Available online at www.sciencedirect.com
SCIENCE DIRECT
ELSEVIER Journal of Economic Dynamics & Control 28 (2004) 1511 – 1539
JOURNAL OF Economic Dynamics & Control
www.elsevier.com/locate/econbase

Taxation, risk-taking and growth: a continuous-time stochastic general equilibrium analysis with labor-leisure choice

Turalay Kenc*

The Business School, Imperial College London, London, UK

Received 15 August 2001; accepted 18 June 2003

Abstract

This paper investigates the equilibrium relationship between taxation, portfolio choice (risk-taking) and capital accumulation. Specifically, it examines how taxes affect risk-taking and capital accumulation. We extend the existing literature by relaxing two crucial assumptions in modelling risk-taking behavior: (i) that the investment opportunity set is fixed and (ii) that there is no distinction between attitudes towards risk and behavior towards intertemporal substitution. We extend the investment opportunity set of individuals through optimally determined human capital; and distinguish intertemporal substitution from attitudes towards risk via a recursive utility function. In the presence of these extensions, the paper successfully derives a closed-form solution to the stochastic growth model with stochastic wage income.

JEL classification: D9; G1; H2

Keywords: Risk-taking; Capital and labor income taxation; Recursive utility; Labor flexibility

1. Introduction

The relationship between taxation, risk-taking and economic growth has been the topic of several papers in recent years (see Sandmo, 1989; Turnovsky, 1993; Smith, 1996b; Asea and Turnovsky, 1998, *inter alia*). Asea and Turnovsky (1998), for example, analyze the effect of taxes on capital income on risk taking and capital accumulation to address the question of whether taxing the return on investment increases the total amount of investment and the total amount of risk undertaken. This paper builds on this

* Tel.: +44-20 7594 92912; fax: +44-20 7823 7685.
E-mail address: t.kenc@ic.ac.uk (T. Kenc).

0165-1889/$ - see front matter
doi:10.1016/j.jedc.2003.10.001

literature and extends it in two important directions. First, we explore the interaction between taxation, risk-taking and capital accumulation in a model with labor supply flexibility. We allow the labor-leisure choice to be endogenously determined within the model. This allows us to examine the impact of taxation on both capital and labor income on the growth rate, the volatility of the growth rate and the shares of equity, government bonds and the safe asset in the optimal portfolio. In order to carry out this analysis the paper successfully derives a closed-form solution to the stochastic growth model with stochastic wage income. Second, we distinguish between attitudes towards risk (the desire to smooth consumption across states of nature) and attitudes towards intertemporal substitution (the desire to smooth consumption across time).

To explore the interaction between taxation, risk-taking and capital accumulation in a model with labor supply flexibility and the separation of risk aversion and intertemporal substitutability, we adopt the continuous-time stochastic dynamic general equilibrium framework developed by (Eaton, 1981) and lately extended by (Grinols and Turnovsky, 1993).[1] Since the analytical framework is essentially an intertemporal CAPM in general equilibrium, the model is ideally suited to the analysis of the interactions between taxation, portfolio choice and growth. This offers an advantage over the models used in the endogenous growth theory literature, which explain the mean of the growth rate, and the real business cycle literature, which explains the variability of the growth rate. Since the former literature is interested in the first moment while the latter deals with the second moment, neither is useful for analyzing portfolio choice issues. Our model does not suffer from this shortcoming. We extend the Eaton–Grinols–Turnovsky (EGT) type of the continuous-time stochastic dynamic general equilibrium model: (i) to incorporate endogenous labor supply along the lines of (Bodie et al., 1992a) and (ii) to relax the parametric restriction on risk aversion and intertemporal substitution imposed by the expected utility function by adopting 'generalized isoelastic preferences' along the lines of Svensson (1989), Grinols (1996) and Obstfeld (1994b). Recently, the first extension has been received a considerable attention in the literature since the optimal choice of labor-leisure generates a flexible investment opportunity set, and thereby affects the optimal portfolio choice and furthermore leads to time-varying portfolio choice as opposed to the atemporal portfolio choice that is established in the classical Merton model (see Merton, 1969). However, mostly these papers use a partial equilibrium analysis (see Elmendorf and Kimball, 2000). To our knowledge, Basak (1999), Turnovsky (2000) and Turnovsky and Chattopadhyay (2003) are the only three papers which model labor in a continuous time general equilibrium framework. Basak (1999) provides analytical comparative static analysis of the effects of labor-leisure choice on consumption, stock market, and other fluctuations without explicitly solving it. There are also other limitations in the model of Basak: there is no capital and his analysis is restricted to the expected utility case. This is not surprising since there is no general

[1] See also Turnovsky (1993) (multi shocks), Grinols and Turnovsky (1994) (exchange rate determination and asset prices), Benavie et al. (1996) (optimal investment policy), Turnovsky and Grinols (1996) (optimal government finance policy and exchange rate management) Corsetti (1997) (exogenous labor and optimal government spending) Grinols and Turnovsky (1998) (debt policy) and Evans and Kenc (2003) (welfare cost of monetary and fiscal policy shocks).

solution to the precautionary savings problem with CRRA utility. Therefore, either the focus of the EGT class of stochastic growth models is restricted to models without wage income or models assume no capital as in Basak (1999). Unlike Basak (1999), Turnovsky (2000) and Turnovsky and Chattopadhyay (2003) explicitly solve the problem and examines the growth and welfare effects of taxation and public expenditure. Our approach is close to that of Turnovsky (2000) and Turnovsky and Chattopadhyay (2003) but we use a different solution method and model labor differently both on the supply side and on the demand side. The significance of the solution method used in this paper is that it is firmly based on the contingent claims pricing idea of finance theory. More precisely, we apply the human capital valuation method developed by Bodie et al. (1992b) in order to derive a closed-form solution by exploiting the fact that in EGT models wage growth is perfectly correlated with the rate of return to the risky asset.

We allow the tax rates on the deterministic components of capital income and labor income to be different from the tax rates on the stochastic components of capital income and labor income. This reflects the possibility that the tax code might include offset provisions which have the effect of taxing the stochastic components of asset returns differently from the expected component. The explicit introduction of stochastic elements brings new insights to bear on familiar questions and issues in economics. First, agents may care not only about the first moment of income or prices but also about their second moment. The first moment is usually the concern of standard deterministic models, such as representative agent and overlapping generations models, in which uncertainty is effectively assumed away. In the finance literature, however, the conditional variance of income or prices is as important as the mean and therefore needs to be considered. Second, it is likely that taxes will affect the variance as well as the mean return to an investment. Since taxes reduce the variance and encourage agents to hold risky assets in a stochastic environment, tax policy design issues arise. Furthermore, our work also shed lights into the two recent issues: (i) the relationship between growth and volatility and (ii) estimate how much current consumption society would be willing to forgo to avoid volatility. The literature on growth is challengingly ambiguous: the relationship is positive or negative depending on the theoretical model used and existing empirical evidence does not resolve the conflict.[2] In the literature following Lucas (1987) there is ongoing development and refinement of his measure of the welfare costs of macroeconomic volatility.[3] Now, to the extent that

[2] In reference to the theoretical literature Blackburn (1999) rather neatly explains these diametrically opposed results in terms of the mechanism responsible for generating technological progress—'whether the activity that generates productivity improvements is a substitute or complement to production' (p. 68). Indicative empirical papers on the relationship between growth and volatility are: Kormendi and Meguire (1985), Saint-Paul (1995) (providing evidence consistent with a positive relationship), Ramey and Ramey (1995) and Martin and Rogers (2000) (providing evidence consistent with a negative relationship).

[3] See Dolmas (1998), Epaulard and Pommeret (2003), Obstfeld (1994a) for a partial equilibrium welfare analysis, Barlevy (2002) and Tallarini (1996, 2000) for a general equilibrium welfare analysis and Turnovsky (1993) for an analytical analysis in a restricted general equilibrium framework using a logarithmic utility function. In this paper, we employ a rich general equilibrium framework and provide a numerical analysis of welfare.

changes in economic welfare are determined by what happens to growth, these two literatures have a common focus. However, the methods and models utilized in these literatures are sufficiently different to have served to create something of a dichotomy between them. In this paper, we utilize a framework which allows us to consider the growth and welfare effects of volatility as jointly determined; and explore the role of preference-related deep parameters in changing the scale and direction of influence of volatility on growth and welfare.

The rest of the paper is organized as follows. Section 2 outlines our continuous time stochastic dynamic general equilibrium model with recursive utility and endogenous labor income. Section 3 considers the optimal saving and portfolio choice decision. We numerically analyze the model in Section 4 while Section 5 discusses possible extensions to the model. Section 6 provides some concluding remarks.

2. The aggregate model

This section describes the building blocks of the model. In particular, it describes the production technology, the behavior of the government sector, government expenditure and finance, and equilibrium in the product market of the small open economy that we consider.[4] We assume that the economy is closed and specializes in the production of a single good. The economy is subject to an exogenous productivity shock and it is this that gives it its stochastic nature.

2.1. Production technology

In the spirit of the Arrow–Romer growth model, consider an economy in which there is a productive sector with a continuum of competitive firms uniformly distributed on the interval $[0,1]$. Each firm has access to the same technology and firms compete in both the factor and the goods markets. Production of the representative firm, i, is given by the constant returns to scale production function

$$\mathrm{d}Y_i(t) = [\xi\,\mathrm{d}t + \eta\,\mathrm{d}y(t)]K_i(t)^{1-\alpha}J_i(t)^{\alpha}, \tag{1}$$

where $\mathrm{d}Y$ is the flow of output, $K(t)$ and $J(t)$ are the capital and labor inputs, respectively, $\mathrm{d}y(t)$ is the increment to a zero mean, unit variance Wiener process and ξ and η $(\xi,\eta > 0)$ denote the instantaneous drift and standard deviation of productivity shocks, respectively.

Following Romer (1986), we consider the external effect of capital on labor productivity, specified by the following definition of labor efficiency units:

$$J_i(t) \equiv L_i(t)K(t), \tag{2}$$

[4] The household sector is considered in Section 3.

where $L_i(t)$ are labor physical units and $K(t)$ is the average economy-wide stock of capital. Defining

$$\mu \equiv [NL_i]^{\alpha}\xi = L^{\alpha}\xi, \tag{3}$$

$$\sigma \equiv [NL_i]^{1-\alpha}\eta = L^{\alpha}\eta \tag{4}$$

and substituting into (1), the production function can be written as an AK function (Rebelo, 1991) with a stochastic linear coefficient,

$$\mathrm{d}Y_i(t) = [\mu\,\mathrm{d}t + \sigma\,\mathrm{d}y(t)]K_i(t). \tag{5}$$

As discussed in Bodie et al. (1992b) the wage rate is modelled as instantaneously riskless:[5] The wage rate is known at the beginning of the period t and is equal to the expected marginal physical production of labor over the period $t + \mathrm{d}t$, namely

$$w = E\left(\frac{\partial[\mathrm{d}Y_i(t)]}{\partial L}\right)_{K=\bar{K},\,L=\bar{L}} = \alpha\mu\bar{K}/\bar{L}. \tag{6}$$

The total rate of return to labor, $\mathrm{d}R_L$, over the period $t + \mathrm{d}t$ is thus given by

$$\mathrm{d}R_L = w\,\mathrm{d}t = \alpha\mu[\bar{K}/\bar{L}]\,\mathrm{d}t.$$

The equilibrium rate of return on capital (equity shares), $\mathrm{d}R_K$ over the period $(t, t + \mathrm{d}t)$ will be generated by an Ito process of the following form:

$$\mathrm{d}R_K = r_K\,\mathrm{d}t + \sigma_K\,\mathrm{d}y, \tag{7}$$

where

$$r_K = E\left(\frac{\partial[\mathrm{d}Y_i(t)]}{\partial L}\right)_{K=\bar{K},\,L=\bar{L}} = (1-\alpha)\mu, \qquad \sigma_K = (1-\alpha)\sigma.$$

2.2. The government sector

Government behavior is characterized by three activities: (i) choosing its expenditure and financing it by (ii) taxation and (iii) issuing bonds. Tax revenue T derives from proportional taxes on both capital income and labor income and is given by

$$\mathrm{d}T = [\tau_K(1-\alpha)\mu + \tau_L\alpha\mu]K\,\mathrm{d}t + [\tau'_K\sigma_K]K\,\mathrm{d}y, \tag{8}$$

where τ denotes the tax rate. Note that tax rates are allowed to differ across the deterministic and stochastic components of capital and labor income.[6] Such differential taxation will arise if, for example, the government explicitly co-insures risky undertakings to a degree beyond that implied by the average tax rate. In addition, a non-proportional income tax with a full-loss offset is better approximated by allowing

[5] I am grateful for the anonymous referee for pointing out this.

[6] Tax rates on the stochastic component of income are superscripted with a prime.

τ'_K to differ from τ_K (or τ_L). τ'_K would then show the government's share in bearing the risk of capital, for example.

Government expenditure $\mathrm{d}G$ is assumed to be proportional to national income and is given by

$$\mathrm{d}G = g\mu K\,\mathrm{d}t + g'\sigma K\,\mathrm{d}y. \tag{9}$$

Like taxation, the proportion of government expenditure to income, g, differs across the deterministic and stochastic components of income.[7]

Recall that the government can also finance its expenditure by issuing bonds. These bonds are assumed to be consols paying an instantaneous real coupon equal to u.[8] Defining $B(t)$ and $P_{\mathrm{B}}(t)$ as the number of consols and their price in terms of the consumption good, the government budget constraint implies

$$uB\,\mathrm{d}t + \mathrm{d}G - \mathrm{d}T = P_B\,\mathrm{d}B. \tag{10}$$

2.3. Product market equilibrium

The final element of the aggregate model is the product market equilibrium relationship, given by

$$\mathrm{d}K = \mathrm{d}Y - C\,\mathrm{d}t - \mathrm{d}G. \tag{11}$$

The growth rate of the capital stock can thus be parameterized as

$$\frac{\mathrm{d}K}{K} = \left[(1-g)\mu - \frac{C}{K}\right]\mathrm{d}t + (1-g')\sigma\,\mathrm{d}y. \tag{12}$$

Define D as the ratio of real debt demanded to the capital stock:

$$(P_B B)^d \equiv DK. \tag{13}$$

The price of bonds P_B adjusts instantaneously to maintain equilibrium in the bond market and thus

$$P_B \equiv DK/B. \tag{14}$$

Given that the policy parameters are time-invariant, technological shocks are the only source of uncertainty. The after-tax equilibrium rate of return on each financial asset will depend only on these shocks and will be generated by an Ito process of the form

$$r_i(t)\,\mathrm{d}t + \sigma_i\,\mathrm{d}y(t), \qquad i = K, B, \tag{15}$$

where $\mathrm{d}y(t)$ is the common stochastic term driving the return on all financial assets.

Given the competitive equilibrium assumption, the return on equity shares will be equal to the net marginal product of capital. The parameters of process (15) for the

[7] As with taxation, the proportion with respect to the stochastic component of income is superscripted with a prime.

[8] Note that the government bond becomes money when u is set to be 0 (Eaton, 1981).

case of equity can be derived from (7) together with (8):

$$r_K(t) = (1 - \tau_K)(1 - \alpha)\mu, \tag{16}$$

$$\sigma_K(t) = (1 - \tau'_K)(1 - \alpha)\sigma. \tag{17}$$

Two observations are in order here. First, the volatility of productivity shocks is absorbed by the presence of labor.[9] Second, the tax rate on the stochastic component of capital income reduces the variance of capital income. In other words, taxing the stochastic component of capital income provides insurance against uncertainty, a result that is well-known in the taxation literature.

In order to determine closed-form expressions for the case of debt, observe that the stochastic process for aggregate financial wealth is given by

$$\frac{\mathrm{d}F}{F} = \left[(1-b)r_K + br_B + (1-b)(1-\tau_L)\alpha\mu - \frac{C}{F}\right] F\,\mathrm{d}t + [(1-b)\sigma_K + b\sigma_B]\,\mathrm{d}y, \tag{18}$$

where $b \equiv D/(1+D)$, the share of government debt in total financial wealth. Since

$$K = (1-b)F \tag{19}$$

and b is constant (this is demonstrated later), $\mathrm{d}K = (1-b)\,\mathrm{d}F$. The stochastic process for capital is also given by expression (12). Equating the coefficients of the deterministic and stochastic components of (12) and (18) implies that

$$r_B(t) = r_K + [(1-\alpha)\tau_K + \alpha\tau_L - g]\mu/b - C/K, \tag{20}$$

$$\sigma_B(t) = \sigma_K + [(1-\alpha)\tau'_K - g']\sigma/b. \tag{21}$$

Expression (21) implies that if $(1-\alpha)\tau'_K = g'$ then capital and debt are equally risky. Capital is the riskier asset if $(1-\alpha)\tau'_K < g'$, otherwise bonds are riskier. Note also that when $(1-\alpha)\tau'_K + \alpha = g'$ the government deficit is non-random because randomness in government expenditure exactly matches random variation in tax revenue. This means that the supply of bonds is non-random. However, the price of bonds exhibits randomness because of variation in the demand for bonds. Since this demand is proportional to the capital stock, a shock to the net rate of return on capital translates into an equivalent shock to capital gains on bonds.[10]

[9] This is very much consistent with pertinent macroeconomic findings on volatility raking such as: that dividends have the most volatile growth rates followed by labor, consumption, then wages, and that consumption and labor comove positively. These findings are mainly established in the real business cycle literature, see Basak (1999) for references.

[10] For an excellent discussion of this, see Eaton (1981).

3. Optimal savings and portfolio choice

The representative consumer holds two traded financial assets (bonds and equity) and owns one non-traded asset (human capital, H). At any point in time the representative consumer's financial wealth derives from government bonds, B, and equity, K. The wealth constraint faced by the individual is

$$W = F + P_H H = P_B B + K + P_H H. \tag{22}$$

Consumers are assumed to purchase output out of both wage income generated from endogenously supplied labor, $L(t)$, and capital income from holding assets. They purchase this output over the instant $\mathrm{d}t$ at the non-stochastic rate $C(t)\,\mathrm{d}t$.

The representative household's objective is to select a portfolio of assets and values for consumption, C, and leisure, ℓ, to maximize the recursive utility function [11]

$$\begin{aligned}
U(t) &= \mathrm{e}^{-\rho t}[Z(t)^{\zeta}\,\mathrm{d}t + \mathrm{e}^{-\rho\,\mathrm{d}t}\tilde{U}(t+\mathrm{d}t)^{\zeta}]^{1/\zeta}, \quad \zeta < 1, \neq 0,\\
\tilde{U}(t+\mathrm{d}t) &= [\mathscr{E}_t U(t+\mathrm{d}t)^{1-\gamma}]^{1/(1-\gamma)}, \quad 0 < \gamma, \neq 1,\\
Z(t) &= C(t)^{\theta}\ell(t)^{1-\theta},
\end{aligned} \tag{23}$$

where $\mathscr{E}$ is the expectation operator conditional on all relevant information available at time t, ρ is the rate of time preference, ζ is the parameter dictating the agent's elasticity of intertemporal substitution, γ is the agent's degree of risk aversion [12] with respect to the aggregator Z and Z is a composite good [13] comprising the consumer good C and leisure, ℓ, [i.e., $Z(t)=C(t)+w(t)\ell(t)$]. With this specification, the agent's elasticity of intertemporal substitution, ε, is given by $1/(1-\zeta)$. The relative importance of the consumer good in full consumption is measured by θ.

The utility function may be interpreted as follows. At each point in time the household receives utility from its total consumption $Z(t)$, and calculates the discounted

[11] This is often referred to as continuous time generalized isoelastic preferences. Svensson (1989) first obtained the continuous-time analogues of recursive utility functions developed by Weil (1990) and Epstein and Zin (1991a,b). Grinols (1996) and Obstfeld (1994b) are early applications of this type of utility functions. Duffie and Epstein (1992a,b) used a stochastic differential equation to obtain a stochastic differential utility function. The utility function adopted in this paper is in the spirit of Svensson (1989).

[12] Eq. (23) impose $\zeta = 0$ and $\gamma = 1$ to rule out the logarithmic expected utility case:

$$\mathscr{E}\int_0^{\infty} \mathrm{e}^{-\rho t}[\theta \log C(t) + (1-\theta)\ell(t)]\,\mathrm{d}t, \qquad \rho \in (0,\infty).$$

Furthermore, if $\gamma = 1$ and $\zeta \neq 0$, then the consumer has non-expected utility preferences with logarithmic tastes about risk. Finally, if $\gamma = 1-\zeta$, the utility function collapses to the expected utility case:

$$\mathscr{E}\int_0^{\infty} \mathrm{e}^{-\rho t}\,\frac{[C(t)^{\theta}\ell(t)^{1-\theta}]^{1-\gamma}}{1-\gamma}\,\mathrm{d}t, \qquad \rho \in (0,\infty), \quad \gamma \in (0,\infty).$$

[13] Since the aggregator Z is defined over multi goods, the utility function exhibits a multivariate risk aversion. However, our constant risk aversion assumption does not alter any fundamental results [see Kihlstrom and Mirman (1981)].

certainty equivalent of random utility in the next instant. This structure does not require the usual restriction that $\zeta+\gamma=1$ and thus allows the consumer to have preferences about the timing of the resolution of uncertainty: if $\gamma>\zeta$, 'early' resolution is preferred since distaste for risk, γ, is stronger than distaste for intertemporal fluctuations, ζ. If $\zeta>\gamma$, 'late' resolution is preferred.

The utility function in (23) is maximized subject to the following stochastic wealth accumulation equation:

$$\mathrm{d}W = W[b(t)\,\mathrm{d}R_B + (1-b(t))\,\mathrm{d}R_K] - C\,\mathrm{d}t - w\ell\,\mathrm{d}t, \tag{24}$$

where $b = P_B B/F$ is the portfolio share of government bonds in financial wealth, $(1-b) = K/F$ the portfolio share of real equities in financial wealth and $\mathrm{d}R_i$ the stochastic after-tax real rate of return on asset i, $i = B, K$.

We normalize the total time endowment to 1 which implies that $0 \leqslant \ell(t) \leqslant 1$ and $\ell(t) + L(t) = 1$.

3.1. Valuing human capital

To value non-traded human capital, the existence of a safe asset is required. Such an asset can be constructed by forming a portfolio with proportions $\hat{b}$ of debt and $1-\hat{b}$ of equity. An expression for $\hat{b}$ is obtained as

$$\hat{b} = \frac{b(1-\tau'_K)(1-\alpha)}{[g' - \alpha\tau'_L - (1-\alpha)\tau'_K]}. \tag{25}$$

Assuming that markets are complete, it is obvious that $\hat{b}$ may be outside $[0,1]$ which implies that negative holdings of bonds or equity may be required to eliminate risk.[14] As long as the actual demand for debt and equity are both positive, the negativity of $\hat{b}$ or $1-\hat{b}$ poses no problem. When debt and equity are held together in these proportions, the stochastic components of their returns exactly offset each other and the return on the composite safe asset is

$$r_S = r_K + \frac{(1-\alpha)(1-\tau'_K)}{[g' - \alpha\tau'_L - (1-\alpha)\tau'_K]}[[(1-\alpha)\tau_K + \alpha\tau_L - g] - bC/K]. \tag{26}$$

We can now write (24) as

$$\mathrm{d}W = \left[r_S + \omega(t)(r_K - r_S) - \frac{C}{W} - \frac{w\ell}{W}\right] W\,\mathrm{d}t + \omega(t)\sigma_K(t) W\,\mathrm{d}y(t), \tag{27}$$

where $\omega(t)$ is the share of total wealth held in the equity in period t.

In equilibrium, the future uncertainty evolution of the wage rate follows a standard geometric Brownian motion process:

$$\frac{\mathrm{d}w}{w} = \mu_w\,\mathrm{d}t + \sigma_w\,\mathrm{d}y, \tag{28}$$

[14] Note that this is the same as the notion of replication used in option pricing.

where μ_w is the instantaneous growth rate of the wage rate which is proportional to the equilibrium growth rate and σ_w is the instantaneous volatility of the growth rate of the wage rate which is proportional to the volatility of the rate of return on capital hence the volatility of the equilibrium growth rate. Expression (17) implies that, after taxes, the volatility of wages and equity volatility covary, that is,

$$\sigma_w = k\sigma_K, \quad k = (1 - \tau'_L)\alpha. \tag{29}$$

Note that $k = 0$ implies that wages are fully deterministic.

The individual's life-cycle decisions can then be set out as a step-by-step process (Bodie et al., 1992a). The description below is in line with the baseline case of fully flexible labor.

Step 1. *Calculation of human capital*. Estimate a value for the investor's human capital by valuing future wage income at each given time t; determine the risk characteristics of the wage flows.

To price the non-traded asset (human capital), we use the contingent claims technique developed by (Merton, 1977).[15] It is based on the replicating portfolio idea that the risks of human capital can be hedged (replicated) by a portfolio of (traded) financial assets. In this way, human capital is valued as if it were a traded asset. But, this valuation relies on the assumption that changes in wages are instantaneously perfectly correlated with the risky asset. In other words, traded assets provide a perfect hedge against wages. Let $H(P(t),t)$ denote the value of human capital at time t conditional on the current price of the risky asset $P(t)$. Let us construct (at an initial instant at $t=0$) a portfolio (whose value is designated by V) by buying θ^1 units of the share in P whose process is a geometric Brownian motion as in (7) and θ^0 units of a risk-free investment at the rate r_S whose unitary value evolves according to: $D(t) = D_0 e^{r_S t}$. We force θ^1 and θ^0 vary over time as a function of P and t only: $\theta^1(P(t),t)$ and $\theta^0(P(t),t)$. In addition, this is a self-financing portfolio. We have

$$V(P(t),t) = \theta^1(P(t),t)P(t) + \theta^0(P(t),t)D(t). \tag{30}$$

In the absence of deposits or withdrawals of funds, the increases $\mathrm{d}V$ in the value of this portfolio can only result from the gains on the constituent elements (appreciations $\mathrm{d}P$ on the risky asset and interest $r\,\mathrm{d}t$ on the risk-free investment):

$$\mathrm{d}V = \theta^1\,\mathrm{d}P + r\theta^0\,\mathrm{d}D \tag{31}$$

By applying Ito's lemma we calculate the increases $\mathrm{d}H$ in the value of the human capital as

$$\mathrm{d}H = \frac{\partial H}{\partial t}\,\mathrm{d}t + \frac{\partial H}{\partial P}\,\mathrm{d}P + \frac{1}{2}\frac{\partial^2 H}{\partial P^2}[\mathrm{d}P]^2 + w(t)\,\mathrm{d}t. \tag{32}$$

The no-arbitrage equilibrium condition requires that

$$\mathrm{d}V = \mathrm{d}H. \tag{33}$$

[15] The certainty equivalent present value method due to Davis and Norman (1990) could also be used to calculate the value of lifetime labor income.

Identifying like terms in expressions dV and dH implies that

$$\theta^1 = \frac{\partial H}{\partial P}. \tag{34}$$

Solving (30) for $\theta^0(t)$ and substituting (34) into the resulting expression we obtain

$$\theta^0 = (V - \theta^1 P)/D = \frac{(V - (\partial H/\partial P)P)}{D}. \tag{35}$$

Substituting (32) and (31) (after plugging Eqs. (34) and (35) into (31)) and grouping together the terms in dt of the resulting expression yield the fundamental partial differential equation:

$$\frac{\partial H}{\partial t} + r_S \frac{\partial H}{\partial P} P + \frac{1}{2}\sigma^2 \frac{\partial^2 H}{\partial P^2} P^2 - r_S H + w(t) = 0. \tag{36}$$

Solving the partial differential equation (36) yields an expression for $H(P(t),t)$. Appropriate substitutions from (7) and (28) leads to the following valuation formula for H:

$$H[w(t),t] = [w(t)/\lambda], \tag{37}$$

$$\lambda = r_S + k(r_K - r_S) - \mu_w. \tag{38}$$

Note $H[w(t),t]$ is the present discounted value of total time endowment (not only wage income).

Step 2. *Calculation of total wealth*. Sum human capital (found in step 1) and financial wealth.

Step 3. *Determine optimal levels of consumption and leisure*. Obtain the optimal level of the composite good which depends on the investor's current total wealth (found in step 2) as well as his remaining years of life (this is a standard stochastic dynamic optimization problem). Once the optimal level of the composite good has been obtained, static optimization is applied to find the optimal levels of consumption and leisure.

Step 4. *Determine the optimal portfolio*. Determine the optimal gross proportion of total wealth to be invested in the risky asset.

Step 5. *Calculation of risk-taking*. Determine the optimum proportion of financial wealth invested in the risky asset, by adjusting the optimal gross proportion in step 4 for the dollar-value equivalent exposure to the risky asset of human wealth. (Bodie et al. (1992a) call this the investor's implicit exposure to risk embodied in his uncertain future labor income).

As a simple illustration, consider a working individual with a financial wealth of $200,000.

Step 1: The value of his human capital is $400,000. Furthermore, suppose that the risk characteristics of his human capital make it equivalent to holding $300,000 in the riskless asset and $100,000 in the risky asset.

Step 2: The value of total wealth (sum of human wealth and financial wealth) is $600,000.

Step 3: The optimal level of the composite good is obtained based on the value of total wealth. Optimal levels of consumption and leisure depends on the current wage.

Step 4: Suppose that the optimal proportion of total wealth invested in the risky asset is 40%—that is, \$240,000 gross investments in the risky asset and \$360,000 in the riskless asset.

Step 5: The optimal explicit investment in the risky asset is therefore \$140,0000 (=240,000–100,000). This implies that his 140,000/200,000 or 70% of his current financial wealth is invested in the risky asset. This optimal share is a function of time since the value of human capital varies with time.

The stochastic optimal control problem we have is such that the household maximizes utility by choosing the optimal full (composite) consumption–wealth ratio and the optimal shares of assets in the portfolio while taking the rates of return on assets and the relevant variances and covariances as given.[16] Details of the solution to this problem are given in the appendix. Defining full (composite) consumption as

$$Z = C + w\ell,$$

the first-order conditions are

$$C = \theta Z \tag{39}$$

and

$$w\ell = (1-\theta)Z. \tag{40}$$

The solution is then

$$Z = MW, \tag{41}$$

where

$$M = \frac{1}{1-\zeta}\left[\rho - \zeta\left(\beta - (1-\theta)\mu_w - \frac{1}{2}\gamma\sigma_W^2 + \frac{1}{2}(1-\theta)[(1-\theta)(1-\gamma)+1]\sigma_w^2 - (1-\gamma)(1-\theta)\sigma_{W_w}\right)\right]. \tag{42}$$

In the no labor case, we have

$$M = \frac{1}{1-\zeta}\left[\rho - \zeta\left(\beta - \frac{1}{2}\gamma\sigma_W^2\right)\right]. \tag{43}$$

In the above equations $\beta = r_S + \omega((1-\tau_K)r_K - r_S)$ represents the rate of return to the total portfolio. In the no labor case, $\beta - \frac{1}{2}\gamma\sigma_W^2$ denotes the risk adjusted rate of return to the total portfolio.

Expressions (41) and (42) reveal that the optimal consumption, work and saving decision depends on both the intertemporal elasticity of substitution and the coefficient

[16] Note, however, that these rates of return, variances and covariances will be determined by the market-clearing (general equilibrium) conditions of the model.

of risk aversion. The optimal portfolio share is given by

$$\omega = \underbrace{\frac{r_K - r_S}{\gamma\sigma_K^2}}_{\text{myopic demand}} - \underbrace{\frac{(1-\gamma)(1-\theta)k}{\gamma}}_{\text{hedging demand}}. \tag{44}$$

This expression indicates that the optimal portfolio depends directly on the risk aversion parameter γ but not directly on the intertemporal elasticity of substitution. However, since r_S appears in the expression, it is obvious that the intertemporal elasticity of substitution indirectly influences the degree of risk-taking through r_S. This stems from the fact that risk-taking behavior is determined within a general equilibrium. The above expression also reveals the effects of wage risk on risk taking in that it increases the demand for the risky asset when the individual investor is risk averse, $\gamma > 1$, implying that labor flexibility increases the demand for the risky asset.

Note, however, that expression (44) only determines the optimal *gross* proportion of total wealth invested in the risky asset. We are interested in calculating the optimum proportion of financial wealth invested in the risky asset. This is obtained by adjusting the optimal gross proportion for the wealth and substitution effects of human wealth on risk-taking:

$$\hat{\omega} = \omega + (\omega - k)H/F, \tag{45}$$

where H denotes human wealth (the discounted present value of future labor income) and F represents financial wealth. From (45) it is apparent that human capital generates a wealth effect on risk-taking ($\omega H/F$) because labor income increases the investment opportunity set. Expression (45) also reveals that the substitution effect of human capital on risk-taking (kH/F) is negative. This confirms the prediction from portfolio theory that the more uncertain are wages, the less is invested in risky assets. Bodie et al. (1992a) refer to this as implicit investment in the risky asset. The individual's explicit investment in the risky asset can, therefore, be defined as the difference between his gross investment minus his implicit investment. It is obvious that ω equals $\hat{\omega}$ when the wealth and substitution effects perfectly offset each other. Another important observation worth making here is that if the wage profile is very risky ($k > \omega$), the demand for the risky asset decreases.

Expression (45) supports the results of Turnovsky (1995), Benavie et al. (1996), Smith (1996b) and Corsetti (1997) who all show that taxes on capital income generate two opposing effects on risk-taking behavior: a negative effect which works through τ_K and a positive effect which works through τ_K'. Since both terms appear in (45), any qualitative result must depend on specific numerical values for the parameters. More importantly, the 'endogenous investment opportunity set' feature of this model can also influence the result. Therefore, with labor supply flexibility we can now examine the effects of labor income taxes on risk taking behavior and hence on the accumulation of human and non-human capital. We do this numerically in the next section.

As a final point to note in this section, from (44) the optimal share of bonds in wealth is

$$b = (1 - \hat{b})(1 - \omega). \tag{46}$$

4. Numerical analysis

This section investigates the effect of changes in taxes on capital income and labor income on the economy. Following Asea and Turnovsky (1998) we measure the tax effects on the economy by looking at four key variables: portfolio shares; the mean growth rate; the variance of the growth rate and welfare. We use analytical comparative statics as the foundation of our numerical analysis.

The mean equilibrium growth rate ψ equals the deterministic component of $\mathrm{d}K/K$ or $\mathrm{d}W/W$. Taking the deterministic part of $\mathrm{d}W/W$ we have the following expression:

$$\psi = \frac{\mu(1-\tau_K) - \delta - \zeta(1-g')\sigma^2(g' - \tau_K' + \gamma(1-g'))/2}{(1-\zeta)}. \tag{47}$$

Similarly, the stochastic component of $\mathrm{d}W/W$ gives us the volatility of the mean equilibrium growth rate:

$$\sigma_W = \omega(1-\alpha)(1-\tau_K')\sigma. \tag{48}$$

Eq. (48) reveals the effect of wage uncertainty on the volatility of total wealth: for a risk averse investor, wage risk increases the volatility of wealth through its positive impact on the demand for risky capital.

The portfolio share of risky capital is calculated using (44). To assess the welfare consequences of tax policy we use the current value function $X(W,w)$, from which equivalent variations are computed as welfare criteria.

Specific values of the preference, technology, government expenditure, tax and shock parameters are used in the computations. These values are reported in Table 1. Values for the coefficient of constant relative risk aversion, the rate of time preference, and the marginal product of capital are chosen in line with the real business cycle literature while the rest (government expenditure, tax and shock parameters and the like) are chosen to be representative of the UK economy.

We also undertake welfare calculations which are based on money metric equivalent variations. This requires the evaluation of the expected lifetime utility associated with the optimal consumption path:

$$\mathscr{E}_0(U) = A\,\frac{W(0)^{1-\gamma}w(0)^x}{1-\gamma}, \tag{49}$$

where

$$A = [\theta^\theta(1-\theta)^{1-\theta}]^{1-\gamma}M^{(\zeta-1)/\zeta(1-\gamma)}. \tag{50}$$

To evaluate the welfare cost of changes in tax policy, we use the following definition:

Definition: The welfare cost of changes in tax policies is the percentage of capital the representative agent is ready to give up at period zero to be as well off under $[\mu(\tilde{\Omega}),\sigma(\tilde{\Omega})]$ as he is under $[\mu(\Omega),\sigma(\Omega)]$, where Ω represents initial tax rates and $\tilde{\Omega}$

Table 1
Parameters and benchmark values of variables

Variable	Symbol	Value
Parameters		
Labor factor share	α	0.650
Marginal product of capital	μ	0.175
Risk aversion parameter	γ	4.000
Elasticity parameter	$1/(1-\zeta)$	0.500
Rate of time preference	ρ	0.045
Variance of output	σ_y^2	0.090
Deterministic capital tax rate	τ_K	0.350
Stochastic capital tax rate	τ_K'	0.350
Deterministic labor tax rate	τ_L	0.300
Stochastic labor tax rate	τ_L'	0.300
Government size	g	0.250
Stochastic government size	g'	0.250
Variables		
Capital	K	100.000
Labor	L	75.000
Mean equilibrium growth rate	ψ	0.015
Variance of growth rate	σ_w^2	0.001
Consumption–wealth ratio	C/W	0.059
Rate of return on equities	r_K	0.085
Rate of return on bonds	r_B	0.096
Rate of return safe asset	r_S	0.072
Portfolio share of equities	$(1-b)$	0.550
Portfolio share of bond	b	0.450
Portfolio share of risky asset	ω	0.172
Value of human capital	H	61.556
Value of total wealth	W	162.374

represents tax rates under the revised policy.[17] The cost, denoted by κ, is

$$\mathscr{E}_0[U(W(0), w(0)\mu(\Omega), \sigma(\Omega)] = \mathscr{E}_0[U((1-\kappa)W(0), w(0)\mu(\tilde{\Omega}), \sigma(\tilde{\Omega})]. \tag{51}$$

Using (49) we can write the cost of policy change as

$$\kappa = 1 - \left(\frac{\delta[\mu(\Omega), \sigma(\Omega)]}{\delta[\mu(\tilde{\Omega}), \sigma(\tilde{\Omega})]}\right)^{1/(1-\gamma)}. \tag{52}$$

Values of other important inputs to the generation of numerical estimates of welfare costs are set out in Table 1. We make no claim as to the empirical realism of these

[17] Our definition of welfare cost in terms of initial capital differs from Lucas' definition of welfare cost: Lucas measured welfare cost in terms of constant amount of consumption on each period that the agent would be willing to sacrifice to avoid consumption risk. Since the growth rate of consumption in our model is endogenous, we use initial capital instead of initial consumption as the reference point along the lines of Turnovsky (1993) and Epaulard and Pommeret (2003). In the presence of endogenous consumption growth. However, Barlevy (2002) still uses Lucas' definition by separating changes in the consumption profile due to changes in the volatility of investment from those due to changes in the level of investment.

numerical values. Rather, we have utilized values that seem plausible. That said, particular mention should be made of the values assigned to the risk aversion parameter and the elasticity of intertemporal substitution. The risk aversion parameter is assigned a value of 4, which is the mid point of the range of conventional estimates (2–6) referred to in Obstfeld (1994b). It should be noted, however, that some authors suggest that values as low as unity or as high as 30 cannot be ruled out (see Epstein and Zin, 1991a; Kandel and Stambough, 1991). The elasticity of intertemporal substitution 0.5 is that used by Obstfeld (1994b) in his calibration exercise and is consistent with what Epstein and Zin (1991a) describe as "a reasonable inference". However, smaller values cannot be ruled out. Hall (1988) and Campbell and Mankiw (1989), for example, suggest an intertemporal substitution elasticity of 0.10. Later in this paper, we explore the sensitivity of our results to different values for the key risk aversion parameter and for a range of correlation coefficients.

The results of the numerical analysis for a 10% rise in one of the four taxes $(\tau_K, \tau_L, \tau'_K, \tau'_L)$ at any one time are reported in Table 2 while Figs. 1 and 2 illustrate the robustness of the responses of these variables for a range of tax rates on the deterministic and stochastic components of capital and labor income, respectively. We now analyze the effect on the variables one by one.

4.1. Portfolio effects

In interpreting the results it is useful to bear in mind that given the stochastic nature of the model, changes in the tax parameters generate variance effects as well as mean effects. In the case of the capital income tax experiment, mean effects are as follows. A rise of 10% in the capital income tax rate, τ_K, reduces the after-tax mean return to capital, thereby inducing investors to shift away from holding capital in their portfolios. It can be seen from Table 2 that for the no labor case, an increase in τ_K decreases the share of equity in the portfolio by almost 10.8%. However, the introduction of labor alters this effect. Allowing for labor supply that is exogenously determined reduces the effect due to the fact that the share of capital in total wealth is reduced. When labor supply is endogenously and optimally determined, the increase in the labor supply (around 1%) as a result of the income effect generated by the reduction in the after-tax mean return to capital yields a slightly higher portfolio effect—a 7.8% decline as opposed to 7%.

When considering tax effects on the share of the risky asset in the portfolio, an interesting result occurs. In the no labor case, for instance, a 10% increase in τ_K increases the portfolio share of the risky asset while the portfolio share of the riskless asset decreases. This rejects the way in which (a) the government sector and its' borrowing are modelled and (b) the relative riskiness of bonds and equity implied by the parameter values chosen. Since we model the demand for government bonds as proportional to the capital stock, a shock in the net rate of return on capital translates into an equivalent shock in the capital gain on bonds. From (20) and (21), the rise in τ_K increases the demand for government bonds by reducing the equilibrium risk premium on debt. This rise in demand is greater than the decline in the demand for equity because the parameter values chosen imply that bonds are less risky than capital.

Table 2
Effects of a rise of 10% in taxes

% of change	τ_K	τ'_K	τ_L	τ'_L
No labor				
Growth rate	−6.9254	4.0985	−0.0000	−0.0000
Variance of growth rate	31.6028	−20.8968	−0.0000	−0.0000
Labor supply	0.0000	0.0000	0.0000	0.0000
C/W	−10.3404	6.6397	0.0000	0.0000
Risk-free rate of return	−30.0647	19.4578	0.0000	0.0000
Portfolio share of equities	−10.7858	2.8519	0.0000	0.0000
Portfolio share of risky asset	14.7182	−5.9984	−0.0000	−0.0000
Equivalent variation	−0.2440	0.1206	0.0000	0.0000
Endogenous labor				
Growth rate	−23.5117	3.5196	−5.2869	0.3760
Variance of growth rate	98.9745	−12.2177	43.8393	−3.7396
Labor supply	1.0283	−0.1892	−0.8085	−0.0283
C/W	−7.6406	1.0820	−2.0688	0.1069
Risk-free rate of return	−14.4822	2.0041	−4.2574	0.2716
Portfolio share of equities	−7.8137	1.2019	−2.0380	0.3346
Portfolio share of risky asset	40.6980	−0.9288	20.1767	−1.8807
Equivalent variation	−0.1158	0.0161	−0.0518	0.0019
Exogenous labor				
Growth rate	−21.1923	3.0277	−2.5727	0.2899
Variance of growth rate	57.4989	−7.6476	21.2834	−2.9863
Labor supply	0.0000	0.0000	0.0000	0.0000
C/W	−6.3361	0.8856	−0.9998	0.0729
Risk-free rate of return	−11.5393	1.5896	−2.0620	0.2009
Portfolio share of equities	−7.0446	1.0906	−1.6523	0.3157
Portfolio share of risky asset	25.4986	1.5693	10.1287	−1.5045
Equivalent variation	−0.1162	0.0163	−0.0171	0.0018

This in turn implies that overall risk falls which ultimately increases the demand for the risky asset.

When labor is introduced, the magnitude of this effect increases for $\gamma > 1$. Since the ‘wealth effect’ is greater with a flexible (endogenous) labor supply, the portfolio share of the risky asset is higher in the flexible labor case than in the fixed (exogenous) labor case, the portfolio share of the risky asset increasing by some 40.7% as opposed to the 25.5% increase that occurs when labor supply is fixed.

In general, increasing the tax rate on the stochastic component of capital income reduces the risk which investors ‘price’ at γ, encouraging the holding of capital, which leads to an increase in the portfolio share of equities. The rise in τ'_K decreases the demand for government bonds by raising the equilibrium risk premium on debt. Since bonds are riskier than capital, the decline in the demand for government bonds is

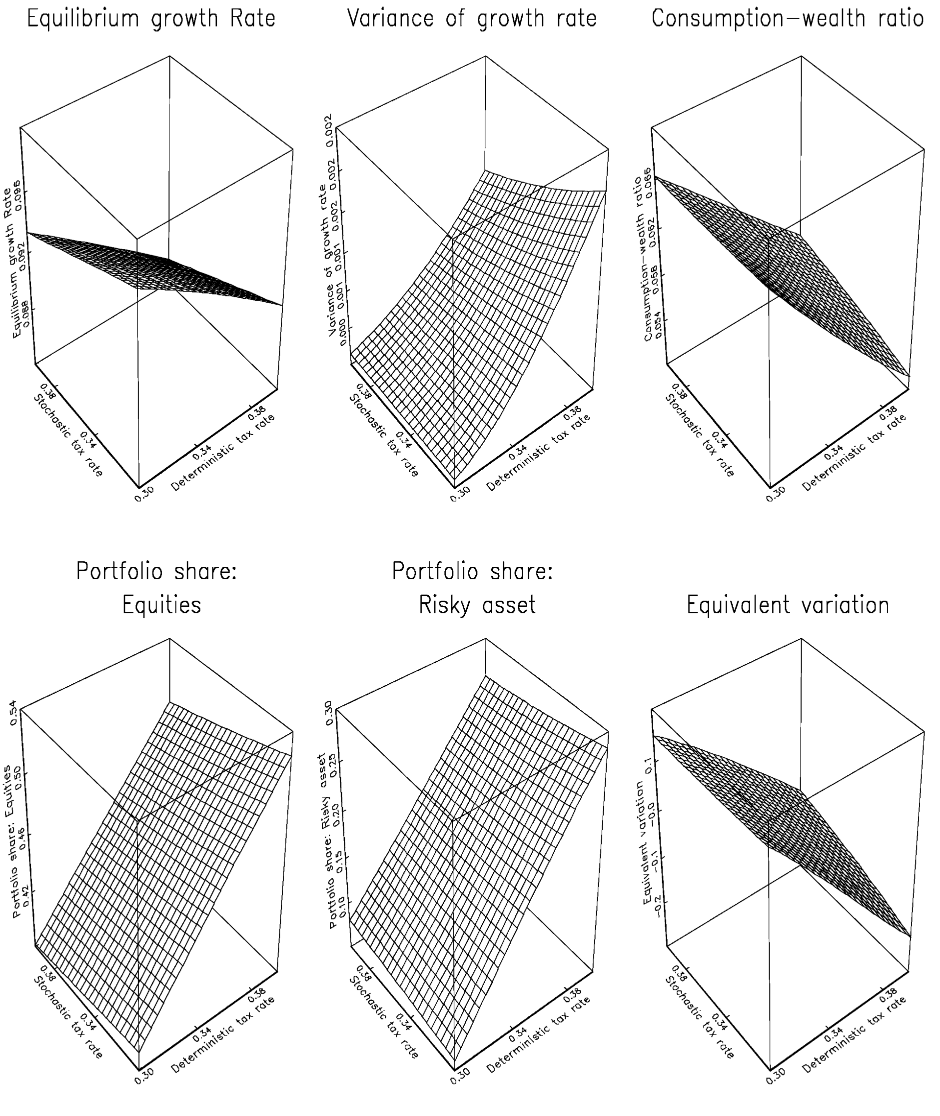

Fig. 1. Capital income tax.

greater than the rise in the demand for equity, implying that overall risk rises. This ultimately decreases the demand for the risky asset.

A rise in the tax rate on labor income, τ_L, increases the after-tax wage bill, leading to a decrease in the return to capital and in turn a decline in the demand for equity. With a higher τ_L, tax revenue for the government increases, decreasing the riskiness of government bonds. This leads to an increase in the demand for government bonds, with a corresponding rise in the portfolio share of the risky asset. The increase in

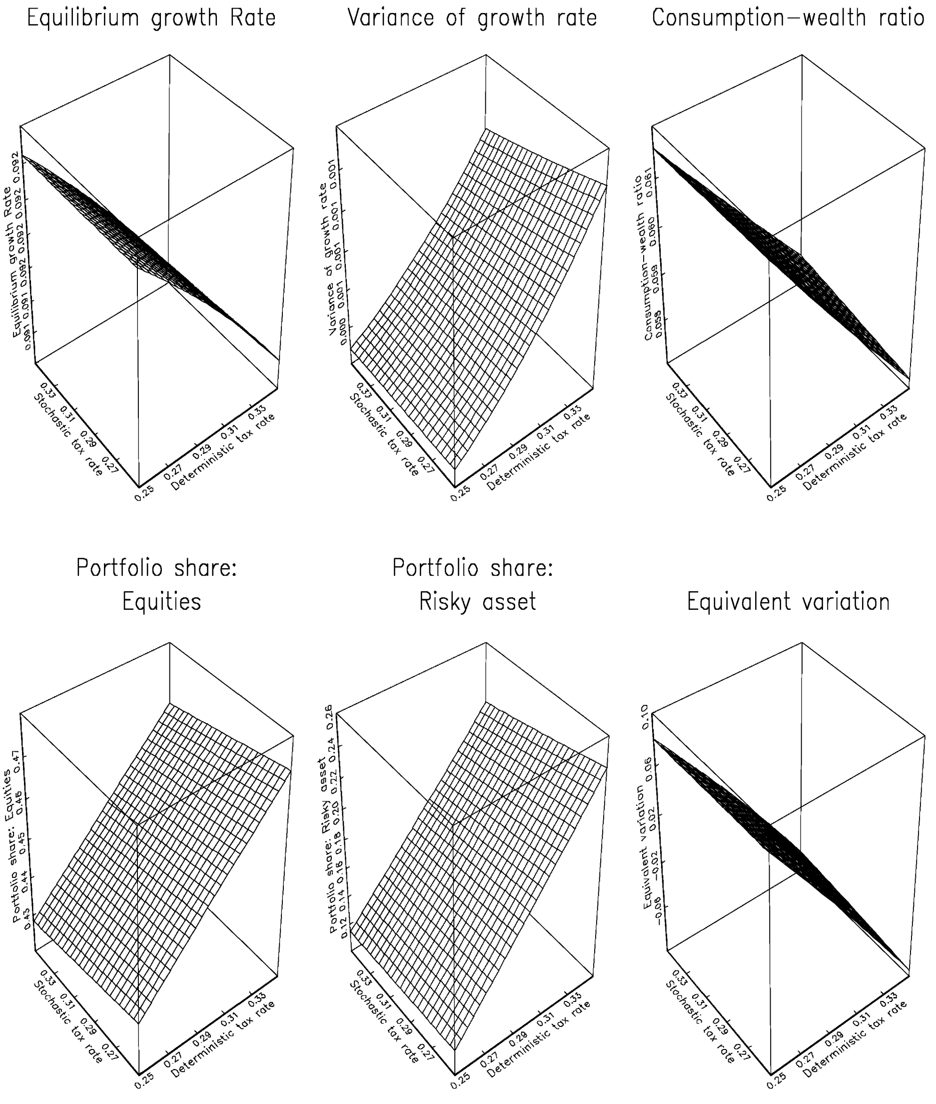

Fig. 2. Labor income tax.

the portfolio share of the risky asset is higher in the flexible labor case (a rise of 20.2%) than the fixed labor case (a rise of 10.1%). This difference arises because of differences in the wealth effect in the model with exogenous versus endogenous labor supply. Note that, as in the case of increasing the tax on the stochastic component of capital income, these effects are reversed when the tax rate on the stochastic component of labor income is increased.

4.2. Effects on the growth rate

The effects of the tax policies on the share of the productive asset (equity) and the unproductive asset (bonds) in the portfolio spillover into growth effects. To illustrate, in the case of no labor the rise in τ_K reduces the after-tax return to capital. This shifts the portfolio away from equity to government bonds which has the effect of reducing the equilibrium growth rate by 6.9%. When we allow labor to enter the model, the reduction in the (aggregate) capital stock lowers the efficiency of labor. This follows from the way in which we model endogenous growth. The effect this have is that when the labor supply is fixed exogenously, increasing τ_K further reduces the growth rate by 21.2%. With flexible labor, the labor supply increases as a result of the income effect and this in turn reduces the growth rate by some 23.5%. An increase in τ_K' on the other hand leads to a shift of assets away from the unproductive asset towards the productive one. This in turn leads to positive growth effects, although the extent of the growth reduces when labor is introduced. In the flexible labor supply model, for example, increasing τ_K' reduces labor supply via the income effect, leading to a lower increase in the growth rate as compared to the no labor case.

The shift of assets away from equity under increases in τ_L gives rise to negative growth effects. This reflects the fact that an increase in the tax rate reduces the supply of labor. The negative growth effect is smaller in the exogenous labor supply model as compared to the endogenous labor supply model (a decrease in growth of 2.6% versus a decrease of 5.3%). An increase in the tax rate on the stochastic component of labor income has the effect of increasing the growth rate. The size of the increase, however, is significantly less than that experienced when τ_K' is increased. The reason for this is that τ_L' indirectly affects the portfolio allocation.

4.3. Variance of the growth rate

Table 2 reveals that the impact of a higher deterministic capital income tax rate on the variance of the growth rate is positive in all three cases. This is because, as discussed earlier, the rise in the tax rate shifts the portfolio towards risky assets, increasing the volatility of the growth rate. The magnitude of the effect differs with respect to the treatment of labor, the impact being much more pronounced when labor supply is determined endogenously.

An increase in τ_K' on the other hand reduces the variance of the growth rate because it implies an increase in the government's share of risk bearing. With labor, the magnitude of this effect is smaller due to the 'wealth effect'. In the cases where labor income is taxed, the volatility effect reflects changes in the portfolio share of the risky asset.

4.4. Welfare

A higher tax rate has three effects on welfare: a growth effect, a portfolio effect and a labor-leisure effect. To the extent that an increase in a particular tax rate reduces the growth rate it will be welfare reducing. However, the portfolio effect may either decrease or increase welfare, depending upon the size of the existing portfolio share of the risky asset, ω, relative to its first-best optimum. Starting from a second-best optimum, in the absence of risk it is possible for an increase in the tax rate to be welfare improving. This will be the case if the higher tax shifts the portfolio in the direction of the first-best optimum, ω. To the extent that an increase in a particular tax rate distorts the labor-leisure choice it will be welfare deteriorating.

In the no labor model, the rise in τ_K generates welfare deterioration: the representative agent would be willing to give up 0.12% of their initial wealth to live in a world with no change in the tax rate. The welfare deterioration reflects changes in the consumption–wealth ratio C/W and the shares of the different assets in the portfolio. Since incorporating labor into the model breaks down the strong mapping between the dynamic behavior of aggregate consumption and stock market wealth, the change in C/W in the models with labor is smaller than that in the model without labor. Note that taxes on the stochastic components of income are welfare-improving, but the effect is very small.

4.5. Sensitivity analysis

The evidence from the experiments conducted here suggests some clear portfolio, growth, volatility and welfare effects from the different tax policies and identifies that growth is the principal driver of those welfare effects. However, it may be inappropriate to place too much weight on the particular values presented in Table 2. In particular, we are mindful that different parameter values capturing the two elements of consumer preferences could bring about significant differences in the optimal consumption/saving decision (Eq. (42)), the mean growth rate (Eq. (47)) and the intertemporal utility of the representative agent—our measure of welfare.

To explore this possibility, we carry out a sensitivity analysis for the first policy and second model case—a rise of 10% in the capital income tax rate, τ_K and endogenous labor. For this experiment, we calculate the portfolio, growth, volatility and welfare effects for a range of values for the two preference parameters—risk aversion and intertemporal substitution. However, values for these two parameters should be chosen in such way that they satisfy both feasibility ($\varepsilon \leqslant 1$) and transversality ($\gamma \geqslant 1$) conditions as demonstrated in Smith (1996a).[18] We therefore assume that the risk aversion parameter takes values between 1 and 17; the elasticity of substitution parameter ε takes values between 0.45 and 0.85.

Fig. 3 reveals that our previous result now needs careful quantification. The result is not robust to the range of parameter values considered here: indeed, a high enough intertemporal substitution parameter (say $\zeta > -0.2$) coupled with a low enough risk aversion (say $\gamma < 5$) could almost eliminate the welfare effect—equivalent variation

[18] In models based on power utility functions (i.e., time-separable isoelastic preferences) transversality condition—the value function does not blow up—is necessary and sufficient for the feasibility condition—consumption and welfare are non-negative—to be satisfied. Smith (1996a) demonstrates that when preferences are generalised isoelastic type (i.e., recursive utility), "*the transversality condition is generally neither necessary nor sufficient for feasibility*".

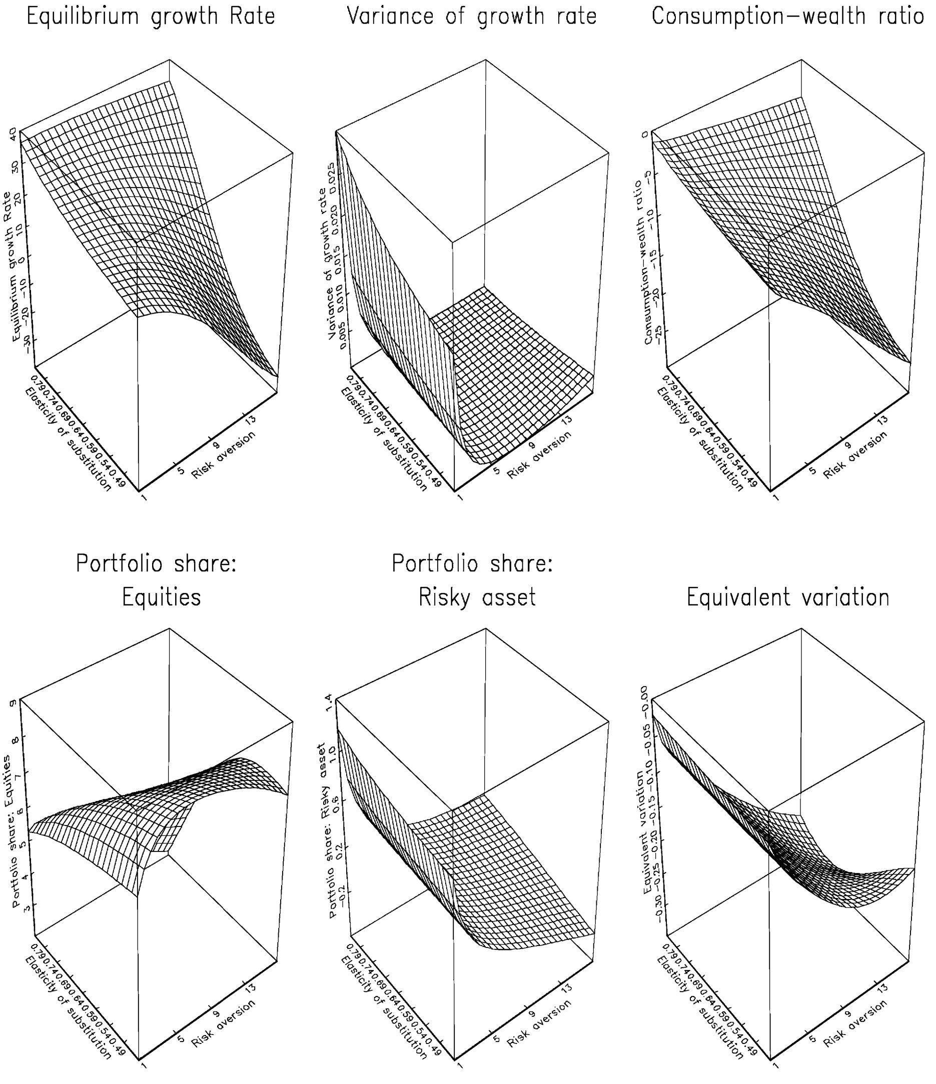

Fig. 3. Sensitivity analysis.

(which can be seen from the bottom right of Fig. 3). A key reason for this is that at low degrees of risk aversion coupled with low degrees of risk aversion the tax rise policy leads both to positive changes in the equilibrium growth rate (top left) and to no changes in the consumption–wealth ratio (top right). As explained above, these two effects enhance welfare. However, for high risk aversion, the tax rise policy generates negative growth effects and leads to negative changes in the consumption–wealth ratio, and thus is welfare deteriorating (bottom left).

5. Extensions to the model

There are several possible extensions to the model that can be considered. There are at least two areas to exploit on the labor income side. The first is to price the discounted present value of future labor income which is subject to uninsurable risk. Recent studies by *inter alia* (Duffie et al., 1997; Koo, 1998) show that, in a partial equilibrium, it is possible to price uninsurable labor income risk. Koo (1998), for example, showed that this can be done by calculating the certainty equivalent present value of lifetime income using a state price density that gives a zero risk premium to any risk orthogonal to the risks in financial assets.

The second extension is to introduce distributional shocks to labor and capital incomes. The constancy of output shares implies that labor and capital incomes are perfectly correlated. Labor's share of income is not constant over time. Rather, it tends to move countercyclically. Ghosh and Pesenti (1994) incorporate these shocks into their model. To maintain analytical tractability they assume that redistributive shocks are temporary and hedgeable. The wage rate and the rate of return to capital are described by

$$w(t) = (1-\alpha)[\mu\,\mathrm{d}t + \sigma\,\mathrm{d}y - \sigma_x\,\mathrm{d}y]\left[\frac{K}{L}\right]^{\alpha},$$

$$r_K(t) = \alpha[\mu\,\mathrm{d}t + \sigma\,\mathrm{d}y + \sigma_x\,\mathrm{d}y]\left[\frac{K}{L}\right]^{\alpha-1},$$

where σ_x is the redistributive shock parameter. Ghosh and Pesenti (1994) point out that an increase in σ_x has two effects on portfolio allocation. First, it raises the volatility of risky assets, leading to lower holdings of the tangency portfolio. Second, and conversely, it reduces fluctuations in labor income relative to capital income and lowers the scope for hedging. Whether the second effect outweighs the first depends upon the relative magnitudes of domestic capital and national wealth.

Finally, the model can be extended to include Tobin's q theory of investment and adjustment costs to investment along the lines of Benavie et al. (1996), thereby permitting analysis of investment and tax issues. With this feature it is no longer the case that the cost of risky capital is only determined by saving-risk behavior. Firms' demand for capital also plays a role because investing is subject to installation costs.

6. Concluding remarks

In this paper, we have developed a stochastic dynamic general equilibrium model of the relationship between taxation, risk-taking and capital accumulation. This model extends the current literature in two ways. First, we extend the model to include the supply of labor. This allows us to analyze the effects of labor income on capital accumulation and the effects of taxation on labor income on economic growth. We allow the supply of labor to be endogenously determined and find that the flexibility and uncertainty this generates for labor income significantly affect risk-taking and capital accumulation. Even allowing the supply of labor to be exogenously fixed significantly influences the results.

Second, we allow the representative household to maximize a stochastic differential utility function that breaks the relationship between the coefficient of relative risk aversion and the elasticity of intertemporal substitution. Using this specification of utility, we are able to show that the optimal consumption and portfolio choice is driven not only by attitudes towards risk but also by the intertemporal elasticity of substitution.

We analyze the model numerically and find that increasing tax rate on the deterministic component of capital income reduces the mean growth rate while increasing its variance. This reduction in the growth rate and increase in the variance of the growth rate is far more pronounced when labor is introduced into the model. Increasing the tax rate on the deterministic component of capital income leads to a reduction in investment in the productive asset (equity) and an increase in investment in the unproductive risky asset (government bonds). These findings support those of Asea and Turnovsky (1998). Increasing the tax rate on the stochastic component of capital income has the opposite effect to an increase in the tax rate on the deterministic component, encouraging investment in equity and increasing the growth rate. Increasing the tax rate on labor income has the same effect as increasing the tax rate on capital income, although the magnitude of the effects is far less pronounced.

We identify two extensions to the model developed in this paper are worth pursuing. First, the model could be extended to incorporate undiversifiable labor income risk. Second, the model could be extended to consider the effect of shocks to the distribution of factor incomes. Both will change the correlation between wage income and capital income and since human capital is a risky asset, it is likely that the proportion of financial wealth invested in this asset will alter.

Finally, it should be noted that the model we develop is relevant for the analysis of international finance issues and monetary issues. It is possible to open up the economy by allowing for international trade and/or international capital mobility. Money can be introduced into the model either through a cash-in-advance constraint or through the structure of utility. With these extensions, this framework is suitable for calibrated analyzes of issues such as the Feldstein–Horioka puzzle, the optimality of interest rate smoothing and the welfare cost of volatility.

Acknowledgements

I am grateful to Lynne Evans, Ian Garrett, Earl Grinols, John Hutton, Michael Reiter and Stephen J. Turnovsky for helpful comments and discussions related to this paper. Also, I thank participants at both the 1998 Money, Macro and Finance (MMF) conference, Imperial College, London and the CEPR/Tilburg University Conference on Dynamic Aspects of Taxation, September 2000 for perceptive comments on previous drafts of this paper. The usual disclaimer applies.

Appendix A. Derivation of the optimal consumption and portfolio choice

The representative consumer's optimization problem is to find the solution to

$$\lim_{dt\to 0^+} \max_{\{Z,\omega\}} \mathrm{e}^{-\rho t}\{Z(t)^{\zeta}\,\mathrm{d}t + \mathrm{e}^{-\rho\,\mathrm{d}t}[\mathscr{E}_t U(t+\mathrm{d}t, t_0)^{1-\gamma}]^{\zeta/(1-\gamma)}\}^{(1-\gamma)/\zeta} \tag{A.1}$$

s.t.

$$\frac{\mathrm{d}W}{W} = \psi\,\mathrm{d}t + \mathrm{d}u_W, \tag{A.2a}$$

$$\psi = r_S + \omega(r_K - r_S) - \frac{C}{W} - \frac{w\ell}{W}, \tag{A.2b}$$

$$\mathrm{d}u_W = \omega(1-\tau_K')\sigma\,\mathrm{d}y, \tag{A.2c}$$

$$\frac{\mathrm{d}w}{w} = \mu_w\,\mathrm{d}t + \sigma_w\,\mathrm{d}y. \tag{A.2d}$$

The Bellman function associated with the problem is then defined as

$$(1-\gamma)X(W(t),w(t)) = \lim_{dt\to 0^+} \max_{\{C,\ell,\omega\}} \mathrm{e}^{-\rho t}\{[C(t)^{\theta}\ell(t)^{1-\theta}]^{\zeta}\,\mathrm{d}t + \mathrm{e}^{-\rho\,\mathrm{d}t}[(1-\gamma)\mathscr{E}_t X \times(W(t+\mathrm{d}t), w(t+\mathrm{d}t))]^{\zeta/(1-\gamma)}\}^{(1-\gamma)/\zeta}. \tag{A.3}$$

Postulate a value function $X(W(t),w(t),t)$ for some constant A of the form

$$X(W(t),w(t),t) = \mathrm{e}^{-\rho t}\,\frac{AW(t)^{1-\gamma}w(t)^{x}}{1-\gamma}.$$

Its current value version is given by

$$V(W(t),w(t)) = \frac{AW(t)^{1-\gamma}w(t)^{x}}{1-\gamma}. \tag{A.4}$$

The expression $\mathscr{E}_t V(W(t+\mathrm{d}t), w(t+\mathrm{d}t))$ can be calculated from the following relationship:

$$\mathscr{E}_t V(W(t+\mathrm{d}t), w(t+\mathrm{d}t)) - \mathscr{E}_t V(W(t),w(t),t) = \mathscr{E}_t(\mathrm{d}V). \tag{A.5}$$

Using Ito's formula one calculates $\mathscr{E}_t(\mathrm{d}V)$ as

$$\mathscr{E}_t(\mathrm{d}V) = \frac{\partial V}{\partial W}\mathscr{E}(\mathrm{d}W) + \frac{\partial V}{\partial w}\mathscr{E}(\mathrm{d}w) + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}\mathscr{E}(\mathrm{d}W)^2 + \frac{1}{2}\frac{\partial^2 V}{\partial w^2}\mathscr{E}(\mathrm{d}w)^2 + \frac{\partial^2 V}{\partial W\,\partial w}\mathscr{E}(\mathrm{d}W)\mathscr{E}(\mathrm{d}w). \tag{A.6}$$

Applying methods of stochastic calculus the above expression is rewritten as

$$\mathscr{E}_t(\mathrm{d}V) = \left(\psi\, W\,\frac{\partial V}{\partial W} + \mu_w w\,\frac{\partial V}{\partial w} + \frac{1}{2}\,\sigma_W^2 W^2\,\frac{\partial^2 V}{\partial W^2} + \frac{1}{2}\,\sigma_w^2 w^2\,\frac{\partial^2 V}{\partial w^2} + \sigma_{Ww} Ww\,\frac{\partial^2 V}{\partial W\,\partial w}\right)\mathrm{d}t. \tag{A.7}$$

After substituting and simplifying, from (A.5) we obtain an expression for $\mathscr{E}_t V(W(t+\mathrm{d}t), w(t+\mathrm{d}t))$

$$\mathscr{E}_t V(W(t+\mathrm{d}t), w(t+\mathrm{d}t)) = \left(\psi\, W\,\frac{\partial V}{\partial W} + \mu_w w\,\frac{\partial V}{\partial w} + \frac{1}{2}\,\sigma_W^2 W^2\,\frac{\partial^2 V}{\partial W^2} + \frac{1}{2}\,\sigma_w^2 w^2\,\frac{\partial^2 V}{\partial w^2} + \sigma_{Ww} Ww\,\frac{\partial^2 V}{\partial W\,\partial w}\right)\mathrm{d}t + \frac{AW(t)^{1-\gamma}w(t)^x}{1-\gamma}. \tag{A.8}$$

Using the definition of the current value function $V(W,w)$ we compute the partial derivatives as

$$V_W = AW^{-\gamma}w^x, \qquad V_w = \frac{xAW^{1-\gamma}w^{x-1}}{1-\gamma}, \tag{A.9}$$

$$V_{WW} = -\gamma AW^{-\gamma-1}w^x, \qquad V_{ww} = \frac{(x-1)xAW^{1-\gamma}w^{x-2}}{1-\gamma}, \tag{A.10}$$

$$V_{Ww} = xAW^{-\gamma}w^{x-1}, \qquad V_{wW} = xAW^{-\gamma}w^{x-1}. \tag{A.11}$$

Finally, we get

$$\mathscr{E}_t V(W(t+\mathrm{d}t), w(t+\mathrm{d}t)) = \left[\left((1-\gamma)\psi + \mu_w x - \frac{1}{2}\,(1-\gamma)\gamma\sigma_W^2 + \frac{1}{2}\,x(x-1)\sigma_w^2 + (1-\gamma)x\sigma_{Ww}\right)\mathrm{d}t + 1\right] \times A\,\frac{W^{1-\gamma}w^x}{1-\gamma}. \tag{A.12}$$

Substituting (A.12) into (A.3)

$$AW(t)^{1-\gamma}w(t)^x = \lim_{\mathrm{d}t\to 0^+}\max_{\{C,\ell,\omega\}}\left\{[C^\theta \ell^{1-\theta}]^\zeta\,\mathrm{d}t + \mathrm{e}^{-\rho\,\mathrm{d}t}\left[(1-\gamma)\left[\left(\psi + \frac{\mu_w x}{1-\gamma} - \frac{1}{2}\,\gamma\sigma_W^2 + \frac{1}{2}\,\sigma_w^2\,\frac{x(x-1)}{1-\gamma} + x\sigma_{Ww}\right)\mathrm{d}t + \frac{1}{1-\gamma}\right]AW^{1-\gamma}w^x\right]^{\zeta/(1-\gamma)}\right\}^{(1-\gamma)/\zeta}. \tag{A.13}$$

Define full (composite) consumption expressed in terms of consumer goods and leisure by

$$Z \equiv C + w\ell.$$

This yields the expenditure shares

$$C = \theta Z, \quad w\ell = (1-\theta)Z,$$

assuming that

$$Z = MW,$$

where M is a constant to be determined and combining them yields

$$C^{\theta}\ell^{1-\theta} = \theta^{\theta}(1-\theta)^{1-\theta}Zw^{-(1-\theta)}. \tag{A.14}$$

Recalling the mathematical properties that $\lim_{x\to 0}(1+x)^y = 1+xy$ and $\lim_{x\to 0} \mathrm{e}^x = 1+x$, we can write the following:

$$\begin{aligned} &A^{\zeta/(1-\gamma)}W(t)^{\zeta}w(t)^{-(1-\theta)\zeta} \\ &\quad = \max_{\{C,\ell,\omega\}} [\theta^{\theta}(1-\theta)^{1-\theta}]^{\zeta}M^{\zeta}W^{\zeta}w^{-(1-\theta)\zeta} + \left[\zeta\left(\psi + \frac{\mu_w x}{1-\gamma} - \frac{1}{2}\gamma\sigma_W^2\right.\right. \\ &\qquad \left.\left. + \frac{1}{2}\,\frac{x(x-1)\sigma_w^2}{1-\gamma} + x\sigma_{Ww}\right)\mathrm{d}t - \rho + 1\right] A^{\zeta/(1-\gamma)}W^{\zeta}w^{-(1-\theta)\zeta}. \end{aligned} \tag{A.15}$$

Notice that $x = -(1-\theta)(1-\gamma)$. Dividing by $A^{\zeta/(1-\gamma)}W(t)^{\zeta}w(t)^{-(1-\theta)\zeta}$ and subtracting 1 implies

$$\begin{aligned} 0 \equiv &\max_{\{C,\ell,\omega\}} ([\theta^{\theta}(1-\theta)^{1-\theta}]^{\zeta}M^{\zeta})/A^{\zeta/(1-\gamma)} \\ &+ \zeta\left(\psi - \mu_w(1-\theta) - \frac{1}{2}\gamma\sigma_W^2 + \frac{1}{2}\sigma_w^2(1-\theta)[(1-\theta)(1-\gamma)+1]\right. \\ &\left. - (1-\gamma)(1-\theta)\sigma_{Ww}\right) - \rho. \end{aligned} \tag{A.16}$$

First-order conditions are

$$\partial/\partial M : \zeta[\theta^{\theta}(1-\theta)^{1-\theta}]^{\zeta}M^{\zeta-1}/A^{\zeta/(1-\gamma)} + \zeta = 0,$$

which, after substituting and simplifying, yields:

$$\begin{aligned} M = \frac{1}{1-\zeta}&\left[\rho - \zeta\left(\beta - \mu_w(1-\theta) - \frac{1}{2}\gamma\sigma_W^2\right.\right. \\ &\left.\left. + \frac{1}{2}(1-\theta)[(1-\theta)(1-\gamma)+1]\sigma_w^2 - (1-\gamma)(1-\theta)\sigma_{Ww}\right)\right], \end{aligned} \tag{A.17}$$

where $\beta = r_S + \omega((1-\tau_K)r_K - r_S)$

$$\partial/\partial\omega : \zeta(\partial\psi/\partial\omega - (1/2)\gamma[\partial\sigma_W^2]/\partial\omega - (1-\gamma)(1-\theta)[\partial\sigma_{Ww}]/\partial\omega) = 0, \tag{A.18}$$

$$A = [\theta^\theta(1-\theta)^{1-\theta}]^{1-\gamma} M^{(\zeta-1)/\zeta(1-\gamma)}. \tag{A.19}$$

References

Asea, P.K., Turnovsky, S.J., 1998. Capital income taxation and risk taking in a small open economy. Journal of Public Economics 68, 55–90.

Barlevy, G., 2002. The cost of business cycles under endogenous growth. Working Paper, Northwestern University.

Basak, S., 1999. On the fluctuations in consumption and market returns in the presence of labor and human capital: an equilibrium analysis. Journal of Economic Dynamics and Control 23, 1029–1064.

Benavie, A., Grinols, E., Turnovsky, S.J., 1996. Adjustment costs and investment in a stochastic endogenous growth model. Journal of Monetary Economics 38, 77–100.

Blackburn, K., 1999. Can stabilisation policy reduce long-run growth. Economic Journal 109, 67–77.

Bodie, Z., Merton, R.C., Samuelson, W.F., 1992a. Labour supply flexibility and portfolio choice in a life cycle model. Working Paper 3954, National Bureau of Economic Research.

Bodie, Z., Merton, R.C., Samuelson, W.F., 1992b. Labour supply flexibility and portfolio choice in a life cycle model. Journal of Economic Dynamics and Control 16, 427–449.

Campbell, J., Mankiw, G., 1989. Consumption, income and interest rate: reinterpreting the time series evidence. In: Blanchard, O., Fischer, S. (Eds.), NBER Macroeconomic Annual, Vol. 4. MIT Press, Cambridge, MA, pp. 185–216.

Corsetti, G., 1997. A portfolio approach to endogenous growth: equilibrium and optimal policy. Journal of Economic Dynamics and Control 21, 1627–1644.

Davis, M.H.A., Norman, A.R., 1990. Portfolio selection with transaction costs. Mathematics of Operational Research 15, 676–713.

Dolmas, J., 1998. Risk preferences and the welfare cost of business cycles. Review of Economic Dynamics 1, 646–676.

Duffie, D., Epstein, L.G., 1992a. Asset pricing with stochastic differential utility. Review of Financial Studies 5, 411–436.

Duffie, D., Epstein, L.G., 1992b. Stochastic differential utility. Econometrica 60, 353–394.

Duffie, D., Fleming, W., Soner, H.M., Zariphopoulou, T., 1997. Hedging in incomplete markets with hara utility. Journal of Economic Dynamics and Control 21, 753–782.

Eaton, J., 1981. Fiscal policy, inflation and the accumulation of risky capital. Review of Economic Studies 98, 435–445.

Elmendorf, D.W., Kimball, M.S., 2000. Taxation of labor income and the demand for risky assets. International Economic Review 41, 801–832.

Epaulard, A., Pommeret, A., 2003. Recursive utility, growth and the welfare cost of volatility. Review of Economic Dynamics 6, 672–684.

Epstein, L.G., Zin, S.E., 1991a. Substitution, risk aversion, and the temporal behaviour of consumption and asset returns: an empirical analysis. Journal of Political Economy 99, 263–286.

Epstein, L.G., Zin, S.E., 1991b. Substitution, risk aversion, and the temporal behaviour of consumption and asset returns. Econometrica 57, 937–969.

Evans, L., Kenc, T., 2003. Welfare cost of monetary and fiscal policy shocks. Macroeconomic Dynamics 7, 212–238.

Ghosh, A., Pesenti, P., 1994. International portfolio diversification, non-tradable human wealth and consumption growth: some puzzles and interpretations. Working Paper 60, Innocenzo Gasparini Institute for Economic Research.

Grinols, E., 1996. The link between domestic investment and domestic savings in open economies: evidence from balanced stochastic growth. Review of International Economics 38, 1–30.
Grinols, E., Turnovsky, S.J., 1993. Risk, financial market, and macroeconomic equilibrium. Journal of Economic Dynamics and Control 17, 1–36.
Grinols, E., Turnovsky, S.J., 1994. Exchange rate determination and asset prices in a stochastic small open economy. Journal of International Economics 36, 75–97.
Grinols, E., Turnovsky, S.J., 1998. Consequences of debt policy in a stochastically growing monetary economy. International Economic Review 39, 495–521.
Hall, R.E., 1988. Intertemporal substitution in consumption. Journal of Political Economy 96, 339–357.
Kandel, S., Stambough, R.F., 1991. Asset returns and intertemporal preferences. Journal of Monetary Economics 27, 39–71.
Kihlstrom, R.E., Mirman, L.J., 1981. Constant, increasing and decreasing risk aversion with many commodities. Review of Economic Studies 48, 271–280.
Koo, H.K., 1998. Consumption and porfolio selection with labour income: a continuous time approach. Mathematical Finance 8, 76–101.
Kormendi, R., Meguire, P., 1985. Macroeconomic determinants of growth: cross-country evidence. Journal of Monetary Economics 16, 141–163.
Lucas, R.E.J., 1987. Models of Business Cycles. Basil Blackwell, Oxford.
Martin, P., Rogers, C.A., 2000. Long term growth and short-term economic instability. European Economic Review 44, 359–381.
Merton, R.C., 1969. Lifetime portfolio selection under uncertainty: the continuous-time case. Review of Economics and Statistics 51, 247–257.
Merton, R.C., 1977. On the pricing of contingent claims and the Modigliani–Miller theorem. Journal of Financial Economics 5, 241–249.
Obstfeld, M., 1994a. Evaluating risky consumption paths: the role of intertemporal substitution. European Economic Review 87, 613–622.
Obstfeld, M., 1994b. Risk-taking, global diversification, and growth. American Economic Review 84, 1310–1329.
Ramey, G., Ramey, V.A., 1995. Cross-country evidence on the link between volatility and growth. American Economic Review 85, 1138–1152.
Rebelo, S., 1991. Long-run policy analysis and long-run growth. Journal of Political Economy 99, 500–521.
Romer, P.M., 1986. Increasing returns and long-run growth. Journal of Political Economy 94, 1002–1037.
Saint-Paul, G., 1995. Productivity growth and the structure of the business cycle. European Economic Review 37, 861–890.
Sandmo, A., 1989. Differential taxation and the encouragement of risk-taking. Economics Letters 31, 55–59.
Smith, W.T., 1996a. Feasibility and transversality conditions for models of portfolio choice with non-expected utility in continuous time. Economics Letters 53, 123–131.
Smith, W.T., 1996b. Taxes, uncertainty, and long-run growth. European Economic Review 40, 1647–1664.
Svensson, L.E.O., 1989. Portfolio choice with non-expected utility in continuous time. Economics Letters 30, 313–317.
Tallarini, T.D., 2000. Risk-sensitive real business cycles. Journal of Monetary Economics 45, 507–532.
Tallarini, T.D.J., 1996. Risk-sensitive real business cycles. Ph.D. Thesis, University of Chicago.
Turnovsky, S.J., 1993. Macroeconomic policies, growth, and welfare in a stochastic economy. International Economic Review 34, 953–981.
Turnovsky, S.J., 1995. Methods of Macroeconomic Dynamics. MIT Press, Cambridge, MA.
Turnovsky, S.J., 2000. Government policy in a stochastic growth model with elastic labor supply. Journal of Public Economic Theory 2, 389–433.
Turnovsky, S.J., Chattopadhyay, P., 2003. Volatility and growth in developing economies: some numerical results and empirical evidence. Journal of International Economics 52, 267–295.
Turnovsky, S.J., Grinols, E., 1996. Optimal government finance policy and exchange rate management in a stochastically growing economy. Journal of International Money and Finance 15, 687–716.
Weil, P., 1990. Non-expected utility in macroeconomics. Quarterly Journal of Economics 105, 29–42.