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The Opportunity Criterion: Consumer Sovereignty Without the Assumption of Coherent Preferences

By ROBERT SUGDEN*

This paper proposes a formulation of consumer sovereignty, for use in normative economics, which does not presuppose individuals' preferences to be coherent. The fundamental intuition, that opportunity and responsibility have moral value, is formalized as an "opportunity criterion" for assessing resource allocation systems. A model of an exchange economy is presented in which rational arbitrageurs compete to make profits by trading with nonrational consumers. In equilibrium, this economy satisfies the opportunity criterion. One interpretation of this result is that, in a competitive environment, the overall effects of money pumps are benign, even if individuals' preferences are unstable or incoherent. (*JEL* D51, D63)

This paper reconsiders one of the oldest questions in economics: What, if anything, is good about allocating resources through markets? Since the paper will focus on perfectly competitive exchange economies, it may seem that I am tackling a question for which the answer is already known. What, the reader may ask, is there to add to the first fundamental theorem of welfare economics, that competitive equilibrium is Pareto efficient?

My starting point is that that theorem tells us what is good about markets, only if we accept two presuppositions. First, since the theorem is framed in terms of Pareto efficiency, we need to accept preference satisfaction as a normative criterion. Second, since it is a postulate of the theorem that each individual has consistent preferences over the set of possible consumption bundles, we need to be confident that individuals really do have such preferences. Among economists, those presuppositions were once accepted almost universally; but current developments in normative and behavioral economics are making each of them seem less secure. In normative economics, there is a growing emphasis on opportunity rather than preference satisfaction as the criterion for assessing economic and social arrangements. Research in behavioral economics is calling into question the descriptive validity of the conventional assumption that individuals act on consistent preferences. In this paper, I propose a form of normative economics that is compatible with both of these developments, while retaining the principle of consumer sovereignty that has been central to the mainstream tradition of welfare economics.

In Section I, I discuss the difficulties that are generated for welfare economics by the growing evidence that individuals' economic behavior deviates systematically from the predictions of rationality-based models. In Section II, I explain and offer supporting philosophical arguments for the normative intuition on which my approach is based: that, for each individual, opportunities to choose between alternative options have value, irrespective of whether or not those choices can be rationalized in terms of stable preferences. In Section III, I formalize this intuition as a formal criterion—the *opportunity criterion*—for assessing the opportunities that individuals possess in an economic system. This criterion does not presuppose that individuals have coherent preferences. In Section IV, I define a condition of *market clearing* which is similar to the Walrasian concept of competitive equilibrium, but which does not refer to prefer-

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ences. I show that if the market-clearing condition is satisfied, so is the opportunity criterion. This result parallels the first theorem of welfare economics, but leaves open the question of whether, in the absence of the usual assumptions about individual rationality, there are any forces which tend to induce market clearing. In Sections V, VI, and VII, I address this question by developing a model in which the opportunities that are available to consumers are provided by profit-seeking arbitrageurs. I define a concept of *free-entry equilibrium* which represents the outcome of competition among such arbitrageurs. I show that, given certain conditions, free-entry equilibrium exists, and that free-entry equilibrium implies the satisfaction of the opportunity criterion. In Section VIII, I offer some interpretations of these results. I claim that, in a certain sense, my approach treats markets as collections of money pumps operated with the intention of extracting value from consumers. The overall effect of these money pumps is benign, not because consumers are induced to form coherent preferences, but because of the effects of competition among arbitrageurs.

I. Why Preference Instability Creates Problems for Welfare Economics

Over the last 20 years, there has been an accumulation of evidence that the behavior of real human decision makers deviates systematically from the predictions of conventional choice theory. It now seems clear that these anomalies are not mere artifacts of particular experimental and survey designs. Beyond this point, opinion among researchers in the field is divided. Some maintain that most anomalies are transient effects which tend to disappear with market experience; others propose that anomalies can be explained by theories in which individuals act on preferences which deviate in various ways from the standard assumptions; still others argue that decision-making behavior is so context-dependent that it is better modeled as the product of a suite of mental routines, and that preferences are "constructed" only in response to specific problems.[1] The jury is still out; but it would surely be prudent to begin to think about what kinds of normative economics would be possible if conventional assumptions about preferences had to be given up.

To explain why the findings of behavioral economics can create problems for welfare economics, it is useful to consider some specific regularities in behavior that have been found in empirical studies. These examples will provide points of reference for the general analysis presented later in the paper.

As a starting point, consider the experimental finding that there is a high degree of stochastic variation in the choices of given individuals. In order to organize the data generated by experiments in which the same individuals confront exactly the same decision problems on two or more occasions, it is necessary to postulate that each individual's preferences are subject to some kind of random variation. From the perspective of *descriptive* theory, this finding does not cause fundamental problems. It is reasonably straightforward to adapt conventional choice theory by adding a "white noise" term to utility functions (John D. Hey and Chris Orme, 1994), or by using a "random preference" model in which the preferences on which an individual acts on any particular occasion are drawn at random from a pool of potential preferences (Graham Loomes et al., 2002). In a *normative* perspective, however, one faces the problem of how to construct a measure of well-being from stochastic preferences. A measure might be based on some central tendency of preferences, but the normative status of such a measure is open to question. If an individual's own perceptions of what she prefers are unstable, what are the grounds for treating her "average" preferences as a measure of her well-being?

Preference instability generates still more difficult problems for welfare economics if an individual's preferences over given outcomes vary systematically according to the viewpoints from which decision problems are assessed, or according to the contextual "framing" of those problems. One particularly significant instance of this general problem is the case of *reference-dependent* preferences. There is a great deal of evidence that individuals' preferences over given outcomes vary according to the "reference point" or status quo position in relation to which those outcomes are viewed (Richard H. Thaler,

[1] Alternative interpretations of this evidence are offered by Charles R. Plott (1996), Ken Binmore (1999), George Loewenstein (1999), and Chris Starmer (1999).

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1980; Amos Tversky and Daniel Kahneman, 1991). Such reference-dependence seems to be implicated in the well-known disparity between willingness-to-pay and willingness-to-accept valuations, which causes severe difficulties for cost-benefit analysis. Another instance of the same general problem is the observation that individuals' time preferences are often better represented by models of *hyperbolic discounting* than by the conventional model of exponential discounting. That is, a person's rate of time preference between any two periods t and $t + 1$, as viewed from any third period $T \leq t$, increases with T (Loewenstein and Jon Elster, 1992). As a final example, there is some evidence that preferences revealed in experimental markets are subject to *shaping* effects: a person's subjective valuation of a given good is greater, the higher the market price at which that person sees it being traded, even if prices convey no information about the good itself (Loomes et al., 2003).

In each of these three cases, behavior can be modeled as revealing preferences that are in some sense context-dependent. The difficulty is that such models are not compatible with the interpretation of "preference" that is standard in welfare economics. Economists often want to make normative comparisons between very different social states—for example, between a future in which international trade is subject to tariffs and one in which it is not. The standard methods of welfare economics hold individuals' preferences constant across the relevant social states, treat those constant preferences as measures of well-being, and ask how far they are satisfied in each state. Such analysis is not possible if individuals' preferences shift according to trivial changes in viewpoint or context. This problem might be resolved if there were some defensible criterion for defining "true" preferences for each individual, such that context-dependent deviations from such preferences could be treated as normatively irrelevant errors. But there seem to be no adequate grounds for privileging any particular set of contextual features as the basis for defining the preferences that are to measure well-being.

II. The Value of Opportunity

In normative economics, there is an increasing interest in criteria of opportunity rather than of preference satisfaction. In opportunity-based theories, value is attached to the size and richness of an individual's *opportunity set*—that is, the set of options from which he is free to choose. Some contributors to this literature argue that opportunity has intrinsic value as an element of well-being in its own right (Amartya Sen, 1992; Kenneth J. Arrow, 1995). Others argue that the extent to which a person satisfies his preferences is not a proper concern of public policy or of a theory of justice. Justice, it is claimed, requires equality of opportunity, not equality of preference satisfaction: each individual must take responsibility for how he uses his opportunities (G. A. Cohen, 1989; John E. Roemer, 1998). These broad currents of thought provide the normative background for the present paper.

I start from an apparently simple normative intuition: it is good that each person is free to get what she wants, in so far as this is possible within the constraints imposed by other people's being free to get what they want. This, in essence, is the intuition that underlies the familiar concept of *consumer sovereignty*. However, consumer sovereignty is normally formulated in terms of the satisfaction of coherent preferences. Each individual, it is usually claimed, is the best or proper judge of her own well-being, and those judgements of well-being are revealed in her preferences. In contrast, my aim is to reformulate the idea of consumer sovereignty in a way that does not require assumptions about the coherence of preferences.

If this aim is to be achieved, we need to find some way of saying that it is good for an individual to have a wide range of alternative options from which to choose, whether or not her choices reveal any internally consistent set of judgements about well-being. We need to recognize that the individual may act on different preferences at different times, in different situations, and in response to different framings of what, according to conventional economic theory, is the same decision problem; but we still need to be able to say that it is good for her that, at each moment, she is free to satisfy whatever preferences she then has.

For example, suppose that my preferences are reference-dependent. Other things being equal, I tend to prefer to keep whatever my current endowments happen to be rather than to exchange them for something else. Living in a

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market economy, I am broadly free to choose for myself what trades I make, subject to my finding willing trading partners. Suppose I have bought a Ford car, which I would not now exchange for a Renault of equal market value. But if instead I had been endowed with the Renault, I would have been just as unwilling to exchange it for the Ford. Do I benefit from the opportunity that the market gives me to choose between these two makes of car? Or would I have been just as well off if a central planning agency had allocated me a Renault and prohibited me from exchanging it? If "benefit" is interpreted in terms of the satisfaction of coherent preferences, the question seems to have no determinate answer: I do not have a stable preference ranking of the cars which can be used to assess whether I am better off with a chosen Ford than with an allocated Renault. Still, it seems true to say that, in allowing me this choice, the market respects my sovereignty as a consumer in a way that the central planning regime does not. And one might think (as I for one do) that this is something worth respecting.

We need to disconnect the concept of consumer sovereignty from the principle that preference satisfaction is a measure of well-being. This proposal requires a fundamental shift in normative thinking: it runs counter to ideas about preferences that are deeply embedded in economics. It is a folk saying in the discipline that, as far as theory is concerned, an individual *is* a preference ordering: everything the theorist needs to know about a person is contained in that person's preferences. Viewed in this perspective, a person who lacks a coherent set of preferences appears as lacking an integrated sense of his own self: there seems to be no firm basis for statements about what it is rational *for him* to do, or about what, in his own judgement, is good *for him*. If we are to make sense of the value of consumer sovereignty in a world in which consumers lack consistent preferences, we need some other way of representing the individual person as a continuing agent. We need to be able to say that opportunity is good for him, without identifying "him" with his preferences.

Before I propose a solution to this problem, it will be useful to consider two of the main theoretical strategies that economists use when analyzing inconsistencies in an individual's preferences across time. In different ways, these strategies conserve the link between agency and coherent preference—the link that my proposal will break.

The first strategy is to use models of *multiple selves*. The individual is modeled as a collection of distinct selves, each with its own coherent preferences; at different times or in different situations, different selves are in control of the individual's actions. Interactions between these selves are represented in much the same way as are interactions between different people. Typically, each self is treated as having its own view about the welfare of the individual as a whole; if those views differ, the selves interact strategically with one another (R. H. Strotz, 1955–1956). This approach generates descriptive theories of behavior that are broadly compatible with conventional principles of rational choice. But this comes at a cost: we are left with no preference-based concept of welfare that applies to the person as a continuing entity.

The second strategy is to invoke the concept of a *metaranking*. A person who sometimes acts on one preference ordering and sometimes on another is assumed to have a coherent higher-level ranking of the various lower-level preference orderings which govern her day-to-day behavior. The self-integration of the continuing person is then represented by her metaranking (Harry G. Frankfurt, 1971; Sen, 1977). This approach allows us to distinguish between those lower-level preferences that the continuing person prefers to have and those that she would prefer not to have. We might then claim that normative analysis should be concerned only with the satisfaction of the former. This way of thinking about people's choices coheres with some of the more high-minded traditions of moral philosophy, but it is fundamentally opposed to what I take to be the spirit of consumer sovereignty as a normative principle. The metaranking approach locates normative authority, not in the day-to-day decisions that individuals make as economic actors, but in each person's supposed higher moral self. A robust concept of consumer sovereignty, I suggest, should not need to invoke such a moralized account of preference. In any case, the concept of higher preference seems out of place in relation to many of the preference inconsistencies that feature in behavioral economics. Take the case of the cars. Endowed with the Ford, I prefer it to the Renault; endowed with the

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Renault, I prefer it to the Ford. But do I have a higher self which identifies with one of those preferences but not with the other? The difference between the two preferences seems to be a matter of perspective—of where I happen to be standing—rather than of "higher" and "lower."

Each of the two strategies I have described represents individual agency in terms of coherent preferences. My alternative proposal is to represent agency in terms of a normative disposition: *responsibility*. The continuing agency of a person across time is to be understood as the continued existence of a self-acknowledged locus of responsibility.

The intuitive idea is that a person is a continuing locus of responsibility—for short, a *responsible agent*—to the extent that, at each moment in her life, she identifies with her own actions, past, present, and future. A responsible agent treats her past actions as her own, whether or not they were what she now desires them to have been. Similarly, she treats her future actions as her own, even if she does not yet know what they will be, and whether or not she expects them to be what she now desires them to be. To treat a past action as one's own is to take *ex post* responsibility for it, rather than attributing it to an alien past self. Thus, a person who identifies with her past actions will not see herself as justified in demanding redistributive transfers to compensate for the foreseeable consequences of imprudent decisions she made in the past.[2] To treat a future action as one's own is to take *ex ante* responsibility for it, rather than conceiving of oneself as the principal in a principal-agent interaction with an alien future self. A person who identifies with her future actions will not want to impose external constraints on her future choices as a way of forcing those choices to match her current conception of what is good for her.[3]

[2] The claim that such demands are unjust is central to some recent normative analyses of opportunity (e.g., Roemer, 1998).

[3] She may form plans and *resolve* to carry them through, relying on her future self to treat such resolutions as reasons for action, as in Edward F. McClennen's (1990) theory of resolute choice. However, she has no wish to *compel* her future self to act in accordance with these resolutions. She takes *ex ante* responsibility for her future self's choices, whether or not those choices will be consistent with her current resolutions.

This conception of responsibility provides philosophical underpinning for the claim that opportunity has value. Consider the set of opportunities that are open to some individual across time. Is it a good thing that this set is larger rather than smaller? In conventional welfare economics, more opportunity is better than less only to the extent that it allows the individual to achieve a more preferred outcome; if the individual lacks coherent preferences, there seems to be no way of answering the question. In a model of multiple selves, the question has to be posed separately in relation to each self, and increases in lifetime opportunity may be judged to be good from the viewpoint of one self and bad from that of another. If appeal is made to metarankings, increases in the opportunities of selves which act on "inferior" preferences may be judged to have negative value. For example, in problems involving self-control, an earlier (or higher) self may approve of restrictions on the opportunities of a later (or lower) self. Similarly, a later self may wish that an earlier self had had less opportunities to act imprudently. But if an individual is understood as a continuing locus of responsibility, any increase in that individual's lifetime opportunity is good for her in an unambiguous sense. The larger her opportunity set is, the more she—construed as responsible agent with a continuing existence through time—is free to do. This is true whether or not her actions across time are consistent with any one set of coherent preferences.

III. The Opportunity Criterion

I now propose a formal normative criterion which encapsulates the intuitions presented in Section II. I begin by considering the opportunity set of a single individual.

I work with a simple model in which there are *m goods,* where $m > 1$. Let $\mathbf{x}_i \equiv (x_{i1}, \ldots, x_{im})$ represent the quantities of those goods held by a given *consumer i*; any such vector is a *bundle.* One such bundle, denoted $\mathbf{z}_i$, is i's *initial endowment.* The model is intended to represent the opportunities that are open to i, over some period of time, to change his holdings of goods through some process of trade. The economic agents with whom i might trade are not modeled explicitly at this stage; they are represented only by the trading opportunities they offer. The aim

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is to formulate a normative criterion for assessing those opportunities.

Formally, opportunities are represented by an *opportunity set* $\mathcal{O}_i \in \mathbb{R}^m_+$, such that $\mathbf{z}_i \in \mathcal{O}_i$. The interpretation is that, for each bundle $\mathbf{x}_i \in \mathcal{O}_i$, there is some sequence of exchanges which leads from $\mathbf{z}_i$ to $\mathbf{x}_i$, such that i can expect to be able to carry out all of those exchanges if he so chooses. I say that *i can expect* to carry out these exchanges, rather than that *i is able* to do so, so as to allow the possibility that other agents are offering trading "opportunities" to i which, if i were actually to take up, those agents could not honor. My reason for allowing this possibility will become clear later. In interpreting the model, I shall make the informal assumption that, at all times, i knows $\mathcal{O}_i$.

Now let us assume the viewpoint of some social planner who has the power to decide, within certain resource constraints, what initial endowment and what trading opportunities i will be given. Specifically, suppose that the resource constraints can be represented by some non-empty *feasible set* of bundles X_i: the resources at the planner's disposal make it possible for i to have any one of the bundles in this set, but do not allow him to have any bundle from outside the set. What opportunity set should i be given?

In this simple case, it is natural to interpret the principle of consumer sovereignty as requiring that i's opportunity set should contain every feasible bundle—that is, as requiring that $X_i \subseteq \mathcal{O}_i$. It will turn out to be convenient to use an equivalent *ex post* characterization of this criterion. Consider any opportunity set $\mathcal{O}_i$. Suppose that, in response to these opportunities, i engages in a sequence of exchanges which takes him to some feasible bundle $\mathbf{x}_i^* \in \mathcal{O}_i$. *Ex post,* in the situation in which i holds $\mathbf{x}_i^*$, i might point to some other bundle $\mathbf{x}_i' \neq \mathbf{x}_i^*$, and ask the planner to justify the fact that his holding is not $\mathbf{x}_i'$. To this demand, one surely adequate response is to point out that $\mathbf{x}_i'$ is not an element of the feasible set. If this is the case, i's not having $\mathbf{x}_i'$ is an unavoidable consequence of resource constraints. A second possible response is to point out that $\mathbf{x}_i'$ is an element of $\mathcal{O}_i$. If this is the case, i's not having $\mathbf{x}_i'$ is attributable to decisions that he has taken knowingly of his own free will: as a responsible agent, he must acknowledge those decisions as his own.

These ideas can be expressed through the following criterion. For the case of a single consumer who ends up with some feasible bundle $\mathbf{x}_i^* \in \mathcal{O}_i$, the *opportunity criterion* is satisfied if and only if, for every conceivable bundle $\mathbf{x}_i \neq \mathbf{x}_i^*$, either $\mathbf{x}_i \notin X_i$ or $\mathbf{x}_i \in \mathcal{O}_i$.

The opportunity criterion can be satisfied without the consumer's acting on coherent preferences. For example, suppose there are just two goods and that i's initial endowment is $\mathbf{z}_i = (10, 10)$. Suppose that feasibility constraints allow i to increase his holdings of good 2 by 4/5 unit for every unit of good 1 he forgoes, and to increase his holdings of good 1 by 4/5 unit for every unit of good 2 he forgoes. (Perhaps any change in i's holdings incurs transport or transaction costs.) Formally, the feasible set is defined by the condition that, for any bundle $\mathbf{x}_i = (x_{i1}, x_{i2})$, $\mathbf{x}_i \in X_i$ if and only if $\max[5x_{i1} + 4x_{i2}, 4x_{i1} + 5x_{i2}] \leq 90$. The condition $X_i \subseteq \mathcal{O}_i$ is satisfied if i is offered the opportunity to buy whatever quantities of good 2 he wishes at a price of 5/4 units of good 1 per unit of good 2, and to sell whatever quantities of good 2 he wishes at a price of 4/5. Suppose i is given these opportunities, and responds in the following way. First, feeling a desire to have more of good 2, he sells five units of good 1 and takes four units of good 2 in exchange. Then, changing his mind and feeling a desire to have more of good 1, he buys back the five units of good 1, giving up 6.25 units of good 2. He now has the holding $\mathbf{x}_i^* = (10, 7.75)$. If we stipulate that all coherent preferences rank larger bundles above smaller ones, this sequence of trades cannot be interpreted as revealing coherent preferences. Still, the opportunity criterion is satisfied. The consumer may acknowledge that, through changing his mind, he has made an unambiguous loss; but that loss was the result of decisions that he made freely, and with which, as a responsible agent, he still identifies.

The idea behind this example, that the consumer simply "changes his mind" about the trades he wants to make, may seem artificial. But, as was pointed out in Section I, many of the regularities that have been found in behavioral economics take the form of changes in an individual's preferences that are induced by shifts in his viewpoint or by changes in the framing of his decision problems. Thus, reversals in preference that appear arbitrary when viewed in the perspective of conventional consumer theory can be the predictable consequences of behavioral mechanisms. For example, suppose that

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good 1 is a claim on consumption in some future period, and that good 2 is a claim on consumption in a still later period. If the individual discounts future consumption hyperbolically, his preference between bundles of these two goods may reverse over time, just as in the example.

I now consider how the opportunity criterion can be generalized to cases involving more than one consumer. Consider an economy with m goods and n consumers. In interpreting this model, I assume that each consumer is concerned only about his own holdings of goods.[4]

An *allocation* $\mathbf{x} \equiv (\mathbf{x}_1, \ldots, \mathbf{x}_n)$ is a profile of bundles of goods, one bundle for each consumer; the *initial allocation* $\mathbf{z} \equiv (\mathbf{z}_1, \ldots, \mathbf{z}_n)$ is a profile of initial endowment bundles. For each consumer i there is an opportunity set $\mathcal{O}_i$. The *feasible set* of allocations X is some set of allocations such that $\mathbf{z} \in X$. The interpretation is that X comprises all those allocations that are feasible, given the resources of the economy. I shall consider only exchange economies with free disposal and without transaction costs. In such an economy, X contains all those allocations $\mathbf{x}$ that satisfy the constraints

$$(1) \quad \sum_i (x_{ij} - z_{ij}) \leq 0; \qquad (j = 1, \ldots, m)$$

Just as, in the one-person case, I did not require that all opportunities were feasible, so, in the n-person case, I do not require that every logically possible n-tuple of choices made by the n consumers from their respective opportunity sets is feasible. I shall be modeling the opportunities that consumers have by virtue of having access to a market. When a market is in equilibrium, consumers severally have opportunities which each chooses not to exercise and which, if all consumers tried to exercise simultaneously, could not be realized. (For example, each of us takes himself to be free to buy as much of any major consumer good as he can afford at the current price; but if all of us simultaneously tried to buy much more than we customarily do of the same good, some of us would find our plans frustrated.)

The opportunity criterion can now be stated in its n-person form. Given a profile of opportunity sets $\mathbf{O} \equiv (\mathcal{O}_1, \ldots, \mathcal{O}_n)$ from which consumers have made choices leading to a feasible allocation $\mathbf{x}^*$, the criterion is satisfied if and only if, for every allocation $\mathbf{x} \neq \mathbf{x}^*$, either $\mathbf{x} \notin X$ or there is some consumer j such that $\mathbf{x}_j \neq \mathbf{x}_j^*$ and $\mathbf{x}_j \in \mathcal{O}_j$. In other words: every feasible allocation other than the one that has in fact come about assigns to some consumer a bundle that that consumer had, but did not take, the opportunity to achieve.

This criterion can be interpreted as a generalization of the idea that, in a one-person economy, that person should be able to take responsibility for the outcome he experiences. The n-person criterion requires that, for every feasible alternative to the actual outcome, *someone* can take responsibility for that alternative not having come about. To see what this means, suppose that, after $\mathbf{x}^*$ has come about, some consumer i asks for a justification of the fact that his holding is $\mathbf{x}_i^*$ and not some other bundle $\mathbf{x}_i'$. In order for this complaint to have substance, i needs to be able to point to some feasible allocation $\mathbf{x}'$ in which his own bundle is $\mathbf{x}_i'$. If $\mathbf{x}_i' \in \mathcal{O}_i$, the fact that $\mathbf{x}_i'$ has not come about is the result of i's own free choices, for which he must take responsibility. But what if, although $\mathbf{x}_i' \notin \mathcal{O}_i$, there is some other consumer j for whom $\mathbf{x}_j' \neq \mathbf{x}_j^*$ and $\mathbf{x}_j' \in \mathcal{O}_j$? If this is the case, then in proposing $\mathbf{x}'$, i is proposing that j ends up with a bundle, different from the one j in facts holds, that j has already had the opportunity to achieve. That is, i is proposing to undo the outcome for j of j's free choices.

In its n-person form, the opportunity criterion is concerned with *efficiency* in the allocation of opportunities, and not with the *distribution* of those opportunities. In this respect, it is the analogue in the domain of opportunity of the Pareto criterion in the domain of preference satisfaction. If the Pareto criterion is satisfied, any feasible alternative to the allocation that has in fact come about involves someone's moving to a less preferred position. If the opportunity criterion is satisfied, any feasible alternative to the allocation that has in fact come about involves undoing an outcome for which some individual is responsible. In conventional welfare economics, the satisfaction of the Pareto

[4] Since preferences do not appear in the model, this condition cannot be stated as a formal property of preferences. It must be treated as a precondition for the applicability of the model to real-world situations.

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criterion is interpreted as a necessary but not a sufficient condition for optimality. My proposal is that the opportunity criterion should be given an analogous status in an opportunity-based form of normative economics.

IV. Market-Clearing Prices and the Opportunity Criterion

I now define a property of initial endowments, opportunity sets, and final outcomes which is sufficient to ensure that the opportunity criterion is satisfied.

Consider any exchange economy with n consumers and m goods. Let $\mathbf{z}$ be the initial allocation. Let $\mathbf{O}$ be the profile of opportunity sets, and let $\mathbf{x}^*$ be a feasible allocation which comes about as a result of each consumer i choosing $\mathbf{x}_i^*$ from $\mathcal{O}_i$. I shall say that the triple $(\mathbf{x}^*, \mathbf{z}, \mathbf{O})$ is *market compatible* if there is some vector of nonnegative *prices* $\boldsymbol{\rho} \equiv (\rho_1, \ldots, \rho_m)$, such that at least one price is strictly positive and the following conditions are satisfied:

A1. *MARKET-CLEARING PRICES. For each good j: either* $\rho_j > 0$ *and* $\Sigma_i (x_{ij}^* - z_{ij}) = 0$, *or* $\rho_j = 0$ *and* $\Sigma_i (x_{ij}^* - z_{ij}) \leq 0$.

A2. *BUDGET-BASED OPPORTUNITY. For each individual i:* $\mathcal{O}_i = \{\mathbf{x}_i\text{: } \Sigma_j \rho_j(x_{ij} - z_{ij}) \leq 0\}$.

The following result establishes that these conditions are sufficient for the satisfaction of the opportunity criterion:

Result 1: For all allocations $\mathbf{x}^*$, $\mathbf{z}$ and for all profiles of opportunity sets $\mathbf{O}$: if $(\mathbf{x}^*, \mathbf{z}, \mathbf{O})$ is market compatible, then $\mathbf{O}$ satisfies the opportunity criterion.

Proofs of results are given in an Appendix. The proof of Result 1 is a simple adaptation of a familiar revealed-preference proof of the first fundamental theorem of welfare economics.

Result 1 tells us that if each good has a single price at which all individuals are free to buy and sell whatever quantities they wish, if individuals are free to dispose of goods if they choose, and if, as a result of choices that individuals make in relation to those prices and disposal options, all markets clear, then the opportunity criterion is satisfied.

This result is a first step towards a normative justification of the market in terms of the opportunity criterion; but it is no more than this. If it is to justify the market, there has to be an argument that shows that market forces induce market-clearing prices. The concept of market clearing encapsulated in A1 is similar to Walrasian equilibrium in conventional general equilibrium theory, but there is one significant difference. Each consumer's bundle $\mathbf{x}_i^*$ is not modeled as a utility-maximizing choice; it is merely whatever that consumer chooses from his opportunity set. Walrasian equilibrium is normally interpreted in terms of consumers who respond to market prices by making utility-maximizing bids to buy and sell goods: the Walrasian auctioneer, who personifies the forces of the market, interacts with *rational* consumers. But is market clearing still an appropriate equilibrium concept if individuals are not assumed to have coherent preferences?

V. Arbitrage: The Intuition Behind the Model

The fundamental idea I now develop is that, whether or not consumers have coherent preferences, arbitrage induces market-clearing prices. At this point, I introduce a new class of actors into the theoretical framework: *traders.* So far, I have modeled consumers as facing "trading opportunities," which have been treated as primitives. I now open that black box: traders are the agents who offer those opportunities. My objective is to model the behavior of traders in determining the opportunities that individuals face. In this Section, I give an intuitive account of this model.

In the model, traders do not consume goods; they merely buy them from, and sell them to, consumers with the objective of making profits. In general, goods are not directly traded against one another. Instead, each of goods 2, ... , m is traded separately against good 1, which will be called *money.* Traders measure profits in terms of increases in their holdings of money; to them, holdings of other goods have no intrinsic value, positive or negative. For consumers, money has intrinsic value in consumption as well as serving as a medium of exchange.

In thinking about the model, it might help to imagine the following scenario. Think of the consumers as the inhabitants of an island, unable to travel to or from the mainland. They hold endowments of the various goods, including

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money. The traders live on the mainland. They are able to travel to and from the island, but the only good that they can transport in either direction is money. They come to the island equipped with stocks of money to be used as working capital. Through trading with consumers, each trader hopes to return to the mainland with more money than she took out.

The distinction between consumers and traders is intended to represent two different roles that economic actors can play. As consumers, individuals attempt to satisfy their subjective desires by acquiring bundles of goods that they want to consume. As traders, individuals attempt to make profits by buying goods at low prices and reselling them at higher prices. The model will attribute a high degree of rationality to traders, in the sense of their being able to foresee opportunities for profit, and to pursue such opportunities consistently. However, it is *not* assumed that traders have consistent preferences in the m-dimensional space of bundles of goods, any more than this is assumed of consumers. The rationality of traders consists in their ability to make accurate predictions of demand and supply, and to solve *objective* maximization problems. All that is assumed about their subjective tastes is that they prefer more money to less.

The real-world counterparts of the traders of the model are professional entrepreneurs and arbitrageurs, who have accumulated experience of the particular markets in which they work and who have been subject to a natural selection process. (Traders who consistently make losses do not accumulate the reserves necessary to engage in large-scale arbitrage.) Thus, I suggest, it is reasonable to make the modeling assumption that traders are well-informed and instrumentally rational in the pursuit of profit.

The market of the model works in the following way. Initially, each trader's holding of each non-money good is zero. Each consumer has a positive holding of each of these goods. Prior to any trade taking place, each trader sets the values of certain parameters which specify the trades she is willing to make. For each non-money good, she sets a selling price (the money price at which she is willing to sell the good to consumers), a buying price (the money price at which she is willing to buy it from consumers), a selling constraint (the maximum total quantity she is willing to sell), and a buying constraint (the maximum total quantity she is willing to buy). Consumers also have a free disposal option for non-money goods.

Trading takes place over an interval of time, during which the prices and constraints set by each trader are fixed. We may think of consumers milling around a public marketplace in which traders are displaying their buying and selling prices. Constraints are not displayed, but if a trader has already sold (or bought) as much as her constraint allows, her selling (or buying) price is no longer displayed. At any time, a consumer may approach a trader and ask to trade on the terms that trader is offering. If a consumer's trading request would require a trader to exceed a constraint, trade takes place up that constraint.

Notice that, in this model, consumers are price-takers but traders are not. Each consumer takes every trader's buying and selling prices as given, and decides how much to trade at those prices. In contrast, each trader sets the prices at which she offers to buy and sell, and allows consumers to choose how much trade takes place. Thus, a trader can exercise control over the quantities that she buys and sells only though the buying and selling constraints that she sets at the beginning of the trading period.[5] This form of control will be important in my analysis of arbitrage. For example, suppose that some good is being sold by one trader at a price that is higher than that at which it is being bought by another. Intuitively, this seems to create an opportunity for pure profit: it is possible for a third trader to buy a small quantity of the good at the lower price and sell exactly the same quantity at the higher price. But such an operation may not be possible if traders cannot set constraints on their sales and purchases.

During the trading period, traders' promises to deliver goods are treated, both by consumers and by other traders, as equivalent to possession of the corresponding goods; thus, at any given time, a trader's holding of a good may be negative. However, a trader incurs a severe penalty if, *at the end of the trading period,* she has negative holdings of any good. (To have negative holdings of any good at the end of the

[5] Such a mechanism is not needed in Walrasian general-equilibrium models, but only because the artificial device of the auctioneer allows all economic agents to be price-takers.

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trading period is to be unable to make deliveries that have been promised.) Consumers are not permitted to have negative holdings at any time. There is some exogenous limit to the length of the trading period, so that the trading process cannot continue indefinitely.

I make just one substantial assumption about the rationality of consumers: *at any given time, in relation to any given good,* consumers buy only at the lowest price currently offered, and sell only at the highest price currently offered. This assumption is essential for the model, since it defines the terms on which the traders compete. However, there is no assumption that the various trades that a given consumer makes are *jointly* consistent with any concept of preference—not even a preference for more money rather than less. For example, suppose that the lowest selling price quoted by traders for some good j is higher than the highest buying price. A consumer who is endowed with some positive amount of good j might choose to sell this endowment at the highest buying price on offer and then, immediately afterwards, choose to buy the same quantity back again at the lowest selling price on offer. These two trades, taken together, imply a loss of money and no compensating gain. Still, each trade, considered in isolation, was made on the best available terms.

VI. Arbitrage: The Formal Model

I now present the formal model. The model focuses on the trading process *as viewed by traders.* Specifically, it represents consumers' initial endowments, and it represents the final outcomes of their aggregated decisions, as these impinge on traders, for given profiles of prices and quantity constraints. Consumers' behavior over time is not modeled explicitly; nor is the distribution of total trades among consumers. By virtue of these features, the model is compatible with a very wide range of possible behavior—both rational and irrational—on the part of consumers. The objective is to investigate properties of markets that are induced by arbitrage, independently of whatever factors govern consumer behavior.

There is a finite number of consumers n. (In interpreting the model, the number of consumers will be treated as large.) For each consumer i, for each good j, there is a finite and strictly positive endowment z_{ij}.[6] At the end of the trading period, each consumer i's holding of each good j, denoted x_{ij}, is equal to z_{ij} plus net purchases made during the trading period, minus disposals during that period; for all i and j, x_{ij} is nonnegative.

There is a finite number of traders N, where $N > 1$. (In interpreting the model, the number of traders will be taken to be large.) Each trader k sets, for each non-money good j, a *selling price* $p^S_{kj} \geq 0$, a *buying price* $p^B_{kj} \geq 0$, a *selling constraint* $c^S_{kj} \geq 0$, and a *buying constraint* $c^B_{kj} \geq 0$. Prices are expressed in money units per unit of good j; constraints are expressed in units of good j. The interpretation is that trader k undertakes to sell good j to consumers at price p^S_{kj} in any quantities that consumers request, subject to the constraint that her *total* sales do not exceed c^S_{kj}; and similarly for buying. The profile of prices and constraints set by all traders for all markets is the *offer configuration.*

For each offer configuration $\mathbf{F}$, there is a determinate *outcome* $\phi(\mathbf{F})$. The outcome of an offer configuration specifies, for each non-money good, how much is bought and sold by each trader, and how much consumers dispose of. The quantity of each non-money good j sold by trader k, as an outcome of offer configuration $\mathbf{F}$, is denoted by $q^S_{kj}(\mathbf{F})$; the total sold by all traders is $q^S_j(\mathbf{F})$. Similarly, $q^B_{kj}(\mathbf{F})$ is the quantity of good j bought by trader k, and $q^B_j(\mathbf{F})$ is the total quantity bought by all traders. The quantity of good j that is disposed of by consumers is denoted $d_j(\mathbf{F})$. All $q^S_{kj}(\mathbf{F})$, $q^B_{kj}(\mathbf{F})$ and $d_j(\mathbf{F})$ are nonnegative. It is convenient to use $p^S_{kj}(\mathbf{F})$ to denote the selling price for good j set by trader k in offer configuration $\mathbf{F}$, $c^S_{kj}(\mathbf{F})$ to denote the selling constraint for good j set by trader k in offer configuration $\mathbf{F}$, and so on. The quantities of money "sold" (i.e., spent in buying non-money goods) and "bought" (i.e., received in exchange for non-money goods) by each trader are defined by the identities:

[6] The assumption that endowments are strictly positive is a familiar simplifying assumption in general-equilibrium theory, where it is used to exclude the possibility that there are discontinuities in excess demand functions as prices move between very low nonzero values and zero itself. It plays an analogous role in the existence theorem that will be proved in this paper.

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$$(2)\quad q^S_{k1}(\mathbf{F}) \equiv \sum_{j=2}^{m} p^B_{kj}(\mathbf{F}) q^B_{kj}(\mathbf{F}); \qquad (k = 1, \dots, N).$$

$$(3)\quad q^B_{k1}(\mathbf{F}) \equiv \sum_{j=2}^{m} p^S_{kj}(\mathbf{F}) q^S_{kj}(\mathbf{F}); \qquad (k = 1, \dots, N).$$

The total quantities of money sold and bought by all traders are denoted by $q^S_1(\mathbf{F})$ and $q^B_1(\mathbf{F})$, respectively.

The following definitions will be used. Consider any offer configuration $\mathbf{F}$ and any non-money good j. A trader k is an *active seller* of good j in $\mathbf{F}$ if $c^S_{kj}(\mathbf{F}) > 0$; she is an *active buyer* of good j if $c^B_{kj}(\mathbf{F}) > 0$; she is *active* in $\mathbf{F}$ if she is an active buyer or seller of at least one non-money good. A price π is an *active selling price* for good j in $\mathbf{F}$ if there is some trader k for whom $p^S_{kj}(\mathbf{F}) = \pi$ and $c^S_{kj}(\mathbf{F}) > 0$; similarly, it is an *active buying price* for good j in $\mathbf{F}$ if there is some trader k for whom $p^B_{kj}(\mathbf{F}) = \pi$ and $c^B_{kj}(\mathbf{F}) > 0$. (That is, a price is active if some trader is willing to trade at that price.) A price π is an *effective selling price* for good j in $\mathbf{F}$ if there is some trader k for whom $p^S_{kj}(\mathbf{F}) = \pi$ and $q^S_{kj}(\mathbf{F}) > 0$; similarly, it is an *effective buying price* for good j in $\mathbf{F}$ if there is some trader k for whom $p^B_{kj}(\mathbf{F}) = \pi$, and $q^B_{kj}(\mathbf{F}) > 0$. (That is, a price is effective if some trade takes place at that price.) An active selling price π for good j is *constrained* if (i) $q^S_{kj}(\mathbf{F}) = c^S_{kj}(\mathbf{F})$ holds for every trader k for whom $p^S_{kj}(\mathbf{F}) = \pi$, and (ii) for every offer configuration $\mathbf{F}'$ which differs from $\mathbf{F}$ only in respect of the selling constraint for good j of some such trader k, $c^S_{kj}(\mathbf{F}') > c^S_{kj}(\mathbf{F})$ implies $q^S_{kj}(\mathbf{F}') > c^S_{kj}(\mathbf{F})$. (That is, an active selling price is constrained if every trader who offers to sell at that price is selling as much as she is willing to sell, and if any relaxation of a selling constraint by any of those traders would induce an increase in sales.) The concept of a constrained buying price is defined in a symmetrical way. An offer configuration is *exploitable* if, for some non-money good, the lowest active selling price is strictly less than the highest active buying price. (That is, an exploitable offer configuration allows consumers to make pure gains by buying cheap and selling dear.)

Implicitly, the outcome function $\phi(\cdot)$ represents whatever factors govern the behavior of consumers over time in response to prices and constraints set by traders. It also represents whatever rationing mechanisms operate when there is excess demand or supply for goods. For the purposes of the model, it is not necessary to specify this function in detail. It is sufficient to impose the following restrictions on it:

B1. *FEASIBILITY. For all offer configurations* $\mathbf{F}$: (*i*) *for all non-money goods* $j = 2, \dots, m$: $\sum_i z_{ij} + q^S_j(\mathbf{F}) - q^B_j(\mathbf{F}) - d_j \geq 0$; (*ii*) $\sum_i z_{i1} + q^S_1(\mathbf{F}) - q^B_1(\mathbf{F}) \geq 0$.

B2. *RESPECT FOR SELLING AND BUYING CONSTRAINTS. For all offer configurations* $\mathbf{F}$, *for all traders k, for all non-money goods j*: (*i*) $q^S_{kj}(\mathbf{F}) \leq c^S_{kj}(\mathbf{F})$; (*ii*) $q^B_{kj}(\mathbf{F}) \leq c^B_{kj}(\mathbf{F})$.

B3. *PRICE SENSITIVITY. For all offer configurations* $\mathbf{F}$, *for all non-money goods j*: (*i*) *for all traders k, h, if* $p^S_{kj}(\mathbf{F}) < p^S_{hj}(\mathbf{F})$ *then* either $q^S_{hj}(\mathbf{F}) = 0$ *or* $q^S_{kj}(\mathbf{F}) = c^S_{kj}(\mathbf{F})$; (*ii*) *for all traders k, h, if* $p^B_{kj}(\mathbf{F}) > p^B_{hj}(\mathbf{F})$ *then* either $q^B_{hj}(\mathbf{F}) = 0$ *or* $q^B_{kj}(\mathbf{F}) = c^B_{kj}(\mathbf{F})$; (*iii*) *if* $p^B_{kj}(\mathbf{F}) > 0$ *and* $q^B_{kj}(\mathbf{F}) < c^B_{kj}(\mathbf{F})$ *are true for any trader k, then* $d_j(\mathbf{F}) = 0$.

B4. *INDEPENDENCE OF INACTIVE PRICES.* (*i*) *Let* $\mathbf{F}$, $\mathbf{F}'$ *be two offer configurations which differ only in respect of the selling price of some trader k for some non-money good j. If k is not an active seller of good j either in* $\mathbf{F}$ *or in* $\mathbf{F}'$, *then* $\phi(\mathbf{F}') = \phi(\mathbf{F})$. (*ii*) *Let* $\mathbf{F}$, $\mathbf{F}'$ *be two offer configurations which differ only in respect of the buying price of some trader k for some non-money good j. If k is not an active buyer of good j either in* $\mathbf{F}$ *or in* $\mathbf{F}'$, *then* $\phi(\mathbf{F}') = \phi(\mathbf{F})$.

B5. *SYMMETRY. For all offer configurations* $\mathbf{F}$, *for all traders k, h, for all non-money goods j*: (*i*) *if* $p^S_{kj}(\mathbf{F}) = p^S_{hj}(\mathbf{F})$ *and* $q^S_{kj}(\mathbf{F}) > q^S_{hj}(\mathbf{F})$, *then* $q^S_{hj}(\mathbf{F}) = c^S_{hj}(\mathbf{F})$; (*ii*) *if* $p^B_{kj}(\mathbf{F}) = p^B_{hj}(\mathbf{F})$ *and* $q^B_{kj}(\mathbf{F}) > q^B_{hj}(\mathbf{F})$, *then* $q^B_{hj}(\mathbf{F}) = c^B_{hj}(\mathbf{F})$.

B6. *CONTINUITY. For all non-money goods j: within the domain of nonexploitable offer configurations,* $q^S_j(\cdot)$, $q^B_j(\cdot)$ *and* $d_j(\mathbf{F})$ *are continuous functions of all* p^S_{hi}, p^B_{hi}, c^S_{hi}, c^B_{hi}.

B7. *BOUNDEDNESS. For each non-money good j: within the domain of nonexploitable offer configurations, there is an upper bound to each of* $q^S_j(\mathbf{F})$ *and* $q^B_j(\mathbf{F})$.

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B8. *INTRINSIC VALUE OF MONEY. There is some* ε, *satisfying* $0 < \varepsilon < 1$, *such that, for all offer configurations* **F**, *for all vectors of prices* $(\pi_2, \ldots, \pi_m)$: *if, in* **F**, *each* π_j *is both an active and unconstrained selling price and an active and unconstrained buying price, and if* $\sum_{j=2}^{m} \pi_j/(1 + \sum_{g=2}^{m} \pi_g) \geq 1 - \varepsilon$, *then* $q_1^S(\mathbf{F}) - q_1^B(\mathbf{F}) > 0$.

The first four of these restrictions are motivated by the informal description of the model. B1 states the feasibility condition that, for each good, consumers' total endowments *plus* traders' net sales *minus* consumers' disposals (in the case of a non-money good) are nonnegative. Notice that, for each good j, the left-hand side of the relevant inequality is identically equal to $\sum_i x_{ij}$, that is, consumers' post-trade holdings of good j. B2 requires that traders' selling and buying constraints are not exceeded. Parts (i) and (ii) of B3 require that if some traders offer higher selling prices or lower buying prices than others, those offering the less favorable prices make trades only if the constraints associated with the more favorable prices are binding. This is an implication of the informal assumption that, at every point in time, consumers trade only at the most favorable prices currently available. Part (iii) of B3 extends this assumption to the disposal option. It requires that this option is used only if all the constraints associated with nonzero buying prices are binding. B4 requires that consumers take no account of the selling prices of traders who are not active sellers, nor of the buying prices of traders who are not active buyers. This is an implication of the informal assumption that, at any time, consumers see only the prices offered by traders whose constraints are not then binding.

The remaining restrictions are simplifying assumptions; they have little substantive economic content. B5 requires that, except as a consequence of binding constraints, traders who quote the same selling (or buying) price for a given good sell (or buy) equal quantities of it. This assumption abstracts from the effects of random variation in the assignment of consumers to traders. B6 requires that small changes in traders' prices and constraints do not induce discontinuous changes in the aggregate behavior of consumers.[7] B7 rules out the possibility that, in response to nonexploitable offer configurations, consumers are willing to buy or sell indefinitely large quantities of any non-money good.[8] B8 requires that if the selling and buying prices of non-money goods are unconstrained and sufficiently high, consumers choose to make trades whose overall effect is to induce a net flow of money from traders to consumers. Intuitively: if money is sufficiently cheap relative to goods, consumers will add to their initial holdings of money. Given the informal assumption that money has intrinsic value, this condition makes only minimal demands on consumers' rationality.

If this model is to serve its intended purpose, the restrictions B1–B8 should be consistent with the conventional theory of rational choice while also permitting a wide variety of the types of deviation from that theory that have been studied by behavioral economists. It is easy to verify that B1–B8 are consistent with the assumption that each consumer's behavior is rationalizable by a preference ordering over bundles of the m goods.[9] I now present some illustrations of the capacity of the model to accommodate deviations from conventional theory.

First, consider an offer configuration in which, for each non-money good j, all traders set the same selling price π_j^S and the same buying price π_j^B, with $\pi_j^S > \pi_j^B$. Suppose that each of these prices is unconstrained. If a consumer acts on a stable preference ordering in which more money is preferred to less, he will not incur unambiguous losses by both buying and selling the same good. However, such behavior might be induced by stochastic variation

[7] B6 allows that small changes in traders' prices might induce discontinuous changes in the *distribution* of trade among traders. It also allows there to be discontinuities at the boundary of the set of exploitable offer configurations. The first type of discontinuity should be expected as a result of the price sensitivity of consumers; the second type would occur if consumers were sufficiently alert to opportunities for arbitrage.

[8] The model imposes no limit on the number of times that a consumer may sell and re-buy the same good. Thus, in the absence of B7, infinite trading volumes would be compatible with finite endowments and strictly positive prices.

[9] This preference ordering can be required to satisfy the conventional properties of continuity, convexity, and local nonsatiation. Because of the boundedness condition B7, there cannot be nonsatiation in all goods everywhere in commodity space, but there can be nonsatiation in money.

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in the consumer's preferences over the trading period, or (as explained in Section II) by systematic changes in preferences over this period as a result of hyperbolic discounting. B1–B8 permit such loss-making combinations of buying and selling.

Now consider the family of offer configurations in which, for each non-money good j, there is a unique price π_j, set by all active traders both as a selling price and a buying price, which is unconstrained both for selling and buying. In relation to such offer configurations, the behavior of a consumer who acts on stable, strictly convex preferences satisfies the weak axiom of revealed preference. (That is, if some consumer i chooses some bundle $\mathbf{y}_i$ in some situation in which another bundle $\mathbf{y}_i'$ is feasible, he will not choose $\mathbf{y}_i'$ when $\mathbf{y}_i$ is feasible.) However, violations of this axiom can be induced by reference-dependent preferences, as modeled by Tversky and Kahneman (1991) and Alistair Munro and Sugden (2003). If i's preferences are reference-dependent, it is possible that, if endowed with some bundle $\mathbf{y}_i$, he would reject an opportunity to exchange it for another bundle $\mathbf{y}_i'$, while if he had been endowed with $\mathbf{y}_i'$, he would have rejected an opportunity to trade in the opposite direction. Such behavior is consistent with B1–B8.

Violations of the weak axiom can also occur if consumers are motivated to buy more of a good by the knowledge that other people are buying it at a high price. This kind of "snob effect" is closely related to the shaping effects discussed in Section I.[10] It is easy to construct demand functions which are continuous in prices and which have the property that compensated demand is *positively* related to price, in contravention of the weak axiom.[11] B1–B8 permit such demand functions.

Now that I have constructed a model market, I define a concept of equilibrium for that model. The concept I use, *free-entry equilibrium,* is intended to correspond as closely as possible with the concept of perfectly competitive equilibrium that microeconomics has traditionally applied to sets of competing firms. The intuitive idea is that an offer configuration is a free-entry equilibrium if no trader trades at a loss, and if traders who are not active in the market cannot make positive profits by entering (other things remaining constant).

As with conventional concepts of competitive equilibrium for firms, there is an implicit assumption that, in equilibrium, traders can predict the consequences for themselves of the alternative trading strategies that are available to them. In the context of my model, this requires that the behavior of consumers is predictable in the aggregate. Notice that this form of predictability does not imply that the choices of individual consumers reveal coherent preferences—for at least two reasons. First, the behavior of an individual may be predictable by virtue of regularities that are consistent with some theory other than that of rational choice. Second, if the number of consumers is large, predictability in the aggregate is compatible with stochastic variation in the behavior of individual consumers.

Formally, I define an offer configuration $\mathbf{F}$ to be a *free-entry equilibrium* if it satisfies the following conditions:

C1. *NONNEGATIVE PROFIT. For each trader* k: $\sum_{j=2}^{m} p_{kj}^S(\mathbf{F})q_{kj}^S(\mathbf{F}) \geq \sum_{j=2}^{m} p_{kj}^B(\mathbf{F})q_{kj}^B(\mathbf{F})$.

C2. *NO SHORTFALLS. For each trader k, for each non-money good j*: $q_{kj}^B(\mathbf{F}) \geq q_{kj}^S(\mathbf{F})$.

C3. *NO PROFITABLE ENTRY. Let* $\mathbf{F}'$ *be any offer configuration which differs from* $\mathbf{F}$ *only in respect of the prices and constraints of some trader k who is not active in* $\mathbf{F}$. *If* $q_{kj}^B(\mathbf{F}') \geq q_{kj}^S(\mathbf{F}')$ *for each non-money good j, then* $\sum_{j=2}^{m} p_{kj}^S(\mathbf{F}')q_{kj}^S(\mathbf{F}') \leq \sum_{j=2}^{m} p_{kj}^B(\mathbf{F}')q_{kj}^B(\mathbf{F}')$.

C4. *EXISTENCE OF POTENTIAL ENTRANT. At least one trader is not active in* $\mathbf{F}$.

C1 requires that no trader makes a loss. Since it is possible to avoid losses by becoming nonactive, this is a natural equilibrium condition. C2 requires that no trader's sales of any good

[10] The snob effect (choosing to buy a good because other people are buying it at a high price) is compatible with price sensitivity (when buying a good, buying it at the lowest available price). Arguably, both of these effects are implicated in the efforts that some producers of goods with "designer" labels make to prevent their products being sold at discounted prices.

[11] For example, suppose there are two goods, with prices p_1 and p_2. A consumer is endowed with quantities z_1 and z_2. The quantities demanded, x_1 and x_2, are given by the demand functions $x_1 = p_1[p_1z_1 + p_2z_2]/[p_1^2 + p_2^2]$ and $x_2 = p_2[p_1z_1 + p_2z_2]/[p_1^2 + p_2^2]$.

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exceed her purchases; this is an implicit representation of the informal assumption that traders are heavily penalized for failure to deliver on the promises they make. C3 requires that no nonactive trader can make a strictly positive profit by becoming active, while satisfying the condition that sales may not exceed purchases. This is a natural equilibrium condition for a market with free entry and exit. C4 is a convenient way of representing the idea, familiar in the theory of perfect competition, that, for every market, there are potential entrants.

The final component of the model is a specification of the opportunities that are made available to each consumer by each offer configuration. In specifying these opportunities, two theoretical problems have to be faced. The first is that, because of the buying and selling constraints set by traders, consumers may be unable to make transactions that *in fact* they want to make at effective prices. To keep the model simple and general, I have not included any explicit representation of the rationing process that operates when buying or selling constraints bind; but, in the absence of such a representation, there is ambiguity about the opportunities available to given consumers when some prices are both effective and constrained. The second problem is that, even if all effective prices are unconstrained, there may be *counterfactual* transactions that consumers may be unable to make at those prices.

I sidestep the first problem by considering only those opportunities that are generated by unconstrained prices. It will turn out that, for the purposes of this paper, an analysis of the opportunities created by constrained prices is not needed. Provided the model is interpreted as representing a market in which there are many consumers, each of whom accounts for only a small share of the total volume of trade, the second problem seems to be of little economic significance. Consider an offer configuration such that all effective prices are unconstrained. Thus, no consumer is constrained in carrying out the trades that he in fact chooses to make. Now consider the effect of a change in the behavior of one consumer i, everything else being held constant. At the level of the market, this will induce only small changes in the total trading requests made by consumers. If, as a result of those changes, some trading constraints start to bind, unsatisfied requests will be small relative to total trading volumes. Since the resulting rationing effects will be spread over all consumers, any effect on i himself can be expected to be very small.[12]

Accordingly, I focus on the opportunities that are made available to each consumer by unconstrained prices, on the implicit assumption that any requests made by the consumer to trade at those prices would be met. Consider any consumer i in relation to any offer configuration $\mathbf{F}$. I define the *unconstrained opportunity set* for consumer i, denoted $\mathcal{O}_i^*(\mathbf{F})$, as the set of all those bundles $\mathbf{x}_i \equiv (x_{i1}, \dots, x_{im})$ that i can reach, given his initial endowment $\mathbf{z}_i$, by buying and selling non-money goods at prices that are active and unconstrained in $\mathbf{F}$, and by free disposal of non-money goods.

VII. Arbitrage and Opportunity

I now investigate the implications of free-entry equilibrium for consumers' opportunities. It is natural to begin by asking whether free-entry equilibrium exists. The following result gives a positive answer to that question:

Result 2: If the outcome function $\phi(\cdot)$ satisfies B1, B2, B3, B6, B7, and B8, then there is an offer configuration which is a free-entry equilibrium.

The next result states what, for the purposes of this paper, are the crucial properties of free-entry equilibrium:

Result 3: For all offer configurations $\mathbf{F}$: if $\mathbf{F}$ is a free-entry equilibrium and if the outcome function $\phi(\cdot)$ satisfies B1, B2, B3, B4, B5, and B6, then for each non-money good j, if there exists any effective selling price or effective buying price for that good, there is a price $\pi_j \geq 0$ such that (i) π_j is both the unique effective selling price and the unique effective buying price for good j, (ii) π_j is unconstrained, both as

[12] In a more sophisticated (but more complex) model, the outcome of each offer configuration would be subject to stochastic variation, and traders would carry inventories to allow them to respond to variations in trading requests. In real markets, the existence of inventories provides each consumer with the assurance that his trading requests, however unexpected, will be met—provided that all other consumers act predictably.

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a selling price and a buying price, and (iii) either $\pi_j > 0$ and $q_j^B(\mathbf{F}) = q_j^S(\mathbf{F})$ or $\pi_j = 0$ and $q_j^B(\mathbf{F}) \geq q_j^S(\mathbf{F})$.

This result tells us that, provided at least some trade takes place in each non-money good, competition among profit-seeking traders combined with price sensitivity on the part of consumers ensures that each good is traded at a single price, the same price for buying and for selling, and that all requests by consumers to trade at those prices are met.

To see the implications of this result for consumers' opportunities, suppose there exists some offer configuration $\mathbf{F}$ such that $\mathbf{F}$ is a free-entry equilibrium and such that some trade takes place in each non-money good. Then, by Result 3, there is a vector $\boldsymbol{\pi} \equiv (\pi_2, \ldots, \pi_m)$ of unconstrained prices at which, given $\mathbf{F}$, consumers are able to buy and to sell. Thus, the unconstrained opportunity set for each consumer is given by:

(4)

$$\mathcal{O}_i^*(\mathbf{F}) = \left\{ \mathbf{x}_i : x_{i1} - z_{i1} + \sum_{j=2}^{m} \pi_j (x_{ij} - z_{ij}) \leq 0 \right\}; \quad (i = 1, \ldots, n).$$

Notice that, in $\mathbf{F}$, there are no active buying or selling prices more favorable to consumers than $\boldsymbol{\pi}$. (This follows from its being a property of free-entry equilibrium that each π_j is the *unique* effective buying and selling price for good j, and from the price-sensitivity condition B3.) Thus, $\mathcal{O}_i^*(\mathbf{F})$ represents the opportunities that are made available to i by *all* prices in a free-entry equilibrium offer configuration $\mathbf{F}$.

It is now possible to adapt Result 1 to generate the core result of this paper:

Result 4: For all offer configurations $\mathbf{F}$: if $\mathbf{F}$ is a free-entry equilibrium, if the outcome function $\phi(\cdot)$ satisfies B1, B2, B3, B4, B5, and B6, and if, for each non-money good j, there is either some effective selling price or some effective buying price, then $\mathbf{O}^*(\mathbf{F}) \equiv (\mathcal{O}_1^*[\mathbf{F}], \ldots, \mathcal{O}_n^*[\mathbf{F}])$ satisfies the opportunity criterion.

So, provided that at least some trade takes place in each non-money good, competition among traders and price sensitivity on the part of consumers are sufficient to ensure that the opportunity criterion is satisfied.

The proviso that at least some trade takes place in each non-money good is unavoidable. To see why, consider the case in which, for some non-money good j, the lowest price at which any consumer in any circumstances would sell that good is higher than the highest price at which any consumer in any circumstances would buy it. Then, clearly, it is compatible with free-entry equilibrium for there to be no active buyers or sellers of good j. If that is the case, individual consumers do not have any opportunities to exchange good j for other goods, and so the opportunity criterion cannot be satisfied. However, this proviso demands very little. So long as some consumer is willing to sell some quantity of good j at some price greater than that at which another consumer is willing to buy some quantity of it, there cannot be a free-entry equilibrium in which there is no trade in that good. Thus, the proviso requires only that there are *some* potential gains from trade in each good.

VIII. Markets as Money Pumps

What, if anything, is good about allocating resources through markets? That question was the starting point for this paper. My analysis supports a particular answer: that competitive markets are efficient mechanisms for providing individuals with opportunities, thereby allowing each person to be responsible for the outcomes he or she experiences. In interpreting the formal results, it is natural to ask what gives markets this (on my account) normatively valuable property. Viewed through the lens of my model, what are the essential features of markets?

I suggest that four features of the model are particularly significant as stylized representations of properties of real markets. First, there is the *profit motive.* There is a significant number of actors who are motivated by a desire to buy cheap and sell dear, and who are alert to opportunities which allow them to do so. Second, there is *free entry.* Potential traders who see opportunities for profit are not prevented from exploiting them. This property is crucial in ensuring that, in equilibrium, no profits can be made through arbitrage. Third, there is *public-*

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ness of transactions. All offers to buy and sell are visible to, and open to, all consumers. It is by virtue of this property that each consumer has opportunities that extend beyond the trades that he is in fact willing to make. Finally, there is *price sensitivity.* Consumers are aware of any differences in the prices offered to them by different traders, and buy and sell only at the most favorable prices currently on offer. It is because of this property that the profit motive leads traders to compete to offer favorable prices to consumers. Summing up, the profit motive (on the part of traders) and price sensitivity (on the part of consumers) supply the motive power for a process of competition. Free entry and publicness are the fundamental rules of the game which govern that process.

In comparison with more familiar models of markets, my model is distinctive in how little it assumes about the rationality of consumers: all it assumes is price sensitivity. Most of the work of generating the valuable properties of markets is done by profit-seeking traders. From the viewpoint of those traders, consumers are essentially passive, responding to traders' offers in a predictable but not necessarily rational fashion. We might say that the body of consumers appears to traders rather as a population of fish appears to a set of competing trawler-owners, or as an oil field appears to a set of competing oil-prospectors.

The metaphor of the oil field prompts another: the traders are operators of *money pumps.* In the literature of decision theory, a money pump is a sequence of trading opportunities offered to a particular individual such that, if all those opportunities are accepted, the resulting sequence of trades generates an unambiguous gain for the trader who offers the opportunities and an unambiguous loss for the person who accepts them. Many theorists have claimed that, in order for an individual to be invulnerable to exploitation by money pumps, he must have preferences which satisfy conventional axioms of rationality, such as transitivity and the independence axiom of expected utility theory. The suggestion has been that the possibility of money pumps forces economic actors to act on preferences which satisfy rationality axioms. It is a presupposition of this literature (shared by those theorists who deny the validity of money pump arguments) that vulnerability to money pumps, if it exists, is a pathology of individual decision-making.[13]

In contrast, my approach does not make any fundamental distinction between money pumps—that is, trading sequences which impose unambiguous losses on "irrational" individual consumers—and what might be called benign arbitrage—that is, trading activities which generate profits by realizing gains from trade between "rational" consumers.[14] In each case, traders make profits by offering to consumers opportunities which those consumers freely choose to take up. In each case, the existence of profit opportunities is a phenomenon of disequilibrium. Competition between traders ensures that neither kind of profit opportunity exists in equilibrium. Nevertheless, the preference inconsistencies which make consumers potential victims of money pumps may persist in equilibrium. For example, a consumer whose preferences are subject to stochastic variation, or who discounts the future hyperbolically, may be willing to make trades at one moment which, later, he would be willing to pay a premium to reverse. According to the conventions of the money-pump literature, such a consumer's preferences are "exploitable"—by which is meant that the consumer could be money-pumped by a monopolist with unlimited ability to set discriminatory prices. In my model, however, traders do not have such monopoly power. In equilibrium, preference inconsistencies are not exploited. What competition erodes is not individuals' propensities to act contrary to the axioms of rational choice theory, but the profit margins that can be achieved by trading with individuals who act in this way.

On the account I am offering, it is as a result of providing a field of open competition for would-be arbitrageurs and money-pump operators that markets provide the opportunities that allow individuals to take responsibility for the

[13] Recent contributions to the theoretical debate about the validity of money pump arguments include Mark J. Machina (1989), McClennen (1990), Paul Anand (1993), Kim C. Border and Uzi Segal (1994), David Kelsey and Frank Milne (1997), and Robin P. Cubitt and Sugden (2001).

[14] Robert F. Nau and Kevin F. McCardle's (1991) analysis of the relationship between arbitrage and rationality follows a similar approach to that of the present paper, but in relation to choice among lotteries with money consequences.

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outcomes they experience. A market, we might say, is a complex system of money pumps, each of which is operated with the intention of extracting value from us, the consumers. Nevertheless, that system gives us opportunity and responsibility—whether or not our preferences meet the standards of rational choice theory.

Appendix

PROOF OF RESULT 1:

Let $(\mathbf{x}^*, \mathbf{z}, \mathbf{O})$ be any market-compatible triple. To prove Result 1, it is sufficient to show that, for every feasible allocation $\mathbf{x} \neq \mathbf{x}^*$, there is some individual i for whom $x_i \neq x_i^*$ and $x_i \in \mathcal{O}_i$. To allow a proof by contradiction, suppose the contrary. Then there exists some feasible allocation $\mathbf{x}' \neq \mathbf{x}^*$ such that, for each i, either (a) $x_i' = x_i^*$ or (b) $x_i' \notin \mathcal{O}_i$. Let $\boldsymbol{\rho}$ be the price vector in relation to which A1 and A2 are satisfied. Trivially, (a) implies $\Sigma_j\, \rho_j(x_{ij}' - x_{ij}^*) = 0$. Since $x_i^* \in \mathcal{O}_i$, (b) implies $\Sigma_j\, \rho_j(x_{ij}' - x_{ij}^*) > 0$. Since $\mathbf{x}' \neq \mathbf{x}^*$, (b) must apply for some individual i. Thus, $\Sigma_i \Sigma_j\, \rho_j(x_{ij}' - x_{ij}^*) > 0$. Since $\mathbf{x}^*$ satisfies A1, we have $\Sigma_i \Sigma_j\, \rho_j(x_{ij}^* - z_{ij}) = 0$. Hence $\Sigma_i \Sigma_j\, \rho_j(x_{ij}' - z_{ij}) > 0$. Since each ρ_j is nonnegative, this implies that, for some good j, $\Sigma_i\, (x_{ij}' - z_{ij}) > 0$, which contradicts the supposition that $\mathbf{x}'$ is feasible.

PROOF OF RESULT 2:

The proof is modeled on standard fixed-point proofs of the existence of competitive equilibrium. Following the structure of such proofs, prices are expressed in normalized form. Consider any vector $\boldsymbol{\pi} \equiv (\pi_2, \dots, \pi_m)$ of nonnegative prices for non-money goods, defined so that each π_j is the money value of one unit of good j. For any such $\boldsymbol{\pi}$, we can construct a vector $\boldsymbol{\rho} \equiv (\rho_1, \dots, \rho_m)$ of nonnegative *normalized* prices for all goods (including money), defined by $\rho_1 = 1/[1 + \Sigma_{h=2}^{m}\, \pi_h]$ and, for $j = 2, \dots, m$, $\rho_j = \pi_j/[1 + \Sigma_{h=2}^{m}\, \pi_h]$. Conversely, we may define an *admissible* normalized price vector as any vector $\boldsymbol{\rho} \equiv (\rho_1, \dots, \rho_m)$ of nonnegative numbers summing to unity and satisfying $\rho_1 > 0$. For every admissible $\boldsymbol{\rho}$, there is a price vector $\boldsymbol{\pi} \equiv (\pi_2, \dots, \pi_m)$ such that $\boldsymbol{\rho}$ is its normalization.

The following procedure can be used to construct a specific offer configuration $\mathbf{F}_\rho$ from any given admissible normalized price vector $\boldsymbol{\rho}$. In this offer configuration, trader 1 is the only active trader. For each non-money good j, fix numbers γ_j^S, γ_j^B such that, for all nonexploitable offer configurations $\mathbf{F}$, if $c_{1j}^S(\mathbf{F}) = \gamma_j^S$ and $c_{1j}^B(\mathbf{F}) = \gamma_j^B$, the buying and selling prices set by trader 1 are unconstrained. That such numbers exist follows from B7. Define the price vector $\boldsymbol{\pi} \equiv (\pi_2, \dots, \pi_m)$ so that $\boldsymbol{\rho}$ is its normalization. Then, for each non-money good j, set $p_{1j}^S[\mathbf{F}_\rho] = p_{1j}^B[\mathbf{F}_\rho] = \pi_j$, $c_{1j}^S(\mathbf{F}_\rho) = \gamma_j^S$, and $c_{1j}^B(\mathbf{F}_\rho) = \gamma_j^B$. Using the outcome function $\phi(\cdot)$, we can determine the values of $q_{1j}^S(\mathbf{F}_\rho)$ and $q_{1j}^B(\mathbf{F}_\rho)$ for each good j. Thus, for each good j, we can define an *excess sales* function $w_j(\boldsymbol{\rho}) \equiv q_{1j}^S(\mathbf{F}_\rho) - q_{1j}^B(\mathbf{F}_\rho)$.

Suppose that, for each good j, either $w_j(\boldsymbol{\rho}) = 0$ or (for $j \neq 1$) $w_j(\boldsymbol{\rho}) < 0$ and $\rho_j = 0$; this will be called the *balanced trade* condition. Then C1 and C2 are satisfied. For an entrant to make positive profit, *either* (a) she must sell a positive quantity of some non-money good j at a price higher than π_j, *or* (b) she must buy a positive quantity of some non-money good j at a price lower than π_j. But, because of B3 and the assumption (licensed by B7) that trader 1's buying and selling prices are unconstrained for all nonexploitable offer configurations, neither (a) nor (b) can be true. Thus, C3 is satisfied. Since only one trader is active, and since $N > 1$ by assumption, C4 is satisfied. Since all of C1–C4 are satisfied, $\mathbf{F}_\rho$ is a free-entry equilibrium. Hence, in order to prove Result 2, it is sufficient to show the existence of some admissible $\boldsymbol{\rho}$ such that the balanced trade condition is satisfied.

Now consider the following class of *tâtonnement* functions $\tau(\cdot)$, each of which constructs a "new" admissible normalized price vector $\boldsymbol{\rho}' = \tau(\boldsymbol{\rho})$ from an "existing" admissible normalized price vector $\boldsymbol{\rho}$. The new price of each good j is defined by $\rho_j' = \rho_j''/\Sigma_j\, \rho_j''$ where $\rho_j'' \equiv \rho_j[1 + \alpha \min(0, w_j[\boldsymbol{\rho}]/\Sigma_i\, z_{ij})]$ and α is a parameter satisfying $0 < \alpha < 1$. This definition has the following features:

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(i) For each good j: $\rho'_j - \rho_j > -\alpha$. (The derivation of this property uses the fact that $w_j(\boldsymbol{\rho}) \geq -\Sigma_i z_{ij}$, which is implied by B1.)

(ii) For each good j: if $w_j(\boldsymbol{\rho}) > 0$ and $\rho_j > 0$, then $\rho'_j > \rho_j$. [Suppose $w_j(\boldsymbol{\rho}) > 0$ and $\rho_j > 0$. Then $\rho''_j = \rho_j$. The identities (2) and (3) entail $\Sigma_j \rho_j w_j(\boldsymbol{\rho}) \equiv 0$. Thus, for some good h, $\rho_h w_h(\boldsymbol{\rho}) < 0$, and hence $\rho''_h < \rho_h$. But, for all goods h, $\rho''_h \leq \rho_h$. Thus, after normalization, we have $\rho'_j > \rho_j$.]

(iii) If the balanced trade condition is not satisfied with respect to $\boldsymbol{\rho}$, then $\boldsymbol{\rho}' \neq \boldsymbol{\rho}$. [If the balanced trade condition is not satisfied, there is some good j such that $w_j(\boldsymbol{\rho}) \neq 0$ and $\rho_j > 0$. But then, because of the identity $\Sigma_j \rho_j w_j(\boldsymbol{\rho}) \equiv 0$, there must be some good g such that $w_g(\boldsymbol{\rho}) > 0$ and $\rho_g > 0$. Using (ii), this implies $\rho'_g > \rho_g$.]

It follows from (iii) that, in order to prove Result 2, it is sufficient to show that $\boldsymbol{\rho}' = \boldsymbol{\rho}$ holds for some admissible $\boldsymbol{\rho}$, that is, that $\tau(\cdot)$ has a fixed point.

The proof can be completed by using Brouwer's Theorem if we can show that, for some value of α, $\tau(\cdot)$ is a continuous mapping from a closed, bounded, and convex set into itself. By B6, for all values of α, $\tau(\cdot)$ is a continuous mapping from the set of all admissible normalized price vectors into itself; and that set is bounded and convex. But, because of the condition $\rho_1 > 0$, it is not closed.[15] However, it is sufficient for the proof to find a subset of the set of all admissible normalized price vectors such that this subset is closed, bounded, and convex and is mapped into itself by $\tau(\cdot)$ for some value of α.

To construct such a subset, fix some value of ε in the range $0 < \varepsilon < 1$, such that $w_1(\boldsymbol{\rho}) > 0$ is true for all $\boldsymbol{\rho}$ satisfying $0 < \rho_1 \leq \varepsilon$. That some such ε exists is entailed by B8. Next, fix the value of α so that $0 < \alpha < \varepsilon$. Now consider the subset of normalized price vectors $\mathbf{P}^* = \{\boldsymbol{\rho}: \rho_1 \geq \varepsilon - \alpha\}$. Notice that $\mathbf{P}^*$ is closed, bounded, and convex. Let $\boldsymbol{\rho}$ be any element of $\mathbf{P}^*$, and let $\boldsymbol{\rho}' = \tau(\boldsymbol{\rho})$. By virtue of B8 and (ii), $\varepsilon - \alpha \leq \rho_1 \leq \varepsilon$ implies $\rho'_1 > \rho_1$ and hence $\boldsymbol{\rho}' \in \mathbf{P}^*$. By virtue of (i), $\rho_1 \geq \varepsilon$ implies $\rho'_1 > \varepsilon - \alpha$ and hence $\boldsymbol{\rho}' \in \mathbf{P}^*$. Thus, for all $\boldsymbol{\rho} \in \mathbf{P}^*$, $\tau(\boldsymbol{\rho}') \in \mathbf{P}^*$.

PROOF OF RESULT 3:

Consider any offer configuration $\mathbf{F}$ which is a free-entry equilibrium. Suppose (this is *Supposition 1*) that, for some non-money good j, there is an effective selling price π^S and an effective buying price π^B such that $\pi^S > \pi^B$. Let k be any trader who is not active in $\mathbf{F}$; the existence of such a trader follows from C4. Let $\mathbf{F}^1$ be an offer configuration which differs from $\mathbf{F}$ only in respect of k's selling and buying prices and constraints for good j. Set $p^S_{kj}(\mathbf{F}^1) = \pi^S$, $p^B_{kj}(\mathbf{F}^1) = \pi^B$, and $c^S_{kj}(\mathbf{F}^1) = c^B_{kj}(\mathbf{F}^1) = \varepsilon$, where $\varepsilon > 0$. By B4 and B6, as $\varepsilon \to 0$, $q^S_j(\mathbf{F}^1) - q^S_j(\mathbf{F}) \to 0$ and $q^B_j(\mathbf{F}^1) - q^B_j(\mathbf{F}) \to 0$. Thus, by B2, B3, and B5, for sufficiently small values of ε, $q^S_{kj}(\mathbf{F}^1) = q^B_{kj}(\mathbf{F}^1) = \varepsilon$. At any such ε, k makes a profit in $\mathbf{F}^1$, contrary to the "no profitable entry" condition C4. Hence, Supposition 1 is false.

Thus, for each non-money good, the lowest effective buying price is at least as high as the highest effective selling price. In conjunction with the "no shortfalls" condition C2, this implies that no trader makes a strictly positive profit in her transactions in any non-money good. But, by virtue of the nonnegative profit condition C1, no trader makes a strictly negative total profit. So each trader makes zero profit in her transactions in each non-money good. Since effective selling prices are never greater than effective buying prices, and since (by C2) there are no shortfalls, it follows that, for each non-money good j, *either* (a) no trade takes place (i.e., there is no effective buying price and no effective selling price for good j) *or* (b) some trade takes place in good j, all such trade—both buying and selling—takes place at the same price π_j, and, for each trader, *either* $\pi_j > 0$ and sales and purchases of good j are equal *or* $\pi_j = 0$ and sales are no greater than purchases. This proves parts (i) and (iii) of Result 3.

Now suppose that (b) holds for some non-money good j, and let π_j be the effective buying and selling price for that good. Suppose (this is *Supposition 2*) that π_j is a constrained selling price. Let k be any trader not active in $\mathbf{F}$, and let $\mathbf{F}^2$ be an offer configuration which differs from $\mathbf{F}$ only in

[15] This condition reflects the special role of money as a medium of exchange in the model. If all trade is conducted in money, the relative prices of non-money goods can be defined only if money has positive value.

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respect of k's selling and buying prices and constraints for good j. Set $p^S_{kj}(\mathbf{F}^2) = p^B_{kj}(\mathbf{F}^2) = \pi_j$, $c^S_{kj}(\mathbf{F}^2) = \varepsilon^S$, and $c^B_{kj}(\mathbf{F}^2) = \varepsilon^B$, where $\varepsilon^S, \varepsilon^B > 0$. By the same reasoning as used in the first part of the proof, for sufficiently small values of ε^S and ε^B, $q^S_{kj}(\mathbf{F}^2) = \varepsilon^S$ and $q^B_{kj}(\mathbf{F}^2) = \varepsilon^B$.

The next step is to show that, for sufficiently small values of ε^S and ε^B, π_j is also a constrained selling price in $\mathbf{F}^2$. Since π_j is a constrained selling price in $\mathbf{F}$, if any trader h who is an active seller of good j in $\mathbf{F}$ relaxes her selling constraint, all other prices and constraints being held constant at their values in $\mathbf{F}$, h's sales of good j increase. By virtue of B4 and B6, for sufficiently small values of ε^S and ε^B, it must also be true that if h relaxes her selling constraint with all other prices and constraints being held constant at their values in $\mathbf{F}^2$, her sales increase. It has been shown that for sufficiently small values of ε^S and ε^B, $q^S_{kj}(\mathbf{F}^2) = \varepsilon^S$ (where k is the trader who is active in $\mathbf{F}^2$ but not in $\mathbf{F}$). This implies that, starting from sufficiently small values of ε^S and ε^B, if k relaxes her selling constraint with all other prices and constraints being held constant at their values in $\mathbf{F}^2$, k's sales increase. Thus π_j is a constrained selling price in $\mathbf{F}^2$.

Now let $\mathbf{F}^3$ be any offer configuration which differs from $\mathbf{F}$ only in respect of k's selling and buying prices and constraints for good j. Set $p^S_{kj}(\mathbf{F}^3) = \pi_j + \delta$, $p^B_{kj}(\mathbf{F}^3) = \pi_j$, $c^S_{kj}(\mathbf{F}^3) = \varepsilon^S$, and $c^B_{kj}(\mathbf{F}^3) = \varepsilon^B$, where $\delta > 0$, where ε^S and ε^B take the same values as in $\mathbf{F}^2$, and such that there is no trader h for whom $\pi_j + \delta > p^S_{hj}(\mathbf{F}^3) > \pi_j$ and $c^S_{hj}(\mathbf{F}^3) > 0$. (Since the number of traders is finite, the condition that there is no such trader is satisfied at all sufficiently small values of δ.) Now compare the trades that k makes in $\mathbf{F}^2$ and $\mathbf{F}^3$. It follows from B6 that, as $\delta \to 0$, $q^B_j(\mathbf{F}^3) - q^B_j(\mathbf{F}^2) \to 0$. Thus, by B2 and B3, at sufficiently low values of ε^B, $c^B_{kj}(\mathbf{F}^3) = \varepsilon^B$. Similarly, as $\delta \to 0$, $q^S_j(\mathbf{F}^3) - q^S_j(\mathbf{F}^2) \to 0$. Since π_j is the only effective selling price for good j in $\mathbf{F}$, no trader in $\mathbf{F}^2$ or $\mathbf{F}^3$ sells good j at any price π'_j such that $\pi'_j \neq \pi_j$ and $\pi'_j < \pi_j + \delta$. Since π_j is a constrained selling price in $\mathbf{F}^2$, sales of good j by traders other than k at price π_j cannot be greater in $\mathbf{F}^3$ than in $\mathbf{F}^2$. Thus, total sales by traders other than k at prices less than $\pi_j + \delta$ are no greater in $\mathbf{F}^3$ than in $\mathbf{F}^2$. By B3, in $\mathbf{F}^3$, total sales at prices greater than $\pi_j + \delta$ are zero unless k's selling constraint is binding, i.e., unless $q^S_{kj}(\mathbf{F}^3) = q^S_{kj}(\mathbf{F}^2)$. Thus, as $\delta \to 0$, $q^S_{kj}(\mathbf{F}^3) - q^S_{kj}(\mathbf{F}^2) \to 0$. Using B4, it follows that there exist strictly positive ε^S, ε^B, and δ, such that $\varepsilon^S = \varepsilon^B$, $q^S_{kj}(\mathbf{F}^3) = \varepsilon^S$, and $q^B_{kj}(\mathbf{F}^3) = \varepsilon^B$. At such values of ε^S, ε^B, and δ, k makes a profit in $\mathbf{F}^3$, contrary to the "no profitable entry" condition C4. Hence Supposition 2 is false: π_j is *not* a constrained selling price. A symmetrical argument shows that π_j is not a constrained buying price. This proves part (ii) of Result 3.

PROOF OF RESULT 4:

Let $\mathbf{F}$ be an offer configuration which is a free-entry equilibrium, and suppose that, for each non-money good, there is either an effective selling price or an effective buying price. Let $\boldsymbol{\pi} \equiv (\pi_2, \dots, \pi_m)$ be the vector of effective and unconstrained prices in $\mathbf{F}$. (That such a vector exists is an implication of Result 3.) Let $\boldsymbol{\rho} \equiv (\rho_1, \dots, \rho_m)$ be the normalization of $\boldsymbol{\pi}$, defined as in the proof of Result 2. Notice that each ρ_j is nonnegative and that ρ_1 is strictly positive. Let $\mathbf{x}^*$ be the allocation which results from the trades and disposals made by consumers in response to $\mathbf{F}$.

By part (iii) of Result 3, for each non-money good $j = 2, \dots, m$: *either* $\rho_j > 0$ and $q^B_j(\mathbf{F}) = q^S_j(\mathbf{F})$ *or* $\rho_j = 0$ and $q^B_j(\mathbf{F}) \geq q^S_j(\mathbf{F})$. In the case of money, we have $\rho_1 > 0$ and [by virtue of the identities (2) and (3)] $q^B_1(\mathbf{F}) = q^S_1(\mathbf{F})$. For each non-money good j, $\Sigma_i\, x^*_{ij} \equiv \Sigma_i\, z_{ij} + q^S_j(\mathbf{F}) - q^B_j(\mathbf{F}) - d_j(\mathbf{F})$. For money (for which the model allows no disposal option), $\Sigma_i\, x^*_{i1} \equiv \Sigma_i\, z_{i1} + q^S_1(\mathbf{F}) - q^B_1(\mathbf{F})$. By B3, for all non-money goods j: $\rho_j > 0$ implies $d_j(\mathbf{F}) = 0$. Hence, for $j = 1, \dots, m$: *either* $\rho_j > 0$ and $\Sigma_i\, (x^*_{ij} - z_{ij}) = 0$ or $\rho_j = 0$ and $\Sigma_i\, (x^*_{ij} - z_{ij}) \leq 0$. Thus, the triple $(\mathbf{x}^*, \mathbf{z}, \mathbf{O}^*[\mathbf{F}])$ satisfies A1 with respect to $\boldsymbol{\rho}$. Rewriting (4) in terms of normalized prices, $\mathbf{O}^*(\mathbf{F}) = \{\mathbf{x}_i: \Sigma_j\, \rho_j(x_{ij} - z_{ij}) \leq 0\}$. Thus, $(\mathbf{x}^*, \mathbf{z}, \mathbf{O}^*[\mathbf{F}])$ satisfies A2 with respect to $\boldsymbol{\rho}$. By Result 1, $\mathbf{O}^*(\mathbf{F})$ satisfies the opportunity criterion.

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