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Journal of Banking & Finance 29 (2005) 1509–1534
Journal of BANKING & FINANCE
www.elsevier.com/locate/econbase

Cash-flow shortage as an endogenous bankruptcy reason

Marliese Uhrig-Homburg *

Chair of Financial Engineering and Derivatives, Universität Karlsruhe (TH), 76128 Karlsruhe, Germany

Received 6 November 2001; accepted 2 June 2004
Available online 7 October 2004

Abstract

This paper develops a simple model for a leveraged firm and endogenizes the firm's bankruptcy point by assuming that equity issuance is costly. Equity-issuance costs reflect the difficulties in issuing new equity for firms that are close to financial distress. The resulting model captures cash-flow shortage as a reason to go bankrupt, though the equity value is positive. I analyze the optimal bankruptcy point as well as corporate bond prices and yield spreads for various levels of equity-issuance costs in order to study the impact of different liquidity constraints. Finally, I discuss the consequences on optimal capital structure.

JEL classification: G13; G32; G33
Keywords: Continuous-time bankruptcy model; Corporate bond pricing; Credit spreads; Equity finance costs; Optimal capital structure

1. Introduction

The modeling of default is crucial for pricing defaultable claims and understanding firms' financing decisions. Basically firms default for one of the following two

* Tel.: +49 721 608 8183; fax: +49 721 608 8190.
E-mail address: uhrig@wiwi.uni-karlsruhe.de

0378-4266/$ - see front matter
doi:10.1016/j.jbankfin.2004.06.026

reasons: either the available cash-flow is insufficient to meet payments to creditors (cash-flow shortage) or the firms' liabilities exceed the firms' assets (overindebtedness). Indeed, according to the bankruptcy laws in many countries, a formal bankruptcy process can only be initiated if one of these two insolvency requirements is fulfilled. [1]

Theoretical models with perfect frictionless markets do not need to distinguish between cash-flow shortage and overindebtedness. A cash-flow shortage occurs only in conjunction with overindebtedness. This means that the impact of different bankruptcy reasons on the values of corporate securities or optimal capital structure cannot be studied reasonably within such models. The aim of this paper is to present a theoretical model of the firm that allows the analysis of these issues. I extend the firm value model of Leland (1994) in order to capture the importance of a cash-flow shortage. By assuming that equity issuance causes transaction costs, I obtain a rationale for defaulting due to liquidity reasons. As a result, my approach combines two extreme forms of liquidity constraints: the immediate default triggered by a cash-flow shortage as in Kim et al. (1993) or Ericsson's (2000) model and the perspective of the endogenous bankruptcy models of Leland (1994) or Leland and Toft (1996) in which equity holders can always overcome liquidity problems. My integrated approach captures any mechanism between these two extreme default-triggering mechanisms. Hence, the model allows to analyze the consequences of different forms of liquidity constraints, contributing to a better understanding of real-world bankruptcy problems.

Compared to a model of an otherwise identical firm that can costlessly issue new equity, my model predicts higher credit spreads and a smaller amount of debt. More precisely, it predicts that even moderate equity issuance costs (i) can already more than halve the under-pricing of corporate bond prices documented in previous studies and (ii) lead to significantly lower leverage ratios, especially when firm risk is high. Since both predictions fit empirically observed spreads and actual leverage ratios better than those of classical structural models without liquidity constraints, the proposed modeling helps to explain these empirical puzzles. The intuition of these results is that the firm seeks to avoid a default for liquidity reasons and therefore it chooses a lower leverage ratio in capital structure optimum. For a given amount of debt, higher equity-issuance costs lead to earlier defaults and therefore make the debt more risky.

My approach fits within a broader literature that builds on Merton's (1974) structural approach but reflects some elements of the complex bankruptcy process in more detail. These models recognize that the default level is a decision of the firm. Leland (1994, 1998) and Leland and Toft (1996) follow an idea outlined in Black and Cox (1976), modeling default as the optimal management decision not to inject

[1] Cash-flow shortage is among the group of required statuses in almost all bankruptcy codes. In contrast, overindebtedness, which in a legal sense means that the total face value of the firm's liabilities exceeds the value of the firm's assets, is mentioned only in some codes such as Germany, Canada, and Japan. The US code allows firms to file for bankruptcy even if they are not insolvent.

new equity with the consequence that the firm fails to meet its debt service requirements. The same default-triggering mechanism is used by Goldstein et al. (2001) within a dynamic capital structure model. Other examples are Mello and Parsons (1992), Mella-Barral and Perraudin (1997), or Mella-Barral (1999), who model default as the option to shut down a firm's activities. The approaches put forward by Anderson and Sundaresan (1996), Anderson et al. (1996), and Fan and Sundaresan (2000) model default as an outcome of the bargaining process between various claimholders.

Within most of the aforementioned approaches, a cash-flow shortage is not an issue. Exceptions are the models of Kim et al. (1993), Anderson and Sundaresan (1996), Anderson et al. (1996), and Ericsson (2000). In these models, default is triggered as soon as the cash flow available for payouts falls below the required debt service payments. However, this way of modeling cash-flow shortage is not fully convincing in a more general modeling environment. Even if borrowing is not feasible and covenants restrict firms from selling assets, equity holders should still try to keep the firm alive, either by injecting new equity themselves or by issuing further equity. Therefore, the presumed bankruptcy-triggering mechanism is model-consistent only if the issue of new equity is forbidden, i.e. equity-issuance costs are infinite. On the other hand, Leland (1994, 1998) and Leland and Toft (1996) endogenously derive the optimal bankruptcy point by assuming that all debt services are met by issuing new equity, which is a costless process. Goldstein et al. (2001) use the same default-triggering idea within a dynamic capital structure model. In addition to the possibility of injecting new equity they consider the possibility of increasing future debt levels. However, this is not done in order to finance debt service in periods of a low payout flow. Rather the firm uses its option to adjust its capital structure when the firm value is sufficiently increased. Despite this possibility, Goldstein et al. still assume that all debt services are met by issuing new equity which is a costless process. Thus, these models cannot reflect cash-flow shortage as an independent bankruptcy reason. In contrast, my variation on Leland's model provides an adequate basis to capture such liquidity problems by accounting for non-zero but finite equity-issuance costs.

In the following, I present a model that captures cash-flow shortage as a reason for a firm to go bankrupt even though the equity value is positive. This is achieved by extending Leland's (1994) model in two important points. Firstly, I explicitly ensure that, in the event of bankruptcy, bondholders do not receive more than the face value of debt – reflecting current bankruptcy law. This allows me to manage defaults at positive equity values in principle. The limitation of the amount paid out to bondholders can lead to situations where the post bankruptcy value of the firm's assets exceeds the face value of debt such that a positive equity value remains. If instead I had followed Leland by assuming that bondholders receive whatever is left of the firm's assets after bankruptcy, the equity value in the case of bankruptcy would have been zero. Secondly, I assume that equity issuance causes transaction costs for firms facing a cash-flow shortage. This provides an incentive for the firm to default already at positive equity values. In an aggregate way, these transaction costs reflect the difficulties in issuing new equity for firms that are close to financial distress. Such firms may be restricted in their ability to raise new funds by issuing equity. For instance, if the current equity holders are not able to inject new equity themselves, because they do not have enough wealth, they would have to convince new investors of the firm's quality. This is presumably much more difficult for a firm that faces a cash-flow shortage than for a healthy firm. I approximate this situation by assuming that firms outside of financial distress can issue equity costlessly.

The remainder of the paper is organized as follows: Section 2 develops a simple model for the leveraged firm. Section 3 analyzes the owner–manager's optimal strategy. Section 4 presents several applications of the model and Section 5 concludes.

2. The Model

Following the lines of Merton (1974), Black and Cox (1976), and others, I consider a firm with risky assets, the value V of which follows the log-normal diffusion process

$$dV = V(\mu - \delta)\,dt + \sigma V\,dz. \tag{1}$$

z is a standard Brownian motion, μ, is the instantaneous expected return on asset value V, δV is the firm's total dollar payout to all securityholders, and σ is the constant proportional volatility of the return on the firm value. In the tradition of the classical capital structure literature pioneered by Modigliani and Miller (1958), I assume the firm's activities to be independent of its capital structure. [2] The exogenous payout δV could be interpreted as the net cash flow resulting from the firm's optimal production and investment decisions. However, deducing this cash flow endogenously from more fundamental assumptions, for example in an environment with stochastic investment opportunities as in Brennan and Schwartz (1984), is beyond the scope of this paper. Instead, I consider a partial equilibrium approach, where the dynamics of the firm's asset value and the ongoing cash flow after investment generated by these assets are treated as exogenous.

Following Leland (1994) I make three simplifying assumptions. Firstly, I assume the existence of a risk-free asset paying a constant rate of interest r. Secondly, I consider a static debt structure. For simplicity, I assume that the firm has only one perpetual bond outstanding. F denotes the face amount and the perpetuity pays a continuous coupon stream cF per year. This second assumption excludes an explicit time dependence of security prices. Thirdly, I assume a simple tax environment where coupon payments are tax-deductible at the firm level. Given a corporate tax rate τ, the firm derives tax benefits from its debt at a rate of $cF\tau$ until default. Since

[2] This assumption rules out a number of important issues discussed in the corporate finance literature, such as the potential bondholder–stockholder conflicts concerning dividend payment, claim dilution, asset substitution, or underinvestment. See, e.g. Smith and Warner (1979).

I only consider tax benefits in this setup, the net cash flow δV should be interpreted as the after-tax net cash flow before interest. [3]

I consider an owner–manager who maximizes equity value. In principle, he has three possibilities for financing debt service. Firstly, the owner–manager could sell assets. Secondly, he could borrow the required money and, thirdly, he could issue additional equity. Since I assume that the firm's activities are independent of the choice of leverage, the first possibility is not feasible in my setting. [4] I also exclude the second alternative in order to keep the debt structure fixed. Consequently, only the third alternative remains, i.e. debt service has to be financed by issuing equity.

If equity is issued at fair conditions and agency costs resulting from conflicts between managers and outside equity holders are not considered, then it is not important whether the required cash is provided by new investors or by the owner–manager himself. Therefore, at each coupon payment date the owner–manager has to solve the following decision problem. Either he can immediately default on his coupon payments, or he can inject new money into the firm in order to pay the bondholders. Given that the owner–manager chooses to default at some level V_B of the firm value, creditors immediately have a claim of F on the firm. This reflects current bankruptcy law in many countries including the US and Germany, and clearly differs from the assumption implicitly made by Leland (1994). Note also that depending on the coupon level c and the market environment, the face value F can be above or below the market value of the perpetuity. In the case of default a fraction $1 - \alpha$ of the firm's value prior to default is assumed to be lost to bankruptcy cost. When $F > \alpha V_B$, creditors take over the firm, which then has value αV_B while the owner–manager receives nothing. If the value of the firm after default exceeds the face amount F, the creditors are reimbursed, and the owner–manager is left with the residual $\alpha V_B - F$. This case can be interpreted as the owner–manager's decision to liquidate his firm voluntarily. In this case, αV_B is the firm's liquidation value. [5]

[3] Instead of modeling the asset value V as the state variable one could alternatively use earnings before interest and taxes (EBIT) or the claim to future EBIT as in Goldstein et al. (2001). Doing so would solve two important problems of dynamic capital structure models. First, the government claim would be modeled as an outflow, leading to more realistic comparative statics with respect to the tax-rate. Second, an all-equity firm would no longer exist. Therefore, the assumption of a traded asset value would not lead to a putative arbitrage opportunity through trading the unleveraged and the optimally leveraged firm simultaneously. Here, the focus is not primarily on taxation aspects. Furthermore, it is not possible to exploit a positive price difference between the optimally leveraged and the all-equity firm value given the single and irreversible capital structure decision within my static debt structure environment. Therefore, such a modeling is not necessary here.

[4] Often, bond indentures contain contractual provisions that restrict firms from selling assets to make payments to bondholders. Black and Cox (1976), Leland (1994), and Leland and Toft (1996) assume that coupon payments must be financed by issuing new equity. In contrast, Merton (1974) assumes that stockholders are allowed to sell assets in order to make these payments.

[5] Mello and Parsons (1992), Mella-Barral and Perraudin (1997), Mella-Barral (1999) and others distinguish between the liquidation value and the going-concern value that a new owner can achieve while in my model both values are αV_B. This distinction is needed in their approach in order to ensure that the optimal liquidation time for an all-equity firm is not infinity as is implicitly assumed in my approach.

In contrast to Black and Cox (1976), Leland (1994), and Leland and Toft (1996), I assume that equity issues entail transaction costs. In order to clarify the nature of these costs I first define a critical asset value $V_S := \frac{c(1-\tau)}{\delta}F$. When V reaches this critical level V_S the current cash flow δV generated by the firm's assets does just cover the after-tax debt services $cF(1-\tau)$. Any further decline in V leads to a negative net cash-flow to the owner–manager. I therefore interpret V_S as the cash-flow shortage point. Now I consider two cases with respect to the current level of the asset value: As long as the asset value V is above V_S, I assume that an equity issue is costlessly possible. However, as soon as the firm value V falls below the critical value V_S equity issues require additional payments (transaction costs) proportional to the amount of cash $K(t) = c(1-\tau)F - \delta V(t)$ required. With a transaction-cost rate of β, transaction costs at time t amount to

$$\beta K(t) = \beta(c(1-\tau)F - \delta V(t)) = \beta\delta(V_S - V(t)). \tag{2}$$

Since transaction costs become relevant only if the current asset value has reached a sufficiently low level, this modeling is consistent with the intuition that firms close to financial distress have more difficulties in raising new capital than lowly leveraged firms. Note in addition that it would not be plausible to assume that the owner–manager pays transaction costs for the equity issue while at the same time receiving a positive cash flow stream δV. It seems more reasonable to assume that the owner–manager uses this cash to pay interest to the bondholders. In a world without transaction costs the payment of the debt service out of the owner–manager's pocket is equivalent to the issuance of new equity. If transaction costs are considered for all asset levels, the above strategy can be used to avoid transaction costs for sufficiently high levels of the asset value.

It can be concluded that at any point in time at which the level of the firm value V is below the critical value V_S, the owner–manager must raise additional cash to prevent default. For this reason, the owner–manager has to pay transaction costs $\beta K(t)$ as long as the firm value is between V_S and the level V_B at which he chooses to default. Note that, similar to previous studies, new equity is issued only to the extent that new cash is needed to cover the required payments. Whether or not the firm faces a cash-flow shortage after the equity issue therefore depends solely on the evolution of the asset value V given by (1).

Concerning the modeling of equity-issuance costs two further-reaching issues are worth noting. First, equity-issuance costs are exogenously given. However, in an economic modeling they could be captured through asymmetric information between the firms' insiders and public-market investors. Although the explicit modeling of informational asymmetries is beyond the scope of this paper, the adverse selection problem of Myers and Majluf (1984) might be a rationale for such transaction costs: As long as the firm has enough internal funds it gets along without external sources. Whenever the cash-flow is so low that project financing by issuing new equity is required, asymmetric information can lead to underinvestment. The equity-issuance costs I assume in this paper could be interpreted as the negative consequences of such an underinvestment. [6] Second, the assumption of equity-issuance costs which are non-zero but finite raises the interesting question whether all residual cash flows after debt service should be paid out as dividends. In a related paper, Fan and Sundaresan (2000) show that it is in fact optimal to pay out all the residual cash flows as dividends in an endogenous bankruptcy model in the spirit of Leland (1994) while a hard cash flow-based covenant as in Kim et al. (1993) can lead to situations where equity holders deny themselves dividends in order to prevent a potentially forced liquidation. If equity holders have the power to precipitate bankruptcy but cash injections are costly, the optimal dividend policy is not at all obvious. On the one hand, there might be reasons to forgo current dividends in order to reduce the expected cost of cash injections in the future. On the other hand, the disadvantages resulting from the fact that retained earnings become accessible by the debt holders upon bankruptcy suggest to choose a high dividend level. I do not explicitly derive the optimal dividend policy in my endogenous bankruptcy model since this would require solving numerically a much more complicated model that uses accumulated earnings as a second state variable and then results in a two-dimensional and highly non-linear optimization problem. However, following the lines of Fan and Sundaresan (2000) I can show for many reasonable parameter choices including those chosen in this paper that even if equity issuance is costly, no interior dividend policy is optimal but again all residual cash flows should be paid out as dividends. Therefore, it is this dividend policy that I presuppose in the following.

3. The owner–manager's optimal decision

The owner–manager chooses the asset value level V_B that triggers bankruptcy. Since his policy is to maximize the equity value E, I first derive the equity value as a function of the default point V_B and then proceed in solving for the value $V_B^\star$ that maximizes E with respect to V_B.

3.1. The equity value as a function of the default point

Until the owner–manager chooses to default at the asset level V_B, he receives the cash flow δV. To keep the firm alive, he has to meet the contractual coupon payments resulting in a cash outflow of $cF(1-\tau)$ after tax benefits. Whenever the asset value is so low that the net cash-flow $\delta V - cF(1-\tau)$ to the owner–manager is negative, new equity must be issued. In this case, equity-issuance costs of $\beta\delta(V_S - V)$ are

[6] Duffie and Lando (2001) integrate an asymmetric information structure in firm value models in order to show that for these models there exists a stochastic intensity which provides a justification for the assumptions typically made in reduced-form models. In particular, they assume that public-market investors cannot observe the firm's assets directly, instead receiving only imperfect information about the firm. This could result in a situation where outside investors hesitate to inject money into a firm with positive equity value, since they are unable to perceive the true equity value. If the firm cannot signal its true value to outside investors, liquidity problems may arise despite a positive equity value.

incurred. Given these cash flows, the equity value $E(V)$ satisfies the following second-order ordinary differential equation:

$$\begin{array}{ll} \frac{1}{2}\sigma^2 V^2 E_{VV} + (r-\delta)VE_V + \delta V - cF(1-\tau) = rE & \text{if } V_S \leqslant V, \\ \frac{1}{2}\sigma^2 V^2 E_{VV} + (r-\delta)VE_V + \delta V - cF(1-\tau) - \beta\delta(V_S - V) = rE & \text{if } V_S \geqslant V, \end{array} \tag{3}$$

where subscripts denote partial derivatives. In deriving this differential equation, no-arbitrage arguments apply that are based on the assumption that either the assets of the firm themselves are a traded asset or that traded assets exist which are perfectly correlated with the firm's assets. Otherwise, I would have to assume risk neutrality or a market price of the firm's risk has to be taken into account.

The general solution to this Euler differential equation (3) is

$$\begin{array}{ll} E(V) = -\dfrac{cF(1-\tau)}{r} + V + A_1 V^{\lambda_1} + A_2 V^{\lambda_2} & \text{if } V_S \leqslant V, \\ E(V) = -\dfrac{cF(1-\tau)}{r} - \dfrac{\beta\delta V_S}{r} + V(1+\beta) + B_1 V^{\lambda_1} + B_2 V^{\lambda_2} & \text{if } V_S \geqslant V, \end{array} \tag{4}$$

where A_1, A_2, B_1, and B_2 are yet unspecified real numbers and λ_1 and λ_2 are the solutions of the characteristic polynomial with $\lambda_1 > 0$ and $\lambda_2 < 0$.

When V becomes large, debt service and tax benefits approach the value of a certain cash-flow stream. Thus, the equity value must satisfy

$$\lim_{V\to\infty} E(V) = V - \frac{cF(1-\tau)}{r}. \tag{5}$$

The lower boundary condition reflects that when the owner–manager chooses to default, he has to pay off debtholders if possible, leaving the equity owner with

$$E(V_B) = \max(0, \alpha V_B - F). \tag{6}$$

From (5) I easily obtain $A_1 = 0$ in (4). The remaining three unknown parameters A_2, B_1 and B_2 are established from (6) and the requirement that $E(V)$ is continuously differentiable at V_S. Solving the resulting system of linear equations in A_2, B_1 and B_2 and rearranging terms leads to:

$$\begin{aligned} E(V, V_B) = {} & V - \frac{cF}{r}\left(1 - \left(\frac{V}{V_B}\right)^{\lambda_2}\right) - \min(F, \alpha V_B)\left(\frac{V}{V_B}\right)^{\lambda_2} \\ & + \frac{\tau cF}{r}\left(1 - \left(\frac{V}{V_B}\right)^{\lambda_2}\right) - (1-\alpha)V_B\left(\frac{V}{V_B}\right)^{\lambda_2} \\ & - \begin{cases} (C_2 V^{\lambda_2}) 1_{V_B \leqslant V_S} & \text{for} \quad V_S \leqslant V, \\ \dfrac{\beta\delta V_S}{r} - \beta V + D_1 V^{\lambda_1} + D_2 V^{\lambda_2} & \text{for} \quad V_S \geqslant V, \end{cases} \end{aligned} \tag{7}$$

with

$$D_1 = \frac{\frac{\beta\delta V_S}{r}\lambda_2 + \beta(1-\lambda_2)}{V_S^{\lambda_1 - 1}(\lambda_1 - \lambda_2)}, \tag{8}$$

$$D_2 = -\frac{\beta\delta V_S}{r} V_B^{-\lambda_2}\left[1+\frac{\lambda_2}{\lambda_1-\lambda_2}\left(\frac{V_B}{V_S}\right)^{\lambda_1}\right] + \beta V_B^{1-\lambda_2}\left[1-\frac{1-\lambda_2}{\lambda_1-\lambda_2}\left(\frac{V_B}{V_S}\right)^{\lambda_1-1}\right] \quad (9)$$

and

$$C_2 = D_2 + V_S^{1-\lambda_2}\frac{\left(1+\frac{\delta\lambda_1}{r}-\lambda_1\right)\beta}{\lambda_1-\lambda_2}. \quad (10)$$

An examination of the firm's equity value in (7) shows that it consists of five parts. The first is the asset value V, which can be interpreted in this setting as the value of the unleveraged firm. The second and third expression reducing equity value in Eq. (7) are the present value of the payments to bondholders in terms of coupon payments cF and the default payment $\min(F,\alpha V_B)$. Thus they represent the value $D(V)$ of the consol bond. The fourth expression in Eq. (7) is the present value $\mathrm{TB}(V)$ of the tax benefits τcF while the fifth expression gives the present value $\mathrm{BC}(V)$ of the bankruptcy costs $(1-\alpha)V_B$. Finally, the last expression in Eq. (7) is the present value $\mathrm{EIC}(V)$ of all future equity-issuance costs. Given this interpretation, Eq. (7) can be expressed as [7]

$$E(V) = V - D(V) + \mathrm{TB}(V) - \mathrm{BC}(V) - \mathrm{EIC}(V). \quad (11)$$

Since the total value of the leveraged firm $U(V)$ is the value of equity plus the value of debt, the value of the unleveraged firm reflects the firm's asset value plus the value of the tax benefits less the values of bankruptcy and equity-issuance costs, i.e.

$$U(V) = V + \mathrm{TB}(V) - \mathrm{BC}(V) - \mathrm{EIC}(V). \quad (12)$$

The basic structure of the closed-form solution (7) for the equity value is comparable to the one obtained in Leland (1994). [8] However, there are two major differences: First, an analogous expression to the minimum function in my formula does not appear in Leland's formula. When bankruptcy occurs in Leland's model, creditors always take over the firm with a value of αV_B while the owner–manager receives nothing. In contrast, I consider the face value F as an additional upper bound for the value that the creditor can receive to reflect current bankruptcy law. This minimum function makes the following analysis slightly more difficult. As a consequence, bankruptcy costs can have an effect on equity value since bankruptcy costs reduce the firm's value and this might not be fully offset by the decrease in the value of the consol bond. More important, bankruptcy is not inevitably involved with the firm being overindebted. The second difference to Leland's formula is due to the equity-issuance costs that reduce equity value when the default-triggering level V_B is below the cash-flow-shortage point V_S. In the following I analyze the consequences of both modeling differences in detail.

[7] To simplify the expressions, I skip the explicit dependence of V_B here.
[8] Cf. Leland (1994, Eq. (13), p. 1221).

3.2. The optimal default point

Let me now determine the optimal bankruptcy-triggering level $V_B^\star$. If $V_B^\star < V_S$, the value $V_B^\star$ that maximizes $E(V, V_B)$ for any level of V solves

$$(1-\lambda_2)(1+\beta) + \frac{cF(1-\tau)+\beta\delta V_S}{r}\lambda_2\frac{1}{V_B^\star} - \beta\left(\frac{\delta\lambda_2}{r}+1-\lambda_2\right)(V_B^\star)^{\lambda_1-1}V_S^{1-\lambda_1} = 0 \quad \text{if } F \geqslant \alpha V_B^\star \tag{13a}$$

or

$$(1-\lambda_2)(1+\beta-\alpha) + \left(\frac{cF(1-\tau)+\beta\delta V_S}{r} - F\right)\lambda_2\frac{1}{V_B^\star} - \beta\left(\frac{\delta\lambda_2}{r}+1-\lambda_2\right)(V_B^\star)^{\lambda_1-1}V_S^{1-\lambda_1} = 0 \quad \text{if } F < \alpha V_B^\star. \tag{13b}$$

(13a) and (13b) are the first-order conditions that result from differentiating (7) with respect to V_B. It can be easily shown that the second derivative of E with respect to V_B is negative at $V_B^\star$. Therefore, I do indeed have a local maximum at $V_B^\star$. If $V_B^\star$ solves (13a), the equity value is zero at $V = V_B^\star$ and the necessary optimality condition for this optimal stopping problem, [9] $\frac{\partial E(V)}{\partial V}\big|_{V=V_B^\star} = 0$, holds. In addition, $E(V)$ is increasing monotonously in V for $V \geqslant V_B^\star$. Hence, the equity value is positive for all $V > V_B^\star$ and approaches the value of zero as V goes to V_B with a smooth transition to the payoff function $E(V) = 0$ at the stopping region.

In contrast, the equity value and the first derivative with respect to V are positive at $V = V_B^\star$ if $V_B^\star$ solves (13b). In this case, I obtain $E(V_B^\star) = \alpha V_B^\star - F > 0$. Although the absolute priority rule is respected here, the value of the equity is not zero given default. The reason for this is that creditors can be fully compensated and a residual value remains for the shareholders if $\alpha V_B^\star - F > 0$. Since $\frac{\partial E(V)}{\partial V}\big|_{V=V_B^\star} = \alpha \geqslant 0$ and $E(V)$ is increasing monotonously in V for $V \geqslant V_B^\star$ my results are again consistent with the limited liability of equity and equity values approach the payoff function in a smooth way when V approaches $V_B^\star$, i.e. the smooth-pasting condition holds.

If $V_B^\star \geqslant V_S$, equity-issuance costs are always zero and I can derive the optimal bankruptcy-triggering level analytically:

$$V_B^\star = \begin{cases} \frac{cF(1-\tau)}{r}\frac{\lambda_2}{\lambda_2-1} & \text{if } F \geqslant \alpha V_B^\star, \\ \left(F - \frac{cF(1-\tau)}{r}\right)\frac{\lambda_2}{(\alpha-1)(\lambda_2-1)} & \text{if } F < \alpha V_B^\star. \end{cases} \tag{14a, b}$$

Again, it can be easily shown that the second derivative of E with respect to $V_B^\star$ is negative, here.

Having determined all possible local maxima, it now remains to find the value $V_B^\star$ where the function $E(V, V_B)$ reaches its global maximum. I turn to this issue in the

[9] See also Leland (1994, p. 1222) for this "smooth-pasting" condition.

next section, and also give economic interpretations for the possible bankruptcy-triggering events.

3.3. Economic interpretations for the endogenous bankruptcy-triggering events

The previous analysis results in four different economic interpretations for the endogenous bankruptcy-triggering events. Assume first that $\frac{1}{\alpha}F < V_{\mathrm{S}}$. For this situation, Fig. 1 displays three possible regions for the optimal bankruptcy-triggering level $V_{\mathrm{B}}^{\star}$. The value $V_{\mathrm{B}}^{\star}$ is either lower than $\frac{1}{\alpha}F$, between $\frac{1}{\alpha}F$ and V_{S}, or above V_{S}. The first two cases indicate a cashflow shortage, while a bankruptcy-triggering level above V_{S} means that the owner–manager goes bankrupt without running out of cash. Bankruptcy-triggering levels below $\frac{1}{\alpha}F$ indicate additionally that the remaining firm value is too low to fully pay off the bondholders and $E(V_{\mathrm{B}}^{\star}, V_{\mathrm{B}}^{\star}) = 0$, while the full payment of F is made if $V_{\mathrm{B}}^{\star}$ lies above $\frac{1}{\alpha}F$ and $E(V_{\mathrm{B}}^{\star}, V_{\mathrm{B}}^{\star}) > 0$. Therefore, a bankruptcy-level below $\frac{1}{\alpha}F$ means that the firm runs out of cash and is simultaneously overindebted.

A bankruptcy-triggering level between $\frac{1}{\alpha}F$ and V_{S} has the interpretation that a cash-flow shortage occurs without the firm being overindebted, and neither a cash-flow shortage nor overindebtedness are relevant if the bankruptcy-triggering level is above V_{S}.

To solve for the optimal bankruptcy-triggering level $V_{\mathrm{B}}^{\star}$, I solve (13a) and (13b) numerically. In addition, I compute the analytical solution (14b). Since $E(V, V_{\mathrm{B}})$ is continuously differentiable at $V_{\mathrm{B}} = V_{\mathrm{S}}$, and the solutions to (13b) and (14b) result

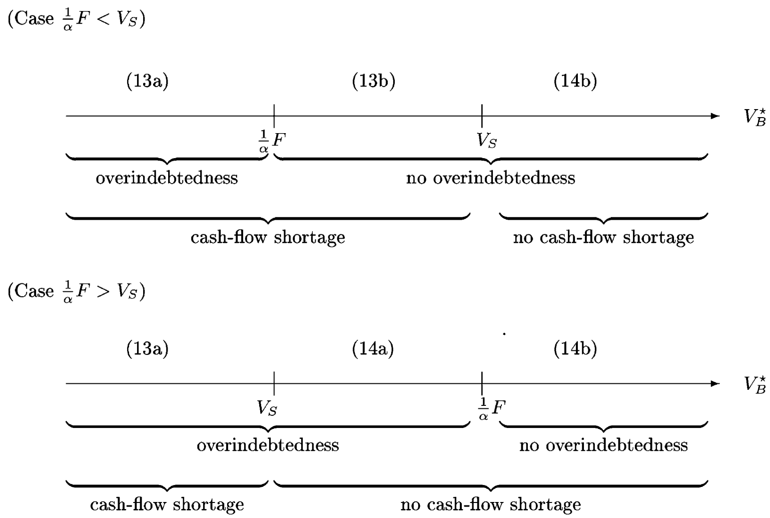

Fig. 1. Possible regions for the optimal bankruptcy-triggering level. This figure displays the possible regions for the optimal bankruptcy-triggering level $V_{\mathrm{B}}^{\star}$ together with the economic interpretation and the relevant first-order conditions in brackets. The case $\frac{1}{\alpha}F < V_{\mathrm{S}}$ is shown at the top of the figure, the case $\frac{1}{\alpha}F > V_{\mathrm{S}}$ at the bottom.

in local maxima if they are admissible, they cannot occur simultaneously. Therefore, I obtain at most two local maxima, one at the region above $\frac{1}{\alpha}F$ and one below $\frac{1}{\alpha}F$, the higher of which is chosen. If I obtain less than two local maxima, the boundary value at $V_{\mathrm{B}} = \frac{1}{\alpha}F$ also has to be considered.

A similar analysis holds for the case $\frac{1}{\alpha}F > V_{\mathrm{S}}$. The possible regions for the optimal bankruptcy-triggering level together with the relevant first-order conditions are also displayed in Fig. 1. Again, (13a) indicates that a cash-flow shortage and overindebtedness occur simultaneously. Eq. (14a) means that the firm is overindebted but has not yet run out of cash, and (14b) indicates as before that neither cash-flow shortage nor overindebtedness are the reasons to go bankrupt.

In summary, the four different first-order conditions (13a), (13b), (14a) and (14b) lead to four different bankruptcy reasons:

Condition (13a): The firm is overindebted and runs out of cash.
Condition (13b): Although the firm is not overindebted it still runs out of cash.
Condition (14a): The firm is overindebted but does not run out of cash.
Condition (14b): Neither is the firm overindebted nor does it run out of cash.

So far, the model allows the owner–manager to go bankrupt whenever it is optimal. Condition (14b) shows that there are situations where the owner–manager decides to default although the firm neither faces a cash-flow shortage nor it is overindebted. In these cases, wealth is transferred from creditors to debtors and some fraction of the asset value is thrown away. So the only reason for the owner–manager to go bankrupt is that he benefits from the early redemption of his consol bond while creditors are worse off. [10] However, if insolvency requirements are not fulfilled, formal bankruptcy procedures cannot be used in many countries. [11] On the other hand, it is very unlikely that renegotiation outside a formal bankruptcy process would lead creditors to accept an early redemption of the consol bond at the face value F in cases where the market value is clearly above F. Therefore a default of a firm that is not overindebted nor short in cash seems implausible.

To prevent such an early default in the following, I do not allow the owner–manager to declare bankruptcy before he runs out of cash or the firm is overindebted although he might be willing to do so. This is simply achieved by imposing the additional boundary conditions that the bankruptcy-triggering level $V_{\mathrm{B}}^{\star}$ never exceeds the maximum level of the cash-flow-shortage point V_{S} and the critical asset value level $V_O = \frac{1}{\alpha}F$ that indicates overindebtedness. Thus, condition (14b) leads no longer to

[10] Ultimately, this decline in value is entirely imposed on the owner–manager since creditors will anticipate the behavior of the owner–manager and therefore require an adequate compensation in form of a higher promised return when they buy the consol bond.

[11] An exception is the US code, which allows firms to file for bankruptcy even if they are not insolvent. In contrast, there are status requirements in Canada, France, Germany, Great Britain, Italy, and Japan. The German law, for example, states that compulsory liquidation presupposes certain preconditions: either the firm cannot repay its creditors, or it is overindebted, or a cash-flow shortage is impending. While in general both debtor and creditor can initiate a formal bankruptcy process in case of the first two triggering events, it is only the debtor who can declare impending inability to repay creditors.

a feasible solution. As a result, I still derive the firm's bankruptcy point endogenously, but the owner–manager's possible operations are now restricted. In what follows, I throughout use this restricted version of my model.

3.4. Numerical analysis of the optimal bankruptcy-triggering level

The results of the previous section show that the optimal bankruptcy-triggering level $V_B^\star$ is, as in Leland's (1994) model, independent of the firm's asset value V. If it is optimal to go bankrupt before a cash-flow shortage occurs ($V_B^\star > V_S$), we learn from (14a,b) that the optimal bankruptcy-triggering level is independent of the transaction-cost rate β. The special case of (14a) corresponds to the situation already discussed in Leland (1994), with an asset value $V_B^\star$ that is increasing in the coupon rate c, decreasing in the risk-free rate r, the tax rate τ, the payout rate δ, and the riskiness of the firm σ, and is independent of the bankruptcy-cost factor α. [12] The results differ, however, if I consider the remaining cases (13a) and (13b).

In order to analyze the bankruptcy point in more detail for these cases, I perform a comparative static analysis. As a base case I consider a firm that generates an ongoing cash flow δV with $\delta = 0.05$ while having a debt-service obligation of cF with coupon level c of 9% and a face value F of \$100. In interpreting these figures one should keep in mind that δV is the dollar payout to all security holders leading to δ-values that are typically above the dividend-yield. The value of 5% is a typical free-cash-flow rate for a German DAX 30 firm estimated from consolidated balance sheets and cash flow statements. Furthermore, I assume that the risk-free rate is 5%, the volatility of the firm's assets is 15%, and the fraction $1 - \alpha$ of the firm's value lost to bankruptcy cost is 30%. [13] For the moment, I assume a corporate tax rate of 0% which simplifies the following discussion. This is feasible since I do not analyze the optimal capital structure here but simply assume some exogenous debt level without requiring it to be optimal. When analyzing the optimal capital structure choice in the next section, I follow Leland (1998) and choose a corporate tax rate of $\tau = 20\%$.

Fig. 2 shows the default point $V_B^\star$ as a function of the transaction-cost rate β for different interest-rate levels r. For the given parameter values, the firm faces a cash-flow shortage as soon as the asset value V falls below $V_S = \frac{c(1-\tau)}{\delta}F = \$180$. Since $\frac{1}{\alpha}F = \$142.86 < V_S = \$180$, the situation considered corresponds to the one displayed at the top of Fig. 1. Four results are worth noting:

Firstly, the optimal bankruptcy-triggering level $V_B^\star$ tends to increase with the transaction-cost rate β. This result is reasonable since the higher the equity-issuance costs, the less attractive it is for the owner–manager to keep the firm alive by injecting new cash. For interest-rate levels $r \geqslant \delta$ $V_B^\star$ approaches the cash-flow-shortage point $V_S = \$180$ from below as equity-issuance costs get very large.

[12] See Leland (1994, p. 1222). The only difference to Leland's formula is that I consider a positive payout rate δ, while in his model $\delta = 0$ holds.

[13] The base case parameter values for σ and α have been chosen within the range of values used in related studies and estimates found in various empirical investigations. See Leland (1994), Fan and Sundaresan (2000), Anderson and Sundaresan (2000), Andrade and Kaplan (1998), and Moody's (2003).

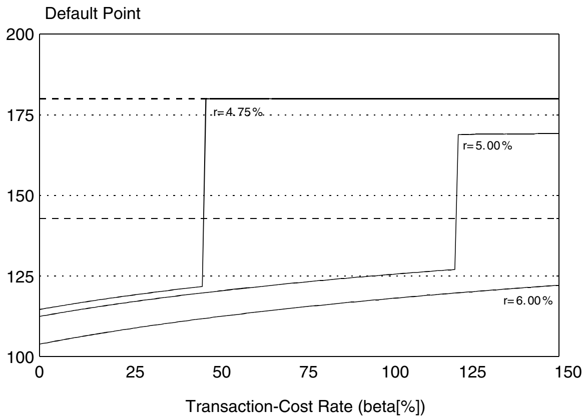

Fig. 2. Default point as a function of transaction costs. This figure shows the endogenous default point $V_B^\star$ as a function of the transaction-cost rate β for different interest-rate levels r. β is the percentage of the proceeds received from the equity issue that has to be paid as transaction costs when the firm faces a cash-flow shortage. The dashed line at \$180 indicates the cash-flow-shortage point. Default levels below the dashed line at \$142.86 indicate overindebtedness. It is assumed that bankruptcy costs are 30% of the asset value $V_B^\star$ at default ($\alpha = 0.7$), the face value $F = \$100$, the coupon rate $c = 9\%$, the asset volatility $\sigma = 15\%$, the payout rate $\delta = 5\%$, and the corporate tax rate $\tau = 0$.

The shape of the function $V_B^\star(\beta)$ leads me to the second interesting result: the discontinuities of the curves. For the interest-rate level $r = 5\%$, for example, there is a discontinuity at $\beta = 120\%$. At this point $V_B^\star$ jumps from a value below $\frac{1}{\alpha}F$ to a value above $\frac{1}{\alpha}F$. Mathematically, there is a change in the relevant first-order condition from (13a) to (13b). Economically, this means a change from a situation in which the owner–manager receives nothing in case of a default (overindebtedness) to the case where the value of the firm after default exceeds the face amount F such that the creditors get back their money, and the owner–manager receives $\alpha V_B^\star - F$.

Thirdly, whenever $V_B^\star$ reaches the level V_S, $V_B^\star$ is not affected by a further increase in transaction costs β. The reason for this is that higher transaction costs typically lead to an earlier default to avoid the payment of these costs. However, if the default level $V_B^\star$ is already at the cash-flow-shortage point, equity-issuance costs do not arise in any case.

Finally, Fig. 2 shows that the bankruptcy-triggering level increases with decreasing risk-free rates. Since the value of the consol bond and the value of the equity-issuance costs are ceteris paribus more sensitive towards changes in the bankruptcy-triggering level if risk-free rates are low, and these values are decreasing with $V_B^\star$ in the given situation, then decreasing interest rates make it more attractive for the owner–manager to default earlier.

Further analysis shows that the bankruptcy-triggering level $V_B^\star$ is independent of the bankruptcy-cost factor α as long as $V_B^\star < \frac{1}{\alpha}F$. As in Leland's (1994) model the equity value is insensitive towards changes in α although higher bankruptcy costs directly reduce the value of the leveraged firm. However, this is precisely offset by a

Table 1
Comparative statics of default level

	Overindebtedness		No overindebtedness
	Cash-flow shortage	No cash-flow shortage	Cash-flow shortage
Equity-issuance cost rate β	$\Uparrow$	0	$\Uparrow$
Bankruptcy costs $1-\alpha$	0	0	$\Downarrow$
Cash flow rate δ	$\Downarrow$	$\Downarrow$	$\Downarrow$
Coupon rate c	$\Uparrow$	$\Uparrow$	$\Uparrow$
Volatility σ	$\Downarrow$	$\Downarrow$	$\Downarrow$
Interest rate r	$\Downarrow$	$\Downarrow$	$\Downarrow$
Tax rate τ	$\Downarrow$	$\Downarrow$	$\Downarrow$

This table describes the comparative statics of the endogenous default level V_B for varying parameter values, $\Uparrow$ ($\Downarrow$) indicates that the optimal default level rises (falls) in the respective parameter. 0 means that there is no effect.

lower debt value. The situation changes if $V_B^\star \geqslant \frac{1}{\alpha}F$. Again, higher bankruptcy costs reduce the value of the leveraged firm, but now, for a fixed bankruptcy level V_B the debt value is unaffected by the bankruptcy costs since the creditors get back the face amount F in any case. Therefore, higher bankruptcy costs make it less attractive for the owner–manager to default, resulting in lower optimal bankruptcy-triggering levels $V_B^\star$. Not surprisingly, a higher payout rate δ leads ceteris paribus to a lower bankruptcy-triggering level and a larger coupon size c increases the optimal bankruptcy-triggering level. Finally, the optimal bankruptcy level $V_B^\star$ decreases as the riskiness of the firm increases. Table 1 summarizes these sensitivities of the optimal default level. An extensive comparative static analysis shows that the sensitivities are robust with respect to the underlying parameters.

In summary, the comparative static results with respect to changes in the coupon size, the firm's riskiness and the risk-free interest rate are in line with the results found in Leland (1994). Differences occur with respect to the bankruptcy cost factor when the firm is not overindebted. Also in contrast to Leland's model, the cash-flow-shortage point is important now. A cash-flow shortage is not a compelling reason to go bankrupt, but since it is costly to overcome a cash-flow shortage, equity-issuance costs shift the optimal bankruptcy level towards the cash-flow-shortage point.

4. Applications

In the following applications I analyze credit spreads implied by my model and I study the impact on optimal capital structure.

4.1. Liquidity constraint and its impact on spreads

In the current corporate bond pricing models, liquidity problems lead either to an immediate default as e.g. in Kim et al. (1993) or they are irrelevant since equity holders are always able to overcome these problems, as in the models of Leland

(1994, 1998) and Leland and Toft (1996). In the first branch of models, bankruptcy is probably triggered too early. However, the second class of models also seems unrealistic since it is well-known that liquidity problems often cause firms to go bankrupt. My model incorporates these two cases. The level of the equity-issuance costs determines the position in between these extremes. Equity-issuance costs of zero correspond to Leland's approach while the model approaches a pure cash-flow-triggered default as in Kim et al. (1993) when equity-issuance costs become large.

The default-triggering mechanism obviously influences the prices of risky debt and therefore also the spreads. Fig. 3 shows the value of the consol bond as a function of the equity-issuance-cost rate β. I consider the base case outlined in Section 3.4 and assume an asset value V of \$200. This asset level ensures that the firm is still alive $(V > V_B^\star)$ for all β-values considered.

Obviously, the value of the consol bond declines when equity-issuance costs rise. If cash-flow shortage is irrelevant, as in the model of Leland (1994), we obtain the highest bond value while the bond price is the lowest for the pure cash-flow-triggered default (indicated by the lower dashed line). The jump in value at $\beta = 120\%$ results from the jump in the bankruptcy-triggering level $V_B^\star$ already discussed in Section 3.4. At this point $V_B^\star$ jumps from a value below $\frac{1}{\alpha}F$ to a value above $\frac{1}{\alpha}F$. For β below 120%, two effects influence the bond value, a timing and a cash-flow effect. Firstly, bankruptcy is triggered earlier when β rises (timing effect). At the same time, bondholders obtain a higher value if bankruptcy occurs (cash-flow effect). Obviously, the timing effect dominates. When $V_B^\star$ jumps above $\frac{1}{\alpha}F$, i.e. the firm is not overindebted,

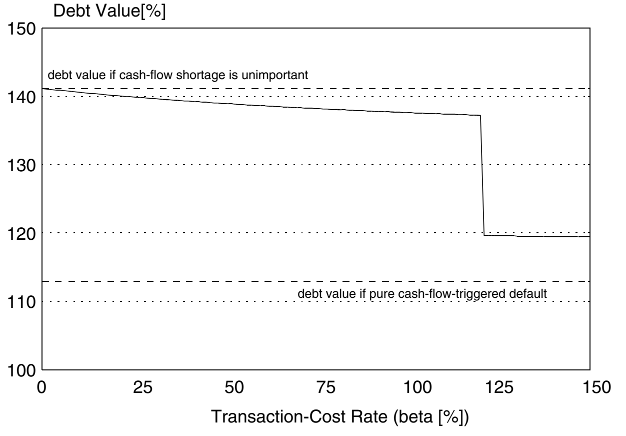

Fig. 3. Debt value as a function of transaction costs. This figure shows the debt value as a function of the transaction-cost rate β. β is the percentage of the proceeds received from the equity issue that has to be paid as transaction costs when the firm faces a cash-flow shortage. The dashed lines indicate the two extreme cases of no transaction costs and infinitely high transaction costs (pure cash-flow triggered default). It is assumed that the bankruptcy costs are 30% of the asset value $V_B^\star$ at default ($\alpha = 0.7$), the asset value $V = \$200$, the face value $F = \$100$, the coupon rate $c = 9\%$, the asset volatility $\sigma = 15\%$, the payout rate $\delta = 5\%$, the risk-free interest-rate $r = 5\%$, and the corporate tax rate $\tau = 0$.

Table 2
Comparative statics of credit spreads

	Overindebtedness		No overindebtedness
	Cash-flow shortage	No cash-flow shortage	Cash-flow shortage
Equity-issuance cost rate β	$\Uparrow$	0	$\Uparrow$
Bankruptcy costs $1-\alpha$	$\Uparrow$	$\Uparrow$	$\Downarrow$
Cash flow rate δ	$\Uparrow$	$\Uparrow$	$\Uparrow$
Coupon rate c	$\Uparrow$	$\Uparrow$	$\Uparrow$
Volatility σ	$\Uparrow$ or $\Downarrow$[a]	$\Uparrow$ or $\Downarrow$[a]	$\Uparrow$ or $\Downarrow$[a]
Interest rate r	$\Downarrow$	$\Downarrow$	$\Downarrow$
Tax rate τ	$\Downarrow$	$\Downarrow$	$\Downarrow$

This table describes the comparative statics of the yield spread of the debt over the risk-free interest rate for varying parameter values, $\Uparrow$ ($\Downarrow$) indicates that the spread rises (falls) in the respective parameter. 0 means that there is no effect.

[a] For very risky firms the spreads are falling in firm risk.

the bondholders always receive the face amount of \$100, and the only question is the timing of this payment. Thus, bond prices decrease in β for β above 120. The earliest default possible in this setting is at a level $V = V_S = \$180$, i.e., the pure cash-flow-triggered default. The corresponding yield spreads over the risk-free rate rise from 137 basis points when cash-flow shortage is unimportant to 300 basis points in case of a pure cash-flow-triggered default.

Table 2 summarizes the comparative statics of credit spreads. Again, the impact of bankruptcy costs depends on the bankruptcy reason. For overindebted firms higher bankruptcy costs increase credit spreads. However, the opposite result can be obtained for firms that are not overindebted. Through lowering the endogenous default level, increasing bankruptcy costs can lead to decreasing credit spreads. Further analysis shows that credit spreads are increasing in the cash flow and the coupon rate. As in Leland's model an increase in the tax rate lowers credit spreads through lowering the bankruptcy level. The same is true for the risk-free rate. More surprising are the comparative statics with respect to the volatility σ. Two opposite effects are important. On the one side, higher firm risk is negative for bond holders since they suffer from low firm values but do not profit from high ones. This asymmetry leads to higher credit spreads. On the other side, higher firm risk lowers the default level and thus decreases credit spreads. For very risky firms, the second effect dominates.

Within classical structural models theoretical credit spreads are found to be too low compared to the empirically observed ones. [14] Since liquidity constraints ceteris paribus increase credit spreads the proposed modeling is one step towards more realistic spreads within structural models. In a recent empirical study of structural models based on individual bond prices reported in Bridge Information Systems' corporate bond data base Lyden and Saraniti (2000) document the extent to which

[14] Standard reference is the empirical work of Jones et al. (1984). See also the more recent studies of Anderson and Sundaresan (2000) and Lyden and Saraniti (2000).

classical models underestimate credit spreads. Assuming a debt structure that gives priority to short-term liabilities, they find a bias of 29 basis points, on average, for the Merton model. Without a doubt, part of this bias is due to a liquidity premium. However, for the remaining part measurement and modeling errors are responsible. Since the impact of bond liquidity on bond prices is not reflected by this model, only the latter part of the bias is of interest here. Fig. 4 gives an idea of the potential improvement that could be achieved through the proposed modeling. For the base case parameters and a positive tax rate of 20% the figure shows the increase in the yield spread when the liquidity constraint changes from $\beta = 0$ (unconstrained model) to positive β-values for varying volatilities. For a volatility of 28%, which is just the average value within the sample of Lyden and Saraniti (2000), a β-value of 10% can explain already 15% and a β-value of 50% almost 60% of the overall bias. To increase spreads by all the 29 basis points a β-value of 110% would be necessary. Thus, liquidity constraints have the potential to improve the fit of classical models substantially.

4.2. Optimal capital structure

Different bankruptcy reasons presumably affect the optimal capital structure and vice versa. To analyze the optimal capital structure choice, I determine the coupon level c and the face value F that maximize the total value of the leveraged firm, i.e.:

$$\max_{c,F}(E + D) = \max_{c,F}(V + \mathrm{TB}(V) - \mathrm{BC}(V) - \mathrm{EIC}(V)). \tag{15}$$

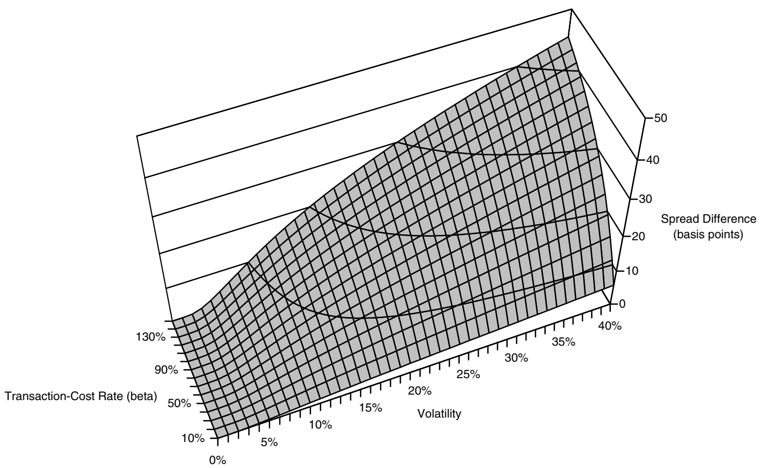

Fig. 4. Potential improvement by liquidity constraints. This figure shows the increase in the yield spread in basis points when the liquidity constraint changes from $\beta = 0$ (unconstrained model) to positive β-values as a function of the transaction-cost rate β and the firm risk σ. β is the percentage of the proceeds received from the equity issue that has to be paid as transaction costs when the firm faces a cash-flow shortage. It is assumed that the bankruptcy costs are 30% of the asset value $V_B^\star$ at default ($\alpha = 0.7$), the asset value $V = \$200$, the face value $F = \$100$, the coupon rate $c = 9\%$, the payout rate $\delta = 5\%$, the risk-free interest-rate $r = 5$, and the corporate tax rate $\tau = 20\%$.

Rather than providing a new capital structure theory the aim is to study the consequences of different bankruptcy reasons on optimal capital structure and the feedback on the actual bankruptcy reason.

Similar to Leland (1994), the optimal capital structure is determined by trading off tax benefits of debt against bankruptcy costs. But, in addition, equity-issuance costs have to be considered as a cost of debt. Furthermore, in contrast to Leland, the above optimization problem is two-dimensional. True, as long as the firm is overindebted when defaulting (i.e. $V_{\mathrm{B}}^{\star} \leqslant \frac{F}{\alpha}$) only the \$-coupon $c \times F$ is relevant. In this case, all payments to claimholders (including the default payment) depend either on $c \times F$ or are independent of both of the variables. As a consequence, the endogenous triggering level $V_{\mathrm{B}}^{\star}$ is also only a function of $c \times F$. However, the situation is different when the firm is not overindebted and yet is in default, since the payments to creditors upon default depend solely on the face value F. Thus, the optimization problem is in fact two-dimensional in this region. Therefore, I first analyze whether firms with optimally structured debt default without being overindebted.

The forces at work are best understood when we fix a face value F and change the coupon level c marginally such that the default-level $V_{\mathrm{B}}^{\star}$ jumps from below $\frac{F}{\alpha}$ to a level above this point. Due to the earlier default, tax benefit decreases and bankruptcy costs increase. Both effects have a negative impact on the value of the leveraged firm. Only lower equity-issuance costs could theoretically offset this. However, extensive numerical analysis show that for all realistic parameter values the first two effects dominate. This implies that firms optimally structure their debt such that cash-flow shortage does not become a bankruptcy reason without overindebtedness.

Thus, I can characterize the capital structure by the \$-coupon $c \times F$. Though cash-flow shortage is not an independent bankruptcy reason in capital structure optimum, liquidity constraints do have an impact on optimal capital structure. Firstly, given the optimal coupon level $c \times F$, the choice of the two variables c and F leading to this \$-coupon is restricted. In fact, there exists a lower bound for the face value F. [15] If firms chose a face value below this critical level, a cash-flow shortage would be the bankruptcy reason. In this sense, the choice of the face value F and the coupon c for a given \$-coupon $c \times F$ can be understood as a commitment not to default for liquidity reasons only. Secondly, liquidity constraints have a numerical impact on the optimal \$-coupon. Fig. 5 presents the forces driving capital structure choice as a function of the \$-coupon for various levels of equity-issuance costs. The forces increasing the firm value have a positive sign while those decreasing the firm value are presented with a negative sign. In addition, the net effect is shown.

Similar to previous models, tax benefit increases first with the \$-coupon due to the higher tax benefit per unit time, but then decreases since the higher default probability reduces the expected time the tax benefit can be enjoyed. [16] The higher default probability also implies a higher value of the bankruptcy costs BC. Concerning

[15] Equivalently, there exists an upper bound for the coupon level c.

[16] Extensive discussion of this issue can be found in Brennan and Schwartz (1978).

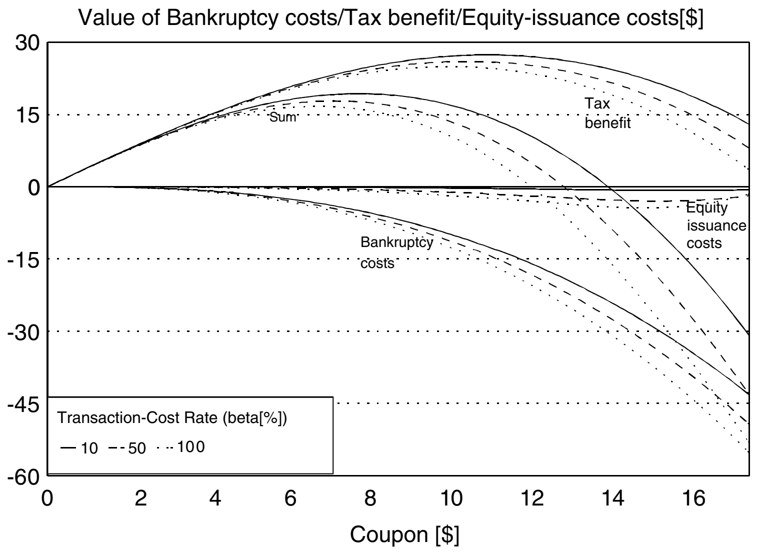

Fig. 5. Forces driving capital structure choice. This figure shows the values of the tax benefit, the bankruptcy costs, and the equity issuance costs as a function of the \$-coupon for various levels of equity-issuance cost rates β. β is the percentage of the proceeds received from the equity issue that has to be paid as transaction costs when the firm faces a cash-flow shortage. The forces increasing the firm value have a positive sign while those decreasing the firm value are presented with a negative sign. In addition, the net effect is shown. It is assumed that the bankruptcy costs are 30% of the asset value $V_B^{\star}$ at default ($\alpha = 0.7$), the asset value $V = \$200$, the asset volatility $\sigma = 15\%$, the payout rate $\delta = 5\%$, the risk-free interest-rate $r = 5\%$, and the corporate tax rate $\tau = 20\%$.

the equity-issuance costs, I have again two effects. First, a higher \$-coupon increases the cash-flow-shortage point V_S implying that equity-issuance costs are incurred earlier. But as with the tax benefit, the higher default probability counteracts this first effect. Summing up these three effects results in a positive net-effect of debt on the firm value that increases first with a higher coupon level, then decreases, and for very high coupon levels gets even negative.

Now it is interesting to analyze the influence of the equity-issuance cost rate β. First, there is a direct effect on the value of the equity-issuance costs itself. A higher value of β increases the equity-issuance costs per unit time. This results in a higher value of EIC for low and medium coupon levels. For high coupon levels this effect is offset by the then increased default probability resulting from the higher endogenous bankruptcy level $V_B^{\star}$. This higher endogenous default-triggering level is also the reason why β reduces the value of the tax benefit while increasing the value of bankruptcy costs. Thus, higher liquidity constraints reinforce the disadvantages of debt and reduce the benefits. As a result, the optimal coupon level $c \times F$ and leverage $\frac{D}{E+D}$ decrease with β. The numerical effects of equity-issuance costs on leverage for varying bankruptcy costs and firm risk are shown in Figs. 6 and 7. Highest leverage is obtained in the extreme case of Leland's (1994) model when equity-issuance costs

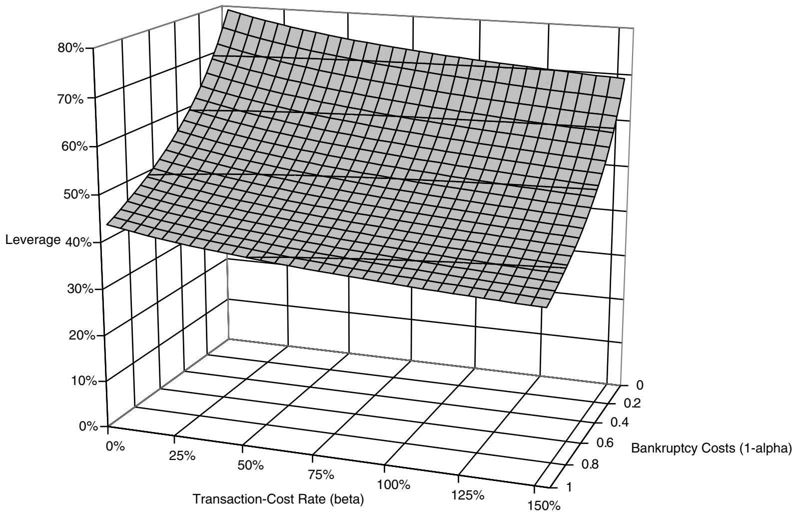

Fig. 6. Leverage as a function of equity-issuance costs and bankruptcy costs. This figure shows the optimal leverage $\frac{D}{E+D}$ as a function of the equity-issuance cost rate β and the bankruptcy costs. Bankruptcy costs are given as a fraction the asset value $V_B^\star$ at default $(1-\alpha)$. β is the percentage of the proceeds received from the equity issue that has to be paid as transaction costs when the firm faces a cash-flow shortage. It is assumed that the asset value is $V = \$200$, the asset volatility $\sigma = 15\%$, the payout rate $\delta = 5\%$, the risk-free interest-rate $r = 5\%$, and the corporate tax rate $\tau = 20\%$.

are zero. For the base case parameters and $\beta = 0\%$ a leverage ratio of 62% results. Increasing the liquidity constraints reduces the leverage ratio to 52% for $\beta = 150\%$. Not surprisingly, leverage decreases in bankruptcy costs. But as Fig. 6 shows, the impact of equity-issuance costs on leverage is not very sensitive to the chosen level of bankruptcy costs. Increasing firm risk also reduces leverage, as illustrated in Fig. 7. Interestingly, the impact of liquidity constraints on leverage becomes more pronounced for more risky firms. Take for example a firm with a volatility level of 40%. A leverage ratio of 48% is optimal when equity-issuance costs are zero. Small equity-issuance costs of 10% lead to a 9% decrease in leverage and moderate equity-issuance costs of $\beta = 50\%$ produce a 30% decrease. The leverage ratio reduces to a value of only 23% for very strong liquidity constraints represented by $\beta = 150\%$. Overall, the results show that compared to a model of an otherwise identical firm that can costlessly issue new equity, my model predicts lower optimal leverage ratios.

Table 3 summarizes the comparative statics of the leverage ratio, the \$-coupon $c \times F$ and the yield spread of the optimally structured debt over the risk-free interest rate. Not surprisingly, equity-issuance costs and bankruptcy costs both reduce the optimal coupon payments leading to lower leverage ratios and spreads while the tax rate has the opposite effects. When comparing these comparative statics of the spreads with those presented in Table 2, one should keep in mind that the promised coupon payments $c \times F$ now also vary while they are fixed in Table 2. As in

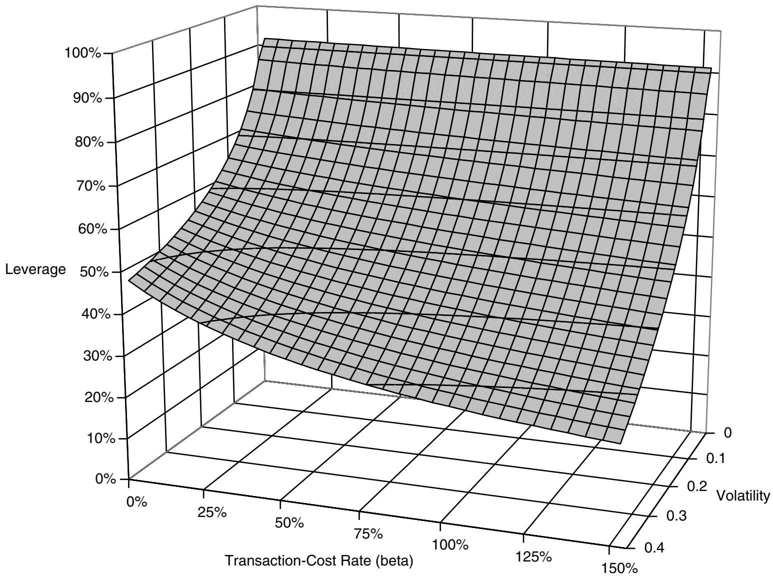

Fig. 7. Leverage as a function of equity-issuance costs and firm risk. This figure shows the optimal leverage $\frac{D}{E+D}$ as a function of the equity-issuance cost rate β and firm risk σ. β is the percentage of the proceeds received from the equity issue that has to be paid as transaction costs when the firm faces a cash-flow shortage. It is assumed that bankruptcy costs are 30% of the asset value $V_B^\star$ at default ($\alpha = 0.7$), the asset value $V = \$200$, the payout rate $\delta = 5\%$, the risk-free interest-rate $r = 5\%$, and the corporate tax rate $\tau = 20\%$.

Table 3
Comparative statics of leverage, coupon, and spreads in capital structure optimum

	Leverage	\$-coupon	Spreads
Equity-issuance cost rate β	$\Downarrow$	$\Downarrow$	$\Downarrow$
Bankruptcy costs $1-\alpha$	$\Downarrow$	$\Downarrow$	$\Downarrow$
Cash flow rate δ	$\Downarrow$	$\Uparrow$[a]	$\Uparrow$
Volatility σ	$\Downarrow$	U-shaped	$\Uparrow$
Interest rate r	$\Uparrow$[b]	$\Uparrow$	$\Downarrow$
Tax rate τ	$\Uparrow$	$\Uparrow$	$\Uparrow$

This table describes the comparative statics of the leverage $\frac{D}{E+D}$, the \$-coupon $c \times F$ and the yield spread of the optimally structured debt over the risk-free interest rate for varying parameter values. $\Uparrow$ ($\Downarrow$) indicates that the value rises (falls) in the respective parameter.

[a] For low cash-flow rates the opposite direction is also possible.

[b] When interest rates and equity-issuance costs are both very high the opposite direction is also possible.

Leland's model, the optimal \$-coupon is a U-shaped function of firm riskiness, while leverage decreases and spreads increase in σ. Further analysis shows that also the cash flow rate decreases leverage and increases spreads. Finally, Fig. 8 presents

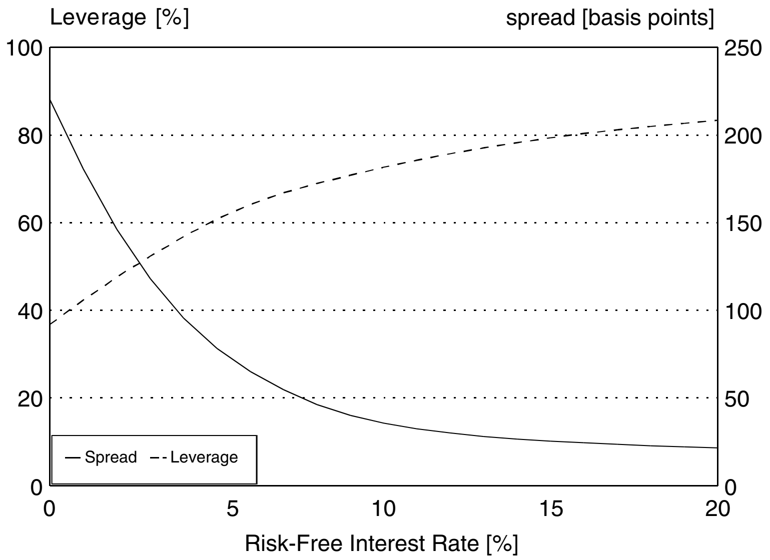

Fig. 8. Leverage as a function of risk-free interest rates. This figure shows the optimal leverage $\frac{D}{E+D}$ as a function of the risk-free interest rate r. It is assumed that equity-issuance cost parameter is $\beta = 0.1$, where β is the percentage of the proceeds received from the equity issue that has to be paid as transaction costs when the firm faces a cash-flow shortage. The bankruptcy costs are 30% of the asset value $V_B^{\star}$ at default ($\alpha = 0.7$), the asset value $V = \$200$, the asset volatility $\sigma = 15\%$, the payout rate $\delta = 5\%$, and the corporate tax rate $\tau = 20\%$.

leverage and spreads as a function of the risk-free interest rate. Although higher risk-free interest-rates raise the promised coupon payments the spreads are decreasing. This relationship is consistent with the findings of extant empirical studies such as Duffee (1998) for the US market or Düllmann et al. (2000) for the German market. Furthermore, from the key innovation of the paper at least two implications can be drawn with respect to the optimal capital structure choice that are consistent with corporate practice. First, observed leverage ratios vary significantly depending on the measure of leverage used as is shown e.g. by Rajan and Zingales (1995). Nevertheless, most classical structural models without liquidity constraints predict optimal leverage ratios that are even above the highest of these empirical values which typically results if the ratio of total liabilities to total assets is used as the definition of leverage. In contrast, my model predicts optimal debt levels that are much more in line with those observed in practice. Second, if we think of equity-issuance costs caused by problems of asymmetric information my results imply that young and/or unknown firms with a higher potential for such problems should ex ante use less debt. In fact, Wohlschieß (1996) empirically studied capital structure choice for German firms and found that firms that are publicly traded for only a short while have significantly less debt than firms with a long history in capital markets.

5. Conclusions

Liquidity problems often lead to bankruptcy. I incorporate this important fact into a theoretical model of a leveraged firm by considering equity-issuance costs that reflect the difficulties in issuing new equity for firms close to financial distress. In this setup, the bankruptcy point is derived endogenously as the optimal decision of an equity-value maximizing owner–manager. A variation of the level of equity-issuance costs makes it possible to capture the importance of a cash-flow shortage in the theoretical model.

The model is useful to study the implications of different degrees of liquidity constraints on the values of corporate securities and on optimal capital structure. (i) Focusing on the model as a simple tool for corporate bond valuation, I show that the degree of liquidity constraint influences yield spreads substantially. Stronger liquidity constraints lead ceteris paribus to higher spreads. (ii) From the analysis of optimal capital structure, I can conclude that cash-flow shortage is not an independent bankruptcy reason. The modeling of cash-flow shortage is important nonetheless. First, not all the combinations of percentage coupon and face value that lead to the optimal $-coupon do also lead to the capital structure optimum. Instead, I find that firms commit themselves not to default solely for liquidity reasons by choosing a sufficiently high face value of debt. Second, higher liquidity constraints lead to lower debt levels. If one interprets equity-issue constraints as being the result of information asymmetry, an implication is that firms with a higher potential for these problems should issue less debt.

The model seeks to reflect real-world default-triggering events more realistically than existing corporate bond-valuation models. However, to keep the expressions for corporate securities simple, several restrictive assumptions had to be made. The model could be extended along the lines of Leland and Toft (1996), who analyze the maturity of debt. More difficult extensions will include stochastic default-free interest rates possibly correlated with the asset value and dynamic restructuring as in Fischer et al. (1989) or Goldstein et al. (2001). Finally, the treatment of retained earnings as an additional state variable seems to be a challenging and promising direction for future research.

Acknowledgement

I thank Martin Bardenhewer, Wolfgang Breuer, Wolfgang Bühler, Darell Duffie, Alexander Kempf, Olaf Korn, William Perraudin, Rainer Schöbel, Miikka Taurén, Marc Windfuhr, Josef Zechner and the participants of the annual meeting of the *Deutsche Gesellschaft für Finanzwirtschaft* 1999, the Economic Workshop at Tübingen, and the Finance Seminar at Birkbeck, London, for many helpful comments and suggestions on an earlier version of this paper. Valuable suggestions from two anonymous referees significantly improved the paper.

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