CAN A FIRM'S EXPECTED MARGINAL TAX RATE EXCEED 100 PERCENT?

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The effects of a progressive marginal tax rate structure are examined when the base of the tax is a firm's realized profit, which depends on the realization of a random variable and the firm's choice of an output level. Even though the firm is risk neutral and the marginal tax rate on any realized outcome is in strictly less than 100 percent, it is shown that the progressive rate structure causes the firm to choose a lower output level than would be chosen in the absence of the tax. This result is in striking contrast to the choice neutrality of such a tax in a world of certainty. A novel reason for this is identified—namely, that uncertainty and the progressive rate structure generate expected marginal tax rates of 100 percent and higher.

Keywords: *uncertainty; progressive marginal tax rates*

1. INTRODUCTION

Consider a profit-maximizing business firm, in a world of certainty, for which per period profit is a concave function of its output level. As compared to a no-tax environment, how does the imposition of a tax on profit alter such a firm's short-run output decision? Under widely applicable assumptions on tax structure, the theoretical answer to this question is well known and often serves as a centerpiece example of the power of marginal analysis. As long as the tax is nondecreasing in profit and has marginal rates strictly less than 100 percent, then, since at least the work of Musgrave (1959), it has been recognized that the tax has no effect on the firm's short-run output decision. But what happens to this short-run neutrality of a tax on profit when a firm faces uncertainty, something that is a feature of any realistic economic environment?

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With uncertainty, a tax on profit falls on realizations that are uncertain at the time of the output choice. Most of what we know about this environment comes from Sandmo (1971), whose primary result was to show that a competitive, risk-averse firm chooses a lower level of output than a risk-neutral firm when facing price uncertainty. But Sandmo also showed that a strictly proportional tax on profit (which entails a full-loss offset) has an ambiguous effect on a risk-averse firm's output, depending on the specific characteristics of relative risk aversion. So we do know that short-run neutrality is not a necessary feature of a tax on profit when the firm is risk averse. It is difficult, however, to disentangle the effects of risk aversion and taxation in the Sandmo model, and it seems natural both to ask about the effect of uncertainty on a risk-neutral firm and to extend the analysis to a progressive tax rate structure, which is a feature at least minimally induced by the absence of full-loss offsets in most corresponding real-world taxes. Surprisingly, there is no such analysis in the extant literature.

The purpose of this article is to show that short-run neutrality does not hold for cases in which a risk-neutral firm faces a tax on profit with increasing marginal rates. The analysis also generalizes the results beyond cases of just competitive price uncertainty. Another feature of the article is to stress some surprising intuition behind the nonneutrality result. The intuition is based on showing that the expected marginal tax rate on expected profit is actually greater than 100 percent at the level of output that would be chosen in the absence of the tax, even though the marginal tax rate on any particular realization of profit is strictly less than 100 percent.

The basic features of the model are described in section 2. Section 3 shows that a progressive tax on profit realizations induces a firm to choose a lower level of output than would be the case with a proportional tax or with no tax. Section 4 explains this result in terms of marginal expected tax rates of 100 percent and greater and then identifies the characteristic inefficiency induced by the marginal tax rate distortion. Section 5 contains concluding comments.

2. THE THEORETICAL MODEL

Suppose a business firm knows that its profit, π, depends on its output level, x, and a random variable θ according to the function $\pi(x; \theta)$, where the realization of θ depends on the particular realization of the state of the world. If the firm gets to observe the realization of θ before choosing its output level, then that choice is made under certainty, and the conditions for the short-run neutrality of a tax on profit hold. Choice under uncertainty, on the other hand, arises when the firm has to choose x prior to observing the realization of θ. In the analysis that follows, there is uncertainty as just described, but the firm does know the distribution of θ and how it affects profit. The firm knows that $\theta \in [0, \bar{\theta}]$ and is distributed by the cumulative density function $F(\theta)$, which is continuous and differentiable with density $f(\theta)$. The firm's choice depends on the way in which π varies with respect to x and θ. With respect to x, the assumptions are that π is continuous, single peaked, and twice differentiable, with $\pi_{xx} < 0$ everywhere. With respect to θ, the assumptions are as follows:

$$\pi_\theta > 0, \pi_{x\theta} > 0.$$

These relationships are illustrated in Figure 1, where $\theta_1 < \theta_2 < \theta_3$. Note that the functional dependence of π on θ described here is considerably more general than the Sandmo (1971) case of competitive price uncertainty, although that is clearly encompassed as a special case. The specification, for instance, covers the possibility of θ being a shifter of downward sloping demand, as long as marginal revenue shifts up when the demand curve does, or θ can equally well be thought of as a cost curve shifter for a firm facing a given structure of demand. In this fairly general framework, we can now characterize the firm's problem and choice in the absence of the tax, which provides a benchmark for the effects of a tax on profit.

The firm is assumed to be risk neutral, so with no tax, the relevant objective is

$$\text{Max } \Pi \equiv \int_0^{\bar{\theta}} \pi(x;\theta)dF.$$

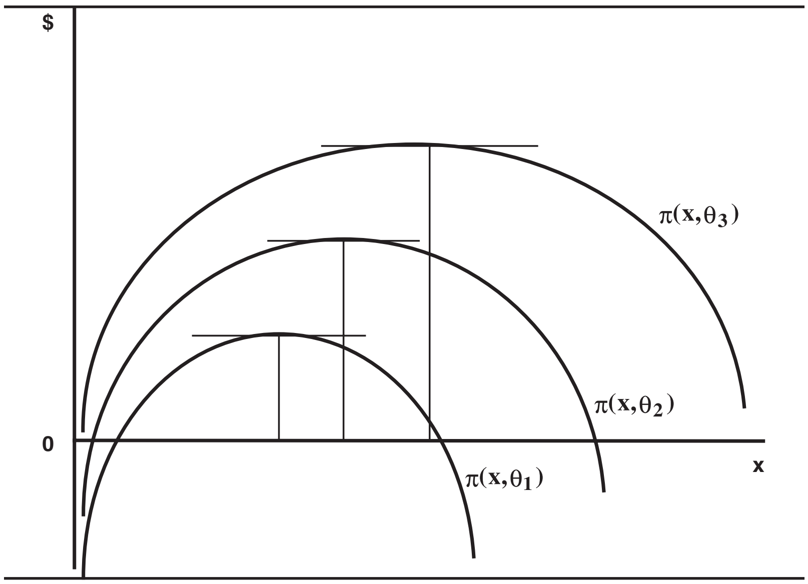

Figure 1: Characteristics of the Profit Function

Define

$$\hat{x} \in \operatorname{argmax}\Pi \rightarrow \int_0^{\bar{\theta}} \pi_x(\hat{x})dF = 0. \tag{1}$$

With the first-order condition of equation (1) providing the benchmark, the question now is as follows: does a progressive tax with π as its base cause the firm to choose a level of x other than $\hat{x}$?

The firm's tax liability is described by the function $T(\pi)$, which has the following properties:

$$T(0) = 0,\ 0 < T' < 1, \text{ and } T'' > 0.$$

The tax structure is progressive with respect to the base in virtue of the increasing marginal rate. The tax is negative for negative realizations of π, but the loss offset is imperfect because of the increasing marginal rates. Given that the tax base is realized π, it is not obvious that the progressive tax structure is neutral with respect to the choice of x because the firm is attempting to maximize the after-tax expectation of π, not after-tax realized π. Indeed, a progressive rate structure is not neutral with respect to the choice of x, as will now be demonstrated.

3. THE NONNEUTRALITY OF A PROGRESSIVE RATE STRUCTURE

In contrast to the no-tax case, the tax on realizations of π means that the firm's objective is now as follows:

$$\text{Max } \Pi^A \equiv \int_0^{\bar{\theta}} [\pi(x;\theta) - T(\pi())] dF.$$

Define

$$x^* \in \text{argmax}\Pi^A \rightarrow \int_0^{\bar{\theta}} \pi_x(x^*)[1 - T'(\pi(x^*))] dF = 0. \qquad (2)$$

Note that if T' is constant, then $(1 - T')$ does not depend on θ and can be taken outside the definite integral, leaving the first-order condition (FOC) as

$$(1 - T) \int_0^{\bar{\theta}} \pi_x dF = 0,$$

which is solved by $x^* = \hat{x}$. But with T' increasing in π and, hence, depending on θ, the tax terms cannot be taken out, and the magnitude of x^* compared to $\hat{x}$ is not immediately transparent. To make such a comparison requires a bit of mathematical work, and compact expectation notation is helpful.

For what follows, note that for given x, we have

$$\int_0^{\bar{\theta}} \pi_x dF = E[\pi_x],$$

$$\int_0^{\bar{\theta}} T'(\pi) dF = E[T'],$$

$$\int_0^{\bar{\theta}} \{T'(\pi_x - E[\pi_x])\} dF = \mathrm{cov}(T', \pi_x).$$

We can immediately rewrite equation (2) as

$$E[\pi_x] - \int_0^{\bar{\theta}} T'\pi_x dF = 0$$

and then add and subtract $\int_0^{\bar{\theta}} T' E[\pi_x] dF$ on the left-hand side (LHS) to obtain

$$(1 - E[T'])\, E[\pi_x] - \mathrm{cov}(T', \pi_x) = 0.$$

The final, revealing form of the FOC is

$$E[\pi_x] = \frac{\mathrm{cov}(T', \pi_x)}{(1 - E[T'])}. \tag{3}$$

In the form of equation (3), the implications of the FOC are easily deduced. First note that $(1 - E[T']) > 0$ because $E[T'] < 1$, by the assumption that the marginal tax rate on any realization of π is less than 1. On the other hand, both π and π_x are increasing in θ, by assumption, while T' is increasing in π. The result is that

$$\mathrm{cov}(T', \pi_x) > 0.$$

This takes us to our main result.

Proposition 1: $x^* < \hat{x}$.

Proof. With no tax, $E[\pi_x(\hat{x})] = 0$, as indicated by equation (1), whereas with increasing marginal tax rates, $E[\pi_x(x^*)] > 0$, as has been deduced from equation (3). $E[\pi]$ is concave in x, which yields the result.

The progressive tax rate structure is not neutral with respect to the choice of x in an environment with uncertainty, even though the marginal tax rate on any particular realization is always less than 100 percent. A continuously increasing marginal tax rate structure is not a necessary condition for this result, and it is possible at this point to provide a sketch of the extension to more realistic piecewise linear tax rate structures.

At a purely formal level, it is clear that the result is driven by the fact that $\mathrm{cov}(T', \pi_x) > 0$. Of course, $T'' > 0$ guarantees that different realizations of θ result in realizations of π that are exposed to different marginal tax rates, but this is more than is required for $\mathrm{cov}(T', \pi_x) > 0$. With a piecewise linear progressive rate structure, the same result obtains if there is enough variation in the possible realizations of π so that some of them fall into different marginal rate brackets. On the other hand, if at $\hat{x}$, all possible realizations of π fall into the same marginal tax rate bracket, then there is no variation in T' and $\mathrm{cov}(T', \pi_x) = 0$, with the result that $x^* = \hat{x}$. Except for the possibility of all possible π realizations falling in the same marginal rate bracket, the nonneutrality result extends to piecewise linear tax rate structures.

4. INTUITION AND INEFFICIENCY

Why would a risk-neutral firm choose less than the expected π-maximizing level of x when the marginal tax rate for any realization of π is less than 100 percent? The deeply intuitive answer to this question is that the marginal tax rate on realized profit is not the relevant one for the firm's ex ante decision on output, whereas the expected marginal tax rate on expected marginal profit is. Using this altered perspective on the problem, we can now show that expected marginal tax rates greater than 100 percent apply to relevant parts of the firm's output choice domain.

4.1. 100 PERCENT-PLUS EXPECTED MARGINAL TAX RATES

As the firm increases x, marginal expected profit is $\int_0^{\bar{\theta}} \pi_x \, dF$. The same increase in x also produces the firm's marginal expected tax liability of $\int_0^{\bar{\theta}} \pi_x T'(\pi) dF$. The firm's stopping rule on x is the point where the increase in Π is just offset by the increase in expected taxes. It is worth reminding ourselves that under a constant marginal tax rate $t \in (0, 1)$, this does not happen until $d\Pi/dx = 0$ because the expected marginal tax liability is $t\int_0^{\bar{\theta}}\pi_x\,dF$, which is always less than $\int_0^{\bar{\theta}}\pi_x\,dF$ when the latter is positive. For a risk-neutral firm, a constant marginal tax rate on realizations is equivalent, behaviorally, to the same constant marginal tax rate on expectations and, hence, the neutrality of this tax on the choice of x. Such is not the case with a progressive rate structure.

The fact that $\int_0^{\bar{\theta}}\pi_x\,dF > 0$ at x^*, under a progressive tax structure, actually implies that expected marginal tax liability is not always less than 100 percent. The reason the firm stops at a level of x where Π is still increasing is because at that point, the increase in Π is just offset by an increase in expected taxes: this meets the definition of a 100 percent expected marginal tax rate on the expected marginal profit. To push this even further, we know that Π^A is strictly concave, so for all $x \in (x^*, \hat{x})$, it is the case that Π^A is decreasing while Π is increasing, meaning expected marginal taxes are more than offsetting increases in Π; that is, in this range, there are 100 percent-plus expected marginal tax rates.

To see what is going on a little more clearly, we need to take a more detailed look at the marginal expected tax liability arising from increases in x and the implied rate on increases in expected π.

The expected marginal tax (henceforth referred to as MT) is given by

$$MT \equiv \int_0^{\bar{\theta}} \pi_x T'(\pi)\,dF,$$

which can be rewritten as

$$MT \equiv E[T']\,E[\pi_x] + \mathrm{cov}(T', \pi_x). \tag{4}$$

To convert MT into a marginal rate (call this the MR), divide it by $E[\pi_x]$ to obtain[1]

$$MR \equiv e[T'] + \frac{\mathrm{cov}(T', \pi_x)}{E[\pi_x]}.$$

We are now in a position to see why expected marginal tax rates attain levels of 100 percent and higher before x reaches $\hat{x}$.

Clearly, $E[T']$ depends on x, but it is always less than 1, so the result is driven by the second term of MR. The covariance in the numerator is positive and does not go to zero as $x \to \hat{x}$. On the other hand, Π is concave and achieves a maximum at $\hat{x}$, where $E[\pi_x] = 0$, so $E[\pi_x]$ becomes ever smaller and goes to zero as x goes to $\hat{x}$ and the second term of MR increases without limit. Before getting to $\hat{x}$, then, MR will reach 1—at x^*, to be precise—and be larger than 1 $\forall x \in (x^*, \hat{x})$. Hence, it is expected marginal tax rates of 100 percent and greater that induce even a risk-neutral agent to choose $x^* < \hat{x}$.

An appreciation of the exact role played by the progressive rate structure is enhanced by disaggregating the marginal effects at $\hat{x}$, where Π is maximized. By definition, $\int_0^{\bar{\theta}} \pi_x(\hat{x})dF = 0$, and this requires that π_x is negative for some realizations of θ and positive for others. Define $\tilde{\theta}$ such that

$$\pi_x(\hat{x}, \tilde{\theta}) = 0,$$

and rewrite the condition as

$$\int_0^{\tilde{\theta}} \pi_x dF + \int_{\tilde{\theta}}^{\bar{\theta}} \pi_x dF = 0. \tag{5}$$

Because π_x is increasing in θ, $\pi_x(\hat{x}, \theta) < 0$ for all $\theta < \tilde{\theta}$ but is positive for all $\theta > \tilde{\theta}$. Now consider the MT at $\hat{x}$, which can be written as

$$MT(\hat{x}) = \int_0^{\tilde{\theta}} \pi_x T' dF + \int_{\tilde{\theta}}^{\bar{\theta}} \pi_x T' dF > 0.$$

The sign is positive as a consequence of π increasing in θ and T' increasing in π for the following reason. In (5), the expectation of the negative realizations of π_x just exactly offset the expectation of the positive realizations, but in MT, each negative realization is weighted by a smaller T' than the T' at each of the positive realizations. So MT is positive at $\hat{x}$, even though expected marginal profit is zero.

In this context, a simple concrete example will make the intuition crystal clear. Suppose that there are only two possible realizations of θ

that are equally likely, with $\overline{\theta} > \underline{\theta}$. Also suppose that $\pi_x(\hat{x}; \overline{\theta}) = \$100$ and $\pi_x(\hat{x}; \underline{\theta}) = -\$100$, so that we have the zero marginal expected profit required at $\hat{x}$. But total realized profit is higher under the realization of $\overline{\theta}$ than it is under the realization of $\underline{\theta}$, and the marginal tax rate on profit is higher at the higher profit realization than it is at the lower one. So let us assume for concreteness that these marginal rates are 30 percent and 20 percent, respectively. Adding the $\hat{x}$th unit of output at the high realization increases taxes by \$30, while adding the same unit at the low realization reduces taxes by \$20. The expected marginal tax for this case is \$5 on an expected marginal profit of zero, and the firm will choose not to produce the $\hat{x}$th unit of output.

An expected marginal tax rate in excess of 100 percent clearly distorts the firm's output choice. This distortion signals the existence of an inefficiency that persists in all short-run settings, regardless of long-run alterations in industry composition. That inefficiency can now be precisely demonstrated.

4.2. THE INEFFICIENCY OF THE PROGRESSIVE RATE STRUCTURE

When the tax authority and firms are risk neutral, a tax on profit with a progressive marginal rate structure is ex ante inefficient, at least from a first-best standpoint, where an individualized proportional tax can be designed for each firm. With a progressive rate structure and the firm choosing $x = x^*$, the expected tax revenue from a firm is given by

$$R = \int_0^{\overline{\theta}} T(\pi(x^*; \theta))dF,$$

and expected after-tax profit is given by

$$\Pi^A(x^*) \equiv \Pi(x^*) - R.$$

The problem is that expected, before-tax profit is not maximized because that requires $x = \hat{x}$, and there are unexploited gains that could be further divided. Assume that $\Pi^A(x^*) \geq 0$, so the firm has no long-run incentive to exit the industry. When that is the case, it is also the case that $\Pi(\hat{x}) > \Pi(x^*) \geq \mathrm{R}$, and there exists a proportional tax rate $t < 1$ such that

$$t\Pi(\hat{x}) = R.$$

Under the proportional tax, the firm does indeed choose $x = \hat{x}$, so expected after-tax profit is

$$\Pi^A(\hat{x}) \equiv \Pi(\hat{x}) - R.$$

With equal expected tax revenue, it is clear that $\Pi^A(\hat{x}) > \Pi^A(x^*)$ because $\Pi(\hat{x}) > \Pi(x^*)$. So clearly, an ex ante Pareto improvement is possible by moving from a progressive to a proportional rate structure for the firm.

5. CONCLUSIONS

A progressive tax on profit distorts a firm's output choice and generates a theoretically demonstrable inefficiency. The important point here is that the inefficiencies arising from a tax on profit are not limited to the long-run consequences of a tax on capital because the inefficiency detailed here obtains in both the short and long runs. Putting aside possible distributional merits of a progressive rate structure, the analysis does perhaps provide an argument against a progressive rate structure, rather than having much additional to say about the consequences of a high or low level of a uniform rate in the long run. Of course, this conclusion is derived from a theoretical result that depends on possible profit realizations falling into different marginal rate brackets and by itself leaves open the question of how it applies to existing real-world approximations to taxes on profit. Nothing definitive can be said here, but there are good reasons for thinking that this study's results do have implications for many existing taxes, despite some well-known facts that might seem to indicate otherwise.

With respect to the U.S. corporation income tax, almost all U.S. public finance textbooks say something very close to the following statement by Rosen (2005, 401): "Most corporate income is taxed at the 35 percent rate, so for our purposes, the system can safely be presented as a flat rate of 35 percent." But for the purposes of this article, it is not safe to present the system as such, even though it is true that most corporate income is indeed taxed at the 35 percent rate. There are two basic reasons for this, the first having to do with the size distribution of pretax profit across corporations. In tax year 2001, there were about 5.14 million corporate tax returns filed, reporting a total pretax profit of about $603.6 billion, but the 11,239 corporations with assets greater than $250 million (0.22 percent of all filers) accounted for 86.8 percent of pretax profit, with the average for these firms being $46.6 million, which is well into the 35 percent bracket.[2] For the remaining 99.78 percent of filers (more than 5 million of them), on the other hand, average pretax profit was $15,597, which is in the 15 percent bracket that starts at zero. This means that a very large number of corporations are in easy reach of negative profits under bad realizations and the progressivity induced by less than full-loss offsets, whereas on the upside, the rate structure is decidedly progressive for multiples of over twenty times the average, with marginal rates going through brackets of 25, 34, and 39 percent up to a profit of $335,000. The conclusion here has to be that there are many firms capable of having profit realizations that put them in different marginal rate brackets.

This possibility is linked to the second reason for not treating the tax as flat rate, which is that just looking at the aggregate of realized profits masks the uncertainty faced by all firms, including those with profit taxed well into the highest bracket.

To illustrate this point, suppose each firm has an independent chance of making profit taxed in the highest bracket with probability 2/3 but risks making zero profit with probability 1/3 at its optimizing output level. After the realizations for all of the firms are added up, 100 percent of all positive corporate profit realized is taxed at the highest rate, but this does not make it safe to present the tax structure as a flat rate. This ignores the fact that many firms have had bad realizations and that all firms faced the prospect of a bad realization with probability 1/3 and an ex ante expected marginal tax rate much higher than the top bracket on realizations.

It is therefore possible to say that the model very likely does have important implications for the contemporary U.S. corporation income tax. And the model's scope of application is broadened by noting that profit is often subject to tax under the individual income tax and that, cross-nationally and historically, there is a lot of variation in the levels and progressivity of the rates at which profit is and has been taxed. Having now noted the possible wide range of application, it is interesting to consider some previously unnoticed implications the model has for an important debate in the theory and evidence of tax shifting.

According to the model, a tax on profit with a progressive rate structure under uncertainty has a short-run output effect and clearly raises the possibility of short-run price effects and tax shifting. For instance, if a progressive tax on profit is imposed de novo on all firms in an industry, then the price of the good must be higher for the given realizations of θ than it would have been in the absence of the tax. In the famous 1960s debate about whether there is short-run shifting of the corporation income tax,[3] no account was taken of the output effect derived in this article. At the time, as exemplified by Musgrave (1959), the view was that empirical evidence of short-run shifting of the corporation tax must imply that firms have some objective other than profit maximization, such as revenue maximization. Katz and Rosen (1985) have since shown that strategic interdependence can result in a tax on profit having short-run output and price effects for profit-maximizing oligopolists. However, does that mean that short-run effects in nonoligopoly environments require us to abandon profit maximization as a firm's objective?

Strictly speaking, the answer is probably yes, but not in a way anywhere near as dramatic as substituting revenue maximization. In a world of uncertainty, we can simply replace the assumption of the firm-maximizing profit for a given realization of the state of the world with the assumption that the firm is maximizing *expected* profit prior to knowing the realization of the state of the world. Under the latter assumption, this study has shown that a progressive tax on realized profit does produce a short-run output effect that will, in turn, produce a price effect and tax shifting. While this produces a number of new questions about the comparative statics of changing tax rates, it is nevertheless a logical extension of classical profit maximization as opposed to a radical change in assumption about a firm's objective.

NOTES

1. As $x \rightarrow \hat{x}$ and $E[\pi_x] \rightarrow 0$, the first term is $E[T']$ by L'Hopital's rule.

2. The size distribution figures are calculated from Table 1 in the Internal Revenue Service (2004). Current and historical marginal tax rates and brackets can be found in the Internal Revenue Service (2003).

3. See Mieszkowsi (1987) for a summary of the issues involved and for additional references.

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