Optimal clearing margin, capital and price limits for futures clearinghouses

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Abstract

We provide a model for a futures clearinghouse to use for setting optimal levels of clearing margin, capital and price limits, which minimizes the costs to clearing firms and simultaneously protects the clearinghouse from default by clearing firms. We show how to estimate the capital requirement, which supports the clearinghouse's residual default risk that is not covered by the clearing margin. We apply our model to the Winnipeg Commodity Exchange and demonstrate that price limits reduce the sum of optimal clearing margin and capital to a level that is substantially lower than that required in the absence of price limits.
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1. Introduction

Futures clearinghouses guarantee the open futures positions of clearing firms, and manage the default risk resulting from this guarantee, by requiring that

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clearing firms deposit margin, by marking open futures positions to market daily, and by calling for additional margin, if clearing firms' daily losses exceed their margin. If a clearing firm's loss is less than its margin, the clearinghouse simply transfers funds from the losing firm to winning clearing firms. However, if a clearing firm's loss exceeds its margin, and it defaults, the clearinghouse will have to fulfill its guarantee. Therefore, the clearinghouse requires a capital contribution from clearing firms, in the form of a security deposit (Chicago Mercantile Exchange, 2003), or as a contribution to a fund known as a 'guaranty' (New York Mercantile Exchange, 2003) or as a 'clearing' (Winnipeg Commodity Exchange Clearing Corporation, 2003; Canadian Derivatives Clearing Corporation, 2003) fund, or by a purchase of shares in the clearinghouse (Board of Trade Clearing Corporation, 2002–2003). If a clearing firm cannot meet its financial obligations to the clearinghouse and defaults, the clearinghouse will use the defaulting firm's margin and its capital, in that order, to meet its obligations. Capital contributions of $875 million in 2003 to the four largest futures clearinghouses in the US, and margin contributions many times larger in magnitude (Rosenzweig, 2003), attest that both margin and capital pose substantial costs to clearing firms. However, the near failure of the Hong Kong Futures exchange in 1987 (Cornford, 1996) because its clearinghouse had insufficient funds, emphasizes the need for adequate levels of margin and capital. Therefore, we investigate how a futures clearinghouse can set optimal levels of the clearing margin, capital and price limits, which will minimize the costs to clearing firms and, simultaneously, provide protection against default risk to the clearinghouse.

We use a framework introduced by Brennan (1986), who points out that competition between futures exchanges explains why very few of the many new futures contracts succeed, and concludes that successful contracts will minimize the total costs of futures trading. Brennan models an exchange's choice of the optimal margin and price limits that will minimize traders' total costs, which include the opportunity cost of the margin, the liquidity cost incurred if a price limit is hit and trading is interrupted, and the legal, reputation and other costs that a trader incurs if he reneges on his futures position. A trader has an incentive to renege if his daily loss exceeds his margin. In the absence of price limits, the exchange could minimize the probability of reneging and thus, the cost of reneging, by setting the required margin at high levels. Margin requirements could be reduced by imposing daily price limits. When a price limit is hit, the trader is unable to observe his 'true' loss, and is forced to conjecture what it is. If he believes that it exceeds his margin, he has an incentive to renege. Brennan explains that a futures contract can be made self-enforcing, a property under which all parties to the contract adhere to "its terms without the threat of legal action", by setting the margin and price limits, such that when a price limit is hit, the expected loss to the trader is less than or equal to his margin. Price limits thus act as a substitute for margin, which can now be set at reduced levels.

We extend Brennan's model to the problem that futures clearinghouses face and set the optimal clearing margin, capital and price limits, which will minimize clearing firms' costs. We describe Brennan's model and our extension in Section 2. The

clearing firm's capital contribution supports the residual default risk of the clearing-house, which arises if the firm's daily loss exceeds its margin. If the margin contributed is low, the capital required is high. We estimate the capital requirement as a function of the margin and the probability distribution of uncensored futures prices.

The residual default risk of the clearinghouse depends on the probability distribution of uncensored futures prices that would prevail in the absence of price limits. This is because a losing clearing firm cannot close out its position when the futures price hits a price limit, so that its potential loss depends on uncensored futures prices rather than on observed futures prices which are censored by price limits. If reliable daily data on the futures contract's underlying asset's spot prices and associated carrying costs are available, a model such as the cost of carry model may be used to estimate uncensored futures prices. If such data are unavailable, then the probability distribution of uncensored futures prices have to be estimated by using observed censored futures prices. Section 3 explains how we conduct this estimation, using a generalized autoregressive conditional heteroskedastic (GARCH) model and maximum likelihood methods.

Previous research on optimal margin setting by futures exchanges propose theoretical models and/or apply these to specific futures contracts (Dutt and Wein, 2003; Broussard, 2001; Cotter, 2001; Chou et al., 2000; Dewachter and Gielens, 1999; Longin, 1999; Kupiec, 1994; Fenn and Kupiec, 1993; Ma et al., 1993; Moser, 1992; Edwards and Neftci, 1988; Gay et al., 1986; Hartzmark, 1986 and Figlewski, 1984). Ackert and Hunter (1994) test a descriptive model that sets price limits in futures markets by minimizing the long-run average cost of market making. Bates and Craine (1999) model the default risk exposure of the CME from stock index futures contracts during the crash of 1987. Gemmill (1994) examines the capital requirements of the International Commodities Clearinghouse in London in 1987. However, to the best of our knowledge, our paper is the first to address the futures clearinghouse's problem of setting the optimal clearing margin and capital for clearing firms and optimal daily price limits for the futures contract.

We apply the model to the clearinghouse of the Winnipeg Commodity Exchange (WCE), which is a good subject for our analysis, since reliable daily data on futures prices are available while such data are unavailable for spot prices. We address the canola futures contract, which is the primary futures contract traded on the WCE, on the basis of trading volume (73% of total futures trading volume) and open interest (63% of total futures open interest). We analyze two periods, extending from 1995 to 1997 and from 1999 to 2001, respectively, which differ on the clearinghouse associated with the WCE and on the relative levels of margin and price limits. Section 4 describes our empirical analysis and results. The results confirm that price limits reduce the sum of optimal margin and capital requirements to a level that is substantially lower than that required in their absence. Optimal margin levels are higher than actual margin levels in both periods. Optimal price limits are higher in the first period, but lower in the second period, than actual price limits. The sum of optimal margin and capital exceeds that of actual margin and capital in the second period. We provide our conclusions in Section 5.

2. Theoretical model on optimal clearing margin, capital and price limits

We explain Brennan's model on how a futures exchange could set the optimal margin and price limits that will minimize traders' costs and then show how we extend it to the futures clearinghouse's problem of setting the optimal clearing margin, capital and price limits.

2.1. Brennan's model

Brennan first explains how an exchange could set margin and price limits that will make a futures contract self-enforcing. Suppose a trader takes a short position in a futures contract at time 0 at a price F_0 and deposits a margin M with the broker. The futures price F_1 is revealed at time 1 and the contract matures at time 2. The trader has an incentive to renege on the contract if his loss at time 1, $F_1 - F_0$, is greater than his margin M. Suppose that a price limit L is imposed, so that trades may occur at time 1, only at prices which lie in the range $F_0 - L$, $F_0 + L$. The trader will have no incentive to renege if his expected loss at time 1, conditional on the upper price limit having been hit at time 1, is less than or equal to his margin M. This is given by

$$E\lfloor \widetilde{F}_1 - F_0 | \widetilde{F}_1 \geqslant F_0 + L \rfloor \leqslant M. \tag{1}$$

In Eq. (1), $\widetilde{F}_1$ is the futures price that would have been observed at time 1 in the absence of the price limit. Similarly, a trader who takes a long position in the contract at time 0, will have no incentive to renege, if his expected loss at time 1, conditional on the lower price limit having been hit, is less than or equal to his margin M. This is given by

$$E\lfloor F_0 - \widetilde{F}_1 | \widetilde{F}_1 \leqslant F_0 - L \rfloor \leqslant M. \tag{2}$$

Brennan notes that since margin is costly, it will never be optimal to set the margin so that the inequality in (1) (or (2)) is satisfied, so that the optimal level of margin as a function of the price limit is determined by equality between the two sides of Eq. (1) (or (2)). L must be set less than M for the price limit to be effective in enforcing the contract. Further, if Eq. (1) (or (2)) does not hold, then imposing a price limit can increase the incidence of reneging, because the short (long) trader has an incentive to renege each time the upper (lower) price limit is hit, rather than when a price increase (decrease) depletes his margin.

Brennan then provides a model which a futures exchange could use to set optimal levels of the margin and the price limit, by minimizing a trader's total costs, which include the opportunity cost of the margin, the liquidity cost incurred if the futures price hits a limit and the market shuts down for the day, and the legal, reputation and other costs associated with contract enforcement that the trader would incur if he reneges on his futures position. He assumes that contract enforcement costs are such that the optimal contract is completely self-enforcing. His optimizing model is described in Eqs. (3) and (4).

$$\text{Minimise} \quad \kappa M + \gamma \frac{\Pr\lfloor |\widetilde{F}_1 - F_0| \geqslant L \rfloor}{\Pr\lfloor |\widetilde{F}_1 - F_0| \leqslant L \rfloor} \tag{3}$$

with respect to M, L such that

$$E\left[\widetilde{F}_1 - F_0 \middle| \widetilde{F}_1 \geqslant F_0 + L\right] = M. \tag{4}$$

In the objective function of Eq. (3), the first term represents the opportunity cost of the margin, and κ is its unit cost. The second term represents the liquidity cost that the trader incurs if a price limit is hit, and γ represents its unit cost. The liquidity cost is 0, if a price limit is never hit during the day, and is infinite, if a price limit is hit, and trading is halted. This motivates Brennan to define the liquidity cost as proportional to the ratio of the probability that a price limit is hit to the probability that no price limit is hit. The second term has a minimum value when L is very large. Assuming that futures price changes belong to a symmetric distribution, the constraint of Eq. (4) ensures that the contract is self-enforcing.

Brennan does not solve this model. Instead, he calculates the trader's total costs, for various values of M and L, using $\kappa = 0.02\%$ and $\gamma = 1$, for a situation in which the trader has access to external information, such as on spot prices, in addition to observed futures prices. κ and γ are the relative weights assigned to the margin, and to the probability of hitting a price limit, respectively, in the objective function. The daily interest rate, which is an estimate of κ, is similar in magnitude to the 0.02% that Brennan assumes. However, there is no clear answer as to what should be the value of γ, since the unit cost that the trader incurs when a price limit is hit is not measurable. If we solve the model of Eqs. (3) and (4) using $\kappa = 0.02\%$ and $\gamma = 1$, since γ is much larger than κ, the objective function is dominated by the necessity to minimize the probability that the price limit is hit, and L becomes very large. The constraint in Eq. (4) then makes M very large as well. The probability of hitting the price limit becomes 0 and the price limit becomes ineffective in reducing the margin required.

2.2. Our model of optimal clearing margin, capital and price limits

Let M_S and $C_S(M_S)$ denote the margin and capital deposited by a clearing firm which initiates a short futures position, and let M_L and $C_L(M_L)$ be the margin and capital deposited by a clearing firm which initiates a long futures position, at the futures price F_{t-1}, at $t-1$. We denote the upper price limit by L_U and the lower price limit by L_L. Our model for setting the optimal values of M_S, $C_S(M_S)$, L_U, M_L, $C_L(M_L)$ and L_L, simultaneously is described by the following:

Minimize $(M_S + C_S(M_S))k + (M_L + C_L(M_L))k$ (5)

with respect to M_S, L_U, M_L, L_L

subject to

$$E\lfloor \widetilde{F}_t - F_{t-1} | \widetilde{F}_t \geqslant F_{t-1} + L_U \rfloor = M_S + C_S(M_S), \tag{6}$$

$$M_S + C_S(M_S) > L_U, \tag{7}$$

$$E\lfloor F_{t-1} - \widetilde{F}_t | \widetilde{F}_t \leqslant F_{t-1} - L_L \rfloor = M_L + C_L(M_L), \tag{8}$$

$$M_L + C_L(M_L) > L_L, \tag{9}$$

$$P(\widetilde{F}_t \geqslant F_{t-1} + L_U) + P(\widetilde{F}_t \leqslant F_{t-1} - L_L) \leqslant p. \tag{10}$$

The objective function of Eq. (5) represents the sum of the opportunity costs of holding margin and capital with the clearinghouse for both clearing firms. k is the per unit opportunity cost of the margin and capital. We explain how we estimate the capital requirement as a function of the margin for both clearing firms in Section 2.2.1. We assume as Brennan does, that the optimal contract is self-enforcing. Constraints (6)–(9) ensure that the margin, capital and price limits are set such that the futures contract is completely self-enforcing, and are explained in Section 2.2.2. We do not incorporate the liquidity cost of hitting a price limit in the objective function, as Brennan does, for the reasons described in the last paragraph of Section 2.1. Instead, we control the liquidity cost by constraining the probability of hitting a price limit to a desired value in Eq. (10), which is addressed further in Section 2.2.3.

2.2.1. Relationship between capital requirement and margin requirement

The clearing firm with a short position at $t - 1$ would have a loss on day t equal to $\widetilde{F}_t - F_{t-1}$ if $\widetilde{F}_t > F_{t-1}$, where $\widetilde{F}_t$ is the equilibrium futures price that would prevail on day t in the absence of a price limit. If this loss is less than or equal to the margin M_S that the firm has deposited, the clearinghouse faces no residual default risk exposure. However, if $\widetilde{F}_t - F_{t-1} > M_S$, the clearinghouse is exposed to a potential loss of $\widetilde{F}_t - F_{t-1} - M_S$. Since the clearinghouse's first line of defense is the clearing firm's margin contribution, and the next, is that firm's capital contribution, the capital level should be set so that it supports the clearinghouse's residual default risk exposure. Thus, if $F_{t-1} = \$100$, $M_S = \$10$, and $\widetilde{F}_t = \$105$, the capital required at t would be \$0. If $\widetilde{F}_t = \$115$, $F_{t-1} = \$100$ and $M_S = \$10$, the capital required would be \$5. Fig. 1 shows the probability density function $f(\widetilde{F}_t)$ of the uncensored futures price $\widetilde{F}_t$. If $\widetilde{F}_t < F_{t-1} + M_S$ the clearinghouse has no residual default risk exposure and no capital is required. If $\widetilde{F}_t = F_A$, which is greater than $F_{t-1} + M_S$, the capital required
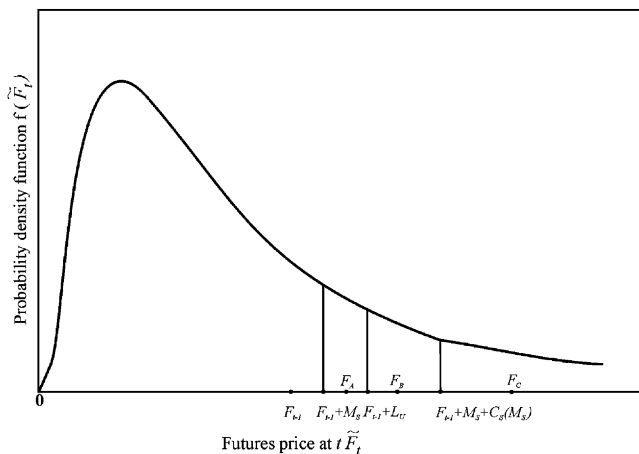

Fig. 1. The relationship between clearing margin M_S and capital $C_S(M_S)$ for a clearing firm with a short futures position and the upper price limit L_U.

is $F_A - (F_{t-1} + M_S)$. If $\widetilde{F}_t = F_B$, the capital required is $F_B - (F_{t-1} + M_S)$. If the capital contribution is to cover the maximum loss that the clearing firm could incur, it could be infinitely large. A more reasonable rule is to set the capital requirement, $C_S(M_S)$, as equal to the expected value of the capital required at t, conditional on capital being required. This is given by

$$
\begin{aligned}
C_S(M_S) &= E[\widetilde{F}_t - (F_{t-1} + M_S)|\widetilde{F}_t > F_{t-1} + M_S] \\
&= \frac{\int_{F_{t-1}+M_S}^{\infty} (\widetilde{F}_t - (F_{t-1} + M_S))f(\widetilde{F}_t)\mathrm{d}\widetilde{F}_t}{\int_{F_{t-1}+M_S}^{\infty} f(\widetilde{F}_t)\mathrm{d}\widetilde{F}_t}.
\end{aligned}
\tag{11}
$$

The clearing firm with a long position at $t-1$ has a loss on day t equal to $F_{t-1} - \widetilde{F}_t$, if $\widetilde{F}_t < F_{t-1}$ and poses a residual default risk exposure to the clearinghouse if $F_{t-1} - \widetilde{F}_t > M_L$, or if $\widetilde{F}_t < F_{t-1} - M_L$. The capital required at t to support this exposure is $F_{t-1} - \widetilde{F}_t - M_L$. We set the capital requirement $C_L(M_L)$ equal to the expected value of the capital required at t, conditional on capital being required, to obtain

$$
\begin{aligned}
C_L(M_L) &= E[F_{t-1} - \widetilde{F}_t - M_L|\widetilde{F}_t < F_{t-1} - M_L] \\
&= \frac{\int_0^{F_{t-1}-M_L} (F_{t-1} - \widetilde{F}_t - M_L)f(\widetilde{F}_t)\mathrm{d}\widetilde{F}_t}{\int_0^{F_{t-1}-M_L} f(\widetilde{F}_t)\mathrm{d}\widetilde{F}_t}.
\end{aligned}
\tag{12}
$$

If $\widetilde{F}_t = F_{t-1}(1 + \tilde{x}_t)$, where $\tilde{x}_t$ is the return on the futures contract at time t in the absence of price limits, and $f(\tilde{x}_t)$ is its probability density function, Eqs. (11) and (12) become

$$
C_S(M_S) = \frac{\int_{\frac{M_S}{F_{t-1}}}^{\infty} (F_{t-1}(1 + \tilde{x}_t) - (F_{t-1} + M_S))f(\tilde{x}_t)\mathrm{d}\tilde{x}_t}{\int_{\frac{M_s}{F_{t-1}}}^{\infty} f(\tilde{x}_t)\mathrm{d}\tilde{x}_t},
\tag{13}
$$

$$
C_L(M_L) = \frac{\int_{-\infty}^{\frac{-M_L}{F_{t-1}}} (F_{t-1} - M_L - F_{t-1}(1 + \tilde{x}_t))f(\tilde{x}_t)\mathrm{d}\tilde{x}_t}{\int_{-\infty}^{\frac{-M_L}{F_{t-1}}} f(\tilde{x}_t)\mathrm{d}\tilde{x}_t}.
\tag{14}
$$

2.2.2. Relationship between margin, capital and price limit for a self-enforcing contract

Consider the clearing firm which has initiated a short position in the contract at $t-1$ at a futures price F_{t-1} and deposited margin M_S and capital $C_S(M_S)$ with the clearinghouse. In the absence of price limits, the firm has no incentive to renege on its futures position, if its loss at t, $\widetilde{F}_t - F_{t-1} < M_S + C_S(M_S)$, but has an incentive to renege if $\widetilde{F}_t - F_{t-1} > M_S + C_S(M_S)$, that is, if $\widetilde{F}_t > F_{t-1} + M_S + C_S(M_S)$. For example, if $\widetilde{F}_t = \$112$, $F_{t-1} = \$100$, $M_S = \$10$ and $C_S(M_S) = \$5$, the firm's loss is $\$12$, while the amount it has on deposit with the clearinghouse is $\$15$, so that it has no incentive to renege. However, if $\widetilde{F}_t = \$120$, $F_{t-1} = \$100$, $M_S = \$10$ and $C_S(M_S) = \$5$, the clearing firm's loss of $\$20$ exceeds the amount it has on deposit of $\$15$, so that the firm would now have an incentive to renege. This is illustrated in Fig. 1. In

the absence of price limits, if the futures price at t is F_B, the clearing firm's loss $F_B - F_{t-1} < M_S + C_S(M_S)$, so that it has no incentive to renege. If the futures price at t is F_C, the firm's loss $F_C - F_{t-1} > M_S + C_S(M_S)$, so that it now has an incentive to renege. As Fig. 1 shows, the probability of reneging, in the absence of price limits, is the area under the density function $f(\widetilde{F}_t)$, to the right of $F_{t-1} + M_S + C_S(M_S)$. This may be made very small by making $M_S + C_S(M_S)$ very large.

Price limits can be used to reduce the required sum of margin and capital. If an upper price limit is hit at t, the clearing firm's potential loss depends on the equilibrium futures price $\widetilde{F}_t$ that would prevail in the absence of price limits. As Fig. 1 shows, if $\widetilde{F}_t$ is between $F_{t-1} + L_U$ and $F_{t-1} + M_S + C_S(M_S)$, as in F_B, the firm's potential loss is less than the amount it has on deposit with the clearinghouse and the firm has no incentive to renege. If $\widetilde{F}_t > F_{t-1} + M_S + C_S(M_S)$, as in F_C, its potential loss exceeds the amount it has on deposit and the firm has an incentive to renege. The firm does not observe $\widetilde{F}_t$ when the upper price limit is hit, and is forced to focus on its expected loss, conditional on the upper price limit having been hit. The clearinghouse can make the contract self-enforcing, by setting the clearing firm's expected loss at t, conditional on the upper price limit being hit, equal to the sum of required margin and capital, as in Eq. (6).

The upper price limit L_U must be set so that it is less than the sum of the margin and capital requirement $M_S + C_S(M_S)$, as in Eq. (7). If $L_U > M_S + C_S(M_S)$, then the clearing firm will be able to observe price increases which exhaust the amounts deposited with the clearinghouse and the price limit will be ineffective in making the contract self-enforcing. The limit will also become ineffective in reducing the sum of required margin and capital to a level below that which is required in the absence of price limits. Eq. (7) ensures that the firm will never experience a realized loss which exceeds the amount it has on deposit with the clearing house. Thus, if $F_{t-1} = \$100$, $M_S = \$10$, $C_S(M_S) = \$5$ and $L_U = \$13$, trading at day t can only be in the futures price range of 87,113 $(F_{t-1} - L_U, F_{t-1} + L_U)$ and trading will be discontinued for the day if the futures price hits either limit. The maximum realized loss that the firm can have is \$13, which is less than the sum of its margin and capital contribution of \$15.

Eqs. (6) and (7) must both hold for the futures contract to be self-enforcing for the short clearing firm. If (6) holds, but (7) does not, the clearing firm's realized loss could exceed the amount it has on deposit with the clearinghouse. If (7) holds, but (6) does not, the firm's expected loss exceeds this amount and it may then renege every time the upper price limit is hit. Fig. 1 shows that the probability of reneging will then be the area under the density function $f(\widetilde{F}_t)$, to the right of $F_{t-1} + L_U$, which is higher than the probability of reneging in the absence of price limits.

Consider the clearing firm which has initiated a long position in the futures contract at $t - 1$ and deposited margin M_L and capital $C_L(M_L)$. The clearinghouse can make the contract self-enforcing for the long clearing firm by setting the sum of margin and capital required equal to the firm's expected loss at t, conditional on the lower price limit being hit, as in Eq. (8), and the sum of margin and capital to a value higher than the lower price limit, as in Eq. (9), so that the firm's realized loss never exceeds the amount on deposit with the clearinghouse.

Eqs. (6) and (8) may be rewritten in terms of $\tilde{x}_t$ and $f(\tilde{x}_t)$ as

$$\frac{\int_{\frac{L_U}{F_{t-1}}}^{\infty}(F_{t-1}(1+\tilde{x}_t)-F_{t-1})f(\tilde{x}_t)\mathrm{d}\tilde{x}_t}{\int_{\frac{L_U}{F_{t-1}}}^{\infty}f(\tilde{x}_t)\mathrm{d}\tilde{x}_t}=M_S+C_S(M_S),\tag{15}$$

$$\frac{\int_{-\infty}^{\frac{-L_L}{F_{t-1}}}(F_{t-1}-F_{t-1}(1+\tilde{x}_t))f(\tilde{x}_t)\mathrm{d}\tilde{x}_t}{\int_{-\infty}^{\frac{-L_L}{F_{t-1}}}f(\tilde{x}_t)\mathrm{d}\tilde{x}_t}=M_L+C_L(M_L).\tag{16}$$

2.2.3. Controlling the liquidity cost of hitting a price limit

Eq. (10) constrains the sum of the probabilities of hitting the upper and lower price limits on any day t to a value less than or equal to an acceptable probability p. The solution to the model depends critically on p. Eq. (10) may be rewritten in terms of $\tilde{x}_t$ and $f(\tilde{x}_t)$ as

$$\int_{\frac{L_U}{F_{t-1}}}^{\infty}f(\tilde{x}_t)\mathrm{d}\tilde{x}_t+\int_{-\infty}^{\frac{-L_L}{F_{t-1}}}f(\tilde{x}_t)\mathrm{d}\tilde{x}_t\leqslant p.\tag{17}$$

2.2.4. Implications of our model

Eqs. (13) and (14) equate capital to the clearinghouse's expected default risk exposure which exceeds margin, while (15) and (16) set the clearing firms' expected loss, conditional on a price limit hit, to the sum of margin and capital. Eqs. (13) and (15), (14) and (16), imply that $M_S = L_U$ and $M_L = L_L$, respectively. If we do not specify a relationship between capital and margin, as in (13) and (14), our model (Eqs. (5)–(10)) provides a unique solution for $M_S + C_S(M_S)$, L_U, $M_L + C_L(M_L)$ and L_L, and an infinite number of combinations of M_S and $C_S(M_S)$, and, of M_L and $C_L(M_L)$, which are optimal. A unique solution for M_S, $C_S(M_S)$, L_U, M_L, $C_L(M_L)$ and L_L, can then be obtained by constraining capital, as we do, or margin.

2.3. Comparable model with no price limits

We can compare the optimal levels of margin and capital from our model to those that a clearinghouse would set in the absence of price limits, if it minimizes the opportunity costs of margin and capital to both short and long clearing firms, with the constraint that the sum of the probabilities that each clearing firm would renege is set less than or equal to an acceptable probability q. This model is summarized by Eqs. (18) and (19) that follow:

$$\text{Minimize}\quad (M_S+C_S(M_S))k+(M_L+C_L(M_L))k\tag{18}$$

with respect to M_S, M_L

subject to

$$P\left(\widetilde{F}_t>F_{t-1}+M_S+C_S(M_S)\right)+P\left(\widetilde{F}_t<F_{t-1}-M_L-C_L(M_L)\right)\leqslant q.\tag{19}$$

$C_S(M_S)$ and $C_L(M_L)$ are again estimated by Eqs. (13) and (14), respectively. The solution to the model depends critically on the value of q. Eq. (19) may be written as

$$\int_{\frac{M_S+C_S(M_S)}{F_{t-1}}}^{\infty} f(\tilde{x}_t)\mathrm{d}\tilde{x}_t + \int_{-\infty}^{\frac{-M_L-C_L(M_L)}{F_{t-1}}} f(\tilde{x}_t)\mathrm{d}\tilde{x}_t \leqslant q. \tag{20}$$

3. Estimation of probability density function of daily unrestricted futures returns

We explain how we estimate the probability density function $f(\tilde{x}_t)$ of the daily unrestricted futures return $\tilde{x}_t$ from observed daily returns which are censored by price limits. If the volatility of the function $f(\tilde{x}_t)$ is large and it has thick tails, then the probability associated with futures price changes which exceed the margin or the price limit will be large and the expected value of a futures price change, given that the price change exceeds the margin or the price limit, will also be large. The form of the density function thus critically affects the choice of the optimal margin, capital and price limits. Suppose that the observed return x_t, which is calculated as $\frac{F_t}{F_{t-1}} - 1$, where F_t is the observed futures price at time t, is constrained by price limits to a lower limit l_t and an upper limit u_t. Then the likelihood function that is based on a sample of the previous n observed futures returns $x_1, x_2, \ldots, x_t, \ldots, x_n$, is given by

$$L = \prod_{x_t=l_t} F(l_t) \cdot \prod_{l_t<x_t<u_t} f(x_t) \cdot \prod_{x_t=u_t}(1 - F(u_t)) \tag{21}$$

where $F(\tilde{x}_t)$ is the corresponding cumulative distribution function given by

$$F(\tilde{x}_t) = \int_{-\infty}^{\tilde{x}_t} f(y)\mathrm{d}y. \tag{22}$$

Eq. (22) captures the probability that on day t, the observed futures price may be at the lower price limit, between the lower and upper price limits, or at the upper price limit.

3.1. Characteristics of the assumed probability density function $f(\tilde{x}_t)$

The empirical analysis confirms that daily returns to the canola futures contract conform to the logistic density function, with skewness close to zero. We assume, therefore, that $f(\tilde{x}_t)$ is a Type III generalized logistic probability density function (Balakrishnan, 1992; Davidson, 1980) given by

$$f(\tilde{x}_t) = \frac{\Gamma(2\theta)}{\sigma_t(\Gamma(\theta))^2} \frac{\mathrm{e}^{\frac{-(\tilde{x}_t-\mu_t)\theta}{\sigma_t}}}{\left(1 + \mathrm{e}^{\frac{-(\tilde{x}_t-\mu_t)}{\sigma_t}}\right)^{2\theta}} \quad -\infty < \tilde{x}_t < \infty, \quad \theta > 0, \tag{23}$$

where

$$\Gamma(\theta) = \int_0^{\infty} \mathrm{e}^{-y} y^{\theta-1}\mathrm{d}y, \quad \theta > 0. \tag{24}$$

θ, μ_t and σ_t are the parameters of the generalized logistic probability density function. The corresponding cumulative distribution function is given by

$$F(\tilde{x}_t) = \frac{\Gamma(2\theta)}{(\Gamma(\theta))^2} \int_0^{p(\tilde{x}_t)} y^{\theta-1}(1-y)^{\theta-1}\mathrm{d}y, \quad -\infty < x_t < \infty, \tag{25}$$

$$p(\tilde{x}_t) = \frac{1}{1 + \mathrm{e}^{\frac{-(\tilde{x}_t - \mu_t)}{\sigma_t}}}. \tag{26}$$

The characteristics of the density function of Eq. (23) are as follows:

Expected value of $\tilde{x}_t = \mu_t$, $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (27)

Variance of $\tilde{x}_t = h_t = 2\Psi'(\theta)\sigma_t^2$, $\qquad\qquad\qquad\qquad\qquad$ (28)

Skewness $= 0$, $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (29)

Kurtosis $= 3 + \dfrac{\Psi'''(\theta)}{2(\Psi'(\theta))^2}$, $\qquad\qquad\qquad\qquad\qquad\qquad$ (30)

where

$$\Psi(\theta) = \frac{\partial \ln \Gamma(\theta)}{\partial \theta}, \;\; \Psi'(\theta) = \frac{\partial \Psi(\theta)}{\partial \theta}, \;\; \Psi''(\theta) = \frac{\partial \Psi'(\theta)}{\partial \theta}, \;\; \text{and} \;\; \Psi'''(\theta) = \frac{\partial \Psi''(\theta)}{\partial \theta}.$$

The kurtosis of Eq. (30) measures the degree of peaked-ness and the thickness of the tails of the density function and depends on θ. When $\theta = 1$, the density function of Eq. (23) reduces to the logistic density function with a kurtosis of 4.2. As θ reduces below 1, the kurtosis increases, and at $\theta = 0.5$, it equals 5. As θ increases above 1, the kurtosis decreases. At $\theta = 5$, it equals 3.22, close to the value of 3 for the normal probability density function.

3.2. Estimation of the parameters by maximum likelihood

If the unrestricted futures price would drop 8% at $t - 1$, but the observed censored futures price is only allowed to drop 5% due to price limits, we would expect that the price would drop 3% on the next day t. Accordingly, we take limit moves into account to estimate the conditional mean μ_t of the unrestricted futures return $\tilde{x}_t$, as the expected value of the return in excess of the return at the price limit, conditional on having hit a price limit at $t - 1$, as follows:

$$\mu_t = E_{t-1}[\tilde{x}_t] \\ = \begin{cases} E[\tilde{x}_{t-1} - l_{t-1} | x_{t-1} = l_{t-1}] = \dfrac{\int_{-\infty}^{l_{t-1}}(\tilde{x}_{t-1} - l_{t-1})f(\tilde{x}_{t-1})\mathrm{d}\tilde{x}_{t-1}}{\int_{-\infty}^{l_{t-1}} f(\tilde{x}_{t-1})\mathrm{d}\tilde{x}_{t-1}}, \\ E[\tilde{x}_{t-1} - u_{t-1} | x_{t-1} = u_{t-1}] = \dfrac{\int_{u_{t-1}}^{\infty}(\tilde{x}_{t-1} - u_{t-1})f(\tilde{x}_{t-1})\mathrm{d}\tilde{x}_{t-1}}{\int_{u_{t-1}}^{\infty} f(\tilde{x}_{t-1})\mathrm{d}\tilde{x}_{t-1}}. \end{cases} \tag{31}$$

$\tilde{x}_{t-1}$ is the unrestricted return and l_{t-1} and u_{t-1} are the lower and upper limit on the observed return x_{t-1} at $t - 1$. If a price limit was not hit on the previous day $t - 1$, μ_t is set equal to 0.

We account for time-variation in the conditional variance h_t of the unrestricted futures return $\tilde{x}_t$, earlier given in Eq. (28), by using a GARCH(1,1) model (Bollerslev, 1986). h_t is updated according to the following equation:

$$h_t = w + \alpha x_{t-1}^2 + \beta h_{t-1} + \gamma d_{t-1}. \tag{32}$$

w, α, β and γ are the parameters of the model, x_{t-1} the observed daily return, and h_{t-1} is the conditional variance at $t - 1$. d_{t-1} is a dummy variable equal to 1 if the futures price hits a price limit on day $t - 1$ and 0 otherwise. This corrects for the underestimation of the conditional variance that would otherwise occur for days for which the previous day is a limit move day.

We use a sample of the previous n daily returns to estimate the parameters for each day t. We fix θ at a value of 0.1, set h_0 equal to the sample variance and maximize the natural log of the likelihood function of Eq. (28) with respect to w, α, β and γ, by using Fortran programs and the Berndt, Hall, Hall and Hausman (BHHH) (1974) algorithm to converge to a final solution. μ_t is set according to Eq. (31), if the price limit is hit on the previous day, and is set to 0, if the price limit is not hit on the previous day. The value of σ_t that is used in calculating the likelihood function is obtained by equating the right-hand sides of Eqs. (28) and (32) for each day t of the n sample days, for each iteration within the algorithm. We repeat this process by increasing θ in steps of 0.1, with a maximum value of 5. The value of θ at which the maximized log-likelihood value is the largest gives us the maximum likelihood estimate of θ. The corresponding estimates of w, α, β and γ are the maximum likelihood estimates of these parameters for that value of θ. This method is called the "profile likelihood method" because the likelihood function is profiled with regard to the shape parameter θ. The estimates obtained in this process are the same as the original maximum likelihood estimates of the five parameters θ, w, α, β and γ, if they exist uniquely.

4. Empirical analysis and results

4.1. Characteristics of data used in the analysis

Data are obtained from the WCE on settlement prices, volume and open interest for the canola futures contract and are used to compute observed daily returns. A time series of daily returns, settlement prices and open interest is created by including data from each nearby futures contract, but excluding observations for its delivery month, when the contract is illiquid, and days with 0 trading volume, when settlement prices are not trading prices. In period 1, which extends from January 3, 1995 to December 31, 1997, the margin that is imposed by the Winnipeg Commodities Clearing Limited (WCCL), the clearinghouse of the WCE in this period, equals the price limit, and the normal margin and price limit are both $10/contract. In

period 2, which extends from January 4, 1999 to December 31, 2001, the margin is updated monthly by the Winnipeg Commodity Exchange Clearing Corporation (WCECC), the clearinghouse of the WCE in this period (WCE, 1998), and set equal to the nearest futures settlement price × 2 × maximum of the standard deviations of daily returns that are computed using data from the nearby contract for the past 20, 90 and 260 days (WCECC, 1999). In period 2, the normal price limit is $10/contract. The WCE expands price limits by 50% for the next 3 trading days, if 2 out of the 3 nearest contracts close either limit up or limit down, and expanded price limits are maintained for subsequent 3 day periods until 2 of the 3 nearest contracts do not close at the limit (WCE, 1999). Margins are expanded by 50% on days of expanded price limits. Table 1 reports statistics for the data. The standard deviations of daily returns indicate greater volatility for the canola futures contract in period 2. In period 1, the margin equals the price limit and the observed futures price change never exceeds the margin in either direction. In period 2, the price change exceeds margin in 5.11% of the days. The observed futures price hit the price limit for a greater proportion of days in period 1 and the proportion of days with expanded price limits is higher in period 1 than 2.

4.2. Analysis and results of maximum likelihood estimation of $f(\tilde{x}_t)$

The time series of observed returns for period 1 and 2 are analyzed using BEST-FIT (a probability distribution fitting software program provided by Palisade Corporation). The chi-squared test statistics resulting from comparison of each time series to 26 possible functions, confirm that each series conforms to the logistic

Table 1
Characteristics of canola futures contract, trading volume, open interest, days when observed futures price change exceeded the margin, days with price limit hits, and days with expanded price limits, for 1995/01/03–1997/12/31 and 1999/01/04–2001/12/31

Variable	1995/01/03–1997/12/31	1999/01/04–2001/12/31
Average futures settlement price ($)	418.37	289.77
Average/standard deviation of return on futures (%)	−0.0220	−0.0301
	0.9654	1.0983
Average/standard deviation of margin ($/tonne)	10.3590	7.2018
	1.2917	2.2496
Average/standard deviation of price limit ($/tonne)	10.3590	10.1036
	1.2917	0.7127
Trading volume	3,810,854	5,943,320
Open interest	512,622	773,907
Total number of days	752	724
Number/% of days that futures price	0	37
change exceeded margin in either direction	0.0000	5.1105
Number/% of days that futures price change	15	6
hit the price limit in either direction	1.9947	0.8287
Number/% of days with expanded price limits	54	15
	7.1809	2.0718

Table 2
Averages and standard deviations of daily maximum likelihood estimates of the parameters of the probability density function $f(\tilde{x}_t)$ of the unrestricted daily futures return $\tilde{x}_t$ and its conditional variance h_t, for 1995/01/03–1997/12/31 and 1999/01/04–2001/12/31

Period/variable	θ	μ_t (%)	σ_t	w	α	β	γ	h_t (%%)	$\ln L$
1995/01/03–1997/12/31									
Parameter									
Average	3.1656	0.0013	0.0109	0.0001	0.1764	0.4269	0.00002	1.2714	97.2334
Standard deviation	1.9415	0.0957	0.0067	0.0001	0.2188	0.3776	0.0001	1.0898	9.4011
Standard error of parameter									
Average			0.0002	0.0003	0.4032	1.6019	0.0070	1.6155	
Standard deviation			0.0003	0.0005	0.4022	4.7256	0.1116	2.4947	
1999/01/04–2001/12/31									
Parameter									
Average	2.7169	0.0020	0.0106	0.0001	0.1442	0.5109	0.00028	1.4772	95.6828
Standard deviation	1.9775	0.0960	0.0076	0.0001	0.2119	0.3666	0.0017	1.2102	10.3709
Standard error of parameter									
Average			0.0001	0.0003	0.3709	1.7890	0.0301	1.8527	
Standard deviation			0.0004	0.0006	0.4156	4.7105	0.4573	5.1223	

density function, with skewness that is close to 0. This explains our assumption that the probability density function of unrestricted futures returns also follows a generalized logistic density function, as in Eq. (23). For each day in both periods, using observed futures returns for the 30 previous days, we maximize the natural log $\ln L$ of the likelihood function L of Eq. (21), and estimate θ, μ_t, σ_t, w, α, β, γ, h_t and thus $f(\tilde{x}_t)$. We use FORTRAN programs, IMSL MATH/Library subroutines, the BHHH algorithm and the profile likelihood method described in Section 3.2, to conduct these estimations.

Table 2 shows averages and standard deviations of daily estimates of θ, μ_t, σ_t, w, α, β, γ, h_t and $\ln L$ and of standard errors of σ_t, w, α, β, γ and h_t, in both periods. Table 3 shows the averages and standard deviations of the expected value, standard deviation, skewness and kurtosis of the estimated density function $f(\tilde{x}_t)$ and of the 30 day samples of observed returns used in the estimation. Two sample t-tests with unequal sample variances confirm that the expected value, standard deviation and kurtosis of $f(\tilde{x}_t)$ are statistically higher at the 95% confidence level than corresponding sample values in both periods.

4.3. Analysis and results of optimal margin, capital and price limits

We next solve for the optimal margin, capital and price limits for each day t in both periods, using our model, and a value for the maximum acceptable probability of hitting the price limit of $p = 1\%$. The daily interest rate on 3-month Canadian Treasury bills is used as the estimate of k. We also solve for the optimal margin and capital in the absence of price limits, using the comparable model and a value

Table 3
Averages and standard deviations of the characteristics of the estimated probability density function $f(\tilde{x}_t)$ of the unrestricted daily futures return $\tilde{x}_t$ and of the samples of 30 previous observed daily futures returns used in the estimation, for 1995/01/03–1997/12/31 and 1999/01/04–2001/12/31

Period	Estimated probability density function $f(\tilde{x}_t)$				Samples of observed futures returns			
	Expected value (%)	Standard deviation (%)	Skewness	Kurtosis	Expected value (%)	Standard deviation (%)	Skewness	Kurtosis
1995/01/03–1997/12/31								
Average	0.0013	1.0602	0.0000	3.7932	−0.0145	0.9440	−0.0020	2.6609
Standard deviation	0.0957	0.3842	0.0000	0.8396	0.1714	0.2389	0.4209	0.6321
1999/01/04–2001/12/31								
Average	0.0020	1.1384	0.0000	3.9830	−0.0260	1.0309	−0.1249	3.0759
Standard deviation	0.0960	0.4259	0.0000	0.9418	0.2051	0.3571	0.6016	1.1272

for the probability of reneging $q = 0.0001\%$. We conduct the optimization by using FORTRAN programs and IMSL MATH/Library subroutines.

Table 4 presents the results for both periods. Part A of Table 4 presents average optimal values of margin, capital and price limits from our model, Part B presents average optimal values of margin and capital for the comparable model, and Part C presents average values of actual margins and price limits. Since the density

Table 4
Comparison of averages of daily estimated optimal margin, capital and price limits, for 1995/01/03–1997/12/31 and 1999/01/04–2001/12/31, from models with and without price limits, to averages of actual margin and price limits

Variable	1995/01/03–1997/12/31	1999/01/04–2001/12/31
A. With price limits		
Optimal margin ($/%)	12.3492	9.5321
	2.9760	3.2347
Optimal capital ($/%)	2.0510	1.6385
	0.4960	0.5556
Optimal price limit ($/%)	12.3493	9.5321
	2.9760	3.2347
Cost of margin and capital ($)	0.0019	0.0014
B. Without price limits		
Optimal margin ($/%)	27.1341	21.4418
	6.5546	7.2728
Optimal capital ($/%)	1.8939	1.5378
	0.4588	0.5214
Cost of margin and capital ($)	0.0038	0.0028
C. Actual values		
Margin ($)	10.3590	7.2018
Price limit ($)	10.3590	10.1036

function $f(\tilde{x}_t)$ is symmetric, and the return to the long clearing firm equals the negative of the return to the short clearing firm, we would expect that the optimal margin (and capital) for the short clearing firm should equal the optimal margin (capital) for the long clearing firm, and that the optimal upper and lower price limit would also be equal. The empirical analysis confirms this. Therefore, the values of margin and capital presented in Table 4 are those for the short clearing firm as well as for the long clearing firm. The values for price limits apply to both the upper and the lower price limit.

Compare the optimal values with price limits (Part A, column 1) and without price limits (Part B, column 1) in period 1. Average optimal margin with price limits ($12.3492) is 45.51% of average optimal margin without price limits ($27.1341). This underlines the importance of price limits in reducing the margin requirement. Average optimal capital with price limits ($2.0510) is higher than the average optimal capital without price limits ($1.8939). This is as expected, since the lower the margin, the higher the capital required. The sum of average optimal margin and capital with price limits ($14.4002) is 49.61% of the same sum ($29.0280) without price limits. The average cost of margin and capital with price limits ($0.0019) is 50.00% of the cost without price limits ($0.0038). Similar results are observed for period 2.

Compare average optimal levels of margin and price limits of our model (in Part A) to actual levels (in Part C). Average optimal margin is higher than average actual margin in both periods. The average optimal price limit is higher in period 1, but lower in period 2 than the average actual price limit. Actual capital requirements are not available from the WCCL for period 1. The WCECC reports a ratio of required capital to margin of 27% in period 2, which implies an average value for the actual capital of $1.9445 (0.27 × average actual margin of $7.2018), which is higher than the average optimal level of $1.6385 for period 2. The sum of the average actual margin and capital of $9.1463 (7.2018 + 1.9445) is less than the average price limit ($10.1036), indicating that the canola futures contract is not self-enforcing in period 2. This sum is also less that of average optimal margin and capital of $11.1706 (9.5321 + 1.6385) in period 2.

4.4. Analysis and results of probabilities of exceeding margin and hitting the price limit

We estimate the probability of a futures price change in excess of the optimal margin and of a futures price change equal to/in excess of the optimal price limit (if any), for each day t, in both periods, using the estimated density function $f(\tilde{x}_t)$. Table 5 presents the results. Part A of Table 5 shows average values of the probabilities using the optimal margin and price limits of our model, Part B presents average values of the probabilities using the optimal margin of the comparable model without price limits, and Part C shows the number and proportion of days that margin is exceeded and a price limit is hit. We also present the probability that the clearing firm will renege, in Parts A and B.

Compare the results in Parts A and B. On average, the probability of a futures price change in excess of the optimal margin is higher in Part A than in Part B, since

Table 5
Comparison of averages of daily estimated probabilities of futures price change exceeding optimal margin, equal to/in excess of price limits and reneging, for 1995/01/03–1997/12/31 and 1999/01/04–2001/12/31, from models with and without price limits

Variable	1995/01/03–1997/12/31	1999/01/04–2001/12/31
A. With price limits		
Probability of futures price change exceeding estimated optimal margin (%)	0.4986	0.4996
Probability of futures price change equal to/in excess of estimated optimal price limit (%)	0.4986	0.4996
Probability of reneging (%)	0.0000	0.0000
B. Without price limits		
Probability of futures price change exceeding estimated optimal margin (%)	0.0003	0.0003
Probability of reneging (%)	0.0001	0.0001
C. Actual number and proportion of days		
Number/% of days futures price change exceeded actual margin	0.0000	19.0000
	0.0000	2.6243
Number/% of days actual price limit is hit	5.0000	3.0000
	0.6649	0.4144

the optimal margin is reduced in the presence of price limits. The probability of reneging is 0 in Part A, since the futures contract is self-enforcing, and the realized loss of the clearing firm never exceeds the sum of the margin and capital it has deposited with the clearinghouse. The probability of reneging in Part B is the probability that the futures price change will exceed the sum of estimated optimal margin and capital and it equals the chosen value of $q = 0.0001\%$.

Compare the results in Parts A and C. In period 1, the actual margin and price limit are equal, so that the futures price change never exceeds margin. In period 2, the actual margin is less than the price limit. In this period, the optimal margin of our model is larger, on average, than the actual margin, and the probability of exceeding optimal margin is less than the proportion of days that the futures price change exceeds actual margin, as expected. On average, the optimal price limit of our model is higher (lower) in period 1 (period 2) than the actual price limit. Correspondingly, on average, the probability of a futures price change equal to/exceeding the optimal price limit is lower (higher) than the proportion of days in which the futures price change hits the price limit in period 1 (period 2).

5. Conclusion

We model a futures clearinghouse's choice of the optimal clearing margin, capital and price limits that will minimize the opportunity costs of the margin and capital to clearing firms, make the futures contract self-enforcing, and control liquidity costs associated with hitting price limits, when the only information available on uncensored futures prices are observed censored futures prices. We explain the

inverse relationship between the margin and the capital requirement, which supports the residual default risk exposure of the clearinghouse from an open futures position of a clearing firm, which exceeds that covered by the clearing margin, and provide an estimate of the capital requirement as the expected value of the clearinghouse's default risk exposure that exceeds the margin.

Our model implies that if the contract is to be self-enforcing, then the sum of margin and capital should equal the expected loss of the short (long) clearing firm on a upper (lower) price limit hit and should exceed the price limit. Further, if the capital requirement should also cover the expected residual default risk exposure of the clearinghouse that is in excess of the margin, then the margin for the clearing firm with a short (long) futures position should equal the upper (lower) price limit. We show that there is a rational basis for price limits, in that they can reduce the sum of optimal margin and capital required to levels that are considerably lower than those required to minimize default risk in their absence.

We apply the model to the WCE, for 1995–1997, in which the WCCL is the WCE's clearinghouse, and for 1999–2001, in which the WCECC is the WCE's clearinghouse. We estimate optimal levels of margin, capital and price limits for the canola futures contract traded on the WCE, using our model, as well as a comparable model which does not use price limits, but sets the probability of reneging by the clearing firms to a minimal value. Our results show that the sum and opportunity cost of optimal margin and capital of our model are approximately half of the corresponding values for the comparable model without price limits.

Observed differences between the levels of optimal margins and price limits prescribed by our model and those actually used in both periods are most likely due to differences between the acceptable probabilities of exceeding margins and price limits that we use and those used by the WCCL and WCECC. However, our model implies equality between the optimal margin and price limit, and we observe equality between actual margins and price limits in the first period. If the WCCL requires additional funds from clearing firms in addition to margin, then the canola futures contract is self-enforcing in this period. In the second period, the sum of actual margin and capital is lower than the price limit, which implies that the canola futures contract is not self-enforcing. This is most likely because the WCECC uses its monitoring of clearing firms' financial statements, in addition to margin, capital and price limits, to manage its default risk.

Our model could be extended to determine how a futures clearinghouse should set the optimal mix of its risk management policy, which includes the margin, capital, price limits and monitoring of clearing firms' financial statements, and to situations in which other information on uncensored futures prices, such as spot prices, are available.

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