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Fiscal policy and fluctuations in a monetary model of growth

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Abstract

We consider an infinite horizon economy with representative agent, aggregate externalities on capital/labor ratio and liquidity constraint on income taxes. We show that the stationary rate of growth can be indeterminate for a wide range of elasticities of intertemporal substitution in consumption. Such a range is bounded from below by a value undergoing a saddle node bifurcation and from above by a value giving raise to a flip bifurcation. It follows that both multiple stationary rates of growth and cycles may emerge. In addition, we carry out a welfare analysis in terms of the optimal level of taxation, since public spending affects consumer's utility, although in a separable way.

1. Introduction

In this paper we consider an infinite horizon economy with representative agent and representative firm in which a given fraction of capital as well as labor income taxes must be paid by money balances in the hand of the households and previously accumulated. In addition, we assume a positive spillover effect due to the average capital/labor ratio available in the economy. Such a spillover effect, opportunely defined, allows for long-run per-capita growth.

Since the relatively recent renewal of interest in growth theory, fuelled by the contributions, among the others, of Romer (1986), Lucas (1988), Barro (1990), and Rebelo (1991), attention has mainly been paid to the determinants of the stationary long-run rates

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of growth. Indeed, most of the quoted contributions consider models where there exists an unique equilibrium coinciding with the stationary growth rate. It follows that equilibrium is determinate and the economy jumps since the beginning of time on its long-run trajectory. Even when there is place for transitional dynamics, as in Lucas (1988), the unique stationary rate of growth displays the saddle path stability and indeterminacy and endogenous fluctuations are, as a consequence, ruled out. Only more recent contributions have focused on the possibility of obtaining endogenous fluctuations around an indeterminate stationary rate of growth. Among these contributions, it is worthwhile to quote that of Cazzavillan (1996) in which public spending affects consumers' utility and that of Bosi (2005) where the introduction of a cash-in-advance constraint on consumption expenditures in a model otherwise à la Barro may make long-run equilibrium indeterminate.

In this paper, the introduction of a fractional liquidity constraint on income taxes may as well make equilibrium indeterminate. Indeed, we show that for a wide range of elasticities of intertemporal substitution in consumption, the growth factor becomes indeterminate and therefore multiple equilibria and sunspot fluctuations may emerge nearby. More in detail, indeterminacy emerges for small elasticities of intertemporal substitution in consumption through a saddle node bifurcation and then disappears through a flip bifurcation for higher ones. This in turn implies on the one hand the possibility of multiple stationary rates of growth and, on the other, the emergence of perpetual oscillations. In addition, we carry out a welfare analysis concerned with the optimal level of taxation, since we assume that public spending affects the utility of the representative household, even if in a separable way.

Although we provide general conditions for the emergence of endogenous fluctuation, we also follow a strategy in order to appraise more in depth all these features, consisting in analyzing the stability properties of a stationary growth rate set equal to zero: then, by continuity, these properties do hold even for sufficiently small positive rates of growth.

The remainder of the paper is as follows. In Section 2 we describe the economy, namely the consumer's behavior and the technology. In Section 3 we define the intertemporal equilibrium, we carry out the steady state analysis as well as the associated welfare one. Section 4 is devoted to the study of local dynamics and conditions for indeterminacy are established. In Section 5 we study the special case where the rate of growth is set equal to zero. Section 6 concludes the paper.

2. The environment

We consider a one-sector discrete-time infinite-horizon model with identical households and identical firms. The representative agent supplies inelastically one unit of labor in each period $t=0,1,\ldots$ and maximizes the discounted stream of utilities

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-1/\sigma} + g_t^{1-1/\sigma}}{1 - 1/\sigma} \tag{1}$$

where $\beta \in (0,1)$ stands for the discount factor, c for consumption, $\sigma > 0$ for the intertemporal elasticity of substitution in consumption and g for public spending, which individuals take as given. When maximizing, agents must respect the budget constraint which writes

$$M_{t+1} + p_t(k_{t+1} - \Delta k_t) + p_t c_t = M_t + \tau_t + p_t(1-\theta)(r_t k_t + w_t) \tag{2}$$

where M stands for money balances, p for the price of the good, k for capital, Δ for $(1-\delta)$ with $\delta \in [0,1]$ the capital depreciation rate, τ for the lump sum transfers of money issued by the government to the agents, $\theta \in [0,1)$ for the tax rate on capital and wage income, r for the real interest rate and w for the real wage.

We assume, in addition, that a share q included between zero and one of the taxes must be paid by money balances held at the beginning of the period. In other words, agents are subject to the additional liquidity constraint

$$q\theta p_t(r_t k_t + w_t) \le M_t \tag{3}$$

The first-order conditions of the program are obtained, by deriving the infinite-horizon Lagrangian function with respect to $\{c_t, M_{t+1}, k_{t+1}, \lambda_t, \mu_t\}_{t=0}^{\infty}$, where λ and μ are the multipliers associated to, respectively, the budget constraint and the liquidity constraint:

$$\beta^t u'(c_t) = p_t \lambda_t \quad \lambda_t = \lambda_{t+1} + \mu_{t+1}$$

$$p_t \lambda_t + p_{t+1}\mu_{t+1} q\theta r_{t+1} = p_{t+1}\lambda_{t+1}[\Delta + (1-\theta)r_{t+1}]$$

jointly with constraints (2) and (3). By eliminating the multipliers, we get the Euler equation

$$\frac{c_{t+1}}{c_t} = \left(\beta \frac{\Delta + [1-\theta(1-q)]r_{t+1}}{1 + q\theta r_{t+1} p_{t+1}/p_t}\right)^{\sigma} \tag{4}$$

Optimal plans must also verify the transversality condition

$$\lim_{t\to+\infty} \beta^t c_t^{-1/\sigma} k_t = 0 \tag{5}$$

The firm j produces the good according to the Cobb-Douglas production function:

$$F(K_j, L_j) \equiv A\kappa^{1-\alpha} K_j^{\alpha} L_j^{1-\alpha} \tag{6}$$

where $A>0$ and $\alpha \in (0,1)$ are productivity parameters, K_j and L_j are, respectively, the amount of capital and labor utilized by the firm j. $\kappa^{1-\alpha}$ is a positive externality of capital intensity $\kappa \equiv K/L$, where K and L denote, respectively, the aggregate capital and the aggregate labor. As in Romer (1986), we assume that the firm does not internalize the effects of its own capital utilization on the productivity of the other firms and takes κ as given. Profit maximization entails $r = \alpha A\kappa^{1-\alpha}(K_j/L_j)^{\alpha-1}$ and $w = (1-\alpha)A\kappa^{1-\alpha}(K_j/L_j)^{\alpha}$. Since we assume the firms to be identical, we have $K/L = K_j/L_j$ for every j. The fact that each worker supplies inelastically one unit of labor and that the population has a unit size implies, at symmetrical equilibrium, $L=1$ and $\kappa = K = k$. The real interest rate turns then out to be constant and the real wage to be linear in capital:

$$r = \alpha A \tag{7}$$

$$w = (1-\alpha)Ak \tag{8}$$

3. Intertemporal equilibrium

Money supply grows, for simplicity, at a constant growth rate $\mu - 1$. If the opportunity cost of money holding (roughly, the nominal interest rate) is positive, then the cash-in-advance constraint (3) binds. By dividing, side by side, the expression of (3) evaluated at time $t+1$ by its expression evaluated at time t, by using the equilibrium factors conditions (7) and (8), and by observing that $M_{t+1}/M_t = \mu$, we obtain the money market equilibrium condition:

$$p_{t+1}/p_t = \mu k_t/k_{t+1} \tag{9}$$

Substituting (9) in (4) and setting the growth factor

$$\gamma_t \equiv k_{t+1}/k_t \tag{10}$$

the Euler equation becomes

$$\gamma_t = \frac{c_t/k_t}{c_{t+1}/k_{t+1}}\left(\beta\frac{\Delta + [1-\theta(1-q)]r}{1+q\theta r\mu/\gamma_t}\right)^{\sigma} \tag{11}$$

Equilibrium in good market is ensured by Walras law: $k_{t+1} - \Delta k_t + c_t = (1-\theta)Ak_t$. Dividing both sides of the last expression by k_t and using definition (10), we obtain the consumption-capital ratio

$$c_t/k_t = \Delta + (1-\theta)A - \gamma_t \tag{12}$$

the expression of which can be substituted into (11) to get the one-dimensional dynamics of the growth factor:

$$\gamma_{t+1} = \varphi(\gamma_t) \equiv \Delta + (1-\theta)A - \frac{\Delta + (1-\theta)A - \gamma_t}{\gamma_t}\left(\beta\gamma_t\frac{\Delta + [1-\theta(1-q)]r}{\gamma_t + q\mu\theta r}\right)^{\sigma} \tag{13}$$

where r is given by (7). We notice that, in each period, the variable k_t is predetermined meanwhile k_{t+1} is not. It follows that γ_t is non-predetermined. As a consequence, we require system (13) to be stable in order to get multiple equilibria and expectations-driven fluctuations.

3.1. Steady state

Dropping the time index from γ in Eq. (13), we obtain the equation

$$\gamma = \left(\beta(\Delta + [1-\theta(1-q)]r\frac{\gamma}{\gamma + q\mu\theta r}\right)^{\sigma} \tag{14}$$

the solution of which gives us the steady state value of γ. We observe that, in absence of taxes, the steady state reduces to $\gamma = [\beta(\Delta + r)]^{\sigma}$ as in the standard endogenous growth models à la Romer (1986). Clearly, the impact of the tax on the growth rate is negative, since the tax receipts are not employed to provide a productive public good.

Two important restrictions have to be taken into account: the consumption positivity and the transversality condition. According to Eq. (12), we notice that, in order for consumption to be positive at the steady state, inequality

$$\Delta + (1-\theta)A > \gamma \tag{15}$$

must be satisfied. The transversality condition (5) is equivalent to

$$R \equiv \Delta + (1-\theta)r > \gamma + \theta q(\mu - 1)r \tag{16}$$

where R is the interest factor. We observe that $\theta > 0$, implies a distortion of the usual transversality condition $R > \gamma$ obtained in the endogenous growth literature (Romer, 1986; Rebelo, 1991).

A sufficient condition to ensure the uniqueness of the steady state is $\sigma < 1$. The LHS of (14) is a 45-line. The RHS is a function of γ, starting from the origin and strictly concave, if and only if

$$\sigma < 1 + 2\gamma/(q\theta r\mu) \tag{17}$$

For $\sigma < 1$, the RHS function crosses the 45-line only once for a strictly positive γ, entailing the existence of a unique non-trivial steady state.

3.2. Welfare analysis

The government budget constraint is simply given by $g_t = \theta A k_t$ and we assume, for simplicity, no legs between the tax levying and the public spending. Along the balanced growth path we have

$$k_t = k_0 \gamma^t \tag{18}$$

$$c_t = c_0 \gamma^t \tag{19}$$

$$g_t = \theta A k_0 \gamma^t \tag{20}$$

where, now, according to Eq. (12), the initial consumption is given by

$$c_0 = [\Delta + (1-\theta)A - \gamma]k_0 \tag{21}$$

The welfare functional W is equivalent to the utility functional of the representative agent. Substituting (18), (19), (20) and (21) into (1), we obtain the welfare evaluated at the steady state, which is a function of the fiscal pressure θ:

$$W(\theta) = \frac{1}{1-1/\sigma} \frac{(\theta A k_0)^{1-1/\sigma}}{1-\beta\gamma(\theta)^{1-1/\sigma}} \left(1 + \left[\frac{\Delta + (1-\theta)A - \gamma(\theta)}{\theta A}\right]^{1-1/\sigma}\right)$$

where $\gamma(\theta)$ comes from the implicit Eq. (14). We observe that the transversality condition (16) ensures the convergence of the series (1). The optimal taxation rate is given by $\theta^* = \arg\max W(\theta)$. There is clearly a trade-off: on the one hand a higher θ reduces the growth rate, on the other one it raises the utility level of the public spending. In order to maximize the welfare evaluated at the steady state with respect to the tax rate we have to solve the first-order equation $W'(\theta)=0$, or, equivalently, we solve for γ the equation:

$$1+\frac{1}{\beta\theta A}\frac{\gamma^{1/\sigma}-\beta(\Delta+A)}{1-([\Delta+(1-\theta)A-\gamma]\big/(\theta A))^{1/\sigma}}$$

$$=\frac{1-\beta\gamma^{1-1/\sigma}}{\beta\gamma^{1-1/\sigma}}\frac{\gamma-(\sigma-1)q\mu\theta r}{\sigma\theta r}\frac{\Delta+[1-\theta(1-q)]r}{q\mu(\Delta+r)+(1-q)\gamma} \tag{22}$$

where

$$\theta=\frac{1}{r}\frac{(\Delta+r)\beta\gamma^{1-1/\sigma}-\gamma}{(1-q)\beta\gamma^{1-1/\sigma}+q\mu} \tag{23}$$

Eventually, the solution γ^* has to be replaced in (23) to obtain θ^*. An explicit rule is obtained in the log case $\sigma=1$. Under this hypothesis, (22) and (23) become, respectively:

$$1+\frac{1}{\beta}\frac{\gamma-\beta(\Delta+A)}{\gamma-[\Delta+(1-2\theta)A]}=\frac{1-\beta}{\beta}\frac{\gamma}{\theta r}\frac{\Delta+[1-\theta(1-q)]r}{q\mu(\Delta+r)+(1-q)\gamma}$$

and

$$\theta=\frac{1}{r}\frac{(\Delta+r)\beta-\gamma}{(1-q)\beta+q\mu} \tag{24}$$

The explicit optimal tax rate is then easily computed as

$$\theta^*=\frac{1}{r}\frac{\beta(\Delta+r)+\left(c_1-\sqrt{c_1^2-4c_2c_0}\right)\big/(2c_2)}{q\mu+(1-q)\beta}$$

where

$$c_0=2\beta^2(\Delta+r)[\beta\Delta(A-r)-qr(\mu-\beta)(\Delta+A)] \tag{25}$$

$$c_1=(1+\beta)(r[q\mu+(1-q)\beta][\Delta+A+\beta(\Delta+r)]-2\beta A(\Delta+r)) \tag{26}$$

$$c_2=2(A-r[q\mu+(1-q)\beta]) \tag{27}$$

One may wonder at this point which is the impact of the credit market imperfection q on the optimal tax rate. To get an explicit solution, we compute, as above, the derivative $\partial\theta^*/\partial q$ in the logarithmic case. We observe that, from (24), one has the following expression:

$$\frac{\partial\theta^*}{\partial q}=-\frac{(\mu-\beta)\theta^*+\gamma^{*\prime}(q)/r}{q\mu+(1-q)\beta}$$

where $\gamma^{*\prime}(q)$ is obtained by totally differentiating

$$c_0(q)+c_1(q)\gamma^*+c_2(q)\gamma^{*2}=0 \tag{28}$$

with respect to q and γ^*. More explicitly:

$$\gamma^*(q) = -\frac{c_0' + c_1'\gamma^* + c_2'\gamma^{*2}}{c_1 + 2c_2\gamma^*}$$

where from (25), (26) and (27)

$$c_0' = -2\beta^2 r(\mu - \beta)(\Delta + A)(\Delta + r)$$

$$c_1' = (1+\beta)r(\mu-\beta)[\Delta + A + \beta(\Delta + r)] \quad c_2' = -2r(\mu - \beta)$$

Consider now a calibration with quarterly data. We set $\alpha = 1/3$, $\beta = 0.99$, $\delta = 0.025$, $\mu = 1.01$, $\sigma = 1$, $A = 0.14964$, $q = 1/3$. We obtain $\theta^* = 9.3126\%$, $\gamma^* = 1.01$, $\partial\gamma^*/\partial q = -1.383 \times 10^{-5} < 0$ and $\partial\theta^*/\partial q = -1.5904 \times 10^{-3} < 0$. The fact that $\partial\gamma^*/\partial q > 0$ is easily interpretable: indeed, an higher distortion in the financial market (namely, an higher q) reduces the rate of growth since a larger share of wealth must be now held in money, which is an unproductive asset. From the following inequality, one also has that $\partial\theta^*/\partial q$ is positive. The economic interpretation is as follows: if the rate of growth becomes larger, it is useful to increase the fiscal pressure, since its positive effect on the consumption of the public good more than offsets the negative impact on capital accumulation.

4. Local dynamics

In order to study the local dynamics of system (13), we must evaluate the derivative $\varphi'(\gamma)$. Straightforward computations give

$$\varphi'(\gamma) = \frac{\Delta + (1-\theta)A}{\gamma} - \sigma\left(1 - \frac{\gamma}{\gamma + q\mu\theta r}\right)\left[\frac{\Delta + (1-\theta)A}{\gamma} - 1\right] \tag{29}$$

Inequality (15) ensures the first term in brackets to be positive. The second term in brackets is also positive. Since γ is a non-predetermined variable, local indeterminacy occurs if and only if $|\varphi'(\gamma)| < 1$. A flip bifurcation occurs at $\varphi'(\gamma) = -1$, while there is room for a saddle node bifurcation at $\varphi'(\gamma) = 1$. As usual, the simplest way to provide explicit bifurcation values is to focus on the elasticity of intertemporal substitution. We notice that (15) ensures:

$$\sigma_T \equiv 1 + \frac{\gamma}{q\mu\theta r} < \left(1 + \frac{\gamma}{q\mu\theta r}\right)\frac{\Delta + (1-\theta)A + \gamma}{\Delta + (1-\theta)A - \gamma} \equiv \sigma_F$$

where σ_T and σ_F are, respectively, the saddle node and the flip bifurcation values for σ. Therefore local indeterminacy arises, iff $\sigma_T < \sigma < \sigma_F$.

We observe that the saddle node bifurcation (and the related multiplicity of steady states) is less likely than the flip one. It happens for ‘pathological’ parameter values. Numerical simulations show that, to observe a saddle node bifurcation, one needs a very high fiscal pressure jointly with a high elasticity of intertemporal substitution.

A sufficient condition to rule out the occurrence of saddle node bifurcations is the uniqueness of the steady state ensured by (17). In particular when $\sigma < 1$, the saddle node bifurcation never occurs.

5. Small growth rates (around zero)

We now investigate the existence of a plausible parametric configuration such that indeterminacy arises for small growth rates around zero ($\gamma \approx 1$), possibly for slightly positive growth rates. In order to carry out such an analysis, we will focus on the case of a zero growth rate, i.e. $\gamma = 1$. By studying this case, by continuity we can appraise the dynamics of system (13) for γ close to one.

Let us set the technological parameter A to obtain $\gamma = 1$ at the steady state. According to Eq. (14) with $\gamma = 1$, we require a restriction in the parameter space

$$r = \alpha A = \frac{1/\beta - \varDelta}{1 - \theta[1 + q(\mu/\beta - 1)]} > 0 \tag{30}$$

or, equivalently, the fiscal pressure has to be bounded from above:

$$\theta < [1 + q(\mu/\beta - 1)]^{-1}$$

Solving (30) for A, we get the productivity parameter ensuring a unit growth factor:

$$A \equiv \frac{1}{\alpha} \frac{1/\beta - \varDelta}{1 - \theta[1 + q(\mu/\beta - 1)]} \tag{31}$$

We must now prove that the value of A provided in (31) is compatible with, (1), the transversality condition, and (2), the positivity of consumption.

(1) Since $\beta < 1$, we have

$$\varDelta + \left(\frac{1}{\beta} - \varDelta\right) \frac{1 - \theta[1 + q(\mu - 1)]}{1 - \theta[1 + q(\mu/\beta - 1)]} > \varDelta + \frac{1}{\beta} - \varDelta = \frac{1}{\beta} > 1$$

and then, according to (30), $\varDelta + [(1-\theta) - \theta q(\mu - 1)]r > 1$. This inequality corresponds exactly to the transversality condition (5) when $\gamma = 1$.

(2) We also observe that

$$\varDelta + \frac{1}{\alpha} \frac{1 - \theta}{1 - \theta[1 + q(\mu/\beta - 1)]} \left(\frac{1}{\beta} - \varDelta\right) > \varDelta + \frac{1}{\alpha}\left(\frac{1}{\beta} - \varDelta\right)$$
$$> \varDelta + \frac{1}{\beta} - \varDelta > \frac{1}{\beta} > 1$$

and then, according to (30), $\varDelta + (1-\theta)r/\alpha = \varDelta + (1-\theta)A > 1$, that is exactly (15) when $\gamma = 1$. When $\gamma = 1$ the bifurcation values of system (29) become

$$\sigma_{\mathrm{T}} \equiv 1 + \frac{1 - \theta[1 + q(\mu/\beta - 1)]}{q\mu\theta\rho}$$

$$\sigma_{\mathrm{F}} \equiv \left(1 + \frac{1 - \theta[1 + q(\mu/\beta - 1)]}{q\mu\theta\rho}\right) \frac{\varDelta + \frac{1-\theta}{1-\theta[1+q(\mu/\beta-1)]} \frac{\rho}{\alpha} + 1}{\varDelta + \frac{1-\theta}{1-\theta[1+q(\mu/\beta-1)]} \frac{\rho}{\alpha} - 1}$$

where, now, $\rho \equiv 1/\beta - \varDelta$.

6. Conclusion

In this paper we have considered an infinite horizon economy with positive externalities in capital/labor ratio and a fractional liquidity constraint on income taxes. The presence of externalities allows for unbounded growth, meanwhile the liquidity constraint may make the rate of growth indeterminate. Actually, this is the case when the elasticity of intertemporal substitution in consumption is included in an interval, the lower and the higher bound of which correspond, respectively, to a saddle node bifurcation and to a flip one. These features taken together entail the possibility of multiple stationary rates of growth and the emergence of two-period cycles.

We have carried out in addition a welfare analysis concerning the optimal level of taxation along the stationary equilibrium. Indeed, public spending affects the utility of the representative agent, although in a separable way.

Besides a general analysis, and in order to get more easily interpretable economic conditions, we have followed a methodology consisting in appraising the stability properties of the economy by normalizing the growth factor to one: then, by continuity, the same features do hold even for positive, although small, stationary rates of growth.

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