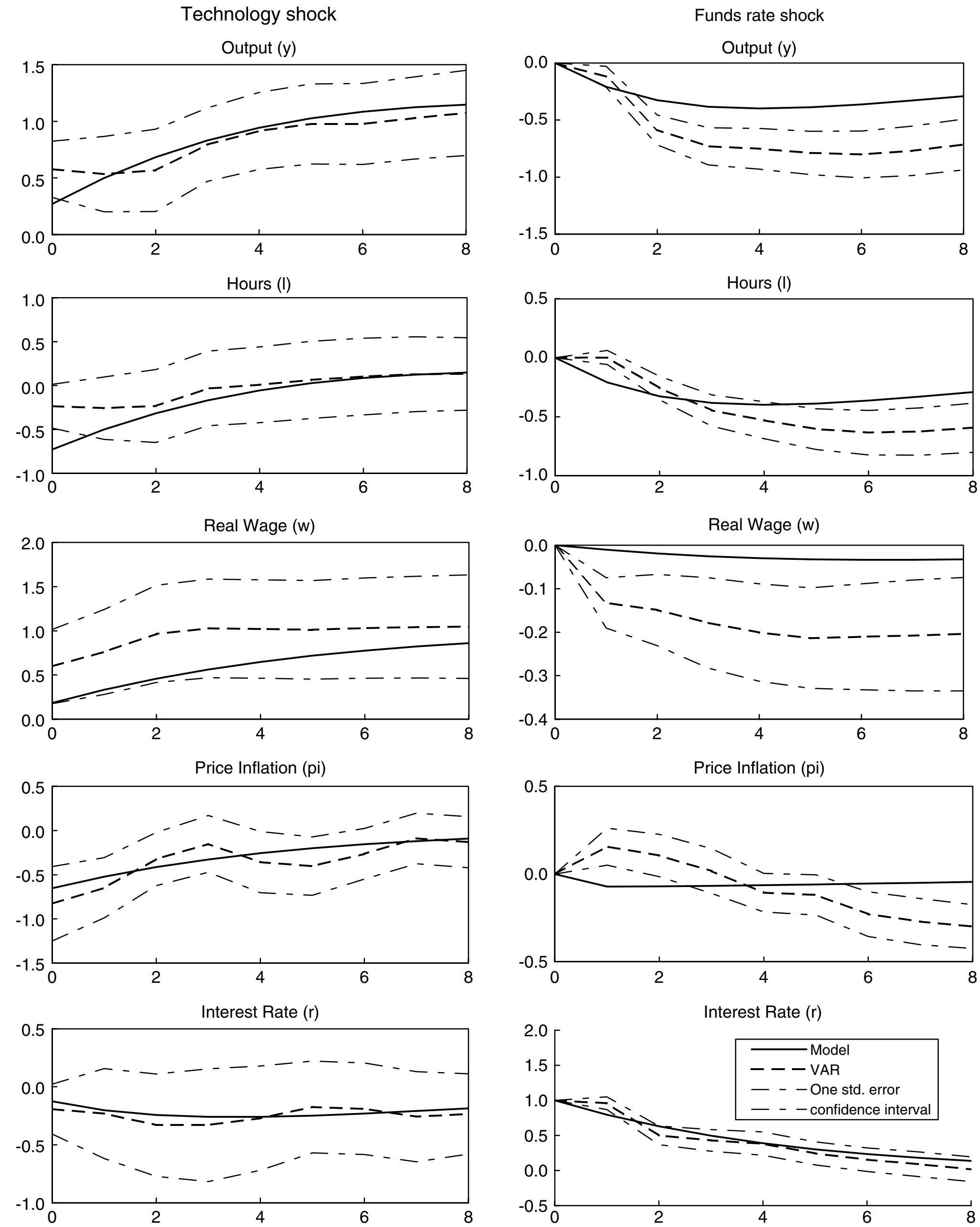

Figure 1. VAR and model responses

responses to a one percentage point funds rate shock. These responses are more precisely estimated than those for the technology shock.

3.2. Model Parameter Estimates

We set the discount factor, β, to 0.9924, normalize the time endowment to unity, and set the steady-state rates of price and wage inflation to zero. The parameters Θ_w and χ_w and Θ_p and χ_p appear only as ratios in the linearized model and are not separately identified. Following LOWW (2005), we set Θ_w and Θ_p to 6 and estimate the coefficients on the driving process in the wage and price Phillips equations, $\kappa_w = \Theta_w/(\chi_w \Pi^2_{w,*})$ and $\kappa_p = \Theta_p/(\chi_p \Pi^2_{p,*})$.

The remaining parameters are estimated by minimizing the squared deviations of the responses of the five variables $[y_t,\ \pi_t,\ w_t,\ l_t,\ r_t]$ implied by our model from their VAR counterparts. To determine the horizon over which to match the IRFs, we apply the information criterion of Hall *et al.* (2007), searching over a minimum horizon of quarters 0 through 4 and a maximum horizon of quarters 0 through 16. This criterion leads us to match the IRFs of the five variables in quarters 0 through 13 following a technology shock in quarter 0, and in quarters 1 through 13 following a funds rate shock (the response in the impact quarter being constrained by the identifying assumption), for a total of 135 moments to match. These moments are weighted inversely proportional to the standard error around the VAR responses, as in Christiano *et al.* (2005). This places more weight on matching the impulse responses to the monetary shock, which, as noted before, are estimated with greater precision than the impulse responses to the technology shock.

In estimating the model using the policy rule (7) we impose the restriction that $\phi_p > 1$ and $\phi_x = 0$.[3] Unrestricted estimation leads to estimates of the indexation parameters γ_w and γ_p very close to or at the upper limit of 1.[4] Because our method of examining parameter uncertainty described below is infeasible when parameters are at boundaries, we fix both of these two parameters at 1. We therefore estimate seven parameters $\{\sigma, \zeta, \eta, \kappa_w, \kappa_p, \phi_r, \phi_p\}$.

The estimated parameters and associated standard errors are shown in the first two columns of Table I. The correlation coefficients of the structural parameter estimates are shown in the final five columns. The covariance matrix of the estimates is computed using the Jacobian matrix from the numerical optimization routine and the empirical estimate of the covariance matrix of the impulse responses from the bootstrap. The estimates of the structural parameters are all statistically significant. The parameters associated with wage and price adjustment costs are estimated with a great deal of precision but the preference parameters, especially σ and ζ, are relatively imprecisely estimated and are very highly correlated with each other, reflecting the difficulty the data have in separating the influences of these parameters. See ELW for further discussion on the issue of parameter identification.

The VAR responses of real wages and inflation differ substantially depending on the source of the shock with rapid responses to technology shocks, and sluggish ones to funds rate shocks. This is a feature that our price and wage specification cannot deliver. Our estimates of κ_w and κ_p imply that wages are slow to adjust to fundamentals, while prices adjust more quickly. These results are driven by the IRFs to the technology shock; indeed, the IRFs to monetary policy shocks alone suggest very gradual price adjustment, consistent with Christiano *et al.* (2005). Despite the greater

[3] Originally we found a slightly negative, but near zero, response to the output gap, perhaps because the model's output gap bears little resemblance to measures used by policymakers. We considered imposing the VAR's estimated rule, as in Rotemberg and Woodford (1997), but encountered convergence problems.

[4] Estimates of γ_w and γ_p are sensitive to the horizon of the IRFs that we match. Matching IRFs of quarters 0 through 4 or 0 through 5 implies γ_w and γ_p estimates close to 0; for longer horizons γ_w and γ_p are at or near 1. The information criterion strongly suggests matching IRFs of quarters 0 through 12 or longer.

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Table I. Parameter estimates

Model parameter	Point estimate	SE	Correlation with				
			σ	η	ζ	κ_w	κ_p
σ	8.209	2.609	1.000	−0.995	0.996	0.716	0.008
η	0.364	0.070		1.000	−0.986	−0.700	−0.019
ζ	1.740	0.179			1.00	0.731	0.019
κ_w	0.006	0.000				1.000	−0.156
κ_p	0.010	0.000					1.000
ϕ_r	0.840	0.001					
ϕ_p	1.000	0.019					

weight placed on matching the more tightly estimated responses of inflation and real wage to the funds rate shock, our model matches better the responses to a technology shock, as shown by the solid lines in Figure 1.

4. WELFARE AND OPTIMAL MONETARY POLICY

In this section we compute the optimal monetary policy responses to technology and preference shocks assuming all model parameters are known. We are admittedly examining a relatively small source of welfare losses in our model. Incorporating other sources of fluctuations would require us to take a stand on the sources of other shocks, which we leave to further research. We assume that the central bank objective is to maximize the unconditional expectation of the welfare of the representative household. We further assume that the central bank has the ability to commit to future policy actions; that is, we examine optimal policy under commitment, as opposed to discretion. We consider only policies that yield a unique rational expectations equilibrium.

4.1. Approximating Household Welfare

We approximate household utility with a second-order Taylor expansion around the deterministic steady state following Rotemberg and Woodford (1997). We denote steady-state values with an asterisk subscript. As shown in Appendix C of ELW, the second-order approximation of the period utility function depends on the squared output gap, the squared quasi-difference of the output gap, the cross-product of the output gap and its quasi-difference, and the squared first difference of the rates of price and wage inflation.

After many steps, the second-order approximation to period utility can be written as

$$\frac{\Xi_{c,t}(C_t - \eta C_{t-1})^{1-\sigma}(\overline{L} - L_{u,t})^{\zeta(1-\sigma)}}{1-\sigma} = \text{T.I.P.} - \mathcal{L} = \text{T.I.P.} - \mathcal{L}_x - \mathcal{L}_p - \mathcal{L}_w$$

where T.I.P refers to terms that are independent of monetary policy and

$$\mathcal{L}_x = (C_* - \eta C_*)^{1-\sigma}(\overline{L} - L_{u,*})^{\zeta(1-\sigma)} \left\{ \frac{1}{2} \cdot \frac{1-\zeta(1-\sigma)}{\zeta} \cdot \left(\frac{1-\beta\eta}{1-\eta}\right)^2 x_t^2 \right.$$

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$$+ \frac{1}{2} \cdot \frac{\sigma}{(1-\eta)^2} \cdot (x_t - \eta x_{t-1})^2$$

$$\left. + (1-\sigma) \cdot \frac{1-\beta\eta}{(1-\eta)^2} \cdot x_t(x_t - \eta x_{t-1}) \right\},$$

$$\mathcal{L}_p = (C_* - \eta C_*)^{1-\sigma}(\overline{L} - L_{u,*})^{\zeta(1-\sigma)} \left\{ \frac{1}{2} \cdot \frac{1-\beta\eta}{1-\eta} \cdot \frac{\Theta_p \Pi_{p,*}}{\kappa_p} \cdot (\pi_{p,t} - \gamma_p \pi_{p,t-1})^2 \right\}, \text{ and}$$

$$\mathcal{L}_w = (C_* - \eta C_*)^{1-\sigma}(\overline{L} - L_{u,*})^{\zeta(1-\sigma)} \left\{ \frac{1}{2} \cdot \frac{1-\beta\eta}{1-\eta} \cdot \frac{\Theta_w \Pi_{w,*}}{\kappa_w} \cdot (\pi_{w,t} - \gamma_w \pi_{w,t-1})^2 \right\}$$

In our welfare calculations, we ignore the T.I.P. terms and report only the terms related to the output gap and price and wage inflation rates. The three elements in $\mathcal{L}_x$ correspond to the period welfare costs associated with output deviating from its natural rate. Due to habit formation, both the level and quasi-difference of the output gap affect welfare. All three preference parameters enter the coefficients of the welfare loss for these terms. The terms in $\mathcal{L}_p$ and $\mathcal{L}_w$ correspond to the welfare loss associated with adjustment costs in changing prices and wages. The coefficients in these terms depend primarily on the model's nominal rigidity parameters; specifically, they are inversely related to the price and wage sensitivity parameters, κ_p and κ_w. The more flexible are prices, the smaller are the welfare costs implied by a given magnitude of inflation fluctuations, and similarly for wages.

Table II reports the implied relative weights on the terms related to the output gap and the first differences of wage and price inflation, where we have normalized the values of the weights by the weight on the price inflation term at the point estimate. The first row reports the sum of the weights on the three terms in the loss associated with the output gap and its quasi-difference. The first column reports the weights computed at the parameter point estimates. The variance in wage inflation has a weight 1.7 times that of price inflation, due to the estimate of κ_w being about 60% that of κ_p. The weights on the variances of the output gap and the quasi-difference of the output gap are notably smaller than that of inflation (due to the high estimated degree of stickiness in wage and price setting) but are higher than typically seen in the literature due to our relatively high estimate of σ.

4.2. Optimal Monetary Policy with no Parameter Uncertainty

In our analysis of optimal monetary policies, it is important to be clear what information the central bank has available in making its decision. We assume the central bank knows the structure of the model. At the time of making its policy decision, the central bank is assumed to observe all past observable data, but not the realization of the current shock. In the case of no parameter

Table II. Relative weights in central bank loss

Weight in loss	Point estimate	Median estimate	70% interval	Mean estimate
ω_x	0.06	0.04	[0.0, 8.7]	8.28
ω_p	1.00	0.92	[0.0, 159.1]	132.45
ω_w	1.71	1.55	[0.0, 270.6]	211.31

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uncertainty, the central bank is able to infer the past values of the natural rates of interest, hours, and output from the observable data.

We first compute the optimal certainty equivalent policy based on the point estimates of the model parameters. The resulting policy and outcomes provide a useful benchmark for the policy rules that we examine. The optimal certainty equivalent policy maximizes the quadratic approximation of welfare, subject to the constraints implied by the linearized model. In computing the welfare loss we assume a discount rate arbitrarily close to zero, so that we are maximizing the unconditional measure of welfare. We compute the fully optimal policy using Lagrangian methods as described in Finan and Tetlow (1999), adapted to take account of assumption of date $t-1$ information in the implementation of monetary policy. The standard deviations of the technology and preference shocks (the only stochastic elements in the model) are set to their corresponding estimated values of 0.64 and 6.17 percentage points, respectively. (See Appendix B of ELW for the calculation of these values.)

The results under the optimal policy are shown in the first column of Table III. The middle portion of the table shows the resulting welfare losses. The first row of this part of the table reports the overall welfare loss, $\mathcal{L}$. Because the units of the welfare loss are difficult to interpret, the next four rows of the table report the welfare losses (and its component parts) in terms of 'consumption-equivalent' units, denoted by $\mathcal{C}$, equal to the percentage point reduction in steady-state consumption (absent fluctuations) that would yield the same welfare loss as implied by fluctuations in the output gap and wage and price inflation rates around their steady-state values. The lower part of the table reports the resulting unconditional standard deviations of the output gap, the first differences of the price and wage inflation rates, and the level of the nominal interest rate.

The 'consumption equivalent' welfare loss is extremely small under the optimal monetary policy with no uncertainty, about 1/200th of 1% of consumption. This reflects the fact that the preference

Table III. Performance of alternative monetary policies

	No uncertainty				Parameter uncertainty			
	Optimal policy	Policy rule coefficients			Optimal policy	Policy rule coefficients		
		Rule 1	Rule 2	Rule 3		Rule 1	Rule 2	Rule 3
r^n				0.84				1.00
x		27.88		0.12		7.02		1.46
π_p		0.01	566.78	391.40		15.39	617.80	480.86
π_w			1000.00	693.78			1000.00	780.89
Welfare losses								
$\mathcal{L}$	717.8	771.1	742.5	719.2	1.98E7	5.20e7	2.20E7	1.99E7
$\mathcal{C}$	0.006	0.007	0.007	0.006	0.007	0.010	0.007	0.007
$\mathcal{C}_x$	0.001	0.002	0.001	0.001	0.001	0.004	0.002	0.002
$\mathcal{C}_p$	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
$\mathcal{C}_w$	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
Standard deviations								
x	0.08	0.10	0.09	0.08	0.09	0.16	0.11	0.10
$\Delta\pi_p$	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
$\Delta\pi_w$	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
r	2.59	2.69	2.38	2.59	2.59	2.22	2.51	2.94

Note: Rule 1 is a version of the standard Taylor rule; rule 2 responds only to wage and price inflation; rule 3 responds to all four variables indicated in the table.

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and technology shocks do not create large tradeoffs between the objectives in the loss. Indeed, were it not for the assumption that policy acts using lagged information, the preference shock would generate *no* welfare loss under optimal policy through its contribution to fluctuations in the output gap and wage and price inflation, while the technology shock would engender only very small welfare losses (reflecting the tradeoff implied by the presence of sticky wages). Under the fully optimal monetary policy, variability in the output gap and the first differences in the rates of wage and price inflation are reduced to nearly zero. The standard deviations of both wage and price inflation are (at annualized rates) about 0.1 percentage point. The optimal policy induces considerable interest rate variability, with the standard deviation of the nominal (annualized) interest rate of over 10 percentage points. This implies that the zero lower bound on nominal interest rate is a relevant concern, but we leave incorporating this constraint to future research.

4.3. Alternative Monetary Policy Rules

We consider three parsimonious monetary policy rules, each of which yields a welfare loss that is very close to the fully optimal policy when parameters are known. The general specification is a Taylor-type policy rule where the nominal interest rate is determined by the lagged values of the central bank estimate of the natural rate of interest and of the output gap ($\hat{r}_t^n$ and $\hat{x}_t$) and the rates of price and wage inflation ($\pi_{p,t}$ and $\pi_{w,t}$):

$$r_t = \pi_{p,t-1} + \phi_{r^n} \hat{r}_{t-1}^n + \phi_x\ \hat{x}_{t-1} + \phi_p\ \pi_{p,t-1} + \phi_w\ \pi_{w,t-1} \tag{8}$$

With known parameters, the central bank estimates of the natural rates are assumed to equal their respective true values. Note that we have assumed that policy responds to the lagged values of these variables, in keeping with our assumption that policy is set using $t-1$ information. We restrict the policy rule coefficient on price inflation to be no smaller than 0.01 and we do not allow any coefficients to exceed 1000.[5]

We consider three policy rules. The first is a version of the standard Taylor rule, where the funds rate is determined by inflation and the output gap. The second rule responds only to wage and price inflation. The third rule is a generalization of the others and is identical to equation (8). This rule is used as a close approximation for the fully optimal rule, but has the advantage that the coefficients are easier to interpret. We compute the optimal coefficients of each rule to maximize unconditional welfare of the representative household using a numerical hill-climber routine. Absent parameter uncertainty, the optimized versions of all three rules yield welfare losses close to that which obtains under the fully optimal policy.

The optimized Taylor rule (rule 1 of Table III) acts like a strict output targeting policy that aims to keep the output gap near zero at all times. This rule has the minimum allowable coefficient on price inflation and a very large coefficient on the output gap. The rule that responds to wage and price inflation (rule 2 of Table III) behaves like a targeting rule that aims to maintain a negative correlation between price and wage inflation, with the latter more tightly controlled. The optimized coefficients exhibit massive responses to wage and price inflation, with the coefficient on wage inflation about 1.76 times larger than for price inflation. This is nearly identical to the ratio of 1.71 of the weights in the objective function of wage to price inflation. The optimized generalized

[5] In the case where this upper bound is a binding constraint, the loss surface is nearly flat in the vicinity of the reported parameter values and increasing the upper bound has only a trivial effect on welfare.

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policy rule (rule 3 of Table III) has a significant response to the natural rate of interest, a modest response to the output gap, and very large responses to the rates of price and wage inflation. This rule behaves much like the rule that targets a combination of wage and price inflation; the ratio of coefficients on wage and price inflation is nearly the same in the two cases. This generalized rule yields a welfare loss that is nearly identical to that under the fully optimal policy.

5. MONETARY POLICY UNDER PARAMETER UNCERTAINTY

We now analyze the performance and robustness of monetary policies under parameter uncertainty where the central bank maximizes expected welfare. The only form of uncertainty that the policymaker is assumed to face is uncertainty regarding model parameters owing to sample variation. We assume that the central bank knows the true model, that the model is estimated using a consistent estimator, and that the central bank is certain that the model and estimation methodology are correct. We abstract from learning and assume that the policymaker's knowledge and uncertainty do not change over time. We assume that private agents know everything, including the central bank's parameter estimates. For a given specification of monetary policy, expected welfare is approximated by numerically integrating the welfare outcomes over a sample drawn from the distribution of the five estimated structural parameters implied by the estimated covariance matrix.

5.1. Natural Rate Uncertainty

We first provide some summary measures of the degree of uncertainty regarding the natural rates of hours, output, and interest owing to parameter uncertainty. In this model the responses of the natural rates to technology and preference shocks depend on three household preference parameters: σ, η, and ζ. For the remainder of the paper, we assume that the distribution of model parameters is jointly normal distributed with mean zero and covariance given by the estimated covariance matrix. We approximate this distribution with a single set of 1000 draws from the estimated covariance matrix, where we truncate the parameter values at the lower ends of their distributions as follows: σ at 0.5, ζ at 0.1, and η at 0.

In general, parameter uncertainty implies uncertainty both about the steady-state values of natural rates as well as their movements over time. For simplicity, we assume that the policymaker, by observing a long time series, is able to estimate the mean level of hours and real interest rates precisely. We assume that the policymaker has no independent knowledge of the time endowment, so perfect knowledge of the mean level of hours has no implications for uncertainty about other preference parameters. We note that under less restrictive assumptions there exist tight links between estimated structural parameters and steady-state values, which affect both model estimation and the analysis of parameter uncertainty. Indeed, Laubach and Williams (2003) find evidence of considerable uncertainty regarding low-frequency components of natural rates of interest and output, suggesting that the assumption that the steady-state levels are known with certainty is untenable in practice.

The responses of natural rates to technology and preference shocks depend on the parameter values describing preferences. We assume that the central bank computes its estimates of natural rates based on the point estimates of the preference parameters and measure natural rate misperceptions as the difference between the natural rate implied by the actual parameter values

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and that implied by the point estimates of the model parameters. Averaging over the 1000 draws from the parameter distribution, the root mean squared deviation of the true natural rate of output and the central bank's estimate (computed using the parameter point estimates) is a modest 0.13 percentage points. The mean first-order autocorrelation of this difference is 0.84. The root mean squared deviation of the true natural rate of interest from the central bank's estimate is a more sizable 1.05 percentage points (at an annual rate), with a mean first-order autocorrelation of 0.35.

5.2. Optimal Monetary Policy under Parameter Uncertainty

To provide a benchmark for policies under uncertainty, we first compute the optimal outcome if the policymaker knew all parameter values and followed the optimal policy in each case. Of course, this outcome (given in the fifth column of Table III) is unattainable, but it provides a benchmark against which to measure the welfare costs of parameter uncertainty.

As can be seen from comparing the first and fifth columns of Table III, the mean welfare loss under the first-best optimal policy is considerably larger than at the parameter point estimates. This is because the mean weights in the welfare loss are much higher than when evaluated at the point estimates (see the final column of Table II). Indeed, as shown by the estimated 70% confidence interval of the loss-function weights in Table II, the variation in the weights implied by parameter uncertainty is enormous. The preference parameters have a large effect on steady-state utility, which affects all loss-function weights and makes them highly correlated. The ratio of the weights, however, varies relatively little over the draws and the consumption-equivalent welfare losses and the variability of key variables are about the same on average as under optimal policy at the parameter point estimates.

We now examine the characteristics and performance of the implementable policy rules introduced in the previous section. We consider the rules that were found to be optimal absent parameter uncertainty and reoptimize their coefficients to minimize the expected welfare loss under parameter uncertainty. As before, in implementing these rules we assume the central bank computes its estimates of natural rates using the parameter point estimates. For each draw from the parameter distribution, the realizations of the natural rates (and all other variables) are generated by the model based on the parameter values drawn.

The optimized Taylor rule assuming no parameter uncertainty does not yield a unique stable rational expectations equilibrium for 17 of the 1000 draws of parameter values drawn from the posterior distribution. The source of the problem is that the response to the output gap is excessively large and this creates instability in the system. The other two policies yield a unique solution for all 1000 draws and deliver mean outcomes that are close to what obtains if the coefficients of the rules are reoptimized under parameter uncertainty.

Relative to the case of no parameter uncertainty, the optimized standard Taylor rule under parameter uncertainty responds far less aggressively to the estimate of the output gap (a coefficient of 7 compared to nearly 28) and more strongly to inflation (a coefficient of over 15 compared to 0.01). This rule yields a unique solution in all 1000 draws from the parameter distribution. The results for this rule are shown in the sixth column of the table. This dramatic change in the characteristics of the optimal Taylor rule reflects the mismeasurement of the natural rate of output under parameter uncertainty. In fact, if the central bank faced parameter uncertainty but somehow knew the true values of the natural rate of output, the optimized Taylor rule would have a very small response to inflation and a very large response to the output gap. However, in the presence of

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natural rate uncertainty, a large response to the output gap generates correspondingly large policy errors. To minimize this source of undesired fluctuations and to offset the effects of the resulting policy errors on inflation, the optimized Taylor rule under parameter uncertainty responds more modestly to the perceived output gap and more strongly to inflation.

Optimized policy rules that respond to wage and price inflation, but not output, are very effective at minimizing the mean welfare loss under parameter uncertainty. The results for this rule are shown in the seventh column of the table. The optimized rule yields a welfare loss close to the first-best allocation, and performs better than the optimized Taylor rule. The coefficients display the same characteristics as in the case of no parameter uncertainty: the coefficient on wage inflation is at its maximum value of 1000, and the ratio of the wage inflation coefficient to the price inflation coefficient is about 1.6, slightly smaller than in the case of no uncertainty. Evidently, responding aggressively to the wage and price inflation rates substitutes for responding to the output gap in this model.

Finally, the generalized rule yields expected welfare nearly the same as the theoretical first-best. Thus, from a Bayesian perspective, knowledge of the exact draw of parameter values has little benefit in terms of expected welfare. This rule responds fully to the natural rate of interest (we impose an upper bound of one on this coefficient), moderately to the output gap, and massively to the rates of wage and price inflation. The results are reported in the final column of the table. Although the response to the output gap is larger than in the case of no parameter uncertainty, the behavior of policy is dominated by the responses to wage and price inflation that are two orders of magnitude larger than the response to the output gap. Interestingly, this rule responds more aggressively to the observable variables than its counterpart derived under no parameter uncertainty. Thus, in this micro-founded DSGE model, parameter uncertainty leads to a more aggressive monetary policy rule, rather than a less responsive one, as in Brainard's (1967) analysis using a traditional model.

6. CONCLUSION

In micro-founded models parameter uncertainty implies joint uncertainty about model dynamics, natural rates, and the welfare costs of fluctuations. This paper has examined optimal policy under parameter uncertainty in a simple model. Our analysis can be extended to models that include additional features of the economy and sources of uncertainty.

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