# Interbank Market Integration under Asymmetric Information

**Xavier Freixas**
Universitat Pompeu Fabra

**Cornelia Holthausen**
European Central Bank

Cross-country bank lending appears to be subject to market imperfections leading to persistent interest rate differentials. In a model where banks need to cope with liquidity shocks by borrowing or by liquidating assets, we study the scope for international interbank market integration with unsecured lending when cross-country information is noisy. We find that an equilibrium with integrated markets need not always exist, and that it may coexist with one characterized by segmentation. A repo market reduces interest rate spreads and improves upon the segmentation equilibrium. However, it may destroy the unsecured integrated equilibrium.

The objective of this article is to study the effects of cross-country asymmetric information on the structure of financial markets. Our main concern is the design of money markets and the role of repo and (unsecured) interbank markets in an international framework, but our results carry over to a more general framework of the analysis of cross-country direct investment, covering the cross-country market both for bonds and equity. The creation of an integrated interbank market is particularly relevant in order for banks to cope efficiently with liquidity shocks. Interbank markets are instrumental in allowing for a smooth working of the payment systems (so that a bank that is lacking liquidity in the payment system is able to borrow from another bank), and in channeling liquidity to the banks and countries that need it most. Both repo and unsecured interbank lending allow banks to cope with liquidity shocks. Still, because unsecured markets are based on peer monitoring, they introduce market discipline, thus playing the role unsecured deposits may play when depositors receive information [Calomiris and Kahn (1991)].

The collapse of a well-functioning unsecured interbank market proved to be crucial in the context of Eastern Asia financial crises where

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interbank lending by foreign countries had played a key role in these countries' funding structure. It also sheds light on the construction of a European interbank market after the creation of the European Monetary Union. Finally, the issue has direct policy implications since, if the interbank market's provision of liquidity is inefficient, this calls for regulatory intervention.

The role of the interbank market to cope with bank specific liquidity shocks and avoid unnecessary liquidation of long-term investments was first acknowledged in Bhattacharya and Gale (1987). Later contributions built upon this role while introducing either moral hazard [Rochet and Tirole (1996)], aggregated liquidity risk [Allen and Gale (2000)], or else by introducing credit risk [Freixas, Parigi, and Rochet (2000)]. These studies yield similar results pointing at the potential contagion provided by an interbank market as well as the effect the network of interbank lending may have on financial fragility. Furthermore, Bhattacharya and Fulghieri (1994) analyze the efficiency of an interbank market in a framework where banks face uncertain timing of liquidity returns, and Holmström and Tirole (1998) discuss the role of liquidity provision by the public sector.

Although our article does not focus on financial crises, we also consider two different interbank lending networks, segmentation versus integration, where the collapse of integration may be interpreted as a financial crisis.

As in Rochet and Tirole, we consider "peer monitoring" as a key factor in improving the efficiency of the interbank market; still we are concerned with asymmetric information and therefore about the quality of signals rather than about moral hazard and the way to discipline borrowers. From that perspective, our work is related to Broecker (1990) and to Flannery (1996) who consider models of asymmetric information and credit risk. An important difference to their work is that the value of a signal is given exogenously in their model, while in our model it is endogenous, as it results from the equilibrium behavior of borrowers.

Our model uses a Diamond and Dybvig type of framework, where consumers are uncertain about the timing of their consumption needs. This generates liquidity shocks, which we assume are present both at the individual and at the aggregate level. To be able to cope with these shocks, banks can invest in a storage technology. Because this technology has a lower return than alternative investment opportunities, it is efficient for banks to use the interbank market. We consider both an unsecured interbank market and a repo market where government bills are traded. In order to introduce credit risk, the model assumes that banks have some risk of failure. As in Rochet and Tirole (1996), banks monitor each other in the interbank market, thus obtaining a signal on the solvency probability of each of their peers. The key assumption of our model is that cross-border information about banks is less precise than home country information. Hence, when a bank tries to borrow from a foreign bank, it does so either because it belongs to a liquidity-short country or else because it has generated a "bad" signal at a domestic level and is therefore unable to borrow in his home country. As it is intuitive, depending on the equilibrium probability distribution of the two types of motivations for borrowing abroad, an integrated interbank market may or may not exist.

Our contribution is to show, using this framework, that having a single currency is no guarantee for having a single uninsured interbank market. Namely, a segmented interbank market is *always* an equilibrium, while the emergence of an integrated international market is only possible when the quality of cross-border information is sufficiently good. A further result is that the integration of markets does not always yield a more efficient outcome.

On the other hand, the repo market provides a perfect medium to channel liquidity between banks and across countries. However, the secured nature of the repo reduces banks' incentives for peer monitoring. As a result, banks with a low probability of solvency are able to obtain liquidity via the repo market and to avoid liquidation, although their liquidation value might exceed their expected continuation value. Surprisingly, we also establish that the combination of both types of markets need not yield a more efficient allocation, as it may lead to the collapse of the unsecured integrated market.

This article is organized as follows. In Section 1 we set up the basic model of interbank credit and the structure of signals. Section 2 analyzes the unsecured interbank market in a two country setting, while Section 3 is devoted to the general and more complex case when the two markets — unsecured and repo — are coexistent. Section 4 extends the analysis by allowing for the introduction of correspondent banking, transnational banking, and too-big-to-fail banks. Section 5 offers some concluding remarks on the policy implications of our results. All proofs are in the Appendix.

## 1. The Model

We consider an economy with two countries.

**Consumers.** In each country, there is a continuum of consumers of a total measure of one, who possess one unit of endowment each at time 0. Consumers are risk averse with twice differentiable concave utility functions and face liquidity shocks as in a Diamond and Dybvig (1983) type of model: they need to consume either at time 1 or at time 2. At time 0, consumers deposit their endowments in a bank, and can withdraw funds at the time they need to consume. Deposits are fully insured by a deposit insurance, so no bank runs occur.[1] We assume that the demand-deposit contract promises them a consumption $c_1$ in period 1 or $c_2$ in period 2.

**Banks' investment.** There is an infinite number of risk neutral, profit-maximizing banks.[2] We assume the population of banks to be exogenously given. In the baseline model, banks are ex ante equal, however, in Section 4 we consider different types of cross-border cooperation in the form of correspondent banking and transnational banks (we do not analyze the free entry equilibrium as this would depend trivially on the sunk cost of setting up a bank domestically and abroad).

Banks receive the consumers' endowments at time 0, and invest them either in a risky technology or in reserves (storage technology). Furthermore, a bank can buy government Treasury bills, which are issued at price $B_0$, and yield 1 in the second period with certainty. Denote the investment in the risky technology $I$, the one in Treasury bills $T_0B_0$ and the one in reserves $s_0 \equiv 1 - I - T_0B_0$.

Each unit invested in the risky technology yields an uncertain payoff $\tilde{R}$ at time 2, where $\tilde{R} = R$ (solvency) with probability $p \geq \frac{1}{2}$, and $\tilde{R} = 0$ (insolvency) with probability $1 - p$. Investment in the technology is assumed to be ex ante efficient in that $pR > 1$. This risky asset can also be (partially) liquidated at time 1, with the following technology: liquidation of $\Delta I$ units gives a liquidation value of $l(\Delta I)$, where $l(\Delta I)$ is increasing and concave. For simplicity we use a logarithmic liquidation function $l(\Delta I) = \ln(\Delta I + 1)$.

Because banks have limited liability they have incentives to forbear no matter how bad the prospects of the risky technology are. As a consequence, in our model bank closure will only occur if it is triggered by a liquidity shock.

**Liquidity shocks.** Our model combines bank-specific liquidity shocks with countrywide ones. Banks are uncertain about the liquidity demand they face at time 1. For a fraction $q$ of all banks, a high fraction of consumers $\pi_H$ is impatient and wishes to withdraw at time 1. A fraction $1 - q$, on the other hand, faces a low liquidity demand $\pi_L$, $\pi_L < \pi_H$. The remaining consumers are patient and withdraw at time 2.

The variable $q$ reflects the countrywide aggregate demand for liquidity and is uncertain as well. We assume aggregate liquidity shocks to occur with probability $\frac{1}{2}$ and restrict our comparative statics analysis to changes in the probability of solvency. Thus, with probability $\frac{1}{2}$, $q = q_B$, in which case a country is in a state of high aggregate liquidity demand, because

[1] The deposit insurance company is assumed to raise funds at unit cost. The deposit insurance is fair in that banks have to bear the expected cost of their failures.

[2] Because of perfect competition in the market for deposits, the banks' optimal deposit contract maximizes depositors' welfare, and bank equilibrium profits are driven to zero.

many banks face high time-1 withdrawals ($\pi_H$). On the other hand, with probability $\frac{1}{2}$ the country faces a low liquidity demand with $q = q_A$, where $q_A < q_B$, so that fewer banks face a high amount of withdrawals. For the sake of simplicity we assume $q_A + q_B$ constant, so that there is no liquidity shock for the two countries taken together. The probability of solvency and liquidity are uncorrelated.

Banks can manage their liquidity needs at time 1 by borrowing or lending in the interbank market, by buying or selling Treasury bills on the repo market, by liquidating assets, and by storing reserves until the next period. If illiquid banks fail to meet their creditors' demand, they are forced into bankruptcy, liquidating all assets. In this case, the proceeds are absorbed by the deposit insurance company.

The timing of the model is the following:

| $t = 0$ | $t = 1$ | $t = 2$ |
|---|---|---|
| – consumers deposit<br>– banks invest | – $q$, $\pi$ and signals observed<br>– liquidation takes place<br>– repo and interbank market operations realized<br>– impatient consumers withdraw | – returns $\tilde{R}$ realized<br>– patient consumers withdraw<br>– interbank loans repaid |

**Information.** At time 0, the ex ante probability of being solvent, $p$, is common knowledge. At time 1, all banks in a given country receive a common, nonverifiable signal $s_D$ about the solvency of each domestic counterpart. The signal can either be good ($\bar{s}$) or bad ($\underline{s}$), it is defined as

$$\text{prob}(s_D = \bar{s}|\tilde{R} = R) = \text{prob}(s_D = \underline{s}|\tilde{R} = 0) = \alpha,$$

where we assume that the signal is informative, that is, $\alpha \in (\frac{1}{2}, 1]$. Our setting thus reflects the existence of "soft" domestic information regarding the individual banks' strategies, their risk-taking behavior, and their accounting strategy (loan-loss provisions, window dressing, and so on). This information is not directly observable by foreign banks, even at a cost.

Denote $\theta \equiv p\alpha + (1-p)(1-\alpha)$, the ex ante probability that the good signal is received about a bank.[3] The above expressions allow us to compute the probability $\bar{p}$ of success, conditional on the bank having produced the signal $s_D = \bar{s}$. It is given by $\bar{p} \equiv \text{prob}(\tilde{R} = R|s_D = \bar{s}) = p\alpha/\theta$ and we denote, symmetrically, $\underline{p} \equiv \text{prob}(\tilde{R} = R|s_D = \underline{s}) = \frac{p(1-\alpha)}{(1-\theta)}$.

The signals received in the foreign country can only be observed with some noise. We make the following assumption, which holds regardless of the bank's solvency.

[3] Because we are dealing with a continuum of banks, the ex ante probability is equal to the ex post fraction of banks of this type.

***Assumption 1.*** *(Noisy cross-country information)*

$$\text{prob}(s_F = \bar{s}|s_D = \bar{s}) = \text{prob}(s_F = \underline{s}|s_D = \underline{s}) = 1-\beta,$$

*where* $\beta \in (0, \frac{1}{2})$ *denotes the probability that the domestic signal is received wrongly in the foreign country. Again, the signal is a common one, that is, all foreign banks receive the same signal about a particular bank.*

The lower the value of $\beta$, the better is the information flow between countries. Each bank is then characterized by a pair $(s_D, s_F)$, denoting the signals that have been received by domestic and foreign banks about this particular bank. Assumption 1 implies that $s_D$ is a sufficient statistics for $(s_D, s_F)$.[4]

Since this assumption plays a crucial role in the whole analysis, it is worth fully developing its implications. To begin with, we think of our assumption as applied to *soft*, not to *hard*, information. Obtaining a balance sheet of a foreign bank is obviously as easy as obtaining it for a domestic one. Our assumption is that a balance sheet interpretation draws on the "cultural" context that is not exportable. To take a few examples, the level of derivatives operations and the way in which they are used to hedge or to speculate could be known domestically but not cross border. Rumors about a banks' financial distress could also be local. Similarly, the level of window dressing that the regulators allow for may be country-specific unwritten information. Whether a bank will be bailed out or not in case of failure is also soft information. These examples seem to indicate that a bank operating in another country (i.e., operating under different supervision and different regulatory rules) is culturally different, an explanation for why we observe so often the lack of success of cost-efficient foreign banks trying to enter another country.

Note that a bank cannot observe its own solvency, but only the signal. Therefore, it has no informational advantage over the other market participants regarding its own solvency. This assumption is required in order to leave aside additional moral hazard problems.

The second assumption we make regarding the signal structure is that when a bad signal is observed it is efficient to close down the bank. That is, we assume that the parameters are such that in equilibrium, the following condition holds:

***Assumption 2.*** *(Efficient closure of bad-signal banks)*

$$\text{prob}(\tilde{R} = R|s_F = \underline{s})R \leq l'(I).$$

Because of concavity of $l(\cdot)$, this assumption implies that it is efficient to liquidate the entire risky technology of a bank when the bad signal has

[4] It can easily be checked that for any signal combination, for example, for $(\bar{s}, \bar{s})$, we have

$$p(\tilde{R} = R|(\bar{s}, \bar{s})) = \frac{p\alpha(1-\beta)}{p\alpha(1-\beta) + (1-p)(1-\alpha)(1-\beta)} = p(\tilde{R} = R|s_D = \bar{s}).$$

been received in the foreign country. Assumption 2 obviously implies that it is also efficient to close down this bank with better quality of information, for example, after observing the domestic bad signal ($\underline{p}R \leq l'(I)$).

On the other hand, from the assumption $pR > 1$ it follows that $\bar{p}R > 1$, so that (partial) liquidation of a high-signal bank is never efficient.

## 2. Cross-country Unsecured Interbank Market Integration

This section focuses on the emergence of unsecured markets for liquidity, leaving aside the repo market. For simplicity, we assume that banks hold no Treasury bills. These will be reintroduced in Section 3. As usual, we start by analyzing banks' time-1 problem of liquidity management, which is the focus of our analysis. In Section 2.3, we will turn to their time-0 investment decision.

We regard two countries with different aggregate liquidity demands. For $q = q_A$, there is excess liquidity, while for $q = q_B$, the liquidity shortage is so high that both lenders and borrowers liquidate. It will be verified later on (Lemma 6) that this is consistent with the optimal investment strategy.

Let us denote $A$ as the country with excess liquidity, and $B$ as the one with a liquidity deficit. We assume that there are no legal or infrastructural barriers to the emergence of an international money market.

### 2.1 Equilibrium structure

Our assumptions imply that each bank is characterized by a pair $(s_D, s_F)$ consisting of the signals observed about this particular bank in the home country and abroad: Banks with liquidity needs can find themselves with one of the four pairs of signals: $(s_D, s_F) = (\bar{s}, \bar{s})$, $(\bar{s}, \underline{s})$, $(\underline{s}, \bar{s})$, or $(\underline{s}, \underline{s})$. The individual signal pair will determine the ability of a bank to borrow in the interbank markets. An interbank market is modeled as an anonymous competitive market where all agents are price takers. The anonymity assumption is justified in our context as there is no cost in diversifying the portfolio of interbank loans, so lenders are exposed to the average credit risk of the interbank market. Equilibrium interest rates are those for which the markets clear. Participating agents know the structure of the equilibrium including the proportion of agents affected by each type of domestic and foreign signal, the bank specific liquidity shocks, and the aggregate liquidity shocks. As a consequence, observing the equilibrium interest rates does not allow to infer any additional information, in particular not about specific banks. Domestic lenders know they are endowed with a superior information structure while foreign borrowers know they face a worse population of borrowers and this is reflected in the equilibrium interest rates. It is crucial that foreign lenders do not have access to the information available to domestic lenders, since otherwise, our source of asymmetric information disappears.

Assumption 2 implies that lending to banks about which the low signal $\underline{s}$ has been obtained has a negative net present value. Consequently, only banks with the signal pair $(\bar{s}, \bar{s})$ are able to choose in which country to borrow. The other banks are constrained to using either the domestic market if they are $(\bar{s}, \underline{s})$, or the foreign market if they are $(\underline{s}, \bar{s})$. Banks of type $(\underline{s}, \underline{s})$ are not able to borrow at all. It is important to realize that in equilibrium, banks with $s_F = \underline{s}$ will not ask for a loan abroad since this loan will always be denied by the foreign lender. This is because if all banks from a country — with a good or a bad foreign signal — ask for a foreign loan, it is impossible for the foreign lender to infer any (domestic) information from this strategy. Hence he has to rely on his own signal and does not lend to the bad-signal bank. On the other hand, when only banks with a good foreign signal $s_F = \bar{s}$ ask for a loan, in equilibrium lenders will be able to update their information on foreign borrowers' solvency on the basis of their strategic behavior and on equilibrium interest rates: When facing the demand for a loan from a foreign bank, lenders know that the bank wishes to borrow abroad for one of the two reasons: either it is of the type $(\underline{s}, \bar{s})$ and thus not able to obtain a domestic loan, or it is of the type $(\bar{s}, \bar{s})$ and chooses to borrow abroad because of a better interest rate. While a lender does not know of which type a particular bank is, he can observe the proportion of each type and thus infer the corresponding probability of solvency of the foreign bank, which we will denote as $p_F$. A lender's decision whether to lend at all to foreign banks and if so, at which interest rate, therefore depends on the proportion of each type of foreign borrower.

Because of the limited access of $(\bar{s}, \underline{s})$ banks to the foreign interbank market and of $(\underline{s}, \bar{s})$ to the local interbank market, insurance against liquidity shocks is only partial. This is illustrated in Figure 1.

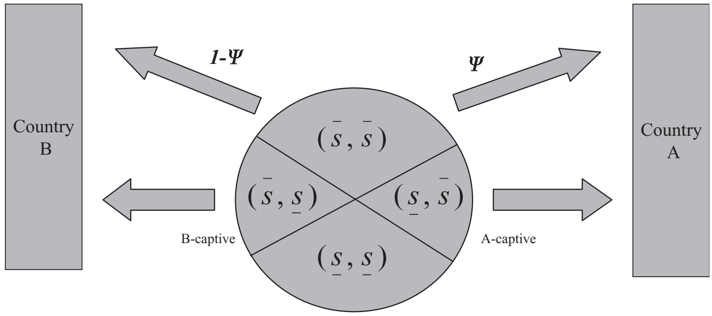


**Figure 1**
**Borrowing choices for country-B banks**
Banks can only obtain a loan in the country where the good signal $\underline{s}$ has been obtained about them. In equilibrium, a fraction $1 - \psi$ of $(\bar{s}, \bar{s})$ banks borrows domestically while $\psi$ borrow abroad.

All in all, there are four credit markets, for each country $i = A$, $B$ a market that clears domestic demand for loans at interest rate $r_i$, and one for the foreign demand for domestic loans at rate $r_{iF}$. Naturally, liquidity supply in the market for domestic and foreign loans is interdependent. For expositional reasons, we add supply and demand for these two markets in the following way: Liquidity demand in country $i$, $\Omega_i$, is the sum of time-1 withdrawals of all agents active in the markets, that is, domestic borrowers and lenders as well as foreign borrowers. Liquidity supply $\Lambda_i$ is the sum of these agents' reserve holdings plus any liquidity they obtain by liquidation. These are given by the following expressions:

$$\Omega_i \equiv \left( \sigma_L^i \pi_L + \sum_{k=D,F} \sigma_k^i \pi_H \right) c_1 \tag{1}$$

$$\Lambda_i \equiv \sum_{k=L,D,F} \sigma_k^i [s_0 + l(\Delta I_k^i)], \tag{2}$$

where the subindex $k$ denotes the different agents active in the local interbank market, $k = L$, $D$, $F$. Here, $L$ denotes lenders, $D$ refers to domestic borrowers, and $F$ to foreign borrowers, and $\sigma_k^i$ is the (equilibrium) measure of each type of bank in country $i$. Here, interbank interest rates are crucial in determining how much of the long run technology the banks will liquidate. Using the banks' first-order conditions, we can establish the following lemma:

***Lemma 1.*** *The supply of liquidity in country i, $\Lambda_i$, is nondecreasing in the domestic interbank interest rate $r_i$.*

*Proof.* See Appendix.

For low levels of interbank interest rates, only reserves are used and the credit market does not develop. For larger levels of interest rates, a market begins to develop where lending banks do not liquidate, but lend out their reserves and have a zero expected yield. When we consider higher interest rates, both borrowing and lending banks liquidate their long-term assets and the expected yield for the lending banks is strictly positive.

Unsurprisingly, the interbank interest rate is an increasing function of the net aggregate demand for liquidity. However, markets do not provide for a perfect smoothing of liquidity shocks: Lending banks liquidate less than borrowing banks. The reason is that the expected cost of borrowing is higher than the expected return from lending. This can be seen by computing the expected return on lending and the expected cost of borrowing conditional on the banks' survival: Lenders obtain an expected return of $\bar{p}(1 + r)$ on loans, while borrowers pay $(1 + r)$ on each unit borrowed. Notice that this discrepancy on the valuation of the returns has its origin on the *lender's* limited liability: Lenders' probability of default plays no role in the calculation of borrowers, while lenders do care about the possibility that borrowers default. The lender undervalues the return on a loan because with some probability it will accrue to the deposit insurance authority.

It is quite intuitive that the equilibrium will be characterized by a unilateral flow of borrowers from the country with high liquidity needs and high interest rates to the country with excess liquidity and low interest rates. The following lemma establishes this point. Denoting by $\psi_i$ the fraction of $(\bar{s}, \bar{s})$ banks in country $i$ that choose to borrow abroad, we prove that $\psi_A = 0$.

***Lemma 2.*** *Borrowers from the excess liquidity country A borrow only in country A* ($\psi_A = 0$). *In addition, in the case of segmented markets* ($\psi_B = \psi_A = 0$), *interest rates in country B are higher than in country A* ($r_A < r_B$).

*Proof.* See Appendix.

This will reduce the number of markets to three, and moreover simplify our notations, since it implies having only one foreign market, the one for borrowers of country $B$ that wish to borrow in country $A$. Thus, denote $\psi \equiv \psi_B$, and $r_F \equiv r_{AF}$ the rate for foreign borrowers in country $A$. A perfect Bayesian equilibrium in the interbank market is a quintuple ($\psi^*$, $r_A^*$, $r_B^*$, $r_F^*$, $p_F^*$), specifying the fraction $\psi^*$ of $(\bar{s}, \bar{s})$ banks from country $B$ borrowing abroad as well as the interest rates demanded and the corresponding rationally updated probability $p_F^*(\psi)$. The variable $\psi$ is crucial for the following analysis, since it describes the behavior of the only types of borrowers that are free to choose in which country to borrow. Accordingly, it characterizes the type of equilibrium that prevails: *Integration*[5] will be defined as the case where in equilibrium $\psi^* > 0$, while *segmentation* is defined as the opposite case where $\psi^* = 0$.

In an integrated equilibrium, with $0 < \psi^* < 1$, two conditions need to be satisfied: on the one hand, lenders in country $A$ should be indifferent between lending to either country:

$$\bar{p}(1 + r_A) = p_F(\psi)(1 + r_F). \tag{3}$$

Also, borrowers able to borrow at home or abroad should be indifferent where to borrow, thus

$$r_B = r_F \tag{4}$$

while in an integrated equilibrium with $\Psi^* = 1$, Equation (4) holds as an inequality: $r_B \geq r_F$.

[5] The case $\Psi^* = 1$ is theoretically possible and corresponds to the (exceptional) case where all banks having a choice as where to borrow prefer to borrow abroad. For the sake of completeness we have chosen to consider this case, and to include it as a form of integration, since indeed, the borrowers are able to use either of the two markets.

Equation (3) allows us to derive $r_F$ as a function of $r_A$. Then, we have two market clearing conditions, Equations (5) and (6), one for each country, to determine the remaining two interest rates $r_A$ and $r_B$.

$$F_A \equiv \Omega_A - \Lambda_A = 0 \tag{5}$$

$$F_B \equiv \Omega_B - \Lambda_B = 0. \tag{6}$$

In the segmented equilibrium, $\psi^* = 0$, $p_F(0) = \mathrm{prob}(\tilde{R} = R|\underline{s}) = \underline{p}$, and $r_F$ is indeterminate. The equilibrium with integration is characterized by the following proposition:

***Proposition 1.*** *Under integration we have* $r_B(\psi^*) \geq r_F(\psi^*) > r_A(\psi^*)$, $p_F(\psi)(1 + r_F^*) = \bar{p}(1 + r_A^*)$. *Under segmentation we have* $r_F(0) > r_B(0)$, $p_F(0) = \mathrm{prob}(\tilde{R} = R|\underline{s}) = \underline{p}$.

*Proof.* See Appendix.

Notice that in our terminology, integration includes the corner solution where $\psi = 1$, and $r_B(1) > r_F(1)$: the interest rate charged abroad is still lower than the domestic one, and all banks who are able to, will borrow abroad.[6] Only "captive" ($\bar{s}$, $\underline{s}$) banks will be forced to borrow domestically at the higher rate $r_B(1)$.

Even in the equilibrium with integration, this integration is only partial as long as there is noise in the flow of information across borders. This implies that first, lenders in country $B$ draw an informational rent from the liquidity shortage. Second, lending banks in country $A$ finance a heterogenous population of foreign banks, consisting of all the high risk banks ($\underline{s}$, $\bar{s}$), and a fraction $\psi$ of the low risk ones ($\bar{s}$, $\bar{s}$).

## 2.2 Integration versus Segmentation

We will now proceed to establish under what conditions segmentation and integration occur, that is, to determine the equilibrium values of $\psi$. We will define all variables as functions of $\psi$. A crucial element in the analysis is the information updating of foreign lenders. Observing the measure of banks trying to borrow abroad, lenders can infer $\psi$ and therefore the fraction of ($\bar{s}, \bar{s}$) and ($\underline{s}$, $\bar{s}$) banks among the foreign borrowers. As is intuitive, a higher fraction of ($\bar{s}, \bar{s}$) banks implies a higher updated probability of solvency $p_F(\psi)$. Moreover, from Equation (3) it follows that the premium on foreign loans, $r_F(\psi) - r_A(\psi)$, is decreasing in $\psi$.

***Lemma 3.*** *A foreign bank's updated probability of solvency $p_F(\psi)$ is increasing in $\psi$. For $\psi > 0$, the premium charged to foreign borrowers $r_F(\psi) - r_A(\psi)$ is decreasing in $\psi$.*

[6] Notice that Equation (4) was stated for the opposite case that $\psi < 1$.

*Proof.* See Appendix.

In order to obtain $\psi^*$, we first derive the demand and supply of liquidity in the domestic and foreign markets for given values of $\psi$. Market clearing then allows us to characterize interest rates in both markets as functions of $\psi$. As a final step, we need to compare interest rates $r_B$ and $r_F$ because these are the ones that $(\bar{s}, \bar{s})$ banks would need to pay in their home respective in the foreign country. A fixed-point argument then determines how many of these banks will choose to borrow in which country, that is, the equilibrium value of $\psi$. As a first result, we have

***Lemma 4.*** *The domestic interest rate in country B, $r_B$, is decreasing in $\psi$.*

*Proof.* See Appendix.

Unsurprisingly, a higher level of cross-country borrowing eases the liquidity shortage in country $B$ and leads to a lower interest rate $r_B$. Regarding interest rates charged in country $A$, the results are not so clear-cut, because two factors influence the interest rate $r_F$: On the one hand, a higher $\psi$ implies a higher demand for liquidity in the local market, hence $r_A$ is nondecreasing in $\psi$.[7] On the other hand, the results from Lemma 3 apply, that is, the quality of foreign borrowers is increasing in $\psi$, which implies a downward tendency for foreign interest rates. As a result, $r_F$ may be in — or decreasing in — $\psi$.

This, together with the results from Lemma 4, has two implications: First, it might be the case that $r_F(\psi) > r_B(\psi)$ for all values of $\psi$. In this case, borrowing abroad is never an equilibrium strategy. Second, if $r_F(\psi) \leq r_B(\psi)$ for some $\psi$, an equilibrium with integration exists. However, nothing precludes the segmentation equilibrium to be obtained as well so that multiple equilibria are possible.

To illustrate these different possible scenarios, Figure 2 shows the interest rates $1 + r_B$ and $1 + r_F$ as functions of $\psi$ for different parameter constellations. In constructing the figure, all equilibrium conditions except Equation (4) are taken into account. The first graph corresponds to a case where interest rates in country $B$ are moderate compared to $r_F$, while in cases (ii) and (iii), $r_B$ is higher relative to $r_A$, and thus to $r_F$.

In all cases, a segmented equilibrium with separated interbank markets (point A) is possible. In case (i), it is the only equilibrium because for all values of $\psi$, the rate charged abroad is strictly higher than the one in country $B$. In this situation, banks of the $(\bar{s}, \bar{s})$-type prefer to borrow domestically in country $B$, so $\psi^* = 0$. Any bank trying to borrow abroad will be denied a loan because it reveals being of the bad type.

In cases (ii) and (iii), the curves $1 + r_B(\psi)$ and $1 + r_F(\psi)$ cross for some $\psi > 0$ so there are values of $\psi$ for which $r_B \geq r_F$. In case (ii), three equilibria

[7] The proof is analogous to the one of Lemma 4.

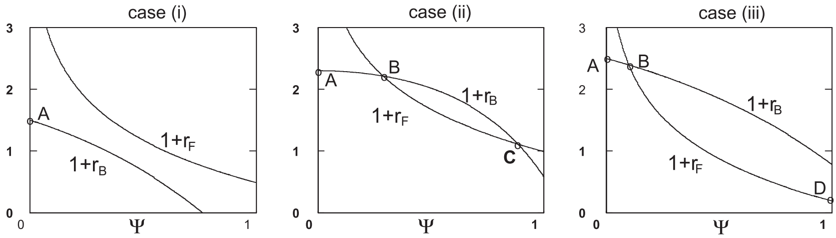


**Figure 2**
**Interest rates for country-*B* borrowers as functions of $\psi$**
Liquidity-short banks in country $B$ can either borrow domestically at rate $r_B$, or in the foreign market at rate $r_F$. Both rates depend on the fraction of $(\bar{s}, \bar{s})$-banks borrowing abroad, $\psi$. The existence of an integrated equilibrium depends on the relative location of both rates.

coexist, two of them with an active international market (at the crossing points B and C): borrowers face the same interest rates in both countries, and they are indifferent as to where to demand a loan. Still, C implies a higher level of integration than B. Finally, in case (iii), there is an equilibrium with integration at point D, where all banks prefer to borrow abroad ($\psi = 1$) and the only banks in country $B$ to borrow domestically are the $(\bar{s}, \underline{s})$ ones.

Figure 2 illustrates under which circumstances an equilibrium with an integrated market can be obtained. Since the foreign interest rate is increasing in the informational premium, integration becomes impossible for very high $\beta$ [case (i)]. Another important parameter is the difference in liquidity needs across countries, $\Delta q \equiv q_B - q_A$: Only when $\Delta q$ is sufficiently large [cases (ii) and (iii)], integration is possible.

The following proposition outlines the results of this market imperfection.

***Proposition 2.*** *(Multiple equilibria). If the difference in aggregate liquidity demand in the two countries is large enough, both the integrated and segmented market equilibrium exist. Otherwise, only the segmented market equilibrium exists.*

*Proof.* See Appendix.

Proposition 2 implies, first that there is a threshold $\overline{\Delta q}$ (which depends on the exogenous parameters $\alpha$, $\beta$, and $\bar{p}$) such that only for $\Delta q \geq \overline{\Delta q}$ an integrated equilibrium exists. Second, and more important for its policy implications, it implies that whenever an integrated market exists the segmented market also exists. So even if integration is possible it is not necessarily reached, and even if integration is reached, the possibility to revert to the segmentation equilibrium is always present. This possible collapse of the integrated equilibrium is somewhat reminiscent of Flannery's (1996) argument for the collapse of a domestic interbank market. In both cases it is the existence of excessively noisy information that drives the collapse of the market.

In this model, segmentation is quite a "robust" equilibrium because there exist self-fulfilling beliefs that support it, independently of the existence of an integrated interbank market equilibrium. This is of concern because usually, but not necessarily, integration will Pareto dominate segmentation, a point that the following proposition establishes.

***Proposition 3.*** *For a given investment portfolio, the integrated equilibrium does not always dominate the segmented equilibrium, but it does so for sufficiently small* $\beta$.

*Proof.* See Appendix.

As is intuitive, for most parameter constellations integration leads to a higher expected welfare than segmentation because of higher cross-country interest rate smoothing. However, if the signals in the foreign country are sufficiently imprecise, foreign lenders make more "mistakes" in granting loans to insolvent banks, which decreases welfare. Still, parameter values for which this effect dominates are hard to find since the integrated equilibrium exists only for small $\beta$.

The analysis shows that a high level of cross-border information (i.e. low $\beta$) is essential for an integrated interbank market to exist. However, even when the difference in information across borders is sufficiently low, there is no guarantee that private market forces reach the most efficient equilibrium.

Furthermore, in all possible equilibria, an inefficiency remains that is due to the informational asymmetry between countries. Because of the concavity of the liquidation technology, the most efficient outcome would involve all banks with the domestic good signal to liquidate the same amount. However, since borrowers from country $B$ pay a premium that reflects the asymmetry in information, they liquidate more than banks in country $A$ in both equilibria.

Proposition 3 was stated for a given investment decision by banks. Obviously, banks may choose a different portfolio depending on the type of equilibrium they expect at time 1. This will have a further impact on welfare. To complete the analysis, we now turn to banks' time-0 investment decision.

### 2.3 The investment decision

At time 0, banks choose how much to invest in reserves or in the risky asset in order to maximize ex ante expected profits $\Pi$. For an interior equilibrium to exist, the following condition is required: $\frac{\partial \Pi}{\partial I} = \frac{\partial \Pi}{\partial s_0}$. While the expected return on the risky asset, $pR$, is exogenously given, the return on reserves depends on the continuation equilibrium at time 1, which reflects the prevailing liquidity conditions.

Without loss of generality, we assume in the following that in the integrated equilibrium, there is still excess liquidity after interbank transactions have taken place, that is, $\bar{p}(1+r_A)=1$ (the other case is derived analogously). Denoting $\mu=\{q_A,\ q_B,\ \pi_L,\ \pi_H,\ \alpha,\ \beta\}$ the set of model parameters, this equilibrium requires the following:

***Lemma 5.*** *(Continuation equilibrium). For every $\mu$ and $s_0$, there exists a time $t=1$ continuation equilibrium characterized by an equilibrium interbank interest rate in the liquidity-short country, $r_B(\mu,s_0)$, which satisfies the equilibrium condition $\Omega_B=\Lambda_B$, that is:*

$$1+r_B(\mu,s_0)=R\exp\left\{\frac{(\sigma_L^B\pi_L+\sigma_D^B(\psi)\pi_H)c_1}{\sigma_L^B+\sigma_D^B(\psi)}-s_0\right\}. \tag{7}$$

*Proof.* See Appendix.

Lemma 5 is derived for an equilibrium with an integrated market, however, by setting $\psi=0$ it applies also to the case of segmented markets.

Now, because of rational expectations on the lottery $r_k(\mu,\ s_0)$ where the randomness comes from $k=A,\ B$ (i.e., the aggregate liquidity shock), we have the following lemma:

***Lemma 6.*** *An equilibrium at time $t=0$ always exists. The interior solution is characterized by an equilibrium interest rate in country B satisfying*

$$E\{\bar{p}(1+r(\mu,s_0))\}=\frac{\sigma_L^A p(R-1)+\sigma_B^A(\bar{p}R-1)+(\sigma_L^B p+\sigma_B^B\bar{p})R}{\sigma_L^B p+\sigma_B^B}. \tag{8}$$

*Proof.* See Appendix.

In order for an integrated equilibrium to exist, notice that it is not possible that $\bar{p}(1+r_A)>R$, because $r_A<r_B$ implies that $\frac{\partial\Pi}{\partial s_0}>\frac{\partial\Pi}{\partial I}$ so that investment in reserves would always dominate. On the other hand, $1+r_B<R$ implies $\frac{\partial\Pi}{\partial s_0}<\frac{\partial\Pi}{\partial I}$ in which case banks would invest all in the risky asset. In other words, for an interior equilibrium, it needs to be the case that some agents liquidate positive amounts at time 1, while others do not. This is always possible, as $q_A<q_B$.

From Lemmas 5 and 6, we can derive the optimal investment in reserves $s_0^*$ that is consistent with rational expectations. It is easy to see that

$$\frac{\partial s_0}{\partial\psi}<0,$$

that is, the investment in reserves is decreasing in the "degree of integration", measured by $\psi$. This result is intuitive, because more integration implies that there is less need for banks to insure themselves against the possibility of being in the liquidity-short country. In particular, this implies that, for the same parameter constellations, if banks expect a segmentation of markets, they will invest more in reserves then they would if the integrated equilibrium was expected. This makes expected welfare in the integrated equilibrium comparatively higher, because investment in reserves is ex ante less efficient than investment in the risky asset, since $pR > 1$. Nevertheless, this result does not allow us to go beyond Proposition 3 and to obtain a general result ranking the two types of equilibria in terms of welfare.

## 3. Coexistence of Unsecured and Repo Markets in an International Setting

Finally, we discuss the general case where both types of markets, repo and unsecured, coexist. In the analysis, the difference between reserves and Treasury bills is crucial. Reserves allow agents to consume at time $t = 1$, while Treasury bills entitle the bearer to liquidity at time $t = 2$. Still, since there is a market for Treasury bills that opens at time $t = 1$, Treasury bills are a close substitute for reserves at the individual level. Nevertheless, at the aggregate level only reserves can be used as consumption at time $t = 1$. For a bank, a unit invested in reserves, that is, in the storage technology, will yield 1; a unit invested in Treasury bills that is sold at time 1 will yield $B_1/B_0$. In equilibrium, banks will have to hold both reserves and Treasury bills and therefore the equilibrium price $B_0$ will have to adjust accordingly.

We assume that the supply of Treasury bills is exogenously given by the government, and that they are issued at a price $T_0$ that induces banks to include them in their equilibrium portfolio. We focus on the more interesting case where the supply of Treasury bills is small so that borrowers cannot rely on the repo market alone to obtain liquidity, and liquidity-short banks without access to unsecured markets have to close down (the opposite case where banks are able to cope with liquidity shocks by selling Treasury bills is trivial). In this way we are able to examine the combined effect of integration through the repo market and market discipline by peer monitoring. The role of Treasury bills in the model will be to allow the transfer of reserves from the liquidity-long country to the liquidity-short one without any risk, at the cost of a lower return.

In this context, as it is intuitive, it is possible to show that the excess liquidity banks of the liquidity-short country will never hold Treasury bills until time 2. This is in line with the fact that these lenders have an informational rent from lending to the interbank market because of more accurate information, so that they can lend the liquidity they obtain through the sale of Treasury bills. Therefore, equilibrium is now obtained with a transfer of liquidity equal to $\Delta s_0$ through the sale of Treasury bills. That is, time-1 equilibrium in the interbank markets requires

$$\Omega_A = \Lambda_A - \Delta s_0 \tag{9}$$

$$\Omega_B = \Lambda_B + \Delta s_0. \tag{10}$$

The following proposition extends Proposition 1 and characterizes equilibrium in the two countries:

***Proposition 4.*** *The continuation equilibrium in the interbank market is characterized by* $r_B^* \geq r_F^* > r_A^*$ and either

1. *Integration if* $0 < \psi^* \leq 1$, $r_B(\psi^*) \geq r_F(\psi^*)$, $p_F(1 + r_F^*) = \bar{p}(1 + r_A^*) = (\frac{1}{B_1}) < \bar{p}(1 + r_B^*)$, *and country-B banks will sell all their Treasury bills to the country-A ones.*
2. *Integration with unsecured markets segmentation if* $r_B^* = r_A^*$ and $\psi^* = 0$ *in which case* $p_F = \text{prob}(\tilde{R} = R|\underline{s})$. *This will occur when the aggregate liquidity shocks are small with regard to the Treasury bill market.*
3. *Segmentation if* $\psi^* = 0$ *and* $r_B^* > r_A^*$, *in which case* $p_F =$ prob $(\tilde{R} = R|\underline{s})$, *and country-B banks will sell all their Treasury bills.*

*Proof.* See Appendix.

Proposition 4 considers a situation where banks with excess liquidity are willing to both buy repos and lend in the unsecured market. Therefore, the price for liquidity on both markets is the same one for lenders in country $A$, that is, $\bar{p}(1 + r_A) = \frac{1}{B_1}$. This implies that for borrowers in both countries, the cost of obtaining liquidity through the repo market, $\frac{1}{B_1}$, is smaller than the cost it faces in the unsecured market, which is $1 + r_s$, $s = A, B$.[8] The borrowing banks will resort to the unsecured market only after having sold all their Treasury bills. Thus, in an international framework, the repo facility will be used to transfer liquidity from the excess liquidity country to the liquidity-short one, while the unsecured market will be used to provide liquidity to domestic and foreign borrowers, with foreign borrowers paying an imperfect information premium $r_F - r_A$.

This immediate effect from the introduction of a repo market is therefore that the liquidity spread between countries is reduced. However, from Proposition 2 we know that a large difference in liquidity needs is essential for the emergence of an integrated equilibrium. The repo market may thus crowd out the integrated unsecured market. This is stated in the following proposition:

***Proposition 5.*** *The introduction of a cross-border repo market implies*

- *for a segmented equilibrium,* $(\psi^* = 0)$, $r_B - r_A$ *is reduced*
- *the integrated equilibrium* $(\psi^* > 0)$ *collapses for sufficiently high* $\Delta s_0$.

*Proof.* See Appendix.

[8] See the discussion on Lemma 1.

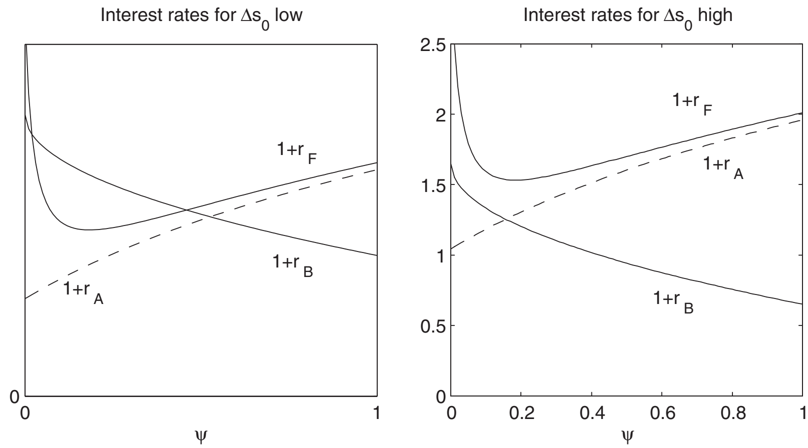


**Figure 3**
**Equilibria with different levels of Treasury bill holdings**

The possible equilibria in the unsecured market are illustrated in Figure 3. The left graph illustrates the case where $\Delta s_0$ is relatively low. Here, equilibria with both separated and integrated unsecured markets exist. In the right graph, on the other hand, $\Delta s_0$ is so high that integration in the unsecured market is not possible any longer.

Finally, let us turn to the welfare effects of the introduction of a repo market. First, as long as the nature of the integrated equilibrium is not altered by the introduction (i.e., the market remains integrated, or remains segmented), a repo market leads to an increase in welfare, because the transfer of liquidity smoothes interest rates across countries.[9]

However, if the integrated unsecured market breaks down, the degree of integration will be reduced again. It would be tempting to jump to the conclusion that the collapse of the integrated equilibrium resulting from an infinitesimal increase in Treasury bills decreases welfare. Still, in the light of Proposition 3, we know that the analysis is more involved. Indeed, the next proposition establishes that the collapse of an integrated equilibrium need not be welfare decreasing.

***Proposition 6.*** *The introduction of a repo market is welfare improving*

- *if the equilibrium type is unchanged (either integrated or segmented), or*
- *if the integrated equilibrium collapses, provided that $\beta$ is sufficiently large.*

[9] If the amount of Treasury bills were sufficient for liquidity-short banks to survive, the repo market would enable some banks to survive (those with a $\underline{s}$-signal), even though from an efficiency point of view, they should close down. This negative effect of a repo market is ruled out, however, under our model assumptions.

*Proof.* See Appendix.

To sum up, for a given investment, the establishment of a repo market is unambiguously welfare improving as long as it does not lead to the collapse of the integrated unsecured market for liquidity. Should this happen, however, the direction of the welfare effects is inconclusive and will depend on the model parameters.

However, we have to again consider the banks' time-0 investment decision to completely assess the welfare effects. The following lemmas are equivalent to Lemmas 5 and 6 from Section 2.3.

***Lemma 7.*** *(Continuation equilibrium). For every $\mu$ and $s_0$, there exists a time $t=1$ continuation equilibrium characterized by an equilibrium interbank interest rate $r_B(\mu, s_0)$ and an equilibrium price for bonds $B_1(\mu, s_0)$ that satisfy the equilibrium condition $\Omega_B = \Lambda_B$, that is:*

$$s_0 = \frac{[\sigma_L^B \pi_L + \sigma_D^B(\psi)\pi_H]}{\sigma_L^B + \sigma_D^B(\psi)} c_1 - \ln\left(\frac{1 + r_B(\mu, s_0)}{R}\right) - T_0 B_1(\mu, s_0) \quad (11)$$

*and*

$$\frac{1}{B_1(\mu, s_0)} = \bar{p}(1 + r_A(\mu, s_0)). \quad (12)$$

***Lemma 8.*** *An equilibrium at time $t=0$ always exists. The equilibrium interest rate in country B satisfies*

$$E\{1 + r(\mu, s_0)\} = \frac{1}{B_0} = \frac{\sigma_L^A p(R-1) + \sigma_B^A(\bar{p}R - 1) + \sigma_L^B p + \sigma_B^B \tilde{p}}{\sigma_L^B p \bar{p} + \sigma_B^B \tilde{p}} \quad (13)$$

*Proof.* See Appendix.

Equations (11) through (13) determine the optimal investment in reserves, $s_0$. Similar to Section 2, we find that $s_0$ is decreasing in $\psi$ so that more reserves are held in a segmented than in an integrated equilibrium. Again, this makes the integrated one relatively more appealing from a welfare point of view.

Notice that banks are indifferent between investing in the risky asset or reserves, so the optimal $T_0$ is determined by the exogenously given supply of Treasury bills. Investment in the risky asset can then be derived as the residual.

## 4. Extensions

Till now we have restricted our focus to a simplified world where banks were either domestic or foreign, correspondent banking services were excluded and the population of banks was homogeneous in regard of their credit risk. In the following three subsections we consider these extensions, starting with correspondent banking, then turning to transnational banks and concluding with the introduction of safe banks.

### 4.1 Correspondent banking

Correspondent banking will develop when some banks are able to borrow from the liquid country and lend to the illiquid one. These are liquidity-long banks from country $B$ with a $\bar{s}$ foreign signal that will borrow at the rate $r_F$, and lend at the domestic rate $r_B$. As correspondent banking develops, some aggregate amount of liquidity $Z$ is channeled into the illiquid country and this will result in new equilibrium interest rates $r_B(Z)$ and $r_F(Z)$.

For correspondent banking to be profitable, the cost of borrowing in country $A$, $1 + r_F(Z)$, cannot exceed the average return from lending to borrowers in country $B$, $\bar{p}(1 + r_{CB}(Z))$, where $r_{CB}(Z)$ represents the interest rate charged by correspondent banks. Therefore, it is required that

$$1 + r_F(Z) \leq \bar{p}(1 + r_B(Z)). \tag{14}$$

Condition (14) implies that $r_F(Z) < r_B(Z)$, requiring that the equilibrium is of the $\psi = 1$ type, which corresponds to the case where all $(\bar{s}, \bar{s})$ banks of country $B$ borrow abroad. In addition, corresponding banking is unable to reduce the spread $r_B - r_F$ to zero as the following proposition establishes

***Proposition 7.*** *Correspondent banking will develop only in the $\psi = 1$ equilibrium provided that the interest rate wedge $r_B(0) - r_F(0)$ is sufficiently large, and in equilibrium the wedge $r_B(Z^*) - r_F(Z^*)$ is strictly positive.*

*Proof.* See Appendix.

This interest rate differential will trigger the arbitrage operated by correspondent banking: Those banks able to borrow abroad would do so, even if they are liquidity long, and then use the liquidity obtained to lend it to those banks in their home country.

At best, the interbank market equilibrium will be such that competition among correspondent banks will lead to the limit point $Z^*$ where there are no more gains from correspondent banking:

$$1 + r_F(Z^*) = \bar{p}(1 + r_{CB}(Z^*)) = \bar{p}(1 + r_B(Z^*)). \tag{15}$$

From this discussion it is clear that correspondent banking has a positive effect on welfare, since it helps channeling liquidity to where it is most needed: It is efficient that banks with the signals $(\bar{s}, \underline{s})$ liquidate as little as possible. This fact together with a concave liquidation technology implies that having correspondent banking is welfare-improving. The difference with respect to the introduction of a repo market is that there is no possible switch to another equilibrium, as correspondent banking requires $\psi = 1$. The inefficiency due to the asymmetry of information across countries is not completely removed since an interest rate differential continues to remain.

## 4.2 Transnational banks

We will define a transnational bank as one which is part of the financial systems in the two countries and thus issues the same domestic signal in both countries. Therefore, there is no cross-country information asymmetry for transnational banks, so that they are able to operate in both markets when the signal they receive is good, and in none when it is bad. This implies that transnational banks borrow from the country with lower interest rates and lend in the country with the higher ones. Contrasting with the corresponding banking case that emerged only for $\psi = 1$, a transnational bank will operate whenever there is an interest rate differential, including the case of $\psi = 0$.

Because in our model there is no limit to the amount a bank can borrow other than its credit appraisal, transnational banks will borrow from country $A$ and lend to country $B$, as correspondent banks do.

The difference with respect to correspondent banking is that in the correspondent banking case, when arbitraging interest rate differentials is not profitable because condition (14) is not met, transnational banks will still choose to borrow from the cheapest source, in country $A$, if they are liquidity-short ($\pi_H$), and to lend at the best rates, in country $B$, if they are liquidity long ($\pi_L$). Consequently, we will have a "variable size market", as transnational banks will choose the market they enter. If we assume there is a measure $\mu$ of transnational banks in each country, the effect we will have is an increase in the supply of loans in the country with a liquidity shortage, and an increase in the demand for loans in the excess liquidity country. Equilibrium will occur with

$$\widehat{\Omega}_B = \Lambda_B \tag{16}$$

$$\widehat{\Omega}_A = \Lambda_A, \tag{17}$$

where the expression for $\widehat{\Omega}_B$ and $\widehat{\Omega}_A$ are given by:

$$\widehat{\Omega}_B = (1-\mu)\Omega_B(\psi) + \mu(\sigma_L^A + \sigma_L^B)\pi_L$$

$$\widehat{\Omega}_A = (1-\mu)\Omega_A(\psi) + \mu(\sigma_D^A + \sigma_F^A + \sigma_D^B)\pi_H.$$

As a consequence, the effect of transnational banks is, even in the absence of cross-country arbitrage, to diminish the liquidity shocks by allowing a fraction of banks to choose the best rates without facing any information asymmetries.

It is also worth noticing that whenever transnational banks are present, correspondent banking is not profitable. This means that obviously, all transnational banks are able to act as correspondent banks. Still, if the amount of transnational banks is sufficiently large, banks with a good foreign signal will be indifferent between borrowing at home or borrowing abroad, as an integrated market with $r_B = r_F$ can be achieved. This is in contrast to the result we obtain with correspondent banking (Proposition 7). If the amount of transnational banks is even larger, then the spread between $r_B = r_F$ and $r_A$ diminishes and the informational rents in the illiquid country begin to be eroded. This is so as a transnational bank is able to obtain liquidity in the country with excess liquidity at interest rate $r_A$ (while a correspondent bank would borrow at $r_F$).

Notice that the presence of transnational banks is similar to an economy with a cross-border repo market: In both cases, the transfer of liquidity is facilitated and this is efficient provided there is no switch to an inferior equilibrium, as established in Proposition 5.

### 4.3 Heterogeneous default risk levels

Assume now that banks are not homogeneous anymore, and that some banks in each country have a lower probability of failure which is common knowledge. For the sake of simplicity we take the extreme case where some banks are perfectly safe while others are risky. We consider this case to be particularly meaningful, as the too-big-to-fail argument implies that some banks have the unlimited support of the regulatory authority, while others face default with a nonzero probability.

The assumption $\bar{p} = 1$ immediately implies that such a bank is able to borrow freely from any market, as a transnational bank. But in addition, Equation (15) implies a complete integration of the markets. The existence of noisy information for foreign banks will imply that lending unsecured to banks abroad is not profitable, as there is no spread to compensate for the risk. In equilibrium, liquidity is transferred from the excess liquidity country to the one experiencing the liquidity shortage by the safe banks at no cost. This equilibrium is efficient, (independently of the existence of an implicit transfer from the government to the too-big-to-fail bank that is out of the focus of our analysis).

This type of equilibrium seems to describe rather well the current situation in the European Monetary Union. While interest rates in the different member states have converged rapidly [see ECB (2000)], the interbank market seems to be characterized by a two-tier structure: In a recent study, Ciampolini and Rhode (2000) report the results of a survey conducted among European Banks according to which only a few large banks are active in the international market while smaller ones are confined to domestic sources for liquidity.

The implications of this finding are far reaching. It implies that a country that lacks the resources (or the credibility) to back up its major banks in case of distress will be at a disadvantage in obtaining liquidity in the international interbank market. For developed countries that are able to bail-out their banks, it may also imply that there is an interest on behalf of the country to build strong "too-big-to-fail" banks in order to compete in the international arena. If this was so, governments might have an interest in promoting national mergers by creating national banks rather than allowing for the creation of transnational banks that they would not be able to support.

## 5. Conclusion

In this article, we analyzed a model of interbank markets in an international context. We focused on the respective roles played by an unsecured money market and a repo market on the domestic and international levels, and developed their welfare properties.

In an economy with unsecured markets, lending takes place on the basis of peer monitoring. This is shown to be efficient, as funds are channeled to the most efficient projects. In repo markets, on the other hand, monitoring plays no role because all loans are collateralized. Therefore, markets are unable to achieve efficient liquidation of unprofitable projects, and insolvent banks forebear. Still, contrary to unsecured markets, a repo market is able to achieve liquidity smoothing across solvent market participants.

In an international context, interbank markets seem to work less efficiently, leading to market imperfections such as liquidity shortages or interest rate differentials. Although these differences could be attributed to exchange rate risk, we argue that the main barrier to an integrated international market is the existence of asymmetric information between different countries.

We have shown that as long as peer monitoring across borders is less efficient than on a domestic level, the integration of unsecured markets can never be perfect. In particular, cross-border lending involves the payment of interest rate premia which reflect the adverse selection of borrowers in the international market. This implies that a perfect liquidity smoothing across borders cannot take place. As a consequence, we show, first, that an equilibrium with an integrated interbank market does not always exist. Second, even if it does exist, at the same time market segmentation is always an equilibrium. Therefore, even with monetary integration or currency pegging, market integration is not necessarily achieved. Interestingly, we also found that in an integrated equilibrium, welfare is not necessarily higher than in the segmented one.

A repo market, on the other hand, is always able to function on an international basis, since it overcomes the problem of asymmetric information. Thus, it is able to achieve liquidity smoothing at least to some degree that is ex ante beneficial to both countries. Still, the welfare effects from a repo facility remain ambiguous: Our analysis shows that a repo market reduces the benefits from peer monitoring and might even impede the integration of markets. Furthermore, even the combination of both types of markets is not necessarily beneficial since integration comes at the cost of a higher degree of inefficient forbearance.

Finally, the effects of correspondent banks, transnational banks with varying degrees of riskiness are analyzed. We show that these institutions can play, in varying degrees, a crucial role in the cross-border liquidity transfer.

## Appendix: Proof of Lemmas and Propositions

*Proof of Lemma 1.* Without loss of generality (w.l.o.g.), assume that $T_0 = 0$, as the repo market is inactive and Treasury bills play no role. For a bank of type $k$, $k = L, D, F$, denote $\Delta I_k$ as the amount liquidated, $L_k^S(L_k^D)$ as the loan given to (demanded by) a bank of type $k$, and $s_k^1$ as the storage of reserves. Country indices are dropped. The time-1 optimization problem for a bank facing withdrawals $\pi_L$ is:

$$\max_{\{\Delta I_L, s_L^1, L_D^S, L_F^S\}} p\{R(I - \Delta I_L) + \bar{p}(1 + r_D)L_D^S + p_F(1 + r_F)L_F^S + s_L^1 - (1 - \pi_L)c_2\}$$
$$\text{s.t. } s_0 + l(\Delta I_L) \geq \pi_L c_1 + s_L^1 + L_D^S + L_F^S,$$

as well as the appropriate nonnegativity constraints. Here, $r_D$ and $r_F$ denote the domestic and foreign interbank market rates, and $p_F$ refers to the expected probability of solvency of foreign banks. Notice that we have assumed that the investment in the risky technology $I$ is so large that the bank fails whenever the risky project is not successful. Similarly, a bank of type $k = D,F$ with withdrawals $\pi_H$ faces the problem

$$\max_{\{\Delta I_k, s_k^1, L_k^D\}} p\{R(I - \Delta I_k) - (1 + r_k)L_k^D + s_k^1 - (1 - \pi_H)c_2\}$$
$$\text{s.t. } s_0 + L_k^D + l(\Delta I_k) \geq \pi_H c_1 + s_k^1.$$

The first-order conditions for the Lagrangian for an $L$-bank, with the multiplier $p\lambda$ for the liquidity constraint are the following:

$$\begin{aligned} \bar{p}(1 + r_D) - \lambda &\leq 0 \\ p_F(1 + r_F) - \lambda &\leq 0 \\ -R + \lambda l'(\Delta I_L) &\leq 0 \\ 1 - \lambda &\leq 0, \end{aligned} \tag{18}$$

while in a state of nature $k = D, F$ (and multiplier $p\mu$) they are characterized by:

$$\begin{aligned} -(1 + r_k) + \mu &\leq 0 \\ -R + \mu l'(\Delta I_k) &\leq 0 \\ 1 - \mu &\leq 0 \end{aligned} \tag{19}$$

In order for $L$-banks to lend to liquidity-short banks from both countries, we need $\bar{p}(1 + r_D) = p_F(1 + r_F)$. If $\bar{p}(1 + r_D) \leq 1$, only reserves are used, $\Delta I_k = 0$ for every $k$, and the interbank markets are inactive. Banks with a liquidity shortage liquidate positive amounts only for $1 + r_k > 1/R$, while excess liquidity banks offer loans for $\bar{p}(1 + r_D) > 1$, and liquidate for $\bar{p}(1+r_D) > 1/R$. The optimal liquidation decisions are

$$l(\Delta I_L) = \max\left\{0, \ln\left(\frac{\bar{p}(1+r_D)}{R}\right)\right\}$$
$$l(\Delta I_D) = \max\left\{0, \ln\left(\frac{1+r_D}{R}\right)\right\}$$
$$l(\Delta I_F) = \max\left\{0, \ln\left(\frac{1+r_F}{R}\right)\right\} = \max\left\{0, \ln\left(\frac{\bar{p}(1+r_D)}{p_F R}\right)\right\}.$$

From Equation (2), it follows that $\Lambda_i$ is nondecreasing in the local interest rate. ■

*Proof of Lemma 2.* We will prove the lemma in two steps. First we prove that only one $\psi_i$ can be nonzero, the one corresponding to the country with high interest rates. Second, we prove that if $\psi_A > 0$, the interest rates differentials between the two countries would imply a contradiction.

1. Assume by way of contradiction that $r_i \geq r_j$ and $\psi_j > 0$. Because country $i$ banks have access to coarser information on foreign borrowers, we have $r_i < r_{iF}$, for $i = A, B$.
   On the other hand, a necessary condition for $\psi_j > 0$ is that $r_{iF} \leq r_j$ (borrowing in country $i$ is attractive for $(\bar{s}, \bar{s})$ borrowers in country $j$) but this would imply $r_j > r_i$, a contradiction, so that $r_i \geq r_j$, implies $\psi_j = 0$.
2. To prove this second point, consider first the interest rates when $\psi_A = \psi_B = 0$. Equilibrium demands $\Omega_i = \Lambda_i(r_j)$ for $i = A, B$. Since the liquidation technology is the same in the two countries, so is the supply function, and as we have $\Omega_A < \Omega_B$ and the supply is increasing in $r_j$, we obtain $r_A < r_B$.

Consider now the case $\psi_A > 0$ and $\psi_B = 0$. In this case, the demand $\Omega'_B$ in country $B$ is larger than in the $\psi_A = \psi_B = 0$ case, while the demand for $\Omega'_A$ is lower than before. This would imply therefore that the market clearing interest rates satisfy again $r_A < r_B$. But then, using the argument in (1), yields a contradiction, since we have proved in (1) that the in order to have $\psi_A > 0$ we need $r_A > r_B$. ■

*Proof of Proposition 1.* A necessary condition for $\psi > 0$ is that borrowing abroad is no more expensive than borrowing domestically, so that $r_B \geq r_F$, with $\psi = 1$ in case of strict inequality. On the other hand, Equation (3) implies $r_F \geq r_A$.

If $\psi = 0$, there is segmentation, since any potential borrower is identified as a $(\underline{s}, \bar{s})$-type and therefore, $p_F = \underline{p}$. But, if $r_F < r_B$, we would have $\psi > 0$, a contradiction. ■

*Proof of Lemma 3.* The updated probability of solvency of a foreign borrower is

$$p_F(\psi) = \frac{\psi \text{prob}(\tilde{R} = R \text{ and } (\bar{s}, \bar{s})) + \text{prob}(\tilde{R} = R \text{ and } (\underline{s}, \bar{s}))}{\psi \text{prob}(\bar{s}, \bar{s}) + \text{prob}(\underline{s}, \bar{s})}$$
$$= \frac{\psi p \alpha(1-\beta) + p(1-\alpha)\beta}{\psi(1-\beta)\theta + (1-\theta)\beta}.$$

Taking into account that $\theta = \alpha p + (1-\alpha)(1-p)$, it is easy to see that $p'_F(\psi) > 0$.

From Equation (3), the foreign premium is given by $r_F - r_A = (1 + r_A)\left(\frac{\bar{p}}{p_F(\psi)} - 1\right)$. Because $p'_F(\psi) > 0$, it is decreasing in $\psi$. ■

*Proof of Lemma 4.* In country $B$, there are $\sigma_A^B = 1 - q_B$ lenders, $\sigma_D^B(\psi) = q_B\theta[1 - \psi(1-\beta)] \equiv q_B\xi^B$ domestic borrowers, and no foreign borrowers, $\sigma_F^B = 0$. Using the first-order conditions (20), we can express liquidity demand and supply, Equations (1) and (2), as functions of $\psi$,

$$\Omega_B(\psi) = (\sigma_L^B \pi_L + \sigma_D^B(\psi)\pi_H)c_1$$
$$\Lambda_B(\psi) = \sigma_L^B\left[s_0 + \ln\left(\frac{\bar{p}(1+r_B)}{R}\right)\right] + \sigma_D^B(\psi)\left[s_0 + \ln\left(\frac{1+r_B}{R}\right)\right].$$

Since $\bar{p}R > 1$, it is never efficient to choose $s_0$ so that $\Omega_B(\psi) > s_0(\sigma_L + \sigma_D^B(\psi))$. This implies that some banks (i.e., domestic borrowers) will always liquidate positive amounts. The interest rate $r_B$ adjusts so that

$$F_B \equiv \Lambda_B(\psi) - \Omega_B(\psi) = 0. \tag{22}$$

Then, $\frac{\partial F_B}{\partial r_B} = \frac{\sigma_L^B + \sigma_D^B}{1+r_B} > 0$ (respectively $\frac{\partial F_B}{\partial r_B} = \frac{\sigma_D^B}{1+r_B} > 0$ if lenders do not liquidate) and $\frac{\partial F_B}{\partial \sigma_D^B(\psi)} = l(\Delta I_D^B) - (\pi_H c_1 - s_0) \equiv X_D^B$ which is negative because $\pi_H$-banks borrow. Then, the Implicit Function Theorem implies $\frac{dr_B}{d\sigma_D^B(\psi)} > 0$, and together with $\frac{d\sigma_D^B(\psi)}{d\psi} < 0$, we obtain

$$\frac{dr_B}{d\psi} = \frac{dr_B}{d\sigma_D^B(\psi)} \frac{d\sigma_D^B(\psi)}{d\psi} < 0. \qquad \blacksquare$$

*Proof of Proposition 2.* The segmented equilibrium exists since for $\psi = 0$, any $(\bar{s}, \bar{s})$ bank would prefer to borrow at rate $r_B$ rather than not obtaining any credit. Therefore, $\psi = 0$ is consistent with their behavior, $F_A = 0$ and $F_B = 0$ determine $r_A$ and $r_B$, respectively, while $r_F$ is indeterminate since there is no cross-border interbank lending.

An integrated equilibrium does not exist if and only if

$$\min_{\psi} \; r_F(\psi) - r_B(\psi) > 0. \tag{23}$$

Let $\psi^*(\Delta q)$ be the solution to Equation (23) with equality. We will show that if $\Delta q$ satisfies

$$\min_{\psi^*(\Delta q)} r_F(\psi^*(\Delta q)) - r_B(\psi^*(\Delta q)) = 0,$$

then for any $\Delta q'$ such that $\Delta q' \le \Delta q$, condition (23) holds, that is, the integrated equilibrium does not exist. We only have to prove that $\frac{d(r_F - r_B)}{d\Delta q} < 0$.

Since $q_B + q_A = \bar{Q}$, we have $dq_A = -dq_B$, so that $d\Delta q = 2dq_B$. Consider first $\frac{dr_B}{d\Delta q}$. In country $B$, $r_B$ solves $\Lambda_B = \Omega_B$ (as in the proof of Lemma 4). From Equation (22), denote $F_B \equiv \Lambda_B - \Omega_B = (1 - q_B)X_L^B + q_B \xi^B X_D^B = 0$ where $X_L^B \ge 0$ and $X_D^B \le 0$. From the proof of Lemma 4, we have $\frac{\partial F_B}{\partial r_B} = \frac{1}{1+r_B}(\sigma_L^B + \sigma_D^B)$. On the other hand,

$$\frac{\partial F_B}{\partial \Delta q} = 2\frac{\partial F_B}{\partial q_B} = 2\{-X_L^B + \xi^B X_D^B\} = -\frac{2}{q_B} X_L^B \le 0$$

where the last equality follows from Equation (22).

Next, consider $\frac{dr_F}{d\Delta q}$. Liquidity demand and supply in country \it$A$ are

$$\Omega_A = (\sigma_L^A \pi_L + \sigma_D^A(\psi)\pi_H + \sigma_F^A(\psi)\pi_H)c_1$$

$$\Lambda_A = \sigma_L^A\left[s_0 + \ln\frac{p_F(1+r_F)}{R}\right] + \sigma_D^A\left[s_0 + \ln\frac{p_F(1+r_F)}{\bar{p}R}\right] + \sigma_F^A\left[s_0 + \ln\frac{1+r_F}{R}\right]$$

with $\sigma_L^A = 1 - q_A$, $\sigma_D^A = q_A\theta$, and $\sigma_F^A = q_B[\psi(1-\beta)\theta + \beta(1-\theta)] \equiv q_B\xi^A$, and where we have used Equation (3). Denote

$$F_A = \Lambda_A - \Omega_A \equiv (1 - q_A)X_L^A + q_A\theta X_D^A + q_B\xi^A X_F^A = 0 \tag{24}$$

with $X_L^A \ge 0$, $X_D^A \le 0$ and $X_F^A \le 0$. Then, $\frac{\partial F_A}{\partial r_F} = \frac{\sigma_L^B + \sigma_D^A + \sigma_F^A}{1+r_F}$ and

$$\frac{\partial F_A}{\partial \Delta q} = 2\frac{\partial F_A}{\partial q_B} = 2\left\{\left[-X_L^A + \theta X_D^A\right]\frac{dq_A}{dq_B} + \xi^A X_F^A\right\} = \frac{2}{q_B}\left\{[X_L^A - \theta X_D^A](q_B + q_A) - X_L^A\right\}$$

where the last equality follows from Equation (24).

Using the Implicit Function Theorem and that $r_F = r_B$ for $\Delta q$ belonging to $F$, we find

$$\begin{aligned}\frac{d(r_F - r_B)}{d\Delta q} &= -\frac{\partial F_A/\partial \Delta q}{\partial F_A/\partial r_F} + \frac{\partial F_B/\partial \Delta q}{\partial F_B/\partial r_B} \\ &= -\frac{2(1+r_F)}{q_B}\left\{\frac{(X_L^A - \theta X_D^A)(q_B + q_A)}{\sigma_L^A + \sigma_D^A + \sigma_F^A} + \frac{X_L^B}{\sigma_L^B + \sigma_D^B} - \frac{X_L^A}{\sigma_L^A + \sigma_D^A + \sigma_F^A}\right\}\end{aligned}$$

The first term is positive. Furthermore, the second term exceeds the third term: first, $X_L^B > X_L^A$ because $r_B > r_A$ implies $l(\Delta I_L^B) > l(\Delta_L^A)$. Second, it is easy to show that $\sigma_L^B + \sigma_D^B < \sigma_L^B + \sigma_D^A + \sigma_F^A$, as it is equivalent to $(q_B - q_A)(1-\theta) + q_A\psi(1-\beta) + q_B\xi^A \geq 0$. Hence, $\frac{d(r_F - r_B)}{d\Delta q} < 0$. ■

*Proof of Proposition 3.* Suppose that the integrated equilibrium exists. Denoting by index $i = A, B$ the country, and $k$ the bank type. Expected welfare is equal to the expected returns on total assets in the economy (notice that the distribution of assets among banks, depositors, and the deposit insurance company is irrelevant because the latter raises funds at unit cost). Expected welfare is

$$W = I(pR - 1) - \frac{1}{2}\sum_{i=A,B}\sum_{k}\sigma_k^i[p_k R\Delta I_k^i - l(\Delta I_k^i)], \tag{25}$$

where $p_k$ denotes the expected probability of solvency for a given type $k$, i.e. $p_k \in \{p, \bar{p}, \underline{p}\}$.

Denote the integrated equilibrium $I$ and the separated one $S$. W.l.o.g. suppose that in country $A$, even in the integrated equilibrium $\bar{p}(1 + r_A) = 1$. This implies that country-$A$ banks liquidate the same amount in both equilibria, so we can neglect them in the welfare comparison. For country $B$, denote interest rates in the segmented equilibrium $r_B(S)$ and those in the integrated one $r_B(I) = r_F(I)$, where $r_B(S) > r_B(I)$. A change from a separated to an integrated equilibrium then affects the following banks in country $B$: $\sigma_L^B \equiv 1 - q_B$ lenders and $\sigma_D^B \equiv q_B\theta$ borrowers with the good domestic signal will face higher interest rates $1 + r_B$. Thus, $\Delta I_k^B(S) > \Delta I_k^B(I)$, $k$ = $L$, $D$. Furthermore, $\sigma_C^B \equiv q_B(1-\theta)\beta$ borrowers with the signal pair $(\underline{s}, \bar{s})$ will go bankrupt in the equilibrium with separation, having to liquidate $\Delta I_C^B(S) \geq \Delta I_D^B(S)$, but obtain a foreign loan with integration and liquidate $\Delta I_D^B(I)$.

For $p_L = p$, $p_D = \bar{p}$, and $p_C = \underline{p}$, we can manipulate the change in welfare from Equation (25) so that

$$\begin{aligned}\Delta W &\equiv W(S) - W(I) \\ &= \sum_{k=L,D,C}\sigma_k^B\{[p_k R\Delta I_k^B(S) - l(\Delta I_k^B(S))] - [p_k R\Delta I_k^B(I) - l(\Delta I_k^B(I))]\} \\ &= \sum_{k=L,D,C}\sigma_k^B\left\{\int_{\Delta I_k^B(I)}^{\Delta I_k^B(S)} l'(\Delta I)d\Delta I - \int_{\Delta I_k^B(I)}^{\Delta I_k^B(S)} p_k R d\Delta I\right\} \\ &= \sum_{k=L,D,C}\sigma_k^B\left\{\int_{\Delta I_k^B(I)}^{\Delta I_k^B(S)}[l'(\Delta I) - p_k R]d\Delta I\right\}.\end{aligned} \tag{26}$$

Our model assumptions imply $\bar{p}R \geq pR \geq l'(\Delta I)$ and hence that the terms corresponding to $k = L$, $D$ are negative, while Assumption 2 implies $\underline{p}R \leq l'(\Delta I)$ so that the term for $k = C$ is positive.

The sign of $\Delta W$ depends therefore on parameters. Suppose first that $\beta \to 0$, so that $\sigma_C^B \to 0$. Then, $\Delta W < 0$, and the break down of the integrated equilibrium decreases welfare.

On the other hand, consider the following set of parameters: $R = 2$, $p = 0.8$, $\alpha = 0.9$, $\beta = 0.45$, $I = 0.5$, $T_0 = 0$, $q_B = 0.55$, $q_A = 0.425$, $\pi_H = 0.8$, and $\pi_L = 0.3$. Calculations show that expected welfare [Equation (25)] is $W^{\text{seg}} = 0.2968$ in the segmented equilibrium. Furthermore, an integrated equilibrium exists for $\psi \approx 0.77$, leading to expected welfare of $W^{\text{int}} = 0.2964 < W^{\text{seg}}$. ■

*Proof of Lemma 5.* From the proof of Lemma 1, we know that in the liquidity-short country, $l(\Delta I_L^B) = \ln(\frac{\bar{p}(1+r_B)}{R})$ and $l(\Delta I_D^B) = \ln(\frac{1+r_B}{R})$. At time 1, liquidity demand and supply in country $B$ are then given as

$$\Omega^B(\mu) = [\sigma_L^B \pi_L + \sigma_D^B(\psi)\pi_H]c_1$$
$$\Lambda^B(\mu, s_0) = \sigma_L^B\left[s_0 + \ln\left(\frac{\bar{p}(1+r)}{R}\right)\right] + \sigma_D^B(\psi)\left[s_0 + \ln\left(\frac{1+r}{R}\right)\right].$$

Market clearing demands $\Omega^B(\mu) = \Lambda^B(\mu, s_0)$, which yields Equation (7). ■

*Proof of Lemma 6.* Existence of the time-0 equilibrium follows from the fact that we are maximizing a continuous function on a compact set. The time-0 maximization problem is

$$\max_{\{I,s_0\}} \Pi = \tfrac{1}{2}(\sigma_L^A\Pi_L^A + \sigma_B^A\Pi_B^A + \sigma_L^B\Pi_L^B + \sigma_B^B\Pi_B^B)$$

such that $I + s_0 = 1$, where $\Pi_k^i$, $i = A, B$, $k = L, B$ denote time-1 expected profits and are defined as

$$\Pi_L^i = \max_{\Delta I_L^i} p\{R(I - \Delta I_L^i) + \bar{p}(1+r_i)L_L^i + s_L^{i1} - (1-\pi_L)c_2\}$$
$$\text{s.t. } L_L^i = s_0 + l(\Delta I_L^i) - \pi_L c_1 - s_L^{i1}$$

for a lender ($L$), or

$$\Pi_B^i = \max_{\Delta I_B^i} \bar{p}\{R(I - \Delta I_B^i) - (1+r_i)L_B^i + s_B^{i1} - (1-\pi_B)c_2\}$$
$$\text{s.t. } L_B^i = \pi_H c_1 + s_B^{i1} - s_0 - l(\Delta I_B^i)$$

for a borrower ($B$), or zero if the bank needs liquidity but has a bad signal. The first-order conditions for the time-0 problem are

$$2\frac{\partial\Pi}{\partial I} = \sigma_L^A pR + \sigma_B^A\bar{p}R + \sigma_L^B pR + \sigma_B^B\bar{p}R \tag{28}$$
$$2\frac{\partial\Pi}{\partial s_0} = \bar{p}(1+r_A)[\sigma_L^A p + \sigma_B^A] + \bar{p}(1+r_B)[\sigma_L^B p + \sigma_B^B].$$

Without loss of generality, we focus on the case $\bar{p}(1+r_A) = 1$, that is, that there is excess liquidity in country $A$ even after the interbank market has taken place. For an interior solution, $\frac{\partial\Pi}{\partial I} = \frac{\partial\Pi}{\partial s_0}$ requires Equation (8). ■

*Proof of Proposition 4.* We know that $p_F(1+r_F^*) = \bar{p}(1+r_A^*) < \bar{p}(1+r_B^*) < 1 + r_B^*$. Now, depending on the Treasury bills yield $\frac{1}{B_1}$, banks will be willing to buy and sell Treasury bills rather than making loans. Notice first that if $\frac{1}{B_1} > 1 + r_B^*$ there is an excess demand in the Treasury bill market as no agent wants to sell Treasury bills, and if $\frac{1}{B_1} < \bar{p}(1+r_A^*)$ there is an excess supply as no one wants to buy them. As a consequence, for possible equilibria we have to focus on the following cases:

1. $p_F(1+r_F^*) = \bar{p}(1+r_A^*) = \frac{1}{B_1} < \bar{p}(1+r_B^*)$
2. $\bar{p}(1+r_A^*) = \frac{1}{B_1} < \bar{p}(1+r_B^*)$
3. $\bar{p}(1+r_A^*) < \frac{1}{B_1} = \bar{p}(1+r_B^*)$
4. $\bar{p}(1+r_A^*) = \frac{1}{B_1} = \bar{p}(1+r_B^*)$

Case 1 corresponds to integration. Country-$B$ banks will all prefer to sell their Treasury bills and use the liquidity thus obtained either to borrow less or to lend more in the interbank market. Case 2 corresponds to segmentation. For a higher Treasury bill yield, we would reach case 3. This, however, cannot be sustained as an equilibrium as we have assumed that the Treasury bill holdings are insufficient to guarantee survival by borrowing on the Treasury bills market only. Consequently, liquidity-short banks from country $A$ will bid higher interest rates rather than being liquidated. If this occurs, the interest rate hike either restores the case

1–2 type of equilibrium or else it involves only the domestic interbank market. In the latter case, we reach case 4 which corresponds to the case where unsecured interbank market segmentation occurs so that there is no strict inequality between $r_A^*$ and $r_B^*$. In case 4, the Treasury bill market leads to interest rate equalization without any cross-country unsecured borrowing. If the yield on interest rates is higher, then only liquidity-short country-$B$ banks will sell Treasury bills. Competition among buyers will then restore the equality of either 1, 2, or 4. ■

*Proof of Proposition 5.* Suppose the cross-country trading of Treasury bills leads to a transfer of liquidity of $\Delta s_0$ as in Equations (10) and (9). It is sufficient to show that $\frac{\partial(r_B - r_A)}{\partial \Delta s_0} < 0$. Consider country $A$. From Equation (9), equilibrium with the market now requires a lower supply of liquidity $\Lambda_A$. Lemma 1 implies that $\frac{\partial r_A}{\partial \Delta s_0} \geq 0$. Consider now country $B$. Notice first that without the Treasury bill market, there is a liquidity shortage in $B$ so that in equilibrium, a positive measure of banks need to liquidate positive amounts. W.l.o.g. assume that $\Delta I_L^B = 0$. From Equations (1), (10), and (20) it follows $1 + r_B = R\exp\{\frac{\Omega_B - \Delta s_0}{\sigma_D^B}\}$ so that $\frac{\partial r_B}{\partial \Delta s_0} < 0$. This proves the first part of the proposition.

To establish the second part, suppose that $T_0$ is so high that an equilibrium of the second type in Proposition 4 is obtained, where $r_A(\psi = 0) = r_B(\psi = 0)$. Since $r_F \geq r_A$, $r_A$ is nondecreasing and $r_B$ nonincreasing in $\psi$ (see Lemma 4), if follows that $r_F(\psi) > r_B(\psi)$ for all $\psi$. ■

*Proof of Proposition 6.* Part 1: Suppose that prior to the establishment of the repo market, the economy is in the segmented equilibrium. Welfare is defined in Equation (25). We need to show that $\frac{\partial W}{\partial \Delta s_0} > 0$. Notice first that for the segmented equilibrium, the equilibrium conditions Equation (9) and (10) can be rewritten

$$l(\Delta I^A) \equiv \sum_{k=L,D} \sigma_k^A l(\Delta I_k^A) = \Omega_A - s_0(\sigma_L^A + \sigma_D^A) + \Delta s_0 \tag{29}$$

$$l(\Delta I^B) \equiv \sum_{k=L,D} \sigma_k^B l(\Delta I_k^B) = \Omega_B - s_0(\sigma_L^B + \sigma_D^B) - \Delta s_0. \tag{30}$$

This implies that

$$\sum_{i=A,B} l(\Delta I^i) = \Omega_A + \Omega_B - \sum_{i=A,B} \sum_{k=L,D} \sigma_k^i s_0$$

is independent of $\Delta s_0$. Therefore, from Equation (25), $\frac{\partial W}{\partial \Delta s_0}$ reduces to

$$\frac{\partial W}{\partial \Delta s_0} = -\frac{1}{2} \sum_{i=A,B} \sum_{k=L,D} \sigma_k^i p_k R \frac{\partial \Delta I_k^i}{\partial \Delta s_0}.$$

Consider first a situation in which there is no repo market, that is, $\Delta s_0 = 0$. Because $\Omega_B > \Omega_A$ by assumption, and $\sigma_L^A + \sigma_D^A - (\sigma_L^B + \sigma_D^B) = (q_B - q_A)(1 - \theta) > 0$, we have $l(\Delta I^B) > l(\Delta I^A)$, that is, in a situation without an active repo market, aggregate liquidation is higher in country $B$ than in country $A$.

Now consider an increase in $\Delta s_0$:

$$\sigma_k^i \frac{\partial \Delta I_k^i}{\partial \Delta s_0} = \sigma_k^i \exp(l(\Delta I_k^i)) \frac{\partial l(\Delta I_k^i)}{\partial \Delta s_0} = (\Delta I_k^i + 1)\sigma_k^i \frac{\partial l(\Delta I_k^i)}{\partial \Delta s_0},$$

which implies, using Equations (29) and (30),

$$\sum_{k=L,D} \sigma_k^A \frac{\partial \Delta I_k^A}{\partial \Delta s_0} = -\sum_{k=L,D} (\Delta I_k^A + 1)$$

$$\sum_{k=L,D} \sigma_k^B \frac{\partial \Delta I_k^B}{\partial \Delta s_0} = \sum_{k=L,D} (\Delta I_k^B + 1).$$

Therefore, and since $r_A < r_B$ implies $\Delta I_k^B > \Delta I_k^A$ for $k = L, D$,

$$\frac{\partial W}{\partial \Delta s_0} = -\frac{1}{2}\sum_{k=L,D} p_k R[\Delta I_k^B - \Delta I_k^A] > 0,$$

as was to be proved. The case of the integrated equilibrium is derived analogously.

Part 2: An integrated equilibrium does not exist if and only if Equation (23) holds. Suppose that the size of the repo market, $\Delta s_0$, is such that the equilibrium value of $\psi$ just satisfies

$$\min_{\psi(\Delta s_0)} r_F(\psi(\Delta s_0)) - r_B(\psi(\Delta s_0)) = 0. \tag{31}$$

We first show that if $\Delta s_0$ satisfies Equation (31), then for any $\Delta s_0'$ such that $\Delta s_0' > \Delta s_0$, condition Equation (23) holds, that is, the integrated equilibrium does not exist. For this, we have to prove that $\frac{d(r_F - r_B)}{d\Delta s_0} > 0$. From Equations (10) and (9), define $\hat{F}_A \equiv F_A - \Delta s_0$ and $\hat{F}_B \equiv F_B - \Delta s_0$ with $F_B$ and $F_A$ as defined in Equations (22) and (24). From the proof of Lemma 4 and Proposition 2, we have $\frac{\partial F_B}{\partial r_B} > 0$ and $\frac{\partial F_A}{\partial r_A} < 0$. The Implicit Function Theorem then indeed yields

$$\frac{dr_B}{d\Delta s_0} = -\frac{\partial \hat{F}_B / \partial \Delta s_0}{\partial \hat{F}_B / \partial r_B} > 0$$

and

$$\frac{dr_F}{d\Delta s_0} = -\frac{\partial \hat{F}_A / \partial \Delta s_0}{\partial \hat{F}_A / \partial r_F} < 0.$$

As a second step, we show under which circumstances welfare is decreased. For $\Delta s_0$ satisfying (31), consider an infinitesimal increase in $\Delta s_0$, $\Delta s_0' = \Delta s_0 + \varepsilon$. The direct effect of this $\varepsilon$-change on welfare will be negligible compared to the effect of the equilibrium switch, since the latter implies a discrete jump in welfare. This change in welfare resulting from the equilibrium switch is given by Equation (26), which holds also for the case that banks hold a small amount of Treasury bills before and after the change (this can be checked easily). It follows that for $\beta$ sufficiently small, the switch to a segmented equilibrium reduces welfare. ■

*Proof of Lemma 7.* At time 1, liquidity demand and supply in country $B$, $\Omega_B$ and $\Lambda_B$, are as in the case without a repo market, and given by Equation (27). However, because banks can obtain liquidity from abroad by selling Treasury bills, equilibrium on the interbank market now requires

$$\Omega_B = \Lambda_B + \Delta s_0, \tag{32}$$

where $\Delta s_0$ denotes the liquidity obtained from selling all Treasury bills, $\Delta s_0 = (\sigma_L^B + \sigma_D^B(\psi))T_0 B_1$. Notice here that because $B_1$ is a function of $1 + r_B$, we cannot find a closed-form solution for the interest rate because the expression is nonlinear in $1 + r_B$. Instead solving Equation (32) for $s_0$, we obtain Equation (11) which defines the equilibrium interest rate implicitly. Finally, Equation (12) results from the discussion on Proposition 4. ■

*Proof of Lemma 8.* The time-0 maximization problem is

$$\max_{\{I, s_0\}} \Pi = \tfrac{1}{2}(\sigma_L^A \Pi_L^A + \sigma_B^A \Pi_B^A + \sigma_L^B \Pi_L^B + \sigma_B^B \Pi_B^B)$$
$$\text{s.t. } 1 = I + s_0 + T_0 B_0$$

where

$$\sigma_L^A = 1 - q_A \qquad \sigma_L^B = 1 - q_B$$
$$\sigma_B^A = q_A \theta \qquad \sigma_B^B = q_B[\theta + (1-\theta)\beta]$$

Time-1 profits for lenders in country $i$, $i = A, B$ are

$$\Pi_L^i = \max_{\Delta I_L^i} p\,\{R(I-\Delta I_L^i) + \bar{p}(1+r_i)L_L^i + s_L^{i1} - (1-\pi_L)c_2 + T_L^{i1}\}$$
$$\text{s.t. } L_L^i = s_0 + l(\Delta I_L^i) - \pi_L c_1 - s_L^{i1} - B_1 \Delta T_L^i$$

where we have defined $\Delta T_L^i \equiv T_L^{i1} - T_0$ — notice that $L_L^A$ denotes the total loans given to domestic and foreign banks: they have the same expected return since $\bar{p}(1+r_A) = p_F(1+r_F)$. For a borrower in country $A$:

$$\Pi_B^A = \max_{\Delta I_B^A} \bar{p}\{R(I-\Delta I_B^A) - (1+r_A)L_B^A + s_B^{A1} - (1-\pi_B)c_2 + T_B^{A1}\}$$
$$\text{s.t. } L_B^A = \pi_H c_1 + s_B^1 - s_0 - l(\Delta I_B^A) - B_1 \Delta T_B^A.$$

For banks in country $B$, these terms are equivalent with one exception: because the population of borrowers in country $B$ is different, its rate of solvency is different. Denote it $\tilde{p} \equiv \left(\frac{\theta\bar{p} + (1-\theta)\beta p)}{(\theta + (1-\theta)\beta}\right)$. The time-0 first-order conditions are

$$2\frac{\partial \Pi}{\partial I} = \sigma_L^A pR + \sigma_B^A \bar{p}R + \sigma_L^B pR + \sigma_B^B \tilde{p}R - \lambda$$
$$2\frac{\partial \Pi}{\partial s_0} = \bar{p}(1+r_A)[\sigma_L^A p + \sigma_B^A] + (1+r_B)[\sigma_L^B p\bar{p} + \sigma_B^B \tilde{p}] - \lambda$$
$$2\frac{\partial \Pi}{\partial T_0} = \bar{p}(1+r_A)B_1[\sigma_L^A p + \sigma_B^A] + (1+r_B)B_1[\sigma_L^B p\bar{p} + \sigma_B^B \tilde{p}] - \lambda B_0.$$

We focus again on the case $\bar{p}(1+r_A) = 1$. Setting $\frac{\partial \Pi}{\partial I} = \frac{\partial \Pi}{\partial s_0} = \frac{\partial \Pi}{\partial T_0}$ yields Equation (13). ■

*Proof of Proposition 7.* Using the fact that $r_F'(Z) \geq 0$ and $r_B'(Z) \leq 0$, condition (14) for $Z = 0$ implies

$$\frac{1-\bar{p}}{\bar{p}}(1 + r_F(0)) \leq r_B(0) - r_F(0).$$

On the other hand, the same inequality will hold in equilibrium replacing 0 by $Z$, so that the wedge $r_B(Z) - r_F(Z)$ has a lower bound. ■

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