

Efficient v.s. equilibrium unemployment with match-specific costs

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Abstract

This paper extends the standard matching model by adding match-specific costs, which can only be partially protected from hold-up because of workers' bargaining power. We show that a decrease in equilibrium unemployment might improve welfare for realistic values of workers' bargaining power.

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1. Introduction

In the standard matching theory where wages are determined by a Nash-bargaining solution, the equilibrium unemployment is efficient only for a specific value of workers' bargaining power (this is referred to as the Hosios condition; see [Hosios \(1990\)](#)).¹ Empirical estimates of the matching function show that the efficient unemployment rate is lower than the equilibrium one if workers' bargaining power is greater than 50%.² But, because most estimates of workers' bargaining power indicate a lower

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¹ To be more precise, the Hosios condition states that the worker's bargaining power is equal to the elasticity of the matching function with respect to unemployment.

² This is a lower bound (see [Blanchard and Diamond \(1989\)](#) and [Broersma and Van Ours \(1999\)](#)).

value than 50%³, one can question the positive effects on welfare of a decrease in equilibrium unemployment.

This note aims at re-examining this conclusion by using a matching model with match-specific costs (such as screening and training costs), which can only be partially protected from hold-up because of workers' bargaining power (see Malcomson (1997) for a survey on hold-up theory). We show that a decrease in unemployment can be welfare improving even when workers' bargaining power is lower than 50%.

The intuition is as follows. The Hosios condition arises because the welfare gain from an additional vacancy equals the gain from filling an additional vacancy multiplied by the marginal increase in the number of matches. However, the gain in terms of profit from creating a vacancy consists of the added firm's profit multiplied by the probability that this vacancy will be filled, which is the average number of vacancies filled. Efficiency requires those two to be equal. Because hold-up reduces the firm's profit gain from creating a vacancy, a lower wage (hence a lower worker's bargaining power) will yield efficiency.

2. Theoretical investigation

2.1. The economic environment

There is a continua of identical risk-neutral workers and firms. Time is continuous. All agents live forever and discount the future at the common rate r . The total mass of workers is normalized to one. Each firm consists of one job. The recruiting cost ω is associated with the opening of a vacancy. In addition, we assume that whenever a vacancy is filled, the firm must incur a per unit fixed cost, Q , in order to begin production.⁴

Matching is frictional and we denote by $x(v, u)$ the flow of new worker–firm matches, where u is the total mass of unemployed workers and v the total mass of vacancies. We denote by n the mass of employees ($u = 1 - n$). In this formulation, x can be viewed as a standard neoclassical production function, which is assumed to be homogenous of degree one, increasing and concave in u and v . Let $\theta = v/u$ denote the labor market tightness. The rate at which vacant jobs are filled is $q(\theta) = x(v, u)/v = x(1, \theta)$ while the rate at which unemployed workers find a job is $p(\theta) = x(v, u)/u = x(\theta, 1) \equiv \theta q(\theta)$. It is easy to verify that $p'(\theta) > 0$, $q'(\theta) < 0$, $p(\infty) = q(0) = \infty$ and $p(0) = q(\infty) = 0$. When a match is formed, the worker–firm pair starts production as soon as the firm has incurred the hiring cost. A firm with a filled job then produces an exogenous flow of output y . The production continues until the job is destroyed at an exogenous rate s . When this event occurs, the worker joins the pool of unemployment.

³ For instance, Cahuc et al. (2004) find that the bargaining power of French unskilled employees is zero, while it is lower than 50% for skilled employees, excepted in the retail (food) and automobile sectors. Abowd and Lemieux (1993), Abowd and Allain (1996) and Cahuc et al. (1997) both find values of workers' bargaining power lower than 50%.

⁴ Following Mortensen and Pissarides (2000), we distinguish recruiting costs, which only include screening costs, from hiring costs, which consist of application, processing and training costs (payroll for instructors, rental of equipment and space). Estimates of turnover costs indicate that training costs for newly hired workers are of significant order (see Hamermesh (1993) and Abowd and Kramarz (1998)).

2.2. The efficient allocation

An allocation is efficient if it maximizes the output net of recruiting and hiring costs. Let assume that the unemployed workers obtain non-labor income z from a fixed home production activity. As a result, the social planner maximizes the following welfare function:⁵

$$W = \int_0^\infty e^{-rt} [ny + (1-n)z - \omega v - Qx(v, 1-n)] dt \quad (1)$$

with respect to v and n , and subject to the law of motion for employment:

$$\dot{n} = x(v, 1-n) - sn \quad (2)$$

Let $\eta(\theta) = -\theta q'(\theta)/q(\theta)$ be the elasticity of the matching function with respect to unemployment. It is easily checked that $\eta'(\theta) \geq 0$. Observe that $x_1(v, 1-n) = q(\theta) + q'(\theta)\theta \equiv (1-\eta(\theta))q(\theta)$ can be thought of as representing the planner's probability that a vacancy will be filled.⁶ Similarly, $x_2(v, 1-n) = \eta(\theta)p(\theta)$ gives the transition rate from unemployment to employment for the planner. We have the following result:

Proposition 1. *If $(y-z)/(r+s) > Q$, an efficient steady state allocation exists and it is given by a pair $(\theta = \theta^*, u = u^*)$ that satisfies:*

$$u = \frac{s}{s + p(\theta)} \quad (3)$$

$$\left[\frac{\omega}{q(\theta)} + (1-\eta(\theta))Q \right] (r+s) = (1-\eta(\theta))(y-z) - \eta(\theta)\omega\theta \quad (4)$$

The left-hand side of Eq. (4) indicates that the planner's valuation of hiring costs is a decreasing function of $\eta(\theta)$. This shows how the planner internalizes the trading externalities: the higher $\eta(\theta)$, the lower the planner's probability that a vacancy will be filled, and thus the lower the hiring costs.

2.3. Equilibrium with wage-bargaining

Wages are determined by bargaining between workers and firms according to the Nash-bargaining rule. The bilateral negotiation aims at splitting the total surplus related to the match between the firm and the worker. Let \mathcal{E} and \mathcal{U} be the expected incomes of employed and unemployed workers, respectively. Similarly, let \mathcal{J} and \mathcal{V} be the value of a filled job and a vacancy, respectively. Both are given by the standard asset value equations (see the Appendix).

Then, as in Caballero and Hammour (1996), we introduce a parameter $\Phi \in (0, 1)$ that captures the share of hiring costs which is specific to the worker and whose quasi-rents cannot be protected by contract. In other words, Φ can be thought of as corresponding to the degree of contractual

⁵ Observe that the unemployed workers' benefit only consists of non-market activities. Allowing for an unemployment insurance scheme with labor taxes would introduce additional sources of inefficiencies that are beyond the scope of this paper.

⁶ f_i stands for the derivative of the function f with respect to its i th argument.

incompleteness. Let $\xi \in (0, 1)$ be the worker's bargaining power. From the Nash-bargaining condition, it is straightforward to show that the equilibrium wage solves the following sharing rule:⁷

$$\mathcal{J} - \mathcal{V} - Q(1 - \Phi) = \left(\frac{1 - \xi}{\xi} \right) [\mathcal{E} - \mathcal{U}] \quad (5)$$

This suggests that the firm's surplus is decreasing both with the worker's bargaining power ξ and with the contractual incompleteness Φ . In equilibrium, this sharing rule implies that (see the Appendix for the calculation details):

$$w = \xi \{y + \omega\theta + Q[\Phi p(\theta) - (1 - \Phi)(r + s)]\} + (1 - \xi)z \quad (6)$$

This wage equation shows that the employed workers (insiders) have an ex-post advantage of the hiring costs that would be lost if the relationship was broken up. That is, the higher the share of these costs, Φ , which are specific to the job-worker pair, the higher the wage. Ultimately, when $\Phi > (r + s)/[r + s + p(\theta)]$, the wage increases with Q .

Since the standard free entry condition requires that all rents from new vacancy creation are exhausted, i.e., $\mathcal{V} = 0$, we obtain (again, see the Appendix for details):

Proposition 2. *If $(y - z)/(r + s) > Q \left[\frac{1 - \xi(1 - \Phi)}{1 - \xi} \right]$, the steady state equilibrium allocation exists and is a pair $(\theta = \theta^b, u = u^b)$ that satisfies Eq. (3) and:*

$$\left[\frac{\omega}{q(\theta)} + (1 - \xi(1 - \Phi))Q \right] (r + s) = (1 - \xi)(y - b) - \xi[\omega\theta + Q\Phi p(\theta)] \quad (7)$$

Corollary 1. *The equilibrium is efficient if and only if:*

$$\xi = \eta(\theta) \left[\frac{y - z + \omega\theta - Q(r + s)}{y - z + \omega\theta + Q\Phi[p(\theta) + r + s] - Q(r + s)} \right] \quad (8)$$

If $\Phi = 0$, it is straightforward to show that the equilibrium is efficient if and only if $\xi = \eta(\theta)$, which is the Hosios condition. In other words, if hiring costs are not specific to the production unit, i.e., can be protected from hold-up, then the Hosios condition delivers efficiency. However, when $\Phi > 0$, the equilibrium is efficient only if $\xi < \eta(\theta)$. Loosely speaking, since hold-up reduces a firm's profit gain from creating a vacancy, a lower wage delivers efficiency. This lower wage is achieved by reducing workers' bargaining power.

Corollary 2. *If $\xi = \eta(\theta)$, $u^b > u^*$.*

In words, whenever the Hosios condition holds and there is contractual incompleteness, hold-up gives rise to an excessive level of equilibrium unemployment.

⁷ This sharing rule is derived from $\max_w (\mathcal{E} - \mathcal{U})^\xi (\mathcal{J} - \mathcal{V} - Q(1 - \Phi))^{1-\xi}$.

3. Quantitative investigation

The numerical analysis accounts for two stylized cases, namely US and Europe (EU hereafter). Our calibration is quarterly. We first assume the conventional constant returns-to-scale Cobb–Douglas matching function, that is $x(v, u) = v^{1-\psi} u^\psi$, which implies that $\eta(\theta) = \psi$. Then, we set $r = 1\%$, $y = 1$, $\psi = 50\%$ (see Blanchard and Diamond (1989)), $Q = \omega = 0.3$ (see Mortensen and Pissarides (2000)).

The four remaining parameters (Φ , ξ , z and s) are calibrated in the following way. For each scenario, we consider two extreme values for the degree of contractual incompleteness, that is $\Phi = 0$ (no hold-up) and $\Phi = 1$ (“complete” hold-up), and alternative values for the worker’s bargaining power, between 0% and 50%. For the US case, z and s are adjusted in order to get an equilibrium rate of unemployment of 6.5% and an unemployment duration of one quarter. For the EU case, z and s are set to obtain the same steady state unemployment but an average unemployment duration which is twice as long, i.e. 6 months. This illustrates the European experience wherein unemployment spells are longer but less frequent than in the US. For each couple (ξ , Φ) and the underlying structural parameters (s , z), we then compute the corresponding efficient unemployment rate.

Fig. 1 shows that, if $\Phi = 0$, the efficient unemployment rate equals the actual (equilibrium) one since the Hosios condition holds ($\xi = 50\% = \psi$). For lower values of the worker’s bargaining power, the efficient unemployment rate is greater than the actual one. Incorporating hold-up with $\Phi = 1$ (the lower curve), it comes that efficiency occurs for a worker’s bargaining power of 33.5% under assumptions closer to the US case and 24.5% under assumptions closer to the EU case. This suggests, for instance, that a realistic bargaining power of EU workers of 30% implies that the equilibrium unemployment is greater than its efficient value. On the contrary, without hold-up, such a bargaining power implies that the equilibrium unemployment is lower than its efficient value. This clearly highlights that measuring the size of hold-up is a crucial preliminary to design labor market policies.

To conclude, it should be recognized that hold-up has been incorporated in a very parsimonious way. Taking into consideration the possibility of specific investment in training would admittedly enrich the analysis but would also add complexities that are beyond the scope of this note.

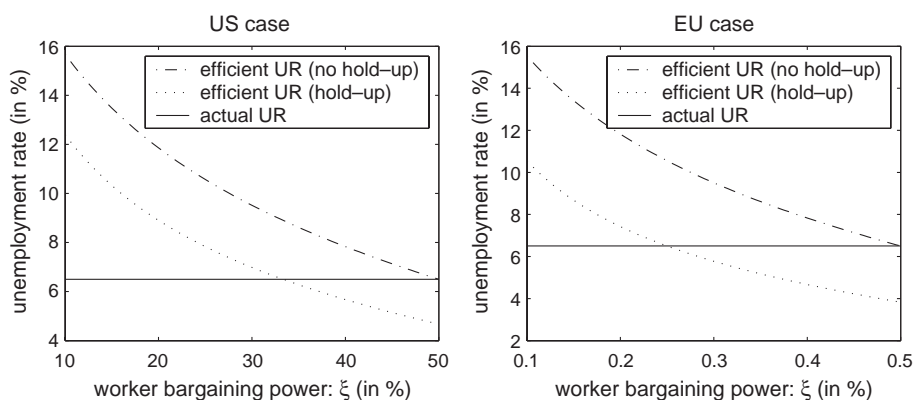


Fig. 1.

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Appendix A. Technical Appendix

Proof of Proposition 1. Recall that condition (4) can be written as:

$$\begin{aligned} \left[\frac{\omega}{q(\theta)} + (1 - \eta(\theta))Q \right] (r + s) &= (1 - \eta(\theta))(y - z) - \eta(\theta)\omega\theta \\ \Leftrightarrow \left[\frac{\omega}{(1 - \eta(\theta))q(\theta)} + Q \right] (r + s) &= y - z - \frac{\eta(\theta)}{(1 - \eta(\theta))} \omega\theta \\ \Leftrightarrow g(\theta) &= z(\theta) \end{aligned}$$

where

$$\begin{aligned} g(\theta) &= \left[\frac{\omega}{(1 - \eta(\theta))q(\theta)} + Q \right] (r + s) \\ z(\theta) &= y - z - \omega \frac{\eta(\theta)}{1 - \eta(\theta)} \theta \end{aligned}$$

To establish the existence of the steady state allocation, we first notice that $g(0) = Q(r + s)$, $g(\infty) = \infty$, $g'(\theta) > 0$, $z(0) = y - z$ and $z'(\theta) \leq 0$. Since $g(\cdot)$ and $z(\cdot)$ are continuously differentiable functions, then if $(y - z)/(r + s) > Q$, there exists a unique steady state efficient allocation $\theta = \theta^*$ that satisfies Eq. (4). \square

Derivation of wage equation

Standard asset value equations are⁸

$$r\mathcal{E} = w - s(\mathcal{E} - \mathcal{U}) \quad (9)$$

$$r\mathcal{U} = z + p(\theta)(\mathcal{E} - \mathcal{U}) \quad (10)$$

$$r\mathcal{J} = y - w - s(\mathcal{J} - \mathcal{V}) \quad (11)$$

$$r\mathcal{V} = -w + q(\theta)(\mathcal{J} - \mathcal{V} - Q) \quad (12)$$

Then, the condition $\mathcal{V} = 0$ and the sharing rule (5) imply that

$$\xi\mathcal{J} = (1 - \xi)(\mathcal{E} - \mathcal{U}) + \xi Q(1 - \Phi)$$

⁸ Since the focus of this article is on steady-states, we suppress time dependence.

$$\Leftrightarrow \xi(r+s)\mathcal{J} = (1-\xi)(r+s)(\mathcal{E}-\mathcal{U}) + (r+s)\xi Q(1-\Phi)$$

$$\Leftrightarrow \xi(y-w) = (1-\xi)[w-z-p(\theta)(\mathcal{E}-\mathcal{U})] + (r+s)\xi Q(1-\Phi)$$

$$\Leftrightarrow \xi(y-w) = (1-\xi)(w-z) - p(\theta)\xi\mathcal{J} + \xi p(\theta)(1-\Phi)Q + (r+s)\xi Q(1-\Phi)$$

Since, from $\mathcal{V} = 0$, we have that $\mathcal{J} = Q + \omega/q(\theta)$, we obtain:

$$w = \xi(y + \omega\theta) + (1-\xi)z + p(\theta)\xi Q - p(\theta)\xi(1-\Phi)Q - (r+s)\xi Q(1-\Phi)$$

It is then straightforward to derive the wage Eq. (6).

Proof of Proposition 2. The combination of the condition $\mathcal{V} = 0$, that implies $Q + \omega/q(\theta) = (y-w)/(r+s)$, and Eq. (6) gives the following condition (equivalent to Eq. (7)):

$$\left[\frac{\omega}{q(\theta)} + Q \right] (r+s) = (1-\xi)(y-z) - \xi[\omega\theta + Q\Phi p(\theta) - Q(1-\Phi)(r+s)]$$

$$\Leftrightarrow \tilde{g}(\theta) = \tilde{z}(\theta)$$

Let us define:

$$\tilde{g}(\theta) = \left[\frac{\omega}{q(\theta)} + Q \right] (r+s)$$

$$\tilde{z}(\theta) = (1-\xi)(y-z) - \xi[\omega\theta + Q\Phi p(\theta) - Q(1-\Phi)(r+s)]$$

Hence, we now have $\tilde{g}(0) = Q(r+s)$, $\tilde{g}(\infty) = \infty$, $\tilde{g}'(\theta) > 0$, $\tilde{z}(0) = (1-\xi)(y-z) + \xi Q(1-\Phi)(r+s)$ and $\tilde{z}'(\theta) < 0$. Since $\tilde{g}(\cdot)$ and $\tilde{z}(\cdot)$ are continuously differentiable functions, if $(y-z)/(r+s) > Q \left[\frac{1-\xi(1-\Phi)}{1-\xi} \right]$, there exists a unique steady state equilibrium allocation with wage bargaining $\theta = \theta^b$ that satisfies Eq. (4). \square

Proof of Corollary 1. Eq. (4) can be written as:

$$\tilde{g}(\theta) = z^\star(\theta)$$

with

$$z^\star(\theta) \equiv (1-\eta(\theta))(y-z) - \eta(\theta)[\omega\theta - Q(r+s)]$$

Accordingly, $\theta^b = \theta^\star$, if and only if $\tilde{z}(\theta) = z^\star(\theta)$. Solving this restriction for workers' bargaining power, we obtain Eq. (8). \square

Proof of Corollary 2. If $\eta(\theta) = \xi$, from the Eq. (4) we know that θ^\star satisfies:

$$\left[\frac{\omega}{q(\theta^\star)} + (1-\xi)Q \right] (r+s) + \xi\omega\theta^\star = (1-\xi)(y-z) \quad (13)$$

Similarly, from Eq. (7), θ^b satisfies:

$$\left[\frac{\omega}{q(\theta^b)} + (1 - \xi)Q \right] (r + s) + \xi\omega\theta^b + \xi Q\Phi[p(\theta^b) + r + s] = (1 - \xi)(y - z) \quad (14)$$

Since $p'(\theta) > 0$, $q'(\theta) < 0$, by comparing Eqs. (13) and (14), it is straightforward to show that $\theta^b < \theta^*$. From Eq. (3), we get $u^b > u^*$. \square

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