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Paths of Accumulation and Growth: Towards a Keynesian Long-period Theory of Output

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ABSTRACT *According to the principle of effective demand, the equilibrium level of aggregate output is a multiple of the expected autonomous demand for the period under consideration. Aggregate demand matches aggregate supply in equilibrium, but the equilibrium may and usually does lie below the output corresponding to full capacity and full employment. However, in the long term firms are presumed to use capacity at the normal or desired degree. Can the principle of effective demand be extrapolated to conclude that the rate of growth of output will depend on the expected rate of growth of autonomous demand? A positive answer would be a significant step towards a Keynesian long-period theory of output. This paper attempts to make an advance in that direction. Starting from a 'prospective accelerator' that incorporates expected increases in autonomous demand and takes account of an excess of capacity in order to eliminate it, this paper shows that the path of autonomous demand determines both the actual and the warranted rates of growth.*

1. Introduction

According to the principle of effective demand (Keynes, 1936; Kalecki, 1971) the equilibrium level of production depends on expected demand: more concretely, it is a multiple of expected autonomous demand for the year or period under consideration. It constitutes a macroeconomic equilibrium in the sense that the entire output produced is demanded, that is, all the savings stemming from current income are matched by investment spending. Yet such an equilibrium may and usually does occur below full capacity and full employment.

When applying the logic of the principle of effective demand to the long term it should be concluded that the rate of growth of output is determined by the expected rate of growth of autonomous demand. But is that extrapolation justified? Full capacity is a condition for economic efficiency and profit maximization,

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so firms will try to achieve it in the long term.[1] Harrod (1939) applied the label 'warranted rate of growth' to the rate that ensures macroeconomic equilibrium with full capacity use through time. He proved that it depends on the propensity to save and technology (the desired capital:output ratio), not on aggregate demand. In addition, he warned that it rendered capitalist economies highly unstable: if entrepreneurs expect aggregate demand to grow at the warranted rate, their expectations will be confirmed ('warranted'), otherwise they will be disappointed and centrifugal forces will destabilize the economy. If Harrod's (1939) conclusions were correct there would be no role for autonomous demand in the long term, which is presumably the realm of growth theory. Fluctuations in demand have been relegated to short-term static analysis and to trade cycle theories, which have traditionally incorporated disequilibria, rigidities and miscalculations.[2]

Hicks (1950) gave an interesting account (to be examined below) of the role of autonomous demand, but it was concerned with the trade cycle and has been dismissed by growth economists. Garegnani (1983, 1992) called attention to the importance of building a long-period theory of output that paralleled the Sraffian long-period theory of prices. Both of these theories are supposed to rely on 'normal' or 'fully adjusted positions', where expectations are realized, capacity is operated at the normal level and no particular 'imperfections' hinder market mechanisms. This paper follows this research project. It aims to show that, even under such ideal conditions, the rate of growth of autonomous demand determines the actual rate of growth of output and the warranted rate.

The paper starts by distinguishing autonomous or modernization investment from induced or expansionary investment. The latter is captured by the 'prospective accelerator', which is examined in section 2. It exhibits two unique traits: it is forward looking (it looks at the expected rate of growth of autonomous demand) and it takes into account undesired inventories and excess capacity in order to eliminate them. In section 3 the paper builds a multiplier-accelerator model similar to Hicks's (1950) supermultiplier, but one that can be applied in disequilibria situations. Then the paper explores the dynamics of the system when the rate of growth of autonomous demand happens to coincide with the initial warranted rate (section 4) and in the more realistic case in which the two rates differ from one another (section 5). The traverse from the old warranted rate to the new one is made possible by over- or under-use of capacity, which causes investment to

[1]On the contrary, full employment cannot be considered a condition for efficiency and profit maximization at the firm level. Nor is it a hard constraint for economies that are growing at a moderate rate. By paying a small premium over the going wage, entrepreneurs can attract additional hours of labour from already employed workers and can draw immigrants and new participants into the workforce. This paper will abstract from such issues and from natural resource constraints in order to focus on capacity constraints.

[2]For example, in one of the most widely used macroeconomics textbooks, Gregory Mankiw (2003) summarized the field's main lessons as follows: 'Lesson 1: In the long run, a country's capacity to produce goods and services determines the standard of living of its citizens. Lesson 2: In the short run, aggregate demand influences the amount of goods and services that a country produces' (p. xxiii).

accelerate or decelerate, changing the structure of production and demand. The paper will enter into the debate triggered off by Garegnani's suggestion.[3] It will also attempt to clarify some puzzles in the Keynesian literature concerning the limits of the multiplier, the plausibility of the accelerator and the instability of the warranted rate of growth.

2. Accumulation and the Prospective Accelerator

Investment should be the cornerstone of any theory of growth. The absence of an investment function is one of the major shortcomings of neoclassical growth theory, which simply assumes that investment will adjust to full capacity savings.[4] Post-Keynesians give due consideration to the investment function, but they have not reached any consensus about it, perhaps because the founding figures of their tradition have provided conflicting signals. Kalecki (1971) adopted a different investment function in each of his papers. Keynes (1936) toyed with two possibly incompatible theories in *The General Theory of Employment, Interest and Money*: the first refers to the marginal productivity of capital and the second to the 'animal spirits' (expectations) of capitalists. A useful strategy for coping with complex issues is to separate the elements. In this case it is proposed to distinguish modernization (or autonomous) investment from expansionary (or induced) investment.

Innovative entrepreneurs who launch new products and new processes carry out modernization, on the lines suggested by Schumpeter (1912). It is not aimed at increasing productive capacity but at transforming it, this being the main vehicle of technical progress.[5] It is the key element of autonomous demand that in any Keynesian system is presumed to be the driving force of the economy. This Schumpeterian insight provides a convenient closure to Keynes's principle of effective demand, by importing objectivity and empirical content to the 'animal spirits' of entrepreneurs. For the purposes of this paper it will be sufficient to

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[3]Garegnani's suggestion has been pursued in Eatwell & Milgate (1983) (see in particular the editors' Introduction and Conclusions) and in the journal *Political Economy* (*s*ee Vianello, 1985; Amadeo, 1986; Ciccone, 1986; Committeri, 1986; Kurz, 1986). A second set of discussions on the topic began a decade later in *Contributions to Political Economy* (see Serrano, 1995; Trezzini, 1995, 1998; Barbosa-Filho, 2000; Park, 2000). All of these interventions will be commented upon below.

[4]Amazingly enough, the investment function of the static IS-LM model disappears in neoclassical growth models.

[5]This paper abstracts from technical change, as is usual in the literature on normal prices and normal output. This decision can be justified by assuming either that (1) in the period being considered the effects of technical change are negligible or (2) technical change is Harrod neutral. The latter assumption implies that, despite the variety of changes occurring at the firm level, in the aggregate a constant desired capital:output ratio k is observed (as will be seen, k is the only technological variable that appears in the accelerator). Note that Harrod neutrality conforms to the stylized facts that Kaldor (1961) observed for the post-war period and can be extended to the second half of the twentieth century. In most advanced countries, technical change has been embodied in new machines that have raised the productivity of labour. The productivity gains have accrued principally to workers, who have consumed most of the wage increases. Both capital and output have grown significantly and proportionally, so that their ratio k has remained fairly constant. The same has happened to the shares of profits and wages in income.

treat as given the level of autonomous demand in the initial year and its expected rate of growth. It should be kept in mind that this rate is independent of the dynamics of actual output and may change from time to time. The demand for new capital goods and durable consumption goods usually follows a sigmoid life cycle: after the introduction of the product, demand at first accelerates, then decelerates and eventually stagnates when the market becomes saturated.

Firms undertake expansionary investment in order to accommodate productive capacity to the expected growth of demand: their aim is to produce already existing goods efficiently, i.e. at normal capacity use. According to the acceleration principle, investment will be a multiple k of the expected growth of demand, k being the desired capital:output ratio (Kurz, 1992). Let us assume, as a first approximation, that the economy at the initial year t is in a fully adjusted position. Firms have produced output Y_t using capacity at its normal level. This implies that the actual capital:output ratio k_t equals the normal or desired one, i.e. k. Output has adjusted to expected aggregate demand ($Y_t = D_t$), which, in turn, is a multiple of the expected autonomous demand Z_t for that year. Suppose that innovative entrepreneurs have been increasing production at a rate g_z and plan to continue on the same path. Aggregate demand and output will follow suit so there will be a coincidence between the rates of growth of autonomous demand, aggregate demand and output. The assumption of full capacity macroeconomic equilibrium implies that these rates also coincide with the warranted rate of growth. In a fully adjusted economy the investment decisions at the end of period t can be formulated as follows:

$$I_t = k(D_{t+1} - Y_t) = k g_z Y_t \qquad (1)$$

Normal conditions imply that there are neither undesired inventories carried from the previous period ($E_{i,t-1} = 0$), nor excess capacity ($E_{k,t} = 0$). In other words, the weight of undesired inventories in income is zero ($\varepsilon_t = E_{i,t-1}/Y_t = 0$) and the degree of capital use is at its normal level ($u_t = Y_t/Y^* = k/k_t = 1$).[6] If this is not the case, firms will try to restore the equilibrium levels by cutting down or increasing investment spending.[7] First, undesired inventories will be subtracted from the demand expected for year t. The actual level of production will be $Y_t = D_t - E_{i,t-1}$. Second, the excess capacity will be subtracted from the investment decided in equation (1). A more general investment equation, suitable for both equilibrium and disequilibrium, would be

$$I_t = k(D_{t+1} - Y_t) - E_{k,t} = k(D_{t+1} - (D_t - E_{i,t-1})) - E_{k,t} \qquad (2)$$

[6]This paper defines $u_t = Y_t/Y^*$. Y_t is the current level of income stemming from the existing stock of capital used at any rate k_t: $Y_t = K_t/k_t$. Y^* is 'capacity income' stemming from the normal use k of the capital stock: $Y^* = K_t/k$. These definitions allow us to write $u_t = k/k_t$.

[7]To simplify the exposition, this paper will comment mostly on the cases where there are undesired inventories and excess capacity. The reader will have no problem reversing the exposition whenever inventories are below normal and capacity is being overutilized.

The paper explains the working of the model in detail in the Appendix. There it becomes clear that a fall in the expected rate of growth of autonomous demand for year t results in excess inventories in year $t-1$. Since these goods do not need to be produced in the following period, some capacity will be redundant in t: $E_{k,t} = kE_{i,t-1}$. Using this result, equation (2) can be rewritten as $I_t = k(D_{t+1} - D_t)$. Equation (1), however, is no longer correct ($I_t \neq kg_z Y_t$). The growth rates of output, aggregate demand and autonomous demand cease to be equal to each other whenever there are undesired inventories and excess capacity. Yet, since autonomous demand is growing steadily at g_z, it may be referred to provided that the necessary adjustments are introduced in either Y_t or k. An expression that relates u with ε, namely $u_t = 1/(1 + \varepsilon_t)$, can be obtained from the equality $E_{k,t} = kE_{i,t-1}$.[8] Then the relationship between production and demand can be expressed in three alternative ways: $D_t = Y_t(1 + \varepsilon_t) = Y_t/u_t = (k_t/k)Y_t$. This allows us to present the investment function in a more compact form:

$$I_t = kg_{\hat{z}} Y_t(1 + \varepsilon_t) = k/u_t g_{\hat{z}} Y_t = k_t g_{\hat{z}} Y_t \tag{3}$$

The preceding formulas refer to investment decisions, which, following Kalecki (1971), should be differentiated from investment expenditures. Sensible entrepreneurs will pace investment expenditures in order to avoid excessive risks or to take advantage of financial conditions. A first rule might be to limit negative gross investment to depreciation. A second rule might be not to invest above the current cash flow. A large investment project adopted on the basis of any of the preceding formulas may take several years to be realized. In addition, it can be sped up or slowed down if interest rates fall below or rise above their 'normal' or 'conventional' level.[9] This paper will use the investment functions given by equations (2) or (3) to make it clear that, even under such rough rules, the economic system is fundamentally stable.

In the next section the investment function will be integrated into a multiplier-accelerator model. At first sight, the accelerator appears to be a perfect companion to the multiplier. Both of them respond to the principle of effective demand and help explain the dynamics of modern capitalism, where quantity adjustments are embedded in mass production systems (Nell, 1998, Chapters 10 and 11). Why then did Keynes (1937), Kalecki (1971) and Robinson (1962) reject the acceleration principle, discrediting it in the eyes of most post-Keynesian economists? This is the first puzzle the paper wants to clarify.

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[8] $E_{i,t-1} = E_{k,t}/k = (K_t - K_t^*)/k$, where $K_t = k_t Y_t$ is the actual stock of capital at t and $K_t^* = kY_t$ is the desired one. Dividing by Y_t obtains $\varepsilon_t = (k_t - k)/k = (k_t/k) - (k/k) = \frac{1}{u_t} - 1$. Therefore $u_t = \frac{1}{\varepsilon_t} + 1$.

[9] A comparison with the usual formulation of investment in the neo-classical synthesis may be helpful. In the IS-LM model total investment is expressed as $TI_t = I_o - bi_t$, where I_o is the autonomous investment (a datum) and i_t is the actual interest rate. In the model here total investment is made up of modernization investment (or 'autonomous investment proper') Z_t and induced investment. This is the I_t captured by the accelerator that can be made flexible in order to take into account possible deviations of current interest rates from the conventional level (i^*). The following expression looks more suitable: $TI_t = Z_t + k_t\, g_{\hat{z}}\, Y_t - b[(i - i^*)/i^*]$).

In a letter to Harrod, dated 12 April, 1937, Keynes argued that the accelerator ('the relation' as it was called) was too mechanical, leaving no room for entrepreneurial expectations:

> So far, we have excluded the possibility of changes in expectations. In fact, however, the rate of investment does not depend on current consumption, but on expectations (though the latter are, of course influenced, perhaps unduly, by current consumption). Thus, unless expectations are of a constant character, one would anticipate short-period changes in the *relation*. (Keynes, 1973, Vol. XIV, p. 172)

In 1939 Kalecki (1971) considered the accelerator too rigid, incompatible with adjustments in the degree of capacity use:

> The argument is apparently based on the unrealistic assumption that the degree of use of equipment is constant while it is clear from trade cycle statistics that it is precisely the fluctuation in the use of equipment which accounts chiefly for changes in output, and the proportionate increase or decrease of equipment is of minor importance. (p. 65)

Robinson (1962) found a problem of circularity:

> The point of view embodied in the acceleration principle suggests that investment keeps up with the expected rate of growth of sales. But the rate of accumulation is itself the main determinant of the rate of growth of income and therefore of sales. (p. 13)

Such criticisms may be justified in relation to the traditional 'retrospective accelerator' (after Samuelson, 1939). In the 'prospective accelerator' presented here it is clear that entrepreneurial expectations play a key role. Keynes's 'animal spirits' have materialized: they refer to the expected rate of growth of the markets for the new products that constitute the vector of autonomous demand. Nor is there any problem of circularity once induced demand is separated from autonomous demand (wherein lies the driving force of the economy). Finally, the accelerator is flexible enough to allow adjustments via inventories and capacity use. As a matter of fact, these are the ordinary mechanisms of adjustment after a shock. The distinction between investment decisions and investment implementation enhances the flexibility of the model even further.

3. Macroeconomic Equilibrium at a Given Moment of Time

At any moment in time, firms adjust the level of production to the expected demand for their products. For the economy as a whole, aggregate demand consists of final consumption (private and public), investment (private and public) and net exports. Macroeconomists typically draw a distinction between the induced and the autonomous components. Autonomous demand contains net exports, real public expenditure, autonomous private consumption and what this paper has called 'modernization investment'. Let us include all the components of autonomous demand, i.e. the expenditures that do not derive systematically from current income and do not increase productive capacity, in Z. Alternatively, we might refer to a closed economy with no government sector in order to express the multiplier as simply as possible and identify Z with modernization investment. 'Induced' or 'expansionary' investment can be encapsulated in the accelerator principle derived in the preceding section. Induced consumption (C) reflects the fact that households devote a portion c of their disposable income to final consumption: $C_t = cY_t$. This hypothesis appears to be empirically sound, with the propensity to consume verified to be high and stable.[10] Taking account of these assumptions and definitions we can express aggregate demand and net output as

$$Y_t = cY_t + k_t g_z Y_t + Z_t \tag{4}$$

By algebraic manipulation of the preceding formula we arrive at the multiplier-accelerator model, which allows us to determine the equilibrium level of output, given the level of expected autonomous demand Z_t and its expected rate of growth g_z:

$$Y_t = \left(\frac{1}{1 - c - k_t g_z}\right) Z_t = \mu^* Z_t \tag{5}$$

The term in parentheses (μ^*) is similar to Hicks's supermultiplier but for one point. In Hicks (1950) the actual capital:output ratio k_t was equal to the desired one, i.e. k. In the model here this only occurs when the economy is fully adjusted.

Figure 1 represents the equilibrium level of output and employment in $t = 0$. On the left-hand side we see the expected autonomous demand at time 0 and its rate of growth. Drawing a parallel to induced demand (D_i, the slope of which represents the value of the supermultiplier), we obtain aggregate demand and net output Y_o. Given labour productivity λ we obtain employment: $L_o = Y_o/\lambda$. And given the capital:labour coefficient (Ω) we obtain the capital stock that is going to be operated: $K_o = \Omega L_o$. In the example here labour employed lies below the full employment level L_{fe}.

The capital required coincides with the existing stock of capital (represented by the circle). However, it could have been otherwise. If autonomous demand were $Z_o/2$, half of the capital installed would have been redundant. Unlike unemployment, excess capacity negatively affects the profitability of the firms and we can expect they will modify investment decisions in order to restore the normal position.

A comparison between the Keynesian simple multiplier and the supermultiplier derived here may throw some light. In the Keynesian model the multiplier is lower ($\mu < \mu^*$) but the multiplicand is greater ($Z^* > Z$) than in the case of

[10]In order to render the model closer to reality the paper could follow Kalecki (1971) and disaggregate the propensity to consume, which is a weighted average of the propensities to consume out of wages (c_w) and out of profits (c_p): $c = c_w(W/Y) + c_p(P/Y)$, where W is wages and P is profits.

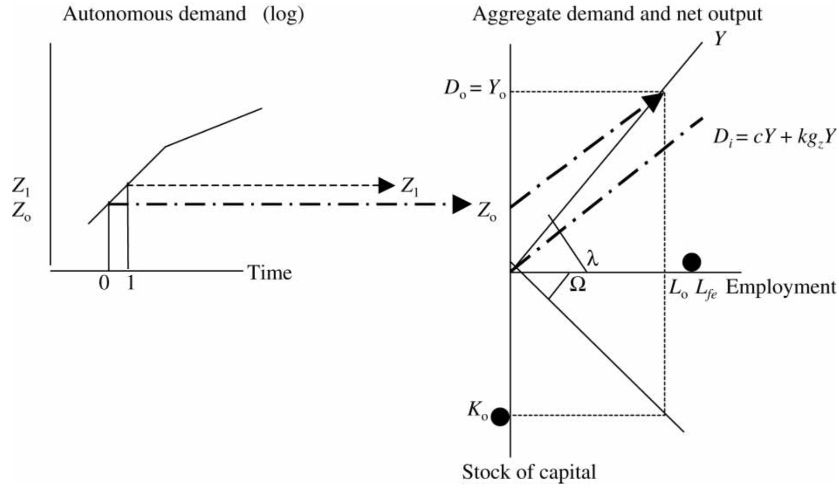

Figure 1. Macroeconomic equilibrium at a given moment in time through the multiplier-accelerator model

the supermultiplier:

$$\begin{aligned} Y_t &= \mu Z^* = \frac{1}{1-c}(I_t + Z_t) \\ Y_t &= \mu^* Z = \frac{1}{1-c-k_t g_z} Z_t \end{aligned} \tag{6}$$

Both procedures should lead to the same result provided they are computed and used properly. In principle it seems more appropriate to limit what we treat as autonomous demand to a minimum, not taking as given what we can explain by further analysis. However, could we legitimately explain investment by the expected rate of growth of capital that is supposed to follow the pace of autonomous demand? That is, could we substitute $g_z K_t$ for I_t in the Keynesian formulation?[11] The substitution would be appropriate whenever the economy is fully adjusted. However, as soon as there are unplanned inventories and excess capacity, such an investment function would be unable to remove them. The opposite situation may provide a second case of coincidence between the two multipliers. Whenever excess capacity is extremely large, induced investment would be zero and both formulas could lead to the same result, namely $Y_t = (1/[1-c])Z_t$.[12]

[11]Substituting in the investment function (K_t/k_t) for Y_t, we can write $Y_t = cY_t + k_t g_z(K_t/k_t) + Z_t = cY_t + g_z K_t + Z_t = (1/[1-c])(g_z K_t + Z_t)$.

[12]Equation (3) should not be applied in a mechanical way. In this case it would be better to consider equation (2): $I_t = k(D_{t+1} - Y_t) - E_{k,t}$. If the excess capacity $E_{k,t}$ is high enough, I_t will approach zero.

4. The Growth Path when Autonomous Demand Grows at the Warranted Rate (Supply Limits to a Demand-led System)

We are now in a good position to analyse the growth path of an economy that follows the expenditure patterns previously described. The three tables in the Appendix will be helpful for this purpose. We take as data the autonomous demand for the initial year Z_o and its expected rate of growth g_z. To begin with, let us suppose that g_z coincides with the warranted rate of growth g_w corresponding to a fully adjusted economy. Under this condition, output would grow along an efficient and steady path. Year after year, capital is operated at normal capacity and the product is entirely absorbed by demand. This growth path is robust in the sense that the economy returns to the warranted rate of growth whenever it is disrupted by a shock.

Table 1 (see the Appendix) simulates a case in which, in year 3, autonomous demand does not grow at all. Investment in period 2 falls to zero generating undesired inventories ($E_{i,2} = 1.1$). In period 3 actual production will be lower than expected aggregate demand, so a part of the installed capital turns out to be redundant (u falls to 0.9). This excess capacity leads to a reduction in investment. However, if autonomous demand resumes its previous rate of growth, the economy will soon progress at full capacity: $g_y = g_w = g_z$.

This result may appear obvious to entrepreneurs and to people with no formal training in economics. However, to most economists it will be a surprise. In his seminal multiplier-accelerator model, Harrod (1939) discovered the existence of a warranted rate of growth, but it rendered the system so unstable that it could be compared to a knife-edge:

> On either side of this line is a ‘field’ in which centrifugal forces operate, the magnitude of which varies directly as the distance of any point in it from the warranted line. Departure from the warranted line sets up an inducement to depart farther from it. The moving equilibrium of advance is thus a highly unstable one. (p. 26)

Such instability runs against empirical evidence, but it seems difficult to discard on theoretical grounds. To avoid ‘explosive results’, Hicks (1950) introduced a ceiling and a floor to the super-multiplier. Kaldor (1951) soon criticized the rationale for this hypothesis. The debate between neoclassical and Post-Keynesian economists triggered by Kaldor (1955–56) and Solow (1956) focused on the variable that could make possible the adjustment of the warranted rate (full capacity growth) to the natural rate (full employment growth), not on the instability resulting from a shift in the rate of growth of demand. In the Post-Keynesian-Sraffian literature prompted by Garegnani (1983, 1992) the instability of the model has been set aside. For example:

> a positive (negative) demand shock can result in an explosive increase (decrease) in the pace of capital accumulation similar to Harrod’s (1939) cumulative deviations from ‘warranted growth’. In fact, I adopted a completely exogenous investment function in the previous sections not only to emphasise the leading role of effective demand, but also to exclude unstable dynamics from the analysis. An explosive and rigid accelerator does not seem to be a good description of the dynamic of real capitalist economies. . .. (Barbosa-Filho, 2000, p. 29)

The robustness of the equilibrium reached by our super-multiplier demonstrates that the instability is not a consequence of the model, but derives from the particular assumption made about firms' investment reaction function. Harrod's position can be summarized in two points: (1) if the desired rate of growth falls below the warranted rate, capacity will be under-used ($u < 1$) and (2) as a consequence, entrepreneurs will reduce the desired rate of growth still further. Point (1) is accepted and point (2) is rejected. In the model here, excess capacity impinges on effective investment, but not on the expected rate of growth of autonomous demand.[13] This rate has an objective and independent pattern of behaviour that is not affected by the fluctuations of actual output.[14]

Of course, the super-multiplier model has some limits and requires some conditions to be applied. Obviously, both the multiplicand and the multiplier have to be positive. The first requirement is the existence of a positive vector of autonomous demand ($Z_t > 0$). When aggregate demand consists solely of induced consumption and induced investment, we could still use a multiplier-accelerator model based on the Keynesian multiplier, but not the supermultiplier. This case is analysed in Table 3 (see the Appendix). We take as given the rate of growth of aggregate demand g_d for computing expansionary investment ($I = k_t g_d D_t$) and output ($Y_t = [1/1 - c)]I_t$).

For the super-multiplier to be positive it is required that $c + k_t g_z < 1$. The maximum rate of growth of autonomous demand, for a constant propensity to consume c, coincides with Harrod's warranted rate, which is defined as

$$\text{maximum } g_z < (1 - c)/k_t \qquad (7)$$

The variability of k_t renders the limits more flexible. A rise in g_z can be offset by a fall in k_t. Yet, the supply limits cannot be completely removed. Overutilization of capacity ($k_t < k$) cannot go beyond the engineering limit. Underutilization of capacity ($k_t > k$) reaches the economic limit when profits become zero. The point to be stressed is that a demand-led system cannot avoid certain supply limits.[15]

[13]Alexander (1949) made a similar claim. Harrod (1973, pp. 19–20) recognized that the objection was sensible enough. Unfortunately, growth economists continued emphasizing the centrifugal forces associated with the knife-edge parable.

[14]The term 'prospective accelerator' has been chosen to emphasize that, in their investment decisions, firms are forward looking. They decide on investment on the basis of the expected increase in aggregate demand, which is a multiple of autonomous demand $I_t = k\Delta D = kg_z Z_t \mu^*$. Then they discount undesired inventories $E_{i,t-1}$ and excess capacity $E_{k,t}$, so the preceding expression becomes $I_t = k[D_{t+1} - (D_t - E_{i,t-1})] - E_{k,t}$. Autonomous demand, however, continues growing at the rate g_z and firms must make up for the goods not produced in t. This explains why in equation (3) undesired inventories ($\varepsilon > 0$) and excess capacity ($u < 1$ or $k_t > k$) push up investment.

[15]Trezzini (1995, 1998) dismissed the Hicksian supermultiplier because it does not grant autonomous demand the possibility for growing at any rate. Note that we are not preventing a single innovative firm from growing as fast as 50, 100 or 200%. The limit applies to the group of innovative firms, the production of which accounts for a non-negligible share of current output. We could even permit the whole group to grow for a time above the limit set in equation (7) by reducing k_t. However, the flexibility of technology has a limit. Labour force and natural resources constraints will also make themselves felt after a point.

5. The Growth Path when the Expected Growth of Autonomous Demand Differs from the Initial Warranted Rate (the Traverse)

The coincidence between the autonomous and the warranted rate was nothing but a didactical recourse to show the (remote) possibility of steady growth at full capacity. If autonomous demand is truly autonomous it can change at any moment and shift below or above the warranted rate. Table 2 (see the Appendix) simulates a permanent fall of g_z after year 3 from 0.05 to 0.04. The reaction of the economy is similar to the previous case, although the final result changes. At year 3 firms face undesired inventories and excess capacity, which prompt them to curb investment. After several periods of adjustment, the economy returns to an efficient path without undesired inventories and with normal capacity use. Note, however, that the actual rate of growth has adjusted to the lower pace of autonomous demand. And so has the warranted rate of growth ($g_y = g_w = g_z = 0.04$). Hicks (1950) called this adjustment process a traverse (see also Lowe, 1976). It implies a change in the structure of demand and of output. In the numerical example here, the share of autonomous demand in income ($z_t = Z_t/Y_t$) rises from 0.10 to 0.12, while the share of expansionary investment in income (σ_t) falls from 0.10 to 0.08.

Once again the result will surprise growth economists. The warranted rate of growth is usually defined by the ratio s/k. Apparently there are only two ways of altering the warranted rate. Neoclassical economists (after Solow, 1956) have pointed to the desired capital : output ratio. Post-Keynesians (following Kaldor, 1955–56) have pointed to the aggregate propensity to save $s = 1 - c$, which supposedly is influenced by changes in distribution. In our example, neither the capital : output ratio, nor the propensity to save, nor distribution has changed, yet the warranted rate of growth has fallen towards the new rate of growth of autonomous demand. Some variable is missing in the traditional presentation of the warranted rate. As a matter of fact, Harrod's presentation is only correct when aggregate demand consists of induced consumption and induced investment. For the general case the formula is a bit different.

Let us rewrite the macroeconomic equilibrium in equation (4) adding an asterisk to emphasize that we refer to full capacity output and substituting k for k_t and $g_{w,t}$ for g_z.

$$Y_t^* = cY_t^* + kg_{w,t}Y_t^* + Z_t \tag{8}$$

Dividing through by Y_t^* and isolating $g_{w,t}$ we obtain the warranted rate of growth at t:

$$g_{w,t} = \frac{1 - c - z_t}{k} = \frac{\sigma_t}{k} \tag{9}$$

Note that in the numerator of the above formula we do not find the share of total savings, but the share of savings available for accumulation. Exported goods cannot be accumulated. Nor do the goods devoted to modernization investment expand capacity. The expression $\sigma_t = I_t/Y_t$ is the share in income of the savings available for accumulation, which we suppose to be actually invested. It is not a behavioural coefficient like c, but an *ex post* relation that will change whenever $z_t = Z_t/Y_t$ changes.

Following Park (2000) we can distinguish the long-period from the short-period warranted rate of growth. The first one (g_w) corresponds to a fully adjusted economy where both capacity k_t and the structure of output (z_t, and σ_t) are at their final equilibrium levels. The short-period warranted rate $g_{w,t}$ is computed by the same equation (9) at a time when z_t and σ_t are in the process of adjustment.[16] The long-period equilibrium values of these variables are precisely those which ensure the equalities $g_{w,t} = g_w = g_y = g_z$. Equating equation (9) to g_z we obtain

$$\begin{aligned} \sigma_t^* &= kg_z \\ z_t^* &= 1 - c - kg_z \end{aligned} \qquad (10)$$

Figure 2, based on the data from Table 2 (see the Appendix), shows the relationship between g_w and the ratio σ/k.[17]

Figure 3, which is also based on data from Table 2, uses the typical Post-Keynesian technological frontiers to show a not so traditional Post-Keynesian result. On the left-hand side we have the distribution frontier, where w stands for the real wage and r for the rate of profit. On the right-hand side appears the growth frontier: $\hat{c}$ stands for the 'income not accumulated' per worker (consumption plus autonomous demand divided by employees) and g_w for the warranted rate of growth of output. In the southwest quadrant we have the Cambridge equation, which relates the rates of growth and profit. When aggregate demand consists of induced consumption and induced investment we write $g_w = rs_p$, where s_p represents savings out of profits: $s_p = (Y-C)/P$. Whenever there is autonomous demand proper we should write $g_w = r\sigma_p$, where σ_p takes account only of the savings available for accumulation: $\sigma_p = (Y-C-Z)/P$. Initially the economy is at point *a*. Output is growing at $g_y = g_z = g_w = 0.05$. After a fall in the rate of growth of autonomous demand, the economy will eventually recover its fully adjusted path at point *b*, where $g_y = g_z = g_w = 0.04$. Being on the technological frontier, both positions can be said to be fully adjusted and efficient, although they represent different structures of production (as Lowe (1976) and Pasinetti

[16]Table 3 in the Appendix examines the case where aggregate demand consists solely of induced consumption and induced demand. Since there is no autonomous demand proper, the adjustment cannot be made via z. It will occur through inventories and capacity. Notice, however, that this does not impair the stability of the system. The share in income of undesired inventories and excess capacity are maintained in reasonable limits.

[17]Trezzini (1995) arrived at equations similar to this paper's equations (9) and (10), though he found a paradox that calls the validity of the model into question: 'A paradox implicit in Hicks's model now also becomes clear: while it is stated that the autonomous demand expansion is the leading factor in economic growth, it is simultaneously stated that the rate of growth is maximum when the components determining economic growth, and therefore their rate of growth, are zero. The origin of this paradox lies exactly in the assumption of normal capacity utilization' (p. 48). Our position is that, within the broad limits analysed in the previous section, g_z can take any value: z_t will adjust to make possible the new path of growth of autonomous demand in normal conditions. 'Normal capacity' is not assumed to obtain over the course of the traverse. On the contrary, changes in capacity use are the first mechanism of adjustment.

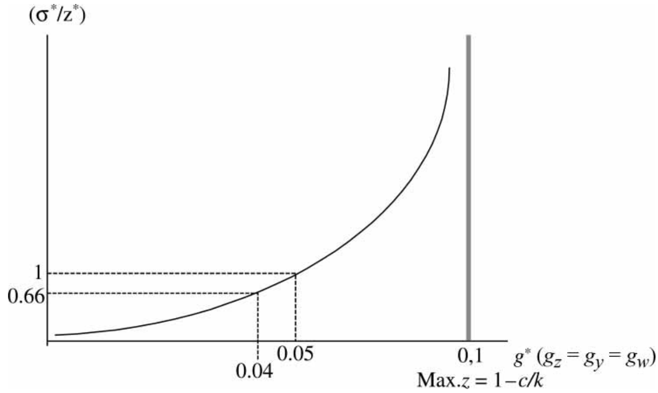

Figure 2. The structure of demand and the warranted rate of growth

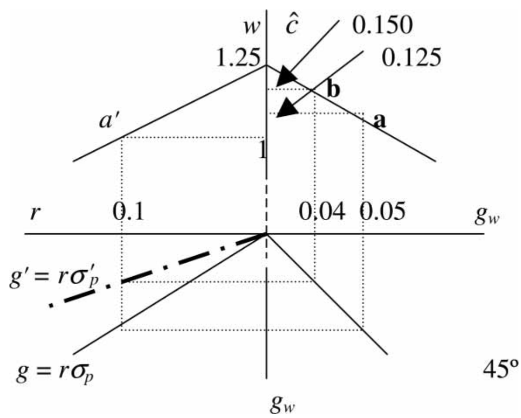

Figure 3. Changes in the structure of production and expenditures after a fall in the rate of growth of autonomous demand

(1993) emphasized). At *b* the ratio of the share of new goods in output has increased at the expense of the share of capital goods, which implies a fall in the share in profits of savings available for accumulation (σ_p falls from 0.5 to 0.4). However, neither the distribution (*w* and *r*), nor the psychological propensities to consume or save (*c, s* and s_p), have changed.

6. Conclusions

According to Garegnani (1983, p. 75)

> a satisfactory long-period theory of output does not require much more than (a) an analysis of how investment determines savings through changes in the level of productive capacity (and not only through changes in the level of utilisation of productive capacity); (b) a study of the factors affecting the long-run levels of investment; and (c) a study of the relation of consumption expenditures and aggregate income.

This paper has tried to answer these and related questions, joining the contributions of Serrano (1995), Trezzini, (1995, 1998), Park (2000) and Barbosa-Filho (2000). The two extreme positions can be summarized in the following way. Serrano (1995) argued that a necessary condition for a long-period equilibrium of output is the normal use of capacity, which is granted by Hicks's (1950) supermultiplier provided entrepreneurs' expectations of autonomous demand are correct. Trezzini (1995, 1998) dismissed the Hicksian supermultiplier because of its rigidity: he argued that it could only be applied when the rate of growth of autonomous demand coincides with the warranted one. A fall (rise) in g_z, he concluded, will lead to a period of underutilization (overutilization) of capacity, which can endure forever. In conclusion, one cannot speak at the same time of a leading role for both autonomous demand and full capacity use.

The analysis here is similar to Trezzini's (1995, 1998) in many respects, but leads to conclusions closer to those of Serrano (1995). The explanation lies in this paper's investment function. The prospective accelerator is forward looking–it looks at the expected increases in autonomous demand, which are supposed to be independent of current production–and it is able to cope with unplanned inventories and excess capacity in order to remove them. The result is a system led by autonomous demand in the short and in the long period. If the growth of autonomous demand is maintained long enough, both the rate of growth of income and the warranted rate would adjust to the former. If the pace of autonomous demand is not so stable, the adjustment might never be completed and capacity would rarely be fully used. However, even in these conditions we are in the realm of a long-period theory of output since the economy gravitates towards fully adjusted positions. The conclusion is akin to the long-period theory of prices. Despite continuous demand shocks there is a tendency for prices to adjust to the costs of production, i.e. to values corresponding to the most efficient technique (for a given real wage) operated in 'normal' conditions. Changes in technology and distribution modify the data of the quantity and price systems, but not the idea of equilibrium or the adjustment mechanisms.

These mechanisms have been explained in different ways, which we could summarize in a simple metaphor. Suppose that we are driving a car on a main road. As soon as we turn onto a narrower and steeper local road, the driver reduces speed by releasing the accelerator pedal. Later, to prevent the car from stalling, he moves into a lower gear, allowing the pedal to recover the normal position. The entry into a motorway permits a higher speed. First, the driver will press the accelerator pedal and later he will move into top gear. The translation of the metaphor is straightforward. The different roads correspond to different paths of autonomous demand. The accelerator pedal stands for the degree of capacity use, which firms try to maintain at the optimum level ($u = 1$ or $k_t = k$): this is the short-run adjustment mechanism. The gear stands for the acceleration or deceleration of investment that brings about a shift in the structure of demand and production, i.e. the share in income of expansionary investment (σ) and autonomous demand (z): this is the mechanism of adjustment in the long period.

The growth path that results from the neoclassical model has been defined as steady and efficient. The opposite features characterize the path that emerges from this paper's post-Keynesian–Sraffian model: it is irregular (cycles embedded in long waves of prosperity and depression), inefficient (capacity use out of equilibrium most of the time) and transformational (changes in the structure of production).

Acknowledgments

This paper has a long history. It draws on De-Juan (1999), which in turn draws on the author's 1989 doctoral dissertation. Edward Nell, to whom I am especially indebted, supervised the thesis. I am also grateful to two anonymous referees for advice on matters of emphasis.

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Appendix

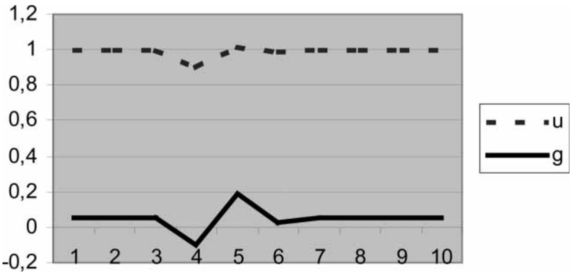
Growth of Autonomous Demand at the Initial Warranted Rate (Adjustment After a Transient Shock)

Symbols

t year.
Z autonomous demand ('modernization investment' in the simplified version of the model).
D aggregate demand as a multiple of autonomous demand: $D_t = [1/(1-c-kg_z)]Z_t$.
Y net output. It adjusts to aggregate demand, subtracting the excess of inventories carried from the past: $Y_t = D_t - E_{i,t-1}$.
K stock of capital: $K_t = K_{t-1} + I_{t-1}$.
E_k excess capacity: $E_{k,t} = K_t - (kY_t)$ ($k = 2$ being the optimal capital : output ratio).
C final consumption: $C_t = cY_t$ ($c = 0.8$ being the aggregate propensity to consume).
I accumulation of capital or induced investment or expansionary investment: it is decided on the last day of the year and can be calculated either by equation (2), i.e. $I_t = k(D_{t+1} - Y_t) - E_{k,t}$ or by the compact formulas of equation (3), i.e. $I_t = (k/u_t)g_z Y_t = k_t g_z Y_t$.
E_i excess inventories: $E_{i,t} = Y_t - C_t - I_t - Z_t$. Note that $E_{k,t} = kE_{i,t-1}$.
z share of autonomous demand in income: $z_t = Z_t/Y_t$.
σ share of induced investment in income: $\sigma_t = I_t/Y_t$.
u degree of capacity use: $u_t = Y_t/Y = Y_t/(K_t/k)$. At full capacity $u_t = 1$.
g_y actual rate of growth: $g_{y,t} = (Y_t - Y_{t-1})/Y_{t-1}$. The warranted rate is $g_{w,t} = (1-c-z_t)/k = \sigma_t/k$.

Table 1. Adjustment after a Transient Shock

t	Z	D	Y	K	E_k	C	I	E_i	z	σ	u	g_y
0	1.000	10.00	10.00	20.00	0.000	8.00	1.00	0.000	0.100	0.100	1.000	0.050
1	1.050	10.50	10.50	21.00	0.000	8.40	1.05	0.000	0.100	0.100	1.000	0.050
2	1.102	11.02	11.02	22.05	0.000	8.82	0.00	1.102	0.100	0.000	1.000	0.050
3	1.102	11.02	9.92	22.05	2.205	4.41	1.10	−0.221	0.111	0.111	0.900	−0.100
4	1.158	11.58	11.80	23.15	−0.441	8.60	1.16	0.441	0.098	0.098	1.019	0.189
5	1.216	12.16	12.11	24.31	0.088	8.13	1.22	−0.009	0.101	0.101	0.996	0.027
6	1.276	12.76	12.77	25.53	−0.018	8.62	1.28	0.002	0.100	0.100	1.001	0.055
7	1.340	13.40	13.40	26.80	0.003	8.94	1.34	0.000	0.100	0.100	1.000	0.049
8	1.407	14.07	14.07	28.14	0.000	9.30	1.41	0.000	0.100	0.100	1.000	0.050
9	1.477	14.78	14.77	29.55	0.000	9.67	1.48	0.000	0.100	0.100	1.000	0.050



Data. We take the initial level of autonomous demand ($Z_o = 1$) and its rate of growth ($g_z = 0.05$) as data. The expected autonomous demand is zero ($Z_3 = 0$) at year 3, so no induced investment is required at the end of period 2. In year 4 autonomous demand resumes its traditional growth path ($g_z = 0.05$). Other data are $k = 2$ and $c = 0.8$.

Comments. The economy grows at the warranted rate during years 0, 1 and 2: $g_{w,o} = \sigma_o/k = 0.05$. This implies $E_i = 0$, $E_k = 0$ and $u = 1$. The freezing of autonomous demand at year 3 implies that, at the end of period 2, no investment is undertaken and undesired inventories amount to $E_{i,2} = 1.102$. They will be subtracted from next year's production, generating excess capacity: $E_{k,3} = K_3 - kY_3 = kE_{i,2} = 2.205$. Firms will adjust investment in order to eliminate the surplus inventories and excess capacity. They succeed after three or four periods, although cycles are unavoidable.

Adjustment When the Growth of Autonomous Demand Falls Permanently Below the Initial Warranted Rate of Growth

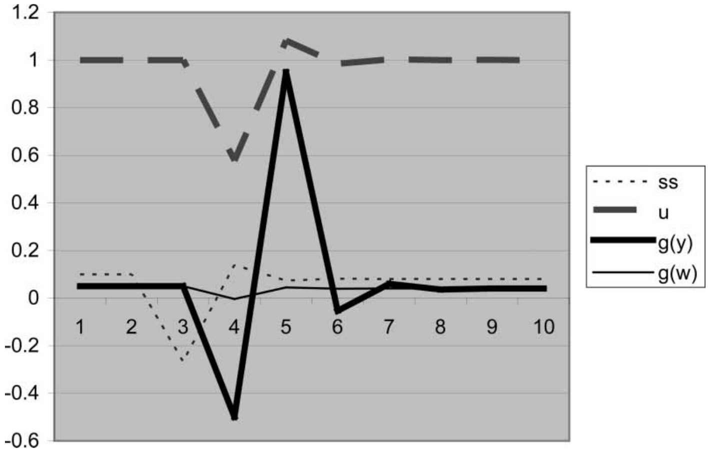
Symbols and data. These are the same as for Table 1 (but note that, in the figure, σ is written as *ss*). The only new variable is r = rate of profit = profits/value of capital. To compute r we have assumed that the real wage always equals unity and that all wages are consumed. Savings may be devoted to accumulation (I) or to autonomous expenditures (Z).
Comments. Initially the rate of growth of autonomous demand coincides with warranted growth: $g_z = g_w = \sigma_0/k = 0.1/2 = 0.05$. g_z falls permanently to 0.04 in year 3. In column D we should introduce the new rate of growth that affects the super-multiplier (after year 3 its value falls from 10 to 8.33).

Investment decisions (I) at the end of period 2 take into account the new growth rate. Accumulation is computed by equation (2): $I_t = k(D_{t+1} - Y_t) - E_{k,t}$ (with $Y_t = D_t - E_{i,t-1}$). The compact formulas of equation (3) are not valid for the investment decisions at the end of year 2, when the value of the super-multiplier changes. However, once I_2 has been computed by the previous method, we can use any of these formulas for the following years: $I_t = (k/u_t) g_z Y_t = k_t g_z Y_t$.

The process of adjustment follows a cyclical pattern, although the cycles are now more pronounced. The degree of capacity use eventually returns to its equilibrium level ($u = 1$). The same is true for the share of inventories in net product, which falls to zero.

Table 2. Adjustment after a Permanent Fall of g_z

t	Z	D	Y	K	E_k	C	I	E_i	z	σ	u	g_y	$g_{w,t}$	r
0	1.000	10.00	10.00	20.00	0.00	8.00	1.00	0.00	0.10	0.10	1.00	0.05	0.050	0.10
1	1.050	10.50	10.50	21.00	0.00	8.40	1.05	0.00	0.10	0.10	1.00	0.05	0.050	0.10
2	1.102	11.02	11.02	22.05	0.00	8.82	−2.94	4.04	0.10	−0.27	1.00	0.05	0.050	0.10
3	1.147	9.55	5.51	22.05	8.08	4.41	0.76	−0.81	0.21	0.14	0.58	−0.50	−0.010	0.06
4	1.192	9.93	10.75	23.15	−1.62	8.60	0.79	0.16	0.11	0.07	1.08	0.95	0.044	0.11
5	1.240	10.33	10.17	24.31	0.32	8.14	0.83	−0.03	0.12	0.08	0.98	−0.05	0.039	0.10
6	1.290	10.75	10.78	25.53	−0.06	8.62	0.86	0.01	0.12	0.08	1.00	0.06	0.040	0.10
7	1.341	11.18	11.17	26.80	0.01	8.94	0.89	0.00	0.12	0.08	1.00	0.04	0.040	0.10
8	1.395	11.62	11.62	28.14	0.00	9.30	0.93	0.00	0.12	0.08	1.00	0.04	0.040	0.10
9	1.451	12.09	12.09	29.55	0.00	9.67	0.97	0.00	0.12	0.08	1.00	0.04	0.040	0.10



z share of autonomous demand in income: $z_t = Z_t/Y_t$.
σ share of induced investment in income: $\sigma_t = I_t/Y_t$.
u degree of capacity use: $ut = Y_t/Y = Y_t/(K_t/k)$. At full capacity $u_t = 1$.
g_y actual rate of growth: $g_{y,t} = (Y_t - Y_{t-1})/Y_{t-1}$. The warranted rate is $g_{w,t} = (1 - \mathrm{c} - z_t)/k = \sigma_t/k$.

In the long run the burden of adjustment impinges on the structure of demand and output: $\sigma_t^* = I_t/Y_t$ falls from 0.10 to 0.08 and $z_t^* = Z_t/Y_t$ rises from 0.1 to 0.12. Note that the starting point was a fall in the rate of growth of autonomous demand, but the economy reacted by decelerating investment much more. This conveys a fall in the warranted rate of growth from 0.05 to 0.04, although the change is slow and cyclical. In the new 'fully adjusted system' the warranted rate will be $g_w = \sigma_t^*/k = 0.08/2 = 0.04 = g_{z,t}$.

The second conclusion to be emphasized is that the traverse to a lower warranted rate of growth has been possible with constant technology ($k = 2$), constant distribution and constant expenditure patterns. The real wage has been set equal to unity. The rate of profit does fall when $u < 1$, but it soon recovers its equilibrium level, $r = 0.10$. At any point in time the propensity to consume is $c = C/Y = 0.8$, the propensity to save is $s = (Y–C)/Y = 0.20$ and the propensity to save out of profits is $s_p = (Y–C)/P = 1$.

Adjustment When the Expected Growth of Aggregate Demand Falls Permanently Below the Initial Warranted Rate of Growth (No Autonomous Demand Proper)

Symbols. These are the same as for Table 1.

Data. There is no autonomous demand proper: aggregate demand = induced consumption + induced investment. We take as a datum the level of aggregate demand at the initial year ($D_o = 10$) and its expected rate of growth $g_d = 0.1$. g_d falls permanently to 0.05 after year 3. Other data are $k = 2$ and $c = 0.8$.

Table 3. Adjustment in a Simple Model without Autonomous Demand Proper

t	D	Y	K	E_k	C	I	E_i	σ	u	g_y
0	10.000	10.00	20.00	0.000	8.00	2.00	0.000	0.200	1.000	0.100
1	11.000	11.00	22.00	0.000	8.80	2.20	0.000	0.200	1.000	0.100
2	12.100	12.10	24.20	0.000	9.68	1.21	1.210	0.100	1.000	0.100
3	12.705	11.49	25.41	2.420	9.20	1.27	1.028	0.110	0.905	−0.050
4	13.340	12.31	26.69	2.057	9.85	1.33	1.128	0.108	0.923	0.071
5	14.007	12.88	28.01	2.257	10.30	1.40	1.175	0.109	0.919	0.046
6	14.708	13.53	29.41	2.350	10.83	1.47	1.236	0.109	0.920	0.051
7	15.443	14.21	30.89	2.471	11.37	1.54	1.297	0.109	0.920	0.050
8	16.215	14.92	32.43	2.594	11.93	1.62	1.362	0.109	0.920	0.050
9	17.026	15.66	34.05	2.724	12.53	1.70	1.430	0.109	0.920	0.050

Comments. The example fits the traditional presentation of the simple Keynesian multiplier. Adjustment begins with the investment decision at the end of year 2, once entrepreneurs realize that the growth of the economy is bound to decelerate. This reaction brings about an undesired surplus of inventories and excess capacity, which entrepreneurs will try to eliminate through their investment decisions. Since there is no autonomous demand proper, the weight of the adjustment will fall entirely on inventories and capacity. Firms will not be able to reach a fully adjusted position. The economy will grow below the new warranted rate ($g_{\hat{w}} = \sigma'/k = 0.109/2 = 0.0545$). Yet, it can be considered as a 'second best' equilibrium. Excess inventories and excess capacity are eliminated every year and stabilize at a certain value: $u' = 0.92$ and $\varepsilon' = E_{i,\ t-1}/Y_t = 0.087$. These equilibrium values can be calculated by means of the following formulas: $\varepsilon' = (1-c-kg)/(1+g+kg)$ and $u' = 1/(1+\varepsilon')$.