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# Population concentration, urbanization, and demographic transition

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#### Abstract

This paper investigates how urbanization and demographic transition interrelate with each other via merits (agglomeration economies) and demerits (congestion diseconomies) of population concentration. It reveals the mechanism by which agglomeration economies and congestion diseconomies affect the fertility rate. Furthermore, analysis also shows that by assuming declines in infant and child mortality rate, the model developed in this paper can replicate the following well-known historical patterns: (i) advances in urbanization, (ii) rises in the wage rate, (iii) declines in fertility, and (iv) rises followed by declines in the population growth rate (the inverted U-shaped demographic transition). © 2005 Elsevier Inc. All rights reserved.

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# 1. Introduction

It is well known that many countries have experienced the inverted U-shaped demographic transition, i.e., rises followed by declines in the population growth rate. Figure 1

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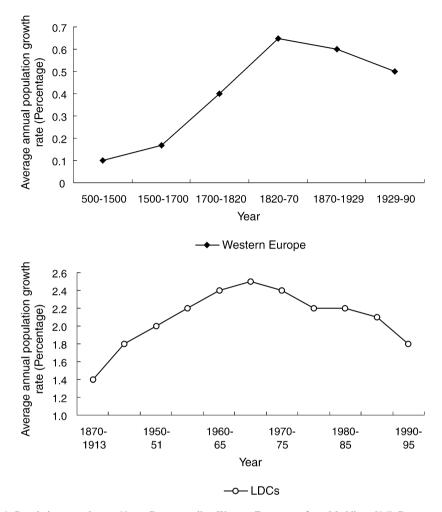


Fig. 1. Population growth rate. *Notes*: Data regarding Western Europe are from Maddison [16]. Data regarding LDCs are from United Nations [21].

shows a few examples. It shows the demographic transitions of Western Europe as well as less developed countries (LDCs). Recently, many studies have attempted to explain this phenomenon from the viewpoint of economics by analyzing how economic activities affect the demographic transition and *vice versa*.

In order to examine the relationship between fertility and economic activities, the following three types of models have been considered. In the first type of model, parents derive utility from their children's utility. We call this type of model the dynasty model (e.g., Razin and Ben-Zion [17], and Caballe [5]). In the second type of model, the need for material support during old age motivate them to bear children, which indicates that child rearing is considered as an investment. We call this type of model the non-altruistic model (e.g., Zhang and Nishimura [22]). Finally, in the third type of model, parents derive utility

from the number of children. We call this type of model the utility from the number of children model (e.g., Eckstein and Wolpin [7]).

Thus far, the inverted U-shaped demographic transition has been intensively analyzed from the viewpoint of the so-called "quantity-quality trade off of children" using the dynasty or utility from the number of children models. This view implies that although parents obtain higher utility with increased number of children or when their children attain a higher level of human capital, each parent is endowed with limited time that he/she can allocate to parenting and educating children. Therefore, there exists a trade off between the quantity and quality of children.

The quantity-quality trade off arguments successfully explain the features of long-run population transition. In this paper, we present a utility-from-the-number-of-children model that describes another mechanism that can account for the long-run population transition. At this point, we focus on the phenomenon of "urbanization."

Many countries that experienced the inverted U-shaped demographic transition were observed to have remarkable advances in urbanization. For example, Bairoch [1] estimated that in 1800, the urbanization rate in England was 23 percent, while this figure rose to 79 percent in 1980. A similar trend was observed in other countries as well. Figure 2 describes this trend with respect to European countries and LDCs.

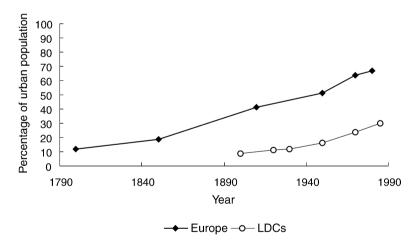


Fig. 2. Level of urbanization. Note: Data are from Bairoch [1, Tables 13.4 and 27.2].

<sup>&</sup>lt;sup>1</sup> Becker et al. [3] is a seminal work that presented a model in which fertility is closely related with human capital accumulation. In their model, parents obtain utility not only from consumption but also from the quantity and quality of their children. Each parent allocates his/her fixed time to working, parenting, and educating children. Hence, there is the quantity–quality trade off of children. Many papers that include Kalemli-Ozcan et al. [12], Kalemli-Ozcan [13,14] developed models having this type of quantity–quality trade off.

Technological progress also effectuates the quantity-quality trade off. It causes a rise in the return from educating children since education and technological progress are complementary to each other. Furthermore, technological progress affects the relative returns between the quantity and quality of children. Galor and Weil [10], as well as Tamura [20] have previously presented models of this type.

Galor [9] presented stylized facts that imply (i) that urbanization and economic development started simultaneously, and (ii) that in the early phases of urbanization and economic development, the population growth rate increased, but it declined in succeeding years. Moreover, Schultz [19] empirically showed that an advance in urbanization reduces fertility. It will then be safe to state that urbanization and demographic transition share a close relationship that is worthy of examination. However, to date, theoretical analysis of the relationship between urbanization and demographic transition has not attracted much attention. To the best of the authors' knowledge, Zhang [23] appears to be the only exception. Zhang [23] constructed a two-sector overlapping generations model that involves the endogenous determination of fertility and education. Theoretically, Zhang showed that, in comparison with the rural sector, better opportunities for earnings and education in the urban sector result in urbanization, which leads to lower fertility since per capita output grows relative to the moving cost. At this point, we will use the concepts of "agglomeration economies" and "congestion diseconomies" to explain the relationship between urbanization and fertility decline from another perspective. Furthermore, the ability of this perspective to replicate the inverted U-shaped demographic transition will also be shown.

Urbanization is a phenomenon caused by large population concentration in smaller areas known as "cities." Urban and regional economists have revealed the positive and negative effects of the geographical concentration of economic agents in an economy. The former effects are termed as "agglomeration economies," which are caused due to various reasons.<sup>2</sup> The latter effects are termed as "congestion diseconomies," which are primarily caused by workers' commuting to business districts.<sup>3</sup>

This paper aims to investigate the relationship between urbanization and demographic transition via agglomeration economies and congestion diseconomies by extending an overlapping generations model of endogenous fertility à la Eckstein and Wolpin [7] to incorporate two sectors, namely, urban and rural sectors. In the model described in this paper, each individual allocates his/her time to working and parenting. Educational investments are excluded from this model. Therefore, the quantity–quality trade off is absent in this model. The urban sector is characterized by agglomeration economies and congestion diseconomies. Due to agglomeration economies, with many urban people, production is more efficient and the wage rate is higher in cities than in rural areas, while the opposite is true with few urban people. Due to the congestion diseconomies, increasing urban population reduces the urban workers' time available for working or parenting, which in turn reduces the urban disposable income.

<sup>&</sup>lt;sup>2</sup> Causes of agglomeration economies include (i) knowledge spillover across firms, (ii) the presence of extensive division of labor, (iii) preference for variety in consumption and increasing returns owing to firm-level economies to scale, (iv) fostering human capital accumulation, (v) expansion of the sector at the origin of innovation, and (vi) heterogeneity of workers and firms. See Duranton and Puga [6] and papers listed therein.

<sup>&</sup>lt;sup>3</sup> See Kanemoto [15] and Fujita [8] for detailed arguments.

<sup>&</sup>lt;sup>4</sup> Sato [18] constructed a similar model: a *n*-sector overlapping generations model having agglomeration economies and congestion diseconomies. While Sato [18] aimed to explain the cross-sectional regional variations in fertility, this paper investigates the historical patterns in fertility, urbanization, and demographic transition.

Analysis reveals the effect of agglomeration economies and congestion diseconomies on demographic transition.<sup>5</sup> Population concentration in the urban sector exerts two forces: a force of raising the urban wage rate due to agglomeration economies and a force of lowering the urban disposable income due to congestion diseconomies. Since having children requires an individual to sacrifice some of his/her working time, the former force bears two conflicting effects on fertility: the positive income effect and the negative effect that raises the opportunity cost of parenting. The latter force bears the negative income effect. In this model, since the first and second effects neutralize each other, only the third effect is effective and an increase in urban population reduces urban fertility to be lower than rural fertility. It is also shown that urbanization progresses as the total population increases due to agglomeration economies. Finally, simulation analysis proves that declines in the infant and child mortality rate accelerate the urbanization process, which in turn raises the average wage rate, reduces the total fertility rate of the country, and generates the inverted U-shaped demographic transition.

Declines in infant and child mortality, rises in income, and declines in fertility were also observed together with the inverted U-shaped demographic transition and advances in urbanization. The results of this paper are consistent with these well-known observations.<sup>6</sup>

The relationship between declines in mortality rate and the inverted U-shaped demographic transition was investigated in Kalemli-Ozcan [13,14]. In the models described in these papers, a decline in infant and child mortality is an engine of economic growth. The key feature of these models is the uncertainty with respect to the survival of each child. Since whether or not each child survives to be an adult is determined stochastically, parents determine the number of children and their education levels in order to maximize the expected utility. Since parents are assumed to be risk-averse, there exists a precautionary demand for children among them. A decline in the mortality rate implies a decrease in the uncertainty, which in turn lowers the precautionary demand. Since these models have the quantity–quality trade off, it increases the educational investments and enhances economic growth.

In our model, a decline in infant and child mortality affects the demographic transition through another channel. In order to analyze this channel, we consider the survival of children as deterministic. Parents know the exact percentage of the surviving children and they take this into consideration when they decide the number of children. Due to this assumption, the fertility rate becomes indirectly dependent on the mortality rate. A decline in the mortality rate increases the total population size, which in turn raises the urbanization rate due to agglomeration economies. As stated above, since the fertility rate is lower in the urban sector than in the rural sector, the national fertility rate decreases. This mechanism brings our model the inverted U-shaped demographic transition in the process of economic development.

<sup>&</sup>lt;sup>5</sup> Behrens [4] also analyzed the relationship between urbanization and demographic transition. It examined the effects of an exogenously determined population growth on the evolution of manufacturing real wages via agglomeration economies and urbanization. In contrast, our study focuses on the influences of agglomeration economies and congestion diseconomies on endogenously determined population and economic growth. Therefore, these two studies are complementary to each other.

<sup>&</sup>lt;sup>6</sup> Some data on these observations are available on request.

Of course, we do not intend to offer the findings described in this paper as the only explanation of the observed facts. Instead, we aim to present the significant effects of agglomeration economies and congestion diseconomies on demographic transition as well as present our findings as one of the probable explanation of the observed facts.

This paper is structured as follows. In Section 2, we introduce the basic structure of the model. Section 3 shows the existence of an equilibrium. In Section 4, simulation analysis is executed. Concluding remarks are provided in Section 5.

## 2. Model

Consider an economy that comprises two sectors: urban and rural. Relevant variables of the urban sector carry the subscript u and those of the rural sector carry the subscript r. Time is discrete and each individual lives for two periods; childhood and adulthood. Let  $N_{it}$  denote the number of adults in sector i (i = u, r) in period t and  $N_t$  the total number of adults in period t:  $N_t = N_{ut} + N_{rt}$ . In the adulthood, individuals choose the sectors they live in, supply labor in those sectors, and decide on consumption and the number of children. At the beginning of period t, each adult in sector i has  $n_{it}/(1-\phi)$  children. Here, only  $1-\phi$  percent of these children are assumed to grow to be adults in period t+1; thus,  $\phi$  represents the infant and child mortality rate.  $\phi$  is assumed to be exogenous and satisfies  $0 < \phi < 1$ . For the analytical simplicity, we assume that parents regard their children's death as deterministic.

 $n_{it}/(1-\phi)$  represents the total fertility rate in each sector. The total fertility rate in the economy is, then, given by

$$f_t = \left(\frac{n_{ut}}{1 - \phi} N_{ut} + \frac{n_{rt}}{1 - \phi} N_{rt}\right) \frac{1}{N_t}.\tag{1}$$

The total population size, which is the sum of the number of adults and that of their children, is described by  $(1 + f_t)N_t$ . At the end of period t, each adult exits the economy. Thus, while  $n_{ut}N_{ut}/(1-\phi)+n_{rt}N_{rt}/(1-\phi)$  children enter the economy at the beginning of period t,  $n_{ut}N_{ut}+n_{rt}N_{rt}$  children survive and  $N_{ut}+N_{rt}$  adults exit the economy at the end of period t. We assume that adults can migrate (with their children) between the sectors without any cost, which implies that the number of surviving children in sector i at the end of period t may not coincide with the number of adults in sector i in period t+1. However, the total number of surviving children at the end of period t must be equal to the total number of adults in period t+1:

$$N_{t+1} = N_{ut+1} + N_{rt+1} = n_{ut}N_{ut} + n_{rt}N_{rt}. (2)$$

This condition is referred to as the law of motion of population.

Individuals are assumed to have an identical utility function of the Cobb–Douglas form and the utility of each individual depends on one's own consumption during adulthood and on the number of surviving children one has:

$$U_{it} = c_{it}^{\alpha} n_{it}^{1-\alpha},\tag{3}$$

where  $c_{it}$  is consumption during adulthood in sector i. There is only one type of goods available in this economy, which we treat as a numeraire.  $\alpha$  is a positive constant and satisfies  $0 < \alpha < 1$ .

We construct a model that is as simple as possible in order to derive results. For this purpose, we assume the Cobb–Douglas utility function. The Cobb–Douglas or log-linear utility function, which is qualitatively identical to the Cobb–Douglas utility function by nature, is widely used in models of endogenous fertility including Becker et al. [3] and Zhang [23].

In order to raise  $n_{it}/(1-\phi)$  children, each individual must spend  $bn_{it}/(1-\phi)$  time. b is a positive constant and common to both sectors. We assume that each individual is endowed with one unit of time. These assumptions require that the number of surviving children  $n_{it}$  must satisfy  $0 \le n_{it} \le (1-\phi)/b$ . Each individual faces the time constraints as follows:

$$l_{it} = l_{wit} + \frac{bn_{it}}{1 - \phi},$$

where  $l_{wit}$  represents the working time,  $l_{it}$  is the disposable time in the i = u, r sector, which can be used for either working or parenting. As will be observed later, congestion diseconomies exist in the urban sector and an increase in urban population decreases the disposable time. In contrast, there are no congestion diseconomies in the rural sector and the disposable time is always equal to the time endowment (i.e.,  $l_{rt} = 1$ ). This time constraint imposes the following budget constraint on each individual:

$$\left(l_{it} - \frac{bn_{it}}{1 - \phi}\right) w_{it} = c_{it},$$

where  $w_{it}$  denote the wage rate per working hour in sector i. The term in parentheses of the left-hand side (LHS) represents the working time of individuals and  $l_{it}w_{it}$  represents the disposable income if workers use all their disposable time for working, that is, the potentially disposable income. The first-order conditions for maximization of the utility function in sector i give

$$c_{it} = \alpha l_{it} w_{it},$$

$$n_{it} = \frac{(1 - \alpha)(1 - \phi)l_{it} w_{it}}{b w_{it}} = \frac{(1 - \alpha)(1 - \phi)l_{it}}{b}.$$
(4)

We assume that the urban wage rate is given by the following wage equation:

$$w_{ut} = B \exp\{\beta N_{ut}\},\tag{5}$$

where B and  $\beta$  are positive constants. This equation represents the existence of agglomeration economies in the urban sector, that is, the greater the urban population, the higher the

<sup>7</sup> In this model, we assume that the wage functions and the congestion diseconomy function take an exponential form. These assumptions are solely for analytical simplicity. As will be observed later, the results of Propositions 1 and 2 can be derived using general form functions.

urban wage rate. Furthermore, population concentration has a negative effect of congestion diseconomies, and the urban disposable time is assumed to be represented as follows:

$$l_{ut} = \frac{1}{\exp\{DN_{ut}\}},\tag{6}$$

where D is a positive constant. The denominator of the right-hand side (RHS) of the above equation represents that the concentration of population reduces the disposable time.<sup>8</sup>

The rural wage rate is assumed to be

$$w_{rt} = G \exp\{-\gamma N_{rt}\},\tag{7}$$

where G and  $\gamma$  are positive constants. Agriculture is usually considered to be the main industry in the rural sector. Then, individuals are required to use land for production. Since it is natural to regard the supply of land as limited, rural productivity and wage rate are assumed to decline with increasing rural workers. We suppose no congestion diseconomies with respect to time in the rural sector, that is,

$$l_{rt} = 1. (8)$$

In this paper, it is assumed that G > B, which implies that agglomeration economies here are such that while with few workers, rural production is more efficient than urban production and the rural wage rate is higher than the urban wage rate, the opposite is true with many workers (see Fig. 3).

From (4)–(6), the number of children in the urban sector is

$$n_{ut} = \frac{(1 - \alpha)(1 - \phi)B \exp\{\beta N_{ut}\}}{bB \exp\{\beta N_{ut}\} \exp\{DN_{ut}\}} = \frac{(1 - \alpha)(1 - \phi)}{b \exp\{DN_{ut}\}}.$$
 (9)

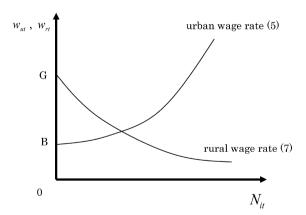


Fig. 3. Wage rate.

<sup>&</sup>lt;sup>8</sup> Our assumption of congestion diseconomies can be regarded as commuting costs. In the urban economics literature, commuting time is considered to increase with population concentration, which reduces the time available for working, parenting, and other activities. Our assumption follows this type of congestion diseconomies. See Fujita [8] and Kanemoto [15], among others.

This equation reveals three effects resulting from the growth in urban population on the number of surviving children in the urban sector  $n_{ut}$ . First, a growth in urban population raises the urban wage rate and bear a positive income effect, which is represented by  $B \exp\{\beta N_{ut}\}$  in the numerator of the middle term of (9). Second, it also causes the negative effect that raises the opportunity cost of rearing children, which is represented by  $B \exp\{\beta N_{ut}\}$  in the denominator of the middle term of (9). Finally, population concentration leads to a reduction in the potentially disposable income due to congestion diseconomies, which is represented by  $\exp\{DN_{ut}\}$  in the denominator of the middle term of (9). In this model, since the first and second effects neutralize each other, only the third effect is effective and an increase in urban population reduces  $n_{ut}$ .

From (4), (7), and (8), the number of children in the rural sector is

$$n_{rt} = \frac{(1 - \alpha)(1 - \phi)}{b}. (10)$$

(9) and (10) imply that for any  $N_{ut} > 0$ ,  $n_{ut}$  and  $n_{rt}$  are determined as interior solutions (i.e.,  $0 < n_{ut} < (1 - \phi)/b$  and  $0 < n_{rt} < (1 - \phi)/b$ ). Furthermore, for any  $N_{ut} > 0$ ,  $n_{ut}$  is smaller than  $n_{rt}$ , and the following proposition holds.

**Proposition 1.** For any  $N_{ut} > 0$ , the total fertility rate  $n_{it}/(1-\phi)$  is lower in the urban sector than in the rural sector.

Under the Cobb–Douglas or log-linear utility function, agents devote an equal share of their disposable time to rearing children. Due to the congestion diseconomies, the disposable time is always smaller in the urban than in the rural sector. Hence, the fertility rate is always lower in the urban than in the rural sector. <sup>10</sup>

From (3) and (4), the indirect utility function is

$$V_{it} = \alpha^{\alpha} \left\{ \frac{(1-\alpha)(1-\phi)}{b} \right\}^{1-\alpha} l_{it} w_{it}^{\alpha}. \tag{11}$$

Since adults are perfectly mobile, migration occurs in order to equate the indirect utility among sectors. Thus, we obtain

$$V_{ut} = V_{rt}. (12)$$

This condition is referred to as the no-migration condition.

<sup>&</sup>lt;sup>9</sup> These two effects were fully discussed in Becker [2].

<sup>10</sup> Under a more general utility function, the wage rate also affects the fertility rate. Due to the agglomeration economies and congestion diseconomies, the urban wage rate is higher than the rural wage rate in an migration equilibrium. This implies that the opportunity costs of rearing children are high in the urban sector than in the rural sector. If this substitution effect and the effect of congestion diseconomies dominate the positive income effect, the urban fertility rate is lower than the rural fertility rate. Under the Cobb—Douglas or log-linear utility function, the negative substitution effect and the positive income effect neutralize each other, and only the effect of congestion diseconomies remains relevant.

# 3. Equilibrium

An equilibrium of the model is summarized by sequences  $\{N_{ut}\}$ ,  $\{N_{rt}\}$ , and  $\{N_t\}$  that satisfy  $N_t = N_{ut} + N_{rt}$ , the law of motion of population (2), and the no-migration condition (12). All other variables are determined adequately if  $\{N_{ut}\}$ ,  $\{N_{rt}\}$ , and  $\{N_t\}$  are determined as positive values.

Substituting (5) to (8) into (11), we obtain the indirect utility in each sector as a function of its number of adults  $N_{it}$ . From this and  $N_t = N_{ut} + N_{rt}$ , (12) becomes

$$B\exp\left\{\left(\beta - \frac{D}{\alpha}\right)N_{ut}\right\} = G\exp\left\{-\gamma(N_t - N_{ut})\right\}. \tag{13}$$

The LHS of this equation represents the indirect utility of an urban individual and the RHS represents that of a rural individual. We assume that the effect of agglomeration economies dominates that of congestion diseconomies (that is,  $\beta - D/\alpha > 0$ ) in the urban sector. Furthermore, we assume that  $\beta - D/\alpha < \gamma$ . This guarantees the stability of an equilibrium with respect to migration.<sup>11</sup>

Equation (13) can be reduced to

$$N_{ut} = \frac{\gamma N_t - \lambda}{Q},\tag{14}$$

where  $\Omega$  and  $\lambda$  are defined as  $\Omega = \gamma - \beta + D/\alpha > 0$  and  $\lambda = \ln G - \ln B > 0$ .

In order for  $\{N_{ut}\}$ ,  $\{N_{rt}\}$ , and  $\{N_t\}$  to be positive, inequalities  $0 < N_{ut} < N_t$  must hold since  $N_{rt} = N_t - N_{ut}$ . These inequalities and (14) yield

$$N_t > \frac{\lambda}{\gamma}$$
 and  $N_t < \frac{\lambda}{\beta - D/\alpha}$ .

From the assumptions G>B and  $\beta-D/\alpha>0$ , we can see that  $\lambda/\gamma=(\ln G-\ln B)/\gamma$  and  $\lambda/(\beta-D/\alpha)=(\ln G-\ln B)/(\beta-D/\alpha)$  are positive. Since  $\gamma$  is assumed to be larger than  $\beta-D/\alpha$ , the above inequalities yield

$$\frac{\lambda}{\gamma} < N_t < \frac{\lambda}{\beta - D/\alpha}.\tag{15}$$

In the range of (15),  $\{N_{ut}\}$ ,  $\{N_{rt}\}$ , and  $\{N_t\}$  are positive. However, in the range of  $N_t \le \lambda/\gamma$ ,  $N_{ut} = 0$  and  $N_{rt} = N_t$ , since the indirect utility of a rural individual is higher than that of an urban individual. Meanwhile, in the range of  $N_t \ge \lambda/(\beta - D/\alpha)$ ,  $N_{ut} = N_t$  and  $N_{rt} = 0$ , since individuals are better off living in cities.

In this paper, the urbanization rate is measured by the proportion of urban adults  $N_{ut}/N_t$ . From the above arguments, we obtain

$$\frac{\partial V_{ut}}{\partial N_{ut}} < \frac{\partial V_{rt}}{\partial N_{ut}}, \quad \text{which is equivalent to} \quad \beta - \frac{D}{\alpha} < \gamma.$$

Such definition of stability is very common to models having multiple sectors (or cities, or regions) and individual's mobility. See Henderson [11] and Kanemoto [15], among others.

<sup>&</sup>lt;sup>11</sup> For an equilibrium to be stable with respect to migration, it is sufficient that an individual is worse off if he/she migrates to the other sector. This requires the following condition to hold true at an equilibrium point.

$$\begin{split} N_{ut} &= \frac{\mathrm{d}(N_{ut}/N_t)}{\mathrm{d}N_t} = 0, \quad \text{if } N_t \leqslant \frac{\lambda}{\gamma}, \\ N_{ut} &= \frac{\gamma N_t - \lambda}{\Omega}, \qquad \frac{\mathrm{d}(N_{ut}/N_t)}{\mathrm{d}N_t} = \frac{\lambda}{\Omega N_t^2} > 0, \quad \text{if } \frac{\lambda}{\gamma} < N_t < \frac{\lambda}{\beta - D/\alpha}, \end{split}$$

and

$$N_{ut} = N_t, \qquad \frac{\mathrm{d}(N_{ut}/N_t)}{\mathrm{d}N_t} = 0, \quad \text{if } N_t \geqslant \frac{\lambda}{\beta - D/\alpha},$$

which proves the following proposition.

**Proposition 2.** If  $\lambda/\gamma < N_t < \lambda/(\beta - D/\alpha)$ , the urbanization rate  $N_{ut}/N_t$  increases as the number of total adults  $N_t$  increases. If  $N_t \leq \lambda/\gamma$ ,  $N_{ut} = 0$  and  $N_{rt} = N_t$  (the pure agricultural economy). If  $N_t \geq \lambda/(\beta - D/\alpha)$ ,  $N_{ut} = N_t$  and  $N_{rt} = 0$  (the pure modern economy).

Proposition 2 states that a sufficiently large amount of people is essential for urbanization and once urbanization begins, the urbanization rate increases as the number of total adults increases. This result is based on the assumptions regarding agglomeration economies that are described in Fig. 3.

Equations (5) and (6) imply the existence of a single city in the economy. This assumption is made for expositional simplicity. Propositions 1 and 2 will hold valid even if multiple cities are assumed in the economy. The key feature here is the existence of agglomeration economies and congestion diseconomies in cities.

As stated in the previous section, due to the assumptions of Cobb–Douglas utility function and congestion diseconomies, the fertility rate is always lower in the urban than in the rural sector. Furthermore, due to the agglomeration economies in the urban sector and decreasing returns in the rural wage function, an increase in the total population enforces the relative advantage of working in the urban sector. Consider one population distribution between sectors. Now suppose temporarily that population increases holding the distribution fixed. Then, the urban wage rate rises due to agglomeration economies and the rural wage rate declines due to decreasing returns. This causes rural to urban migration since the effect of agglomeration economies is assumed to dominate that of congestion diseconomies (that is,  $\beta - D/\alpha > 0$ ). The above arguments imply that population growth results in advances in urbanization.

Since these mechanisms remain unaffected by the introduction of other cities, Propositions 1 and 2 still hold true in a model with multiple cities. These mechanisms remain unaffected by the functional form of equations from (5) to (7) as well. Therefore, even if we consider functional forms other than an exponential form, Propositions 1 and 2 still hold true.

Finally, we consider the law of motion of population (2). From  $N_t = N_{ut} + N_{rt}$  and (14), (2) in the range of  $\lambda/\gamma < N_t < \lambda/(\beta - D/\alpha)$  can be rewritten as follows:

$$\begin{split} N_{t+1} &= K \left\{ N_t + \left( \exp\{-DN_{ut}\} - 1 \right) N_{ut} \right\} \\ &= K \left\{ \frac{\lambda}{\Omega} + \left( 1 - \frac{\gamma}{\Omega} \right) N_t - \frac{1}{\Omega} (\lambda - \gamma N_t) \exp\left\{ \frac{D}{\Omega} (\lambda - \gamma N_t) \right\} \right\}, \end{split}$$

where K is defined as  $K = (1 - \alpha)(1 - \phi)/b > 0$ . In the range of  $N_t \le \lambda/\gamma$ , all agents live in the rural sector, in which the number of children is given by (10). Therefore, (2) becomes

$$N_{t+1} = K N_t. (16)$$

Finally, in the range of  $N_t \ge \lambda/(\beta - D/\alpha)$ , all individuals live in cities, in which the number of children is given by (9). Therefore, (2) in this range is

$$N_{t+1} = K N_t \exp\{-DN_t\}.$$

Summarizing the above arguments, we have the following law of motion of population:

$$N_{t+1} = \begin{cases} KN_t, & \text{if } N_t \leqslant \frac{\lambda}{\gamma}, \\ K\{\frac{\lambda}{\Omega} + (1 - \frac{\gamma}{\Omega})N_t \\ -\frac{1}{\Omega}(\lambda - \gamma N_t) \exp\{\frac{D}{\Omega}(\lambda - \gamma N_t)\}\}, & \text{if } \frac{\lambda}{\gamma} < N_t < \frac{\lambda}{\beta - D/\alpha}, \\ KN_t \exp\{-DN_t\}, & \text{if } N_t \geqslant \frac{\lambda}{\beta - D/\alpha}. \end{cases}$$

$$(17)$$

The model can be shown to have a stationary equilibrium by showing that the difference equation (17) has a stationary state since  $N_{ut}$  and  $N_{rt}$  are determined by (14) and  $N_t = N_{ut} + N_{rt}$  for given  $N_t$ . Examination of (17) yields the following proposition. <sup>12</sup>

**Proposition 3.** If  $1 < K < \exp\{\lambda D/(\beta - D/\alpha)\}$  and  $K\{1 - (1 + \exp\{-2\})\gamma/\Omega\} > -1$ , the model has a stable stationary equilibrium that satisfies  $\lambda/\gamma < N_t < \lambda/(\beta - D/\alpha)$  and a continuum of equilibria that converge to it.

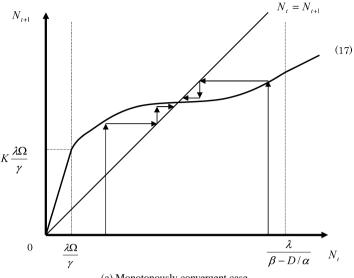
Proposition 3 describes the conditions under which our model has a stable stationary equilibrium. Figure 4 describes a stationary equilibrium and its stability. Figure 4(a) represents a monotonously convergent case and Fig. 4(b) represents a cyclically convergent case.

In both cases,  $N_t$  converges to the unique stationary value from any initial value of  $N_0$ . If the parameter values change, the unique stationary value of  $N_t$  also changes, and the economy starts to converge to new stationary equilibrium.

In the following section, we show by simulation how this convergence progresses when the infant and child mortality rate  $\phi$  declines and show that the converging process can replicate the historical patterns presented in the introduction of this paper.

A decline in  $\phi$  raises the stationary value of  $N_t$ . In (17), a decline in  $\phi$  raises K, which implies an upward shift in the line representing the dynamics of  $N_t$ . This upward shift raises the stationary value of  $N_t$  (see Fig. 4). An increase in  $N_t$  reduces  $N_{rt} = \{(D/\alpha - \beta)N_t + \lambda\}/\Omega$  and raises  $N_{ut} = (\gamma N_t - \lambda)/\Omega$ . This increases the nominal wages in both sectors because we assume that the technology of the rural sector is decreasing returns to scale and that there are agglomeration economies in the urban sector. This also lowers the total fertility rate since it is always lower in the urban sector than in the rural sector, and the urban fertility rate is a decreasing function of urban population due to congestion diseconomies.

<sup>12</sup> Proof of this proposition is available upon request.



(a) Monotonously convergent case.

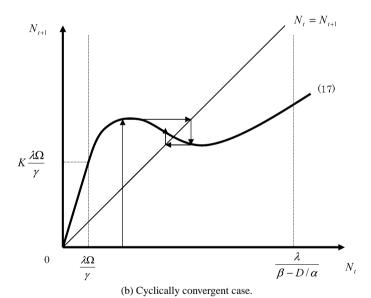


Fig. 4. Stationary equilibrium.

# 4. Analysis

In this section, we show via simulation that declines in the infant and child mortality rate  $\phi$  accelerate urbanization (cause rises in  $N_{ut}/N_t$ ), which in turn increases the average wage rate in the economy  $w_t = (w_{ut}N_{ut} + w_{rt}N_{rt})/N_t$  and reduces the total fertility rate in the economy  $f_t$ . It is also shown that the population growth rate  $g_t$  generates an inverted U-shaped curve: there are rises in the population growth rate, which are followed by declines in it. Note that  $g_t = \{(1 + f_t)N_t - (1 + f_{t-1})N_{t-1}\}/\{(1 + f_{t-1})N_{t-1}\}.$ 

Parameter values are as follows: the utility parameter  $\alpha = 0.7$ ; the cost parameter of child rearing b = 0.2; the parameters in wage equations  $\beta = 0.5$ , B = 1,  $\gamma = 0.5$ , G = 1.2; the parameter of cost of living D = 0.5; the mortality rate  $\phi = 0.3$ .<sup>13</sup> As seen in the Appendix, these parameter values satisfy all the constraints that have been stated so far. In the stationary equilibrium under these parameters, the number of adults N is 0.646, the average wage rate w is 1.102, the total fertility rate of the country f is 1.429, the urbanization rate  $N_u/N$  is 0.508, and the total population size (1 + f)N is 1.569.

Suppose that the country is in the stationary equilibrium stated above. Consider now that there is remarkable progress in medical technology and declines in the exogenously determined mortality rate  $\phi$  from 0.3 to 0.1 by 0.02 per period. When  $\phi$  stops declining at time 10, the country begins to converge to the new stationary equilibrium. This motion is described in Fig. 5.

Declines in mortality rate in Fig. 5(a) lead to rises in the percentage of children who grow to be adults. As seen above, this applies to both sectors and increases the population in the economy, which raises the urbanization rate as described in Proposition 2 (Fig. 5(c)). Concentration of working people in the urban sector raises the average wage rate due to agglomeration economies in the urban sector and decreasing returns to scale in rural pro-

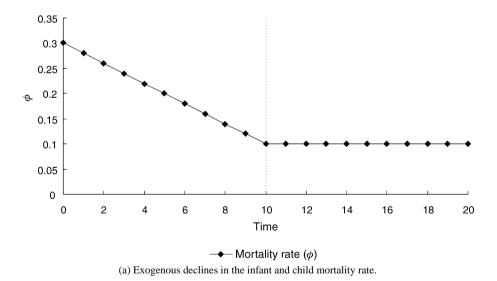
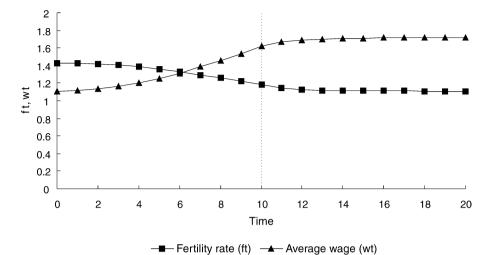
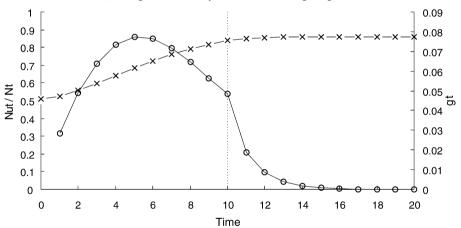


Fig. 5. Simulation analysis.

 $<sup>^{13}</sup>$  We have checked the robustness of the results provided in this paper against other parameter values. Considering the set of parameter values described in the text as a bench mark, we allowed each parameter to individually deviate the bench mark. We changed the parameter values in the following intervals by 0.1: 0.1 to 0.9 with respect to  $\alpha$ , 0.1 to 0.9 with respect to  $\beta$ , 0.1 to 1.1 with respect to 1.1 to 1.1 with respect to 1.1 to 1.1 to 1.1 with respect to 1.1 to 1.1 to 1.1 with respect to 1.1 to 1.1 with



(b) Changes in the fertility rate and in the average wage rate.



-x - Urbanization rate (Nut/Nt) - Population growth rate (gt)

(c) Changes in the urbanization rate and in the population growth rate.

Fig. 5. (Continued).

duction, as seen in Fig. 5(b). Since the total fertility rate is lower in the urban sector than in the rural sector as described in Proposition 1, the total fertility rate of the economy decreases with urbanization (Fig. 5(b)). As seen in Fig. 5(c), demographic transition in terms of the population growth rate follows an inverted U-shaped curve in this process. The initial rises in population are attributed to declines in the infant mortality rate. This increase in population strengthens the agglomeration economies in the urban sector, and become the engine of urbanization. Since the total fertility rate in the urban sector is always lower than in the rural sector, the total fertility rate in the economy declines as urbanization advances. The initial declines in the mortality rate and the subsequent declines in the fertility rate

result in an inverted U-shaped demographic transition. Thus, the model described in this paper can replicate the observed patterns that have been stated in the introduction of this paper.

# 5. Concluding remarks

This paper explored the effects of agglomeration economies and congestion diseconomies on demographic transition. Analysis showed that by assuming declines in the infant and child mortality rate, the model can simultaneously replicate the well-known historical patterns such as advances in urbanization, rises in the average wage rate, declines in the total fertility rate, and the inverted U-shaped demographic transition.

These results imply the existence of a close relationship among demographic transition, urbanization and economic development. However, these phenomena have been studied separately in earlier studies. This paper presented one possible story in which urbanization plays a pivotal role in the process of economic development accompanied by the inverted U-shaped demographic transition.

Our model is very simple and can be extended to many directions. Here, we give one example. In Galor and Weil [10], they showed, using a model having human capital accumulation and technological progress, that economic development generally has three stages: Malthusian regime, post-Malthusian regime, and Modern growth regime. It would be an important issue of future research to examine whether or not the mechanism described in this paper can explain this regime changes.

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# Appendix A. Constraints on parameter values

Values of parameters are as follows: the utility parameter  $\alpha=0.7$ ; the cost parameter of rearing a child b=0.2; the parameters in wage equations  $\beta=0.5$ , B=1,  $\gamma=0.5$ , G=1.2; the parameter of cost of living D=0.5; the infant mortality rate  $\phi=0.3$ , which declines to 0.1 by 0.02 per period.

The constraints that are required to be satisfied are: (i) G > B, (ii)  $\beta < D/\alpha$ , (iii)  $1 < K = (1 - \alpha)(1 - \phi)/b < \exp\{\lambda D/(\beta - D/\alpha)\}$ , and (iv)  $K\{1 - (1 + \exp\{-2\})\gamma/\Omega\} > -1$ . (i) is satisfied via assumption. (ii) is satisfied since  $D/\alpha = 0.42$ , etc. (iii) is satisfied because  $1.05 \le K \le 1.35$  for  $\phi \in [0.1, 0.3]$  and  $\exp\{\lambda D/(\beta - D/\alpha)\} = 2.15$ , etc. Finally,  $K\{1 - (1 + \exp\{-2\})\gamma/\Omega\} = -0.34$ , etc., and (iv) is satisfied.

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