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Notes, Comments, and Letters to the Editor

Correlated information, mechanism design and informational rents

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Abstract

Crémer and McLean (Econometrica 56 (1988) 1247–1258) obtain a sufficient and necessary condition for full surplus extraction in Bayesian–Nash equilibrium—the rank condition, which McAfee and Reny (Econometrica 60(2) (1992) 395–421) later generalize for continuous type spaces. This paper shows that, if the principal does not know how noisy is the agent's signal—or equivalently, when signals available to an agent can be ranked by Blackwell's informativeness and, an agent's informativeness is independent of others' information, the rank condition fails to hold. Conversely, when rank condition fails and informational rents are left to an agent, the model can be interpreted as if, the principal were uncertain about the informativeness of the agent's signal.

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1. Introduction

Crémer and McLean [3] obtain a sufficient and necessary condition for full surplus extraction in Bayesian–Nash equilibrium. This paper shows that if the principal does not know how noisy is the signal of an agent—or equivalently, when signals available to an agent can be ranked by Blackwell's informativeness measure, and moreover, the quality of the signal of an agent is independent of the others' agents information then, the rank condition fails to hold. Conversely, whenever informational rents are left to an agent, it is

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always possible to interpret the model as if, the principal were uncertain on how noisy is the agent's signal.

Accounts of why the principal fails to extract all the surplus rely on risk-aversion and limited liability [8], or competition among sellers [7]. Another complementary explanation pursued here argues that the optimal mechanism requires too much knowledge from the principal. It is rather unrealistic that, the amount of effort each potential buyer devotes to gather information or the quality of the buyer's information would be perfectly known. As Crémer and McLean [3, p. 1254] originally pointed,

> In 'nearly all' auctions the seller should be able to extract the full surplus, which implies that asymmetry of information between sellers and buyers should be of no practical importance. Economic intuition and informal evidence suggest that this result is counterfactual, and several explanations can be suggested. First, the assumption that a common knowledge probability distribution exists is very strong.

As in [5], this paper is concerned with the rent extraction problem in a given Bayesian game Γ. The principal's objective is to extract the maximum of expected equilibrium payoffs agents get at Γ. The paper main result is that the principal may fail to extract all the payoffs when an agents' quality of information is uncertain. The intuition for the result is outlined below.

For simplicity, let us say agents are with strictly positive probability poorly or well informed. In order to extract the surplus, the principal offers menus of contracts, possibly distinct menus, to each agent. By choosing a contract from the menu, an agent gains the right to participate into the game. Thus, a contract is simply an entry fee. The monetary transfers stipulated by the contract are contingent on the contractual choices of other agents. Moreover, since the choices of agents depend on their private information, from an agent's perspective, a contract is a lottery with a payoff determined by the realization of the others' types.

A well-informed agent can mimic the contractual choice of a poorly informed agent without incurring a higher expected entry fee: she can simply add noise to her private signal and randomize over the equilibrium choices that the poorly informed makes. As a result, the principal is unable to screen agents and charge her a higher expected entry fee. In the ensuing game, however, she is able to leverage on her information advantage. The entry fee does not depend on the strategies played during the game, and so, at the game, she can always guarantee to herself a higher expected payoff than the poorly informed gets. Consequently, the optimal mechanism must leave her informational rents.

Two conditions are crucial for the above accounting of the existence of informational rents: Information must be valuable at the game the agents play; The signals' distributions available to an agent must be weekly ranked in order of informativeness by Blackwell's criterion.

2. Rent extraction in a nutshell

This section introduces most of the notation, and revisits the results of [3,5], henceforth C&M–M&R.

Consider a finite Bayesian game $\Gamma = (N, (C_i)_{i\in N}, (T_i)_{i\in N}, (p_i)_{i\in N}, (u_i)_{i\in N})$ where, N is the set of players, and for each player $i \in N$: C_i is the set of possible actions, T_i is the set of types, p_i is the belief over the types of other players (conditional on i's own type), and u_i is the payoff function.

Let σ be a Bayes–Nash equilibrium of Γ, and $u_i^\sigma = (u_i(\sigma|t_i))_{t_i\in T_i}$ be the corresponding vector of *interim* equilibrium payoffs of player i.

The principal problem is to design a mechanism to extract the rents of i: find a finite collection of stochastic entry fees, $f_i^k : T_{-i} \to R$ with $k \in T_i$, in order to maximize

$$\sum_{t_i\in T_i} \left(\min_k \sum_{t_{-i}\in T_{-i}} f_i^k(t_{-i}) p_i(t_{-i}|t_i) \right) p_i(t_i) \tag{1}$$

subject to the participation constraints,

$$\forall t_i \in T_i, \quad \min_k \sum_{t_{-i}\in T_{-i}} f_i^k(t_{-i}) p_i(t_{-i}|t_i) \leqslant u_i^\sigma(t_i). \tag{2}$$

The basic idea is exactly as in C&M–M&R: The principal offers the menu of stochastic entry fees to player i. By picking an entry fee from the menu, player i is allowed to play the game Γ. The payments stipulated by an entry fee do not depend upon the action she chooses to play later on in the game. Conditional on her type, she must select the entry fee with the lowest expected payment, provided it is individually rational.

C&M–M&R identify a property of information structures sufficient for full surplus extraction. The property is also necessary if payoff structures are arbitrary. The condition requires that, for any player, there is no type whose beliefs are convex combination of beliefs of other types of the player herself.

Theorem 1 (*Crémer and McLean [3] and McAfee and Reny [5]*). *The optimal mechanism yields full surplus extraction for any u_i^σ if and only if $\forall\, t_i \in T_i,\ \nexists\, (\lambda_\tau)_{\tau\in T_i} \geqslant 0,\ \lambda_{t_i} < 1$ such that*

$$p_i\,(t_{-i}|t_i) = \sum_{\tau\in T_i} \lambda_\tau\ p_i\,(t_{-i}|\tau) \quad \forall\, t_{-i} \in T_{-i}. \tag{C1}$$

The condition above is often referred to as the *rank condition* or *linear independence of beliefs* condition. It says the matrix of beliefs, whose entries are $p_i(t_{-i}|t_i)$, has full rank. Put simply, the rows $p_i(\cdot|t_i)$ are linearly independent.

McAfee and Reny [5] offer the following interpretation for it: Consider λ as a prior distribution on the agent's types. The induced distribution on t_{-i} is $\sum_{\tau\in T_i} \lambda_\tau\ p_i\,(\cdot|\tau)$. If the equality in (C1) holds, then learning that the agent's type is t_i provides no new information about t_{-i}. The rank condition requires that learning the agent's type is always informative about t_{-i}.

When the beliefs of a type are not a convex combination of other types' beliefs, there is a hyperplane that separates the beliefs of this type from the beliefs of other types. In other words, there is a contingent entry fee $g_i^{t_i}(\cdot)$ such that $\langle g_i^{t_i},\ p_i(\cdot|t_i)\rangle = 0$ and $\langle g_i^{t_i},\ p_i(\cdot|\tau_i)\rangle > 0$ for all $\tau_i \neq t_i$. The inner product of the entry fee and beliefs of a type is just the expected payment the type incurs. Consequently, by offering the menu of entry fees $\{g_i^{t_i}\}_{t_i \in T_i}$, the principal is able to induce the players to reveal their beliefs. A type who chooses an entry fee intended to another type incurs positive payments but, no payments are incurred by choosing the entry fee tailored to one's own type. In the language of [6], belief extraction is feasible.

In order to extract payoffs, the principal constructs another menu, $\{f_i^{t_i} = u_i^{\sigma}(t_i)\mathbf{1} + \omega_i g_i^{t_i}\}_{t_i \in T_i}$, where any type that chooses the entry fee $f_i^{t_i}$ pays $u_i^{\sigma}(t_i)$, the interim equilibrium payoffs of type t_i with certainty and, in addition, pays the expected penalty of misreporting. The penalty is zero if no misreporting occurs but otherwise, the principal can make it arbitrarily large by increasing ω_i.

3. Blackwell's ranking and informational rents

Most applications identify the 'types' of a player with realizations of a given single-dimensional signal. In this paper, the types of a player describe the particular extraction of a signal x *as well as* the probability distribution from where the player's signal is drawn from, which is parameterized by θ.

From the point of view of the principal, θ is not known with certainty, possibly, due to covert acquisition of information, which the principal cannot, directly or indirectly, control. In this paper, however, the underlying model of acquisition of information is left un-modelled. For simplicity, player i has at her disposal only two signals: $X^{\overline{\theta}}$ or $X^{\underline{\theta}}$.

Signals are ranked in order of informativeness accordingly Blackwell's criterion:[1] $X^{\overline{\theta}}$ is more informative, about others players' types t_{-i}, than signal $X^{\underline{\theta}}$, if and only if, there are scalars $B_{(x,y)} \geqslant 0$ such that, for all y realizations of $X^{\underline{\theta}}$ and all $t_{-i} \in T_{-i}$,

$$\frac{p\left(t_i = (y, \underline{\theta})|t_{-i}\right)}{p\left(t_{i2} = \underline{\theta}\right)} = \sum_{x \in \text{supp } X^{\overline{\theta}}} B_{(x,y)} \frac{p\left(t_i = (x, \overline{\theta})|t_{-i}\right)}{p\left(t_{i2} = \overline{\theta}\right)}, \tag{C2}$$

where p is a common-prior distribution and

$$\sum_{y \in \text{supp } X^{\underline{\theta}}} B_{(x,y)} = 1.$$

More precisely, (C2) says signal $X^{\underline{\theta}}$ is a garbling of $X^{\overline{\theta}}$ and, the degree of informativeness of i's signal is independent of others' information.[2] Intuitively, $X^{\underline{\theta}}$ is a further randomization over the possible outcomes of $X^{\overline{\theta}}$, with the $B_{(x,y)}$ being the weights employed by the 'randomization' as depicted in Fig. 1.

The next proposition establishes that equivalence of the Blackwell's ordering of signals and the rank condition (C1).

[1] See [2].

[2] I am grateful to a referee for pointing (C2) also requires independence of θ and t_{-i}.

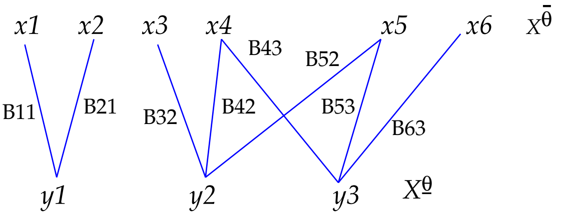

Fig. 1. $X^{\underline{\theta}}$ is obtained by adding 'noise' to $X^{\overline{\theta}}$.

Proposition 2. *The equality in* (C1) *holds if and only if it is possible to write the set of types of player i as* $T_i = \{x_1, \ldots, x_{n_i}\} \times \{\underline{\theta}, \overline{\theta}\}$ *and find signals* $X^{\overline{\theta}}$ *and* $X^{\underline{\theta}}$ *such that*: (1) *The distribution of types can be expressed as* $p(t_i, t_{-i}) = \Pr(X^{t_{i2}} = t_{i1}, t_{-i}) \Pr(t_{i2})$ *and* (2) *The signal* $X^{\overline{\theta}}$ *is sufficient for the signal* $X^{\underline{\theta}}$ *regarding* t_{-i}.

Proof. First, we prove that if (C2) holds then the equality in (C1) holds. Let $\widehat{\lambda}_{(x,y)} = B_{(x,y)} \frac{p\left(t_i=(x,\overline{\theta})\right)}{p(t_i=(y,\underline{\theta}))}$, where $B_{(x,y)}$ are the coefficients implied by Blackwell's ranking (C2). Bayes' rule implies that,

$$\begin{aligned}
\frac{p\left(t_{-i}|t_i = (y,\underline{\theta})\right)}{p(t_{i2} = \underline{\theta})} &= \frac{p(t_{-i})}{p(t_i = (y,\underline{\theta}))} \frac{p\left(t_i = (y,\underline{\theta})|t_{-i}\right)}{p(t_{i2} = \underline{\theta})} \\
&= \frac{p(t_{-i})}{p(t_i = (y,\underline{\theta}))} \sum_{x \in \mathrm{supp}\ X^{\overline{\theta}}} B_{(x,y)} \frac{p\left(t_i = (x,\overline{\theta})|t_{-i}\right)}{p\left(t_{i2} = \overline{\theta}\right)} \\
&= \frac{p(t_{-i})}{p(t_i = (y,\underline{\theta}))} \sum_{x \in \mathrm{supp}\ X^{\overline{\theta}}} B_{(x,y)} \\
&\quad \times \frac{p\left(t_{-i}|t_i = (x,\overline{\theta})\right)}{p\left(t_{i2} = \overline{\theta}\right)} \frac{p\left(t_i = (x,\overline{\theta})\right)}{p\left(t_{-i}\right)} \\
&= \sum_{x \in \mathrm{supp}\ X^{\overline{\theta}}} B_{(x,y)} \frac{p\left(t_i = (x,\overline{\theta})\right)}{p(t_i = (y,\underline{\theta}))} \frac{p\left(t_{-i}|t_i = (x,\overline{\theta})\right)}{p\left(t_{i2} = \overline{\theta}\right)} \\
&= \sum_{x \in \mathrm{supp}\ X^{\overline{\theta}}} \widehat{\lambda}_{(x,y)} \frac{p\left(t_{-i}|t_i = (x,\overline{\theta})\right)}{p\left(t_{i2} = \overline{\theta}\right)},
\end{aligned}$$

which is essentially the equality in (C1), with $\lambda_{(x,y)} = \widehat{\lambda}_{(x,y)} \frac{p(t_{i2}=\underline{\theta})}{p(t_{i2}=\overline{\theta})}$.

To prove the converse, consider $\mathcal{T}_i$, a subset of T_i maximal with respect the following property: beliefs about t_{-i} of any type $t_i \in \mathcal{T}_i$ cannot be expressed as a convex combination of beliefs of types in the set $\mathcal{T}_i \setminus \{t_i\}$. For finite type spaces, $\mathcal{T}_i$ can be obtained by a constructive proof while, for infinite T_i, Zorn's lemma is needed.

The beliefs of types in $\mathcal{T}_i$ form a base for the beliefs of types in T_i. As a result, beliefs of any type in $T_i \backslash \mathcal{T}_i$ are a convex combination of beliefs of types in $\mathcal{T}_i$. Finally, to construct the more informative signal, $X^{\overline{\theta}}$, set the joint distribution:

$$\Pr\left(X^{\overline{\theta}} = x, t_{-i}\right) = \begin{cases} 0 & \text{for } x \notin \mathcal{T}_i, \\ \dfrac{p(x, t_{-i})}{\Pr(\mathcal{T}_i)} & \text{for } x \in \mathcal{T}_i \ . \end{cases} \tag{3}$$

The less informative signal has support on $T_i \backslash \mathcal{T}_i$ and its construction is analogous. To verify that $X^{\overline{\theta}}$ is sufficient for $X^{\underline{\theta}}$, use Bayes' rule in (C1) to obtain Blackwell's ranking, with $B_{(x,\tau)} = \lambda_\tau \frac{p(x)}{p(\tau)} \frac{1-p(\mathcal{T}_i)}{p(\mathcal{T}_i)}$. $\square$

Observe that the proof of Proposition 2 can be easily adapted for the case of continuous distributions.

It is well known that for independent information, the rank condition fails. In this case, the corresponding signals obtained by Proposition 2 are equivalent in the sense of Blackwell: $X^{\overline{\theta}}$ and $X^{\underline{\theta}}$ generate identical beliefs. Consequently, θ must be payoff relevant if full surplus extraction were to fail.

Neeman [6] points that, if beliefs do not determine preferences, that is, if there are types with identical beliefs but distinct valuations, then the rank condition fails. Again, the corresponding signals implied by Proposition 2 are Blackwell equivalent and, θ must be payoff relevant if full surplus extraction were to fail.

When the parameter θ is purely informational, not payoff relevant, can full surplus extraction fails? To answer this question, the VNM utility of player *i* is hereafter assumed to be independent of *i*'s type. In sum, $u_i(c, t_{-i})$ where *c* is the action profile and t_{-i} is the profile of other players' types. The expected utility remains function of the player's type.

Condition (C2) of Proposition 2 implies that full extraction is not achievable for *arbitrary* payoffs. Yet, equilibrium payoffs at Γ are not arbitrary when θ *only affects beliefs but not the VNM utility function*. One should ask, what restrictions (C2) imposes on payoffs? The answer is: for all $y \in \text{supp } X^{\underline{\theta}}$,

$$u_i^\sigma(y, \underline{\theta}) \leqslant \sum_{x \in \text{supp } X^{\overline{\theta}}} \lambda_{(x,y)} \ u_i^\sigma(x, \overline{\theta}) = \lambda_y^\top \ u_i^\sigma(\overline{\theta}). \tag{C3}$$

The inequality follows from directly from Blackwell's theorem: holding constant the strategies of others σ_{-i} in the game Γ, player *i* faces a decision problem. As a result, conditional on observing the more informative signal, her expected payoff must be weekly higher than conditional on observing the less informative signal.

The condition below says information is valuable at the game Γ. The condition is necessary for the existence of informational rents and, Corollary 4 shows that it is also sufficient.

Condition 3. *There exists at least one poorly informed type* $(y, \underline{\theta})$ *such that C3 holds as a strictly inequality.*

The following corollary establishes the existence of informational rents.

Corollary 4. *Assume Condition* 3 *and also that no type is excluded by the optimal mechanism, then player i obtains a higher expected surplus at the optimal mechanism when her signal is* $X^{\overline{\theta}}$ *than the surplus she gets when her signal is* $X^{\underline{\theta}}$.

Proof. Once more, the proof follows directly from Blackwell's Theorem: for a given the optimal menu of fees and the choices of the other players, the agent faces a decision problem. Consequently, the well informed is guaranteed to pay on expectation no more than the poorly informed pays. In the following game Γ, however, (C3) and Condition 3 assure that the well informed has a strictly higher surplus. □

An Example. Let Γ be the game given below where s_1 and s_2 are chance moves made by Nature, with equal probability. Player 1 observes Nature's move with probability $0 < \rho < 1$ and so, $T^1 = \{\underline{\theta}, \overline{\theta}_1, \overline{\theta}_2\}$. Player 2 gets a noisy signal about Nature's move, $t_2 \in T^2$, which satisfies $0 < p(s_2|t_2) < \frac{2}{3}$. Since we are interested in extracting 1's surplus, player 2, in this example, is basically a dummy.

s_1	C	D
C	8, 8	0, 0
D	0, 0	0, 0

s_2	C	D
C	2, 2	0, 6
D	6, 0	0, 0

We consider the following Nash equilibrium: $\sigma_1(\underline{\theta}) = \sigma_1(\overline{\theta}_1) = \sigma_2(t_2) = C$ and $\sigma_1(\overline{\theta}_2) = D$ and so, $u_1^{\sigma} = (5, 8, 6)^{\top}$. Let $f_1^{t_1}(\cdot) \in \mathrm{R}^{\#T^2}$ be the entry fee designed for type t_1 and, let $p(\cdot|t_1) \in \mathrm{R}^{\#T^2}$ be the beliefs of type t_1 of player 1 about types of player 2.

Since, $p(\cdot|\underline{\theta}) = \frac{1}{2}p(\cdot|\overline{\theta}_1) + \frac{1}{2}p(\cdot|\overline{\theta}_2)$ it follows that full extraction, $\langle f_1^{\underline{\theta}},\ p(\cdot|\underline{\theta})\rangle = 5$, $\langle f_1^{\overline{\theta}_1},\ p(\cdot|\overline{\theta}_1)\rangle = 8$ and $\langle f_1^{\overline{\theta}_2},\ p(\cdot|\overline{\theta}_2)\rangle = 6$, is not achievable. Furthermore, the principal cannot extract the surplus of the informed types only, $\overline{\theta}_1$ and $\overline{\theta}_2$, without violating the participation constraint of the uninformed type, $\underline{\theta}$.

There are many optimal schedules in this example. Any optimal menu has to extract the surplus of types $\underline{\theta}$ and $\overline{\theta}_k$ leaving informational rents to type $\overline{\theta}_{-k}$. Moreover, the informed agent's expected payment equals the uninformed agent's payment. Therefore, the principal's expected revenue is 5 and, expected information rent is 2.

Finally, if acquiring information is costly, to support player 1 randomization as an equilibrium outcome, $0 < \rho < 1$, the information acquisition cost has to match the expected informational rent.

4. Conclusion

The type space considered in this paper is quite parsimonious. It is a simple short-cut to point that uncertainty about information quality matters. Clearly, it remains unrealistic to assume that the set of possible signals are available to the agents is known with certainty. To describe a more realistic environment, a richer language capable of describing higher order beliefs, as employed by Epstein and Peters [4] or [1], is called for.

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