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Some Relationships between T-Accounts, Input-Output Tables and Social Accounting Matrices

GRAHAM PYATT

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ABSTRACT *Some relationships between the T-accounting format for presenting commodity balances; input-output (IO) tables; and social accounting matrices are discussed in this paper. The starting point is to recognize that IO tables do not contain all the information that is needed to complete a social accounting matrix (SAM), or, therefore, for the modelling of phenomena that depend on having a fully articulated SAM, such as the interdependence of the distribution of income and the structure of production. There is a need, therefore, to establish the character of the extra information that is required and this can be achieved by imposing the requirement that a SAM should be consistent with the basic cash identity that is fundamental to all social accounting. A second agenda is to develop the argument that, while T-accounts can, in principle, provide a database equivalent to that of a SAM, in practice, they are typically found to be an imperfect substitute. It is important, therefore, in designing a database, to go beyond the confines of an (extended) IO system and T-accounts. SAMs provide an appropriate framework for doing so.*

KEYWORDS: *(Extended) input-output tables; national accounts; social accounting matrices (SAMs)*

1. Introduction

Economic statistics have made slow progress over the past 50 years. Foundations were laid prior to World War II both for national income accounting and input-output (IO) analysis, and their subsequent integration has been a theme running through various stages of development, culminating in the *System of National Accounts* (SNA) set out in by the United Nations (UNSO, 1968). An important stimulus for this effort was the demonstrable value of having an IO table to facilitate the computation of national income estimates via the commodity balance approach.

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Other topics which have long been recognized as candidates for development within the mainstream of the national income accounts are the flow of funds, asset accounts (or national balance sheets) and the distribution of income. Of these, the flow of funds has probably received most attention, while concern for the distribution of income was effectively divorced from the mainstream, with the introduction of a separate *System of Social and Demographic Statistics* (UNSO, 1975). As a result, the analysis of income distribution within its macro-economic context has been constrained by a lack of adequate data, or progress has depended on unofficial efforts to overcome the limitations of official statistics. In particular, the development of social accounting matrices (SAMs) and extended IO tables has been important in building foundations for analytic work.

However, SAMs and extended IO tables are not equivalent and one key difference can be explained by analogy. The essence of IO is not that production activity is disaggregated into different industries, but that these industries are related, one to the other, through transactions between them, i.e. through the buying and selling of raw materials, and that the structure of production is conditioned by these linkages. By the same token, the essence of a SAM, in this context, is not the disaggregation of institutions into different household types plus various categories of companies, government and the rest of the world, all of which is on offer through an extended IO approach. Rather, the essential detail is to be found in the matrix of transactions and transfers between the different types of institution. These details include the unrequited transfers which characterize the social security system and direct taxation, all types of private remittance and all property income flows. The pattern of these transfers conditions the distribution of income in exactly the same way as the pattern of interindustry transactions conditions the structure of production. From this, it must follow that theoretical models of income distribution which omit this detail, such as the multiplier model of Miyazawa and Masegi (1963), are necessarily problematic as explanations of income distribution, as are the databases that support them. This point is elaborated in Pyatt (1999), where it is argued that there is a long-standing misunderstanding of what Kurtz *et al.* (1998) refer to as 'Miyazawa multipliers'.¹

The contention in Pyatt (1999) is that these multipliers do not explain the distribution of income as it is normally understood in this context, i.e. the distribution of the income generated by current production across the various institutions that comprise a society, namely (socio-economic) groups of households, different types of company (such as private versus public enterprises) and different branches of government. Instead, if problematic assumptions are to be avoided, the Miyazawa multipliers have to be seen as explanations of the factorial distribution of income, which is the distribution of total value added (the GDP) as payments to different types of labour, fixed capital and other real assets that contribute their (factor) services to the generation of the GDP.

One reason for writing this paper is to respond to the possibility that such misunderstanding may arise from a failure to appreciate the strategic differences that exist between different data systems, i.e. between SAMs, commodity balances and extended IO tables. Therefore, the initial concern in this paper is to provide a formal demonstration of similarities and differences between these various data systems and, in particular, to identify the information that is required to complete a SAM, and why it is required, given an (extended) IO table as the starting point. There are various ways in which this demonstration can be achieved and the approach adopted has been chosen to highlight some particular aspects.

The exposition begins in Section 2 with the representation of commodity balances in what is known as the *T*-accounting format that has been promoted in an Inter-Secretariat Working Group report (ISWG, 1993) which replaces the earlier, matrix-based SNA set out in UNSO (1968). Then, in Section 3, this *T*-accounting representation is translated into a matrix format, which distinguishes between products, activities and factor services in order to describe the important characteristics of production activities and it adds to this detail a set of institutional accounts that complete the definition of the rows and columns of the matrix.

Three features of this matrix are important here. First, it will be evident from inspection that the lower right-hand quadrant of the matrix is not fully specified by the information that can be extracted from the commodity balances. Second, it will be shown that this same information is sufficient to complete the three other quadrants, and that an extended IO table can be derived from them. Third, it will be shown that, provided that the lower right-hand quadrant of the matrix is completed with care, the matrix as a whole can be turned into a SAM. Much of the initial motivation of this paper derives from the need to identify precisely the information that is necessary to achieve this closure.

Of course, some readers will be well aware of the information requirements of a complete SAM. However, the way in which these requirements are identified in Section 4 may still be of some interest. Specifically, the approach in Section 4 involves starting with the basic cash identity, articulated in the form of a matrix that records all cash transactions between all pairs of institutions, which is then augmented by details of the increases that accrue over the accounting period in the cash balances that are held by each of the institutions (other than the monetary authority). This augmented matrix is a SAM, and the main result to be established is that, if we remove from this SAM all transactions to which at least one party is a production activity, then the matrix that remains can be used to 'fill in' the lower right-hand quadrant, i.e. to complement the detail in an IO table, so as to obtain a fully articulated SAM. This implies that four types of information are required to close the IO table previously derived in Section 3, covering all current transfers between institutions; the flow of funds; trade in second-hand goods; and changes in cash balances.

Having demonstrated the way in which a complete SAM can be developed from an initial set of *T*-accounts for commodity balances, it is of some interest to show that one can move in the opposite direction and derive the *T*-accounts from an initial SAM. This possibility is demonstrated in Section 5. It should mean that the choice between a SAM and a full set of *T*-accounts is a matter of taste. However, in practice, the combination of *T*-accounts and (extended) IO tables has failed to respond to the needs of analysis, especially in the area of income distribution analysis, as previously noted. Therefore, while the order in which ideas are developed in this paper has its own internal logic, there is an alternative ordering which is also of interest, because it supports the view that, relative to the other data systems considered here, i.e. IO tables, commodity balances and *T*-accounts, the SAM is more fundamental. Hence, it can be maintained that SAMs are the preferred system, with other systems being essentially derivative and/or otherwise inferior.

The argument is straightforward in so far as *T*-accounts, commodity balances and IO tables can all be derived from a SAM, while derivation in the opposite direction, starting with commodity balances or an IO table, is not possible. The proposition is contentious, therefore, only in so far as a system of *T*-accounts can

constitute a complete information system in the same sense that a SAM is complete. Therefore, the case for preferring a SAM must depend first on its relative parsimony; second, on the insights to be gained by seeing economic structure presented in a SAM format; and, third, on advantages that arise in practice, as reflected, not least, in the fact that it is now more or less standard procedure for an important class of numerical models to be implemented on the basis of their underlying SAMs. This case is to be argued in Sections 6 and 7, drawing on the various constructions that are to be developed in Sections 2–5.

In Section 6, the argument is quite general, while, in Section 7, the most recent SNA, as set out in ISWG (1993), is used as an example of a *T*-accounting system within which problems arise in practice. Two problems in particular are noted, i.e. the unsatisfactory translation of asset accounts from the *T*-accounting format into a SAM; and the inadequate treatment of income generation and the distribution of income, as argued in Keuning (1998), for example. It is noted that both problems have been resolved by analysts working in the SAM tradition. Some final comments and conclusion are presented in Section 8.

2. Commodity Balances and *T*-accounts

A set of *T*-accounts, as illustrated in Table 1, provides a familiar starting point for the development of IO tables and their analysis. These accounts show, in whatever detail they may be specified, the balance of demand and supply for each good or service during a particular accounting period. For that reason, the table is sometimes referred to as a 'supply/use (or disposition) table'.² A basic characteristic of such a table is that, for any row, the entries that appear to the left of the central column add up to the same total as do those on the right-hand side. Hence, the aggregate supply of each commodity is matched by aggregate demand or uses. These common totals define the entries in the central column of the table, which is the trunk of the *T*-accounts.

Table 1 can have any number of rows, depending on the level of detail or disaggregation that is required within each of the broad categories 'products', 'factor services' and 'domestic institutions'. Here, as elsewhere in this paper, such detail is to be understood but will not be elaborated. Therefore, unless otherwise stated, the various submatrices which provide the building blocks of Table 1 are to be interpreted as being of no particular size. However, legitimate inferences about the conformity of these matrices can be drawn from the way that they are assumed to fit together in Table 1 and subsequently.

The category 'products' covers all goods and non-factor services which are recognized as being produced, and the first stratum (row) of the table shows the balance of demand and supply for each of the individual (groups of) products, valued at market prices.³ The main sources identified on the left-hand side of the table are 'the rest of the world', which supplies imports, and the various domestic production activities, which generate their own products, the value of which must be increased by import duties and indirect taxes (less subsidies) in order to arrive at the market value of aggregate supplies. These values are accounted for on the right-hand side by demand (both intermediate and final), with exact balance being guaranteed by the fact that the domestic element of final demand includes net increases in stocks of raw materials and finished products plus an appropriate allowance for work in progress, all of which contribute, by definition, to the

Table 1. Commodity balances in an extended T-accounting format

Supply				Demand		
Rest of the world	Domestic		Totals	Domestic		Rest of the world
	Institutions	Production activities		Production activities	Institutions	
$S'_{1,1}$ Imports of goods and non-factor services (c.i.f.)	$S'_{4,1}$ Indirect taxes on products plus import duties	$S'_{2,1}$ Gross outputs of products, valued at producer prices	Total supply of (demand for) new products at market prices	$S_{1,2}$ Use of raw material (= intermediate demand)	$S_{1,4}$ Domestic final demand for new products	$S_{1,5}$ Exports of goods and non-factor service (f.o.b.)
$S'_{3,3}$ Factor income paid abroad (= imports of factor services)	$S'_{4,3}$ Factor services supplied plus VAT receipts of government	0	Total supply of (demand for) factor services at market prices	$S_{3,2}$ Payments for factor services (including VAT)	0	0
0	<u>Some receipts of domestic institutions</u>	0	Some receipts of domestic institutions	$S_{4,2}$ Indirect taxes on activities plus operating surpluses	0	$S_{4,5}$ (part) Factor income received from abroad
<i>Total domestic expenditure on foreign goods and services</i>	<i>Total income generated by contributions to current production</i>	Gross outputs of products, valued at producer prices	Totals	Gross outputs of products, valued at producer prices	<i>Domestic final demand (at market prices)</i>	<i>Total foreign expenditure on domestic goods and services</i>

difference between total output and the aggregate absorption of products via intermediate (raw material) and final uses.

The remaining entries for goods and services are to be found in the second row of Table 1. They describe the flow of factor services through domestic markets. These services are supplied by the various institutions in our economy, i.e. by domestic households, companies and branches of government, or by foreign institutions (the rest of the world), in the sense that these institutions own the assets which generate the factor services that are supplied to production activities. In return, these same institutions receive payment for the factor services that they provide. Such payments are entered on the left-hand side of the second row of Table 1. Their origin in services rendered to domestic production activities, and paid for by them, is accounted for on the right-hand side of the same row.

The entries in the third row of Table 1 cover three items. First, we have details of any indirect taxes levied on activities. Second, 'factor income received from abroad' is recorded here: it corresponds to any payments made by foreign production activities to domestic institutions on account of factor services rendered. The third set of entries in this row of Table 1 records the 'operating surpluses' that are realized by production activities after paying for those factor services that they hire from the market. Therefore, operating surpluses are defined here as the return to the proprietors of particular trading activities for (a) providing some factor services free of charge, such as working capital, plant and machinery that is owned by the company and, therefore, does not have to be rented, their own labour, etc.; and (b) any supernormal profits (or losses) that may result from trading.

It is somewhat unusual to show such details separately: normal practice is to treat operating surpluses as payments for one or more factor service and, therefore, to include them in the second row of the table, along with any indirect taxes that may be levied on production activities, such as licence fees, permits, etc. One of the advantages of the more extended treatment in Table 1 is that it helps to make the point that the operating surplus of an activity is a balancing item that accounts for the difference between the value added by it (gross output less the actual use of raw materials) and the amount that it must pay to others for factor services (excluding interest) and in indirect taxes. From this, it follows, as a matter of definition, that the total costs of activities, as itemized in the columns on the right-hand side of Table 1 that are headed 'domestic production activities', must be equal to the gross outputs of these same activities, details of which are to be found in the corresponding columns on the left-hand side of the table.

A further implication of the inclusion in Table 1 of a third row of accounts—for 'domestic institutions'—is to require the entries within this row on the left-hand side of the table to offset those on the right-hand side. This is achieved under the column for domestic institutions by the inclusion of a diagonal matrix, the elements of which are equal to the sum of the entries within each row on the right-hand side. The diagonal matrix that is determined by this procedure is identified by underlining in Table 1.

This treatment guarantees that, within the third row of the table, the various row entries that appear on the two sides of the table will balance each other in aggregate. However, it does not imply that the column totals for domestic institutions on the two sides of the table will be equal. And there is no reason why they should be: there is no necessary reason why the final demand generated by an institution should be equal to the aggregate income that it receives as payment for the contributions that it makes to production (either in our economy or in the rest

of the world). Similarly, there is no reason why total domestic expenditure on foreign goods and services should be equal to total foreign expenditure on goods and services that are supplied by our economy. Hence, while the left- and right-hand columns for each production activity must have common totals, this is not the case for either foreign or domestic institutions: discrepancies can and will arise as between the column totals for institutions on the two sides of the table. For that reason, these totals are shown in *italics*.

A qualification of this conclusion is of some interest. It concerns the fact that the discrepancies previously referred to are subject to an aggregate constraint under which, in total, they must be equal to zero. This follows from the fact that, for every row in the main body of the table, the two sides of the *T*-accounts must balance, which has the implication that the final row of Table 1 (which records the column totals) articulates the accounting identity

$$\begin{aligned}
 &\text{Income generated by contributions to current production} \\
 &\quad + \\
 &\quad \text{Imports of goods and services} \\
 &\quad = \\
 &\quad \text{Domestic final demand} \\
 &\quad + \\
 &\quad \text{Exports of goods and services}
 \end{aligned}
 \tag{1}$$

Finally, it can be noted that the inclusion of all types of indirect taxes (which should be interpreted as being net of subsidies) adds important detail to the table. Indirect taxes on products are included in the product accounts on the left-hand side, as a receipt of government and, on the right-hand side, as an implicit element in the cost of products to their eventual buyers. Indirect taxes on activities, such as licence fees and the cost of concessions, are treated as a receipt of government and a cost of those activities that have to pay them. Similarly, value-added tax is a source of revenue for the government and a cost for every activity that is obliged to pay this tax (which, typically, is levied in proportion to the value added that an activity generates).

3. Closure of the IO System to Form a SAM

Most of the details of Table 1 are carried over without modification to Table 2, where they are reassembled in a matrix format. This translation is made explicit by the use of the notation S_{ij} for submatrices of Table 2.⁴ The only exception to this general treatment arises in relation to the diagonal matrix that appears on the left-hand side of Table 1 at the intersection of those rows and columns which record receipts of, and expenditures by domestic institutions. It will emerge from the argument to be developed in the following that this exception is of no particular significance. Meanwhile, it can be noted here that, if this element of Table 1 were to be carried over, then it would appear in Table 2 in row (iv), column (iv), where it would have the effect of creating a set of non-zero entries along the main diagonal of the table.

It is evident from inspection that, if the shaded quadrant in Table 2 is ignored, then Table 2 is an IO table which gives complete details of the demand for products and factor services, and their supply.

It is also evident from the table that, if the shaded quadrant is taken into account, then Table 2 is a square matrix which partially satisfies the criteria that would have to be met if the table were to be a SAM. First, it satisfies the condition

Table 2. IO table set in the format of a SAM

	(i) Domestic production			(v) Institutions		Totals
	(i) Products	(ii) Activities	(iii) Factor services	(iv) Domestic	(v) Rest of the world	
Domestic production						
(i) Products	0	$S_{1,2}$ Use of raw materials	0	$S_{1,4}$ Domestic final demand (at market prices)	$S_{1,5}$ Exports of goods and non-factor services (f.o.b.)	Total demand for products at market prices
(ii) Activities	$S_{2,1}$ Gross outputs of products (at producer prices)	0	0	0	0	Gross outputs of products, valued at producer prices
(iii) Factor services	0	$S_{3,2}$ Payments for factor services (including VAT)	0	0	0	Total demand for factor services at market prices
Institutions						
(iv) Domestic	$S_{4,1}$ Indirect taxes on products plus import duties	$S_{4,2}$ Indirect taxes on activities plus operating surpluses	$S_{4,3}$ Factor services supplied plus VAT receipts of government	$S_{4,4}$	$S_{4,5}$ (part) Factor income received from abroad	Total income generated by contributions to production
(v) Rest of the world	$S_{5,1}$ Imports of goods and non-factor services (c.i.f.)	0	$S_{5,3}$ Factor income paid abroad (= imports of factor services)	$S_{5,4}$	$S_{5,5}$	Total domestic expenditure on foreign goods and services
Totals	Total supply of products at market prices	Gross outputs of products, valued at producer prices	Total supply of factor services at market prices	Total expenditure on products at market prices	Total foreign expenditure on domestic goods and services	

that all rows and columns should be symmetrically defined. Second, to the extent that the matrix is complete, it conforms to the convention that an entry in the body of the table should represent (a) a receipt for whichever account is identified by the row in which that entry is located; and (b) an expenditure for the account identified by the column in which the entry in question is to be found. Therefore, the detail in Table 2 constitutes an example of single-entry accounting. Third, we can note that, if a matrix \mathbf{S} is to be a SAM, then it must have the property that

$$(\mathbf{S} - \mathbf{S}')\mathbf{i} \equiv 0 \quad (2)$$

i.e. all row and columns pairs must have identical totals. For those rows and columns of Table 1 which are complete, equation (2) is satisfied as a direct consequence of the way in which Table 1 is constructed. Thus, for the product accounts, the element of domestic final demand that covers 'changes in stocks' serves to guarantee that these accounts will balance: their row and column totals must be equal. For the production activity accounts, 'operating surpluses' act as the balancing items. For the 'factor services' accounts, the column entries serve only to allocate the aggregates which accumulate in the rows. Hence, the accounts for 'factor services' must also balance.

Since the entries in Table 2 do not violate any of the necessary conditions for the table as a whole to be a SAM, it must follow that, if the detail that is missing in the lower right-hand quadrant of the table is completed appropriately, then Table 2 can be developed into a SAM, i.e. that for an appropriate specification of the matrices $\mathbf{S}_{4,4}$, $\mathbf{S}_{4,5}$, $\mathbf{S}_{5,4}$ and $\mathbf{S}_{5,5}$, Table 2 can be developed into a complete SAM.

In order to identify what such an appropriate specification might be, it can be noted that the accounts (iv), which refer to domestic institutions, will necessarily satisfy equation (2) if

$$(\mathbf{S}_{4,4} - \mathbf{S}'_{4,4})\mathbf{i} + (\mathbf{S}_{4,5} - \mathbf{S}'_{5,4})\mathbf{i} = \text{Final demand for new products} \quad (3)$$

- Revenue from indirect taxes and subsidies
- Operating surpluses received
- Payments received for factor services rendered

i.e. if their total receipts match their total outlays.

Similarly, in relation to the rest of the world accounts, the analogous requirement is that

$$(\mathbf{S}_{5,4} - \mathbf{S}'_{4,5})\mathbf{i} + (\mathbf{S}_{5,5} - \mathbf{S}'_{5,5})\mathbf{i} \equiv \text{Expenditure by the rest of the world on our exports} \quad (4)$$

- Revenue of the rest of the world from our imports
- Factor income received from our economy

As the next step in the argument, it is possible to develop the implications of these two conditions for the general case in which there are many domestic institutions, each with its own, separate accounts and with the 'rest of the world' being similarly disaggregated. However, little is lost for present purposes by assuming that there is no disaggregation of the rest of the world; and there is some gain from a pedagogic point of view.

By assuming that the rest of the world is not disaggregated, it follows that a full accounting of transactions with the rest of the world requires only one row in

Table 2 and one column.⁵ And from that, it follows that, if equation (3) is satisfied, then equation (4) will also be satisfied, so that, from an analytic perspective, it is sufficient to identify the circumstances in which equation (3) will be satisfied in order to ensure that Table 2 is a SAM.

This useful result depends on the fact that, for any square matrix \mathbf{S} , all corresponding rows and columns will have matching totals if such a condition is satisfied for all pairs but one. Thus, the (aggregate) account for the rest of the world will balance if, in addition to the production accounts, the separate accounts for each domestic institution are also balanced. Hence, if the separate accounts which describe transactions with the rest of the world are consolidated into a single account, then equation (3) is a sufficient condition for Table 2 to be a SAM.

The implications of equation (3) will be developed in the following with the assistance of Table 3. However, before we come to that, two points about the equation as it appears should be noted. First, it can be noted that factor income received from abroad appears in equation (3) alongside any factor income received by domestic institutions in payment for services supplied to domestic production activities. Hence, these two types of income can be combined at this point. Second, it can also be noted that, if the diagonal element on the left-hand side of the third row in Table 1 had been carried forward to Table 2, then it would have made a contribution to the diagonal of matrix $\mathbf{S}_{4,4}$. However, since this matrix enters into equation (3) in the form $(\mathbf{S}_{4,4} - \mathbf{S}'_{4,4})\mathbf{i}$, it follows that equation (3) is independent of the diagonal elements of matrix $\mathbf{S}_{4,4}$. Hence, for present purposes, there would be no point in carrying forward this diagonal element from Table 1 to Table 2.

Turning now to the detail of Table 3, we can begin by noting that the first row of the table repeats the information contained in equation (3), with (net) receipts or revenues on the left-hand side and outlays or expenditures on the right.

The second row of Table 3 reproduces the details of account (ii) in Table 2. It can be read as a definition of the entry 'operating surpluses received by domestic institutions', which appears as an entry in Table 3 both on the left-hand side, in row 1, and on the right-hand side, in row 2. These two entries cancel each other out, therefore. Similarly, row 3 of Table 3 provides a definition of the 'gross outputs of production activities at producer prices' in terms of sales at market prices, indirect taxes and changes in stocks. Row 4 provides a similar elaboration of 'final expenditures by domestic institutions on new products at market prices'.

In row 5, the relationship between intermediate use of products and purchases for intermediate use is defined. In row 6, the implications of indirect taxes on factor services are noted. To complete the detail, a disaggregation of 'purchases of raw materials by production activities at market prices' is provided in row 7.

The reason for assembling all these details in this particular way is to arrive at the expression for $(\mathbf{S}_{4,4} - \mathbf{S}'_{4,4})\mathbf{i} + (\mathbf{S}_{4,5} - \mathbf{S}'_{5,4})\mathbf{i}$ as in row 9. This is reached in two steps, the first of which, in row 8, is simply to aggregate all the entries in rows 1–7 on the two sides of Table 3, after cancelling out those entries (shown in italics) that appear on both sides of the table. This intermediate result is then summarized in row 9 as

$$\begin{aligned}
 (\mathbf{S}_{4,4} - \mathbf{S}'_{4,4})\mathbf{i} + (\mathbf{S}_{4,5} - \mathbf{S}'_{5,4})\mathbf{i} = & \text{Indirect tax payments less revenues received} \\
 & + \text{Net purchases of new goods and services} \\
 & \quad \text{(at market prices) by domestic institutions} \quad (5) \\
 & \quad \text{on their own account or on behalf of} \\
 & \quad \text{production activities}
 \end{aligned}$$

Table 3. The implications of equation (3)

Revenues	Expenditures
1 $(S_{4,4} - S'_{4,4})i + (S_{4,5} - S'_{5,4})i$ Indirect tax revenues received (by government) Operating surpluses received by domestic institutions Payments to domestic institutions for factor services rendered to production Factor income received from abroad by domestic institutions	<i>Final expenditures by domestic institutions on new products at market prices, i.e. excluding net purchases of second-hand goods</i>
2 <i>Gross outputs of new products by production activities, valued at producer prices</i>	<i>Operating surpluses received by domestic institutions</i> <i>Use of raw materials (valued at market prices) by production activities</i> <i>Purchases of factor services (at market prices) by production activities</i> <i>Indirect taxes on production activities</i>
3 Sales of products by production activities, valued at market prices <i>Value of increases in work in progress and stocks of finished products held by production activities</i>	<i>Gross outputs of new products by production activities, valued at producer prices</i> Indirect taxes on the products of production activities
4 <i>Final expenditures by domestic institutions on new products at market prices, i.e. excluding net purchases of second-hand goods</i> Net purchases of second-hand goods	Purchases by domestic institutions of products at market prices for final use <i>Value of the increases in stocks of raw materials held by production activities</i> <i>Value of increases in work in progress and stocks of finished products held by production activities</i>
5 <i>Use of raw materials (valued at market prices) by production activities</i> <i>Value of increases in stocks of raw materials held by production activities</i>	<i>Purchases of raw materials (at market prices) by production activities</i>
6 <i>Purchases of factor services (at market prices) by production activities</i>	Purchases of factor services (at supplier's prices) by production activities Indirect taxes on the factor services hired by production activities
7 <i>Purchases of raw materials (at market prices) by production activities</i>	Purchases of imports (c.i.f.) by production activities from the rest of the world Indirect taxes on the imports of each production activity Purchases of raw materials of domestic origin (at market prices)
8 $(S_{4,4} - S'_{4,4})i + (S_{4,5} - S'_{5,4})i$ Indirect tax revenues received (by government) Payments to domestic institutions for factor services rendered Sales of products by production activities, valued at market prices Net purchases of second-hand goods	Indirect taxes paid on activities, products, factor services and imports Purchases of factor services (at supplier's prices) by production activities Purchases by domestic institutions of products for final use at market prices Purchases of imports (c.i.f.) by production activities from the rest of the world Purchases of raw materials of domestic origin (at market prices)
9 $(S_{4,4} - S'_{4,4})i + (S_{4,5} - S'_{5,4})i$	Indirect tax payments <i>less</i> revenue received Net purchases of new goods and services at market prices

Table 4. Representation of a SAM as an augmented transactions matrix

Institutions	Institutions			Totals
	The monetary authority	Government	All other institutions	
The monetary authority	0	0	Increases in cash balances ^{a,b}	Increases in the base money supply ^b
Government	Increase in the base money supply ^b	Cash transactions	Cash transactions	Total expenditure
All other institutions	0	Cash transactions	Cash transactions	Total cash receipts
Totals	Increase in the base money supply ^b	Total expenditure	Total cash receipts	Totals

Notes: ^a Includes deposits of the commercial banking sector with the central bank/monetary authority.

^b All increases are aggregate accruals over the accounting period and may be negative.

^c The matrix of all cash transactions is referred to as matrix **T** in the main text.

4. The Cash Identity

The cash identity on which social accounting is founded builds on the fact that, for any institution, other than the government and its monetary authority, the difference between aggregate receipts and expenditures must be equal to the increase in cash balances. Hence, by complementarity, the aggregate of government expenditures minus receipts is equal to the aggregate of the increase in the cash balances of other institutions. Table 4 illustrates this accounting identity.

The third row and column in Table 4 show that, for all institutions other than the government and its monetary authority, the differences between total cash receipts and expenditures that arise as a consequence of their involvement in transactions must emerge as a change in cash balances (which are defined to include all credits with the central bank, such as the credits (deposits) of the commercial banking sector). In aggregate, these increases in cash balances constitute the increase in the base money supply, which is a form of purchasing power for the government that complements the revenue that the government receives as a consequence of cash transactions. In other words, total government expenditure is financed either through cash transactions (tax revenue, sale of bonds, etc.) or by expansion of the base money supply.

The matrix in Table 4 is a SAM because it satisfies equation (2), such that every pair of corresponding row and column totals must be equal. The construction of a SAM requires, therefore, that the matrix **T** of all cash transactions should be augmented by details of any increase in the base money supply, disaggregated to show the complementary increases in cash balances for each institution.

To demonstrate the relevance of this cash identity in the form of Table 4 to the closure of the matrix in Table 2, we need to investigate further the nature of the cash transactions which are recorded in the matrix **T**. Figure 1 gives an initial indication of the variety of transaction types that are likely to be involved. At the most detailed level shown in the figure, nine types of transaction are distinguished. However, this is purely illustrative and considerable further disaggregation is possible. Nevertheless, the detail shown in the figure is sufficient for present purposes. It suggests that the matrix **T**, which covers all these transactions in

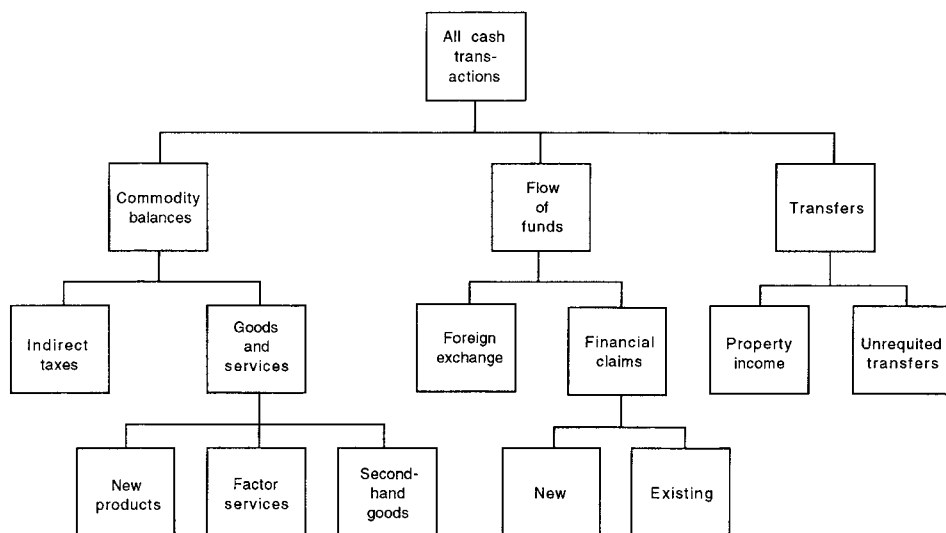


Figure 1. Disaggregation of the matrix **T** of cash transactions.

aggregate, can be disaggregated, in the first instance, to recognize the three main categories of transactions that are distinguished in the figure, i.e. all those different transactions which, in combination, generate 'commodity balances', 'the flow of funds' and 'transfers'.

Further disaggregation within the category 'commodity balances' suggests two main branches which refer, first, to the collection and eventual payment to the government of indirect taxes; and, second, to the buying and selling of goods and services. Similarly, there are two main types of transaction covered by the flow of funds. One is the buying and selling of foreign exchange, and the other derives from the creation of new financial claims and trading in those that already exist. Finally, there are two branches distinguished within the category 'transfers'. One refers to the property income (interest and dividend) payments which service financial claims, and the other covers a range of unrequited transfers, such as personal gifts, charitable contributions and direct taxes on wealth and income.

Each of the six boxes which are represented at this second level of disaggregation in Fig. 1 is a matrix that records, in aggregate, all transactions within the accounting period that fall within a particular category. In every case, the rows and columns are symmetrically defined with reference to the same set of institutions, and the addition of all six matrices yields the matrix **T** of all transactions in total.

Consider now what would happen if we formed a variant of the SAM in Table 4, in which the matrix **T** was replaced by those elements of **T** which contribute either to the flow of funds or to transfers. The resulting matrix would not be a SAM, because all transactions related to commodity balances would not be taken into account. Indeed, to be more specific, the row totals of the matrix obtained via the suggested aggregation would differ from the corresponding column totals by an amount for each institution that was equal to the difference between those particular outlays and receipts that are the result of transactions that fall within the category 'commodity balances'.

An implication of this last result is that, if the lower right-hand quadrant of Table 2 were to be filled by a matrix of all flow of funds transactions plus transfers

plus details of the current operations of the monetary authority, then we would come very close to satisfying equation (5). Therefore, we would be near to the closure of the IO table in Table 2, so as to form a SAM. Moreover, the discrepancies that would remain could be shown to depend on a failure to include transactions in second-hand goods in the lower right-hand quadrant, i.e. on those transactions in which the seller is not a production activity.⁶ Specifically, the details in the final row of Fig. 1 imply that

$$\begin{aligned}
 & \text{Net indirect tax receipts} \\
 & \quad + \\
 & \text{Net revenue from the sale of new products} \\
 & \quad + \\
 & \text{Net revenue from the sale of second-hand goods} \\
 & \quad + \\
 & \text{Net revenue from the sale of financial claims (the flow of funds)} \\
 & \quad + \\
 & \text{Net revenue from transfers} \\
 & \quad = \\
 & \text{Net increases in cash balances}
 \end{aligned} \tag{6}$$

which (because of equation (5)) can be rearranged as

$$\begin{aligned}
 & \text{Net revenue from the sale of second-hand goods} \\
 & \quad + \\
 & \text{Net revenue from the sale of financial claims} \\
 & \quad + \\
 & \text{Net revenue from transfers} \\
 & \quad + \\
 & \text{Net decreases in cash balances} \\
 & \quad = \\
 & (\mathbf{S}_{4,4} - \mathbf{S}'_{4,4})\mathbf{i} + (\mathbf{S}_{4,5} - \mathbf{S}'_{5,4})\mathbf{i}
 \end{aligned} \tag{7}$$

Therefore, it follows that the closure of an IO table to complete a SAM requires that each of the four considerations listed as contributing to the final expression in equation (7) has to be taken into account.⁷

A point of some historical interest is that only three of the four categories of information were missing from the original formulation of an IO table in Leontief (1936); current transfers were included. As Leontief (1936, p. 106) explained: 'The theoretical basis of the subsequent statistical analysis is rather simple. The economic activity of the whole country is visualised as if it was covered by one huge accounting system.' However, because some categories of information are missing, Leontief's system of accounts is not fully articulated: 'the expenditure and revenue accounts ... cover explicitly all the commodity and service transactions but not the so-called capital items' (p. 106). Hence, '... the credit and the debit sides of the account for each particular enterprise or household will not necessarily balance ...' (p. 106).

In subsequent expositions of his framework, Leontief dropped the record of current transfers between households and enterprises from his database, and it is only over time that analysts such as Lager (1988) have come to realize that this detail needs to be restored in order to model the distribution of income in relation to, and as a determinant of, the structure of production.⁸

5. The (Re)Construction of *T*-accounts from a SAM

The previous analysis demonstrates the possibility of progressing from a set of *T*-accounts for commodity balances to a fully articulated SAM. To round off that discussion, it is of some interest to show that one can proceed in the opposite direction, i.e. from a SAM to *T*-accounts. Table 5 illustrates the procedure.

The right-hand side of Table 5 shows a SAM in schematic form, using the same notation as previously, and the left-hand side reproduces the same matrix, but in transposed form. Hence, equation (2), which states that all corresponding row and column totals of a SAM must be equal, translates in Table 5 into the condition that the sum of those entries in any row of the table that appear to the left-hand side of the central column (or trunk) of the *T*-accounts must be equal to the sum of those entries in the same row that appear to the right-hand side. This construction also serves to illustrate that, while a SAM is essentially a single-entry form of accounting, the *T*-accounting approach is inherently double entry.

Several steps are now needed to transform Table 5 into the version of *T*-accounting that is illustrated in Table 1. The first step is simply to eliminate two of the rows and two columns on each side of the table.

The essential concern in Table 1 was to capture all the details that are to be found in the first and third rows of Table 5. Also, as can be seen from inspection, none of this detail is lost if those columns in Table 5 that relate to products and factor services are eliminated on both sides of the central column. Moreover, the (horizontal) row for production activities and institutions could also be eliminated at this point. However, it is apparent (again by inspection) that, if the row which refers to domestic institutions is retained, then full information can be retained on the production activity accounts, the details of which are captured in column form on the two sides of the central trunk. Hence, by eliminating all the detail which is shaded in Table 5, a full accounting for products, activities and factor services is retained.

The table that results from these eliminations has the same format as Table 1, but the row entries that describe transactions by domestic institutions are different. An explanation of the differences can be constructed in two parts.

First, the elimination of whole columns on both sides of Table 5 removes three entries which contribute to the overall balance in the original table of transactions that involve domestic institutions. Therefore, to restore this balance, some adjustments have to be made. The most straightforward way to do this is to add two diagonal matrices to the original matrix, as explained in the notes to the table; one on the left-hand side, to compensate for the elimination of $S'_{1,4}$, and the other on the right-hand side, to compensate for the loss of both $S_{4,1}$ and $S_{4,3}$.

The version of Table 5 that is obtained as a result of these adjustments is more elaborate than Table 1. Nevertheless, it is a *T*-accounting system. Moreover, the extra detail that it contains relative to Table 1 is sufficient to provide most of the information that is needed to fill in the lower right-hand quadrant of Table 2. All that is missing is the detail of the matrix $S_{5,5}$. Also, as we have noted previously, this omission is of no consequence if the account for the rest of the world is not disaggregated, because, in that event, $S_{5,5}$ reduces to a scalar which appears on the main diagonal of Table 2. Therefore, in such circumstances, the *T*-accounting format shown in Table 5 is sufficient to obtain a complete SAM representation of the relevant transactional relationships.

From these arguments, it follows that Table 1 can be obtained from Table 5 by

Table 5. (Re)Construction of *T*-accounts from a SAM

(v)	(iv)	(iii)	(ii)	(i)		(i)	(ii)	(iii)	(iv)	(v)
Institutions		Domestic production				Domestic production			Institutions	
Rest of the world	Domestic	Factor services	Activities	Products	Totals	Products	Activities	Factor services	Domestic	Rest of the world
S'_{5,1}	S'_{4,1}	0	S'_{2,1}	0	Total demand for products at market prices	0	S_{1,2}	0	S_{1,4}	S_{1,5}
0	S'_{4,2}	S'_{1,2}	0	S'_{1,2}	Gross outputs of products, valued at producer prices	S_{2,1}	0	0	0	0
S'_{5,3}	S'_{4,3}	0	0	0	Total demand for factor services at market prices	0	S_{3,2}	0	0	0
S'_{5,4}	S'_{4,4}^a	0	0	S'_{1,4}	Total revenue of domestic institutions	S_{4,1}	S_{4,2}	S_{4,3}	S_{4,4}^a	S_{4,5}
S'_{5,5}	S'_{4,5}	0	0	S'_{1,5}	Total receipts of the RoW from our economy	S_{5,1}	0	S_{5,3}	S_{5,4}	S_{5,5}

Note: ^a To preserve the balance of the third row of the table, it is necessary to add both a diagonal matrix \hat{a} in column (iv) on the left-hand side, where $\mathbf{a} = S'_{4,1}\mathbf{i}$; and a second diagonal matrix \hat{b} in column (iv) on the right-hand side, where $\mathbf{b} = S_{4,1}\mathbf{i} + S_{4,3}\mathbf{i}$.

further elimination of detail. Specifically, the matrices $\mathbf{S}_{4,4}$ (and its transpose) and $\mathbf{S}_{5,4}$ need to be suppressed, and the matrix $\mathbf{S}_{4,5}$ has to be replaced by that part of it which captures details of the factor income earned abroad by domestic institutions. The balance of the account for institutions can now be restored by adding a diagonal matrix on the left-hand side, which is equal to

$$(\mathbf{S}_{4,4} - \mathbf{S}'_{4,4})\mathbf{i} + (\mathbf{S}_{4,5} - \mathbf{S}'_{5,4})\mathbf{i} \quad (6)$$

if any factor income received from abroad can be ignored. It will be recalled that this same expression has arisen previously in the context of Table 3. This is not a coincidence.

6. A Hierarchy among Alternative Data Systems

There is logical hierarchy among the alternative data systems considered in this paper, which can be demonstrated by considering a three-step process in which commodity balances are derived from an initial SAM via a system of *T*-accounts, and the associated IO table is derived from the commodity balances. This can be shown as follows.

Let the initial SAM be denoted by a matrix \mathbf{S} . Then, a system of *T*-accounts, such as that illustrated in Table 5, can be generated as a new matrix of the form $(\mathbf{S}' : \mathbf{y} : \mathbf{S})$, where the elements of the vector \mathbf{y} are the equivalent row and column totals of the SAM.⁹ Hence, the possibility of expanding a SAM into a complete system of double-entry accounts is established.¹⁰

As a second step, this complete system of double-entry accounts can be reduced to a commodity balance system, by eliminating those rows of the matrix $(\mathbf{S}' : \mathbf{y} : \mathbf{S})$ which do not record details of the supply and disposition of goods and services. This necessarily involves some loss of information.

An option at this point is to aggregate or combine some columns of the matrix that is obtained as a result of the previous step, i.e. of the censored version of the matrix $(\mathbf{S}' : \mathbf{y} : \mathbf{S})$. This will have no effect on the balance of the remaining rows. And it may or may not involve some further loss of information, since many elements on both sides of any given row of $(\mathbf{S}' : \mathbf{y} : \mathbf{S})$ will normally be zero.¹¹

As a result of these first two steps, a commodity balance table, such as that shown previously as Table 1, can be obtained. Hence, the feasibility of starting with a SAM \mathbf{S} , duplicating the data to form double-entry *T*-accounts, and then extracting from the *T*-accounts those data that are necessary to construct commodity balances can be formalized.

The final step, from commodity balances to an IO table, is illustrated by the transformation from Table 1 to Table 2, with Table 2 being an IO table which necessarily contains less information than does the original SAM, because information has been lost en route when rows of the *T*-accounts are eliminated. In addition, there may or may not have been further loss of information, depending on whether or not some columns of the *T*-accounts are combined before making the final IO table.

The fact that there are two possible reasons for a loss of information in the process of forming an IO table from a complete set of *T*-accounts, i.e. the censoring of rows and the aggregation of columns, provides one way of explaining why it is misleading to characterize a SAM as an IO table in which the consolidated account for institutions has been disaggregated to allow the final demands of different household types to be shown separately. All that can be achieved by such disaggregation is to undo any loss of information that has occurred en route from the

complete *T*-accounts to the IO table, as a result of some columns of the *T*-accounts being aggregated. Therefore, it has no bearing on the loss of information that necessarily results from the elimination of rows of the *T*-accounts. Therefore, the difference between a SAM and an IO table is more fundamental than a disaggregation of the consolidated account for institutions. In other words, an extended IO table is not a SAM.

This argument needs to be qualified in so far as, if all institutional transactions are consolidated into a single account, then the resulting IO table will be a complete SAM, because all transactions between institutions will be eliminated as a result of the consolidation. However, such an eventuality does not constitute an important caveat: it is analogous to a situation in which there can be no interindustry transactions, because all production activities have been consolidated into a single production account (see Batey & Rose, 1990). There is, therefore, no escape from the proposition that details both of interindustry transactions and of interinstitutional transactions are necessarily a part of a SAM and, hence, of those (complete) models of the interdependence of income distribution and production structure which a SAM can support. Any other approach that fails to take into account the consequences of interinstitutional transactions (the lower right-hand quadrant of the SAM) cannot support a satisfactory model of the interdependence of income distribution and production structure. As noted in the Introduction, I have developed this point in Pyatt (1999), by showing that the well-known Miyazawa model makes sense as a model of the factorial distribution of income but, as a model of the distribution of income among institutions, it is problematic.

The first of the three steps described, which involves the formation of the *T*-accounts ($\mathbf{S}' : \mathbf{y} : \mathbf{S}$) from an initial SAM, suggests that, in avoiding unnecessary duplication, the SAM gains over the *T*-accounting format in terms of elegance and parsimony. However, since there is no loss of information involved in the mapping of a SAM into the complete set of *T*-accounts as given by ($\mathbf{S}' : \mathbf{y} : \mathbf{S}$), no options are foreclosed if the *T*-accounting format is preferred to the SAM, if only on the grounds of familiarity. Either way, it should always be possible to derive the underlying SAM from a complete system of *T*-accounts, so that, in this sense, the two are equivalent, i.e. it should be possible to reconstruct each of these data systems from the other.

7. SAMs and the SNA

In practice, it is not so easy to derive the underlying SAM from a set of *T*-accounts, and this can be illustrated with reference to the 1968 and 1993 versions of the SNA (see ISWG, 1993; UNSO, 1968). From the outset, the 1968 version was formulated as a SAM that is more elaborate than those considered in this paper, because it includes a complete set of asset accounts. In contrast, the 1993 SNA has been formulated, in the first instance, as a complete set of *T*-accounts, which also includes asset accounts. However, SAMs are discussed at some length in ISWG (1993) in order to establish continuity with the earlier version, as set out in UNSO (1968). The initial discussion revolves around Table 2.5 in ISWG (1993).¹² Subsequent discussion is to be found in Chapter 20 of ISWG (1993), where several SAMs are presented, some of which include asset accounts, while others do not.

In relation to Table 2.5, the authors of ISWG (1993) recognize that their attempt to include asset accounts in a SAM is not entirely satisfactory and that further work on the matter is called for. Specifically, they say (ISWG, 1993, p. 57):

The [SAM presented in Table 2.5] is considered provisional because improvements are needed, particularly in the presentation of the accumulation accounts and balance sheets ... The difficulty is related to the use of the main feature of the matrix presentation i.e. that each item which is presented twice in the [*T*-accounts]—as use and resource, asset and liability—is only included once in the matrix: at the intersection of the column of the account in which it is a use or asset and the row of the account in which it is a resource or liability, or vice versa.

This confirms the point that it is not a trivial matter to move from a set of *T*-accounts to a SAM, despite the fact that, logically, it should be possible. Indeed, the authors of the 1993 SNA could have reproduced the SAM arrangement of asset accounts that is promoted in the 1968 SNA. They evidently rejected this option, perhaps because several commentators have found this aspect of the 1968 SNA difficult to follow. Alternatively, the reason could be that the *T*-accounts for assets that follow from the 1963 SNA proposals according to the mapping from **S** to (**S'**:**y**:**S**) are different from the preferred presentation of asset accounts as set out in Table 2.5 of ISWG (1993). In Pyatt (1996), I have taken up the challenge of trying to include asset accounts in a SAM framework in a way that can be related directly to the *T*-accounting format of the 1993 SNA. Therefore, an alternative to the treatment in the 1968 and 1993 SNAs is available.¹³

Turning now to the discussion of SAMs in Chapter 20 of ISWG (1993), the views of Keuning (1998) on the relationship between *T*-accounts and SAMs is of some interest. Keuning (1998, pp. 438–439) writes:

Unfortunately the linkage between the [commodity balance table] i.e. a detailed overview of production and sales, on the one hand, and the institutional sector accounts (i.e. income distribution, expenditures, etc.) on the other hand, does not receive much attention ... in the 1993 SNA's central framework itself. Specifying this linkage—that is, accounting for the generation of income (value added) by means of labour and capital inputs—is key to a better understanding of differences in welfare and productivity among nations and over time. Instead of the trivial “generation of income” account in the 1993 SNA, this account should record the actual remuneration of homogeneous groups of production factors, paid by industries (the demand side of factor markets) and received by institutional sectors (the supply side of factor markets). The 1993 SNA's central framework distinguishes neither various types of labour input (by sex, skill level, etc.) nor various types of capital input (by asset type) in production. In view of the important role of (human, financial, other tangible, natural and fixed) capital in economics, this is a serious omission. It is ironic that, in an era in which management gurus and economic policy advisors alike emphasise the pivotal role of human resources, the core national accounts provide no insight whatsoever [into] the qualifications of the actual and potential labour force. For a remedy, one must look to the SAM as elaborated in Chapter 20 of the 1993 SNA ...

Keuning's argument, as set out here, focuses attention on the need to disaggregate value added in order to capture details of the relative importance of different factors of production in generating the totals. Therefore, his argument is the natural complement to the case for disaggregating final demand to capture the relative

importance of different institutions in particular markets. Indeed, the circular flow from final demand to the generation of factor incomes, the expenditure of which drives final demand, is basic to all economies. A SAM provides a straightforward way of capturing the details of this circular flow, as emphasized by Keuning, who could have added that a SAM presents these details in a way which makes the circularity explicit: the three submatrices of the SAM that record the relevant details are located within the overall SAM, in a pattern that corresponds to a circular permutation matrix. Therefore, the SAM makes it possible to see what is happening within an economy, in a literal sense, with which the *T*-accounts cannot compete.¹⁴

While Keuning is correct to emphasize the importance of the mapping of value added from production activities to different factors of production, his discussion might also have addressed two additional concerns (as might the discussion in Chapter 20 of ISWG (1993)). First, in order to construct the story of the circular flow, it is necessary to have a submatrix within the SAM that maps factor incomes received back to the institutions that provide the relevant factor services. Therefore, Keuning has chosen to emphasize just one of three mappings, all of which are necessary ingredients of a complete picture and, hence, of a SAM which is designed to include separate accounts for the factors of production. These mappings are the mapping of value added from activities to factors in payment for their services (as flagged by Keuning); the mapping of factor income to the institutions which provide factor services; and the mapping of the income of institutions into demand for goods and non-factor services. Here, we can note that the second of these mappings depends on the distribution, across institutions, of ownership of the factors of production and, hence, on savings (past and present) which are used to finance the acquisition of real assets. Therefore, this mapping is one way in which the distribution of wealth influences the distribution of income and the structure of production and, in turn, is influenced by them.

The second point that might usefully have been considered in greater detail in Chapter 20 or elsewhere in ESWG (1993) is one that has already been referred to as being of central concern in this paper. The three submatrices of the SAM discussed above already are necessary to describe the circular flow of income in an economy, but they are not sufficient. The additional requirements are, in no particular order, details of interindustry transactions and details of the transfers that take place between institutions. The first of these requirements is, of course, satisfied by an IO table. The second calls for the additional information that is necessary to develop an IO table into a fully articulated SAM. Most of the present paper has been taken up with the identification of these additional details which, as already noted, include unrequited transfers (direct taxation, social security payments, remittances, etc.) and property income transfers of rent, interest and the payment of dividends by the corporate sector to their shareholders. Both types of transfer are, of course, essential ingredients of an understanding of the way in which income distribution and production structures are interwoven. Moreover, reference to the corporate sector makes the further point that there is more involved here than the disaggregation of the household sector.

8. Some Comments in Conclusion

The analysis in this paper has taken as its initial objective the precise identification of differences between and the interrelationships among different data systems, i.e.

T-accounts, commodity balances, (extended) IO tables and SAMs, using formal (mathematical) constructs. A second objective has been to show that, among these alternatives, the SAM is more fundamental, in the sense of being a sufficient statistic for the other data systems and, in practice, has distinct advantages of two types. One of these advantages is the reduction of the risk of confusion, such as that which attends the treatment of capital accounts in ISWG (1993). This is because, being more fundamental in the formal sense of a sufficient statistic, SAMs encourage recourse to first principles, and this can be a distinct advantage in trying to unravel the accounting structure that underlies a particular model or analytic framework. A second advantage in practice is the emphasis on consistency and the importance of complete articulation, both of which are of the essence in trying to understand feedback systems, in general, and the interdependence of the distribution of income and the structure of production, in particular. The limitations of extended IO analysis in this area, such as that of Miyazawa and Masegi (1963), illustrates this point. Here, we have noted that a satisfactory treatment of such phenomena ultimately requires a complete statement of the circular flow from commodity demands to factor demands via gross outputs and, hence, back to the institutions that generate the final demand for commodities. This circular flow is the essence of the multiplier process and the interdependences of income distribution and the structure of production. However, this whole process is significantly modified both by the interdependence of production activities via IO and by the interdependence of institutions via the transfers and property income flows that have been discussed in this paper.

Finally, it can be recalled that the way in which these details have been identified in this paper builds on the approach in Pyatt (1991). Therefore, the starting point for the construction of a SAM is a simple matrix of all cash transactions between a closed set of institutions, augmented by changes in cash balances. The presentation in Section 3 has provided a more formal statement of this core from which all social accounting develops. *A fortiori* this is also the core of all *T*-accounting systems, including the 1993 SNA; the core of IO tables; and the core of multiplier analysis that is properly based on a complete articulation of the transactional relationships between and among institutions and their respective production, consumption and accumulation activities. Thus, the formalization of this core (in Table 4) and its elaboration into different types of transaction (as in Fig. 1) can be considered as a new introduction to the fundamentals of social accounting. Beyond that, it is hoped that this paper will serve to undermine two persistent fallacies:

- (1) that *T*-accounting and SAMs are equivalent, conceptually and in practice;
- (2) that SAMs are no more than extended IO tables in which the household accounts have been disaggregated.

The fact is that the various differences between an extended IO system and a SAM are critical to a proper understanding of the interrelationships between the structure of production and the distribution of income among institutional sectors. Therefore, the completion of the lower right-hand corner of the SAM is an essential prerequisite for any analysis which is sensitive to this interdependence.

Notes

1. See the editors' introduction to Kurtz *et al.* (1998) for an interpretation of Miyazawa multipliers that is challenged in Pyatt (1999).

2. In ISWG (1993) the format is referred to as a resource/use table.
3. Since products are the outputs of production activities, the recognition of an output as being a product depends on whether the activity that produced it is considered to be a production activity.
4. Note that, when these matrices appear on the left-hand side of Table 1, they are in transposed form.
5. Hence, matrix $\mathbf{S}_{3,3}$ is a scalar and $(\mathbf{S}_{3,3} - \mathbf{S}'_{3,3})\mathbf{i} = \mathbf{o}$.
6. Transactions in second-hand goods are sales of goods by someone other than their original producer.
7. There is a special case in which the preceding, rather elaborate argument is unnecessary. This arises when all the institutional accounts in Table 2 are consolidated into a single account. The rule by which equation (2) is satisfied for all accounts if corresponding row and column totals are equal for all but one account now implies that the balance of all production accounts will guarantee the balance of the single, consolidated account for institutions.
8. The way in which this same problem arises in interregional analysis is in relation to the transfers that take place between institutions in different regions. In this context, Batey and Rose (1990, p. 46) have noted that:

The most problematic aspect of inter-regional income distribution analysis ... is on the data side. Transboundary income flows have often been neglected in both regional and interregional models [and] this results in an implicit "no cross-payments" assumption associated with intermediate transactions [which, in turn] results in a serious overstatement of regional multipliers ... complete resolution of this problem will be difficult because it involves obtaining data on absentee ownership of land and the spatial delineation of stock portfolios. Still, it is crucial to the accurate assessment of the impacts of public policy on both total income and its distribution.

9. That is to say that, if all the elements of the vector \mathbf{i} are equal to unity, then $\mathbf{Si} = \mathbf{y} = \mathbf{S}'\mathbf{i}$.
10. As far as I am aware, this simple construction of a system of T -accounts from an initial SAM has not been noticed previously in the literature.
11. The temptation to aggregate or combine columns is strong, in so far as T -accounts are often designed for presentation on a printed page, the width of which might otherwise have to be considerable.
12. This particular SAM is too large to fit on a single page, so it has been printed on a folded sheet that is to be found in a pocket inside the back cover of the volume.
13. The main elements of this demonstration can be found in Pyatt (1991).
14. Of course, SAMs which do not have explicit factor accounts are possible. However, even in those cases in which the factor accounts are suppressed (using the method of apportionment discussed in Pyatt (1989) or in some other way), the correspondence of the SAM structure to that of a circular permutation matrix remains.

References

- Batey, P. W. J. & Rose, A. Z. (1990) Extended input-output models: Progress and potential, *International Regional Science Review*, 13, pp. 27-49.
- Inter-Secretariat Working Group (1993) *System of National Accounts, 1993* (Commission of the European Communities—Eurostat, Brussels/Luxembourg; International Monetary Fund, Washington, DC; Organisation for Economic Co-operation and Development, Paris; United Nations, Statistical Office, New York; World Bank, Washington, DC).
- Keuning, S. (1998) A powerful link between economic theory and practice: National accounting, *Review of Income and Wealth*, 44, pp. 437-446.
- Kurtz, H. D., Dietzenbacher, E. & Lager, C. (eds) (1998) *Input-Output Analysis*, Vols I-III (Cheltenham, Edward Elgar).
- Lager, C. (1988) The use of a social accounting matrix for comparative static economic modelling, in: M. Ciaschini (ed.) *Input-Output Analysis: Current Developments*, pp. 75-89 (London, Chapman and Hall).
- Leontief, W. (1936) Quantitative input and output relations in the economic system of the United States, *Review of Economic Statistics*, 18, pp. 105-125.
- Miyazawa, K. (1976) *Input-Output Analysis and the Structure of Income Distribution* (Berlin, Springer).
- Miyazawa, K. & Masegi, S. (1963) Interindustry analysis and the structure of income distribution, *Metroeconomica*, 15, pp. 89-103.

- Pyatt, G. (1989) The method of apportionment and accounting multipliers, *Journal of Policy Modelling*, 11, pp. 111–130.
- Pyatt, G. (1991) Fundamentals of social accounting, *Economic Systems Research*, 3, pp. 315–341.
- Pyatt, G. (1996) Monetary and financial flows in a system on national accounts, *Annali di Statistica*, Anno 125, Series X, Vol. 11 (Rome, Istituto Nazionale di Statistica).
- Pyatt, G. (1999) *Some Early Multiplier Models of the Relationship between Income Distribution and Production Structure*, Mimeo.
- UNSO (1968) *A System of National Accounts, Studies in Methods, Series F*, No. 2, Rev. 3 (New York, United Nations Statistical Office).
- UNSO (1975) *Towards a System of Social and Demographic Statistics, Studies in Methods, Series F*, No. 18 (New York, United Nations Statistical Office).