Product-Mix Decisions Under Activity-Based Costing With Resource Constraints And Non-proportional Activity Costs

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Abstract

Companies adopting activity-based costing as a management and decision-making tool should be aware of its potential drawbacks. Activity-based costing relies heavily on the assumption of proportional activity cost structures. Further, it ignores resource and technological constraints. I argue in this article that both problems can be addressed through the integration of the activity-based costing with operational decisions based on Goldratt's Theory of Constraints, (TOC). The proposed approach offers the added benefit of mitigating the short-run bias of the TOC approach in product-mix decisions.

Introduction

Activity-Based Costing is widely recognized as a management decision tool in identifying profitable and non-profitable products. It provides a systematic approach to analyzing overhead and fixed operating costs and to identifying non-value added activities, processes and products. Over the course of the last decade many corporations have successfully used ABC systems to overhaul their costing procedures and to streamline their operating processes.

Further development of activity-based costing may, however, be hampered by two theoretical drawbacks. First, activity-based costing, as currently practiced, ignores resource constraints. ABC advocates correctly argue that overhead and operating costs of companies have to be clearly understood and non-value added processes eliminated. However, under ABC the distinction between value added and non-value added activities and products are made absolute as if management faces no constraints. In practice, in most companies struggle to manage their constrained resources so they can increase production and sales turnover. This kind of operational focus may result in product-mix decisions that may appear irrational from an ABC perspective. Products with low profit margins may continue to be produced where highly profitable products, as identified by the company's ABC system, may be dropped. This disconcerting paradox is the result of ABC's inability to account for resource constraints.

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The solution to this problem should be sought in the theory of constraints. The fundamental thesis of TOC is that production processes are interdependent and the performance of any production system is determined by its slowest process (see Goldratt and Cox, 1986; and Goldratt and Fox, 1987). According to TOC, managers should focus their attention on removing bottleneck processes. TOC can thus be viewed as a *short-run* optimization procedure for managing resources and opening bottlenecks with the goal of maximizing throughput (defined as the difference between money coming into the company with money paid to outside suppliers, subcontractors and salespeople). However, contrary to the ABC, TOC treats overhead and operating expenses as given. The two approaches appear to complement one another in addressing short-run operational and long-run cost management problems. In a recent article on integrating the two approaches, Robert Kee (1995) writes: "The strengths of ABC and TOC are complementary in nature. The strengths of each model overcomes a major limitation of the other."

Taking advantage of this complementarity and of the mixed-integer programming technique[1], Kee (I 995) integrates the two approaches and explicitly introduces technological, and resource constraints into the ABC framework. However, Kee falls short of utilizing the full potential of the mixed-integer programming technique in overcoming yet another important limitation of ABC approach, namely, its underlying proportionality assumption. The proportionality assumption implies that average costs remain constant as volume increases. However, where economies of scale are present average costs may decrease, and where production is at or close to maximum capacity average costs may be increasing. Thus, the proportionality of costs for each and every single activity in a company is a strong with little empirical support that seriously curtails the application of ABC (see Noreen, 199 1; Noreen and Soderstrom, 1994; Datar and Gupta, 1994). The present article extends Kee's work by introducing product-mix decisions and non-linear activity costs into the ABC framework. The proposed approach offers a readily accessible improvement over both ABC and TOC in strategic decision-making and cost management. The proposed approach will be described using the following numerical illustration.

A Numerical Illustration

Company XYZ produces two products: X and Y. Both products go through four processing stages (departments). The company aims at maximizing its throughput under a TOC framework. The initial data of the problem including maximum capacity of each department and the usage of its resources by the two products are shown in Table 1. Also shown are a graph of the resource constraints and the feasible production region of XYZ and potential product-mixes A, B, C, D and E. XYZ's optimum product-mix and its maximized throughput can be determined using basic linear programming. The mathematical formulation of this problem, hereafter called Problem 1, is shown in Table 2. The optimum product-mix of XYZ includes 9,000 units of X and 4,000 units of Y (Point D). The maximum throughput, obtained at point D, is $305,000.

Observe that under TOC (Problem 1), operating expenses are treated as fixed. This treatment forces throughput to be independent of operating expenses. However, as the ABC literature has correctly emphasized many operating costs, frequently believed to be fixed, vary with respect to certain cost drivers. Ignoring variable operating expenses may have serious consequences for product-mix decisions. To demonstrate this point, assume that the largest share of XYZ's operating expenses are due to a single activity (say, the setup activity) with a variable or mixed cost structure. Based on the information in Table 1, product-mix D requires 49 setups with a total cost of $107,800 and an average cost of $2,200. Product X, however, consumes

TABLE 1
XYZ's Products and Departments

	Product X	Product Y	Maximum capacity per period (hrs)
Throughput per unit	$25	$20	
Batch size	200	1,000	
Number of setups per batch	1	1	50
Processing time in department 1 (hours per unit)	1	4	40,000
Processing time in department 2 (hours per unit)	4		61,000
Processing time in department 3 (hours per unit)	6	5	74,000
Processing time in department 4 (hours per unit)	4	1	40,000

five times more setup activities compared to product Y. Consequently, as output-mix varies so does the total number of the setups required. Given a variable set up cost structure, changing the company's output-mix will change its operating expense and profitability. The TOC approach, however, by treating operating expenses as fixed may fail to identify the optimal product-mix and to maximize profits. The solution to this problem is to incorporate activity-based costing into the TOC framework. The integration of ABC and TOC is shown as Problem 2 in Table 3.

In Table 3 setup costs are assumed to be proportional to the number of setups performed[2]. Thus, Problem 1 should be modified to reflect the differential consumption of setup activities by X and Y. The integrated problem maximizes what this study will refer to as modified throughput". Modified throughout equals throughput less setup costs. Observe that two new variables and four new constraints are added to the initial program. The new variables, S1 and S2, represent total number of setups for X and Y, respectively. These are specified as positive integers in the last two constraints. The batch-size con-

TABLE 2
XYZ's Optimization Problem
And Feasible Production Region

Problem 1 - Maximize throughput: $25 X + $20 Y

Subject to:	
Dept. 1 Constraint:	$X+4Y \leq 40{,}000$
Dept. 2 Constraint:	$4X+5Y \leq 61{,}000$
Dept. 3 Constraint:	$6X+5Y \leq 74{,}000$
Dept. 4 Constraint:	$4X+Y \leq 40{,}000$
	$X \geq 0$, $Y \geq 0$
Optimum solution:	Product-Mix D
	X= 9,000 units
	Y= 4,000 units
Maximized throughput:	$305,000

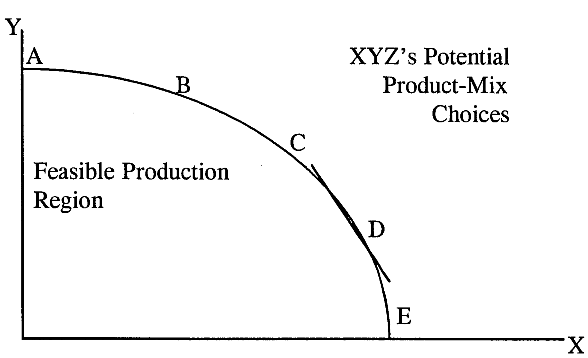

TABLE 3
XYZ's Optimum Product-Mix Under The TOC-ABC Approach

Problem 2- Maximize " modified throughput": $25 X + $20 Y - $2,200 (S1 +S 2), where, S1 and S2 represent the number of setups for the X and Y product lines, respectively.

Subject to:

Dept. 1 Constraint:	$X+4Y \leq 40{,}000$
Dept. 2 Constraint:	$4X+5Y \leq 61{,}000$
Dept. 3 Constraint:	$6X+5Y \leq 74{,}000$
Dept. 4 Constraint:	$4X+Y \leq 40{,}000$
X Batch sizes:	$X-200\ S1 \leq 0$
Y Batch sizes:	$Y-1000\ S2 \leq 0$
Non-negativity constraints:	$X \geq 0$, $Y \geq 0$
Integer constraints:	S1 & S2 are positive
Optimum solution:	Product-mix B
	X= 4,000 units
	Y= 9,000 units
	S1= 20
	S2= 9
Throughput:	$280,000
Maximized modified throughput:	$216,200

straints ensure consistency between the output level, and the number of setups for each product line. For instance, since the batch size for product X is 200 units and each run requires one setup, the number of setups has to equal the number of batches (i.e., $S1 \geq {}^{x}/_{200}$). This yields the fifth constraint. The sixth constraint has a similar interpretation.

The integrated ABC-TOC problem can be solved using mixed-integer programming. Table 3 shows the optimum solution with variable setup costs. The inclusion of setup costs into the TOC problem results in product-mix B becoming optimal. Product-mix B is made up of 4,000 units of X and 9,000 units of Y. This is the exact opposite of what was considered optimal under the TOC approach- While X is the most profitable product identified using the throughput measure, Y exhibits the highest profit-margin using the modified throughput measure. This result establishes the complementarity of the two approaches and the fact that any decision based solely on one or the other approach may be misleading. Also, notice that regardless of the number of activities involved, the above procedure is easy to implement and computer spreadsheets, such as Microsoft Excel, are capable of solving such problems with accuracy and speed.

Non-proportional Activity Costs

The programming framework developed here can be extended by relaxing the assumption of proportional activity cost. Two distinct cost structures will be considered here: (a) a linear structure including a fixed component and (b) a nonlinear structure based on a cumulative average-time learning model. Non-proportional activity cost structures have, for the most part, been ignored by the activity-based costing literature. Where activity costs include a fixed component, the ABC literature suggests that the fixed component should be filtered to a higher category of activity costs.[3] For instance, any fixed costs with respect to unit-related activities are evaluated against batch-related and product-sustaining activities to see if they still behave as fixed costs. If so, they will be included in facility-sustaining activity costs and treated as period costs. The variable component would, as usual, be approximated by a proportional cost structure. However, this ad hoc arrangement is fairly restrictive and reduces the relevance of the costing procedure and of product-mix decisions. Management may be interested in charging all activity costs to products to be able to better identify its profitable products. Treating fixed activity costs as period cost complicates this task. Ideally, the decision-maker should be able to include the best attainable estimate of the activity costs (proportional or otherwise) in the formulation and solution of the product-mix problems. Fortunately, a solution to this problem exists.

TABLE 4
XYZ's Optimum Product-Mix under the TOC-ABC Approach With Non-Proportional Activity Costs

Problem 3 - Maximize "modified throughputs: \$25 X + \$20 Y- \$500 (S1 +S2) - \$83,300.

Subject to:

Dept. 1 Constraint:	$X+4Y \leq 40{,}00$
Dept. 2 Constraint:	$4X+5Y \leq 61{,}000$
Dept. 3 Constraint:	$6X+5Y \leq 74{,}000$
Dept. 4 Constraint:	$4X+Y \leq 40{,}000$
X Batch sizes:	$X-200\ S1 \leq 0$
Y Batch sizes:	$Y-1000\ S2 \leq 0$
Non-negativity constraints:	$X \geq 0$, $Y \geq 0$.
Integer constraints:	S1 and S2 are positive
Optimum solution:	Product-Mix C
	X= 6,500 units
	Y= 7,000 units
	S1= 33
	S2= 7
Throughput:	\$302,500.
Maximized modified throughput:	\$199,200

Continuing the above numerical example, assume that the setup costs of XYZ are made up of \$83,300 fixed costs and of unit variable costs of \$500.[4] Table 4 illustrates the revised problem (Problem 3) and its solution. Under the revised setup cost structure, it can be observed that the optimum product-mix is at point C instead of the extreme point B. The new mix includes 6,500 units of X and 7,000 units of Y. In problem 3, the declining average cost of each setup increases the relative profitability of product X and causes a partial reversal of the shift to product-mix B that was observed in Problem 2.

A similar outcome will be observed when the setup costs follow a learning model. Under this scenario, shown in Problem 4, Table 5, the high setup costs of product X is partially offset by cost savings obtained through the learning experienced in the setup activity. The cost of the first setup in XYZ company Is \$7,701 and setup activity exhibits a learning effect of 80%[5]. In comparison to Problem 3, optimum product-mix remains unaffected at point C, but modified throughput increases from \$199,200 to \$208,558. Once again, the declining average cost of the setup activity due to the learning effect partially offsets the cost advantages of product Y over product X. The flexibility of the proposed technique is worth noting. Since the problem is solved by spreadsheet or mathematical software using numerical methods, just about any type of activity cost function can be accommodated and the system design need not be restricted to proportional cost functions.

Summary and Conclusion

Table 6 presents a summary of the XYZ's product-mix choices and potential throughput for each choice under the four competing setup cost formulations. It demonstrates the potential for an incorrect product-mix decision under the Theory of Constraint. Using the TOC approach, product-mix C is identified as the optimal choice (Problem 1). Incorporating activity-based costing into the decision problem and accounting for variable total setup costs, it is shown that product-mix B is the optimal choice (Problem 2). TOC's incorrect choice stems primarily from its treatment of setup and other operating costs as fixed costs. ABC's solution, in turn, may be incorrect due to its highly restrictive assumption of proportionality of total (or constancy of average) activity costs. Non-proportional cost structures are examined in Problems 3 and 4. In both situations, average setup costs exhibit a declining pattern making product-mix C the optimal choice. The mixed-integer programming method can therefore be readily utilized to improve product-mix decisions in the context of TOC problems.

Companies adopting activity-based cost-

TABLE 5
XYZ's Optimum Product-Mix Under The TOC-ABC Approach With Activity Costs Exhibiting a Learning Effect

Problem 4 - Maximize "identified throughput": $25 X + $20 Y- $7,701 (S1 +S2)$^{.678}$

Subject to:

Dept. 1 Constraint:	$X+4Y \leq 40{,}000$
Dept. 2 Constraint:	$4X+5Y \leq 61{,}000$
Dept. 3 Constraint:	$6X+5Y \leq 74{,}000$
Dept. 4 Constraint:	$4X+Y \leq 40{,}000$
X Batch sizes:	$X-200\ S1 \leq 0$
Y Batch sizes:	$Y-1000\ S2 \leq 0$
Non-negativity constraints:	$X \geq 0$, $Y \geq 0$.
Integer constraints:	S1 & S2 are positive
Optimum solution:	Product-mix C
	X= 6,500 units
	Y= 7,000 units
	S1 = 33
	S2= 7
Throughput:	$302,500
Maximized modified throughput:	$208,558

ing as a management and decision-making tool face certain drawbacks. Activity-based costing relies heavily on the assumption of proportional cost structures and ignores resource constraints. These problems limit the power of ABC as a management decision tool. Both problems can, however, be solved through an integration of the activity-based costing with the Theory of Constraints and the application of the method of mixed-integer programming. Additionally, the technique provides a solution to the short-term focus of the TOC and the adverse consequences of it for strategic decision-making.

Suggestions for Future Research

Future research in the area of cost system design can benefit from empirical documentation of diverse activities in different industries. Additionally, field studies providing examples of operational constraints to be incorporated in cost systems can considerably enrich the ABC and TOC literature. 🕮

Footnotes

1 Mixed-integer programs are mathematical programs involving real and integer variables. For a theoretical outline of this technique see, Hillier and Lieberman (1990). Mixed-integer programs can be readily set up and solved using most linear programming software. However, some spreadsheet software, such as Microsoft Excel, provides an easy and widely accessible alternative. The mix-integer programming technique has previously been used in the accounting literature to examine similar issues. For a recent example see Kee (1995).

2 Notice that this standard procedure for determining activity drivers rates is based on the following strong and restrictive assumptions. First, the activity costs include no fixed component. Alternative, it may be assumed that the fixed component has is treated as period costs and not charged to any product. Second, the variable component of activity costs exhibit proportionality or constancy of average costs. Third, all activities included in a single activity pool satisfy the above properties. Furthermore, the single activity driver identified for the activity pool perfectly explains cost variations for each activity included in that pool.

3 See, for instance, Robin Cooper, "Cost Classification in Unit-Based and Activity-Based Manufacturing Systems," Emerging Practices in Cost Management, ed. Barry J. Brinker, (Boston: Warren, Gorham & Lamont, 1990)

4 As in Problem 1, the new cost structure will yield an average cost of $2,200 for 49 setups. In effect, it is being assumed that the designer of the cost system is relying

TABLE 6
XYZ's Modified Throughput For Alternative Product-Mixes

	Product Mix A	Product Mix B	Product Mix C	Product Mix D	Product Mix E
Volume of X	0	4,000	6,500	9,000	10,000
Volume of Y	10,000	9,000	7,000	4,000	0
Setups for Product line X	0	20	33	45	50
Setups for Product line Y	10	9	7	4	0
Throughput - Problem 1	200,000	280,000	302,500	305,000	250,000
Total setup costs are fixed.					
Modified throughput - Problem 1	92,200	172,200	194,700	197,200	142,200
Modified throughput - Problem 2	178,000	216,200	214,500	197,200	140,000
Proportional setup costs.					
Modified throughput - Problem 3	111,700	182,200	199,200	197,200	141,700
Non-proportional setup cost.					
Modified throughput - Problem 4	163,304	204,463	208,558	197,200	140,712
Non-proportional and nonlinear setup costs.					

on a single observation of setup costs to extrapolate outside the relevant range. While descriptive of the real-world practice, this is clearly an unreliable cost estimation procedure since an infinite number of cost functions may satisfy a single cost observation. When more than one observation is used, care must be taken to ensure that they span a wide range of activity levels to avoid the same problem. The ABC literature is generally silent on the issue of proper procedures for estimating activity costs and driver rates.

5 Assuming an 80% learning effect, average cost of each setup is: \$7,701 $(S1+S2)^{-0.321928}$ where, -0.321928 is given by log(.80) over log(2). The total cost of (S1 +S2) setups included in the objective function of Problem 4, is: \$7,701 $(S1+S2)^{-0.321928}$ $(S1+S2)$ = \$7,701 $(S1+S2)^{0.678}$

References

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3 Hillier, F.S., and G. J. Lieberman *Introduction to Operations Research* (McGraw-Hill, NY: 1990.

4 Robert Kee "Integrating Activity-Based Costing with the Theory of Constraints to Enhance Production-Related Decisions." *Accounting Horizons* (December): 48-6 1.

5 Noreen, E. 199 1. Conditions under which activity-based costing systems provide relevant costs, *Journal of Management Accounting Research* (Fall): 159-168.

6 ________ and N. Soderstrom. 1994. Are overhead costs strictly proportional to activity') *Journal of Accounting and Economics* (1994): 255-278.