# Characterizing Pareto Improvements in an Interdependent Demand System

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## Abstract

Interdependent preferences generally imply Pareto inefficiency. For a general demand system, we provide a characterization of Pareto improvements. For a prominent parametric specification, the Linear Expenditure System, we characterize in detail the welfare loss associated with interdependent preferences. Using an estimated empirical model of this kind, we calculate the compensating variation corresponding to the welfare loss.

## 1. Introduction

Interdependent preferences (IP)—preferences which depend on other people's consumption—can be seen as consumption externalities: since in a competitive market economy with IP consumers usually do not coordinate their decisions, the resulting Nash equilibrium in general is not Pareto optimal. In particular, if a consumer's consumption of a particular good has an effect on other consumers' utility, then in general the price of the good does not reflect the marginal social costs of consumption. See Davis and Whinston (1965), who discuss this issue by pointing out the difference between the first-order conditions of the competitive market equilibrium and the Pareto optimum.

Given that the equilibrium in general is Pareto inefficient, a number of papers have studied the properties of optimal tax rates that are obtained if a government supports a Pareto optimum by maximizing a (utilitarian) social welfare function. In order to keep the analysis manageable, these studies typically focus on situations in which, among other assumptions, (a) all consumers have identical utility functions, and/or (b) the consumption externality of a

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commodity appears in the utility functions as a term, which depends only on some measure of the aggregate consumption of the commodity in question (i.e., not on the individual consumption of the commodity by all other consumers), and/or (c) there is only one commodity that creates a consumption externality. See e.g., Boskin and Sheshinski (1978), Frank (1985), Persson (1995), and Sandmo (1975), which employ (a), (b), and (c); see Diamond (1973) and Oswald (1983), which employ (b) and (c); see Sadka (1978), which employs (b).[1]

The present paper analyzes a more general case. In Section 2, we consider a general demand system with IP. We assume that the utility of each consumer depends on both his own consumption bundle and the individual consumption bundles chosen by all other consumers. The utility functions are not necessarily identical for all consumers, and the number of goods is arbitrary. We show that in this general case, under a relatively weak condition with respect to the degree of conspicuousness of the goods, there exist Pareto improvements of an appealing type over the Nash equilibrium. Next, we focus in Section 3 on a prominent parametric demand system, the Linear Expenditure System (LES). The LES has been used frequently in work on IP, see e.g., Gaertner (1974), Pollak (1976), Pollak and Wales (1992), and Kapteyn et al. (1997). We characterize a Pareto-optimal consumption pattern for the LES case and the size of the welfare loss from IP, and we derive a closed form expression of (Pigouvian) taxes and subsidies that eliminate this welfare loss.[2] Finally, using estimation results obtained with Dutch data for a LES demand system reported by Kapteyn et al. (1997), we derive an upper bound of the compensating variation corresponding to the welfare loss from IP. Section 4 concludes.

## 2. Conditions for Pareto Improvements

Consider a society with $N$ consumers and $G$ goods. Let $x_{gn} \geq 0$ be the quantity of good $g$ consumed by consumer $n$; $g = 1, \ldots, G$, $n = 1, \ldots, N$. Consumer $n$ has an income $y_n$; all consumers face the same prices $p \equiv (p_1, \ldots, p_G)'$. Prices and incomes are assumed to be given and positive; as in the empirical microeconometric work on IP, we do not explicitly model the production side of the economy. Let $x_n \equiv (x_{1n}, \ldots, x_{Gn})'$, $x \equiv (x_1', \ldots, x_N')'$, and

[1] A related but different branch of literature has analyzed the inefficiency of IP in signaling games in which consumers purchase goods to signal their wealth, see e.g., Corneo and Jeanne (1997) and Ireland (1994). Other studies have investigated the implications of IP for economic growth; see e.g., Cole, Mailath, and Postlewaite (1992) and Rauscher (1997). These authors investigate whether or not "status seeking" accelerates economic growth and analyze the implications for the taxation of capital accumulation.

[2] Strictly speaking, the welfare loss we analyze is the loss associated with noncooperative Nash behavior in the presence of IP. For the purpose of briefness we refer to this as "the welfare loss from IP."

$y \equiv (y_1, \ldots, y_N)'$. The preferences of consumer $n$ are represented by a utility function $U_n: \mathbb{R}_+^{GN} \to \mathbb{R}$, where $\mathbb{R}_+^{GN}$ is the nonnegative orthant of $\mathbb{R}^{GN}$. An allocation $x$ is admissible if, for all $n$, $\sum_{g=1}^{G} p_g x_{gn} = y_n$, i.e., if it satisfies the budget constraint for all consumers. Let $S_n \equiv \{x_n \in \mathbb{R}_+^G \mid \sum_{g=1}^{G} p_g x_{gn} = y_n\}$ for all $n$, and $S \equiv S_1 \times S_2 \times \cdots \times S_N$. Given $x \in S$, we define $x \backslash t_n \equiv (x_1', \ldots, x_{n-1}', t_n', x_{n+1}', \ldots, x_N')'$, where $t_n$ is an arbitrary vector in $S_n$, for all $n$. An allocation $\hat{x} \in S$ is a Nash equilibrium for $(p, y)$ if for all $n$ we have

$$U_n(\hat{x}) = \max_{t_n \in S_n} U_n(\hat{x} \backslash t_n). \tag{1}$$

Next, given $x \in S$, we define subset $G(x)$ of the goods $\{1, 2, \ldots, G-1\}$ according to

$$g \in G(x) \Leftrightarrow \frac{\partial U_n(x)}{p_g \partial x_{gk}} < \frac{\partial U_n(x)}{p_G \partial x_{Gk}}, \qquad \forall n = 1, \ldots, N; \;\; \forall k \neq n, \;\; k = 1, \ldots, N.$$

Note that if $\frac{\partial U_n(x)}{p_g \partial x_{gk}} < 0$ (respectively, $>0$), then it represents in the allocation $x$ the decrease (respectively, increase) in the utility of consumer $n$ as a result of a small increase in the amount of money spent on the consumption of good $g$ by consumer $k \neq n$. We may say that in $x$, for a given consumer $n$, good $g$ is more conspicuous than good $h$ if $\frac{\partial U_n(x)}{p_g \partial x_{gk}} < \frac{\partial U_n(x)}{p_h \partial x_{hk}}$ for *all* other consumers $k \neq n$; i.e., a change in the amount of money spent by *any* other consumer on good $g$ has a greater negative (or a smaller positive) effect on the utility of $n$ than a change in the amount of money spent on good $h$. So, $G(x)$ is the set of goods, which are more conspicuous than good $G$ for *each* consumer in $x$. If $G(x)$ is nonempty, then there is at least one pair of goods for which the ordering of the degree of conspicuousness is the same for each consumer. Observe that the set $G(x)$ depends on the ordering of the goods; in particular, it depends on the choice of the good $G$.

We now make following assumptions.

ASSUMPTION 1: *$U_n(x)$ is twice continuously differentiable and bounded for all $x \in S$ and all $n$.*

ASSUMPTION 2: *$U_n(x \backslash t_n)$ is strictly concave with respect to $t_n \in S_n$ for all $x \in S$ and all $n$.*

PROPOSITION 1: *Let Assumptions 1 and 2 hold. We then have:*

*(i) There exists a Nash equilibrium $\hat{x} \in S$ as defined in (1).*

*(ii) If $\hat{x}_{gn} > 0$ for all $g = 1, \ldots, G$ and $n = 1, \ldots, N$ and, moreover, the set $G(\hat{x})$ is nonempty, then there exists an allocation $x^* \in S$ such that, for all $n$,*

- $0 < x_{gn}^* < \hat{x}_{gn}, \quad \forall g \in G(\hat{x})$
- $x_{gn}^* = \hat{x}_{gn}, \quad \forall g \in \{1, \ldots, G-1\} \backslash G(\hat{x})$
- $x_{Gn}^* > \hat{x}_{Gn}$
- $U_n(x^*) > U_n(\hat{x})$.

*Proof:* See the Appendix.

Thus, there exist Pareto improvements over the Nash equilibrium, provided that we are able to order the goods in such a way that there exists at least one good, which is more conspicuous than good $G$ for all consumers in the Nash equilibrium. The Pareto improvements are of an appealing type, i.e., they can be achieved by consuming less of the goods that are more conspicuous than good $G$ and consuming more of good $G$ itself.

## 3. A Parametric Case: The Linear Expenditure System

We now turn to the LES. The utility function of consumer $n$ without preference interdependence is then specified as

$$U_n = \sum_{g=1}^{G} \gamma_g \ln(x_{gn} - b_{gn}), \tag{2}$$

with $\gamma_g > 0$, $\forall g$, $\sum_{g=1}^{G} \gamma_g = 1$, and $x_{gn} > b_{gn}$, $\forall g$, $\forall n$. The budget constraint is $\sum_{g=1}^{G} p_g x_{gn} = y_n$. Maximization of $U_n$ with respect to $x_{gn}$, $\forall g$, under the budget constraint yields the demand equations

$$x_{gn} = b_{gn} + \frac{\gamma_g}{p_g}\left(y_n - \sum_{h=1}^{G} p_h b_{hn}\right). \tag{3}$$

We assume throughout that $y_n$ is high enough such that in (3) we have $x_{gn} > \max\{0, b_{gn}\}$, $\forall g$, $\forall n$.

Following Pollak (1976) and Kapteyn et al. (1997), preference interdependence is introduced by making the quantities $b_{gn}$ dependent on the consumption by others in the following way:

$$b_{gn} = b_{g0} + \beta_g \sum_{k=1}^{N} w_{nk} x_{gk}, \tag{4}$$

where $w_{nk}$ is a reference weight, representing how strongly consumer $n$ is affected by consumer $k$'s consumption. The weights satisfy $w_{nn} = 0$, $w_{nk} \geq 0$, and $\sum_{k=1}^{N} w_{nk} = 1$. Notice that $\beta_g w_{nk}$ denotes the effect of the consumption of good $g$ by consumer $k$ on the consumption of good $g$ by consumer $n$. Alternatively, we also remark that $\sum_{k=1}^{N} w_{nk} x_{gk}$ represents mean expenditures on good $g$ in the reference group of consumer $n$, where this reference group is defined as the set of consumers $k$ for whom $w_{nk} > 0$. The parameter $b_{g0}$ is a good-specific constant and $\beta_g$ a good-specific coefficient. The latter parameter is also a measure of the conspicuousness of good $g$. Kapteyn et al. (1997) impose $0 \leq \beta_g < 1$. While values within this range may be plausible, the analysis below only requires $-1 < \beta_g < 1$ (in particular, we need this assumption in the derivation of the Nash equilibrium (A6) in the Appendix).

### 3.1. The Nash Case

Substitution of (4) into (3) gives consumer $n$'s reaction functions, i.e., his optimal demands as a function of the consumption of others. The (one-shot) Nash equilibrium $\hat{x}$ for this case with heterogeneous consumers can be obtained analytically due to the linearity of the model, see the Appendix [in particular (A6)].

In the Appendix we also show that

$$\frac{\partial U_n(\hat{x})}{p_g \partial x_{gk}} = \frac{-\beta_g w_{nk}}{c_n}, \qquad \forall g, \forall n, \forall k \neq n, \tag{5}$$

where $c_n$ is a positive term, independent of $g$. Similar to Section 2, we can define a set $G(\hat{x})$. Suppose, for example, that $\beta_1 \geq \cdots \geq \beta_{G-1} > \beta_G$. Then, using (5), we have $G(\hat{x}) = \{1, \ldots, G-1\}$. In turn, similar to (ii) of Proposition 1, we can easily show that there exists an admissible allocation $x^*$ such that, for all $n$, we have (i) $0 < x^*_{gn} < \hat{x}_{gn}$, for $g = 1, \ldots, G-1$; (ii) $x^*_{Gn} > \hat{x}_{Gn}$; and (iii) $U_n(x^*) > U_n(\hat{x})$.

In the special case when consumers have identical preferences with equal reference weights $w_{nk} = \frac{1}{N-1}$, $\forall n \neq k$, and equal incomes $y_n = \bar{y}$, $\forall n$, we have $x_{gn} = x_g^{\text{Nash}}$, $\forall n$, $\forall g$, with

$$x_g^{\text{Nash}} = \frac{b_{g0}}{1-\beta_g} + \frac{\gamma_g/(1-\beta_g)}{p_g \sum_{h=1}^{G} \gamma_h/(1-\beta_h)} \left( \bar{y} - \sum_{h=1}^{G} \frac{p_h b_{h0}}{1-\beta_h} \right); \tag{6}$$

see the Appendix.

### 3.2. A Pareto-Efficient Allocation

It is instructive to compare Equation (6) with the Pareto-efficient demand obtained by maximizing the utilitarian social welfare function

$$\sum_{n=1}^{N} \sum_{g=1}^{G} a_n \gamma_g \ln \left( x_{gn} - b_{g0} - \beta_g \sum_{k=1}^{N} w_{nk} x_{gk} \right) \tag{7}$$

with respect to $x_{gn}$, $\forall n$, $\forall g$, subject to

$$\sum_{g=1}^{G} p_g x_{gn} = y_n, \qquad n = 1, \ldots, N \tag{8}$$

for any choice of weights $a_n$'s satisfying $a_n > 0, \forall n$. Inspection of the first-order conditions of the problem reveals that for the general LES case, the solution cannot be written in closed form. However, if we take the special case with $w_{nk} = \frac{1}{N-1}$, $\forall n \neq k$, and $y_n = \bar{y}$, $\forall n$, and further take $a_n = 1, \forall n$, then we have the symmetric solution with $x_{gn} = x_g^{\text{Pareto}}$, $\forall n$, $\forall g$, where

$$x_g^{\text{Pareto}} = \frac{b_{g0}}{1-\beta_g} + \frac{\gamma_g}{p_g}\left(\bar{y} - \sum_{h=1}^{G} \frac{p_h b_{h0}}{1-\beta_h}\right); \tag{9}$$

see the Appendix. We remark that putting $a_n = 1$, $\forall n$, is reasonable in this symmetric case; cf. Sandmo (1975) and Boskin and Sheshinski (1978).

While the subsistence quantities in the Nash and Pareto cases (Equations (6) and (9), respectively) are identical, the marginal budget shares, i.e., the proportions that are used to spend the "supernumerary income" $(\bar{y} - \sum_{h=1}^{G} \frac{p_h b_{h0}}{1-\beta_h})$, differ. In the Pareto case, the marginal budget shares "ignore" the conspicuous nature of goods; in the Nash case it is taken into account.

Comparison of (6) and (9) reveals that

$$x_g^{\text{Nash}} \gtrless x_g^{\text{Pareto}} \Leftrightarrow \frac{1}{1-\beta_g} \gtrless \sum_{h=1}^{G} \frac{\gamma_h}{1-\beta_h}. \tag{10}$$

Note that $1/(1-\beta_g)$ is also a measure of the conspicuousness of good $g$. So, Equation (10) states that the Nash quantity of good $g$ is larger (smaller) than the Pareto quantity if and only if it is more (less) conspicuous than the weighted average conspicuousness of all goods.

### 3.3. The Welfare Loss

Let $x^{\text{Nash}} \equiv (x_1^{\text{Nash}}, \ldots, x_G^{\text{Nash}})'$ and $x^{\text{Pareto}} \equiv (x_1^{\text{Pareto}}, \ldots, x_G^{\text{Pareto}})'$. Below we will sometimes use the notation $x^{\text{Nash}}(p, \bar{y})$ and $x^{\text{Pareto}}(p, \bar{y})$ to emphasize that these consumption vectors depend on the prices and incomes. Using (6) and (9) it is straightforward to verify that

$$\begin{aligned} &U_n(x^{\text{Pareto}}, \ldots, x^{\text{Pareto}}) - U_n(x^{\text{Nash}}, \ldots, x^{\text{Nash}}) \\ &\quad = \ln\left(\sum_{g=1}^{G} \frac{\gamma_g}{1-\beta_g}\right) - \ln\left(\prod_{g=1}^{G}\left(\frac{1}{1-\beta_g}\right)^{\gamma_g}\right) \geq 0 \end{aligned} \tag{11}$$

by Jensen's inequality for sums (see e.g., Berck and Sydaeter 1991, p. 28). So, the utility difference only depends on the ratio of the weighted arithmetic and geometric means of $1/(1-\beta_g)$, $g = 1, \ldots, G$, with weights $\gamma_g$.

The welfare loss can also be expressed in terms of a compensating variation. Suppose that consumers behave according to the Nash equilibrium. Then the income level $y^*$ necessary to achieve the utility level associated with the social optimum is defined by

$$\begin{aligned} &U_n(x^{\text{Nash}}(p, y^*), \ldots, x^{\text{Nash}}(p, y^*)) \\ &\quad = U_n(x^{\text{Pareto}}(p, \bar{y}), \ldots, x^{\text{Pareto}}(p, \bar{y})). \end{aligned} \tag{12}$$

For the LES case this yields

$$y^* = S + (\bar{y} - S) \cdot \frac{\sum_{g=1}^{G} \frac{\gamma_g}{1-\beta_g}}{\prod_{g=1}^{G} \left( \frac{1}{1-\beta_g} \right)^{\gamma_g}}, \tag{13}$$

where $S = \sum_{g=1}^{G} \frac{p_g b_{g0}}{1-\beta_g}$ denotes the subsistence expenditures. For $S \geq 0$, we have

$$1 \leq y^*/\bar{y} \leq \frac{\sum_{g=1}^{G} \frac{\gamma_g}{1-\beta_g}}{\prod_{g=1}^{G} \left( \frac{1}{1-\beta_g} \right)^{\gamma_g}}. \tag{14}$$

The upper bound will be used in the empirical illustration in Subsection 3.5. With $-1 < \beta_g < 1$, the range of the right-hand side of (14) is $[1, \infty)$.

### 3.4. Optimal Tax Rates

A government is able to eliminate the welfare loss arising from Nash behavior by means of (budget neutral) taxes and subsidies. Recall that given the price vector $p$ and the income $\bar{y}$ available to each consumer, the Pareto-optimal allocation equals $x^{\text{Pareto}}(p, \bar{y})$ as given in (9). The problem of the government is to determine taxes and subsidies, or equivalently a new price vector $p^* \equiv (p_1^*, \ldots, p_G^*)'$ say, which incorporates the taxes and subsidies, such that (i) $x^{\text{Nash}}(p^*, \bar{y}) = x^{\text{Pareto}}(p, \bar{y})$, and (ii) $\sum_{g=1}^{G} p_g^* x_g^{\text{Nash}}(p^*, \bar{y}) = \bar{y}$. Notice that (i) assumes that all consumers behave Nash given the price vector $p^*$, whereas (ii) reflects budget neutrality of the taxes and subsidies. From (6) and (9), it follows that the prices $p^*$ are defined by

$$\frac{\gamma_g/(1-\beta_g)}{p_g^* \sum_{h=1}^{G} \gamma_h/(1-\beta_h)} \left( \bar{y} - \sum_{h=1}^{G} \frac{p_h^* b_{h0}}{1-\beta_h} \right) = \frac{\gamma_g}{p_g} \left( \bar{y} - \sum_{h=1}^{G} \frac{p_h b_{h0}}{1-\beta_h} \right), \tag{15}$$

for $g = 1, \ldots, G$, which implies that

$$\frac{p_g^*}{p_h^*} = \frac{p_g(1-\beta_h)}{p_h(1-\beta_g)}. \tag{16}$$

Hence, as a result of the taxes and subsidies, the price ratio consumers are facing between the two goods $g$ and $h$ has increased (decreased) if good $g$

is more (less) conspicuous than good $h$. Using budget neutrality, the closed form solution of $p_g^*$ $(g = 1, \ldots, G)$ turns out to equal

$$p_g^* = p_g \cdot \frac{\bar{y}}{T(1-\beta_g)} \tag{17}$$

with

$$T = \sum_{g=1}^{G} \frac{\gamma_g}{(1-\beta_g)} \left( \bar{y} - \sum_{h=1}^{G} \frac{p_h b_{h0}}{1-\beta_h} \right) + \sum_{g=1}^{G} \frac{p_g b_{g0}}{(1-\beta_g)^2};$$

see the Appendix.

### 3.5. An Empirical Illustration

We conclude with an empirical illustration based on estimation results reported in Kapteyn et al. (1997).[3] In particular, we derive an upper bound of the compensating variation corresponding to the welfare loss from IP.

Kapteyn et al. have estimated with Dutch data an empirical model with IP using virtually the same functional form specifications as in the LES case examined here. The 2813 households in their cross section were assigned to one of 75 social groups; households belong to the same social group if their heads have the same educational attainment (three categories), have the same age (five categories), and have the same job type (five categories). Let $N_s$ denote the number of households in social group $s$. We have $w_{nk} = \frac{1-\kappa}{N_s - 1}$ if $k$ is in $n$'s social group, and $w_{nk'} = \frac{\kappa}{N - N_s}$ if $k'$ is not in $n$'s social group. The parameters $\beta_g$ and $\gamma_g$ were assumed not to vary across households and social groups. There are seven expenditure categories.

Kapteyn et al. report the estimation results given in Table 1, where

$$r_g = \frac{(1-\kappa)(\beta_g - \pi)\gamma_g}{1-(1-\kappa)\beta_g}. \tag{18}$$

The estimates of the $\gamma_g$'s are all significantly different from zero, while four out of the six estimated $r_g$'s are significantly different from zero. The values of $\gamma_g$ and $r_g$ for the remaining seventh category "other expenditures" follow from $\sum_{g=1}^{7} \gamma_g = 1$ and $\sum_{g=1}^{7} r_g = 0$, and are equal to 0.010 and 0.003, respectively. The parameters $\kappa$ and $\pi$ could not be identified from the data. Kapteyn et al. remark that there is ample empirical evidence that consumers mainly

[3]It was only in the 1990s when IP were incorporated in empirical microeconometric analysis. Other examples are Alessie and Kapteyn (1991), Blomquist (1993), Woittiez and Kapteyn (1998), and Aronsson, Blomquist, and Sacklén (1999). The slow rate of accumulation of empirical evidence on IP may be related to difficulties of identification, see e.g., Manski (1993).

Table 1: Estimation results of Kapteyn et al. (1997)

| **Expenditure Category** | $\gamma_g$ | $r_g$ |
|---|---|---|
| Food | 0.131 | −0.021 |
| Housing | 0.274 | 0.052 |
| Clothing | 0.081 | −0.003 |
| Medical care | 0.094 | 0.018 |
| Education and entertainment | 0.172 | −0.004 |
| Transportation | 0.238 | −0.045 |

compare themselves with others who are similar. This suggests that the size of $\kappa$ is rather small. Therefore, assume $\kappa = 0$, i.e., preferences are interdependent within social groups, but not across groups. Even with $\kappa$ known, $\pi$ is unidentified. However, the right-hand side of (14) turns out to be independent of $\pi$, which can be seen as follows. With $\kappa = 0$, (18) implies

$$\beta_g = \frac{r_g + \pi \gamma_g}{r_g + \gamma_g}. \tag{19}$$

Therefore,

$$\sum_{g=1}^{G} \frac{\gamma_g}{1 - \beta_g} = \sum_{g=1}^{G} \frac{r_g + \gamma_g}{1 - \pi} = \frac{1}{1 - \pi}$$

and

$$\prod_{g=1}^{G} \left( \frac{1}{1 - \beta_g} \right)^{\gamma_g} = \prod_{g=1}^{G} \left( \frac{r_g + \gamma_g}{\gamma_g (1 - \pi)} \right)^{\gamma_g} = \frac{1}{1 - \pi} \prod_{g=1}^{G} \left( \frac{r_g + \gamma_g}{\gamma_g} \right)^{\gamma_g}.$$

Using (14) and inserting the estimated values for $r_g$ and $\gamma_g$, we find that, for $S \geq 0$,

$$y^* / \bar{y} \leq \prod_{g=1}^{7} \left( \frac{\gamma_g}{r_g + \gamma_g} \right)^{\gamma_g} = 1.013. \tag{20}$$

(The term $1 - \pi$ does not cancel out when $\kappa \neq 0$.) So, households who behave according to a Nash equilibrium require an income increase of at most 1.3% to be as well off as they would be if social welfare were maximized.[4]

Finally, we have computed numerically the upper bound for the compensating variation for positive values of $\kappa$ and different values of $\pi$. The values of $\kappa$ and $\pi$ are chosen as follows. First, we remark that a parameter similar to $\kappa$ arises in a related demand system analyzed by Alessie and Kapteyn

[4]The empirical estimates do not allow to calculate the optimal taxes and subsidies of (17), as this requires knowledge of the unidentified parameters $\pi$ and $b_{g0}$, $g = 1, \ldots, G$.

(1991). These authors argue that in their model 0.08 is an upper bound for this parameter. Motivated by this, we also focus here on cases with $\kappa = 0.01, 0.02, \ldots, 0.08$. Second, given $\kappa$, $\pi$ (which we take as multiples of 0.1) is constrained by the requirement, made by Kapteyn et al. (1997), that $0 \leq \beta_g < 1$ for all $g$. Taking this into account, it turns out that the upper bound for the compensation variation increases, but only to a limited extent. The maximum value found is 4.8% (if $\kappa = 0.08$ and $\pi = 0.9$).

## 4. Concluding Remarks

It is important to note that a low outcome for the compensating variation does not necessarily imply that preferences are only moderately interdependent. First, the size of the potential welfare increase also depends on the variation in conspicuousness across goods. If all goods are highly, but also approximately equally conspicuous, the welfare increase and the corresponding compensating variation will be small. For example, if $\beta_g = \beta \; \forall g$, then the right-hand side of (14) equals 1, no matter the value of $\beta$. Moreover, for various reasons the percentage obtained here is only a rough estimate of the true compensation variation. For example, a further disaggregation of goods is likely to increase the estimated compensating variation. The category "transportation," for instance, includes expenditures on public transportation as well as expenditures on private cars. The former subcategory is likely to exhibit a low degree of conspicuousness, whereas cars are probably among the most conspicuous goods. Including separate equations for both subcategories in the demand system will increase the variation in conspicuousness across goods and probably the estimated compensating variation. Moreover, it should be mentioned that expenditure data alone do not allow for complete identification of the welfare loss, for essentially the same reason why they do not allow for the identification of constant welfare equivalence scales. Interdependence may simply lower the level of experienced utility without affecting the allocation very much.

In many countries luxury goods (which generally exhibit a high degree of conspicuousness) are currently taxed at a higher VAT rate than are necessities. In most countries in the European Community new cars—probably the most conspicuous of all goods—are taxed with an additional "special users tax" at a rate of approximately 30%. Preference interdependence provides a welfare theoretic basis for such a tax (in addition to usual motivations such as internalizing the costs of environmental damage and government fund raising). To determine the optimal level of the tax an empirical analysis along the lines followed in the previous section is required. In addition to a further disaggregation of goods, using more flexible functional forms, and considering dynamic aspects, future research of this type should include the collection of more direct data on reference groups, for example, based on experimental or subjective approaches.

## Appendix: Proofs

*Proof of Proposition 1:* Part (i) directly follows from Friedman (1991, p. 72), by noting that the set $S_n$ is compact and convex for all $n$. In order to prove part (ii) we first introduce some notation. Let $S_n^{-G} \equiv \{z_n \in \mathbb{R}_+^{G-1} | z_{gn} > 0 \text{ for } g = 1, \ldots, G-1, \text{ and } y_n - \sum_{g=1}^{G-1} p_g z_{gn} > 0\}$, for all $n$. Let $S^{-G} \equiv S_1^{-G} \times S_2^{-G} \cdots \times S_N^{-G}$, and $z \equiv (z_1', \ldots, z_N')'$ be an arbitrary vector in $S^{-G}$. For all $n$, let the function $V_n : S^{-G} \to \mathbb{R}$ be defined as

$$V_n(z) \equiv U_n\Bigg(z_{11}, \ldots, z_{G-1,1}, \frac{1}{p_G}\left[y_1 - \sum_{g=1}^{G-1} p_g z_{g1}\right], \\ z_{12}, \ldots, z_{G-1,2}, \frac{1}{p_G}\left[y_2 - \sum_{g=1}^{G-1} p_g z_{g2}\right], \ldots, \\ z_{1N}, \ldots, z_{G-1,N}, \frac{1}{p_G}\left[y_N - \sum_{g=1}^{G-1} p_g z_{gN}\right]\Bigg).$$

Next, using the notation of Section 2, given an arbitrary $x_n \in S_n$, we define the $G-1 \times 1$ vector $x_n^{-G} \equiv (x_{1n}, \ldots, x_{G-1,n})'$, for all $n$. Given an arbitrary $x \in S$, we define the $G-1 \times N$ vector $x^{-G} \equiv ((x_1^{-G})', \ldots, (x_N^{-G})')'$.

According to (ii) of Proposition 1, $\hat{x}$ is a Nash equilibrium with $\hat{x}_{gn} > 0$ for all $g$ and $n$. Using the above definitions, this implies that for all $n$

$$V_n(\hat{x}^{-G}) = \max_{z_n \in S_n^{-G}} V_n\Big(\big((\hat{x}_1^{-G})', \ldots, (\hat{x}_{n-1}^{-G})', z_n', (\hat{x}_{n+1}^{-G})', \ldots, (\hat{x}_N^{-G})'\big)'\Big).$$

As a result, we must have

$$\frac{\partial V_n(\hat{x}^{-G})}{\partial x_{gn}} = 0, \quad n = 1, \ldots, N; \quad g = 1, \ldots, G-1. \tag{A1}$$

We also know that there is a nonempty subset $G(\hat{x})$ of the set $\{1, \ldots, G-1\}$ such that for all $n$

$$\frac{\partial V_n(\hat{x}^{-G})}{\partial x_{gk}} = \frac{\partial U_n(\hat{x})}{\partial x_{gk}} - \frac{\partial U_n(\hat{x})}{\partial x_{Gk}} \frac{p_g}{p_G} < 0, \quad \forall k \neq n, \ \forall g \in G(\hat{x}). \tag{A2}$$

It follows from Equations (A1) and (A2) and the fact that in $\hat{x}$ the budget constraint is satisfied for each consumer, that there exist $x_{gn}^*$ with $g = 1, \ldots, G-1$ and $n = 1, \ldots, N$, such that

- $0 < x_{gn}^* < \hat{x}_{gn}$ if $g \in G(\hat{x})$,
- $x_{gn}^* = \hat{x}_{gn}$ if $g \in \{1, \ldots, G-1\} \backslash G(\hat{x})$,
- $y_n - \sum_{g=1}^{G-1} p_g x_{gn}^* > 0$,
- $V_n(x_{11}^*, \ldots, x_{G-1,1}^*, \ldots, x_{1N}^*, \ldots, x_{G-1,N}^*) > V_n(\hat{x}^{-G})$.

Next, we define for all $n$

$$x^*_{Gn} \equiv \frac{1}{p_G}\left[y_n - \sum_{g=1}^{G-1} p_g x^*_{gn}\right] > 0$$

and summarize all elements $x^*_{gn}$ in the obvious way in the allocation $x^* \in S$. Note that $U_n(x^*) = V_n((x^*)^{-G}) > V_n(\hat{x}^{-G}) = U_n(\hat{x})$. ■

## Derivation of the Nash Equilibrium $\hat{x}$

If we substitute Equation (4) into (3), we end up with the following system of equations that must be satisfied by the Nash equilibrium:

$$\begin{aligned} x_{gn} = b_{g0} + \beta_g \sum_{k=1}^{N} w_{nk} x_{gk} \\ + \frac{\gamma_g}{p_g}\left[y_n - \sum_{h=1}^{G} p_h b_{h0} - \sum_{h=1}^{G}\sum_{k=1}^{N} \beta_h w_{nk} p_h x_{hk}\right], \quad \forall g, \forall n. \end{aligned} \tag{A3}$$

In order to derive an explicit expression of the Nash equilibrium, we now introduce the following notation: $x^g \equiv (x_{g1}, \ldots, x_{gN})'$ $\forall g$, $X \equiv (x^1, \ldots, x^G)$, $b_0 \equiv (b_{10}, \ldots, b_{G0})'$, $y \equiv (y_1, \ldots, y_N)'$, $\beta \equiv (\beta_1, \ldots, \beta_G)$, $\gamma \equiv (\gamma_1, \ldots, \gamma_G)'$, $p \equiv (p_1, \ldots, p_G)'$, $P \equiv \mathrm{diag}(p_1, \ldots, p_G)$, and $B \equiv \mathrm{diag}(\beta_1, \ldots, \beta_G)'$. Further, let $W \equiv (w_{ij})$ denote the $N \times N$ matrix with as elements the reference weights $w_{ij}$, let $\iota_N$ denote an $N \times 1$ vector of ones, and $I_G$, $I_N$, and $I_{GN}$ denote identity matrices of dimensions $G \times G$, $N \times N$, and $GN \times GN$, respectively. We also recall that vec $X$ denotes the $GN \times 1$ vector obtained by stacking the columns of matrix $X$ one underneath the other. Notice that vec $X$ only differs from the allocation $x$ introduced in Section 2 with regard to the ordering of the goods $x_{gn}$.

The above notation enables us to rewrite (A3) as

$$\begin{aligned} (I_N - \beta_g W + \gamma_g \beta_g W)\, x^g + \frac{\gamma_g}{p_g}\sum_{\substack{h=1 \\ h \neq g}}^{G} \beta_h p_h W x^h \\ = \left(b_{g0}\iota_N + \frac{\gamma_g}{p_g} y - (b_0' p)\frac{\gamma_g}{p_g}\iota_N\right), \quad \forall g, \end{aligned} \tag{A4}$$

or, using the Kronecker product, as

$$\begin{aligned} \left(I_{GN} - B \otimes W + P^{-1}\gamma\beta' P \otimes W\right) \mathrm{vec}\, X = b_0 \otimes \iota_N + P^{-1}\gamma \otimes y \\ - (b_0' p) P^{-1}\gamma \otimes \iota_N. \end{aligned} \tag{A5}$$

We remark that Kapteyn et al. (1997) derive a similar equation for the special case where $p_g = 1$ for all $g$. However, our expression allows for arbitrary prices $p_g$. Assuming that $0 \leq \beta_g < 1$ for all $g$, Kapteyn et al. (1997, Lemma 2) further prove that in case all prices are equal to unity the $GN \times GN$ matrix between parentheses on the left-hand side of (A5) is nonsingular. One can verify that their proof easily extends to our more general case with arbitrary prices and $-1 < \beta_g < 1$ for all $g$. Consequently, the explicit expression of the Nash equilibrium is given by

$$\begin{aligned}\operatorname{vec} X = &\left(I_{GN} - B \otimes W + P^{-1}\gamma\beta' P \otimes W\right)^{-1} \\ &\times \left(b_0 \otimes \iota_N + P^{-1}\gamma \otimes y - (b_0' p) P^{-1}\gamma \otimes \iota_N\right). \qquad \text{(A6)}\end{aligned}$$

The vector $\hat{x} = \hat{x}(p, y)$, which is used in the paper to denote the Nash equilibrium can be read off directly from (A6).

### Derivation of Equation (5)

Using the utility function underlying the LES with IP, we obtain

$$\frac{\partial U_n(x)}{\partial x_{gk}} = \frac{-\gamma_g \beta_g w_{nk}}{\left[x_{gn} - b_{g0} - \beta_g \sum_{k=1}^{N} w_{nk} x_{gk}\right]}, \quad \forall g, \forall n, \forall k \neq n.$$

Next, we see from (A3) that in the Nash equilibrium $\hat{x}$ there must hold

$$\hat{x}_{gn} = b_{g0} + \beta_g \sum_{k=1}^{G} w_{nk} \hat{x}_{gk} + \frac{\gamma_g}{p_g} c_n, \quad \forall g, \forall n,$$

with

$$c_n \equiv \left[y_n - \sum_{h=1}^{G} p_h b_{h0} - \sum_{h=1}^{G} \sum_{k=1}^{N} \beta_h w_{nk} p_h \hat{x}_{hk}\right].$$

Observe that $c_n$ is independent of $g$. Moreover, because $\hat{x}_{gn} > b_{gn}$, $\forall n, \forall g$, we conclude that $c_n > 0$, $\forall n$. Finally, we see that

$$\frac{\partial U_n(\hat{x})}{p_g \partial x_{gk}} = \frac{-\gamma_g \beta_g w_{nk}}{p_g (\gamma_g / p_g) c_n} = \frac{-\beta_g w_{nk}}{c_n}, \quad \forall g, \forall n, \forall k \neq n,$$

which equals (5) of Subsection 3.1.

### Derivation of Equation (6)

Assume that $w_{nk} = \frac{1}{N-1}$, $\forall n \neq k$, and $y_n = \bar{y}$, $\forall n$. We then obtain from (A5) that $x_{gn} = x_g^{\text{Nash}}$, $\forall n, \forall g$. Writing $x^{\text{Nash}} = (x_1^{\text{Nash}}, \ldots, x_G^{\text{Nash}})'$, it turns out that in this case (A6) can be reduced to

$$x^{\text{Nash}} = \left(I_G - B + P^{-1}\gamma\beta' P\right)^{-1} \left(b_0 + (\bar{y} - b_0' p) P^{-1}\gamma\right).$$

Remark that the inverse matrix on the right-hand side of this equation is the inverse of a summation of a diagonal matrix and a matrix of rank 1. Therefore, we can write this inverse matrix as

$$\left(I_G - B + P^{-1}\gamma\beta' P\right)^{-1} = (I_G - B)^{-1} - \frac{(I_G - B)^{-1}P^{-1}\gamma\beta' P(I_G - B)^{-1}}{1 + \beta' P(I_G - B)^{-1}P^{-1}\gamma};$$

see e.g., Rao (1973, p. 33). Using this, it can be verified by straightforward manipulations that

$$\begin{aligned} x_g^{\text{Nash}} &= \left(\frac{1}{1-\beta_g}\right)\left[b_{g0} + \frac{\gamma_g}{p_g}(\bar{y} - b_0' p)\right] \\ &\quad - \frac{\gamma_g/(p_g(1-\beta_g))}{\sum_{h=1}^{G}\gamma_h/(1-\beta_h)}\left[\sum_{h=1}^{G}\frac{\beta_h p_h}{1-\beta_h}\left(b_{h0} + \frac{\gamma_h}{p_h}(\bar{y} - b_0' p)\right)\right], \quad \forall g, \end{aligned}$$

which in turn can be further simplified as

$$x_g^{\text{Nash}} = \frac{b_{g0}}{1-\beta_g} + \frac{\gamma_g/(1-\beta_g)}{p_g\sum_{h=1}^{G}\gamma_h/(1-\beta_h)}\left(\bar{y} - \sum_{h=1}^{G}\frac{p_h b_{h0}}{1-\beta_h}\right), \quad \forall g. \qquad \text{(A7)}$$

We remark that (A7) corresponds to (6) of Subsection 3.1.

## Derivation of Equation (9)

The Lagrange function associated with the problem (7) and (8) equals

$$\begin{aligned} L(x, \mu_1, \ldots, \mu_N) &= \sum_{n=1}^{N}\sum_{g=1}^{G} a_n\gamma_g \ln\left(x_{gn} - b_{g0} - \beta_g\sum_{k=1}^{N} w_{nk}x_{gk}\right) \\ &\quad - \sum_{n=1}^{N}\mu_n\left(\sum_{g=1}^{G} p_g x_{gn} - y_n\right), \end{aligned}$$

where the real scalars $\mu_1, \ldots, \mu_N$ denote the Lagrange multipliers. Taking the partial derivatives with respect to $x_{gn}$ and the Lagrange multipliers, we obtain the first-order conditions

$$\frac{a_n\gamma_g}{\left[x_{gn} - b_{g0} - \beta_g \sum_{k=1}^{N} w_{nk} x_{gk}\right]} - \sum_{\substack{q=1\\ q\neq n}}^{N} \left[\frac{a_q\gamma_g\beta_g w_{qn}}{x_{gq} - b_{g0} - \beta_g \sum_{k=1}^{N} w_{qk} x_{gk}}\right] = \mu_n p_g, \quad \forall g, \forall n$$

$$\sum_{g=1}^{G} p_g x_{gn} = y_n, \quad \forall n.$$

We observe that without further simplifications, it is not possible to derive an explicit solution of the social welfare maximization problem. Therefore, let us assume that $w_{nk} = \frac{1}{N-1}$, $\forall n \neq k$, $y_n = \bar{y}$, $\forall n$, $a_n = 1$, $\forall n$, and focus on the symmetric solution with $x_{gn} = x_g$, $\forall n$, $\forall g$, and $\mu_n = \mu$, $\forall n$. In that case, the first-order conditions simplify to

$$\frac{\gamma_g}{x_g - b_{g0} - \beta_g x_g} - \frac{\gamma_g\beta_g}{x_g - b_{g0} - \beta_g x_g} = \mu p_g, \quad \forall g$$

$$\sum_{g=1}^{G} p_g x_g = \bar{y}.$$

From these equations, it follows that in the present case the solution $x^{\text{Pareto}} = (x_1^{\text{Pareto}}, \ldots, x_G^{\text{Pareto}})'$, say, of the problem is given by

$$x_g^{\text{Pareto}} = \frac{b_{g0}}{1-\beta_g} + \frac{\gamma_g}{p_g}\left(\bar{y} - \sum_{h=1}^{G} \frac{p_h b_{h0}}{1-\beta_h}\right), \quad \forall g. \tag{A8}$$

Notice that (A8) corresponds to (9) of Subsection 3.2.

## Derivation of Equation (17)

First, notice that it follows from Equation (16) that

$$p_g^* = \left(\frac{p_g}{1-\beta_g}\right)\left(\frac{p_1^*(1-\beta_1)}{p_1}\right), \quad \forall g. \tag{A9}$$

Using (A8), (A9), and budget neutrality of the taxes and subsidies, we derive that

$$\sum_{g=1}^{G} p_g^* \left( \frac{b_{g0}}{1-\beta_g} + \frac{\gamma_g}{p_g} \left( \bar{y} - \sum_{h=1}^{G} \frac{p_h b_{h0}}{1-\beta_h} \right) \right)$$
$$= \sum_{g=1}^{G} \left( \frac{p_g}{1-\beta_g} \right) \left( \frac{p_1^*(1-\beta_1)}{p_1} \right) \left( \frac{b_{g0}}{1-\beta_g} + \frac{\gamma_g}{p_g} \left( \bar{y} - \sum_{h=1}^{G} \frac{p_h b_{h0}}{1-\beta_h} \right) \right) = \bar{y}.$$

In turn, this implies that

$$\frac{p_1^*(1-\beta_1)}{p_1} = \frac{\bar{y}}{T}, \tag{A10}$$

where

$$T = \sum_{g=1}^{G} \frac{\gamma_g}{(1-\beta_g)} \left( \bar{y} - \sum_{h=1}^{G} \frac{p_h b_{h0}}{1-\beta_h} \right) + \sum_{g=1}^{G} \frac{p_g b_{g0}}{(1-\beta_g)^2}.$$

Combination of (A9) and (A10) yields (17) of Subsection 3.4.

## References

ALESSIE, R. J. M., and A. KAPTEYN (1991) Habit formation, interdependent preferences, and demographic effects in the almost ideal demand system, *Economic Journal* **101**, 404–419.

ARONSSON, T., S. BLOMQUIST, and H. SACKLÉN (1999) Identifying interdependent preferences in an empirical model of labor supply, *Journal of Applied Econometrics* **14**, 607–626.

BERCK, P., and K. SYDSAETER (1991) *Economists' Mathematical Manual*. Berlin: Springer-Verlag.

BLOMQUIST, S. (1993) Interdependent behavior and the effect of taxes, *Journal of Public Economics* **51**, 211–218.

BOSKIN, M. J., and E. SHESHINSKI (1978) Optimal redistributive taxation when individual welfare depends upon relative income, *Quarterly Journal of Economics* **92**, 589–601.

COLE, H. L., G. J. MAILATH, and A. POSTLEWAITE (1992) Social norms, savings behavior, and growth, *Journal of Political Economy* **100**, 1092–1125.

CORNEO, G., and O. JEANNE (1997) Conspicuous consumption, snobbism and conformism, *Journal of Public Economics* **66**, 55–71.

DAVIS, O. A., and A. B. WHINSTON (1965) Welfare economics and the theory of second best, *Review of Economic Studies* **32**, 1–14.

DIAMOND, P. A. (1973) Consumption externalities and imperfect corrective pricing, *Bell Journal of Economics and Management Science* **4**, 526–538.

FRANK, R. H. (1985) The demand for unobservable and other nonpositional goods, *American Economic Review* **75**, 101–116.

FRIEDMAN, J. W. (1991) *Game Theory with Applications to Economics*. New York: Oxford University Press.

GAERTNER, W. (1974) A dynamic model of interdependent consumer behavior, *Zeitschrift für Nationalökonomie* **34**, 327–344.

IRELAND, N. J. (1994) On limiting the market for status signals, *Journal of Public Economics* **53**, 91–110.

KAPTEYN, A., S. A. VAN DE GEER, H. VAN DE STADT, and T. WANSBEEK (1997) Interdependent preferences: An econometric analysis, *Journal of Applied Econometrics* **12**, 665–686.

MANSKI, C. F. (1993) Identification of endogenous social effects: The reflection problem, *Review of Economic Studies* **60**, 531–542.

OSWALD, A. J. (1983) Altruism, jealousy and the theory of optimal non-linear taxation, *Journal of Public Economics* **20**(1), 77–87.

PERSSON, M. (1995) Why are taxes so high in egalitarian societies? *Scandinavian Journal of Economics* **97**, 569–580.

POLLAK, R. A. (1976) Interdependent preferences, *American Economic Review* **66**, 309–320.

POLLAK, R. A., and T. J. WALES (1992) *Demand System Specification and Estimation*. New York: Oxford University Press.

RAO, C. R. (1973) *Linear Statistical Inference and Its Applications*. New York: Wiley.

RAUSCHER, M. (1997) Conspicuous consumption, economic growth, and taxation, *Journal of Economics* **66**, 35–42.

SADKA, E. (1978) On the optimal taxation of consumption externalities, *Quarterly Journal of Economics* **92**, 165–174.

SANDMO, A. (1975) Optimal taxation in the presence of externalities, *Swedish Journal of Economics* **77**(1), 86–98.

WOITTIEZ, I. B., and A. KAPTEYN (1998) Social interactions and habit formation in a model of female labour supply, *Journal of Public Economics* **70**, 185–205.