J Econ Inequal (2013) 11:517–533
DOI 10.1007/s10888-012-9232-5

# Wages of regular and irregular workers, the price of education, and income inequality

**Hideki Nakamura**

Received: 18 August 2010 / Accepted: 13 September 2012 / Published online: 29 September 2012


**Abstract** In this paper, we develop a model characterized by skill-biased technological change and increasing costs of education to investigate income inequality. Irregular workers cannot escape poverty by commencing investment in education because wage inequality between regular and irregular workers widens and the price of education increases with the average level of education. Moreover, if the productivity of elementary education is low relative to that of higher education, middle-income individuals are eventually unable to pursue higher education because the threshold for education expenditure rises with the price of education. Thus, income inequality may widen, even among regular workers.

**Keywords** Regular and irregular workers • Skill-biased technological change • Price of education • Income inequality

**JEL Classification** I20 • O11 • O15 • O30

## 1 Introduction

In this paper, we explore the income dynamics of three income groups in a model characterized by increasing costs of education and skill-biased technological change. Low-income individuals that are also irregular workers cannot escape poverty by commencing investment in education because wage inequality between regular and irregular workers widens and the price of education increases with the average level of education. If the productivity of elementary education is low relative to that of higher education, the increasing costs of education would hinder middle-income

H. Nakamura (✉)
Faculty of Economics, Osaka City University, 3-3-138 Sugimoto, Sumiyoshi,
Osaka 558-8585, Japan
e-mail: hnakamur@econ.osaka-cu.ac.jp

individuals from pursuing higher education because the threshold for education expenditure rises with the price of education.

The reason for studying the effects of skill-biased technological change and increasing costs of education on income inequality is the extent of the widening income inequality currently confronting Japan. A large increase in income inequality has recently been identified.[1] The depth of the Japanese middle-income class has also been decreasing, while the relative poverty ratio has been increasing. The observed increase in income inequality could be closely related to the recent increase in the employment of irregular workers (part-time and temporary employees). For the most part, the wage gap between regular and irregular workers in Japan has been gradually widening.[2] Technological progress could be an important cause of the increasing wage gap between regular and irregular workers because of skill-biased technological change (see Koba [6] and Yamaguchi [24]).

The intergenerational transmission of earnings mobility through education investment could also be crucial in the increase in income inequality in Japan. All other things being equal, educational background could affect the relative employment of regular and irregular workers. Furthermore, a growing wages gap has been found in Japan, even among regular workers, according to differences in sex, firm size, and education, including university quality (see Uni [23], Tachibanaki and Yagi [20], and Shigeno and Matsuura [17]). However, at the same time, the cost of education has been increasing.[3] The children of upper-class or high-income parents could then have an advantage when applying to well-known universities (see Kobayashi [7], Tachibanaki and Matsuura [19], and Tachibanaki and Yagi [20]).

This paper theoretically explores how the employment system and the increasing costs of education affect income inequality. This paper also concerns the reasons for the decline in the population share of the Japanese middle class. Even though we refer to low-, middle-, and high-income groups, these groups are homogeneous except for the initial level of education of their members. We posit that individuals in the low-income group, unlike the middle- and high-income groups, cannot initially receive higher education. The initial education level of the high-income group is higher than that of the middle-income group. Individuals with no higher education then become irregular workers, whereas individuals with higher education become regular workers. Thus, regular workers have more labor efficiency units than irregular workers, due to earlier investment in education. We consider two education sectors (elementary and higher education) and a goods sector.

We make three major assumptions. First, we consider skill-biased technological change with the assumption that an increase in the average education level increases the rate of technological progress. The assumption of skill-biased technological change enables us to consider the sort of wage inequality between regular and

[1] See Tachibanaki [18]. For example, Uni [23] found that the increasing wage inequality has caused the recent increase in income inequality. Abe [1] also identified increases in the poverty ratio within age groups.

[2] See Ohta [15]. The employment of irregular workers has greatly increased in Japan since the latter half of the 1990s, with irregular workers representing 34.9 % of all workers in 2010. The average lifetime wages of temporary staff and part-timers, respectively, were only about one half and one quarter of those of regular workers in 2006.

[3] In 2012, the average ratio of education costs to income was 37.7 %.

irregular workers observed in Japan. Second, while we consider altruistic motives for education investment, we allow zero expenditure for higher education. This implies that children receive higher education only if their parents can afford it. This does not necessarily imply convex education expenditure with respect to the parent's education level, because elementary education provides the basic income. Finally, we consider the price of higher education. We assume that diminishing returns exist in the human capital stock of teachers and the number of teachers per student, and hence that the price of education increases with the average education level.

In our model, the threshold for the commencement of education investment rises more rapidly than the income level of low-income individuals because of the widening wage inequality and the increasing costs of education. Thus, it becomes more difficult for low-income individuals to become regular workers when the average level of education increases. However, the education level dynamics of middle- and high-income individuals are unaffected by the wage rate of regular workers because higher education teachers are also regular workers. If the productivity of elementary education is low relative to that of higher education, then middle-income individuals are eventually unable to pursue higher education.[4] The low productivity of elementary education implies convex education expenditure with respect to the education level of parents, while the threshold for education expenditure rises with the price of education. In contrast, the relatively higher productivity of higher education implies a large demand for education among high-income individuals. As the price of education rises more rapidly than the income of middle-income individuals, they are eventually unable to keep pace with the increase in the price of education.

Some studies have investigated the connection between educational attainment and wage inequality between skilled and unskilled workers. For example, Maoz and Moav [8] showed a positive correlation between the intergenerational mobility and wage equality observed in developed economies. By considering ability-biased technological transitions, Galor and Moav [2] illustrated that the evolutions of technology, educational attainment, and wage inequality were consistent with the observed pattern in the United States. This paper takes into account the wage system and the increasing costs of education commonly observed in Japan. We identify the different effects of the wage system and increasing costs of education on three income groups.[5]

This paper also relates to a literature that explores the effects of public and private education systems on income inequality and growth (see, for example, Glomm and Ravikumar [5] and Takii and Tanaka [21]). We consider public elementary education and private higher education because almost all children in Japan attend public elementary schools and because we wish to take into account the increasing costs of

[4]The Japanese government recently implemented a less strenuous form of elementary education known as *Yutori Kyoiku*, which reduced the number of school days and the amount of coursework. This reform was not without criticism because of the argument that the quality of public education has correspondingly deteriorated. See, for example, Nishimura [14].

[5]Nakajima and Nakamura [11, 12] also examined the effect of the price of education on income inequality. However, they did not take into account the wages of regular and irregular workers in either study, nor did they consider the three income groups analyzed in this paper. Furthermore, the result in this paper does not depend on tax systems. However, the result in Nakajima and Nakamura [12] depends on lump-sum taxes.

higher education.[6] We find that the productivity gap between elementary and higher education plays a crucial role in the determination of income inequality between the middle- and high-income groups.

The remainder of the paper is organized as follows. Section 2 details the model used. Using this model, Section 3 examines whether low-income individuals can escape poverty and become regular workers. In this section, we also examine the education dynamics for middle- and high-income individuals. We conclude our paper in Section 4 with a brief summary.

## 2 Model

Our model comprises a small, open, overlapping-generations economy. The world interest rate is assumed to be stationary. Individuals live three periods. In the first period, individuals equally receive elementary education. If parents decide to invest in higher education for their children, they can receive it too in the first period. In the second period, individuals with no higher education work as irregular workers, whereas individuals with higher education work as regular workers. In this period, individuals decide their own consumption and savings levels as well as the level of education for their children. In the third period, individuals consume their incomes.

We denote the high-, middle-, and low-income groups as $h$, $m$, and $l$, respectively. The initial education levels of these groups are denoted $e_{\mathrm{h},-1}$, $e_{\mathrm{m}.-1}$, and $e_{\mathrm{l},-1}$, respectively. We assume that $e_{\mathrm{h},-1} > e_{\mathrm{m},-1} > 0$ and $e_{\mathrm{l},-1} = 0$. The population of each generation is assumed to be constant at $L$. We also assume that the initial numbers of the high-, middle-, and low-income groups are, respectively, $\lambda_{\mathrm{h},-1}L$, $\lambda_{\mathrm{m},-1}L$, and $(1 - \lambda_{\mathrm{h},-1} - \lambda_{\mathrm{m},-1})L$.

### 2.1 Individuals

We first describe the relationship between education and human capital formation. For simplicity, we assume that the human capital of an individual is of the following linear type:

$$h(e_{\mathrm{it}}) = \bar{e}(\delta) + \gamma e_{\mathrm{it}}, \tag{1}$$

where $i = l, m$, and $h$. We assume $0 < \gamma$, $0 < \delta < 1$, $\bar{e}'(\delta) > 0$, and $\bar{e}''(\delta) < 0$. $h(e_{\mathrm{it}})$ is the human capital stock level, $\bar{e}(\delta)$ is the elementary education level, $\delta$ is the income tax rate, and $e_{\mathrm{it}}$ is the higher education level.

Elementary education provides the basis of the human capital stock. All individuals in the high-, middle-, and low-income groups receive the same level of compulsory elementary education. In contrast, individuals can receive different levels of higher education because it is elective.[7]

[6] In 2011, public elementary school students accounted for 98.6 % of all elementary school students in Japan. The corresponding share for junior high school students was 93.2 %.

[7] In Japan, compulsory education encompasses elementary and junior high school. Almost all students can also attend high school because of government aid. Thus, in our model, higher education corresponds to only university education and supplementary private education.

Next, we consider the utility maximization problem of an individual born in period $t-1$. We assume that parental preferences depend on consumption in periods $t$ and $t+1$ and on the education level of their children. Parents must pay income taxes used for elementary education. Parents' disposable income is used for consumption in period $t$, for education expenditure for their children, and for savings. The utility maximization problem of an individual born in period $t-1$ is as follows:

$$\max_{c^y_{\text{it}}, e_{\text{it}}, c^o_{\text{it}+1}} \beta_1 \ln c^y_{\text{it}} + \beta_2 \ln(\theta + e_{\text{it}}) + (1-\beta_1-\beta_2) \ln c^o_{\text{it}+1}, \tag{2}$$

$$s.t. \qquad (1-\delta) I_{\text{it}} = c^y_{\text{it}} + p_{\text{t}} e_{\text{it}} + \frac{c^o_{\text{it}+1}}{R}, \tag{3}$$

where $i = l, m$, and $h$. We assume $0 < \beta_1$, $0 < \beta_2$, $\beta_1 + \beta_2 < 1$, and $0 < \theta$. $I_{\text{it}}$ is the income level, $c^y_{\text{it}}$ and $c^o_{\text{it}+1}$ are the consumption levels in the second and third periods, respectively, $R$ is the gross interest rate, and $p_{\text{t}}$ is the price of education.

Parents obtain a higher utility level as the education level of their children increases.[8] Because even low-income individuals receive elementary education, which forms the basis of human capital stock, we allow a zero education expenditure by assuming $\theta$.[9]

The first-order conditions of the utility maximization problem imply that the expenditure for education is a convex function with respect to the income level. When the condition

$$\beta_2(1-\delta) I_{\text{it}} - (1-\beta_2)\theta p_{\text{t}} \leq 0, \tag{4}$$

holds, there is no education expenditure for the children. This condition holds easily when the income level is low and the price of education is high. Thus, the price of education may act as a threshold for educational expenditure.

Using Eqs. 2 and 3, we have the following:

$$c^y_{\text{it}} = \frac{\beta_1}{1-\beta_2}(1-\delta) I_{\text{it}}, \tag{5}$$

$$p_{\text{t}} e_{\text{it}} = 0, \tag{6}$$

$$c^o_{\text{it}+1} = R \frac{1-\beta_1-\beta_2}{1-\beta_2}(1-\delta) I_{\text{it}}. \tag{7}$$

Individuals expend their disposable income on consumption and savings. The consumption levels in periods $t$ and $t+1$ increase with the income level.

When the income level is high enough to satisfy

$$\beta_2(1-\delta) I_{\text{it}} - (1-\beta_2)\theta p_{\text{t}} > 0, \tag{8}$$

[8] Alternatively, we could assume that the utility function depends on the human capital stock level of children. Such an assumption implies substitutable relationships between elementary and higher education in the utility function. However, in Japan, wealthy parents can usually afford higher education for their children, regardless of their elementary education.

[9] Considering altruistic bequest motives, Moav [10], Galor and Moav [3], and Nakajima and Nakamura [11] also allowed for zero education expenditure. Galor and Zeira [4] considered indivisibilities in education investment and showed persistent inequality.

it becomes possible for parents to invest in education for their children. Using Eqs. 2 and 3, the first-order conditions of the utility maximization problem are as follows:

$$c_{\mathrm{it}}^{y} = \beta_1[(1-\delta)I_{\mathrm{it}} + \theta p_{\mathrm{t}}], \tag{9}$$

$$p_{\mathrm{t}}e_{\mathrm{it}} = \beta_2(1-\delta)I_{\mathrm{it}} - (1-\beta_2)\theta p_{\mathrm{t}}, \tag{10}$$

$$c_{\mathrm{it+1}}^{o} = R(1-\beta_1-\beta_2)[(1-\delta)I_{\mathrm{it}} + \theta p_{\mathrm{t}}]. \tag{11}$$

The income level and the price of education determine the demand for education. Education expenditure increases with a rise in the level of income. However, its expenditure decreases with a rise in the price of education. The consumption levels in periods $t$ and $t+1$ increase with the rise in the income level.

### 2.2 Education sectors

We first describe the elementary education sector. The government employs teachers and other staff to supply education. We assume that high-, middle-, and low-income individuals are employed according to their proportions in the population; that is, the numbers of high-, middle-, and low-income individuals in period $t$ are $\delta\lambda_{\mathrm{ht-1}}L$, $\delta\lambda_{\mathrm{mt-1}}L$, and $\delta\lambda_{\mathrm{lt-1}}L$, respectively. While $\delta$ represents the income tax rate, it also represents the share of public employees in the population. The government funds elementary education by collecting income taxes to pay the wages of public employees.

Although an increase in the tax rate decreases disposable income directly, it also raises the productivity of elementary education. The quantity represented by $(1-\delta)\bar{e}(\delta)$ becomes part of disposable income. Thus, we assume that the government determines the tax rate to maximize the following term:

$$\max_{\delta}(1-\delta)\bar{e}(\delta). \tag{12}$$

The tax rate is set to

$$\bar{e}'(\delta)(1-\delta) = \bar{e}(\delta). \tag{13}$$

High productivity of elementary education implies a high basic income.

Next, we consider the institution of higher education. We assume that only individuals with the highest education levels can become teachers. Thus, teachers are high-income individuals and regular workers. When the human capital of a teacher is high and the number of teachers per student is large, education progresses well.[10] However, there are diminishing returns for both the human capital stock of teachers and the number of teachers per student. Thus, the average education level per student is assumed to be subject to the following production function:

$$e_{\mathrm{at}} = (h(e_{\mathrm{ht-1}})\tau_{\mathrm{t}})^{\alpha}, \tag{14}$$

[10] By considering a model in which the outputs partially depend on customers as inputs, Rothschild and White [16] showed that the prices charged to customers for the net amount they obtain from the firm are competitive and support an efficient allocation. Considering teacher quality and class size, Tamura [22] examined the condition for convergence.

where we assume $0 < \alpha < 1$. $e_{\mathrm{at}}$ is the average education level per student received in period $t$, $\tau_{\mathrm{t}} \equiv L_{\mathrm{t}}^{T}/L_{\mathrm{t}}^{S}$ is the number of teachers per student, and $L_{\mathrm{t}}^{T}$ and $L_{\mathrm{t}}^{S}$ are the numbers of teachers and students, respectively, in period $t$. When middle- and high-income individuals receive higher education, the average education level is given by $e_{\mathrm{at}} = \lambda_{\mathrm{ht}} e_{\mathrm{ht}} + \lambda_{\mathrm{mt}} e_{\mathrm{mt}}$.

Tuition revenue pays the wages of teachers. The balanced budget for this is written as

$$p_{\mathrm{t}} e_{\mathrm{at}} L_{\mathrm{t}}^{S} = w_{\mathrm{rt}} h(e_{\mathrm{ht-1}}) L_{\mathrm{t}}^{T}, \tag{15}$$

where $w_{\mathrm{rt}}$ is the wage rate of regular workers.

The left-hand side of Eq. 15 is tuition revenue, and the right-hand side is the wage cost. We determine the price of education using a zero-profit condition. Using Eqs. 14 and 15, the price of education can be represented as

$$p_{\mathrm{t}} = w_{\mathrm{rt}} {e_{\mathrm{at}}}^{(1-\alpha)/\alpha}. \tag{16}$$

A low $\alpha$ value implies that the price increases greatly with a rise in the average education level. Furthermore, the price of education and the wage rate of regular workers are positively related.

### 2.3 Goods sector

Production takes place according to a neoclassical constant returns-to-scale production technology that is subject to technological progress. The production function is

$$Y_{\mathrm{t}} = F(K_{\mathrm{t}}, A_{\mathrm{t}} H_{\mathrm{t}}) \equiv A_{\mathrm{t}} H_{\mathrm{t}} f(k_{\mathrm{t}}), \tag{17}$$

where $k_{\mathrm{t}} \equiv \frac{K_{\mathrm{t}}}{A_{\mathrm{t}} H_{\mathrm{t}}}$. $K_{\mathrm{t}}$ is the quantity of physical capital inputted in period $t$, $H_{\mathrm{t}}$ is the quantity of efficiency units of the composite labor inputted in period $t$, and $A_{\mathrm{t}}$ is the technological level in period $t$.

By assuming skill-biased technological progress, we represent composite labor as a weighted sum of the quantities of the efficiency units of regular workers and irregular workers:[11]

$$H_{\mathrm{t}} = L_{\mathrm{rt}} + (1 - o g_{\mathrm{t}}) L_{\mathrm{nrt}}, \tag{18}$$

where we assume $0 < o < 1$ and $0 < g_{\mathrm{t}} < 1$. $g_{\mathrm{t}} \equiv \frac{A_{\mathrm{t}} - A_{t-1}}{A_{t-1}}$ is the rate of technological progress. $L_{\mathrm{rt}}$ and $L_{\mathrm{nrt}}$ are the quantities of the efficiency units of regular and irregular workers inputted in period $t$, respectively.

High technological progress implies a low quantity of irregular workers in terms of efficiency units. We assume that technological progress is an increasing function of the average higher education level of workers:

$$g_{\mathrm{t}} = g(e_{\mathrm{at-1}}), \tag{19}$$

[11] We consider skill-biased technological change because of the empirical facts found by Koba [6] and Yamaguchi [24]. Equation 18 is essentially the same as the assumption of Galor and Moav [2]. Maoz and Moav [9] created skill-biased growth without assuming a skill-biased technological change.

where we assume $g'(e_{\mathrm{at}-1}) > 0$ and $g''(e_{\mathrm{at}-1}) < 0$. The initial technological level is historically given.

We assume that physical capital completely depreciates in one period. Using Eqs. 17, 18, and 19, the first-order conditions of the cost minimization problem are then as follows:

$$R = f'(k), \tag{20}$$

$$w_{\mathrm{rt}} = A_{\mathrm{t}}[f(k) - f'(k)k], \tag{21}$$

$$w_{\mathrm{nrt}} = A_{\mathrm{t}}(1 - og(e_{\mathrm{at}-1}))[f(k) - f'(k)k], \tag{22}$$

where $w_{\mathrm{nrt}}$ is the wage rate of irregular workers.

The capital per labor in efficiency units is constant because of the assumption of stationary world interest rate. Thus, the wage rates of regular and irregular workers change according to level of technology and the rate of technological progress. Furthermore, there exists a wage gap between regular and irregular workers because of skill-biased technological progress. The wage gap between these types of workers increases with the average education level.

Labor market equilibrium conditions imply

$$L_{\mathrm{rt}} = (1 - \delta)\lambda_{\mathrm{mt}-1}h(e_{\mathrm{mt}-1})L + [(1 - \delta)\lambda_{\mathrm{ht}-1}L - L_{\mathrm{t}}^{T}]h(e_{\mathrm{ht}-1}), \tag{23}$$

$$L_{\mathrm{nrt}} = (1 - \delta)(1 - \lambda_{\mathrm{ht}-1} - \lambda_{\mathrm{mt}-1})\bar{e}(\delta)L. \tag{24}$$

## 3 Dynamics of the education levels of the low-, middle-, and high-income groups

### 3.1 Can the low-income group escape poverty?

We first examine the condition under which low-income individuals can begin to receive higher education. The income level of low-income individuals is represented by $I_{\mathrm{pt}} = \bar{e}(\delta)w_{\mathrm{nrt}}$. We assume that low-income individuals who are also irregular workers cannot initially afford higher education for their children:

$$\beta_2(1 - \delta)\bar{e}(\delta)w_{\mathrm{nr0}} < (1 - \beta_2)\theta p_0. \tag{A1}$$

We then have the following proposition.

**Proposition 1** *Suppose that* $e_{\mathrm{l},-1} = 0$*. Under Assumption A1, low-income individuals can never begin to invest in education as long as the average education level increases.*

*Proof* Using Eqs. 4, 16, 21, and 22, low-income individuals cannot commence education investment as long as the following condition holds:

$$\frac{\beta_2(1 - \delta)\bar{e}(\delta)}{(1 - \beta_2)\theta} < \frac{1}{1 - og(e_{\mathrm{at}-1})}e_{\mathrm{at}}{}^{(1-\alpha)/\alpha}. \tag{25}$$

Evaluating Eq. 25 at $t = 0$, we have Eq. A1. The right-hand side of Eq. 25 increases with increases in $e_{\mathrm{at}-1}$ and $e_{\mathrm{at}}$. Thus, Eq. A1 implies Eq. 25. □

The price of education increases with the average education level of students, represented by $e_{\mathrm{at}}$. The wage gap between regular and irregular workers increases with the average education level of workers, represented by $e_{\mathrm{at}-1}$. Because the threshold for the commencement of education investment rises with the average education levels, low-income individuals never escape poverty by receiving education because of both technological progress and the increasing costs of education.

How does the average education level then evolve? The income levels of high- and middle-income individuals are represented by $I_{\mathrm{ht}} = h(e_{\mathrm{ht}-1})w_{\mathrm{rt}}$ and $I_{\mathrm{mt}} = h(e_{\mathrm{mt}-1})w_{\mathrm{rt}}$, respectively. The supply condition for education in Eq. 16 and the demand condition in Eq. 10 imply the dynamics of their average education level:

$$u(e_{\mathrm{at}}) = v(e_{\mathrm{at}-1}), \tag{26}$$

where

$$u(e_{\mathrm{at}}) \equiv {e_{\mathrm{at}}}^{1/\alpha} + (1-\beta_2)\theta(\lambda_{\mathrm{ht}-1}+\lambda_{\mathrm{mt}-1}){e_{\mathrm{at}}}^{(1-\alpha)/\alpha},$$

$$v(e_{\mathrm{at}-1}) \equiv \beta_2(1-\delta)(\lambda_{\mathrm{mt}-1}+\lambda_{\mathrm{ht}-1})\bar{e}(\delta) + \beta_2(1-\delta)\gamma e_{\mathrm{at}-1}.$$

Using Eqs. 10, 14, and 16, the number of teachers per student is represented as

$$\begin{aligned} &h(e_{\mathrm{ht}-1})\tau_{\mathrm{t}} + (1-\beta_2)\theta(\lambda_{\mathrm{mt}-1}+\lambda_{\mathrm{ht}-1})(h(e_{\mathrm{ht}-1})\tau_{\mathrm{t}})^{1-\alpha} \\ &\quad - \beta_2(1-\delta)[(\lambda_{\mathrm{mt}-1}+\lambda_{\mathrm{ht}-1})\bar{e}(\delta) + \gamma e_{\mathrm{at}-1}] = 0. \end{aligned} \tag{27}$$

We discuss the case in which $\alpha < 1/2$ and there is a steady state.[12] Equation 26 can be rewritten as follows:

$$e_{\mathrm{at}} = \Gamma(e_{\mathrm{at}-1}), \tag{28}$$

where $\Gamma(0) > 0$, $\Gamma'(e_{\mathrm{at}-1}) > 0$, $\Gamma''(e_{\mathrm{at}-1}) < 0$, and $\lim_{e_{\mathrm{at}-1}\to\infty} \Gamma'(e_{\mathrm{at}-1}) = 0$.

Because both teachers and the parents that pay for higher education for their children are regular workers, their wage rates do not affect the dynamics of the education level. Figure 1 depicts the dynamics. The average education level in period $t-1$ determines the average education level in period $t$. Therefore, a low level of income implies a low level of demand for education, so the price of education is also low. Starting from $e_{\mathrm{a},-1}$ in Fig. 1, the average education level increases monotonically, regardless of the education level of the middle- and high-income individuals. The number of teachers also increases because of the increase in the average education level. The price of education also continues to increase. The average education level converges to ${e_{\mathrm{a}}}^*$.

Using Eqs. 13 and 26, the comparative statistics are as follows:

$$\frac{\partial {e_{\mathrm{a}}}^*}{\partial \alpha} > 0,\ \frac{\partial {e_{\mathrm{a}}}^*}{\partial \gamma} > 0,\ \frac{\partial {e_{\mathrm{a}}}^*}{\partial \beta_2} > 0,\ \frac{\partial {e_{\mathrm{a}}}^*}{\partial \theta} < 0,\ \frac{\partial {e_{\mathrm{a}}}^*}{\partial \bar{e}(\delta)} > 0,\ \frac{\partial {e_{\mathrm{a}}}^*}{\partial \delta} < 0.$$

[12] Multiple steady states can emerge when $\alpha > 1/2$. Detailed results with the existence of multiple steady states are available from the author upon request.

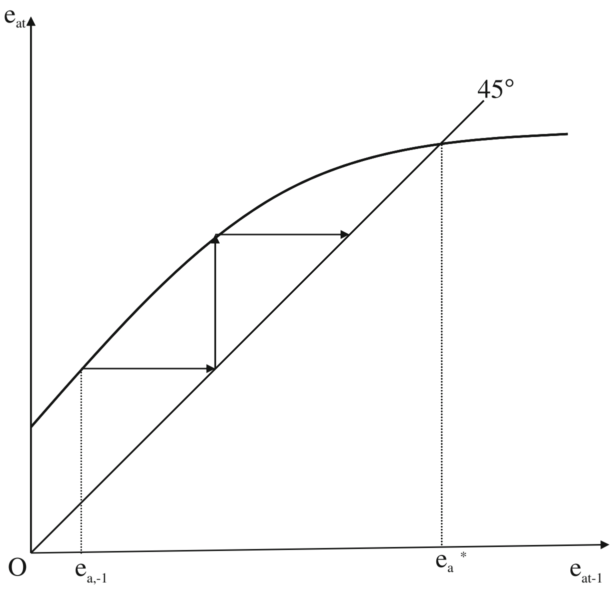


**Fig. 1** Dynamics of the average education level represented as Eq. 28

Assuming that $e_{\mathrm{a}}{}^{*} > 1$, an increase in the efficiency of the education institution increases the steady-state education level. Furthermore, an increase in the productivity of higher education, the productivity of elementary education, and the weight of child's education level in the utility function all increase the level of education. In contrast, a rise in the threshold for education expenditure, represented by $\theta$, decreases the education level. An increase in the income tax rate also decreases the level of education. Although the income tax rate maximizing the basic income level arises from elementary education, an increase in the income tax rate decreases the level of income arising from higher education.

3.2 Can the middle-income group catch up with the high-income group?

Using Eqs. 1, 10, and 16, the education dynamics for high- and middle-income individuals are respectively as follows:

$$e_{\mathrm{ht}} = \frac{\beta_2(1-\delta)\gamma e_{\mathrm{ht}-1} + b\,(e_{\mathrm{at}})}{e_{\mathrm{at}}^{(1-\alpha)/\alpha}}, \tag{29}$$

$$e_{\mathrm{mt}} = \frac{\beta_2(1-\delta)\gamma e_{\mathrm{mt}-1} + b\,(e_{\mathrm{at}})}{e_{\mathrm{at}}^{(1-\alpha)/\alpha}}, \tag{30}$$

where

$$b\,(e_{\mathrm{at}}) \equiv \beta_2(1-\delta)\bar{e}(\delta) - (1-\beta_2)\theta e_{\mathrm{at}}{}^{(1-\alpha)/\alpha}.$$

Individuals in the high- and middle-income groups face the same intercept point for education expenditure, represented by $b\,(e_{\mathrm{at}})$. The value of $b\,(e_{\mathrm{at}})$ decreases with an increase in the price of education. The value of $b(e_{at})$ becomes negative when the productivity of elementary education is low relative to the price of education. A negative $b(e_{at})$ value implies that education expenditure is a convex function with respect to the education level of the parents.

The dynamics of the education levels of high- and middle-income individuals crucially depend on the sign of $b(e_a{}^*)$. We have $e_a{}^* = (\lambda_h + \lambda_m)e^*$. We define $\Omega(e_a) \equiv u(e_a) - v(e_a)$ to examine its sign. Obviously, $\Omega(e_a{}^*) = 0$. We also define the education level as $\tilde{e}_a$ to make the intercept point for education expenditure equal zero (i.e., $b(\tilde{e}_a) = 0$). The price of education evaluated using $\tilde{e}_a$ corresponds to the threshold for education expenditure with no higher education. Evaluating $\Omega(e_a)$ using $\tilde{e}_a$, we have

$$b(e_a{}^*) \underset{<}{\overset{>}{=}} 0 \quad \Leftrightarrow \quad \Omega(\tilde{e}_a) = \tilde{e}_a \beta_2 (1-\delta)[\frac{\bar{e}(\delta)}{(1-\beta_2)\theta} - \gamma] \underset{<}{\overset{>}{=}} 0. \tag{31}$$

If the productivity of elementary education is low relative to that of higher education, we obtain $\Omega(\tilde{e}_a) < 0$. Because $\Omega(e_a)$ is an increasing function with respect to $e_a$, $\tilde{e}_a < e_a{}^*$ holds. When the productivity of higher education is high, the effect of the education level on the income level becomes strong. Thus, higher productivity in higher education increases the demand for education. Conversely, lower productivity in elementary education implies a low intercept point for education expenditure. The inequality $\tilde{e}_a < e_a{}^*$ implies $b(e_a{}^*) < 0$ because $b(e_a)$ is a decreasing function with respect to $e_a$. We then have a convex education expenditure function in the steady state. Thus, the disposable incomes of parents with a low level of higher education may be inadequate for education expenditure.

We assume the following inequality:

$$\frac{\bar{e}(\delta)}{(1-\beta_2)\theta} < \gamma. \tag{A2}$$

This implies the case in which the productivity of elementary education is low relative to that of higher education. We then obtain the following proposition.

**Proposition 2** *Suppose that* $e_{h,-1} > e_{m,-1} > 0$. *Under Assumption A2, middle-income individuals cannot continue receiving higher education.*

*Proof* As shown in Appendix B, the steady state is a saddle point. Thus, the gap in the level of education between high- and middle-income individuals widens and middle-income individuals cannot continue receiving higher education. □

Let us consider the dynamics for which Assumption A2 holds. Figure 2 provides the phase diagram.[13] Point $B$ is a saddle point. The saddle path is always on the 45-degree line regardless of the numbers of high- and middle-income individuals because of the homogeneity among individuals, except for their initial education levels. Consider the case in which the starting point is represented as $C$. The education levels of both the middle- and high-income groups are positive and the difference

[13]See Appendix A. The phase diagram is applicable as long as both the high- and middle-income individuals receive higher education.

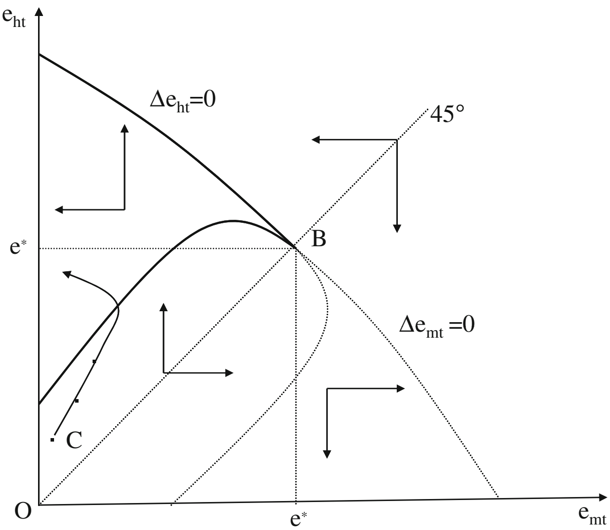


**Fig. 2** Phase diagram for case in which Eq. A2 holds

between the levels is small. The education levels of the high- and middle-income groups temporarily increase in accordance with Eqs. 29 and 30, respectively. The price of education also increases. The price of education depends on the weighted sum of the demand for education by middle- and high-income individuals. When the difference in income levels between the high- and middle-income groups is small, the price of education is not too high for the middle-income group to afford higher education for their children.

The burden of the price of education on incomes differs between high- and middle-income individuals. As shown in Fig. 3, the ratio of the income level of the middle-income group to the price level, represented by $\beta_2(1-\delta)I_{\text{mt}}/p_{\text{t}}$ decreases with an increase in the average education level. However, the income ratio of the high-income group increases. The price of education increases more rapidly than the income level of middle-income individuals because of the large demand for education by high-income individuals. The middle-income individuals then cannot afford the increasing cost of education, so their level of education eventually decreases. The price of education continues to increase because of the more than offsetting increase in the education level of high-income individuals. However, the price of education is relatively low for high-income individuals. Thus, the widening inequality between high- and middle-income individuals becomes an increasingly serious problem.[14]

The education level of middle-income individuals will eventually fall to zero; thus, they will become irregular workers. Only high-income individuals can then continue to receive higher education. The supply condition for education and the demand condition for high-income individuals imply the dynamics of their level of education:

$$e_{\text{ht}}{}^{1/\alpha} + (1-\beta_2)\theta e_{\text{ht}}{}^{(1-\alpha)/\alpha} = \beta_2(1-\delta)\bar{e}(\delta) + \beta_2(1-\delta)\gamma e_{\text{ht}-1}. \tag{32}$$

[14]Income inequality between the high- and low-income groups always widens because low-income individuals cannot commence education investment.

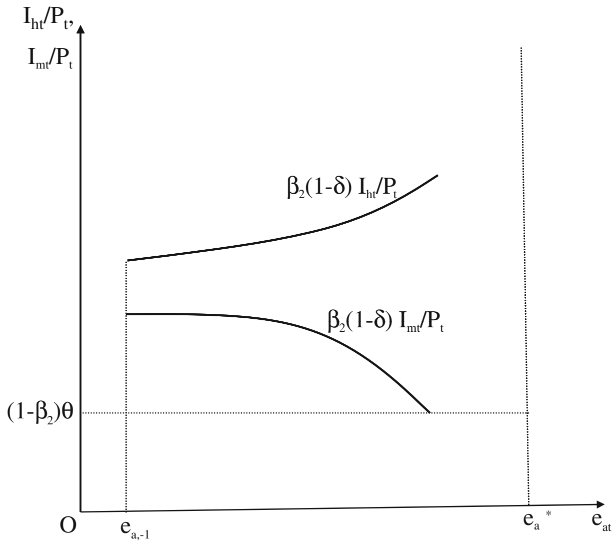


**Fig. 3** Relationships between the price of education and the income levels

The dynamics are qualitatively the same as those in Eq. 26. Because only high-income individuals accumulate human capital, the rate of technological progress converges to $g(\lambda_h e^*)$ where $e^*$ is the steady-state education level. When the ratio of high-income group to the total population is low, the rate of technological progress may be lower than that where middle-income individuals can also receive higher education. This implies that the income level of the high-income group increases more slowly because of slower economic growth.[15]

The ratio of high-income individuals to the population continues to take a value of $\lambda_{h,-1}$, that is, its ratio remains invariant. When middle-income individuals receive education, the ratio of middle-income individuals to the population is $\lambda_{m,-1}$. However, the middle-income group vanishes when middle-income individuals cannot afford education for their children. The ratio of middle-income individuals then becomes zero, that is, we have $\lambda_{mt-1} = 0$. Thus, the ratio of low-income individuals becomes $1 - \lambda_{h,-1}$.

Are there any effective policies that will both reduce income inequality and enhance economic growth, even with the relatively low productivity of elementary education? To address this, we consider a policy of subsidizing the institution of higher education. We assume that the government collects lump-sum taxes and uses them to reduce tuition fees. If education expenditure is no longer a convex function because of the subsidies, middle-income individuals can continue to receive higher education.[16] Thus, it will be possible to attain income equality between the high- and middle-income groups. Because the middle-income individuals also receive higher

[15] If the productivity of elementary education is high relative to that of higher education, we have a stable steady state with the inequality, $b(e_a{}^*) > 0$. Middle-income individuals will be able to catch up with high-income individuals in the long run.

[16] Alternatively, if higher education institutions are established in accordance with income groups, middle-income individuals can continue to receive education.

education, the rate of technological progress may be higher. However, it would be more difficult for low-income individuals to escape poverty. This is because the technological level may increase more rapidly compared with the case in which middle-income individuals cannot receive higher education.

## 4 Concluding remarks

By developing a model characterized by skill-biased technological change and increasing costs of education, we investigated the evolution of income inequality between three income groups. We found that low-income individuals who are also irregular workers cannot commence education investment because of the increasing costs of education and skill-biased technological progress. We also found that if the productivity of elementary education is low relative to that of higher education, even if middle-income individuals temporarily received higher education, they cannot pursue it because the threshold for education expenditure rises with the price of education. Therefore, even middle-income individuals may become trapped in poverty. High-income individuals also may suffer damage from increasing income inequality because of the slowing of technological progress.

When income inequality widens, the importance of public education grows. The government should then take action to prevent any deterioration in the quality of elementary education.[17] The high productivity of elementary education can assist middle-income individuals to accumulate human capital. However, such a policy would be ineffective in helping low-income individuals who are also irregular workers in escaping poverty as long as the wage systems differ between regular and irregular workers.

## Appendix A: Phase diagram

We explain the dynamics represented by Eqs. 29 and 30. We rewrite them as

$$\Delta e_{\mathrm{ht}} = \frac{\beta_2(1-\delta)h(e_{\mathrm{ht}-1})}{(h(e_{\mathrm{ht}-1})\tau_{\mathrm{t}})^{1-\alpha}} - (1-\beta_2)\theta - e_{\mathrm{ht}-1}, \tag{33}$$

$$\Delta e_{\mathrm{mt}} = \frac{\beta_2(1-\delta)h(e_{\mathrm{mt}-1})}{(h(e_{\mathrm{ht}-1})\tau_{\mathrm{t}})^{1-\alpha}} - (1-\beta_2)\theta - e_{\mathrm{ht}-1}. \tag{34}$$

Differentiating Eq. 33 totally and using Eqs. 10, 14, and 15, we find that, for the steady states in which $e_{\mathrm{h}} = e_{\mathrm{m}} \equiv e$ and $e_{\mathrm{a}} = (\lambda_{\mathrm{h}} + \lambda_{\mathrm{m}})e$,

$$\frac{de_{\mathrm{h}}}{de_{\mathrm{m}}}|_{\Delta e_{\mathrm{ht}}=0} = -\frac{\lambda_{\mathrm{m}} Z(e)e}{\lambda_{\mathrm{h}} Z(e)e + b(e_{\mathrm{a}})}, \tag{35}$$

[17] Using a regression approach, Nakamura [13] identified some of the factors affecting the scholastic achievement of elementary and junior high school students.

where

$$Z(e) \equiv \beta_2(1-\delta)h(e)(1-\alpha)(h(e)\tau)^{-1}\frac{\beta_2(1-\delta)\gamma}{1+(1-\alpha)(1-\beta_2)\theta(\lambda_h+\lambda_m)(h(e)\tau)^{-\alpha}}.$$

Differentiating Eq. 34 totally, on the other hand, we have:

$$\frac{de_h}{de_m}|_{\Delta e_{mt}=0} = -\frac{\lambda_m Z(e)e + b(e_a)}{\lambda_h Z(e)e}. \tag{36}$$

We consider the case in which $b(e_a{}^*) < 0$. We first prove that the relationship described in Eq. 37 is equivalent to that described in Eq. 38:

$$\beta_2(1-\delta)\gamma \overset{<}{\underset{>}{-}} \frac{1}{\alpha} e_a{}^{1/\alpha-1} + (1-\beta_2)\theta(\lambda_h+\lambda_m)\frac{1-\alpha}{\alpha} e_a{}^{1/\alpha-2}. \tag{37}$$

$$Z(e)e_a + b(e_a) \overset{>}{\underset{<}{-}} 0. \tag{38}$$

Equation 37 implies the relationship between the slopes of $u(e_a)$ and $v(e_a)$ in Eq. 26.

Using Eqs. 14 and 27, Eq. 37 can be rewritten as

$$\frac{\beta_2(1-\delta)\gamma - (\lambda_h+\lambda_m)Z(e)}{(h(e)\tau)^{1-\alpha}} \overset{<}{\underset{>}{-}} 1. \tag{39}$$

Using Eqs. 14 and 27, we obtain Eq. 38.

The equivalence between Eqs. 37 and 38 implies the following inequality:

$$Z(e^*)e_a{}^* + b(e_a{}^*) > 0. \tag{40}$$

Thus, assuming that $\lambda_h Z(e)e + b(e_a) > 0$ and $\lambda_m Z(e)e + b(e_a) > 0$ for $e_a = e_a{}^*$, the slopes represented by Eqs. 35 and 36 become negative for $e_a = e_a{}^*$.[18]

Using Eqs. 35 and 36, we can state the difference between the slopes of lines $\Delta e_{rt} = 0$ and $\Delta e_{pt} = 0$:

$$\frac{de_h}{de_m}|_{\Delta e_{ht}=0} - \frac{de_h}{de_m}|_{\Delta e_{mt}=0} = \frac{b(e_a)[Z(e)e + b(e_a)]}{\lambda_h Z(e)e[\lambda_h Z(e)e + b(e_a)]}. \tag{41}$$

Using Eqs. 40, 41, and $b(e_a{}^*) < 0$, we obtain the following for $e_a = e_a{}^*$:

$$\frac{de_h}{de_m}|_{\Delta e_{ht}=0} < \frac{de_h}{de_m}|_{\Delta e_{mt}=0}.$$

If $b(e_a{}^*) > 0$ holds, the slopes represented by Eqs. 35 and 36 take negative values in the steady state. We also have:

$$\frac{de_h}{de_m}|_{\Delta e_{ht}=0} > \frac{de_h}{de_m}|_{\Delta e_{mt}=0}.$$

[18] Even if the slopes were positive, our dynamics would not change qualitatively.

**Appendix B: Properties of a steady state**

We linearize the system represented by Eqs. 29 and 30 around the steady states to investigate the stability:

$$\begin{pmatrix} \frac{\partial e_{\mathrm{ht}}}{\partial e_{\mathrm{ht}-1}}|_s & \frac{\partial e_{\mathrm{ht}}}{\partial e_{\mathrm{mt}-1}}|_s \\ \frac{\partial e_{\mathrm{mt}}}{\partial e_{\mathrm{ht}-1}}|_s & \frac{\partial e_{\mathrm{mt}}}{\partial e_{\mathrm{mt}-1}}|_s \end{pmatrix} = \begin{pmatrix} A(e) - \lambda_{\mathrm{h}} B(e) & -\lambda_{\mathrm{m}} B(e) \\ -\lambda_{\mathrm{h}} B(e) & A(e) - \lambda_{\mathrm{m}} B(e) \end{pmatrix},$$

where $s$ means that each of partial derivatives is evaluated at the steady states,

$$A(e) \equiv \frac{\beta_2(1-\delta)\gamma}{(h(e)\tau)^{1-\alpha}} \quad \textit{and} \quad B(e) \equiv \frac{Z(e)}{(h(e)\tau)^{1-\alpha}}.$$

We represent eigenvalues by $\xi$. The implied characteristic polynomial is therefore

$$(\xi - A(e))[\xi - (A(e) - B(e))].$$

We first investigate one eigenvalue, $A(e)$. Using Eqs. 15 and 26, we have

$$\begin{aligned} A(e) - 1 &= \frac{1}{h(e)\tau}\big[\beta_2(1-\delta)\gamma e_{\mathrm{a}} - \beta_2(1-\delta)(\lambda_{\mathrm{h}} + \lambda_{\mathrm{m}})h(e) \\ &\qquad + (1-\beta_2)\theta(\lambda_{\mathrm{h}} + \lambda_{\mathrm{m}}){e_{\mathrm{a}}}^{(1-\alpha)/\alpha}\big] \\ &= \frac{\lambda_{\mathrm{h}} + \lambda_{\mathrm{m}}}{h(e)\tau}[-b(e_{\mathrm{a}})]. \end{aligned}$$

This result implies the following relationship:

$$A(e) \underset{>}{\overset{<}{-}} 1 \quad \Leftrightarrow \quad b(e_{\mathrm{a}}) \underset{<}{\overset{>}{-}} 0. \tag{42}$$

Next, Eq. 39 represents the other eigenvalue, $A(e) - B(e)$. $A(e) - B(e) < 1$ holds for $e_{\mathrm{a}} = {e_{\mathrm{a}}}^*$. Therefore, when $b({e_{\mathrm{a}}}^*) < 0$, one eigenvalue is greater than unity, and the other is less than unity. That is, the steady state is a saddle point. When $b({e_{\mathrm{a}}}^*) > 0$, on the other hand, we have two positive eigenvalues that are less than unity. That is, the steady state is stable.

Using Eqs. 29 and 30, the gap in education levels between the high- and middle-income groups is represented as

$$e_{\mathrm{ht}} - e_{\mathrm{mt}} = \frac{\beta_2(1-\delta)\gamma}{{e_{\mathrm{at}}}^{(1-\alpha)/\alpha}}(e_{\mathrm{ht}-1} - e_{\mathrm{mt}-1}). \tag{43}$$

The coefficient in Eq. 43 is equal to $A(e)$ at a steady state. When $b({e_{\mathrm{a}}}^*) > (<)0$, the coefficient in Eq. 43 is less (greater) than unity, and thus, the gap in the education levels between the high- and middle-income groups narrows (widens).

**Acknowledgements** I would like to thank Tetsuya Nakajima, Masamichi Kawano, Ken Tabata, Keisaki Higashida, Kunio Urakawa, Nobuaki Hori, and the seminar participants at Kwansei Gakuin University and Kyushu University for their valuable comments.

## References

1. Abe, A.: The present poverty and its causes. In: Oshio, T., Tajika, E., Fukawa, T. (eds.) Nihon no Shotokubunpai: Kakusakakudai to Seisakunoyakuwari, pp. 111–137. University of Tokyo Press, Tokyo (2006). In Japanese
2. Galor, O., Moav, O.: Ability-biased technological transition, wage inequality, and economic growth. Q. J. Econ. **115**, 469–498 (2000)
3. Galor, O., Moav, O.: From physical to human capital accumulation: inequality and the process of development. Rev. Econ. Stud. **71**, 1001–1026 (2004)
4. Galor, O., Zeira, J.: Income distribution and macroeconomics. Rev. Econ. Stud. **60**, 35–52 (1993)
5. Glomm, G., Ravikumar, B.: Public versus private investment in human capital: endogenous growth and income inequality. J. Polit. Econ. **100**, 818–834 (1992)
6. Koba, T.: Wage gaps and dual labor markets. Master's thesis, University of Kobe (2002). In Japanese
7. Kobayashi, M.: Opportunity for Higher Education in Japan: An Evaluation of Policies. University of Tokyo Press, Tokyo (2009). In Japanese
8. Maoz, Y.D., Moav, O.: Intergenerational mobility and the process of development. Econ. J. **109**, 677–697 (1999)
9. Maoz, Y.D., Moav, O.: Social stratification, capital-skill complementarity, and the nonmonotonic evolution of the education premium. Macroecon. Dyn. **8**, 295–309 (2004)
10. Moav, O.: Income distribution and macroeconomics: the persistence of inequality in a convex technology framework. Econ. Lett. **75**, 187–192 (2002)
11. Nakajima, T., Nakamura, H.: The price of education and inequality. Econ. Lett. **105**, 183–185 (2009)
12. Nakajima, T., Nakamura, H.: How do elementary and higher education affect human capital accumulation and inequality?: a note. Macroecon. Dyn. **16**, 151–158 (2012)
13. Nakamura, H.: Why does scholastic achievement differ across prefectures in Japan?. J. Asian Econ. **23**, 99–106 (2012)
14. Nishimura, K.: Valuation of 'yutori kyoiku' by economics. In: Ito, T., Nishimura, K. (eds.) Kyouikukaikaku no Keizaigaku, pp. 13–27. Nihonkeizaihyoronsha, Tokyo (2003). In Japanese
15. Ohta, K.: Non-regular employment and gaps in wage incomes. Nihon Rodo Kenkyu Zasshi **557**, 94–107 (2006). In Japanese
16. Rothschild, M., White, L.J.: The analytics of the pricing of higher education and other services in which the customers are inputs. J. Polit. Econ. **103**, 573–586 (1995)
17. Shigeno, Y., Matsuura, K.: Between non-standard employment and unemployment, aging, education: taking into account for coresidence and family background. Kaikei Kensa Kenkyu **41**, 55–65 (2010). In Japanese
18. Tachibanaki, T.: Confronting Income Inequality in Japan: A Comparative Analysis of Causes, Consequences, and Reform. MIT Press, Cambridge (2005)
19. Tachibanaki, T., Matsuura, T.: Economics of Inequality in Academic Background. Keisoshobo, Tokyo (2009). In Japanese
20. Tachibanaki, T., Yagi, T.: Education and Inequality: Why do Individuals Aim to go to Famous Schools?. Nihonhyoronsha, Tokyo (2009). In Japanese
21. Takii, K., Tanaka, R.: Does the diversity of human capital increase GDP?: A comparison of education systems. J. Public Econ. **93**, 998–1007 (2009)
22. Tamura, R.: Teachers, growth, and convergence. J. Polit. Econ. **109**, 1021–1059 (2001)
23. Uni, H.: Increasing wage inequality in Japan and some causes. Keizai Riron **45**, 20–30 (2008). In Japanese
24. Yamaguchi, M.: Some macroeconomic causes in increasing wage differential between regular workers and part-timers. Nihon Rodo Kenkyu Zasshi **554**, 94–107 (2006). In Japanese