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NOTES AND COMMENTS

MULTILATERAL CONTRACTING WITH EXTERNALITIES

BY ARMANDO GOMES¹

This paper proposes a model for multilateral contracting, where contracts are written and renegotiated over time, and where contracts may impose externalities on other agents. Equilibria always exist and the equilibrium value function is linear and monotonically increasing on the contracts. If the grand coalition, or contracting among all agents, is inefficient, we show that bargaining delays arise in positive-externality games and equilibrium inefficiency may remain bounded away from zero even as bargaining frictions converge to zero. Otherwise, if the grand coalition is efficient, there are no bargaining delays, convergence to the grand coalition occurs in a finite number of contracting rounds, and the outcome becomes efficient as players become more patient.

KEYWORDS: Contracts, externalities, renegotiation, coalitional bargaining.

1. INTRODUCTION

WHEN AGENTS CONTRACT, they often impose externalities on other agents. For example, positive externalities may arise in public goods provision problems, as when regions contract on pollution control and neighboring regions outside the contract enjoy a cleaner environment; negative externalities may arise in vertical contracting, as when an upstream firm contracts to supply intermediate goods to downstream firms and other downstream firms outside the contract face more intense competition. More generally, Segal (1999) points out that, in the class of principal–agent problems, multilateral contracting often involves externalities.

This paper proposes a model for the multilateral contracting process where (i) contracts may impose externalities on other agents, (ii) the contracting process is dynamic, allowing agents to contract over time, and (iii) contracts may be renegotiated or rewritten over time. These three features seem to be present in a multitude of economic situations where several contractual agreements are possible, with agents simultaneously competing and cooperating with each other to choose among their best available opportunities. The novelty of the paper is that it combines all these elements, and explores their economic implications with a focus on issues related to the existence of equilibria, presence of bargaining delays, and efficiency of allocations.

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The multilateral contracting game is a dynamic game among several agents in which, at every period, a randomly chosen agent makes contractual offers to an endogenously selected group of agents, who either accept or decline the offer. Because of contracting externalities, contractual offers specify monetary transfers among signatories contingent on who contracted with whom. The equilibrium concept we study is Markov perfect equilibrium, where the state space is the contractual structure (set of contracts written).

This natural formulation of the contracting problem results in an infinite state space stochastic dynamic game. There are no known general existence results for this class of games (see Chakrabarti (2001)). However, exploring the special structure of the game, we prove existence by constructing an equilibrium with special properties. The equilibrium value function is shown to be a linear and monotonically increasing function of the contract's contingent payoff and it can be expressed as the sum of two separate components: the surplus a player can extract from having the opportunity to be the proposer, plus her status quo value, which depends on other players' actions.

The model provides some new economic insights into multilateral contracting issues. Specifically, we show, by means of an example, that in bargaining games with positive externalities, equilibria exhibit bargaining delays—that is, periods in which no contracting takes place—when the grand coalition (or contracting among all players) is inefficient or not feasible. Negotiations resemble a “war of attrition” in which players delay the formation of coalitions so they can free-ride on other players' coalition formation decisions. We also show that equilibrium inefficiency may remain bounded away from zero even as bargaining frictions, such as the interval of time between rounds of negotiations or the risk of breakdown in negotiations, shrink to zero. One economic implication of the result is that industry consolidation by means of mergers may take an inefficiently long time to complete if antitrust regulators forbid mergers to monopoly, so that only partial mergers are feasible, and partial mergers create positive externalities on rival firms.

In contrast, we show that whenever the grand coalition is efficient, there are no bargaining delays and ultimately the efficient grand coalition forms in a finite number of contracting rounds in all equilibria. Therefore, when bargaining frictions are insignificant, the Coasian conjecture holds despite the possibility of widespread externalities in the economy. We also derive a sufficient condition that ensures Pareto efficiency, even when bargaining frictions are not insignificant, because the equilibrium entails an immediate move to the efficient grand coalition.

The remainder of the paper is organized as follows: Section 2 presents the model, Section 3 addresses the existence equilibria, Section 4 studies the efficiency properties of the model, and Section 5 concludes. We discuss the related literature after presenting the model in Section 2 and compare our findings with the literature in Section 4 after deriving our key results.

2. THE MODEL

The set of players is denoted by $N = \{1, 2, \dots, n\}$, and its subsets $C \subset N$ represent the players who have written contracts or formed coalitions. The partitions of N into disjoint coalitions, also referred to as coalition structures (c.s.), $\pi = \{C_1, \dots, C_m\}$, describe who contracted with whom. The set of all feasible partitions is denoted by Π .

A given underlying economic situation specifies the payoff per period $u_C(\pi)$ of a subset C of players who contract for all possible coalition structures π that may arise as a result of other players' contracting decisions; this set of coalition structures is defined as $\Pi_{-C} = \{\pi' \in \Pi : \text{such that } C \in \pi'\}$. The payoff structure $u_C(\pi)$ is referred to in the literature as a *partition function* (see Bloch (1996), Ray and Vohra (1999), and Montero (1999)) and allows for contracting externalities. For example, in our study a game involves *positive (negative) externalities* if contracting by other players increases (decreases) excluded coalitions' payoffs (see also Yi (1997)). Formally, we say that the game has positive (negative) externalities if $u_C(\pi) \leq (\geq) u_C(\pi')$ for all π' coarser than π (i.e., π' involves further contracting by other players) and $\pi, \pi' \in \Pi_{-C}$.

All players are assumed to have von Neumann–Morgenstern preferences and to discount future payoffs at a rate $\delta \in [0, 1)$. The discounted present value of the payoff flow is $U_C(\pi) = u_C(\pi)/(1 - \delta)$. We also analyze the limit equilibrium when players become ever more patient or, equivalently, the time between offers shrinks to zero. When performing this type of analysis, we hold the discounted present values $U_C(\pi)$ constant, adjusting the payoff flow accordingly.

The multilateral contracting game (MCG) is a dynamic game where, at every period, a player is randomly chosen to offer a new contract to a set of players who accept or decline the offer. In our interpretation, contracts are binding agreements that specify monetary transfers among signatories conditional on the contracting decisions of players outside the contract; a contract can be revoked or rewritten only by unanimous consent. The details of contracts and of the contracting/recontracting process are described below.

Contracts that can be written by a subset C of players specify the individual per-period payoffs of each player $i \in C$ contingent on all possible coalition structures formed by the remaining $N \setminus C$ players.² Formally, the contracts y_C that a subset of players C can write are described by the per-period payoffs $y_{i,C}(\pi) \in R$ that each $i \in C$ gets as a function of the period's c.s. $\pi \in \Pi_{-C}$ (note that this c.s. depends on players' $N \setminus C$ contracting decisions). The contract $y_C = (y_{i,C})_{i \in C}$ is composed of all the contracts $y_{i,C}$ and it satisfies the budget balancing condition $\sum_{i \in C} y_{i,C}(\pi) = u_C(\pi)$, where $u_C(\pi)$ is the per-period aggregate payoff of contracting players. At any period of the game, the contracts

²The coalition structure, or the set of players who have contracted, is assumed to be verifiable information.

written by all players are described by $y = (y_C)_{C \in \pi}$ —the *contractual state*—and we let Y denote the set of all contractual states.³ It is convenient to use the single-period reward function $u_i(y)$ to describe player i 's payoff when the contractual state is y (i.e., $u_i(y) = y_{i,C}(\pi)$).

The *rules of recontracting* are formally stated as follows. Let us define, for any $S \subset \pi$, the set $\mathcal{S} \subset N$ as the set of players that belong to the coalitions in S , i.e., $\mathcal{S} = \bigcup_{C \in S} C$. Say that $y = (y_C)_{C \in \pi}$ is the contractual state; then contracts y_C with $C \in S$ may be replaced by a new contract, z_S , as long as all players in S whose contracts are being rewritten approve of the contractual change. Therefore, the recontracting possibilities are those where players in \mathcal{S} can change the contractual state from y to $z = (z_S, y_{-S})$, where $y_{-S} = (y_C)_{C \in \pi \setminus S}$.⁴

The game starts at period $k = 0$ when no players have contracted (let y^0 denote this initial contractual state). The *multilateral contracting game* is the dynamic game with the following extensive form: Say that the current period is k and the contractual state is y , where π describes who contracted with whom. One of the players $i \in N$ is randomly chosen with probability $p_i(\pi)$ to be the proposer. Player i then proposes to a subset \mathcal{S} , where $S \subset \pi$ and $i \in S$, to write a contract z_S . In other words, player i chooses an offer (S, z_S) . Players in \mathcal{S} then respond sequentially in a fixed order (the order of response turns out to be irrelevant) by accepting or rejecting the offer. If the offer (S, z_S) is accepted by all players in \mathcal{S} , then the contractual state changes immediately to $z = (z_S, y_{-S})$. Otherwise, the contractual state remains equal to y . Each player receives a single-period payoff equal to $u_i(z)$ or $u_i(y)$, respectively, depending on whether the offer was accepted or not, and the game continues as described above in the next period $(k + 1)$.

A related model that will be useful in establishing the existence of equilibria in the MCG is the coalition bargaining game (CBG). This game is similar to the MCG except that the proposer offers a lump-sum payoff transfer to players to form coalitions, and upon acceptance of the offer, responders leave the game and the proposer makes all decisions thereafter. At any period, the state of the game is described by a coalition structure π . Players making decisions for coalition $C \in \pi$ are, for simplicity, labeled coalition C and receive the single-period reward $u_C(\pi)$ and are proposers with probability $p_C(\pi) = \sum_{j \in C} p_j(\pi)$.

The extensive form of the *coalition bargaining game* is as follows (the initial coalition structure is π^0 , the finest partition of N): At the beginning of a period, say that the c.s. is π . A coalition $C \in \pi$ is randomly chosen to be the proposer with probability $p_C(\pi) = \sum_{j \in C} p_j(\pi)$ and chooses an offer (S, t) , where $S \subset \pi$, $C \in S$, and $t = (t_B)_{B \in S}$ are the amounts paid to each coalition B , $t_B \in R$, and $t_C = -\sum_{B \in S \setminus C} t_B$ is a budget-balancing condition.⁵ Coalitions $S \setminus C$

³To ensure the convergence of infinite sums of discounted payoffs, we impose the restriction that Y be a compact set.

⁴The set S is a subset of π and $\pi \setminus S$ refers to the standard set exclusion notation.

⁵A proposer may pass up his chance to propose, leading to delays in forming coalitions. Formally, this happens when he chooses $S = \{C\}$.

respond sequentially in a fixed order (again the order of response is not relevant), either accepting or rejecting the offer. If responders are not unanimous in accepting the offer, then the c.s. does not change. If they are unanimous, then each coalition $B \in S \setminus C$ receives transfers t_B and leaves the game, and the c.s. changes to $\pi S = \{S\} \cup \pi \setminus S$. The game repeats in the next period.

Equilibrium concept: We study the Markov perfect equilibria. In general, players' strategies in the k th period of the game may depend upon the entire history of play, but in Markov perfect equilibrium players follow *Markovian strategies*. Markovian strategies are such that the k th period strategy (of both proposers and responders) depends on the "state" of the game at the end of period $k - 1$, but not on other aspects of the $k - 1$ history of the game. For the MCG, the relevant state s is described by the contractual state, $s = y$, and for the CBG it is described by the c.s., $s = \pi$.

Players' strategies are represented by σ_i , and a strategy profile $\sigma = (\sigma_i)_{i \in N}$ is a *Markov perfect equilibrium* (MPE) if and only if it is Markovian and σ is a subgame perfect equilibrium. So, after every history of play, σ_i is a best-response strategy for player i , where deviations are not constrained to be Markovian strategies, when other players play according to σ_{-i} . The players' strategy space is the set of all finite mixed strategies (i.e., finite mixtures of pure strategies). Mixed strategies will turn out to be important to guarantee the existence of equilibria, and the finiteness restriction is a technical assumption imposed to allow us to represent players' values using finite sums rather than integrals.

For σ Markovian strategy, let $\sigma_i(s)(\tau)$ be the probability that player i chooses offer τ when he is chosen to be the proposer at state s . Also let $v_i(\sigma|s)$ and $v_i(\sigma|s, j)$ be player i 's continuation value under σ in the subgame starting with state s , respectively, before the proposer's choice and after player j is chosen to be the proposer. Note that the value is

$$(1) \quad v_i(\sigma|s) = E_{\sigma} \left[\sum_{k=0}^{\infty} \delta^k u_i(s_k) | s \right],$$

where, $(s_k)_{k=0}^{\infty}$ is the stochastic process induced by σ starting from the subgame with state s . Similarly, let $\mu(\sigma)(s, s')$ and $\mu(\sigma)(s, s'|j)$ be the probability of a transition from state s to s' , respectively, unconditional and conditional on the proposer's choice (note that transition probabilities are related by $\mu(\sigma)(s, s') = \sum_{j \in N} p_j(\pi) \mu(\sigma)(s, s'|j)$).

Related Literature

The literature on coalition formation problems is extensive, and we refer here to the research that in our view is most closely related to our work.⁶

⁶Other related topics include Perry and Reny (1994) on continuous time bargaining, Hart and Mas-Colell (1996) on implementation of the Shapley value in nontransferable utility games, and Stole and Zwiebel (1996) and Westermarck (2003) on intrafirm bargaining issues.

One sizeable strand of the literature, starting with Rubinstein (1982), studies coalitional bargaining games without externalities as a noncooperative game of offers and counteroffers (a characteristic function describes the underlying game). In most studies, such as Chatterjee et al. (1993), Moldovanu and Winter (1995), Krishna and Serrano (1996), and Okada (1996), once a coalition forms it leaves the game. Renegotiation of coalitions are allowed by Gul (1989), Seidmann and Winter (1998), and Okada (2000).

Other researchers have extended the analysis to coalitional games with externalities (the underlying game is described by a partition function). Some studies in this area were conducted by Bloch (1996), Ray and Vohra (1999), Montero (1999), Yi (1997), and Bloch and Gomes (2004).⁷ We use a similar framework, the main distinction being that we are the first to study externalities in the presence of renegotiations.

More recently, Gomes and Jehiel (2004) followed our framework to study the problem of coalition formation in a setting where coalitions may break up, which is not possible in our paper where the renegotiation possibilities only allow for expansions of coalitions.⁸ However, in Gomes and Jehiel (2004) the economy has a finite number of states, while the contracting process in coalitional bargaining games with externalities naturally leads to a problem with an infinite number of states (see also the discussion in Section 4 for other relevant differences among the models). While the possibility of coalitions breaking up is essential to study certain problems, such as legislative bargaining, we will show that interesting economic insights in coalitional bargaining games emerge in games with renegotiations without the possibility of breakup.

3. CHARACTERIZATION OF EQUILIBRIA

In this section, we characterize the Markov perfect equilibria, proving the existence of equilibria and establishing basic properties. The MCG is a stochastic game with both infinite state and action spaces. The existence of equilibria for stochastic games with infinite state spaces is an important open problem in the literature (see, for example, Chakrabarti (2001)), so no general existence result can be applied. Our approach explores the relationship between the MCG and the CBG. We first characterize the CBG equilibria and then we show how to make an equilibrium of the CBG into an equilibrium of the MCG.

⁷Externalities have also been addressed by Jehiel and Moldovanu (1995a, 1995b, 1999) using a setting in which a seller owns an indivisible object to be sold to one of several potential buyers. Buyers have different valuations for the good and their valuations depend on who acquires the object.

⁸In Gomes and Jehiel (2004) the underlying game is described by an effectivity function—the economy is described by a finite number of states with an abstract transition rule that prescribes the moves across states that each coalition can implement. Konishi and Ray (2003) also studied the problem of coalition formation using effectivity functions, but their framework is different from ours, among other features, because they use coalition or group deviations as the basic unit of analysis, whereas we use individuals as the basic unit.

3.1. Coalition Bargaining Game

Let us first consider the properties of the CBG. In any Markov perfect equilibrium σ , the minimum offer (S, t) that a responder C is willing to accept at state π is one with $t_C = \delta v_C(\sigma|\pi) + u_C(\pi)$, because if she rejects the offer, she derives a flow of utility $u_C(\pi)$ in the rejection period and her expected utility in the subgame starting next period with state π is $v_C(\sigma|\pi)$, which has a present value equal to $\delta v_C(\sigma|\pi)$. It is convenient for future reference to define the value function at the rejection stage as

$$(2) \quad \hat{v}_C(\sigma|\pi) = \delta v_C(\sigma|\pi) + u_C(\pi).$$

The subgame perfection condition at the offering stage also implies that proposer C 's problem is to choose the offer that maximizes

$$(3) \quad v_C(\sigma|\pi, C) = \max_{(S, t)} \hat{v}_S(\sigma|\pi S) - \sum_{B \in S \setminus C} t_B$$

$$\text{s.t. } t_B \geq \hat{v}_B(\sigma|\pi) \quad \text{for all } B \in S \setminus C,$$

$$C \in S \subset \pi,$$

where $\hat{v}_S(\sigma|\pi S)$ is C 's expected utility when coalition S forms and the state changes to πS , and t_B are the transfers to each responder. Note that the proposer chooses the offer $t_B = \hat{v}_B(\sigma|\pi)$ for all $B \in S \setminus C$, which is the minimum that responders are willing to accept.

Coalition C , conditional on being chosen proposer, can extract a surplus equal to $e_C(\sigma|\pi) = \max_{S \ni C} \hat{v}_S(\sigma|\pi S) - \sum_{C \in S} \hat{v}_C(\sigma|\pi)$ (note that the conditional value is $v_C(\sigma|\pi, C) = \hat{v}_C(\sigma|\pi) + e_C(\sigma|\pi)$). The proposer's surplus is nonnegative because one of the options available to the proposer is not to propose anything (i.e., $S = C$), in which case the surplus is zero. It is convenient for future reference to introduce notation that represents the proposer's surplus

$$(4) \quad e_C(\sigma|\pi) = \max_{S \ni C} e(\sigma|\pi, S) \quad \text{and}$$

$$e(\sigma|\pi, S) = \hat{v}_S(\sigma|\pi S) - \sum_{C \in S} \hat{v}_C(\sigma|\pi).$$

We allow proposers to use mixed (behavior) strategies. Typically, in the coalition bargaining literature, only pure strategy equilibria are considered (e.g., Seidmann and Winter (1998), Chatterjee et al. (1993), Okada (1996, 2000), Montero (1999)), but allowing for mixed strategies (specifically at the proposing stage) is essential to obtain existence (see also Ray and Vohra (1999) for a related result).

Using the Kakutani fixed point theorem (see the Appendix), Proposition 1 derives the existence of MPE for the CBG. In addition, the proposition pro-

vides a blueprint for constructing MPE, which is helpful in the process of finding equilibrium in applications. In particular, the proposition's constructive approach will be used to obtain equilibria in the example considered in Section 4.

PROPOSITION 1: *There exist MPE for all coalition bargaining games. Moreover, an equilibrium can be constructed as follows. Let $\hat{v}_C(\pi) \in R$ be a set of values and let $\hat{\sigma}_C(\pi)$ be probability distributions over $\Sigma_C(\pi) = \{S \subset \pi : C \in S\}$, both satisfying (i) and (ii) below:*

(i) *The support of $\hat{\sigma}_C(\pi)$ satisfies*

$$(5) \quad \text{supp}(\hat{\sigma}_C(\pi)) \subset \arg \max_{S \ni C} e(\pi, S),$$

where $e(\pi, S) = \hat{v}_S(\pi S) - \sum_{C \in S} \hat{v}_C(\pi).$

(ii) *The following system of equations holds (where $e_C(\pi) = \max_{S \ni C} e(\pi, S)$):*

$$(6) \quad \hat{v}_C(\pi) = \delta \sum_{B \in \pi} \sum_{S \subset \pi} p_B(\pi) \hat{\sigma}_B(\pi)(S) (\mathbb{I}_{[C \in S]} \hat{v}_C(\pi) + \mathbb{I}_{[C \notin S]} \hat{v}_C(\pi S)) \\ + \delta p_C(\pi) e_C(\pi) + u_C(\pi).$$

Then there exists an MPE σ defined as follows. Proposers' strategies are $\sigma_C(\pi)(S, t) = \hat{\sigma}_C(\pi)(S)$ if $t_B = \hat{v}_B(\pi)$ for all $B \in S \setminus C$, and $t_C = -\sum_{B \in S \setminus C} t_B$ and $\sigma_C(\pi)(S, t) = 0$ otherwise. The strategies of responders' $B \in S \setminus C$ are to accept any offer (S, t) proposed by coalition C at state π if $t_B \geq \hat{v}_B(\pi)$ and to reject it otherwise. Moreover, the value function of σ satisfies $\hat{v}_C(\sigma|\pi) = \hat{v}_C(\pi)$.

Key when solving for the equilibrium is to find strategies and values that satisfy items (i) and (ii) above. We discussed above the intuition for condition (i) and we now discuss the intuition for (ii). The motivation for the implicit equation (6), which expresses coalition C 's value, is that from any state π , either (a) another player makes an acceptable offer to S , leading to a transition to πS , in which case if $C \notin S$, C 's value is $\hat{v}_C(\pi S)$ and if $C \in S$, then C 's value is $\hat{v}_C(\pi)$, or (b) player C is the proposer, in which case, C 's value is $\hat{v}_C(\pi) + e_C(\pi)$. The proof in the Appendix shows that there always exist values \hat{v}_C and $\hat{\sigma}_C$ that satisfy conditions (i) and (ii) of Proposition 1, and the associated strategy σ is MPE.

We now show that the implicit equation (6) for the value function can be inverted to yield an explicit expression for the value in terms of two additive components: the surplus from being the proposer plus the player's status quo value.

COROLLARY 1: *In any MPE of the CBG the value function can be expressed as*

$$(7) \quad \hat{v}_C(\pi) = \sum_{\pi' \in \Pi_{-C}} M_C(\pi, \pi') (\delta p_C(\pi') e_C(\pi') + u_C(\pi'))$$

or in matrix form as $\hat{v}_C = M_C(\delta p_C e_C + u_C)$, where M_C is a nonnegative matrix.

PROOF: First note that the implicit equation (6) can be written in matrix form as

$$[I - \delta \mu_C] \hat{v}_C = \delta p_C e_C + u_C,$$

where μ_C is a Π_{-C} stochastic matrix,⁹ and $p_C e_C$ and u_C are Π_{-C} vectors with π th coordinate equal to $p_C(\pi) e_C(\pi)$ and $u_C(\pi)$, respectively. The matrix $[I - \delta \mu_C]$ is a (row) dominant diagonal matrix, where I is the Π_{-C} identity matrix. By a well known theorem (see Takayama (1985, p. 381)), matrix $[I - \delta \mu_C]$ is invertible and, moreover, the inverse $M_C = [I - \delta \mu_C]^{-1} = \sum_{k=0}^{\infty} \delta^k \mu_C^k$ is nonnegative, where μ_C^k is the k th power of matrix μ_C . *Q.E.D.*

The first part of a coalition's value is the expected surplus from being the proposer $M_C \delta p_C e_C$; the second part is the status quo value $M_C u_C$. Note that because of externalities, the status quo value, which is the value obtained by making no proposals and rejecting all offers, depends on other players' actions.¹⁰

3.2. Multilateral Contracting Game

We will search for an equilibrium of the MCG with the same stochastic process of coalition formation as an equilibrium of the CBG. We expect that in the MCG equilibrium any proposer who approaches players to make a contract will offer each one of them a contract that will make them just indifferent between acceptance and rejection.¹¹ Letting $v_i(\sigma|y)$ be the value function associated with strategy σ , then the value at which a player is indifferent between acceptance and rejection is $\hat{v}_i(\sigma|y) = \delta v_i(\sigma|y) + u_i(y)$.

⁹Observe that we previously defined $\Pi_{-C} = \{\pi \in \Pi : \text{such that } C \in \pi\}$, and μ_C is defined, for all $\pi, \pi' \in \Pi_{-C}$, as $\mu_C(\pi, \pi') = \sum_{B \in \pi} \sum_{S \subset \pi} p_B(\pi) \hat{\sigma}_B(\pi)(S) \mathbb{I}_{[C \in S]}$ (the probability that C is included in the offer), $\mu_C(\pi, \pi') = \sum_{B \in \pi} p_B(\pi) \hat{\sigma}_B(\pi)(S)$ for $\pi' = \pi S$ and $C \notin S$ (the probability of a move from π to πS not including C), and $\mu_C(\pi, \pi') = 0$ otherwise.

¹⁰Contracting by other players moves the game to new states according to the k -step transition probability μ_C^k , resulting in a status quo value $\sum_{\pi' \in \Pi_{-C}} M_C(\pi, \pi') u_C(\pi')$. If coalition C deviates (rejecting all offers and not making any offers), then there is a probability $\mu_C^k(\pi, \pi')$ that the state moves from π to π' in k periods, and so C 's expected reward in period k is $\sum_{\pi' \in \Pi_{-C}} \mu_C^k(\pi, \pi') u_C(\pi')$.

¹¹We also expect that the surplus e_C of a coalition C proposing in the CBG will be equal to the surplus of each $i \in C$ proposing in the MCG—a result motivated by the unanimity requirement among coalition members for contractual changes.

Unfortunately, the CBG equilibrium yields coalition C 's value, $M_C \delta p_C \times (e_C + u_C)$, but not the values of each individual player's $i \in C$ (and knowing such values is essential to specify an MCG equilibrium strategy). Nonetheless, in the CBG and MCG, respectively, coalition C is proposer with probability p_C and each player is proposer with probability p_i (where $p_C = \sum_{i \in C} p_i$), and coalition C 's payoff is u_C and each individual player's payoff is $y_{i,C}$ (where $u_C = \sum_{i \in C} y_{i,C}$).

A candidate for the value function arises from the analogy between the two games pointed out above and the coalition value formula (7). We postulate the existence of an MCG equilibrium strategy whose associated value function is equal to

$$(8) \quad \hat{v}_i(y) = \sum_{\pi' \in \Pi_{-C}} M_C(\pi, \pi') (\delta p_i(\pi') e_C(\pi') + y_{i,C}(\pi')),$$

where $\hat{v}_C(\pi) = \sum_{i \in C} \hat{v}_i(y)$.¹² The key result of this section is the following proposition.

PROPOSITION 2: *The MCG has an MPE. Specifically, there exists an MPE with strategy profile σ defined as follows, where $\hat{v}_i(y)$ is defined by (8): At any contractual state $y = (y_C)_{C \in \pi}$ responder j 's strategy is to accept an offer (S, z_S) if and only if $\hat{v}_j(z_S, y_{-S}) \geq \hat{v}_j(y)$; proposer i 's strategy is to offer contracts (S, z_S) that satisfy $\hat{v}_j(z_S, y_{-S}) = \hat{v}_j(y)$ for all $j \in S \setminus i$ with probability $\hat{\sigma}_C(\pi)(S)$, where C are the players with whom i contracted. The value function of σ satisfies $\hat{v}_i(\sigma|y) = \hat{v}_i(y)$. Moreover, $\hat{v}_i(\sigma|y^0) = \hat{v}_i(\pi^0)$ and $\sum_{j \in C} v_j(\sigma|y) = \hat{v}_C(\pi)$.*

The strategy profile σ defined above is such that responders reject any offer that yields them a value lower than $\hat{v}_i(\sigma|y)$ —the value they get by rejecting the offer—and proposers offer contracts to responders that make them indifferent between acceptance and rejection.

We sketch the proof of Proposition 2. First, we show that the strategy profile σ is well defined, which requires showing that there always exists an offer (S, z_S) that satisfies $\hat{v}_j(z_S, y_{-S}) = \hat{v}_j(y)$. Then we show that the value function $\hat{v}_i(\sigma)$ associated with the strategy profile σ satisfies $\hat{v}_i(\sigma|y) = \hat{v}_i(y)$. This holds because $\hat{v}_i(\sigma)$ is the solution of the functional equation

$$(9) \quad \hat{v}_i(\sigma|y) = \delta \sum_{y' \in Y} \mu(\sigma)(y, y') \hat{v}_i(\sigma|y') + u_i(y) \quad \text{for all } y \in Y,$$

where $\mu(\sigma)(y, y')$ are the transition probabilities. Therefore $\hat{v}_i(\sigma)$ is a fixed point of the mapping $T: B(Y) \rightarrow B(Y)$, where $B(Y)$ is the complete metric

¹²Note that the only difference between (7) and (8) is that the coalition proposer probabilities p_C are replaced by the individual proposer probabilities p_i , and the coalition payoff u_C is replaced by the contractual payoffs $y_{i,C}$ of each agent.

space of all bounded functions $g: Y \rightarrow R$ and where T is the mapping defined by $T(g)(y) = \delta \sum_{y' \in Y} \mu(\sigma)(y, y')g(y') + u_i(y)$. We show that the contraction mapping theorem applies to T , so this mapping has a unique fixed point (Lucas and Stokey (1989)). The proof concludes by showing that $\hat{v}_i(y)$, defined by (8), is a fixed point of T .

The MCG can potentially have other MPE equilibria. Under what conditions can we say that the equilibria will be like the one described in Proposition 2? We show in the Appendix that if players' value functions and transition probabilities depend only on the set of players who have contracted, but not specifically on what contracts other players may have written, then the equilibrium is as above (see the formal statement and proof in the Appendix).

A corollary of our results is that players' values come from two separate additive components: the surplus from being the proposer ($M_C \delta p_i e_C$) and the status quo value ($M_C y_{i,C}$).

COROLLARY 2: *The MCG has MPE with value function $\hat{v}_i(y)$ equal to $M_C(\delta p_i e_C + y_{i,C})$, where C is the set of players with whom i contracted. Moreover, the equilibrium value is a linear function of the contract $y_{i,C}$ and is also monotonically increasing on this contract, that is, $\hat{v}_i(y') \geq \hat{v}_i(y)$ if $y'_{i,C} \geq y_{i,C}$.*

Therefore, for any given MPE, we can compare players' values at any two different subgames or contractual states. We have shown that values are linear on the contracts' payoffs and, moreover, because M_C is a nonnegative matrix (see Corollary 1), players' values are also monotonically increasing on the contracts' payoffs.

4. BARGAINING DELAYS AND EFFICIENCY PROPERTIES

4.1. Efficiency

We explore in this section the efficiency properties of the equilibrium. We show that whenever the current state is inefficient and the grand coalition is efficient, there are *no bargaining delays* in equilibrium. Bargaining delays happen when negotiations do not evolve or when no changes in the state space take place during a period of time. Moreover, all equilibria converge to the efficient state after a finite number of periods, so all equilibria are asymptotically efficient as the time between offers shrinks to zero. Therefore, the Coasian conjecture—that an efficient outcome eventually should arise—holds, despite the possibility of widespread externalities in the game.

PROPOSITION 3: *In any multilateral contracting game where the grand coalition is efficient, bargaining delays never happen. More precisely, for any $\delta \in [0, 1)$, starting from an inefficient state, negotiations move to a new state with probability 1. Therefore, the equilibrium path converges to the efficient grand coalition after a finite number of periods.*

Note that convergence occurs in at most $\#N - 1$ periods, where $\#N$ is the number of players. The key reason why asymptotic Pareto efficiency holds is that agents can renegotiate agreements and the grand coalition, being efficient and stable, acts as an attractive sink to which the system converges. While several intermediate inefficient contracts can be written along the equilibrium path, the efficient contract is ultimately written after a finite number of renegotiations. Whenever these intermediate inefficient contracts entail only a negligible amount of inefficiency in the interim, which is the case when the time interval between negotiations is very short (or $\delta \rightarrow 1$), an almost Pareto efficient outcome arises.

Even when contracting entails a move to the efficient outcome in a finite number of steps, as necessarily is the case when an efficient grand coalition exists, there still may be significant inefficiencies when convergence is not immediate and bargaining frictions are relevant (which is the case when players are impatient and the time interval between rounds of negotiations is not small, or there is a significant exogenous risk of breakdown in negotiations as in Binmore, Rubinstein, and Wolinsky (1986)).

We provide below a sufficient condition that guarantees the existence of an equilibrium that entails an immediate move to an efficient grand coalition. This sufficient condition ensures the existence of a Pareto efficient outcome regardless of bargaining frictions.

PROPOSITION 4: *Consider a game where the grand coalition is efficient and satisfies, for all c.s. π and $S \subset \pi$ with $S \neq \pi$,*

$$(10) \quad U_S(\pi_S) + p_S(\pi_S) \left(U_N(\pi_N) - \sum_{B \in \pi_S} U_B(\pi_S) \right) \\ \leq \sum_{C \in S} \left(U_C(\pi) + p_C(\pi) \left(U_N(\pi_N) - \sum_{B \in \pi} U_B(\pi) \right) \right).$$

Then, for all $\delta < 1$, there exists MPE σ with an equilibrium value

$$(11) \quad v_C(\sigma|\pi) = U_C(\pi) + p_C(\pi) \left(U_N(\pi_N) - \sum_{B \in \pi} U_B(\pi) \right)$$

*and equilibrium dynamics where the grand coalition forms immediately from any state.*¹³

The equilibrium value (11) is equal to the coalition's status quo payoff plus its share (based on its proposer probability) of the payoff increase associated

¹³In the MCG, the corresponding equilibrium value is $v_i(\sigma|y) = Y_i(\pi) + p_i(\pi)(U_N(\pi_N) - \sum_{B \in \pi} U_B(\pi))$ at any state $y = (y_C)_{C \in \pi}$, where the status quo discounted present value at y is $Y_i(\pi) = y_{i,C}(\pi)/(1 - \delta)$.

with formation of the grand coalition, $U_N(\pi_N) - \sum_{B \in \pi} U_B(\pi)$. To prove the result that the grand coalition forms immediately from any state, it is sufficient to prove that the surplus $e(\sigma|\pi, N) \geq e(\sigma|\pi, S)$ for all π and S , which we show is true whenever (10) holds (see the Appendix).

4.2. Bargaining Delays

An important assumption is that the grand coalition is efficient or that contracting among all agents is the most efficient allocation. Informally, this condition holds in complete or incomplete contracting settings whenever it is possible for all players to write a contract that is more efficient than any set of partial contracts (note that this condition may hold even in the presence of externalities).

However, contracting among all players may not be efficient or feasible in all instances. For example, antitrust regulators often disallow mergers that monopolize an industry, so only partial mergers may be feasible. We show below, by means of an example, that bargaining delays naturally arise in the coalition formation process if partial mergers are profitable and create positive externalities, but mergers to monopoly are forbidden.¹⁴

EXAMPLE—Bargaining Delay: Consider a game among three symmetric firms where the three-firm coalition is not feasible, so that only the three pairwise coalitions $\{i, j\}$ can form. The stand-alone payoffs are normalized to zero, and any two-firm coalition produces a payoff $U_2 = U_{ij}([ij|k])$ and the excluded firm payoff is $U_1 = U_k([ij|k])$. Assume that $U_1 > U_2 > 0$, so that the game is one with positive externalities, and let δ be arbitrarily close to 1.¹⁵

We show in the Appendix that all MPE equilibria exhibit bargaining delays. That is, it takes several periods for the economy to move from the initial state to the efficient state $[ij|k]$ where two players form a coalition (the aggregate value in this state is $U_1 + U_2 > 0$). The intuition for the result—why players choose to delay the formation of coalitions—is that there is a chance that other players will be the ones forming coalitions—it is not an equilibrium that coalitions never form—and coalitions create positive externalities for excluded players. So the equilibrium is one in which there is a “war of attrition” with players who attempt to free-ride on the coalition formation decision of others.

¹⁴This insight can help explain why industries with excess capacity may take an inefficiently long time to restructure through capacity-reducing mergers and why profitable mergers driven by market power may take a long time to occur. In both cases, mergers increase profits for all firms in the industry (i.e., the game is one with positive externalities).

¹⁵Symmetry is only used for simplicity and the results are robust to local perturbations of the parameters. The Cournot model of Perry and Porter (1985) originates the parameters values used in the example.

Note that in the example, even though the process ultimately converges to the most efficient feasible state, convergence occurs very slowly, creating bargaining inefficiencies even when the discount rate is arbitrarily close to 1. The following symmetric equilibrium strategy, in which firms refrain from merging with high probability, illustrates the point. Let the proposers' strategy be to offer to form a coalition with any other player with probability

$$\sigma = \frac{3(1 - \delta)(U_2)}{2\delta(2U_1 - U_2)}$$

that pay $U_2/2$ (and make no offers with probability $1 - 2\sigma$) and let the responders' strategy be to reject any offer below $U_2/2$ and accept it otherwise. In the Appendix, we show that this strategy is MPE, and players' values at the initial state are $\hat{v}_i = U_2/2$. There is then a significant ($U_1 - U_2/2 > 0$) amount of inefficiency, regardless of how close to 1 the discount rate is: as $\delta \rightarrow 1$, it takes an increasing number of periods for the process to converge to the efficient state because $\sigma \rightarrow 0$. Observe that this contrasts to the case in which the grand coalition is efficient, and the process converges to the efficient grand coalition in at most $\#N - 1$ periods.

Related Literature

We now compare our results with the existing literature. Chatterjee et al. (1993) were the first to show that inefficiencies, such as bargaining delays and lack of formation of an efficient grand coalition, can occur in coalition bargaining. Okada (1996) modified Chatterjee et al. (1993), allowing the proposer to be chosen randomly, rather than in a predefined order, and showed that in superadditive settings there are no bargaining delays. In both models, coalitions cannot renegotiate (i.e., once formed, they leave the game). Seidmann and Winter (1998) showed that the grand coalition forms when renegotiations of coalitions are allowed (see also Okada (2000)). Our contribution extends these results to multilateral games with externalities.

Proposition 3 is also related to results in Gomes and Jehiel (2004). They find that for discount rates arbitrarily close to 1 (very patient players), the economy is efficient if there is at least one state that is efficient and negative-externality-free (in the sense that a move away from that state does not hurt the players whose consent is not required for the move). This property is satisfied by an efficient grand coalition in our game, and thus their result suggests more general circumstances in which efficiency holds. However, note that in our bargaining delay example, states $[ij|k]$ are also efficient and negative-externality-free, and we exhibited an equilibrium that is not asymptotically efficient. Our results, though, are not in contradiction with Gomes and Jehiel (2004), because Assumption A2 of their model does not hold in our model. Basically, in the context of our example, A2 means that it is possible for the

proposer to form a pairwise coalition that requests compensation from the excluded player. This possibility rules out inefficiencies and delays because the proposer is able to credibly extract from the excluded player the positive externalities that the coalition is creating. However, in many circumstances, such as mergers in oligopolistic industries, which motivated our example, such types of transfers are illegal.

There is an extensive literature that focuses on explaining bargaining delays. One strand of this literature shows that the presence of uncertainty and incomplete information gives rise to delays because players use costly delays as a mechanism to convey high valuation (see the survey by Kennan and Wilson (1993)). However, models that explain delays in complete information settings are rare.¹⁶ In a context similar to ours, Ray and Vohra (1999, 2001) and Jehiel and Moldovanu (1995b) explore the role of externalities for bargaining delays. Ray and Vohra (1999, 2001) allow players to commit to withdraw from negotiations and they show that in equilibrium they opt to do so to free-ride on the coalition formation decisions of players who remain in the game. This behavior gives rise to inefficient equilibria, but not to multiperiod bargaining delays of the sort we exhibited above. Thus our results complement those of Ray and Vohra.

Jehiel and Moldovanu (1995b) study bargaining delays using a setting in which a seller owns an indivisible object to be sold to one of several potential buyers. Their main results are that (i) when externalities are positive, there exist no subgame perfect equilibria (SPNE) in pure strategies with bounded recall that exhibits delay; (ii) when externalities are negative, there may be cycles in all SPNE with bounded recall. The differences in results are due to the fact that we allow for mixed strategy equilibria, while Jehiel and Moldovanu (1995b) focus on pure strategies. In our previous example, there are also no delays if strategies are pure. The intuition for the disappearance of delay is similar to the one provided by Jehiel and Moldovanu (1995b): delays with positive externality are sustained by the belief of each player that a coalition (not involving themselves) is going to form. We have exhibited a mixed strategy in which this type of belief is consistent, but with pure strategies it never is. So our analysis complements Jehiel and Moldovanu (1995b) because we point out that allowing for mixed strategies can originate delays in the presence of positive externalities.¹⁷

5. CONCLUSION

This paper explored the economic implications of externalities in multilateral contracting settings with infinitely many contracting possibilities. We

¹⁶Some papers that also study bargaining delays are Dekel (1990), Fernandez and Glazer (1991), Fershtman and Seidmann (1993), and Ma and Manove (1993).

¹⁷Note that Jehiel and Moldovanu (1995b) also show that delays arise in positive-externality finite-horizon games.

characterized the model's equilibria and showed that they always exist and have nice properties, such as linearity and monotonicity on the contracts. We showed that there may be bargaining delays in positive-externality games if a comprehensive agreement is not possible, because players may prefer to delay contracting so that they can potentially free-ride on other players' contracting decisions. However, if a comprehensive agreement is efficient and feasible, then there are no bargaining delays and the outcome is asymptotically Pareto efficient despite the existence of externalities: even though interim contracting inefficiencies may exist, they are irrelevant in terms of efficiency.

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APPENDIX

PROOF OF PROPOSITION 1: To show that there exist payoffs and probability distribution that satisfy (5) and (6), consider the correspondence $\mathcal{F}: X \times \Sigma \rightrightarrows X \times \Sigma$: (domain) X is the set of points $x = (x_C(\pi))$ such that $x_C(\pi) \geq \min_{\pi' \in C} \{U_C(\pi')\}$ and $\sum_{C \in \pi} x_C(\pi) \leq \max_{\pi \in \Pi} \{\sum_{C \in \pi} U_C(\pi)\}$; and Σ is the set of probability distributions $\sigma = (\sigma_C(\pi))$, where $\sigma_C(\pi) \in \Sigma_C(\pi)$; (def. \mathcal{F}) $(y, \rho) \in \mathcal{F}(x, \sigma)$ if and only if

$$\text{supp}(\rho_C(\pi)) \subset \arg \max_{S \ni C} \left(x_S(\pi S) - \sum_{C \in S} x_C(\pi) \right)$$

and

$$\begin{aligned} y_C(\pi) = & \delta \sum_{B \in \pi} \sum_{S \subset \pi} p_B(\pi) \sigma_B(\pi)(S) (\mathbb{I}_{[C \in S]} x_C(\pi) + \mathbb{I}_{[C \notin S]} x_C(\pi S)) \\ & + \delta p_C(\pi) e_C(\pi) + u_C(\pi), \end{aligned}$$

where

$$e_C(\pi) = \max_{S \ni C} \left(x_S(\pi S) - \sum_{C \in S} x_C(\pi) \right).$$

A standard argument proves that all the conditions of the Kakutani fixed point theorem hold: $\mathcal{F}(X \times \Sigma) \subset \mathcal{F}(X \times \Sigma)$, $X \times \Sigma$ is a compact and convex finite dimensional set, $\mathcal{F}(x, \sigma)$ is convex (and nonempty), and \mathcal{F} has a closed graph (i.e., is upper hemicontinuous). So by the Kakutani fixed point theorem, there is a payoff and a distribution probability that satisfy (5) and (6).

The value function of the strategy profile σ satisfies $v_C(\sigma|\pi) = v_C(\pi)$. The proof is by induction on the number of players in the c.s. π . The result holds for π with one player. Suppose it holds for all c.s. with less than m players and let π be a c.s. with m players. Let us compute $v_C(\sigma|\pi)$ for player $C \in \pi$: when C proposes to S , C 's continuation value is $\hat{v}_C(\pi S) - \sum_{B \in S, B \neq C} \hat{v}_B(\pi) = e_C(\pi) + \hat{v}_C(\pi)$ if $S \neq \{C\}$ and $\hat{v}_C(\sigma|\pi)$ if $S = \{C\}$; when $B \neq C$ proposes to S , C 's continuation value is $\hat{v}_C(\pi)$ if $C \in S$, $\hat{v}_C(\pi S)$ if $C \notin S$ and $S \neq \{B\}$, and $\hat{v}_C(\sigma|\pi)$ if $S = \{B\}$. Putting this together and rearranging terms, we have that

$$\begin{aligned} \hat{v}_C(\sigma|\pi) &= \delta p_C(\pi) e_C(\pi) + u_C(\pi) + \delta \sum_{B \in \pi} \sum_{S \subset \pi} p_B(\pi) \\ &\quad \times \sum_{S \neq B} \sigma_B(\pi)(S) (\mathbb{I}_{[C \in S]} \hat{v}_C(\pi) + \mathbb{I}_{[C \notin S]} \hat{v}_C(\pi S)) \\ &\quad + \sigma_B(\pi)(\{B\}) \hat{v}_C(\sigma|\pi), \end{aligned}$$

which is a linear equation with only one solution, $\hat{v}_C(\sigma|\pi) = \hat{v}_C(\pi)$. By the single-period deviation principle, it follows that σ is also an MPE. *Q.E.D.*

PROOF OF PROPOSITION 2: Let Y_π be the set of states y with c.s. π . Normalize the partition function so that all $u_C(\pi) > 0$ (this is without loss of generality). Let the state space Y be such that $y \in Y$ if and only if $y_{i,C}(\pi) \geq l_{i,C}(\pi) = -\delta p_i(\pi) e_C(\pi)$.

The following claim implies that the strategy σ is well defined.

CLAIM 1: For any $y \in Y_\pi$, $\sum_{i \in C} \hat{v}_i(y) = \hat{v}_C(\pi)$ and $\hat{v}_i(y) \geq 0$, and, reciprocally, for all vectors $(v_i(\pi))_{i \in C}$ such that $v_i(\pi) \geq 0$ and $\sum_{i \in C} v_i(\pi) = \hat{v}_C(\pi)$, there exist $y \in Y_\pi$ such that $\hat{v}_i(y) = v_i(\pi)$ for all $i \in C$.

(\Rightarrow) The statement holds because $\sum_{j \in C} p_j(\pi) = p_C(\pi)$, $\sum_{j \in C} y_{j,C}(\pi) = u_C(\pi)$, and $\hat{v}_i(y)$ is the π th coordinate of vector $[I - \delta \mu_C]^{-1}(\delta p_i e_C + y_{i,C})$ and $y_{i,C}(\pi) \geq -\delta p_i(\pi) e_C(\pi)$.

(\Leftarrow) Consider the vector $\alpha_{i,C} = v_i(\pi)/u_C(\pi) \geq 0$ for all $i \in C$. The contract $y = (y_C, y_{-C}) \in Y_\pi$, where $y_{i,C}(\pi) = (1 - \alpha_{i,C}) l_{i,C}(\pi) + \alpha_{i,C}(u_C(\pi) - \sum_{j \in C, i \neq j} l_{j,C}(\pi))$ is such that $\hat{v}_i(y) = v_i(\pi)$ for all $i \in C$, which proves the claim.

CLAIM 2: The mapping $T: B(Y) \rightarrow B(Y)$ is a contraction mapping with modulus δ . Consider the metric space $B(Y)$ of the all bounded functions endowed with the sup norm $\|g\| = \sup_{y \in Y} |g(y)|$; $B(Y)$ is a complete metric space. For any $g, h \in B(Y)$,

$$\|T(g) - T(h)\| = \sup_{y \in Y} \left| \delta \sum_{y' \in Y} \mu(\sigma)(y, y') (g(y') - h(y')) \right| \leq \delta \|g - h\|,$$

so the contraction mapping theorem applies to T and thus it has a unique fixed point $\hat{v}_i(\sigma|y)$.

CLAIM 3: We have $T(\hat{v}_j)(y) = \hat{v}_j(y)$.

Let π be the c.s. associated with y . It is easy to see that for all $z \in Y$ such that $\mu(y, z|i) > 0$, then $z = (z_S, y_{-S})$ for some $S \subset \pi$ and $\hat{v}_j(z)$ is equal to $e_C(\pi) + \hat{v}_j(y)$ if $i = j \in S$, $\hat{v}_j(y)$ if $j \in S \setminus i$, and $\hat{v}_j(z)$ if $j \notin S$. Note that $T(\hat{v}_j)(y) = \delta \sum_{i \in N} \sum_{z \in Y} p_i(\pi) \mu(\sigma)(y, z|i) \hat{v}_j(z) + u_j(y)$ and, using σ 's definition,

$$\begin{aligned} T(\hat{v}_j)(y) &= \delta \sum_{i \in N} p_i(\pi) \sum_{S \subset \pi} \sigma_{B_i}(\pi)(S) (\mathbb{I}_{[j \in S]} \hat{v}_j(y) + \mathbb{I}_{[j \notin S]} \hat{v}_j(z)) \\ &\quad + \delta p_j(\pi) e_C(\pi) + u_j(y), \end{aligned}$$

where B_i is the unique $B_i \in \pi$ such that $i \in B_i$. Combining $\sum_{i \in B} p_i(\pi) = p_B(\pi)$ for all $B \in \pi$ and (7), we finally have that $T(\hat{v}_j) = \hat{v}_j$.

The single-period deviation principle and the fact that $\hat{v}_j(\sigma|y) = \hat{v}_j(y)$, implies that σ is MPE: Responders surely cannot improve by deviating from σ and, moreover, any acceptable offer (S, z_S) by proposer i is such that i 's payoff is $\hat{v}_i(\sigma|z_S, y_{-S}) \leq e_C(\pi) + \hat{v}_i(\sigma|y)$, which is i 's payoff if he conforms with strategy σ , and so i cannot improve by a single-period deviation from σ . Q.E.D.

PROPOSITION—Sufficient Condition: *Let σ be an MPE of the MCG that satisfies the following statements:*

(i) *The aggregate values $\sum_{j \in C} v_j(\sigma|y)$ are equal for all states $y \in Y_\pi$ and any fixed $C \in \pi$ ($\sum_{j \in C} v_j(\sigma|y) = \sum_{j \in C} v_j(\sigma|y')$ for any $y, y' \in Y_\pi$).*

(ii) *The probabilities $\sum_{z \in Y_{\pi'}} \mu(\sigma)(y, z)$ of a transition from any state $y \in Y_\pi$ to a contractual state in $Y_{\pi'}$ are all the same (i.e., $\sum_{z \in Y_{\pi'}} \mu(\sigma)(y, z) = \sum_{z \in Y_{\pi'}} \mu(\sigma)(y', z)$ for any $y, y' \in Y_\pi$). Then $\hat{v}_i(\sigma|y) = \hat{v}_i(y)$ and the strategy σ is as the strategy defined by Proposition 2.*

PROOF: Let $\hat{\sigma}_C(\pi)(S) = \sum_{z: \pi_z = \pi S} \mu(\sigma)(y, z|i)$, where $y \in Y_\pi$, and $\hat{v}_C(\pi) = \sum_{j \in B} \hat{v}_j(\sigma|y)$, $e(\pi, S) = \sum_{j \in S} \hat{v}_j(\sigma|z_S, y_{-S}) - \sum_{j \in S} \hat{v}_j(\sigma|y)$, and $e_C(\pi) = \max_{C \in S \subset \pi} e(\pi, S)$. By (i) and (ii), all terms are well defined.

CLAIM: For all $y \in Y_\pi$ and $z \in Y$ such that $\mu(\sigma)(y, z|i) > 0$, then $z = (z_S, y_{-S})$ for some $S \subset \pi$ and $\hat{v}_j(\sigma|z)$ is equal to $e_C(\pi) + \hat{v}_j(\sigma|y)$ if $i = j$ and $\hat{v}_j(\sigma|y)$ if $j \in S \setminus i$.

Note that combining the claim, (9), and (7) implies that $\hat{v}_j(\sigma|y) = \hat{v}_j(y)$, as we want to show.

PROOF OF CLAIM: By induction. The claim obviously holds when there is only one coalition in π . Suppose the claim holds for all c.s. with less than m coalitions and consider a c.s. π with m coalitions. For all coalitions π'

coarser than π , the claim and its implication apply, so for all $z \in Y_{\pi'}$, then $\hat{v}_j(\sigma|z) = \hat{v}_j(z)$ is linear and monotonic in z_j . This implies that, for $j \in \mathcal{S} \setminus i$, $\hat{v}_j(\sigma|z)$ cannot be greater than $\hat{v}_j(\sigma|y)$, because otherwise the proposer could improve by offering an acceptable offer that is worth slightly less to the responder (which is possible because of linearity and monotonicity of $\hat{v}_j(\sigma|z)$). Also, $\hat{v}_j(\sigma|z) = e_C(\pi) + \hat{v}_j(\sigma|y)$ if $i = j \in \mathcal{S}$: $\hat{v}_i(\sigma|z) \leq e_C(\pi) + \hat{v}_i(\sigma|y)$, because $\hat{v}_i(\sigma|z_S, y_{-S}) \geq \hat{v}_i(\sigma|y)$ for all $j \in \mathcal{S} \setminus i$, so

$$\begin{aligned} & \hat{v}_i(\sigma|z_S, y_{-S}) - \hat{v}_i(\sigma|y) \\ &= \sum_{j \in \mathcal{S}} \hat{v}_j(\sigma|z_S, y_{-S}) - \sum_{j \in \mathcal{S} \setminus i} \hat{v}_j(\sigma|z_S, y_{-S}) - \hat{v}_i(\sigma|y) \\ &\leq \sum_{j \in \mathcal{S}} \hat{v}_j(\sigma|z_S, y_{-S}) - \sum_{j \in \mathcal{S}} \hat{v}_j(\sigma|y) = e(\pi, \mathcal{S}). \end{aligned}$$

Moreover, $\hat{v}_i(\sigma|z)$ cannot be smaller than $e_C(\pi) + \hat{v}_i(\sigma|y)$ because otherwise the proposer could improve (because of linearity and monotonicity of $\hat{v}_j(\sigma|z)$). Q.E.D.

PROOF OF PROPOSITION 3: Let $U_N(\{N\}) = U$ and let σ be any MPE of the CBG: (i) Consider any c.s. π that is inefficient (i.e., $\sum_{C \in \pi} U_C(\pi) < U$). This implies that

$$\sum_{C \in \pi} \hat{v}_C(\sigma|\pi) = \sum_{C \in \pi} \delta v_C(\sigma|\pi) + (1 - \delta) U_C(\pi) < U,$$

because $\sum_{C \in \pi} v_C(\sigma|\pi) \leq U$, as well. Now assume, by contradiction, that there is a coalition, say B , that proposes an offer that is rejected with positive probability. The maximum surplus of player B then must be zero, because that is the surplus B gets if the offer is rejected, and B 's equilibrium offer maximizes his surplus (see Section 3). However, if B proposes $S = \pi$ (so that $\mathcal{S} = N$), j can get a surplus of at least $U - \sum_{C \in \pi} \hat{v}_C(\sigma|\pi) > 0$, which is a contradiction. (ii) The number of coalition structures is finite and it is impossible to go back to a c.s. that has been played before, so (i) implies that inefficient c.s. are played only a finite number of times, and, thus, Pareto efficiency is reached in the limit when δ converges to 1. By the equivalence results of Section 3, the proposition can also be extended to the MCG. Q.E.D.

PROOF OF PROPOSITION 4: Consider a probability distribution $\tilde{\sigma}$ equal to $\tilde{\sigma}_i(\pi)(N) = 1$ and let

$$\begin{aligned} v_C(\pi) &= U_C(\pi) + p_C(\pi) \left(U_N(\pi_N) - \sum_{B \in \pi} U_B(\pi) \right), \\ \hat{v}_C(\pi) &= \delta v_C(\pi) + (1 - \delta) U_C(\pi), \end{aligned}$$

and

$$e(v^\delta|\pi, S) = \hat{v}_S(\pi S) - \sum_{B \in S} \hat{v}_B(\pi).$$

The strategy profile σ as defined in Proposition 1 is an MPE because the system of equations (6) holds (immediate) and the inequalities (5) also hold because the surplus is $d(\delta) = e(v^\delta|\pi, S) - e(v^\delta|\pi, N) \leq 0$: $d(\delta)$ is a linear function $d(\delta) = (d(1) - d(0))(\delta - 1) + d(1)$ of δ and $d(1) = v_S(\pi S) - v_S(\pi) \leq 0$ because it is equivalent to condition (10); the slope is increasing,

$$\begin{aligned} d(1) - d(0) &= p_S(\pi S) \left(U_N(\pi_N) - \sum_{B \in \pi S} U_B(\pi S) \right) \\ &\quad + \left(1 - \sum_{B \in S} p_B(\pi) \right) \left(U_N(\pi_N) - \sum_{B \in \pi} U_B(\pi) \right) \geq 0 \end{aligned}$$

because the grand coalition is efficient.

Q.E.D.

PROOF OF THE BARGAINING DELAY EXAMPLE: By contradiction, suppose that there is an equilibrium σ with no bargaining delays. This equilibrium is Pareto efficient, $\sum_i v_i = U_1 + U_2$, where v_i are the values at the initial state (note that after any two-firm coalition forms, the coalition value is U_2 and the excluded firm value is U_1). The no-delay condition implies that any proposer i prefers to merge with the firm with lowest value ($U_2 - \min_{j \neq i} v_j \geq v_i$). However, if firms' values are ranked $v_1 < v_2 < v_3$ (a similar argument applies if inequalities are not strict), then $\min_{j \neq 1} v_j = v_2$, where firm 2's value satisfies $v_2 \geq U_1$: firm 2's value is at least U_1 , because if firm 2 makes no proposals and accepts no offers, firms 1 and 3 will form a coalition when 3 proposes. Combining these results, we have

$$3U_2 \geq \sum_i v_i + \sum_i \min_{j \neq i} v_j = U_1 + U_2 + \sum_i \min_{j \neq i} v_j \geq U_1 + U_2 + U_1,$$

which contradicts $U_1 > U_2$.

The strategy profile proposed is MPE by Proposition 1: it satisfies both the support restriction (i), because the surplus $e_{ij} = U_2 - 2\hat{v} = 0$, and the system of equation (ii), because, by construction, $\hat{v}_i = \delta(\frac{1}{3}\hat{v}_i + \frac{2}{3}(\sigma(\hat{v}_i + U_1) + (1 - 2\sigma)\hat{v}_i))$.

Q.E.D.

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