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Factor price uncertainty, technology choice and investment delay

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Abstract

This paper develops a theory of putty-clay investment under factor price uncertainty using a Brownian motion framework. Ex ante the firm faces a choice of technologies that differ by their relative factor intensities, but ex post technologies are Leontief. The presence of competing technologies and factor price uncertainty can cause delay of profitable investments for a monopolist firm facing a one-time investment decision. Furthermore, uncertainty can cause an existing firm to wait for more extreme operating cost differentials before switching technologies. These delays in investment are present even without considering the effect of uncertainty on the firm's choice of scale.

JEL classification: L1; D8

Keywords: Technology choice; Firm investment; Brownian motion

0. Introduction

Firms often must account for uncertainty in future input prices when deciding on their investment strategies. It is clear that general uncertainty in production costs yields parallel results to models where firms face uncertain demand. Such models have been well studied in the literature within a Brownian motion framework (e.g. Dixit, 1995; Dixit and Pindyck, 1994; Abel and Eberly, 1996, 1997) and this research has explained how fixed or irreversible costs to investment can create ranges of inaction in a firm's optimal behavior. However, fluctuations in absolute input prices are not the only uncertain aspect of input prices affecting the investment decision. This paper shows that

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uncertainty of relative input prices can also play a role in investment inaction through its effect on the firm's choice of technology. If firms face fixed costs of changing the factor intensities used in production, zones of investment inaction will exist over ranges of relative prices.

This paper focuses on investment decisions given 'putty-clay' technology, i.e., technology characterized by initial flexibility in the scale and use of inputs, but less flexible after irreversible capital investments have been made.[1] Recent work has examined putty-clay technology and its potential role in explaining recessions or business cycles. Gilchrist and Williams (2000) emphasize the importance of capacity constraints involved with a putty-clay technology in capital and labor. In their model permanent shocks to factor prices produce prolonged and asymmetric investment and employment responses that are consistent with business cycle evidence. Wei (2003) examines whether the OPEC oil crisis could explain the stock market collapse in the early 1970s by forcing firms to abandon capital used in energy-intensive technologies.[2] In contrast to these studies that focus on the level of input prices, I examine the role of *uncertainty* regarding relative input prices in explaining delayed investment. If the extent of input uncertainty varies over time, this link may contribute an additional consideration to our understanding of investment dynamics.

Several studies have addressed the putty-clay aspect of technology and its implications on investment under uncertainty. Using a general equilibrium vintage capital model, Gilchrist and Williams (2002) show how greater uncertainty in the productivity of putty-clay investments can lead to reduced investment at the micro-level (through smaller project sizes), and smaller increases in investment at the aggregate level (through entry) than in standard vintage capital models. Kon (1982) shows how output price uncertainty could lead to investment in more labor-intensive technologies. Using a two-period setup, Hiebert (1990) demonstrates how uncertainty regarding wages could lead to less investment in capital. His result, however, was driven by a change in the firm's desired scale. This paper will show that even when scale of production is inflexible, investment can be lowered by input price uncertainty.

While the effect of input price uncertainty on decisions of scale are well-known, the focus here is its effect on technology choice. In order to isolate this effect from the demand/scale decision of the firm, the analysis assumes that the firm's quantity produced is constant. This allows the firm's problem to essentially be reduced to one of cost minimization. Though this particular setup is meant as an abstraction, it resembles the problem of a price-regulated monopolist facing excess demand. An example, would be an electric power company considering building a new plant, where the price is set by regulators and the firm must produce the full demand (Christensen and Greene, 1976).

[1] It is not difficult to think of cases where such a model is fitting. One example could be a farmer deciding whether to invest in a tractor/thresher, or to choose a more labor-intensive technology using hand tools and bullocks. Dixit (1994) poses the problem of a power plant choosing between a coal-burning furnace or an oil-burning furnace. Similarly, a manufacturer may have to choose either a high-tech process line requiring relatively more skilled labor, or a series of machines that can be manually operated by unskilled workers.

[2] In a related paper, Atkeson and Kehoe (1999) model energy-use using a putty-clay technology and show that it fits the cross-sectional relationship between energy prices and capital stocks better than a standard technology.

The firm chooses between a capital-intensive technology (nuclear or hydro-electric) or a raw materials-intensive technology (coal-burning) and considers the fluctuating price of the raw material in making its investment decision. A similar example is posed by Dixit (1995).[3]

A second technical abstraction is chosen with the intent of isolating the effect of a putty-clay technology on the firm's investment decision: the firm chooses between different Leontief technologies. Thus, while the firm can substitute between factors ex ante, it faces fixed-proportion technology ex post.

This paper analyzes the optimal policies of a firm under two distinct models. In both models, the firm faces a fixed cost of purchasing capital, but operating costs are stochastic. Although the models could represent various scenarios with different inputs, including the example given above, the analysis is posed as a choice between capital and labor. The first model describes a 'monopolist' facing an irreversible decision on an option to invest. The firm is a monopolist, not in the sense of being a price-setter, but in the sense of being the sole possessor of a (potentially) positive net-present value investment opportunity.

Section 1 illustrates this problem. Section 2 follows a similar setup, but it analyzes a firm that is already producing and must decide whether to switch technologies. The models' conclusions are summarized in Section 3.

1. One-time investment decision problem

As explained above, I abstract from demand uncertainty and scale decisions by assuming the firm faces a fixed demand, q, at a constant output price,[4] p. The investment opportunity involves the fixed cost of an initial capital purchase. The price of capital is normalized to one. Capital does not depreciate.[5] Thus, the firm has no further capital expenses beyond the initial investment,[6] but must pay labor each period at a wage, w. The wage process follows a geometric Brownian motion with drift, μ,

[3] Another example would be a firm that has sole possession of a technology to produce a good below the (constant) marginal cost of the current producer. If the market for the good were also perfectly inelastic, the firm would price just below the cost of the competitor and capture the full market.

[4] This assumption could be relaxed. It is conceptually straightforward, although algebraically tedious, to model a firm that chooses its scale given a more realistic demand function. This would not qualitatively change the results, however. The results are driven by the form of the technology, which is putty-clay in a specific sense: neither capital nor quantities can be freely adjusted after the original investment decision is made.

[5] Identically, depreciation could occur through an exogenous Poisson failure rate, δ. The results of this section would still hold with a new interpretation of $\rho = \tilde{\rho} + \delta$ as an augmented discount rate.

[6] The model could be easily generalized to allow for non-wage production costs proportional to the stock of capital $\tilde{K}$ (at a rate, r). The $r\tilde{K}$ term could represent the cost of replacing depreciated capital, maintenance costs, or even energy costs that are proportional to the amount of capital used. These non-wage production costs would also be included as irreversible costs in the decision (i.e. $K = (1 + r/\rho)\tilde{K}$). Alternatively, one could think of the up front cost $\tilde{K}$ as a capital installation cost (proportional to the amount of capital) and r as the rental rate of capital.

and variance, σ:

$$\frac{\mathrm{d}w}{w} = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}z,$$

where z is a Wiener process.

1.1. Single technology

For expositional purposes, first consider the problem of a firm deciding whether or not to invest in a project with only a single available production technology. The technology requires K_1 units of capital and L_1 units of labor to produce the desired quantity, q. The firm has no shutdown option. The expected value of investing in the technology at wage w, hereafter referred to as the *investment value*, is

$$V_1(w) = \frac{pq}{\rho} - \frac{wL_1}{\rho - \mu} - K_1, \tag{1}$$

where ρ is the rate at which the firm discounts future profits. Assuming $\rho > \mu$ keeps the discounted expected value of labor costs bounded. I also assume $pq/\rho - K_1 > 0$, i.e., the investment is profitable at some positive wage.

Since the investment value is a function of the wage rate, the firm will choose to invest at a sufficiently low wage, but wait if the wage is too high. If the firm had a 'now or never' opportunity to invest, and the wage were below the threshold $\tilde{w}$ defined by $V_1(\tilde{w}) = 0$, the firm would clearly invest. When the firm has the option of waiting, however, it realizes that the wage could rise to an unprofitable level in the future, and so it may want to wait to see future movements in w. Since the firm could always invest immediately, the value function of a firm with the option to invest *or* wait, $V(w)$ (hereafter referred to as the *value function*) is weakly greater than the investment value $V_1(w)$ at every wage. Indeed, at $\tilde{w}$ (the indifference wage for the firm with a 'now or never' opportunity) the value of waiting will be positive. The firm will wait for a lower wage $w^* < \tilde{w}$, before making the irreversible investment.

For wages below w^*, the firm invests and so the value function, $V(w)$, equals the investment value, $V_1(w)$. For the inaction region above w^*, the firm earns no current period return from the potential investment, so the value function simply equals an option value function, $F(w)$ (hereafter referred to as the *investment option value*). The Bellman equation for $F(w)$ is simply $E(\mathrm{d}F) = \rho F\,\mathrm{d}t$. By the application of Ito's lemma, $F(w)$ is described by

$$\rho F(w) = \mu w F'(w) + \tfrac{1}{2}\sigma^2 w^2 F''(w) \quad \text{for } w \in [w^*, \infty). \tag{2}$$

The investment option value solving this differential equation is of the form

$$F(w) = C_1 w^{\beta_1} + C_2 w^{\beta_2}, \tag{3}$$

where β_1 and β_2 are the roots of the quadratic equation:

$$\tfrac{1}{2}\sigma^2\beta^2 + \left(\mu - \tfrac{1}{2}\sigma^2\right)\beta - \rho = 0. \tag{4}$$

It can be easily shown that one root (β_1) is positive, while the other (β_2) is negative. Furthermore, the upper bound on the trend in the wage ($\mu < \rho$) implies $\beta_1 > 1$.

Since the value of the investment is bounded above by pq/ρ, it can be shown that $C_1 = 0$. The constants C_2 and w^* are fixed by smooth pasting and value matching conditions, two standard conditions in the literature (see Dixit and Pindyck, 1994). The value matching condition states that the value function must be continuous at w^* and therefore $F(w^*) = V_1(w^*)$. The value-matching condition is therefore

$$C_2(w^*)^{\beta_2} = \frac{pq}{\rho} - \frac{w^* L_1}{\rho - \mu} - K_1. \tag{5}$$

The smooth-pasting condition states that the first derivative of the value function must also be continuous at w^*, so $F'(w^*) = V_1'(w^*)$. Though formal explanations for this condition exist (see Dixit and Pindyck, 1994), intuitively a kink in the value function at w^* can be ruled out for two reasons. First, an upward-pointing (i.e., $V_1'(w^*) > F'(w^*)$) kink can be ruled out because otherwise at some point just inside the inaction region (i.e., $\tilde{w} = w^* + \varepsilon$), $V_1(\tilde{w})$ would exceed $F(\tilde{w})$. Second, a downward-pointing (i.e., $V_1'(w^*) < F'(w^*)$) could also never be optimal because of the high variation in the Brownian motion wage process. By waiting a little longer (i.e., $\mathrm{d}t$) the firm could see the next movement (of a larger $\sqrt{\mathrm{d}t}$ order) of the wage process, and the discounted expectation (weighted-average) of wages on either side of the downward-pointing kink exceeds the value of investing at the kink.

Substituting in to the smooth-pasting condition yields

$$\beta_2 C_2(w^*)^{\beta_2 - 1} = \frac{-L_1}{\rho - \mu}. \tag{6}$$

Note that (5) and (11) can be solved to yield

$$C_2 = \frac{1}{1-\beta_2}\left(\frac{pq}{\rho} - K_1\right)\left[\frac{\beta_2(\rho-\mu)}{L_1(1-\beta_2)}\left(K_1 - \frac{pq}{\rho}\right)\right]^{\beta_2}, \tag{7}$$

$$w^* = \frac{\beta_2}{1-\beta_2}\left(\frac{\rho-\mu}{L_1}\right)\left(K_1 - \frac{pq}{\rho}\right). \tag{8}$$

Given our assumptions C_2 and w^* are both positive. A graphical representation of the investment value, investment option value, inaction region, and resulting value function is shown in Fig. 1. The tangency point at w^* satisfies value-matching and smooth-pasting. The gray line denotes the value function $V(w)$ which coincides with the investment option value $F(w)$ in the inaction region, and the investment value $V_1(w)$ in the investment region. For illustration, the function $F(w)$ is extended to the left of w^* as a dashed line, while the thin solid straight line is the extension of $V_1(w)$ to the left of w^*.

1.2. Choice of technologies

Now consider a firm choosing between the initial technology for producing q (Technology 1, defined by the input requirements K_1 and L_1), and a second more labor-intensive technology (Technology 2, defined by $K_2 < K_1$ and $L_2 > L_1$). The firm must now decide not only *when* but also *in which technology* to invest.

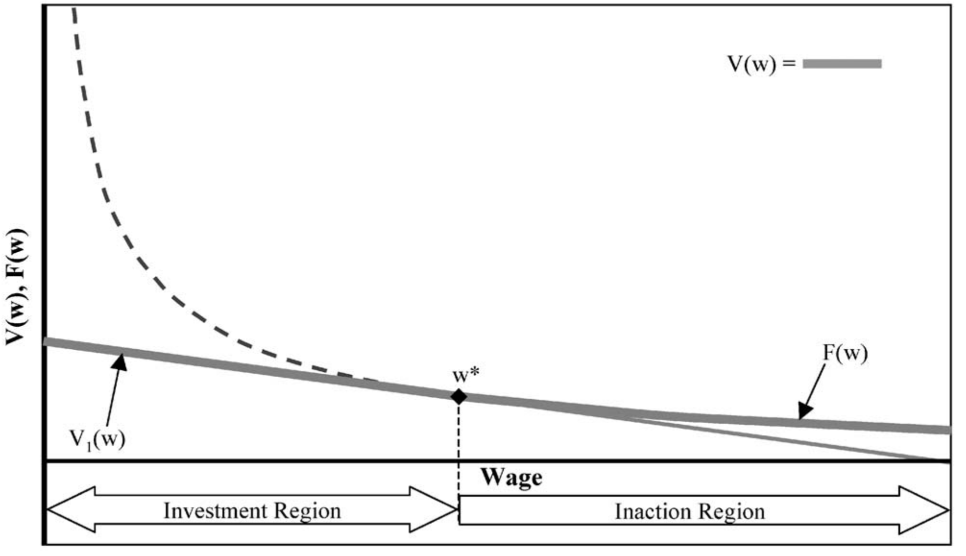

Fig. 1. Case of single technology: investment value, investment option value, and value function.

Following Eq. (1), the investment value for Technology 2 is

$$V_2(w) = \frac{pq}{\rho} - \frac{wL_2}{\rho - \mu} - K_2. \tag{9}$$

Since both technology values are functions of the wage rate, the choice of technology could also depend on the wage. The low labor requirements of Technology 1 may be preferred at high wages, but the low capital requirements of Technology 2 might be better if the wage were low1. Again, if the firm had a 'now or never' opportunity to invest and the wage were low enough to be profitable, the firm would clearly pick the technology with the highest investment value at that wage.

With the option of waiting to invest, the firm again has an inaction region at high wages ($w > w^*$) and an investment option value function of the form in Eq. (3) satisfying Eq. (2).

The method for determining the constants C_2 and w^* now involves both technologies, however. It can be illustrated by first considering the case of Technology 1 alone (presented in the previous section) and the analogous case of Technology 2 alone. Let F_i represent the investment option value for technology i. Note, both F_1 and F_2 must satisfy the identical differential Eq. (2). Furthermore, the roots of the quadratic Eq. (4) depend only the parameters of the wage process (not on the technology), so F_1 and F_2 must have the same exponential form, with only the coefficients differing. Thus, F_1 and F_2 are log-parallel with different C_2 and w^*, denoted $C_{2,i}$ and $w^{*,i}$.

The values of C_2 and w^* are determined by choosing the investment option value (and corresponding $w^{*,i}$) with the greater $C_{2,i}$. The exponential investment option value curve is always weakly greater than the relevant linear investment value curve with equality only at the tangency point, w^*. For values above this critical wage, the value function of the option to invest in a range of technologies, $V(w)$, is simply the investment option value of the first technology to be selected as the wage falls, $F(w)$.

Depending on the input requirements and the price process, at very high wages the firm could wait to invest in either Technology 2 or Technology 1. These two scenarios are depicted graphically in Figs. 2 and 3, respectively.

Fig. 2(a) is an overlay of the single-technology value functions for Technology 1 and Technology 2. The case depicted is, $C_{2,2} > C_{2,1}$, so the investment option value is greater for Technology 2 (thin dashed line) than for Technology 1 (thin dash-dotted line). In this case, the entire value function for Technology 2 (long shaded dashes) dominates the value function for Technology 1 (short shaded dashes), so that the presence of an option to choose Technology 1 is irrelevant to the firm.

Fig. 2(b) shows that the resulting value function $V(w)$ (shaded) is therefore identical to the single technology case and $w^* = w^{*,2}$. At wages below w^*, Technology 2 is always preferred to Technology 1. In the inaction region above w^*, the investment option value for Technology 2 exceeds both the investment value and the investment option value of Technology 1. Hence, the firm waits to invest in Technology 2. The resulting value function is the shaded line and coincides with $V_2(w)$ to the left of w^*, and with $F(w) = F_2(w)$ to the right. [7]

The case in Fig. 3, where $C_{2,1} > C_{2,2}$ and the firm waits to invest in Technology 1, is slightly more complicated. Fig. 3(a) again presents an overlay of the single-technology value functions. Since $C_{2,1} > C_{2,2}$, at high wages the investment option value is now higher for Technology 1. Thus, for wages above w^+, the value of having only an option to invest in Technology 1 dominates, while for wages below w^+ the value function for Technology 2 dominates.

The value function for a choice to invest in either of the two technologies, however, is not just the upper envelope of these two value functions. This upper envelope would produce a kink in the value function at w^+, which can be ruled out for the same reason that smooth-pasting at w^* is assured. That is, the firm *does not* make a final decision to invest in Technology 1 for wages above w^+, or wait for only Technology 2 at wages below w^+. Instead, at the kink at w^+, it pays for the firm to keep open the option to wait for either technology because the discounted expectation of weighting a small amount of time is positive (i.e., the discounted weighted average of the values on either side of the kink exceeds the value of investing at the kink). The firm therefore waits for more extreme differences in the values of Technology 2 and Technology 1, before making a decision to either invest in Technology 1 or invest in Technology 2. Thus, a second region of inaction must exist between some w_2^*, a lower bound where the firm decides to invest in Technology 2, and some w_1^*, an upper bound where the firm invests in Technology 1. Within this region of inaction, the firm hesitates between two profitable investments in order to simply delay the irreversible choice of technology. (Note that the thresholds w_1^* and w_2^* bound the inaction region of a firm deciding between two technologies. They are distinct from $w^{*,1}$ and $w^{*,2}$,

[7] The addition of any technology j, as long as it were more capital intensive than Technology 2 and yielded a $C_{2,j} < C_{2,2}$, would not change the value function or the firm's optimal behavior.

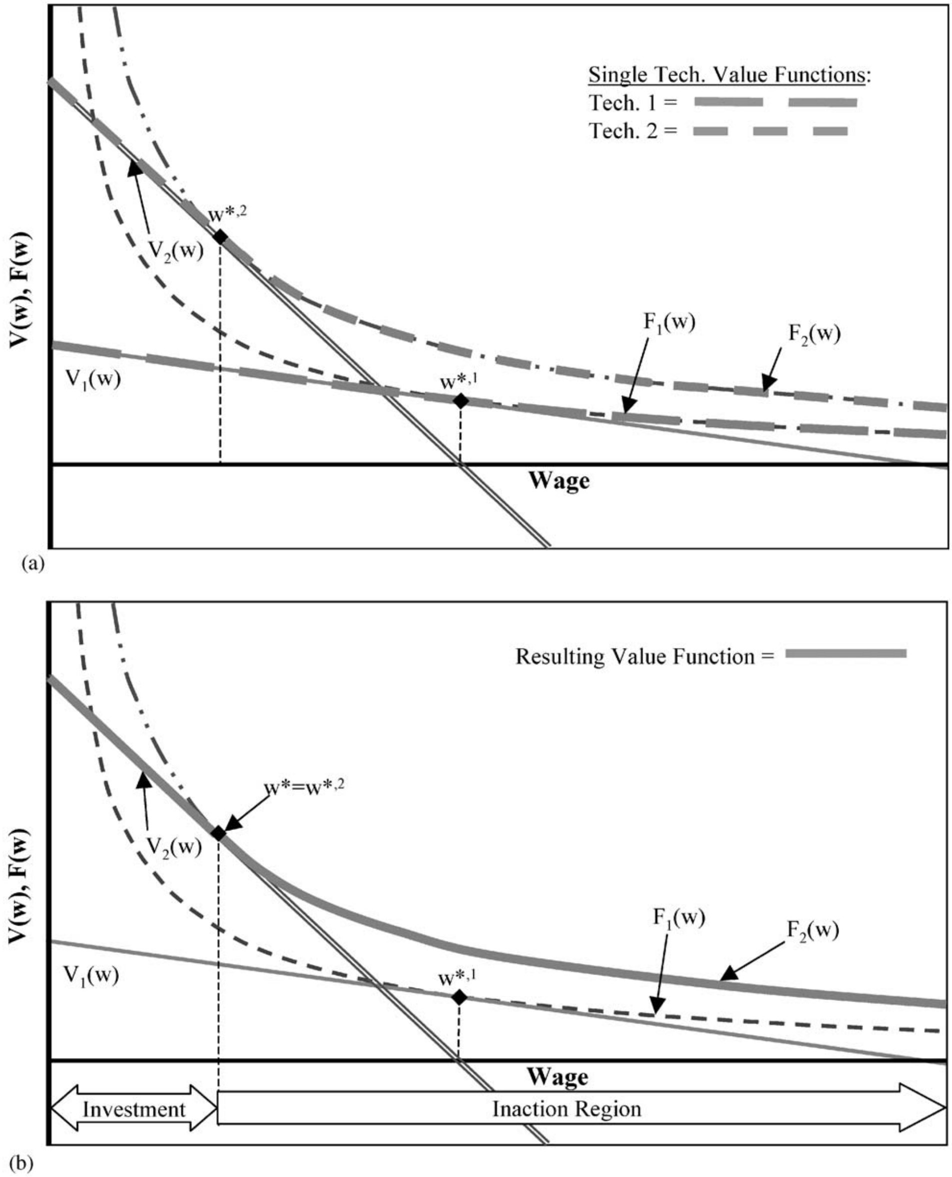

Fig. 2. (a) Labor-intensive investment option value dominates: overlay of single-technology value functions. (b) Labor-intensive investment option value dominates: resulting value function with choice of technologies.

the thresholds to invest in the single-technology problem where the sole technology is Technology 1 and Technology 2, respectively.)

Within this second region of inaction, the option value (denoted $F_{12}(w)$ and hereafter referred to as the *technology choice option value*) again satisfies the differential

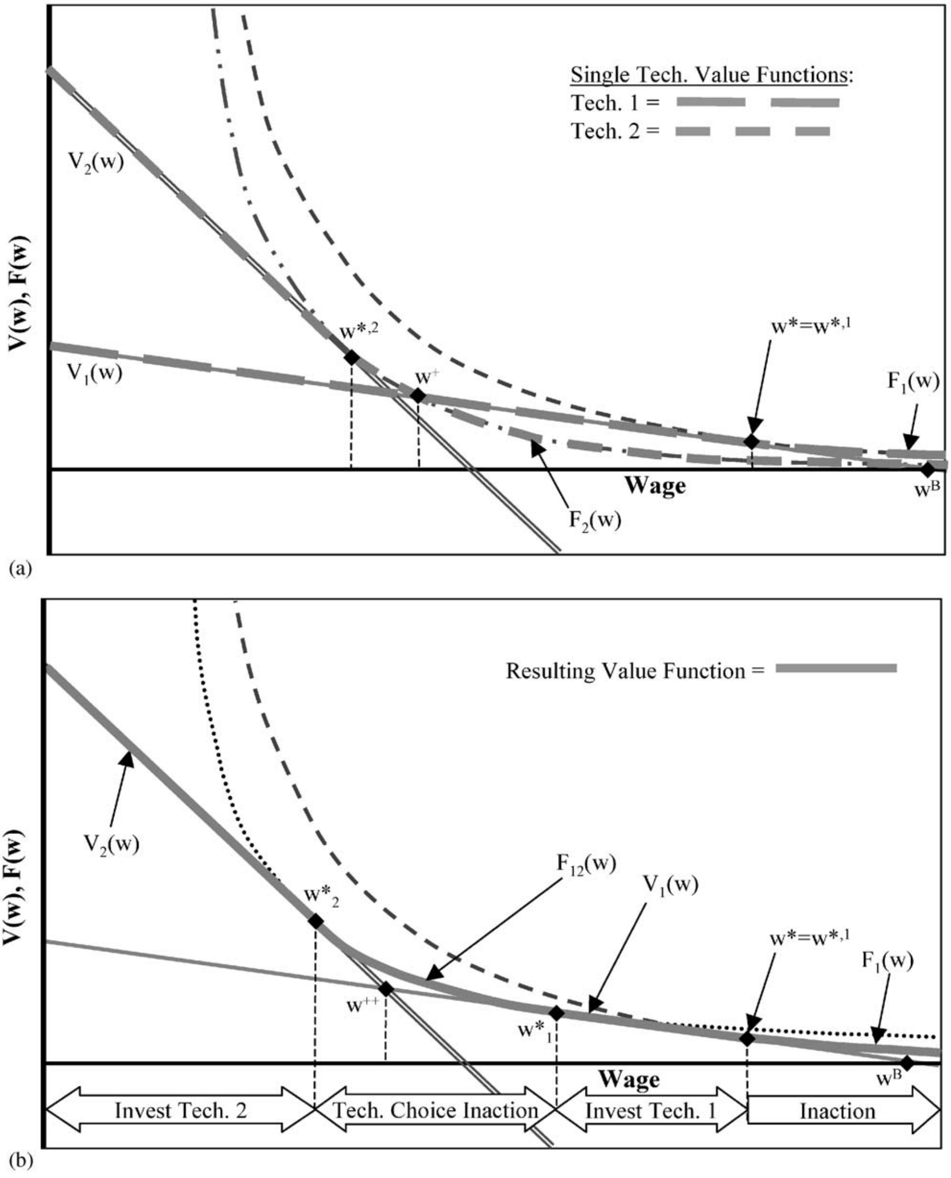

Fig. 3. (a) Capital-intensive investment option value dominates: overlay of single-technology value functions. (b) Capital-intensive investment option value dominates: resulting value function with choice of technologies.

equation in (2) and has the form

$$F_{12}(w) = D_1 w^{\beta_1} + D_2 w^{\beta_2} \quad \text{for } w \in [w_2^*, w_1^*]. \tag{10}$$

Since this inaction region is bounded by finite threshold wages, the earlier argument cannot be used to set either D_1 or D_2 to zero. In fact, both terms will be non-zero; the second term is decreasing in w and represents the decreasing value of the option of Technology 2 as wages rise. It is tempered by the first term, which reflects the value of the option to choose Technology 1 over Technology 2 at higher wages. The four constants w_1^*, w_2^*, D_1 and D_2 are again determined by the value matching and smooth pasting conditions:

$$D_1(w_1^*)^{\beta_1} + D_2(w_1^*)^{\beta_2} = \frac{pq}{\rho} - \frac{w_1^* L_1}{\rho - \mu} - K_1,$$

$$D_1(w_2^*)^{\beta_1} + D_2(w_2^*)^{\beta_2} = \frac{pq}{\rho} - \frac{w_2^* L_2}{\rho - \mu} - K_2 \tag{11}$$

and

$$\beta_1 D_1(w_1^*)^{\beta_1 - 1} + \beta_2 D_2(w_1^*)^{\beta_2 - 1} = \frac{-L_1}{\rho - \mu},$$

$$\beta_1 D_1(w_2^*)^{\beta_1 - 1} + \beta_2 D_2(w_2^*)^{\beta_2 - 1} = \frac{-L_2}{\rho - \mu}. \tag{12}$$

Hence, the firm's problem involves two inaction regions. In the first inaction region, characterized by its lower bound w^*, the firm waits to invest until the foregone profits from not investing outweigh the uncertainty risk of the wage rising. In the second inaction region, between w_1^* and w_2^*, both technologies are profitable. Given wage uncertainty, however, the firm waits for the investment value of one technology to be significantly higher than the value of the other before choosing a technology.

The resulting value function $V(w)$ for the choice of technologies and relevant option values are depicted in Fig. 3(b). Again, for illustration, the functions $F_{12}(w)$ (dotted line), $F_1(w)$ (dashed line), $V_1(w)$ (thin solid line), and $V_2(w)$ (double solid line) have been extended beyond their relevant regions of the value function. Note that $V(w)$ coincides with

- the investment value for Technology 2, $V_2(w)$, for wages below w_2^*;
- the technology choice option value, $F_{12}(w)$, in the investment choice inaction region for intermediate wages between w_2^* and w_1^*;
- the investment value for Technology 1, $V_1(w)$, for wages between w_1^* and w^*; and
- the investment option value, $F(w) = F_1(w)$, for wages above w^*.

Note that the value function is nowhere equal to the investment option value for Technology 2 (i.e., $F_2(w)$, shown in Fig. 3(a), but omitted in Fig. 3(b)), since at high wages the firm waits to invest in Technology 1.

How would the problem change if there were no uncertainty? Without uncertainty the only benefit of waiting comes from the drift component of the wage process. If wages are falling ($\mu < 0$), it is possible that the firm might delay and wait for lower wages before investing. The value function $F_{\sigma=0}(w)$ satisfies

$$\rho F_{\sigma=0}(w) = \mu w F'_{\sigma=0}(w).$$

Integration yields

$$F_{\sigma=0}(w) = E w^{\rho/\mu},$$

where E, the constant of integration, can be found by the value matching condition at the upper bound wage of investment, w^*. The point of investment is the break-even contemporaneous wage. We find this wage for both technologies:

$$w^{*,i} = \frac{\rho K_i - pq}{L_i}.$$

The technology with the largest constant of integration is the technology that is eventually chosen:

$$E = \max_i E_i = \max_i \frac{((pq/\rho) - ((\rho K_i - pq)/(\rho - \mu)) - K_i)}{((\rho K_i - pq)/L_i)^{\rho/\mu}}.$$

If $\mu \geqslant 0$, the situation can be easily described by returning to the graphical analysis in Figs. 2 and 3. In a world without uncertainty and with a non-negative drift in the wage, the exponential option value curves do not exist and the firm's problem is completely described by the two linear technology value functions in Fig. 3(b). At wages below the intersection w^{++}, the firm chooses Technology 2 and makes the smaller capital investment. At higher wages, the firm invests in more capital and chooses Technology 1. At wages above the break even wage, w^B, for Technology 1, the value of investing in either technology is negative and the firm chooses not to invest at all. Thus, the only inaction region lies above this break even wage.

In summary, regardless of the value of μ, without uncertainty there exists only one inaction region and it lies above a critical wage w^*.

What effect does uncertainty have on the firm's investment decision? Since uncertainty raises the option value of waiting (functions $F(w)$ and $F_{12}(w)$, higher uncertainty can only reduce the range of wages at which investment is made. The following claims are made regarding the level of uncertainty.

Claim 1. *An positive upper bound w^* for investing exists, and is decreasing in σ. Hence, the investment inaction region expands with uncertainty.*

Claim 2. *w_2 is decreasing in σ, and w_1 is increasing in σ. So, if the technology choice inaction region exists (i.e., $C_{2,1} > C_{2,2}$), it expands with uncertainty.*

Claim 1 is shown formally in the appendix. Analytical expressions of $\partial w_2/\partial\sigma$ and $\partial w_1/\partial\sigma$ produced using the implicit functions defined Eqs. (11) and (12) cannot be signed because their non-linearity in σ (through β_1 and β_2) complicates the expressions. This is typical in this (see Dixit and Pindyck, 1994) Brownian motion literature.

Claim 2 is quite intuitive and has been verified numerically[8] for a range of parameter values, however. This numerical appendix is available upon request.

The effects of uncertainty in Claims 1 and 2 can be easily explained graphically. When a small amount of uncertainty is introduced[9] (i.e. σ is increased), the firm's problem will resemble the scenario in Fig. 3(b). The high wage waiting region expands to the left of w^* and a small second waiting region appears around the kink. Again, the intuition is simple. When the expected wage path is stochastic, it always pays for a firm at these critical points of indifference to wait for more information before investing.

As σ continues to increase, the two investment option value curves $F_1(w)$ and $F_2(w)$ – characterized by $C_{2,1}$ and $C_{2,2}$, respectively, and shown in Fig. 3(a) – increase and begin converging. These investment option values increase because uncertainty raises the chance of the wage falling dramatically while waiting to invest. Convergence comes from the fact that a decline in wages increases the value of the labor-intensive technology disproportionately. Thus, w^* is decreasing in σ, and the high-wage inaction region expands.

Following a similar reasoning, the technology choice option value $F_{12}(w)$ in Fig. 3(b) increases as well. That is, just as $F_2(w)$ (the investment option value of waiting for Technology 2) increases relative to $F_1(w)$ (the investment option value of waiting for Technology 1), the value of the option to choose Technology 2 over Technology 1 increases[10] with uncertainty. Of course, as the value of waiting for Technology 2 (i.e. $F_2(w)$ in Fig. 3(a)) increases, the relative value of investing in Technology 1 over waiting for Technology 2 decreases.[11] Overall, however, the increasing option value of Technology 2 dominates and $F_{12}(w)$ is increasing in σ. As this technology choice option value increases, w_2^* decreases, w_1^* increases, and the technology choice inaction regions expands.

At some knife-edge σ, the three option value curves coincide (i.e. $F_1(w) = F_2(w) = F_{12}(w)$).[12] For σ greater than this value, $C_{2,2} > C_{2,1}$ and Fig. 2 describes the situation. The chance of wages dropping in the near future are now high enough that it never pays to invest in the capital-intensive technology.

[8] For numerical solutions, the system of four non-linear equations in four unknowns is first simplified by noticing the linearity in D_1 and D_2. One can eliminate these constant terms and greatly simplify the problem to a system of two non-linear equations in two unknowns. Nevertheless, the option value functions in the equations involve both positive and negative exponential terms that explode toward infinity as w decreases toward zero or increases toward infinity. Hence, iterative convergence to numerical solutions requires good initial values.

[9] This is a comparative static analysis, not a dynamic one.

[10] This is captured by the second element of $F_{12}(w)$ in Eq. (10). The constant D_2 decreases, but the exponent β_2 is increasing in σ. The overall effect on $D_2 w^{\beta_2}$ over the range (w_2^*, w_1^*) is positive.

[11] This is captured by the first element of $F_{12}(w)$ in Eq. (10). The exponent β_1 is decreasing in σ. In addition, the constant D_1 is eventually decreasing in σ. The overall effect is for $D_1 w^{\beta_1}$ to decrease in σ over the relevant range (w_2^*, w_1^*).

[12] At this point, the value of investing in Technology 1 never exceeds the option value of investing in Technology 2. Hence, $D_1 = 0$. For higher values of σ beyond this point, the value of the option to not invest exceeds the value of the option to invest in Technology 1. Hence, $F_{12}(w)$ becomes irrelevant to the problem.

To summarize, wage uncertainty can only enlarge the range (or ranges) of inaction. Uncertainty causes firms to delay investment in otherwise profitable projects for two reasons: (1) so that the profitability of the project can be reasonably assured and (2) to try to assure that the most profitable technology is chosen. The first type of investment delay is present in an ordinary optimal stopping problem, while the second is introduced by the presence of competing technologies.[13]

2. Option of switching technologies

The model above assumed that firms make a one-time choice of technology and they cannot switch technologies. This section considers a firm that has already paid initial capital costs, but has the option of switching technologies. Again, for simplicity, we assume the firm has no shut-down option and only the two-technology case is modeled. Technology 1 is again capital-intensive, while Technology 2 is labor-intensive. That is, $K_1 > K_2$ and $L_2 > L_1$. It is assumed that, perhaps because of adverse selection problems or thin markets for specialized capital, capital investment is not fully reversible. That is, a firm selling used capital, K, can only recover λK, where $\lambda \in (0,1)$. Let K_{21} denote the cost of switching from Technology 2 to Technology 1, and K_{12} denote its analog. Then

$$K_{21} = K_1 - \lambda K_2 > K_2 - \lambda K_1 = K_{12}. \tag{13}$$

Consider a firm with Technology 2 and the option of switching to Technology 1. The value of the firm optimally continuing with its current technology (i.e. within the inaction region) satisfies

$$\rho V_2(w) = pq - wL_2 + \mu w V_2'(w) + \tfrac{1}{2}\sigma^2 w^2 V_2''(w).$$

The solution to this differential equation is

$$V_2(w) = \frac{pq}{\rho} - \frac{wL_2}{\rho - \mu} + G_{2,1} w^{\beta_1} + G_{2,2} w^{\beta_2}$$

with β_1 and β_2 again defined by Eq. (4). A firm using Technology 2 and paying no labor costs ($w = 0$) will remain using Technology 2 forever, since labor costs will remain at zero and the cost of switching to Technology 1, K_{21}, is positive. Thus, at $w = 0$, the option of switching to Technology 1 has no value and, therefore, $G_{2,2} = 0$.

[13] Extending this analysis to the case of any finite number of possible (K_i, L_i) technologies is straightforward using the following approach. First, any technology whose investment value V_i is dominated by other available technologies may be eliminated from the analysis. The second step is to determine the investment option value curve and upper bound wage w^* by choosing the maximum value of $C_{2,i}$ (and corresponding $w^{*,i}$) as done in the two technology example. Any technology that is more capital-intensive than the one chosen at w^* can also be ignored (see footnote 7). Finally, the technology choice option values (i.e. the options of choosing between two technologies) should then be determined for every combination of the remaining technologies. The upper envelope of all of these option value curves and corresponding investment values lines constitutes the value function.

Similarly, for a firm using Technology 1, the value function within the continuation region is of the form

$$V_1(w) = \frac{pq}{\rho} - \frac{wL_1}{\rho - \mu} + G_{1,1}w^{\beta_1} + G_{1,2}w^{\beta_2}.$$

Since a firm will continue with the capital-intensive technology as wages increase, and the value of having this technology must be bounded, $G_{1,1} = 0$.

The two value functions are therefore

$$V_1(w) = \frac{pq}{\rho} - \frac{wL_1}{\rho - \mu} + G_{1,2}w^{\beta_2}, \tag{14}$$

$$V_2(w) = \frac{pq}{\rho} - \frac{wL_2}{\rho - \mu} + G_{2,1}w^{\beta_1}. \tag{15}$$

There will exist two wages w_{12}^* and w_{21}^* ($w_{12}^* \neq w_{21}^*$ because of the irreversible fixed cost) at which the firm will switch from Technology 1 to Technology 2 and from Technology 2 to Technology 1, respectively. After canceling out the revenue terms (i.e., pq/ρ), the value matching conditions simplify to

$$-\frac{w_{12}^*L_1}{\rho - \mu} + G_{1,2}(w_{12}^*)^{\beta_2} = -\frac{w_{12}^*L_2}{\rho - \mu} + G_{2,1}(w_{12}^*)^{\beta_1} - K_{12}, \tag{16}$$

$$-\frac{w_{21}^*L_1}{\rho - \mu} + G_{1,2}(w_{21}^*)^{\beta_2} - K_{21} = -\frac{w_{21}^*L_2}{\rho - \mu} + G_{2,1}(w_{21}^*)^{\beta_1}. \tag{17}$$

In other words, at the value matching wage, the cost of continuing with the present technology equals the cost of the alternative technology including the capital switching cost.

The two smooth pasting conditions for the switches simplify to the same expression, which I write in terms of $w_{ij}^* \in \{w_{12}^*, w_{21}^*\}$:

$$\frac{(L_2 - L_1)}{\rho - \mu} + \beta_2 G_{1,2}(w_{ij}^*)^{\beta_2 - 1} - \beta_1 G_{2,1}(w_{ij}^*)^{\beta_1 - 1} = 0. \tag{18}$$

Positive solutions to these conditions exist as stated below. Hence, regions of switching and investment exist together with regions of inaction. These inaction regions for Technology 1 and Technology 2 overlap, and as we saw in the one-time decision case, the presence of wage uncertainty here can only increase the chance of delaying investment and the size of the inaction ranges.

Claim 3. $w_{21}^* > w_{12}^* > 0$, *if and only if* $K_{12} < 0$. *That is, if the net capital cost of switching to Technology 2 is negative, switching regions exist for firms with either technology and they overlap.*

Claim 4. w_{21}^* *is increasing in* σ *and* w_{12}^* *is decreasing in* σ. *Thus, the switching regions compress and the regions of inaction, and the amount of overlap of these regions, expand with uncertainty.*

Claim 3 is shown formally in the Appendix. Again, analytical representations of $\partial w_{21}^*/\partial\sigma$ and $\partial w_{12}^*/\partial\sigma$ are extremely complicated expressions that cannot be signed, but Claim 4 has been verified numerically.

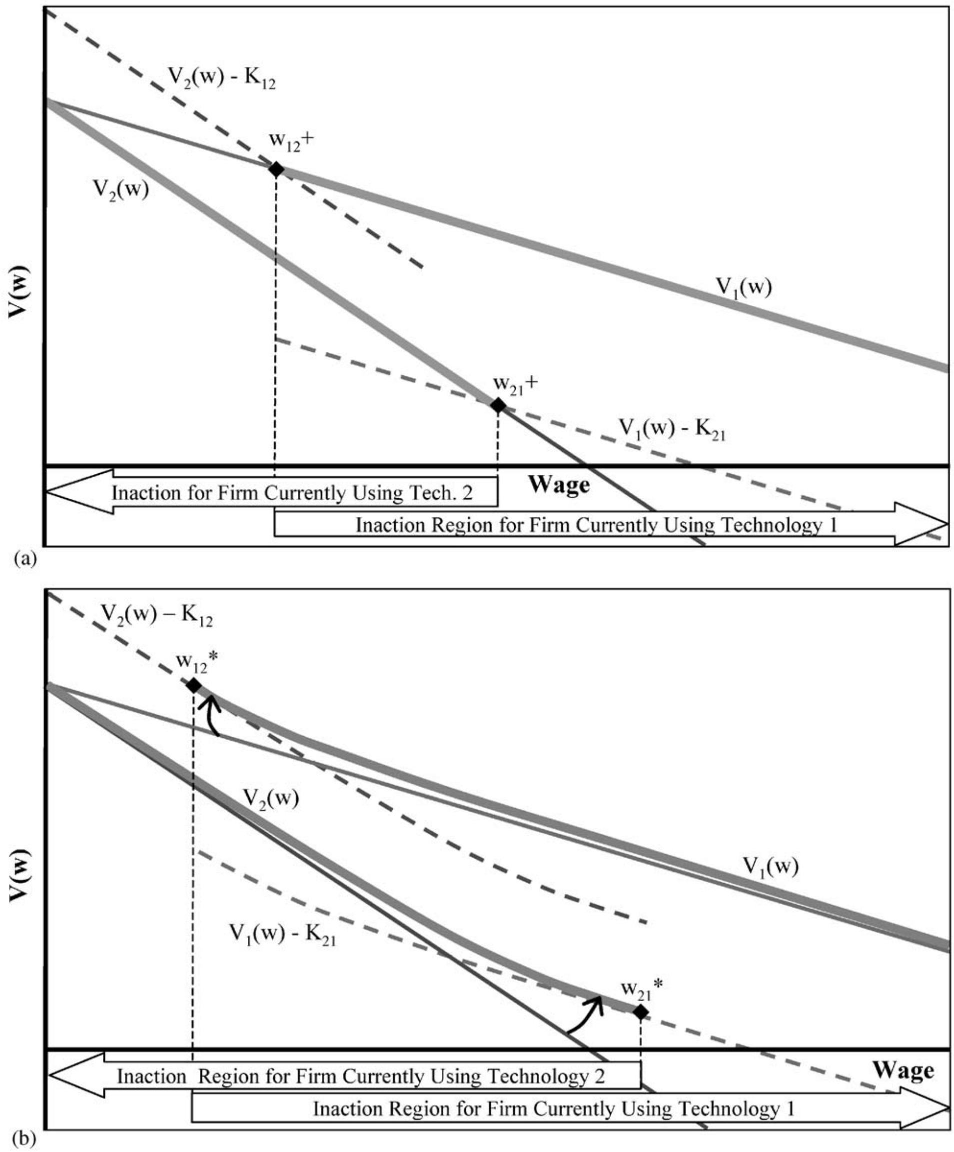

Fig. 4. (a) Value functions for two technologies with fixed wage and switching option. (b) Value functions for two technologies with wage uncertainty and switching option.

The effects of uncertainty are demonstrated graphically in Fig. 4. (For presentation purposes, the drift in the wage process is assumed to be zero.) Fig. 4(a) shows the value functions for firms with either technology when wages are constant. The solid lines indicate the value of producing with a technology already possessed. These values meet at $w = 0$, since operating costs are zero for both technologies at this point. The dashed lines indicate the value of the alternative technology after paying net capital investment costs to switch. Thus, a firm producing with Technology 1 compares the value of continuing with its technology (solid shaded line), with the value of Technology 2 net of the (negative) net cost K_{12} that must be paid (dashed line) to switch. If the value of the alternative technology net of switching costs is greater, the firm changes technologies. Because of the irreversible costs to switch technologies, the ranges of inaction of the two technologies overlap (i.e., $w_{21}^* > w_{12}^*$), even when the wage is fixed.

Fig. 4(b) shows how this overlap grows with uncertainty, σ. The value functions are now curved lines, due to the option value that comes from the wage uncertainty. The arrows in Fig. 4(b) show that these value functions are curved toward each other at extreme wages, relative to the no uncertainty value functions (redrawn from Fig. 4(a) for illustration). Again, the firm switches technologies when the difference between the two value functions (shaded curved lines) lie below the value of the alternative technologies net of switching costs (dashed lines). As σ rises, the option value to holding onto a given technology increases, the value functions curve even farther away from the no uncertainty lines, and the firm waits for a larger differential in technology values before making the irreversible investment.

3. Conclusion

This paper used standard continuous-time stochastic models to analyze the problem of a firm choosing a 'putty-clay' technology in the face of uncertain input prices. The framework demonstrates how the presence of uncertainty and the irreversibility of technology investment creates an option value to both a potential entrant making a monopolistic one-time decision or a firm already producing under one technology but considering changing technologies.

The models also has similar implications about investment delay. For a firm with a monopoly option to invest, the presence of uncertainty and fixed costs causes the firm to delay investment under ranges of input prices. With the option to switch technologies, firms may change their technology in response to changes in relative wages, but the presence of investment irreversibility causes these switches to be less frequent. Furthermore, uncertainty causes the firms to wait for more extreme operating cost differentials before switching. These effects are present even without considering the effect of decisions on plant scale.

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Appendix A

Proof of Claim 1. The existence of a positive w^* is obvious since (1) and (9) are positive as $w \to 0$ and negative as $w \to \infty$.

I show now that w^* is increasing in σ. Note that (7) and (8) can be generalized for two technologies $i = 1, 2$:

$$C_{2,i} = \frac{1}{1-\beta_2}\left(\frac{pq}{\rho} - K_i\right)\left[\frac{\beta_2(\rho-\mu)}{L_i(1-\beta_2)}\left(K_i - \frac{pq}{\rho}\right)\right]^{\beta_2}, \tag{A.1}$$

$$w^{*,i} = \frac{\beta_2}{1-\beta_2}\left(\frac{\rho-\mu}{L_i}\right)\left(K_i - \frac{pq}{\rho}\right). \tag{A.2}$$

The argument then consists first of showing that $w^{*,i}$ is decreasing in σ for $i = 1, 2$. We must also eliminate the possibility that the relevant w^* switches from i to j, where $w^{*,j} > w^{*,i}$. More specifically, $w^{*,2} > w^{*,1}$, so we must rule out the case where both $w^{*,1}$ and $w^{*,2}$ decrease, but w^* increases if the relevant stopping technology switches from Technology 1 to 2. This is done by showing that $C_{2,1}$ increases faster in σ than $C_{2,2}$.

The effect of σ on (A.1) and (A.2) is through its presence in the expression for β_2. Differentiating the implicit function Q defined by (4) with respect to σ yields

$$\frac{dQ}{d\sigma} = \frac{\partial Q}{\partial \beta_2}\frac{d\beta_2}{d\sigma} + \frac{\partial Q}{\partial \sigma} = 0. \tag{A.3}$$

Since, for $\beta = \beta_2$, $\partial Q/\partial\beta < 0$ and $\partial Q/\partial\sigma = \sigma\beta(\beta - 1) > 0$ by Eq. (A.3) $d\beta_2/d\sigma > 0$. Hence

$$\frac{\partial w^{*,i}}{\partial \sigma} = \frac{\partial w^{*,i}}{\partial \beta_2}\frac{\partial \beta_2}{\partial \sigma} < 0.$$

Since β_2 is negative and increasing in σ. Taking derivatives of (A.1), it can also be shown that $C_{2,1}$ and $C_{2,2}$ are increasing in σ.

What remains to be shown is that $C_{2,1}$ increases faster in σ than $C_{2,2}$. Denoting the ratio $C_{2,2}/C_{2,1}$ as CR and differentiating yields

$$\frac{\partial CR}{\partial \sigma} = \left[\frac{((pq/\rho) - K_2)}{((pq/\rho) - K_1)}\frac{L_1}{L_2}\right]^{\beta_2}.$$

This expression is greater than one since $K_1 > K_2$, $L_2 > L_1$ and β_2 is negative. Thus CR is decreasing in σ and the proof is complete. □

Proof of Claim 3. We proceed by showing contradictions in the assumptions of no switching wages, only a w^*_{21} switching wage, or only a w^*_{12} switching wage.

(A) *Ruling out no switching wages*: Assume w_{12}^* and w_{21}^* do not exist. Then $G_{1,2} = G_{2,1} = 0$, since this term represents the value of the option to switch from Technology 1 to 2. We then have switching from Technology 1 to 2 has negative value at all wages, and vice versa. These expressions are

$$\frac{pq}{\rho} - \frac{wL_1}{\rho-\mu} > \frac{pq}{\rho} - \frac{wL_2}{\rho-\mu} - K_{12},$$
$$\frac{pq}{\rho} - \frac{wL_2}{\rho-\mu} > \frac{pq}{\rho} - \frac{wL_1}{\rho-\mu} - K_{21}.$$

This creates a contradiction if and only if $K_{12} < 0$. This also verifies the no uncertainty case.

(B) *Ruling out no* w_{12}^*: Assume w_{21}^* exists, but not w_{12}^*, then $G_{1,2} = 0$ and $G_{2,1} > 0$. We again have switching from Technology 1 to 2 has negative value at all wages. The new expression

$$\frac{pq}{\rho} - \frac{wL_1}{\rho-\mu} > \frac{pq}{\rho} - \frac{wL_2}{\rho-\mu} + G_{2,1}w^{\beta_1} - K_{12}.$$

This creates a contradiction since the left-hand side goes to infinity as $w \to \infty$, while the right-hand side goes to negative infinity.

(C) *Ruling out no* w_{21}^*: Assume w_{12}^* exists, but not w_{21}^*, then $G_{1,2} > 0$ and $G_{2,1} = 0$. We then have switching from Technology 2 to 1 has negative value at all wages. The expression

$$\frac{pq}{\rho} - \frac{wL_2}{\rho-\mu} > \frac{pq}{\rho} - \frac{wL_1}{\rho-\mu} + G_{1,2}w^{\beta_2} - K_{21},$$
$$\frac{-w(L_2 - L_1)}{\rho-\mu} - G_{1,2}w^{\beta_2} > -K_{21}$$

creates a contradiction as $w \to \infty$, since K_{21} is finite.

Hence, two positive roots exist and (17) ensure $w_{21}^* > w_{12}^*$, if w_{12}^* exists, so we can rule out $w_{21}^* = w_{12}^*$. $\square$

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