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FORWARD MARKETS AND THE BEHAVIOUR OF THE COMPETITIVE FIRM WITH PRODUCTION FLEXIBILITY*

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ABSTRACT

This paper examines the production and hedging decisions of the competitive firm under output price uncertainty when a forward market for its output is available. The firm possesses production flexibility in that it makes its production decision after the resolution of the output price uncertainty, albeit subject to a capacity constraint on production. We show that the firm optimally acquires a higher level of capacity investment than an otherwise identical firm with no production flexibility. We further show that production flexibility allows the firm to implicitly hedge against its output price risk exposure by the *ex post* production decision. The firm as such under-hedges its output price risk exposure in the forward market wherein the forward price contains a non-positive risk premium.

Keywords: forward markets, production flexibility
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I. INTRODUCTION

The seminal work of Sandmo (1971) has inspired numerous contributions to the theory of the competitive firm under output price uncertainty (see, e.g., Batra and Ullah, 1974; Chavas, 1985; Machnes, 1993;

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Wong, 1996; to name just a few). One important strand of this literature examines the behaviour of the competitive firm when a forward/futures market exists for hedging purposes (see, e.g., Danthine, 1978; Holthausen, 1979; Feder, Just, and Schmitz, 1980; Broll, 1992). Two notable results emanate. First, the 'separation theorem' states that the production decision of the competitive firm is affected neither by the risk attitude of the firm nor by the incidence of the underlying output price uncertainty. Second, the 'full-hedging theorem' states that the competitive firm should completely eliminate its output price risk exposure by adopting a full hedge if the forward/futures market is unbiased.

In contrast to the extant literature which assumes that the competitive firm makes its production and hedging decisions simultaneously, we follow the approach of Turnovsky (1973), Hartman (1976), Holthausen (1976), and Epstein (1978) that some production decisions can be delayed until the output price uncertainty has been resolved. Specifically, we consider a scenario in which the competitive firm produces its output only after knowing the realized output price, albeit subject to a capacity constraint on production (see Sherman and Visscher, 1978; Dotan and Ravid, 1985). Anticipating the *ex post* production decision, the firm chooses its capacity investment and forward position prior to the resolution of the output price uncertainty.

With production flexibility, we show that the separation theorem fails to hold in that the firm's optimal capacity investment depends on the firm's preferences and on the underlying output price uncertainty. In particular, the firm optimally chooses a higher level of capacity investment than an otherwise identical firm with no production flexibility. We further show that the full-hedging theorem does not hold with production flexibility. The reason is that production flexibility allows the firm to implicitly hedge against its output price risk exposure by the *ex post* production decision. This implicit real hedge crowds out the explicit forward hedge, thereby making under-hedging optimal whenever there is a non-positive risk premium embedded in the forward price.

Besides production flexibility, export flexibility in that exporting firms make their export decisions after the resolution of exchange rate uncertainty is another important strand of the literature on *ex post* flexibility to which this paper adds. As argued by Ben-Zvi and Helpman (1992), international transactions in segmented markets are better described by export flexibility. One can as such view exports as options. It then follows from the option pricing theory that international trade may increase with exchange rate volatility so long as exporting firms are not too risk averse (see Broll, 1999; Broll and Eckwert, 1999). In the context of export flexibility and currency forward markets, Eldor and Zilcha (1987) and Wong (2001, 2002) examine the interplay between real and financial hedging and show the optimality of an under hedge. Should the exporting firm be competitive, Broll and Wahl (1997) show how the firm can completely eliminate its exchange rate risk exposure by writing currency call options.

The rest of this paper is organized as follows. Section II delineates the model of the competitive firm with production flexibility. Section III characterizes the firm's optimal capacity investment when a forward market for its output is available. Section IV goes on to derive the firm's optimal forward position. Section V concludes.

II. THE MODEL

Consider the competitive firm which produces a single output, q, according to a variable cost function, $c(q)$, where $c(0)=c'(0)=0$, $c'(q)>0$, and $c''(q)>0$. Prior to the resolution of output price uncertainty, the firm has to purchase z units of capital goods at a constant unit cost, γ. The acquired amount of capital goods, z, determines a capacity constraint on production. Following Sherman and Visscher (1978) and Dotan and Ravid (1985), we model such a constraint by $q \leq z$. The choice of z is referred to as the firm's capacity investment decision.

The prevailing output price, p, is a random variable distributed according to a probability density function, $g(p)$, over support $[\underline{p}, \overline{p}]$, where $0 \leq \underline{p} < \overline{p} \leq \infty$. To reduce its exposure to the output price risk, the firm has access to a forward market wherein h units of the firm's output can be sold (purchased if negative) forward at a pre-determined price, f. The choice of h is referred to as the firm's hedging decision.

The firm possesses production flexibility in that it makes its production decision after the resolution of the output price uncertainty. Given the observed output price, p, and the capacity constraint on production, $q \leq z$, it is easily verified that the firm's optimal output is z if $p \geq c'(z)$, and is $q(p)$ if $p < c'(z)$, where $q(p)$ solves $c'[q(p)] = p$. In words, the firm's capacity constraint on production is binding if the realized output price is sufficiently high; otherwise, the firm optimally produces at the point where its marginal cost of production equals the realized output price.

The firm has a von Neumann–Morgenstern utility function, $u(\pi)$, defined over its profits, π, with $u'(\pi) > 0$ and $u''(\pi) < 0$, indicating the presence of risk aversion. The firm is an expected utility maximizer and has to solve the following capacity investment and hedging decisions prior to the resolution of the output price uncertainty:

$$\max_{z,\,h}\ EU = \int_{\underline{p}}^{c'(z)} u'[\pi_1(p)]g(p)\ \mathrm{d}p + \int_{c'(z)}^{\overline{p}} u'[\pi_2(p)]g(p)\ \mathrm{d}p \tag{1}$$

$$\text{s.t. } \pi_1(p) = pq(p) - c[q(p)] - rz + (f-p)h, \tag{2}$$

$$\pi_2(p) = pz - c(z) - rz + (f-p)h. \tag{3}$$

Equations (2) and (3) are the firm's profits when the capacity constraint on production is lax and binding, respectively.

Using Leibniz's rule, the first-order conditions for program (1) are given by

$$-\int_{\underline{p}}^{c'(z^*)} u'[\pi_1^*(p)]rg(p)\ \mathrm{d}p + \int_{c'(z^*)}^{\overline{p}} u'[\pi_2^*(p)][p - c'(z^*) - r]g(p)\ \mathrm{d}p = 0, \quad (4)$$

$$\int_{\underline{p}}^{c'(z^*)} u'[\pi_1^*(p)](f - p)g(p)\ \mathrm{d}p + \int_{c'(z^*)}^{\overline{p}} u'[\pi_2^*(p)](f - p)g(p)\ \mathrm{d}p = 0, \quad (5)$$

where an asterisk (*) indicates an optimal level. The second-order conditions for program (1) are satisfied given risk aversion and the strict convexity of $c(q)$.

III. OPTIMAL CAPACITY INVESTMENT DECISION

In this section, we shall examine the firm's optimal capacity investment. As a benchmark, we consider a hypothetical case wherein the firm possesses no production flexibility. That is, the firm's capacity constraint on production is always binding. In this case, program (1) becomes

$$\max_{z,\,h} \int_{\underline{p}}^{\overline{p}} u'[\pi_2(p)]g(p)\ \mathrm{d}p, \quad (6)$$

where $\pi_2(p)$ is defined in equation (3). The well-known separation theorem applies in that the firm's optimal capacity investment, z^0, solves $c'(z^0) = f - r$. Furthermore, if the forward market is unbiased, $f = E(p)$, the full-hedging theorem applies in that the firm's optimal forward position, h^0, is a full hedge, $h^0 = z^0$.

Now, we return to the original decision problem of the firm with production flexibility. Adding equations (4) and (5) and rearranging terms yields:

$$f - c'(z^*) - r = \frac{\int_{\underline{p}}^{c'(z^*)} u'[\pi_1^*(p)][p - c'(z^*)]g(p)\ \mathrm{d}p}{\int_{\underline{p}}^{c'(z^*)} u'[\pi_1^*(p)]g(p)\ \mathrm{d}p + \int_{c'(z^*)}^{\overline{p}} u'[\pi_2^*(p)]g(p)\ \mathrm{d}p}. \quad (7)$$

Since the integrand in the numerator on the right-hand side of equation (7) is over interval $[\underline{p}, c'(z^*))$, it follows that $c'(z^*) > f - r$. The strict convexity of $c(q)$ then implies that $z^* > z^0$, thereby invoking the following proposition.

Proposition 1: The optimal capacity investment of the competitive firm with production flexibility is higher than that of an otherwise identical firm with no production flexibility, $z^* > z^0$.

To see the intuition of Proposition 1, we rewrite equations (2) and (3) as

$$\pi_1(p) = \pi_2(p) + c(z) - c[q(p)] - p[z - q(q)], \tag{8}$$

$$\pi_2(p) = fz - c(z) - rz + (p - f)(z - h). \tag{9}$$

Suppose that there is no production flexibility. Then, the firm's capacity investment, z, affects its output price risk exposure only through the last term on the right-hand side of equation (9). The firm as such could have completely eliminated its output price risk exposure had it chosen $h = z$ within its own discretion. In other words, the degree of output price risk exposure to be assumed by the firm should be totally unrelated to its capacity investment decision. Thus, z^0 is the one that maximizes $fz - c(z) - rz$, thereby rendering the separation theorem. However, if the firm possesses production flexibility, there is an additional risk term on the right-hand side of equation (8), $c(z) - c[q(p)] - p[z - q(p)]$, which depends on z. Since $c'(z) - p < 0$, to reduce this additional risk component the firm is induced to invest beyond z^0. In words, the separation theorem fails to hold because the firm has to use its optimal capacity investment as an additional hedging instrument. In this case, the firm's preferences and the underlying output price uncertainty play an important role in determining the optimal capacity investment.

IV. OPTIMAL HEDGING DECISION

In this section, we shall characterize the firm's optimal forward position, h^*. To this end, we differentiate EU in program (1) with respect to h and evaluate the resulting derivative at $z = h = z^*$ to yield

$$\frac{\partial EU}{\partial h} = \int_{\underline{p}}^{c'(z^*)} u'[\hat{\pi}_1^*(p)](f - p)g(p)\,\mathrm{d}p + \int_{c'(z^*)}^{\overline{p}} u'[\pi_2^*(f)](f - p)g(p)\,\mathrm{d}p, \tag{10}$$

where $\hat{\pi}_1^*(p) = pq(p) - c[q(p)] - rz^* + (f - p)z^*$ and $\pi_2^*(f) = fz^* - c(z^*) - rz^*$. If the sign of the right-hand side of equation (10) is negative (positive), equation (5) and the second-order conditions would imply that $h^* < (>) z^*$.

The following proposition shows that the right-hand side of equation (10) is unambiguously negative when the forward price, f, does not exceed the expected output price, $E(p)$.

Proposition 2: If a non-positive risk premium is embedded in the forward price, $f \leq E(p)$, then the optimal forward position of the competitive firm with production flexibility is an under hedge, $h^* < z^*$.

Proof: Consider first the case where $f \geq c'(z^*)$. Rearranging terms of equation (10) yields

$$\frac{\partial EU}{\partial h} = u'[\pi_2^*(f)][f - E(p)] + \int_{\underline{p}}^{c'(z^*)} \{u'[\hat{\pi}_1^*(p)] - u'[\pi_2^*(f)]\}(f-p)g(p)\ \mathrm{d}p. \tag{11}$$

Since $\hat{\pi}_1^*(p) - \pi_2^*(f) = c(z^*) - c[q(p)] - p[z^* - q(p)]$ and $c'[q(p)] = p$, it follows from risk aversion and the strict convexity of $c(q)$ that $u'[\hat{\pi}_1^*(p)] < u'[\pi_2^*(f)]$ for all $p \in [\underline{p}, c'(z^*))$. Thus, the second term on the right-hand side of equation (11) is unambiguously negative. Inspection of equation (11) reveals that $\partial EU/\partial h < 0$ whenever $f \leq E(p)$.

Now, consider the case where $f < c'(z^*)$. Rearranging terms of equation (10) yields

$$\begin{aligned}\frac{\partial EU}{\partial h} &= u'[\hat{\pi}_1^*(f)][f - E(p)] + \int_{\underline{p}}^{c'(z^*)} \{u'[\hat{\pi}_1^*(p)] - u'[\hat{\pi}_1^*(f)]\}(f - p)g(p)\ \mathrm{d}p \\ &\quad + \{u'[\pi_2^*(f)] - u'[\hat{\pi}_1^*(f)]\} \int_{c'(z^*)}^{\overline{p}} (f - p)g(p)\ \mathrm{d}p. \end{aligned} \tag{12}$$

Since $\partial\hat{\pi}_1^*(p)/\partial p = q(p) - z^* < 0$, it follows from risk aversion that $u'[\hat{\pi}_1^*(p)] < u'[\hat{\pi}_1^*(f)]$ for all $p \in [\underline{p}, f)$ and that $u'[\hat{\pi}_1^*(p)] > u'[\hat{\pi}_1^*(f)]$ for all $p \in (f, c'(z^*)]$. Thus, the second term on the right-hand side of equation (12) is unambiguously negative. Since $\hat{\pi}_1^*(f) - \pi_2^*(f) = c(z^*) - c[q(f)] - p[z^* - q(f)]$ and $c'[q(f)] = f$, it follows from risk aversion and the strict convexity of $c(q)$ that $u'[\hat{\pi}_1^*(f)] < u'[\pi_2^*(f)]$. Thus, the third term on the right-hand side of equation (12) is also unambiguously negative. Inspection of equation (12) reveals that $\partial EU/\partial h < 0$ whenever $f \leq E(p)$. ■

To see the intuition of Proposition 2, suppose that $f = E(p)$. If the firm possesses no production flexibility, it follows from the full-hedging theorem that the optimal forward position is a full hedge, $h = z$. Production flexibility, however, allows the firm to implicitly hedge against its output price risk exposure by the *ex post* production decision. Specifically, for bad realizations of $p \in [\underline{p}, c'(z))$, the firm optimally produces less than its capacity constraint on production would allow. This reduces its output price risk exposure for all $p \in [\underline{p}, c'(z))$. A full hedge via forward trading becomes suboptimal in this interval. In other words, the implicit real hedge crowds out the explicit forward hedge, thereby making an under hedge optimal. Should $f < E(p)$, there is a speculative motive which induces the firm to purchase its output forward. Thus, the under-hedging strategy when $f = E(p)$ is reinforced when $f < E(p)$.

V. CONCLUSIONS

This paper has examined the optimal production and hedging decisions of the competitive firm under output price uncertainty when a forward market for its output is available. Production flexibility allows the firm to make its production decision after the resolution of the output price uncertainty, albeit subject to a capacity constraint on production. We have shown that the separation theorem fails to hold in that the firm uses its optimal capacity investment as an additional hedging instrument. In particular, the optimal capacity investment of the firm is higher than that of an otherwise identical firm with no production flexibility. We have further shown that production flexibility renders the firm to implicitly hedge against its output price risk exposure by the *ex post* production decision. This implicit real hedge crowds out the explicit forward hedge, thereby invalidating the full-hedging theorem. The firm as such under-hedges its output price risk exposure whenever there is a non-positive risk premium embedded in the forward price.

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