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# A STOCHASTIC BARGAINING PROCESS AND CORRESPONDING ONE-SHOT SOLUTION CONCEPT

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A stochastic bargaining process is first introduced to solve two-person conflicts. The outcome of the process depends on the initial offers of the two players, as well as on the offer-dependent breakdown probabilities. After the convergence (in most cases the finiteness of the process) is verified, a one-shot solution is introduced. The existence of a unique solution is then proven, and its relation to the non-symmetric Nash solution is discussed.

## 1. Introduction

This paper is concerned with finding solutions for two-person conflicts. Based on the pioneering work of Nash (1950) many researchers have suggested solution concepts and methods for conflict resolution. The axiomatic approach requires the solution to satisfy certain conditions which are called axioms. In most cases the existence and uniqueness of such a solution is proven. The original set of axioms of Nash (1950) has been modified and generalised by several authors. For example, non-symmetric Nash solutions have been examined by Harsanyi and Selten (1972) and solutions satisfying individual monotonicity have been introduced by Kalai and Smorodinsky (1975). This solution has been further generalised by Anbarci (1995) as the reference function solution. Among the most commonly known solution concepts mentioned are the egalitarian solution of Kalai (1977), the super-additive solution of Perles and Maschler (1981), the equal sacrifice solution of Chun (1988) and the equal area solution of Anbarci (1993). Nash (1953) has shown that the equilibrium set coincides with the set of Pareto solutions if the problem is considered as a two-person non-cooperative game. One might consider bargaining as a single player decision problem, when the strategy selection of the other player is considered random. If uniform distribution is assumed and each player maximises his/her own expected payoff, then the optimal selection is equivalent to the Nash solution. Similar equivalence holds for applying the principle of Zeuthen (1930) in determining the order in which concessions are made. One of the most popular bargaining method is the alternating offer process (Rubinstein, 1982), which has been extended by Howard (1992). In the model of Anbarci (1995), the payoffs of the players depend not only on their offers to themselves but also on how generous their offers are.

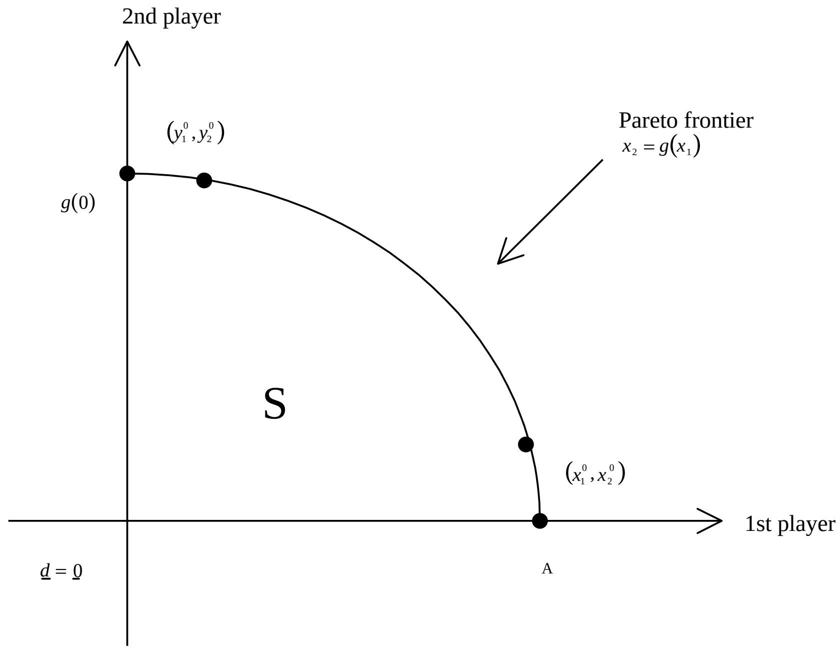


Fig. 1 Illustration of conflict.

The bargaining process to be introduced in this paper is a combination of the ideas of Rubinstein (1982) and Anbarci (1995), when the breakdown probabilities depend on how good the offers are. We will prove that the process converges under realistic conditions, and that it is finite in most cases. Based on this idea a one-shot solution concept similar to that of Anbar and Kalai's (1978) will be introduced. It is based on the relative power of player 2 above player 1. We will also show the existence of a unique solution in an important special case.

## 2. Problem Formulation and the Bargaining Process

Our bargaining problem is defined by a pair $(S, \underline{d})$, where $S \subseteqq R^2$ is a closed, comprehensive feasible payoff set (that is, $\underline{x} \in S$ and $\underline{x}' \leqq \underline{x}$ imply that $\underline{x}' \in S$), and $\underline{d}$ is the disagreement payoff vector. If the players reach an agreement, then the agreed payoffs are received by the players, and in the case of breakdown, they receive the corresponding components $d_1$ and $d_2$ of the disagreement payoff vector. In this paper, we will assume that the payoff coordinates are shifted so that $d_1 = d_2 = 0$. We also assume that the Pareto frontier of $S$ is given by the graph of a continuous, strictly decreasing function $g : [0, A] \mapsto R$ such that $A > 0$ and $g(A) = 0$. Here, we could have chosen $A = 1$ by scaling $x_2$, but this slight simplification would not make further discussion less complicated. It is assumed that at the initial time period $t = 0$, the initial offers of the two players are given as $(x_1^0, x_2^0)$ and $(y_1^0, y_2^0)$ where $0 \leqq y_1^0 < x_1^0 \leqq A$ and $x_2^0 = g(x_1^0)$, $y_2^0 = g(y_1^0)$.

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If at some time period $t$, player 1 makes a concession, then he/she selects a point $(x_1^t, x_2^t)$ on the Pareto frontier between the previous offers $(x_1^{t-1}, x_2^{t-1})$ and $(y_1^{t-1}, y_2^{t-1})$. That is, $x_1^{t-1} \geqq x_1^t \geqq y_1^{t-1}$ and $x_2^t = g(x_1^t)$. It is assumed that the probability that player 2 accepts this offer depends on how good this offer is for him or her:

$$P_2(x_2^{t-1}, y_2^{t-1}, x_2^t) = h_2\left(\frac{x_2^t - x_2^{t-1}}{y_2^{t-1} - x_2^{t-1}}\right) \tag{1}$$

where

(i) $h_2 : [0,1] \mapsto R$ is a strictly increasing, continuous function such that $h_2(0) = 0$ and $h_2(1) = 1$.

If no concession is made by player 1, then $x_2^t = x_2^{t-1}$, making the acceptance probability zero, and if $x_2^t = y_2^{t-1}$, then player 1 accepts the last offer of player 2 which will certainly lead to an agreement among the players. We also assume that in selecting the offer, player 1 is *myopic* and maximises his/her expected payoff occurring in this step only, and does not take into account possible later negotiation steps. This approach is realistic when there is a large uncertainty in future payoff values and even in the possibility of continuing the negotiation. The expected payoff of player 1 is given as

$$E_1(x_1) = x_1 h_2\left(\frac{g(x_1) - x_2^{t-1}}{y_2^{t-1} - x_2^{t-1}}\right), \tag{2}$$

since the disagreement payoffs are equal to zero. Since this function is continuous on the closed interval $[y_1^{t-1}, x_1^{t-1}]$, there is an expected payoff maximising offer $x_1^t$. In order to guarantee the uniqueness of $x_1^t$, we introduce the following conditions:

(ii)$_1$ Function $g$ is differentiable and has a negative derivative on the interval $[0, A]$.
(ii)$_2$ The product $x_1 g'(x_1)$ is non-increasing for $x_1 \in [0, A]$.
(ii)$_3$ Function $h_2$ is differentiable on $[0, 1]$.
(ii)$_4$ The ratio $h_2'(u)/h_2(u)$ strictly decreases for $u \in (0, 1]$.

Notice that these conditions are satisfied if $A = 1$, $g(x_1) = 1 - x_1$ and $h_2(u) = u^p$ $(p > 0)$.

**Lemma 2.1.** *Under conditions* (i) *and* (ii)$_1$–(ii)$_4$ *player* 1 *has a unique expected payoff maximising offer.*

**Proof.** Simple differentiation shows that

$$\begin{aligned} E_1'(x_1) &= h_2\left(\frac{g(x_1) - x_2^{t-1}}{y_2^{t-1} - x_2^{t-1}}\right) + x_1 h_2'\left(\frac{g(x_1) - x_2^{t-1}}{y_2^{t-1} - x_2^{t-1}}\right)\frac{g'(x_1)}{y_2^{t-1} - x_2^{t-1}} \\ &= h_2(u)\left[1 + \frac{h_2'(u)}{h_2(u)}\frac{x_1 g'(x_1)}{y_2^{t-1} - x_2^{t-1}}\right] \end{aligned} \tag{3}$$

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with

$$u = \frac{g(x_1) - x_2^{t-1}}{y_2^{t-1} - x_2^{t-1}}\,.$$

Notice first that $E_1(x_1)$ is zero for $x_1 = x_1^{t-1}$ and is positive for $x_1 \in [y_1^{t-1}, x_1^{t-1})$, so the optimal $x_1^t$ occurs for $x_1^t < x_1^{t-1}$, that is player 1 must make some concessions. The sign of $E_1'(x_1)$ is the same as the sign of the bracketed factor for $u > 0$. Conditions $(\mathrm{ii})_4$, $(\mathrm{ii})_2$ and the assumption that $g$ strictly decreases imply that the bracketed factor is strictly decreasing in $x_1$. Furthermore, $E_1'(x_1^{t-1}) \leqq 0$. Therefore $x_1^t$ is unique and can be obtained in the following way. If $E_1'(y_1^{t-1}) \leqq 0$, then $x_1 = y_1^{t-1}$ is the optimal choice, otherwise $x_1^t$ is the unique solution of the equation $E_1'(x_1) = 0$ in the interval $(y_1^{t-1}, x_1^{t-1})$. Since the bracketed factor is strictly monotonic, its root can be obtained easily by applying the bisection or the secant method. □

Interchanging the role of the two players, the same idea applies to player 2 in cases when he/she makes concessions. Let $G$ denote the inverse of function $g$ which exists and is strictly decreasing. The expected payoff of this player is given as

$$E_2(y_2) = y_2 h_1\left(\frac{G(y_2) - y_1^{t-1}}{x_1^{t-1} - y_1^{t-1}}\right) \tag{4}$$

where

(iii) $h_1 : [0,1] \mapsto R$ is a strictly increasing, continuous function such that $h_1(0) = 0$ and $h_1(1) = 1$.

The continuity of Eq. (4) on $[x_2^{t-1}, y_2^{t-1}]$ guarantees the existence of at least one expected payoff maximising selection $y_2^t$. This value is unique, if the following conditions hold:

$(\mathrm{iv})_1$ The ratio $g(x_1)/g'(x_1)$ is non-decreasing for $x_1 \in [0, A]$.
$(\mathrm{iv})_2$ Function $h_1$ is differentiable on [0,1].
$(\mathrm{iv})_3$ The ratio $h_1'(u)/h_1(u)$ strictly decreases for $u \in (0, 1]$.

Notice that assumption $(\mathrm{iv})_1$ implies a similar condition as $(\mathrm{ii})_1$ for function $G$ on the interval $[0, g(0)]$, and assumptions $(\mathrm{iv})_2$, $(\mathrm{iv})_3$ are equivalent to conditions $(\mathrm{ii})_3$ and $(\mathrm{ii})_4$ if functions $h_1$ and $h_2$ are interchanged.

Based on the above described expected payoff maximising rule, the negotiation process can be defined in several different ways. One possible alternative is the following. At time period $t = 1$, each player makes concession simultaneously. If $x_1^1 \leqq y_1^1$, then bargaining terminates and each player receives his/her demanded payoff (or any point of the Pareto frontier between $(x_1^1, x_2^1)$ and $(y_1^1, y_2^1)$ might be selected as final solution). Otherwise, negotiation continues at $t = 2$ in the same way starting from the latest offers, and so on. Another way of defining the bargaining

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process is the following. Assume that player 1 makes concessions in each odd (or even) step, and player 2 makes new offers in each even (or odd) step, and in each step the new offer is selected accordingly to the above rule. The two alternative bargaining processes can be examined in the same way. In the foregoing discussion, the second process will be considered.

Next, we will show that under realistic conditions the process is convergent, that is sequences $\{x_1^t\}$ and $\{y_1^t\}$ have a common limit, and the process is even finite if $h_2(u)/h_2'(u)$ or $h_1(u)/h_1'(u)$ is bounded from above. The fact that for $t \geqq 2$, $y_1^0 < x_1^t < x_1^{t-1}$ and $x_1^0 > y_1^t > y_1^{t-1}$ imply that both sequences $\{x_1^t\}$ and $\{y_1^t\}$ converge unless the process is finite. Let $x_1^*$ and $y_1^*$ denote the limits and let $x_2^* = g(x_1^*)$ and $y_2^* = g(y_1^*)$. In the case of an infinite process, $x_1^t$ and $y_2^t$ are both interior optima implying that $E_1'(x_1^t) = 0$. This equation can be rewritten as

$$x_1^t = -\frac{h_2\left(\frac{g(x_1^t)-x_2^{t-1}}{y_2^{t-1}-x_2^{t-1}}\right)(y_2^{t-1}-x_2^{t-1})}{h_2'\left(\frac{g(x_1^t)-x_2^{t-1}}{y_2^{t-1}-x_2^{t-1}}\right)g'(x_1^t)}. \tag{5}$$

We then assume that

(v) $\lim_{u\to 0+} \frac{h_2(u)}{h_2'(u)} = 0$.

Notice that this condition holds for the special choice of $h_2(u) = u^p$ $(p > 0)$.

The main result of this section can be formulated as follows.

**Theorem 2.1.** *Under assumptions* (i), (ii)$_1$–(ii)$_4$, (iii), (iv)$_1$–(iv)$_3$, *and* (v) *the process is convergent.*

**Proof.** If the process terminates after finitely many steps, then for some $t > 0$, $x_1^t = y_1^t$ which is a special case of convergence. Next, we assume that the process is infinite and $x_1^* \neq y_1^*$. Then for all $t$, when player 1 makes concessions, Eq. (5) is satisfied. Next we show the existence of a positive number $\varepsilon_0$ such that for all $x_1 \in (y_1^2, x_1^1)$, $g'(x_1) \leqq -\varepsilon_0$, that is, $g'(x_1^t)$ is bounded away from zero for $t \geqq 2$. Assume on contrary that such $\varepsilon_0$ does not exist. Then for all $\varepsilon > 0$, there is an $x_\varepsilon \in (y_1^2, x_1^1)$ such that $|x_\varepsilon g'(x_\varepsilon)| \leqq \varepsilon$. Therefore assumption (ii)$_2$ implies that $y_1^2 g'(y_1^2) = 0$. Since $y_1^2 > 0$ (after one concession, player 2 must give a positive offer to player 1), this may occur only if $g'(y_1^2) = 0$, which contradicts assumption (ii)$_1$. Letting $t \to \infty$ in Eq. (5) shows that $x_1^t \to 0$, which is an obvious contradiction again. $\square$

Notice that in the case when $g'$ is continuous, the existence of $\varepsilon_0$ is obvious.

Interchanging the role of the players, it is obvious that the assertion of the theorem remains valid if condition (v) is replaced by the similar assumption

(vi) $\lim_{u\to 0+} \frac{h_1(u)}{h_1'(0)} = 0$.

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Before formulating the result on the finiteness of the process, we introduce the following assumption:

(vii) There is a positive constant $a$ such that for all $u \in [0, 1]$,

$$\frac{h_2(u)}{h_2'(u)} \leqq a\,.$$

**Theorem 2.2.** *Assume that in addition to the conditions of Theorem* 2.1, *assumption* (vii) *holds. Then the process is finite.*

**Proof.** From Eq. (5), we have

$$0 \leqq x_1^t \leqq \frac{a}{\varepsilon_0}(y_2^{t-1} - x_2^{t-1})$$

where $\varepsilon_0$ was defined in the proof of the previous theorem. Letting $t \to \infty$ and using the fact that Theorem 2.1 implies that $y_2^* = x_2^*$ we conclude that $x_1^* = 0$, which is an obvious contradiction. □

Interchanging the role of the players we can easily see that the assertion of the theorem remains valid if condition (vii) is replaced by the following:

(viii) There is a positive constant $\bar{a}$ such that for all $u \in [0, 1]$,

$$\frac{h_1(u)}{h_1'(u)} \leqq a\,.$$

## 3. A Special Case

Select

$$h_1(u) = u^{p_1} \quad \text{and} \quad h_2(u) = u^{p_2}$$

with some positive constants $p_1$ and $p_2$. These values can be considered to be the power indices of the players, since larger value of $p_1$ (or $p_2$) forces player 2 (or player 1) to make larger concessions. In this special case, conditions (i), (ii)$_3$–(ii)$_4$, (iii), (iv)$_2$–(iv)$_3$, (v), (vi), (vii) and (viii) obviously hold. For all $t > 0$, the values of $x_1^t$ and $y_2^t$ can be obtained as follows. Define

$$H_1(x_1) = x_1 g'(x_1) p_2 + g(x_1) - x_2^{t-1}$$

and

$$H_2(y_2) = y_2 G'(y_2) p_1 + G(y_2) - y_1^{t-1}\,.$$

Equation (5) is now equivalent to $H_1(x_1) = 0$. Conditions (ii)$_2$, (iv)$_4$ and the assumption on $g$ imply that both functions $H_1$ and $H_2$ are strictly decreasing in

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intervals $[y_1^{t-1}, x_1^{t-1}]$ and $[x_2^{t-1}, y_2^{t-1}]$, respectively. If $H_1(y_1^{t-1}) \leqq 0$, then $x_1^t = y_1^{t-1}$, and negotiation terminates. Otherwise, $x_1^t$ is the unique solution of equation

$$H_1(x_1) = 0 \tag{6}$$

in interval $(y_1^{t-1}, x_1^{t-1})$. Similarly, if $H_2(x_2^{t-1}) \leqq 0$ then $y_2^t = x_2^{t-1}$ is the optimal choice for player 2, and negotiation terminates. Otherwise, $y_2^t$ is the unique solution of equation

$$H_2(y_2) = 0 \tag{7}$$

in interval $(x_2^{t-1}, y_2^{t-1})$.

## 4. A One-Shot Solution

Assume now that the individual power indices $p_1$ and $p_2$ are unknown, but their ratio, $\alpha = p_2/p_1$ is given. The value of $\alpha$ can be imagined as the relative power index of player 2 over player 1. Let $(x_1^0, x_2^0)$ and $(y_1^0, y_2^0)$ denote again the initial offers of the two players. The one-shot solution is a point $(z_1, z_2)$ on the Pareto frontier between the initial offers which is optimal for both players with some particular values of $p_1$ and $p_2$ such that $\alpha = p_2/p_1$. This solution is obviously interior and satisfies Eqs. (6) and (7), that is

$$z_1 g'(z_1) p_2 + g(z_1) - x_2^0 = 0, \qquad z_2 \frac{p_1}{g'(z_1)} + z_1 - y_1^0 = 0\,.$$

Simple algebra shows that these equations are equivalent to a single equation

$$\frac{x_2^0 - g(z_1)}{z_1 g'(z_1)} - \alpha \frac{y_1^0 - z_1}{\frac{g(z_1)}{g'(z_1)}} = 0\,. \tag{8}$$

Let $H(z_1)$ denote the left-hand-side. Then $H(x_1^0) < 0$, $H(y_1^0) > 0$ and $H$ strictly decreases in $z_1$. Therefore there is a unique solution in interval $(y_1^0, x_1^0)$.

## 5. Numerical Examples

This section presents two examples. The first one shows a finite process, and the second case illustrates an infinite process, when condition (viii) is violated.

**Example 5.1.** Define

$$S = \{(x_1, x_2) | x_1, x_2 \geqq 0, x_2 \leqq (2 - x_1)^2 - 1\}\,,$$

where $A = 1$ and $g(x_1) = (2 - x_1)^2 - 1$. Function $g$ is strictly convex showing that assumption $(\text{ii})_2$ is much weaker than assuming the concavity of $g$. Assume that the initial offers of player 1 and player 2 are $(0.9, 0.21)$ and $(0.1, 2.61)$, respectively. $h_1(u) = h_2(u) = \sqrt{u}$ is further selected for computing acceptance probabilities. Simple calculation shows that regardless which player makes the first concession, the process always terminates after four steps. Tables 1 and 2 show the consecutive offers of the two players.

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Table 1. Offers when player 1 starts concessions.

| $t$ | $x_1^t$ | $x_2^t$ | $y_1^t$ | $y_2^t$ |
|---|---|---|---|---|
| 0 | 0.9 | 0.21 | 0.1 | 2.61 |
| 1 | 0.575 | 1.030 | | |
| 2 | | | 0.358 | 1.696 |
| 3 | 0.375 | 1.640 | | |
| 4 | | | 0.375 | 1.640 |

Table 2. Offers when player 2 starts concessions.

| $t$ | $x_1^t$ | $x_2^t$ | $y_1^t$ | $y_2^t$ |
|---|---|---|---|---|
| 0 | 0.9 | 0.21 | 0.1 | 2.61 |
| 1 | | | 0.358 | 1.696 |
| 2 | 0.575 | 1.030 | | |
| 3 | | | 0.549 | 1.107 |
| 4 | 0.549 | 1.107 | | |

Consider next the special case of Sec. 3 with the same Pareto frontier and select, for example, $\alpha = 0.5$ to get the one-shot solution $z_1 \approx 0.46165$ and $z_2 \approx 1.3665$. We mention that in this case, Eq. (8) has the special form

$$\frac{x_2^0 - (2 - z_1)^2 + 1}{z_1 2(z_1 - 2)} - \alpha \frac{y_1^0 - z_1}{\frac{(2-z_1)^2 - 1}{2(z_1 - 2)}} = 0\,,$$

and it was solved by using the bisection method.

**Example 5.2.** Select now

$$S = \{(x_1, x_2) | x_1, x_2 \geqq 0, x_2 \leqq 1 - x_1\}\,,$$

where $A = 1$ and $g(x_1) = 1 - x_1$. Function $g$ is now linear (that is, not strictly concave). Select $h_1(u) = h_2(u) = 2u - u^2$. We will now show that the process cannot terminate infinitely many steps by proving that at each step, the expected profit maximising offer is interior. Assume that player 1 makes a concession. Since $h_1 \equiv h_2$ the case of player 2 can be discussed in the same way. It is sufficient to show that (using the notation of the proof of Lemma 2.1) $E_1'(y_1^{t-1}) > 0$.

From Eq. (3), we have

$$E_1'(y_1^{t-1}) = h_2(1) \left[1 + \frac{h_2'(1)}{h_2(1)} \frac{y_1^{t-1} g'(y_1^{t-1})}{y_2^{t-1} - x_2^{t-1}}\right] = 1\,,$$

since $h_2(1) = 1$ and $h_2'(1) = 0$.

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## 6. Relation to the Non-Symmetric Nash Solution

Consider now the special case when $y_1^0 = x_2^0 = 0$, that is, both players offer the disagreement payoff to each other as the initial offers. In this special case, Eq. (8) can be rewritten as

$$\frac{g(z_1)}{z_1 g'(z_1)} = -\sqrt{\alpha}\,, \tag{9}$$

where we use the assumption that $g'$ is negative.

It is well-known [see for example, Harsanyi and Selten (1972)] that the non-symmetric Nash solution is the unique optimal solution of the problem.

$$\text{Maximise } z_1^\gamma g(z_1)^{1-\gamma}, \qquad \text{subject to } 0 \leqq z_1 \leqq A \tag{10}$$

with some $0 < \gamma < 1$. At $z_1 = 0$ and $z_1 = A$ the objective function is zero, otherwise it is positive. Therefore the optimal solution is interior and

$$\frac{d[z_1^\gamma g(z_1)^{1-\gamma}]}{dz_1} = \gamma z_1^{\gamma-1} g(z_1)^{1-\gamma} + z_1^\gamma (1-\gamma) g(z_1)^{-\gamma} g'(z_1) = 0\,.$$

This equation can be rewritten as

$$\frac{g(z_1)}{z_1 g'(z_1)} = -\frac{1-\gamma}{\gamma}\,. \tag{11}$$

Comparing Eqs. (9) and (11) and using the fact that both equations have a unique solution in $(0, A)$, we conclude that the solutions coincide with

$$\alpha = \left(\frac{1-\gamma}{\gamma}\right)^2\,. \tag{12}$$

That is, in this special case the one-shot solution and the non-symmetric Nash solution are equivalent to each other. This equivalence does not hold if the initial offers are selected in a different way. Hence, the one-shot solution can be considered as a further generalisation of the non-symmetric Nash solution, when the solution also depends on the initial offers.

## 7. Conclusions

A sequential bargaining process is introduced in this paper, which has two new elements compared to earlier concepts. First, it is assumed that the breakdown probabilities depend on how good the latest offers are. Second, the sequence of consecutive offers also depends on the initial offers of the players.

For myopic players, we could show that under general conditions the process converges, and it even converges in a finite number of steps under slightly more restrictive assumptions. If the breakdown probabilities are characterised by power functions, then the exponents can be considered as the power indices of the two players.

A one-shot solution concept has been introduced based on the relative power of player 2 over player 1, and under general conditions the existence of a unique solution is proven. It is an interesting result that if the players initially offer the disagreement payoffs to each other, then the one-shot solution coincides with a non-symmetric Nash solution.

Numerical examples illustrate the bargaining process and solution concepts. Our conditions on the payoff space do not require its convexity, therefore the methodology introduced in this paper can be applied more generally than earlier concepts and methods known from the literature.

The extension of the bargaining process and solution concept for non-myopic players will be the subject of a future paper.

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