Journal of Economic Behavior & Organization
Vol. 55 (2004) 223–236

JOURNAL OF
Economic Behavior
& Organization

www.elsevier.com/locate/econbase

On the microeconomics of specialization

Dimitrios Diamantaras[a], Robert P. Gilles[b,*]

[a] *Department of Economics, Temple University, Philadelphia, PA 19122, USA*
[b] *Department of Economics, Virginia Polytechnic Institute and State University (0316), Blacksburg, VA 24061, USA*

Received 16 March 2003; accepted 15 July 2003
Available online 31 July 2004

Abstract

We consider individual consumer–producers who operate within a perfectly competitive market economy with transaction costs, presenting several propositions that characterize the optimal production and consumption plans of such a consumer–producer. First, we show that under rather sparse conditions on production technologies and consumer preferences, there exists a solution to the consumer–producer optimization problem if transaction costs are asymptotically high. Second, under strengthened properties on consumer preferences, there exists such a solution, even in the absence of transaction costs. Third, we discuss the conditions under which a consumer–producer specializes to different degrees. These results generalize the existing results in the literature.

Keywords: Consumer–producer; Specialization; Division of labor; Increasing returns

1. Introduction

A classic question in economics concerns the endogenous division of labor (e.g., Smith, 1776). The standard neoclassical framework has avoided this problem by introducing an exogenous dichotomy between consumers and producers. Recently, there has been a renewed interest in this issue, in particular through the study of markets with transaction costs and consumer–producer agents. Here, a *consumer–producer* is an individual who produces,

* Corresponding author. Tel.: +1 540 231 4069; fax: +1 540 231 5097.
E-mail addresses: dimitris@temple.edu (D. Diamantaras); rgilles@vt.edu (R.P. Gilles)

doi:10.1016/j.jebo.2003.07.007

trades, and consumes goods in a perfectly competitive market economy: this individual is subject to the hypothesis of price-taking behavior. In this paper, we provide general conditions under which consumer–producers configure optimally their economic activities of consumption, production and trade. This forms a foundation for the endogenous emergence of a social division of labor.

The main idea in our model is based on the Smith–Young approach to the relationship of specialization, the social division of labor, and increasing returns to scale (Smith, 1776; Young, 1928). Smith argued that the social division of labor is limited by the extent of the market so that the benefits of specialization to an individual are determined largely by the existing social division of labor in the economy. (This is also known as the *Smith Theorem*.) Young introduced a synergetic argument by stating that the extent of the market also depends upon the level of social division of labor. Thus, the presence of increasing returns to scale leads to specialization and further social division of labor. In turn, a high level of social division of labor leads to increasing economies of specialization that form further incentives to specialize and deepen the social division of labor.

As mentioned above, the main tool in our model is the notion of the consumer–producer. In the standard neoclassical, general equilibrium treatments (Debreu, 1959; McKenzie, 1959), consumers and producers are separate decision-making entities. Extensions of this approach to incorporate increasing returns are possible (see Villar, 2000, and the references therein), but they do not address or formalize the Smith–Young specialization hypothesis.

In a seminal contribution, Rader (1964) studied a general equilibrium model with consumer–producers, endowing each individual with a very general production set, an initial vector of commodities, and a preference relation. He proved the existence of competitive equilibrium and the two fundamental welfare theorems in his model. However, he did not address the Smith–Young specialization hypothesis.

Yang and his research group linked the notion of the consumer–producer to the Smith–Young specialization hypothesis (see Yang, 2001, for a survey and further references). In this approach, the “New Classical” framework, Yang introduces increasing returns to labor in each individual’s production set, which leads to specialization in trade and production. Following the discussion of Yang and Ng (1993) which used specific functional forms, Wen (1998, 2000) discussed more general conditions under which specialization occurs. Subsequently, Yao (2002) has refined and extended her results. In this paper, we are able to weaken these conditions even further.

In Gilles et al. (2003) and Gilles and Diamantaras (2003), we have discussed a general equilibrium model that combines consumer–producers with social production and transaction costs. In this work, we view the configuration of production as a collective decision. We have shown existence of equilibrium and the two fundamental welfare theorems. In particular, Gilles and Diamantaras allow for the endogenous determination of the set of tradeable goods, thus making the home production of non-tradeable goods vital for individuals’ consumption.

In the present paper, we do not address the formulation of a general equilibrium theory; we only report results on the fundamental properties of the behavior of an individual price-taking consumer–producer. Developing these basic results into a fully developed general equilibrium theory can be done along the lines of Rader (1964) and/or

Gilles and Diamantaras (2003). Sun et al. (2004) have developed such a model along the lines of Rader.

We view the endogenous division of labor as resulting from two fundamental characteristics of the primitives of the model of the consumer–producer. First, there are increasing returns to scale related to the use of production technologies by consumer–producers. Second, transaction costs related to the use of the market mechanism limit the use of market contracts. The combination of these two fundamental characteristics can be expected to lead to the desired specialization of individuals in trading as well as production activities and, thus, to a social division of labor in the economy. Ideally, due to increasing returns to scale, individual consumer–producers are expected to specialize their production activities by producing only one commodity to be sold on the market. Furthermore, intuitively, transaction costs make it unprofitable for individuals to buy and sell the same commodity on the market simultaneously. The combination of these two conjectures leads us to the *endogenous establishment of a complete division of labor, in production as well as trade*. Our analysis in this paper is aimed at the question whether these conjectures hold for the formally described decision environment.

In this paper, we present an exhaustive analysis and characterization of the solutions to the consumer–producer's utility maximization problem. The existing literature does not make clear exactly which results are valid under what conditions on the primitives of the model. This paper intends to fill this gap, presenting some generalized results that characterize optimal consumption–production plans.

Our most important contribution is to study the problem methodically and reach the most general conditions known under which the problem has a solution that involves specialization. First of all, we discuss the existence of a solution. This has not been done in the literature for the kind of model we study. It turns out that the existence is less straightforward than other authors may perhaps have thought. We show two existence results, Theorems 1 and 4.

Furthermore, we generalize the formulation of the consumer–producer maximization problem as found in the literature. Unlike Yang (2001) and the related literature, we incorporate the choice of labor supply in the problem for the first time. Moreover, we avoid a previously imposed and rather unnatural restriction on self-production (Rader, 1964; Sun et al., 2000; see the discussion in the second paragraph before Theorem 1 below).

Our analysis of the fundamental consumer–producer optimization problem leads us to the following three insights.

- There are two fundamentally different sets of conditions under which there exists a solution to the consumer–producer optimization problem. Our first existence result (Theorem 1 in this paper) states that under very weak regularity conditions there exists such a solution if the market transaction costs are *asymptotically* sufficiently high. Second, we can replace the condition of asymptotically high transaction costs by a stronger condition on the consumer–producer's preferences and a weak regularity condition on the market transaction costs (Theorem 4). We emphasize that these additional regularity conditions remain mild and that the resulting existence theorem is widely applicable, also to cases without transaction costs. Furthermore, conditions of this kind are necessary for existence, as we show by means of Example 2.

- Next we address the specialization of the consumer–producer's trading activities. We show in Theorem 3 that under rather weak regularity conditions the consumer–producer does not buy and sell the same commodity. This property holds even in the absence of transaction costs. Notably, we do not use convexity or even continuity assumptions in this theorem as have been used for similar results in the literature.
- We show in Theorem 5 that, under slightly stronger conditions on the individual's production technology, the individual does not sell more than one commodity. The main assumption is that of weakly increasing returns to labor. This property provides a foundation of the desired complete, endogenous division of labor at the social level. The market-equilibrating processes guide each consumer–producer to specialize trading in that good for which that individual possesses a comparative advantage.

This paper is structured as follows. In Section 2 we discuss the consumer–producer optimization problem and our first existence result, formulated for markets with asymptotically infinite transaction costs. In Section 3 we formulate and prove the announced characterizations of the solutions to the consumer–producer optimization problem. In the process we derive a second existence result for markets with asymptotically finite transaction costs. Finally, Section 4 concludes with the discussion of some potential applications.

2. The consumer–producer optimization problem

Consider an individual consumer–producer who operates as a price-taker within a competitive market economy with a set of m tradeable goods, denoted by $M = \{1, \dots, m\}$. There is also one non-tradeable good, the individual's time, which is used for either leisure or labor. (As mentioned before, the introduction of leisure into the consumer–producer optimization problem is novel.) Labor is the only input for the individual's production technology for all goods. The individual is endowed with one unit of time. The individual's preferences are represented by a utility function $u : \mathbb{R}^{m+1}_{+} \to \mathbb{R}$. The individual's final consumption of good i is denoted by x_i, the total amount of labor time devoted to the home-production of certain goods is denoted by L, so a typical value of the utility function appears as $u(x_1, \dots, x_m, 1 - L)$.

The individual is able to produce every one of the m tradeable goods by employing her labor. For each tradeable good $i \in M$, this is represented by a production function $f_i : [0, 1] \to \mathbb{R}_+$.

The individual acts as a price-taker in all m competitive markets where she can buy and sell all tradeable goods. There are transaction costs related to the use of these markets modelled in the “iceberg" tradition. For each $i \in M$, we introduce a *transaction efficiency function* $n_i : \mathbb{R}_+ \to \mathbb{R}_+$ such that if the individual purchases $x_i \geqslant 0$ of good i on the market, the individual obtains $n_i(x_i)$ for net consumption.

For each tradeable $i \in M$, $x_i^d \geqslant 0$ denotes the amount purchased on market i and $x_i^s \geqslant 0$ denotes the amount sold on market i. We denote the corresponding m-dimensional vectors by x^d and x^s. The amount of labor devoted to the production of good $i \in M$ is indicated by $\ell_i \geqslant 0$, and the m-dimensional vector of these amounts by ℓ. The net consumption of good

$i \in M$, x_i, now equals $x_i = f_i(\ell_i) - x_i^s + n_i(x_i^d)$. For all $i \in M$, $p_i \geqslant 0$ denotes the market price of good i.

2.1. The individual's optimization problem

The individual solves the following maximization problem, to be referred to as (P):

$$\max_{\{x^s, x^d, \ell\} \geqslant 0} \quad u(x_1, \dots x_m, 1 - L)$$

subject to

$$x_i = f_i(\ell_i) - x_i^s + n_i(x_i^d) \geqslant 0, \text{ for every } i \in M, \tag{1}$$

$$L = \sum_{i \in M} \ell_i \leqslant 1, \tag{2}$$

$$\sum_{i \in M} p_i x_i^d \leqslant \sum_{i \in M} p_i x_i^s. \tag{3}$$

In the consumer–producer decision problem (P) constraint (1) imposes a consumption feasibility condition for each tradeable good. Constraint (2) describes the labor time constraint of the individual regarding the employment of her production technology. Finally, constraint (3) imposes a budget constraint on the individual's consumption and production possibilities.

We call a point $\{x^s, x^d, \ell\}$ that satisfies the feasibility conditions (1), (2), and (3) a *configuration*, and a solution point of (P) is an *optimal configuration*.

Throughout this paper, we shall introduce several sets of assumptions on the primitives of the model in order to establish certain properties of optimal configurations. Our first assumption introduces mild regularity conditions on the available production technologies, trading technologies, and consumer preferences.

Assumption 1 (Regularity conditions).

(a) For all $i \in M$, the production function f_i is continuous.
(b) For all $i \in M$, $f_i(0) = 0$.
(c) For all $i \in M$, the transaction efficiency function n_i is continuous.
(d) For all $i \in M$ and for all $x \in \mathbb{R}_+$, it holds that $n_i(x) \leqslant x$.
(e) For all $i \in M$, we have a strictly positive price, i.e., $p_i > 0$.
(f) The utility function u is continuous.

For an elaboration of Assumption 1(c) and (d) on the transaction technologies see Gilles et al. (2003), Gilles and Diamantaras (2003) and Yang (2001). We are able to show existence under the regularity conditions imposed by Assumption 1 and the additional hypothesis that transaction costs are asymptotically high.

Assumption 2 (Asymptotically infinite transaction costs). For every $i \in M$, it holds that $x - n_i(x) \to \infty$ as $x \to \infty$.

Assumption 2 imposes sufficiently high transaction costs within *each* market. These high transaction costs make large trades unattractive and, thus, guarantee the existence of an optimal configuration.

Note that we do not impose the restriction that $x_i^s \leqslant f_i(\ell_i)$ on (P), unlike, for example, Rader (1964; p. 144) and Sun et al. (2000, p. 11, display (2)). This condition imposes an exogenous constraint on the traded quantities of each good. With such a trade restriction in place, Assumption 2 would not be needed for the existence proof. However, such an a priori restriction on the individual's trading abilities seems implausible and unrealistic; what prevents the purchase and immediate resale of any amount of that good? Also, it is preferable to allow precisely this kind of re-trading, since in our model we want to allow the introduction of intermediary traders instead of ruling it out a priori. (See also Yang, 2001, Chapter 7.)

Also, we do not employ a net trade formulation as, for example, used by Sun et al. (2004). Such a formulation avoids the basic re-trade problem at the level of the foundations of the model and, thus, seems an inadequate description of transaction costs in a competitive market.

Theorem 1. *Under* Assumptions 1 *and* 2 *there exists an optimal configuration.*

Proof. The point $(x^s, x^d, \ell) = (0, 0, 0)$ is feasible for (P). Indeed, constraints (2) and (3) of (P) are obviously satisfied by this point. Constraint (1) of (P) in combination with Assumption 1(d) and (b) yields $x_i = 0$ for all $i \in M$, because by Assumption 1(d) and non-negativity of n_i it follows that $n_i(0) = 0$, and from 1(b) we have $f_i(0) = 0$. Finally, the consumption plan $(x_1, \ldots, x_M, 1 - L) = (0, \ldots, 0, 1)$ belongs to the domain of u.

Next we show that the feasible set of (P) is bounded. Suppose *per absurdum* that it is unbounded. Then there exists a sequence (x^{sk}, x^{dk}, ℓ^k), $k = 1, 2, \ldots$ in the feasible set that is unbounded above. By constraint (2), the ℓ^k sequence is bounded, so either the sequence x^{sk} or the sequence x^{dk} is unbounded above. Therefore, there exists a good $i \in M$ such that either the sequence x_i^{sk} or the sequence x_i^{dk} is unbounded above. We now show that constraints (1) and (3) lead to the desired contradiction.

If x_i^{sk} is unbounded above, then for the same good x_i^{dk} must also be unbounded above; otherwise at some point constraint (1) would be violated. If x_i^{dk} is unbounded above, then there must be some good j for which x_j^{sk} is unbounded above as well, or constraint (3) would be violated. It now follows that for some good $i \in M$, both sequences x_i^{sk} and x_i^{dk} are unbounded above.

Using constraints (1) and (2), we have

$$x_i^k = f_i(\ell_i^k) - x_i^{sk} + n_i(x_i^{dk}) = f_i(\ell_i^k) + (x_i^{dk} - x_i^{sk}) + (n_i(x_i^{dk}) - x_i^{dk}) \geqslant 0. \tag{4}$$

By Assumption 2 the difference $n_i(x_i^{dk}) - x_i^{dk}$ is unbounded below. Thus, $x_i^{dk} - x_i^{sk}$ must be unbounded above, or by (4) x_i^k will eventually become negative, since $f_i(\ell_i^k)$ is bounded. However, if $x_i^{dk} - x_i^{sk}$ is unbounded above, by constraint (3) there will be another good, h, such that x_h^{sk} is unbounded above. Since we can then repeat this reasoning for h and since there is only a finite number of goods, we conclude that eventually every sequence x_i^{sk} and x_i^{dk}, for all $i \in M$, is unbounded above.

This implies that for every $i \in M$ it holds that $x_i^{dk} > x_i^{sk}$ for large enough k, but this contradicts constraint (3), recalling that by 1(e), $p_i > 0$. Hence, our hypothesis that the feasible set is unbounded is false.

Also, Assumption 1(a) and (c) implies that the feasible set of (P) is closed. Hence, by Assumption 1(f), the Weierstrass Theorem implies the existence of at least one solution of (P). □

At first it might appear puzzling that we use Assumption 2; intuitively one suspects that even with an unbounded feasible set, Assumption 1 should be sufficient to guarantee that the maximization problem (P) has a solution. This would be based on the boundedness of the available labor. Indeed, for a solution not to exist, the individual should want to perform an infinite progression of larger and larger purchases of one good, financed by larger and larger sales of another good. This would run up against the upper bound for the production of the good sold, $f_i(1)$, so the possibility suggests itself of the same good being bought and sold in unbounded amounts. However, this is not optimal, as buying and selling the same good offers the agent no otherwise unavailable options.

In this context this argument is, however, invalid. An explicit description of this process appears after Theorem 4 below as Example 2. It shows that some hypothesis is needed in addition to Assumption 1 in order to prove existence of an optimal configuration. This discussion brings us to one of the central statements of the theorem of optimal configuration seminally investigated by Yang and Ng (1993) and formally stated by Wen (1998, p. 174, Claim 1; see also Wen, 2000).

3. Characterization results

In this section we strengthen the existing statements of Wen's (1998) two first results. We are able to show these results under weaker conditions than used in Wen (2000) and Yao (2002).

We call a configuration *weakly specialized* if the individual does not buy and sell the same good. We start by investigating under which conditions optimal configurations are weakly specialized.

Assumption 3.

(a) For all $i \in M$ we have that $n_i(0) = 0$, and for all $x \geqslant 0$ and all $y \geqslant 0$, $n_i(y) + x \geqslant n_i(x + y)$.
(b) The utility function u is non-decreasing.

To our understanding Assumption 3(a) is the most general condition under which the optimal configuration can be shown to be weakly specialized. That an optimal configuration is weakly specialized has been shown by Wen (1998) for a twice differentiable, convex transaction efficiency function and by Yang (2001) for a linear transaction efficiency function. Assumption 3(a) is much more general. Wen (2000) makes an assumption almost identical to Assumption 3A(a) below and removes the differentiability assumption, but does not include leisure in the utility function. Finally, she does not discuss the existence of a solution to her analogue of problem (P).

First, Assumption 3(a) implies the transaction efficiency function regularity property stated as Assumption 1(d). Indeed, Assumption 1(d) is simply Assumption 3(a) for the case that $y = 0$.

Second, the following discussion shows that, under Assumption 1(d), Assumption 3(a) is strictly weaker than convexity as imposed by Wen (1998, 2000). To make this point, we first show that convexity implies Assumption 3(a).

Theorem 2. *Under* Assumption 1(c) *and* (d), *for every good* $i \in M$, *if the transaction efficiency function* n_i *is convex, then it satisfies* Assumption 3(a).

Proof. Let $i \in M$ and $0 < \varepsilon < 1$ be arbitrary. Then from the convexity of n_i and Assumption 1(d) we have

$$n_i(x+y) \leqslant (1-\varepsilon)n_i\left(\frac{x}{1-\varepsilon}\right) + \varepsilon n_i\left(\frac{y}{\varepsilon}\right) \leqslant x + \varepsilon n_i\left(\frac{y}{\varepsilon}\right). \tag{5}$$

Now by continuity of the transaction efficiency function as imposed by Assumption 1(c) and the fact that (5) holds for arbitrary ε, we now can take the limit $\varepsilon \to 1$ and we may conclude that $\varepsilon n_i(y/\varepsilon) \to n_i(y)$. This implies that the function n_i indeed satisfies Assumption 3(a). □

Third, we show that Assumption 3(a) is strictly weaker than convexity by means of the following example. This example also shows that Assumption 3(a) holds for some concave and continuous transaction efficiency functions and even for some discontinuous functions.

Example 1.

(a) Consider a transaction efficiency function $n : \mathbb{R}_+ \to \mathbb{R}_+$ given by $n(x) = x$ if $0 \leqslant x \leqslant 1$ and $n(x) = \sqrt{x}$ if $x > 1$. This transaction efficiency function is continuous and concave. It also satisfies Assumptions 1(d) and 2. We proceed by showing that it satisfies Assumption 3(a). Let $x, y \geqslant 0$. Then we distinguish three different cases. If $x + y \leqslant 1$ the function n is linear and thus 3(a) is trivially satisfied. If $x + y \geqslant 1$ and $y \leqslant 1$, it holds that $n(x+y) - n(y) = \sqrt{x+y} - y \leqslant x$ since $\sqrt{x+y} \leqslant x + y$. Finally, if $x + y > 1$ and $y > 1$, then from $\sqrt{x+y} + \sqrt{y} > 1$ it follows that

$$n(x+y) - n(y) = \sqrt{x+y} - \sqrt{y} = \frac{x}{\sqrt{x+y} + \sqrt{y}} < x.$$

This shows that n satisfies Assumption 3(a).

(b) Next consider a transaction efficiency function $n : \mathbb{R}_+ \to \mathbb{R}_+$ given by $n(x) = x$ if $0 \leqslant x \leqslant 1$ and $n(x) = (1/2)x$ if $x > 1$. Clearly this function is discontinuous at $x = 1$. Also, it is non-convex and non-concave, yet it satisfies Assumptions 1(d), 2, and 3(a). It is trivial to check Assumptions 1(d) and 2. Regarding Assumption 3(a), for $x + y \leqslant 1$, this is obvious. For $x + y > 1$, the only non-trivial case to check is for $y \leqslant 1$. For this case, we have $n(y) + x = y + x > (1/2)(x + y) = n(x + y)$.

The following theorem shows that for any configuration there is a weakly specialized configuration that yields at least as much utility.

Theorem 3. *Under Assumption 3, if in any configuration the individual buys and sells the same good, then there is a configuration in which the individual does not buy and sell the same good and in which the individual is at least as well off.*

Proof. The structure of our proof is largely based on the logical construction introduced by Wen (1998, Claim 1).

Suppose there is a feasible point of (P), $X^0 = (x^{s0}, x^{d0}, \ell^0)$, at which the individual buys and sells the same good, say good j. Thus, we assume that $x_j^{s0} > 0$ and $x_j^{d0} > 0$. By Assumption 3(a) we have that $n_j(0) = 0$. Furthermore, we recall that Assumption 3 implies that Assumption 1(d) is satisfied for each transaction efficiency function n_j as well.

Case 1. Suppose $x_j^{s0} > x_j^{d0} > 0$. Let $x_j^{d*} = 0$, $x_j^{s*} = x_j^{s0} - x_j^{d0} > 0$, and, leaving all other components of the point X^0 unchanged, denote the new point by $X^* = (x^{s*}, x^{d*}, \ell^0)$. The point X^* is feasible, as can be easily checked. The individual's consumption of good j at the new point is $x_j^* = f_j(\ell_j^0) - x_j^{s*} + n_j(x_j^{d*}) = f_j(\ell_j^0) - x_j^{s0} + x_j^{d0} + n_j(x_j^{d*})$. Also, $n_j(x_j^{d*}) = n_j(0) = 0$. Thus, by Assumption 1(d), $x_j^* = f_j(\ell_j^0) - x_j^{s0} + x_j^{d0} \geqslant f_j(\ell_j^0) - x_j^{s0} + n_j(x_j^{d0}) = x_j^0$. By Assumption 3(b), $u(x^*, 1 - L^0) \geqslant u(x^0, 1 - L^0)$, and the assertion is shown.

Case 2. Suppose $x_j^{d0} \geqslant x_j^{s0} > 0$. Let $x_j^{s*} = 0$, $x_j^{d*} = x_j^{d0} - x_j^{s0} \geqslant 0$, and, leaving all other components of the point X^0 unchanged, denote the new point by $X^* = (x^{s*}, x^{d*}, \ell^0)$. The point X^* is feasible, as can be easily checked. The individual's consumption of good j at the new point is $x_j^* = f_j(\ell_j^0) - x_j^{s*} + n_j(x_j^{d*}) = f_j(\ell_j^0) + n_j(x_j^{d0} - x_j^{s0})$. Apply now Assumption 3(a) with $x = x_j^{s0}$ and $y = x_j^{d0} - x_j^{s0}$ to find $x_j^* \geqslant f_j(\ell_j^0) - x_j^{s0} + n_j(x_j^{d0}) = x_j^0$. As before, by Assumption 3(b), $u(x^*, 1 - L^0) \geqslant u(x^0, 1 - L^0)$, and again the assertion is proven. □

Theorem 3 as stated is slightly stronger than the weakly specialized optimal configuration property discussed in the introduction. That an individual specializes her trade scheme in such a fashion is subject to the following immediate conclusion from Theorem 3.

Corollary 1 to Theorem 3. *Under Assumption 3, if in an optimal configuration the individual buys and sells the same good, then there is another optimal configuration in which the individual does not buy and sell the same good.*

The following more restrictive assumptions allow a stronger conclusion for Corollary 1 to Theorem 3.

Assumption 1A(d). For all $i \in M$, $n_i(0) = 0$, and for all $x > 0$, $n_i(x) < x$.

Assumption 3A.

(a) For all $i \in M$, all $x > 0$, and all $y > 0$, $n_i(y) + x > n_i(x + y)$.
(b) The utility function u is strictly increasing.

Assumption 3A does not imply Assumption 1A(d), unlike the case for Assumption 3, which implies Assumption 1(d). Therefore, the corollary below is tight.

Corollary 2 to Theorem 3. *Under* Assumptions 1A(d) *and* 3A, *the individual does not sell and buy the same good in any optimal configuration.*

The proof of this assertion is a simple adaptation of the proof of Theorem 3.

From Theorem 3, we can now show existence not relying on Assumption 2.

Theorem 4. *Under* Assumptions 1 *and* 3, *there exists an optimal configuration.*

Proof. By Assumption 3, Theorem 3 implies that for every configuration there exists a point that lies in a subset of the feasible set of (P) given by

$$\Phi = \{(x^s, x^d, \ell) \in \mathbb{R}^{3m}_+ | (x^s, x^d, \ell) \text{ satisfies } (1),\ (2), \\ \text{and } (3) \text{ and } x^s_j \leqslant f_j(1) \text{ for all } j \in M\},$$

and that yields at least as much utility. This is true because if $x^s_j > f_j(1)$, then $x^d_j > 0$ by feasibility constraint (1) of (P), and then Theorem 3 applies and allows a choice of a feasible point that satisfies $x^s_j \leqslant f_j(1)$.

From the feasibility constraint (3) of (P) it follows that $\sum_{i \in M} p_i x^d_i \leqslant \sum_{i \in M} p_i f_i(1)$. From this and Assumptions 1(a), (c), (d), and (e), it follows that the set Φ is a compact set in $\mathbb{R}^{3m}_+$. By the continuity of u, Assumption 1(f), there exists a point $\hat{X} = (\hat{x}^s, \hat{x}^d, \hat{\ell}) \in \Phi$ at which the function u attains a restricted maximum over Φ. We now show that $\hat{X}$ indeed is a global maximum of u, and thus is a solution to (P).

Suppose to the contrary that there is some feasible point $X^0 = (x^{0s}, x^{0d}, \ell^0)$ that results in a strictly higher utility: it holds that $u(x^0, 1 - L^0) > u(\hat{x}, 1 - \hat{L})$. Then by Theorem 3 there is a feasible point $X^* = (x^{*s}, x^{*d}, \ell^*) \in \Phi$ such that $u(x^*, 1 - L^*) \geqslant u(x^0, 1 - L^0) > u(\hat{x}, 1 - \hat{L})$. This contradicts the optimality of $\hat{X}$ over Φ. Hence, we may conclude that $\hat{X}$ is indeed a global solution to (P). □

The following example shows that Assumptions 3(a) or 2 are needed to show existence; Assumption 1 is not enough by itself or even in combination with Assumption 3(b).

Example 2. Let $m = 2$ and $u(x_1, x_2, 1 - L) = x_1 x_2$ for all x_1, x_2 and L. Let the transaction efficiency functions be given by $n_1(x^d_1) = 0$ for all $x^d_1 \geqslant 0$, and

$$n_2(x^d_2) = \begin{cases} 0, & \text{if } 0 \leqslant x^d_2 < 1, \\ x^d_2 - \frac{1}{x^d_2}, & \text{otherwise.} \end{cases}$$

Let $f_1(\ell_1) = \ell_1$ for all ℓ_1, $f_2(\ell_2) = 0$ for all ℓ_2, and $p_1 = p_2 = 1$.

Note that Assumptions 1 and 3(b) are met, but Assumptions 2 and 3(a) are not satisfied in this situation. If the individual chooses autarky, the utility level achieved is 0, and clearly this is not optimal since by trading the individual can get a positive utility level. In non-autarkic situations, it is always optimal to set $\ell_1 = 1$ and $\ell_2 = 0$. Given this, consider the following production-trading choice, which can be described by some $y > 1$: $x^s_1 \in [0, 1]$, $x^s_2 = y$. The budget constraint becomes $x^s_1 + y = x^d_2$, since given the transaction efficiency function for good 1, the individual will never want to buy any positive amount of it. By this trade, the individual consumes the amounts $x_1 = 1 - x_1^s$ and $x_2 = x_1^s + y - 1/(x_1^s + y) - y = x_1^s - 1/(x_1^s + y)$. The utility function takes the value $(1 - x_1^s)(x_1^s - 1/(x_1^s + y))$, and has no maximum in y for any value of x_1^s. Therefore, there is no optimal configuration.

As Wen (1998) shows, one can establish further properties beyond the property stated in Theorem 3. We call a configuration *strongly specialized* if the individual does not sell more than one good in this configuration. We show that, under slightly stronger conditions than before, for any configuration there is a strongly specialized configuration that yields at least as much utility.

Note that in the next theorem, unlike in Wen (1998, 2000), Yang (2001) and Yao (2002), leisure is included in the list of arguments of the utility function.

Assumption 4. For all $i \in M$, the production function f_i is convex.

Assumption 4 imposes weakly increasing returns to scale on the individual's production technology.

Theorem 5. *Under* Assumptions 1(e), 3(b) *and* 4, *for any configuration there exists a configuration in which the individual sells no more than one good and obtains at least as much utility.*

For the proof we need an auxiliary result, which is stated and shown in Yao (2002):

Lemma 1 (Yao, 2002, Proposition 1). *Assume that* $f : \mathbb{R} \to \mathbb{R}$ *is convex* (*concave*). *Then, for any* $x \in \mathbb{R}$, $\Delta x > 0$, $a > 0$, *it holds that*

$$\frac{f(x + a + \Delta x) - f(x + a)}{\Delta x} \geqslant (\leqslant) \frac{f(x + a) - f(x)}{a}.$$

Proof of Theorem 5 (Adapted from Yang, 2001, pp. 333–334 and Yao, 2002, pp. 419–420). Suppose that utility is maximized at a point where the individual sells two or more goods. We will show that utility does not decrease if the individual reduces the number of goods sold.

Let goods 1 and 2 be sold. Let L_1 and L_2 be the amounts of labor allocated to the production of goods 1 and 2, respectively. Let x_1 and x_2 be the amounts produced of goods 1 and 2, respectively, and $x_1^s > 0$ and $x_2^s > 0$ the respective amounts sold. Define the amounts of labor ℓ_1 and ℓ_2 by $f_i(\ell_i) = x_i - x_i^s \geqslant 0$ for $i = 1, 2$, and let $\Delta\ell_i = L_i - \ell_i > 0$ for $i = 1, 2$. Then we have

$$x_i^s = f_i(\ell_i + \Delta\ell_i) - f_i(\ell_i) > 0, \quad \text{for } i = 1, 2.$$

The contribution to the individual's income from selling the two goods 1 and 2 is

$$p_1 x_1^s + p_2 x_2^s = p_1[f_1(\ell_1 + \Delta\ell_1) - f_1(\ell_1)] + p_2[f_2(\ell_2 + \Delta\ell_2) - f_2(\ell_2)].$$

We will show that by selling only one of these two goods, but not changing the self-consumption of them and not changing any other decision, the individual can achieve at least as much income. Define k_1 and k_2 by

$$k_1 = \frac{p_1[f_1(\ell_1 + \Delta\ell_1) - f_1(\ell_1)]}{\Delta\ell_1} > 0,$$

and

$$k_2 = \frac{p_2[f_2(\ell_2 + \Delta\ell_2) - f_2(\ell_2)]}{\Delta\ell_2} > 0.$$

From their definitions it is clear that k_1 and k_2 represent discrete approximations of the marginal revenues for the two goods involved. Without loss of generality we may assume that good 1 has the lower marginal value; we assume that $k_1 \leqslant k_2$.

The contribution to income from selling these two goods can now be written as

$$p_1[f_1(\ell_1 + \Delta\ell_1) - f_1(\ell_1)] + p_2[f_2(\ell_2 + \Delta\ell_2) - f_2(\ell_2)] = k_1\Delta\ell_1 + k_2\Delta\ell_2 > 0.$$

From $k_1 \leqslant k_2$ we now have

$$\begin{aligned} &p_2[f_2(\ell_2 + \Delta\ell_2 + \Delta\ell_1) - f_2(\ell_2)] \\ &\quad = p_2[f_2(\ell_2 + \Delta\ell_2 + \Delta\ell_1) - f_2(\ell_2 + \Delta\ell_2)] + p_2[f_2(\ell_2 + \Delta\ell_2) - f_2(\ell_2)] \\ &\quad = \Delta\ell_1 p_2 \frac{f_2(\ell_2 + \Delta\ell_2 + \Delta\ell_1) - f_2(\ell_2 + \Delta\ell_2)}{\Delta\ell_1} + \Delta\ell_2 p_2 \frac{f_2(\ell_2 + \Delta\ell_2) - f_2(\ell_2)}{\Delta\ell_2} \\ &\quad \geqslant (\Delta\ell_1 + \Delta\ell_2)k_2 \geqslant k_1\Delta\ell_1 + k_2\Delta\ell_2 > 0. \end{aligned} \tag{6}$$

For the next-to-the-last step, we used the lemma to get

$$\frac{f_2(\ell_2 + \Delta\ell_2 + \Delta\ell_1) - f_2(\ell_2 + \Delta\ell_2)}{\Delta\ell_1} \geqslant \frac{f_2(\ell_2 + \Delta\ell_2) - f_2(\ell_2)}{\Delta\ell_2}.$$

The implication of (6) is that the individual can generate at least as much income by selling only good 2, in the process neither changing her consumption of any good, nor changing the total labor used for production of the goods. This enables the individual to purchase at least as much of the goods for consumption or to enjoy more leisure. Hence, she can achieve at least the maximum utility by selling only good 2. □

A simple example shows that Assumption 4 is necessary for Theorem 5 to hold.

Example 3. Consider a case with three goods, and for goods 1 and 2 let the production functions be the same and be given by

$$f_i(\ell_i) = \begin{cases} \ell_i, & \text{if } 0 \leqslant \ell_i \leqslant \frac{1}{2}, \\ \frac{1}{2}, & \text{if } \frac{1}{2} \leqslant \ell_i \leqslant 1. \end{cases}$$

For good 3, we set the function $f_3(\ell_3) = 0$ for all $\ell_3 \geqslant 0$ to describe the self-production possibilities. We introduce a utility function that attaches no value to anything but good 3: $u(x_1, x_2, x_3, 1 - L) = x_3$. Let $n_1(x) = n_2(x) = 0$ for all $x \geqslant 0$ and $n_3(x) = (1/2)x$ for all $x \geqslant 0$. One can easily determine that there is an optimal configuration for every price vector. Furthermore, for any solution it has to hold that $x_1^s = x_2^s = 1/2$. Finally, the production functions for goods 1 and 2 do not satisfy Assumption 4.

4. Concluding remarks

The specialization issues studied in this paper bear directly on the analysis of the endogenous division of labor in a competitive market economy, whether in the presence of transaction costs or not. We established two characterizations of optimal configurations under relatively mild regularity conditions, namely Theorems 3 and 5. These results have wide-ranging applicability. Some of these applications have already been investigated in the literature. Yang (2001) discusses a number of them, including models on the endogenous determination of transaction intermediaries (middlemen), money, growth (with special emphasis to learning-by-doing), the theory of the firm, and other applications.

It is worthwhile to explore the general properties of general equilibrium consumer–producer models. Our own work on transaction cost economies (Gilles and Diamantaras, 2003; Gilles et al., 2003) encompasses such a model in a more abstract sense, and it suggests considerable room for fruitful investigations of the properties of such models. Romer (1994) emphasizes the importance of the emergence of new commodities in the process of economic development. Historically, specialization in trade and production was the crucial factor in the development of new products. This suggests that the Smith–Young specialization hypothesis has bearing on the study of economic development. As individuals specialize, they may learn how to create new products. The role of learning-by-doing in the creation of new products thus receives a microeconomic foundation. (For an example of such an approach we refer to Yang and Borland, 1991.)

Our results also provide a foundation for the study of a new economic domain for social choice theory, namely a domain of economies populated by consumer–producers.

The development of a complete theory of the endogenous emergence of a social division of labor is an important avenue of future research. We expect that such a theory can be based on the two fundamental specialization properties that have been stated and shown here and that it will significantly enhance the applicability of general equilibrium theory.

Ackmowledgements

We would like to thank Hans Haller for some very fruitful exchanges regarding this paper and the subject of New Classical Economics in general. We also thank two referees, Yew-Kwang Ng, Xiaokai Yang, and participants at various seminars for their comments, most notably Karl Vind who made a very useful comment about the literature at the Sixth International Meeting of the Society for Social Choice and Welfare in Pasadena, CA.

References

Debreu, G., 1959. Theory of Value. Wiley, New York.

Gilles, R.P., Diamantaras, D., 2003. To trade or not to trade: economies with a variable number of tradeables. International Economic Review 44, 1173–1204.

Gilles, R.P., Diamantaras, D., Ruys, P.H.M., 2003. Optimal design of trade institutions. Review of Economic Design. 8, 269–292.

McKenzie, L.W., 1959. On the existence of general equilibrium for a competitive market. Econometrica 27, 54–71.

Rader III, J.T., 1964. Edgeworth exchange and general economic equilibrium. In: Yale Economic Essays, vol. 4. Input-Output Publishing Company, Newcastle-upon-Tyne, UK, pp. 132–180 (reprinted in Rader, T., 1996. Economics with No Special Technology; Posthumous Essays and Selected Articles).

Romer, P., 1994. New goods, old theory, and the welfare costs of trade restrictions. Journal of Development Economics 43, 5–38.

Smith, A., 1776. An Inquiry into the Nature and Causes of the Wealth of Nations. University of Chicago Press, Chicago (Reprint).

Sun, G.-Z., Yang, X., Yao, S., 2000. Toward a theory of impersonal networking decisions and endogenous structure of the division of labor. Working Paper.

Sun, G.-Z., Yang, X., Zhou, L., this issue. General equilibria in large economies with transaction costs and endogenous specialization. Journal of Economic Behavior and Organization.

Villar, A., 2000. Equilibrium and Efficiency in Production Economies, 2nd ed. Springer Verlag, Berlin.

Wen, M., 1998. An analytical framework of consumer–producers, economies of specialization and transaction costs. In: Arrow, K.J., Ng, Y.-K., Yang, X. (Eds.), Increasing Returns and Economic Analysis. St. Martin's Press, New York, NY, pp. 170–185.

Wen, M., 2000. The dichotomy between production and consumption decisions and economic efficiency. Paper presented at the Australian Economic Theory Workshop, January 1998, Division of Economics, RSPAS, Australian National University, revised 2000.

Yang, X., 2001. Economics: New Classical Versus Neoclassical Frameworks. Blackwell Publishers, Malden, MA.

Yang, X., Borland, J., 1991. A microeconomic mechanism for economic growth. Journal of Political Economy 99, 460–482.

Yang, X., Ng, Y.-K., 1993. Specialization and Economic Organization: A New Classical Microeconomic Framework. North-Holland, Amsterdam.

Yao, S., 2002. Walrasian equilibrium computation, network formation, and the Wen theorem. Review of Development Economics 6, 415–427.

Young, A.A., 1928. Increasing returns and economic progress. The Economic Journal 38, 527–542.