From private banking to central banking: ingredients of a welfare analysis*

Neil Wallace†

March 31, 2004

Abstract

An earlier analysis that compares allocations achievable using inside (private) money to those achievable using outside (government) money is extended. It is shown that outside-money allocations are a subset of inside-money allocations provided that the inside money issued by one issuer can be distinguished from that issued by others. If that recognizability assumption is weakened, then the subset result could conceivably be over-turned. Even so, the analysis suggests that the outside-money arrangement, with its uniform money, should be managed so as to attain some of the benefits of inside money.

JEL classification: E 40

Key words: inside money, outside money, monitoring, matching model

1 Introduction

Central banks, at least in the U.K. and the U.S., emerged as monopoly issuers of banknotes from systems in which there were many private banks issuing banknotes. The emergence was accompanied by debates about how private banks should be regulated and about how central banks should behave. In the U.K., the debates were between what were called the *banking* and the

*Prepared for the conference, *Models of Monetary Economies II: The Next Generation*, May 21-22, 2004, Minneapolis.

†Department of Economics, The Pennsylvania State University; <neilw@psu.edu>.

currency schools, with the former advocating some version of laissez-faire in intermediation and the latter advocating some version of hard money or what much later came to be called monetarism. In the U.S., the issues were similar, but advocates of the banking school position tended to be labeled advocates of the *real-bills doctrine*. In this paper, I revisit those issues by setting out a framework in which an arrangement that resembles private banknote issue can be compared with one that resembles a central bank monopoly on note issue.

My starting point is the model that Cavalcanti and I used to compare inside and outside money (see [1] and [2]). That model adopts the pairwise-meeting setting in [11] and [13] and contains the assumption that some people are (perfectly) monitored and others are not monitored at all. For well-known reasons (see, for example, [6]), the non monitored people are the source for a demand for payable-to-the-bearer, tangible media of exchange. Here, I adopt the above features of the Cavalcanti and Wallace analysis, but depart from it in the following ways.

First, I add a centralized meeting that alternates in time with the pairwise meetings of [11] and [13]. The centralized meeting, which can be interpreted as everyone meeting the central bank, is used solely for risk sharing: no production or consumption occurs and, therefore, no utility is realized. Second, I allow the non monitored people to hide money. Third, I let individual money holdings be general, instead of restricting such holdings to be in the set $\{0, 1\}$. Finally, and, most significantly, I make different assumptions regarding the kind of threats that are allowed.

Cavalcanti and Wallace threaten monitored people who defect with permanent autarky. In contrast, I assume that any such defector joins the ranks of the non monitored people and suffers no further punishment. I also assume that defection by a single person does not give rise to punishment of a significant portion of the entire economy and I assume that people can distinguish the money issued by one monitored person from that issued by other monitored people. The last assumption, which I label the recognizability assumption, allows the money issued by a defector to be rejected without shutting down a significant portion of the entire economy.

Under those assumptions, I show that inside money has the same kind of advantage over outside money that Cavalcanti and I found; that is, the set of outside-money incentive-feasible allocations is a subset of the set of inside-money incentive-feasible allocations. Under their assumptions, Cavalcanti and Wallace showed that the converse is not true. Here, it also seems not to be true, but I cannot show it. In particular, the presence of the centralized meeting enlarges the set of outside-money allocations. In fact, it enlarges it so much that if defection could be punished by permanent autarky and if individual money holdings were assumed to be bounded, then the converse would be true.

After presenting the subset result, I discuss whether the recognizability assumption is reasonable and speculate about the consequences of abandoning it. One consequence of weakening it, I suggest, is greater vulnerability of inside-money to counterfeiting. That greater vulnerability could offset the advantage that inside money has when the recognizability assumption holds. However, even if it does, the features of the model that give rise to that advantage have implications for the kind of risk-sharing that should occur in the centralized meeting under outside-money—under a central-bank monopoly on note issue.

One qualification should be noted at the outset. Two questions arise in any model of money. Why are people using money when trades could conceivably be accomplished with some version of borrowing and lending, and why are people using money when trades could conceivably be accomplished by trading assets with higher rates of return? Here I deal only with the first. I avoid the second troublesome and unsettled question by assuming that all real objects are perishable. We will see that quite a bit can be said about inside and outside money without dealing with it.

2 A model

As noted above, the background environment is borrowed from [11] and [13]. Time is discrete. There is a non-atomic, unit measure set of each of $K \geq 3$ specialization types of infinitely lived people and there are K distinct, produced, and perishable goods at each date. A specialization-type k person, $k \in \{1, 2, ..., K\}$, produces only good k and consumes only good $k+1$ (modulo K). Each person maximizes expected discounted utility with discount factor $\beta \in (0, 1)$. For a specialization-type k person, utility in a period is $u(q_{k+1}) - q_k$, where $q_{k+1} \in \mathbb{R}_+$ is consumption of good $k+1$ and $q_k \in \mathbb{R}_+$ is production of good k. The utility function $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly concave, strictly increasing, continuously differentiable and satisfies $u(0) = 0$ and $u'(\infty) = 0$. In addition, $u'(0)$ is sufficiently large.

Each period is divided into two parts, which can be labeled morning and afternoon. The morning is reserved for random pairwise meetings. A meeting between specialization types k and $k+1$ is called a single-coincidence meeting. Other meetings are called no-coincidence meetings.[1] In the afternoon, everyone is together and engages in risk-sharing in ways that are spelled out below.

The following monitoring assumption is borrowed from [1]. The set of each specialization type is partitioned in an exogenous way into two sets: the fraction m_1 are monitored and the fraction m_2 (with $m_1 + m_2 = 1$) are not. That is, the history of each monitored person is common knowledge, while that of each non monitored person is private to the person. It is as if each monitored person wears a computer chip that transmits everything that the person does to everyone else. In contrast, the only thing known about a non monitored person is the person's type.

Finally, to permit a discussion of private money, each person has a printing press capable of turning out indivisible and identical durable objects. The items turned out by the printing press of any one person are distinguishable from those turned out by other peoples' printing presses and from central bank money.

2.1 A class of symmetric allocations

Throughout, I consider only a limited class of allocations. First, as is fairly standard in the kind of model set out above, nothing depends on specialization type. Therefore, in what follows type will be used to designate only whether the person is monitored: 1 is for a monitored person, 2 is for a non monitored person. Second, the allocations are designed to permit a comparison of two rather special arrangements: either all valuable money is uniform central bank money—which, perhaps, misleadingly, I label outside money; or there is, in addition, valuable money issued solely by type-1 people—an arrangement I label inside money. Moreover, in the inside-money arrangement, all the valuable monies are perfect substitutes and no type-1 person begins either a morning or an afternoon with anything other than the person's printing press.

[1] Some economists have expressed concern about the assumption that meetings are random That concern is misplaced. First, the role of such randomness is to generate random earning opportunities and random consumption opportunities. Such randomness could be generated by preference shocks. Second, settings in which every meeting is a single-coincidence meeting have similar implications (see Wallace and Zhu [14]).

Given that limited purpose, I assume that a person's state, in addition to the person's type, is an element in the set of non negative integers, denoted $\mathbb{Z}$. For a type-2 (non-monitored) person, $z \in \mathbb{Z}$ is the amount of valuable money held and is assumed to be private information to the person. For a type-1 (monitored) person, z has the same interpretation in the outside-money arrangement, but is simply a label in the inside-money arrangement. In either case, for a type-1 person, z is assumed to be publicly known.[2] I let $\mathbb{I} = \{1, 2\}$, the set of types, and let $\mathbb{S} = (\mathbb{I} \times \mathbb{Z})$. An allocation describes what happens in pairwise meetings and what happens in the centralized meeting as functions of type, state, and date.

I start with pairwise meetings. For reasons described below, I assume that nothing happens in no-coincidence meetings. As regards single-coincidence meetings, the state of a meeting is $(s_c, s_p) \in \mathbb{S}^2$, where the first component describes the consumer and the second the producer as they enter the meeting. Although the discreteness of the set $\mathbb{Z}$ implies that there is a role for lotteries, for what is done here, adding lotteries seems not to matter. Therefore, to keep the notation simple, I assume that both output and state transitions are deterministic. I let $y_t(s_c, s_p)$ denote date-t output, $c_t(s_c, s_p)$ the date-t, end-of-meeting state for the consumer, and $p_t(s_c, s_p)$ the date-t, end-of-meeting state for the producer. That is, $y_t : \mathbb{S}^2 \rightarrow \mathbb{R}_+$ is date-t output and $c_t : \mathbb{S}^2 \rightarrow \mathbb{Z}$ and $p_t : \mathbb{S}^2 \rightarrow \mathbb{Z}$ are the date-t, end-of-meeting states for consumers and producers, respectively.

In the centralized meeting, there are new states that depend for each person only on the state of the person. Thus, I let $h_t : \mathbb{S} \rightarrow \mathbb{Z}$ denote that new end-of-afternoon state. The state transitions in the centralized market would seem to make superfluous the state transitions that could conceivably happen in no-coincidence meetings. In no-coincidence meetings, type-2 people could be given money by type-1 people, but it would seem equivalent to have that happen in the centralized market.

Letting $x_t \equiv (y_t, c_t, p_t, h_t)$, an allocation is a sequence $\{x_t\}_{t=0}^{\infty}$. It describes what happens in the economy in the following sense. Given a date 0 probability distribution over $\mathbb{I} \times \mathbb{Z}$, the assumption that pairwise meetings are random determines a distribution over the kinds of meetings that occur. Then, x_0 describes what happens in those meetings and in the subsequent centralized meeting, and implies a date-1 probability distribution over $\mathbb{I} \times \mathbb{Z}$;

[2] I work with a discrete set of individual states solely in order to avoid dealing with measure-theoretic concepts.

and so on. Such evolution is described in more detail below.

2.2 Incentive-feasible allocations

The heart of the model is the description of the subset of allocations that are incentive feasible. For type-2 who are not monitored, the constraints—truth-telling and participation constraints—are what they would be in a static model. As regards type-1 people, I assume, as noted above, that each such person has the option to disappear into the ranks of the type-2 people at any time. It is as if each such person can at any time throw away or disable the device that monitors the person. In what follows, I take it for granted that a type-1 person who defects would want to join the ranks of the type-2 people: If the person continued to be monitored, then harsher punishment could be inflicted on the person.

The sequence of actions is as follows. In a single-coincidence meeting at date t, the two people see each other's type in the set $\mathbb{I}$. In addition, the states in $\mathbb{Z}$ of all the type-1 people are seen. Then the people who are type-2 people in the meeting report their money holdings —simultaneously if there are two of them. The only lying that is permitted is under-representation of holdings.[3] Then the allocation—in particular, y_t, c_t, and p_t—has a suggested outcome for the meeting. Then the consumer and producer simultaneously say *yes* or *no* to the suggested outcome. If either says *no*, then both people leave the meeting with no output produced and with no state transitions. Moreover, if a type-1 person says *no*, then that is a defection that converts the person permanently into a type-2 person starting in the afternoon. If a type-2 person says *no*, then that has no future consequences beyond autarky in the current meeting. In addition, a type-1 person can defect from an inside-money allocation after trading by failing to destroy any money received in a trade. The afternoon is similar and simpler because each person is dealt with individually. A type-1 person gets to say *yes* or *no* to the suggested state transition: saying *no* leaves the person's state unchanged, but is a defection that converts the person permanently into a type-2 person starting the next morning. A type-2 person reports the person's state in $\mathbb{Z}$, again with only under-representation possible. Then the person gets to say *yes* or *no* to the suggested state transition. As in the morning, saying *no* for a type- 2 person

[3] The inability to over-state money holdings gives meaning to the notion that money is tangible.

has no future consequences.

Before I express the above conditions in terms of allocations, I should make two comments. First, the above scheme permits only individual defections in pairwise meetings. I would prefer to allow all cooperative pairwise defections. However, that calls for dealing with the pairwise core under asymmetric information, a task which I am not prepared to undertake here.[4] Second, I allow only limited punishments (threats). I rule out all punishments that involve positive-measure responses to defection by a single person. Thus, for example, reversion to autarky for the entire economy is not an allowable punishment for individual defection. However, as spelled out below, in inside-money arrangements, defection by a type-1 person is accompanied by making worthless the money that that person can print.

In order to express the truth-telling and participation constraints in terms of allocations, it is convenient to describe explicitly the economy's law of motion and to define discounted expected values. Let $\pi_t = (\pi_t^1, \pi_t^2)$, where $\pi_t^i : \mathbb{Z} \to [0,1]$ and $\pi_t^i(z)$ is the pre-morning date-t fraction of type-i people in state z and let $\theta_t = (\theta_t^1, \theta_t^2)$, where $\theta_t^i : \mathbb{Z} \to [0,1]$ and $\theta_t^i(z)$ is the pre-afternoon date-t fraction of type-i people in state z. Then,

$$\theta_t^i = \pi_t^i T_t^i, \tag{1}$$

where the entry in row j and column j' in the matrix T_t^i, the probability for a type i person of leaving a date-t meeting in state j' given that the person was in state j entering the meeting, is given by

$$\begin{aligned} T_t^i(j,j') &= \frac{K-2}{K}\delta_{jj'} + \frac{m_1}{K}\sum_z \pi_t^1(z)[c_t(j';i,j,1,z) + p_t(j';1,z,i,j)] \\ &\quad + \frac{m_2}{K}\sum_z \pi_t^2(z)[c_t(j';i,j,2,z) + p_t(j';2,z,i,j)]. \end{aligned} \tag{2}$$

Here, $\delta_{jj'} = 1$ if $j = j'$ and 0 otherwise and, in an abuse of notation, $c_t(j'; s_c, s_p) = 1$ if $c_t(s_c, s_p) = j'$ and 0 otherwise, and similarly for $p_t(j'; 1, z, i, j)$. And

$$\pi_{t+1}^i = \theta_t^i H_t^i, \tag{3}$$

[4]Notice, however, that prevention of over-issue of inside money in meetings between type-1 consumers and type-2 producers is not dependent on ruling out cooperative defections. Such a producer would not be tempted to produce a great deal in exchange for a lot of money if the producer knows that such a trade is a defection that will render the money obtained useless.

where the entry in row j and column j' of the matrix H_t^i is given by

$$H_t^i(j,j') = h_t(j';i,j), \tag{4}$$

where $h_t(j';i,j) = 1$ if $h_t(i,j) = j'$ and 0 otherwise. Also, let $r_t^i : \mathbb{Z} \to \mathbb{R}$, where $r_t^i(j)$, the date-t morning "return" to a type-i person in state j, is given by

$$\begin{aligned} r_t^i(j) &= \frac{m_1}{K}\sum_z \pi_t^1(z)\{u[y_t(i,j,1,z)] - y_t(1,z,i,j)\} + \\ &\quad \frac{m_2}{K}\sum_z \pi_t^2(z)\{u[y_t(i,j,2,z)] - y_t(2,z,i,j)\}. \end{aligned} \tag{5}$$

Let $v_t = (v_t^1, v_t^2)$, where $v_t^i : \mathbb{Z} \to \mathbb{R}$ and $v_t^i(z)$ is the date-t pre-morning discounted expected utility for a type i person in state z and let $w_t = (w_t^1, w_t^2)$, where $w_t^i : \mathbb{Z} \to \mathbb{R}$ and $w_t^i(z)$ is the date-t pre-afternoon discounted expected utility for a type i person in state z. We have

$$v_t^i = r_t^i + T_t^i w_t^i \text{ and } w_t^i = \beta H_t^i v_{t+1}^i. \tag{6}$$

To express the truth-telling and participation constraints compactly, another bit of intermediate notation is helpful. Let

$$G_t^c(i,z,s) = u[y_t(i,z,s)] + w_t^i[c_t(i,z,s)], \tag{7}$$

the date-t morning payoff to a type i, state z consumer in a meeting with an s producer, and let

$$G_t^p(s,i,z) = -y_t(s,i,z) + w_t^i[p_t(s,i,z)], \tag{8}$$

the date-t morning payoff to a type i, state z producer in a meeting with an s consumer.

The constraints that pertain to type-2 people are identical for outside- and inside-money allocations. Those that pertain to the monitored people are different. Therefore, I give the former first and then give the latter separately for each kind of allocation.

The truth-telling constraints for type-2 people in pairwise meetings with other type-2 people are

$$\max_{z' \le j} \sum_z \pi_t^2(z) G_t^c(2,z',2,z)] = \sum_z \pi_t^2(z) G_t^c(2,j,2,z), \tag{9}$$

and

$$\max_{z' \leq j} \sum_{z} \pi_t^2(z) G_t^p(2, z, 2, z') = \sum_{z} \pi_t^2(z) G_t^p(2, z, 2, j), \tag{10}$$

where the first is for a consumer and the second is for a producer. These must hold for all $j \in \mathbb{Z}$. The participation constraints for these meetings are

$$\min\{G_t^c(2, j, 2, z), G_t^p(2, z, 2, j)\} \geq w_t^2(j), \tag{11}$$

which must hold for all $(j, z) \in \mathbb{Z} \times \mathbb{Z}$. The constraints for type-2 people in meetings with type-1 people can be written

$$\max_{z' \leq j} G_t^c(2, z', 1, z) = G_t^c(2, j, 1, z)] \geq w_t^2(j) \tag{12}$$

and

$$\max_{z' \leq j} G_t^p(1, z, 2, z') = G_t^p(1, z, 2, j) \geq w_t^2(j) \tag{13}$$

where, in each case, the equality is truth-telling and the inequality is the participation constraint. These, also, must hold for all $(j, z) \in \mathbb{Z} \times \mathbb{Z}$. For the centralized meeting, we have

$$\max_{z \leq j} v_{t+1}^2[h_t(2, z)] = v_{t+1}^2[h_t(2, j)] \geq v_{t+1}^2(j) \tag{14}$$

for all $j \in \mathbb{Z}$.

For both inside and outside-money allocations, I permit creation of money in the centralized meeting. For type-2 people, all state transitions have non negative transfers because any type-2 person can always report state 0. More generally, in order to induce truth-telling, all transfers to type-2 people must be weakly increasing in a type-2 person's state.

I begin with a definition of incentive-feasible, inside-money allocations.

Definition 1 *A sequence $\{y_t, c_t, p_t, h_t\}_{t=0}^{\infty}$ is an incentive-feasible, inside-money allocation if there exists $\{v_t, w_t, \theta_t, \pi_t\}_{t=0}^{\infty}$ such that (1)-(14) hold and such that*

$$\min\{G_t^c(1, z, s), G_t^p(s, 1, z)\} \geq w_t^2(0) \text{ for all } (s, z) \in \mathbb{S} \times \mathbb{Z}, \tag{15}$$

$$w_t^1[p_t(2, z, 1, z')] \geq w_t^2[z - c_t(2, z, 1, z')] \text{ for all } (z, z') \in \mathbb{Z} \times \mathbb{Z}, \tag{16}$$

$$w_t^1(z) \geq w_t^2(0) \text{ and } v_{t+1}^1[h_t(1, z)] \geq v_{t+1}^2(0) \text{ for all } z \in \mathbb{Z}, \tag{17}$$

and

$$c_t(2, z, 2, z') + p_t(2, z, 2, z') = z + z' \text{ for all } (z, z') \in \mathbb{Z} \times \mathbb{Z}. \tag{18}$$

Condition (15) is the requirement that a type-1 person says *yes* to the trade the allocation specifies. If the person says *no*, then the person begins the afternoon as a type-2 person with no money. That is the alternative because saying *no* implies no trade and because the type-1 person entered the meeting with no other money. Condition (16) says that the type-1 person is willing to destroy any money received in trade with a type-2 person. This is written only for the case when the trading partner is a consumer because if w_t^2 is strictly increasing, then a type-2 producer does not surrender money.[5] (By the way, an allocation could call for type-2 people to surrender money to type-1 people when they receive output even though the money is to be immediately destroyed. Taking money from type-2 people in such trades can enhance its value and that can, in turn, weaken the constraints involving production by type-2 people.) Condition (17) is the requirement that a type-1 person not defect just prior to the centralized meeting and that the person says *yes* to the afternoon state transition. Condition (18) says that money holdings are preserved in meetings among type-2 people.

Implicit in these constraints is that a defector's printing press becomes useless. The interpretation is that the money turned out by that press is no longer valued; that is, that everyone stops accepting it. Of course, this may hurt those who are holding that money at the time of the defection.[6]

Now I turn to outside-money allocations. Here, as noted above, the only valued money is a uniform money that no individual person can create, and the state for a type-1 person is interpreted as the person's holding of that money. In accord with that, I restrict state transitions in single-coincidence meetings so that the amount of money in all meetings is preserved.

Definition 2 *A sequence $\{y_t, c_t, p_t, h_t\}_{t=0}^{\infty}$ is an incentive-feasible, outside-money allocation if there exists $\{v_t, w_t, \theta_t, \pi_t\}_{t=0}^{\infty}$ such that (1)-(14) hold and such that*

$$\min\{G_t^c(1,z,s), G_t^p(s,1,z)\} \geq w_t^2(z) \text{ for all } (s,z) \in \mathbb{S} \times \mathbb{Z}, \tag{19}$$

$$w_t^1(z) \geq w_t^2(z) \text{ and } v_{t+1}^1[h_t(1,z)] \geq v_{t+1}^2(z) \text{ for all } z \in \mathbb{Z}, \tag{20}$$

[5] The ability of type-2 people to under-represent money holdings implies that w_t^2 is weakly increasing. And unless money is worthless, it must be strictly increasing. It should be understood that condition (16) is void if $z - c(2, z, 1, j) < 0$.

[6] If the printing presses printed dated notes, then the printing press could be rendered useless without affecting the notes issued earlier.

and

$$c_t(s_c, s_p) + p_t(s_c, s_p) = z + z' \textit{ for all } (s_c, s_p) \in \mathbb{S} \times \mathbb{S}. \tag{21}$$

Constraints (19) and (20) in definition 2 mimic (15) and (17) in definition 1, except that in definition 2 a defector keeps the outside money the person has. Constraint (21) is the same as constraint (18) in definition 1—except that it applies to all meetings, not just those between type-2 people.

3 A subset result

I can now prove that the set of incentive-feasible outside-money allocations is a subset of the incentive-feasible inside-money allocations.

Proposition 1 *If $\{y_t, c_t, p_t, h_t\}_{t=0}^{\infty}$ is an incentive-feasible, outside-money allocation, then it is an incentive-feasible, inside-money allocation.*

Proof. As noted above, because type-2 people can hide money, w_t^2 and v_t^2 are weakly increasing. It follows that if (19) and (20) in definition 2 hold, then (15) and (17) in definition 1 hold. And, obviously, (21) implies (18). Thus, it remains to show that (16) holds. We have

$$p_t(2, z, 1, z') = z + z' - c_t(2, z, 1, z') \geq z - c_t(2, z, 1, z'), \tag{22}$$

where the equality follows from (21). But then

$$w_t^1[p_t(2, z, 1, z')] \geq w_t^2[z + z' - c_t(2, z, 1, z')] \geq w_t^2[z - c_t(2, z, 1, z')], \tag{23}$$

where the first inequality follows from the first inequality in (20) and the second from the fact that w_t^2 is weakly increasing. ■

The advantages of inside money are evident from definitions 1 and 2. First, under inside money, in meetings between type-1 consumers and type-2 producers, there is no need to preserve the sum of the states in the meeting. Second, under inside money a defector gets to begin the next period, as a type-2 person, with no more than the money earned in the last pairwise meeting. The ability under outside money of a defector to use any outside money held seems to be the crucial difference between inside- and outside-money allocations. If the defection payoffs were the same, then type–1 people could be given sufficiently high transfers in the centralized market and these would overcome the constraint that outside money cannot be created in meetings between type-1 consumers and type-2 producers. However, as the model is formulated, such transfers have to be limited because they increase defector payoffs.

Ultimately, we are interested in good allocations. The simplest criterion is a representative-agent criterion, interpreted as expected utility prior to the assignment of types and prior to the assignment of states; namely, $W = \sum_{i=1}^{2} m_i(\pi_0^i v_0^i)$. A little manipulation of (1)-(6) implies that

$$W = \frac{1}{K}\sum_{t=0}^{\infty}\left\{\beta^t \sum_{z}\sum_{z'}\left[\sum_{i=1}^{2}\sum_{j=1}^{2}[m_i m_j \pi_t(z)\pi_t(z')g(y_t(i,z,j,z'))]\right]\right\}, \quad (24)$$

where $g(x) \equiv u(x) - x$. That is, as one would surmise, representative-agent welfare is just the discounted sum of the expected value of the excess of utility of consumption over the disutility of production over all single-coincidence meetings. And, of course, if x^* denotes the maximizer of $g(x)$, then an upper bound on W is $g(x^*)/K(1-\beta)$, the value of W achieved if x^* is produced in every single-coincidence meeting.[7]

Proposition 1 implies that the search for good allocations can be limited to inside-money allocations. Beyond that, I can offer only a few vague conjectures.

According to the model, it is desirable to insulate a person's current prospects from the particular realizations of consumption and production opportunities that the person has experienced. The ability to do that for non monitored people is very limited. As noted above, state transitions in the centralized market for such people are limited to non negative transfers which must be weakly increasing in a person's money holdings. Because making such transfers strictly increasing seems to run counter to the goal of insuring the non monitored, a surmise is that such transfers should be lump-sum transfers. However, such transfers tend to be inflationary, and, therefore, have to be limited.[8]

As regards monitored people, it may seem that nothing should depend on their states, something that was actually assumed in [1]. However, that may not be best. If nothing depends on the state of monitored people and if

[7]Kocherlakota [7] describes a mechanism that achieves the upper bound in this setting. However, his mechanism is vulnerable to cooperative defection by pairs in meetings.

[8]See Molico [9], Deviatov and Wallace [4], and Deviatov [3] for analyses of transfers in versions of the model with no monitored people.

they create money when they are consumers in meetings with non monitored producers, then in order to limit inflation monitored people must produce and acquire (and destroy) money when they are producers in meetings with non monitored people. But, if nothing depends on the state of monitored people, then such production is a gift and is likely to be constrained by participation constraints. Some dependence on states for monitored people could loosen such constraints.

4 Weakening the recognizability assumption

Given proposition 1, why did we see the evolution of central-bank monopolies on note issue? One possible answer is that the model set out above is in crucial respects off the mark. One questionable feature is the recognizability assumption. Recognizability has for long been on the standard list of desirable properties of money. Like the other properties on this list—for example, divisibility and portability—its appearance suggests that the property is in some sense scarce. Recognizability means the ease with which a genuine object can be distinguished from fakes or counterfeits. Hence, if we are to depart from the recognizability assumption made above, then we have to model counterfeiting and its relationship to inside money and to outside money.

One formulation that permits a weakening of the recognizability assumption is in Williamson and Wright [16]. They model the degree of recognizability of goods in terms of the probability that a person receives a completely informative signal rather than a completely uninformative signal about whether the good is genuine. Nosal and Wallace [10] apply that formulation to outside money. After describing their model, I will hint at how to adapt it to the setting described above.

In many respects, the model in [10] is a special case of the setting described above. There is no centralized meeting, there is a single outside money, there are no monitored people, and individual money holdings are in the set $\{0, 1\}$. At the beginning of each date, each person can produce a unit of counterfeit money at a cost in terms of utility. Counterfeits are perishable. This gives sellers an unambiguous incentive to avoid receiving counterfeits and implies that the stock of counterfeits is not a state variable. In a meeting between a buyer and a seller, first the pair receives a signal: with probability ϕ, the signal is informative and reveals whether the buyer's money is genuine or counterfeit; with probability ϕ, the signal is completely informative. Then the buyer makes a take-it-leave-it lottery offer.

Under those assumptions, Nosal and I show the following: if a positive linear function of the cost of counterfeiting and the probability of getting the informative signal is high enough, then there is a monetary equilibrium in which no counterfeiting occurs; otherwise, there is no monetary equilibrium that satisfies the Cho-Kreps refinement. The Cho-Kreps refinement is applicable because counterfeits will be accepted only if the uninformative signal is realized and only if there is pooling in that circumstance. But if there is such pooling, then a holder of genuine money could profitably defect by offering a lower probability of giving up money in exchange for less output, a defection that is not profitable for a holder of counterfeit money because such a person cares only about output received.

One main assumption I would make in adapting that model to the setting studied above would be to assume that the probability of receiving the informative signal is lower the greater the number of distinct objects to be recognized. While admittedly a brute force assumption, it goes well with the Williamson-Wright formulation of the recognizability problem. And to open the way to applying the Cho-Kreps refinement, I would assume that in meetings among non monitored people, buyers make take-it-leave-it lottery offers, possibly a schedule of offers that is a function of the money holdings of the seller taking into account the seller's ability to hide money. In all other meetings, the kind of *yes-no* game described above would continue to be played. In other respects, versions of the Nosal-Wallace assumptions can be made—although describing the technology for producing counterfeits is necessarily more complicated when the $\{0, 1\}$ set of allowable holdings is generalized.[9] In any case, in such a model there ought to be a region in the parameter space, possibly including the assumption that m_2 is large, for which the following is true: there are incentive-feasible outside-money (uniform money) allocations with valued money in which no counterfeiting occurs, and the only incentive-feasible inside-money allocations are those which emulate outside-money allocations by not having monitored people issue money.[10]

[9]In addition to a more complicated technology for producing counterfeits, in a more general setting complications arise because buyers and sellers can conceivably have some genuine money and some counterfeit money.

[10]Williamson [15] obtains a result of this kind in a very different setting. Although Williamson attaches the label *private money* to some of the assets in his model, they

Of course, there are any number of ways of weakening the recognizability assumption. So long as it is weakened to retain the plausible notion that a system with many distinct monetary objects is more vulnerable to counterfeiting than one with a single uniform object, the advantages that inside money has in terms of the constraints in definitions 1 and 2 could be overcome. If so, then we would want to describe good outside-money arrangements.

As regards non monitored people, I have nothing to add to what was said above. For monitored people, a good outside-money arrangement would tend to have centralized-market state transitions that shift money away from those with a lot of it to those with little of it. That is a way to achieve some separation of a person's current prospects from the realizations they have experienced.[11]

5 Concluding remarks

A weakness of the above presentation is that somewhat general ideas are exposited against the background of a very specific model.[12] Thus, the subset result—incentive-feasible outside-money allocations are a subset of incentive-feasible inside-money allocations—would seem to be quite general. For example, I suspect that it survives the introduction of private-information

are better labeled capital because they are producible assets with real, random pay-offs. In fact, those assets have a return distribution that dominates that on outside money. Despite that, in the absence of counterfeiting, Williamson finds a steady state in which both assets are held. I suspect that the indivisibility of assets and the unit bound on individual holdings in his model accounts for that result.

[11] Labeling the h function as the actions of a central bank may seem gratuitous. However, I have not imposed the restriction that the stock of outside money is constant. If it is changing, then that calls for something like central bank activity. Also, the form of a good h function is very dependent on the background environment. Settings with additional private information—about idiosyncratic taste or technology shocks—would tend to make good h functions more closely resemble the operations of a central bank discount window.

[12] There are potential uses of a particular background environment. It would be helpful in showing that the set of implementable inside-money allocations includes allocations with trade. (Indeed, to show that, it may also be necessary to replace the set of individual states by a bounded set.) And the background environment would be helpful in showing that the converse of the subset result does not hold.

preference or technology shocks of the sort studied in Green [5].[13]

Aside from the subset result, the other ideas that seem not to be special to the particular model are the following. Non monitored people are the source of a demand for transferable, tangible objects. If monitored people can create such objects in their meetings with non monitored people, objects I label inside money, then their spending is freed from dependence on their recent trades. But, if inside money is to work well, then issuers cannot be allowed to defect and to continue issuing. That, in turn, requires that the inside money of one issuer be distinguishable from inside money issued by others. But because the inside monies get passed around among the non monitored people, such recognizability seems problematic. If sufficiently problematic, then it may be best to have a uniform money and a single issuer and to manage the uniform-money system so as to attain some of the desirable features of inside-money.

Although none of the above ideas is new, putting them together provides a somewhat new perspective on the development of central banking and the 19-th century debates about the regulation of private banking and the management of a central bank monopoly on note issue . Even the background environment seems reasonable for the purpose to which it is put. According to Lloyd Mints (see [8], pages 10,11), Adam Smith invented the *real bills doctrine* in the following passage:

> When a bank discounts to a merchant a real bill of exchange drawn by a real creditor upon a real debtor, and which, as soon as it becomes due, is really paid by that debtor; it only advances to him a part of the value which he would otherwise be obliged to keep by him unemployed and in ready money for answering occasional demands ([12], page 323).

Smith's real debtor is like a monitored person in the model. Such a person's ability to produce plays the backing role implicit in Smith's description of a *real bill*. And, although the required results are not yet in hand, there is a strong presumption that the model favors the banking school rather than the currency school. The model suggests that there is a role for inside money under the recognizability assumption. However, it gives no support to

[13] A limited analysis of good inside-money allocations in the presence of extreme, private-information, productivity shocks is in [1].

the notion that inside money can be fruitfully studied using the competitive framework.

The ideas set out above are not only relevant to monetary history. Versions of the inside-outside money issues debated in the 19-th century are still with us. How should central-bank discount windows operate? What role should central banks have in providing intra-day credit? And how should stored value—the modern day equivalent of private banknotes—be regulated? All of these questions are closely related to the 19-th century debates about inside and outside money.

References

[1] R. Cavalcanti and N. Wallace, A model of private banknote issue. *Rev. of Econ. Dynamics* 2 (January 1999), 104-36.

[2] R. Cavalcanti and N. Wallace, Inside and outside money as alternative media of exchange. *J. of Money, Banking, and Credit.* (August, 1999). part 2. 443-57.

[3] A. Deviatov, Money creation in a random-matching model. manuscript, 2003, http://www.nes.ru/˜deviatov.

[4] A. Deviatov and N. Wallace, Another example in which lump-sum money creation is beneficial. *Advances in Macroeconomics*, Vol. 1: No. 1, (2001), Article 1.

[5] E. J. Green, Lending and the smoothing of uninsurable income. In *Contractual Arrangements for Intertemporal Trade* ed by E.C. Prescott and N. Wallace. University of Minnesota Press, Minneapolis 1987, 3-25.

[6] N. Kocherlakota, Money is memory. *J. of Econ. Theory* **81** (August 1998), 232-51.

[7] Kocherlakota, The two-money theorem. *International Econ Rev*, **43** (2002), 333-46.

[8] L. Mints, *A History of Banking Theory*, University of Chicago Press, 1945.

[9] M. Molico, The distribution of money and prices in search equilibrium. Unpublished Ph.D. dissertation. The University of Pennsylvania, 1997.

[10] E. Nosal and N. Wallace, A model of counterfeiting, manuscript, 2004.

[11] S. Shi, Money and prices: a model of search and bargaining. *J. of Econ. Theory* **67** (1995), 467-498.

[12] A. Smith, *The Wealth of Nations*, The University of Chicago Press, 1976.

[13] A. Trejos and R. Wright, Search, bargaining, money and prices. *J. of Political Economy* **103** (1995) 118-141.

[14] N. Wallace and T. Zhu, A commodity-money refinement in matching models. *J. of Econ. Theory*, (forthcoming).

[15] S. Williamson, Private money and counterfeiting, *Econ. Quarterly,* Federal Reserve Bank of Richmond, 88/3, (Summer 2002), 37-57.

[16] S. Williamson and R. Wright, Barter and monetary exchange under private information. *Amer. Econ. Rev.* **84** (1994), 104-123.