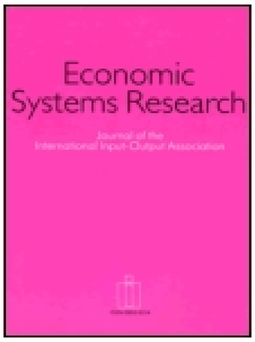


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Miguel Ángel Tarancón & Pablo Del Río

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# Projection of Input–Output Tables by means of Mathematical Programming based on the Hypothesis of Stable Structural Evolution

MIGUEL ÁNGEL TARANCÓN & PABLO DEL RÍO

*Facultad de Ciencias Jurídicas y Sociales de Toledo, Universidad de Castilla-La Mancha, Toledo, Spain*

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ABSTRACT *The high costs involved in the construction of input–output tables (IOTs) using survey methods, makes the development of mathematical projection techniques attractive. An adjustment and projection method of IOTs, based on mathematical programming techniques, is proposed here. The flexibility and ability to include information on elements and aggregates is one of this method's main advantages over alternative adjustment methods. Among the information included in the adjustment, the most relevant is related to the evolution hypothesis of the production structure under stable conditions. This leads to the inclusion of intervals for coefficients. A set of adjusted tables, consistent with their own internal structure, is obtained after an interactive and iterative process that reconciles all information sources. A projection of IOTs in Spain for the period 1995–1998 is undertaken to examine the accuracy of the method.*

KEY WORDS: Input–output projection, mathematical programming, structural evolution

## 1. Introduction

Many authors have focused their research efforts on the projection of input–output tables (IOTs). The high costs (and delays) involved in the construction of tables using survey methods, makes the development of mathematical projection techniques worth considering. The inclusion of information on the elements of IOTs and the relationships therein, is not allowed in traditional adjustment techniques. In contrast, mathematical programming

*Correspondence Address:* Miguel Ángel Tarancón, Econometrics Area, Facultad de Ciencias Jurídicas y Sociales de Toledo, Universidad de Castilla-La Mancha, C/Cobertizo de San Pedro Mártir s/n, Toledo 45071, Spain. Email: miguelangel.tarancon@uclm.es

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techniques allow for the integration of a wide range of information. Therefore, these techniques provide a much more flexible modelling framework.

The information on the stability of the IOT structure is very relevant. Empirical studies have shown the evolution of the technical coefficients to be in line with a stable structure. This paper builds on this idea and introduces intervals for the technical coefficients that are based on structural evolution. These are incorporated in the adjustment model and in the projection of IOTs.

This paper also provides an interactive and iterative solution method of the resultant mathematical model. The researcher thus obtains a final solution that is approved by experts or commissioners at a higher decision level. This is in clear contrast to automatic adjustment methods, which run the risk of obtaining mathematically feasible solutions that are not acceptable for experts (e.g. because they contradict structural patterns).

The next section provides an overview of IOT adjustment techniques and methodologies. Section 3 introduces the hypothesis of a stable structure, which is used as the basis for the inclusion of intervals in the adjustment model. These intervals are then developed in Section 4, while Section 5 provides the resulting linear programming model. This model is solved in an interactive and iterative manner, following the methodological steps that are described in Section 6. Section 7 compares the proposed adjustment method to the RAS method. An application of the method is then given in Section 8, where the projection of the Spanish IOTs for 25 EUROSTAT sectors is obtained for the period 1995–1998. Section 9 presents the main conclusions.

## 2. Input–Output Projection Techniques and Mathematical Programming

The different techniques that have been proposed are traditionally based on the adjustment of a reference table to information for the margins of the updated matrix.[1] The RAS technique is the one most commonly used.[2] It is based on the iterative correction of rows and columns of coefficients or values in the input–output matrix. This process is equivalent to the execution of the following optimization model.

$$\text{Min} \sum_{ij} x_{ij}^{*} \log\left(\frac{x_{ij}^{*}}{x_{ij}^{0}}\right) \tag{1}$$

subject to:

$$\sum_{i} x_{ij}^{*} = q_j \quad \text{and} \quad \sum_{j} x_{ij}^{*} = v_i \tag{2}$$

where $x_{ij}^{*}$ is an element of the adjusted intermediate deliveries matrix $\mathbf{X}^{*}$, $x_{ij}^{0}$ is an element of the reference intermediate deliveries matrix $\mathbf{X}^{0}$, $q_j$ gives the intermediate purchases by sector $j$, and $v_i$ the intermediate sales of sector $i$. The technical coefficients matrix $\mathbf{A}^{*}$ is obtained as

$$a_{ij}^{*} = \frac{x_{ij}^{*}}{w_j} \tag{3}$$

with $w_j$ the output in sector $j$.

The RAS technique has been widely applied empirically, but the following precautions should be kept in mind. The iterative process requires knowledge of the margins of the adjusted matrix. These margins are not always known with certainty, because they require knowledge of total intermediate purchases and sales per sector or, alternatively, outputs, values added, and final demands. Typically, all of this is published with a certain degree of delay. Therefore, estimates of the margins are used.

Second, the RAS process cannot be developed with interval estimates of the margins. Hence, point estimates are used, which may carry an implicit error. Such errors are transmitted during the RAS adjustment and affect all elements of the adjusted matrix, because there are no limitations to the variability of individual elements. Thus, some elements may contain inadmissible errors. Third, if information for the margins is not available, the adjustment process must also include the primary inputs and final demands, as well as the sectoral outputs (Beutel, 1983, p. 314), which exceeds the RAS procedure.

The development of mathematical programming techniques has led to the specification of more flexible adjustment methods based on optimization tools. The work carried out by Dorfman *et al.* (1958) or Matuszewski *et al.* (1964), can be considered as pioneering efforts of this approach (see also Carter, 1970; Bachem and Korte, 1979; Harrigan, 1983). In Spain, the model developed within the framework of the ECONOMETRIST project (Martínez Aguado *et al.*, 1998) generates intervals around the technical coefficients. These intervals are based on sensitivity analyses. This approach is the starting point for the methodology in this paper.

## 3. The Hypothesis of a Stable Structural Evolution

IOTs are both a system of matrices of interrelated values and a representation of a productive system. From the available input combinations, producers choose the one that provides the highest profit margin (Sevaldson, 1974). Many empirical papers have examined the changes in the production structure as represented by the changes in the technical coefficients of the IOT. In general, these studies note the relative stability of the coefficients, which follow a stable evolution within a predefined margin.[3]

Recently, Azid (2002, pp. 2–5) suggested that a change in the table's technical coefficients is a reflection of technological innovation. Such innovation brings about an accumulation of new productive capital. However, it is unlikely that a rapid substitution of this accumulated new capital level will take place. Therefore, it may be expected that the new technology, and thus the new productive capital, will be slowly deployed and will coexist with the technologies currently in place (and with the existing capital). This implies a slow and gradual, but constant, change of the technical coefficients. In the same vein, Antonelli (1999) argues that technical process is accumulative. Therefore, several technologies belonging to the same industry or activity branch will coexist. This makes technical coefficients evolve on the basis of the industry's technological paths. From Dosi's (2000, p. 16) point of view, these paths imply a 'relatively coherent change pattern in technical coefficients'.

A conclusion that may be drawn from these studies, is that the evolution of technical coefficients (which form the structure of the IOT) takes place in a slow and stable manner. This is the so-called *hypothesis of stable structural evolution.* It reflects that, although the coefficients change according to (in most cases) well-defined paths, this change will take place so slowly that the basic structural patterns will not exhibit a drastic rupture. Therefore, structural patterns – characterizing the economy – will be maintained along more or less broad intervals (Szyrmer, 1985). The hypothesis of a stable structural evolution may be useful for projecting IOTs. A mathematical programming adjustment model will be developed that allows for the inclusion of the hypothesis.

## 4. The Operative Approach of the Stable Structural Evolution Hypothesis: Systems of Trend Equations and the Schintke and Stäglin (1988) Algorithm

Several possibilities exist in order to include the stable structural evolution hypothesis. The alternative used here will mainly depend on the statistical information available. From an empirical point of view, several studies have analysed technical coefficients' evolution. Scherer (1982) and Pulido (1986) coincide in pointing out how the diffusion of innovations in various industries, and therefore the evolution of the technical coefficients that represents such a diffusion, follows an S-shaped pattern. More recently, Okuyama *et al.* (2000) proposed an autoregressive behaviour of technical coefficients based on the estimation of a VAR model. Pulido and Fontela (1993) model the behaviour of technical coefficients through mathematical techniques and statistics. Therefore, trend models that reflect technical coefficients' evolution through time may be proposed.

However, series of homogeneous tables that allow estimating the evolution of input–output coefficients and values are not always available. In such cases, structural algorithms based on a sensitivity analysis may be used as an alternative. The Schintke and Stäglin (1988) algorithm, for example, allows the construction of intervals that are coherent with the stable evolution hypothesis.

The choice of the Schintke and Stäglin algorithm can be justified on the following grounds. First, such intervals guarantee that the adjusted IOT's structure covers the basic patterns that characterize the underlying economic system. Second, the hypothesis behind the algorithm is generally accepted and may be formulated as follows. 'The need for an IOT structure coefficient to remain stable beyond a point in which a variation, higher to the preset percentage, takes place in the production of any financial sector for a same level of final demand'.[4]

The algorithm provides variability intervals with a width that is inversely proportional to the relevance of the coefficient within the production structure. The degree of relevance of a coefficient can be obtained by solving the following equation.

$$\omega_{ij}(p) = a_{ij}\left(p\alpha_{ji} + 100 \operatorname*{Max}_{k=1,\ldots,n} \frac{\alpha_{ki}w_j}{w_k}\right) = a_{ij}\left(p\alpha_{ji} + 100\frac{\alpha_{ii}w_j}{w_i}\right) \tag{4}$$

with $\omega_{ij}$ the degree of relevance of the coefficient $a_{ij}$; $p$ the maximum percentage of absolute variation of the production of any sector (i.e. 'acceptable limit' of error); $x_{ij}$ intermediate delivery from sector $i$ to sector $j$; $a_{ij}$ technical coefficient defined as $x_{ij}/w_j$; $\alpha_{ij}$ element of the inverse matrix $(\mathbf{I}\text{-}\mathbf{A})^{-1}$; and $w_j$ output of sector $j$.

The sensitivity of the coefficient (which provides its maximum and minimum margins of variability) is

$$r_{ij} = \frac{p}{\omega_{ij}} \tag{5}$$

with $r_{ij}$ the maximum value (limit) of variation of the technical coefficient $a_{ij}$ (as a percentage) that does not lead to changes greater than $p$ in the production of $i$. This variability percentage will be extremely small in the case of the IOT's most important coefficients. Such small variabilities determine, as was mentioned above, the table's basic structural patterns.

Under stable conditions, the evolution of technical coefficients (no matter whether it is obtained from the estimation of trend equations or from using variability intervals that determine the IOT's basic structural patterns) must be included in the adjustment process using a model with an adequate flexibility. This model, based on mathematical programming, is described in the next section.

## 5. The Mathematical Model

The mathematical adjustment model that constitutes the core of the methodology is the so-called ANAIS model (Vázquez and Tarancón, 1999). It is implemented as a mathematical optimization model with the following specification.

### 5.1. The Objective of the Model

The objective of the model is the individual and global minimization of the relative discrepancies between the elements of the reference IOT and the adjusted IOT.

$$\text{Min} \sum_{ij} \left| \frac{x_{ij}^0 - x_{ij}^*}{x_{ij}^0} \right| + \sum_{ik} \left| \frac{y_{ik}^0 - y_{ik}^*}{y_{ik}^0} \right| + \sum_{dj} \left| \frac{z_{dj}^0 - z_{dj}^*}{z_{dj}^0} \right| \tag{6}$$

with $x_{ij}^*$ ($x_{ij}^0$) indicating the elements of the adjusted (reference) matrix of intermediate deliveries $\mathbf{X}^*$ ($\mathbf{X}^0$); $y_{ik}^*$ ($y_{ik}^0$) the elements of the adjusted (reference) final demand matrix $\mathbf{Y}^*$ ($\mathbf{Y}^0$); and $z_{dj}^*$ ($z_{dj}^0$) the elements of the adjusted (reference) primary inputs matrix $\mathbf{Z}^*$ ($\mathbf{Z}^0$).

The objective function, which is not linear, can be modified. We describe this modification for the elements of $\mathbf{X}$ matrix. Let

$$pox_{ij},\ nex_{ij} \geq 0 \tag{7}$$

$$r_{ij}^x x_{ij}^0 \geq 0 \tag{8}$$

$$x_{ij}^0 = x_{ij}^* + pox_{ij} - nex_{ij} \tag{9}$$

$$nex_{ij} \leq r_{ij}^x x_{ij}^0 \tag{10}$$

$$pox_{ij} \leq r_{ij}^x x_{ij}^0 \tag{11}$$

Thus, we can define the following equation

$$x_{ij}^0(1 - r_{ij}^x) \leq x_{ij}^* \leq x_{ij}^0(1 + r_{ij}^x) \tag{12}$$

The same procedure is developed for the $y_{ik}$ (final demands) and $z_{dj}$ (primary inputs). So, the model is redefined as

$$\text{Min}\left( \sum_{ij} r_{ij}^x x_{ij}^0 + \sum_{ik} r_{ik}^y y_{ik}^0 + \sum_{dj} r_{dj}^z z_{dj}^0 \right) \tag{13}$$

subject to:

$$x^0_{ij}(1 - r^x_{ij}) \leq x^*_{ij} \leq x^0_{ij}(1 + r^x_{ij}) \quad \forall i, j \tag{14}$$

$$y^0_{ik}(1 - r^y_{ik}) \leq y^*_{ik} \leq y^0_{ij}(1 + r^y_{ik}) \quad \forall i, k \tag{15}$$

$$z^0_{dj}(1 - r^z_{dj}) \leq z^*_{dj} \leq z^0_{dj}(1 + r^z_{dj}) \quad \forall d, j \tag{16}$$

The model does not take into consideration the separate relative discrepancies, but that of its weighted sum. Next, we consider the possibility of the relative discrepancies being smaller than a certain level *rmax.* This leads to the following hierarchical multi-objective linear programming problem.

$$\text{Min}\, rmax \tag{17}$$

$$\text{Min}\left(\sum_{ij} r^x_{ij}x^0_{ij} + \sum_{ik} r^y_{ik}y^0_{ik} + \sum_{dj} r^z_{dj}z^0_{dj}\right) \tag{18}$$

subject to equations (7)–(11), (14)–(16), and to

$$r^x_{ij}x^0_{ij} \leq rmax \quad \forall i, j \tag{19}$$

$$r^y_{ik}y^0_{ik} \leq rmax \quad \forall i, k \tag{20}$$

$$r^z_{dj}z^0_{dj} \leq rmax \quad \forall d, j \tag{21}$$

### *5.2. Other Restrictions*

First, *consistency equations* are the set of identities that constitute the accounting structure of the IOT.

$$\sum_{ik} y^*_{ik} = \sum_{dj} z^*_{dj} \tag{22}$$

$$\sum_{j} x^*_{ij} + \sum_{k} y^*_{ik} = w^*_i \tag{23}$$

$$\sum_{i} x^*_{ij} + \sum_{d} z^*_{dj} = w^*_j \tag{24}$$

Second, *coherence equations* are employed to avoid variations in the coefficients that are mathematically possible but difficult to accept from the production perspective.

$$a^-_j \leq \frac{x^*_{ij}}{w^*_j} \leq a^+_j \tag{25}$$

Third, *variability intervals on deliveries in the IOT* are used to ensure that they are coherent with their foreseeable evolution paths.

$$x^-_{ij} \leq x^*_{ij} \leq x^+_{ij} \tag{26}$$

$$y^-_{ik} \leq y^*_{ik} \leq y^+_{ik} \tag{27}$$

$$z^-_{dj} \leq z^*_{dj} \leq z^+_{dj} \tag{28}$$

Fourth, *variability intervals for economic aggregates.* The values of economic aggregates in the IOT may be obtained from provisional figures (provided by the national statistical offices) or from forecasts derived from the macroeconomic models.

$$\left[\sum_k y^*_{ik}\right]^- \leq \sum_k y^*_{ik} \leq \left[\sum_k y^*_{ik}\right]^+ \tag{29}$$

$$\left[\sum_j z^*_{dj}\right]^- \leq \sum_j z^*_{dj} \leq \left[\sum_j z^*_{dj}\right]^+ \tag{30}$$

$$\left[\sum_j w^*_j\right]^- \leq \sum_j w^*_j \leq \left[\sum_j w^*_j\right]^+ \tag{31}$$

Obviously, any further information on the behaviour of elements (either values or coefficients) and on the relationships between them (in the form of restrictions or intervals) can be added to this basic specification.

## 6. Adjustment and Coherence of the IOT to be Projected

The mathematical model provides information and modifies its specification along different methodological stages. In the following, the modelling process will be described by distinguishing each of these steps.

The first step is the *initial specification of the mathematical model*, which involves the basic design of the optimization model adapted for the specific application at hand. All the available information on the accounting and structural restrictions, and on the intervals is included.

The second step covers the *treatment of incompatibilities.* The initial information that enters the initial specification of the model may have different origins or sources. In addition to the uncertainty involved in the information as obtained from forecasts, opinions and advances, this may result in incompatibilities within the information set. This may imply that the model has no feasible solution. This second step takes the form of an interactive and iterative process in which incompatibilities are eliminated by the researcher adapting the model's specification in a controlled manner.

For instance, one of the model's restrictions could be

$$x^*_{ij} \leq x^{\text{sup}}_{ij} \tag{32}$$

This equation is not met if, after trying to solve the model, we find for the resulting adjusted value that $x^*_{ij} > x^{\text{sup}}_{ij}$. Consequently, the model will not have a feasible solution, and performing a sensitivity analysis is not possible. In order to avoid this, a supplementary variable *sv* is added and equation (32) is replaced by

$$x^*_{ij} \leq x^{\text{sup}}_{ij} + sv_{x^*_{ij} \leq x^{\text{sup}}_{ij}} \tag{33}$$

$$sv_{x^*_{ij} \leq x^{\text{sup}}_{ij}} \geq 0 \tag{34}$$

Such supplementary variables are added to all restrictions that have not been met. The initial objective is replaced by the objective of minimizing the sum of the supplementary variables. That is

$$\text{Min} \sum_{sv} \tag{35}$$

subject to the model's adapted restrictions (i.e. including supplementary variables and their positivity conditions).

This adapted specification ensures a feasible solution, as well as the possibility of performing a sensitivity analysis for all specified restrictions. This sensitivity analysis identifies those restrictions that, when relaxed, decrease the objective. One by one the restrictions are successively converted into variables. In this way, their value may be modified when the solution to minimizing the sum of supplementary variables is executed.

As a consequence, the researcher obtains an indication for the possible relaxation in each restriction, as well as the resulting decrease in the objective function. The researcher or expert may now evaluate all possibilities and choose which restriction's relaxation is most adequate.[5] The selected relaxation is applied and then the process is repeated, in an iterative manner, until the objective (35) becomes zero. The initial model will now have a feasible solution if the selected restriction relaxations are applied.

The third step is the *adjustment stage.* After the previous step, the model has a feasible solution. In this step, the multi-objective (17) and (18) is incorporated in order to achieve an optimal solution, which leads to an adjusted and consistent IOT that includes all information and satisfies all criteria. The fourth step covers the *feedback stage.* Some of the initial information may have been modified in the meantime. The new information will then be integrated into the database.

The process as outlined above is applied in the next two sections. First, the goodness of fit of our model is compared to that of RAS. Second, the set of Spanish IOTs for the 1995–1998 period are obtained.

## 7. A Comparison between RAS and ANAIS

This section compares the results from an IOT adjustment provided by the optimization model and the RAS technique, with the purpose of underlining the advantages and disadvantages of ANAIS. To this end, the 1994 Spanish IOT technical coefficients matrix will be estimated, using RAS and ANAIS, and compared with the actual 1994 IOT. The following information has been used. The 1993 IOT was used as the reference table; total intermediate purchases and sales per sector in 1994; actual sectoral outputs for 1994; and the 1986–1993 series of homogeneous tables, which enables the estimation of technical coefficients' trend equations.

The RAS adjustment technique (Stone *et al.*, 1963) is based on the iterative correction of the elements of the adjustment matrix until the row- and column-sums equal the prespecified margins. Once the iterative process ends, the following matrix is obtained

$$\mathbf{A}^* = \hat{\mathbf{r}}\mathbf{A}^{1993}\hat{\mathbf{s}} \tag{36}$$

with

$$\sum_i a_{ij}^* w_j^{1994} = q_j^{1994} \quad \text{and} \quad \sum_j a_{ij}^* w_j^{1994} = v_i^{1994} \tag{37}$$

where $\mathbf{A}^*$ is the adjusted technical coefficients matrix for 1994; $\mathbf{A}^{1993}$ the reference technical coefficients matrix; $w_j^{1994}$ the 1994 output in sector $j$; $q_j^{1994}$ the 1994 intermediate purchases by sector $j$; $v_i^{1994}$ the 1994 intermediate sales by sector $i$; and $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$ the diagonal matrices with correction factors for the rows and columns.

On the other hand, the specification of the optimization model is

$$\text{Min } rmax \tag{38}$$

$$\text{Min } \sum_{ij} r_{ij}^a a_{ij}^{1993} \tag{39}$$

subject to equations (7)–(11), (37), and

$$a_{ij}^- \leq a_{ij}^* \leq a_{ij}^+ \tag{40}$$

$$r_{ij}^a a_{ij}^{1993} \leq rmax \quad \forall i, j \tag{41}$$

$$a_{ij}^{1993}(1 - r_{ij}^a) \leq a_{ij}^* \leq a_{ij}^{1993}(1 + r_{ij}^a) \tag{42}$$

The intervals $[a_{ij}^-; a_{ij}^+]$ for the technical coefficients integrate the stable structural evolution hypothesis in two ways. First, through each coefficients' trend interval estimation,[6] based on the 1986–1993 IOT historic series. The functional form selected for all coefficients has been logarithmic, since it decreases the influence of existing erratic observations. We have used

$$\ln(a_{ij}^*)_t = a + bt + u_t \tag{43}$$

with $t$ the trend variable; $a$ and $b$ the parameters of the equation; and $u_t$ the error.

Second, owing to the limited number of available observations, the intervals are relatively broad. Therefore, for the most important coefficients, according to Schintke and Stäglin's (1988) algorithm, intervals have been substituted that were derived from the algorithm itself, i.e. by applying equation (5). These have a smaller range.

Four alternative scenarios have been performed for both RAS and ANAIS. In case A, the matrix is adjusted to the actual 1994 sectoral totals of intermediate purchases and sales (i.e. $q_j^{1994}$ and $v_i^{1994}$ are known with certainty). In case B, the matrix is adjusted to sectoral totals of intermediate purchases and sales estimations that include a maximum error of 2%.[7] In case C, the maximum error is 5%, and in case D it is 10%. Clearly, cases B, C and D try to simulate a situation in which the true margins for the adjustment matrix are not available, and only estimates can be used.

Table 1 shows – for each of the four cases – the mean absolute errors obtained for the technical coefficients, as well as for the table's most important coefficients, according to

**Table 1.** Mean absolute errors (as %)

| | Case A | Case B | Case C | Case D |
|---|---|---|---|---|
| Mean absolute error | 4.46<br>(1.68) | 5.67<br>(2.59) | 5.89<br>(4.28) | 10.69<br>(8.62) |
| Weighted mean absolute error | 15.55<br>(7.90) | 16.12<br>(13.04) | 20.41<br>(23.97) | 33.55<br>(34.86) |
| Mean absolute error, important coefficients | 2.42<br>(1.39) | 2.38<br>(2.50) | 3.29<br>(4.11) | 5.73<br>(7.50) |
| Weighted mean absolute error, important coefficients. | 87.60<br>(48.88) | 84.98<br>(80.41) | 108.46<br>(145.15) | 171.72<br>(200.55) |

*Note:* The first values are the ANAIS results, the second values (in parentheses) are the RAS results.

equation (5). These errors are weighted using the values from equation (4) so as to evaluate the accuracy in relation to their individual importance. The results indicate that the mean absolute error produced by RAS is lower than the one produced by ANAIS in all cases. However, the difference between the methods decreases when the maximum error included in the margins (of intermediate costs and sales) increases. If the absolute errors are weighted, using equation (4), the mean absolute error is less for ANAIS than for RAS when the maximum error is 2% or larger (cases C and D). This indicates a better adjustment with ANAIS of those coefficients whose individual importance is larger.

If only the table's most important coefficients are taken into account – i.e. applying the Schintke and Stäglin algorithm with a maximum variability percentage of 10% – the mean absolute error is smaller for RAS only if the margins are known with certainty (case A). In the other cases, in which an implicit error exists in the matrix margins, ANAIS performs better. In the case of weighted errors, ANAIS performs better in cases C and D.

One might conclude from the previous exercise that RAS offers better results, whenever the margins of the adjusted matrix are known with certainty (case A). This might suggest that using RAS is preferable to ANAIS. However, several remarks should be made in this respect. First, the quality of ANAIS adjustments may be expected to improve when more and longer series of homogeneous IOTs become available. They allow one to estimate the paths of coefficients' evolution with more precision, leading to more restrictive prediction intervals.

Second, in cases where the margins are not available with certainty, point estimates that carry an implicit error (such as in cases B, C and D) have to be used. In these cases, RAS seems to be more sensitive with respect to errors than ANAIS. This is because in RAS there are no intervals that limit the variability of the adjusted elements, errors cumulate. When the maximum error level for the margins increases, the mean absolute error increases at a greater speed for RAS than for ANAIS. In particular, if we restrict our attention to the most important coefficients, ANAIS outperforms RAS if the margins contain errors.

Third, ANAIS can also reach a solution with interval estimates for the margins, instead of point estimates. In addition, the application in the next section illustrates that ANAIS can be applied to entire IOTs, not just matrices of intermediate deliveries. Fourth, ANAIS is not an automatic adjustment technique, but rather an iterative process guided by the researcher or an expert. Therefore, it is possible to avoid an accumulation of errors in certain coefficients that is unacceptable. Such unacceptable errors are identified in the '*treatment of incompatibilities*' phase.

## 8. A Multi-period Projection of the 1995–1998 Spanish IOTs

As an application of the methodology designed above, the projections of the Spanish IOTs have been derived. The tables are annually adjusted and the following data are available with certainty: a series of homogeneous IOTs (1986–1994) in the EUROSTAT classification into 25 sectors (or activity branches); and a series of main economic aggregates from the Spanish IOTs (1986–1997). They include value added; private and public consumption; capital formation; stock fluctuations; imports; exports; import-related taxes; and value added taxes. A series of Quarterly Accounts (1986–1998) for certain economic aggregates.

The main problem found is the lack of sectoral data for outputs (for which even the total is unavailable) and final demands. Only the sectoral values added are available for 1995. Therefore, the adjustment process must be applied to the entire IOT, given the main macro aggregates and preserving coherence to the structural patterns. With regard to the validation of the resulting tables, no tables for 1995–1998 have been published afterwards that could be used to validate the adjustment quality in terms of errors. Instead, the structural consistency of the adjusted tables will be examined in order to check whether the basic structural patterns of the 1986–1994 tables are maintained in the adjusted tables. The purpose of this exercise is to examine the following.

First, whether the 1986–1994 published tables support the hypothesis of a stable structural evolution (i.e. of basic structural patterns). Second, whether the 1995–1998 projected tables maintain, to a large extent, the structural patterns shown in the 1986–1994 tables. To this end, a series of algorithms – establishing the IOT's basic structural patterns – are applied to the published and the adjusted tables in order to check whether the patterns are similar.

### *8.1. Model Specification and Solution*

The adjustment model is formulated as a multi-criteria optimization model with a two-level hierarchy, see equations (17)–(21), (14)–(16) and (7)–(11). The identities of the IOT's accounting structure are added as identity restrictions, equations (22)–(24). In addition, the values of the economic aggregates provided by the National Accounts, and the sectoral gross values added are included using equations (29)–(31).[8] Intervals are set for the technical coefficients (i.e. the elements of matrix **A**) using the coherence specification in equation (25). The intervals are calculated by applying the algorithm in equation (5) to the previous year's published/adjusted table and allow a 1% variation in sectoral output.

Finally, intervals are added to the evolution paths of the IOT's individual elements on the intermediate transactions matrix **X**, the final demand matrix **Y** and the primary inputs matrix **Z**, see equations (26)–(28). The purpose of using these intervals is to obtain a certain degree of cross-temporal coherency throughout the series of published and adjusted tables, always within the limits set by structural evolution. In this case, the intervals are determined by the estimation of a system of trend equations, named IPE (Individualized Projection of Elements). The equations estimate each element's trend on the basis of the series of homogeneous IOTs. The IPE equations thus determine the range for the IOT's individual elements that matches the structural evolution. The intervals are thus consistent with the elements' historical structural patterns. For each year of adjustment, the system provides the projected value of the IOT's element. Based on this projection, its interval is established by relating its range to the adjustment's goodness of fit of the corresponding IPE equation. Three basic specifications have been used for the IPE equations: linear, parabolic, and autoregressive. Table 2 shows the IPE equations' specifications for each adjustment year.

The intervals of both the elements in the IOTs and of the technical coefficients enter the mathematical adjustment model's initial specification for each year in the projection period. This model will be subject to the *treatment of incompatibilities* process (see earlier). The purpose of this stage is to adapt the adjustment model's initial specification (using a controlled modification process of intervals) in order to obtain an optimization

**Table 2.** IPE equations specification (1995–1998)

| | 1995 | | 1996 | | 1997 | | 1998 | |
|---|---|---|---|---|---|---|---|---|
| | No eqs. | (%) | No eqs. | (%) | No eqs. | (%) | No eqs. | (%) |
| *Linear Projection*: | *517* | *71.51* | *520* | *69.89* | *520* | *69.89* | *517* | *69.40* |
| With dummy variable(s) | 24 | 3.32 | 24 | 3.23 | 24 | 3.23 | 25 | 3.36 |
| With AOD* | 6 | 0.83 | 7 | 0.94 | 4 | 0.54 | 4 | 0.54 |
| Others | 487 | 67.36 | 489 | 65.73 | 492 | 66.13 | 488 | 65.50 |
| *Parabolic Projection*: | *174* | *24.07* | *192* | *25.81* | *190* | *25.54* | *197* | *26.44* |
| With dummy variable(s) | 54 | 7.47 | 54 | 7.26 | 53 | 7.12 | 55 | 7.38 |
| With AOD | 0 | 0.00 | 0 | 0.00 | 0 | 0.00 | 0 | 0.00 |
| Others | 120 | 16.60 | 138 | 18.55 | 137 | 18.41 | 142 | 19.06 |
| *Autoregressive Projection*: | *32* | *4.43* | *32* | *4.30* | *34* | *4.57* | *31* | *4.16* |
| With dummy variable(s) | 23 | 3.18 | 22 | 2.96 | 24 | 3.23 | 22 | 2.95 |
| With AOD | 0 | 0.00 | 0 | 0.00 | 0 | 0.00 | 0 | 0.00 |
| Others | 9 | 1.24 | 10 | 1.34 | 10 | 1.34 | 9 | 1.21 |
| *Projections Total*: | *723* | *100.00* | *744* | *100.00* | *744* | *100.00* | *745* | *100.00* |

*AOD = autoregressive outline in the disturbance.

model with a feasible solution that ensures consistency with respect to economic aggregates, the table's own structural patterns, and the information regarding the evolution paths of the IOT's elements and their relationships.

Table 3 shows the number of modifications made to the IOT elements' intervals in this process. Once the modifications are made, a specification with a feasible solution of the ANAIS adjustment model is obtained. In the following stage, the multi-criteria function, consisting of equations (17) and (18), is optimized to obtain the adjusted table in each year.

**Table 3.** Interval modifications (1995–1998)

| Intervals on: (*) | 1995 | 1996 | 1997 | 1998 |
|---|---|---|---|---|
| X | 47 | 40 | 6 | 9 |
| PC | 20 | 22 | 16 | 2 |
| KF | 21 | 8 | 17 | 16 |
| SV | 3 | 0 | 0 | 1 |
| EX | 18 | 6 | 3 | 2 |
| VA | 0 | 30 | 18 | 12 |
| TR | 1 | 0 | 0 | 0 |
| IM | 54 | 23 | 2 | 2 |
| IT | 4 | 0 | 0 | 1 |
| TX | 6 | 9 | 4 | 1 |
| TOTAL | 174 | 138 | 66 | 46 |

(*) X indicates the intermediate deliveries. The final demand matrix Y is composed of vectors PC (private consumption), KF (gross capital formation), SV (changes in stocks), EX (exports), TR (transfers between sectors), IM (imports), IT (import-related taxes), and TX (indirect taxes). The primary inputs matrix Z consists of the vector VA (gross value added).
*Source:* own elaboration.

## *8.2. Evaluation of the Adjusted Tables' Coherence*

The aim is to project the Spanish IOTs under structural coherence conditions. Therefore, the 1995–1998 tables must maintain the basic structural patterns detected in the 1986–1994 series of published tables. To reach this objective, the basic structural features of all published and projected tables were identified by applying certain techniques. First, select a group of coefficients to study their evolution. In this case, the most important coefficients were identified using the Schintke and Stäglin (1988) algorithm. Note that the number and distribution of important coefficients is in itself an indicator of the tables' basic structural pattern's stability condition. Second, using these coefficients, the main productive routes for each IOT are identified using qualitative input–output analysis. This allows us to determine whether the main connections between branches remain invariable, despite the coefficients' change. This constitutes a basic structural feature of the economic system represented by the IOT series.

### *Identification of important coefficients*

The Schintke and Stäglin (1988) algorithm is used to identify the most important technical coefficients in each table of the analysed series. A coefficient is considered important if its admissible variability margin $r_{ij}$ is below 5%, as calculated in equation (5). The results are given in Table 4.

The results indicate that, whenever a coefficient is identified as important in published tables (nine times), it is also projected as important in the four adjusted tables. There is only one exception: the coefficient $a_{1,15}$ was only identified once in an adjusted table. Certain coefficients were identified as important in some published tables but not in the adjusted tables. These coefficients are found in the rows for the extractive sector (2 and 3), the heavy sector (6), and the food sector (11). It is important to point out that these coefficients were identified as important in the oldest tables but not in the more recent ones. This can be interpreted as a consequence of the stable structural evolution towards the detrimented services sector, a common symptom of economies that are soon to reach a high development level.[9]

In contrast, some coefficients identified as important in all projected tables only appear as such in some of the published tables. This could be due to the same reason above. That is, the published tables where these coefficients were identified as important were the most recent ones. These cases were especially found in the service sectors (17, 21, 23, and 24). This leads to the idea that the projected tables reflect the structural trend of increasing the importance of industries in the tertiary sector.

Summing up, the study of the identification of important coefficients reveals that there is a basic structural pattern that is maintained across published tables (1986–1994). The percentage of important coefficients ranges from 8 to 9%, its tendency is slightly declining, and the evolution tends to concentrate these coefficients in the service sector. This evolution is continued in the projected tables (1995–1998).

### *Identification of productive paths*

The identification of productive paths tries to determine the main connections between sectors. These connections are a central feature of a productive system (Antille *et al.*, 2000, pp. 8–11). The methodology used for finding these connections is qualitative input–output analysis. Schnabl (1994) proposes that the intermediate transactions

**Table 4.** Distribution of important coefficients, published and adjusted IOTs (1986–1998)

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | *9 (4)* | | | | 9 (1) | | | | | | | | | | |
| 2 | | *9 (4)* | *9 (4)* | *9 (4)* | *9 (4)* | | (1) | | | | | | 2 | 8 (4) | | | | | *9 (4)* | *9 (4)* | 3 | | | | |
| 3 | | | *9 (4)* | | | *9 (4)* | 6 | 1 | 5 | | | | | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | | | | | *9 (4)* | | | | | | | | | *9 (4)* | | | | | | | | | | | |
| 6 | | | | | | | 5 | | | | | | | | | | | | | | | | | | |
| 7 | | | | | | | | | | | | | | | | | | | | | | | | | |
| 8 | | | | | | | | *9 (4)* | | | | | | | | | | | | | | | | | |
| 9 | | | | | | | | *9 (4)* | | | | | | | | | | | | | | | | | |
| 10 | | | | | | | | | | *9 (4)* | | | | | | | | | | *9 (4)* | | | | | |
| 11 | *9 (4)* | | | | | | | | | | | | | | | | | 6 | | | | | | | |
| 12 | | | | | | | | | | | | *9 (4)* | | | | | | | | | | | | | |
| 13 | | | | | | | | | | | | | *9 (4)* | | | | | | | | | | | | |
| 14 | | | | | | | | | | | | | | | | | | | | | | | | | |
| 15 | | | | | | | | | | | | | | | *9 (4)* | | | | | | | | | | |
| 16 | | | | | | | | | | | | | | | | | | | | | *9 (4)* | | | | |
| 17 | | | *9 (4)* | 3 (4) | | *9 (4)* | *9 (4)* | *9 (4)* | 4 (4) | | | 2 (4) | *9 (4)* | *9 (4)* | *9 (4)* | | | | *9 (4)* | 5 | *9 (4)* | | | | |
| 18 | | | | | | | | | | | | | | | | | | | | *9 (4)* | | | | | |
| 19 | | | | | | | | | | | | | | | | | | | | | | | | | |
| 20 | | | | | | | | | | | | | | | | | | | | | | | | | |
| 21 | | | | | | | | | | | | | | | | | | | | 5 (4) | | | | | |
| 22 | | | | | | | | | | | | | | | | | | | | | | | | | |
| 23 | | | | | | | | | | | | | | 1 (3) | | | | | | | | | | | |
| 24 | | | 3 (4) | 4 (4) | 5 (4) | 4 (4) | *9 (4)* | *9 (4)* | *9 (4)* | 3 (4) | | 3 (4) | 6 (4) | *9 (4)* | *9 (4)* | | | | | *9 (4)* | *9 (4)* | *9 (4)* | *9 (4)* | | |
| 25 | | | | | | | | | | | | | | | | | | | | | | | | | |

*Note:* The number of times the coefficient has been identified as important in the series of tables published in 1986–1994 is shown in each cell. The number of times the coefficient has been identified as important in the series of 1995–1998 projected tables is indicated in parentheses. Those coefficients identified as important in all tables are given in italics.

matrix **X** be subject to a 'filter' in order to generate a matrix **W** with elements being either 1 or 0, depending on whether they exceed the preset filter or not. Aroche-Reyes (1996) suggests using Schintke and Stäglin's (1988) view on tolerable limits. The advantages of using this filter are that it is adjustable and that it takes into account (from the start) the set of direct and indirect relationships inherent in the technical coefficients matrix **A**, which helps avoid problems related to transitivity (de Mesnard, 1995).

Unitary elements of **W** indicate that the coefficient associated with that element is important. However, the way in which the transaction between sectors $i$ and $j$ (inherent to the important coefficient $a_{ij}$) transmits its 'system production variation power' across intersectoral relationships is yet to be determined. A possible method would be to find the transactions that originate important coefficients in the following stages of the productive process. With this purpose, the binary matrices that show productive relationships initiated by those coefficients that have exceeded the filter are calculated as

$$\mathbf{W}^{(k)} = \mathbf{W} \otimes \mathbf{W}^{(k-1)} \tag{44}$$

with $\mathbf{W}^{(1)} = \mathbf{W}$. The Boolean multiplication is such that $w_{ij}^{(k)} = 1$ if the element $(i, j)$ of the ordinary matrix product $\mathbf{W}\mathbf{W}^{(k-1)}$ is positive. Therefore, the productive relationships (originated by the important coefficients in the successive stages) are summarized in the following Boolean sum matrix (Schnabl, 1994: p. 54).

$$\Psi = \mathbf{W}^{(1)} \oplus \mathbf{W}^{(2)} \oplus \cdots \oplus \mathbf{W}^{(\mathrm{k})} \oplus \cdots \tag{45}$$

where the Boolean sum is such that $\psi_{ij} = 1$ if the element $(i, j)$ of the ordinary matrix sum $(\mathbf{W}^{(1)} + \mathbf{W}^{(2)} + \cdots + \mathbf{W}^{(k)} + \cdots)$ is positive. This element $\psi_{ij}$ indicates an 'important' relationship between the buying sector $j$ and the selling sector $i$.

Based on the matrix in equation (45), the qualitative demand model is developed. This model is useful to determine the main connections between activity branches.

$$\mathbf{w}(\mathbf{bin}) = \Psi \otimes \mathbf{y}(\mathbf{bin}) \tag{46}$$

where **y(bin)** is a binary column vector that indicates the existence of variations in the final demand of sector $i$ (in which case $y(bin)_i = 1$); and **w(bin)** is a binary column vector whose elements represent the sectors (i.e. the impacted branches) with an output that changes significantly due to the aforementioned demand variations. Table 5 shows the result of applying this model to the IOTs that have been published and adjusted, taking as 'important' coefficients those which have been identified in the previous subsection.

Several conclusions may be drawn from Table 5. First, the number of connections between branches (or sectors) causing an impact ('impacting branches') and those affected by an impact ('impacted branches') varies from 56 to 60, both in the published and in the projected tables. Second, a tendency to substitute the connections of impacted branches belonging to the primary and basic industrial sectors (more specifically, branches 1, 2, and 3) for branches belonging to the service sector, especially branch 24, can be noticed.

As was mentioned in the previous subsection, a major conclusion from the observation of the main productive routes (connections) is that the basic productive relationships are maintained and follow the evolution attributed to the economic tertiarization process, both

**Table 5.** Identified productive paths (1986–1998)

| Impact | 1986 | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1, 11 | 1, 11 | 1, 11 | 1, 11 | 1, 11 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 2, 3, 17 | 2, 3, 17 | 2, 3, 17 | 2, 3, 17 | 2, 3, 17 | 2, 3, 17 | 2, 3, 17, 24 |
| 4 | 2 | 2 | 2 | 2 | 2 | 2, 24 | 2, 17, 24 |
| 5 | 2, 5 | 2, 5 | 2, 5 | 2, 5 | 2, 5, 24 | 2, 5, 24 | 2, 5, 24 |
| 6 | 2, 3, 17 | 2, 3, 17 | 2, 3, 17 | 2, 3, 17 | 2, 3, 17 | 2, 3, 17, 24 | 2, 3, 17, 24 |
| 7 | 2, 3, 6, 17, 24 | 2, 3, 6, 17, 24 | 2, 3, 6, 17, 24 | 2, 3, 6, 17, 24 | 2, 3, 6, 17, 24 | 2, 3, 17, 24 | 17, 24 |
| 8 | 2, 3, 8, 9, 17, 24 | 2, 3, 8, 9, 17, 24 | 2, 3, 8, 9, 17, 24 | 2, 3, 8, 9, 17, 24 | 2, 3, 8, 9, 17, 24 | 8, 9, 17, 24 | 8, 9, 17, 24 |
| 9 | 2, 3, 17, 24 | 2, 3, 17, 24 | 2, 3, 17, 24 | 2, 3, 17, 24 | 2, 3, 17, 24 | 17, 24 | 17, 24 |
| 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10, 24 |
| 11 | 1, 11 | 1, 11 | 1, 11 | 1, 11 | 1, 11 | 1, 11 | 1, 11 |
| 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12, 24 |
| 13 | 2, 13, 17 | 2, 13, 17 | 13, 17 | 13, 17 | 13, 17 | 13, 17, 24 | 13, 17, 24 |
| 14 | 2, 5, 17, 24 | 2, 5, 17, 24 | 2, 5, 17, 24 | 2, 5, 17, 24 | 2, 5, 17, 24 | 2, 5, 17, 24 | 2, 5, 17, 24 |
| 15 | 1, 11, 15, 24 | 1, 11, 15, 17, 24 | 1, 11, 15, 17, 24 | 1, 11, 15, 17, 24 | 1, 11, 15, 17, 24 | 1, 11, 15, 17, 24 | 1, 11, 15, 17, 24 |
| 16 | — | — | — | — | — | — | — |
| 17 | — | — | — | — | — | — | — |
| 18 | 1 | 1, 11 | 1, 11 | 1, 11 | 1, 11 | 1, 11 | — |
| 19 | 2, 17 | 2, 17 | 2, 17 | 2, 17 | 2, 17 | 2, 17 | 2, 17 |
| 20 | 1, 2, 10, 11, 18, 24 | 1, 2, 10, 11, 18, 24 | 1, 2, 10, 11, 18, 24 | 1, 2, 10, 11, 18, 24 | 1, 2, 10, 11, 16, 17, 18, 24 | 1, 2, 10, 11, 16, 17, 18, 21, 24 | 2, 10, 16, 17, 18, 21, 24 |
| 21 | 2, 16, 17, 24 | 2, 16, 17, 24 | 16, 17, 24 | 2, 16, 17, 24 | 16, 17, 24 | 16, 17, 24 | 16, 17, 24 |
| 22 | 24 | 24 | 24 | 24 | 24 | 24 | 24 |
| 23 | 24 | 24 | 24 | 24 | 24 | 24 | 24 |
| 24 | — | — | — | — | — | — | — |
| 25 | — | — | — | — | — | — | — |
| Number of routes | 56 | 58 | 57 | 58 | 60 | 59 | 57 |

(*continued*)

| Impact | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 |
|---|---|---|---|---|---|---|
| 1 | 1, 11 | 1, 11 | 1, 11 | 1, 11 | 1, 11 | 1, 11 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 2, 3, 17, 24 | 2, 3, 17, 24 | 2, 3, 17, 24 | 2, 3, 17, 24 | 2, 3, 17, 24 | 2, 3, 17, 24 |
| 4 | 2, 17, 24 | 2, 17, 24 | 2, 17, 24 | 2, 17, 24 | 2, 17, 24 | 2, 17, 24 |
| 5 | 2, 5, 24 | 2, 5, 24 | 2, 5, 24 | 2, 24 | 2, 24 | 2, 24 |
| 6 | 2, 3, 17, 24 | 2, 3, 17, 24 | 2, 3, 17, 24 | 2, 3, 17, 24 | 2, 3, 17, 24 | 2, 3, 17, 24 |
| 7 | 17, 24 | 17, 24 | 17, 24 | 17, 24 | 17, 24 | 2, 17, 24 |
| 8 | 8, 9, 17, 24 | 8, 9, 17, 24 | 8, 9, 17, 24 | 8, 9, 17, 24 | 8, 9, 17, 24 | 8, 9, 17, 24 |
| 9 | 17, 24 | 17, 24 | 17, 24 | 17, 24 | 17, 24 | 17, 24 |
| 10 | 10, 24 | 10, 24 | 10, 24 | 10, 24 | 10, 24 | 10, 24 |
| 11 | 1, 11 | 1, 11 | 1, 11 | 1, 11 | 1, 11 | 1, 11 |
| 12 | 12, 17, 24 | 12, 17, 24 | 12, 17, 24 | 12, 17, 24 | 12, 17, 24 | 12, 17, 24 |
| 13 | 13, 17, 24 | 13, 17, 24 | 13, 17, 24 | 13, 17, 24 | 13, 17, 24 | 13, 17, 24 |
| 14 | 2, 5, 17, 23, 24 | 2, 5, 17, 24 | 2, 5, 17, 24 | 2, 5, 17, 23, 24 | 2, 5, 17, 23, 24 | 2, 5, 17, 23, 24 |
| 15 | 1, 11, 15, 17, 24 | 1, 11, 15, 17, 24 | 1, 11, 15, 17, 24 | 15, 17, 24 | 15, 17, 24 | 15, 17, 24 |
| 16 | — | — | — | — | — | — |
| 17 | — | — | — | — | — | — |
| 18 | — | — | — | — | — | — |
| 19 | 2, 17 | 2, 17 | 2, 17 | 2, 17 | 2, 17 | 2, 17 |
| 20 | 2, 10, 16, 17, 18, 21, 24 | 2, 10, 16, 17, 18, 21, 24 | 2, 10, 16, 17, 18, 21, 24 | 2, 10, 16, 17, 18, 21, 24 | 2, 10, 16, 17, 18, 21, 24 | 2, 10, 16, 17, 18, 21, 24 |
| 21 | 16, 17, 24 | 16, 17, 24 | 16, 17, 24 | 16, 17, 24 | 16, 17, 24 | 16, 17, 24 |
| 22 | 24 | 24 | 24 | 24 | 24 | 24 |
| 23 | 24 | 24 | 24 | 24 | 24 | 24 |
| 24 | — | — | — | — | — | — |
| 25 | — | — | — | — | — | — |
| Number of routes | 59 | 58 | 58 | 56 | 56 | 57 |

in the historical tables (1986–1994) and in the projected tables (1995–1998). This means that the projection has provided structurally consistent tables that show a coefficient change, which is coherent with the hypothesis of stable evolution defended in this paper.

## 9. Concluding Remarks

An IOT adjustment method based on the specification and solution of a mathematical programming model has been proposed in this paper. Compared with other available methods, the IOT adjustment method has several advantages. On the one hand, it is highly flexible. That is, it is able to incorporate information that differs from the usual information for the margins and it is also able to include information in the form of variability intervals. On the other hand, the method allows the possibility of reaching a solution that can be controlled by the researcher. The ultimate goal is to avoid obtaining projections that, although mathematically feasible, are structurally difficult to accept.

In this context, the stable structural evolution hypothesis is worth integrating within the process. This hypothesis is based on empirical evidence, which suggests a relatively stable evolution of the IOT. That is, the productive structure of the table evolves slowly and in a stable manner while it maintains its structural patterns (featuring the represented productive system) within wide time intervals. Setting variability intervals on the technical coefficients of the IOT has been used as the operative approach to integrate this hypothesis in the model. These intervals have been calculated by estimating trend equations or by using the Schintke and Stäglin algorithm.

At the same time, the methodology proposed in this paper includes an interactive and iterative procedure, based on a sensitivity analysis. This modifies the initial specification of the mathematical model in order to achieve the necessary coherence between all the sources of information included. This process is ‘supervised’ so that solutions that are not feasible (according to the researcher’s judgement) are avoided. At the end of the monitored adjustment process, we obtain a IOT consistent with the various types of information that have been included (i.e. consistency with respect to economic aggregates, the table’s own structural patterns, and the information regarding the evolution paths of the IOT’s elements and their relationships).

Our simulation results indicate that the RAS method achieves better global results if the true margin values for the IOT are available. On the other hand, the ANAIS technique offers greater flexibility when adding non-marginal information or data on variability intervals.

We have applied the ANAIS method in order to obtain the Spanish IOTs for the years 1995 to 1998. These tables are coherent with their own internal structure, as shown by the application of the algorithms (which identify their structural characteristics) to the tables of the years 1990 to 1998.

This application raises two issues that might be dealt with in future research. First, a more thorough analysis of the behaviour of the ANAIS model (e.g. at a more detailed level of sector classification) as well as a comparison with the results from other biproportional adjustment methods should be undertaken. Second, the inclusion in the adjustment process of price effects should be analysed. The adjustment of tables in nominal prices may lead to significant errors. This might be due to the instability caused by the variations in relative prices, especially in certain sectors such as energy. More specifically, the technical coefficient $a_{ij}$ is defined as a function of prices and quantities

$$a_{ij} = \frac{p_i x_{ij}^q}{p_j w_j^q} \tag{47}$$

where $x_{ij}^q$ and $w_j^q$ are, respectively, intermediate purchases and real output, and $p_i$ and $p_j$ are the prices of the goods produced by the sectors $i$ and $j$.

The paper shows that a large variation in the price relations may change the value of the coefficients, although the input/output ratio remains constant. So, it would be interesting to build deflation vectors, at least for each demand activity, and include them in the adjustment model. This could substantially improve the results of the projections of the tables, because it would minimize this source of error, leading to less fluctuations in the value of the coefficients and, therefore, to the possibility of building more restrictive variability intervals.

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## Notes

1. For an extensive and exhaustive review of these techniques (generally called biproportional adjustment methods), see Lahr and de Mesnard (2004) and Kratena and Zakarias (2004).
2. Certain variants of these techniques allow for including into the adjustment process information other than just the margins. This is the case for V-Ras (Shishido *et al.*, 1991), or the UBS model (Wright *et al.*, 1997). Junius and Oosterhaven (2003) develop an RAS version that allows for the inclusion of negative entries in the adjustment process.
3. See Middelhoek (1970), Vaccara (1970) and Fontela and Pulido (1991).
4. Usually, this percentage is 1%.
5. The selection of one or another restriction's relaxation will depend on the researcher's criteria. Mathematically feasible solutions that are less desirable (or infeasible) from an economic point of view should be avoided. This requires that the adjustment process is guided by an expert, instead of applying an automatic procedure. Generally speaking, the modification that maximizes the ratio between the decrease in the supplementary variables' sum and the change in the restriction might be an appropriate choice.
6. The estimation has been performed for a confidence interval of 90%.
7. The errors were established randomly. Appendix A shows the results.
8. Also, the *transfers between activity branches* are included. This aggregate retrieves secondary production from the various sectors, which are then reassigned to the appropriate sector. The sum of the IOT's vector elements is 0 by default.
9. The economy's tendency towards the tertiary sector is an interesting fact that may be viewed as a consequence of the evolution of an economic system in development. An empirical study applied to the Spanish case is given by Rodríguez-González and Cañada-Vicinay (2000). The authors study the structural change undergone by the Spanish economy from 1980 to 1994, reaching similar conclusions. That is, an Increment of the detrimented tertiary sector at the cost of the basic industry.

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## Appendix A. 1993 IOT adjustment to 1994 margins with implicit errors

| | Intermediate Purchases (q vector) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Case A | | Case B | | Case C | | Case D | |
| sector | value | error | value | error | value | error | value | error |
| 1 | 16164437 | 0.00 | 16477933 | 1.94 | 15877728 | −1.77 | 15880397 | −1.76 |
| 2 | 12929045 | 0.00 | 13050579 | 0.94 | 12829311 | −0.77 | 12569545 | −2.78 |
| 3 | 8319210 | 0.00 | 8147983 | −2.06 | 8171652 | −1.77 | 8428432 | 1.31 |
| 4 | 5935067 | 0.00 | 6050172 | 1.94 | 5770309 | −2.78 | 6498886 | 9.50 |
| 5 | 9567205 | 0.00 | 9752753 | 1.94 | 9301619 | −2.78 | 9203277 | −3.80 |
| 6 | 6727778 | 0.00 | 6723782 | −0.06 | 6541014 | −2.78 | 6403009 | −4.83 |
| 7 | 4883728 | 0.00 | 4832019 | −1.06 | 4895006 | 0.23 | 4997825 | 2.34 |
| 8 | 2160687 | 0.00 | 2180997 | 0.94 | 2273960 | 5.24 | 2034273 | −5.85 |
| 9 | 5099065 | 0.00 | 5146997 | 0.94 | 4906406 | −3.78 | 5009464 | −1.76 |
| 10 | 16984722 | 0.00 | 17144380 | 0.94 | 17875140 | 5.24 | 17033899 | 0.29 |
| 11 | 30094960 | 0.00 | 29475543 | −2.06 | 30164455 | 0.23 | 28026231 | −6.87 |
| 12 | 7654454 | 0.00 | 7726407 | 0.94 | 7288523 | −4.78 | 7911615 | 3.36 |
| 13 | 6251818 | 0.00 | 6373067 | 1.94 | 6579567 | 5.24 | 5886047 | −5.85 |
| 14 | 4380976 | 0.00 | 4334590 | −1.06 | 4171537 | −4.78 | 4931659 | 12.57 |
| 15 | 4945825 | 0.00 | 4893458 | −1.06 | 5205108 | 5.24 | 5365054 | 8.48 |
| 16 | 23131670 | 0.00 | 22886751 | −1.06 | 23648787 | 2.24 | 21304877 | −7.90 |
| 17 | 16868703 | 0.00 | 17027271 | 0.94 | 16231350 | −3.78 | 16572287 | −1.76 |
| 18 | 22933606 | 0.00 | 22919985 | −0.06 | 23906027 | 4.24 | 24642864 | 7.45 |
| 19 | 8803187 | 0.00 | 8885937 | 0.94 | 8382339 | −4.78 | 9459294 | 7.45 |

(*continued*)

**Appendix A.** *Continued*

| | Intermediate Purchases (q vector) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Case A | | Case B | | Case C | | Case D | |
| sector | value | error | value | error | value | error | value | error |
| 20 | 3358197 | 0.00 | 3322641 | −1.06 | 3433271 | 2.24 | 3196088 | −4.83 |
| 21 | 1282632 | 0.00 | 1307508 | 1.94 | 1337018 | 4.24 | 1246968 | −2.78 |
| 22 | 862116 | 0.00 | 870220 | 0.94 | 890030 | 3.24 | 794031 | −7.90 |
| 23 | 10183279 | 0.00 | 10075458 | −1.06 | 10308862 | 1.23 | 9900127 | −2.78 |
| 24 | 17401891 | 0.00 | 17739386 | 1.94 | 17093233 | −1.77 | 18520782 | 6.43 |
| 25 | 20481982 | 0.00 | 20060420 | −2.06 | 20323986 | −0.77 | 21589308 | 5.41 |
| sum | 267406236 | | 267406236 | | 267406236 | | 267406236 | |

| | Intermediate sales (v vector) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Case A | | Case B | | Case C | | Case D | |
| sector | value | error | value | error | value | error | value | error |
| 1 | 23054482 | 0.00 | 23022404 | −0.14 | 23224222 | 0.74 | 23787481 | 3.18 |
| 2 | 29903826 | 0.00 | 29264974 | −2.14 | 29237996 | −2.23 | 28158563 | −5.84 |
| 3 | 11486561 | 0.00 | 11585285 | 0.86 | 11684575 | 1.72 | 11966834 | 4.18 |
| 4 | 10248621 | 0.00 | 10336705 | 0.86 | 9919211 | −3.21 | 10677133 | 4.18 |
| 5 | 14441936 | 0.00 | 14710279 | 1.86 | 14548266 | 0.74 | 13165055 | −8.84 |
| 6 | 9120785 | 0.00 | 9199176 | 0.86 | 8647471 | −5.19 | 9045307 | −0.83 |
| 7 | 5749931 | 0.00 | 5741931 | −0.14 | 5565118 | −3.21 | 5990345 | 4.18 |
| 8 | 2919290 | 0.00 | 2915228 | −0.14 | 2883121 | −1.24 | 2924375 | 0.17 |
| 9 | 6396349 | 0.00 | 6323575 | −1.14 | 6632956 | 3.70 | 6727866 | 5.18 |
| 10 | 8563311 | 0.00 | 8551396 | −0.14 | 8372642 | −2.23 | 8406663 | −1.83 |
| 11 | 19601463 | 0.00 | 19574190 | −0.14 | 19939366 | 1.72 | 20617389 | 5.18 |
| 12 | 4609378 | 0.00 | 4556935 | −1.14 | 4597792 | −0.25 | 4617408 | 0.17 |
| 13 | 8618700 | 0.00 | 8434574 | −2.14 | 8597037 | −0.25 | 8202028 | −4.83 |
| 14 | 7040075 | 0.00 | 7170886 | 1.86 | 6674738 | −5.19 | 6417629 | −8.84 |
| 15 | 3663956 | 0.00 | 3732035 | 1.86 | 3763304 | 2.71 | 3303305 | −9.84 |
| 16 | 9513649 | 0.00 | 9690420 | 1.86 | 9489736 | −0.25 | 10197337 | 7.19 |
| 17 | 18766164 | 0.00 | 18740053 | −0.14 | 19275005 | 2.71 | 19362820 | 3.18 |
| 18 | 4802976 | 0.00 | 4748330 | −1.14 | 4601165 | −4.20 | 4426435 | −7.84 |
| 19 | 9351899 | 0.00 | 9245498 | −1.14 | 9697834 | 3.70 | 10117645 | 8.19 |
| 20 | 1683321 | 0.00 | 1697788 | 0.86 | 1745588 | 3.70 | 1635665 | −2.83 |
| 21 | 4429207 | 0.00 | 4511505 | 1.86 | 4199357 | −5.19 | 4081969 | −7.84 |
| 22 | 6782386 | 0.00 | 6908408 | 1.86 | 7033272 | 3.70 | 6522432 | −3.83 |
| 23 | 7591131 | 0.00 | 7732181 | 1.86 | 7721993 | 1.72 | 7528312 | −0.83 |
| 24 | 39066839 | 0.00 | 39012482 | −0.14 | 39354471 | 0.74 | 39526241 | 1.18 |
| 25 | 0 | 0.00 | 0 | 0.00 | 0 | 0.00 | 0 | 0.00 |
| sum | 267406236 | | 267406236 | | 267406236 | | 267406236 | |

**Appendix B. R-25 classification by Eurostat**

| R25 | Activity branches |
|---|---|
| 01 | Agriculture, forestry and fishery products |
| 02 | Fuel and power products |
| 03 | Ferrous and non-ferrous ores and metals |
| 04 | Non-metallic mineral products |
| 05 | Chemical products |
| 06 | Metal products except machinery |
| 07 | Agricultural and industrial machinery |
| 08 | Office and data processing machines |
| 09 | Electrical goods |
| 10 | Transport equipment |
| 11 | Food, beverages, tobacco |
| 12 | Textiles and clothing, Elater and footwear |
| 13 | Paper and printing products |
| 14 | Rubber and plastic products |
| 15 | Other manufacturing products |
| 16 | Building and construction |
| 17 | Recovery, repair services, wholesale, retail |
| 18 | Lodging and catering services |
| 19 | Inland transport services |
| 20 | Maritime and air transport services |
| 21 | Auxiliary transport services |
| 22 | Communication services |
| 23 | Services of credit and insurance institutions |
| 24 | Other market services |
| 25 | Non-market services |