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Collective contests with externalities

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Abstract

This paper examines collective contests associated with externalities. The collective contest is modelled as a two-stage game in which intra-group sharing rules and individual outlays are determined sequentially. Depending upon the restrictions on the intra-group sharing rules and the extent of externalities, we identify three kinds of Nash equilibria, and compare them with the outcome of the contest between individuals. This paper also proposes a real rent-dissipation rate as a measure of social waste when externalities are present. The externalities are shown to have significant effects on the relationship between the number of players and the real rent-dissipation rate. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

Various economic and social interactions may be viewed as contests in which players compete by expending resources to win a prize such as fame, patent, promotion, government facilities, and so on. Many authors have extensively examined characteristics of contests in diverse settings. Examples include Tullock (1980), Dixit (1987), Katz et al. (1990), Ursprung (1990), Hirshleifer (1991),

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Leininger and Yang (1994), Baik and Shogren (1994, 1995), and Riaz et al. (1995).[1] Most papers in the literature have assumed that the size of the prize (rent) is fixed regardless of the level of aggregate efforts expended in the contest.[2]

Many contests are associated with externalities. Reflecting this, several authors have examined implications of the externalities on contest outcomes. Buchanan (1980) has examined the efficiency implications of rent-seeking behavior in situations where external diseconomies exist. Congleton (1989) has analyzed status-seeking contests which generate positive or negative externalities to individuals not actively involved in the contests. One of his examples is a competitive footrace which generates entertainment for speculators. However, his model does not allow for the possibility that the participants in the contests are also affected by the externalities.

Extending the argument of Congleton (1989), this paper examines individual and collective contests in which aggregate efforts generate externalities to the participants in the contests. Examples of such contests are found in Olympic games. Most participants in Olympic games might enjoy a feeling of pride or accomplishment regardless of whether they are medal-winners or not. The level of pride of the participants in the contests might be related to the level of aggregate efforts made in the contests. Another example is a rent-seeking contest. It is possible that costs of rent-seeking or lobbying are affected by behavior of other lobbyists or rent-seekers. For example, the more lobbyists are in the Washington, DC area, the more elegant hotels and restaurants are established in the area. If hotel or restaurant industries have increasing returns to scale, then unit costs of lobbying or rent-seeking will be reduced. Lobbyists and government officials would also have an enlarged choice set. In his paper related to the present one, Chung (1996) has analyzed contests in which aggregate efforts generate externalities in the form of an increase in the size of a rent. The present paper focuses on contests in which externalities are generated through reduction in costs. Contests are often associated with negative externalities as well. Tullock (1988, p. 479) notes that ''rent-seeking activity which is directed at obtaining government favors will have as a by-product an increase of public ignorance in the area concerned.'' This can be viewed as negative externality associated with rent-seeking activity. Rent-seeking activities would also decrease efforts in other productive activities. Then, the more efforts are made in rent-seeking contests, the more negatively the players are affected.

This paper is organized as follows. Section 2 examines the individual contest with externalities, as a comparative benchmark case. We propose a real rent-dis-

[1] The characteristics of contests are extensively analyzed in the literature of rent-seeking initiated by seminal work of Tullock (1967), Krueger (1974), and Posner (1975). For a recent survey of the literature, see Nitzan (1994).

[2] There are some exceptions. See, for example, Appelbaum and Katz (1987), Long and Vousden (1987), and Chung (1996).

sipation rate as a suitable measure of social waste resulting from rent-seeking contests with externalities. We find that the externalities significantly alter the relationship between the number of players and the real rent-dissipation rate. Section 3 sets out the model of collective contests with externalities. The collective contest is modelled as a two-stage game in which intra-group sharing rules and individual outlays are determined sequentially. Depending upon the restrictions on the intra-group sharing rules and the extent of externalities, we identify three kinds of Nash equilibria. We then compare them with the benchmark case. Concluding remarks are offered in Section 4.

2. The individual contest with externalities

Consider a contest in which N risk-neutral players compete to win a private-good rent. The rent is worth S. To win the rent, player i expends x_i in units commensurate with the rent. As in Tullock (1980), player i's probability of winning the rent, Π_i, is given by his or her relative outlay to aggregate outlays:

$$\Pi_i = x_i/X, \tag{1}$$

where $X(=\Sigma_i x_i)$ denotes the aggregate outlays.

We assume that the contest is associated with externalities in that each player's real cost of rent seeking is affected by the aggregate outlays. Specifically, the real cost of rent-seeking of player i is assumed to be given by $(x_i - \beta X)$. Then, the expected payoff of participation in the contest for player i, V_i, is given by

$$V_i = \Pi_i(S - (x_i - \beta X)) + (1 - \Pi_i)(-(x_i - \beta X)) = S\Pi_i - x_i + \beta X, \tag{2}$$

where $\beta < 1/N$. The meaning of the upper bound of β will be discussed shortly. The payoff function in Eq. (2) can be viewed as that in the case where each player simply obtains some proportion of aggregate efforts as externality effect, irrespective of whether he is a winner or not. This interpretation seems valid in athletic contests such as Olympic games. This contest can also be viewed as a rent-seeking contest in which the size of the rent increases with aggregate efforts, as in Chung (1996). In this case, the sum of the rent and the aggregate externality effect is given by $(S + \beta NX)$. However, in Eq. (2), the increased portion βNX is equally shared by the players, whereas all the rent accrues to the winner in Chung (1996).

When $\beta > 0$, positive externality is present in the contest. On the other hand, if $\beta < 0$, the contest is associated with negative externality. The lower bound of β can be obtained from the individual rationality condition that the expected payoff of participation in the contest is non-negative. For example, if $N > 2$, the lower bound of β is given by $-1/N(N-2)$. When $\beta = 0$, V_i is identical to the conventional one.

The problem of the risk-neutral player i is:

$$\underset{x_i}{\text{Max}}\ V_i = S\Pi_i - x_i + \beta X$$

Each player is assumed to behave in a Cournot–Nash way. That is, each player decides the level of his effort or outlay, taking all the other players' decisions as given. The first-order condition for the above maximization problem is

$$\partial V_i / \partial x_i = S(X - x_i)/X^2 - 1 + \beta = 0, \quad \text{for } i = 1, \dots, N. \tag{3}$$

The second-order condition is satisfied. From Eq. (3) it follows that

$$S(X - x_i)/X^2 = (1 - \beta), \quad \text{for } 1 = 1, \dots, N. \tag{4}$$

Summing Eq. (4) over i and simplifying, we obtain

$$X* = S(N-1)/N(1-\beta). \tag{5}$$

It is easy to find that the aggregate outlays increase with the externality parameter β. With a higher value of β each player spends more on the contest, thereby increasing the aggregate outlays.

In the literature the rent-dissipation rate t $(=X/S)$ is used as a measure of social waste resulting from rent seeking. However, this nominal rent-dissipation rate is no longer a valid measure of social waste resulting from rent seeking when externalities are present. With externalities it is appropriate to make a distinction between nominal and real rent-dissipation rates. This distinction is analogous to that between nominal and real exchange rates. For each participant, the real cost of rent-seeking is given by $x_i - \beta X$. The sum of real costs is $(1 - \beta N)X$. The upper bound on β, $\beta < 1/N$, is obtained from the condition that the sum of real costs $(1 - \beta N)X$ is positive. When β is positive, the aggregate real cost is smaller than the aggregate nominal cost X. If β is negative, however, the aggregate real cost is greater than the aggregate nominal cost. We define the real rent-dissipation rate t^r as the ratio of the real cost to the rent, i.e., $t^r = (1 - \beta N)X/S$. The nominal and the real rent-dissipation rates, t and t^r, are, respectively, given by

$$t = (N-1)/N(1-\beta), \tag{6}$$
$$t^r = (1-\beta N)(N-1)/N(1-\beta).$$

The difference between the two rates is $t - t^r = \beta(N-1)/(1-\beta)$. For a given β, the larger N is, the greater the difference between the two rates. Even for a small β, the difference between the two rates is large if N is sufficiently large. This means that the real cost of rent-seeking can be significantly different from the nominal cost of rent-seeking. Differentiation of t^r with respect to N gives

$$\partial t^r / \partial N = (1 - \beta N^2)/(1-\beta)N^2.$$

If $\beta \leq 1/N^2$, $\partial t^r/\partial N \geq 0$. In this case an increase in players increases the real rent-dissipation rate. When $\beta \geq 1/N^2$, then $\partial t^r/\partial N \leq 0$. For any value of β, if the number of players is large enough, the real rent-dissipation rate decreases as the number of players further increases.

We now examine the relationship between the degree of externality and the nominal and the real rent-dissipation rates. Straightforward calculation shows that

$$\partial t/\partial\beta > 0,\ \partial^2 t/\partial\beta^2 > 0,\ \partial t^{\mathrm{r}}/\partial\beta < 0, \text{ and } \partial^2 t^{\mathrm{r}}/\partial\beta^2 < 0.$$

Note that the nominal rent-dissipation rate increases with the externality parameter β. On the other hand, the real rent-dissipation rate is inversely related to the externality parameter β. Of course, the two rent-dissipation rates are the same when there does not exist externality at all, i.e., when $\beta = 0$.

The externality parameter β may not be easy to observe. This raises a serious problem regarding assessment of rent-seeking costs. The problem is more serious when N is large. Being ignorant of the true value of β, one cannot derive correct welfare implications by simply observing the rent and the nominal rent-seeking expenditures.

3. Collective contests with externalities

This section examines a collective contest between two groups associated with externalities. The collective contest might be modelled in two different ways. One way is to consider an imperfectly-discriminating winner-take-all contest in which the probability of winning is assigned to each group. The other way is to examine a share contest in which the two groups compete for a share of the rent, as in Baik and Lee (1997). We will examine both models of collective contests. In fact, under risk-neutrality the two models are equivalent to each other.

Each group consists of a fixed number of identical risk-neutral members, denoted by n_i, for $i = 1,2$. Without loss of generality we assume that group 1 is greater than or equal to group 2, i.e., $n_1 \geq n_2$. The size of total population N, $(N = n_1 + n_2)$, is assumed to exceed 2. Member k of group i contributes X_{ki} to his own group. Group i expends in the aggregate X_i $(= \Sigma_k\ X_{ki})$. Total outlay of the two groups is denoted by X $(= X_1 + X_2)$. Negative value of X_{ki} or X_i is allowed, provided that $X > 0$. That is, an individual or a group in the aggregate can take away resources contributed by other individuals or group, as long as total expenditure X is positive.

3.1. Winner-take-all contest vs. share contest

Nitzan (1991a) has shown that a pure strategy Nash equilibrium does not exist with certain values of intra-group sharing-rule parameters in a collective winner-take-all contest. To resolve the issue of non-existence of a pure strategy equilibrium, we now offer an extended version of a probability-assigning process. When both groups expend non-negative outlays and $X > 0$, group i's probability of winning the prize, Π_i, is given by

$$\Pi_i = X_i / X, \tag{1'}$$

where $X_i \geq 0$, for $i = 1,2$, and $X > 0$. If $X_1 = X_2 = 0$, we assume that $\Pi_1 = \Pi_2 = 1/2$. When $X_i < 0$, $X_j > 0$, and $X > 0$, we assume that the probability of winning is assigned to each group as follows: Group j with positive outlays wins the prize with probability 1, whereas group i, which in the aggregate has taken away resources, loses with probability 1. That is,

$$\Pi_j = 1, \text{ if } X_i < 0,\ X_j > 0, \text{ and } X > 0$$
$$\Pi_i = 0, \text{ if } X_i < 0,\ X_j > 0, \text{ and } X > 0.$$

Moreover, we assume that the losing group i has to pay the penalty in the amount of $(-X_i/X)S$ to the winning group. This scheme is similar to the British rule system of loser pays in the sense that the losing group has to pay the penalty, (see Baik and Shogren, 1994). Note that the amount of penalty is proportional to the ratio of the amount of resources taken away by the losing group relative to the total outlay X.

Each group distributes the prize or penalty among its members according to the prespecified intra-group sharing rule. The proportion of the rent or penalty accruing to individual k of group i is denoted by f_{ki}. The rent as well as the penalty is fully distributed so that $\Sigma_k f_{ki} = 1$, for $i = 1,2$. The proportion f_{ki} is assumed to be determined by his own outlay and by outlays of the other members of group i. Following Nitzan (1991a,b), the present paper focuses on a family of intra-group distribution rules given by

$$f_{ki} = (1 - \alpha_i) X_{ki}/X_i + \alpha_i/n_i. \tag{7}$$

Nitzan (1991a,b) and Lee (1995) have adopted the restriction that α_i should belong to the closed unit interval [0,1]. However, the parameter α_i may have a negative value or exceed one, as in Baik and Lee (1997). A negative value of α_i implies that group i places great emphasis on members' efforts. On the other hand when α_i exceeds one, voluntary contributions are discouraged. The distribution rule given by Eq. (7) is compatible with any value of α_i since $\Sigma_k f_{ki} = 1$ for any value of α_i.

The expected payoff to member k of group i when $X_1 \geq 0$, $X_2 \geq 0$, and $X > 0$, is given by

$$\begin{aligned} V_{ki} &= \Pi_i\big(Sf_{ki} - (X_{ki} - \beta X)\big) + (1 - \Pi_i)\big(-(X_{ki} - \beta X)\big) \\ &= (X_i/X) f_{ki} - X_{ki} + \beta X \end{aligned} \tag{8}$$

If group i in the aggregate takes away resources ($X_i < 0$) and $X > 0$, group j wins the prize and in addition receives the penalty paid by group i. The expected payoff to member k of the winning group j in this case is given by

$$\begin{aligned} V_{kj} &= 1\big[\{S + (-X_i/X)S\} f_{kj} - (X_{kj} - \beta X)\big] + 0\big[-(X_{kj} - \beta X)\big] \\ &= (X_j/X) S f_{kj} - X_{kj} + \beta X, \end{aligned} \tag{8'}$$

where the numbers one and zero in front of brackets denote winning and losing probabilities, respectively. Note that the functional form of Eq. (8′) is identical to that of Eq. (8). Notice also that the reimbursement as well as the prize is fully distributed to the members of group j. The losing group i collects the penalty to be paid to the winning group from its members according to the prespecified distribution rule f_{ki}. The expected payoff to member k of the losing group i is

$$\begin{aligned} V_{ki} &= 1\left[(X_i/X)Sf_{ki} - (X_{ki} - \beta X)\right] + 0\left[Sf_{ki} - (X_{ki} - \beta X)\right] \\ &= (X_i/X)Sf_{ki} - X_{ki} + \beta X. \end{aligned} \tag{8''}$$

Again the functional form of Eq. (8″) is identical to that of Eq. (8). Thus, without worrying about the existence of an equilibrium, Eq. (8) can be used for the analysis of the winner-take-all contest.

In the share contest between two groups, group i's share of the prize Π_i is given by the ratio of its outlay X_i relative to the total outlay X:

$$\Pi_i = X_i/X. \tag{1'}$$

If $X_i < 0$ and $X > 0$, then $\Pi_i < 0$. In such a case, group i in the aggregate takes away resources contributed by the other group, and pays the penalty in the amount of $(-X_i/X)$S, when the rent is distributed. The other group j receives both the rent and the penalty paid by group i. It is easy to find that the functional form of individual payoff in the share contest is equivalent to Eq. (8). Thus, as long as X is positive, the existence issue raised by Nitzan (1991a) can be assumed away with suitable interpretation of the process of the contest both in the winner-take-all contest and in the share contest. The collective contest in this paper may be viewed either as a winner-take-all contest or as a share contest under risk-neutrality.

3.2. Two-stage contest

We now model the collective contest as a two-stage game. We assume that the intra-group sharing-rule parameters α_1 and α_2 are determined prior to individual decisions on the amount of contribution. In the first-stage the representative of each group chooses the value of the intra-group sharing-rule parameter which maximizes the group's aggregate payoffs, respectively. [3] In so doing, they suitably take the actions of the members in the second stage into account. In the second stage all the members of the groups choose their outlays simultaneously and independently. All of this is common knowledge to the members of the two groups. The solution to this game is assumed to be subgame-perfect.

[3] Maximizing aggregate payoffs of the group is equivalent to maximizing individual payoff in the symmetric equilibrium in which each member obtains the same payoff. Thus, each member will agree to the sharing rule proposed by the representative.

To obtain the solution, we work backwards by analyzing the second-stage actions of individuals in the first place. Substituting Eq. (1′) and Eq. (7) into Eq. (8), we obtain

$$V_{ki} = S(1-\alpha_i)X_{ki}/X + S\alpha_i X_i/n_i X - X_{ki} + \beta X,$$
$$\text{for } k=1,\ldots,n_i,\ i=1,2. \tag{9}$$

Individual k of group i maximizes V_{ki} by suitably choosing the level of X_{ki}. The first-order condition for maximization of V_{ki} is

$$\partial V_{ki}/\partial X_{ki} = S(1-\alpha_i)(X-X_{ki})/X^2 + S\alpha_i(X-X_i)/n_i X^2 - 1 + \beta = 0. \tag{10}$$

The second-order conditions are satisfied. The N equations given by Eq. (10) determine the Nash equilibrium values of X_{ki} for given values of α_1, α_2, and β, under a weak restriction that X be positive. Summation of Eq. (10) over relevant range of k and simplification gives, for $i=1,2$,

$$X_i/X = \alpha_i + (1-\alpha_i)n_i - ((1-\beta)n_i X/S). \tag{11}$$

Simultaneous solution of Eq. (11), for $i=1,2$, gives:

$$X_1(\alpha_1,\alpha_2,\beta) = SQ[n_1 n_2(\alpha_2-\alpha_1) + n_2\alpha_1 - n_1\alpha_2 + n_1]/N^2(1-\beta), \tag{12}$$

$$X_2(\alpha_1,\alpha_2,\beta) = SQ[n_1 n_2(\alpha_1-\alpha_2) - n_2\alpha_1 + n_1\alpha_2 + n_2]/N^2(1-\beta), \tag{13}$$

and

$$X(\alpha_1,\alpha_2,\beta) = SQ/N(1-\beta), \tag{14}$$

where $Q=[N-n_1\alpha_1-n_2\alpha_2+\alpha_2-1]$.

A positive value of Q assures a positive value of X. We suitably restrict our attention to the set of α_1 and α_2 which guarantees positive value of Q. This set indeed encompasses a wide range of α_1 and α_2. Note that the value of β has no effect on the area of α_1 and α_2 in which Q is positive. For given values of α_1 and α_2, positive externality tends to increase aggregate efforts made in the contest.

The first-stage interaction between the representatives of the two groups is now analyzed. The representative of each group maximizes the sum of aggregate payoffs V_i $(=\Sigma_k V_{ki})$ by suitably choosing α_i, respectively. The task of each representative is, respectively:

$$\underset{\alpha_1}{\text{Maximize}}\ V_1(\alpha_1,\alpha_2\ \beta) = \Sigma_k V_{k1}(\alpha_1,\alpha_2,\beta) = SX_1(\alpha_1,\alpha_2,\beta)$$
$$/X(\alpha_1,\alpha_2,\beta) - X_1(\alpha_1,\alpha_2,\beta) + \beta n_1 X(\alpha_1,\alpha_2,\beta) \tag{15}$$

$$\underset{\alpha_2}{\text{Maximize}}\ V_2(\alpha_1,\alpha_2,\beta)=\Sigma_k V_{k2}(\alpha_1,\alpha_2,\beta)=SX_2(\alpha_1,\alpha_2,\beta)$$

$$/X(\alpha_1,\alpha_2,\beta)-X_2(\alpha_1,\alpha_2,\beta)+\beta n_2 X(\alpha_1,\alpha_2,\beta) \tag{16}$$

The first-order conditions for Eq. (15) and Eq. (16) are, respectively:

$$\partial V_1/\partial\alpha_1=(n_1-1)S\big[(n_1-n_2)\{(n_2-1)\alpha_2+1\}-2n_2(n_1-1)\alpha_1\big]$$

$$/N^2+\beta n_1 S(1-n_1)/N(1-\beta)=0, \tag{17}$$

$$\partial V_2/\partial\alpha_2=(n_1-1)S\big[(n_2-n_1)\{(n_1-1)\alpha_1+1\}-2n_1(n_2-1)\alpha_2\big]$$

$$/N^2+\beta n_2 S(1-n_2)/N(1-\beta)=0. \tag{18}$$

The second-order conditions are satisfied. In the case with no restriction, simultaneous solution of Eqs. (17) and (18) gives the set of the equilibrium intra-group sharing-rule parameters $(\alpha_1*,\ \alpha_2*)$. The set can be obtained through application of the Kuhn–Tucker conditions when there is the restriction on values of α_1 and α_2.

3.3. *Characteristics of Nash equilibria*

Depending upon the restrictions on values of α_1, α_2, and β, we identify three kinds of Nash equilibria. One is the case when α_1 and α_2 belong to the closed interval [0,1] and the externality parameter β is small. The second case is when α_1 and α_2 belong to the closed interval [0,1] and β is sufficiently large. The third case is the one without any restriction on values of α_1 and α_2.

Case 1. $\alpha_i\in[0,1]$, for $i=1,2$, and $\beta\le(n_1-n_2)/((n_1)^2+n_1n_2+n_1-n_2)$ [4]

This is the case when the externality parameter β is small and/or the two groups are dissimilar in size. In this case, from Eq. (14), X is always positive, and thus a pure strategy Nash equilibrium exists. If $n_2=1$, α_2* can be any value in the interval [0,1], while $\alpha_1*=1/2(1-\beta)$. When $n_2>1$, group 2 will set α_2 as low as possible since $(\partial V_2/\partial\alpha_2)<0$. Hence it follows that $\alpha_2*=0$. On the other hand, group 1 solves Eq. (17) and obtains

$$\alpha_1*=(n_1-n_2)/2n_2(n_1-1)-\beta n_1 N/2n_2(n_1-1)(1-\beta). \tag{19}$$

It is easy to find that α_1* belongs to the closed interval [0,1]. From α_1* given in Eq. (19), we find that positive externality renders the intra-group sharing rule of

[4] Note that $(n_1-n_2)/((n_1)^2+n_1n_2+n_1-n_2)<1/N$.

the larger group to be more effort-based. Since the larger group can recover larger proportion of total efforts in the form of positive externality, the larger group encourages voluntary contribution by making the intra-group sharing-rule to be more effort-based than in the case without positive externality. As a result, the larger group is more likely to win the contest than the smaller one. Simple calculation confirms that

$$\Pi_1 * = X_1 * / X * = 1/2 + \beta n_1 / 2(1-\beta). \tag{20}$$

Note that $\Pi_1 *$ is greater than $1/2$ when β is positive. On the other hand, negative externality makes the intra-group sharing rule of the larger group to be more egalitarian. Then the smaller group is more likely to win the prize.

We now examine the effect of externality on nominal and real rent-dissipation rates in this case. Simple calculation shows that

$$t = X * / S = \{(2n_2 - 1) + (n_1 - 2n_2 + 1)\beta\} / 2n_2(1-\beta)^2, \tag{6'}$$

$$t^{\mathrm{r}} = (1 - \beta N)\{(2n_2 - 1) + (n_1 - 2n_2 + 1)\beta\} / 2n_2(1-\beta)^2.$$

For each value of β, the real rent-dissipation rate t^{r} is lower than that in the benchmark case. As in the individual contest, the nominal rent-dissipation rate is an increasing function of the externality parameter β. The real rent-dissipation rate is inversely related to the parameter β when β is non-negative. However, when β is negative, it is not clear whether the real rent-dissipation rate is positively related to β or not.

Case 2. $\alpha_i \in [0,1]$, for $i = 1,2$, and $\beta > (n_1 - n_2)/((n_1)^2 + n_1 n_2 + n_1 - n_2)$

In this case positive externality is strong enough and/or the two groups are similar in size. Both groups distribute the prize solely based on relative effort. That is, $\alpha_1 * = \alpha_2 * = 0$. Inserting these values into $X_1 *$, $X_2 *$, and $X *$, we find that $X_1 * = S(N-1)n_1/N^2(1-\beta)$, $X_2 * = S(N-1)n_2/N^2(1-\beta)$, and $X * = S(N-1)/N(1-\beta)$. Note that the total and individual outlays are equal to those in the individual contest. Thus, the outcome of the collective contest is equivalent to that of the individual contest. The probability of winning, or the share of the rent, is proportional to the size of each group.

Case 3. No restriction on values of α_1 and α_2

The reaction functions of the two groups in this case are given by Eqs. (17) and (18). When $n_2 > 1$, simultaneous solution of them gives

$$\alpha_1 * = (n_1 - n_2)/N(n_1 - 1) - \beta(2n_1 - n_2)/(1-\beta)(n_1 - 1)$$

$$\alpha_2 * = (n_2 - n_1)/N(n_2 - 1) - \beta(2n_2 - n_1)/(1-\beta)(n_2 - 1).$$

Note that α_2* is always smaller than zero, if $n_2 > 1$. When $\beta \geq (n_1 - 1)/[(2n_1 - n_2)N + n_1 - 1]$, α_1* is smaller than zero as well. If $n_2 = 1$, α_2* can have any value. Inserting these values into X_1*, X_2*, and $X*$, we find

$$\Pi_1* = X_1*/X* = n_2/N + \beta(n_1 - n_2)/(1 - \beta), \tag{20'}$$

$$X* = S[(N-1) + \beta N/(1-\beta)]/N(1-\beta) \tag{21}$$

Note that $n_2/N < \Pi_1* < 1/2$. While the larger group is less likely to win the contest, the disadvantage of the larger group is to some extent alleviated compared to the case without externality. The nominal and the real rent-dissipation rates are, respectively, given by

$$t = (N + \beta - 1)/N(1-\beta)^2$$

$$t^{\mathrm{r}} = (N + \beta - 1)(1 - \beta N)/N(1-\beta)^2$$

Notice that at each level of positive externality, the corresponding nominal and real rent-dissipation rates are higher than those in the benchmark case. Thus, when the collective contest is associated with positive externalities, the rent-dissipation rate can be higher than in the individual contest. Simple calculation shows that $t - t^{\mathrm{r}} = (N + \beta - 1)\beta/(1-\beta)^2$ and $\partial t^{\mathrm{r}}/\partial N = (-\beta N^2 - \beta + 1)/N^2(1-\beta)^2$. Thus when N is large, there is a large difference between the two rates. Also, an increase in N decreases the real rent-dissipation rate, if $\beta \geq 1/(N^2 + 1)$.

4. Concluding remarks

This paper has examined individual and collective contests associated with externalities. Depending upon the restrictions on the intra-group sharing rules and the extent of externalities, we have identified three kinds of Nash equilibria in the collective contest. We have found that the rent is more dissipated in the collective contest than in the individual contest if the contest is associated with positive externalities, and if there is no restriction on the intra-group sharing rules. We have also introduced a distinction between nominal and real rent-dissipation rates. The two rates are shown to be inversely related. Since many contests are associated with externalities, one should be very cautious when deriving welfare implications from the observation of the size of the rent and the rent-seeking expenditures.

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