Computational Public Economics

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1. Introduction

In announcing this special issue of the *Journal of Public Economic Theory*, we were presented with a difficult challenge. Computational Economics is a large tent. It includes agent-based modelers, numerical analysts, simulators, calibrators, experimentalists, network theorists, creators of artificial agents, and mechanism designers. We knew that it would be impossible to exhibit the advancements in these areas in one issue of a journal. Therefore, our principal goal was to demonstrate to the readers of this journal the extent to which computational work was theoretical, that computational modeling was not just "a bunch of simulations," and to show how computational theory is another way in which we can gain insight into economic phenomena. As one of us wrote, now almost a decade ago, computational methods are primarily a complement, and not a substitute for mathematical theory (Judd 1997).

The opportunity to edit a special issue of a journal devoted to public economics was particularly exciting in light of our individual research interests, but even more so because the strengths of computational methods are well suited to problems in public economics. Advocates of computational approaches praise their ability to deal with robustness, heterogeneity, nonlinearity, geographic space, dynamics, and learning. Most of these are defining characteristics of public economics. First, it is difficult to examine public finance problems in a robust and quantitatively informative manner if we restrict our analyses to closed-form solutions. Public finance analyses often focus on complex trade-offs among contradictory factors. Theoretical analyses generally focus on qualititative models that say little about the quantitative importance of their results or on highly simplified versions that are unlikely to be robust to even small changes in specifications. Only computational analyses examining a wide variety of alternative specifications can produce robust insights into most important public finance questions.

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Second, public economics covers environments in which the standard private good, general equilibrium assumptions do not hold; environments that contain goods which are non-rival or non-excludable or goods whose production obeys increasing returns to scale in production. In these environments, unlike in an economy with private goods, the microlevel incentives of agents do not align with global efficiency. The canonical example of a problem from public economics is the pure public good provision problem in which private incentives lead to underprovision. In light of the probable market failure, public goods must be provided by some alternative institution or mechanism. The environments created by these mechanisms are likely to be more "complex" than standard economic environments. We might not expect a collection of agents to be able to learn an equilibrium as easily as they might in a standard market setting.

Third, since public economics also considers economies with goods that create externalities, we might expect to see systems that have multiple equilibria. This is true in local public-goods models in which agents choose locations, and the agents at each location vote upon the level and types of public goods and services. Agent-based models and computational models are adept at handling geographic space, and networks for that matter. Mathematical models can be used to show that efficient equilibria exist, but they often say little about which equilibria are selected or how the selection depends upon the behavior of the economic actors. Understanding the dynamics and the selection of equilibria (if equilibria are even attained) in such environments is an important problem that computational methods are well suited to address.

Fourth, in addition to formal economic mechanisms to overcome externalities and free rider problems, there are also informal arrangements such as reciprocity norms and cooperative norms (Axelrod 1997). The ability of agent-based models to show the emergence of cooperation was just one reason why computational methods have taken off in recent years. Agent-based models of norms and higher order strategies often exhibit epochs of cooperation and punishment. These periods of cooperative and non-cooperative regimes more accurately reflect what we see in the real world than does the stark efficiency suggested by mathematical models.

Fifth, one of the often-mentioned advantages of agent-based models and computational methods is that they can handle heterogeneous agents with greater facility. In a single period private goods market, heterogeneity does not have a huge effect. If anything, reaching equilibrium may be easier than with homogeneous agents because demand and supply curves will be smoother with heterogeneous individual demand and supply functions. In public-goods economies that is not true. Homogenous demand for a public good makes the model easier to solve. All the agents desire the same level of the public good making the problem easier. When the agents prefer different levels, the problem becomes more difficult because they must all accept a common level. Thus, the effects of heterogeneity are likely to be larger in public-goods environments. When we move from a static equilibrium model to a dynamic model, these difficulties will only increase. To put this in another way, while it is not easy to solve for the dynamic equilibrium of an economy with private goods and a market, it is even harder to do so for an economy with private goods and public goods that includes both a market and a tax mechanism.

Finally, as we mentioned the presence of externalities, local public goods, and free rider problems can create multiple equilibria. But another way to think of this is that these environments create rugged utility landscapes. Computing equilibria, particulary efficient equilibria will be much more challenging in these environments than in a standard economic environment.

The papers that we have chosen for this special issue have much to recommend themselves. There are four general insights that we would like you to keep in mind while reading these papers. The first is that computational and mathematical theory are complementary. The second is that the devil is in the details. These two insights will become obvious as you read through the papers. The third is that there is a growing convergence between the numerical analysts, mathematical theorists, and agent-based modelers. The fourth is that that although a theorem may be true, the transparency of computations often reveals deeper truths that may have been obscured. For this last reason, we call for a new revelation principal that asks that the processes by which equilibria arise should be revealed in addition to their existence.

2. Complements

When we prove a theorem characterizing the set of equilibria in a model or class of models, we like to think that we have explained what will happen in that model—the likely end result. However, there are important differences between (1) proving the existence of an equilibrium, (2) being able to solve for it, and (3) constructing a set of agents who can learn the equilibrium. A complete understanding of a system requires all three steps. Just knowing an equilibrium exists is not enough. We must know where it is and whether agents, be they people, firms, organizations, or governments, can attain it. This is where computational modeling enters. Numerical analysts focus on (2), while agent-based modelers work on (3). It is obvious why finding the equilibria and having agents find the equilibria matters. Otherwise, we might be predicting that a system consisting of a ball dropped onto a spinning disc of infinite radius is at its equilibrium—an unlikely outcome.

We see this interplay between the mathematical and computational theory in the papers by Arifovic and Ledyard, and by Page and Tassier. Both these papers attempt to construct models of learning agents that locate equilibria in the Groves-Ledyard mechanism. Even though these are agent-based modeling papers, they include numerical analyses in the background. The equilibria had to be computed using numerical techniques so that the modelers can say with certainty that what the agents converge to are in fact the equilibria of the model. In the Page and Tassier paper, the utility functions allow for a tremendous number of equilibria, and they in fact find even more than were previously thought to exist. The effort to locate these new equilibria involved substantial numerical analysis even though that part of the investigation is not part of the final paper.

Another kind of complementarity is displayed in the paper by Cardak. He examines how the distribution of abilities interacts with the distribution of income under alternative institutions for financing education. He first proves key existence and characterization results. However, they could not produce the kind of sharp comparisons he wanted. Therefore, he used numerical methods to derive important results about whether public financing of education would reduce income inequality. A significant finding is that his results were different from previous theoretical analyses that made unappealing simplifying assumptions.

3. The Devil Is in the Details

When we say that the devil is in the details, we mean two things: First, there are many details that are swept under the rug when we write a theorem but which cannot be ignored when writing a numerical approximation or an agent-based model. These include technical considerations like higher order derivatives and ratios of agent types, to practical matters like time and place and whether to include them. Second, and this is especially true in the numerical-analysis models, computational modeling is as much as an art as a science. Performing a high-dimensional numerical integration is not just a matter of plugging some parameters into an algorithm and turning a crank, and in agent-based models, the choice over agent behavioral patterns, how fast do they learn?, how much foresight do they have?, etc . . . has an enormous impact on how the model performs.

In the Ledyard and Arifovic paper and the Page and Tassier paper, we see a basic interplay between learning rate and model performance. When students learn game theory, they are taught that the equilibrium is at the intersection of the best response functions. While that is true, it is misleading in the following sense: if agents best respond, they may never locate the equilibrium because they may consistently over- and undershoot it. Obtaining an equilibrium often, or dare we say usually, requires some sort of dampened response by all agents or a partial updating procedure, where only some of the agents respond in any one time step. In sum, there is an art to constructing these models, and much of that art is in the details.

4. Convergence

The field of computational economics is growing in size and importance. A decade ago, if you would have asked someone for a definition of computational economics, they would have said that the field consists of two disparate sets of researchers pursuing distinct methodologies. The first of these groups, the numerical analysts, were people who use nonlinear optimization techniques to locate optimal response functions and equilibria in environments in which analytic solutions are difficult if not impossible to attain. The second group, the agent-based modelers, pursued an agenda that questioned the standard approach to economics (Tesfatsion 1997). These agent-based modelers constructed worlds with heterogenous actors with endogenous interaction structures who relied on adaptive rules. These models sometimes achieved equilibria, sometimes they generated simple or elaborate patterns, and sometimes they appeared to exhibit perpetual novelty. Ironically, the success of each of these disparate agendas has resulted in a partial convergence in methodology and research questions and it has also brought both communities in closer touch with mathematical theorists.

First, and perhaps most importantly, numerical analysts have convinced many mathematical modelers of the benefits of including numerical simulations of their models. These simulations can then be calibrated to data so that the magnitudes of effects can be considered as well as the directional impacts revealed by comparative static analysis. This has led to an increase in the number of computational models of mathematical models. Once the decision has been made to make a computational model, the benefits from tweaking the model to include geography, heterogeneity, or bounded computational ability are hard to resist. And in doing so, the numerical analyst becomes an agent-based modeler. At the same time, agent-based modelers have been striving to understand the phenomena that their models generate. This has led them to look at simplifications of their models with a goal of proving analytical results. This convergence has and will result in a deeper understanding of economic phenomenon.

The paper by Zambrano in this issue is an example of such work. The bar problem, or what is sometimes called the El Farol Problem is considered by many to be one of the foundational models of complex adaptive systems. Agents evolve diverse rules for deciding when to go to the bar. Zambrano constructs a mathematical model to show how and why the El Farol model works. His analysis, while powerful, does not prove the primacy of mathematics, but instead shows the complementary nature of computational and mathematical theory and is a shining example of how a combination of mathematical and computational theories can lead to a richer understanding.

The Zanbrano paper, with the exception of the computations which compare the agent-based model with the mathematical model would be considered by many to be a standard mathematical theory paper. He has written the paper to reveal the computational underpinnings of his results. For quite a while, computations have been a dirty secret among many mathematical theorists. It is time to be open about computations' value as engines of intuitions. Almost all theories spring from examples. We move from the particular to the general. Computers enable us to construct more and better examples than do chalk and slate and paper and pencil. We should not abandon simple examples, but the economy, at least the part we are trying to understand, is complex (Anderson, Arrow, and Pines 1988, Arthur, Durlauf, and Lane 1997); if it were not, we would not be spending so much time contemplating it.

The Cardak, Chatterjee et al., and Pestieau et al. papers are also excellent examples of how the computational approach can significantly increase the range of our understanding. Cardak's model is much more flexible and realistic than earlier analyses and presents a much more robust set of results on education and inequality.

Pestieau et al. examines basic questions in tax policy and tax enforcement. Standard analyses of tax policies assume one kind of information problem. Mirrlees (1971) assumes that the government can perfectly observe income but not ability. The tax-evasion literature assumes that it is difficult to measure income but simplifies other observability issues. Both these simple cases are useful for generating some key insights but the real world contains both kinds of observational problems. Pestieau et al. examines a model that integrates questions of optimal tax rates and tax enforcement policies. This kind of multi-dimensional information is very difficult to analyze theoretically because there is no obvious pattern of binding incentive constraints. One-dimensional models are very special and their insights seldom generalize to multidimensional models. A numerical approach can directly analyze multi-dimensional models. The results in Pestieau et al. show that neither simple one-dimensional model does an acceptable job by itself.

Chatterjee et al. examine tax policies in stochastic-growth models. Much of this literature assumes that utility is intertemporally separable. In particular, these models assume that risk aversion is inversely proportional to the intertemporal elasticity of substitution in consumption. Chatterjee et al. reexamines basic issues in tax incidence in dynamic models and finds that results are significantly sensitive to alternative reasonable specifications.

These latter two papers, as well as the Zambrano paper demonstrate that the distinctions between mathematics, numerical computation, and agent-based models may be stark, but the questions that they can ask overlap and therefore inform and complement one another.

5. A New "Revelation" Principal

The revelation principal (Myerson 1992) had a large impact on the theoretical study of market and non-market institutions. It states that any equilibrium from any mechanism is also the equilibrium of a type revelation mechanism. Therefore, if we only care to know what the efficient equilibria are we can restrict attention to direct revelation games. What we propose, and what should be clear from the papers in this journal, is in effect, a new revelation principal. This principal states that to the extent possible *all* equilibria should be revealed as should the processes by which those equilibria can be attained.

We see the importance of finding the equilibria and attaining them in all these papers, but at the risk of self-promotion, we ask that you look closely at the Page and Tassier paper. In their orginal paper, Groves and Ledyard proved that all equilibria of their model were efficient. That is true. But what is also true is that they assumed an unbounded message space, without it, with only a few dozen agents there are billions of other equilibria, none of which are efficient. Now, we might say, all the better reason to keep the message space unbounded. But, if we do so, then almost any learning dynamic leads the agents off to infinity. To put it bluntly, even though all equilibria are efficient, they would not be found in most cases. The payoff structure creates incentives to run for the border. And when no border exists, the agents keep running. Note that this finding in no way contradicts the mathematical theorem. The equilibria are efficient. Learning just does not find them.

The Zambrano paper reveals a similar gap in the mathematics. One conception of mixed strategies is of people using idiosyncratic decision rules to avoid correlation in action. What Zambrano shows is that these idiosyncratic rules can do better than random mixing. The original mathematical theory is not wrong, it just did not allow for the possibility that arose in the agent-based model and may well arise in the real world.

6. Conclusions

The key fact is that computational methods can effectively examine questions that have been and will probably remain outside the reach of conventional theoretical and paper and pencil techniques. This is particularly true when computation is a partner with sound theory, and will become more important in economics as theoretical and computational techniques are combined with increasingly powerful computers. The papers in this special issue are just a few excellent examples illustrating the potential of the computational approach. We hope that they encourage others to pursue heretofore unapproachable problems using computational techniques and to complement their mathematical analyses with revealing computations.

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