Journal of Banking & Finance 35 (2011) 182–192

Contents lists available at ScienceDirect

Journal of Banking & Finance

journal homepage: www.elsevier.com/locate/jbf

Asset-liability management under time-varying investment opportunities

Robert Ferstl [a,*], Alex Weissensteiner [b]

[a] Department of Finance, University of Regensburg, Germany
[b] School of Economics and Management, Free University of Bolzano, Italy

ARTICLE INFO

Article history:
Received 15 February 2010
Accepted 20 July 2010
Available online 24 July 2010

JEL classification:
C61
G11

Keywords:
Asset-liability management
Predictability
Stochastic programming
Scenario generation
VAR process

ABSTRACT

Stochastic linear programming is a suitable numerical approach for solving practical asset-liability management problems. In this paper, we consider a multi-stage setting under time-varying investment opportunities and propose a decomposition of the benefits in dynamic re-allocation and predictability effects. We use a first-order unrestricted vector autoregressive process to model asset returns and state variables and include, in addition to equity returns and dividend-price ratios, Nelson/Siegel parameters to account for the evolution of the yield curve. The objective is to minimize the Conditional Value at Risk of shareholder value, i.e., the difference between the mark-to-market value of (financial) assets and the present value of future liabilities.

1. Introduction

One of the classical problems in finance is the derivation of optimal dynamic investment strategies over a finite planning horizon, where the uncertainty is modeled with stochastic processes driving asset returns and state variables. Early works go back to the pioneering papers of Samuelson (1969) and Merton (1969). The use of a geometric Brownian motion, i.e., a constant risk premia, ensures analytical tractability. If such a process describes reality well, investors should hold a constant asset allocation over time for utility functions of the CRRA class. On the one hand, this result contrasts to the prevailing common practice that recommends more risk-taking for longer investment horizons. On the other hand, extensive empirical literature finds predictability in asset returns. A typical specification regresses an independent lagged predictor, e.g., dividend-price ratio, earnings-price ratio, interest rates and spreads, on the stock market return or on the equity premium. Beginning with contributions of Keim and Stambaugh (1986) and Campbell and Shiller (1988), several authors discuss the question again actively in the past decade (e.g., Cochrane and Piazzesi, 2005; Ang and Bekaert, 2007; Goyal and Welch, 2008; Campbell and Thompson, 2008).

Many papers analyze the impact of time-varying investment opportunities on the optimal strategy of a utility maximizing investor and find deviations from a pure myopic policy. However, analytical results such as those in Kim and Omberg (1996) and Wachter (2002) are the exception rather than the rule. The overwhelming part of the literature employs numerical methods. In addition to approximate analytical approaches (see, e.g., Campbell et al., 2003), two main types of numerical solution techniques can be found in the literature: While the first approach discretizes the state space and solves the problem by backward induction (e.g., Barberis, 2000), the second method is simulation-based (e.g., Brandt et al., 2005).

In this paper we propose stochastic linear programming (SLP) for an asset-liability management (ALM) problem of a pension fund when investment opportunities are time-varying. We choose the multi-period SLP approach for its attractive combination of efficiency and flexibility. While many other numerical methods focusing on more general insights are confined to a small number of risky assets (often only one), do not allow for constraints on the asset allocation, ignore transactions costs and taxes, and cannot include coherent risk measures (e.g., Conditional Value at Risk) or exogenous cash flows (e.g., heritage or labor income), the SLP approach enables us to incorporate such important real-world aspects. Furthermore, the recursive structure of such multi-period optimization problems encourages the development of efficient and robust (linear and non-linear)

* Corresponding author. Tel.: +49 941 943 2693; fax: +49 941 943 812693.
E-mail addresses: robert.ferstl@wiwi.uni-regensburg.de (R. Ferstl), alex.weissensteiner@unibz.it (A. Weissensteiner).

0378-4266/$ - see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.jbankfin.2010.07.028

solvers. For successful applications to related problems see, e.g., Gondzio and Kouwenberg (2001) and Consiglio et al. (2006). In accordance with other authors (e.g., Campbell et al., 2003; Brandt et al., 2005), we use a first-order unrestricted vector-autoregressive process VAR(1) to model asset returns and other state variables. Given that the evolution of the whole term structure plays a decisive role in an ALM context, a parametric approach is appropriate in maintaining computational tractability. Following Boender et al. (2005), we use the Nelson and Siegel (1987) exponential component framework. Further, we propose to combine the historical parameter estimates with log equity returns and log dividend-price ratios to calibrate a VAR(1) process as the basis for the scenario generation. To exploit short-run asset return predictability, we implement a multi-stage optimization setting.

The key parameter, which has to be kept under control in an ALM optimization task, is the difference between the mark-to-market value of (financial) assets and the present value of future liabilities (or, in general, cash flows). For example, a negative value indicates that the pension plan is underfunded. This difference between assets and liabilities can also be interpreted as the shareholder value (SV) of the benefit owners. Therefore, we minimize the Conditional Value at Risk (*CVaR*) of the SV under the constraint that some expected value for the SV is attained at the end of the planning horizon. In addition to the classical budget, inventory and asset allocation constraints, we restrict the maximum drawdown in SV between two adjacent stages to lie above some prespecified level for each path in our scenario tree. This amount represents the maximum loss a sponsor of the plan is willing to suffer during one period. Other articles using *CVaR* in financial optimization tasks are, e.g., Rockafellar and Uryasev (2002), Mulvey and Erkan (2006) and Quaranta and Zaffaroni (2008).

This paper contributes to the existing literature in the following ways: we propose a parsimonious method for scenario generation that accounts for bond and equity return predictability with a small number of explanatory factors. Therefore, we combine log equity returns and log dividend-price ratios with Nelson/Siegel parameters in a VAR(1) process. The model captures important stylized facts found in the finance literature, both for the equity returns and for the evolution of the yield curves. We show how to exploit the predictability effects within a multi-period asset-liability management setting formulated as SLP and provide a sensitivity analysis on various factors influencing the optimal solution (e.g., for the target SV and for the maximum drawdown constraint). Furthermore, due to the contradictory results in the existing literature on forecasting equity returns, we propose a decomposition of the predictability effects. For that purpose, we introduce a second VAR process that covers only the bond return predictability. A comparison of our findings enables us to quantify the difference in terms of expected final SV, and to decompose the effects in benefits from the possibility to re-allocate the portfolio strategically over time and benefits from the equity return predictability in the initial setting. Although the magnitude of the different outcomes in our analysis is inevitably linked to the particular parameters chosen in our examples, we can provide an economic explanation of the results.

The paper is organized as follows: in Section 2 we present the notation and the stochastic linear programming formulation. Section 3 explains the scenario generation procedure, which includes the estimation of Nelson/Siegel parameters, the calibration of the VAR(1) model, and the generation of arbitrage-free asset returns. In Section 4 we present a numerical example and discuss the results as well as the economic implications of the proposed strategy. An analysis of the predictability effects is provided in Section 5 and Section 6 concludes the paper.

2. Model

2.1. Asset-liability management problem

We consider the following asset-liability management model, which is formulated as a multi-stage stochastic linear program with recourse, see Fig. 1.

A company plans to minimize the Conditional Value at Risk (*CVaR*) of its shareholder value V_T^s at the end of a planning horizon T by making asset allocation decisions at discrete time stages $t = 0, \ldots, T-1$. It can choose between $i = 1, \ldots, N$ assets where $i = 1$ is an equity and $i = 2, \ldots, N$ are zero-coupon bonds. The wealth in asset i after each investment is given by W_0^i and $W_t^{i,s}$.

Further, we assume that a deterministic cash flow L_t takes place at times m_t with $t = 0, \ldots, T, \ldots, \mathcal{T}$ that can occur after the end of the planning horizon.[1] This kind of setting is typical for a defined benefit pension fund where the future payouts to its contributors are fixed. The simplest way to hedge the interest-rate risk is by constructing a portfolio of zero-coupon bonds with appropriate maturities. However, this approach would not take into account the predictability of asset returns and other state variables within the planning horizon. Therefore, we consider scenarios $s = 1, \ldots, \mathcal{S}$ and construct a scenario tree consisting of the stochastic asset returns. We model the uncertainty by a VAR(1) process incorporating stock returns, dividend-price ratios and level, slope and curvature of the term structure of interest rates. A detailed description of the scenario generation procedure follows in Section 3.

2.2. Stochastic linear programming formulation

We define the following additional notation:*Indices*

s — trajectory that corresponds to a unique path in the scenario tree with $s = 1, \ldots, \mathcal{S}$

i — available asset (class) in the market with $i = 1, \ldots, N$

User-specified parameters

α — confidence level for *VaR* and *CVaR*

θ — target (minimum) expected shareholder value over the planning horizon

γ — maximum loss in the shareholder value between two adjacent stages

l^i, u^i — lower and upper bounds on the portfolio weights of asset i

Deterministic input data

w_0^i — initial value of the asset i before transactions

τ_P^i, τ_S^i — proportional transaction costs of the asset i for purchases and sales

$L_t\ (t = 0, \ldots, T, \ldots \mathcal{T})$ — deterministic cash flow at stage t

Scenario-dependent data

p^s — probability of scenario s, we generate trees with equiprobable scenarios using the algorithm described in Section 3.3

$R_t^{i,s}\ (1 \leqslant t \leqslant T)$ — gross returns of asset i at stage t, see Section 3.3

$\boldsymbol{\beta}_t^s\ (0 \leqslant t \leqslant T)$ — Nelson/Siegel parameter vector for the yield curve at stage t, see Section 3.1

$\delta(\boldsymbol{\beta}_t^s, m_t^\tau)\ (0 \leqslant t \leqslant T)$ — discount factor at stage t calculated by $e^{-y(\boldsymbol{\beta}_t^s, m_t^\tau) m_t^\tau}$, where m_t^τ indicates the time to maturity (in years) from t to τ and $y(\boldsymbol{\beta}_t^s, m_t^\tau)$ the corresponding yield

$\mathcal{L}_t^s\ (0 \leqslant t \leqslant T)$ — scenario-dependent present value at stage t of all future cash flows, which is calculated within the planning horizon:

$$\mathcal{L}_t^s = \sum_{\tau=1}^{\mathcal{T}-t} L_{t+\tau} \delta(\boldsymbol{\beta}_t^s, m_t^\tau) \quad \forall s,\ 0 \leqslant t \leqslant T. \tag{1}$$

[1] Note that t is a time index and m_t is the corresponding time interval in years between zero and stage t.

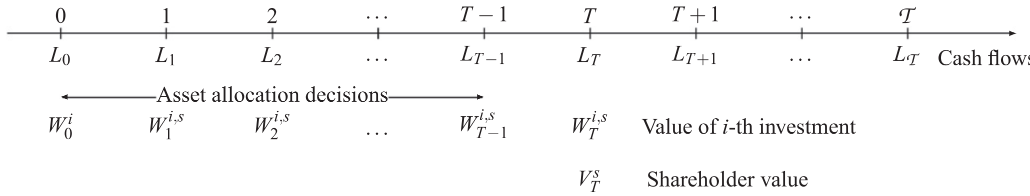

Fig. 1. Overview of the decision model.

Decision variables
$P_0^i \geqslant 0, P_t^{i,s} \geqslant 0$ $(1 \leqslant t \leqslant T-1)$ purchases of asset i at stage t
$S_0^i \geqslant 0, S_t^{i,s} \geqslant 0$ $(1 \leqslant t \leqslant T-1)$ sales of asset i at stage t

Auxiliary variables
$W_0^i, W_t^{i,s}$ $(0 \leqslant t \leqslant T)$ wealth in asset i at stage t
V_t^s $(0 \leqslant t \leqslant T)$ shareholder value, i.e., the sum of the mark-to-market value of the financial assets and the present value of the future cash flows, at stage t
ϕ *VaR* at the optimal solution required for *CVaR* definition
$\psi_T^{+s} \geqslant 0$ shareholder value shortfall in excess of *VaR* at stage t
$\psi_T^{-s} \geqslant 0$ shareholder value surplus in excess of *VaR* at stage t

The split variable formulation of the multi-stage stochastic program with recourse is listed in (2)–(15). In the objective function (2) we minimize the Conditional Value at Risk $CVaR_\alpha$ of the shareholder value (over the decision and auxiliary variables specified above) with confidence level $\alpha \in]0,1[$, which is a convex function of the assets in the portfolio and a coherent risk measure. For discrete distributions, $CVaR_\alpha$ can be expressed by a linear programming formulation according to Rockafellar and Uryasev (2000), Rockafellar and Uryasev (2002).

$$CVaR_\alpha = \min\left\{\phi + \frac{1}{1-\alpha}\sum_{s=1}^{\mathcal{S}} p^s \psi_T^{+s}\right\} \tag{2}$$

$$\text{subject to}: W_0^i = w_0^i + P_0^i - S_0^i \quad \forall i, \tag{3}$$

$$W_t^{i,s} = R_t^{i,s} W_{t-1}^{i,s} + P_t^{i,s} - S_t^{i,s} \quad \forall i,s;\ 1 \leqslant t \leqslant T-1, \tag{4}$$

$$\sum_{i=1}^{N} P_t^{i,s}\left(1+\tau_P^i\right) = \sum_{i=1}^{N} S_t^{i,s}\left(1-\tau_S^i\right) + L_t \quad \forall i,s;\ 0 \leqslant t \leqslant T-1, \tag{5}$$

$$\sum_{i=1}^{N} W_t^{i,s} > 0 \quad \forall s\ 0 \leqslant t \leqslant T-1, \tag{6}$$

$$l^i \leqslant \frac{W_t^{i,s}}{\sum_{i=1}^{N} W_t^{i,s}} \leqslant u^i \quad \forall i,s;\ 0 \leqslant t \leqslant T-1, \tag{7}$$

$$V_t^s = \sum_{i=1}^{N} W_t^{i,s} + \mathcal{L}_t^s \quad \forall s;\ 0 \leqslant t \leqslant T-1, \tag{8}$$

$$V_T^s = \sum_{i=1}^{N} R_T^{i,s} W_{T-1}^{i,s} + L_T + \mathcal{L}_T^s \quad \forall s, \tag{9}$$

$$V_t^s \delta\left(\boldsymbol{\beta}_{t-1}^s, m_{t-1}^\tau\right) + \gamma \geqslant V_{t-1}^s \quad \forall s;\ 1 \leqslant t \leqslant T;\ \tau = 1, \tag{10}$$

$$\psi_T^{+s} = -V_T^s - \phi + \psi_T^{-s} \quad \forall s, \tag{11}$$

$$\sum_{s=1}^{S} p^s V_T^s \geqslant \theta, \tag{12}$$

$$\psi_T^{+s} \geqslant 0, \quad \psi_T^{-s} \geqslant 0\ \forall s, \tag{13}$$

$$P_t^{i,s} \geqslant 0, \quad S_t^{i,s} \geqslant 0 \quad \forall i,s; \quad 0 \leqslant t \leqslant T-1, \tag{14}$$

$$P_t^{i,s} = P_t^{i,s'}, \quad S_t^{i,s} = S_t^{i,s'} \quad \forall i,s;\ 0 \leqslant t \leqslant T-1, \tag{15}$$

$\forall s, s'$ with identical past up to time t.

While the first-stage decision variables have to fulfill the inventory equations in (3), forcing the mark-to-market value of each asset to equal the initial holdings w_0^i plus purchases minus sales, the second-stage inventory equations in (4) also account for the stochastic gross returns $R_t^{i,s}$ on the holdings of the previous period $W_{t-1}^{i,s}$.[2] The budget equation in (5) contains the scenario-dependent decision variables for purchases and sales of available securities, the proportional transaction costs τ_P^i and τ_S^i as well as the cash flows L_t.

We restrict the total wealth in (6), i.e., the sum over all holdings, so that it remains positive at all time stages where asset allocation decisions are taken. However, the mark-to-market value of an individual asset can become negative. The lower bounds l^i and the upper bounds u^i defined in (7) restrict the feasible portfolio weights. These are particularly relevant for institutions which have to meet regulatory requirements (e.g., investment directives imposed by law).

We denote the shareholder value in (8) as sum of the total mark-to-market values of the assets plus the present value of all future cash flows. Note that at the current stage the cash flow is already included through the inventory equations, but these are only defined until $T-1$. Therefore, we add L_T to the shareholder value at the end of the planning horizon in (9). Constraint (10) ensures that the maximum loss in the shareholder value between two adjacent stages is below a given level γ. This leads to a stronger consideration of the interest-rate risk within the planning horizon, as the uncertain changes in the yield curve affect also the present value of the cash flows (in addition to the asset returns). Given the shareholder value at stage t in (9), the portfolio shortfall in excess of Value at Risk used in the objective function is $\psi_T^{+s} = \max\left[0, -V_T^s - \phi\right]$.[3] To determine the value of the maximum operator in the linear programming formulation we introduce two non-negative auxiliary variables ψ_T^{+s} and ψ_T^{-s}, see (11) and (13). In (12) we enforce that the expected shareholder value exceeds some prespecified level θ.

Condition (14) ensures the non-negativity of the decision variables for purchases and sales. Further, so-called "non-anticipativity constraints" are imposed in (15) to guarantee that a decision made at a specific node is identical for all scenarios leaving that node.

3. Modeling uncertainty

3.1. Term structure of interest rates

In an ALM context, where the main objective is to manage the shareholder value, the term structure of interest rates plays a central role. While the evolution of the yield curve influences returns for the different bond holdings, it also determines the present value of future cash flows. We propose to use the Nelson/Siegel model here for two reasons. On the one hand, this parsimonious

[2] Given that the first-stage decisions are deterministic, we omit the scenario index s in (3).

[3] Note that with this formulation $\psi_T^{+s} \geqslant 0$, i.e., a positive $CVaR_\alpha$ indicates a negative expected wealth conditional on a shortfall event.

parametric model can represent the entire yield curve with only a few parameters, restricting the size of the scenario tree and ensuring computational tractability of the SLP. On the other hand, some extensions which include the Nelson/Siegel model as a special case may not be superior in out-of-sample forecasting due to their potential overfitting of in-sample data (see, e.g., Diebold and Li, 2006).

The three-factor model for the spot rates can be written as:

$$y(\boldsymbol{\beta}_t, m) = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda_t m}}{\lambda_t m}\right) + \beta_{3,t}\left(\frac{1-e^{-\lambda_t m}}{\lambda_t m} - e^{-\lambda_t m}\right), \quad (16)$$

where $y(\boldsymbol{\beta}_t, m)$ indicates the (continuously compounded) spot rate for maturity m at stage t given the parameter vector $\boldsymbol{\beta}_t = [\beta_{1,t}, \beta_{2,t}, \beta_{3,t}]$. Due to the fixing of the loadings, the factors $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ can be interpreted as the level, slope and curvature of the term structure of interest rates. The first factor determines the long-term level of the spot rates as $y(\boldsymbol{\beta}_t, \infty) = \beta_{1,t}$, while the instantaneous yield depends on both the level and the slope factor by $y(\boldsymbol{\beta}_t, 0) = \beta_{1,t} + \beta_{2,t}$. This is due to the following facts: the factor loading of $\beta_{2,t}$ starts from a value of one and decreases asymptotically to zero for long maturities. The factor loading for the curvature is hump-shaped, approaching zero for very short and very long maturities. The parameter λ_t determines where the factor loading of the curvature achieves its maximum. Decreasing λ_t ensures a better fit for long maturities, while increasing this value enhances the fit for short maturities. Following Diebold and Li (2006), we fix λ_t at 0.1148, which minimizes the mean squared error in our data set. In this way, the estimation of the remaining parameters $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ reduces to an ordinary least squares (OLS) regression with the advantage of better numerical stability. We include these estimated Nelson/Siegel parameters then in the VAR(1) process presented below.

3.2. Time-varying investment opportunities

We model time-varying investment opportunities with an unrestricted, stationary VAR(1) process, where stationarity refers to time-invariant expected values, variances and covariances. In this paper we use the following $(K \times 1)$ parameter vector ξ_t (with $K = 5$):

$$\xi_t = \begin{bmatrix} r_t^1 \\ d_t - p_t \\ \boldsymbol{\beta}_t \end{bmatrix},$$

where $r_t^1 \equiv \log\left(R_t^1\right)$ refers to the log equity return, $d_t - p_t$ to the log dividend-price ratio and $\boldsymbol{\beta}_t$ to the (3×1) vector of Nelson/Siegel parameters. While many papers use the dividend-price ratio as predictor for equity returns (see, e.g., Barberis, 2000; Campbell et al., 2003; Brandt et al., 2005), Boender et al. (2005) propose modeling the evolution of yield curves with the Nelson/Siegel approach. We combine these two concepts in order to obtain a joint model for explaining bond and equity return predictability.

The idea behind a vector-autoregressive process is that an economic variable is not only related to its predecessors in time, but also depends linearly on past values of other variables. The functional form of a VAR(1) process can be written as:

$$\xi_t = \mathbf{c} + \mathbf{A}\xi_{t-1} + \mathbf{u}_t, \quad (17)$$

where $\mathbf{c}$ is the $(K \times 1)$ vector of intercepts, $\mathbf{A}$ is the $(K \times K)$ matrix of slope coefficients and $\mathbf{u}_t$ the $(K \times 1)$ vector of i.i.d. innovations with $\mathbf{u} \sim N(0, \boldsymbol{\Sigma}_u)$. The covariance matrix $\boldsymbol{\Sigma}_u$ is given by $\mathbb{E}(\mathbf{u}\mathbf{u}^\top)$. Thus, we allow the shocks to be cross-sectionally correlated, but assume that they are homoscedastic and independently distributed over time. If all eigenvalues of $\mathbf{A}$ have modulus less than one, as in our empirical example below, the stochastic process in (17) is stable with unconditional expected values $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Gamma}$ for the steady state at $t = \infty$ (see, e.g., Lütkepohl, 2005):

$$\boldsymbol{\mu} := (\mathbf{I} - \mathbf{A})^{-1}\mathbf{c},$$
$$\mathrm{vec}(\boldsymbol{\Gamma}) := (\mathbf{I} - \mathbf{A} \otimes \mathbf{A})^{-1}\mathrm{vec}(\boldsymbol{\Sigma}_u),$$

where $\mathbf{I}$ refers to the identity matrix, the symbol $\otimes$ is the Kronecker product and "vec" transforms a $(K \times K)$ matrix into a $(K^2 \times 1)$ vector by stacking the columns.

To estimate the intercepts and slope parameters of the VAR(1) process via OLS we use quarterly data from 1987.Q3 to 2007.Q4. We take the stock returns, which refer to the S&P 500 index, and the corresponding dividend-price ratios from the Goyal and Welch (2008) data set, while we estimate Nelson/Siegel parameters $\boldsymbol{\beta}_t$ from US spot rates provided by the Federal Reserve Board. The Schwarz Criterion (also known as Bayesian Information Criterion) indicates the autoregressive order of one. Table 1 reports the corresponding parameters (values for the t-statistics and p-values in parenthesis), where $|z|$ indicates the modulus of the eigenvalues of the characteristic polynomial.

The sample period of 80 quarters in our data set with five regression parameters and a confidence level of 95% gives a critical (absolute) t-value of 1.99.

In line with the current literature, the dividend-price ratio with a coefficient of 0.97 has very high persistent dynamics and — compared to other regressors — displays a large t-value for predicting equity returns (in Campbell et al. (2003) the corresponding t-value is equal to 2.32, and Brandt et al. (2005) report 0.87). Looking at the

Table 1
VAR(1) parameters, t-statistics in () and p-values in [].

	c	r_{t-1}^1	$d_{t-1} - p_{t-1}$	$\beta_{1,t-1}$	$\beta_{2,t-1}$	$\beta_{3,t-1}$	$R-$ sq.
r_t^1	0.3369	−0.0667	0.0691	−0.5306	−0.8973	−0.6467	0.1023
	(1.5523)	(−0.6537)	(1.7824)	(−0.4655)	(−1.4729)	(−1.8773)	
	[0.1248]	[0.5154]	[0.0788]	[0.6430]	[0.1450]	[0.0644]	
$d_t - p_t$	−0.1068	0.0990	0.9689	−0.4644	0.8860	0.4066	0.9636
	(−0.4888)	(0.9642)	(24.8347)	(−0.4046)	(1.4443)	(1.1719)	
	[0.6265]	[0.3381]	[0.0000]	[0.6870]	[0.1529]	[0.2450]	
$\beta_{1,t}$	0.0133	0.0157	0.0021	0.8936	0.1027	0.0465	0.8662
	(0.8837)	(2.2212)	(0.7773)	(11.2910)	(2.4274)	(1.9462)	
	[0.3797]	[0.0294]	[0.4394]	[0.0000]	[0.0176]	[0.0554]	
$\beta_{2,t}$	0.0039	0.0022	0.0014	0.0165	0.7954	−0.0593	0.7824
	(0.1741)	(0.2053)	(0.3535)	(0.1396)	(12.6126)	(−1.6627)	
	[0.8623]	[0.8379]	[0.7247]	[0.8894]	[0.0000]	[0.1006]	
$\beta_{3,t}$	0.0021	−0.0506	−0.0001	0.0430	−0.2618	0.7994	0.7818
	(0.0408)	(−2.0683)	(−0.0110)	(0.1571)	(−1.7900)	(9.6638)	
	[0.9676]	[0.0421]	[0.9913]	[0.8756]	[0.0775]	[0.0000]	
$\|z\|$		0.1043	0.6879	0.9289	0.9516	0.9264	

corresponding p-value, this parameter is significant at a 10% level. Further, the Granger-Causality test indicates that the lagged dividend-price ratio is a significant variable (at a 10% level) for predicting equity returns. The R-squared is 10.23%, compared to 8.6% as reported in Campbell et al. (2003).

The first lags of the Nelson/Siegel coefficients $\beta_{i,t-1}$ show the usual strong persistence of the term structure, and the parameters are statistically significant at a 1% level. Additionally, we observe a strong dependence of $\beta_{1,t}$ and $\beta_{3,t}$ on the lagged stock returns. Both coefficients are statistically significant at a 5% level. Campbell et al. (2003) report similar findings between excess bond returns and lagged excess stock returns for quarterly and annual data sets. Given our estimated VAR(1) process, an increase in the lagged equity return leads, certeris paribus, to an upward-shift in the term structure with the ensuing decline of bond returns. Although the negative coefficient in Campbell et al. (2003) for the quarterly data supports this finding, we defer a more thorough investigation to future research. However, the Granger–Causality test points out that the lagged stock return is a significant variable (at a 5% level) to predict $\beta_{1,t}$ and $\beta_{3,t}$.

As in Campbell et al. (2003) and the literature mentioned therein, unexpected log stock returns are strongly negatively correlated with shocks to the log dividend-price ratio (cross-correlation of -0.98). The residual series pass the multivariate normality test using Cholesky orthogonalization (by Ltkepohl). In Table 2 we indicate the mean vector $\boldsymbol{\mu}$ in the steady state. The expected simple return per annum for equities equals 7.29%, while the term structure of interest rates is increasing and concave.

3.3. Scenario generation

We try to exploit predictability in the asset returns by using four decision stages with time intervals of three months, i.e., reallocations at $m_t \in \{0, 0.25, 0.5, 0.75\}$. The multivariate process in (17) evolves in discrete time, and the corresponding probability distribution is approximated with a few mass points in terms of a so-called scenario tree. Although different approaches have been discussed in the literature, we focus here on the technique proposed by Høyland et al. (2003) to match the first four conditional moments and the correlations of the process. This method uses an iterative procedure that combines simulation, Cholesky decomposition and various transformations to achieve the correct correlations without changing the marginal moments. More nodes emanating from one predecessor node (i.e., a higher branching factor) facilitate the matching of moments but increase the number of scenarios. In this application we use a constant branching factor of ten with four decision stages, resulting in a tree with a total number of scenarios $\mathcal{S}$ equal to 10^4.

For the scenario generation, in addition to the parameter estimation in Section 3.2, we have to set the starting values of our VAR(1) process. Two choices are suitable: currently realized parameters *versus* steady state values. On the one hand, for practical applications the SLP literature proposes a rolling-forward approach (see, e.g., Dempster et al., 2003), where at each stage the process parameters are re-estimated and a new scenario tree is generated. To predict and exploit returns in such a context, the starting values should coincide with the most current realizations. On the other hand, numerical results based on this special setting do not allow to draw general conclusions. Therefore, it is common to start the investigation with the unconditional expected values of the estimated process (e.g., Campbell et al., 2003). We follow this second approach by starting from the steady state in the optimization examples presented in Section 4.

Further, the scenario tree must satisfy the no-arbitrage condition. Arbitrage opportunities are present in a market whenever investors, without spending own money and without taking risk, have a probability greater than zero to earn a positive portfolio return. For a discussion on the existence of an equivalent risk-neutral measure given a set of discrete scenarios, which guarantees the absence of arbitrage, see, e.g., Topaloglou et al. (2008). Therefore, we apply the arbitrage-check proposed by Klaassen (2002), which accounts for the return of traded assets in different successor nodes. However, the simulated process in (17) does not only model asset returns, but also includes state variables in form of the dividend-price ratio as well as the Nelson/Siegel parameter vector. The last one is important for two reasons: First, the parameters determine the whole term structure of interest rates, and in this way the present value of the given cash flows at each stage and each scenario. Second, changes in the yield curve drive the realized gross returns $R_t^{i,s}$ in (4) and (9) of the different bond holdings. To check for arbitrage opportunities during the construction of our scenario tree, we also have to account for these potential bond returns.

For zero-coupon bonds the returns can be easily calculated. We define $P^s(t,m)$ as the market price of the m-year maturity zero bond at stage t and in scenario s. Then the gross return a short time period Δt later is given by:

$$R^s(t+\Delta t, m) = \frac{P^s(t+\Delta t, m-\Delta t)}{P^s(t,m)} = \frac{e^{my(\boldsymbol{\beta}_t^s, m)}}{e^{(m-\Delta t)y(\boldsymbol{\beta}_{t+\Delta t}^s, m-\Delta t)}},$$

where $y(\boldsymbol{\beta}_t^s, m)$ defines the term structure at stage t and scenario s, see (16). Moving to the log holding-period returns results in:

$$\begin{aligned} r^s(t+\Delta t, m) &= my(\boldsymbol{\beta}_t^s, m) - (m-\Delta t)y(\boldsymbol{\beta}_{t+\Delta t}^s, m-\Delta t) \\ &= \Delta t y(\boldsymbol{\beta}_t^s, m) - (m-\Delta t)\left[y(\boldsymbol{\beta}_{t+\Delta t}^s, m-\Delta t) - y(\boldsymbol{\beta}_t^s, m)\right]. \end{aligned} \tag{18}$$

Eq. (18) shows that the continuously compounded return is a weighted sum given by the yield at the beginning of the period $y(\boldsymbol{\beta}_t^s, m)$ multiplied by the holding period Δt minus the yield change $\left[y(\boldsymbol{\beta}_{t+\Delta t}^s, m-\Delta t) - y(\boldsymbol{\beta}_t^s, m)\right]$ times the remaining maturity of this zero bond $(m-\Delta t)$.

We calculate these scenario-dependent returns of the different tradeable bonds using (18) and include them, together with the returns modeled directly by the VAR(1) process (e.g., the equity returns), in the arbitrage-check proposed by Klaassen (2002).[4]

The evolution of the yield curve in our scenario tree satisfies the most important stylized facts (see, e.g., Diebold and Li, 2006):

1. The unconditional expected yield curve is increasing and concave.
2. The yield curve assumes a variety of shapes through time, including upward sloping, downward sloping, humped, and inverted humped. Further, Table 3 reports percentiles for spot rates with different maturities for cumulative probabilities $p_z \in \{0.025, 0.5, 0.975\}$ across all scenarios at $m_t = 0.75$. The upper and lower bounds make evident that although a variety of shapes is possible, the evolution of the term structure is well-behaved and economically sound.

Table 2
Unconditional expected values $\boldsymbol{\mu}$ for the steady state.

r^1	$d-p$	β_1	β_2	β_3
0.0176	−4.0620	0.0496	−0.0166	0.0405

[4] During the generation of the scenario tree, a total of 12 candidates of successor bundles was rejected due to this check. More concisely, using the definition of Klaassen (2002), arbitrage of the "first type" ("second type") occured nine (three) times.

Table 3
Percentiles for the term structure of interest rates at $m_t = 0.75$.

Maturity	1	5	10	15	20	25	30
$p_z = 0.025$	1.3675	2.7359	3.5978	4.0027	4.2080	4.3255	4.3735
$p_z = 0.5$	3.6287	4.5353	5.1225	5.3985	5.5038	5.5245	5.5112
$p_z = 0.975$	5.8009	6.0899	6.4587	6.6398	6.6753	6.6532	6.5837

3. Yield level dynamics, reflected by the parameter for $\beta_{1,t-1}$, are much more persistent than spread dynamics of $\beta_{2,t-1}$, see Table 1.

Finally, Table 4 illustrates the unconditional expected value $\overline{R}_t^{i,s}$ and the standard deviation $\sigma\left(R_t^{i,s}\right)$ of the generated (quarterly) returns for the different assets at the various stages. The values follow economic intuition. A higher market risk is compensated by a higher expected return. Due to starting the scenario generation from the steady state, the expected return remains constant over time. Further, note that after the first period the return of the short-term bond changes in the successor nodes, i.e., the unconditional standard deviation also differs from zero for this asset.

4. ALM results

4.1. Initial setting

In this section, we present a numerical example with a cash flow structure typical for a defined benefit pension scheme. A company with a planning horizon of one year accumulates cash inflows (fund raising period) and takes asset allocation decisions at the beginning of each quarter. After that, a long period of cash outflows follows, representing the defined pension payments (see Fig. 2). Due to our assumed age distribution of the benefit owners, the major liabilities will occur between the years three and eleven.

The company can choose from the following assets: an equity investment (in the present case, this is the S&P 500 index), and three zero-coupon bonds with maturities of 3 months, 5 years and 10 years. The short-term zero-coupon bond can be seen as an equivalent to a cash account. Because its maturity perfectly matches the intervals between two rebalancing stages, the riskless return is known at the beginning of the period and equals the spot rate for that maturity. We use transaction costs for purchases and sales of $\tau_P = \tau_S = [1\%, 0\%, 0.5\%, 0.5\%]$.[5] These are set to zero for the three-month bond. Further, the company has no initial holdings in any of the assets. For the first numerical experiment, we use the lower and upper bounds $l = [0\%, -30\%, 0\%, 0\%]$ and $u = [130\%, 100\%, 130\%, 130\%]$ in the asset allocation constraint (7). In such a setting, in which modest leverage with a short position in the riskless bond is allowed, we mimic a *prudent* version of a so-called "130/30" strategy. Compared to the traditional long-only approach, long-short strategies expand alpha opportunities for active portfolio management. Various papers over the last decade report benefits from such extensions (see, e.g., Grinold and Kahn, 2000; Johnson et al., 2007). We determine a feasible value for the target SV θ by the following rule of thumb:

$$\theta = \left[\sum_{i=1}^{N} w_0^i + \sum_{\tau=0}^{T-1} L_\tau \delta(\boldsymbol{\beta}_0, m_\tau)\right] \delta(\boldsymbol{\beta}_0, m_T)^{-1} e^{v m_T} + L_T + \mathbb{E}\left[\mathcal{L}_T^s\right],$$

where v denotes a required excess return for our SLP strategy. The term in brackets is the sum of initial holdings and the present value of all future cash flows until $T-1$. It is converted into a terminal value at stage T. The remaining two terms are the cash flow L_T at the end of the planning horizon, and the expected present value of all remaining cash flows occurring after the planning horizon. By setting $v = 4.00\%$, we obtain a target value θ equal to 34.78. The initial shareholder value is 21.71. The bound on the maximum drawdown of the shareholder value in (10) is set to $\gamma = 35$. We implement the scenario generation procedure in MATLAB and we formulate the optimization problem in AMPL. The solution time on a MacBook Pro 2.4 GHz Intel Core 2 Duo, 4 GB RAM with MOSEK is approximately 57 s with the interior-point solver.

Fig. 3 shows the distribution of the shareholder value at the end of the planning horizon. The first-stage solutions of the SLP are $W_0 = [31.51\%, -30.00\%, 0.00\%, 98.49\%]$ with $CVaR_{0.95} = 26.17$ and $VaR_{0.95} = 14.18$. The minimum shareholder value is -108.96 (i.e., the worst possible scenario for our company). Because the risk measures VaR_α and $CVaR_\alpha$ can become positive or negative, it may be convenient to calculate so-called deviation measures. These are introduced in Rockafellar et al. (2006), and indicate the difference between the risk measure and the mean of the distribution. The *CVaR* Deviation $CVaR_{0.95}^{\Delta} = 60.96$ and the *VaR* Deviation $VaR_{0.95}^{\Delta} = 48.97$ can easily be calculated as shown in Fig. 3. By definition these are always positive and should be used, for example, in Sharpe-like ratios.

Table 5 shows the risk-return tradeoff between the expected final SV and the *CVaR* along with the corresponding frontier portfolios. Here, we directly change θ beginning from the feasible starting value that was calculated by setting the excess return v. We can see that even for low targets our company prefers to hold the maximum allowed short position in bond 1. We interpret this as hedging demands in our asset-liability management setting. Given the long-term payouts illustrated in Fig. 2, one of the main sources of risk in SV are low interest-rate levels, which increase the present value of future liabilities $\mathcal{L}_t^s$ and decrease the SV in (8). To hedge against such unfavorable scenarios the optimal policy proposes a strong exposure to the long-term bond, which will benefit from low interest rates. Even for a high target of $\theta = 36.25$ the minimum shareholder value is bounded by the maximum drawdown constraint. Given the initial SV of 21.71, the minimum possible value after four periods with $\gamma = 35$ is approximately 122 (including compounding effects). Further, Table 5 shows that with an increasing target the company gradually shifts from the long-term bond to the equity investment.

4.2. Constant-mix strategies

Depending on the degree of risk aversion of an investor, different constant-mix strategies are a popular piece of advice for portfolio allocation decisions as reported, e.g., in Canner et al. (1997). For comparison with our results in Section 4.1, we consider such investment policies here. We assume that a pension fund manager ignores the interest-rate risk for future cash flows as well as the predictability in asset returns, or is faced with bounds on the allowed asset allocation, which are set too tight.

Table 6 shows the results for different constant-mix strategies. While the "Bond 1 only" strategy mimics a seemingly riskless investment in the cash account, the other two strategies are balanced portfolios with different bond to stock ratios. Apparently, these policies are suboptimal in a risk-return sense. For similar risk in terms of $CVaR_{0.95}$, the expected shareholder value θ is, in all cases, considerably below the corresponding values in Table 5. Different to the conclusion of Canner et al. (1997) for pure asset management, the cost of non-optimization can be high in an ALM context, i.e., pension fund managers may be warned upon copying such simple strategies.

4.3. Maximum drawdown constraint

The maximum drawdown of SV is restricted by constraint (10). This amount is the highest potential loss a sponsor of the pension

[5] The vector indicated by square brackets contains the values for the assets $i = 1, \ldots, N$.

Table 4
Unconditional expected value and standard deviation of asset returns (in %).

$m_t \backslash i$	$\bar{R}_t^{i,s}$				$\sigma\left(R_t^{i,s}\right)$			
	1	2	3	4	1	2	3	4
0.25	1.7588	0.8450	1.3262	1.4752	6.6825	0.0000	2.4206	4.3393
0.5	1.7588	0.8450	1.3262	1.4753	6.8425	0.1701	2.4671	4.4185
0.75	1.7589	0.8450	1.3263	1.4755	6.8732	0.2345	2.5009	4.4663
1	1.7589	0.8450	1.3262	1.4754	6.8927	0.2757	2.5204	4.4939

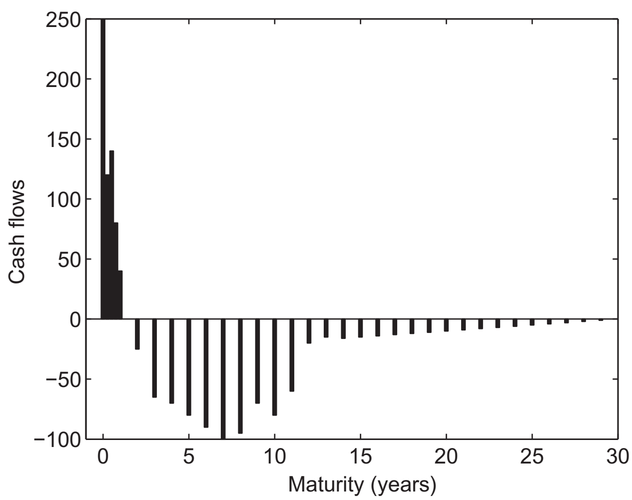

Fig. 2. Cash flow structure.

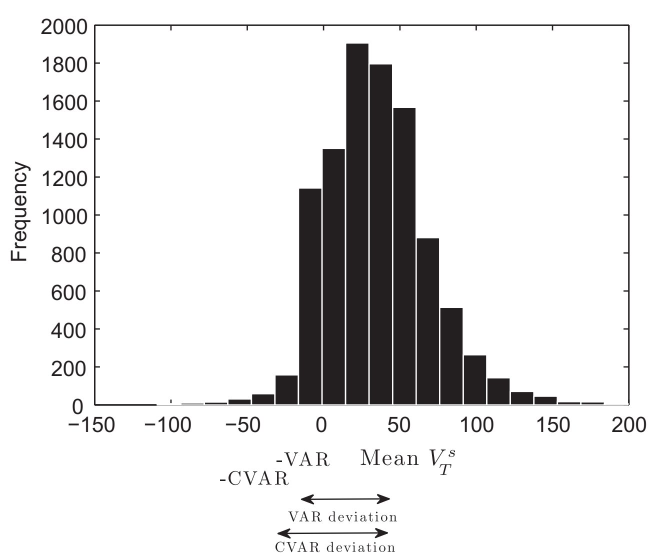

Fig. 3. Histogram of shareholder value V_T^s at the end of planning horizon.

Table 5
First-stage asset allocation with shareholder value constraint.

θ	$CVaR_{0.95}$	$VaR_{0.95}$	MinV_T^s	$W_0^1\%$	$W_0^2\%$	$W_0^3\%$	$W_0^4\%$
35.25	30.64	17.26	−113.25	49.63	−30.00	0.00	80.37
35.50	33.37	19.61	−115.98	61.21	−30.00	0.00	68.79
35.75	36.35	21.98	−119.55	76.31	−30.00	0.00	53.69
36.00	39.68	24.32	−121.74	85.60	−30.00	0.00	44.40
36.25	45.13	28.31	−121.74	85.60	−30.00	0.00	44.40

plan is willing or able to suffer during one period. In our base case of Section 4.1, we set $\gamma = 35$ and the target $\theta = 34.78$. Fig. 4 illustrates a slack variables analysis at each rebalancing stage and each scenario. If the slack variable equals zero the boundary is active and will restrict the solution. We can see that the constraint becomes binding, starting from $m_t = 0.5$ as the uncertainty in the asset returns increases. Further, to study the impact of this shareholder value constraint on the optimal investment policy, we show in Table 7 results for the optimization task when (10) is disabled. As expected, a better objective function value (i.e., a lower $CVaR_\alpha$) is found without binding boundaries. For example, the θ of 36 from Table 5 (active maximum drawdown constraint) can be compared to the θ of 40 in the first row of Table 7, showing that the expected SV can be increased by 11% given a similar risk in terms of *CVaR*. However, in contrast to Table 5, the minimum possible SV at the end of the planning horizon (Min V_T^s) worsens due to unfavorable scenarios. Values for θ below 40.5, although further reducing *CVaR*, do not lead to noticeable changes in the first-stage asset allocation.

5. Predictability effects

The VAR(1) process in (17) implies predictability in all asset returns and affects the optimal solution. In the present section, we further investigate the impact of time-varying investment opportunities on our results. This issue is actively discussed in the literature as forecasting asset returns out-of-sample is by no means an easy task. While there is strong empirical evidence supporting bond return predictability (e.g., Cochrane and Piazzesi, 2005; Kessler and Scherer, 2009; Wright and Zhou, 2009), the findings are much weaker with contradictory results for equity returns (see, e.g., Ang and Bekaert, 2007; Goyal and Welch, 2008; Campbell and Thompson, 2008). Hence, for the purpose of comparison we model predictability only for the bond returns (see, e.g., Koijen et al., 2010), and estimate a new VAR(1) process using only the Nelson/Siegel coefficient series. Table 8 reports the corresponding parameters (values for the *t*-statistics and *p*-values in parenthesis). Again, the first lags of the Nelson/Siegel coefficients $\beta_{i,t-1}$ show a strong persistence of the term structure, the high *t*-values indicate statistical significance of the parameters and the process is stable (all eigenvalues have modulus less than one).

We model the evolution of equity returns by a Brownian motion with quarterly mean 1.87% and standard deviation 7.80% (estimated from our data set), assuming no correlation with the noise of the Nelson/Siegel coefficients. Compared to our "full preditability" VAR(1) model reported in Table 1, we denote this reduced process in the following analysis as "partial predictability" setting. Differences in the results are then due to the predictability of equity returns included in the first case. For both processes we do not consider parameter uncertainty or learning, which are discussed in, e.g., (Barberis, 2000 or Xia, 2001).

5.1. Predictability in an ALM context

Table 9 compares the solution of the initial setting in Section 4.3 (without the maximum drawdown constraint) to the first-stage

Table 6
First-stage asset allocation of alternative strategies.

Strategy	θ	$CVaR_{0.95}$	$VaR_{0.95}$	MinV_T^s	$W_0^1\%$	$W_0^2\%$	$W_0^3\%$	$W_0^4\%$
Equal weights	16.39	39.63	26.55	−101.04	25.00	25.00	25.00	25.00
Bond 1 only	8.84	69.55	50.91	−148.94	0.00	100.00	0.00	0.00
30/70	16.85	42.84	29.58	−105.52	30.00	23.33	23.34	23.33

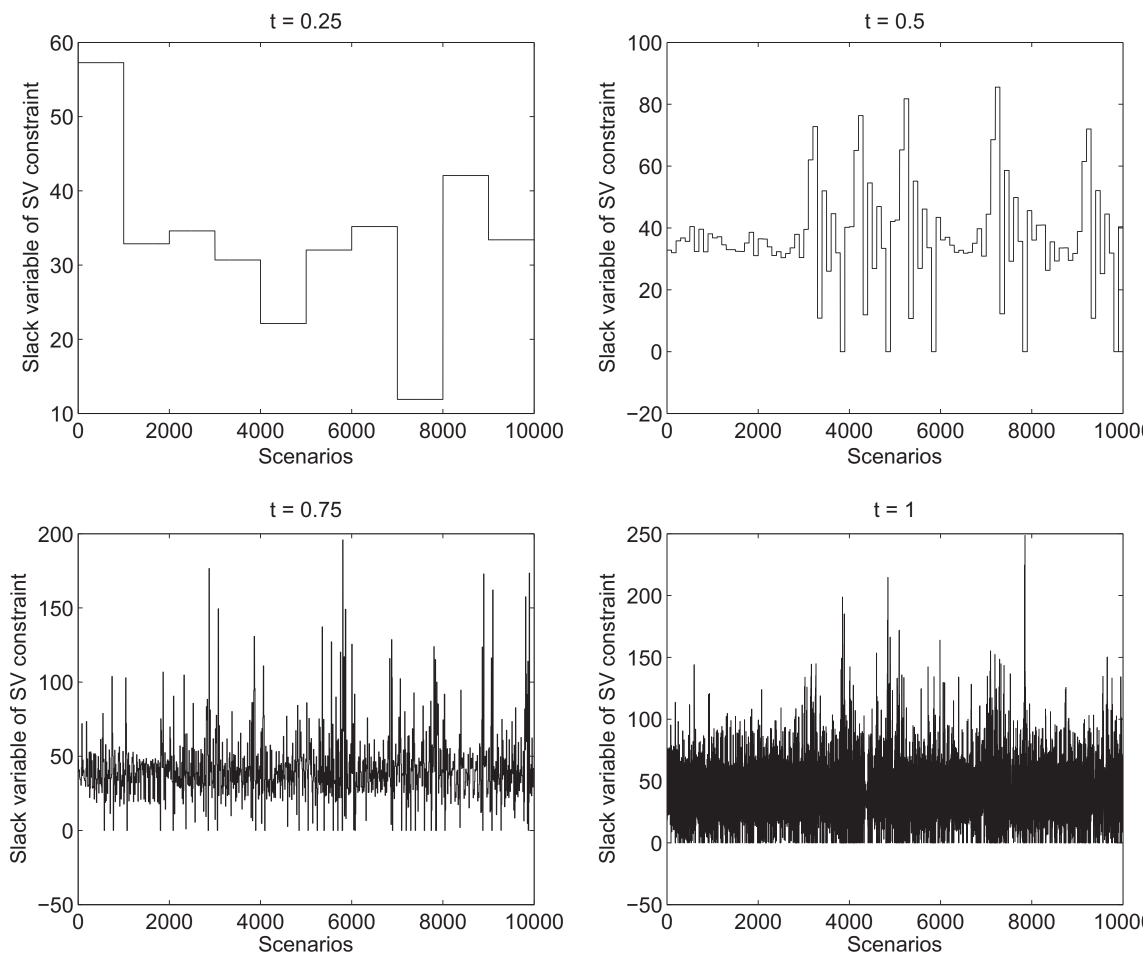

Fig. 4. Slack variables for shareholder value constraint.

Table 7
First-stage asset allocation without shareholder value constraint.

θ	$CVaR_{0.95}$	$VaR_{0.95}$	MinV_T^s	$W_0^1\%$	$W_0^2\%$	$W_0^3\%$	$W_0^4\%$
40.00	40.54	22.99	−168.55	0.00	−30.00	0.00	130.00
40.50	43.53	24.64	−168.78	0.00	−30.00	0.00	130.00
41.00	46.84	27.77	−171.16	9.41	−30.00	0.00	120.59
41.50	50.25	30.34	−174.59	23.01	−30.00	0.00	106.99
42.00	53.86	32.64	−177.35	33.28	−30.00	0.00	96.72

Table 8
VAR(1) parameters, t-statistics in () and p-values in [].

	c	$\beta_{1,t-1}$	$\beta_{2,t-1}$	$\beta_{3,t-1}$	$R-$ sq.
$\beta_{1,t}$	0.0022	0.9453	0.1083	0.0534	0.8566
	(0.6943)	(19.9129)	(2.5565)	(2.8431)	
	[0.4896]	[0.0000]	[0.0126]	[0.0057]	
$\beta_{2,t}$	−0.0038	0.0505	0.7997	−0.0519	0.7820
	(−0.8212)	(0.7375)	(13.0965)	(−1.9193)	
	[0.4141]	[0.4631]	[0.0000]	[0.0587]	
$\beta_{3,t}$	0.0015	0.0343	−0.2593	0.8151	0.7691
	(0.1319)	(0.2102)	(−1.7814)	(12.6402)	
	[0.8954]	[0.8341]	[0.0788]	[0.0000]	
$\|z\|$		0.9657	0.9302	0.6642	

asset allocation optimized with the partial predictability scenarios. We set $\theta = 2$ in order to obtain roughly similar results in terms of *CVaR* (approximately 26). One can immediately verify that the ability to forecast equity returns substantially improves the expected final shareholder value (34.78 vs. 2). However, as the future cash flows influence the magnitude of this effect, we defer a quantification in terms of "percentage benefit" from the equity return predictability to the next section, in which a pure asset management strategy (i.e., a setting without future cash flows) is considered. Furthermore, comparing the first-stage asset allocations shows: The partial predictability case focuses on hedging interest-rate risk by investing in the long-term bond, and the full predictability case considers more equities.

Table 10 presents the intertemporal asset allocations for full and partial predictability. We report for each decision stage the

Table 9
First-stage asset allocations.

Full predictability				Partial predictability			
Multi-period model							
θ	$CVaR_{0.95}$	$VaR_{0.95}$	$\text{Min}V_T^s$	θ	$CVaR_{0.95}$	$VaR_{0.95}$	$\text{Min } V_T^s$
34.78	26.17	14.18	−108.96	2.00	26.40	26.01	−34.90
$W_0^1\%$	$W_0^2\%$	$W_0^3\%$	$W_0^4\%$	$W_0^1\%$	$W_0^2\%$	$W_0^3\%$	$W_0^4\%$
31.51	−30.00	0.00	98.49	15.06	−30.00	0.00	114.94

Table 10
Intertemporal asset allocation.

t	Full predictability				Partial predictability			
	$\overline{W}_t^1\%$	$\overline{W}_t^2\%$	$\overline{W}_t^3\%$	$\overline{W}_t^4\%$	$\overline{W}_t^1\%$	$\overline{W}_t^2\%$	$\overline{W}_t^3\%$	$\overline{W}_t^4\%$
Asset allocation (mean across all scenarios)								
0.25	35.41	−30.00	0.00	94.59	24.80	−30.00	1.27	103.93
0.5	31.08	−25.95	10.61	84.25	22.21	−26.60	22.04	82.36
0.75	22.82	−4.10	7.36	73.92	15.16	−0.88	10.52	75.20
Asset allocation (lower 10% of scenarios)								
0.25	13.82	−30.00	0.00	116.18	0.00	−30.00	0.00	130.00
0.5	34.86	−30.00	16.21	78.93	8.86	−30.00	10.41	110.72
0.75	5.17	−22.93	18.55	99.22	1.32	−15.00	13.31	100.36
Asset allocation (upper 10% scenarios)								
0.25	69.23	−30.00	0.00	60.77	6.96	−30.00	0.00	123.04
0.5	61.61	4.95	14.83	18.61	37.88	−27.39	21.27	68.25
0.75	29.78	44.15	5.38	20.68	41.92	9.83	9.68	38.57

Table 11
First-stage asset allocation without external cash flows.

θ	$CVaR_{0.95}$	$VaR_{0.95}$	$\text{Min}V_T^s$	$W_0^1\%$	$W_0^2\%$	$W_0^3\%$	$W_0^4\%$
260.00	−258.15	−258.27	256.81	0.55	75.50	23.95	0.00
263.00	−256.40	−256.55	248.25	6.21	46.57	47.23	0.00
266.00	−254.35	−254.50	244.88	8.60	10.50	80.90	0.00
269.00	−251.63	−251.81	241.61	19.94	−30.00	110.06	0.00
272.00	−246.92	−247.40	227.71	34.18	−30.00	49.55	46.27

mean values $\overline{W}_t^i\%$ across all scenarios as well as those for the 10% of the scenarios with the lowest total wealth and the 10% with the highest total wealth. As expected, our results indicate that in the worst case a more prudent asset allocation is appropriate by favoring long-term bonds (to hedge the interest-rate risks from our cash flows), while in scenarios with higher wealth more risk can be taken by holding higher equity positions.

Due to the predictability in the equity returns, the mean allocation to equities over all decision stages is higher, i.e., equities become more attractive compared to the other assets.

5.2. Predictability in a pure asset management context

The objective of this section is twofold. First, to isolate the impact of the cash flows, we consider a pure asset management problem and compare the results to our previous findings from the ALM context. Second, we use this setting to deepen our discussion and to quantify the benefits of equity return predictability in our data set. We begin with the first issue.

In the results of Table 5, we find for all levels of θ a short position in the riskless bond W_0^2 (in all cases the lower bound of −30% become active) and a heavy investment in the long-term bond W_0^4. We interpret these hedging demands as a consequence of active ALM. The huge risk of low interest-rate levels induced by the long time series of payouts can be mitigated by long-term bond investments. Further, equity investments W_0^1 are taken into account only when a higher target θ is required.

To compare these ALM results with a pure asset management approach, we take the setting of the base case from Section 4.1 without future cash flows (i.e., $L_t = 0, \forall t > 0$) and disable the constraints on the scenario-dependent maximum loss in the SV. Given that the interest-rate risk for payouts no longer prevails in the optimization task, the short- and medium term bonds become attractive investments for low levels of θ, see Table 11. By increasing this target, more wealth is allocated to the assets with higher expected returns, i.e., to long-term bonds and equities.

In the rest of this section, we analyze the impact of stock return predictability in more detail using the “full” and the “partial” processes from Section 5.1. Furthermore, as the benefit of predictability is inevitably connected to the possibility of re-allocating the portfolio at the decision stages, in addition to our proposed *multi-period* model in Section 2, we define a *one-period* model that is nested in (2)–(15): we set L_t, $P_t^{i,s}$ and $S_t^{i,s}$ equal to zero $\forall t > 0$, restrict condition (7) at the first stage, where the only decision is taken, and disable the constraint in (10). In this way, we can decompose the effects in benefits from the dynamic re-allocation and benefits from the equity return predictability.

Table 12 compares the multi-period to the one-period model for both full and partial predictability. While the upper left panel reports results from Table 11, we set the target θ for the other panels

Table 12
First-stage asset allocations without external cash flows.

Full predictability				Partial predictability			
Multi-period model							
θ	$CVaR_{0.95}$	$VaR_{0.95}$	$\text{Min}V_T^s$	θ	$CVaR_{0.95}$	$VaR_{0.95}$	$\text{Min}V_T^s$
266.00	−254.35	−254.50	244.88	261.00	−254.34	−254.46	251.50
$W_0^1\%$	$W_0^2\%$	$W_0^3\%$	$W_0^4\%$	$W_0^1\%$	$W_0^2\%$	$W_0^3\%$	$W_0^4\%$
8.60	10.50	80.90	0.00	12.12	51.49	36.39	0.00
One-period model							
θ	$CVaR_{0.95}$	$VaR_{0.95}$	$\text{Min}V_T^s$	θ	$CVaR_{0.95}$	$VaR_{0.95}$	$\text{Min}V_T^s$
260.30	−254.28	−255.36	251.41	258.20	−254.31	−255.01	252.01
$W_0^1\%$	$W_0^2\%$	$W_0^3\%$	$W_0^4\%$	$W_0^1\%$	$W_0^2\%$	$W_0^3\%$	$W_0^4\%$
5.82	64.04	30.14	0.00	3.18	78.64	18.19	0.00

Table 13
Intertemporal asset allocation without external cash flows.

t	Full predictability				Partial predictability			
	$\overline{W}_t^1\%$	$\overline{W}_t^2\%$	$\overline{W}_t^3\%$	$\overline{W}_t^4\%$	$\overline{W}_t^1\%$	$\overline{W}_t^2\%$	$\overline{W}_t^3\%$	$\overline{W}_t^4\%$
Asset allocation (mean across all scenarios)								
0.25	11.54	3.12	70.33	15.01	9.90	50.42	35.54	4.14
0.5	20.71	31.25	33.19	14.84	14.56	51.34	24.76	9.34
0.75	20.62	57.90	4.64	16.84	13.99	72.25	0.70	13.06
Asset allocation (lower 10% of scenarios)								
0.25	0.01	78.94	21.05	0.00	0.27	84.82	14.91	0.00
0.5	0.20	79.69	6.66	13.45	0.59	90.67	8.74	0.00
0.75	0.13	98.54	0.32	1.01	0.10	99.07	0.09	0.74
Asset allocation (upper 10% scenarios)								
0.25	52.48	13.04	34.46	0.02	27.24	26.13	46.62	0.00
0.5	89.69	−20.44	6.44	24.31	43.38	12.42	16.89	27.31
0.75	86.37	−0.16	0.00	13.78	61.00	7.23	0.00	31.77

in such a way in order to obtain a similar risk in terms of *CVaR*. This enables us to compare the results of the different settings in terms of expected final SV θ and to interpret the percentage differences (as the planning horizon T is equal to one year) as "extra return". For the partial predictability process, the difference in θ between the multi-period and the one-period model indicates the advantage of the dynamic strategy, here equal to 1.08% p.a. (=261/258.2 − 1, see the target values in the right panels). When adding the possibility to forecast equity returns as well (full predictability case in the upper left panel), this extra-return increases to 3.02% p.a. (=266/258.2 − 1), i.e., compared to the dynamic strategy in the partial predictability case, more than half of the benefit is due to the equity return predictability (i.e., benefit of 3.02% vs. 1.08%). We used the high transaction costs reported in Section 4.1 in all calculations. Clearly, lower transaction costs would make bond and equity investments even more attractive, improving the multi-period SLP results reported here.

In Table 13 we compare the intertemporal asset allocation across the scenarios for full and partial predictability. We once again see a "flight-to-safety" in the lower 10% of the scenarios which is represented, in the case without cash flows, by allocating more money to the short-term bond (equivalent to a cash account). In the upper 10% of the scenarios more wealth is invested in equities.

As before, equities become (on average) more attractive with higher holdings when including stock return predictability.

6. Conclusion

In this paper, we address the question of time-varying investment opportunities with the main focus on an asset-liability management problem typical for a pension fund. We consider multi-stage stochastic linear programming as an appropriate numerical method, because it can handle features that reflect real investment practice very effectively. The contribution comprises a discussion of several methodological issues important for the setup of the problem and the analysis of the solution. For the scenario generation we propose a joint model for bond and equity return predictability that is able to explain typical stylized facts with a small number of factors. In addition to log equity returns and log dividend price ratios, we use the parametric Nelson/Siegel model to capture the dynamics of the yield curve. This ensures a compact size of the scenario tree, which is important for the computational tractability of the stochastic optimization problem. We provide a realistic numerical example, in which we minimized the Conditional Value at Risk of final shareholder value, and we check the robustness of our results with a sensitivity analysis for the most important input parameters. The current literature reports strong empirical evidence for predictability of bond returns, whereas for equities the conclusions are contradictory. Therefore, we consider a detailed decomposition of the effects that are exploited by our model. In addition to quantifying the benefits of a multi-stage model with equity return predictability, our analysis shows that the results confirm economic intuition. The framework can easily be extended to include other assets or additional constraints of interest to an investor. As our findings suggest, a more detailed focus on predictability effects implied by the underlying econometric model should be valuable for existing decision-support models used in current investment practice.

Acknowledgements

The authors thank Michael Hanke, an anonymous referee and Ike Mathur (the managing editor) for very helpful comments and suggestions. Furthermore, we thank participants of the XI Workshop on Quantitative Finance at the University of Palermo. All remaining errors are of course our own.

References

Ang, A., Bekaert, G., 2007. Stock return predictability: Is it there? The Review of Financial Studies 20, 651–707.
Barberis, N.C., 2000. Investing for the long run when returns are predictable. The Journal of Finance 55, 225–264.
Boender, G., Dert, C., Heemskerk, F., Hoek, H., 2005. A scenario approach of ALM. In: Zenios, S.A., Ziemba, W.T. (Eds.), Handbook of Asset and Liability Management, vol. 2. North-Holland, Amsterdam.
Brandt, M.W., Goyal, A., Santa-Clara, P., Stroud, J.R., 2005. A simulation approach to dynamic portfolio choice with an application to learning about return predictability. The Review of Financial Studies 18, 831–873.
Campbell, J.Y., Shiller, R.J., 1988. The dividend-price ratio and expectations of future dividends and discount factors. The Review of Financial Studies 1, 195–228.
Campbell, J.Y., Thompson, S.B., 2008. Predicting excess stock returns out of sample: Can anything beat the historical average? The Review of Financial Studies 21, 1509–1531.
Campbell, J.Y., Chan, Y.L., Viceira, L.M., 2003. A multivariate model of strategic asset allocation. Journal of Financial Economics 67, 41–80.
Canner, N., Mankiw, N.G., Weil, D.N., 1997. An asset allocation puzzle. The American Economic Review 87, 181–191.
Cochrane, J., Piazzesi, M., 2005. Bond risk premia. The American Economic Review 95, 138–160.
Consiglio, A., Saunders, D., Zenios, S.A., 2006. Asset and liability management for insurance products with minimum guarantees: The UK case. Journal of Banking & Finance 30, 645–667.
Dempster, M.A.H., Germano, M., Medova, E.A., Villaverde, M., 2003. Global asset liability management. British Actuarial Journal 9, 137–216.
Diebold, F.X., Li, C., 2006. Forecasting the term structure of government bond yields. Journal of Econometrics 130, 337–364.
Gondzio, J., Kouwenberg, R., 2001. High performance computing for asset-liability management. Operations Research 49, 879–891.
Goyal, A., Welch, I., 2008. A comprehensive look at the empirical performance of equity premium prediction. The Review of Financial Studies 21, 1455–1508.
Grinold, R.C., Kahn, R.N., 2000. The efficiency gains of long-short investing. Financial Analysts Journal 56, 40–53.
Høyland, K., Kaut, M., Wallace, S.W., 2003. A heuristic for moment-matching scenario generation. Computational Optimization and Applications 24, 169–185.
Johnson, G., Ericson, S., Srimurthy, V., 2007. An empirical analysis of 130/30 strategies: Domestic and international 130/30 strategies add value over long-only strategies. The Journal of Alternative Investments 10, 31–42.
Keim, D., Stambaugh, R.F., 1986. Predicting returns in the stock and bond markets. Journal of Financial Economics 14, 357–390.
Kessler, S., Scherer, B., 2009. Varying risk premia in international bond markets. Journal of Banking & Finance 33, 1361–1375.
Kim, T.S., Omberg, E., 1996. Dynamic nonmyopic portfolio behavior. The Review of Financial Studies 9, 141–161.
Klaassen, P., 2002. Comment on "Generating scenario trees for multistage decision problems". Management Science 48, 1512–1516.
Koijen, R.S., Nijman, T.E., Werker, B.J.M., 2010. When can life-cycle investors benefit from time-varying bond risk premia? The Review of Financial Studies 23, 741–780.
Lütkepohl, H., 2005. Introduction to Multiple Time Series Analysis. Springer, Berlin.
Merton, R.C., 1969. Lifetime portfolio selection under uncertainty: The continuous-time case. Review of Economics and Statistics 51, 247–257.
Mulvey, J.M., Erkan, H.G., 2006. Applying CVaR for decentralized risk management of financial companies. Journal of Banking & Finance 30, 627–644.
Nelson, C.R., Siegel, A.F., 1987. Parsimonious modeling of yield curve. Journal of Business 60, 473–489.

Quaranta, A.G., Zaffaroni, A., 2008. Robust optimization of conditional value at risk and portfolio selection. Journal of Banking & Finance 32, 2046–2056.
Rockafellar, R.T., Uryasev, S., 2000. Optimization of conditional value-at-risk. Journal of Risk 2, 21–41.
Rockafellar, R.T., Uryasev, S., 2002. Conditional value-at-risk for general loss distributions. Journal of Banking & Finance 26, 1443–1471.
Rockafellar, R., Uryasev, S., Zabarankin, M., 2006. Generalized deviations in risk analysis. Finance and Stochastics 10, 51–74.
Samuelson, P.A., 1969. Lifetime portfolio selection by dynamic stochastic programming. Review of Economics and Statistics 51, 239–246.
Topaloglou, N., Vladimirou, H., Zenios, S.A., 2008. Pricing options on scenario trees. Journal of Banking & Finance 32, 283–298.
Wachter, J., 2002. Portfolio and consumption decisions under mean-reverting returns: An exact solution. Journal of Financial and Quantitative Analysis 37, 63–91.
Wright, J.H., Zhou, H., 2009. Bond risk premia and realized jump risk. Journal of Banking & Finance 33, 2333–2345.
Xia, Y., 2001. Learning about predictability: The effects of parameter uncertainty on dynamic asset allocation. The Journal of Finance 56, 205–246.