Dynamic Price Discrimination with Asymmetric Firms
Author(s): Yongmin Chen
Source: *The Journal of Industrial Economics*, Vol. 56, No. 4 (Dec., 2008), pp. 729-751
Published by: Wiley
Stable URL: http://www.jstor.org/stable/25483437

THE JOURNAL OF INDUSTRIAL ECONOMICS 0022-1821
Volume LVI December 2008 No. 4

DYNAMIC PRICE DISCRIMINATION WITH ASYMMETRIC FIRMS*

YONGMIN CHEN†

This paper considers variants of a dynamic duopoly model where one firm has a stronger market position than its competitor. Consumers' past purchases may reveal their different valuations for the two firms' products. Price discrimination based on purchase histories tends to benefit consumers if it does not cause the weaker firm to exit; otherwise it can harm consumers. The effect of price discrimination also depends on firms' cost differences, market competitiveness, and consumers' time horizon. The stronger firm may price below cost in the presence of consumer switching costs, with the purpose and effect of eliminating competition.

I. INTRODUCTION

IN MARKETS WITH REPEATED PURCHASES, firms sometimes engage in price discrimination based on consumers' past purchases. For example, telephone companies sometimes offer lower prices to customers who switch from a competitor's service; credit card companies sometimes offer lower interest rates to consumers who transfer balances from competitors; and an electricity company sometimes offers lower rates to a rival's customers. The discriminatory pricing in these examples has the common feature that the prices depend on consumers' past purchases and thus incorporate explicit dynamic considerations. Furthermore, the information about a consumer's purchase history takes a particularly simple form, namely whether the consumer has been purchasing from a competitor. Such dynamic pricing practices, sometimes also called behavior-based price discrimination, have received much attention in the recent economics, literature (e.g., Chen [1997], Villas-Boas [1999], Fudenberg and Tirole [2000] and Taylor [2003]). Unlike the traditional economic analysis about price; discrimination, a common theme of this new literature is that dynamic price

*I thank Mark Armstrong, Mats Bergman, James Dana, Samuel Raisanen, two referees, the Editor and participants of the Seminar on Pros and Cons of Price Discrimination at the Swedish Competition Authority, the Workshop on Consumer Search and Switching Cost at Erasmus University Rotterdam, the session on 'Recent Developments in Antitrust Analysis' at the 2007 ASSA meetings in Chicago, and the 2007 Summer Workshop on Asian Competition Policy at Shanghai University of Finance and Economics for helpful discussions and suggestions.

†Author's affiliation: Department of Economics, University of Colorado, Boulder, Colorado, 80309, U.S.A.
e-mail: Yongmin.chen@colorado.edu

discrimination tends to intensify competition and benefit consumers.[1] This literature, however, has focused on markets where firms are *ex ante* symmetric and the number of firms is fixed exogenously.

In some recent antitrust cases, the issue of dynamic price discrimination has arisen in markets with asymmetric firms. For instance, in *AKZO*, the European Court of Justice upheld the principle established by an earlier decision of the European Commission that it is abusive for a dominant firm to offer selectively low prices to customers of a small competitor while maintaining substantially higher prices for its existing customers. The Court viewed such behavior as showing AKZO's adopting a strategy with the intention of damaging its (smaller) competitor.[2] As another example, in 2005 the Swedish Competition Authority sued Teliasonera, a dominant firm in the Swedish telecom market, for having selectively offered better terms to a rival's customers. The agency alleges that the company has abused its dominant market position by engaging in discriminating practices, in violation of the Swedish Competition Act. These cases indicate a view by antitrust authorities that dynamic price discrimination in some asymmetric markets is anticompetitive and harmful to consumers, in contrast to the results from the existing economics literature with symmetric firms. Therefore, it is desirable to extend the economics literature on dynamic price discrimination to markets with asymmetric firms, in the interest of both economic theory and antitrust analysis.

This paper provides an economic analysis of dynamic price discrimination with asymmetric firms. We consider a market where there are two competing firms, one of which possesses a stronger brand (due either to higher quality in a vertical sense or to a more desirable product position in a horizontal sense), which enables it to have a stronger (and possibly dominant) market position. Consumers have different brand preferences, and they may also incur costs if they switch suppliers.[3] Time is discrete and there are $T \geqslant 3$ periods. The weaker firm may choose to exit the market after the two initial periods, due to the fixed cost of remain in the market; thus the number of active firms is endogenous. We study and compare the (subgame perfect) equilibrium of this market under uniform price (henceforth *UP*), where a firm can set only a

[1] Consumers can also benefit from price discrimination in static oligopoly (e.g., Holmes [1989]; and Corts [1998]). Fudenberg and Villas-Boas [2006] surveys behavior-based price discrimination. For a more general treatment of recent developments in the economics of price discrimination, see Armstrong [2006] and Stole [2007].

[2] Judgment of the Court of Justice of 3 July, 1991, in case C-62/86, AKZO Chemie BV v Commission, European Court Reports 1991, I-3359. See Chen [2005] for more discussion of this case, and of another relevant case, *Irish Sugar*.

[3] Dynamic price discrimination can occur both due to consumers' different brand preferences and due to consumer switching cost. In the former, Consumers' past purchases reveal information about their preferences towards different firms' products, enabling the firms to segment the consumers. In the latter, consumers are segmented from their past purchases even if they *ex ante* have no brand preferences. We consider both factors in this paper.

single price to all consumers at each period, and the (perfect Bayesian) equilibrium of this market under discriminatory price (henceforth DP), where a firm can charge different prices to consumers who have or have not purchased from the rival. Our analysis offers several insights:

First, dynamic price discrimination tends to benefit consumers if it does not cause the weaker firm to exit. As in the existing literature, under DP, competition is intensified for the more price-sensitive consumers after they are identified by past purchases. While this effect is likely to dominate on consumer welfare, there could be countervailing effects: the less price-sensitive consumers may pay more; and more subtly, attracted by the future benefits of low prices, some otherwise highly price sensitive consumers may initially become less sensitive to price differences between firms, resulting in a higher price for them early on. A long time horizon (i.e., both sufficiently high T and high discount factor δ) is sufficient for consumers to benefit from DP, or to reap the benefits of intensified competition under DP, if the number of firms does not decrease.

Second, dynamic price discrimination can harm consumers, if it causes the weaker firm to exit, after which the market is monopolized and, as a result, price increases. Since consumers may benefit from the lower price under DP before the weaker firm's exit, there could be a trade off between short-term gain and long-term loss in consumer welfare under DP. A sufficient condition for consumers to be worse off when DP causes exit is that consumers have a long time horizon, provided that the monopoly price by the stronger firm is higher than the equilibrium price under UP.

Third, the effects of dynamic price discrimination on consumers also depend importantly on the competitiveness of the market under uniform price and on the (marginal) cost difference between the two firms. If the degree of competition under UP is low, then DP is more likely to benefit consumers by intensifying competition. Furthermore, under DP the marginal cost of the stronger firm, holding the other firm's cost constant, affects consumer welfare in a non-monotonic way: Starting from the other firm's marginal cost level, increasing this cost initially reduces consumer welfare until the cost reaches some critical level, at which a further increase benefits consumers, and after that, consumer welfare again decreases as this cost increases.

Fourth, in the absence of consumer switching cost, equilibrium price is weakly above average (variable) cost under price discrimination (which arises sorely due to consumers' differences in brand preferences), even if discriminatory price causes exit of the weaker firm and harms competition. In the presence of switching cost, however, the stronger firm may offer below-cost prices to its weaker rival's customers, and such pricing can only serve the purpose of driving the rival from the market, with the effect of eliminating competition. This suggests a sufficient, but not necessary, condition for identifying anticompetitive dynamic price discrimination by a dominant firm is that the firm engages in below-cost pricing in selling to its rival's customers.[4] This underscores the importance of consumer switching cost in understanding firm behavior and market dynamics.[5]

Our paper is related to Armstrong and Vickers [1993], who consider a model where a dominant incumbent faces an endogenous degree of competition in one of its two markets. Price discrimination enables the incumbent to react more aggressively to entry, resulting in less entry, which tends to harm consumers, with the welfare effect depending on the relative efficiencies of the incumbent and the entrant. While our analysis shares some similar intuition to Armstrong and Vickers [1993], in our model consumers are strategic and their segmentation occurs endogenously, as a result of their past purchases. Thus, we incorporate dynamic considerations that are not present in Armstrong and Vickers [1993].[6] Our paper is also related to the economic theory and legal policy of predatory pricing, of which Bolton, Brodley and Riordan [2000] provide a comprehensive treatment. For a price to be considered predatory, it is generally necessary that the price be below some measure of cost and that the predator can later raise prices sufficiently to recoup the loss from below-cost pricing. Our analysis shows that when a dominant firm is able to identify a smaller rival's customers and to offer them a lower price, the price can be predatory without being below cost.

The rest of the paper is organized as follows. Section II describes and analyzes our basic model, which illustrates the trade off between UP and DP in a highly simplified framework. Section III extends the basic model to introduce increased competition under uniform price, which allows us to explore the effects of dynamic price discrimination more generally. Section IV extends the basic model in another direction by allowing consumer switching cost, and identifies a key factor in determining whether and when anti-competitive below-cost pricing might arise in equilibrium. Section V concludes. The appendix contains proofs for all propositions and corollaries in the paper.

II. BASIC MODEL

Time is discrete and is indexed as 1, 2, ..., T, where $T \geqslant 3$. There are two firms, A and B, with constant marginal costs c_A and c_B, respectively,

[4] We shall sometimes call the firm with the stronger market position the 'dominant' firm, when it has substantial market power.

[5] The competitive effects of switching costs have been studied extensively in the recent economics literature (e.g., Klemperer [1987a], Farrell and Shapiro [1988], and Farrell and Klemperer [2004]). Switching costs can alter the prices of an incumbent facing entry threat, and can explain limit pricing behavior, as in Klemperer [1987b]. Our results show that switching costs can also alter the prices of a firm when there is the possibility of exit by its rival.

[6] For antitrust policy considerations, we focus on the effects of price discrimination on consumer welfare. While the possibility of dynamic price discrimination can also affect entry, our paper focuses on the effect of such practice on exit, motivated by antitrust cases where a dominant firm and its smaller competitor are both in the market in which dynamic price discrimination occurs.

where $c_A \geqslant c_B$. For firm B, there is a fixed cost k to stay in the market for each of the periods $t = 3, \ldots, T$.[7] Other costs for A and B are normalized to zero.

There are two types of consumers, H and L. The total mass of consumers is equal to one, with the population size of types H and L being α and $1-\alpha$, respectively. A consumer's type is her private information. Each type H consumer demands one unit of the product with reservation price V_H, and will only purchase from A. Each type L consumer will purchase $q = G(p)$ from the firm with the lower price, p, where

$$G(p)\begin{cases} >0 & if \quad p<V_H \\ =0 & if \quad p \geqslant V_H \end{cases}.$$

Each L consumer receives surplus $s(p)$, and $s'(p) \equiv -G(p) < 0$ for $p < V_H$.[8] The discount factor for all consumers and for both firms is δ.

This formulation captures in a highly simplified way the key features of a market where consumers have different brand preferences and one firm possesses a stronger brand that potentially enables it to have a dominant position. One vertical-differentiation interpretation of the model is that A's product has higher quality than B's (consistent with $c_A \geqslant c_B$): H consumers have sufficiently higher valuation for high quality than for low quality; but L consumers do not place a premium on high quality, and their purchases are also more price sensitive (having a downward-slopping individual demand curve). We shall thus sometimes call H and L consumers the high and low-value segments of the market, respectively. Alternatively, a horizontal-differentiation interpretation of the model is that the two firms are positioned at two separate locations with all consumers residing at A's location (thus A is at an advantageous market position, but with higher marginal cost). H consumers have sufficiently high transportation cost so that they only consider purchasing from A; while L consumers have zero transportation cost, and their demand is also more elastic. We shall call A the stronger firm and B the weaker firm.

Define, for $j = A, B$,

$$\pi_j(p) = (p - c_j)G(p), \tag{1}$$

$$p_j^m = \arg\max_p \pi_j(p). \tag{2}$$

[7] Our assumption that there is no fixed cost for firm A is made only for ease of exposition. Also, we assume k occurs only after $t = 2$, so that our focus is on exit instead of entry decisions. We could alternatively assume that B needs to pay the fixed costs of the first two periods as an entry cost, and our results would be essentially the same as long as this entry cost is below B's equilibrium profit for the first two periods.

[8] We could allow each H consumer's demand to be downward-sloping as well, which would make the analysis less convenient, without changing our main results.

Then p_j^m and $\pi_j(p_j^m)$ would be firm j's monopoly price and profit with respect to L consumers, if j were to be a monopolist selling only to L consumers. Notice that $p_j^m < V_H$.

Throughout the paper, we assume:

A1: For $j = A, B$, p_j^m exists uniquely, $c_A < p_B^m \leqslant p_A^m$, $\pi_j'(p) > 0$ for $p < p_j^m$, and $k \leqslant (1-\alpha)\pi_B(p_B^m)$.

We note that *A1* contains assumptions that are mostly self explaining. With $k \leqslant (1-\alpha)\pi_B(p_B^m)$, we ensure that under uniform price firm B will not exit the market in equilibrium, which allows us to highlight how dynamic price discrimination may affect the exit decision of firm B.

We shall consider only pure strategies when equilibrium in pure strategies exists; we later expand the strategy set to include mixed strategies when there is no pure strategy equilibrium.

The nature of our analysis depends on the comparison between $(p_B^m - c_A)\left[\alpha + (1-\alpha)G(p_B^m)\right]$ and $\alpha(V_H - c_A)$. For the basic model, we assume:

$$\text{(C1)} \qquad (p_B^m - c_A)\left[\alpha + (1-\alpha)G(p_B^m)\right] \leqslant \alpha(V_H - c_A),$$

which says that A's profit from selling to the H consumers alone at V_H is no less than its profit from selling to all consumers at p_B^m. The purpose of (C1) will become clear shortly.[9]

We start our analysis by considering the equilibrium under uniform price, or *UP*. Each firm can only set a single price at each period under *UP*. Let $\{p_{tA}^u, p_{tB}^u\}$ be A's and B's prices at period t. The proper solution concept here is subgame perfect Nash equilibrium (SPNE).

At period T, the last period, the unique Nash equilibrium is $\{p_{TA}^{u*}, p_{TB}^{u*}\} = \{V_H, p_B^m\}$, with all H and L consumers purchasing from A and B, respectively. Clearly, B is maximizing its profit with p_B^m. Also, A cannot do better than charging V_H to sell only to the H consumers, because if it attempts to sell to the L consumers by undercutting B, its profit would be reduced due to condition (C1). The equilibrium profit for firm B in period T, without including k, is

$$(1-\alpha)\pi_B(p_B^m).$$

By assumption *A1*, $(1-\alpha)\pi_B(p_B^m) \geqslant k$, and hence it is optimal for B to stay in the market in period T. It follows that the unique subgame perfect equilibrium is for B to stay in the market all periods and for A and B to charge respectively the monopoly prices for the H and L consumers, V_H and p_B^m, at each period; or $\{p_{tA}^{u*}, p_{tB}^{u*}\} = \{V_H, p_B^m\}$ for $t = 1, \ldots, T$.

[9] The case where (C1) does not hold will be analyzed in the next section.

Remark 1. Under UP, the basic model has a unique SPNE. At this equilibrium, B remains in the market, $\{p_{tA}^{u*}, p_{tB}^{u*}\} = \{V_H, p_B^m\}$, and H and L consumers purchase from A and B respectively for $t = 1, \ldots, T$.

We next analyze equilibrium under discriminatory price, or DP. Under DP, firms can offer different prices to consumers who have different purchase histories.

Suppose that at the beginning of some $t \geqslant 2$, β_{tA} consumers have only purchased from A in the past, β_{tB} consumers have made *at least one purchases* from B in the past, and $1 - \beta_{tA} - \beta_{tB}$ have not made purchases in the past. Call them consumers with purchase history a, b, and o, respectively. Under DP, we assume that firm A and B each can offer *up to* three different prices at every $t \geqslant 2$ to these three groups of consumers:

$$\left(p_{tA}^a, p_{tA}^b, p_{tA}^o\right) \text{ and } \left(p_{tB}^a, p_{tB}^b, p_{tB}^o\right).$$

A strategy of firm j under DP is a price at $t = 1$, p_{1j}, together with a sequence of prices $\{(p_{tj}^a, p_{tj}^b, p_{tj}^o) : t = 2, \ldots, T\}$ if j is in the market; and for firm B there is also the decision of whether to exit at the end of $t \geqslant 2$.[10]

Firms have their beliefs about the probability that each consumer group belongs to type H at t, and we denote this probability that firm $j \in \{A, B\}$ assigns to group $i \in \{a, b, o\}$ by $(\mu_{tj}^a, \mu_{tj}^b, \mu_{tj}^o)$. Each consumer makes her purchase decision that maximizes her discounted sum of surplus.

A perfect Bayesian equilibrium (PBE), which is the proper solution concept for games under dynamic price discrimination, is a pair of two firms' strategies, a system of beliefs, and the purchase strategies of all consumers such that the strategies of all players are sequentially rational given the belief system, and the beliefs are consistent with players' strategies and with Bayes' rule whenever possible. At any $t \geqslant 2$, if B believes that consumers with history a are all H customers, B cannot gain by offering any price to these consumers (recall that an H consumer will only purchase from A); thus, in this case, we will omit p_{tB}^a in describing B's strategy. Also, if in equilibrium all consumers make purchases at any t, as they will in our model, p_{tj}^o will not be offered along the equilibrium path and is only relevant for out of equilibrium deviations. Unless otherwise stated, all equilibria in our analysis are supported by $\mu_{tj}^o = 1$ and $p_{tj}^o = V_H$ off the equilibrium path for $t \geqslant 2$; and to economize notations we shall also omit p_{tj}^o in describing equilibrium strategies.

As usual for games with price competition by firms having different costs, we shall assume that a firm will not bid any price for which its payoff is lowered if it succeeds in selling to the consumers at that price. This

[10] As it will become clear, once B exits, it has no incentive to re-enter the market, even if entry cost is zero. For ease of exposition, when we say B exits at the end of t, we also mean that B remains out of the market for the rest of the game.

assumption is implied by the standard refinement that firms do not play weakly dominated strategies.[11]

Proposition 1. (i) If $(1-\alpha)\pi_B(c_A) \geqslant k$, the basic model has a unique PBE under DP. At this equilibrium, B stays in the market for all periods,

$$\begin{aligned} \{p_{1A}^*, p_{1B}^*\} &= \{V_H, p_B^m\}, \\ \{(p_{tA}^{a*}, p_{tA}^{b*}), p_{tB}^{b*}\} &= \{(V_H, c_A), c_A\} \text{ for } t = 2, \ldots, T; \end{aligned} \tag{3}$$

all H and L consumers purchase from A and B respectively in all periods.

(ii) If $(1-\alpha)\pi_B(c_A) < k$, the basic model has a unique PBE under DP. At this equilibrium, B exits at the end of $t = 2$,

$$\begin{aligned} \{p_{1A}^*, p_{1B}^*\} &= \{V_H, p_B^m\}, \\ \{(p_{2A}^{a*}, p_{2A}^{b*}), p_{2B}^{b*}\} &= \{(V_H, c_A), c_A\}, \\ (p_{tA}^{a*}, p_{tA}^{b*}) &= (V_H, p_A^m) \text{ for } t = 3, \ldots, T; \end{aligned} \tag{4}$$

all H and L consumers purchase from A and B respectively in $t = 1, 2$, and all consumers purchase from A in $t = 3, \ldots, T$.

Thus, in $t = 1$, A and B will charge respectively the monopoly prices for the H and L consumers; the H consumers will purchase from A while the L consumers will purchase from B. If B does not exit the market, its competition with A will drive the price for the L consumers down to c_A in the subsequent periods; and if B exits at the end of $t = 2$, the price for the L consumers is c_A in $t = 2$ but rises to p_A^m afterwards.

We note that our assumption that H consumers are loyal to A and will only purchase from A significantly simplifies the analysis, but it is not essential for our results. If H consumers could also consider purchasing from B, our results would be essentially the same if, for instance, (i) each H consumer also has a downward-slopping demand curve and V_H is the monopoly price, and (ii) an H consumer has higher surplus from A's product under V_H than from B's product under p_B^m due to vertical product differentiation.

Denote the discounted sum of aggregate consumer surplus under UP and DP by W^u and W^d, respectively. We have:

[11] But this refinement can also lead to nonexistence of equilibrium in price games with a continuous strategy space. To preserve equilibrium existence for our game under the dominance refinement, we can consider a sequence of price games with discretized strategy spaces (prices). An equilibrium exists under the dominance refinement for each game in the sequence. Let the limit of this sequence be our game, as the mesh of the partition goes to zero. The equilibrium of our game under the dominance refinement is then the limiting equilibrium of the sequence.

Corollary 1. In the basic model, $W^d > W^u$ if $(1-\alpha)\pi_B(c_A) \geqslant k$ or if $c_A = c_B$; and $W^d < W^u$ if $(1-\alpha)\pi_B(c_A) < k$, $c_A > c_B$, and both T and δ are sufficiently large. Furthermore, firm B is always worse off under DP.

The results here capture the basic (potential) conflict for consumer welfare under dynamic price discrimination with asymmetric firms. On the one hand, DP intensifies competition (only for L consumers here), which by itself benefits consumers. This is an insight from the existing studies of dynamic price discrimination, where the focus has been on markets with a given number of *ex ante* symmetric firms. On the other hand, the intensified competition under DP can cause the weaker firm to exit the market, resulting in a price increase afterwards and potentially hurting consumers. In the simple model here, consumer welfare is higher under DP if it does not cause firm B to exit or if A's marginal cost is not higher than B's; while consumer welfare is lower under DP only if it causes firm B to exit, A has higher marginal cost, and the time horizon is sufficiently long (both T and δ are large).

Interestingly, because of this conflict, consumer welfare under DP, W^d, is non-monotonic in c_A. When c_A starts from some high value such that $(1-\alpha)\pi_B(c_A) > k$, marginal decreases in c_A increase W^d. As c_A goes down further to reach the point at which $(1-\alpha)\pi_B(c_A) = k$, a marginal decrease in c_A reduces W^d, due to the exit effect. W^d rises again as c_A decreases further beyond that point.

We note that firm A's lowest price in equilibrium is c_A, which is A's marginal as well as average (variable) cost.

Remark 2. In equilibrium, firm A does not engage in below-cost pricing, even when A's more aggressive pricing under DP induces firm B to exit and harms consumers.

Our basic model has two noteworthy features. First, due to condition (C1), competition is weak, in the sense that under uniform price the firm with a stronger brand relinquishes the low-value consumer segment to the weaker competitor, so that there is no head-to-head competition between the two firms. Price discrimination eliminates this market segmentation and intensifies competition. This formulation makes the best case for DP in its effect on consumer welfare. Second, consumer preferences are constant over time; namely past purchases do not affect consumer preferences towards either firm. The basic model provides a benchmark for our analyses in the next two sections that further consider the effects of increased competition and of consumer switching cost (under which a consumer's preference towards different firms is affected by her past purchase).

III. INCREASED COMPETITION

This section considers the effects of increased competition by introducing effective competition to the model under UP. The main difference from the basic model is that condition (C1) does not hold. That is, in this section we instead assume

$$\text{(C2)} \qquad \left(p_B^m - c_A\right)\left[\alpha + (1-\alpha)G\left(p_B^m\right)\right] > \alpha(V_H - c_A).$$

In this case, let $\underline{p} \in (c_A, p_B^m]$ be such that

$$\text{(5)} \qquad \left(\underline{p} - c_A\right)\left[\alpha + (1-\alpha)G(\underline{p})\right] = \alpha(V_H - c_A).$$

Notice that the unique existence of $\underline{p}$ is guaranteed since $\pi_j'(p) > 0$ for $p < p_j^m$. For this section, we further assume:

A2. (i) $s(\underline{p}) < s\left(p_B^m\right) + \delta s(c_A)$, and (ii) $k \leqslant (1-\alpha)\pi_B(\underline{p})$.

Part (i) of *A2* is easily satisfied if δ is not too small (notice that $s(\underline{p}) < s(c_A)$), which is a natural assumption since we are mainly interested in situations where the future is relatively important. Part (ii) strengthens part of Assumption *A1*, again ensuring that firm B will not exit in equilibrium under *UP*.[12]

Everything else is the same as in the basic model. Again, we first consider uniform price and then discriminatory price.

III(i). *Uniform Price*

Now, it is easy to see that there will be no pure strategy Nash equilibrium for the stage game at each period. Firm B will charge a price no higher than p_B^m, and it is willing to charge a lower price than A. This motivates A to charge V_H, which means B should change p_B^m. But then due to condition (C2), A has the incentive to undercut B. In fact, for any B's price between $(\underline{p}, p_B^m]$, A has the incentive to undercut. Therefore, we consider equilibrium in mixed strategies, where B randomizes prices on $[\underline{p}, p_B^m]$, while A randomizes on $[\underline{p}, p_B^m] \cup \{V_H\}$.

Consider the following pair of mixed strategies. Firm A chooses $p \in [\underline{p}, p_B^m] \cup \{V_H\}$ according to $F_A(p)$; and Firm B chooses $p \in [\underline{p}, p_B^m]$ according to $F_B(p)$. Firm A's expected profit from choosing any $p \in [\underline{p}, p_B^m]$ is

$$\begin{aligned}&\alpha(p - c_A)F_B(p) + (p - c_A)[\alpha + (1-\alpha)G(p)][1 - F_B(p)] \\ &\quad = \alpha(V_H - c_A).\end{aligned}$$

That is,

$$\begin{aligned}(p - c_A)(1-\alpha)G(p)F_B(p) = (p - c_A)&[\alpha + (1-\alpha)G(p)] \\ &- \alpha(V_H - c_A),\end{aligned}$$

[12] We note that if $c_A = c_B$, as $k \to 0$ and $\alpha \to 0$, the model has its limit the symmetric Bertrand duopoly under market demand $G(p)$.

and thus

$$F_B(p) = \frac{(p - c_A)(1 - \alpha)G(p) - \alpha(V_H - p)}{(p - c_A)(1 - \alpha)G(p)} \text{ if } \underline{p} < p < p_B^m.$$

Firm B's expected profit from choosing any $p \in [\underline{p}, p_B^m]$ is

$$0 \cdot F_A(p) + (1 - \alpha)\pi_B(p)[1 - F_A(p)] = (1 - \alpha)\pi_B(\underline{p}).$$

Thus

$$F_A(p) = \frac{\pi_B(p) - \pi_B(\underline{p})}{\pi_B(p)} = 1 - \frac{\pi_B(\underline{p})}{\pi_B(p)} \text{ if } \underline{p} \leqslant p \leqslant p_B^m.$$

Define

$$(6) \qquad F_A(p) = \begin{cases} 1 & \textit{if} \quad p \geqslant V_H \\ 1 - \frac{\pi_B(\underline{p})}{\pi_B(p_B^m)} & \textit{if} \quad p_B^m \leqslant p < V_H \\ 1 - \frac{\pi_B(\underline{p})}{\pi_B(p)} & \textit{if} \quad \underline{p} \leqslant p < p_B^m \\ 0 & \textit{if} \quad p < \underline{p} \end{cases},$$

$$F_B(p) = \begin{cases} 1 & \textit{if} \quad p \geqslant p_B^m \\ 1 - \frac{\alpha(V_H - p)}{(1-\alpha)\pi_A(p)} & \textit{if} \quad \underline{p} < p < p_B^m \\ 0 & \textit{if} \quad p < \underline{p} \end{cases}.$$

Notice that $F_A(p)$ has its only mass point at $p = V_H$, and $F_B(p)$ has its only mass point at p_B^m. The equilibrium in mixed strategies is characterized in Proposition 2 below.

Proposition 2. Assume (C2) holds. Then there is a unique SPNE in mixed strategies under UP, at which firm B stays in the market all periods, with firms A and B pricing according to probability distribution functions $(F_A(p), F_B(p))$ in each period. The equilibrium profits per period for A and B are $\pi_A^* = \alpha(V_H - c_A)$ and $\pi_B^* = (1 - \alpha)\pi_B(\underline{p})$.

The expected equilibrium prices of A and B at each period are:

$$p_A^u \equiv \int p \, dF_A(p) < V_H, \; p_B^u \equiv \int p \, dF_B(p) < p_B^m.$$

Since H consumers will only purchase from A while L consumers will purchase form the firm with the lower price, and since there is positive probability $p_A < p_B$, at each period the expected price of H consumers is $p_H^u = p_A^u$, and the expected price of L consumers is $p_L^u = p^u < p_B^u$.

Let the probability distribution of the lower price of the two firms at each period be $F_{\min}(p)$. The expected discounted sum of aggregate consumer surplus is

$$W^u = \left[\alpha\left(V_H - p_A^u\right) + (1-\alpha)\int s(p)dF_{\min}(p)\right]\left(1+\delta+\ldots+\delta^{T-1}\right)$$
$$> \left[\alpha\left(V_H - p_A^u\right) + (1-\alpha)\int s(p)dF_B(p)\right]\left(1+\delta+\ldots+\delta^{T-1}\right)$$
$$\geqslant \left[\alpha\left(V_H - p_A^u\right) + (1-\alpha)s\left(p_B^u\right)\right]\left(1+\delta+\ldots+\delta^{T-1}\right),$$

where the first inequality is due to $\int s(p)dF_B(p) < \int s(p)dF_{\min}(p)$, and the second inequality is because $s\,(p)$ is convex and thus

$$\int s(p)dF_B(p) \geqslant s\left(\int pdF_B(p)\right) = s\left(p_B^u\right).$$

On the other hand, we have

$$W^u = \left[\alpha\left(V_H - p_A^u\right) + (1-\alpha)\int s(p)dF_{\min}(p)\right]\left(1+\delta+\ldots+\delta^{T-1}\right)$$
$$< \left[\alpha\left(V_H - p_A^u\right) + (1-\alpha)s\left(\underline{p}\right)\right]\left(1+\delta+\ldots+\delta^{T-1}\right).$$

III(ii). *Effects of Price Discrimination*

We first characterize equilibrium under price discrimination.

Proposition 3. (i) If $(1-\alpha)\pi_B(c_A) \geqslant k$, there is a PBE under DP, at which the equilibrium prices are given by (4), B stays in the market for all periods, and all H and L consumers purchase from A and B respectively in all periods.

(ii) If $(1-\alpha)\pi_B(c_A) < k$, there is a PBE under DP, at which the equilibrium prices are given by (5), B exits at the end of $t = 2$, all H and L consumers purchase from A and B respectively in $t = 1, 2$, and all consumers purchase from A in $t = 3, \ldots, T$.

Notice that Proposition 3 is the same as Proposition 1 except that we have not established equilibrium uniqueness in Proposition 3. Potentially, under (C2) there might be equilibria where the (complete) separation of consumers does not occur in $t = 1$. For instance, we have not been able to rule out the possibility of an equilibrium where firm A prices slightly above $\underline{p}$ in $t = 1$ and sells to all L consumers, followed by A and B's prices $\left\{V_H, p_B^m\right\}$ in $t = 2$, and price discrimination occurs afterwards. However, the equilibrium in Proposition 3 involves most price discrimination, in the sense that price discrimination occurs at the earliest possible time and lasts the longest. We shall focus our analysis on this equilibrium.

We can compare the equilibrium outcomes under conditions (C2) and (C1). With uniform price the market is more competitive and equilibrium prices lower under (C2), while with discriminatory price the equilibrium outcome is the same under either (C2) and (C1). Thus, different from the basic model, price discrimination now raises price for the H consumers in all periods. Furthermore, price discrimination also raises price for the L consumers in $t = 1$ while lowers price for them in future periods; and the reason for the higher price in $t = 1$ is that L consumers become less price sensitive in $t = 1$ under DP, because they take into account the future prices they will be offered, which depend on their current purchases.

The fact that equilibrium price under DP is not (weakly) lower than under UP for every period, even when there is no exit, makes the comparison of consumer welfare for the two pricing regimes more complicated. But unambiguous comparisons are available if both T and δ are sufficiently large.

Corollary 2. Suppose that Condition (C2) holds. When T and δ are sufficiently large,

$$W^d \begin{cases} > W^u & \text{if} \quad (1-\alpha)\pi_B(c_A) \geqslant k \\ < W^u & \text{if} \quad (1-\alpha)\pi_B(c_A) < k \end{cases}.$$

Since increased competition, in the sense of lower p_A^u and p_B^u, increases consumer welfare under UP but does not change consumer welfare under DP, $W^d - W^u$ is lower with increased competition. In other words:

Remark 3. When the market is more competitive under uniform pricing, consumers gain less (or lose more) from price discrimination.

As in the basic model, the lowest price firm A charges is c_A. Thus in equilibrium firm A again does not engage in below cost pricing.

IV. WITH CONSUMER SWITCHING COST

We now extend the basic model in another direction: suppose that a consumer incurs switching cost $\sigma > 0$ each time she changes her supplier. We assume that $\sigma < s(p_A^m)$ (thus σ is not to too large), and a stronger version of (C1) is satisfied:

$$\text{(C1}'\text{)} \qquad \max_{c_A \leqslant p \leqslant \bar{p}} \{\alpha(p - c_A) + (1-\alpha)\pi_A(p)\} \leqslant \alpha(V_H - c_A),$$

where $\bar{p} \in (p_B^m, V_H)$is defined by

$$s(\bar{p}) = s(p_B^m) - \sigma.$$

Notice that (C1′) implies (C1). Condition (C1′), which ensures that A will charge V_H under UP, simplifies our analysis but is not essential for the main insight of this section. Everything else is the same as in the basic model.

Under uniform price, the solution to the game is straightforward. In the last period, $t = T$, regardless of how the L consumers purchase in $T - 1$, the unique Nash equilibrium is for A to charge V_H and for B to charge p_B^m, with all H and L consumers purchasing from A and B, respectively. Notice that by condition (C1′), A is better off charging V_H than charging any price not exceeding $\bar{p}$ in T, and $s(p_B^m) - \sigma > s(p)$ for $p > \bar{p}$. From backward induction, we thus have:

Remark 4. In the presence of consumer switching cost, there is a unique SPNE under UP, where the equilibrium outcome is the same as in the basic model of Section 2.

Again, competition under UP is weak: there is no head-to-head competition between the two firms due to (C1′).

Next we turn to the analysis under discriminatory price. Define $c_1 < c_2$ by the following conditions:

$$s(c_1) = s(c_A) + \sigma, \tag{7}$$

$$s(c_2) = s(c_A) - \sigma. \tag{8}$$

Then, by the monotonicity of $s(\cdot)$, c_i exist uniquely and $0 < c_1 < c_A < c_2$, provided that c_A is not too small and σ not too large, which we shall assume. Notice that c_1 is the price B needs to charge in order to have an L consumer switch from A to B in T, and c_2 is the price B will charge an L consumer in T who has purchased from B in $T-1$, provided that the L consumer has revealed her type and A's price for the consumer is c_A.

There are three possible cases to consider: (i) $k \leqslant (1-\alpha)\pi_B(c_1)$,which implies that $c_1 > c_B$; (ii) $k > (1-\alpha)\pi_B(c_2)$, and (iii) $(1-\alpha)\pi_B(c_1) < k \leqslant (1-\alpha)\pi_B(c_2)$.

We first consider cases (i) and (ii). Define $\tilde{p}$ by

$$s(\tilde{p}) - [s(c_A) - \sigma] = \delta[s(c_A) - s(c_2)] = \delta\sigma,$$

or $s(\tilde{p}) = s(c_A) - \sigma(1-\delta)$. Then $\tilde{p}$ exists uniquely and $c_A < \tilde{p} < c_2$, where $\tilde{p}$ is the price by B in $T-1$ that will make an L consumer indifferent between purchasing from B and switching to A at price c_A, provided that the consumer purchased from B in $T-2$ and B remains in the market.

Proposition 4. (i) If $k \leqslant (1-\alpha)\pi_B(c_1)$, there is a unique PBE, where B always remains in the market,

$$\begin{aligned} \{p_{1A}^*, p_{1B}^*\} &= \{V_H, p_B^m\}, \\ \{(p_{tA}^{a*}, p_{2A}^{b*}), (p_{tB}^{b*},)\} &= \{(V_H, c_A), \tilde{p}\} \text{ for } t = 2, \ldots, T-1, \\ \{(p_{TA}^{a*}, p_{3A}^{b*}), (p_{TB}^{b*},)\} &= \{(V_H, c_A), c_2\}; \end{aligned}$$

all H and L consumers purchase from A and B respectively in all periods.

(ii) If $k > (1-\alpha)\pi_B(c_2)$, there is a unique PBE, where B exits at the end of $t=2$,

$$\{p_{1A}^*, p_{1B}^*\} = \{V_H, p_B^m\},$$
$$\{(p_{2A}^{a*}, p_{2A}^{b*}), p_{2B}^{b*}\} = \{(V_H, c_A), \tilde{p}\},$$
$$(p_{tA}^{a*}, p_{tA}^{b*}) = (V_H, p_A^m) \text{ for } t = 3, \ldots, T;$$

all H and L consumers purchase from A and B respectively in $H = 1, 2$, and all consumers purchase from A in $t = 3, \ldots, T$.

Thus, in $t = 1$, again A and B will charge respectively the monopoly prices for the H and L consumers; the H consumers will purchase from A while the L consumers will purchase from B. But different from Proposition 1, if B does not exit the market, its competition with A will drive the price for the L consumers down only to $\tilde{p}$ in $t = 2, \ldots, T-1$ and to c_2 in T. Since $c_A < c_2$ and $c_A < \tilde{p}$, the presence of consumer switching costs raises prices for the mobile consumers. If B exits at the end of $t = 2$, the price for the L consumers is $\tilde{p}$ in $t = 2$ (comparing to c_A when there is no switching cost), but again rises to p_A^m afterwards.

We next turn to case (iii). Let $p_a < c_A$ and $p_b \leqslant c_B$ satisfy

$$\pi_A(p_a) + \delta\pi_A(p_A^m) = 0 \tag{9}$$

$$(1-\alpha)\pi_B(p_b) + \delta[(1-\alpha)\pi_B(c_2) - k] = 0. \tag{10}$$

Then p_a and p_b exist uniquely, again by $\pi_j'(p) > 0$ for $p < p_j^m$ from assumption $A1$; p_a is the lowest price at which A is willing to sell to L consumers in order to earn the monopoly profit from them next period, and p_b is the lowest price at which B is willing to sell to the L consumers in order to sell to them at price c_2 next period. Notice that both $c_A - p_a$ and $p_b - p_a$ increase as $\delta\pi_A(p_A^m)$ increases. Define

$$s(\hat{p}) = s(p_b) + \sigma(1-\delta). \tag{11}$$

Then $\hat{p}$ is the price by A in $T-1$ that will make an L consumer indifferent in $T-1$ between purchasing from A at $\hat{p}$ and purchasing from B at p_b, provided that the consumer purchased from B in $T-2$ and will purchase from A in T. We note that there are parameter values under which $0 < p_a < \hat{p} < p_b$. It is always true that $\hat{p} < p_b$, and its is likely that $0 < p_a < \hat{p}$ if c_A is not too small and $c_A - c_B$ is not too large.

Proposition 5. Assume $(1-\alpha)\pi_B(c_1) < k \leqslant (1-\alpha)\pi_B(c_2)$. If in addition $0 < p_a < \hat{p}$, then there is a PBE at which firm B exits at the end of $t = 2$,

$$\{p_{1A}^*, p_{1B}^*\} = \{V_H, p_B^m\},$$

$$\{(p_{2A}^{a*}, p_{2A}^{b*}), p_{2B}^{b*}\} = \begin{cases} \{(V_H, \hat{p}), p_b\} & \textit{if} \quad T \text{ is an odd number} \\ \{(V_H, c_A), \tilde{p}\} & \textit{if} \quad T \text{ is an even number} \end{cases},$$

$$(p_{tA}^{a*}, p_{tA}^{b*}) = (V_H, p_A^m) \text{ for } t = 3, \ldots, T;$$

all H consumers purchase from A in all periods; all L consumers purchase from B in $t = 1$ and purchase from A for all $t > 2$; and in $t = 2$ all L consumers purchase from A if T is an odd number but from B if T is an even number.

It may seem not intuitive that the equilibrium prices at $t = 2$ should depend on whether T is an odd or even number. To understand this result, notice that under condition (iii), B will remain in the market in T if the L consumers have purchased from it in $T - 1$; otherwise it will not be in the market in T. But since $p_a < \hat{p}$, A is willing to charge a sufficiently low price ($\hat{p}$) to the L consumers in $T - 1$ so that it is not profitable for B to stay in the market for $T - 1$. By backward induction, if T is an odd number, B will stay in the market for $t = 3$ (and exit at $t = 4$ if $T > 3$) if all L consumers purchased from B in $t = 2$, which motivates A to lower its price sufficiently (to $\hat{p}$) in $t = 2$ to attract the L consumers in $t = 2$, ensuring that B will exit at the end of $t = 2$. Even though this is costly for A in $t = 2$, A is compensated by the monopoly profit in $t = 3$ (and beyond). On the other hand, if T is an even number, B will exit in $t = 3$ by backward induction, regardless of whether the L consumers have purchased from it in $t = 2$. In this case, A does not need to incur the loss in $t = 2$ by pricing below c_A to the L consumers, and hence B will sell to the L consumers in $t = 2$ after they have purchased from it in $t = 1$.

Notice that since $\hat{p} < p_b \leqslant c_B \leqslant c_A$, firm A may price below its average variable (marginal) cost in $t = 2$. Thus, with switching costs, it is possible that the dominant firm will engage in equilibrium below-cost pricing under DP, with the purpose and effect of eliminating competition.[13]

We can further compare consumer welfare for all parameter values.

Corollary 3. Assume $0 < p_a < \hat{p}$. In the presence of consumer switching costs, if $k \leqslant (1-\alpha)\pi_B(c_1)$, DP increases consumer welfare; if $(1-\alpha)\pi_B(c_1)$

[13] In other models with consumer switching costs, the equilibrium prices can be below marginal cost even under uniform pricing (see, e.g., Farrell and Klemperer [2007]). With a given number of symmetric firms, this below-cost pricing is a form of aggressive competition that typically does not raise antitrust concerns, in contrast to the below-cost pricing here under discriminatory pricing.

$< k$, DP may decrease consumer welfare, and A may engage in below-cost pricing with the purpose and effect of eliminating competition.

Thus, as in the basic model, if DP does not cause the weaker firm to exit, it benefits consumers, due to intensified competition; otherwise DP can harm consumers.

Dynamic pricing based on purchase history occurs usually for two reasons. First, consumers may have inherent preference diversity for different firms' products and past purchases reveal such preferences. second, consumers may have costs to switch suppliers, which is another source of product differentiation that separates consumers under repeated purchases. Our analysis suggests that the nature of dynamic price discrimination in markets with asymmetric firms depends on whether or not switching cost is present. If DP is based purely on consumers' inherent brand preferences, below-cost pricing does not occur as an equilibrium strategy. However, in the presence of switching cost, consumers have the incentive to purchase from their current supplier, and a firm thus benefits from having a higher market share. Consequently, a dominant firm sometimes has the incentive to engage in 'predatory' below-cost pricing. It loses in the short run for pricing below cost, but its low price deprives the rival of the market share needed to remain profitably in the market. The dominant firm is then able to increase its future prices and profits sufficiently to recoup the short-term losses.

V. CONCLUSION

The effects of dynamic price discrimination on competition and consumer welfare change dramatically from symmetric to asymmetric markets, where the issue of endogenous industry structure arises naturally. The intensified competition under price discrimination, which tends to benefit consumers in markets with symmetric firms, can cause the exit of a weaker firm and harm consumers in markets with asymmetric firms. A sufficient condition for dynamic price discrimination to benefit consumers is that it does not result in fewer firms and that consumers have a long time horizon. When price discrimination induces exit, there may be a trade off for consumer welfare between current price reductions and future price increases, and price discrimination can reduce consumer welfare. The less efficient the dominant firm is, and/or the more competitive the market is under uniform price, the less (more) consumers tends to benefit (lose) from discriminatory price. Dynamic price discrimination may or may not appear as predatory in the usual sense, depending crucially on whether there is consumer switching cost: if dynamic pricing is based purely on consumer's brand preferences, it will not involve below-cost prices, even when it causes exit and harms consumers; in the presence of consumer switching costs, the dominant firm may price below cost in selling to the rival's customers, with the purpose and effect of eliminating competition.

Our analysis is based on variants of a duopoly model that are highly stylized, the choice of which is motivated by both analytical convenience and antitrust concerns. The equilibrium feature that only the stronger firm has the strict incentive to engage in purchase history based price discrimination is clearly due to our simplifying assumption that the high value consumers will only purchase from the stronger firm, but it also reflects the observation that it is the dominant firm that price discriminates in some actual antitrust cases involving asymmetric firms. That there may be market segmentation under uniform price, where the stronger and weaker firms sell to different segments of the market without head-to-head competition, simplifies the analysis and highlights the intensifying-competition effect of price discrimination; but it also seems realistic in some markets, and Section III shows how this assumption can be relaxed. The assumption that the low value consumers consider the two firms' products as perfect substitutes is again for analytical convenience; it is possible to introduce some product differentiation for this group of consumers as well, but this substantially complicates the analysis, especially with many periods, due to the need by both firms to draw inferences about the types of this group of consumers based on past purchases.[14] Finally, instead of finitely many periods, one could alternatively pursue an infinite-horizon model, possibly with overlapping generations of consumers. The analysis would be much more complicated, but it appears that the main insight of our analysis can still be valid. In particular, dynamic price discrimination could lower the equilibrium price in an infinite-horizon model (see, e.g., Villas-Boas [1999]), which would lead to a similar trade off under the endogenous market structure as explored in our model.

For antitrust policies, our analysis suggests that when a stronger firm engages in below-cost pricing that targets a weaker rival's customers, there is compelling reason to consider such pricing behavior as predatory and harming competition. Without below-cost pricing, dynamic price discrimination by a stronger firm can still have an exclusionary purpose and effect; but it is also possible that the price discrimination benefits consumers. In such situations, the competitive effects can be determined only in the context of each case through detailed economic analysis. In particular, discriminatory price by the stronger firm would be anticompetitive when there is a 'dangerous probability' that it will result in the

[14] In fact, instead of two type consumers, we could alternatively consider a Hotelling-type model where firms' asymmetry arises due to consumers' different transportation costs (preference intensities) towards the two firms, as in Chen [2006], or due to consumers' valuation differences from the two firms and/or from firms' cost differences. Such a model, however, appears much less tractable, without the clear advantage of offering additional insights.

competitor's exit. Otherwise, it is unlikely to raise significant antitrust concerns.

APPENDIX

The appendix contains the proofs for Propositions 1–3 and Corollaries 1 and 2. The proofs for Propositions 4 and 5, and for Corollary 3 are on the Journal's website, *www.essex.ac.uk/jindec/*

Proof of Proposition 1. (i) If in $t = 1$ both firms follow the equilibrium prices $\{p^*_{1A}, p^*_{1B}\}$and consumers make the purchases as described, then A and B's prices $\{(p^{a*}_{tA}, p^{b*}_{tA}), p^{b*}_{tB}\}$ at $t = 2, \ldots, T$, and only these prices, are sequentially rational, following the equilibrium belief that consumers with histories a and b are H and L consumers, respectively, while the off-equilibrium belief and price are $\{\mu^o_{tj} = 1, p^o_{tj} = V_H\}$. It thus suffices to show that no player can profit from any deviation in $t = 1$, and that there can be no equilibrium where $\{p^*_{1A}, p^*_{1B}\} \neq \{V_H, p^m_B\}$. First, given the equilibrium prices and the behavior of other consumers, and given both firms' belief and prices off the equilibrium path, each consumer's purchase in $t = 1$ is optimal. Second, if A lowers its price sufficiently below p^m_B so that all L consumers purchase from it in $t = 1$, then at $t = 2$ A and B would face essentially the same game as in $t = 1$, but one period is lost and A's profit in $t = 1$ is below the equilibrium profit, since by (C1)

$$(p^m_B - c_A)\left[\alpha + (1-\alpha)G(p^m_B)\right] < \alpha(V_H - c_A).$$

Thus A cannot profit from any deviation that lowers its price at $t = 1$, and obviously it cannot benefit from raising its price above V_H. Third, B cannot benefit from deviating its price away from p^m_B at $t = 1$. Finally, there can be no other equilibrium, since if there were another equilibrium, $\{\tilde{p}^*_{1A}, \tilde{p}^*_{1B}\}$, it must be that $\{\tilde{p}^*_{1A}, \tilde{p}^*_{1B}\} \neq \{V_H, p^m_B\}$. But then either A or B or both firms could increase payoff by deviating to $\{V_H, p^m_B\}$.

(ii) For any $t = 2, \ldots, T$, if all H and L consumers purchase from A and B respectively in $t = 1$, B remains in the market in t, the unique equilibrium outcome in t is $\{(p^{a*}_{2A}, p^{b*}_{2A}), p^{b*}_{tB}\} = \{(V_H, c_A), c_A\}$; and it follows that, since $(1-\alpha)\pi_B(c_A) < k$, B will stay out of the market for $t = 3, \ldots, T$. Thus, it suffices to show that there is a unique equilibrium in which, with the equilibrium outcome in the continuation game, we have that $\{p^*_{1A}, p^*_{1B}\} = \{V_H, p^m_B\}$, and all H and L consumers purchase from A and B in $t = 1$, respectively. With the similar reasoning as in (i), neither any consumer, nor firm A or firm B, can benefit from any deviation in $t = 1$, and there can be no equilibrium with $\{p^*_{1A}, p^*_{1B}\} \neq \{V_H, p^m_B\}$. *Q.E.D.*

Proof of Corollary 1. We first notice that the H-consumers receive the same surplus (equal to zero) under both UP and DP. Thus, the change in aggregate consumer surplus is the same as the change in consumer surplus by the L-consumers.

Next, if $(1-\alpha)\pi_B(c_A) \geqslant k$, since $c_A < p^m_B$,

$$\begin{aligned} W^d - W^u &= (1-\alpha)\{[s(p^m_B) + s(c_A)(\delta + \ldots + \delta^{T-1})] \\ &\quad - [s(p^m_B)(1 + \delta + \ldots + \delta^{T-1})]\} \\ &= (1-\alpha)[s(c_A) - s(p^m_B)](\delta + \ldots + \delta^{T-1}) > 0 \text{ for any } c_A \geqslant c_B. \end{aligned}$$

Next, if $(1-\alpha)\pi_B(c_A)<k$,

$$\begin{aligned}
&W^d - W^u \\
&\quad = (1-\alpha)\left[s(p_B^m) + \delta s(c_A) + s(p_A^m)(\delta^2 + \ldots + \delta^{T-1})\right] \\
&\quad - (1-\alpha)\left[s(p_B^m)(1+\delta+\ldots+\delta^{T-1})\right] \\
&\quad = (1-\alpha)\delta\left[s(c_A) - s(p_B^m)\right] - \left[s(p_B^m) - s(p_A^m)\right](\delta^2 + \ldots + \delta^{T-1}).
\end{aligned}$$

Thus, if $c_A = c_B$, $W^d - W^u = (1-\alpha)\delta\left[s(c_A) - s(p_B^m)\right] > 0$, and if $c_A > c_B$

$$W^d - W^u \to (1-\alpha)\delta\left\{\left[s(c_A) - s(p_B^m)\right] - \left[s(p_B^m) - s(p_A^m)\right]\frac{\delta}{1-\delta}\right\} < 0$$

for $T \to \infty$ and $\delta \to 1$.

Finally, it is straightforward that firm *B's* discounted sum of profits is lower under *DP* than under *UP*. *Q.E.D.*

Proof of Proposition 2. First consider period T. For firm A, with $p = V_H$ or with any $p \in [\underline{p}, p_B^m]$, *A's* expected profit given $F_B(p)$ is

$$\begin{aligned}
&\alpha(p - c_A)F_B(p) + (p - c_A)[\alpha + (1-\alpha)G(p)][1 - F_B(p)] \\
&\quad = (p - c_A)[\alpha + (1-\alpha)G(p)] - (1-\alpha)(p - c_A)G(p)F_B(p) \\
&\quad = (p - c_A)[\alpha + (1-\alpha)G(p)] - (1-\alpha)\pi_A(p)\left[1 - \frac{\alpha(V_H - p)}{(1-\alpha)\pi_A(p)}\right] \\
&\quad = (p - c_A)\alpha + (1-\alpha)\pi_A(p) - (1-\alpha)\pi_A(p) + \alpha(V_H - p) = \alpha(V_H - c_A),
\end{aligned}$$

and *A's* profit is lower with any other price. Thus, given $F_B(p)$, it is optimal for A to randomize on $p = V_H$ and on $p \in [\underline{p}, p_B^m]$ according to $F_A(p)$, with zero probability density placed on any other price.

For firm B, with any $p \in [\underline{p}, p_B^m]$, *B's* expected profit given $F_A(p)$ is

$$\begin{aligned}
&0 + (1-\alpha)(p - c_B)G(p)[1 - F_A(p)] = (1-\alpha)\pi_B(p)[1 - F_A(p)] \\
&\quad = (1-\alpha)\pi_B(p)\frac{\pi_B(\underline{p})}{\pi_B(p)} = (1-\alpha)\pi_B(\underline{p}),
\end{aligned}$$

and *B's* profit is lower with any other price. Thus, given $F_A(p)$, it is optimal for B to randomize on $p \in [\underline{p}, p_B^m]$according to $F_B(p)$, with zero probability density placed on any other price.

Notice that $F_A(\underline{p}) = 0$, $F_A(V_H) = 1$, and $F_A(p)$ has only one mass point at V_H with probability measure $\frac{\pi_B(\underline{p})}{\pi_B(p_B^m)}$. Notice also that $F_B(\underline{p}) = 0$, $F_B(p_B^m) = 1$, and $F_B(p)$ has only one mass point at p_B^m with probability measure $\frac{\alpha(V_H - p_B^m)}{(1-\alpha)\pi_A(p_B^m)} \in (0,1)$. Hence $F_A(p)$ and $F_B(p)$ are genuine cumulative distribution functions. Thus, if B stays in the market, $(F_A(\cdot), F_B(\cdot))$ constitutes a mixed-strategy Nash equilibrium in period T. The equilibrium profits for A and B in $t = T$ are $\pi_A^* = \alpha(V_H - c_A)$ and $\pi_B^* = (1-\alpha)\pi_B(\underline{p})$. Furthermore, there can be no other equilibrium. This is because if there were another equilibrium, $(\tilde{F}_A(\cdot), \tilde{F}_B(\cdot))$, the equilibrium payoff for A and B must still be $\alpha(V_H - c_A)$ and

$(1-\alpha)\pi_B(\underline{p})$, respectively. Writing down the expressions for expected profits of A and B, we would have $(\tilde{F}_A(\cdot), \tilde{F}_B(\cdot)) = (F_A(\cdot), F_B(\cdot))$. Thus the equilibrium in T is unique.

Since by assumption $A2$, $k \leqslant (1-\alpha)\pi_B(\underline{p})$, B will indeed stay in the market in period T. Following backward induction, there is a unique subgame perfect Nash equilibrium in mixed strategies for the game, in which B stays in the market for all periods, A and B price according to $(F_A(\cdot), F_B(\cdot))$ in each period, and their equilibrium profits at each period are π_A^* and π_B^*. *Q.E.D.*

Proof of Proposition 3. (i) If in $t = 1$ both firms follow the equilibrium prices and consumers make the purchases as described, their actions in $t = 2, \ldots, T$, and only these actions, are sequentially rational, following the equilibrium belief that consumers with histories a and b are H and L-consumers, respectively, while the off-equilibrium belief and price are $\{\mu_{tj}^o = 1, p_{tj}^o = V_H\}$. It thus suffices to show that no player can profit from any deviation in $t = 1$.

First, given the prices on and off the equilibrium paths, each consumer's purchase in $t = 1$ is optimal. Second, if A deviates to a lower price that would attract some (or all) L-consumers, in order for the deviation to potentially benefit A, it must not reduce A's profit for period $t = 1$. Thus the lowest potentially profitable deviating price at $t = 1$ by A is $\underline{p}$. But since from $(A2)$,

$$s(\underline{p}) - s(p_B^m) < \delta[s(c_A) - s(V_H)] = \delta s(c_A),$$

with each L-consumer believing that she will be offered V_H and c_A in $t = 2$ for purchasing from A and B in $t = 1$, respectively, no L-consumer will purchase from A at $t = 1$ for any deviating price A is willing to offer. Thus A cannot profit from any deviation at $t = 1$. Finally, B cannot benefit from changing its price away from p_B^m at $t = 1$.

(ii) For any $t = 2, \ldots, T$, if all H and L-consumers purchased from A and B respectively in $t = 1$, and B remains in the market in t, the unique equilibrium outcome in t is $(p_{tj}^*, p_{tj}^{b*}, p_{tj}^{o*}) = (V_H, c_A, V_H)$. In this case, it follows that, since $(1-\alpha)\pi_B(c_A) < k$, B must stay out of the market and A will charge $(p_{tA}^*, p_{tA}^{b*}, p_{tA}^{o*}) = (V_H, p_A^m, V_H)$ for $t = 3, \ldots, T$. Thus, it suffices to show that there is an equilibrium in which, following the equilibrium outcome in the continuation game, $\{p_{1A}^*, p_{1B}^*\} = \{V_H, p_B^m\}$ and all H and L-consumers purchase from A and B respectively in $t = 1$.

First, each consumer is optimizing in $t = 1$ given the firms' strategies. Next, A is willing to lower its price at most to p in $t = 1$ in order to sell to all consumers in that period. But no L consumer will purchase from A at such a price, with the expectation that she will be offered V_H and C_A in $t = 2$, respectively, for purchasing from A and B in $t = 1$, since

$$s(\underline{p}) - s(p_B^m) < \delta[s(c_A) - s(V_H)] = \delta s(c_A).$$

Hence A cannot benefit from any deviation in $t = 1$. Finally, B cannot benefit from changing its price at $t = 1$. *Q.E.D.*

Proof of Corollary 2. (i) If $(1-\alpha)\pi_B(c_A) \geqslant k$, firm B will remain in the market for all periods. Denote consumer welfare, industry profit and total social surplus in period t by w_t^j, π_t^j, and z_t^j, for $j = d, u$. Then, for $t \geqslant 2$,

$$\pi_t^d = [\alpha(V_H - c_A) + (1-\alpha)\pi_B(c_A)],$$

$$\pi_t^u = [\alpha(V_H - c_A) + (1-\alpha)\pi_B(\underline{p})],$$

and hence, $\pi_t^d - \pi_t^u = (1-\alpha)[\pi_B(c_A) - \pi_B(\underline{p})] < 0$.Notice also that for $t \geqslant 2$, $z_t^d > z_t^u$ since output is higher under DP and expected unit cost is lower under DP. Therefore

$$w_t^d - w_t^u \equiv w^d - w^u > 0 \text{ for } t = 2, \ldots, T.$$

Hence,

$$W^d - W^u = w_1^d - w_1^u + \left(w^d - w^u\right)\delta\left(1 + \delta + \ldots + \delta^{T-2}\right)$$
$$\to w_1^d - w_1^u + \left(w^d - w^u\right)\frac{\delta}{1-\delta} > 0 \text{ when } T \to \infty,\ \delta \to 1.$$

(ii) If $(1-\alpha)\pi_B(c_A) < k$,firm B will stay out of the market for $t \geqslant 3$.

$$w_t^u = \alpha\left(V_H - p_A^u\right) + (1-\alpha)\int s(p)dF_{\min}(p) > \alpha\left(V_H - p_A^u\right) + (1-\alpha)\int s(p)dF_B(p)$$
$$\geqslant \alpha\left(V_H - p_A^u\right) + (1-\alpha)s\left(p_B^u\right)\forall t,$$

while $w_1^d = (1-\alpha)s\left(p_B^m\right)$, $w_2^d = (1-\alpha)s(c_A)$, and $w_t^d = (1-\alpha)s\left(p_A^m\right)$for $t > 2$. Thus,

$$w_t^d - w_t^u \leqslant (1-\alpha)s\left(p_A^m\right) - \left[\alpha\left(V_H - p_A^u\right) + (1-\alpha)s\left(p_B^u\right)\right] < 0 \text{ for } t > 2.$$

Hence, when $T \to \infty$, $\delta \to 1$,

$$W^d - W^u \leqslant w_1^d - w_1^u + \delta\left[w_2^d - w_2^u\right]$$
$$+ \delta^2\left[(1-\alpha)s\left(p_A^m\right) - \left[\alpha\left(V_H - p_A^u\right) + (1-\alpha)s\left(p_B^u\right)\right]\right]\left(1 + \delta + \ldots + \delta^{T-3}\right)$$
$$\to w_1^d - w_1^u + \delta\left[w_2^d - w_2^u\right] + \delta^2\left[(1-\alpha)s\left(p_A^m\right) - \left[\alpha\left(V_H - p_A^u\right) + (1-\alpha)s\left(p_B^u\right)\right]\right]\frac{1}{1-\delta} < 0.$$

Q.E.D.

REFERENCES

Armstrong, M., 2006, 'Recent Developments in the Economics of Price Discrimination,' in Blundell, R., Newey, W. and Persson, T. (eds.), *Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress*, Volume 2 (Cambridge University Press, Cambridge).

Armstrong, M. and Vickers, J., 1993, 'Price Discrimination, Competition and Regulation,' *The Journal of Industrial Economics*, 41, pp. 335–360.

Bolton, P.; Brodley, J. and Riordan, M., 2000, 'Predatory Pricing: Strategic Theory and Legal Policy,' *Georgetown Law Journal*, 88, pp. 2239–2330.

Chen, Y., 1997, 'Paying Customers to Switch,' *Journal of Economics and Management Strategy*, 6, pp. 877–897.

Chen, Y., 2005, 'Oligopoly Price Discrimination by Purchase History,' in *Pros and Cons of Price Discrimination*, (Swedish Competition Authority, Stockholm, Sweden).

Chen, Y., 2006, 'Marketing Innovation,' *Journal of Economics and Management Strategy*, 15, pp. 101–123.

Corts, K., 1998, 'Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment,' *Rand Journal of Economics*, 29, pp. 306–323.

Farrell, J. and Shapiro, C., 1988, 'Dynamic Competition with Switching Costs,' *RAND Journal of Economics*, 19, pp. 123–137.
Farrell, J. and Klemperer, P., 2007, 'Coordination and Lock-In: Competition with Switching Costs and Network Effects,' in Armstrong, M and Porter, R (eds.), *Handbook of Industrial Organization*, Vol. III (North Holland, Amsterdam, the Netherlands).
Fudenberg, D. and Tirole, J., 2000, 'Customer Poaching and Brand Switching,' *RAND Journal of Economics*, 31, pp. 634–657.
Fudenberg, D. and Villas-Boas, J. M., 2006, 'Behavior-Based Price Discrimination and Customer Recognition,' in Hendershott, T. J. (ed.), *Handbook on Economics and Information Systems* (Elsevier, Amsterdam, The Netherlands).
Holmes, T., 1989, 'The Effects of Third-Degree Price Discrimination in Oligopoly,' *American Economic Review*, 79, pp. 244–250.
Klemperer, P., 1987a, 'The Competitiveness of Markets with Switching Costs,' *RAND Journal of Economics*, 18, pp. 138–150.
Klemperer, P., 1987b, 'Entry Deterrence in Markets with Consumer Switching Costs,' *Economic Journal*, 97, pp. 99–117.
Stole, L., 2007, 'Price Discrimination and Imperfect Competition,' in Armstrong, M and Porter, R. (eds.), *Handbook of Industrial Organization*, Vol. III, forthcoming.
Taylor, C., 2003, 'Supplier Surfing: Price Discrimination in Markets with Repeat Purchases,' *RAND Journal of Economics*, 34, pp. 223–246.
Villas-Boas, J. M., 1999, 'Dynamic Competition with Customer Recognition,' *RAND Journal of Economics*, 30, pp. 604–631.

Supporting Information

Additional Supporting Information may be found in the online version of this article.