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Depletion of natural resources, technological uncertainty, and the adoption of technological substitutes

Richard E. Just [a,*], Sinaia Netanyahu [b], Lars J. Olson [a]

[a] *Department of Agricultural and Resource Economics, University of Maryland, College Park, MD 20742-5535, USA*
[b] *Department of Industrial Engineering and Management, Ben-Gurion University of the Negev, Beer-Sheva, Israel*

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Abstract

Given an existing but unadopted backstop technology, this paper investigates the impact of uncertainty in the discovery date of superior backstop technologies on the rate of exhaustible resource depletion and adoption timing. Contrary to studies with a single backstop technology, the elevated rate of resource price increase due to uncertainty persists beyond discovery of a backstop technology. The option value of waiting can justify putting off, possibly indefinitely, what would otherwise be a sound investment in current backstop technology. In other circumstances the resource is depleted more rapidly and adoption occurs sooner than if future discoveries were not anticipated.

JEL classification: Q32; O31

Keywords: Backstop technology; Resource depletion; Uncertain discovery of technology; Desalination; Increasing resistance

* Corresponding author. Tel.: +1 301 405 1289; fax: +1 301 314 9879.
E-mail address: rjust@umd.edu (R.E. Just).

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1. Introduction

This paper considers adoption of an existing backstop technology in problems of exhaustible resources when discovery of superior technologies is anticipated.[1] This problem spans two rather separate literatures.

The exhaustible resource literature on backstop technologies explores the implications of anticipated discovery of a single backstop technology for optimal resource depletion. Nordhaus (1973) first showed that, when the date of discovery of a backstop technology is anticipated with certainty, the resource is fully exhausted by the date of discovery (assuming marginal production cost exceeds extraction cost). Dasgupta and Heal (1974) considered uncertainty in the discovery date and found conditions under which optimal resource depletion could be studied by optimizing a model with certain non-discovery and a suitably chosen discount factor.[2] Dasgupta and Stiglitz (1981), using the certainty equivalent resource stock as a standard of comparison, found that uncertainty in the discovery date leads to greater (less) conservation if the initial stock is large (small) compared to the certainty equivalent stock. These seminal papers have been subsequently generalized, for example, by adding a pre-discovery stage that endogenizes R&D spending, which in turn influences the discovery process (Dasgupta et al., 1977; Kamien and Schwartz, 1978; Hung and Quyen, 1993). All of these studies, however, consider only a one-time discovery of a single technology with known characteristics where no backstop technology currently exists.

An essentially independent literature considers adoption of successive technology discoveries without reference to exhaustible resource issues. Verifying earlier heuristic arguments, Kamien and Schwartz (1972) found that uncertainty in the discovery date of a superior technology tends to retard both scrapping of outmoded technology and adoption of the best current technology. Balcer and Lippman (1984) generalized this literature for the case where multiple sequential discoveries are anticipated with random times between discoveries. Assuming only a single switch in technology is possible, they found that discovery date uncertainty can hasten rather than retard adoption of the best current technology, and that, as time passes and no improved technology is discovered, previously unprofitable technology may become profitable and be adopted. Farzin et al. (1998) argued that the option value of delaying adoption is neglected by the earlier net present value approach in this literature and considered alternatively the dynamic programming approach of Dixit and Pindyck (1994). Allowing multiple switches in technology, they found a greater delay in adoption is optimal but only in cases where adoption of potential future discoveries is infeasible (the Balcer–Lippman case).[3]

[1] A backstop technology is a technology that produces an inexhaustible perfect substitute for the exhaustible resource.

[2] In some of this literature, the backstop is not a technology but an alternative event that releases the economy from the resource constraint, such as discovery of further resource stocks (Dasgupta and Heal, 1974) or nationalization of the resource stock (Long, 1975, Section 3d), which eliminates its value for private decision makers.

[3] Farzin et al. (1998) also considered uncertainty not only with respect to the discovery date but also with respect to the degree of efficiency improvement. As a result, many of their results focus on trigger efficiency levels. Incidentally, Doraszelski (2001) corrects an error by Farzin et al. (1998) related to their anomalous result regarding adoption of the ultimate and most efficient technology.

This paper builds on the more recent applications of dynamic programming in the latter multiple-technology literature to consider the option value of delayed investment in the exhaustible resource problem of the former literature. Rather than considering the possible discovery of a single technology, we assume an unadopted backstop technology exists but that technologies may yet be discovered. As expected, the results show that optimal planning will delay investment due to the option value of preserving flexibility to invest in future technologies. But this option value causes the resource to be depleted more rapidly and the shadow value of the resource to rise faster than the discount rate suggested by Hotelling's rule as long as any future technology discovery is anticipated. Surprisingly, this option value can be so great that adoption of current technology is delayed indefinitely, and this delay can occur whether the fixed investment in the backstop technology is high or low.

The paper is organized as follows. Section 2 provides a motivation for this problem by discussing several exhaustible resource problems where the possibility of future discoveries seems to or could have major impacts on adoption of existing backstop technologies. Section 3 develops the model and analyzes the resource-extraction/technology-adoption problem. A dynamic framework is used in which technological progress is modeled as a Poisson process. Following Farzin et al. (1998), a finite number of discoveries are assumed possible, which generates a finite rather than (excessively complicated) infinite dimensional system of differential equations. Because of fixed setup costs of adoption, the maximization problem is potentially non-concave unlike the standard problem in the single backstop technology literature. Also, unlike the standard problem in the multiple successive technologies literature, the value of continuing with the current technology is not constant when an exhaustible resource is involved. Because adoption does not necessarily take place only when a new technology is discovered in these circumstances, two cases must be considered. In the first, a superior technology is adopted immediately upon discovery, making both the resource and existing backstop technology obsolete. Under this scenario, an uncertain discovery date of a superior technology leads to uncertainty with respect to the economic life span of the existing technology. In the second case, the resource remains viable for a time when the superior technology is discovered, in which case the superior technology is not adopted immediately but rather alters the marginal value of the remaining resource stock. The analysis characterizes the effects on resource extraction and technology adoption decisions prior to the discovery date of the superior technology. To avoid extensive mathematical derivations that are now well-understood, we begin by modeling the Hamilton–Jacobi–Bellman equation, which enables more intuitive proofs than are customary, and thus conserves space and maximizes readability. Section 4 summarizes the results and implications.

2. Motivation

Following the 1970's oil shortages, nuclear fission was viewed as a major future energy source. By 1996, nuclear power provided only about 17% of electricity generation worldwide (Energy Information Administration, hereafter EIA, 1998, p. 87) and further development of nuclear power was viewed as dubious, even though it may be the most cost-efficient technology currently available. But nuclear power requires large amounts of capital, lengthy construction processes, and a high cost of spent fuel disposal as well as risk of operating accidents and environmental obstacles. Without doubt, some of the slowing of development of nuclear power facilities has been due to delays in construction caused by environmental concerns and lobbies. However, given uncertainties regarding spent fuel disposal and future technologies for power generation (e.g., solar power), the results of this paper suggest that nuclear power development may be slowing, at least in part, due to technological uncertainty and the probability that capital will not be recovered (EIA, 1998, p. 88).

Similarly, desalination technologies offer an alternative to meet increasing demands for fresh water in arid regions that have abundant coastal seawater. However, desalination plants require large amounts of capital and high energy inputs for operation. Currently, desalination technologies are used primarily only where fossil fuel is abundant (e.g., some OPEC countries) or where fresh water is extremely limited (e.g., arid islands).[4] In other water-scarce regions, e.g., the near Middle East, questions have arisen about why current desalination technologies that appear to be economically feasible have not been exploited. A major issue that has arisen in some of these policy debates is whether postponement of investment is advisable given anticipation of a more energy-efficient desalination technology (Ben-Gurion University, 1994). The results of this paper establish the economic rationale for this view.

Another problem that has been treated as an exhaustible resource is the growing problem of resistance. For example, bacteria have a well-recognized increasing resistance to antibiotic use, and insects and weeds have a growing resistance to pesticides. Because resistance increases as a function of the level of treatment, basic economic analysis has studied these problems as exhaustible resources (where the pool of non-resistant bacteria, insects, or weeds is eventually completely depleted) to determine the optimal rate of use and resistance development (see Hueth and Regev, 1974; Brown and Layton, 1996). More generally, Laxminarayan and Brown (2001) have studied the problem of optimal combinations of use when two antibiotics are available. But none of these studies formally consider anticipation of future backstop technology development.

While the pesticide resistance problem was well-recognized some three decades ago, a major issue that troubled policymakers and environmental economists alike was the slow rate of adoption of integrated pest management (reduced- or non-pesticide methods of pest control). Based on the results of this paper, the slow rate of adoption of this backstop technology can be explained by decision makers' anticipation of future technologies, which were not present in typical economic and policy studies at the time. Indeed, pesticide companies have been able to stay ahead of increasing pest resistance by developing alternative pesticides, many of which are less susceptible to increasing resistance. A similar willingness to "overuse" antibiotics may also be explained by anticipation of better future pharmaceutical alternatives. The results of this paper thus suggest shifting of the debate toward better assessment of the likelihood of future pharmaceutical development, and determining whether perceptions among decision makers are well founded or not, because these issues determine whether a higher rate of use if optimal.

[4] For a survey on existing water desalination technologies and their use world wide, see Awerbuch (1984) and Dabbagh et al. (1994).

Finally, the problem of global warming is viewed as an exhaustible resource problem where various environmental groups argue that known technology for controlling emissions has been underutilized. The results of this paper suggest that optimal public policy will allow global warming to continue more rapidly if the development of technologies to cope with the resulting climate and conditions is anticipated, and that the rate of global warming will be higher as the anticipated rate of discovery is higher. The results also imply that delayed adoption of known emissions control technology is optimal if the discovery of superior technologies is likely, and that the delay will be longer as the rate of discovery is higher.

3. The model and results

Let $x(t)$ denote the stock of an exhaustible resource and $q_0(t)$ represent the rate of resource extraction at time t. Extraction incurs a constant unit extraction cost of c_0.[5] At date 0, a backstop technology exists that is capable of providing a perfect substitute for the resource. Potentially, a finite number of additional superior technologies may be discovered in the future, which each generate discrete rather than continuous technical progress according to the motivation provided by Dasgupta and Heal (1974).[6] These backstop technologies are ordered by vintage, $i = 1, \ldots, N$, where $i = 1$ represents the existing vintage (i.e., technology i is discovered before technology $i + 1$). As suggested by Dasgupta and Heal (1974) and has been widely exploited in the literature on backstop technologies (e.g., Dasgupta and Stiglitz, 1981; Hung and Quyen, 1993), technological discovery is assumed to be governed by a sequence of Poisson processes. Also, following Dasgupta and Heal (1974), Dasgupta and Stiglitz (1981), and many subsequent authors, the probability distribution of discovery is assumed exogenous, i.e., uninfluenced by endogenous policy or R&D.[7] Let τ_i be the discovery date of technology i. Then the distribution of τ_i is exponential with parameter η_i.[8]

[5] While Nordhaus (1973) considered extraction cost to be increasing in cumulative extraction, the common practice in the backstop technology literature has been to assume either constant or zero extraction costs (Dasgupta and Heal, 1974; Dasgupta and Stiglitz, 1981; Hung and Quyen, 1993).

[6] Discrete technological discovery is the standard assumption in both the exhaustible resource literature on backstop technology adoption and the general technological progress literature on multiple technology discoveries. See Dasgupta and Heal (1974) and Kamien and Schwartz (1978) for justification. Tsur and Zemel (2003) present an exception that investigates continuous technological progress in the form of lowering the cost of using the resource.

[7] For studies where the probability or time of discovery is endogenous, see Kamien and Schwartz (1978) and Dasgupta et al. (1977). In their models, the discovery date of the new technology is uncertain but endogenously affected by R&D effort. Given the complication of successive technological discoveries in this study, endogenizing discovery is best left as a topic for future research. Note, however, that our model, where fixed setup costs apply to adoption, is not much different than the endogenous R&D model of Hung and Quyen (1993) where R&D costs are incurred instantaneously followed by a Poisson process of discovery.

[8] If the three Poisson postulates are satisfied, e.g., the probability of discovery of the next technology in a time interval of length h is given by $\eta h + o(h)$, then the time, τ, required for discovery of the next technology, given that it is yet undiscovered, has an exponential distribution with probability density function $f(\tau|\eta) = \eta\, e^{-\eta\tau}$, $\tau > 0$ (zero otherwise). For further interpretation below, note that η is the rate of discovery and $1/\eta$ is the expected time required for the next discovery (see Hogg and Craig, 1994, pp. 132–133).

The rate of production from a technology is represented by $q_i(t)$, where $q_i(t) = 0$ if a technology is not in use. The adoption date of technology i is denoted by T_i and is the first date at which $q_i(t) > 0$. Each technology i is characterized by a constant marginal production cost, c_i, and a fixed setup cost, K_i, that is incurred if and when technology i is adopted.[9] Fixed setup costs of adopting new technology were not considered in the backstop technology literature by Dasgupta and Heal (1974) and Dasgupta and Stiglitz (1981) even though they are critical in the general literature on successive technology adoption (Balcer and Lippman, 1984; Farzin et al., 1998).

The sum of resource extraction plus production from all technologies in current use represents the total rate of output, $Q(t) = \sum_i q_i(t)$. Inverse demand for the good is given by $P(Q)$, consumers' utility is $U(Q) = \int_0^Q P(S)\,\mathrm{d}S$, and consumer surplus is $W(Q) = U(Q) - P(Q)Q$. The risk-free discount rate is δ. The static profit maximizing output for technology i equates price with marginal production or extraction cost and is given by $Q_i^* = P^{-1}(c_i)$. Because producing beyond the output where price is equal to marginal cost is never optimal, the optimal intertemporal rate of production from the resource is bounded above by Q_0^*.We assume the following.

Assumption 1. U is increasing and strictly concave, $\lim_{Q\to 0} P(Q) = \infty$ and $\lim_{Q\to 0} W(Q) = 0$.

Assumption 1 formalizes the notions that demand is downward sloping and that welfare is increasing in Q.[10] It also allows for an indefinite increase in price in the event that no backstop technology is adopted.[11] For each technology, we also assume that the discounted stream of benefits in perpetuity are sufficient to offset the setup cost of adoption.

Assumption 2. $W(Q_i^*) - \delta K_i > 0, \quad i = 1, \ldots, N.$

Assumption 2 is a necessary but not sufficient assumption for technologies to be potentially viable. Technologies that violate Assumption 2 are irrelevant because they would never be adopted. Under uncertainty, a technology may satisfy Assumption 2 but still not be economically viable because its expected lifetime is too short to recover the setup cost.

[9] A standard assumption in the backstop technology literature is that the characteristics of undiscovered technologies, such as their production costs, are perfectly anticipated. While the assumption of constant marginal costs for successive technologies is also restrictive, it is used extensively in the backstop technology literature (e.g., Dasgupta and Stiglitz, 1981). It is also, in a broad sense, congruent with the restrictiveness of the literature on successive technology adoption. For example, Balcer and Lippman (1984) assume that the profit function is linear in the technological level, while Farzin et al. (1998) assume a constant output elasticity, fixed prices of variable inputs and output, and that technological progress is represented by a multiplicative efficiency parameter.

[10] This demand assumption attains a greater level of generality than many other studies. For example, Dasgupta and Heal (1974) require bounded isoelastic utility for their backstop technology results; Dasgupta et al. (1977) assume Cobb–Douglas technology and isoelastic utility; Dasgupta and Stiglitz (1981) assume constancy of the marginal utility of income and, for some results, isoelastic demand; and Farzin et al. (1998) require, in effect, perfectly elastic demand.

[11] This part of Assumption 1 corresponds to Dasgupta and Heal (1974) notion of essentiality of the resource prior to discovery of a backstop technology, which has been used subsequently in various forms by Kamien and Schwartz (1978) and Dasgupta and Stiglitz (1981). It can be relaxed by imposing a joint restriction on the choke price of demand, the discovery rate, and the marginal production and setup cost of a technology.

At each point in time, the decision problem is to choose the rate of extraction from the exhaustible resource and the rates of production from technologies that have been discovered. The objective is to maximize the expected discounted stream of net benefits over time subject to the transition equation for the exhaustible resource and the history of technology discovery. Expectations are over the timing of the discovery of new technologies. At each point in time, the state is determined by (x, i, J), where x is the resource stock, i indexes the latest technology that has been discovered, and J is the set of technologies that have already been adopted (the set of technologies for which setup costs are sunk). Define $V(x, i, J)$ to be the maximum expected value obtainable from the state (x, i, J) at time t. Let $I_k(t) = 1$ if technology k is adopted at time t and $I_k(t) = 0$ otherwise.

The decision problem can then be analyzed as a continuous-time stochastic dynamic programming problem, which captures the option value emphasized by Farzin et al. (1998) for multiple technology problems. The Hamilton–Jacobi–Bellman equation is[12]

$$V(x,i,J) = \max_{q_j(t),I_k(t)} \frac{U(\sum_j q_j(t)) - \sum_j c_j q_j(t) - \lambda_i(t) q_0(t) + \eta_{i+1} V(x,i+1,J)}{\delta + \eta_{i+1}} - I_k(t) K_k, \tag{1}$$

subject to

$$\begin{aligned} &k \leq i(t), \\ &q_j(t) \geq 0, \\ &q_j(t) = 0, \quad \text{if } j \notin J(t). \end{aligned}$$

The economic interpretation of (1) is as follows. The terms in $U(\sum_j q_j(t)) - \sum_j c_j q_j(t) - \lambda_i(t) q_0(t)$ measure the direct and indirect contributions of production decisions to lifetime welfare. Cumulative production provides direct consumer benefits of $U(\sum_j q_j(t))$. The production cost for each technology j is $c_j q_j(t)$. Production using the natural resource involves an extraction cost of $c_0 q_0(t)$ and an opportunity or user cost $\lambda_i(t) q_0(t)$. The term $\eta_{i+1} V(x, i+1, J)$ gives the expected value associated with the discovery of a new technology. As shown by Dasgupta and Heal (1974), when technological progress is exponentially distributed and both existing capital and the resource lose all value at the time of adoption of a new technology, the optimal response is to augment the discount rate by the discovery rate of new technology. The optimal value, $V(x, i, J)$, thus represents a perpetuity whose payoff is the sum of these components, discounted at the rate $\delta + \eta_{i+1}$. This elevated discount rate determines, in part, the opportunity cost of holding the resource under uncertainty.

The potential for discovery of new technologies has the following additional implications for the value function. First, discovery of a superior technology changes the value of the current resource stock even if the superior technology is not adopted immediately. Second, the higher the rate of technological discovery, the less current production decisions and the more optimal decisions under yet-to-be-discovered technologies contribute to the optimal value.

[12] The derivation of (1) follows standard arguments; see, e.g., Rishel (1975) or Fleming and Soner (1993).

The shadow price $\lambda_i(t)$ is equal to the marginal value of the resource stock, $V_x(x(t), i(t), J(t))$, and reflects the opportunity cost, or foregone future value, associated with a unit of extraction.[13] During the interval between discoveries, the evolution of λ_i can be characterized by examining the change in the marginal value of the stock. The differential of V_x is $\mathrm{d}V_x(x, I, J) = V_{xx}(x, i, J)\,\mathrm{d}x = V_{xx}(x, i, J)(-q\,\mathrm{d}t)$. Using the envelope theorem, this is equal to $\{\delta V_x(x, i, J) + \eta_{i+1}[V_x(x, i, J) - V_x(x, i+1, J)]\}\,\mathrm{d}t$, which leads to the dynamic optimality conditions

$$\frac{\lambda_i'}{\lambda_i} = \delta + \eta_{i+1}\left(1 - \frac{\lambda_{i+1}}{\lambda_i}\right), \qquad \frac{\lambda_N'}{\lambda_N} = \delta, \quad i = 1, \ldots, N-1. \tag{2}$$

This is an N-dimensional system of differential equations where the first $N - 1$ equations are similar to the optimality condition found by Dasgupta and Stiglitz (1981) for the period prior to discovery of a single backstop technology. The system has a recursive structure under which the rate of change in the shadow price of technology i depends on the rates of change in the shadow prices of all remaining, undiscovered technologies. When the last technology has been discovered the system reduces to Hotelling's rule for a standard exhaustible resource allocation problem.

While Dasgupta and Heal (1974) and Dasgupta and Stiglitz (1981) showed that the discount rate must be augmented by the rate of discovery for the period prior to discovery of a backstop technology, Eq. (2) makes clear that an elevated discount rate applies as long as any future technology improvement is anticipated. After the discovery of technology i and prior to the discovery of technology $i + 1$, λ_i increases at a rate between δ and $\delta + \eta_{i+1}$. Unlike either the standard deterministic model or the single backstop technology model, two opportunity costs apply to holding the resource stock in situ. The first is the pure rate of time preference, represented by δ, which in the absence of technological progress yields Hotelling's rule. The second opportunity cost is the probability weighted loss incurred if the marginal stock value declines due to technological progress. This cost applies whether from a state of no backstop technology (Dasgupta and Heal, 1974; Dasgupta and Stiglitz, 1981) or from an existing backstop technology as shown here.

A "risk premium" of $\eta_{i+1}(\lambda_i - \lambda_{i+1})/\lambda_i$ is required as an additional incentive to hold the stock. When technological progress is incremental in the sense that λ_{i+1} is close to λ_i, the rate of increase in the shadow value of the resource is approximately δ. But when technological progress occurs in large discrete jumps that cause significant reductions in the shadow value of the resource, the risk of holding the stock in situ is higher and the shadow value increases at a rate closer to $\delta + \eta_{i+1}$. The faster the discovery rate of new technologies, the more significant is the difference between incremental and discrete technological change. The finding that (2) is bounded by δ and $\delta + \eta_{i+1}$ also parallels Dasgupta and Stiglitz (1981) result for the period prior to discovery of a backstop technology.

The optimal allocation of extraction and production across technologies can be determined by maximizing the right-hand side of (1) with respect to $q_i(t)$. The first-order necessary conditions are

$$\begin{aligned} &P(Q(t)) - c_0 - \lambda_i(t) \leq 0, \\ &P(Q(t)) - c_0 - \lambda_i(t) = 0, \quad \text{if } q_0(t) > 0, \end{aligned} \tag{3a}$$

[13] For notational simplicity the dependence of λ_i on J is suppressed.

$$P(Q(t)) - c_j \leq 0,$$
$$P(Q(t)) - c_j = 0, \quad \text{if } q_j(t) > 0, j \in J(t). \tag{3b}$$

A direct consequence of these conditions is the following lemma.[14]

Lemma 1. *If $c_i \neq c_j$ for all i, j then, at each point in time, output is produced using either the resource or a single technology. This is the production method in J that has the lowest marginal social cost, and is the sum of marginal extraction cost and marginal user cost in the case of use of the resource.*

The economic intuition for Lemma 1 is that, in the absence of capacity constraints, the resource can supply $Q(t)$ at a marginal social cost of $c_0 + \lambda_i$, while each technology $j \in J$ can supply $Q(t)$ at a marginal (social) cost of c_j. Since the outputs of technologies are perfect substitutes, demand will be met using the technology with the lowest marginal social cost.

The first-order condition (3a) can also be used to give further economic meaning to the dynamic optimality condition (2). Consider a time interval where resource extraction is positive. Then (3a) implies $\lambda'_i = \mathrm{d}P(\cdot)/\mathrm{d}t$, which is the total rather than partial differential of P with respect to t. Hence, the dynamic optimality condition is equivalent to

$$\begin{aligned}\frac{\mathrm{d}P(\cdot)}{\mathrm{d}t} &= (\delta + \eta_{i+1})[P(Q(x,i,t)) - c_0] - \eta_{i+1}[P(Q(x,i+1,t)) - c_0] \\ &= \delta[P(Q(x,i,t)) - c_0] + \eta_{i+1}[P(Q(x,i,t)) - P(Q(x,i+1,t))]\end{aligned}$$

where $Q(x, i, t)$ and $Q(x, i + 1, t)$ represent the optimal extraction rate when the resource stock is x and the current best available technology is i and $i + 1$, respectively. The rate of increase in the price of output must be equivalent to the discounted price minus marginal extraction cost plus the expected change in price at t. When the discovery of superior technologies is uncertain, the opportunity cost of holding the resource over an interval $(t, t + \mathrm{d}t)$ is higher than without uncertainty. Optimal allocation adjusts for this by increasing the current price of output at a faster rate to offset the probability that the price of output will jump down if a discovery occurs.

Henceforth, we focus on the case with one existing backstop technology and one undiscovered superior backstop technology, for which the discovery rate is simply denoted by η.[15] This involves little loss of generality because our main goal is to characterize how the prospect of a superior technology influences current decisions about extraction and the adoption of an existing backstop technology. Two conditions must be satisfied for the analysis to be interesting. First, the resource must offer some cost advantage over the existing backstop technology. Otherwise, extraction would cease and the existing backstop technology would be adopted immediately. Second, to be economically viable, the

[14] This lemma is largely a consequence of the constant cost assumptions. Most studies in this literature have found such assumptions adequate even though it is well known that technology mixing with resource use will occur under increasing cost assumptions (Smith, 1974; Hung and Quyen, 1993).

[15] Formally, this is equivalent to $\eta_2 = \eta$ and $\eta_i = 0$ for all $i > 2$.

undiscovered technology must be superior to the existing backstop technology. To satisfy these two conditions we assume the following.

Assumption 3. $c_0 < c_1$ and $W(Q_2^*) - \delta K_2 > W(Q_1^*)$.
The first part of this assumption ensures that the resource is initially preferred to the existing backstop technology. The second part implies that the lifetime welfare gain of a switch from the existing backstop technology to the superior technology is sufficient to offset the setup cost incurred when the superior technology is adopted. It also implies that $c_1 > c_2$.

We now proceed to analyze the extraction/adoption problem. At any time prior to the adoption of the existing technology and the discovery of the superior technology, the Hamilton–Jacobi–Bellman equation system (1) can be written as

$$V(x(t),1,0) = \max_{q_j(t),I_1(t)} \frac{U(\sum_j q_j(t)) - \sum_j c_j q_j(t) - \lambda_1(t) q_0(t) + \eta V(x(t),2,0)}{\delta + \eta} - I_1(t) K_1,$$

$$V(x(t),2,0) = \max_{q_j(t),I_2(t)} \frac{U(\sum_j q_j(t)) - \sum_j c_j q_j(t) - \lambda_2(t) q_0(t)}{\delta} - I_2(t) K_2,$$

subject to

$$q_j(t) \geq 0,$$
$$q_j(t) = 0, \quad \text{if } j \notin J(t).$$

The shadow prices of the resource stock evolve according to the system of simultaneous differential equations [see Eq. (2)]

$$\lambda_2' = \delta \lambda_2, \tag{4a}$$

$$\lambda_1' = (\delta + \eta)\lambda_1 - \eta \lambda_2. \tag{4b}$$

Eq. (4a) represents the optimal path for the resource shadow price after technology 2 has been discovered. The method of elimination can be used to solve for the time path for the resource shadow price prior to the discovery of technology 2. Eq. (4b) implies that $\lambda_2 = [(\delta + \eta)/\eta]\lambda_1 + \lambda_1'/\eta$ and $\lambda_2' = [(\delta + \eta)/\eta]\lambda_1' - \lambda_1''/\eta$. Substituting these expressions into (4a) yields the second-order linear differential equation $\lambda_1'' - (2\delta + \eta)\lambda_1' + \delta(\delta + \eta)\lambda_1 = 0$, for which the solution is of the form $\lambda_1(t) = c_1\, e^{(\delta+\eta)t} + c_2\, e^{\delta t}$, where c_1 and c_2 are constants of integration. From (4b) it follows that $c_2 = \lambda_2(0)$ and $c_1 = \lambda_1(0) - \lambda_2(0)$. Hence the general solution to the system of equations in (4) is

$$\lambda_2(t+s) = \lambda_2(t)\, e^{\delta s}, \tag{5a}$$

$$\begin{aligned} \lambda_1(t+s) &= [\lambda_1(t)\, e^{\delta s}]\, e^{\eta s} + \lambda_2(t+s)(1 - e^{\eta s}) \\ &= [\lambda_1(t)\, e^{\delta s}]\, e^{\eta s} + [\lambda_2(t)\, e^{\delta s}](1 - e^{\eta s}) \\ &= \lambda_2(t+s) + [\lambda_1(t) - \lambda_2(t)]\, e^{(\delta+\eta)s}. \end{aligned} \tag{5b}$$

Hence, the shadow price λ_2 increases exponentially at rate δ as indicated in (5a). This system of shadow prices is conditioned on the set of technologies that have been discovered at any point in time and the solution changes continuously as the state evolves over time.

The initial values, $\lambda_1(t)$ and $\lambda_2(t)$, are determined by the current resource stock and the transversality conditions

$$W(q_0(T_2)) = W(Q_2^*) - \delta K_2, \tag{6a}$$

$$W(q_0(T_1)) = W(Q_1^*) - (\delta + \eta)K_1, \tag{6b}$$

that govern the expected timing of adoption for each technology.[16] Note that (5a) and (6a) are conditioned on discovery of the superior technology and can be solved independently of (5b) and (6b). Similarly, (5b) and (6b) are conditioned on the event that the superior technology has not been discovered. The current resource stock provides the initial condition for both (5a) and (5b).

The next lemma characterizes the effect of discovery on the rate of output, output price, and the shadow price of the resource.

Lemma 2. *The rate of output does not decrease at the time of discovery.*

The following argument can be used to establish Lemma 2. Let τ be the time of discovery of the superior technology and define $Q_- = \lim_{t\uparrow\tau}Q(t)$ and $Q_+ = \lim_{t\downarrow\tau}Q(t)$, where $Q(t)$ is optimal. Because the superior technology dominates the existing technology under Assumption 3, two cases must be considered. First, suppose that superior technology is adopted at the time of discovery. By Lemma 1, this only occurs if $c_2 \le c_1$ and $c_2 \le c_0 + \lambda_2(\tau_+)$, which implies $Q_+ = Q_2^* \ge Q_-$. Second, suppose that resource extraction takes place at time τ_- and continues at time τ_+. Then the optimal value after discovery of the superior technology is at least as great as before discovery so that $V(x(\tau), 1, 0) = [W(q_0(\tau_-)) + \eta V(x(\tau), 2, 0)]/(\delta + \eta) \le V(x(\tau), 2, 0) = W(q_0(\tau_+))/\delta$. Simplifying yields $W(q_0(\tau_-)) \le W(q_0(\tau_+))$ so that $q_0(\tau_-) \le q_0(\tau_+)$.

The economic implication of Lemma 2 is that discovery of the superior technology leads to a reduction in both the price of output and the shadow price of the resource, even if the superior technology is not immediately adopted. This reduction in shadow price is similar and has the same intuitive implications as the result found by Dasgupta and Stiglitz (1981) at the time of discovery of the initial backstop technology. In this case, the two possibilities for the adoption of the superior technology are as follows. First, it may be superior to both the resource and the existing backstop technology. Alternatively, the resource may remain economically viable for some period following discovery of the superior technology.

Case 1. The superior technology is adopted immediately upon discovery.

If the instantaneous social welfare associated with adoption of the superior technology exceeds the maximum instantaneous welfare from resource extraction, then the superior technology dominates both the exhaustible resource and the existing backstop technology and will be adopted immediately upon discovery. Formally, this entails the following.

Assumption 4. $W(Q_2^*) - \delta K_2 > W(Q_0^*)$.
which also implies $c_0 > c_2$. The next result follows directly from Assumption 4.

[16] Following Leonard and Long (1992), we do not present a rigorous proof of transversality conditions because, as is well known, such proofs are extensive and center around small differences in assumptions.

Lemma 3. *If Assumptions 1–4 hold, then the superior technology is adopted immediately upon discovery and the resource is abandoned.*

This result that adoption can occur before the resource is fully exhausted and that the resource is then never fully exhausted contrasts sharply with earlier studies that assume either zero extraction costs (e.g., Dasgupta and Heal, 1974; Dasgupta and Stiglitz, 1981; Hung and Quyen, 1993) or perfect foresight (Nordhaus, 1973). Not unexpectedly, if a new technology is, by chance, discovered that can produce a substitute for less than the resource extraction cost, the value of the remaining resource stock falls immediately to zero. This result may also appear to contradict Kamien and Schwartz (1978), who find no effect of extraction cost on their qualitative results, but their results characterize only the time path of consumption and investment before discovery of a single backstop technology.

Now consider the resource-extraction/technology-adoption problem before discovery of the superior technology has occurred. In this case, the problem is when to adopt the existing backstop technology given a risk that both the resource and the existing backstop technology will be made economically obsolete by a superior technology discovery. The value of any remaining resource stock becomes zero at the time of discovery of the superior technology so that $\lambda_2 = 0$ in (4) and (5), and

$$\lambda_1' = (\delta + \eta)\lambda_1,$$

which implies

$$\lambda_1(t + s) = \lambda_1(t)\, \mathrm{e}^{(\delta+\eta)s}.$$

Prior to the discovery of the superior technology, the shadow price of the resource increases over time at a rate $\delta + \eta$ to reflect both the opportunity cost of time and the probability that the stock loses its entire value in the event that the superior technology is discovered.

Case 2. The superior technology is held in abeyance upon its discovery.

Unless the superior technology is economically superior to the exhaustible resource in terms of both its marginal cost of production and instantaneous net welfare (net of interest on the setup cost), it will not be adopted immediately upon discovery, provided a positive resource stock remains. Instead, the exhaustible resource continues to be utilized until the resource price (marginal extraction cost plus marginal user cost) rises to or above c_2 and the instantaneous welfare gains from adoption of the superior technology are sufficient to cover the setup cost of adoption. Prior to discovery of the superior technology, the shadow value of the stock rises over time at a rate between δ and $\delta + \eta$ [see Eq. (2)] because discovery implies only a partial rather than a total decline in the marginal value of the resource stock. For this case assume the following.

Assumption 5. $c_0 < c_2$.

Lemma 4. *If Assumptions 1–3 and 5 hold, then the optimal extraction path exhausts the resource prior to the adoption of a backstop technology.*

The proof proceeds as follows. If the discovery of superior technology has occurred and a positive resource stock exists, the problem is one of choosing the optimal extraction path with a known backstop technology. The optimal extraction path satisfies $q_0(t) \geq Q_2^*$. The resource shadow price increases at the rate of discount until the stock is exhausted just at the point where $W(q_0) = W(Q_2^*) - \delta K_2$. The superior technology is then adopted. Now suppose the discovery of the superior technology has not occurred and that the existing backstop technology is adopted before the resource is exhausted. Then there exist $t_1 < t_2$ such that $Q(t_1) = Q_1^*$ and $Q(t_2) = q_0(t_2) = q > 0$. But this cannot be optimal because, for small $\varepsilon > 0$ and any possible discovery time of the superior technology, the alternative $q_0(t_1) = \varepsilon, q_1(t_1) = Q_1^* - \varepsilon, q_0(t_2) = q - \varepsilon, q_1(t_2) = \varepsilon$ produces the same total output at a lower discounted cost.

The reason why resource extraction never resumes after adoption of a backstop technology is simple. Any extraction that occurs after the backstop technology is adopted can be reallocated to earlier periods to provide increased output at a lower social cost. Such a reallocation is welfare improving so it cannot be optimal to resume extraction once it is halted. This is analogous to Herfindahl's (1967) result on the order of extraction of multiple resource deposits. Combining Lemmas 3 and 4 demonstrates that, at the first time a backstop technology is adopted, the resource is either abandoned if Assumption 4 holds, or exhausted if Assumption 5 holds.

For both Cases 1 and 2, it is informative to compare the optimal resource extraction path under uncertainty with respect to discovery of superior technologies to the optimal path under certainty with respect to discovery of superior technologies. Two key differences arise. First, under uncertainty, the shadow price of the resource increases at a rate between δ and $\delta + \eta$. The risk that the resource is made obsolete by the discovery of a superior technology causes resource extraction to decline at a faster rate under uncertainty. Second, the extraction rate at the time of adoption of an existing backstop technology is less under uncertainty than under certainty.

A central result of this paper is that the risk that the existing backstop technology might become obsolete before its setup cost can be recovered creates an incentive to delay investment in the existing backstop technology. But at the same time, the rate of resource depletion is increased because of the possibility of discovering a better technology that might render the resource worthless thereafter. These two incentives have opposite effects on the optimal time for adoption of the existing backstop technology. Ceteris paribus, the latter provides an incentive for faster extraction and earlier adoption while the former provides an incentive to extract longer and delay adoption. Either incentive can dominate, as we now show.

Proposition 1. *Suppose* Assumptions 1–3 *hold and* $0 > W(Q_1^*) - (\delta + \eta)K_1$, *i.e., the setup cost is positive and the discount rate is sufficiently high. Then the existing backstop technology is never adopted. The initial extraction rate and price are such that planned resource stocks are strictly positive for all t, but approach zero as* $t \to \infty$. *Prior to the discovery of a superior technology, the resource price net of marginal extraction cost increases at a rate between* δ *and* $\delta + \eta$. *If* Assumption 4 *holds (instantaneous welfare with adoption exceeds that with extraction), this continues until discovery of a superior technology, at which time extraction ceases, production switches to the superior technology, and the output price immediately drops to* $P(Q_2^*) = c_2$. *If* Assumption 5 *holds (extraction cost is lower than production cost under the superior technology), then* *extraction continues for some period beyond discovery of the ultimate technology. During this period, the resource price net of marginal extraction cost increases at rate* δ. *The resource is exhausted and the superior technology is adopted at time* T_2 *when* $W(q_0(T_2)) = W(Q_2^*) - \delta K_2$, *which is increasing in the discount rate and both the setup cost and production cost of the superior technology, and decreasing in the resource extraction cost. In contrast, if no superior technology is anticipated, then the resource price net of marginal extraction cost increases at rate* δ. *The resource is exhausted and the existing backstop technology is adopted in finite time* T_1 *when* $W(q_0(T_1)) = W(Q_1^*) - \delta K_1$, *which is increasing in the discount rate and both the setup cost and production cost of the existing technology, and decreasing in the resource extraction cost.*

The proof of Proposition 1 proceeds in two parts. First, note that the discovery rate is high enough that the lifetime of the existing backstop technology is not expected to be long enough to recover the setup cost. Under these circumstances, the existing backstop technology will never be adopted under uncertainty about discovery of a superior technology, even if extraction becomes arbitrarily close to zero. The remainder of Proposition 2 involves a straightforward characterization of the shadow price and adoption time, given that the first technology will not be adopted.

These results are interesting for the case where discovery of a superior technology takes far longer than initially anticipated. By comparison, if no superior technology is expected, then the existing backstop technology will be adopted in finite time because $W(Q_1^*) - \delta K_1 > 0$ under Assumption 2. Proposition 1 thus illustrates how the mere prospect of rapid discovery can be sufficient for infinite delay in the adoption of an existing backstop technology that would otherwise be economically viable.

Proposition 2. *If* Assumptions 1–3 *hold and* $K_1 = 0$, *then, prior to discovery of a superior technology, the resource price net of marginal extraction cost increases at a rate between* δ *and* $\delta + \eta$ *with a planned terminal extraction rate* $q_0(T_1) = Q_1^* = P^{-1}(c_1)$. *In addition, from any stock level, the extraction rate is higher and resource exhaustion and adoption of the existing backstop technology occur sooner, compared to the case where no superior technology is anticipated.*

A heuristic proof of Proposition 2 is as follows. When no setup cost is required to adopt the existing backstop technology, the terminal extraction rate under uncertainty coincides with the terminal extraction rate where no superior technology is expected. In each scenario, resource prices increase until they reach the marginal cost of the existing technology, at which time the resource stock is exhausted and the existing technology is adopted. However, the shadow price of the resource increases at a faster rate when discovery of a superior technology is uncertain. For these two conditions to hold simultaneously, the initial extraction rate must be higher and the existing technology must be adopted sooner when discovery of a superior technology is uncertain than when no superior technology is expected. As long as the superior technology remains undiscovered, the extraction rate under uncertainty declines at a faster rate, so eventually it falls below the extraction rate of the case where no superior technology is expected. Under Assumption 4, if the superior technology is discovered at any point along the extraction path under uncertainty, then extraction ceases and the resource price drops immediately to c_2. Under Assumption 5, if the superior technology is discovered prior to exhaustion of the resource, then the extraction rate increases and the resource price falls at the time of discovery. Extraction follows a time path of more rapid depletion toward a terminal extraction rate of Q_2^*.

Although surprising at a superficial level, the result of Proposition 2 that the existing backstop technology may not be adopted even if it has a zero fixed setup cost applies to the plausible case where the extraction cost is so low that continued extraction is relatively more attractive. More surprising is the result that the extraction rate is higher and the resource is exhausted sooner regardless of the stock level. This result appears to contradict Proposition 3 of Dasgupta and Stiglitz (1981) where opposite results are obtained depending on whether stocks are high or low. Note, however, that Dasgupta and Stiglitz compare to the certainty case where the discovery is made at the expected discovery date whereas the results here compare to the case where no further discovery is anticipated.

Comparing to the multiple technology discovery literature that does not consider exhaustible resources, Kamien and Schwartz (1972) found that uncertainty retards adoption whereas Balcer and Lippman (1984) found that uncertainty can hasten adoption. Balcer and Lippman's potential hastening result, however, rests on the assumption that the technology can only be switched once. Here, we find that both retarding and hastening of adoption is possible based on the role of an exhaustible resource. While uncertainty retards adoption at least initially because of the option value of waiting, the higher rate of depletion of the resource can lead to earlier adoption if the resource is thereby exhausted before a dominating technology is discovered.

Taken together, Propositions 1 and 2 yield the surprising result that the shadow price of the resource can rise faster than if no superior technology is anticipated when the setup cost of the existing technology is either high or low (a zero setup cost is the special case of Dasgupta and Stiglitz, 1981). Based on the results of Hung and Quyen (1993), who find that technological uncertainty causes more (less) rapid depletion if R&D costs are high (low), one might expect a similar result here (where the fixed setup cost represents the R&D required for immediate discovery). The difference is explained by the non-zero extraction cost in this paper. For a given demand, the discounted setup cost may either more or less than offset the difference in lifetime discounted extraction cost and lifetime discounted production cost in determining the value of various technological choices.

While we do not exploit the model further here, an examination of the factors that determine $W(Q_i^*) - \delta K_i$, $i = 1, \ldots, N$, can also reveal a host of other results, with varying similarities to the multiple technology literature where no exhaustible resource is involved. Once any backstop technology is adopted, the problem of this paper becomes simply a multiple technology problem without an exhaustible resource similar to the problem analyzed by Balcer and Lippman (1984) and Farzin et al. (1998). The framework here is closest to Farzin et al. (1998), although we impose no limit on the number of technological advances that can be adopted among those discovered. A direct consequence of Proposition 1, for the case prior to adoption of the superior technology, is that (i) the reduction in unit cost necessary for adoption is inversely related to the discount rate, which means a higher discount rate leads to later adoption, (ii) a smaller improvement in the technology (a higher c_2) leads to later adoption, (iii) a lower extraction cost of the resource leads to later adoption, and (iv) both very high and very low extraction cost can delay technology adoption (Assumptions 4 and 5 cases of Proposition 1). These results parallel the implications of Farzin, Huisman, and Kort's model for switches prior to the last but differ in (i) and (ii) from their results for the last switch after which they permit no further switching even though technological progress may occur.[17]

4. Concluding remarks

This paper examines the effect of technological uncertainty on resource extraction and technology adoption when a backstop technology capable of substituting for the resource already exists but discovery of a superior technology is expected. The results show that uncertainty about the discovery of superior technologies affects both the rate of depletion of the resource and the speed of adoption of the existing backstop technology. In each case, the shadow price of the resource increases at a faster rate, implying more rapid depletion, when discovery of a superior technology remains uncertain compared to the case where no superior technology is expected. This increase depends directly on the rate (or probability) of discovery (η), i.e., the resource is depleted more rapidly when the probability of discovery is high. With respect to speed of adoption, adoption of the existing backstop technology can occur *later* than when no superior technology is anticipated if the setup cost of the existing technology is sufficiently high. This tendency is amplified by a higher probability of discovery. On the other hand, adoption of the existing backstop technology occurs *sooner* than when no superior technology is anticipated if the setup cost of the existing technology is sufficiently low, regardless of the expected time required for discovery. This can occur because the higher extraction rate caused by anticipation of a superior technology eventually leads to lower extraction if the discovery has not yet occurred.

This paper obtains three surprising results. First, the adoption time for an existing backstop technology can (but not necessarily) be delayed compared to the case where no superior technology is expected regardless of whether its fixed setup costs are large or small. Second, adoption of the existing backstop technology can possibly be delayed indefinitely even though it would be adopted if no superior technology were anticipated. This is more likely if the setup cost and/or the cost of production of the existing backstop technology and/or the probability of discovery are sufficiently high. Third, the results imply that Hotelling's rule, whereby the shadow price of the resource increases at the rate of discount, does not apply until the ultimate technology is discovered and no future discoveries are anticipated. Otherwise, the optimal shadow price increases faster and the early rate of extraction is higher than when new technologies are not anticipated. Realistically, a world in which new technologies are no longer anticipated is hard to

[17] Farzin et al. (1998) measure technological progress by a multiplicative parameter in the production function rather than a reduction in cost. They find for the last technology switch that (i) the trigger efficiency level is inversely related to the discount rate so that higher discount rates lead to more rapid adoption, (ii) a smaller improvement in technology leads to a lower trigger efficiency level, (iii) if a more efficient technology is currently in place then adoption of a new technology is less likely, and (iv) both very high and very low current efficiency may produce slower rates of technology adoption.

imagine, which means that Hotelling's rule is permanently too conservative. The current backstop technology literature only shows this error to be temporary.

These results have practical implications for a number of resource conservation problems. A naive view is that backstop technologies for an exhaustible resource should be employed as soon as the current resource price rises to the sum of the unit cost of the backstop technology plus the capitalized setup cost. The results of this paper offer an explanation of what some have viewed as unjustified delays in adoption of resource-conserving technologies. For example, the delay in adoption of water desalination technology in some arid regions such as the Middle East may be explained by large setup costs for existing technology and the prospect for improved technology. The results also offer an explanation for what some have viewed as an excessive rate of depletion of resources when known technologies could reduce depletion. For example, a delay in adoption of known technologies to combat global warming may be explained by anticipation of future technological progress, whether well- or ill-founded.

Several assumptions of this paper should be borne firmly in mind and can hopefully be relaxed in future research. These include the assumptions of exogeneity of technology discovery, known efficiency of anticipated technologies, constant costs of extraction and production, independence of setup costs from previous adoption decisions, and total elimination of dependence on the resource by adoption of any technology. While all of these assumptions are common in the literature, specific studies have successfully relaxed most. Research affecting discovery can be endogenized by incorporating assumptions used by Dasgupta et al. (1977) and Kamien and Schwartz (1978). Uncertainty about efficiency of new technologies can be added following Farzin et al. (1998) analysis. An extraction cost increasing in cumulative extraction can be added following Nordhaus (1973) and increasing costs of production can be added following Smith (1974) and Hung and Quyen (1993), which possibly permits joint use of extraction and backstop production as well as joint use of alternative backstop technologies. Dependence of setup costs on previous technology adoption can be considered following Balcer and Lippman (1984). Because each of these generalities have already been explored to some extent, they were not incorporated in this paper in order to obtain sharp results. How many of these generalities can be successfully considered jointly along with the generalities of this paper is a subject for future research. Another obvious direction for future work is to consider technologies that improve resource use efficiency but do not eliminate resource use. From a policy perspective, empirical research is needed to assess the likelihood of technology adoption and to determine accordingly where existing measures of resource conservation are too conservative or too liberal.

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