American Economic Journal: Microeconomics 2015, 7(2): 188–214
http://dx.doi.org/10.1257/mic.20110106

Affiliation and Entry in First-Price Auctions with Heterogeneous Bidders: An Analysis of Merger Effects†

By Tong Li and Bingyu Zhang*

We study the effects of mergers in timber sale auctions in Oregon. We propose an entry and bidding model within the affiliated private value (APV) framework and with heterogeneous bidders, and establish existence of the entry equilibrium and existence and uniqueness of the bidding equilibrium when the joint distribution of private values belongs to the class of Archimedean copulas. We estimate the resulting structural model, and study merger effects through counterfactual analyses using the structural estimates. We evaluate how merger effects depend on affiliation, entry, and the auction mechanism and find that the seller may benefit from some mergers. (*JEL* C57, D44, G34, L11, L73)

Mergers have long been of public policy interest and drawn much attention in the industrial organization literature. Most of the work in this area has adopted the traditional Nash-Cournot model, which allows firms to have heterogeneous capacities. For theoretical contributions, see Farrell and Shapiro (1990), Perry and Porter (1985), among others; for empirical work, see Pesendorfer (2003). Auctions have been used as a means for price determination in a competitive setting under incomplete information environment, but they may be vulnerable to collusion.

Asymmetry is an essential part of any merger analysis in auctions, as even with symmetric bidders, once there is a merger, the postmerger bidders are asymmetric. To conduct meaningful merger analysis in auctions, asymmetric game-theoretic auction models are necessary. This complicates merger analysis in a first-price setting, because even within the independent private value (IPV) paradigm, there is no closed-form expression for the Nash-Bayesian equilibrium. As a result, most of the work on merger effects in auctions has focused on either open auctions or second

* Li: Department of Economics, Vanderbilt University, VU Station B #351819, Nashville, TN 37235 (e-mail: tong.li@vanderbilt.edu); Zhang: Freddie Mac, 1551 Park Run Drive, McLean, VA 22101 (e-mail: zhangbingyu@gmail.com). We are grateful to three referees for their constructive comments that have greatly improved the paper. We thank Luke Froeb, Ken Hendricks, Joel Horowitz, Ali Hortaçsu, Chuck Manski, Elie Tamer, Frank Wolak, and participants in seminars at Northwestern Univeristy, University of Toronto, Georgetown University, University of Maryland, University of Washington, University of Southern California, University of California at Santa Barbara, Johns Hopkins University, and at the 2nd Conference on Auctions and Procurements in Penn State as well as the 10th World Congress of the Econometric Society for comments and discussions. We also thank Elizabeth TeSelle for proofreading the article. Li gratefully acknowledges support from the National Science Foundation (SES-0922109). Earlier versions of the paper were circulated under the title “Affiliation and Entry in First-Price Auctions with Heterogeneous Bidders.” The views expressed here are those of the authors and do not necessarily represent Freddie Mac or its board of directors’ views.

† Go to http://dx.doi.org/10.1257/mic.20110106 to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.

price auctions, where the dominant strategy is to bid one's value, or on first price auctions with some special value distributions that make the analysis tractable or make simulation of bidding strategies possible. See, e.g., Thomas (2004), Waehrer and Perry (2003), Brannman and Froeb (2000), among others. Waehrer (1999) studies merger effects in first-price and open auction mechanisms within the IPV framework under the assumption that the merged bidders' value distributions can be ranked because of the size of merged groups. Another related strand of the literature is on collusion and bid rigging problems.[1]

In this paper, using timber sale auctions organized by the Oregon Department of Forestry (ODF), we study merger effects within a general framework where bidders are heterogeneous and entry is endogenous. Bidder heterogeneity is motivated by previous work studying the timber auctions in Oregon that shows that hauling distance plays an important role in bidding (Brannman and Froeb 2000) and entry (Li and Zhang 2010) decisions. This means that bidders are asymmetric. Also, all the previous work in studying merger effects in auctions has ignored endogenous entry. However, recent empirical work in auctions in general and in timber auctions in particular (e.g., Athey, Levin, and Seira 2011; Bajari and Hortaçsu 2003; Kransnokutskaya and Seim 2011; Li and Zheng 2009, 2012; Roberts and Sweeting 2013) has demonstrated that bidders' participation and entry decisions are part of the decision-making process. Furthermore, Li and Zhang (2010) find a small but strongly significant level of affiliation among potential bidders' private information (either private values or entry costs). In this paper we attempt to study merger effects in the timber auctions organized by the ODF within a general framework in which potential bidders are affiliated and heterogeneous, and they make entry decisions before submitting bids.

We develop an entry and bidding model for asymmetric bidders within the APV paradigm. On the theoretical front, we extend the results by Lebrun (1999, 2006) for the IPV case with asymmetric bidders and without entry to our case. We establish existence and uniqueness of the bidding equilibrium and existence of the entry equilibrium for a general class of joint distribution of affiliated private values.

Because of the general framework we adopt, the merger effects reflect the interaction of affiliation, entry, and asymmetry, as well as competition. The effect of the number of potential bidders on winning bids and seller's revenue is clear in an IPV model with symmetric bidders and without entry. The effect is less clear in a more general setting, such as the IPV model with entry and symmetric bidders (Li and Zheng 2009, 2012), and the APV model without entry (Pinkse and Tan 2005). In particular, Li and Zheng (2009, 2012) show that in the relationship between the number of potential bidders and the seller's expected revenue, in addition to the usual "competition effect," there is an opposite effect which they term the "entry effect." On the other hand, Pinkse and Tan (2005) postulate that in a conditionally independent private value model, a special case of the APV paradigm, in addition to the "competition effect," there is an opposite effect they term the "affiliation effect." Zhang (2008) shows that in the APV model with entry and symmetric bidders, all

[1] See, e.g., Porter and Zona (1993) and Pesendorfer (2000) for empirical studies of collusion in auctions.

three effects are at work: the competition effect, the entry effect, and the affiliation effect. While we expect these three effects to remain in the APV framework with entry and asymmetric bidders, it is challenging to pinpoint them with asymmetric bidders. The merger effects are closely related to how auction outcomes change with the set of potential bidders, i.e., the number and identity of potential bidders when they are heterogeneous. However, theory yields ambiguous predictions, and we rely on structural analysis to gain insight on this issue.

We develop a structural framework to estimate our entry and bidding model.[2] Because there is no closed form solution for equilibrium bidding functions, we rely on a numerical approximation procedure. Moreover, the structural analysis of auctions with asymmetric bidders has focused on the case with two types of bidders (Athey, Levin, and Seira 2011; Campo, Perrigne, and Vuong 2003; and Kransnokutskaya and Seim 2011). Our model allows for all potential bidders to differ, because in our data, asymmetry is driven by differences in bidders' hauling distances. To simplify the structural analysis, we adopt the copula approach to model the joint distribution of private values and the joint distribution of entry costs. The copula approach, which models the joint distribution based on the marginal distributions, has several advantages in our setting. First, within the class of Archimedean copulas, we establish existence and uniqueness of the bidding equilibrium. Second, by using Clayton copulas to model both the joint distribution of private values and the joint distribution of entry costs, affiliation is imposed. Third, the copula approach makes the structural estimation of entry and bidding models more tractable. Finally, we can conduct counterfactutal analysis of merger effects, as the dependence parameter remains unchanged after merger.

We study merger effects through a set of counterfactual analyses using the estimated structural parameters. We consider two hypothetical mergers, one between the two most competitive firms, and one between the two least competitive firms to estimate effects on the seller's revenue, bidders' welfare, and on allocative efficiency. In both cases, we quantify the effects that can be attributed to entry, affiliation, or the auction mechanism (first-price auction versus open auction).

This paper contributes to the literature of the structural analysis of auction data. The structural approach has been extended to the APV setting by Li, Perrigne, and Vuong (2000, 2002); Campo, Perrigne, and Vuong (2003); and Hubbard, Li, and Paarsch (2012). This paper is the first one to estimate a structural model within the APV paradigm taking into account entry. Recent work has considered participation and entry within the IPV framework. Bajari and Hortaçsu (2003) are an exception, as they consider a common value (CV) model.[3]

Affiliation can be viewed as positive dependence among bidders' private information arising from some auction-specific common component that is unobserved by the bidders, such as species composition or wood quality. Also, as discussed in Haile (2001), there could be years of delay before harvesting, and logs are usually

[2] To the best of our knowledge, Brannman and Froeb (2000), who consider oral timber auctions within an IPV paradigm without entry, is the only paper assessing merger effects in auctions using a structural approach.

[3] Recent work on estimating entry and bidding models within the IPV framework includes Athey, Levin, and Seira (2011); Li (2005); Li and Zheng (2009, 2012); Kransnokutskaya and Seim (2011); and Roberts and Sweeting (2013) to name a few.

harvested at the end of the contract, meaning that bidders could face common shocks affecting future demand, and thus, affecting their valuations. As a result bidders' values can be affiliated through this common factor, among others. There has been recent interest in modeling unobserved auction heterogeneity (e.g., Athey, Levin and Seira 2011; Kransnokutskaya 2011; Li and Zheng 2009; Balat 2013; and Roberts and Sweeting 2013), which is an auction-specific common component that is observed by the bidders but not observed by the econometrician. While both of these modeling approaches can have similar implications for observables, such as dependence among bids (conditional on the auction characteristics observed by the econometrician), the information environment differs, with different behavioral implications. Bidders' values in the latter framework are independent conditional on their information set. The information set can contain auction characteristics both observed and unobserved by the econometrician. The affiliated private value model assumes that bidders' values are affiliated even after conditioning on the information set. We also allow for unobserved auction heterogeneity from an econometric viewpoint.

This paper is organized as follows. Section I describes the data. In Section II we propose the asymmetric APV model with entry. Section III is devoted to the structural analysis of the data, and Section IV describes a set of counterfactual analyses studying merger effects. Section V concludes.

I. Data

The data we study in this paper are from the timber auctions organized by the ODF between January 2002 and June 2007. Before an auction is advertised, the ODF "cruises" the selected tract of timber and obtains information of the tract, such as the composition of the species, the quality grade of the timber and so on. Based on the information it obtains, the ODF sets its appraised price for the tract, which serves also as the reserve price. After the "cruise," a detailed bid notice is usually released four to six weeks prior to the sale date, which provides information about the auction, including the date and location of the sale, species volume, quality grade of the timber, and appraised price, as well as other related information. Potential bidders acquire their own information or private values through different ways and decide whether and how much to bid. Bids are submitted in sealed envelopes that are opened at a bid opening session at the ODF district office offering the sale. The sale is awarded to the bidder with the highest bid. All the sales are therefore first price sealed bid scale auctions.

The original data contain 415 sales in total. Among them, some sales have more than one bid species, which are deleted from our sample because of the "skewed bidding" issue discussed in Athey and Levin (2001). We focus on the sales in which Douglas fir is the only bid species and drop the sales with other than Douglas fir as bid species, because Douglas fir is a majority species. Considering the time that our estimation program could take, we focus on the auctions with at most eight potential bidders. The resulting final sample has 203 sales and 1,074 observed bids.

For each sale, we directly observe some sale-specific variables, including the location and the region of the sale, appraised price, appraised volume measured in thousand board feet or MBF, length of the contract, and trunk diameter. Noting that the bid species is often a combination of a mixture of several grades of quality, we use numbers 1, 2, …, up to 18 to denote the letter-grades used by ODF, so that the final grade of a sale is the weighted average of grades with volumes of grades as the weight. In addition to sale-specific variables, as shown in Brannman and Froeb (2000) and Li and Zhang (2010), hauling distance is an important bidder-specific variable that affects bidders' bidding and entry decisions. However, hauling distance is not observed directly. We use the hauling distance variable constructed in Li and Zhang (2010), who convert the location of a tract into latitude and longitude through the Oregon Latitude and Longitude Locator and find the distances between the tract and the mills of firms by using Google Maps.[4]

The key information related to entry is the identities of potential bidders, which are not observed. Unlike some procurement auctions, where information on bidders who have requested bidding proposals is available and can be used as a proxy for potential bidders (Li and Zheng 2009), we do not have such information in our case, as is usual for timber sale auctions. Therefore we follow Athey, Levin, and Seira (2011) and Li and Zheng (2012) to construct potential bidders. Specifically, we first divide all sales in the original dataset into 146 groups, each of which contains all sales held in the same district in the same quarter of the same year. The union of all observed bidders in one group form the set of potential bidders for all sales in that group. In other words, all auctions in the same group have the same set of potential bidders.[5] Summary statistics of the data are given in Table 1. Notably, the entry proportion, which is calculated as the ratio of the number of actual bidders and the number of potential bidders, is about 0.66 on average, meaning that while there is strong evidence of entry pattern from the potential bidders, on average more than half of the potential bidders would participate in the auction.

II. The Model

In this section we propose a two-stage game-theoretic model to characterize the timber sales, extending the models in Athey, Levin, and Seira (2011) and Krasnokutskaya and Seim (2011) with two groups of bidders within an IPV paradigm to the APV paradigm that allows potential bidders to be different from each other. Specifically, motivated by the finding of Brannman and Froeb (2000) that the hauling distance plays a significant role in bidders' bidding decision in oral timber auctions in Oregon, and the finding of Li and Zhang (2010) using the same data studied in this paper that the hauling distance is important in potential bidders' entry decision and potential bidders are affiliated through their private information (either private values or entry costs), we consider a first-price sealed-bid auction within the APV paradigm with asymmetric bidders.

[4] If the lumber processing location is listed, then we use it for the firm's address; otherwise, we use the location of the headquarters as the firm's address.

[5] Li and Zhang (2010) have conducted robustness checks by using alternative ways of defining potential bidders and they find that the results for testing for affiliation are similar.

Table 1—Summary Statistics of Bidder- and Auction-Specific Covariates

	Observation	Mean	SD
Bid	1,074	384.5844	103.7889
Potential bidders	203	5.8276	1.5690
Actual bidders	203	3.6946	1.7250
Entry proportion	203	0.6550	0.2826
Appraised price	203	331.291	94.322
Distance	1,183	75.2779	45.6976
Volume	203	3,318.468	2,674.112
Duration	203	780.010	199.965
Grade	203	10.326	0.461
Trunk diameter	203	16.722	4.812

In the model, a single object is auctioned off to N heterogeneous and risk-neutral potential bidders, who are affiliated in their private information. For each auction, a reserve price, r, is announced prior to the letting. Bidder i has a private entry cost k_i, including the cost of obtaining private information and bid preparation, and does not obtain his private value v_i until after he decides to participate in the auction. We allow private values to be affiliated among bidders, and entry costs to be affiliated as well, that is $v_1, \ldots, v_N$ jointly follow a distribution $F(\cdot, \ldots, \cdot)$ with support $[\underline{v}, \overline{v}]^N$, and $k_1, \ldots, k_N$ jointly follow a distribution $G(\cdot, \ldots, \cdot)$ with support $[\underline{k}, \overline{k}]^N$. It is assumed that the lower support $\underline{v}$ is a bit smaller than the reserve price, e.g., $\underline{v} = r - \varepsilon$, where ε is a small positive number. We make this assumption in order to guarantee the uniqueness of the bidding equilibrium, which is discussed later in equilibrium characterization. Moreover, we assume that each bidder's entry cost and private value are independent.[6] Affiliation is a concept describing the positive dependence among random variables, which was first introduced into the study of auctions by Milgrom and Weber (1982).[7] Intuitively, affiliation means that large values for some of the components in a random vector make other components more likely to be large than small. We also denote the marginal distribution and density of bidder i's private value by $F_i(\cdot)$ and $f_i(\cdot)$ and marginal distribution and density of bidder i's entry cost by $G_i(\cdot)$ and $g_i(\cdot)$, respectively, and assume that $f_i(\cdot)$ is continuously differentiable and bounded away from zero on $[\underline{v}, \overline{v}]$. The subscript of distribution function implies that all potential bidders are of different types due to the different hauling distances.

[6] Athey, Levin, and Sera (2011) and Kransnokutskaya and Seim (2011) make the same assumption. Roberts and Sweeting (2013) consider the selection effect caused by dependence between the signal drawn before entry and the private value in English auctions with two groups of bidders. Nonparametric identification of such models has recently been studied in Gentry and Li (2014). Extending the approach adopted in this paper to the general selective entry model within the APV paradigm and with asymmetric bidders is left for future research.

[7] It is equivalent to the concept of multivariate total positivity of order 2 (MTP_2) in the multivariate statistics literature.

A. *Bidding Strategy*

Because the entry decision of a bidder, say bidder i, is based on his pre-entry expected profit, which depends on his bidding strategy, we first describe the bidding strategy of bidder i. As in Athey, Levin, and Seira (2011) and Kransnokutskaya and Seim (2011), we assume that bidder i knows the number and the identity of active competitors in the bidding stage,[8] and thus bidder i's bidding strategy is determined by the first order condition of the following maximization problem,

$$\pi_i(v_i \mid a_{-i}) = \max(v_i - b_i) \Pr(B_j < b_i \mid v_i; a_{-i}),$$

where B_j denotes the maximum bid among other active bidders[9] and

$$a_{-i} \in A_{-i} = \{(a_1, \ldots, a_N) \mid a_j \in \{0, 1\}, j = 1, \ldots, N, j \neq i\}$$

is one possibility of the 2^{N-1} combinations of entry behavior of $N - 1$ other potential bidders, where $a_j = 1$ if bidder j participates. Use $n_{a_{-i}}$ to denote the number of active bidders of the combination a_{-i}. As usual we consider a continuously differentiable and strictly increasing bidding strategy, $b_i = s_i(v_i)$. Therefore the first order condition is

$$(1) \quad -F_{V_{-i}|v_i}(s_j^{-1}(b_i), j \neq i \mid v_i) + (v_i - b_i) \sum_{j \neq i}^{n_{a_{-i}}} \frac{\partial F_{V_{-i}|v_i}(s_j^{-1}(b_i), j \neq i \mid v_i)}{\partial v_j} \frac{\partial s_j^{-1}(b_i)}{\partial b_i} = 0,$$

where $F_{\mathbf{V}_{-i}|v_i}$ denotes the joint distribution of $\mathbf{V}_j$, $j \neq i$ conditional on $V_i = v_i$ and $s_j^{-1}(\cdot)$ is the inverse function of the bidding function of bidder j. A set of equations (1) with boundary conditions $s_i^{-1}(\underline{v}) = \underline{v}$ for $i = 1, \ldots, n$ form a system of differential equations characterizing the equilibrium bids for all n active bidders.

B. *Entry Decision*

In the initial participation stage, each potential bidder i only knows his own entry cost, the joint distributions of entry costs and the joint distribution private values. Therefore the entry decision of bidder i is determined by his pre-entry expected

[8] This assumption could be reasonable for markets where participants have more accurate prior information on the number/identity of competitors, such as timber auctions (Athey, Levin, and Sera 2011) or procurement auctions (Kransnokutskaya and Seim 2011). See Kransnokutskaya and Seim (2011) for more discussion on this assumption.

[9] Active bidders are those who participate in the auction, but do not necessarily submit bids, as their private values could be below the reserve price. Actual bidders are those who submit bids. In the data we only observe actual bidders. Since in our data the probability that the private value is below the reserve price is very small, we assume that the reserve price exceeds the lower support $\underline{v}$ by a small value. Therefore the set of active bidders and the set of actual bidders are almost identical.

profit from participation, Π_i. Specifically, he participates in the auction only if his entry cost is less than Π_i. The ex ante expected profit Π_i is given by

$$(2) \qquad \Pi_i = \sum_{a_{-i} \in A_{-i}} \int_{\underline{v}}^{\overline{v}} \pi_i(v_i \mid a_{-i})\, dF_i(v_i) \Pr(a_{-i} \mid a_i = 1),$$

where $\Pr(a_{-i} \mid a_i = 1)$ is a function of p_i, $i = 1, \ldots, N$, where p_i is bidder i's entry probability. As a result, the pre-entry expected profit is the sum of 2^{N-1} products of the post-entry profits and corresponding probabilities with the unknown private value integrated out. On the other hand, in equilibrium the entry probabilities are given by $p_i = \Pr(K_i < \Pi_i) = G_i(\Pi_i)$, for all i.

Note that although the number of potential bidders does not directly affect the bidding strategy in the bidding stage, it affects the number and the identities of active bidders, which in turn have impact on the bidding strategy.

C. *Characterization of the Equilibrium*

Existence and uniqueness of the Bayesian Nash equilibrium with asymmetric bidders has been a challenging problem studied in the recent auction theory literature. See, e.g., Lebrun (1999, 2006) and Maskin and Riley (2000, 2003) within the IPV framework, Lizzeri and Persico (2000) within the APV framework and two types of bidders, and Reny and Zamir (2004) within the affiliated value framework and with asymmetric bidders. The analysis of our model is further complicated by the introduction of affiliation and entry, as well as that we allow all potential bidders to be different from each other. To address the issue of existence and uniqueness in our case, we use the theory of ordinary differential equations (ODE). As is well known, the common ODE takes the following form, $y' = h(y, x)$. As the first order condition, equation (1), does not present that form explicitly for any general copulas, we look at the case where the joint distribution of bidders' private values is characterized by the family of Archimedean copulas, which simplifies the first order conditions to the common form of ODE. For the copula concept and the characterization of Archimedean copulas, see Nelsen (1999). The copula approach can provide a flexible way of modeling joint dependence of multivariate variables using the marginal distributions.

Specifically, by Sklar's theorem (Sklar 1973), for a joint distribution $F(x_1, \ldots, x_N)$, there is a unique copula C, such that $C(F_1(x_1), \ldots, F_N(x_N)) = F(x_1, \ldots, x_N)$, where $F_i(\cdot)$ is the marginal distribution of X_i. For the Archimedean copulas, the copula C can be expressed as $C(u_1, \ldots, u_N) = \phi^{-1}(\phi(u_1) + \cdots + \phi(u_N))$, where ϕ is a generator of the copula and is a decreasing and convex function from $[0, 1]$ to $(0, \infty]$ with $\phi(1) = 0$. The Archimedean family includes a wide range of copulas. It is worth noting that an Archimedean copula does not imply affiliation. It is shown by Müller and Scarsini (2005) that an Archimedean copula defines an affiliated distribution if and only if $(-1)^N \phi^{-1(N)}(\cdot)$ is log-convex. For a Clayton copula that has a generator $\phi(u) = \frac{1}{q}(u^{-q} - 1)$, this is equivalent to $q > 0$. Let $C_i(u_1, \ldots, u_N)$ be the partial derivative of the copula with respect to u_i. It is shown in Hubbard, Li, and Paarsch (2012) that $F_{\mathbf{X}_{-i}|x_i}(x_1, \ldots, x_N) = C_i(F_1(x_1), \ldots, F_N(x_N))$. Therefore, with ϕ a generator for the Archimedean copula, we have

$$F_{\mathbf{X}_{-i}|x_i}(x_1, \ldots, x_N) = \phi^{-1'}\big(\phi(F_1(x_1)) + \cdots + \phi(F_N(x_N))\big)\, \phi'(F_i(x_i)).$$

Thus, the first order condition (1) determining the equilibrium bids can be written as follows:

$$(3) \quad \frac{ds_i^{-1}(b)}{db} = \frac{\phi'^{2}(\phi^{-1}\big(\sum_k \phi\big(F_k(s_k^{-1}(b))\big)\big)}{(n-1)\,\phi'\big(F_i(s_i^{-1}(b))\big) f_i(s_i^{-1}(b))\, \phi''(\phi^{-1}\big(\sum_k \phi\big(F_k(s_k^{-1}(b))\big)\big)} \times \left[\frac{n-2}{s_i^{-1}(b) - b} - \sum_{k \neq i} \frac{1}{s_k^{-1}(b) - b}\right].$$

Note that with the copula specification for the joint entry cost distribution, the entry probabilities in (2) can be expressed in terms of the joint entry cost distribution. For example, the probability that bidder 1 up to bidder $i-1$ participate in the auction while bidder $i+1$ up to bidder N do not, given the participation of bidder i, $\Pr(a_{-i}; p_1, \ldots, p_N | a_i = 1)$, can be expressed as follows,

$$(4) \quad \Pr(a_1 = \cdots a_{i-1} = 1, a_{i+1} = \cdots a_N = 0 | a_i = 1)$$

$$= \frac{\Pr(a_1 = \cdots a_i = 1,\, a_{i+1} = \cdots a_N = 0)}{\Pr(a_i = 1)},$$

where

$$\Pr(a_1 = \cdots a_i = 1,\, a_{i+1} = \cdots a_N = 0)$$

$$= C^{\{k\}}(p_1, \ldots, p_i, 1, \ldots, 1; q_k) - \sum_{i+1 \le j \le N} C^{\{k\}}(p_1, \ldots, p_i, p_j, 1, \ldots, 1; q_k)$$

$$\cdots + (-1)^{N-i} C^{\{k\}}(p_1, \ldots, p_N; q_k),$$

$\Pr(a_i = 1) = C^{\{k\}}(1, \ldots, 1, p_i, 1, \ldots, 1; q_k)$, and $C^{\{k\}}$ denotes the copula for the entry cost.

Equilibrium of the model consists of two parts: entry equilibrium and bidding equilibrium. Based on the choice of Archimedean copulas for the joint distribution of private values, existence of the equilibrium can be established. Moreover, with some additional conditions, the bidding equilibrium is unique. The next proposition describes the equilibrium formally.

PROPOSITION 1: *Assume* (a) *the marginal distribution of entry cost of bidder* i, G_i *is continuous over* $[\underline{k}, \overline{k}]$ *for all* i; (b) *the marginal distribution of private value of bidder* i, F_i *is differentiable over* $(r, \overline{v}]$ *with a derivative* f_i *locally bounded away from zero over this interval for all* i; (c) *the joint distribution of private values follows an Archimedean copula;* (d) v_i *and* k_i *are independent.*

(i) Bidding Equilibrium. *In the bidding equilibrium, bidder i adopts a continuously differentiable and strictly increasing bidding function* $b_i = s_i(v)$ *over* $(r, \overline{v}]$. *The inverse functions of* s_i *for all* i, $s_1^{-1}, \ldots, s_n^{-1}$ *are the solution of the system of differential equations* (3) *with the following boundary conditions:*

$$s_i^{-1}(r) = r \tag{5}$$

$$s_i^{-1}(\eta) = \overline{v}. \tag{6}$$

for some η.

(ii) Uniqueness of Bidding Equilibrium. *Moreover, if* $F_i(r) > 0$ *and* $\frac{\phi'^2(u)}{\phi''(u)}$ *is bounded and decreasing in* u, *then the bidding equilibrium is unique.*

(iii) Entry Equilibrium. *In the entry equilibrium, bidder i chooses to participate in the auction if his entry cost is less than the threshold* $\Pi_i(p)$ *and stay out otherwise, where* $p = (p_1, \ldots p_N)$ *and* p_i *is the entry probability of bidder i and is determined by*

$$p_i = G_i(\Pi_i(p)). \tag{7}$$

It can be easily shown that our model meets assumptions A.1 and A.2 in Reny and Zamir (2004), and that the existence of a monotone pure-strategy bidding equilibrium follows Theorem 2.1 in the same paper. We then need to prove that in such bidding equilibrium, at the upper bound of private value, bidders share the same bid, that is the existence of η in the second boundary condition, equation (6). The proof is provided in the first part of Appendix A. At η, $s^{-1}(b)$ is locally Lipschitz in $(r, \eta]$, and $\phi'^2(u)/\phi''(u)$ is bounded, therefore the right hand side of equation (3) is locally Lipschitz in $(r, \eta]$. It then follows that the Lipschitz uniqueness theorem of ordinary differential equations, s_i^{-1} is unique over $(\underline{v}, \eta]$. The condition that $\phi'^2(u)/\phi''(u)$ is decreasing in u is needed in the proof, and can be readily verified that it is met for a Clayton copula.

As is seen here, the existence of the entry equilibrium is equivalent to the existence of the entry probability p_i, given by the equation (7). Since Π_i is continuous in p_i and thus G_i is continuous over $[0, 1]$, there exists a solution p_i of equation (7), according to Brouwer's fixed point theorem. The formal proofs are provided in Appendix A. There could be more than one set of p satisfying equation (7), which raises the possibility of multiple equilibria. Following Kransnokutskaya and Seim (2011), we verify the uniqueness of the entry equilibrium within the estimation routine by trying different initial values to see whether we obtain similar results.

III. The Structural Analysis

We estimate the model proposed in the last section using the timber sales data. Our objective is to recover the underlying joint distributions of private values and entry costs using observed bids and number of actual bidders. The structural inference in our case is complicated because of the generality of our model that accounts for affiliation, asymmetry, and entry. Our approach circumvents the complications arising from the estimation of our model and makes the structural inference tractable. First, to model the affiliation in a flexible way, we adopt the copula approach in modeling the joint distribution of private values and the joint distribution of entry costs.[10] Second, since we allow bidders to be asymmetric, the system of differential equations consisting of equation (3) that characterizes bidders' Bayesian-Nash equilibrium strategies does not yield closed-form solutions. To address this problem we adopt a numerical method based on Marshall et al. (1994) and Gayle (2004). Third, because of the various covariates we try to control for, and the relatively small size of the dataset, the nonparametric/semiparametric method does not work well here.[11] Therefore, we adopt a fully parametric approach.

A. *Specifications*

We use the Clayton copula to model the joint distributions of both private values and entry costs. With the generator of Clayton copula given above, the joint distribution of private value is specified as

$$F(v_1, \dots v_n) = \left(\sum_i F_i(v_i)^{-q_v} - n + 1\right)^{-1/q_v},$$

and the joint distribution of entry costs is specified as

$$G(k_1, \dots, k_n) = \left(\sum_i G_i(k_i)^{-q_k} - n + 1\right)^{-1/q_k},$$

where q_v and q_k are dependence parameters and F_i and G_i are the marginal distributions of private value and entry cost, respectively. The F_i is specified as a truncated exponential distribution $F_{V_{\ell i}}(v \mid \mathbf{x}_{\ell i}; \boldsymbol{\beta}) = \left[\exp\left(-\frac{1}{\lambda_{v\ell i}}\underline{v}\right) - \exp\left(-\frac{1}{\lambda_{v\ell i}}v\right)\right] / \left[\exp\left(-\frac{1}{\lambda_{v\ell i}}\underline{v}\right) - \exp\left(-\frac{1}{\lambda_{v\ell i}}\overline{v}\right)\right]$, and G_i is assumed to be exponential $G_{K_{\ell i}}(k \mid \mathbf{x}_{\ell i}; \boldsymbol{\beta}) = 1 - \exp\left(-\frac{1}{\lambda_{k\ell i}}k\right)$ for bidder i of the ℓ-th auction, $\ell = 1, \dots, L$, where L is the

[10] Hubbard, Li, and Paarsch (2012) use the copula approach to model affiliation within the symmetric APV framework without entry and propose a semiparametric estimation method.

[11] Gentry and Li (2014) address nonparametric identification of the IPV model with selective entry when the number of actual bidders and bids are observed. With affiliated private values in our case, however, nonparametric identification is an open question that is left for future research. On the other hand, Hubbard, Li, and Paarsch (2012) propose a semiparametric method in estimating the symmetric APV model without entry, which can be extended to our case.

number of auctions, $\lambda_{v_{\ell i}}$ and $\lambda_{k_{\ell i}}$ are the parameters in both exponential distribution and equal $\exp(\boldsymbol{\beta}\mathbf{x}_{\ell i})$ and $\exp(\boldsymbol{\alpha}\mathbf{x}_{\ell i})$, respectively, and $\mathbf{x}_{\ell i}$ is a vector of covariates that are auction specific or bidder specific. Note that in our case, $\mathbf{x}_{\ell i}$ includes variables such as hauling distance, volume, duration, grade, and trunk diameter.[12] Auction specific covariates are used to control heterogeneity of auctions and the hauling distance is used to capture asymmetry among bidders. The specification of the truncated marginal distribution of private value makes the numerical method adopted to solve the equilibrium bids possible.[13] In practice, $\overline{v}$ is assumed to be equal to \$1,000/MBF. We then model the joint distributions of private values and entry costs in auction ℓ as Clayton copula with different dependence parameters q_v and q_k. The use of the Clayton copula offers several advantages. First, it guarantees the existence and uniqueness of the equilibrium as discussed in Section IIC. Second, it preserves the same dependence structure when the number of potential bidders changes. Third, it is relatively easy to draw dependent data from the Clayton copula, as it has a closed form that can be used to draw data recursively.

Note that in these specifications, the asymmetry across potential bidders is captured by the inclusion of the hauling distance variable in $\mathbf{x}_{\ell i}$, while both α and β are kept constant across different bidders. This enables us to estimate a relatively parsimonious structural model and at the same time control for the asymmetry among bidders.

B. *Estimation Method*

Because of the complexity of our structural model, we employ the indirect inference method to estimate the model. Initially proposed in the nonlinear time series context by Smith, Jr. (1993) and developed further by Gourieroux, Monfort, and Renault (1993) and Gallant and Tauchen (1996), the indirect inference method is simulation based and obtains the estimates of parameters by minimizing a measure of distance between the estimates for the auxiliary parameters of an auxiliary model using the original data and simulated data. More specifically, let $\boldsymbol{\theta}$ denote the vector of parameters of interest, γ be the parameters of the auxiliary model, $\hat{\gamma}_T$ and $\hat{\gamma}_{ST}^{(p)}(\theta)$ be the estimates of the auxiliary model using the original data and the p-th simulated data out of P sets of simulated data from the model given a specific $\boldsymbol{\theta}$, respectively. Then the estimator of $\boldsymbol{\theta}$, denoted by $\hat{\boldsymbol{\theta}}_{ST}$, is defined as

$$(8) \qquad \hat{\boldsymbol{\theta}}_{ST} = \arg\min_{\boldsymbol{\theta}} \left[\hat{\gamma}_T - \frac{1}{P}\sum_{p=1}^{P} \hat{\gamma}_{ST}^{(p)}(\theta)\right]' \Omega \left[\hat{\gamma}_T - \frac{1}{P}\sum_{P=1}^{P} \hat{\gamma}_{ST}^{(p)}(\theta)\right],$$

where Ω is a symmetric semi-positive definite matrix. Therefore, to implement the indirect inference method, we have to draw data from the model for a given $\boldsymbol{\theta}$, which

[12] Here we do not introduce unobserved auction heterogeneity into the model, as Li and Zhang (2010) show that it does not have a significant effect in bidders' entry behavior.

[13] Here we need an upper bound for the private value to make the algorithm of finding the equilibrium bids possible.

involves calculating the equilibrium bids and the thresholds of the entry costs. We reply on a numerical approximation procedure, which is an extension of the methods in Marshall et al. (1994) and Gayle (2004) to find the equilibrium bids; the algorithm to find the equilibrium bids is illustrated in detail in Appendix B.[14] We use an iteration procedure to find the equilibrium entry probabilities as described below.

Specifically, we adopt a two-step indirect inference method. In the first step we apply the indirect inference method to the observed bids. The assumptions that the entry cost and private values are independent and that the active bidders know the number of the actual bidders at the time of bidding enable us to recover the distribution of private values with only the observed bids. With the estimated distribution of private values, we apply the indirect inference method again to the observed entry behavior in the second step and estimate the distribution of entry costs. One practical issue in the second step estimation is that we need to compute the equilibrium entry probabilities determined by equation (7) in order to evaluate the objective function in the indirect inference method. Therefore the second step estimation should include two loops, namely, the inner loop, which is the one solving for the equilibrium entry probabilities, and the outer loop, which is the one solving the optimization problem that is computationally intensive. To address this issue, we change the order of loops. Specifically, we first estimate the distribution of entry costs using the indirect inference together with any given entry probabilities.[15] With the estimated distribution, we then update the entry probabilities and estimate the distribution again. We repeat these two steps until the estimates and entry probabilities converge.[16]

The auxiliary model, which is usually simpler than the original model and easier to estimate as well, plays an important role in the indirect inference method. In this paper, following Li (2010), we employ a relatively simple and easy-to-estimate auxiliary model to make the implementation tractable and the inference feasible. Specifically, since we use the number of actual bidders and bids in the estimation of entry and bidding model, respectively, our auxiliary model includes two separate regressions: a linear regression of the observed bids and a Poisson regression of the number of actual bidders, which have the following specifications,

$$b_\ell = \gamma_{10} + \mathbf{X}_\ell'\gamma_{11} + (\mathbf{X}_\ell')^2\gamma_{12} + \ldots + (\mathbf{X}_\ell')^m\gamma_{1m} + \varepsilon_{1\ell},$$

$$\Pr(n_\ell = k) = \frac{\exp(-\lambda_\ell)\,\lambda_\ell^k}{k!},$$

$$\lambda_\ell = \exp\left(\gamma_{20} + \mathbf{X}_\ell'\gamma_{21} + (\mathbf{X}_\ell')^2\gamma_{22} + \ldots + (\mathbf{X}_\ell')^m\gamma_{2m}\right),$$

[14] Note that our algorithm is based on finding roots of a polynomial as are Marshall et al. (1994) and Gayle (2004). It is possible that sometimes there are multiple roots even after we remove the nonreal ones and the ones over the upper bound. In this case, we pick the largest one, as from our experiments we have found that the smaller ones are likely to be below the reserve price.

[15] The initial entry probabilities we use are determined by a reduced-form probit model, which should be close to the equilibrium if the observed entry behavior is the result of our game-theoretic model.

[16] To rule out the possibility of multiple equilibria, we have tried different initial values and obtained almost the same results.

where b_ℓ is the average bid of auction ℓ, and $\mathbf{X}_\ell$ denotes the vector of auction-specific covariates of auction ℓ and the average of bidder-specific covariates. In practice, $\mathbf{X}_\ell$ is a 6×1 vector including hauling distance, volume, duration of a contract, timber grade, trunk diameter, and the number of potential bidders, and m is chosen to be 2, which makes our model over-identified.

As is clear from the discussion above, the indirect inference method is computationally convenient in dealing with such complex two-stage entry and bidding models as in our case, because it relies on routine estimation of the two auxiliary regression models and also simulation of the structural model from the trial values of the parameters, which is facilitated by our copula approach. An issue arising from the implementation of the second step indirect inference method is the discontinuity of the objective function of equation (8) because of the discrete dependent variable (the number of actual bidders) in the auxiliary model that makes a gradient-based optimization algorithm invalid. We address this issue by using simplex, a nongradient-based algorithm. Alternatively, one can follow Keane and Smith, Jr. (2003) to smooth the objective function using a logistic kernel.

C. *Estimation Results*

Table 2 reports the estimation results. For the (marginal) private value distribution, all the estimated parameters have the expected signs. Of particular interest is the parameter of the hauling distance variable, which is used to control for heterogeneity across bidders. It has a significantly negative coefficient, meaning that bidders are asymmetric and that the longer the hauling distance is, the less is the private value mean. Furthermore, the average marginal effect of the hauling distance variable is about -0.512, meaning that a one mile increase in the distance would reduce the private value mean by \$0.512/MBF, while everything else is fixed. Another parameter of particular interest is the dependence parameter q_v in private values, which turns out to be relatively small ($q_v = 0.127$) but significant. To get some idea of how large the dependence is with $q_v = 0.127$, we use a measure called Kendall's τ (Nelsen 1999), which lies between $[-1, 1]$ and is used to measure the concordance of two random variables. Concordance is not really the same concept as affiliation, but measures the positive dependence in a similar way. Kendall's τ is defined as the probability of concordance minus the probability of discordance:

$$\tau_{X,Y} = \Pr\left[(X_1 - X_2)(Y_1 - Y_2) > 0\right] - \Pr\left[(X_1 - X_2)(Y_1 - Y_2) < 0\right].$$

For the Clayton copula with two random variables, $\tau = q/(q+2) = 0.060$. Therefore $q_v = 0.127$ implies that the event of any two bidders' private values being concordant is about 6.0 percent more likely than the event of being discordant.

Two points regarding the estimates in the distribution of entry costs are worth noting. First, the hauling distance variable is significant and positive in the entry cost distribution and its marginal effect is 0.699. Second, the dependence level among the entry costs is 0.534, implying a Kendall's τ of 0.21 in the two bidders case, much higher than implied in the distribution of private values. This indicates that

Table 2—Estimation Results

	Private value distribution		Entry cost distribution	
	Coefficient	SE	Coefficient	SE
Hauling distance	−0.473	0.044	1.261	0.231
Volume	0.103	0.045	0.071	0.079
Duration	−0.028	0.085	0.030	0.122
Grade	1.365	0.051	1.208	0.482
Trunk diameter	0.187	0.118	−0.086	0.311
Dependence parameter	0.127	0.051	0.534	0.116

the affiliation among the entry behaviors is mainly driven by the affiliation among the entry costs.[17]

IV. Counterfactual Analyses of Merger Effects

With the estimated structural parameters, we can exploit the rich environment of our model to study merger effects through counterfactual analysis. Specifically, we will assess the merger effects on the winning bid (thus the seller's welfare), on bidders' welfare, and on allocative efficiency.

As discussed previously, the literature on merger effects in auctions has been limited, and mainly focused on studying predictions from theory within the IPV framework without taking entry into account. In this paper we attempt to broaden the scope by studying merger effects within the APV framework, taking both entry and bidders' asymmetry into account. In particular, we compare the merger effects in our model with auctions within the IPV paradigm, with open auctions, and with auctions without entry. These could shed light on how these assumptions interact with mergers. Comparing the APV model with entry with the IPV model with entry would offer insight on how affiliation effect could come into play in the final merger effect; looking into the difference of merger effects in first-price auctions with entry and open auctions with entry both within the APV paradigm can indicate how merger effects could differ between the two mechanisms, and which one may be more susceptible to exercise of market power. Lastly, how entry affects mergers is studied by comparing the APV model with entry and without entry.

For all the effects discussed below, we consider two types of mergers: "best merger," a merger of two bidders with the longest hauling distance; and "worst merger," a merger of two bidders with the shortest hauling distance. "Worst" and "best" refer to whether the merger increases or decreases competition among bidders. Intuitively, "worst merger" creates a new bidder who has the largest private value distribution that dominates other bidders, which discourages competition,

[17] Using a reduced-form test with the bidders' entry behavior observed in the same data, Li and Zhang (2010) find that there is affiliation among bidders' private information (private values and/or entry costs). With the structural approach here, we are able to quantify the extent to which private values are affiliated and entry costs are affiliated, respectively. We are able to identify the dependence parameter in the joint private value distribution from the joint distribution of the observed bids, and we use the potential bidders' joint entry decision to identify the dependence parameter in the joint entry cost distribution.

while "best merger" creates a bidder with a relative larger private value distribution than the distribution premerger, which could make other bidders more aggressive.[18] Therefore, these two types of extreme mergers can provide a range that bounds the effects of any other merger. Note that the merger considered in this paper is assumed to be exogenous.

Assume that bidder 1 and bidder 2 merge without loss of generality and the merged bidder is denoted by bidder m. The private value V_m is defined as $V_m = \max(V_1, V_2)$.[19] Similarly, the entry cost K_m of the merged bidder is defined as $K_m = \min(K_1, K_2)$. Therefore, the marginal distributions of private values and entry costs of the merged bidder are defined as $F_m(v_m) = C(F_1(v_m), F_2(v_m); q_v)$, and $G_m(k_m) = \widetilde{C^{\{k\}}}(1 - G_1(k_m), 1 - G_2(k_m); q_k)$ in terms of copula, where $\widetilde{C^{\{k\}}}$ is the survival copula associated with $C^{\{k\}}$. It can be easily shown that the dependence (affiliation) parameters remain the same after merger.

We conduct counterfactual analyses on 169 auctions whose numbers of potential bidders are no more than 7. Within each paradigm, we report twenty-fifth percentile, median, and seventy-fifth percentile for the effects on each outcome. We simulate 1,000 auctions based on covariates of each auction, conduct "best" and "worst" mergers for these 1,000 auctions, and compare the premerger and postmerger end outcomes. The results are summarized in Table 3. In the table, the change in the winning bid is listed as percentage change and the change in the bidders' payoff is listed as absolute change, as the payoff before merger could be negative and the percentage change makes no sense in this case. The change in the allocation efficiency in the table is the percentage point change.

A. *Merger Effects on the Seller's Welfare*

The seller's welfare is essentially the winning bid if the auction is successful and the seller's value is assumed to be zero. It is interesting to note that the results from Table 3 indicate that while the seller would be worse off with the "worst merger," he may benefit from the "best merger." Specifically, in the "worst merger" case the merger lowers the expected winning bid in all types of auctions, while the "best merger" raises it on average. This is also the case in terms of the most twenty-fifth percentiles, medians, and seventy-fifth percentiles. Most percentiles in the "worst merger" case are negative, while they are positive in the "best merger" case. Asymmetry plays a role in this result, as by the nature of the two types of mergers, the "worst merger" could discourage competition, while the "best merger" could make other bidders more aggressive.

As pointed out by a referee, the finding that the seller may benefit from mergers is the first one for asymmetric bidders in the literature, while it is often true in the

[18] Of course, this is not all that merger brings to competition. Both types of merger change the number of bidders, which is another main source of competition in the market. In this sense, the names may not precisely describe the effect on competition.

[19] Note that most of the work on mergers models mergers in this way. See, e.g., Waehrer (1999); Dalkir, Logan, and Masson (2000); Waehrer and Perry (2003); among others. McAfee and McMillan (1992) use this type of merger to study efficient collusion.

Table 3—Comparison of Merger Effects

	Winning bid[d]		Internal payoff[e]		External payoff[e]		Efficiency[f]	
APV with entry	Worst[a]	Best[b]	Worst	Best	Worst	Best	Worst	Best
25th percentile[c]	−8.53	−1.10	−14.785	3.980	−3.322	−40.134	−0.158	−0.066
Median[c]	−5.84	0.97	20.367	17.498	12.879	−25.556	−0.135	−0.044
75th percentile[c]	−2.23	4.85	50.628	35.155	38.610	−13.910	−0.109	−0.027
Mean[c]	−4.23	2.44	−1.844	20.884	30.371	−26.461	−0.139	−0.047
IPV with entry								
25th percentile	−9.56	−2.20	−0.777	9.834	−8.529	−54.882	−0.171	−0.097
Median	−6.91	1.12	27.971	26.261	9.315	−33.806	−0.147	−0.071
75th percentile	−3.32	6.47	55.099	45.349	30.251	−18.201	−0.119	−0.052
Mean	−7.75	3.32	16.217	26.272	15.057	−35.529	−0.157	−0.110
Open auction								
25th percentile	−14.03	0.47	0.568	15.813	−0.381	−51.225	0	0
Median	−8.74	1.87	0.881	25.937	−0.231	−34.001	0	0
75th percentile	−5.57	3.57	1.238	43.475	0.032	−19.534	0	0
Mean	−10.64	2.13	0.942	33.552	−0.177	−36.991	0	0
APV without entry								
25th percentile	−4.25	−1.50	−39.899	−29.535	12.932	0.292	−0.147	−0.081
Median	−1.00	0.00	−24.565	−19.985	26.083	20.217	−0.119	−0.047
75th percentile	0.14	4.38	−12.155	−8.464	33.505	29.411	−0.088	−0.024
Mean	−1.97	1.42	−29.217	−22.412	24.256	15.869	−0.128	−0.060

Notes:
[a] "Worst merger" refers to a merger of two bidders with the shortest hauling distance.
[b] "Best merger" refers to a merger of two bidders with the longest hauling distance.
[c] Quantiles and mean here refer to the simulated outcomes over 169 auctions.
[d] For the winning bid column, the percentage changes are reported.
[e] For both internal and external payoffs, the absolute changes are reported.
[f] For efficiency, the changes in percentage points are reported.

symmetric case modeled in Levin and Smith (1994) because more potential bidders could reduce revenues. Intuitively, in the symmetric case and within the IPV paradigm in Levin and Smith (1994), reducing the number of potential bidders can increase revenues when there is an entry stage because the remaining firms know they have a higher chance of winning if they enter, and thus are more likely to participate, and because they may also bid more aggressively when they enter. While such an intuition still applies to the asymmetric case considered here, the affiliation effect also could make bidders more aggressive with fewer bidders, explaining why the seller can benefit from a merger even in the APV case without entry.[20]

A comparison between the APV models with entry and without entry indicates that the seller could benefit less from the "best merger" and suffers less from the "worst merger" in the case without entry than in the entry case. This can be viewed as another indication that because of asymmetry, the extent to which the affiliation effect, the entry effect, and the competition effect are at work could differ between the two types of mergers.

[20] We thank the editor for his insightful comments.

B. *Merger Effects on Bidders' Welfare*

To study the merger effects on bidders' welfare, we consider the internal welfare, namely the welfare of the merging bidders, and the external welfare, the welfare of the nonmerging bidders, in a way similar to Farrell and Shapiro (1990), who study horizontal mergers in a Cournot oligopoly model. If the merger is considered endogenous, the change in the internal welfare has to be nonnegative. Because in this paper, the merger is assumed to be exogenous, the change in the internal welfare could be negative. A winning bidder's welfare is defined as the difference between the private value and the winning bid minus the entry cost, while the welfare of a nonwinning bidder is equal to the negative entry cost, if he participates in the auction, and zero if he does not. Note that for each scenario, the internal welfare and external welfare in the premerger cases are different for two types of mergers, as the merging and nonmerging bidders are different in both types of mergers.

The merger affects the bidders' payoff through several channels. On one hand, the merger creates a stronger bidder compared with the two merging bidders. After the "worst merger," the merged bidder has more monopoly power, while in the "best merger," the merged bidder has a larger probability of winning. From this perspective, the merging bidders should benefit from both types of mergers. On the other hand, a stronger bidder makes other bidders more aggressive, so that they bid more than they would without merger. This would lower the payoff of the merging bidders, but it is less important in the "best merger," since the two merging bidders were the least competitive bidders. Mergers could also affect entry behavior, and thus the number of actual bidders. The "best merger" makes two merging bidders more likely to participate. Assuming there is little impact on nonparticipating bidders' entry behavior[21], the "worst merger" reduces one active bidder while the "best merger" increases one active bidder with a nontrivial probability. The change in the number of actual bidders might have an ambiguous effect on the bids due to the "affiliation effect" and the "entry effect." More specifically both "affiliation effect" and "entry effect" raise bids in the "worst merger" while they could lower bids in the "best merger." The final effect on bidders' payoff depends on the relative magnitudes of these effects.

In our model, internal welfare decreases in the "worst merger" case and increases in the "best merger" case. For the IPV model with entry, it increases in both "worst merger" case and "best merger" case (to more extent than in the APV model with entry). This comparison indicates that with merger and a resulting one fewer potential bidders, the affiliation effect is at work, which makes the merging bidders bid more aggressively. On the other hand, the internal welfare decreases in both cases with the APV model without entry; this could be possibly due to a stronger affiliation effect or/and a weaker competition effect in the case without entry than in the entry case.

In both our model and the IPV model with entry, the "worst merger" increases the external welfare while the "best merger" decreases it, because in the "worst merger"

[21] This is actually found in our part of counterfactual analysis, which is not shown in this paper.

the entry probabilities of other bidders are smaller, and consequently less entry cost is incurred, and in the "best merger" bidders have to bid more aggressively when facing a stronger bidder. The percentage increase in the "worst merger" case in our model is more than in the IPV case, and the percentage decrease in the "best merger" case in our model is less than in the IPV case, meaning that the affiliation effect could make the merger effect on external welfare more positive.

It is also worth noting that for internal welfare, which is the payoff for the merging bidders, the "worst merger" case yields a reduction in the first-price auction case while it leads to an increase in the open auction case. On the other hand, for the "best merger" case, the internal welfare increases in both first-price auction case and the open auction case; however, the increase in the former is less than in the latter. These findings imply that in such a general setting as the affiliated private value model with entry considered here, first-price auctions are likely to be less susceptible to the exercise of market power than open auctions, which have been found in the IPV case without entry (Robinson 1985; Waeher 1999, among others.)

C. *Effect on Allocative Efficiency*

An auction is efficient if the bidder with the highest private value wins the auction. Efficiency is always achieved in auctions with symmetric bidders. When the bidders are asymmetric, there could be some loss in the allocative efficiency. This subsection examines how two types of mergers affect the efficiency within the four types of auctions. First, there is no efficiency loss in the open auction, as bidders bid their private values. Second, as is shown in Table 3, in the other three types of auctions, the merger effect on efficiency is negative, and more negative in the "worst merger" case than in the "best merger" case. The "worst merger" results in a more dominant bidder, leading to a higher level of asymmetry, which possibly lowers efficiency. On the other hand, the "best merger" makes two merging least competitive bidders have a larger probability of winning, thus leading to an efficiency loss. In both cases, it is worth noting that the efficiency loss is less in the APV model with entry than in the IPV model with entry, meaning that affiliation could possibly mitigate the efficiency loss resulting from mergers. This is intuitive, as "affiliated bidders" means that they are statistically similar in terms of their private values; potential efficiency loss resulting from asymmetry could be offset by affiliation.

V. Conclusion

In this paper we study merger effects in timber sales auctions in Oregon. We develop an entry and bidding model with heterogeneous bidders within the APV framework, and establish existence and uniqueness of the Bayesian-Nash equilibrium. We adopt the structural approach to obtain the estimates for the structural parameters in the bidders' private values distribution and entry costs distribution. We are able to quantify the extent to which the potential bidders' private values and entry costs are affiliated, respectively, and find that the affiliation among bidders' private information found in Li and Zhang (2010) is mainly driven by the affiliation among bidders' entry costs. We then use the structural estimates to conduct counterfactual analysis to study the merger effects, and offer insight on how affiliation, entry, and auction mechanism could affect mergers from both the seller's and bidders' perspectives. We find that in some cases the seller may benefit from mergers; first-price auctions are likely to be less susceptible to the exercise of market power than open auctions; mergers in first-price auctions could lead to efficiency loss, the extent of which depends on the types of mergers; affiliation could possibly mitigate the efficiency loss resulting from mergers.

To the best of our knowledge, our paper is the first one that uses the structural approach to study the merger effects based on a general model that allows (heterogeneous) bidders' values to be affiliated and also takes their entry decision into account. Because of the general setting we consider, the analysis of the merger effects is complicated by the interactions of affiliation, asymmetry, and entry. The structural approach we propose offers a promising way to disentangle these effects through counterfactual analysis in addressing policy-related issues.

Appendix A: Proof of the Proposition

The proof of Proposition adapts Lebrun (1999, 2006). We first need the following lemma.

LEMMA 1: *Consider a continuously differentiable and strictly increasing bidding strategy. Assume* $\frac{\phi^{-1'}(\cdot)}{\phi^{-1''}(\cdot)}$ *is decreasing, or equivalently* $\frac{\phi'^{2}(\cdot)}{\phi''(\cdot)}$ *is decreasing. If* $\tilde{\eta} > \eta$ *and* $\tilde{s}_i^{-1}(b)$ *and* $s_i^{-1}(b)$ *for all i are two solutions of the system of differential equations* (3) *with boundary condition* (6) *over* $(\tilde{\gamma}, \tilde{\eta}]$ *and* $(\gamma, \eta]$, *respectively, then the inverse bidding functions satisfy the following condition:* $\tilde{s}_i^{-1}(b) < s_i^{-1}(b)$ *for all b in* $(\max(\gamma, \tilde{\gamma}), \eta]$, *where* $\gamma > \underline{v}$.

PROOF:

Since we know that $\tilde{s}_i^{-1}$ is strictly increasing over $(\gamma, \eta]$, we have $\tilde{s}_i^{-1}(\eta) < \tilde{s}_i^{-1}(\tilde{\eta}) = \overline{v} = s_i^{-1}(\eta)$. Define g in $[\max(\gamma, \tilde{\gamma}), \eta]$ as follows:

$$g = \inf\left\{b \in \left[\max(\gamma, \tilde{\gamma}), \eta\right] \middle| \tilde{s}_i^{-1}(b') < s_i^{-1}(b'), \text{ for all } i \text{ and all } b' \in (b, \eta]\right\}.$$

We want to prove that $g = \max(\gamma, \tilde{\gamma})$. According to the definition of g, $\eta > g$. Suppose that $g > \max(\gamma, \tilde{\gamma})$. By continuity, there exists i such that $\tilde{s}_i^{-1}(g) = s_i^{-1}(g)$. From the definition of g, we also have $\tilde{s}_j^{-1}(g) \leq s_j^{-1}(g)$ for all j. Moreover, there exists $j \neq i$ such that $\tilde{s}_j^{-1}(g) < s_j^{-1}(g)$, because if all the solutions coincide at the point g and therefore coincide in $(g, \eta]$ due to the fact that the right hand side of equation (3) is locally Lipschitz at $(g, \eta]$, which contradicts the fact that at η, $\tilde{s}_i^{-1}(\eta) < s_i^{-1}(\eta)$.

From equation (3), we know $ds_i^{-1}(b)/db$ is a strictly decreasing function of $s_j^{-1}(b)$, for all $j \neq i$, since $\frac{\phi'^{2}(u)}{\phi''(u)}$ is decreasing in u. Consequently, $d\tilde{s}_i^{-1}(g)/db > ds_i^{-1}(g)/db$. Therefore there exists $\delta > 0$ such that $\tilde{s}_i^{-1}(b) > s_i^{-1}(b)$, for all b in $(g, g + \delta)$. This contradicts the definition of g.

A. *Proof of Proposition*

PROOF:

First we prove the first part of the proposition by showing that there exists an η, such that $s_i^{-1}(\eta) = \overline{v}$.

Bidding Equilibrium.—Let $i, 1 \leq i \leq n$ denote the bidder who has the highest bid, denoted by η', at the upper bound of private value $\overline{v}$ and $j, 1 \leq j \leq n$ denote the bidder who has the second highest bid, denoted by η, at the upper bound of private value $\overline{v}$. So $\eta' \geq \eta$.

For bidder i, we know that

$$(\overline{v} - \eta')\Pr(B_{-i} < \eta' | \overline{v}) \geq (\overline{v} - \eta)\Pr(B_{-i} < \eta \mid \overline{v}). \tag{A1}$$

It is obvious that $\Pr(B_{-i} < \eta' | \overline{v}) = 1$.

Since the joint distribution of private values follows Archimedean copulas, we have

$$\begin{aligned}
\Pr(B_{-i} < \eta | \overline{v}) &= \Pr(b_j < \eta, b_k < \eta, k \neq i, j | v_i = \overline{v}) \\
&= \Pr(b_k < \eta, k \neq i, j | v_i = \overline{v}) \\
&= \phi^{-1'}\Big(\sum_{k \neq i,j} \phi\big(F_k(s_k^{-1}(\eta))\big) + \phi\big(F_j(s_j^{-1}(\eta))\big) \\
&\qquad + \phi\big(F_i(\overline{v})\big)\Big)\,\phi'\big(F_i(\overline{v})\big) \\
&= \phi^{-1'}\Big(\sum_{k \neq i,j} \phi\big(F_k(s_k^{-1}(\eta))\big) + \phi(1) + \phi(1)\Big)\,\phi'(1),
\end{aligned}$$

and

$$\begin{aligned}
\Pr(B_{-j} < \eta | \overline{v}) &= \Pr(b_i < \eta, b_k < \eta, k \neq i, j | v_j = \overline{v}) \\
&= \phi^{-1'}\Big(\sum_{k \neq i,j} \phi\big(F_k(s_k^{-1}(\eta))\big) + \phi\big(F_j(\overline{v})\big) \\
&\qquad + \phi\big(F_i(s_i^{-1}(\eta))\big)\Big)\,\phi'\big(F_j(\overline{v})\big) \\
&= \phi^{-1'}\Big(\sum_{k \neq i,j} \phi\big(F_k(s_k^{-1}(\eta))\big) + \phi(1) + \phi\big(F_i(s_i^{-1}(\eta))\big)\Big)\,\phi'(1).
\end{aligned}$$

If $F_i(s_i^{-1}(\eta)) < 1$, then $\phi\Big(F_i(s_i^{-1}(\eta))\Big) > \phi(1)$ and $\Pr(B_{-i} < \eta | \overline{v}) > \Pr(B_{-j} < \eta | \overline{v})$ since $\phi'(1) < 0$ and $\phi^{-1'}(x)$ is increasing in x. Therefore

$$\begin{aligned}(\overline{v} - \eta')\Pr(B_{-j} < \eta' | \overline{v}) &= (\overline{v} - \eta')\Pr(B_{-i} < \eta' | \overline{v}) \\ &\geq (\overline{v} - \eta)\Pr(B_{-i} < \eta | \overline{v}) \\ &> (\overline{v} - \eta)\Pr(B_{-j} < \eta | \overline{v}),\end{aligned}$$

where the first equality follows from $\Pr(B_{-i} < \eta' | \overline{v}) = 1$ and $\Pr(B_{-j} < \eta' | \overline{v}) = 1$, the second inequality follows from (A1), and the last one follows from $\Pr(B_{-i} < \eta | \overline{v}) > \Pr(B_{-j} < \eta | \overline{v})$ if $F_i(s_i^{-1}(\eta)) < 1$. But this is impossible because the optimal bid of bidder j at $\overline{v}$ is η, therefore we have $F_i(s_i^{-1}(\eta)) = 1$ and $\eta' = \eta$.

Uniqueness of the Bidding Equilibrium.—Suppose that there exist two equilibria and thus two different values η and $\tilde{\eta}$ such that the respective solutions $s_i^{-1}(b)$ and $\tilde{s}_i^{-1}(b)$ are also solutions of the system of differential equations for all i. Without loss of generality, we assume that $\eta < \tilde{\eta}$. The value of $\ln\Big(\Pr\big(v_j < s_j^{-1}(b_i), j \neq i | v_i\big)\Big)$ at $b_i = \eta$ is strictly larger than that of $\ln\Big(\Pr\big(v_j < \tilde{s}_j^{-1}(b_i), j \neq i | v_i\big)\Big)$ at the same point. We have shown that $\tilde{s}_i^{-1}(b) < s_i^{-1}(b)$ for all b in $(r, \eta]$. When b converges to r, $s_i^{-1}(r) = r$.

On the other hand, since the maximization problem is equivalent to the following problem

$$\max_{b_i} \ln(v_i - b_i)\Pr(b_j < b_i, j \neq i | v_i),$$

the first order condition can be written as follows

$$\frac{d\ln\Big(\Pr\big(v_j < s_j^{-1}(b_i), j \neq i | v_i\big)\Big)}{db} = \frac{1}{s_i^{-1}(b_i) - b_i}.$$

Therefore $\frac{d\ln\left(\Pr\left(v_j < s_j^{-1}(b), j \neq i | v_i\right)\right)}{db} < \frac{d\ln\left(\Pr\left(v_j < \tilde{s}_j^{-1}(b), j \neq i | v_i\right)\right)}{db}$. Thus, the difference between these two logarithms increases as b decreases towards $\underline{v}$. On the other hand, $\ln\big(\Pr(v_j < r, j \neq i | v_i)\big)$ is a finite value since $F_j(r) > 0$. Therefore for two solutions, $\ln\Big(\Pr\big(v_j < s_j^{-1}(b_i), j \neq i | v_i\big)\Big)$ cannot both converge to the same finite value as b decreases towards r. Therefore η and $\tilde{\eta}$ coincide and the equilibrium is unique.

Entry Equilibrium.—The entry probability p_i is determined by equation (7). Let $\mathbf{p} = (p_1, \ldots, p_n) \in [0,1]^n$ and $G_p = \big(G_1 \circ \Pi_1(\mathbf{p}), \ldots, G_n \circ \Pi_n(p)\big)$. Since

$s_i(v)$ and G_i is continuous, the pre-entry expected profit Π_i and $G_i \circ \Pi_i$ is continuous in $\mathbf{p}$. Therefore $G_p : [0,1]^n \to [0,1]^n$ and is continuous in $\mathbf{p}$. A fixed point of $\mathbf{p}$ follows Brouwer's fixed point theorem.

Appendix B: Solving Equilibrium Bids

A. *Equilibrium Bids*

Note that with the choice of the Clayton copula, the first order condition given in equation (3) can be written as follows,

$$(1+q)(v_i - b)\sum_{j \neq i} \frac{dF_j^{-q}\left(s_j^{-1}(b)\right)}{db} = -q\left(\sum_{k=1}^{n} F_k^{-q}\left(s_k^{-1}(b)\right) - n + 1\right).$$

Define $F_i^{-q}\left(s_i^{-1}(b)\right) = l_i(b)$, then $v_i = F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right)$, and F.O.C. becomes

$$(1+q)\left(F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right) - b\right)\sum_{j \neq i} l_j'(b) = -q\left(\sum_{k=1}^{n} l_k(b) - n + 1\right).$$

Rewrite all terms in the equation as polynomials

$$l_i(b) = \sum_{j=0}^{\infty} a_{i,j}(b - b_0)^j,$$

$$l_i'(b) = \sum_{j=0}^{\infty} (j+1)\, a_{i,j+1}(b - b_0)^j,$$

$$l_i^{-\frac{1}{q}}(b) = \sum_{j=0}^{\infty} g_{i,j}(b - b_0)^j,$$

$$F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right) = \sum_{j=0}^{\infty} p_{i,j}(b - b_0)^j,$$

$$F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right) - b = \sum_{j=0}^{\infty} \tilde{p}_{i,j}(b - b_0)^j,$$

$$F_i^{-1}(x) = \sum_{j=0}^{\infty} d_{i,j}(x - x_0)^j,$$

$$x_i^{-\frac{1}{q}} = \sum_{j=0}^{\infty} c_{i,j}(x - x_0)^j,$$

where $\tilde{p}_{i,0} = p_{i,0} - b_0,\ \tilde{p}_{i,1} = p_{i,1} - 1$, and $\tilde{p}_{i,j} = p_{i,j}$ for $j > 1$.

Computation of $p_{i,j}, g_{i,j}$: following the lemma in Appendix C in Marshall et al. (1994), we have

$$\text{(B1a)} \qquad p_{i,J} = \sum_{r=1}^{J} d_{i,r}\theta_{i,r,J} - z_J,\ p_{i,0} = F_i^{-1}\Big(l_i^{-\frac{1}{q}}(b_0)\Big)$$

$$\text{(B1b)} \qquad \theta_{i,r,J} = \sum_{s=1}^{J-r+1} g_{i,s}\theta_{i,r-1,J-s},\ \theta_{i,0,0} = 1,$$

$$\text{(B1c)} \qquad g_{i,J} = \sum_{r=1}^{J} c_{i,r}\varphi_{i,r,J},$$

$$\text{(B1d)} \qquad \varphi_{i,r,J} = \sum_{s=1}^{J-r+1} a_{i,s}\varphi_{i,r-1,J-s},\ \varphi_{i,0,0} = 1\,.$$

Computation of $a_{i,j}$: from the FOC, we have

$$(1+q)\left(\sum_{j=0}^{\infty}\tilde{p}_{i,j}(b-b_0)^j\right)\sum_{j\neq i}\sum_{s=0}^{\infty}(s+1)\,a_{j,s+1}(b-b_0)^s$$

$$= -q\Big(\sum_{k=1}^{n}\sum_{s=0}^{\infty}a_{k,s}(b-b_0)^s - n + 1\Big)$$

$$(1+q)\left(\sum_{j=0}^{\infty}\tilde{p}_{i,j}(b-b_0)^j\right)\sum_{s=0}^{\infty}(s+1)\left(\sum_{j\neq i}a_{j,s+1}\right)(b-b_0)^s$$

$$= -q\Big(\sum_{s=0}^{\infty}\Big(\sum_{k=1}^{n}a_{k,s}\Big)(b-b_0)^s - n + 1\Big)$$

$$(1+q)\sum_{s=0}^{\infty}(s+1)\left(\sum_{r=0}^{s}\tilde{p}_{i,s}\sum_{j\neq i}a_{j,s+1-r}\right)(b-b_0)^s$$

$$= -q\Big(\sum_{s=0}^{\infty}\Big(\sum_{k=1}^{n}a_{k,s}\Big)(b-b_0)^s - n + 1\Big)$$

Comparing the coefficients of $(b-b_0)^s$ we have

$$\text{(B2a)} \qquad (1+q)(s+1)\left(\sum_{r=0}^{s}\tilde{p}_{i,s}\sum_{j\neq i}a_{j,s+1-r}\right) = -q\Big(\sum_{k=1}^{n}a_{k,s}\Big), \quad \text{for } s > 0$$

$$(\text{B2b}) \qquad (1+q)\,p_{i,0}\sum_{j\neq i} a_{j,1} = -q\left(\sum_{k=1}^{n} a_{k,0} - n + 1\right), \quad \text{for } s = 0.$$

Algorithm:

(i) $d_{i,j}, c_{i,j}$ for $j = 1, \ldots, J$, can be computed by Taylor expansion. In practice, $J = 3$.

(ii) Decide $a_{i,0}, \tilde{p}_{i,0}, \theta_{i,0,0}$, and $\varphi_{i,0,0}$ by the boundary conditions.

(iii) Calculate $\tilde{p}_{i,1}$ from equations (B1) given $a_{i,0}, \tilde{p}_{i,0}, \theta_{i,0,0}$, and $\varphi_{i,0,0}$.

(iv) Calculate $a_{i,1}$ from equations (B2) given $\tilde{p}_{i,1}$.

(v) Repeat steps 3 and 4 until $a_{i,j}, j = 1, \ldots, J$ are calculated.

Now since we have found the coefficients of the Taylor expansion of the inverse bidding function up to the J-th order, we are able to find the equilibrium bid for a given private value for bidder i through the obtained Taylor expansion at an appropriate point. One issue regarding the algorithm is the boundary conditions. From the proposition we know that there are two boundary conditions associated with the equilibrium. Note that lower boundary condition cannot be used although the boundary condition at the lower bound of bids is known to us, due to the singularity problem. Therefore we have to use the condition at the upper bound, which is unknown to us. To address this problem we follow the method described in Marshall et al. (1994) and Gayle (2004) to find the common η first. Roughly speaking, it is to find an η which generates the best equilibrium bids at point $\overline{v}$ according to the algorithm described above. For details see Marshall et al. (1994) and Gayle (2004).

REFERENCES

Athey, Susan, and Jonathan Levin. 2001. "Information and Competition in U.S. Forest Service Timber Auctions." *Journal of Political Economy* 109 (2): 375–417.

Athey, Susan, Jonathan Levin, and Enrique Seira. 2011. "Comparing open and Sealed Bid Auctions: Evidence from Timber Auctions." *Quarterly Journal of Economics* 126 (1): 207–57.

Bajari, Patrick, and Ali Hortaçsu. 2003. "The Winner's Curse, Reserve Prices, and Endogenous Entry: Empirical Insights from eBay Auctions." *RAND Journal of Economics* 34 (2): 329–55.

Balat, Jorge. 2013. "Highway Procurement and the Stimulus Package: Identification and Estimation of Dynamic Auctions with Unobserved Heterogeneity." http://www.banrep.gov.co/sites/default/files/eventos/archivos/sem_4_cali.pdf.

Brannman, Lance, and Luke M. Froeb. 2000. "Mergers, Cartels, Set-Asides and Bidding Preferences in Asymmetric Oral Auctions." *Review of Economics and Statistics* 82 (2): 283–90.

Campo, Sandra, Isabelle Perrigne, and Quang Vuong. 2003. "Asymmetry in First-Price Auctions with Affiliated Private Values." *Journal of Applied Econometrics* 18 (2): 179–207.

Dalkir, Serdar, John W. Logan, and Robert T. Masson. 2000. "Mergers in Symmetric and Asymmetric Noncooperative Auction Markets: The Effects on Prices and Efficiency." *International Journal of Industrial Organization* 18 (3): 383–413.

Farrell, Joseph, and Carl Shapiro. 1990. "Horizontal Mergers: An Equilibrium Analysis." *American Economic Review* 80 (1): 107–26.

Gallant, A. Ronald, and George Tauchen. 1996. "Which Moments to Match?" *Econometric Theory* 12 (4): 657–81.

Gayle, Wayne-Roy. 2004. "Numerical Analysis of Asymmetric First Price Auctions with Reserve Prices." http://repec.org/sce2005/up.18137.1108489875.pdf?origin=publication_detail.

Gentry, Matthew, and Tong Li. 2014. "Identification in Auctions with Selective Entry." *Econometrica* 82 (1): 315–44.

Gourieroux, C., A. Monfort, and E. Renault. 1993. "Indirect inference." *Journal of Applied Econometrics* 8 (S1): S85–118.

Haile, Philip A. 2001. "Auctions with Resale Markets: An Application to U.S. Forest Service Timber Sales." *American Economic Review* 91 (3): 399–427.

Hubbard, Timothy P., Tong Li, and Harry J. Paarsch. 2012. "Semiparametric Estimation in Models of First-Price, Sealed-Bid Auctions with Affiliation." *Journal of Econometrics* 168 (1): 4–16.

Keane, Michael, and Anthony A. Smith, Jr. 2003. "Generalized Indirect Inference for Discrete Choice Models." http://econ.ucsb.edu/~doug/245a/Papers/Generalized%20Indirect%20Inference.pdf.

Krasnokutskaya, Elena. 2011. "Identification and Estimation in Highway Procurement Auctions under Unobserved Auction Heterogeneity." *Review of Economic Studies* 78 (1): 293–327.

Krasnokutskaya, Elena, and Katja Seim. 2011. "Bid Preference Programs and Participation in Highway Procurement Auctions." *American Economic Review* 101 (6): 2653–86.

Lebrun, Bernard. 1999. "First Price Auctions in the Asymmetric *N* Bidder Case." *International Economic Review* 40 (1): 125–42.

Lebrun, Bernard. 2006. "Uniqueness of the Equilibrium in First-Price Auctions." *Games and Economic Behavior* 55 (1): 131–51.

Levin, Dan, and James L. Smith. 1994. "Equilibrium in Auctions with Entry." *American Economic Review* 84 (3): 585–99.

Li, Tong. 2010. "Indirect Inference in Structural Econometric Models." *Journal of Econometrics* 157 (1): 120–28.

Li, Tong, Isabelle Perrigne, and Quang Vuong. 2000. "Conditionally Independent Private Information in OCS Wildcat Auctions." *Journal of Econometrics* 98 (1): 129–61.

Li, Tong, Isabelle Perrigne, and Quang Vuong. 2002. "Structural Estimation of the Affiliated Private Value Auction Model." *RAND Journal of Economics* 33 (2): 171–93.

Li, Tong, and Bingyu Zhang. 2010. "Testing for Affiliation in First-Price Sealed-Bid Auctions Using Entry Behavior." *International Economic Review* 51 (3): 837–50.

Li, Tong, and Bingyu Zhang. 2015. "Affiliation and Entry in First-Price Auctions with Heterogeneous Bidders: An Analysis of Merger Effects: Dataset." *American Economic Journal: Microeconomics*. http://dx.doi.org/10.1257/mic.20110106.

Li, Tong, and Xiaoyong Zheng. 2009. "Entry and Competition Effects in First-Price Auctions: Theory and Evidence from Procurement Auctions." *Review of Economic Studies* 76 (4): 1397–1429.

Li, Tong, and Xiaoyong Zheng. 2012. "Information Acquisition and/or Bid Preparation: A Structural Analysis of Entry and Bidding in Timber Sale Auctions." *Journal of Econometrics* 168 (1): 29–46.

Lizzeri, Alessandro, and Nicola Persico. 2000. "Uniqueness and Existence of Equilibrium in Auctions with a Reserve Price." *Games and Economic Behavior* 30 (1): 83–114.

Marshall, Robert C., Michael J. Meurer, Jean-Francois Richard, and Walter Stromquist. 1994. "Numerical Analysis of Asymmetric First Price Auctions." *Games and Economic Behavior* 7 (2): 193–220.

Maskin, Eric, and John Riley. 2000. "Asymmetric Auctions." *Review of Economic Studies* 67 (3): 413–38.

Maskin, Eric, and John Riley. 2003. "Uniqueness of Equilibrium in Sealed High-Bid Auctions." *Games and Economic Behavior* 45 (2): 395–409.

McAfee, Preston R., and John McMillan. 1992. "Bidding Rings." *American Economic Review* 82 (3): 579–99.

Milgrom, Paul R., and Robert J. Weber. 1982. "A Theory of Auctions and Competitive Bidding." *Econometrica* 50 (5): 1089–1122.

Müller, Alfred, and Marco Scarsini. 2005. "Archimedean copulae and positive dependence." *Journal of Multivariate Analysis* 93 (2): 434–45.

Nelsen, Roger B. 1999. *An Introduction to Copulas.* New York: Springer Science.

Perry, Martin K., and Robert H. Porter. 1985. "Oligopoly and the Incentive for Horizontal Merger." *American Economic Review* 75 (1): 219–27.

Pesendorfer, Martin. 2000. "A Study of Collusion in First-Price Auctions." *Review of Economic Studies* 67 (3): 381–411.

Pesendorfer, Martin. 2003. "Horizontal Mergers in the Paper Industry." *RAND Journal of Economics* 34 (3): 495–515.

Pinkse, Joris, and Guofu Tan. 2005. "The Affiliation Effect in First-Price Auctions." *Econometrica* 73 (1): 263–77.

Porter, Robert H., and J. Douglas Zona. 1993. "Detection of Bid Rigging in Procurement Auctions." *Journal of Political Economy* 101 (3): 518–38.

Reny, Philip J., and Shmuel Zamir. 2004. "On the Existence of Pure Strategy Monotone Equilibria in Asymmetric First-Price Auctions." *Econometrica* 72 (4): 1105–25.

Roberts, James W., and Andrew Sweeting. 2013. "When Should Sellers Use Auctions?" *American Economic Review* 103 (5): 1830–61.

Robinson, Marc S. 1985. "Collusion and the Choice of Auction." *RAND Journal of Economics* 16 (1): 141–45.

Sklar, Abe. 1973. "Random variables: Joint distribution functions and copulas." *Kybernetika* 9 (6): 449–60.

Smith, A. A., Jr. 1993. "Estimating Nonlinear Time-Series Models Using Simulated Vector Autoregressions." *Journal of Applied Econometrics* 8 (S1): S63–84.

Thomas, Charles J. 2004. "The Competitive Effects of Mergers between Asymmetric Firms." *International Journal of Industrial Organization* 22 (5): 679–92.

Waehrer, Keith. 1999. "Asymmetric Private Values Auctions with Application to Joint Bidding and Mergers." *International Journal of Industrial Organization* 17 (3): 437–52.

Waehrer, Keith, and Martin K. Perry. 2003. "The Effects of Mergers in Open-Auction Markets." *RAND Journal of Economics* 34 (2): 287–304.

Zhang, Bingyu. 2008. "Affiliation Effect, Competition Effect, and Entry Effect in First-Price Auctions." Unpublished.

This article has been cited by:

1. James W. Roberts, Andrew Sweeting. 2016. Bailouts and the Preservation of Competition: The Case of the Federal Timber Contract Payment Modification Act. *American Economic Journal: Microeconomics* **8**:3, 257-288. [Abstract] [View PDF article] [PDF with links]