Econometrica, Vol. 61, No. 2 (March, 1993), 423–430

NOTES AND COMMENTS

PURE STRATEGY DOMINANCE

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1. INTRODUCTION

IN RECENT YEARS THE BAYESIAN APPROACH to rationality in noncooperative normal-form games has received some attention (e.g. Bernheim (1984), Pearce (1984), Tan and Werlang (1988)). This approach is based on the postulate that rational players should treat the strategic uncertainty that exists in games in the same way as nonstrategic uncertainty is treated in standard decision theory (Savage (1972)).

In this approach a player is hence called "rational" if he forms a subjective probability measure (a "subjective belief") which is defined on the set of strategy combinations of the other players, and if he then chooses a strategy that maximizes his expected utility whereby expected values are calculated using the subjective belief.

A result that is fundamental to the Bayesian approach to games is then that a strategy maximizes expected utility for some subjective belief, and can hence be chosen by a rational player, if and only if it is not dominated by another strategy of that player. A formal statement of this result is, e.g., Lemma 3 in Pearce (1984).

In the result just described no restrictions are imposed on players' beliefs. Hence nothing is assumed about players' knowledge. In a second step it is customary to consider the assumption that players are rational in the above sense, and that this fact, and players' preferences, are common knowledge.

It has been shown that under this assumption a strategy can be chosen if and only if it survives the iterated deletion of dominated strategies. A formal statement of this result is, e.g., given in Theorems 5.2 and 5.3 in Tan and Werlang (1988).[2]

If one considers the precise formulation of the two results described above one notices two interesting points. The first is that in the formal statements of these results players' preferences are described by their von Neumann Morgenstern (vNM)-utility functions. These are hence taken as exogenously given, and, in the second result, also taken to be common knowledge. The second point is that for the results to be true it is necessary that not only pure but also mixed strategies be taken into consideration.

Taking vNM-utility functions as given is of course conventional in noncooperative game theory; however in the context of the Bayesian approach it is not completely natural, since in Bayesian decision theory subjective probabilities and vNM-utilities are derived simultaneously from one and the same preference ordering (see Savage (1972)). Taking vNM-utilities, in addition, to be common knowledge is of course a very restrictive assumption. In particular this is much more than just assuming that players' preferences over the pure strategy outcomes of the game are common knowledge.

That mixed strategies enter the picture is surprising since in a Bayesian context mixed strategies are always redundant in the sense that when a mixed strategy maximizes expected utility then the same is true for all pure strategies in its support. For the characterization of expected utility maximization in terms of dominance, mixed strategies are however necessary in the conventional approach. This is because a pure strategy that

[1] This note is a revised version of a section in my paper "Bayesian Optimisation and Dominance in Normalform Games" (Basel, 1989). I would like to thank an editor, and two anonymous referees for their advice. In particular, the editor's comments led to a drastic improvement in the presentation of the proof of the Lemma in Section 3. I am grateful to the "Schweizerischer Nationalfonds" for financial support.

[2] Bernheim (1984) and Pearce (1984) show closely related results in slightly different frameworks.

	l	r
t	1	1
m	0	4
b	4	0

FIGURE 1

does *not* maximize expected utility is sometimes not dominated by any pure strategy but only by a mixed strategy. An example is given in Figure 1 (a similar example is on p. 6 of Fudenberg and Tirole (1991)).

In this example there are two players, 1 and 2. Player 1 chooses rows and player 2 chooses columns. In the intersection of any pair of a row and a column we have indicated the vNM-utility of player 1. The strategy t of player 1 is not dominated by any pure strategy. It is dominated only by the mixed strategy that assigns probability 0.5 to m and probability 0.5 to b.

The purpose of this paper is to develop an alternative formalization of Bayesian rationality in games. In our approach the only part of players' preferences that is taken as exogeneously given is their (ordinal) preferences over pure strategy outcomes. Rationality is then defined by requiring that a rational player must form a subjective belief, and then maximize the expected value of some vNM-utility function that agrees with his ordinal preferences.

We shall show that also in our approach a strategy maximizes the expected value of some admissible vNM-utility function for some subjective belief, and can hence be chosen by a rational player, if and only if it is not dominated by another strategy of that player. This result thus parallels the first of the two traditional results explained above. There are however two important differences between the traditional result and our result.

The first is that in our result, in contrast to the traditional result, it suffices to consider pure strategies only. Hence what we show is that a pure strategy can be chosen by a rational player if and only if it is not dominated by another *pure* strategy.

The second difference is less important, but also interesting. It is that the notion of dominance which is used in our new result is somewhat different from the notion of dominance that is used in the traditional result. In the traditional result the appropriate notion of dominance is "strong dominance." A strategy "strongly dominates" another one if it yields strictly better outcomes, whatever the other players do. An alternative notion is "weak dominance." A strategy "weakly dominates" another one if it never yields worse outcomes, and if it sometimes yields better outcomes. For our result we introduce a new notion of dominance that is intermediate between "strong dominance" and "weak dominance."

An alternative interpretation of our result is as follows: Consider a normal-form game with exogenous vNM-utility functions. Then a strategy maximizes for some subjective belief the expected value of this utility function *or of some monotonically increasing transformation of this utility function* if and only if it is not dominated (in our sense) by another pure strategy.

Some intuition for the fact that rationality in our approach can be characterized in terms of a dominance relation among pure strategies only, whereas the traditional approach requires consideration also of mixed strategies, can be obtained as follows: If one considers Bayesian rationality with exogenous and fixed vNM-utility functions, then in the corresponding definition of dominance lotteries must certainly play some role. This is what is accomplished in the conventional approach through the inclusion of mixed strategies. If one formalizes Bayesian rationality starting from preferences over pure strategy outcomes only, then lotteries won't play a role, and hence mixed strategies need not be considered.

In this note we also extend our approach to the case of common knowledge. We assume that it is common knowledge that players are rational in our sense, and also that players' preferences over pure strategy outcomes are common knowledge. Under this assumption a strategy can be chosen if and only if it survives iterated deletion of dominated strategies. Again, only pure strategies need be considered, and the relevant notion of dominance is our modification of the notion of "strong dominance." The argument behind this result is the same as the argument behind the analogous result in the traditional framework.

The motivation for presenting the new approach to Bayesian rationality in games follows from what was said earlier: Firstly it seems that it is somewhat more in line with a purely decision-theoretic view of game theory to treat vNM-utility functions and beliefs in the same way rather than taking one (utility functions) as exogeneous and fixed, letting only the other one (beliefs) vary, as the traditional approach does. Secondly, our version of the common knowledge assumption seems to be less demanding than the traditional version. Finally, our analysis helps to understand better the role of mixed strategies in the study of Bayesian rationality in games.

The organization of this note is as follows. In Section 2 the framework is introduced. Section 3 contains the main result, and Section 4 a proof. Sections 3 and 4 focus on the case in which nothing is assumed about players' knowledge about each other. In Section 5 we briefly discuss the case in which players' rationality, and their preferences, are common knowledge.

2. FRAMEWORK

We consider a normal-form game which is played by a finite set of players $i \in I = \{1, 2, \dots, I\}$ $(I \geqslant 2)$. Every player $i \in I$ has a nonempty, finite set of strategies $s_i \in S_i$. We shall write S for the Cartesian product $\prod_{i \in I} S_i$. Generic elements of S are denoted by s. Moreover if $i \in I$ we shall denote by S_{-i} the Cartesian product $\prod_{j \neq i} S_j$. Generic elements of S_{-i} are denoted by s_{-i}.

It will be convenient to introduce a separate, finite set A of possible outcomes of the game. Generic elements of A will be denoted by $a, b, c, \dots$. We shall think of the elements of A as specifications of all those features of the world that matter for the players' preferences. The description of the game can then be continued by introducing a function $g: S \to A$ that maps every possible strategy combination into an outcome of the game. Of course one might as well take S and A to be identical. However the current approach simplifies in some ways the notation below.

Each player i has a preference relation R_i over A. "$a R_i b$" will be interpreted as: "i finds outcome a at least as good as outcome b." For every i R_i is assumed to be complete, reflexive, and transitive. For every i one can derive from R_i two other relations: firstly the relation P_i which describes strict preference: $a P_i b$ if and only if $a R_i b$ and not $b R_i a$; secondly the relation I_i which describes indifference: $a I_i b$ if and only if $a R_i b$ and $b R_i a$.

3. RATIONALITY AND DOMINANCE

In this note we are interested in the strategy choices of players who have the preferences introduced in the previous section, and who behave "rationally" in the Bayesian sense. As usual we take "rationality" to mean expected utility maximization. More precisely we call a player "rational" if he maximizes the expected value of a vNM-utility function that agrees with his preferences over pure strategy outcomes. Thereby expected values are calculated using some subjective probability measure over the other players' strategy choices.

Formally, for given $i \in I$ a function $u_i : A \to IR$ is called a "utility function for player i" (we drop the prefix "vNM") if and only if for all $a, b \in A : u_i(a) \geqslant u_i(b) \Leftrightarrow aR_i b$. Moreover, a "subjective probability measure for player i" is a probability measure μ_i on S_{-i}. The strategy choices in which we are interested can then be described as follows:

Definition 1: Let $i \in I$ and $s_i \in S_i$. Then s_i is called *rational* if there are a utility function u_i and a subjective probability measure μ_i for player i such that for all $\tilde{s}_i \in S_i$:

$$\sum_{s_{-i} \in S_{-i}} \left[u_i(g(s_i, s_{-i}))\mu_i(s_{-i}) \right] \geqslant \sum_{s_{-i} \in S_{-i}} \left[u_i(g(\tilde{s}_i, s_{-i}))u_i(s_{-i}) \right].$$

We wish to characterize strategies that are "rational" in the sense of Definition 1 as (in a sense that will be made precise) "undominated" strategies. In a preliminary step we consider strategies that maximize expected utility if the support of a player's belief is pre-determined.

Definition 2: Let $i \in I$ and let $\tilde{S}_{-i}$ be a nonempty subset of S_{-i}. A strategy $s_i \in S_i$ is called *rational given* $\tilde{S}_{-i}$ if there are a utility-function u_i and a subjective probability measure μ_i for player i with support $\tilde{S}_{-i}$ such that for all $\tilde{s}_i \in S_i$

$$\sum_{s_{-i} \in S_{-i}} \left[u_i(g(s_i, s_{-i}))\mu_i(s_{-i}) \right] \geqslant \sum_{s_{-i} \in S_{-i}} \left[u_i(g(\tilde{s}_i, s_{-i}))\mu_i(s_{-i}) \right].$$

We now show that a strategy is rational given some set $\tilde{S}_{-i}$ if and only if it is not weakly dominated given that only strategies in $\tilde{S}_{-i}$ are taken into account. Thereby "weak dominance" is defined as follows:

Definition 3: Let $i \in I$, and let $\tilde{S}_{-i}$ be a nonempty subset of S_{-i}. Then a strategy $s_i \in S_i$ is called *weakly dominated given* $\tilde{S}_{-i}$ if there is a strategy $\tilde{s}_i \in S_i$ such that for all $s_{-i} \in \tilde{S}_{-i}$: $g(\tilde{s}_i, s_{-i}) R_i g(s_i, s_{-i})$ with strict preference for at least one $s_{-i} \in \tilde{S}_{-i}$.

Our first result is the following lemma.

Lemma: *Let $i \in I$, and let $\tilde{S}_{-i}$ be a nonempty subset of S_{-i}. Then a strategy $s_i \in S_i$ is rational given $\tilde{S}_{-i}$ if and only if it is not weakly dominated given $\tilde{S}_{-i}$.*

The proof of this result is in the next section. The analogue of this result in the traditional approach to Bayesian rationality and dominance is Lemma 4 in Pearce (1984). It is stated in a framework in which players' preferences are described by exogenously given vNM-utility functions, and not, as in our approach, only by an ordering of the pure strategy outcomes. Moreover, in Pearce's result allowance is made for the possibility that a strategy may be dominated only by a mixed strategy, not by a pure strategy. The dominance notion used by Pearce is, as in our result, weak dominance.

We use the Lemma to derive a characterization of strategies that are "rational" in the sense of Definition 1. For this we have the following definition:

Definition 4: Let $i \in I$ and $s_i \in S_i$. Then s_i is called *dominated* if, for every nonempty subset $\tilde{S}_{-i}$ of S_{-i}, s_i is weakly dominated given $\tilde{S}_{-i}$.

Observe that in this definition the dominating strategy is allowed to depend on $\tilde{S}_{-i}$. It is due to this fact that the notion of dominance introduced in Definition 4 excludes more strategies than the traditional notion of "strong dominance."[3] On the other hand it obviously excludes less strategies than the notion of "weak dominance."

[3] "Strong" dominance was defined in the Introduction.

	l	r
t	b	b
m	b	a
b	a	b

FIGURE 2

PROPOSITION: *Let $i \in I$. Then a strategy $s_i \in S_i$ is rational if and only if it is not dominated.*

The "only-if-part" of this result is obvious. The "if-part" is an immediate consequence of the Lemma.

The Proposition is our main result. We emphasize that the result cannot be further simplified. In particular, it is not true that a strategy is "rational" in the sense of Definition 1 if and only if it is not "strongly dominated" by an alternative pure strategy. In Figure 2 we give a counterexample. In this example there are two players, 1 and 2, and two possible outcomes, a and b. Player 1 chooses rows and player 2 chooses columns. In the intersection of any pair of a row and a column we have indicated the outcome that results.

Suppose that player 1 has preferences $a P_1 b$. Then his strategy t is dominated in the sense of Definition 4. The argument is as follows: If attention is restricted to the right hand choice of player 2, i.e. if $\tilde{S}_{-1} = \{r\}$, then the strategy m dominates t given $\tilde{S}_{-1}$. If $\tilde{S}_{-1} = \{l\}$, then b dominates t given $\tilde{S}_{-1}$. Finally, if $\tilde{S}_{-1} = \{r, l\}$, then both m and b dominate t given $\tilde{S}_{-1}$. Hence t is dominated in the sense of Definition 4.

On the other hand clearly the top strategy is not strongly dominated by any pure strategy. Hence in our approach the notion of dominance introduced in Definition 4 cannot be replaced by the simpler notion of "strong dominance".[4]

We conclude this section with a note on related literature. As was mentioned in the introduction in the literature on Bayesian rationality in games almost always vNM-utility functions have been taken as exogenous and fixed. An exception is, however, Ledyard (1986). Although at first sight this paper deals with rather different issues, Corollary 5.1 in that paper can be regarded as a characterization of expected utility maximizing strategies in terms of dominance in a framework in which only preferences over pure strategy outcomes are exogenous. Ledyard considers the case in which players' beliefs have full support. Hence his result is related to our Lemma.

A difference, however, is that in Ledyard's result a player's vNM-utility function is not necessarily a correct representation of the player's preferences over pure strategy outcomes. This is ensured only for fixed s_{-i} (compare Ledyard (1986, p. 70)). This is natural in his context because he considers "direct revelation games," and hence players' strategies are their types; thus his assumption means that he allows a player's vNM-utility to depend on the other players' types. However in the current context such an approach would not be well motivated.

[4]A result that is exactly analogous to the Proposition is true also in the traditional framework in which vNM-utility functions are taken as exogenous. The details of this are in the discussion paper version of this paper. The result is obtained if one modifies Definition 4, allowing for the possibility that the dominating strategies are mixed strategies. The result is interesting, because, together with Lemma 3 of Pearce (1984) which we described in the Introduction, it implies that in the traditional framework the dominance notion of Definition 4 and strong dominance are equivalent. In our framework this is not true. This is an important difference between our framework and the traditional framework.

4. PROOF OF THE LEMMA

The "only-if-part" is obvious. To prove the "if-part" we shall prove the following statement that clearly implies the "if-part:"

Let $i \in I$, let $\tilde{S}_i$ be a nonempty subset of S_i, let $\tilde{S}_{-i}$ be a nonempty subset of S_{-i}, and let s_i be a strategy in $\tilde{S}_i$. Suppose also that for every strategy $\tilde{s}_i \in \tilde{S}_i$ with $\tilde{s}_i \neq s_i$ there exists at least one $s_{-i} \in \tilde{S}_{-i}$ such that $g(s_i, s_{-i}) \neq g(\tilde{s}_i, s_{-i})$. Finally suppose that s_i is not weakly dominated given $\tilde{S}_{-i}$ by a strategy in $\tilde{S}_i$. Then there exist a utility function u_i and a subjective probability measure μ_i for player i with support $\tilde{S}_{-i}$ such that s_i is the *unique* maximizer of expected utility in $\tilde{S}_i$.

The proof of this assertion is by induction over the cardinality $\#\tilde{S}_{-i}$ of $\tilde{S}_{-i}$. For $\#\tilde{S}_{-i} = 1$ the proof is trivial. So assume $\#\tilde{S}_{-i} > 1$, and suppose the assertion had been proved for the case in which $\tilde{S}_{-i}$ has any smaller cardinality. Define $\hat{S}_{-i} \equiv \{\hat{s}_{-i} \in \tilde{S}_{-i} | g(s_i, s_{-i}) R_i g(s_i, \hat{s}_{-i})$ for all $s_{-i} \in \tilde{S}_{-i}\}$, $\hat{a} \equiv g(s_i, \hat{s}_{-i})$ for $\hat{s}_{-i} \in \hat{S}_{-i}$, and $\hat{S}_i \equiv \{\hat{s}_i \in S_i | \hat{a} P_i g(\hat{s}_i, s_{-i})$ for some $s_{-i} \in \tilde{S}_{-i}\}$.

Hence $\hat{a}$ is the worst outcome that can occur when player i chooses s_i. $\hat{S}_{-i}$ is the set of all strategy combinations of the other players for which this outcome occurs. $\hat{S}_i$ is the set of all strategies of player i that can lead to outcomes worse than $\hat{a}$.

Suppose first that $\tilde{S}_{-i} \setminus \hat{S}_{-i} \neq \varnothing$. Obviously: $\#(\tilde{S}_{-i} \setminus \hat{S}_{-i}) < \#\tilde{S}_{-i}$. Note that $s_i \in \tilde{S}_i \setminus \hat{S}_i$. Observe also that there cannot be any strategy $\tilde{s}_i \in \tilde{S}_i \setminus \hat{S}_i$ that either satisfies $g(\tilde{s}_i, s_{-i}) = g(s_i, s_{-i})$ for all $s_{-i} \in \tilde{S}_{-i} \setminus \hat{S}_{-i}$, or that weakly dominates s_i given $\tilde{S}_{-i} \setminus \hat{S}_{-i}$. This is because according to our construction such a strategy would weakly dominate s_i given $\tilde{S}_{-i}$.

The facts collected in the preceding paragraph imply that we can apply the inductive assumption, and conclude that there are a utility function u_i and a subjective probability measure μ_i with support $\tilde{S}_{-i} \setminus \hat{S}_{-i}$ such that s_i is the unique expected utility maximizer in $\tilde{S}_i \setminus \hat{S}_i$.

Now define for any $\delta > 0$ a utility function u_i^δ by

$$u_i^\delta(a) = \begin{cases} u_i(a) & \text{if } aR_i\hat{a}, \\ u_i(a) - \delta & \text{if } \hat{a}P_i a. \end{cases}$$

Define also for any $\varepsilon \in (0;1)$ a subjective probability measure μ_i^ε by

$$\mu_i^\varepsilon(s_{-i}) = \begin{cases} (1-\varepsilon)\mu_i(s_{-i}) & \text{if } s_{-i} \in \tilde{S}_{-i} \setminus \hat{S}_{-i}, \\ \varepsilon/\#\hat{S}_{-i} & \text{if } s_{-i} \in \hat{S}_{-i}. \end{cases}$$

Now if we first choose ε small enough, s_i will remain the unique maximizer of the expected value of u_i in $\tilde{S}_i \setminus \hat{S}_i$ if expected values are calculated using μ_i^ε rather than μ_i. This is true because expected utility is continuous in the subjective probability measure.

Once we have chosen ε we keep this ε, and hence also the corresponding μ_i^ε, fixed, and next vary δ. We now claim that if one chooses δ large enough, s_i will become the unique maximizer of the expected value of u_i^δ in the whole set $\tilde{S}_i$. To see this note that the choice of δ does not affect the expected utility of s_i or of other strategies in $\tilde{S}_i \setminus \hat{S}_i$, but that on the other hand by increasing δ the expected utility of all strategies in $\hat{S}_i$ can be made arbitrarily small.

The thus constructed utility function u_i^δ and subjective probability-measure μ_i^ε then have the required properties.

Consider next the case that $\tilde{S}_{-i} \setminus \hat{S}_{-i} = \varnothing$. Then let μ_i be any probability measure with support $\tilde{S}_{-i}$. Let u_i be any utility function, and define u_i^δ as above. If we then choose δ large enough, the utility function u_i^δ and the subjective probability measure μ_i will have the required properties, as above.

This concludes the proof. It is interesting to note that roughly speaking the utility function that is constructed in this proof exhibits rapidly decreasing marginal utility (as outcomes get "better"). Moreover the constructed probability measure assigns much larger probability to those choices of the other players that in combination with s_i yield "good" outcomes than to those that yield "bad" outcomes.

5. COMMON KNOWLEDGE OF RATIONALITY AND ITERATED DOMINANCE

5.1. *Common Knowledge*

In the analysis of the preceding sections no assumption with respect to the players' subjective beliefs was made. Hence nothing was assumed for the players' knowledge about each other. In this section we shall briefly consider the case that players are "rational" in the sense of Definition 1, and that this fact, together with players' preferences over pure strategy outcomes, is common knowledge among the players.

Under this assumption a strategy can be chosen if and only if it is one of those strategies that remain after iterated deletion of dominated strategies. Here, "iterated elimination of dominated strategies" is defined as follows: In a first step all dominated strategies are eliminated. Then, in the resulting reduced game, again all dominated strategies are eliminated, etc. This procedure continues until one reaches a reduced game in which no strategies are dominated. In each step a strategy is considered "dominated" if it is dominated in the sense of Definition 4 by some of the remaining pure strategies.

We do not formalize and prove this result here, since this can be done in an exactly analogous way to that used in the traditional framework (Tan and Werlang (1988)).

5.2. *The Order of Elimination*

A further question in this context arises as follows: Consider again the procedure of iterated elimination of dominated strategies. Above we have required that in each step *all* dominated strategies be eliminated. One can, however, think of alternative ways of proceeding, eliminating in each step only *some*, but not necessarily *all*, dominated strategies. Of course one would still only stop once a reduced game had been reached in which no strategy is dominated.

The question arises whether these alternative ways of proceeding will lead to different final sets of strategies. In other words: Will the final strategy sets of the players depend on the order of elimination? If this were the case, then the intuitive plausibility of iterated elimination of dominated strategies would be reduced. Also a practical problem would arise in that, when solving specific games using iterated elimination of dominated strategies, it would be necessary to verify that in each step one actually eliminates all, not just some, dominated strategies.

It can be shown that for the dominance notion introduced here the order of iterated elimination of dominated strategies does not affect the outcome of the procedure. A formal proof of this fact is given in the discussion paper version of this paper.

An analogous result holds in the traditional framework in which players' preferences are described by vNM-utility functions. In this framework the relevant procedure is iterated deletion of strongly dominated strategies, allowing for the possibility that the dominating strategy is mixed. Also for this procedure, the order of elimination is irrelevant. Versions of this result are proven in Gilboa et al. (1990) and Stegeman (1990). The proof in our framework is analogous to their proofs.

It may be useful to mention the two facts on which the proof is built. The first is that if a strategy is dominated, then there must be strategies that dominate it and that are not themselves dominated. The second fact is that if a strategy is dominated given some subset of all other players' strategies, then it is also dominated given any further subset of this subset.

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Manuscript received July, 1990; final revision received July, 1992.

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