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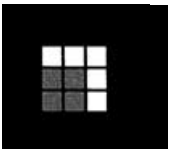

Why Do We Have Urban Density Controls?

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Almost all urban land use controls reduce permitted densities. This article analyzes restrictions on residential densities in a conventional model of density–distance functions. Density controls force development to extend farther than in competitive equilibrium, thus increasing commuting distances and dwelling costs. Residents benefit if, as is likely, they prefer lower densities than in competitive equilibrium. But there is a limit to the extra commuting and housing costs that nevertheless make residents better off. Theoretical and numerical analyses are presented to show that likely parameter values almost certainly result in reductions in residents' welfare.

Virtually all the myriad residential land use controls that pervade U.S. metropolitan areas are intended to limit densities in one way or another: minimum lot size controls, setback requirements, height limitations, subdivision regulations, rationing of municipal services, etc. (see Fischel 1985). States have sovereign control of land use within their borders, but for most practical purposes, states have assigned control over land use to incorporated municipalities within metropolitan areas.[1]

Density controls are popularly associated with suburbs, but many central cities have a panoply of controls.[2] The highest income parts of Chicago, stretching along the lakefront from the loop Central Business District (CBD) northward

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[1] Federal controls on land use increase every decade, but they mostly affect density in only gross ways. Within many metropolitan areas, there are some unincorporated places where counties typically control land use.

[2] Chicago is the laboratory for much of this article, but there is nothing special about the Chicago metropolitan area except its size. Although there is no metric to measure the stringency of land use controls, my judgment is that they are most stringent near the east and west coasts and less stringent in the Midwest and South. Green and Malpezzi (2003) present a general index of city-specific housing regulations and conclude that Chicago is the least regulated of the 56 cities in their sample (see pp. 149–150). California metropolitan areas have not only stringent density controls but also growth boundaries, beyond which development is prohibited. That accounts for their uniquely high housing values. In the Green and Malpezzi list, the five California areas included are all among the country's nine most regulated areas.

to the Evanston border, have had permitted densities reduced by about a third during the last 30 years or so (see Mills 2000).

This article concerns residential development and density controls. However, qualitatively similar controls are imposed on businesses, and similar analysis could be applied to them.

Scholarly papers that study density controls appear every year. Many papers contain estimates of effects of a set of density controls at some time and place. Recent high-quality analyses are Fu and Somerville (2001), Glaeser, Gyourko and Saks (2003) and Thorsnes (2000). Other papers present theoretical analyses of density controls; see Pasha (1996) and Crone (1983). Most theoretical papers, however, simply refer to possible externalities from high-density residential settlements. Fujita (1989) has done a sophisticated analysis of density controls. However, he takes no account of the effects of controls on commuting distances. The focus of this article is whether markets optimize urban density.

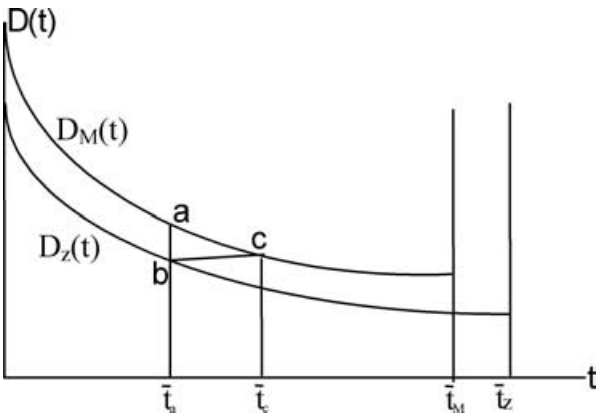
An idea that underlies most thinking and writing about density controls, including mine, is that they may be intended to exclude low-income and/or minority people from high-income suburbs. That notion can hardly apply to central cities as a whole but could well apply to parts of cities that have stringent density controls. A sequence of extraordinary papers by Epple and others (Epple and Platt (1998) and the references therein) shows that, with reasonable assumptions, local fiscal actions may lead to voluntary segregation even in the absence of land use controls.

Economic Analysis of Density Controls

Start with the simplest monocentric urban model, which I believe captures the essence of the issue. Similar analysis could be applied to the area around a suburban center. Assume realistically that land, structural capital and housing markets within a metropolitan area are competitive. Housing is produced with land and structural capital. The best estimates (see Thorsnes 1997) of the elasticity of substitution in housing production place it close to one, but the precise value is not important in this article. The rental rate on structural capital is exogenous to the metropolitan area, but the metropolitan area rental rates on land and housing are in competitive equilibrium so that dwelling owners, whether occupiers or landlords, receive competitive returns on their equity.

There is an unmodeled transportation system that moves workers between their dwellings and centrally located jobs at constant money and time cost per mile of travel. It has long been known that the long-run, competitive equilibrium density–distance function is as shown by $D_{\mathrm{M}}(t)$ in Figure 1. Here M refers to

Figure 1 ■ Effects of density controls.



the competitive market and t to the miles from the employment center (CBD). $\bar{t}_M$ is the distance from the CBD to the edge of the metropolitan area, the farthest distance at which urban landowners can outbid rural landowners (perhaps farmers) for land. $D(t)$ refers to housing density; $D(t) = K(t)/L(t)$, where $K(t)$ is structural capital and $L(t)$ is land, both at t. $L(t)$ is exogenous, but increasing in t, determined by the topography of the metropolitan area. $H(t) = F(K(t), L(t))$ is the total housing at t and $H(t) = (\text{POP}(t))h(t)$, where POP($t$) is population resident at t and $h(t)$ is housing per capita at t. Asymmetrically, $L(t)$ is exogenous, but land rent is endogenous, whereas $K(t)$ is endogenous but structural capital rent is exogenous.

Residents have conventional convex indifference curves between housing and other consumer goods and services. Utility increases in housing per capita $h(t)$ and decreases in t, other things being equal because of time and money costs of commuting.

It has long been known that the basic model holds even if residents have different incomes. Residents self-segregate, so that those in a given distance interval have similar incomes. Under known conditions, high-income residents self-segregate in suburbs. The same holds for density preferences. Residents with strong preferences for low density, segregate themselves where density satisfies their preferences, presumably in distant suburbs.

Density controls relate directly or indirectly to $K(t)/L(t)$. Height limitations and setback requirements relate directly to $K(t)/L(t)$, but restrictions to single-family housing relate in part to $K(t)/L(t)$ and in part to POP(t)/$L(t)$. As a close approximation, I assume that density controls relate to $K(t)/L(t)$.

I assume that density controls are uniformly binding throughout the metropolitan area. That is contrary to the fact that controls are more typically binding in high-income than in low-income communities and neighborhoods. If controls are uniformly binding, then the density function is $D_Z(t)$, where Z refers to "zoned," a euphemism for controls. If controls are binding, then $D_Z(t)$ must be below $D_M(t)$ for all t. $\bar{t}_Z$ is the radius of the metropolitan area with density controls.

Let a and c be two points on $D_M(t)$. c is at a larger t than a. Residents' utility may decrease with increase in density, and it must if the notion of market failure is plausible. The stronger the preference for low density, the lower is a relative to c. Whatever the preference for low density, households are indifferent among all points on $D_M(t)$. Lower housing prices and density at c than at a just compensate households for the higher transportation cost at c than at a. Individual households cannot affect density at a given t, but they can affect their neighborhood density by their choice of t.

It is easy to show that density controls almost certainly make all residents worse off in the long-run equilibrium. Let b be the point on $D_Z(t)$ at the same t as a, but at the density of c. Then the price per unit of housing must be greater at b than that at a. Housing supply is less at b than at a because of density controls, but demand is greater at b than at a because residents prefer low density. The higher housing price at b than that at a does not prove that residents are worse off at b than at a, because the low density at b may compensate for the higher housing price than at a.

But there is no free lunch. At any t in the interval $0 \leq t \leq \bar{t}_M$, fewer residents are housed out to t under D_Z than under D_M. In a closed-city model, all the residents excluded from distances $0 \leq t \leq \bar{t}_M$ by density controls must live in the interval $\bar{t}_M \leq t \leq \bar{t}_Z$. Because a is indifferent to c, and as b has the same density as c, residents are made worse off by the density control, if the resulting increased commuting distance exceeds the distance from b to c. If the maximum extra commuting distances with controls exceeds $t_c - t_a$ $(=c - b)$, then without controls, residents could have achieved the same density, at shorter commuting distance and at lower housing prices. If the housing price at b exceeds the housing price at a, then the extra distance that residents can commute and be better off at b than at a is even less than the distance from b to c. The distance from b to c is upper bound to the extra commuting distance that controls can impose on residents, and nevertheless make them better off.

With normal curvature for $D(t)$, $t_c - t_a$ increases in t_a. Thus, $t_c - t_a$ is largest when $t_a = \bar{t}_M$. If $t_c < \bar{t}_Z$, when $t_a = \bar{t}_M$, then some residents are forced to commute farther with controls than they can and nevertheless be better of under

D_Z than under D_M. If any resident is worse off, then all residents are worse off because utility is equalized at all t under D_M and at all t under D_Z. Thus, $t_c < \bar{t}_Z$ when $t_a = \bar{t}_M$ is sufficient for everyone to be worse off under Z than under M.

The appendix presents calculations that show that for plausible parameter values, $t_c < \bar{t}_Z$ if $t_a = \bar{t}_M$ and the density functions are negative, exponential $D(t) = D_0 e^{-\alpha t}$, where α is the percent decrease in density per mile of distance from the CBD and D_0 is the central density. The calculations assume that $D_Z(t) = \Phi D_M(t), 0 < \Phi < 1$, so that $1 - \Phi$ is the uniform percentage decrease in density imposed by controls.[3]

If the metropolitan area is "open," then residents choose among metropolitan areas so that utility is equated among all metropolitan areas. If all metropolitan areas have similar density controls, no residential reallocation among metropolitan areas results from controls. If employers are also free to move among metropolitan areas, then their locations depend on controls they are subject to in relevant metropolitan areas. The basic conclusion that density controls decrease residents' welfare nevertheless holds in likely circumstances (see Hamilton 1978).

The conclusion is inescapable: government density controls distort resource allocation, cause sprawl (excess suburbanization) and reduce residents' utility levels.

An immediate implication of the foregoing explains a conflict that is apparently increasingly common in suburban communities that have density controls. As has been seen, the legal limit on K/L raises the housing prices because of zero profit conditions in equilibrium. Thus, K/L is less than appropriate if density controls bind. Residents attempt to increase K/L by expanding, by rehabilitating older dwellings or by demolishing them ("teardown") and replacing them with larger structures. Such activities improve the community's housing stock and provide a better approximation to competitive equilibrium, but they undermine the purpose of density controls. The response is lawsuits and government regulations thatattempt to specify just what can and cannot be done to the existing structures. Landmark or historic district designations are typically part of the same syndrome. Thus, local governments place themselves in the absurd position of partially outlawing improvements in the housing stock. In addition, they motivate excessive demolition of the housing stock.

[3] These calculations assume that $D_M(t)$ would follow the same pattern for $\bar{t}_M < t < \bar{t}_Z$ as it does for $t < \bar{t}_M$.

Road congestion strengthens the finding that density controls reduce residents' utility. With density controls, more workers commute from any distance to the CBD on their way to work than without density controls. $D_Z(t) < D_M(t)$ implies that more workers live beyond any t with controls than without. Thus, controls cause and/or worsen congestion.

Of course, residential density controls cause excess suburbanization not only of housing but also of businesses insofar as business locations respond to residential locations of employees and/or customers.

Some people whom I have told about my result have passed it off with the comment that it "only holds in the very long run." Of course, neither markets nor government can alter a metropolitan area's housing stock much during a short time. But the long run may not be as long as some observers think. Density controls in the City of Chicago have been stringent for about 30 years. During that time, residential density has fallen considerably in the North Lake Front neighborhoods. Furthermore, no high-rise buildings have been built on the North Lake Front for more than two decades. Finally, employment has suburbanized massively in Chicago during the last 30 years, undoubtedly at least in part as a result of forced population suburbanization.

Why Do We Have Density Controls?

Under the conventional neoclassical assumptions employed in the previous section, density controls make residents uniformly worse off and cause excessive residential land use ("sprawl") in metropolitan areas. It appears that only malevolent governments would institute and enforce such controls. That seems unlikely in a country in which local government jurisdictions are mostly small and appear to be responsive to residents' strongly felt wishes. That conclusion is supported by vast theoretical and empirical literatures based on the Tiebout and median voter models.

What follows is discussion of defenses of density controls that are common and are thought by many to be plausible.

The most common defense of density controls is that "we just want low density." However, as the preceding analysis has shown, police power enforcement of densities below market equilibrium is likely to be more costly than can be justified in models of rational behavior and competitive markets. And of course, those excluded by controls cannot vote or lobby in controlled communities.

The second most common defense of density controls is the notion that "if our community abandoned its controls, we would be flooded with high-rise

apartments and low-income residents." Several issues are germane to this argument.

The first comment is that the claim is correct. As has been shown, density controls make the legal limit to K/L less than the equilibrium ratio. It follows that if a community relaxed its controls, K/L would increase, perhaps substantially and quickly.

Of course, the argument involves a fallacy of composition. If one community abandoned controls, the effect on its density and income distribution would be much greater than if all communities abandoned their controls. However, if all communities have density controls, it follows from the analysis in the preceeding section that artificially low K/L ratios are pervasive and they would increase everywhere out to $\bar{t}_M$ if controls were abandoned, but by less than if just one community abandoned controls.

Concern about an invasion of low-income people if controls were abandoned is more difficult to deal with. There can be no doubt that exclusion of families whose dwellings can pay only modest taxes and have large numbers of children who are especially expensive to education is a motivation in communities that prohibit small lots, large apartments or, in some cases, any apartments. And of course, density controls are sometimes code words for racial exclusion.[4] But the fallacy of composition must be nearly perfect in this case. There are only so many poor or minority people in a metropolitan area, and the influx into a single community that relaxed controls would be much greater than the influx per community if all relaxed controls. It seems unlikely that density controls cause substantially greater poverty, but they probably produce some, according to mismatch and concentrated poverty papers. However, in many metropolitan areas, density controls are more stringent in many suburbs than in parts of central cities predominantly inhabited by low-income and minority residents. In that case, pervasive relaxation of suburban controls would increase the suburban minority population. But the effect would not be great in most metropolitan areas. The percentage of Chicago's metropolitan population that is made up of minorities is little greater than the percentage in the entire country. Finally, it is quite possible that if all communities in the Chicago metropolitan area relaxed density controls, but communities in other metropolitan areas did not,

[4] It is beyond doubt that racial discrimination still exists in U.S. housing markets; see Yinger (1995). I am confident that it becomes less each decade. Indeed, minority suburbanization increases each decade. As racial discrimination in housing has been illegal for nearly 40 years and has entailed serious risks of both civil and criminal prosecution for perhaps 20 years, it is tempting to believe that local governments have been induced to do what private parties can no longer safely do.

the result would be some increase in minority migration to the Chicago area from elsewhere.

I do not believe that the profession has yet made its peace with a series of path-breaking papers by Epple and others (see Epple and Platt (1998) and references therein). Oversimplifying complex analysis, these papers show that population is likely to self-segregate by income and other characteristics among communities in a metropolitan area by virtue of plausible median voter-determined tax-transfer functions of local governments. These papers involve competitive housing markets but no government density controls of any kind. It is unclear how far Epple's results extend in a spatial context, but they leave me with the belief that density controls are less important as an exclusionary device to enforce segregation by income than many urban specialists appear to have believed.

Understanding why communities in a metropolitan area do not abandon density controls one at a time is not the same as understanding why communities institute them to begin with. Suppose a metropolitan area with a given set of communities, none of which has density controls. Then, based on neoclassical assumptions, none would rationally introduce controls. If one did, residents would move elsewhere to lower housing cost communities. It is much easier to understand why a community would not want to be the first to abandon controls than it is to understand why a community would want to be the first to introduce them.

The competitive actual or implicit (in the case of owner occupancy) rent on a dwelling equals the dwelling's mortgage interest plus the forgone income on the owner's equity plus insurance, maintenance and repair costs less capital gain. From the dwelling owners' viewpoint, all cost components should be after tax. Tax treatment is of course materially different for owner occupiers from tenants (and landlords), but the differences matter little for the present purposes.

If density controls are instituted or made more stringent in a metropolitan area, there is, pursuant to the results of the previous section, a one-time capital gain for existing owners. Many people appear to believe that the capital gain is a benefit. However, it is not, except in special circumstances. Most of the components of housing cost are proportionate to dwelling value. The largest component, mortgage interest plus forgone income on owner's equity, is precisely proportionate to dwelling value. Thus, capital gain is an illusory benefit because it is offset by about the same increase in actual or implicit rental cost. If there are other metropolitan areas that have no or less stringent controls, a dwelling owner can sell and move to a lower cost metropolitan area. Residents of the Los Angeles and San Francisco areas sometimes sell their high-priced homes there and move to Phoenix, Las Vegas or Missoula, where density controls are much less stringent and housing is, partly as a result, much cheaper, and they live

happily ever after. If many Californians followed that strategy, there would be no abnormal California capital gains to cash in. Another deterrent to migration from California is that high house prices there to some extent capitalize the advantages of California life, as any Californian will tell you tirelessly. As has been pointed out, if other metropolitan areas have similar density controls, migration conveys no benefit. If a resident moves from a large to a small dwelling after density controls have become stringent, perhaps motivated by imminent retirement or the empty nest syndrome, then controls carry benefit. If one has a short life expectancy, and one's children or other heirs about whom one cares live in the same metropolitan area or in one with similar controls, then capital gain is mostly an illusory benefit.

Further analysis may provide insight into the popularity of density controls by asking who benefits from them. In some metropolitan areas, certainly including Chicago, especially the City of Chicago, members of the city council represent specific districts and have almost dictatorial control over density controls in their district. In Chicago, almost the entire City's high-income residents live in the greater loop or in four community areas (about 70,000 residents each) that line the lakefront north from the Chicago River. Lincoln Park, where I lived for 12 years, is the City's highest income community area, the Loop is second and River North, between the Loop and Lincoln Park, is third. Most of the other community areas north to the Evanston border also have residents with median incomes well above the City median (but not above incomes of residents of some north shore and western suburbs (see Mills 2000).

Until 2004, Chicago's last comprehensive zoning plan was in 1957. Since then, *ad hoc* downzoning has reduced legal densities by 25–50% in the north shore community areas. Most such communities have strong homeowners' associations. A common scenario is that a developer proposes to build a condo building on or near the lakefront that current controls may allow to be 40 stories. The local homeowners' association lobbies its city council representative who gets the council to restrict the proposed building to 15–25 stories.[5] The developer then abandons the proposal.

[5] One of life's minor ironies is the sight of lakefront residents descending from their 42nd-story condos to complain to their councilman that a proposed nearby 40-story building will ruin the neighborhood by blocking sunlight, congesting streets and sidewalks and by overloading the neighborhood's private schools. Arguments are ritualistic and undocumented by evidence. But the city councilman can count votes. Most ironic is the claim of threatened street congestion. First, city law has for many years required an off-street parking space per dwelling unit, but older and mostly smaller residences lack the off-street parking and thus contribute to road crowding. Second, lakefront residents typically commute to loop jobs and shops by public transit. Third, if satisfactory housing cannot be found in Lincoln Park, residents may locate to northern suburbs and commute by car down the heavily congested Lake Shore Drive to loop jobs and shops.

The motivation for such downzoning is patently political, not substantive. Chicago's population fell by more than 20% from 1950 to 2000 and losses to suburbs were disproportionately of high-income residents. (I am not writing an essay on suburbanization. I am simply claiming that downzoning is without citywide merit.)

Why do neighborhood groups persuasively advocate downzoning? The theory that leaps from the page is that *ad hoc* downzoning reduces housing supply that would compete with neighborhood residents' dwellings. It is the same theory that explains why suburban communities restrict teardowns and additions to existing dwellings.

If competitive downzoning is rampant for long enough, the result is a city of streets but no buildings. Of course, things do not go so far in this or in other countries. Market signals, reform-minded citizens and self-interested democracy limit the extent of such destructive government actions. And there are limits to how much downzoning the courts, and perhaps even the city government, will tolerate. Chicago is now engaged in basic reform of zoning, but the results to date have been worse than the status quo.

How much socially destructive density control can be explained by this competitive downzoning syndrome? I do not know. I believe that downzoning explains some, but I doubt that the explanation can get us far. Neighborhood lobbying for *ad hoc* strengthening of density controls is possible only if density controls already exist. However, as has been pointed out, downzoning in Chicago's north lakefront communities has been at least 25% during a half century in which the city's population decreased by 20%.[6] That is patently irrational from a citywide point of view.[7] Finally, downzoning can be profitable to competing neighborhood owners only if it exceeds that in nearby neighborhoods. If everybody downzones, everybody loses.

A final defense of density controls, which is difficult to offer with a straight face, is that it preserves open space. As has been shown, density controls increase urban land use, and even low-density metropolitan suburbs are of higher density than rural land uses. In addition, density controls promote private urban open space, but it is much less valuable for recreational and amenity uses than for public open space. Federal, state and local governments can and do

[6] The frequent claim that population decreases permit costless downzoning does not hold water. Since 1998, when the city's population finally began to recover, demands for downzoning have increased.

[7] Chicago is certainly not intolerably dense. It is only the fourth densest U.S. city, after New York, San Francisco and Jersey City. It is less dense than many cities in Europe and Asia that function quite well.

purchase as much open space for public use as taxpayers are willing to pay for. Unfortunately, all levels of government preserve open space by simply banning or restricting development. That imposes open space preservation costs on landowners, tempting governments to provide or require excessive open space.

I cannot provide a coherent explanation of density controls. Some people benefit from every government intervention, no matter how destructive it may be. If one is convinced that "special" interests dominate "public" interest strongly enough in U.S. governments, then there are no surprises in government actions. I am not so pessimistic.

Evidence

The analysis to this point has been theoretical and qualitative, with occasional anecdotes. If the pernicious effects of density controls are substantial, then they should show up in data. The difficulty is that there is no natural metric of the stringency of controls. Indeed, the diversity of land use controls, like"nuisance" taxes, is probably intended in part to obscure their stringency. Thus, researchers are even more dependent than usual on proxies to measure the effects of density controls on house values and on other variables.

The earliest empirical study that I have seen is of Davis (1963), who was a pioneer in empirical urban analysis. Hamilton (1978) is, to this day, one of the most imaginative empirical studies; it conjectures that density controls may have increased housing prices by 50% in Baltimore.

White (1975) and empirical studies in Mills and Oates (1975) all concluded that density controls increase housing prices.

The more recent empirical studies include Thorsnes (2000) and Fu and Somerville (2001). The most recent studies are Glaeser and Gyourko (2002) and Glaeser, Gyourko and Saks (2003).

The above studies are extremely diverse in data and techniques of analyses. But the consistency of findings is impressive. Every careful study that I have seen concludes that density controls increase housing prices. Almost all authors point out that adverse effects on housing prices are not conclusive proof that density controls are socially undesirable. Most authors refer to possible uncompensated damages from high densities.

The conclusion of this article is that density controls reduce residents' welfare for plausible parameter values. Mild controls reduce the welfare somewhat and severe controls reduce the welfare severely.

What Should Be Done?

Suppose everyone who matters reads and agrees with this article? What should the government do?

Neither governments nor private parties can or should alter density functions in the short run. During the last half century, U.S. metropolitan areas have developed on the basis of explicit rules and implicit social contracts that favor low densities. In a democracy, governments should not suddenly abandon a social contract based on long experience. But governments could and should cease a half century of movement in the wrong direction and start heading in the right direction.

A start would be to permit high-rise dwellings in selected locations, especially in suburbs. A moratorium could be placed on increases in the stringency of density controls. Governments could enact a succession of 5-year plans for selective relaxation of density controls. Serious reform would need to be at state or federal levels.

Any proposed changes in the direction of density controls should be preceded by an educational program. One does not need to attend many cocktail parties in high-income neighborhoods to understand how much residents revel in the 10–15% per-year increases in housing prices. Almost no one seems to understand that the benefits are mostly illusory, or that they result substantially from government restrictions on housing supply. Inevitably, high and rapidly increasing housing prices keep minorities, who are most vulnerable to high prices in good neighborhoods, out of the best neighborhoods and school districts. Government density controls impair the ability of developers to provide housing that is affordable to minorities in desirable communities. Even residents who deplore the inability of minorities to live in high-income communities typically urge more government controls to coerce the inclusion of low-income housing. They fail to realize that government controls are the problem, not the solution.

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Appendix

This appendix presents numerical analysis that indicates that uniform density controls make residents of a metropolitan area uniformly worse off. Notation in the text is employed here.

The common exponential density function can be written as $D(t) = D_0 e^{-\alpha t}$, where D_0 is the central density, α is the percent decline in density per mile from the center, t is the distance from the center and $D(t)$ is the density at t. Total population from the center to $\bar{t}$ is

$$\text{POP}_{\bar{t}} = \theta \int_0^{\bar{t}} t e^{-\alpha t}\, dt,$$

where $0 < \theta \leq 2\pi$ is part of the circle available for residences.

Density controls reduce density from $D(t)$ to $\Phi D(t)$ at each t, where $0 < \Phi < 1$, and $1 - \Phi$ is the percent reduction in density imposed by controls. In the closed-city model, total metropolitan population is the same with and without controls, so

$$\int_0^{\bar{t}_M} te^{-\alpha t}\, dt = \Phi \int_0^{\bar{t}_Z} te^{-\alpha t}\, dt. \tag{A.1}$$

Of course, $\bar{t}_Z > \bar{t}_M$ for $\Phi < 1$, as assumed in the text.

Integrating Equation (A.1) by parts gives

$$1 - (1 + \alpha \bar{t}_M)e^{-\alpha \bar{t}_M} = \Phi\left[1 - (1 + \alpha \bar{t}_Z)e^{-\alpha \bar{t}_Z}\right],$$

or

$$1 - \Phi = (1 + \alpha \bar{t}_M)e^{-\alpha \bar{t}_M} - \Phi(1 + \alpha \bar{t}_Z)e^{-\alpha \bar{t}_Z}, \tag{A.2}$$

where $\bar{t}_M$ depends on D_0 and Φ as well as on α, but the interest here is on the relationship between $\bar{t}_M$ and $\bar{t}_Z$, so D_0 and Φ have been canceled out to make $\bar{t}_M$ exogenous.

The upper bound to the extra distance that residents can travel and be better off under Z than under M is $t_c - t_a$. That distance is maximal at $t_a = \bar{t}_M$, so the upper bound is

$$e^{-\alpha t_c} = \Phi e^{-\alpha t_M}. \tag{A.3}$$

Equation (A.3) equates density at t_c under M with density at $\bar{t}_M$ under Z. $t_c - \bar{t}_M$ is the maximum extra travel distance that makes residents as well off under Z as under M, only if $p_b = p_a$ (*i.e.*, if housing price at b equals that at a). If $p_b > p_a$, then the maximum extra travel distance is shorter.

Equation (A.2) determines $\bar{t}_Z$ for given $\bar{t}_M$ and Equation (A.3) determines t_c for given $\bar{t}_M$. Residents are uniformly worse off under Z than under M, if $\bar{t}_c < \bar{t}_z$ because the inequality implies that some residents must commute more extra miles than that can make them better off, if Z rules instead of M.

For computing, two inequalities must be satisfied. All three terms in Equation (A.2) are positive, and for $\bar{t}_Z > \bar{t}_M$ the second term on the right is less than the first term. Thus, for a solution to exist, we must have

$$1 - \Phi < (1 + \alpha \bar{t}_M)e^{-\alpha \bar{t}_M}. \tag{A.4}$$

In addition, for given Φ and α, fewer people reside at any t under Z than under M. Even if the metro area has infinite radius, the total population that can reside there is

Table A1 ■ Simulation results.

$\bar{t}_M$	Φ	α	$\bar{t}_Z$	t_c	$\bar{t}_Z/\bar{t}_M$	$t_c/\bar{t}_Z$
6	0.90	0.40	9.20	6.30	1.53	0.68
5	0.90	0.20	5.40	5.55	1.08	1.03
5	0.85	0.20	5.60	5.60	1.12	1.00
5	0.80	0.20	5.90	5.90	1.18	1.00
5	0.90	0.30	5.50	5.35	1.10	0.97
5	0.90	0.40	5.65	5.26	1.13	0.93
5	0.80	0.40	6.60	5.56	1.32	0.84
5	0.90	0.50	5.90	5.21	1.18	0.88
5	0.85	0.40	6.05	5.41	1.21	0.89
5	0.80	0.50	7.50	5.40	1.50	0.72

$$\text{POP} \leq \frac{\Phi\theta D_0}{\alpha^2}. \tag{A.5}$$

What are the plausible values of the parameters? If Φ is sufficiently small, the commuting distances become arbitrarily long. I presume that Φ in the range $0.80 \leq \Phi \leq 0.95$ captures plausible, or at least interesting, values. Most estimates of α are in the range $0.2 \leq \alpha \leq 1.5$. More recent estimates are smaller, partly because of increases in incomes and reduced per-mile commuting costs. But to the extent that businesses follow customers and/or employees to the suburbs and to the extent that density controls are imposed on businesses as well as on residences, then business and residential suburbanization reinforce each other. The implication is that α falls as controls become more stringent. I suppose that controls have become most stringent during the past 25–30 years. This suggests that estimates of α before about 1975 are better estimates of competitive equilibrium values than are more recent estimates.

Table A1 shows calculations for 10 sets of parameter values. The first row shows calculations for plausible Chicago metropolitan area values in the mid-1970s. The first three columns show the 10 sets of parameter values. Columns 4 and 5 show the values of $\bar{t}_Z$ and t_c calculated from Equations (A.2) and (A.3). Column 6 is a measure of sprawl caused by controls, the ratio of the metropolitan area's radius under controls to that in the competitive model. Column 7 is the crucial result. It shows the relationship between t_c and $\bar{t}_Z$, that is, the amount by which the actual maximum commuting distance exceeds or falls short of the maximum distance that could make the residents better off with density controls. For α equal to the implausible value of 0.2, controls have no deleterious effect. For $\alpha > 0.2$, controls make the residents worse off, and relatively worse off the greater is the α. As should be expected, more stringent controls, that is, smaller Φs, make residents worse off than less stringent controls.