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Predicting price spikes in electricity markets using a regime-switching model with time-varying parameters

Timothy D. Mount [*,1], Yumei Ning [1], Xiaobin Cai [1]

Department of Applied Economics and Management, Cornell University, Ithaca, NY 14853, USA

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Abstract

This paper shows that a stochastic regime-switching model can represent the volatile behavior of wholesale electricity prices associated with price spikes effectively. The structure of the model is very flexible because the mean prices in the two regimes and the two transition probabilities are functions of the load and/or the implicit reserve margin. Using price data from the single settlement market in PJM (May 1999 to May 2000), the results show that the estimated switching probability from the low to the high regime predicts price spikes well if the reserve margin is measured accurately.

JEL classification: C32; C53; L94; Q40

Keywords: Price spikes; Regime-switching; Time-varying parameters; Probability of switching; Reserve margin

1. Introduction

The restructuring of electricity markets in the USA has brought about fundamental changes in the behavior of prices in the spot market. Spot prices have been more volatile with price spikes occurring frequently during the summer. For example, a restructured wholesale market was launched in the Pennsylvania–New Jersey–Maryland (PJM) Power Pool on April 1, 1997, which was the first Independent System Operator in the nation. Prior to April 1999, offers to sell electricity in the PJM auction were cost-based. After April 1, 1999, PJM's market participants could submit market-based offers with a price cap of $1000/MWh. Fig. 1 plots the daily average

* Corresponding author. Tel.: +1 607 255 4512; fax: +1 617 255 9984.
E-mail address: tdm2@cornell.edu (T.D. Mount).

[1] Timothy Mount is a Professor and Yumei Ning and Xiaobin Cai are former Ph.D. Graduate Research Assistants in the Department of Applied Economics and Management, Cornell University, Ithaca, New York.

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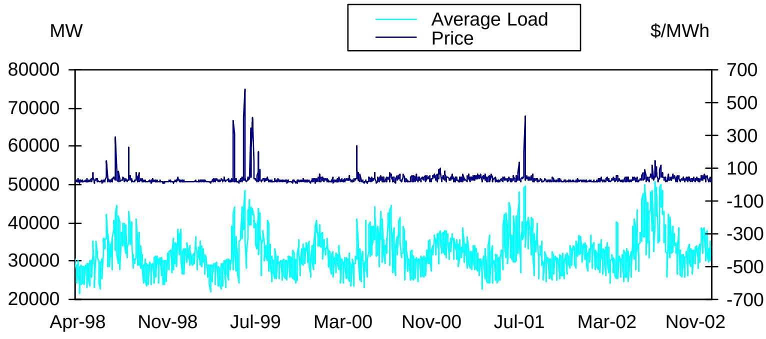

Fig. 1. Daily on-peak price and daily average load in PJM West (April 1, 1998–December 31, 2002).

on-peak price for PJM west from Jan 1, 1998 to December 31, 2002. It shows that price spikes can reach levels that are over an order of magnitude higher than the typical daily price.

In recent years, there have been fewer price spikes in the PJM market due, in part, to the introduction of mitigation procedures after the energy crisis in California (2000–2001) when spot prices increased to unprecedented levels. Since that time, as Bushnell (2003) explains, regulators in the USA have been willing to modify the behavior of suppliers in the wholesale auctions more directly than regulators in other markets. For instance, the Australian electricity market, which also uses a uniform price auction, places fewer restrictions on suppliers, and pries spikes, are a standard feature of this market.

The discussion in the next section explains why price spikes are a typical feature of a deregulated market for electricity, and why this type of price behavior can be represented effectively using a regime-switching model. The primary objective of this paper is to estimate a model that captures the volatility inherent in deregulated markets for electricity and to predict the occurrence of price spikes. One potential application of this model is to improve the forecasting accuracy of price spikes and help buyers and suppliers hedge against upside price risk. Another potential application is to provide more realistic simulations of the volatility of spot prices that can be used to determine the financial value of electricity derivatives.

There have been a number of studies of electricity prices since the markets were deregulated in the U.S.A. For commodities in general, Schwartz (1997) investigated several stochastic models for commodity prices and showed that the prices exhibited strong mean reversion for two commercial commodities. Pilipovic (1997) observed that electricity prices also exhibited mean reversion. Kaminsky (1997), Barz and Johnson (1998), Deng (1998), Deng et al. (2001), and Kamat and Oren (2001) pointed out that including jumps in the model of electricity price improve the accuracy.

Skantze et al. (2000) developed a fundamental model of electricity price dynamics by incorporating the unique characteristics of electricity prices, such as seasonality, lack of load elasticity, stochastic supply outages, strong mean reversion, and stochastic growth of load and supply. Bessembinder and Lemmon (2002) presented an equilibrium model of electricity spot prices in which the price volatility depends on the volatility of system demand and production cost.

Deng (1998) proposed different regime-switching models to capture the price spikes using a continuous-time framework. Huisman and Mahieu (2003) assumed that the electricity price is in one of three different regimes at each point in time, and that prices moved from one regime to another using constant Markov transition probabilities. Ethier and Mount (1999) showed that the behavior of electricity spot prices in different deregulated markets can be described by a stochastic regime-switching model. However, in their analysis, the mean prices and transition probabilities were constant and estimated for each season separately.

The main contribution of our paper is to present a more general model of regime-switching by making key parameters functions of time-varying variables. This model predicts the observed price spikes better than other models because the estimated transition probabilities are affected by current market conditions. The model provides quantitative measures of the volatility of spot prices, long run mean prices, and most importantly, the probability that a price spike will occur.

The paper is organized as follows. In Section 1, a stochastic regime-switching model with time-varying parameters is presented. Following Hamilton (1994), an algorithm for computing maximum likelihood estimates is described in Section 2. The model is estimated in Section 3 using daily price data for PJM from May 1, 1999 to May 31, 2000, corresponding to the period when a single-settlement market was in operation. Using the load and the implicit reserve margin (total offered capacity-load) as explanatory variables, the results show that price spikes can be predicted accurately. However, these results are sensitive to how accurately the explanatory variables are measured, particularly the reserve margin. A summary of the results and conclusions are given in Section 4.

2. A stochastic regime-switching model of price behavior

Data for the actual offers submitted into the PJM market are released after a delay of 6 months. The data for the spring and summer of 1999 are interesting because they represent the transition from "cost-based" offers to "market-based" offers. The resulting price behavior in the summer was highly volatile. The offer curves (actual supply) and the forecasted loads (demand) for three typical days (the first Tuesdays of April, May and July) are shown in Fig. 2 (Mount et al., 2000). The total capacity offered into the market in April is substantially higher than the

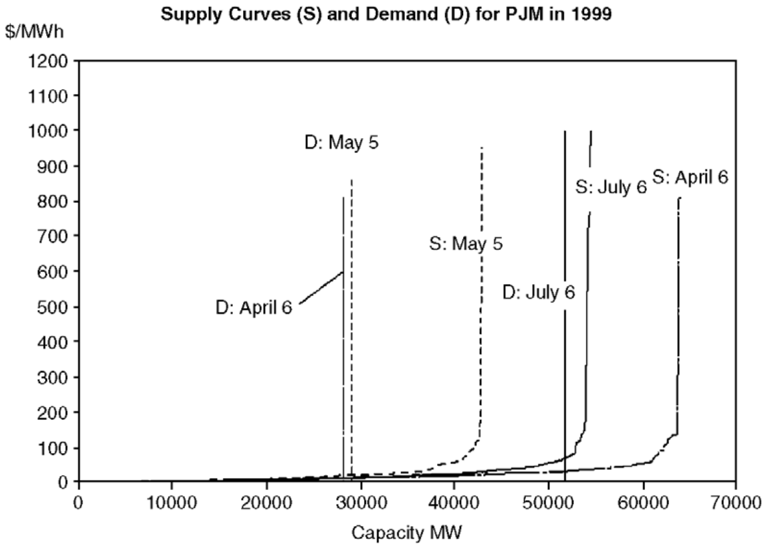

Fig. 2. Supply and demand curves for PJM in 1999.

quantities offered in July and August, although the load in April is relatively low.[2] All three offer curves exhibit a distinct kink after which offers rise almost vertically. The actual load (demand) on July 6 resulted in a substantial price spike with some hourly prices reaching the market price cap level of $1000/MWh. The "hockey stick" shape of the offer curves provides direct evidence that market prices are set in one of two possible regimes. Whenever the forecasted load is higher than the kink in the offer curve, price spikes will occur much more frequently.

The behavior of prices in Fig. 1 implies that the standard stochastic model of geometric Brownian motion (i.e. the logarithm of price is a random walk) used in financial analyses is not appropriate. Ethier and Mount (1999) show that Hamilton's (1989) Markov regime-switching model can be used to capture the behavior of electricity spot prices. Adding Markov regime-switching between a high-price regime and a low-price regime allows for stochastic jumps of prices. Each regime is a mean reverting AR(1) process. One regime represents the relatively flat part of the offer curve (the low-price regime), and the other regime represents the steeply sloped part (the high-price regime). In their analysis, a model with constant parameters is estimated separately for each season. They show that the estimated parameters have distinct seasonal patterns and conclude that a model with time-varying coefficients would be more appropriate. In addition, Diebold et al. (1994) have argued that treating transition probabilities as fixed parameters is restrictive. Consequently, the contribution of this paper is to extend the original regime-switching model by making key parameters functions of time varying variables.

In Hamilton's original model, the current mean of the dependent variable, y_t (the logarithm of price), is a function of parameters in both the current regime and the lagged regime. The regression relationship for y_t can be written as follows:

$$y_t - \mu_{S_t} = \Phi\left(y_{t-1} - \mu_{S_{t-1}}\right) + \varepsilon_t \tag{1}$$

where $S_t = 1$ or 2 identifies the regime; μ_{S_t} is the mean for regime S_t; $0 \le \phi < 1$ is the AR coefficient; ε_t is an unobserved residual that is $N(0, \sigma_{S_t}^2)$.

Specifying that the current price is a function of both the current and lagged mean creates computational complexity for estimation (particularly when more than two states are included in the model). Hamilton (1994), Gray (1996), Kim and Nelson (1999) and Ning (2001) propose a computationally more tractable model in which the current price is a function of the lagged price, but not the lagged mean (i.e. it is not a function of parameters in the lagged state S_{t-1}). This alternative model actually has a higher likelihood value and a lower root mean squared error compared to the original model in (1) using price data for PJM (Ning 2001, Chapter 2).

The most important feature of the new model is that the key parameters are functions of observed explanatory variables. A general specification for the model can be written as follows:

Conditional distributions

y_t is $N(\mu_{1t}, \sigma_1^2)$ if $S_t = 1$
y_t is $N(\mu_{2t}, \sigma_2^2)$ if $S_t = 2$

where y_t is the logarithm of price; μ_{it} is the mean for regime $S_t = i$; σ_i^2 is the variance for regime $S_t = i$.

[2] In April, all capacity had to be offered into the auction, but in May and July, suppliers could withhold some capacity from the auction.

In regression form, the model for y_t is mean reverting and a function of a vector of additional regressors. The form of the mean corresponds to the following geometric lag model:

$$y_t = \mu_{it} + \varepsilon_{it} = \alpha_i + \phi_i y_{t-1} + \gamma_i x_t + \varepsilon_{it} \tag{2}$$

where α_i, ϕ_i and γ_i are unknown parameters for regime $S_t = i$; x_t is an explanatory variable (additional variables can be included); ε_{it} is an unobserved residual that is $N(0, \sigma_i^2)$.

The implication of (2) is that the conditional mean of y_t, $E[y_t | x_t, y_{t-1}, S_t = i] = \mu_{it}$, varies with time.

The final component of the model specifies the transition probabilities for switching from one regime to the other regime as logistic functions. Using this function ensures that predicted probabilities are between 0 and 1. The full set of four transitions probabilities are:

$$Pr[S_t = 1 | S_{t-1} = 1] = P_{1t}$$

$$Pr[S_t = 2 | S_{t-1} = 1] = 1 - P_{1t}$$

$$Pr[S_t = 2 | S_{t-1} = 2] = P_{2t}$$

$$Pr[S_t = 1 | S_{t-1} = 2] = 1 - P_{2t}$$

Note that the regime-switching model simplifies to a model with binomial price spikes if $P_{1t} = 1 - P_{2t}$. The probability of staying in the same regime is specified as a logistic function:

$$P_{it} = Pr[S_t = i | S_{t-1} = i] = \frac{\exp(c_i + d_i z_t)}{1 + \exp(c_i + d_i z_t)} \qquad \text{for } i = 1, 2 \tag{3}$$

where c_i and d_i are unknown parameters for regime $S_t = i$, and z_t is an explanatory variable, which may be the same as x_t in (2) (once again, more than one explanatory variable can be included).

In (2) and (3), both x_t and z_t refer to the information available at time t. The models estimated in Section 4 use either the actual levels of x_t and z_t or day-ahead forecasts of these variables. The objective is to compare the predictability of price spikes in models with and without complete information.

One restrictive feature of the specification in (2) is that the variances for each regime are constant, although Hamilton's standard model allows for GARCH residuals. There are some underlying reasons for specifying constant variances in each regime. The first is that the empirical evidence for GARCH behavior in Fig. 1, for example, is quite limited compared to price behavior for other forms of energy (see Duffie and Gray, 1995). The second reason is that there are distinct analytical advantages from having constant variances in each regime when evaluating financial derivatives, such as the price of an option to sell.

Price volatility is a crucial feature for understanding spot price behavior of electricity, and in particular, when doing financial valuation of electricity derivatives. Consequently, it is important to know the overall variance of the spot price. In this model, the overall (or unconditional) variance of y_t varies with time because the probabilities of being in one or the other regime and the conditional means for each regime vary with time. This can be seen from the following equation.

$$\begin{aligned} \mathrm{Var}[y_t | \Phi_t] &= E\left[y_t^2 | \Phi_t\right] - E[y_t | \Phi_t]^2 \\ &= \rho_t\left(\mu_{1t}^2 + \sigma_1^2\right) + (1 - \rho_t)\left(\mu_{2t}^2 + \sigma_2^2\right) - \left(\rho_t \mu_{1t} + (1 - \rho_t)\mu_{2t}\right)^2 \end{aligned} \tag{4}$$

where $\Phi_t = [y_1, y_2, \ldots, y_{t-1}, x_1, x_2, \ldots, x_t]$ represents the information available to make a one-step ahead forecast of y_t in both regimes, $\rho_t = \Pr[S_t = 1|\Phi_t]$ is the conditional probability of y_t being in state 1. If state 1 represents the high-price regime with $\mu_{1t} > \mu_{2t}$ and $\sigma_1^2 > \sigma_2^2$, then the variance of y_t will be larger when ρ_t is larger. Predictions of ρ_t, as well as estimates of the parameters in (2) and (3), are standard outputs from the model.

3. Estimation

The log-likelihood function of the regime-switching model in (2) and (3) can be constructed recursively. After specifying a startup value for the probability process, the whole series of regime probabilities for period t given information at period $t-1$ can be derived recursively. More specifically, $\rho_{it|t-1}$, the probability of being in regime i given the information in the previous period, is:

$$\rho_{it|t-1} = Pr(S_t = i|\Phi_{t-1}) \qquad \text{for } i = 1, 2, \qquad \left(\text{Note, } \rho_{1t|t-1} = 1 - \rho_{2t|t-1}\right) \tag{5}$$

When new information from the current period is available, these probabilities can be updated to:

$$\rho_{it|t} = \rho_{it} = Pr(S_t = i|\Phi_t) \qquad \text{for } i = 1, 2, \quad (\text{Note, } \rho_{1t} = 1 - \rho_{2t}) \tag{6}$$

The likelihood value for an observed value of y_t in a given regime can be written:

$$f_{it} = f(y_t|S_t = i, y_{t-1}, x_t; \theta_i) \qquad \text{for } i = 1, 2 \tag{7}$$

where $\theta_i = \{\alpha_i, \phi_i, \gamma_i, \sigma_i\}$, for $i = 1, 2$. Under normality, the expression for f_{it} is:

$$f_{it} = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{1}{2}\left(\frac{(y_t - \mu_{it})^2}{\sigma_i^2}\right)\right] \qquad \text{for } i = 1, 2 \tag{8}$$

where μ_{it} is a function of θ_i as defined in Eq. (2).

The conditional likelihood value for an individual observation can be written as a weighted average of the likelihood in (8) for the two regimes as follows:

$$g(y_t|y_{t-1}, x_t, z_t; \phi_i) = f_{1t}\rho_{1t|t-1} + f_{2t}\rho_{2t|t-1} \tag{9}$$

where $\phi_i = \{\theta_i, c_i, d_i\}$, for $i = 1, 2$.

The logarithm of the likelihood for the full sample of T observations can be written as the following product of the conditional probabilities in (9) as follows:

$$\log[\text{Likelihood}(y_1, y_2, \ldots, y_T)] = \sum_{t=1}^{T} \log[g(y_t|y_{t-1}, x_t; \phi)]$$

$$= \sum_{t=1}^{T} \log\left[f_{1t}\rho_{1t|t-1} + f_{2t}\rho_{2t|t-1}\right] \tag{10}$$

This expression has the same basic structure used in time series analysis to derive recursive residuals.

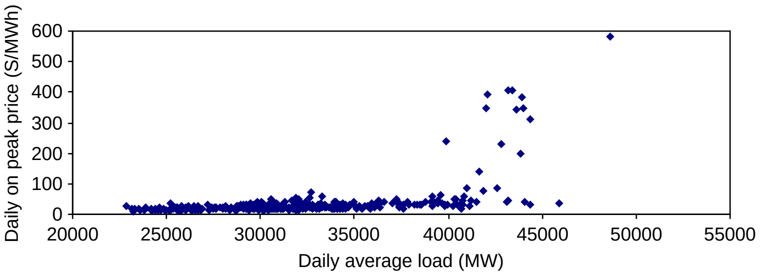

Fig. 3. Daily on-peak price and daily average load in PJM West (May 1, 1999–May 31, 2000).

Hamilton (1994) and Gray (1996) have shown that the optimal inference and forecast of the updated regime probabilities can be determined from the components of the conditional likelihood in (9):

$$\begin{bmatrix} \rho_{1t-1} \\ \rho_{2t-1} \end{bmatrix} = \begin{bmatrix} f_{1t-1}\rho_{1t-1|t-2} / \left(f_{1t-1}\rho_{1t-1|t-2} + f_{2t-1}\rho_{2t-1|t-2} \right) \\ f_{2t-1}\rho_{2t-1|t-2} / \left(f_{1t-1}\rho_{1t-1|t-2} + f_{2t-1}\rho_{2t-1|t-2} \right) \end{bmatrix}. \tag{11}$$

The updated conditional regime probabilities are weighted averages of the probabilities in (11) using the transition probabilities in (3) for regime-switching:

$$\begin{bmatrix} \rho_{1t|t-1} \\ \rho_{2t|t-1} \end{bmatrix} = \begin{bmatrix} \rho_{1t-1}P_{1t} + \rho_{2t-1}(1 - P_{2t}) \\ \rho_{1t-1}(1 - P_{1t}) + \rho_{2t-1}P_{2t} \end{bmatrix} \tag{12}$$

where P_{it} is the probability of staying in regime i defined in (3), for $i = 1, 2$.

This algorithm can be initiated by giving values of ρ_{i0} and the population parameter vector ϕ_i in (9), for $i = 1, 2$. The model is estimated by maximum likelihood using non-linear optimization routines in the computer package GAUSS.

3.1. The estimated model

The first part of this section discusses the factors that govern electricity price behavior in a deregulated market followed by a detailed specification of the model. Since the price fluctuations observed in PJM during the summer of 1999 represent a relatively unregulated market, the analysis is based on these prices (see Fig. 1). The demand and supply curves shown in Fig. 2 imply that high prices occur when demand is high relative to the level of capacity offered into the market. Consequently, price spikes are associated with high demand (Fig. 3) and low reserve margins (Fig. 4).

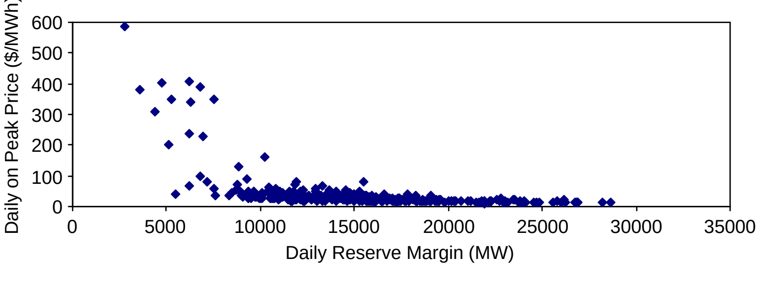

Fig. 4. Daily on-peak average price and daily average reserve margin in PJM West (May 1, 1999–May 31, 2000).

For our analysis, the reserve margin (R) is defined as follows:

$$R = \frac{\text{TOC}}{\text{LOAD}} - 1 \tag{13}$$

where TOC is the total offered capacity. High load or low TOC makes R small, and either condition may lead to prices spikes. In the extreme case when the reserve margin approaches zero, all available supplies, including the most expensive units, have to be dispatched in order to meet load. Since the market clearing price is set by the most expensive unit dispatched, a price spike will occur. When the reserve margin is below 20%, Fig. 4 shows that the price increases dramatically and price spikes are more likely to happen.

Based on the above discussion, the means of the prices in the two regimes and the transition probabilities were specified as functions of the load and the reserve margin. After testing different function forms, the model described in Section 2 was specified as follows:[3]

(Model 1):

The mean prices:

$$y_t = \alpha_i + \phi_i y_{t-1} + \gamma_i R_t + \omega_i \log(L_t) + \varepsilon_{it} \qquad \text{for } i = 1, 2$$

The transition probabilities:

$$P_{it} = Pr[S_t = i | S_{t-1} = i] = \frac{\exp[c_i + d_i(1/R_t) + e_i(L_t/10\,000)]}{1 + \exp[c_i + d_i(1/R_t) + e_i(L_t/10\,000)]} \qquad \text{for } i = 1, 2$$

where y_t is the logarithm of price at time t, R_t is the reserve margin at time t, L_t is the load in MW at time t, and $\varepsilon_{it} \sim N(0, \sigma_i^2)$ for $i=1, 2$. This model does not include variables for weekends and seasonal cycles because the load accounts for these effects.

The data used for estimation were obtained from the PJM Independent System Operator (ISO) for the period from May 1, 1999 to May 31, 2000. The PJM ISO allowed market-based offers from April 1, 1999 and changed to a two-settlement market on June 1, 2000. April 1999 is viewed as a transition period because withholding capacity was not permitted, so it is excluded from the sample. Daily on-peak average spot prices and daily on-peak average loads are used to estimate the model's parameters. On-peak prices correspond to the typical 7×16 forward contract for both weekdays and weekends. These average prices are much less volatile than hourly prices. Weekend observations are included in this analysis because prices exhibit a strong weekly cycle and the price on Monday is likely to be conditional on the price on Sunday rather than the price on Friday.

The total capacity offered in the day-ahead market tends to overestimate the true TOC in (13) because some of the offered capacity may not be available in the real-time market. For example, the capacities of hydro plants are offered at 0 \$/MWh in the day-ahead market but their real-time dispatched capacities vary with weather conditions.[4] Once the market clearing price and load are realized, it is possible to calculate the amounts of offered capacity with offers above (OCA) and below (OCB) the market clearing price. Typically, OCB is larger than the load (L), and the difference corresponds to offered capacity that was not available. The total offered capacity

[3] In practice, generators are not obligated to dispatch their day-ahead capacity even if their offered prices are lower than the real-time market clearing price (see Mansur, 2001).

[4] The different specifications of the model included, for example, using a quadratic form of load instead of linear form for the transition probabilities.

Table 1
Maximum likelihood estimates of Model 1

Variables	Model 1	
	Low-price regime	High-price regime
Parameters for the mean prices		
α (intercept)	1.1432 (1.0782)[a]	2.2885 (6.9228)
ϕ (adjustment)	0.1070**	0.1173* (0.0660)
γ (reserve margin)	−1.2696** (0.0792)	−5.3944** (0.6738)
ω (logarithm of load)	0.2297** (0.1039)	0.3118 (0.6449)
σ^2 (variance)	0.1819** (0.0087)	0.3486** (0.0368)
Parameters for the transition probabilities		
c (intercept)	7.1596** (1.4647)	1.0024** (0.7801)
d (1/reserve margin)	−1.3416** (0.3613)	0.1199 (0.1812)
e (load/10 000)		
Likelihood value	397.9114	

[a] Standard errors in parentheses.
* Estimate is significant at the 10% level.
** Estimate is significant at the 5% level.

(TOC) used to determine R in (13) is (L+OCA), rather than (OCA+OCB). Using this specification, the reserve margin is R=OCA/L.[5]

Model 1 is specified in terms of ex-post information from the real-time market. Consequently, price spikes are predicted using data that are not available in the day-ahead market. Model 1 provides a benchmark for other models discussed below that use forecasts of the load and reserve margin to simulate the conditions that the ISO and generators face in a day-ahead market.

After Model 1 was estimated, some additional restrictions were imposed on the transition probabilities. Load (L_t/10 000) does not add significant explanatory power to either transition probability and these coefficients were set to zero. A Likelihood Ratio test does not reject this restricted model. The maximum likelihood estimates of the restricted Model 1 are summarized in Table 1.

The adjustment coefficients in both price regimes are significant but relatively small compared to our prior expectations. The mean prices for both regimes respond relatively quickly to changes in the load and the reserve margin. The reserve margin is negatively and significantly related to the mean prices in both regimes, and the effect is much more important in the low-price regime. The load is positively related to the mean prices in both regimes, but is not significant in the low-price regime. As expected, the residual volatility of the high-price regime is larger than the volatility of the low-price regime ($\sigma_1 > \sigma_2$). Since the dependent variables are in logarithmic form, the typical proportional error is 80% in the high-price regime and 53% in the low-price regime, which is given by $100(e^{\sigma_i} - 1)$, for i=1, 2.

The probability of staying in the low-price (high-price) regime is higher (lower) for larger values of the reserve margin as expected.[6] Since there are relatively few price spikes in the data, the information for estimating the switching probabilities is limited. In spite of this, the

[5] It is possible that some capacity with offers higher than the market clearing price is also unavailable. This amount of capacity cannot be calculated from the available data and it is treated as part of available capacity in this paper.
[6] The derivative of (3) with respect to the reserve margin (R) is $P_{it}(P_{it} - 1)d_i/R^2$, for i=1, 2.

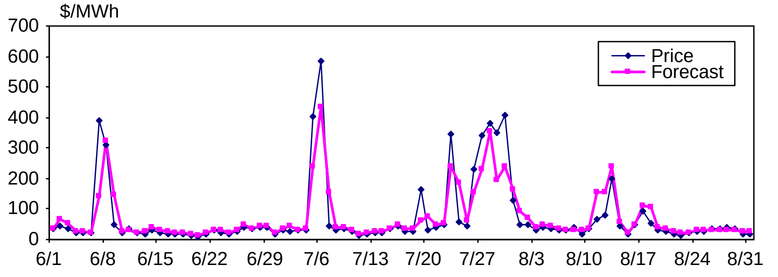

Fig. 5. Daily on-peak average spot prices and the forecasts in PJM West for summer 1999.

coefficient of the reserve margin is highly significant in the low-price regime. Note that the estimated model is quite different from a model with binomial price spikes because $P_{1t} \neq 1 - P_{2t}$.

3.2. Forecasting spot prices

Fig. 5 shows the actual daily average peak prices and the forecasts for the summer of 1999 (from June 1, 1999 to August 31, 1999). Since the dependent variable is the logarithm of price, the unbiased forecast of the price in a given regime i is $\exp(\hat{y}_i + \sigma_i^2/2)$, where $\hat{y}_i$ is the predicted value in (2) and σ_i^2 is the variance. The forecasts of low prices are generally close to the actual prices, but the forecasts of the highest prices are generally lower than the actual prices. The reason for this is that there is uncertainty about which price regime is in effect on a given day, and the forecasts of the prices correspond to a weighted average of the predicted prices in the two regimes, using the probability weights in (12) (derived from the probabilities in (3) and (11)).

Fig. 6 shows the smoothed probabilities of being in the high-price regime for the summer of 1999. The smoothed probabilities are conditional on all information in the sample, and they represent ex-post predictions of whether the observed price was in the high-price regime. The smoothed probabilities are calculated using an algorithm developed by Kim (1993). Generally, the probabilities are close to 1 or 0, and the correspondence between the high probabilities and the price spikes is obvious (see Table 2). Even small price spikes have probabilities that are close to 1.

3.3. Predicting price spikes

The most important policy implication of the model is the relationship between the probability of switching to the high-price regime and the reserve margin. The probability of

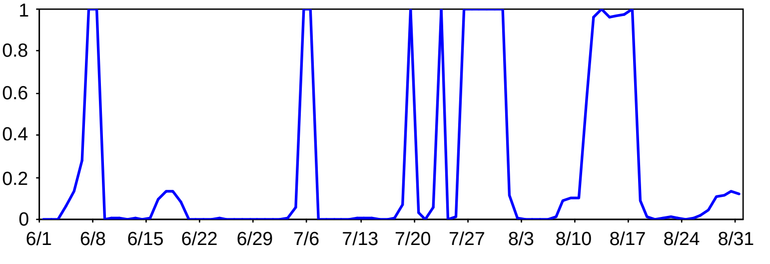

Fig. 6. Smoothed probabilities of being in the high-price regime (PJM West for summer 1999).

Table 2
The switching probabilities for days with high prices

Rank (by price)	Group	Date	Price	Switching probability	Smoothed probability
High prices with high switching probabilities					
1	A, B	07/06/99[a]	585.06	1.0000	1.0000
2	A, B	07/30/99[a]	407.25	0.8946	1.0000
3	A, B	07/05/99[b]	402.53	0.9942	1.0000
4	A, B	06/07/99[b]	389.84	0.7680	1.0000
5	A, B	07/28/99[a]	381.25	0.9999	1.0000
6	A, B	07/29/99[a]	348.45	0.5831	1.0000
7	A, B	07/23/99	347.09	0.9817	1.0000
8	A, B	07/27/99[a]	339.30	0.8988	1.0000
9	A, B	06/08/99[a]	309.46	0.9985	1.0000
10	A, B	07/26/99[b]	229.05	0.7557	1.0000
11	A, B	08/13/99	201.17	0.9882	1.0000
High prices with low switching probabilities					
12	A	07/19/99	163.01	0.2390	1.0000
13	A	07/31/99[a]	127.72	0.3057	0.9999
High switching probabilities with low prices					
15	B	08/12/99	80.63	0.6574	0.8625
16	B	08/11/99	67.28	0.7920	0.0231

[a] High price occurred in the previous day.
[b] First price spike in a sequence of price spikes.

switching to the high-price regime ($(1-P_{1t}$ in (3)) is negatively and significantly related to the reserve margin. Fig. 7 shows that price spikes are likely to happen whenever the reserve margin is less than 20%. This feature is consistent with the data shown in Fig. 4. Fig. 8 gives the corresponding estimated daily probabilities of switching from the low-price regime to the high-price regime from May 1, 1999 to May 31, 2000. The high switching probabilities occur in the summer of 1999 and in May, 2000. These are the most important probabilities for predicting price spikes because they are based on information that is available at time t. In contrast, the smoothed probabilities in Fig. 6 are based on information for the complete sample of data.

The two switching probabilities in (3) are used to update the probabilities of being in the high-price and low-price regime in (12). These are the weights for the corresponding mean prices used to derive the price forecasts in Fig. 5. Fig. 9 plots the day-ahead probability of being in the high-price regime. The legend “probability LH” in Fig. 9 refers to the joint probability of being in the high-price regime following the low-price regime, and “probability HH” refers to the

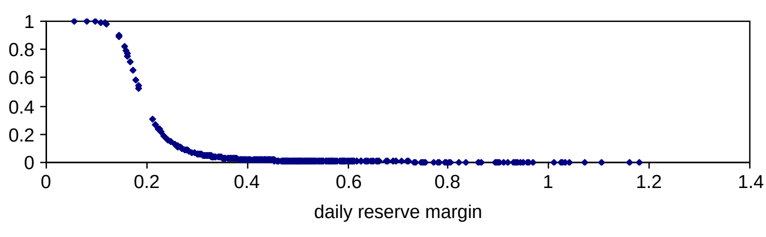

Fig. 7. Probability of switching from low-price regime to high-price regime versus reserve margin for PJM West.

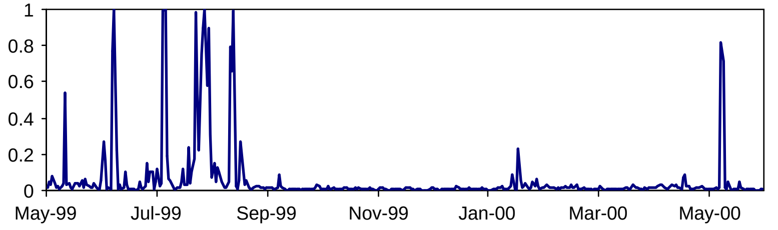

Fig. 8. Daily probabilities of switching from low-price regime to high-price for PJM West.

joint probability of staying in the high-price regime. These probabilities correspond to the components of (12) and can be written:

$$\text{Probability LH} = \rho_{1t-1}(1 - P_{1t})$$

$$\text{Probability HH} = \rho_{2t-1}P_{2t}$$

The sum of the two probabilities measures the probability weight for the predicted prices in the high-price regime in the day-ahead price forecast. Fig. 9 shows that the switching probability for the first observation in a sequence of price spikes plays a very important role in forecasting the subsequent price spikes. The next step in the analysis is to determine how well the high switching probabilities in Fig. 8 predict actual price spikes.

The analysis of price spikes will focus on the 13 observations with prices greater than \$100/MWh. These 13 observations, denoted as Group A, are compared with another 13 observations with the highest switching probabilities, denoted as Group B. The eleven highest prices in Group A also have high switching probabilities and belong to Group B. Overall, there are 15 different observations that are ranked by price in Table 2. Group A includes the observations with rank 1–13. Group B includes the observations with ranks 1–11, 15 and 16.

All of the 13 estimated switching probabilities in Group B are greater than 0.5. Since 11 of these correspond to the 11 highest prices in Group A, the switching probability does predict price spikes greater than \$200/MWh correctly when the switching probability is greater than 0.5. Two price spikes below \$200/MWh occur when the switching probability is less than 0.5. In contrast to the switching probabilities, all of the smoothed probabilities for Group A are 1.0.

Seven of the high prices in Group A occur on days following a high price. Three prices correspond to the first day in a sequence of price spikes and three prices are isolated price spikes. Hence, only six price spikes follow low prices in the sample. In spite of this, the switching probability (from low-price regime to the high-price regime) is still a good predictor of a price spike regardless of whether the price follows a high or a low price. The corresponding switching

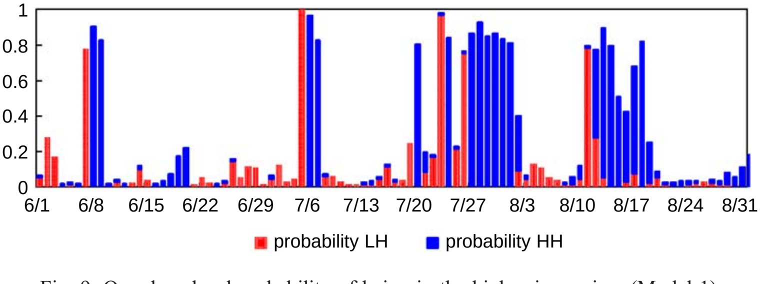

Fig. 9. One-day-ahead probability of being in the high-price-regime (Model 1).

probability from the high-price regime to the low-price regime is always less than 0.25. Consequently, this switching probability is not a useful predictor of when a sequence of price spikes will end.

3.4. Forecasting with incomplete information

In reality, the information about the real-time load and reserve margin is not known until the real-time market has closed. Normally, the Independent System Operator (ISO) announces the forecasted load to suppliers 1 day ahead. Therefore, the forecasted load is the information that can be used to forecast prices for the next day. The ISO also knows the forecasted reserve margin, which can be calculated by subtracting the forecasted load from TOC. Additional models were estimated using these forecasted values instead of the true values.

The accuracy of the forecasted reserve margin is affected by the accuracy of the forecasted load and by the discrepancy between the day-ahead TOC and the actual available capacity. The discrepancy comes from the fact that the actual available capacity is normally less than TOC. In general, the ISO has more information than the suppliers when they make their price forecasts for the next day. Since the ISO does not provide information about the TOC to suppliers, the suppliers must obtain this type of information from other sources. In our analysis, we assume that information about the TOC is only available to the ISO.

Model 1 in Table 1 represents the model for the ISO with complete information about the load and reserve margin. Model 2 is the equivalent model for suppliers with actual load as the only source of information. Model 3 is the day-ahead model for the ISO based on forecasts of load and reserves. Model 4 is the equivalent day-ahead model for suppliers based on forecasted load only. Since Models 2, 3 and 4 do not predict price spikes nearly as well as Model 1, Model 5 was specified for the ISO to illustrate the benefit of having more accurate forecasts of the reserve margin and load.

Since the actual daily forecasts of average load in PJM are not saved in the public archives, a time-series model was fitted to daily average load data to capture all seasonal, weekly cycles, holiday and weather effects (see Ning et al., 2000). The adjusted R^2 for this model is 95%, indicating a good fit, and the predicted load from this model is used as the forecasted load in Models 2–5.

The specifications for Models 2–5 are listed in Appendix A. Since these models use less information than Model 1, weekend dummies are added to compensate for the lost information.

Table 3
Summary of the predictive performance of the five estimated models

Model	Variables[a]		Likelihood	RMSE	Switching probabilities[b]			
	R	L			>70%		>50%	
					Yes	No	Yes	No
1	T	T	397.9119	0.32	10	1	11	3
2	T	–	279.2802	0.53	3	1	10	4
3	F	F	278.0936	0.51	9	3	11	5
4	F	–	268.1197	0.74	4	1	9	3
5	(T+F)/2	(T+F)/2	350.1396	0.41	9	3	11	6

[a] L is load, R is reserve margin. T is the true value and F is the forecasted value.
[b] Number of price spikes predicted correctly (YES) and incorrectly (NO). The actual number of price spikes is 13.

The maximum likelihood estimates for Model 2–5 are listed in Appendix B. The likelihood values for Models 2–5 are much lower than the value for Model 1, and the rankings from the largest to the smallest value are 1, 5, 2, 3, 4. In other words, the rankings (1, 5, 3) for the three ISO models are consistent with the accuracy of the information used. This is also true for the rankings (2, 4) of the two suppliers' models. The likelihood values are summarized in Table 3.

Fig. 10 shows the actual prices and the forecasted prices for the summer 1999 given by all five models (the forecasts for Model 1 in Figs. 5 and 10 are the same). The forecasts for Model 2–5 are very similar to the forecasts for Model 1. The root mean squared errors (RMSE) for Models 1–5 are 0.32, 0.53, 0.51, 0.74 and 0.41, respectively (see Table 3). The RMSE for Models 2 and 4 are relatively large because they predict some high prices that do not actually occur. Overall, the accuracy of forecasts from Models 2, 3 and 4, with incomplete information, are similar to each other. The additional information about the

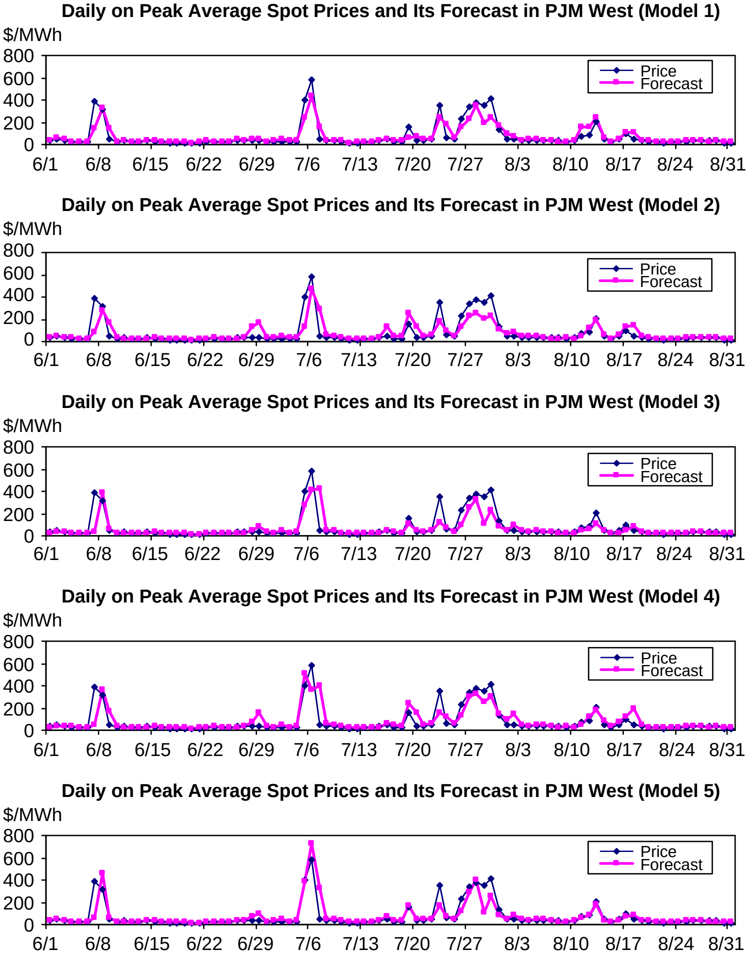

Fig. 10. Daily on-peak average spot prices and its forecast in PJM West for summer 1999.

forecasted reserve margin used by the ISO in Model 3 only provides a slight improvement over Model 4.

In Model 5, the forecasting errors for the both the load and the reserve margin are cut in half, and this results in a large reduction in the RMSE compared to Model 3. Accurate information about the reserve margin, in particular, is valuable for forecasting, and this is the main reason why Model 1 performs better than the other models.

The switching probabilities from the low-price regime to the high-price regime for all five models are similar to the results shown in Fig. 8 for Model 1 (see Fig. 11). The main difference is that there are fewer high values using the two suppliers' models (2 and 4). The results are summarized in Table 3 in terms of the number of correct predictions and incorrect predictions of the 13 observed price spikes identified in Table 2. Using cutoff switching probabilities of both >70% and >50% to predict a price spike, Model 1 performs much better that any of the other four models because the other models have either fewer correct

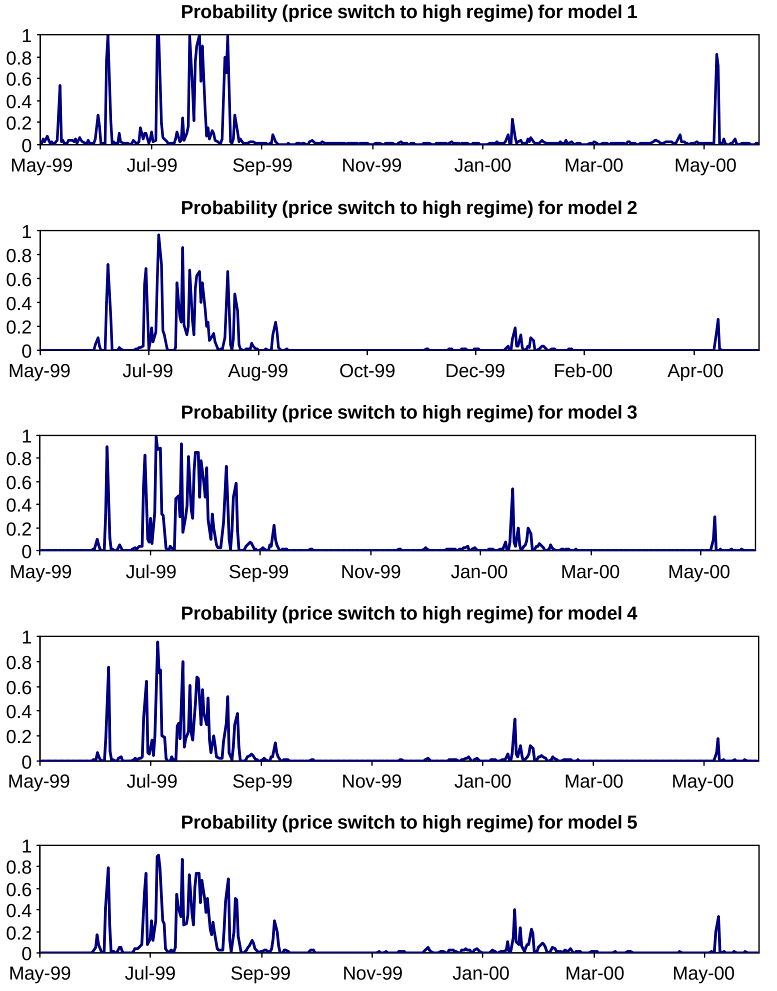

Fig. 11. Probabilities that prices switches to high-price regime generated from Model 1 to Model 5.

predictions and/or more incorrect predictions. In particular, the numbers of correct predictions using >70% as the criterion are much smaller for the two suppliers' models (2 and 4). For predicting price spikes, the forecasted reserve margin provides valuable information for the two ISO models (3 and 5) compared to Models 2 and 4. However, there are more incorrect predictions using Models 3 and 5 compared to Model 1.

4. Conclusions

This paper shows that a stochastic regime-switching model with time-varying parameters can capture the type of volatile price behavior observed in many deregulated spot markets for electricity. The mean prices in two price regimes and the transition probabilities are specified as functions of the offered reserve margin and the system load. The high-price regime corresponds to the observed price spikes that typically occur during the summer months. In addition, the structure of the model is consistent with the actual "hockey stick" shape of the offers submitted by suppliers into the PJM market. Most capacity is offered at relatively low prices, and a few units are offered at much higher prices up to the price cap ($1000/MWh in PJM). Specifying Markov-switching in the models allows the high-price regime to be more persistent than is the case with a simple binomial jump process. Using on-peak daily data for PJM, the analysis shows that the model replicates the observed price volatility well. Consequently, this type of model is potentially useful for evaluating forward contracts and investment decisions in electricity markets. Standard financial models of price do not allow for the unusual asymmetric type of volatility (i.e., infrequent price spikes) found in deregulated electricity markets.

From a policy perspective, it is important to predict price spikes. The probability of switching to the high regime is negatively and significantly related to the reserve margin. Whenever the level of reserve margin is lower than 20%, a price spike is likely to occur. There were 13 price spikes in the data set with prices higher than $100/MWh, and 11 of them have switching probabilities greater than 0.5. Two other observations with probabilities greater than 0.5 do not correspond to price spikes. As a result, the price spikes can be predicted accurately using correct market information. However, the accuracy of the prediction is sensitive to the accuracy of the explanatory variables. Using 1-day-ahead forecasts of the load and reserve margin leads to fewer correct predictions. To predict price spikes effectively, accurate information about the reserve margin, in particular, is needed.

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Appendix A. Model specifications for Model 2 to Model 5

A.1. Model 2 (actual load)

Mean:

$$y_t = \alpha_i + \phi_i y_{t-1} + \omega_i \log(L_t) + \kappa_i(\log(L_t)^*\mathrm{WKD}_t) + \varepsilon_{it} \qquad \text{for } i = 1{,}2$$

Transition probability:

$$P_{it} = Pr[S_t = i|S_{t-1} = i] = \frac{\exp[c_i + e_i(L_t/10\,000)]}{1 + \exp[c_i + e_i(L_t/10\,000)]} \qquad \text{for } i = 1,2$$

where $\varepsilon_{it} \sim N(0, \sigma_i^2)$, for $i = 1, 2$. L_t is actual load.

A.2. Model 3 (forecast reserve margin and forecasted load)

Mean:

$$y_t = \alpha_i + \phi_i y_{t-1} + \gamma_i R_t + \omega_i \log(L_t) + \kappa_i(\log(L_t)^*\mathrm{WKD}_t) + \varepsilon_{it} \qquad \text{for } i = 1,2$$

Transition probability:

$$P_{it} = Pr[S_t = i|S_{t-1} = i] = \frac{\exp[c_i + d_i(1/R_t) + e_i(L_t/10\,000)]}{1 + \exp[c_i + d_i(1/R_t) + e_i(L_t/10\,000)]} \qquad \text{for } i = 1,2$$

where $\varepsilon_{it} \sim N(0, \sigma_i^2)$, for $i = 1, 2$. R_t is forecasted reserve margin and L_t is forecasted load.

A.3. Model 4 (forecasted load)

Mean:

$$y_t = \alpha_i + \phi_i y_{t-1} + \omega_i \log(L_t) + \kappa_i(\log(L_t)^*\mathrm{WKD}_t) + \varepsilon_{it} \qquad \text{for } i = 1,\ 2$$

Transition probability:

$$P_{it} = Pr[S_t = i|S_{t-1} = i] = \frac{\exp[c_i + e_i(L_t/10\,000)]}{1 + \exp[c_i + e_i(L_t/10\,000)]} \qquad \text{for } i = 1,\ 2$$

where $\varepsilon_{it} \sim N(0, \sigma_i^2)$, for $i = 1, 2$. L_t is forecasted load.

A.4. Model 5 (forecasted error for reserve margin and load is reduced by 50%)

Mean:

$$y_t = \alpha_i + \phi_i y_{t-1} + \gamma_i R_t + \omega_i \log(L_t) + \kappa_i(\log(L_t)^*\mathrm{WKD}_t) + \varepsilon_{it} \qquad \text{for } i = 1,\ 2$$

Transition probability:

$$P_{it} = Pr[S_t = i|S_{t-1} = i] = \frac{\exp[c_i + d_i(1/R_t) + e_i(L_t/10\,000)]}{1 + \exp[c_i + d_i(1/R_t) + e_i(L_t/10\,000)]} \qquad \text{for } i = 1,\ 2$$

where $\varepsilon_{it} \sim N(0, \sigma_i^2)$, for $i = 1, 2$. R_t is the average of actual reserve margin and forecasted reserve margin. L_t is the average of actual load and forecasted load.

In all the models above, WKD_t is the dummy for weekend, where 1 is for weekend and 0 otherwise.

Appendix B. Maximum likelihood estimates of regime-switching models

Variables	Model 2 (generators: real time)	Model 3 (ISO: day-ahead)	Model 4 (generator: day-ahead)	Model 5 (reduced forecasted error)
Parameters for means regression				
α_1 (intercept)	−8.1408** (1.5041)	2.8125** (0.1429)	−6.1625** (1.5586)	2.2815* (1.4106)
α_2 (intercept)	−63.2431** (15.9604)	4.7618** (0.8651)	−52.7812** (10.5825)	−22.5413 (17.5217)
ϕ_1 (adjustment)	0.2840** (0.0385)	0.3049** (0.0333)	0.3193** (0.0396)	0.1867** (0.0293)
ϕ_2 (adjustment)	0.1665* (0.1017)	0.4256** (0.1479)	0.2114** (0.0986)	0.2243** (0.0855)
γ_1 (reserve margin)		−1.1106** (0.1267)		−1.5125** (0.1145)
γ_2 (reserve margin)		−4.5698** (2.0552)		−5.5452** (1.5841)
ω_1 (logarithm of load)	1.0121** (0.1506)		0.8099** (0.1554)	0.1089 (0.1351)
ω_2 (logarithm of load)	6.3596** (1.5259)		5.3590** (1.0184)	2.6456* (1.6322)
κ_1 (weekend)	−0.0054** (0.0037)		−0.0073** (0.0038)	
κ_2 (weekend)	−0.0577* (0.0341)	−0.0892** (0.0474)	−0.0764** (0.0260)	−0.0619** (0.0299)
σ_1^2 (volatility)	0.2698** (0.0108)	0.2741** (0.0112)	0.2737** (0.0108)	0.2284** (0.0089)
σ_2^2 (volatility)	0.4277** (0.0600)	0.5497** (0.1250)	0.4533** (0.0627)	0.3746** (0.0652)
Parameters for transition probability				
c_1 (intercept)	24.9289** (6.9170)	24.5746** (9.0428)	5.3590** (1.0184)	19.6202** (5.0489)
c_2 (intercept)	0.8151** (0.5378)	−29.4867** (14.6147)	0.9835** (0.4853)	−33.1739* (20.7864)
d_1 (1/reserve margin)				
d_2 (1/reserve margin)				
e_1 (load/10000)	−5.8281** (1.6967)	−5.9567** (2.5117)	−4.9361** (1.3082)	−4.6989** (1.2834)
e_2 (load/10000)		7.0696** (3.4848)		7.9270* (4.9237)
Likelihood value	279.2802	278.0936	268.1197	350.1396

** Estimate is significant at 5% level.

* Estimate is significant at 10% level.

1 denotes low-price regime and 2 denotes high-price regime. Coefficients presented here are for restricted models.

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