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Forecasting stock return volatility at the quarterly frequency: an evaluation of time series approaches

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The last decade has seen substantial advances in the measurement, modelling and forecasting of volatility which has centered around the realized volatility literature. To date, most of the focus has been on the daily and monthly frequencies, with little attention on longer horizons such as the quarterly frequency. In finance applications, forecasts of volatility at horizons such as quarterly are of fundamental importance to asset pricing and risk management. In this article we evaluate models for stock return volatility forecasting at the quarterly frequency. We find that an autoregressive model with one lag of quarterly realized volatility with an in-sample estimation period of between 60 and 80 quarters produces the most accurate forecasts, and dominates other approaches, such as the recently proposed mixed-data sampling (MIDAS) approach.

Keywords: financial risk management; high-frequency returns; realized volatility; time-series modelling

JEL Classification: C53; G19

I. Introduction

Forecasting return volatility at horizons such as the quarterly frequency plays an important role in asset pricing and financial risk management. For example, often asset allocation decisions requiring volatility forecasts are made on a quarterly basis. Quarterly forecasts of return volatility implied from option prices are heavily used by market participants, though these implied volatilities are based on market prices which may be subject to mispricing and are not always readily available. Often volatility forecasts based on historical time series of returns are also utilized, based on autoregressive specifications following Engle (1982) and Bollerslev (1986). More recently, improvements in volatility forecast accuracy have been achieved by utilizing the realized volatility measurement and modelling approaches of Andersen and Bollerslev (1998), Barndorff-Nielsen and Shephard (2002) and Andersen *et al*. (2003). The realized volatility literature has primarily focused on short horizon volatility forecasting ranging from daily to monthly frequencies as there is a high degree of predictability at these frequencies, see for example, Maheu and McCurdy (2002), Koopman *et al*. (2005), Andersen *et al*. (2007), Corsi (2009) and Martens *et al*. (2009). Ghysels *et al*. (2009) explore longer range return volatility forecasting, demonstrating predictability at the quarterly horizon, and showing that the mixed-data sampling (MIDAS) approach, introduced by Ghysels *et al*. (2005, 2006), has superior forecasting performance relative to commonly utilized models such as GARCH.

In this article, we focus on evaluating the forecasting performance of simple autoregressive models of stock return quarterly realized volatility, against MIDAS

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approaches. The assets analysed are stocks from the Dow Jones Industrial Average (DJIA) index due to the availability of reliable 30 minute returns for quarterly realized volatility measurement for the purposes of measuring forecast accuracy. All stocks display significant first order auto-correlation in quarterly realized volatility. Some stocks display first order auto-correlation in quarterly realized volatility of 0.5, showing strong time dependencies and predictability. We demonstrate superior forecasting performance from a simple autoregressive model with one lag of quarterly realized volatility AR(1), which dominates the MIDAS approach. The quarterly realized volatility for the AR (1) model estimation is computed from daily returns and thus this quarterly forecasting procedure can be applied to a wide class of assets. The MIDAS volatility forecasts are generated from various specifications, including the beta model, restricted beta model, exponential model restricted exponential model and the hyperbolic model. Over our sample of stocks, we find that the simple autoregressive model with one lag of quarterly realized volatility has a lower mean-squared-forecast-error and mean-absolute-forecast-error than the MIDAS forecasts. Since the MIDAS models are also more complicated to estimate, than the simple autoregressive models, often involving nonlinear estimation methods, we conclude in favour of the simple autoregressive model forecasts.

The results of this article highlight an important principle in forecasting, in that relatively parsimonious models often deliver the most reliable forecasts. In the ARCH literature, initiated by Engle (1982), the GARCH(1,1) model proposed by Bollerslev (1986) is the simplest and in general delivers the most accurate forecasts, relative to other more complicated ARCH models, such as IGARCH, EGARCH and FIGARCH. In the realized volatility literature, Andersen *et al.* (2003) demonstrate the dominance of parsimonious models for short horizon volatility forecasts finding the leading model to be a simple autoregressive model of daily realized volatility. This dominance was demonstrated over an extensive range of commonly used time series forecasting models for volatility, including nonlinear models. However, Ghysels *et al.* (2006) with MIDAS models and Corsi (2009) with a Heterogeneous Autoregressive (HAR) model do find some improvements in short horizon volatility predictions in their samples, relative to simple autoregressive specifications of daily realized volatility.

This article is organized as follows. Section II reviews realized volatility measurement and Section III describes our data set of DJIA stocks. Section IV discusses the forecasting approaches and Section V details the empirical results of our study. Section VI contains concluding remarks.

II. Volatility Measurement

In this section we briefly discuss the theoretical justification behind realized volatility measurement and computational issues. First, consider the following class of continuous-time jump diffusions, in which the logarithmic price process $\{p(t)\}_{t\geq 0}$ follows

$$\mathrm{d}p(t) = \mu(t)\mathrm{d}t + \sigma(t)\mathrm{d}W(t) + J(t)\mathrm{d}q(t) \tag{1}$$

where $W(t)$ is standard Brownian motion, $\sigma(t)$ is the volatility process, $\mu(t)$ has bounded and finite variation, $J(t)$ is the jump size and $q(t)$ is a counting process such that $\mathrm{d}q(t) = 0$ when there is no jump and $\mathrm{d}q(t) = 1$ when there is a jump. Next, note that the quadratic variation of the return $r(t) = p(t) - p(t-1)$, is

$$QV_t = \int_{t-1}^{t} \sigma^2(s)\mathrm{d}s + \sum_{t-1<s\leq t} J^2(s) \tag{2}$$

If the process is sampled N times per period on an equally spaced grid, we can then define the $\delta = 1/N$ period returns as $r_{t,i} = p(t + i\delta) - p(t + (i-1)\delta)$, $i = 1, 2, ..., N$. Realized volatility is defined as the sum of squared intraperiod returns sampled at frequency δ,

$$RV_{t+1} = \sum_{i=1}^{N} r_{t,i}^2 \tag{3}$$

As the return sampling frequency tends to infinity, and with zero mean returns, the realized volatility estimator approaches the quadratic variation of the return, following conventional arguments, see Andersen *et al.* (2003) and Barndorff-Nielsen and Shephard (2004). This result of RV_t being a consistent estimator of QV_t as $N \to \infty$ is the theoretical motivation behind realized volatility measurement; however, there are important considerations in regard to the return sampling frequency when realized volatility is computed in practice. First, market microstructure noise (e.g. discreteness of prices and bid/ask bounce) can result in inaccurate high frequency return measurement, thus a balance needs to be reached between a sufficiently high N and a reliable $r_{t,i}$. Bollerslev *et al.* (2007) suggest that a sampling frequency of 22.5 minutes mitigates the effect of 'noise' for all of their 40 stocks. Thus, taking a cautious approach with our Dow stocks, we choose a sampling frequency of 30 minutes for our intraday returns. For the purposes of *ex-post* volatility measurement in forecast evaluation, this article computes realized volatility over a calendar quarter using 30 minute intraday returns and overnight returns. That is, for each trading day in the quarter, we compute the 30 minute intraday returns and overnight return. These returns are then squared and summed over the quarter.

III. Data

Our data set consists of stocks from the DJIA index. Daily data from 1 January 1975 to 31 July 2008, consisting of stock returns, open and close prices, are obtained from the Center for Research in Security Prices (CRSP), with adjustments made for corporate actions, such as dividends, splits, etc. High-frequency data from 1 August 1997 to 31 July 2008, consisting of 30-minute intraday price data, sampled from 9:30 am to 3:30 am, are obtained from Price-Data (www.grainmarketresearch.com). The following stocks, Home Depot, Citigroup, Microsoft, AT&T Inc., Chevron Corp., Verizon Communication and Exxon Mobil Corp., are excluded due to incomplete return time series over the study period, leaving 23 stocks for our analysis.

IV. Forecasting Approaches

We focus our study on three forecasting approaches for quarterly volatility. These are forecasts from constant volatility models, autoregressive realized volatility models and MIDAS models. Constant volatility models are chosen as an initial benchmark and also because these are often used by practitioners, see for example, common estimates of Value-at-Risk (VaR). Autoregressive realized volatility models are chosen based on their recent popularity in forecasting short range volatility, see Andersen *et al.* (2003). MIDAS models are chosen as Ghysels *et al.* (2009) recently demonstrate these models dominating other commonly used forecasting approaches for quarterly volatility such as GARCH. We now briefly discuss each of these three approaches.

Constant volatility models

Constant volatility models forecast volatility from an average volatility measurement over a prior time period. Our constant volatility quarterly forecasts are computed as the average quarterly realized volatility computed over the prior l quarters, where the quarterly realized volatility is computed from daily returns. With the *ith* stocks quarterly volatility forecasting equation being:

$$\sigma^2_{i,t+1} = \frac{1}{l}\sum_{k=0}^{l-1}\sigma^2_{i,t-k} \tag{4}$$

Autoregressive realized volatility models

The most commonly estimated model in the realized volatility literature is the autoregressive model of logged realized volatility with p lags defined as:

$$\ln(\sigma^2_{i,t+1}) = \phi_{i,0} + \sum_{k=1}^{p}\phi_{i,k}\ln(\sigma^2_{i,t+1-k}) + \epsilon_{i,t+1} \tag{5}$$

In our model estimations, $\sigma^2_{i,t}$ is the quarterly realized volatility computed from daily returns for stock i during quarter t and a forecast of $\sigma^2_{i,t+1}$ is obtained from the forecast of $\ln(\sigma^2_{i,t+1})$, assuming log-normality.

MIDAS models

The MIDAS approach, introduced by Ghysels *et al.* (2005, 2006) has recently been advocated as an approach to quarterly volatility forecasting that dominates other commonly used approaches, see Ghysels *et al.* (2009). In the MIDAS approach to quarterly volatility forecasting, the forecasting regression can be formulated as follows:

$$\sigma^2_{i,t+1} = \mu_i + \phi_i\sum_{j=0}^{j^{\max}} b_i(j,\theta)r^2_{t-j} + \epsilon_{i,t+1} \tag{6}$$

where $\sigma^2_{i,t+1}$ is a measure of quarterly volatility (in our model estimations quarterly realized volatility is computed by the sum of squared daily returns within the quarter) and the regressors, $r^2_{t-j}, j = 0, \ldots, j^{\max}$ are measured at a higher frequency (in our our applications we use daily squared returns, following Ghysels *et al.* (2009)). The weighting function, $b_i(j,\theta)$, is parameterized by a low-dimensional parameter vector θ. The intercept μ_i, slope ϕ_i and weighting parameters θ are typically estimated with a Gaussian likelihood as quasi-maximum likelihood estimation, which we follow in our model estimations.

The following weighting functions have been suggested by Ghysels *et al.* (2005, 2006) and Ghysels *et al.* (2009) and are empirically evaluated in Section V.

(1) Exponential:

$$b_i(j,\theta_1,\theta_2) = \frac{\exp\{\theta_1 j + \theta_2 j^2\}}{\sum_{k=0}^{\infty}\exp\{\theta_1 k + \theta_2 k^2\}} \tag{7}$$

which can produce a variety of different decay patterns.

(2) Beta:

$$b_i(j,\theta_1,\theta_2) = \frac{f(\frac{j}{j^{\max}},\theta_1;\theta_2)}{\sum_{k=1}^{j^{\max}} f(\frac{k}{j^{\max}},\theta_1;\theta_2)} \tag{8}$$

where:

$$f(x,a,b) = \frac{x^{a-1}(1-x)^{b-1}\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \tag{9}$$

$$\Gamma(a) = \int_0^{\infty} e^{-x}x^{a-1}\mathrm{d}x \tag{10}$$

which can also produce a variety of different decay patterns.

(3) Hyperbolic:

$$b_i(j,\theta) = \frac{g(\frac{j}{j^{\max}},\theta)}{\sum_{k=1}^{j^{\max}} g(\frac{k}{j^{\max}},\theta)} \tag{11}$$

where $g(j,\theta) = \Gamma(j+\theta)/(\Gamma(j+1)\Gamma(\theta))$ which can be written equivalently as $g(0,\theta) = 1$ and $g(j,\theta) = (j+\theta-1)g(j-1,\theta)/j$, for $j \geq 1$. This specification is not as flexible as the beta weighting function.

In addition to the above three functional forms, two restricted versions of the exponential and beta specifications have also been suggested. 'Exp Rest' is when the constraint of $\theta_2 = 0$ is imposed on the exponential specification and 'Beta Rest' is when the constraint of $\theta_1 = 1$ is imposed on the beta specification. These restricted specifications lead to a slowly decaying pattern of the weighting functions. For further details, see Ghysels *et al*. (2009).

V. Empirical Results

Figure 1 displays the quarterly realized volatility for our 23 DJIA stocks, computed both from 30-minute and daily returns. There is a close correspondence between these two volatility measures, though the volatility measure from daily returns displays an upward bias. Figures 2 and 3 display the Autocorrelation Functions (ACFs) and Partial Autocorrelation Functions (PACFs) of quarterly logarithmic realized volatility, computed from daily returns, for each stock over the sample period from 1997Q4 to 2008Q2. The quarterly logarithmic realized volatility have gradual declining autocorrelations for all stocks in our sample and most of the PACFs cut off at lag 1.

We next empirically evaluate the three approaches to quarterly volatility forecasting, discussed in the previous section, by generating one-quarter-ahead forecasts and then rolling the in-sample estimation period forward one quarter. One quarter ahead forecasts from these models over the forecast evaluation period from 1997Q4 to 2008Q2 (43 out-of-sample periods) are measured against the quarterly realized volatility computed from 30-minute returns and overnight returns. The prediction accuracy is evaluated on the basis of mean squared error (MSE) and mean absolute error (MAE) for each stock. The MSE and MAE are computed as follows:

$$\text{MSE} = \frac{1}{m}\sum_{r=1}^{m}(\widetilde{\sigma_{i,r}^2} - \widehat{\sigma_{i,r}^2})^2 \tag{12}$$

$$\text{MAE} = \frac{1}{m}\sum_{r=1}^{m}|\widetilde{\sigma_{i,r}^2} - \widehat{\sigma_{i,r}^2}| \tag{13}$$

where $\widetilde{\sigma_{i,r}^2}$ represents the one-quarter-ahead volatility forecast made at the end of quarter $r-1$ for stock i and $\widehat{\sigma_{i,r}^2}$ denotes the realized volatility computed from 30-minute returns and overnight returns in quarter r for stock i. An important difference in our forecast evaluation, with Ghysels *et al*. (2009), is that our $\widehat{\sigma_{i,r}^2}$ is computed from over 750 returns, whereas the Ghysels *et al*. (2009) $\widehat{\sigma_{i,r}^2}$ is computed from only 60 returns. Since we have chosen highly liquid assets, we are able to report both MSE and MAE to demonstrate that our conclusions are not sensitive to the choice of these two popular loss functions. Ghysels *et al*. (2009) do not report MAE results due to concerns over measurement error in their $\widehat{\sigma_{i,r}^2}$ affecting this loss function, as demonstrated by Patton (2011).

The constant model forecasts are from in-sample estimation periods of 1, 2, 3, 4, 5 and 6 quarters. Extended in-sample estimation periods for the constant model are not considered due to the well known time-variation in volatility. While the AR model forecasts are from in-sample estimation periods of 20, 30, 40, 50, 60, 70 and 80 quarters. Finally, the MIDAS model forecasts are from an in-sample estimation period of 68 quarters, with MIDAS lag lengths of 40, 60, 80, 100, 150 and 200 trading days. The AR and MIDAS models in-sample estimation periods are chosen based on the available historical data.

Tables 1 and 2 display the performance of the one-quarter-ahead volatility forecasts. The values of MSE and MAE are the average values over all 23 stocks for each model. The AR(1) model with an in-sample estimation period of 70 quarters produces the lowest MSE and MAE as 2.5123 and 0.9903, respectively. AR models with in-sample estimation periods of 60, 70 and 80 quarters produce very similar forecast performance. Not surprisingly given the well known time-variation in volatility, the constant model forecasts perform poorly. Lastly, the MIDAS models demonstrate varying degrees of performance. The unrestricted beta, restricted beta and unrestricted exponential model forecast errors are worse than some of the constant models. Whereas the restricted exponential and hyperbolic model forecast errors are less than the constant models. Relative to the other MIDAS specifications, the hyperbolic model with a lag length of 100 trading days produces the lowest MSE and MAE as 2.7146 and 1.0802, respectively.

Given the lower MSE and MAE results for the AR (1) model, its parsimonious specification and ease of

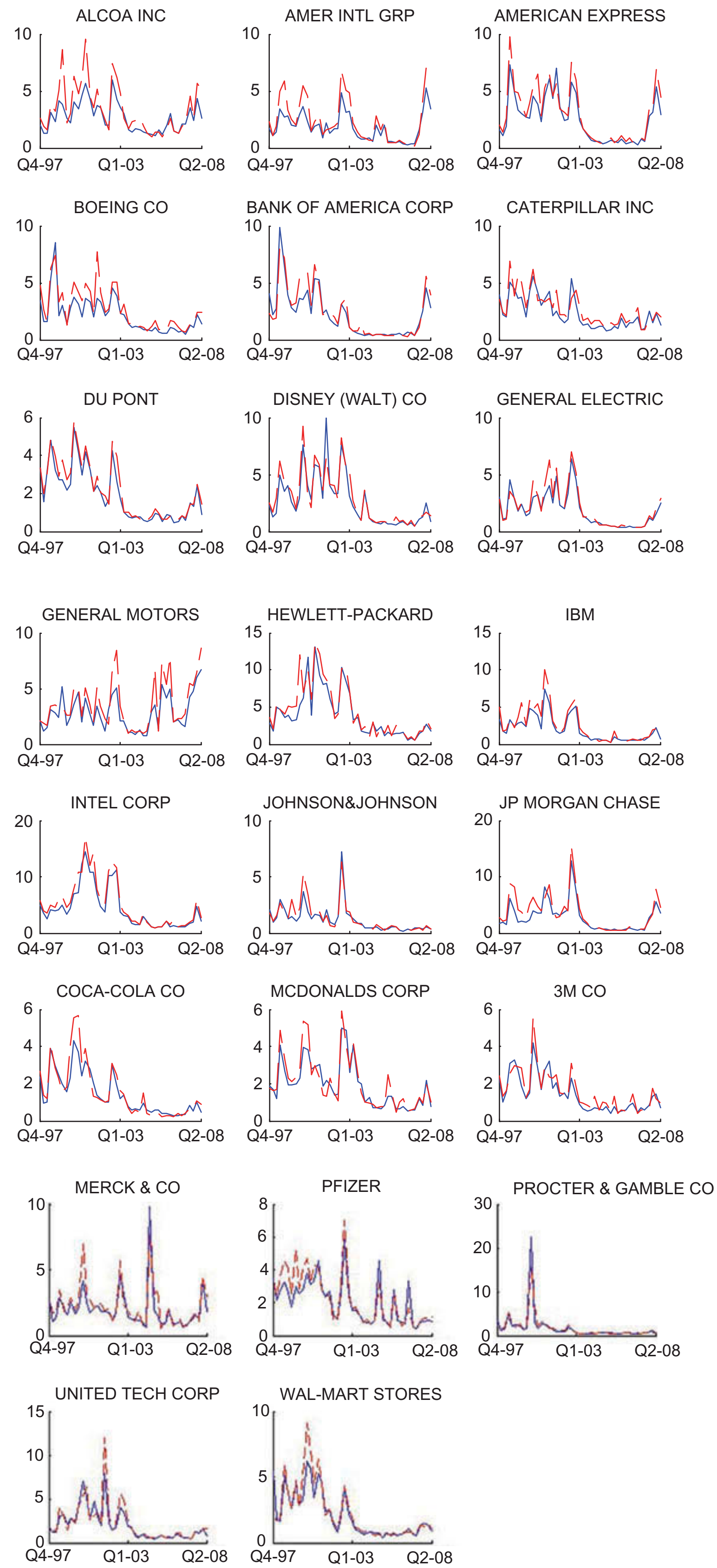

Fig. 1. Quarterly realized volatility
Notes: The solid line is the proxy for the true realized volatility which is computed from 30-minute returns and overnight returns. The dotted line is the realized volatility computed from daily returns. The sample covers the period from 1997Q4 to 2008Q2, 43 quarters in total. Both volatility measures are multiplied by 100.

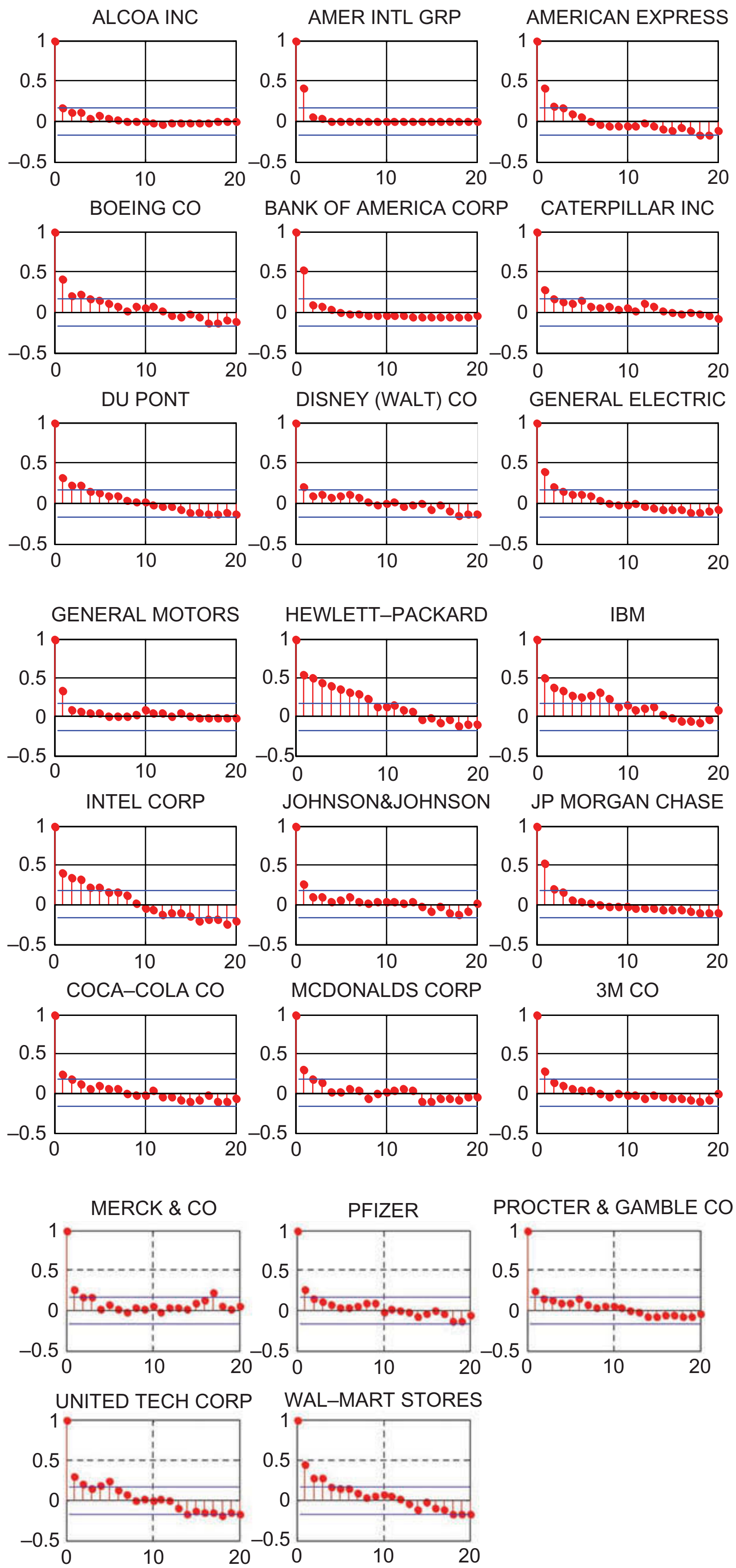

Fig. 2. Autocorrelation functions for logarithmic quarterly realized volatility

Notes: The quarterly realized volatility is computed from daily returns and the sample covers the period from 1997Q4 to 2008Q2. The marked confidence bands are at the 95% level.

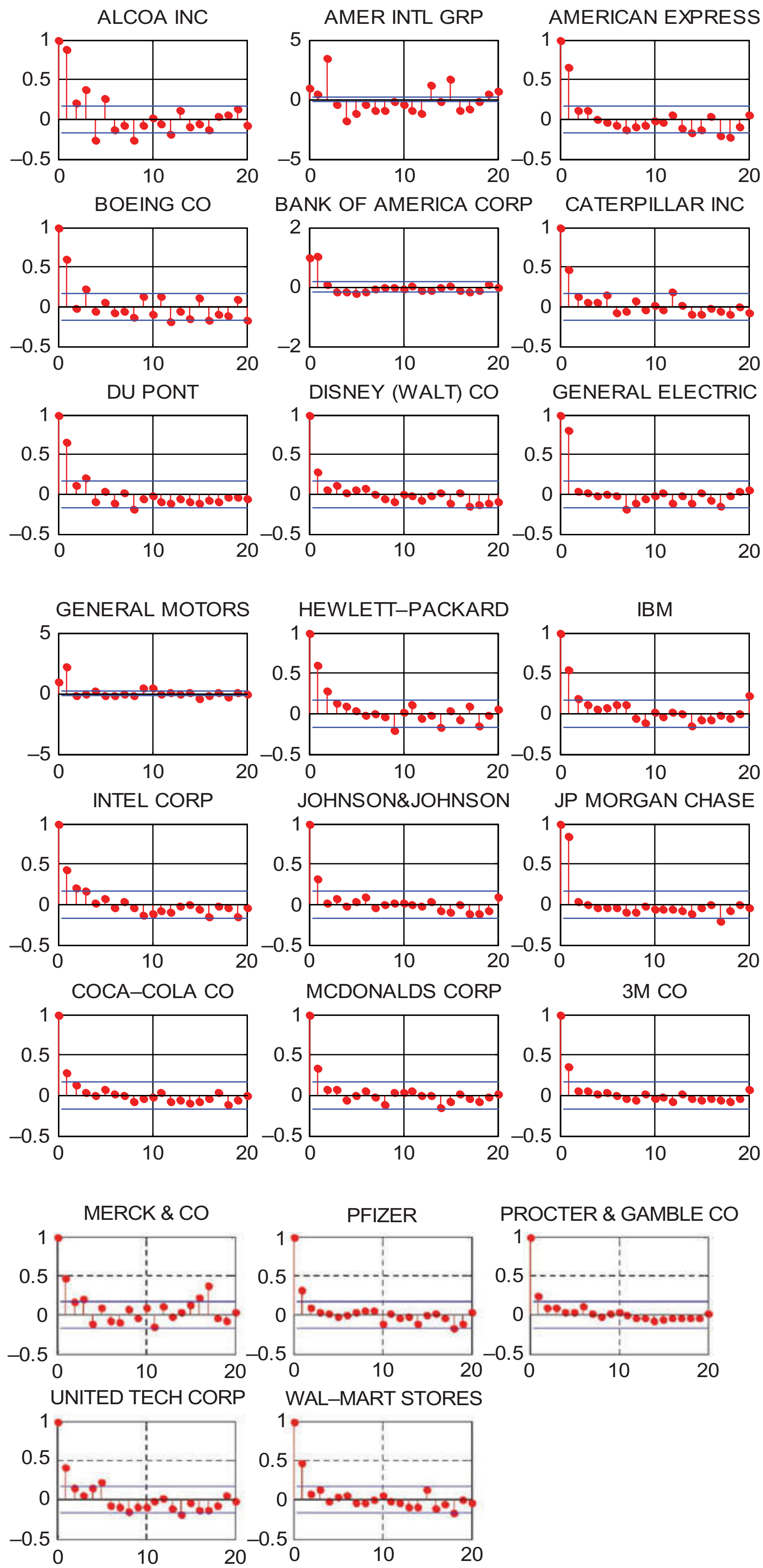

Fig. 3. Partial autocorrelation functions for logarithmic quarterly realized volatility
Notes: The quarterly realized volatility is computed from daily returns and the sample covers the period from 1997Q4 to 2008Q2. The marked confidence bands are at the 95% level.

Table 1. Average MSE of one-quarter-ahead volatility forecasts

A		In-sample size 1Q	2Q	3Q	4Q	5Q	6Q
Constant model		3.8085	3.4697	3.3156	3.2622	3.2757	3.1914
B	In-sample size 20Q	30Q	40Q	50Q	60Q	70Q	80Q
AR(1)	3.0848	2.9676	2.7900	2.5969	2.5262	**2.5123**	2.5263
AR(2)	3.6076	3.1639	2.9240	2.7473	2.6097	2.5694	2.5420
AR(3)	4.1923	3.3301	2.9829	2.8288	2.7289	2.6849	2.6263
AR(4)	5.7534	3.6887	3.1576	2.8935	2.7470	2.7176	2.6599
AR(5)	9.6888	4.1559	3.3163	2.9525	2.7800	2.7838	2.7023
C MIDAS lag length			MIDAS specification BETA	BETA REST	EXP	EXP REST	HYPERB
40			3.7946	3.2419	3.3573	3.3314	2.8993
60			3.5392	3.2159	3.4558	3.2205	2.7218
80			3.5257	3.2190	3.5150	3.5670	3.0740
100			3.5352	3.1776	3.4518	3.2003	2.7146
150			3.5675	3.1660	3.3944	3.2325	2.7134
200			3.5165	3.0004	3.4479	3.2185	2.7332

Notes: For the AR models, the in-sample estimation period is expressed in number of quarters. The constant model forecast is the average of realized volatility over the past quarters. The MIDAS specifications include the unrestricted beta model denoted by BETA, the restricted beta model, denoted by BETA REST, the unrestricted exponential model, denoted by EXP, the restricted exponential model, denoted by EXP REST and the hyperbolic model, denoted by HYPERB. The MIDAS estimation is based on data over the past 68 quarters. The forecast evaluation period is from 1997Q4 to 2008Q2. Values are computed by averaging over stocks and the minimum value is highlighted in bold.

Table 2. Average MAE of one-quarter-ahead volatility forecasts

A		In-sample size 1Q	2Q	3Q	4Q	5Q	6Q
Constant model		1.1790	1.1583	1.1495	1.1560	1.1760	1.1758
B	In-sample size 20Q	30Q	40Q	50Q	60Q	70Q	80Q
AR(1)	1.1470	1.1202	1.0673	1.0088	**0.9844**	0.9903	0.9920
AR(2)	1.2279	1.1707	1.0963	1.0450	1.0142	1.0107	1.0028
AR(3)	1.2962	1.2008	1.1092	1.0642	1.0326	1.0290	1.0175
AR(4)	1.4036	1.2683	1.1468	1.0757	1.0324	1.0294	1.0211
AR(5)	1.6438	1.3567	1.1982	1.1096	1.0516	1.0563	1.0418
C MIDAS lag length			MIDAS specification BETA	BETA REST	EXP	EXP REST	HYPERB
40			1.2186	1.1502	1.2145	1.1653	1.1023
60			1.1755	1.1444	1.2512	1.1648	1.0691
80			1.1780	1.1481	1.2446	1.1805	1.0999
100			1.1850	1.1410	1.2339	1.1632	1.0802
150			1.1715	1.1395	1.2394	1.1660	1.0840
200			1.1584	1.1262	1.2445	1.1778	1.0817

Notes: For the AR models, the in-sample estimation period is expressed in number of quarters. The constant model forecast is the average of realized volatility over the past quarters. The MIDAS specifications include the unrestricted beta model denoted by BETA, the restricted beta model, denoted by BETA REST, the unrestricted exponential model, denoted by EXP, the restricted exponential model, denoted by EXP REST and the hyperbolic model, denoted by HYPERB. The MIDAS estimation is based on data over the past 68 quarters. The forecast evaluation period is from 1997Q4 to 2008Q2. Values are computed by averaging over stocks and the minimum value is highlighted in bold.

estimation, it dominates the other models for quarterly volatility forecasting. Table 3 and 4 display the MSE and MAE for the AR(1) one-quarter-ahead volatility forecast for each stock, with in-sample estimation periods ranging from 20 to 80 quarters. Over the 23 stocks evaluated, 22 of these stocks have the lowest

Table 3. MSE of one-quarter-ahead AR(1) volatility forecasts

	In-sample size						
Company	20Q	30Q	40Q	50Q	60Q	70Q	80Q
ALCOA INC	2.9559	2.5147	2.0081	1.7017	1.5224	1.5071	**1.4297**
AMER INTL GRP	2.0587	1.8562	1.667	1.381	1.2694	1.2026	**1.1657**
AMERICAN EXPRESS	3.1789	2.5205	2.47	2.2761	2.1428	2.1307	**2.1128**
BOEING CO	2.7732	2.639	2.3018	2.0735	2.025	1.9207	**1.9089**
BANK OF AMERICA CORP	3.6612	3.1982	2.8009	2.7186	2.6712	2.6891	**2.6649**
CATERPILLAR INC	1.6342	1.6259	1.5686	1.4212	1.4076	1.3761	**1.3747**
DU PONT	1.258	1.1595	1.1254	1.099	1.0452	1.0039	**1.0031**
DISNEY (WALT) CO	4.2478	4.0283	3.6786	3.3942	3.3399	**3.2502**	3.3251
GENERAL ELECTRIC	1.3313	1.395	1.4268	1.2081	1.1261	1.1289	**1.1227**
GENERAL MOTORS	2.8533	2.6792	2.5548	2.461	2.4065	2.3852	**2.2528**
HEWLETT-PACKARD	5.7152	5.8366	5.7045	5.3126	**5.2884**	5.3263	5.5681
IBM	1.9014	1.7567	1.7917	1.8391	1.7388	1.6931	**1.661**
INTEL CORP	6.8833	6.2645	5.6497	**5.6457**	5.8801	5.9941	6.2241
JOHNSON & JOHNSON	1.2057	1.1956	1.1618	1.1278	**1.12**	1.1311	1.1376
JP MORGAN CHASE	5.3491	5.1831	5.3796	5.0412	4.6055	**4.5255**	4.5262
COCA-COLA CO	0.6945	0.7316	0.6145	0.5708	**0.49**	0.5114	0.5145
MCDONALDS CORP	1.0199	1.0371	0.9711	0.9388	**0.9196**	0.9227	0.9484
3M CO	0.7533	0.78	0.7409	0.6219	**0.6103**	0.6221	0.6178
MERCK & CO	3.0365	2.9111	2.8639	2.7208	2.7186	**2.703**	2.7776
PFIZER	1.7945	1.5958	1.5046	1.4237	1.3492	1.3289	**1.3084**
PROCTER & GAMBLE CO	12.02	12.4463	11.7686	10.6715	**10.6074**	10.6884	10.7097
UNITED TECH CORP	2.8316	3.3286	2.9907	2.7475	2.5163	**2.4267**	2.433
WAL-MART STORES	1.7938	1.5706	1.4263	1.3329	**1.3034**	1.3152	1.3177

Notes: The in-sample estimation period is expressed in number of quarters. The forecast evaluation period is from 1997Q4 to 2008Q2 and the minimum value for each stock is highlighted in bold.

Table 4. MAE of one-quarter-ahead AR(1) volatility forecasts

	In-sample size						
Company	20Q	30Q	40Q	50Q	60Q	70Q	80Q
ALCOA INC	1.3575	1.2710	1.1588	1.0700	1.0238	1.0142	**0.9888**
AMER INTL GRP	1.126	1.0593	0.9837	0.9008	0.8639	0.8521	**0.835**
AMERICAN EXPRESS	1.3201	1.1746	1.1461	1.0848	**1.0397**	1.0708	1.0627
BOEING CO	1.2721	1.3288	1.2219	1.1303	1.0993	1.0722	**1.0706**
BANK OF AMERICA CORP	1.2812	1.1316	1.0464	1.0027	**1.0012**	1.0211	1.0085
CATERPILLAR INC	1.0525	1.0568	1.0144	0.9533	0.9451	0.9444	**0.936**
DU PONT	0.8913	0.8215	0.7928	0.7445	**0.7152**	0.7154	0.7221
DISNEY (WALT) CO	1.5079	1.4961	1.3627	1.2453	1.2105	**1.1943**	1.2272
GENERAL ELECTRIC	0.8204	0.8616	0.8265	0.7566	**0.7221**	0.7425	0.7379
GENERAL MOTORS	1.3997	1.3305	1.2915	1.278	1.2522	1.2717	**1.2274**
HEWLETT-PACKARD	1.7187	1.8001	1.7098	1.5764	1.5524	**1.5422**	1.5948
IBM	1.0019	0.9841	0.9894	0.9993	0.9438	0.9399	**0.9272**
INTEL CORP	1.7826	1.7412	**1.6665**	1.6678	1.7455	1.8215	1.8877
JOHNSON & JOHNSON	0.6522	0.6493	0.6188	**0.6054**	0.6112	0.6194	0.6305
JP MORGAN CHASE	1.5077	1.4621	1.5374	1.4756	1.3924	1.3733	**1.3606**
COCA-COLA CO	0.5782	0.561	0.518	0.5111	**0.4776**	0.4984	0.5059
MCDONALDS CORP	0.7699	0.7617	0.7173	0.7055	**0.6905**	0.6982	0.7053
3M CO	0.6968	0.7146	0.6899	0.6088	0.6039	0.6064	**0.6003**
MERCK & CO	1.1546	1.1273	1.0788	0.99	0.968	0.9569	**0.9457**
PFIZER	1.0601	1.0006	0.9571	0.9249	**0.9035**	0.9074	0.9114
PROCTER & GAMBLE CO	1.3431	1.3114	1.2228	1.0425	**1.0163**	1.0499	1.06
UNITED TECH CORP	1.1878	1.2526	1.1752	1.108	1.0583	1.0431	**1.0416**
WAL-MART STORES	0.8998	0.8679	0.821	0.8215	**0.8041**	0.8217	0.8296

Notes: The in-sample estimation period is expressed in number of quarters. The forecast evaluation period is from 1997Q4 to 2008Q2 and the minimum value for each stock is highlighted in bold.

MSE for in-sample estimation periods between 60 and 80 quarters, with the remaining stock having lowest MSE for an in-sample estimation period of 50 quarters. This demonstrates the general applicability of the AR(1) model with an in-sample estimation period of between 15 and 20 years, across all the stocks in our study.

VI. Conclusion

There is a long tradition in the forecasting literature of utilizing parsimonious time series models. Often these models produce the most accurate forecasts. This principle is widely adopted in short range volatility forecasting with the popular GARCH(1,1) model and the more recent autoregressive models of daily realized volatility advocated by Andersen *et al*. (2003). In this article we find that for longer range volatility forecasts at the quarterly frequency, an autoregressive model with one lag of quarterly realized volatility with an in-sample estimation period of between and 60 and 80 quarters is the dominant forecasting model. MIDAS models are considered, though they are found to generate no improvements in forecasts at the quarterly horizon.

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