A BAYESIAN APPROACH TO FOREIGN EXCHANGE FORECASTING

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ABSTRACT

This paper shows how the loss function approach of Bayesian statistics can be used to develop more useful forecasts in foreign exchange markets. These forecasts may be needed for a variety of applications such as speculation in forward markets, economic or accounting hedging, and estimating future costs or revenues. In the classical (i.e., mean squared error) method the same negative values are assigned to both positive and negative forecast errors, and, as a result, an unbiased forecast is seen as being optimal. The Bayesian approach proposed here, on the other hand, explicitly accounts for consequences of positive or negative forecast errors and for the possible presence of a bias in forward exchange rates. As a result, the optimal forecast is usually *not* equal to the conditional expected value of the future spot rate. The approach proposed here can be adopted regardless of the economic model used by forecasters. The International Fisher Effect is employed to illustrate how this paper's model can be used in practice and how its outcome may differ from the mean squared approach. The results show that for the sample period the forecasts based upon this paper's loss function slightly outperformed those based upon the mean squared errors approach.

I. INTRODUCTION

Forecasts of future spot exchange rates are used as inputs in many financial decisions. For instance, to implement and to calculate the expected cost of variance-minimizing and many other hedging strategies, the hedger needs to provide an

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estimate of the future spot rate.[1] A speculator takes open positions in foreign currencies and therefore needs an estimate of the future spot rate. Also, every financial planning method requires estimates of future revenues and expenses; and if these are affected by currency movements, the investor[2] will need forecasts of future spot exchange rates. These optimal forecasts may not be calculated by the final user. There are many forecasting firms that provide their clients with such estimates (See Levich (1980, 1983)).

This paper deals with the question of obtaining optimal forecasts of future spot rates. It is assumed that the investor has already identified a model suited for his/her particular problem (for example, a particular hedging model), but to use the model, he/she needs a forecast of the future spot rate. Furthermore, it is assumed that the investor has already selected an *economic* model that is believed to explain the behavior of the exchange rate.[3] Therefore, the issue addressed in this paper is the criteria that the investor should use in obtaining such optimal forecasts.

Ordinary least square regression combined with a symmetric quadratic loss function is the most common method of forecasting. In this approach the optimal forecast is equal to the conditional expected value of the future spot rate (See Raiffa and Schlaifer (1968) and Greene (1990)). However, depending on the intended application of the forecast, the conditional expected value may or may not be the best estimate.[4] For example, a forecast of the future spot exchange rate may be compared with the current forward rate to determine a speculator's optimal forward position.[5] If the comparison shows that the foreign currency must be sold in the forward market (i.e., forecast is less than the forward), the speculator will profit if and only if the actual future spot rate is less than the forward rate. Clearly, unbiasedness is less important in this case than is the probability of correctly forecasting the position of the future spot rate relative to the current forward rate.[6] Moreover, investors would often welcome profits but would dislike losses at an *increasing* rate (e.g., large losses may lead to bankruptcy). This indicates that a symmetric loss function may not be appropriate.

This paper develops a Bayesian approach to the forecasting of foreign exchange rates.[7] It presents a loss function that accounts for the possible differential impacts of positive and negative forecasts errors on the investor's welfare. Furthermore, the loss function is set up to correct the conditional expected value of the future spot rate for the presence of risk premium in the forward rate. The case of a symmetric quadratic loss function can be handled as a special case of this paper's loss function. Finally, the loss function proposed here can be used to evaluate the performance of foreign exchange forecasting services.[8]

The plan of the paper is as follows. Part II presents the loss function and obtains a closed- form solution for the optimal forecast. Part III uses the International Fisher Effect as an example of how the proposed solution may be applied, and Part IV offers some concluding comments. Some technical aspects of the paper are discussed in appendices A, B, and C.

II. THE MODEL

Let $\tilde{S}$ denote the future value of the foreign currency and let x denote the optimal forecast of $\tilde{S}$. In general, the optimal forecast is selected such that the expected value of a loss function, $L(\tilde{S}, x)$, is minimized.[9] That is,

$$x=arg\,(\min_{x}\ E_s[L(\tilde{S},x)]), \tag{1}$$

where $E_s[\cdot]$ denotes expectation with respect to density function of $\tilde{S}$. It is well known that if the loss function is quadratic (i.e., $L(\tilde{S},x)=(x-\tilde{S})^2$), then the optimal forecast will be equal to the conditional expected value of $\tilde{S}$; i.e., $x=E_s[\tilde{S}]$. This is the classical criterion for determining the optimality of a forecast.

This paper proposes a more general loss function. Let F denote the current forward exchange rate. The following loss function is proposed for obtaining optimal estimates of the future spot rate:

$$L(\tilde{S},x) = g(F-x)(\tilde{S}-F)+b\text{Exp}[a(x-\tilde{S})]-c(x-\tilde{S})-b, \tag{2}$$

where a, b, c, and g are the parameters of the loss function and their values are selected by the investor. Now the investor will choose x such that the expected value of this loss function is minimized.

To explain how this loss function is developed, each term is examined separately. First, consider the first term on the right-hand side of equation (2). The optimal forecast may be used by the investor to determine his/her optimal position in the forward market, and this term is intended to capture the important properties that the estimate must have in this case. To see this, consider the rule that must be followed by the investor when the optimal forecast, x, is greater than the forward rate, F. The investor must buy in the forward market and sell in the future spot market (i.e., when the contract matures). Depending on the value of the actual spot rate,

(a) the investor earns positive cash flow if and only if $\tilde{S} > F$, (3a)

(b) the investor earns negative cash flow if and only if $\tilde{S} < F$. (3b)

Similarly, when the optimal forecast is smaller than the forward rate, the optimal strategy would be to sell in the forward market. Here,

(a) the investor earns positive cash flow if and only if $\tilde{S} < F$, (4a)

(b) the investor earns negative cash flow if and only if $\tilde{S} > F$. (4b)

By combining the above results, it can be seen that in general:

(a) the investor earns positive cash flow if and only if $(F-x)(\tilde{S}-F)<0$ (5a)

(b) the investor earns negative cash flow if and only if $(F-x)(\tilde{S}-F)> 0$ (5b)

Therefore, one aspect of the investor's objective would be to choose a forecast, x, such that the expected value of $(F-x)(\tilde{S}-F)$ is minimized. Notice that the parameter g of the loss functions is intended to measure the significance that the investor attaches to speculation in the forward market; i.e., if $g = 0$, then speculation is not important.

Now consider the last three terms that appear in equation (2); i.e., $b\text{Exp}[a(x-\tilde{S})]-c(x-\tilde{S})-b$. As was suggested in the introduction, the investor's loss function may be asymmetric for many reasons. For instance, the negative effects of losses could increase at an increasing rate because large losses may lead to bankruptcy. These three terms are intended to capture this aspect of the investor's objective. This function is approximately symmetric when a is a very small positive number, $b=a^{-1}$, and $c=1$. The function will be asymmetric if $b \neq a^{-1}$. Three different plots of this function are displayed in Figure 1.

The investor's problem is to find the value of x that minimizes the expected value of the loss function. This requires the investor to know the probability density function of the future spot rate. In practice, however, this has to be estimated. The most common approach to this problem is to develop an *economic model* and then estimate its parameters using historical data. Suppose the following linear *economic model* is developed (e.g., this could be based upon the Purchasing Power Parity, International Fisher effect, or other models of exchange rate determination):

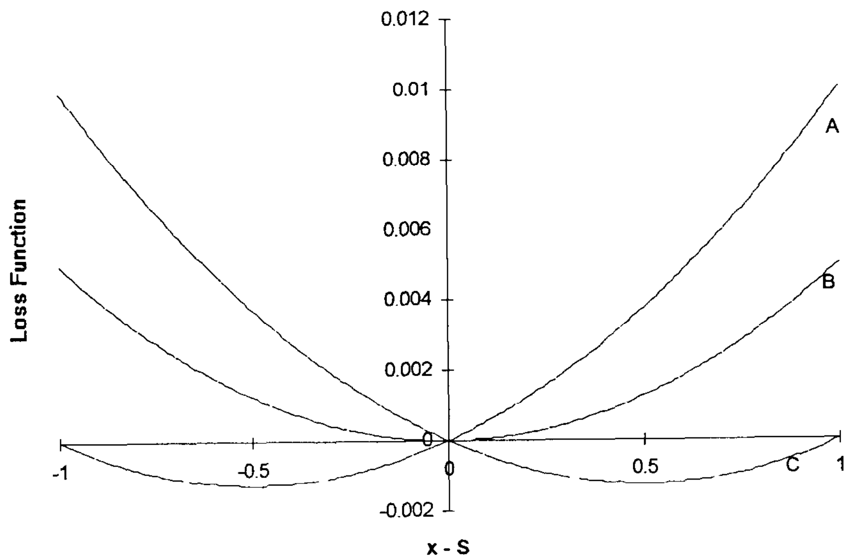
$$\tilde{S} = Y\beta + \tilde{\varepsilon}, \tag{6}$$

where Y is a 1 by k vector of variables that are believed to explain the behavior of $\tilde{S}$, and β is a k by 1 vector of parameters that need to be estimated. The error term, $\tilde{\varepsilon}$, is assumed to be normally distributed with mean zero and variance σ^2. By estimating the parameters of this *economic model*, a density function can be obtained for the foreign exchange rate.

Let S_T and Y_T, respectively, denote a T by 1 and a T by k vectors of T observations on past values of the spot exchange rate and the explanatory variables. Also, let $\hat{\beta}$ denote the ordinary least square estimates of the k parameters of the model. Once the model is estimated, a new set of current values is observed for the explanatory variables denoted by y. Given the vector y, the Bayesian approach can be used to show that the future spot exchange rate has a t-distribution with mean $y\hat{\beta}$ and variance

$$\sigma^2 = \frac{(T-k)\nu^2}{T-k-2}[1 + y(Y_T'Y_T)^{-1}y'], \tag{7}$$

where ν^2 is the ordinary least square estimate of the variance of the error term (See Zellner, 1971).[10] Now that an estimate of the density function of the exchange rate is available, this paper's loss function can be used to obtain an optimal forecast of the future spot rate.



A: $a = 0.01, b = 0.5 + 1/a, c = 1$
B: $a = 0.01, b = 1/a, c = 1$
C: $a = 0.01, b = 0.5 - 1/a, c = 1$

Figure 1

P-1: Given the loss function presented in equation (2), when the sample size T is large, the investor's optimal estimate of the future spot exchange rate will be

$$x = a^{-1}\ln[g(y\hat{\beta} - F) + c] - a^{-1}\ln[\mathrm{ab}] + \mathrm{y}\hat{\beta} - \frac{a}{2}\hat{\sigma}^2. \tag{8}$$

Proof: If the sample size T is large, the t-distribution can be approximated by a normal distribution. Thus, the investor's objective is

$$\min_x E_s\{g(F - x)(\tilde{S} - F) + b\mathrm{Exp}[a(x - \tilde{S})] - c(x - \tilde{S}) - b\}, \tag{9a}$$

subject to $\tilde{S} \sim N(y\hat{\beta}, \hat{\sigma}^2)$. The first-order-condition for this optimization is

$$E_S\{a \times b \times \mathrm{Exp}[a(x - \tilde{S})] - c - g(\tilde{S} - F)\} = 0. \tag{9b}$$

Noting that $\tilde{S}$ is normally distributed, the expectation of the left-hand side can be obtained. The result will be[11]

$$a \times b \times \mathrm{Exp}\left[a(x - y\hat{\beta}) + \frac{a^2}{2}\hat{\sigma}^2\right] - c - g(y\hat{\beta} - F) = 0, \tag{9c}$$

and solving this expression for x, equation (8) is obtained.

The optimal forecast presented here has many appealing properties. First, suppose a is a very small positive number, $ab = c = 1$, and $g = 0$. If one substitutes these values in equation (8), it can be seen that the optimal forecast, x, is approximately equal to $y\hat{\beta}$, which is the optimal forecast when a quadratic loss function is used. In this case, our optimal forecast would coincide with the conditional expected value of $\tilde{S}$.

Second, consider the case where $c = 1$, $g = 0$, and ab is positive but slightly different from unity. Then a bias equal to $-a^{-1}\ln[ab] - a\hat{\sigma}^2/2$ is added to the conditional expected value of $\tilde{S}$. For instance, if $a > 0$ and $ab > 1$, then the conditional expected value of $\tilde{S}$ is reduced. Though in this case the forecast is no longer an unbiased estimate of the future spot rate, it will be optimal for an investor who wants to reduce the probability of overestimating the future spot rate. Also, as can be seen, the bias is related to $\hat{\sigma}^2$, which indicates that the added bias will be larger when the OLS regression has a lower explanatory power. On the other hand, if the investor wants to reduce the probability of underestimating the future spot rate, then he/she could choose the parameters such that $a < 0$ and $ab > 1$.

Third, consider the case where $ab = c = 1$ and a is a very small positive number and $g \neq 0$. Then the optimal forecast is[12]

$$x \approx a^{-1}g(y\hat{\beta} - F) + y\hat{\beta}. \tag{10}$$

Since g is *not* set equal to zero, this forecast is intended to be used as a guide in deciding the investor's optimal position in the forward market. As a result, a bias representing the expected return from forward contracts, $(y\hat{\beta}-F)$, and the importance of the forward speculation component in the loss function, $a^{-1}g$, is added to the optimal forecast that would be obtained under a quadratic loss function. In this case, however, the investor is not particularly interested in increasing or decreasing the probability of under- or over-estimating the future spot rate. As can be seen, this paper's loss function is sufficiently general to be applied to a variety of cases and yet it is simple enough to provide analytically tractable results. In the next section, it is shown how this loss function may be used in practice.

III. AN EXAMPLE

As can be seen from equation (8), to implement the Bayesian approach of this paper, one needs estimates of the first two moments of the probability distribution of the future spot exchange rate. In general, these moments are not known and thus have to be estimated using historical data. This section employs the International Fisher Effect (IFE) as the economic model that is used to estimate the probability distribution of the future spot rate. The IFE is chosen as the economic model for its simplicity and its explanatory power. For a discussion of the IFE see Solnik (1991).[13]

The international Fisher Effect can be written as

$$\tilde{S}_{t+1} = \beta_0 + \beta_1 Y_t + \tilde{\varepsilon}_{t+1}, \tag{11}$$

where β_0 and β_1 are parameters of the model and

$$Y_t = S_t \times \frac{1 + i_{1t}}{1 + i_{2t}}. \tag{12}$$

Table 1
Regression of the Spot U.S. Dollar-U.K. Pound Exchange Rate Against Interest Rates Differentials[a]

Estimation Period	$\hat{\beta}_1$	$\hat{\beta}_2$	$v^2 \times 100^b$	$\hat{\sigma}^2\ x100^c$	R^2
Jan 80–Dec 90	0.050018	0.971491	0.356177	0.3661556	0.960
Feb 80–Jan 91	0.053320	0.969233	0.356549	0.3671227	0.959
Mar 80–Feb 91	0.050125	0.970830	0.369366	0.3795773	0.956
Apr 80–Mar 91	0.057046	0.966137	0.360853	0.3693581	0.956
May 80 –Apr 91	0.065706	0.960468	0.351667	0.3598954	0.956
June 80 –May 91	0.072560	0.955689	0.351245	0.3594573	0.953
July 80–June 91	0.077530	0.952863	0.354185	0.3625360	0.951
Aug 80–July 91	0.087861	0.946207	0.347151	0.3552629	0.950
Sep 80–Aug 91	0.094313	0.942448	0.350255	0.3584376	0.945
Oct 80–Sep 91	0.107703	0.933876	0.342353	0.3507195	0.942
Nov 80–Oct 91	0.103299	0.936850	0.342907	0.3512733	0.938
Dec 80–Nov 91	0.111869	0.931851	0.350358	0.3592173	0.932

Notes: [a]This table presents the results of regressing the spot U.S. Dollar-U.K. Pound exchange rate against the interest rates differential of U.S. Dollar and U.K. Pound. Given the covered interest rate parity, the explanatory variable is equal to the forward rate; i.e.,

$$\tilde{S}_{t+1} = \beta_1 + \beta_2 Y_t + \tilde{\varepsilon}_t,$$

where $Y_t = S_t(1+i_{US,t})/(1+i_{UK,t})$ also represents the current forward rate.

[b]v^2 is the OLS estimate of the variance of $\tilde{\varepsilon}$.

[c]$\sigma^2 = \frac{(T-k)v^2}{T-k-2}\left[1 + y\left(Y_T'Y_T\right)^{-1} y'\right].$

Table 2
Bayesian Forecasts of the U.S. Dollar-U.K. Pound Exchange Rate

Date	*Actual Spot Rate*	*Prev. Month's Forward Rate*	*Cond. Expected Value*[a]	*Bayesian Forecast -1*[b]	*Bayesian Forecast -2*[c]	*Bayesian Forecast -3*[d]
Jan 91	1.9655	1.9174	1.9139	1.7590	2.0774*	1.8922
Feb 91	1.9085	1.9547	1.9444*	1.7432*	2.4243	1.8911*
Mar 91	1.7365	1.8984	1.8934*	1.6713*	2.1242	1.8650*
Apr 91	1.7085	1.7263	1.7266	1.6992*	1.7659	1.7173*
May 91	1.6995	1.6997	1.6991*	1.6766*	1.7303	1.6908*
June 91	1.6180	1.6915	1.6896*	1.6093*	1.7787	1.6755*
July 91	1.6845	1.6104	1.6128*	1.7175*	1.5168	1.6172*
Aug 91	1.6775	1.6769	1.6752	1.6036	1.7555*	1.6620
Sep 91	1.7490	1.6702	1.6686	1.6022	1.7438*	1.6559
Oct 91	1.7393	1.7409	1.7350*	1.4734*	2.0053	1.7028*
Nov 91	1.7630	1.7315	1.7258	1.4713	1.9889*	1.6943
Dec 91	1.8695	1.7549	1.7473	1.4081	2.0952*	1.7073

Notes: *A positive return would have been earned if these figures had been used to determine optimal position in the forward market.

[a]The conditional expected value is calculated using the OLS regression results reported in Table 1. For example, January 1991 value is equal to, $\hat{\beta}_{1t} + \hat{\beta}_{2t} Y_t$, where t = Dec 90.

[b]The Bayesian forecasts were obtained by substituting the results of Table 1 into equation (8). That is, January 1991 values are equal to

$$x = a^{-1}\ln[g(Z_t - F) + c] - a^{-1}\ln[ab] + Z_t - \tfrac{a}{2}\sigma_b^2.$$

where $Z_t = \hat{\beta}_{1t} + \hat{\beta}_{2t} Y_t$, and t = Dec 90. The Bayesian forecasts of this column were obtained assuming that a = 0.1, b = 10, c = 1, and g = 10. These represent a set values for an investor who wants to use these results to speculate in the forward market and who is not particularly concerned with increasing or decreasing the probability of underestimating the future spot rate.

[c]These were obtained assuming that a = -0.1, b = -10.02, c = 1, and g = 10. They represent a set of values for an investor who wants to use these results to speculate in the forward market, but also wants to reduce the probability of underestimating the future spot rate.

[d]These were obtained assuming that a = 1, b = 1.01, c = 1, and g = 10. They represent a set of values for an investor who is not particularly interested in speculating in the forward market, but wants to reduce the probability of overestimating the future spot rate.

Here, i_1 and i_2 denote short-term interest rates of the two countries. This model is applied to the case of U.S. Dollar-U.K. Pound exchange rate. According to the IFE, β_0 is expected to equal zero and β_1 is expected to equal 1.[14] Notice that it is not assumed that the IFE holds (i.e., the forward rate is an unbiased estimate of the future spot rate). The estimated values of β_0 and β_1 are used in the decision process and if the presence of a risk premium is reflected in the estimated values of these two parameters, then the final decision will reflect that.

Monthly nominal exchange rates, one-month forward rates, and monthly Eurocurrency rates for U.S. Dollar and U.K. Pound denominated deposits for the period of January 1980 to January 1992 were obtained from *Data Resources Incorporated.* Initially, the model is estimated using the first 132 observations covering the period January 1980 to December 1990. The estimated parameters are then used in Table 2 to produce four forecasts for January 1991—one corresponding to the classical mean squared errors approach and the other three based upon various forms of this paper's loss function. Next, the model is re-estimated using the data covering February 1980 to January 1991 (i.e., using the next 132 observations) and then used these estimates to produce four new forecasts for February 1991. This procedure was repeated 12 times to produce 12 sets of forecasts for the calendar year of 1991. The results of these regressions are summarized in Table 1. As expected, the R^2 is high for each of the 12 regressions (between 0.93 and 0.96), and the coefficients are not significantly different from their theoretical values.

Table 2 presents the current spot rate, the previous month's forward rate, the conditional expected value of current spot rate, given the information available up to the previous month, and the last month's Bayesian forecasts of the current exchange rate. The conditional expected values can be directly obtained from Table 1 through the OLS regression (i.e., they are equal to $\hat{\beta}_1 + \hat{\beta}_2 Y_t$). The Bayesian forecasts are calculated by substituting the results of Table 1 in equation (8) using three different sets of numerical values for a, b, c, and g. The forecasts appearing in "Bayesian Forecasts-1" column were obtained by assuming that a = 0.1, b = 10, c = 1, and g = 10. These forecasts would be optimal for an investor who wants to use them to speculate in the forward market and who is not particularly concerned with increasing or decreasing the probability of under- or over-estimating the future spot rate (i.e., the investor has an approximately symmetric loss function).

The forecasts appearing in "Bayesian Forecasts-2" column were obtained assuming that a = -0.1, b = -10.02, c = 1, and g = 10. These would be optimal for an investor who wants to use them to speculate in the forward market and at the same time wants to reduce the probability of underestimating the future spot rate.

Finally, the forecasts appearing in "Bayesian Forecasts-3" column were obtained assuming that a = 1, b = 1.01, c = 1, and g = 10. These would be optimal for an investor who is less interested in speculating in the forward market than in reducing the probability of overestimating the future spot rate.

It appears from Table 2 that the conditional expected values are more accurate than the estimates appearing in "Bayesian Forecasts-1" and "Bayesian Forecasts-2" columns. However, as discussed in the previous section, accuracy (i.e., unbiasedness) is not the most critical property of the forecast—especially when the outcome of the model is to be used for speculation in the forward market. For example, Table 2 shows that if an investor were to use the "Bayesian Forecasts-1" to determine his/her strategy in the forward market, then he/she would have earned a positive cash flow in 7 of the 12 months covered in this example. On the other hand, if the investor had used the conditional expected value to determine his/her optimal strategy, he/she would have earned a positive cash flow in 6 of the 12 months.[15] If the investor had used the numerical values of "Bayesian Forecasts-2" he/she would have made money only in 5 of the 12 months studied here. This investor, at the same time, would have reduced the probability of *underestimating* the future spot rate. As was mentioned in the introduction, this would be the optimal forecasting policy when losses from underestimating the future spot rate are to be minimized because they may lead to bankruptcy. These examples demonstrate the major advantage of the Bayesian forecast: that the parameters of the loss function can be adjusted depending on the potential application of the forecast.

IV. CONCLUSION

This paper shows how the Bayesian approach can be used for forecasting foreign exchange rates. The paper proposes a flexible loss function that emphasizes two important properties of foreign exchange forecasts. If unbiasedness is crucial, the loss function can generate a forecast identical to the conditional expected value of the future spot rate. On the other hand, if it is important to correctly forecast the position of the future spot rate relative to the current forward rate, the loss function will add a bias to the conditional expected value of the future spot rate. The bias will be a function of the risk premium in the forward rate and the significance that the investor attaches to forward market speculation.

This paper's Bayesian loss function can be adopted regardless of the economic model used by the forecaster. The International Fisher Effect is employed to demonstrate this point. Using this simple economic model, four different forecasts were produced: one based upon the classical mean squared errors and three based upon the loss function discussed in this paper. During the sample period, the Bayesian forecasts slightly outperformed the classical forecast.

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APPENDIX A. A SIMPLE HEDGING MODEL

This appendix presents a simple hedging model based on the work of Anderson and Danthine (1983), Howard and D'Antonio (1984), Adler and Detemple (1988), Duffie (1989), Duffie and Richardson (1991) and others.

Let V denote the current known value of an asset. The end of period value of this asset will be $V\tilde{R}$ where $\tilde{R}$ is one plus the random rate of return on the asset, which is assumed to be normally distributed. The investor will choose a hedge ratio h such that the expected utility of the end-of-period wealth is maximized. That is,

$$\max_{h} E[U(V\tilde{R} + hV(\tilde{S} - F))]. \tag{A1}$$

In equation (A1), $U(\cdot)$ is the investor's utility function, which is assumed to be strictly concave, while the end-of-period wealth is defined as sum of the value of the risky investment, $V\tilde{R}$, and any gains or losses from holding a position of the size hV in forward exchange contracts.

The first order condition is

$$E[U'(V\tilde{R} + hV(\tilde{S} - F)) \times (\tilde{S} - F)] = 0, \tag{A2}$$

where the prime is used to denote partial derivative. Using the definition of covariance, this equation can be written as

$$EU'(\cdot) \times E[\tilde{S} - F] + Cov[U'(\cdot), \tilde{S}] = 0. \tag{A3}$$

Since $\tilde{R}$ and $\tilde{S}$ are assumed to be normal random variables, Rubinstein's lemma can be used to expand the covariance term. The result is[16]

$$E[U'(\cdot)] \times E[\tilde{S} - F] + E[U''(\cdot)V](Cov[\tilde{S}, \tilde{R}] + hCov[\tilde{S}, \tilde{S}]) = 0. \tag{A4}$$

Finally, the above equation can be solved for h, and if T denotes the investor's measure of relative risk tolerance,

$$h = T\frac{E[\tilde{S}] - F}{Var[\tilde{S}]} - \frac{Cov[\tilde{R}, \tilde{S}]}{Var[\tilde{S}]}. \tag{A5}$$

This hedging ratio is fairly general. For example, if $T=0$, then h will be the same as the optimal hedging ratio when the investor's objective is to minimize variance. On the other hand, if $U(\cdot)$ is an exponential function, then $t=\alpha/V$, where α is the measure of absolute risk tolerance. Finally, if it is assumed that $U(w)=(w-w_0r)/Std[w]$, where w is the end of period value of the portfolio, w_0 is its current value, and r is the risk-free rate, then Howard and D'Antonio (1984) results are obtained. The results of other hedging criteria can be obtained by making suitable assumptions about the shape of $U(\cdot)$ (See Duffie & Richardson, 1991).

This paper's objective in presenting this example was not to discuss optimal hedging strategies but rather to show how the optimal forecast developed in this paper can be used in carrying out a hedging strategy. In this case, the optimal forecast will replace $E[\tilde{S}]$ in equation (A5).

APPENDIX B. CONDITIONS REQUIRED BY WEST (1988)

As was mentioned in the text, the empirical evidence provided here is not meant as a proof that this paper's loss function provides superior forecasts of future spot rates.[17] The empirical example is provided to show how the results of this paper can be used in practice. Furthermore, as has been mentioned, the procedure described in this paper can be used to supplement any econometric model. Thus, the International Fisher Effect used in the empirical section is selected to serve as an example. However, we need to know that the limited evidence presented here is not adversely affected by certain econometric problems.

It is well known that the time series of most exchange rates have unit roots (i.e., they are nonstationary time series in their levels; e.g., see Meese and Singleton (1982)). This could make the results of running regression at levels meaningless. However, West (1988) has shown that even when $\tilde{S}_t$ and $\tilde{Y}_t$ are nonstationary one can proceed to use the OLS approach to run the regression

$$\tilde{S}_{t+1} = \beta_0 + \beta_1 \tilde{Y}_t + \tilde{\varepsilon}_{t+1}, \tag{B1}$$

if two conditions are satisfied. First, both processes must possess unit roots in their levels and be stationary in their first difference. Second, the unconditional mean of $\tilde{Y}_t$ must be non-zero.

Here, Dickey and Fuller (1979) approach is used to test for the presence of unit roots. For example, to test for unit roots in $\tilde{S}_t$, the following regression is run:

$$\Delta\tilde{S}_t = \alpha_0 + \alpha_1 t + \alpha_2 \tilde{S}_{t-1} + \tilde{u}_t, \tag{B2}$$

The test fails to reject the presence of unit roots if α_2 is not significantly different from zero.[18]

Using data from January 1980 to December 1991, the above time series were tested for the presence of unit roots. For the time series of $Y_t = S_t(1+i_{1t})/(1+i_{2t})$ the estimate of α_2 is -0.003022 with a p-value of 0.95998.[19] Thus, one cannot reject the null hypothesis that Y_t has unit roots. The estimate of the unconditional mean of Y_t was significantly different from zero and equal to 1.69. For the time series of S_{t+1} the estimate of α_2 was 0.00256 with the p-value of 0.97339, indicating that S_{t+1} has unit roots as well.

These results clearly indicate that the conditions specified in West (1988) are satisfied. Thus the estimates reported in the empirical section of the paper are consistent and inferences can be made based upon these estimates.

APPENDIX C. THE EFFECTS OF PRIOR BELIEFS

While estimating the parameters of the economic model, one may use prior beliefs about their values. For example, if an IFE based economic model is used, then one's prior belief about β would indicate that its expected value is $\{0,1\}$. This section briefly discusses how such prior beliefs can be incorporated into the estimation and forecasting process.

Consider equation (7) of this paper.

$$\tilde{S}_T = Y_T\beta + \tilde{\varepsilon}_T, \tag{C1}$$

where $\tilde{S}_T$ is a T by 1 vector, Y_T is a T by k matrix, β is a k by 1 vector, and $\tilde{\varepsilon}_T$, is T by 1 vector of normally distributed error terms with mean zero and variance σ^2. Suppose the investor's prior experience leads him/her to believe that β has a normal distribution with mean Θ and variance Φ, while σ^2 has a gamma distribution with parameters λ and α. That is,

$$\Pr(\beta|\sigma) \propto \sigma^{-k}\mathrm{Exp}\left(-\frac{1}{2\sigma^2}(\beta-\theta)'\Phi^{-1}(\beta-\theta)\right), \tag{C2}$$

$$\Pr(\sigma) \propto \sigma^{d+1}\mathrm{Exp}\left(-\frac{h\lambda^2}{2\sigma^2}\right), \tag{C3}$$

where the degree of freedom $h=T\text{-}k$. Given the observation vectors S_T and Y_T, define

$$\begin{aligned}
\beta_a &= (Y_T'Y_T)^{-1}Y_T'S_T, \\
\Phi_a &= (Y_T'Y_T)^{-1}, \\
v_a^2 &= \frac{1}{h}(S_T - Y_T\beta_a)'(S_T - Y_T\beta_a)
\end{aligned}$$

The posterior probability distribution of (β,σ) will be normal-gamma with the parameters (Zellner, 1971; Greene, 1990; Varian, 1974).

$$\begin{aligned}
\Phi_b^{-1} &= \Phi_a^{-1} + \Phi^{-1}, \\
\beta_b &= \Phi_b(\Phi_a^{-1}\beta_a + \Phi^{-1}\theta), \\
v_b^2 &= \frac{1}{T+h}[(hk + \beta_a'\Phi_a^{-1}\beta_a) + (S_T'S_T - \beta_b'\Phi_b^{-1}\beta_b)].
\end{aligned}$$

Notice that given a new set of observed values for y, the distribution of the future spot rate will be a t-distribution. Similar to the case discussed in the text, when the number of observations, T, is large, the t-distribution can be approximated by a normal distribution. Given the loss function developed in the paper, the optimal forecast, x, will be

$$x = a^{-1}\ln[g(y\beta_b - F) + c] - a^{-1}\ln[ab] + y\beta_b - \frac{a}{2}\sigma_b^2, \tag{C4}$$

where

$$\sigma_b^2 = \frac{(T+h)v_b^2}{T+h-2}[1 + y(Y_T'Y_T)^{-1}y'].$$

NOTES

1. For a discussion of various hedging strategies see Anderson and Danthine (1983), Howard and D'Antonio (1984), Adler and Detemple (1988), Duffie

(1989), Shapiro (1992), and Hammer (1992); also see Appendix A for an example of a hedging strategy that requires a forecast of the future spot rate as one of its inputs.

2. The investor may or may not be the person who forecasts the future spot rate. This term is used to refer to both the person who uses the forecast and the person who calculates the forecast.
3. This could be a well-known model such the Purchasing Power Parity, the Monetary approach, or the International Fisher relationship, or a proprietary model.
4. Levich (1980, 1983), Hansen and Hodrick (1980), Dufey and Mirus (1971), Bilson (1983), and Cumby and Modest (1987) have pointed out potential problems associated with evaluating the performance of a foreign exchange forecast only in terms of unbiasedness and forecasting errors.
5. The results of this paper will apply to speculation in forward and futures markets. Thus, throughout the paper the term *futures rate* can be substituted for the term *forward rate.*
6. Consider this example. Suppose the forward price of a foreign currency is \$1.00, and alternative forecasts of the future spot price are (a) \$1.1; and (b) \$0.95. The actual future spot rate turns out to be \$1.01. Though the second forecast is more accurate, it gives the wrong "sell" signal.
7. For a general discussion of Bayesian statistics see Zellner (1971). For some discussions of various applications of Bayesian approach see Fienberg and Zellner (1974) and Bawa, Brown, and Klein (1979). Jorion (1985) shows how the Bayesian approach can be used to obtain more robust estimates of the variance-covariance matrix of asset returns. This estimate can then be used to determine the composition of the *efficient frontier* in the international context.
8. The issue of optimal properties of forecasts of foreign exchange rates has been mostly discussed in the context of the evaluation of forecasting services (e.g., see Levich, 1980, 1983; Bilson, 1983).
9. The consepts of a loss function is very similar to the concept of a utility function. In a sense this paper discusses the application of a more general utility function instead of the quadratic function which is the most widely used.
10. Here, it is assumed that the investor has diffused prior beliefs about β and α It is shown in Appendix C that if (β,σ) have a normal-gamma distribution, then the posterior distribution of β will still be normal and the future spot rate will have a t-distribution. Thus, prior beliefs can be easily incorporated into this paper's results.
11. Note that if $u \sim N(\mu,\delta^2)$, then $E[\mathrm{Exp}[a\tilde{u} + b]] = \mathrm{Exp}\left[a\tilde{\mu} + b + \frac{1}{2}a^2\delta^2\right]$
12. Note that in this case we use the common procedure that $\ln(1+z) \approx z$ when z is small.
13. The evidence presented below is not meant as a proof that this paper's loss function approach generally provides a "better" forecast than the classical mean squared errors approach. It is already known that different types of forecasts are needed for different applications. That is, from the Bayesian sta-

tistics point of view there is no such a thing as the universally optimal estimate. The optimal forecast will depend upon the values the forecaster selects for the parameters of the loss function, and this is the major advantage of using the Bayesian approach. It must be noted, however, that the econometric model is estimated only *once*. Only the way that the estimated parameters of the economic model are adjusted will depend on the values assigned to the variables of the loss function.

14. It is well known that time series of most exchange rates have unit roots (i.e., they are nonstationary time series). This could make the results of running the above regression meaningless. However, as shown by West (1988), if both $\tilde{S}_t$ and $\tilde{Y}_t$ possess unit roots and the unconditional mean of $\tilde{Y}_t$ is non-zero, then standard OLS method can be used to obtain consistent estimates and make inferences regarding the values of coefficients. The time series of $\tilde{S}_t$ and $\tilde{Y}_t$ have been examined and they satisfy the above conditions. See Appendix B.
15. Using the Bayesian forecast, the investor would have earned a positive return for the months of February, March, April, May, June, July, and October. Using the conditional expected value, the investors would not have taken the right position in the forward market in April.
16. According to this lema if x, y, and z are normal random variables then

$$Cov[f(x,y),z]=E[\partial f/\partial x]Cov[x,z]+E[\partial f/\partial y]Cov[y,z].$$

17. In fact, it is impossible to provide such a proof because the optimality of an estimate depends upon the loss function being used. Thus, the question is reduced to asking which loss function is better, and this clearly depends upon the individual and the prevailing circumstances. This is rather similar to asking which utility function is better. Of course, there is no answer.
18. One can run the above regression without the time trend. Both versions were estimated with the results being virtually identical. Also the above equation was estimated using annual data, and the results were not significantly different.
19. The p-value represents the probability of a t-distributed random variable with 143 degrees of freedom to exceed the observed statistics; i.e., it is the lowest significance level for which the null will be rejected.

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