Regional Science and Urban Economics 32 (2002) 475–500
www.elsevier.com/locate/econbase

A model of regulation in the rental housing market

Pascal Raess, Thomas von Ungern-Sternberg*

HEC, Departement d'Econometrie et d'Economie Politique, University of Lausanne, 1015 Lausanne, Switzerland

Received 11 November 1999; received in revised form 3 May 2001; accepted 6 September 2001

Abstract

This paper develops a theoretical model to study the effects of regulation on the rental housing market. Our model emphasises the following specific features of the housing market: product heterogeneity and search costs play a central role, switching (moving) costs are substantial, and the possibilities to price discriminate are important. We show that with short-term rental contracts rents will increase at the time of renegotiation as a result of the 'hold-up' problem. Tenancy rent control which limits the owners' possibilities to increase rents for a certain number of years leads to lower equilibrium rents and higher social welfare. Our model strongly suggests that a policy which consists of indexing rents may be socially preferable to short-term contracts. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Rental Housing; Monopolistic competition; Regulation; Search; Hold-up

JEL classification: D83; L59

1. Introduction

We develop a model to study the possible effects of regulatory intervention in the rental housing market. The problem we focus on was summarized as follows by Arnott in the Palgrave Dictionary (1998): ''The standard rental contract in North America is for 1 year. At the end of the year, the landlord is free not to

*Corresponding author. Fax: +41-21-692-3365.
E-mail address: thomas.vonungern-sternberg@hec.unil.ch (T. von Ungern-Sternberg).

0166-0462/02/$ – see front matter
PII: S0166-0462(01)00093-X

renew the lease (while in principle a tenant can negotiate a longer lease, in practice he cannot) and can raise the rent at will''. While tenants frequently stay several years in the same apartment, they usually have only 1 year leases, which must then be renegotiated. This could cause problems if the rent setting mechanisms are different at the time of initial contracting and renegotiation. Casual empiricism suggests that this might well be the case: in the latter stage the landlord has more bargaining power, because the tenant has sunk costs of moving out of an apartment to which he has got used. In several countries, e.g., France, Switzerland and Germany, the legislator has reacted to this situation by introducing asymmetric regulation. The rules determining the conditions for rent renegotiation with a sitting tenant are considerably more severe than those governing new leases.

To study the effects of this kind of intervention one needs to work within the framework of a model which takes into account at least some of the imperfections of the housing market. Our model focuses on four kinds of deviations from the standard model of perfect competition.[1]

First, houses are differentiated in a large number of dimensions.[2] It thus seems natural to model the housing market within a setting of monopolistic competition. We use the circular road model developed by Salop (1979). In this model of horizontal product differentiation, product heterogeneity is limited to one single dimension, differences in location. The distance between a tenant's most preferred location and his actual location can be interpreted as the cost of mismatch. A more realistic approach might well be to permit differentiation according to more than one dimension, but for reasons of tractability we restrict product differentiation to one dimension.

Second, informational problems play an exceptionally important role in the housing market. The descriptions of apartments that appear in advertisements are usually so summary that one has to visit them to have a clear idea just what they are like. We model the opacity of the rental market by introducing search costs.[3] Again this is a strong simplification of the real world, but it is still an improvement over the assumption of complete information that is frequently made.[4]

[1] In the literature on rent control, Arnott (1995) distinguishes between traditional textbook analysis with a price ceiling and modern analysis. The latter assumes either perfect or imperfect competition. Our model can be classified in the imperfect competition part of the literature.

[2] For alternative ways of modelling heterogeneity in housing models, cf. Rosen (1974) or Sweeney (1974a,b).

[3] If housing were a more homogeneous commodity, informational problems would surely be less important, since it would be easier to quote a standardised price. Courant (1978) and Yinger (1981) were the first to build models of the housing and real estate broker markets, respectively, taking search problems into account. More recently, Arnott (1989), Wheaton (1990) and Igarashi and Arnott (2000) modeled housing heterogeneity with search.

[4] Modelling search with multi-dimensional heterogeneity is feasible but would introduce a further source of non-linearities, making it difficult to explicitly solve the model.

Third, and most important, tenants usually do not know, at the time they rent an apartment, how long they will stay there. As a result, there is often a strong discrepancy between the duration of the initial rental contract and the length of time the tenant actually stays in the flat. The rental contract has to be renewed or renegotiated periodically. The key element of our model is the fact (or assumption) that the price setting mechanism is different at the time a tenant first enters an apartment and the moment he renegotiates his rental contract. When a tenant is looking for an apartment there is competition between owners to attract him. When the contract is being renegotiated, the owner is in a much better bargaining situation. The tenant has large fixed costs associated with moving (search costs and moving costs). There are also the psychological costs of losing neighbours and having to give up a social environment and habits one may have acquired. When a rental contract is renegotiated, there is a classical hold-up problem. While it is true that the owner also has some fixed costs when taking a new tenant, it seems to us that these are much lower than those faced by a tenant. Thus, on the whole it is the tenant who is in a more difficult bargaining position. When the market vacancy rate is low, which is usually the case in large urban centers where rental housing markets concentrate, the weakness of the tenant's bargaining position is reinforced. Moreover, as by assumption there is only one type of tenant in our model, there is no room for 'good' tenants that would have a substantive bargaining power compared to 'bad' tenants. We thus model the process of renegotiation as one where the owner can make the tenant a take-it or leave-it offer.[5]

Finally the landlord may learn something about the tenant's characteristics once he has moved in. This opens the possibility for ex post price discrimination. The landlord may substantially increase the rent when he finds out that his tenant really likes the apartment.

Our model thus studies the interaction among elements of monopolistic competition, search, bargaining with one-sided sunk costs and price discrimination.[6] Regulation in our model takes the form of specifying the time and rules of rent renegotiation, once the initial rental contract has expired. More specifically, regulation sets the number of years over which the landlord cannot increase (renegotiate) the rent once a new tenant has moved in. We investigate how a landlord sets prices in such a setting. We also study how search costs, the number of apartments and social welfare are affected as the duration over which the initial rent has to be maintained changes.

At this point, two remarks are in order:

[5] Allowing for (generalised) Nash bargaining would have been more satisfactory approach. However, the model would be difficult to solve analytically.

[6] We assume there is no depreciation in the housing stock. Maintenance, renovation and up- or downgrading are thus excluded from the model (and as a consequence the incentives problems of who actually carries out maintenance or renovation).

- First, our model does not explain why tenants and landlords do not sign longer-term contracts in the first place. There seems to us to be at least three potential reasons why this might be the case. First, at the time a tenant moves in, he often does not know how long he will stay in the apartment, and he might thus be unwilling to sign a long run contract. Second, in reality tenants also differ in their types. A central problem to landlords when renting out a unit is to attract tenants who can pay their rent (good tenants), and avoid tenants with financial problems (bad tenants). If, at the time of initial contracting, landlords do not know their tenants' financial status, they might well prefer to enter a short-term agreement, and thus facilitate the possible eviction of insolvent tenants. Finally, an adverse selection problem might arise with two type tenants: if landlords keep good tenants but evict bad types, only bad tenants would ask for long-term initial contracts.
- The second remark concerns the intertemporal profile of rents for a sitting tenant. Because of the renegotiation of initial contracts, tenants have 'low' initial rents and 'high' post renegotiation rents. However, the empirical literature points towards an opposite time profile of rents. A positive gap between market rents and rents for sitting tenants in comparable units tends to appear with time. These rent discounts seem to be growing with tenancy duration. A number of reasons can be put forward for this observation:

First, there may be measurement errors: apartments are typically renovated before a new tenant moves in. Apartments which have been occupied by the same tenant for a long time have thus typically suffered some wear and tear, which would justify a rent rebate. It is not clear to what extent the statistics take this into account. Second, there may be different types of owners. Some wish to maximise the rents they collect from their units. They will charge high rents, and accept the transactions costs of having frequent changes of tenants. Others prefer to have stable tenants, and are willing to accept lower rents to this end. Schlicht (1983) argues that as time passes the mismatch between tenants and their units tends to increase, so the rent rebate might have to increase over time. Third, there may be different types of tenants. Good (solvent) tenants get a rebate relatively to bad tenants who cannot pay their rent on a regular basis. On average the good tenants keep an apartment longer than the bad tenants.

The existence of rent discounts is quite well established in the empirical literature.[7] We believe that introducing some of the ideas outlined above into our model would certainly make it more realistic, but that it would not modify the

[7] Note that the rent discount phenomenon is not driven by the nature of the rent regulation legislation. Börsch-Supan (1986) reports comparable rent discounts in Germany and the US, although legislations are very different.

essential conclusions. Furthermore, the model would become analytically untractable.

The contribution of the paper is the way tenancy rent control is modelled, i.e., by specifying the time and rules of rent renegotiation. It clearly distinguishes itself from the traditional way of modeling rent control, which the large majority of researchers do via first-generation controls (nominal upper-limit on the rent level). Igarashi and Arnott (2000), although the model closest to ours in the way the housing market is modelled (monopolistic competition and search) is no exception. If we look at present legislations in various countries such as Germany, France or Switzerland, they are tenancy rent control policies: the emphasis is no longer on a nominal rent freeze, but rather on tenure security (to avoid economic eviction) and on upper limits for annual rent increases.[8] We think our assumptions for modelling regulation captures the salient features of modern legislations. In other words, the paper identifies a new argument for rent regulation, namely to give the tenant more bargaining power at the renegotiation stage.

Our main results are as follows: even with rational expectations and perfect foresight, it is not true that the level of rents and social welfare are unaffected by rent regulation. Quite to the contrary. In our model, where initial rents are freely set by the landlords, there is a strong positive relation between rent regulation and social welfare. The longer the period for which the landlord has to maintain the rent he has initially agreed upon, the lower is the expected present value of rents and the higher social welfare.[9]

The rest of the paper is structured as follows: Section 2 formally sets out the assumptions of the model and solves it for short-term contracts (renegotiation of rental contracts occurs at the end of every year). Section 3 solves the general case where renegotiation of rental contracts occurs after T years. This more general model admits short-term contracts ($T=1$) and long-term contracts ($T=\infty$) as special cases. Section 4 analyses the welfare issues of the general model. Section 5 gives some concluding comments.

[8]In a recent paper by Basu and Emerson (forthcoming in the *Economic Journal*) the legislation is assumed to fix nominal rather than real rents. Our assumption that real rents are fixed seems to us to be more realistic.

[9]In the modern literature, theorists base their support for well-designed second-generation controls on broadly two arguments. In monopolistic competition models (with heterogeneity and search), the idea is that rent control prevents landlords from exploiting all their market power (see, e.g., Igarashi and Arnott, 2000). In our model the intuition is that rent control prevents the landlord from exploiting the 'hold-up' problem and price discrimination. The second type of argument comes from the contract models (Hubert, 1995). There are good and bad type tenants (observed by landlords after one period) and landlords economically evict bad types after one period. Moving is costly and an evicted tenant will relocate anyway. Rent control prevents economic eviction and thus eliminates the negative externality on other landlords.

2. Short-term contracts

2.1. Landlords

The basic model we use is the ‘circular road’ model of monopolistic competition developed by Salop (1979). N risk neutral landlords, each possessing a apartments, enter the market and locate uniformly on a circle of unit circumference. Thus there are aN apartments in the market and the density of apartments on the circle is also aN, whereas the number of competitors is N (large).[10] The flats are identical in their characteristics: the only source of product differentiation is their location on the circle.

The annual cost of each apartment is f, whether it is occupied or vacant. Owners compete in prices to attract tenants and entry occurs until their expected profits are equal to zero.

2.2. Tenants

There is a population of L (large) tenants, with $L < aN$. At the end of each year, each tenant has a probability λ of dying. The life expectation is thus $1/\lambda$ at any point in the consumer’s life (the process is memoryless). In each year λL tenants die and λL new tenants are born. The population is equal to L in any year, consisting of λL ‘new’ tenants and $(1-\lambda)L$ ‘old’ tenants.

Each tenant has a most preferred location, and these locations are uniformly distributed on the circle. The distance between the most preferred location and the actual location of the unit where he lives is the amount of mismatch and is denoted by d. The unit mismatch cost along the circle is denoted by t. U is the utility derived from occupying a flat for 1 year. We assume that U is sufficiently high so that each tenant ‘consumes’ one flat in each year. When a tenant wants to switch apartments, he incurs fixed moving costs F.

2.3. Search

In the real world it is more difficult to find a flat which corresponds to one’s preferences when there are relatively few vacancies, i.e., when the market vacancy rate is low. When modelling the tenants’ search process we wish to capture this crucial element. There are two simple ways of achieving this result: the first is to have the tenants search not just among the set of all empty flats, but among the set of all flats (including the ones that are occupied). The cost of searching for an

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[10] We introduce the a variable because we want to model competition among N landlords (and not among aN apartments). Note that a is exogenous whereas N is endogenous. Each landlord will always have some empty apartments; as a result the number of competitors active in the market is always equal to N. This considerably simplifies the equations.

empty flat then increases as the vacancy rate decreases. The second alternative is to let the tenants search only among the set of empty flats, but to assume that the (per unit) search costs are inversely proportional to the vacancy rate. For technical reasons we will adopt this second approach. If we denote the vacancy rate by V, the per unit search costs are thus equal to s/V.

Ideally, tenants want to live in a location as close as possible to their most preferred location; but search is costly; this trade-off defines the tenant's optimal interval of reservation $\bar{d}_i$.[11] Denote by $\bar{d}_i$ the tenant i's distance of reservation on both sides of his most preferred location. Tenant i's interval of reservation is then of length $2\bar{d}_i$.

The (instantaneous) search process is as follows: we assume that each tenant randomly contacts one of the N landlords. The landlord then randomly selects one of his a apartments and proposes it to the tenant. The information the landlord transmits to the tenant is the rental price he charges and the location of the flat. With these two pieces of information the tenant accepts or refuses the flat. If he accepts, he moves into the flat; if he refuses he keeps on searching.

We assume that at the time tenants search, landlords have already entered the market and set their initial (Nash equilibrium) rents. As we are interested in the symmetric equilibrium, all landlords charge the same price and tenants rationally anticipate this.

2.4. Prices

There are two prices in this model: new tenants pay the 'first year' price p_1; the price charged after the renegotiation stage is called the 'second year' price and denoted by p_2. In each period, λL tenants pay p_1 and $(1-\lambda)L$ tenants pay p_2.

In the market for new tenants, landlords compete in prices. The equilibrium concept is the standard Nash Equilibrium in prices. Each owner sets the rent as an optimal reply to the rents charged by other owners.

In short-term contracts, where tenant protection is nonexistent, rents can be renegotiated at the end of the 'first year'. We assume that at the renegotiation stage the owner makes the tenant a take-it-or-leave-it offer. In doing so he knows the tenant's expected lifetime, moving costs and most preferred location. The 'second year' price is influenced by the following factors: first, the fixed moving costs and search costs that a tenant would have to incur, were he to move, allows for a rent increase known as the hold-up. Second, by assumption, the owner observes his tenant's most preferred location at the end of the 'first year': this is an important piece of information as it tells the owner the intensity of his tenants' preferences. The owner can thus price discriminate among tenants of different types during renegotiation, something he could not (by assumption) do with new tenants. The

[11] Lippman and McCall (1976) have shown that the optimal search behavior in such a setting is a sequential search (stopping rule).

'second year' price will be a function of the tenant's distance from his most preferred location; $p_2 = p_2(d)$.

Since the landlord has rational expectations, he can derive some information on the distribution of tenants' preferences at the time of initial contracting. The landlord knows the tenants' search behavior and thus infers that a tenant that accepts his flat is at most at a distance $\bar{d}_i$ of his most preferred location. At the time of initial contracting, landlords rationally expect their new tenants' preferences to be uniformly distributed on the interval $[-\bar{d}_i, \bar{d}_i]$.

2.5. Equilibrium

The model analysed is a four stage game. We use the concept of Perfect Nash Equilibrium and are interested in the properties of the steady state, symmetric and rational expectations equilibrium.

In the first stage of the game the number of owners, N (each owning a apartments), is determined by a zero profit condition. In the second stage the owners compete in prices to attract tenants by setting their (Nash Equilibrium) 'first year' rents, p_1. In the third stage, new tenants (and those who should decide to leave their apartment) search for an apartment. In the fourth and last stage, the old tenant and the landlord renegotiate the rental contract and set $p_2(d)$. Fig. 1 depicts the sequence taking place in each period of the game.

This game is played in each period and is repeated an infinite number of times. We are interested in the steady state equilibrium, and solve the game by backward induction. For simplicity we ignore discounting.

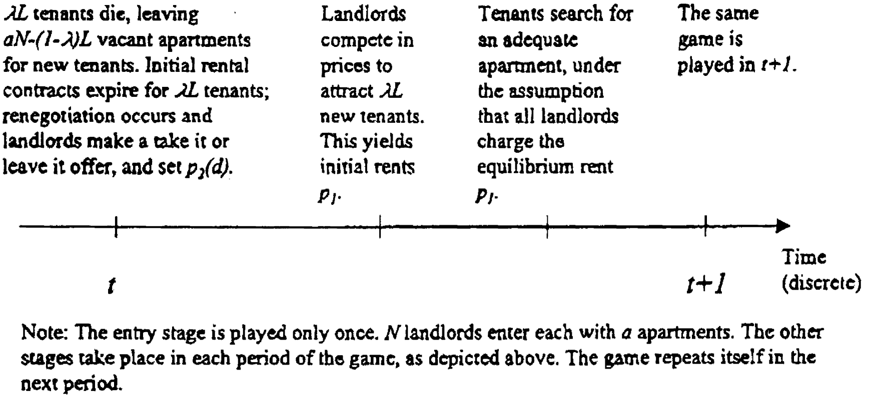

Fig. 1. Period t of the game.

2.6. Solving the model

2.6.1. The fourth stage: renegotiation

Since the probability of a tenant dying at the end of any given year is by assumption constant (and equal to λ), the incentives for a tenant to leave an apartment at the end of any year in time are always the same (in a steady state). Similarly for an owner, the incentives to increase the rent at any end of year are also constant. As a result, the conditions of renewal renegotiated at the end of the 'first year' will be maintained for all the following years. Both the owner's and the tenant's outside options do not change over time (by construction): tomorrow's problem is the same as today's. In essence the short-term contract model is thus a two period model. The reason why we do not limit the tenant's lifetime to two periods is because in Section 3 the longer lifetime allows us to do comparative statics with respect to the duration of rental protection.

The problem the landlord solves when renegotiating at the end of the 'first year' with each of his tenants, is the following: maximize the rent increase given the tenant's alternative to move. At the end of the 'first year', if the tenant stays in his flat, located at distance d of his most preferred location, he has expected (lifetime) surplus U^S:

$$U^S = \frac{1}{\lambda}(U - td - p_2(d)) \tag{1}$$

He gets the utility of occupying a flat minus mismatch cost minus the rent paid, multiplied by his expected lifetime.

The tenant's outside option is moving. If he does so, the costs he incurs are of four types: (a) expected rent paid per year $E(P)$, (b) expected mismatch costs per year $E(Mis)$, (c) expected search costs $E(S)$ and (d) certain moving cost F. The utility of occupying a flat U, expected rent $E(P)$, and expected mismatch costs $E(Mis)$ are incurred in each year, whereas expected search costs $E(S)$ and certain moving cost F are incurred only once. If the tenant were to move at the end of the 'first year' his expected utility U^M, valued at the moment of the renegotiation, would thus be

$$U^M = \frac{1}{\lambda}(U - E(P) - E(Mis)) - E(S) - F \tag{2}$$

The tenant will stay in his current flat if and only if the surplus derived from staying is at least as large as the outside option of moving, i.e., if and only if $U^S \geq U^M$. The landlord, knowing this, will set $p_2(d)$ so that the tenant is just indifferent between staying and moving: this is the take-it-or-leave-it offer. $p_2(d)$ is thus defined by the condition $U^S = U^M$:

$$p_2(d) = E(P) + E(Mis) + \lambda E(S) + \lambda F - td \tag{3}$$

$p_2(d)$ is a decreasing function of d with slope $-t$. As of the 'second year' the owner extracts all the surplus a tenant gets from living in a unit closer to his most preferred location. This is, of course, a form of price discrimination.

The fact that the owner can practice price discrimination as of the 'second year' has consequences for the tenant's search behavior. He will rationally anticipate that finding a flat well matched to his preferences will yield him an additional surplus only for the 'first year'.

2.6.2. The third stage: search

Tenants can (potentially) search for a new flat for two different reasons: because they have only just entered the market and need a flat, or because they might decide to leave their current flat and move to a new one. Given the assumption of a constant probability of dying, the search problems of these two groups of tenants are the same. As we look for a symmetric equilibrium in our model, we take the price distribution of 'first year' rents as degenerate, and tenants rationally anticipate this.The tenant's interval of reservation $\bar{d}_i$ is defined by the following trade-off. If the interval of reservation is high then the expected mismatch cost is high, but the expected number of searches is low.

As the probability of finding an adequate flat with any given draw is equal to $2\bar{d}_i$, the expected number of searches is $1/2\bar{d}_i$. As unit search cost is s/V, total expected search cost is $s/2\bar{d}_iV$. Now tenant i's problem is to choose $\bar{d}_i$ that maximizes his intertemporal expected utility $E\tilde{U}_i$, as seen at the moment he starts searching. We will see that the landlords' optimal 'second year' pricing behavior is such that a tenant who has once moved in will not have an incentive to move (until he dies). Tenant i's intertemporal expected utility $E\tilde{U}_i$ is therefore

$$E\tilde{U}_i = \frac{1}{\lambda}U - p_1^e - \left(\frac{1}{\lambda} - 1\right)p_2^e(d) - \frac{1}{\lambda}\frac{t\bar{d}_i}{2} - \frac{s}{2\bar{d}_iV} \tag{4}$$

The first term is the utility derived from living in the flat. The second and third terms are the expected rents paid during lifetime. The fourth term is expected mismatch cost and the last is expected search cost.

The searching tenant is forward-looking in the sense that he rationally anticipates the fourth stage renegotiation. He knows the landlord will exploit the hold-up problem and practice price discrimination from the 'second year' onwards. As the tenant maximises his expected utility, he expects, on average, to be located in the middle of his interval of reservation, at $\bar{d}_i/2$. Because $p_2(d)$ is linear in d, we have $p_2^e(d) = p_2(\bar{d}_i/2)$. After substitution of (3) into (4) we have:

$$\begin{aligned} E\tilde{U}_i = {} & \frac{1}{\lambda}U - p_1^e - \left(\frac{1}{\lambda} - 1\right)\left(E(P) + E(Mis) + \lambda E(S) + \lambda F - \frac{t\bar{d}_i}{2}\right) \\ & - \frac{1}{\lambda}\frac{t\bar{d}_i}{2} - \frac{s}{2\bar{d}_iV} \end{aligned} \tag{5}$$

Differentiation of $E\tilde{U}_i$ with respect to $\bar{d}_i$ yields the first-order condition:

$$\frac{\partial E\tilde{U}_i}{\partial \bar{d}_i} = \left(\frac{1}{\lambda} - 1\right)\frac{t}{2} - \frac{1}{\lambda}\frac{t}{2} + \frac{s}{2\bar{d}_i^2 V} = 0 \tag{6}$$

Solving (6), we have

$$\bar{d}_i = \sqrt{\frac{s}{tV}} \tag{7}$$

tenant i's optimal interval of reservation.[12] Unsurprisingly, as unit search cost s/V increases, searching becomes less attractive and the tenant is willing to accept a flat in a larger interval. As unit mismatch cost t increases, the intensity of preferences is stronger and the tenant will accept a flat only from a smaller interval. Note that $\bar{d}_i$ is independent of λ because the owner extracts all the surplus the tenant gets from staying in the flat from the 'second year', after the contract is renegotiated (cf. Section 2.6.1).

2.6.3. The second stage: 'First year' rental contracts

To attract tenants, landlords compete in prices. They maximize profits with respect to p_1, the 'first year' rental price, given the tenants' search behavior. As we look for a (symmetric) Nash Equilibrium in p_1, we consider a deviant landlord setting p_{1i}, given that the other landlords set p_1.

When a deviant owner reduces its price from p_1 to p_{1i}, tenants at the margin (i.e., just beyond $\bar{d}^e$, the tenant's interval of reservation as anticipated by the landlord), are willing to accept this flat: in exchange for the higher mismatch costs they get a lower rent.

The question then is: for a given unilateral price decrease Δp by landlord i, what is the interval Δd from which he attracts new tenants? We know that from the 'second year' onwards, the owner extracts the entire tenant's surplus through the hold-up and price discrimination mechanisms. It is thus only in the 'first year' that a tenant can get a utility higher than some critical value, denoted by $\tilde{U}$. The marginal tenant (with most preferred location at distance $\bar{d}^e$ from his residence place) would accept to rent from landlord i only if $U - t\bar{d}^e - p_1 \geq \tilde{U}$. If landlord i now decreases his price to $p_{1i} = p_1 - \Delta p$, some tenants beyond $\bar{d}^e$ would accept his apartment if $U - td_i - p_{1i} \geq \tilde{U}$. Thus by decreasing the price to p_{1i}, landlord i would gain customers from an interval of length $d_i - \bar{d}^e = (p_1 - p_{1i})/t$, or $\Delta d = -\Delta p/t$, on each side. Rewriting this expression yields $d_i = \bar{d}^e + (p_1 - p_{1i})/t$; this we denote it by $d_i(p_{1i})$.

We now have all the elements to specify deviant landlord i's expected profit function (per year). The demand function can be written as $2\int_0^{d_i(p_{1i})} L/2\bar{d}_i N \, \mathrm{d}d$.

[12] The second-order condition for a maximum holds.

Landlord i's expected profit function combines the demand function and the expected rent paid per period, which reads:

$$E\pi_i = 2 \int_0^{d_i(p_{1i})} \frac{L}{2\bar{d}_i N}(\lambda p_{1i} + (1-\lambda)p_2(d))\mathrm{d}d - af \tag{8}$$

Differentiating $E\pi_i$ with respect to p_{1i} yields the first-order condition:

$$\frac{\mathrm{d}E\pi_i}{\mathrm{d}p_{1i}} = \frac{L}{\bar{d}_i N}(\lambda p_{1i} + (1-\lambda)p_2(d_i(p_{1i})))\frac{\mathrm{d}d_i(p_{1i})}{\mathrm{d}p_{1i}} + \int_0^{d_i(p_{1i})} \frac{L}{\bar{d}_i N}\lambda \,\mathrm{d}d = 0 \tag{9}$$

As we look for a symmetric Nash Equilibrium, we have $p_{1i} = p_1$ and thus $\underline{d}_i(\underline{p}_{1i} = \underline{p}_1) = \bar{d}^e$. But $\bar{d}^e = \bar{d}_i$ and $p_1^e = p_1$, using rational expectations. We also set $\bar{d} = \bar{d}^e = \bar{d}_i$. Substituting $\bar{d}_i$ from (7) and of $p_2(d)$ from (3) into (9) yields the first year rent as a function of the structural parameters of the model (for a given number of firms N, which will be determined in the next Section). We proceed similarly for $p_2(d)$:

$$p_1 = \lambda\sqrt{\frac{st}{V}} - (1-\lambda)F \tag{10}$$

$$p_2(d) = (1+\lambda)\sqrt{\frac{st}{V}} + \lambda F - td \tag{11}$$

2.6.3.1. Interpretation

(a) Average Rent Paid. As a useful benchmark let us start off by studying the per-year average rent (located at d), denoted by $\bar{p}(d)$.

$$\bar{p}(d) = \frac{p_1 + \left(\frac{1}{\lambda} - 1\right)p_2(d)}{\frac{1}{\lambda}} = \sqrt{\frac{st}{V}} - (1-\lambda)td \tag{12}$$

One first notes that $\bar{p}(d)$ is independent of F, the fixed moving cost. In spite of the fact that the 'second year' rent increases with the fixed moving cost F, the average (per year or lifetime) rent does not. Owners anticipate that they can increase their rent from the 'second year' onward, and this intensifies 'first year' competition. The resulting rent reduction in the 'first year' just offsets the increase in 'second year' rents. Second, the average rent is a function of the structural parameters of the model. The higher search costs s/V or mismatch costs t, the higher the landlord's monopoly power and the higher the average rent. Third, average rent decreases with distance but only by a factor $(1-\lambda)$ because the owner can price discriminate only from the second year and onward.

It is useful to turn to the discussion of the ‘second year’ and ‘first year’ rents taken individually.

(b) ‘Second Year’ Rents. ‘Second year’ rents increase in moving costs F. The greater the moving cost, the more expensive it is for the tenant to move at the end of the ‘first year’ and the more the owner is able to increase rents. F enters the equation for ‘second year’ rents with a coefficient λ, which can be explained as follows: as the probability of dying λ decreases, the importance of the hold-up problem decreases, because the fixed costs of moving can be spread over a longer expected lifetime.

Slightly less obviously, ‘second year’ rents are also an increasing function of search costs s/V. The reason is that they play the same role as fixed moving costs at the time of renegotiation. The higher search costs, the more expensive it is to move at the end of the ‘first year’, and the greater the hold-up problem, i.e., the rent increase the owner can obtain at the end of year one.

The mismatch cost t enters the price for two reasons. First, the higher t, the greater the tenant’s intensity of preferences, and the higher the owner’s monopoly power. Second, $p_2(d)$ decreases with mismatch costs $(-td)$; in the presence of fixed costs of moving the owners can practice ‘second year’ price discrimination when renegotiating their contracts. This insight is often neglected in the discussion about tenant protection. In practice it seems plausible that the owners can learn quite a lot about the tenants once they have moved in: whether they like the flat or the schools in the area, whether they have made friends with neighbours, etc. It is hard to see why in an unregulated housing market the owners could not make use of this information to extract some of the tenant’s surplus.

(c) ‘First Year’ Rent. The ‘first year’ rent p_1 is increasing in search costs. In models of monopolistic competition, an increase in search costs leads to less competition and higher equilibrium prices.[13] One notes, however, that the term in search costs is an increasing function of λ. The greater is the probability of dying, the higher the influence of search costs on ‘first year’ prices; in the present model search costs have two effects. First, as mentioned, they reduce the intensity of ‘first year’ competition, and second they act as a fixed cost of moving house at the end of the ‘first year’, which allows the owner to increase ‘second year’ prices. Owners anticipate the increase in ‘second year’ prices through search costs, and this increases the intensity of ‘first year’ price competition. As a result of these two effects going in opposite directions, first period prices still do increase with search costs, but this effect gets weaker as life expectancy increases. The $-(1-\lambda)F$ term is due to competition to attract new tenants as owners know they will be able to increase prices from the second year and onwards by a fraction λF.

To get some further insights in the mechanisms that arise in the model, we define the mark-up between the ‘first’ and ‘second year’ rents.

[13]See von Ungern-Sternberg (1982).

(d) The Mark-up. The mark-up $m(d)$ is defined as follows:

$$m(d) = p_2(d) - p_1 = \sqrt{\frac{st}{V}} + F - td \tag{13}$$

It represents the extra price paid by a tenant located at distance d from his most preferred location, from year two and onwards. It is increasing in F because of the hold-up mechanism. s/V and t, which can be interpreted as the landlord's monopoly power increase the mark-up. The $-td$ term is the price discrimination mechanism. The mark-up is independent of λ: whatever the tenant's life expectation, he faces the same mark-up.

Evaluating the mark-up at $\bar{d}$, i.e. for a tenant with the highest mismatch at equilibrium yields the lowest rent increase a tenant can get:

$$m(\bar{d}) = F \tag{14}$$

The mark-up is therefore strictly positive for all tenants. Again this is due to the hold-up: all tenants pay a higher rent (of at least F) from the 'second year' on. Tenants with a smaller mismatch than $\bar{d}$ experience a higher rent increase through price discrimination. Recall however that average rents are independent of F (cf. (a)).

Finally, one can ask whether a landlord having a tenant located at $\bar{d}$ could have an incentive to increase the rent to such an extent that this tenant would leave. The answer is no because if the tenant were to leave, the landlord would simply have one more vacant apartment on his hands. He would not increase his expected revenue as the probability of getting another tenant is determined by the first period rent p_1 (cf. demand function) and not the number of vacant apartments (recall that by assumption each landlord always has some vacant apartments). Thus, even if an owner has a tenant at maximum distance in equilibrium, he earns more by keeping him as long as $p_2(\bar{d}) > 0$. This relationship clearly always holds as $p_2(\bar{d}) = \lambda\sqrt{st/V} + \lambda F > 0$.

2.6.4. The first stage: entry

We now take the last step in solving the model: we determine N, the number of owners, through the free entry condition. In the symmetric, steady state, rational expectations equilibrium we have characterized, the market vacancy rate is $aN - L/aN$ and the occupancy rate is L/aN. The expected revenue a landlord gets from a tenant is a weighted average of first and 'second year' rents. Thus a landlord's expected profit per year is:

$$E\pi = a\left(\frac{aN - L}{aN}*0 + \frac{L}{aN}\left(\lambda p_1 + (1-\lambda)p_2\left(\frac{\bar{d}}{2}\right)\right) - f\right) = 0 \tag{15}$$

where f is the fixed cost of one apartment per year and a the number of apartments of a landlord. $p_2(d)$ is evaluated at $\bar{d}/2$ as it is, on average, the location of a tenant.

This condition defines aN, or equivalently aN/L. Substituting p_1 and $p_2(\bar{d}/2)$ from (10) and (11), (15) reduces to:

$$\frac{aN}{L} = \frac{\lambda + 1}{2f}\sqrt{\frac{st}{V}} \tag{16}$$

But the market vacancy rate V (as seen at the moment of search) is a function of N: $V \equiv aN - (1-\lambda)L/aN$. After substitution of V we can rearrange (16) and write

$$\left(\frac{aN}{L}\right)^2 - (1-\lambda)\frac{aN}{L} - \frac{st(1+\lambda)^2}{4f^2} = 0 \tag{17}$$

This equation has two solutions:

$$\left(\frac{aN}{L}\right)',{}'' = \frac{(1-\lambda) \pm \sqrt{(1-\lambda)^2 + \frac{st(1+\lambda)^2}{f^2}}}{2} \tag{18}$$

One solution (with the minus term) is always negative and thus uninteresting for our model. The second one is always positive as λ is a probability. However, for the solution to make sense we require more than non-negativity: we require $(aN/L)'' > 1$. This condition is equivalent to having $st/f^2 > 4\lambda/(1+\lambda)^2$.[14] This condition always holds for some values of the parameters since $4\lambda/(1+\lambda)^2$ is between 0 and 1 for all $\lambda \in]0,1]$. The condition $st/f^2 > 1$ is thus sufficient to insure that $(aN/L)'' > 1$. Let us now derive some comparative statics results.

Comparative statics. aN increases in L (market size), search and mismatch costs (monopoly power). N decreases with fixed cost f. λ has an ambiguous effect on entry: a high λ increases rents and thus entry, but a high λ means a short life expectancy and thus few years of tenant surplus extraction, which depresses entry.

3. *T* years contracts

In this section we generalise the results of the previous section. Initial rents can now only be renegotiated after T years. When $T = 1$, we have the short-term contract of the previous Section. When $T \to \infty$ we have long-term contracts. We also derive comparative statics results with respect to T. Given the similarity of the structure of this general model with short-term contracts, we limit ourselves to only the essential equations. The interested reader can find a more detailed formulation in Raess and von Ungern-Sternberg (1999).

[14] Our model makes sense only if there is a positive vacancy rate in the market. Our search process would not makes sense if this were not the case. The condition above ensures that N is greater than L for an average rent which permits landowners to cover their fixed cost f.

3.1. The fourth stage: renegotiation

In any year after the T ‘first years’ the renegotiation problem is the same for both the tenant and the landlord. Thus the renegotiation realized after T years will hold forever after. However, the model is no longer a 2-year one (in essence) since we can now do comparative statics with respect to the duration of rent control T.

The problem the landlord faces at the renegotiation stage is to maximize the rent increase given the tenant’s outside option to move. He thus extracts all the surplus the tenant derives from occupying his apartment, from period T onwards. Thus $p_2(d)$ is again given by (3).

3.2. The third stage: search

The tenant’s search problem is in essence the same as in the short-term case (cf. Section 2.6.2). Tenant i maximises his intertemporal expected utility, as seen from the moment when he starts searching. By assumption, the landlord’s pricing behaviour during rent renegotiation is such that the tenant has no incentive to move. His intertemporal expected utility, $E\tilde{U}_i$, is therefore:

$$E\tilde{U}_i = \frac{1}{\lambda}U - \left(\left(\frac{1}{\lambda} - \frac{(1-\lambda)^T}{\lambda}\right)p_1^e + \frac{(1-\lambda)^T}{\lambda}p_2^e(d)\right) - \frac{1}{\lambda}\frac{t\bar{d}_i}{2} - \frac{s}{2\bar{d}_i V} \quad (19)$$

The only difference with short-term contracts (4) is that tenant i pays p_1^e for the T ‘first years’ and $p_2^e(d)$ forever after. However, he pays the ‘first year’ rent p_1^e or the ‘second year’ rent $p_2^e(d)$ only if he still is alive; this is the reason for the coefficients of p_1^e and $p_2^e(d)$.

The searching tenant is forward-looking and rationally anticipates the renegotiation stage after T years: he knows how landlords set rents after renegotiation.The tenant expects, on average, to be located in the middle of his interval of reservation, at $\bar{d}_i/2$, which means $p_2^e(d) = p_2(\bar{d}_i/2)$. Substituting (3) (evaluated at $\bar{d}_i/2$) into (19) and differentiating we obtain the first-order condition:

$$\frac{\partial E\tilde{U}_i}{\partial \bar{d}_i} = \frac{(1-\lambda)^T}{\lambda}\frac{t}{2} - \frac{1}{\lambda}\frac{t}{2} + \frac{s}{2\bar{d}_i^2 V} = 0 \quad (20)$$

Tenant i’s optimal interval of reservation is thus:

$$\bar{d}_i = \sqrt{\frac{\lambda s}{tV(1-(1-\lambda)^T)}} \quad (21)$$

Search and mismatch costs play the same roles as previously. Note that $\bar{d}_i$ is a function of λ as the tenant gets a positive surplus from staying in the flat up to the Tth year. It is also interesting to note that T, the duration of the initial contract enters $\bar{d}_i$ in two ways: through V and through $(1-(1-\lambda)^T)$. But before we can sign the effect of a change in T on $\bar{d}_i$, we must solve for V or equivalently for N, the number of entrants.

3.3. The second stage: T 'first years' rental contracts

Landlord i's maximisation problem is isomorphic to the case of short-term contracts. We thus adopt the same approach as in Section 2.6.3. Landlord i's expected profit function per period reads:

$$E\pi_i = 2\int_0^{d_i(p_{1i})} \frac{L}{2\bar{d}_i N}\big((1-(1-\lambda)^T)p_{1i} + (1-\lambda)^T p_2(d)\big)\mathrm{d}d - af \tag{22}$$

Evaluating the first-order condition at the symmetric equilibrium we obtain:

$$p_1 = \sqrt{\frac{\lambda st}{V}(1-(1-\lambda)^T)} - \frac{(1-\lambda)^T\lambda}{1-(1-\lambda)^T}F \tag{23}$$

$$p_2(d) = \sqrt{\frac{\lambda st}{V}}\left(\sqrt{1-(1-\lambda)^T} + \frac{1}{\sqrt{1-(1-\lambda)^T}}\right) + \lambda F - td \tag{24}$$

3.3.1. Interpretation

The interpretation of the 'first' and 'second year' rents, here before T and after T, is basically the same as in the short-term case (cf. Section 2.6.3): with regards to price discrimination, search, mismatch and fixed moving costs, the same mechanisms are at work. We therefore focus our discussion on the effects of the duration of tenant protection on rents.

(a) 'First Period' Rents (or rents before T). As T increases, p_1 increases:[15] when the duration of tenant protection increases, renegotiation is postponed. As life expectancy is constant $(1/\lambda)$ there are fewer periods for tenant surplus extraction, thus decreasing competition to attract new tenants, which leads to higher initial rents. Two terms can be identified in (23). The first is $\sqrt{\lambda st/V\ (1-(1-\lambda)^T)}$ and is a function of search and mismatch costs. We call it the monopoly power term. The second is $-(1-\lambda)^T\lambda/1-(1-\lambda)^T\ F$ and represents the hold-up. Taken individually, both terms increase as the duration of tenant protection increases, the first because competing landlords know that postponing renegotiation leaves them less time for surplus extraction which drives initial rents up. The hold-up term also increases initial rents, as initial rent reductions decrease when renegotiation is postponed.

(b) 'Second Period' Rents (or rents after T). As T increases $p_2(d)$ decreases,[16] because when T increases the tenant's outside option improves. If he moves, there

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[15] This result is shown in Appendix A.

[16] This result is also shown in Appendix A.

are more periods with 'first year' rents, which disciplines the landlord's pricing behaviour during renegotiation. Rents after T can be decomposed in four elements. The first (monopoly power term) is common to p_1 and suggests $p_2(d)$ is p_1 plus a mark-up (see below). The second term is $\sqrt{\lambda st/V(1-(1-\lambda)^T)}$ and decreases as T increases. That is where the disciplinary effect of the tenant's improved outside option comes in as T rises. The third term, λF is the hold-up and drives rents up, but is independent of T. Finally, a fourth term, $-td$ is price discrimination. From the Appendix C, we know that $\partial \bar{d}/\partial T < 0$. This means that in equilibrium tenants decrease their optimal interval of reservation when tenant protection rises, which increases rents after T, through price discrimination. But the global effect on rents after T is negative when T rises; the disciplinary effect thus dominates the three other terms.

(c) The Mark-up. The mark-up is defined as the difference in rents before and after renegotiation. Formally, we have:

$$m(d) = p_2(d) - p_1 = \sqrt{\frac{\lambda st}{V(1-(1-\lambda)^T)}} + \frac{\lambda}{1-(1-\lambda)^T}F - td \tag{25}$$

$m(d)$ decreases as the duration of tenant protection increases. This is the result of two effects going in the same direction. First, as T increases, 'first year' rents increase through less competition to attract new tenants. Second, rents after renegotiation decrease. The difference can thus only be decreasing. Mathematically, $m(d)$ is a sum of three terms. The first, $\sqrt{\lambda st/V(1-(1-\lambda)^T)}$ is the disciplinary effect on rents after T, and decreases with T. The second, $\lambda/1-(1-\lambda)^T F$ is the hold-up and is also decreasing in T. The only mechanism which increases the mark-up as T rises is price discrimination: as T increases, tenants decrease their optimal interval of reservation which allows for a higher rent increase after T. The mark-up therefore decreases rapidly as T increases.

If we evaluate the mark-up at $\bar{d}$, i.e., the lowest possible mark-up at equilibrium, we have:

$$m(\bar{d}) = p_2(\bar{d}) - p_1 = \frac{\lambda}{1-(1-\lambda)^T}F > 0 \tag{26}$$

This inequality holds for all $\lambda \in]0,1]$. It is the hold-up in action: at the end of the T years, all tenants get a rent increase of at least $\lambda/1-(1-\lambda)^T\ F$.

Finally, as in the short term contract case, one can ask whether a landlord having a tenant located at $\bar{d}$ would have an incentive to increase the rental price such that this tenant would leave. The answer is no because if the tenant were to leave, the landlord would simply have one more vacant apartment for ever. The reason is the same as in the short-term case (cf. Section 2.6.3); having one more vacant apartment does not affect the probability of attracting a new tenant. This probability is affected only by p_1 (cf. demand function). The landlord earns more by keeping a tenant located at $\bar{d}$ as long as $p_2(\bar{d}) > 0$. This relationship clearly always holds as $p_2(\bar{d}) = \sqrt{\lambda st(1-(1-\lambda)^T)/V + \lambda F} > 0$.

The derivation of further comparative statics results unfortunately tends to get quite messy in terms of algebra. We therefore prefer to simply state the results. The interested reader can refer to Raess and von Ungern-Sternberg (1999) where all the algebra is set out explicitly.

3.4. The first stage: entry

The number of landlords entering the market is once again determined by the free entry condition. Just as in Eq. (15) the landlords expected profit is determined by three components: the probability of an apartment being vacant ($aN - L/aN$), the probability of an apartment being occupied by a 'first year' tenant (L/aN $(1-(1-\lambda)^T)$) and the probability of the apartment being occupied by a 'second year' tenant (L/aN $(1-\lambda)^T$). The resulting expression is once again a quadratic equation with only one positive solution.[17]

3.4.1. Comparative statics

We briefly state the comparative statics results for aN, the equilibrium number of owners. With respect to market size, fixed costs, search and mismatch costs, the results are as before. λ, the probability of dying has an ambiguous effect on entry: a high λ increases rents and thus entry, but a high λ means a short live expectation and thus fewer expected years of surplus extraction, which depresses entry.

The last and most interesting variable for comparative static analysis is T. It is shown in Appendix B that $\partial(aN)/\partial T < 0$. As the duration of tenant protection rises, the number of entrants decreases, or equivalently, the vacancy rate decreases. As a corollary average rents paid by tenants decrease when T increases, i.e., $\partial\bar{p}(\bar{d}/2)/\partial T < 0$. The reasons are as follows. Combining (23) and (24) the average rent can be written as

$$\begin{aligned}\bar{p}\left(\frac{\bar{d}}{2}\right) &= (1-(1-\lambda)^T)p_1 + (1-\lambda)^T p_2\left(\frac{\bar{d}}{2}\right) \\ &= \sqrt{\frac{\lambda st}{V}(1-(1-\lambda)^T)}\left(1 + \frac{(1-\lambda)^T}{1-(1-\lambda)^T} - \frac{1}{2}\,\frac{(1-\lambda)^T}{1-(1-\lambda)^T}\right)\end{aligned} \tag{27}$$

[17] The interested reader is again referred to Raess and von Ungern-Sternberg (1999).

Of course, (27) could be further simplified, but we want to look at the three mechanisms affecting average rents. First, we note that average rents are independent of fixed moving costs, as initial rent decreases are such that they just compensate rent increases due to moving costs after renegotiation. The first term in (27) is the monopoly power term: as competition to attract tenants is reduced when the duration of tenant protection rises, average rents increase. The second term is the disciplinary effect, with a coefficient $(1-\lambda)^T$. The landlords' response to an improved outside option for tenants as T increases is a lower rent increase at the moment of renegotiation and this decreases average rents. The third term is the price discrimination effect. It increases average rents as T rises, because tenants' response to longer tenant protection is a reduction in their optimal interval of reservation. However, the term that dominates in the derivative $\partial\bar{p}(\bar{d}/2)/\partial T$ is the disciplinary effect arising from the improved tenants' outside option. The stronger the tenant protection, the better the situation of the tenant from the rental price point of view. The situation of a landlord is unaffected as his expected profit is always zero, whatever the duration of tenant protection: it is only the number of landlords who enter that is affected by T.

The situation can then be summarized as follows. If the duration of tenant protection is high, average rents are low and the number of apartments is low, leaving consumers with little choice of dwellings. However, the average matching quality is good because tenants' search is extensive. On the other hand, if tenant protection is nonexistent, average rents are high, as is entry. This leaves tenants with more choice for dwellings, but mismatch is important because tenants' search is low. This naturally leads us to do a welfare analysis of the different types of contracts.

4. Welfare analysis

To study how social welfare depends on the duration of tenant protection we have to study the social cost function. Social cost does not include rents paid by tenants to landlords as they constitute a transfer with no effects on welfare. There are thus three components in the social cost function: landlords' fixed costs of dwellings, tenants' search and mismatch costs. In each period, each landlord incurs fixed costs af for his stock of dwellings. A fraction λL of the tenants (the 'new' tenants) bear search costs of $s/2\bar{d}V$. L tenants each have expected mismatch costs $t\bar{d}/2$. Mathematically, the social cost function (per period), denoted by SC, reads:

$$SC = aNf + \lambda L\frac{s}{2\bar{d}V} + L\frac{t\bar{d}}{2} \tag{28}$$

The question we want to answer is the following: how is social cost affected by a change in T, the duration of tenant protection? Before we sign this derivative, we turn to the individual analysis of the three components of the social cost function.

4.1. Landlords' fixed costs

It has already been pointed out that $\partial aN/\partial T < 0$. The reason is the following: as T increases, average rents paid to landlords decrease (the disciplinary effect dominates) and the number of entrants falls (zero profit condition). The vacancy rate therefore also falls, which means that total fixed costs of landlords fall as the duration of protection increases.

4.2. Tenants' search costs

We want to sign the expression $\partial(\lambda Ls/2\bar{d}V)/\partial T$. After substitution of $\bar{d}$ (21), it can be rewritten as

$$\frac{\partial(\lambda L \frac{s}{2\bar{d}V})}{\partial T} = \frac{\partial\left(\frac{L}{2}\sqrt{\frac{\lambda st(1-(1-\lambda)^T)}{V}}\right)}{\partial T} \tag{29}$$

which is easily signable. If T increases, V falls and $(1-(1-\lambda)^T)$ increases. It directly follows that the derivative in (29) is positive. This means that tenants' search costs increase with T, for two reasons. First, when T increases, the tenants enjoy the surplus from occupying their dwelling for a longer period, which gives them incentives to search more intensively. Second, as the duration of tenant protection raises, entry or equivalently the vacancy rate are negatively affected, which further increases tenants search costs.

4.3. Tenants' mismatch costs

Finally, we want to concentrate on the $\partial(L(t\bar{d}/2))/\partial T$ term. By substituting $\bar{d}$ from (21), we get

$$\frac{\partial\left(L\frac{t\bar{d}}{2}\right)}{\partial T} = \frac{\partial\left(\frac{L}{2}\sqrt{\frac{\lambda st}{V(1-(1-\lambda)^T)}}\right)}{\partial T} \tag{30}$$

Signing this expression is not straightforward as there are two effects going in opposite directions. First, when T increases, tenants enjoy a surplus from occupying their flat for longer. This gives them an incentive to search for a better match, i.e., to decrease their interval of reservation. This is the $(1-(1-\lambda)^T)$ term and as T rises, mismatch costs decrease. However, increasing tenant protection also negatively affects the vacancy rate V, and this tends to increase mismatch costs. The total effect thus seems ambiguous. However, it is shown in Appendix C that $\partial(L(t\bar{d}/2))/\partial T < 0$. An increase in the duration of tenant protection unambiguously lowers tenants' mismatch costs. The direct effect of the duration of protection on mismatch costs dominates the indirect effect through the vacancy rate.

To see whether social cost increases or decreases with the duration of tenant protection one thus has to check which of these three effects dominates. It can be shown that $\partial SC/\partial T < 0$.[18] Social cost unambiguously decreases as the duration of tenant protection increases. In other words, as T increases, the rise in search costs is more than offset by the fall in both landlords' fixed costs and tenants' mismatch costs.

5. Conclusion

If one wishes to understand the mechanisms at work in imperfectly competitive markets, it is necessary to have models which capture the most important characteristics of the market under examination. In the present paper we have analysed a model of the rental housing market which concentrates on the following special features of this market: monopolistic competition (product heterogeneity), imperfect information (tenant search), switching costs (the hold-up problem) and ex post price discrimination.

We have shown that government intervention which prevents landlords from fully exploiting the potential of the hold-up problem and price discrimination would lead to lower equilibrium rents and higher social welfare. This conclusion is similar to the one reached by other housing economists using models of imperfect competition or contracts. They also emphasize the role for government intervention to correct (at least partially) the perverse effects of the special characteristics present in the housing market.

In our model the driving forces behind this result are as follows: Tenant protection (an increase in the duration of rental contracts) retards the moment when owners can use their bargaining power to extract the tenants' surplus and thus induces tenants to search more intensively for a well matched apartment. This more intensive search leads to lower equilibrium prices and a lower vacancy rate.

[18] As the algebra for this result is quite long, it is not reproduced here. However, it is shown in Raess and von Ungern-Sternberg (1999).

The net effect is an increase in social welfare. The most innovative part of our model is the fact that we have explicitly incorporated the possibility of ex post price discrimination into our model, i.e., the fact that owners may exploit information they have acquired about their tenants when negotiating rent increases.

A more general way of formulating the essential message of our model would be as follows: There are two ways to protect tenants from being exploited at the stage of rent renegotiation: either they are offered legal protection or they must have sufficiently attractive outside options (a high vacancy rate). Having a large stock of unoccupied apartments at any given point in time is however associated with a very high capital cost. It is thus unsurprising that intelligent regulatory intervention can achieve the same end at a considerably lower social cost.

Certain adjustments have to be made to apply our model to the real world. In particular, our model does not incorporate inflation or depreciation. Real rents are thus constant after renegotiation. A natural real world interpretation of our analysis would be that rents should be indexed by the consumer price index, or a rule of the type ‘RPI-X’.[19] The term X would capture productivity gains in the housing sector and/or take into account the fact that the quality of an apartment depreciates over time.

Appendix A

Comparative statics: p_1 and $p_2(d)$

We first show that $\partial p_1/\partial T > 0$, and then that $\partial p_2(d)/\partial T < 0$. p_1 is given by (23) and we define $x = (1-\lambda)^T$. Differentiating p_1 with respect to T yields:

$$\frac{\partial p_1}{\partial T} = \sqrt{\lambda s t}\frac{1}{2}\left(\frac{1-x}{V}\right)^{-1/2}\frac{-\frac{\partial x}{\partial T}V-(1-x)\frac{\partial V}{\partial T}}{V^2} - \lambda F\frac{\frac{\partial x}{\partial T}(1-x)-x\left(-\frac{\partial x}{\partial T}\right)}{(1-x)^2} \tag{A.1}$$

This derivative is easy to sign. As $x=(1-\lambda)^T$, $\partial x/\partial T = (1-\lambda)^T \ln(1-\lambda) < 0$ since λ is a probability. We already showed that $\partial V/\partial T < 0$. We thus have all the elements to unambiguously sign (A.1): $\partial p_1/\partial T > 0$.

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[19] RPI, Retail Price Index. RPI-X is the rule used for regulating public utilities in UK.

We now turn to the determination of the sign of $\partial p_2(d)/\partial T$. Similarly here, we rewrite $p_2(d)$ given in (24), with $x=(1-\lambda)^T$ and differentiate this expression with respect to T:

$$\frac{\partial p_2(d)}{\partial T}=\sqrt{\lambda st}\left(-\frac{1}{2}\right)V^{-3/2}\left\{\frac{\partial V}{\partial T}\left[(1-x)^{-1/2}+(1-x)^{1/2}\right]\right.$$
$$\left.+V\frac{\partial x}{\partial T}\left[-(1-x)^{-3/2}+(1-x)^{-1/2}\right]\right\} \tag{A.2}$$

The problem is to sign the term in the brackets {}. We have already determined the sign of this expression in the welfare analysis in Raess and von Ungern-Sternberg (1999); it is positive. Multiplying it by the negative term $\sqrt{\lambda st}(-1/2)V^{-3/2}$ directly yields the result $\partial p_2(d)/\partial T<0$.

Appendix B

Comparative statics: $\bar{p}(\bar{d}/2)$ and aN/l

We show that $\partial\bar{p}(\bar{d}/2)/\partial T<0$. Note this is equivalent to showing that $\partial(aN/L)/\partial T<0$, because of the free entry condition, that rewrites $\bar{p}(\bar{d}/2)=f^{*}aN/L$. aN/L is explicitly calculated in Raess and von Ungern-Sternberg (1999) and differentiation with respect to T yields after simplification:

$$\frac{\partial\left(\frac{aN}{L}\right)}{\partial T}=\frac{\lambda st(2-(1-\lambda)^T)}{4f^2\sqrt{(1-\lambda)^2+\dfrac{\lambda st(2-(1-\lambda)^T)^2}{f^2(1-(1-\lambda)^T)}}}*\frac{(1-\lambda)^{2T}\ln(1-\lambda)}{(1-(1-\lambda)^T)^2}$$
$$<0 \tag{B.1}$$

This expression is easy to sign: as $\lambda\in[0,1]$, all the terms are positive, except for $\ln(1-\lambda)$ that is negative. Thus $\partial(aN/L)/\partial T<0$ which directly implies $\partial\bar{p}(\bar{d}/2)/\partial T<0$.

Appendix C

Comparative Statics: Mismatch costs

We want to sign $\partial(Lt\bar{d}/2)/\partial T$. Substituting V and differentiating with respect to T, (30) can be rewritten:

$$\frac{\partial\left(L\frac{t\bar{d}}{2}\right)}{\partial T}=-\frac{L}{4}\sqrt{\frac{(\lambda st)^3}{V^3(1-(1-\lambda)^T)^5}}*$$
$$\frac{(1-\lambda)^T\ln(1-\lambda)(2-(1-\lambda)^T)}{\left[(1-\lambda)+\sqrt{(1-\lambda)^2+\frac{\lambda st(2-(1-\lambda)^T)^2}{f^2(1-(1-\lambda)^T)}}\right]^2 f^2\sqrt{(1-\lambda)^2+\frac{\lambda st(2-(1-\lambda)^T)^2}{f^2(1-(1-\lambda)^T)}}}*$$
$$\left[(1-\lambda)^{T+1}-(2-(1-\lambda)^T)\sqrt{(1-\lambda)^2+\frac{\lambda st(2-(1-\lambda)^T)^2}{f^2(1-(1-\lambda)^T)}}\right] \tag{C.1}$$

The first line of this expression is negative. The second line of this expression is also negative. It can be shown that simplification of the third line reduces to:

$$-4\frac{(1-\lambda)^2}{(2-(1-\lambda)^T)^4}(1-(1-\lambda)^T)^2-\frac{\lambda st}{f^2}<0$$

which always holds. Thus, $\partial(L(t\bar{d}/2))/\partial T$ is the product of three negative terms which is negative. In words, an increase in the duration of tenant protection T reduces mismatch costs as it reduces tenants' optimal interval of reservation.

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