

The effect of external finance on the equilibrium allocation of capital[☆]

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Abstract

We develop an equilibrium model to understand how the efficiency of capital allocation depends on outside investor protection and the external financing needs of firms. We show that when capital allocation is constrained by poor investor protection, an increase in firms' external financing needs may improve allocative efficiency by fostering the reallocation of capital from low to high productivity projects. We also find novel empirical support for this prediction.

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1. Introduction

Recent research shows that a country's financial system affects economic growth (see, e.g., King and Levine, 1993a; Levine and Zervos, 1998; Rajan and Zingales, 1998; Demircuc-Kunt and Maksimovic, 1998; Beck et al., 2000a). What are the characteristics of a country's financial structure that are more conducive to economic development? A number of authors suggest that banks do a better job than atomistic markets at raising large amounts of capital (Gershenkron, 1962), monitoring, and gathering information about firms. Other authors emphasize the potential advantages of markets versus banks in effectively allocating capital (e.g., Allen, 1993). Levine (1997) argues that it is the quality of financial services and not the identity—market or bank—of the provider that is the key dimension of a financial system. La Porta et al. (1997, 1998) suggest that the efficiency of the legal system in protecting outside investors is the main variable driving the quality of the financial system.¹

In this paper we contribute to the debate on the real effects of financial structure by suggesting an additional factor that influences economic efficiency: the external financing needs of firms.² In particular, we analyze how this factor affects the efficiency of capital allocation.³

We develop a model in which capital allocation is constrained by the extent to which outside investors are legally protected against expropriation by the manager or other insiders (La Porta et al., 1997, 1998). When investor protection is low, there is a limit on the fraction of cash flows that entrepreneurs can credibly commit to outside investors (*limited pledgeability* of cash flows). Because of this friction, the economy has a limited ability to direct capital to its best users: high productivity projects cannot pledge a sufficiently high return to attract capital from low productivity projects. Our main result is that, if we start from the scenario in which capital reallocation is constrained by limited pledgeability, higher external financing needs can improve allocative efficiency by increasing the liquidation of low productivity projects and thereby making more capital available to high productivity projects.

The need for external finance can improve the allocation of capital even in instances in which it introduces inefficiencies at the firm level. In the presence of limited pledgeability, outside investors prefer to liquidate a mediocre project because they can only get a fraction of the future return. However, in the absence of external

¹See Allen and Gale (2000) and Demircuc-Kunt and Levine (2001) for a broad discussion of different aspects of financial structure.

²Consistent with our focus on the relation between external finance and economic efficiency, Goldsmith (1969) considers the relation between internal and external finance as a defining characteristic of financial structure: "Of particular importance for the study of financial structure is the distribution of total sources and funds of various sectors and subsectors by form and by partner. The tool here is the sources-and-uses-of-funds statement. . . [that] makes possible a determination of the share of internal financing (savings) and external financing (borrowing or issuance of equity securities)" (Goldsmith, 1969, p. 29).

³The empirical results in Beck et al. (2000b) suggest that capital allocation is an important channel through which finance affects economic growth.

finance, insiders prefer not to liquidate such projects because they keep the full return. For a firm to attract sufficient capital when its financing needs are large, the firm has to commit to liquidate. Thus, firms that need to raise external finance are liquidated more often than internally financed firms (as in [Diamond, 1991](#)). While excessive liquidation might be suboptimal for a firm in isolation, it may be socially beneficial because the released capital flows from mediocre to high productivity projects, and thereby improves capital allocation.

An important result of our model is that limited investor protection constrains the efficiency of capital allocation. Consistent with this prediction, [Wurgler \(2000\)](#) finds evidence that measures of financial development and outside investor protection are positively related to the efficiency of capital allocation. However, this result is not particular to our paper. [Levine \(1991\)](#), [Bencivenga et al. \(1995\)](#) and [Shleifer and Wolfenzon \(2002\)](#) also predict that financial frictions such as limited pledgeability reduce allocative efficiency. Rather, the novel empirical predictions of our theory relate to the role of external finance.

Controlling for the level of investor protection, the external financing needs of firms in a given country should have an independent effect on the efficiency of capital allocation. We show that as long as the degree of pledgeability is not too low, the resources that are released by liquidation induced by external financiers will find their way to more productive activities. This argument suggests that countries whose firms have higher financing needs enjoy a more efficient allocation of capital.

We present new evidence that is consistent with this implication. We build a proxy for the aggregate need for external finance of a country's manufacturing sector using [Rajan and Zingales \(1998\)](#) industry-level measure of external finance dependence, and using the shares of the different industries in total manufacturing sector output (the country's industrial structure). We then run cross-country regressions of [Wurgler's \(2000\)](#) measure of the efficiency of capital allocation (the elasticity of industry investment to industry value added) on our country-level measure of external finance dependence and controls.

We show that the need for external finance is positively correlated with the efficiency of capital allocation, even after controlling for variables such as economic and financial development or measures of investor protection. This correlation also seems to be driven by the component of the investment elasticity that is more directly related to the model's predictions, the within-year component. This component provides a measure of how investment responds to value-added shocks across different industries in the same country, and thus better captures the idea of capital *reallocation* that we model. Finally, we show that the correlation remains significant after instrumenting our index of external financing needs with measures of a country's commodity endowments ([Easterly and Levine, 2003](#)), which we argue are exogenous determinants of a country's industrial structure, and thus of its external financing needs. These results are consistent with the model's main predictions.

Our model complements earlier equilibrium models that examine the effect of impediments to financial market operations on the efficiency of capital allocation (e.g., [Levine, 1991](#); [Bencivenga et al., 1995](#)). There are two aspects of our analysis that distinguish our paper. First, we study the effect of differences in external

financing needs on the allocation of capital. Second, we focus on limited pledgeability of cash flows as the main source of imperfections, while these earlier papers have focused on transaction costs of equity market transactions. Our focus on pledgeability is crucial for our results. While models without limited pledgeability but with transaction costs on financial transactions also generate an inefficient equilibrium allocation of capital, they do not predict the same effects of external finance that we obtain in this paper. Limited pledgeability generates a conflict of interests between insiders and outsiders—while insiders prefer to continue mediocre projects, outside investors prefer to liquidate them. As the aggregate level of external finance increases and control shifts to outside investors, liquidation increases. Other frictions to financial market transactions affect all of the firm's investors equally, and thus do not generate a similar conflict of interest.

The model presented in this paper is not the first to analyze the effect of limited pledgeability on the allocation of capital. Shleifer and Wolfenzon (2002) also employ the same type of friction. However, they do not consider the effect of external financing needs on the allocation of capital, and, more importantly, they do not focus on the *reallocation* of capital across ongoing projects of different productivities as we do here. Our results on external financing needs clearly require a model in which capital can be reallocated among existing projects of different productivities, and thus the setup in Shleifer and Wolfenzon (2002) cannot generate our results.

Our paper is also related to the literature that analyzes the effects of financial intermediaries on the allocation of capital (e.g., Boyd and Smith, 1992; King and Levine, 1993b; Galetovic, 1996). This literature, however, has also not focused on the effects of external financing needs, or on capital reallocation under limited pledgeability.

Finally, our paper is related to previous arguments about the benefits of external finance for firm performance. For example, Jensen (1986) argues that the presence of debt mitigates managerial agency costs. Our argument differs in three dimensions. First, our model is an equilibrium model, thus we can analyze the effect of external finance on economy-wide outcomes, and not just on the particular firm that seeks external finance. Second, the benefit of external finance that we propose is a social benefit, not a private one. That is, in our framework, increased external finance does not benefit the recipient firms, but rather the other firms in the economy, as the probability of liquidation of externally financed firms increases. In contrast, Jensen suggests that debt benefits shareholders of the recipient firm itself. Finally, Jensen's argument applies largely to firms with dispersed ownership, in which managers may be free to use the firm's excess cash flows to invest in potentially negative-NPV projects. In contrast, in our model external finance is socially beneficial even when introduced in firms characterized by 100% entrepreneurial ownership.

In the next section we characterize the effect of limited pledgeability and of changes in external financing needs on the equilibrium allocation of capital. Section 2.1 contains an intuitive description of the main effect. Section 3 summarizes the empirical implications of our model, and presents some evidence consistent with these implications. Section 4 presents our final remarks. All the proofs are contained in the appendix.

2. Limited pledgeability, external finance and the equilibrium allocation of capital

In this section we develop our theoretical framework to analyze the effect of both limited pledgeability and firms' external financing needs on the equilibrium allocation of capital.

There are two types of agents in the model. First, there is a set J with measure one of entrepreneurs, each with one project opportunity (projects are described below). There is also a set of investors with no project opportunities. All agents are risk-neutral and do not discount the future. There is a single good in this economy, capital. At the initial date, t_0 , the aggregate amount of capital $1 + K$ (with $K > 1$) is held by the two types of agents. Each entrepreneur has an endowment of $1 - Z$ units (with $0 \leq Z < 1$). The remaining $K + Z$ units of capital is distributed among investors.

The timing of events in the model is as shown in Fig. 1. At date t_0 , entrepreneurs issue financial contracts to raise the capital necessary to pay the investment cost of their projects. Information about the profitability of projects arrives at a later date, t_1 , and cash flows are realized at the final date, t_2 . In light of the new information, at date t_1 , capital can be reallocated among projects before cash flows are realized. The efficiency of this reallocation market is the main object of our analysis.

Technologies: Capital can be stored with no depreciation. In addition, capital can be invested in two types of technologies that pay off in period t_2 . We refer to the first type of technology as projects. Each project is of infinitesimal size. At date t_0 , a project requires an investment of one unit of capital. At date t_1 , a project can be liquidated, in which case the entire unit of capital is recovered. If a project is not liquidated, it can receive additional capital or can simply be continued with no change until date t_2 . At date t_2 , projects generate cash flows.

Projects have different productivities. Each project's type, s , can take one of three values: H (high productivity projects), M (medium productivity projects), and L (low productivity projects) with probabilities p_H , p_M , and p_L (with $p_H + p_M + p_L = 1$), respectively. A project's type is not known at date t_0 (not even to the entrepreneur who owns it) but it is revealed to all agents in the economy at the beginning of date t_1 . The probability distribution is independent across projects such that, at date t_1 , exactly a fraction p_H , p_M , and p_L of the projects are of type H , M , and L , respectively. For simplicity, we assume drastic decreasing returns to scale. Depending on a project's type, each project generates Y_H , Y_M , or Y_L (with

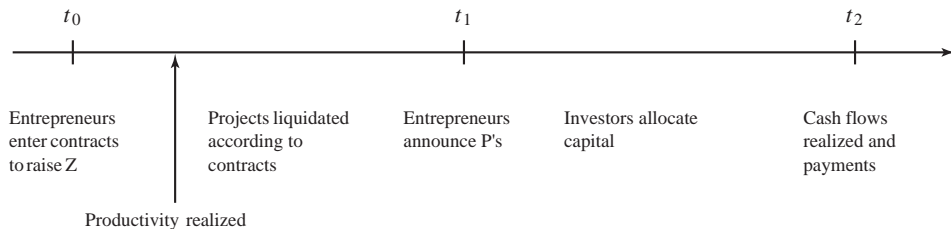


Fig. 1. Timing of events.

$Y_H > Y_M > Y_L \equiv 0$) per unit invested, but only for the first two units, i.e., the unit invested at t_0 and the unit that is potentially invested at t_1 . Additional units invested generate no cash flows.

The second technology, which we refer to as general technology, is available to all agents in the economy at date t_1 . We define $x(\omega)$ as the per-unit payoff of the general technology with ω being the aggregate amount of capital invested in it.

Assumption 1. The general technology satisfies:

- a. $\partial[\omega x(\omega)]/\partial\omega \geq 0$
- b. $x'(\omega) \leq 0$
- c. $x(1 + K) > 1$.

We assume that total output is increasing in the amount invested in the general technology, Assumption 1(a), but that the per-unit payoff is decreasing, Assumption 1(b). Assumption 1(c) guarantees that no capital is invested in storage at date t_1 .

To rank the productivity of the projects and the general technology, we make the following assumption.

Assumption 2. $Y_M > x(0)$.

The most productive technologies are the high productivity projects, followed by projects of medium productivity. By Assumption 2, medium productivity projects are more productive than the general technology regardless of the amount invested in the latter (since $x(0) \geq x(\omega)$ by Assumption 1(b)). Finally, the low productivity projects are the least productive technologies in the economy.

Capital market at date t_0 : In the capital market at date t_0 , entrepreneurs raise Z from outside investors in exchange for a set of promised payments. To focus on imperfections brought about by limited pledgeability of payoffs (described below), we assume that initial contracts can be made fully contingent on dates and type of project. A contract offered by entrepreneur j is defined by a vector $\{q_s^j, D_{1s}^j, D_{2s}^j\}_{s=L,H,M}$, where q_s^j is the probability that the project is liquidated when the realized productivity is s , D_{1s}^j is the payment to the investor at date t_1 when the project is liquidated (since projects generate no cash flow at date t_1 , the payment D_{1s}^j can be positive only when there is liquidation), and D_{2s}^j is the payment to the investor at date t_2 .

Entrepreneurs approach investors and make them a take-it-or-leave-it offer. The assumption that entrepreneurs have all the bargaining power can be justified by the fact that there is excess supply of capital at date t_0 . Since initial investors can always store their capital from date t_0 to date t_1 and then invest it in the reallocation market from date t_1 to date t_2 , they accept the contract only if it offers them a payoff of at least R^*Z at date t_2 , where R^* is the expected return in the date- t_1 reallocation market.

Capital market at date t_1 : At date t_1 , the type of the project is realized and liquidation occurs according to the specified probabilities in the contract. After liquidation occurs, date- t_1 investors (investors that stored capital from date t_0 to date

t_1 , and agents holding the proceeds from liquidated projects) allocate their capital between continuing projects and the general technology. The total amount of capital in the hands of date- t_1 investors is $K + T$, where T is the capital released from liquidated projects. That is, $T = \int_{j \notin C} dj$ with C being the set of projects that continue.

Each continuing project j announces P^j , the amount it promises to pay at date t_2 for the first unit of capital it raises at date t_1 . Projects have no use for additional units of capital and thus offer to pay zero for those units. Next, date- t_1 investors allocate their capital to either the projects or to the general technology. An allocation in the capital market can be described by r^j , the probability that project $j \in C$ gets capital, and ω , the amount allocated to the general technology. An equilibrium allocation must be consistent with date- t_1 investor maximization and satisfy the market clearing condition

$$\omega + \int_{j \in C} r^j dj = K + T. \quad (1)$$

Limited pledgeability of cash flows: We consider an imperfection at the firm level: firms cannot pledge to outside investors the entirety of cash flows generated by projects. In particular, for date- t_2 cash flows, we assume that only a fraction λ of the returns of the second unit invested is pledgeable.⁴ For the date- t_1 reallocation market, this assumption implies the following pledgeability constraint for a continuing project j with realized productivity s

$$P^j \leq \bar{P}_s^j \equiv \lambda Y_s - D_{2s}^j. \quad (2)$$

We also assume that the liquidation proceeds are fully pledgeable (we justify this assumption below).

The limited pledgeability assumption can be justified as being a consequence of poor investor protection, as shown in Shleifer and Wolfenzon (2002). In their model, insiders can expropriate outside investors, but expropriation costs limit the optimal amount of expropriation that the insider undertakes. Higher levels of outside-investor protection (i.e., higher costs of expropriation) lead to lower expropriation and consequently higher pledgeability. To justify the full pledgeability of cash flows in the case of liquidation, we make the natural assumption that liquidation proceeds are easier to verify than project returns.

Limited pledgeability also arises in other contracting frameworks. For example, limited pledgeability is a consequence of the inalienability of human capital (Hart and Moore, 1994). Entrepreneurs cannot contractually commit never to leave the firm. This leaves open the possibility that an entrepreneur could use the threat of withdrawing his human capital to renegotiate the agreed upon payments. If the entrepreneur's human capital is essential to the project, he will get a fraction of the date- t_2 cash flows. A natural assumption is that the entrepreneur's human capital is not needed to liquidate the firm. This justifies the fact that while the entire

⁴The assumption that the cash flows from the first unit are not pledgeable is made only for simplicity. It will become clear our main results do not hinge on this assumption (see footnote 11).

liquidation proceeds are pledgeable, only a fraction of the date- t_2 cash flows are. Limited pledgeability is also an implication of the [Holmstrom and Tirole \(1997\)](#) model of moral hazard in project choice. In that model, project choice cannot be specified contractually. As a result, investors must leave a sufficiently high fraction of the payoff to entrepreneurs to induce them to choose the project with low private benefits but high potential profitability. Since in our framework liquidation can be specified contractually, there is no need to leave anything to the entrepreneur when liquidation occurs. Again, this justifies the full pledgeability of liquidation proceeds.

2.1. Illustration of the main effect

In this section we present an informal argument to illustrate the main mechanism of the model. In the next section we formally characterize the equilibrium. The reader who is not interested in the formal derivation of the equilibrium can safely skip Section 2.2 after reading the current section, and proceed directly to Section 2.3.

It is useful to think about the date- t_1 reallocation market in terms of demand and supply of funds.⁵ On the supply side, there is a total of $K + T$ to be allocated with T being the capital from liquidated projects. This amount is determined by the contract that entrepreneurs enter at date t_0 to raise the necessary capital Z . Since continuation of low productivity projects generates no cash, these projects are always liquidated. The liquidation proceeds are supplied to the market since the entrepreneur has no use for them. As a result, aggregate liquidation proceeds are at least p_L . [Fig. 2](#) depicts the demand and supply schedules for two different supply scenarios. The supply of capital (vertical line) in Panel A is $K + p_L$ while total supply in Panel B is higher.

The demand for capital arises from high and medium productivity projects, and from the general technology. To raise capital at date t_1 , firms pledge part of the project's date- t_2 cash flows. Due to limited pledgeability, firms with high (medium) productivity projects can offer a maximum of λY_H (λY_M) per unit of capital (for the sake of the argument, we assume that $D_{2H} = D_{2M} = 0$) and the general technology offers $x(\omega)$. The aggregate demand schedule (the downward sloping lines in Panels A and B) shows the capital demanded for different levels of market return. If we assume that $x(\omega) > \lambda Y_H$ for low ω , as we do in [Fig. 2](#), some capital must be allocated to the general technology before the projects receive additional capital (the general technology can initially offer a higher return than good and medium projects). As additional capital is allocated to the general technology, the return it offers (the required return on capital) decreases. When this return reaches λY_H , high productivity firms can start attracting capital. Additional capital is allocated to high productivity firms until they all get one unit. This explains the flat region

⁵In this supply and demand framework the existence of an equilibrium is not guaranteed. In the formal characterization of the equilibria, we model the date- t_1 capital market using the non-cooperative game described above to guarantee the existence of an equilibrium. However, the intuition for both cases is very similar.

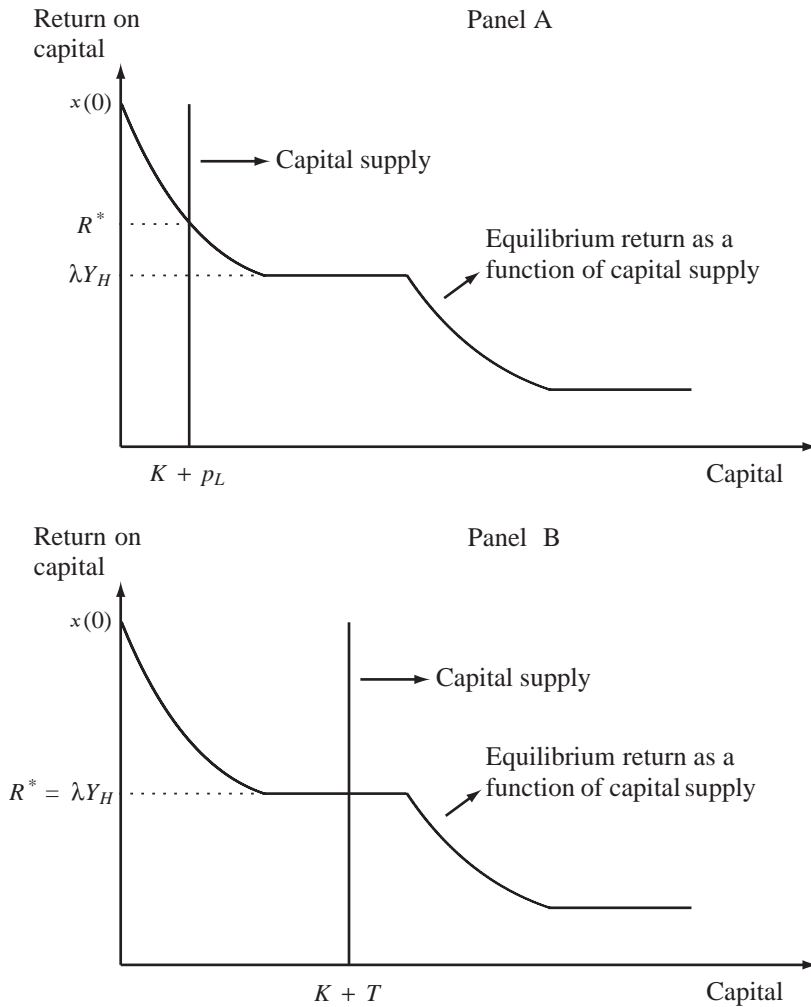


Fig. 2. Equilibrium in the date- t_1 reallocation market. The vertical line represents the total capital available for reallocation, which is the sum of capital stored from date t_0 , K , and the liquidation proceeds. In Panel A, the amount of liquidation is p_L , whereas in Panel B, liquidation proceeds, T , are higher. The downward sloping line represents the equilibrium return, R^* , as a function of the supply of capital. The figure assumes that $x(0)$, the return on the general technology when $\omega = 0$, is higher than λY_H , firm H 's pledgeable income.

of the aggregate demand. The rest of the demand schedule is derived in a similar way.

In Panel A, the market equilibrium rate is $R^* = x(K + p_L)$. In this case, date- t_1 investors allocate their capital to the general technology. Even though high productivity firms are the best users of capital, they cannot attract any capital

because the return in the market is higher than the maximum return they can credibly offer ($\lambda Y_H < R^*$). Panel A assumes that the supply of funds is $K + p_L$. That is, only low productivity projects are liquidated. Notice that, since high productivity firms do not attract new capital in equilibrium, it would be socially optimal to liquidate medium productivity projects and supply their capital to the high productivity projects. However, this reallocation does not happen voluntarily. Firms with medium productivity projects generate cash flows of Y_M if they continue. If these projects are liquidated and the unit recovered is invested in the market, they receive $R^* = x(K + p_L)$. Since $x(K + p_L) < Y_M$, firms with medium productivity projects do not liquidate but rather continue.

In sum, limited pledgeability distorts capital allocation in two ways. First, high productivity firms cannot attract the capital that is available for allocation. And second, given that high productivity firms cannot pledge enough cash, firms with medium productivity projects choose not to liquidate and supply their capital to the high productivity firms.

How can firms with medium productivity projects be forced to liquidate their projects? As we argue in detail in the next section, this is precisely the role of external finance at date t_0 . If the initial financing requirement is high enough, the only way firms are able to repay initial investors is by having some liquidation at date t_1 . Moreover, ex ante, it is less costly to commit to liquidate a project when its realized productivity is medium rather than high. Thus, if liquidation is necessary for initial financing, the optimal contract will require medium projects to be liquidated first. This situation is depicted in Panel B, wherein the supply of capital is higher than in Panel A ($K + T > K + p_L$) because of increased date- t_0 financing requirements. In this case, the additional capital drives down the market return on capital so that high productivity firms attract some funds. In other words, some of the capital from the liquidated medium projects finds its way to the high productivity projects, potentially improving the aggregate payoff.

2.2. Characterization of the equilibrium

We solve the model backwards. Since at date t_2 no decisions are taken, we start by solving the equilibrium in the date- t_1 reallocation market. Then we solve for the optimal date- t_0 contracts.

The equilibrium in the date- t_1 reallocation market: We consider any set C of projects that continue and any arbitrary contract for each of these projects. For a continuing project j with realized productivity s , the relevant part of the contract is $\bar{P}_s^j = \lambda Y_s - D_{2s}^j$, the maximum amount it can offer. In the following discussion we drop the subscript s for simplicity.

The equilibrium in the date- t_1 reallocation market is given in the following lemma.

Lemma 1. *There is a level R^* such that date- t_1 investors allocate an amount $\omega^* = x^{-1}(R^*)$ to the general technology and allocate capital to projects according to the*

following rule:

$$r^j = \begin{cases} 0 & \text{if } P^j < R^* \\ 1 & \text{if } P^j > R^* \text{ or, if } P^j = R^* \text{ and } \bar{P}^j > R^* \\ \bar{r} & \text{if } P^j = \bar{P}^j = R^* \end{cases} \quad (3)$$

with $\bar{r} \in [0, 1]$ chosen to satisfy the market clearing condition in Eq. (1).⁶ Projects with $\bar{P}^j \geq R^*$ offer to pay R^* , and projects with $\bar{P}^j < R^*$ are indifferent among all feasible offers.

The equilibrium strategy of date- t_1 investors is to allocate their capital first to the projects offering the highest return and then to the other projects in decreasing order of the offered return. This implies that there is a cutoff value R^* such that technologies offering strictly more than R^* get capital with certainty and those offering strictly less than R^* do not get capital. Because investors are indifferent among all technologies offering exactly R^* , any rule that allocates capital to a subset of these technologies is consistent with date- t_1 investor maximization behavior. However, the only rule for which an equilibrium exists is one in which all the projects that could have offered a greater return (projects with $\bar{P}^j > R^*$) receive capital with probability one.⁷ The definition of R^* guarantees that the market clears.

Given the allocation rule in Eq. (3), it is optimal for all projects with $\bar{P}^j \geq R^*$ to offer exactly $P^j = R^*$. These projects benefit by raising capital at this price because they generate Y_s out of this unit of capital but pay only $R^* \leq \bar{P}^j < Y_s$. They have no incentive to decrease the offer, because offers of less than R^* receive no capital. Also, they do not benefit by raising the offer. First, according to the allocation rule in Eq. (3), the subset of projects with $\bar{P}^j > R^*$ receive capital with certainty and thus they have no reason to increase their offer. Second, the subset of projects with $\bar{P}^j = R^*$ do not necessarily receive capital with probability one but, due to limited pledgeability, they cannot increase their offer. Finally, the general technology receives capital up to the point where its return is R^* .

Since, in equilibrium, all date- t_1 investors receive a return of R^* , we refer to R^* as the equilibrium return of the reallocation market.

Date- t_0 contracts: To check that a set of contracts $\{q_s^j, D_{1s}^j, D_{2s}^j\}_{s=L,M,H}$ for $j \in J$ is an equilibrium, we first use Lemma 1 to find the equilibrium return and allocation rule (R^* and r) at date t_1 that is generated by the proposed set of contracts. We then check that, for these R^* and r , no entrepreneur has an incentive to deviate at date t_0 by offering a different contract.

We focus on symmetric equilibria in which all entrepreneurs offer the same contract. We characterize the equilibrium contracts as a function of the degree of limited pledgeability λ and the financing requirement Z . Since we consider symmetric

⁶The level of R^* can be found by solving $x^{-1}(R^*) + \int_{\{j \in C \mid \bar{P}^j > R^*\}} dj \leq K + T \leq x^{-1}(R^*) + \int_{\{j \in C \mid \bar{P}^j \geq R^*\}} dj$.

⁷If a project with $\bar{P}^j > R^*$ receives capital with probability less than one when it offers $P^j = R^*$, then it will be optimal for this project to offer $R^* + \varepsilon$ and to receive capital for sure. Clearly, in this case an equilibrium does not exist since ε can always be made infinitesimally smaller.

equilibria, $\bar{P}_s^j(r_s^j)$ is the same for all firms that continue, and can take one of three values depending on realized productivity, \bar{P}_H , \bar{P}_M , and \bar{P}_L (r_H , r_M , and r_L). In the text below, we focus on the most interesting cases, those in which pledgeability is sufficiently low such that the high productivity projects are capital constrained (receive capital with probability less than one). The remaining cases are described in the Appendix. We also assume an upper bound for the financing requirement, $Z \leq p_L + p_M$. We discuss later what happens for higher Z .

Lemma 2. *There is a function $\lambda_1(Z)$ such that the equilibrium contracts offered at date t_0 are:*

- a. For $Z \leq p_L$, $(q_L, D_{1L}, D_{2L}) = (1, Z/p_L, 0)$; $(q_M, D_{1M}, D_{2M}) = (0, 0, 0)$; $(q_H, D_{1H}, D_{2H}) = (0, 0, 0)$. That is, entrepreneurs liquidate their projects with probability 1 when productivity is low and do not liquidate their projects when productivity is medium or high. In this case, $T = p_L$.
- b. For $p_L < Z \leq p_L + p_M$ and $\lambda \leq \lambda_1$, $(q_L, D_{1L}, D_{2L}) = (1, 1, 0)$, $(q_M, D_{1M}, D_{2M}) = \left(\frac{Z-p_L}{p_M}, 1, 0\right)$; $(q_H, D_{1H}, D_{2H}) = (0, 1, 0)$. That is, entrepreneurs liquidate their projects with probability 1 when productivity is low and also liquidate their projects with positive probability when productivity is medium. Entrepreneurs never liquidate high productivity projects. In this case, $T = Z$.

In order to explain the above lemma, we first derive a condition on the equilibrium return R^* . By the market clearing condition, ω^* must satisfy $0 < \omega^* \leq 1 + K$. By Lemma 1, $R^* = x(\omega^*)$ and thus it follows that $x(1 + K) \leq R^* < x(0)$. By Assumptions 1 and 2

$$Y_L < R^* < Y_M < Y_H. \quad (4)$$

Since the participation constraint of the date- t_0 investor binds and payoffs can be transferred one-to-one from the entrepreneur to the investor, the entrepreneur's maximization problem is equivalent to maximizing the combined payoff to him and the date- t_0 investor subject to the participation constraint of the date- t_0 investor binding. By Eq. (4), the entrepreneur benefits from liquidating a low productivity project because continuing the project yields a total payoff of Y_L , whereas the unit recovered from liquidation yields R^* when invested in the market. Similarly, total payoff is reduced if the entrepreneur liquidates either the medium or the high productivity project. The contract offered, however, can involve some liquidation of medium or high productivity projects if it is necessary to satisfy the date- t_0 participation constraint.

If the capital requirements are low, $Z \leq p_L$, then liquidation of the low productivity project is sufficient to satisfy the participation constraint of the initial investor. By liquidating only the low productivity project, the entrepreneur can offer the date- t_0 investor an expected payment of Z at date t_1 . These funds can be invested in the reallocation market to yield ZR^* at date t_2 . This explains part a) of Lemma 2.

However, if the capital requirements are sufficiently high ($Z > p_L$), liquidation of medium or high productivity firms might be the only way to pay back the initial

investor. Of course, a more efficient way to compensate the initial investor is by paying him out of the date- t_2 payoff ($D_{2s} > 0$) since this avoids liquidation of medium or high productivity projects. However, this is not always possible. When the pledgeability parameter λ is very low, firms cannot secure capital in the reallocation market and, since only the returns of the second unit are pledgeable, it is impossible to offer a fraction of the date- t_2 payoffs. But the problem is even more severe. The equilibrium has $D_{2H} = 0$, even when type- H projects get capital in the reallocation market, as long as they get capital with probability less than one. The lower is D_{2H} (i.e., the higher is \bar{P}_H), the more likely a project is to raise capital since it can offer a higher return in the reallocation market (see Lemma 1). If all the projects were offering $D_{2H} > 0$ and getting capital with probability less than one, an entrepreneur would be better off deviating and offering a contract with a slightly lower D_{2H} (i.e., a slightly higher \bar{P}_H) so as to be able to outbid all the other entrepreneurs in the date- t_1 reallocation market. Thus, when high productivity firms are constrained ($r_H < 1$), or equivalently, when $\lambda \leq \lambda_1$, competition for capital at date t_1 drives D_{2H} to zero. By the same logic, $D_{2M} = 0$ in this range.

Given that D_{2H} and D_{2M} are zero in the range of λ considered, initial investors must be paid entirely out of date- t_1 liquidation proceeds. Liquidation of medium and high productivity firms reduce the combined payoff. Thus, when liquidation occurs, the entire liquidation proceeds should be given to the initial investors as this reduces the probability of liquidation. Since the medium project is less productive, it is better to start liquidating it first. It is only when liquidation of the low and medium productivity projects is not sufficient to satisfy the investor participation constraint that the H projects start being liquidated. When $Z \leq p_L + p_M$, liquidation of the L and M projects is sufficient to pay back the investor.⁸ Notice that q_M is increasing in Z . The higher the financing requirement, the more frequent the M project needs to be liquidated.

All other cases are described in the appendix. For $\lambda > \lambda_1$, H projects can pledge a sufficiently high return to receive capital with probability one. Since H projects are no longer capital constrained, there is no benefit in decreasing D_{2H} to outbid other firms. Thus, it becomes possible to sustain an equilibrium with $D_{2H} > 0$. For even higher levels of λ it is possible to sustain equilibria with $D_{2M} > 0$. Finally, when the degree of pledgeability is very large, the entrepreneur is able to pay the investor out of date- t_2 proceeds only.

Lemma 2 describes the equilibrium contract for each pair (λ, Z) . Using these contracts in Lemma 1 we obtain the equilibrium allocation and return r_H, r_M , and R^* . Since for parts (a) and (b) of Lemma 2, $D_{2s} = 0$, the contracts in Lemma 2 imply that $T = p_L$ in case (a) and $T = Z$ in case (b).

Panels A and B of Fig. 2 correspond to cases (a) and (b) of Lemma 2. Panel A corresponds to case (a) of Lemma 2, where pledgeability is low ($\lambda Y_H < x(K + p_L)$). In this case all the capital is allocated to the general technology and $R^* = x(K + p_L)$. H projects cannot offer a sufficiently high return to attract capital ($\lambda Y_H < R^* =$

⁸When $Z > p_L + p_M$, some liquidation of the H project is necessary. In this case, we would have full liquidation of M projects ($q_M = 1$) and partial liquidation of H projects ($q_H = (Z - p_L - p_M)/q_M$) in order to satisfy the financing requirement.

$x(K + p_L)$) and, at the same time, M projects are continued and their capital is not supplied to high productivity projects (because $Y_M > R^*$). Panel B corresponds to case (b) of Lemma 2, with $x(K + Z) \leq \lambda Y_H < x(K + Z - p_H)$. In this case, $\lambda Y_H = R^*$ and H firms are able to attract some capital. However, Z is not large enough to allow all H firms to raise one unit of capital.

2.3. Comparative statics in λ and Z

We now analyze what happens to the aggregate payoff when pledgeability and the financing requirements change. In the following proposition, the threshold λ_1 is the same as in Lemma 2 (this threshold is such that for $\lambda < \lambda_1$, good firms are capital constrained).

Proposition 1. *The aggregate payoff*

- *is non-decreasing in the level of pledgeability, λ ,*
- *for $Z < p_L$, it is unaffected by the financing requirement Z , and*
- *for $p_L \leq Z < p_L + p_M$, there exists a $\lambda_0 < \lambda_1$ such that the aggregate payoff decreases with Z for low λ ($\lambda < \lambda_0$), increases with Z for intermediate levels of λ ($\lambda_0 \leq \lambda < \lambda_1$), and decreases with (or is unaffected by) Z for high λ ($\lambda \geq \lambda_1$).*

Better pledgeability allows projects to raise capital more easily in the reallocation market. Since this capital would have ended up invested in the general technology, an increase in pledgeability leads to higher aggregate output.

For $Z < p_L$, only low productivity projects are liquidated in equilibrium. In this range, an increase in the financing requirement does not affect the probability that medium (q_M) or high productivity projects (q_H) are liquidated. Since the total amount of capital from liquidation proceeds remains unchanged, aggregate output is not affected.

For $Z \geq p_L$, increases in the initial financing requirement make it necessary to liquidate medium productivity projects more often. Since the return of the medium project is higher than that of the general technology, this liquidation creates a social inefficiency when the released capital ends up invested in the general technology. This happens when pledgeability is so low ($\lambda < \lambda_0$) that H firms are not able to offer a sufficiently high return to attract the released capital. In such a case, the reallocation market cannot materialize and external finance only has costs. If pledgeability is higher ($\lambda > \lambda_0$), H firms are able to attract some of the released capital, thereby increasing the aggregate payoff. At pledgeability levels above λ_1 , all H firms receive one unit in the reallocation market and thus an increase in liquidation of medium projects cannot raise aggregate output.^{9,10}

⁹A similar result holds when $Z > p_L + p_M$. In such a case, either H firms are liquidated in equilibrium (for lower λ), or they are not capital constrained (for higher λ). Thus, if all H firms continue they cannot be capital constrained, leaving no room for external finance to improve the allocation of capital.

¹⁰The fact that external finance cannot be beneficial when the pledgeability parameter is higher than λ_1 is driven by the fact that our model only has three types of technologies. When all the high productivity firms

The reason higher financing requirements improve the reallocation of capital is that, because of limited pledgeability, medium productivity firms do not voluntarily liquidate their projects to invest in high productivity firms unless forced to do so by outside investors. Outside investors require liquidation as this is the only way they can get their money back.¹¹ Even though this forced liquidation by outside investors is privately inefficient from the perspective of an individual firm, it is beneficial for the economy as a whole because the additional capital supplied to the external market finds its way to a better user.

It is important to note that the potential benefits of an increase in external finance are a positive externality that higher liquidation generates to other firms in the economy. The firm that seeks outside finance does not benefit from it because the market does not fully compensate this firm for the capital that it releases. In other words, introducing an outside investor that requires liquidation when the firm is of medium productivity is suboptimal for a firm in isolation. Firms seek external finance only when they do not have sufficient funds to invest.

3. Empirical implications and tests

In this section we summarize the empirical implications of our model, compare them to the available empirical evidence, and present new empirical tests of one of our novel implications.

In the model, increases in pledgeability allow projects to raise capital more easily in the reallocation market. Since this capital would have ended up invested in the general technology, an increase in pledgeability leads to improved capital allocation. Levine (1991), Bencivenga et al. (1995), and Shleifer and Wolfenzon (2002) also predict that financial frictions reduce allocative efficiency. Wurgler's (2000) finding that the efficiency of capital allocation is positively correlated with investor protection and financial development supports this prediction.

A novel implication of our model is that the external financing needs of firms in a given country should have an independent effect on allocative efficiency. We show that

(footnote continued)

receive one unit of capital, further reallocation is not possible. With a continuum of productivities, external finance would always allow capital to flow from lower to higher productivity firms. The potential benefit would only disappear when all firms receive the efficient amount of capital.

¹¹In the model we assume for simplicity that the returns from the first unit of capital invested cannot be pledged. Since in the interesting range where high productivity firms are capital constrained, medium productivity firms receive no additional capital, it is clear why an outside investor prefers to liquidate a medium productivity firm rather than to allow such a firm to continue. However, this result would hold more generally if we allowed the first unit to be pledged. In order to see this, suppose that the same fraction λ of both units could be pledged. In all possible equilibria in which high productivity firms are capital constrained, the equilibrium return would satisfy

$$R^* \geq 2\lambda Y_H > \lambda Y_M.$$

Thus, since $R^* > \lambda Y_M$, it would still be true that outside investors strictly prefer to liquidate a medium productivity firm that cannot get additional capital in the external market.

as long as the degree of pledgeability is not too low, the resources that are released by the higher liquidation induced by external financiers will find their way to more productive activities. This argument suggests that controlling for investor protection, higher financing needs will be associated with a more efficient allocation of capital.

One might worry that the external financing requirement of firms in different countries is endogenous. In particular, it would be natural to expect a firm's external financing requirement to be directly related to the level of investor protection. In countries with worse investor protection, firms might end up having lower financing requirements either because they optimally choose to operate in industries with lower capital needs, or because they retain more internal funds to meet financing requirements.¹² Such possibilities would not invalidate our main argument. Even when an increase in retained earnings, for example, is an optimal response to an environment of low investor protection, it does not necessarily follow that such an increase in savings improves the equilibrium allocation of capital, because higher savings might reduce liquidation and hamper capital reallocation.

Furthermore, to the extent that external financing needs have an exogenous component, one should be able to uncover the effect of external finance on the allocation of capital by relating the efficiency of capital allocation to the external financing needs of firms, after controlling for other determinants of allocative efficiency. Previous literature has argued that because of technological reasons, the need for external finance at the industry level will be to some extent exogenous, and can be measured by the actual use of external finance in a relatively well-developed capital market (Rajan and Zingales, 1998). To the extent that the industrial structure of different countries also has an exogenous component (determined, for example, by historical and geographical considerations), we predict that a country's aggregate demand for external finance should be positively related to the efficiency of capital allocation.

With the above caveat in mind, we test the hypothesis that external financing needs increase the efficiency of capital allocation by regressing Wurgler's (2000) measure of the allocative efficiency (the industry elasticity of investment to value added, η) on the aggregate external financing need of the manufacturing sector (EFN) and controls:

$$\eta_c = \alpha + \beta EFN_c + \delta \text{Controls}_c \quad (5)$$

where the subscript refers to country c .

Our measure of the aggregate external financing need EFN_c is

$$EFN_c = \sum \omega_{ic} EFN_i \quad (6)$$

where ω_{ic} is industry's i share in total manufacturing output in country c (data from UNIDO),¹³ and EFN_i is the external finance dependence of industry i from Rajan

¹²Dittmar et al. (2003) show evidence that firms hold more cash in countries where shareholder protection is poor. Their interpretation of the evidence, however, is that firms in countries with poor investor protection hold more cash to increase managerial discretion, in conflict with shareholder interests.

¹³We thank Matias Braun for providing this data.

Table 1

Summary statistics—index of external finance dependence (EFN)

This table displays summary statistics for the index of external finance dependence. This index is computed as a weighted average of the industry-level measures of external finance dependence taken from [Rajan and Zingales \(1998\)](#). The weights are the industrial shares in total manufacturing output. Data is from UNIDO, Industrial Statistics Database.

Country	EFN	Country	EFN	Country	EFN
Australia	0.261	Hong Kong	0.377	Norway	0.262
Austria	0.265	India	0.284	Pakistan	0.157
Bangladesh	0.160	Indonesia	0.191	Panama	0.179
Barbados	0.199	Iran	0.276	Peru	0.175
Belgium	0.193	Ireland	0.316	Singapore	0.417
Bolivia	0.144	Israel	0.387	Spain	0.254
Canada	0.273	Italy	0.307	Swaziland	0.172
Chile	0.120	Japan	0.367	Sweden	0.307
Colombia	0.208	Jordan	0.068	Tanzania	0.183
Cyprus	0.150	Kenya	0.223	Trinidad	0.157
Denmark	0.301	Korea	0.281	Turkey	0.177
Ecuador	0.246	Kuwait	0.138	UK	0.310
Egypt	0.208	Macao	0.243	US	0.346
El Salvador	0.184	Malawi	0.207	Uruguay	0.153
Ethiopia	0.131	Malaysia	0.274	Venezuela	0.151
Fiji	0.201	Malta	0.252	Zimbabwe	0.186
Finland	0.268	Mexico	0.239		
France	0.300	Morocco	0.219		
Germany	0.327	Netherlands	0.293		
Greece	0.229	New Zealand	0.256	Mean	0.211
Guatemala	0.218	Nigeria	0.248	STD	0.079

and [Zingales \(1998\)](#). We show summary statistics on our index of external financing needs in [Table 1](#). We have data for 58 countries.

We use two alternative sets of control variables. First, we control for financial and economic development. Wurgler uses 1960 per capita *GDP* to measure economic development, and he uses the sum of the ratios of stock market capitalization to *GDP* and of private credit to *GDP* to measure financial development. These ratios are the averages of three years, 1980, 1985, and 1990. By construction, Wurgler's measure of financial development (which we call *FD*) is likely to be highly correlated with the extent of external financing needs. In fact, the correlation in our sample is 0.59. Thus, we use alternative measures of financial development that attempt to more directly capture the efficiency of the financial system, as opposed to its size, and would thus not be mechanically linked with external finance. Specifically, we use bank *net interest margin* to measure the efficiency of the banking sector and *turnover* to measure the liquidity of the stock market. [Demirguc-Kunt and Levine \(2001\)](#) suggest that bank *net interest margin* (measured as the 1990–1995 average bank interest income minus interest expense over total assets) captures the inefficiency of the banking sector, and [Demirguc-Kunt and Levine \(1996, 2001\)](#) suggest using *turnover* (the 1980–1995 average values of trades of domestic equities as a share of

the value of domestic equities) to capture stock market liquidity. Because these two variables capture different aspects of financial efficiency, we use them together as controls when estimating Eq. (5), as an alternative to the variable *FD*. We obtain *turnover* and *net interest margin* data from the CD included in [Demirguc-Kunt and Levine \(2001\)](#).

The second set of control variables that we use are measures of the legal protection of outside investors. Because the model suggests that external finance influences the efficiency of capital allocation beyond the effect of pledgeability, the regressions that control for investor protection might capture the model's prediction more directly. We use [La Porta et al. \(1998\)](#) measures of shareholder and creditor rights (*share rights* and *creditor rights*), and also a measure of the effectiveness of law enforcement ([La Porta et al. \(1998\)](#) *rule of law* variable). We also use the index of effective investor rights constructed by [Wurgler \(2000\)](#), which is equal to the sum of *share rights* and *creditor rights*, multiplied by *rule of law*. This index, which we call *RIGHTS*, captures the intuition that it is the interaction between pro-investor laws and their enforcement that matters for financial variables. Finally, because these measures of investor protection are not available for some countries in our sample, we use countries' legal origins to proxy for investor protection in alternative specifications, instead of the measure of investor rights. [La Porta et al. \(1998\)](#) have shown that legal origin explains cross-country variation in the measures of investor rights and law enforcement.

One important consideration is that, because our model focuses on the efficiency of capital reallocation across firms, we expect the effect of external financing needs to be especially strong for the within-year measure of the efficiency of capital allocation. The within-year measure is the elasticity of industry investment to value-added controlling for year fixed effects. As explained by [Wurgler \(2000\)](#), the overall elasticity of investment to value added is also affected by the between-year component, which measures how overall investment in the manufacturing sector responds to time series variation in value added for the whole manufacturing sector in the country. Thus, the within-year component of the elasticity provides a cleaner measure of how capital responds to value-added shocks across different industries in the same country. This measure is much closer to the idea of capital reallocation in our model. To the extent that the elasticity of investment to value-added is positively correlated with external finance, we expect this correlation to be driven mostly by the within-year component.¹⁴

In [Table 2](#) we check the extent to which the aggregate external financing need of a given country is correlated with the efficiency of capital allocation, after controlling for economic and financial development (the first set of controls). In column (1) we replicate Wurgler's main finding that overall financial development (measured by the size of capital markets and intermediary's assets) is positively correlated with the efficiency of capital allocation. Column (2) shows evidence that *EFN* is also positively correlated with the efficiency of capital allocation, when controlling for *GDP* (column 2). When we include both *FD* and *EFN* in the same regression together

¹⁴Jeff Wurgler provides data on within- and between-year components for each country on his website.

Table 2

Efficiency of capital allocation as a function of financial development and external finance dependence

The dependent variable is Wurgler's (2000) measure of the efficiency of capital allocation, the elasticity of industry investment to value-added. In columns (1)–(4), we use the overall elasticity of investment to value-added. In columns (5)–(8), we use the between-year elasticity of investment to value-added, and in columns (9)–(12), we use the within-year elasticity to measure the efficiency of capital allocation. The first independent variable, *EFN*, is the aggregate external financing needs summarized in Table 1. The second independent variable, *FD*, is Wurgler's (2000) measure of financial development, which is equal to the log of one plus the average sum of stock market capitalization and private credit to GDP (1980–1990 averages). The third independent variable, *GDP*, is the 1960 value of log per capita GDP. The next independent variable, *turnover*, is the ratio of stock market turnover to GDP. The last independent variable, *net interest margin*, is measured as the 1990–1995 average bank interest income minus interest expense over total assets. The last two variables come from Demircuc-Kunt and Levine (2001). Robust *t*-statistics are in parentheses.

	Dependent variable: η overall				Dependent variable: η between-year				Dependent variable: η within-year			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
<i>EFN</i>		1.354*** (2.89)	0.763 (1.60)	0.996* (1.92)		1.086 (1.47)	0.320 (0.33)	2.073 (1.08)		1.145*** (2.90)	0.673* (1.96)	1.152** (2.15)
<i>FD</i>	0.323*** (3.09)		0.217* (1.77)		0.419** (2.15)		0.425* (1.72)		0.213** (2.59)		0.010 (0.98)	
<i>GDP</i>	0.154*** (5.07)	0.159*** (5.78)	0.143*** (4.42)	0.161*** (4.54)	0.090 (0.88)	0.107 (1.16)	0.064 (0.61)	0.128 (0.96)	0.053* (1.76)	0.046 (1.47)	0.045 (1.39)	0.022 (0.53)
<i>Turnover</i>				0.154 (1.05)				−0.157 (−0.32)				−0.020 (−0.11)
<i>Net interest margin</i>				−1.179 (−0.89)				5.340 (0.74)				−0.472 (−0.35)
Intercept	0.162*** (3.01)	0.014 (0.14)	0.051 (0.53)	0.104 (0.75)	0.432*** (2.98)	0.397* (1.69)	0.388 (1.62)	0.128 (0.96)	0.091* (1.78)	−0.052 (−0.60)	0.001 (0.02)	0.001 (0.01)
Num. observations	61	58	57	50	53	57	51	50	61	58	57	50
<i>R</i> ²	0.52	0.50	0.52	0.49	0.15	0.10	0.14	0.10	0.21	0.23	0.21	0.18

***, **, * indicate statistical significance at 1%, 5%, and 10% (two-tail) test levels, respectively.

with *GDP* (column 3), the significance of both variables is drastically reduced. This result suggests that because *FD* and *EFN* are highly correlated by construction, it is hard to disentangle the effect of changes in external financing needs from variations in Wurgler's measure of financial development *FD*. In column (4) we use our alternative measures of financial development (*turnover* and *net interest margin*) to try to mitigate this problem. Consistent with our expectations, the effect of external finance on allocative efficiency and its statistical significance are higher in this alternative specification.

In columns (5) to (12), we separately examine the two components of the elasticity of investment to value-added, the within- and between-components. As explained above, the within-year component better captures the intuition of our theory, and thus we expect *EFN* to have a stronger effect on this component. In contrast, as discussed by Wurgler (2000), overall financial development affects both the between and within components. We replicate Wurgler's result in columns (5) and (9). In columns (6)–(8) we show that *EFN* is not significantly related to the between-year component, even when we do not control for *FD*. In contrast, in columns (10)–(12) we show that *EFN* is significantly correlated with the within-year component of the elasticity, as suggested by our theory. This is true even when including both *EFN* and *FD* in the same regression (column 11). Again, significance increases when using *turnover* and *net interest margin* instead of *FD* (column 12). Thus, consistent with the intuition behind our theory, external financing needs seem to affect mostly the within-year component of the elasticity of investment to value-added.

In Table 3 we use our alternative set of control variables that capture the extent of the legal protection of outside investors. Again, we use the overall elasticity of investment to value-added (columns (1)–(3)), and also the between (columns (4)–(6)) and within-year components (columns (7)–(9)). The results are consistent with those reported in Table 2. External finance seems to affect mostly the within-year component of the elasticity of investment to value-added. This conclusion also holds when controlling for measures of investor protection and legal origin, as shown in columns (7)–(9). Notice that statistical significance increases when using legal origin instead of measures of investor protection, partly due to the increase in the number of observations in the regressions (from 40 to 58 countries).

As discussed above, an important caveat in interpreting these regression results is the potential endogeneity of a country's external financing need. For example, external financing needs might be determined by a country's overall level of financial development and outside investor protection. In countries with low financial development and/or poor protection of outside investors, firms might optimally choose to operate in industries with lower capital needs, which might therefore grow faster and come to dominate the economy. While the regressions in Tables 2 and 3 provide some evidence that external financing needs have an effect on allocative efficiency over and above that which can be explained by financial development and investor protection, we also attempt to provide additional evidence of an independent effect of external finance by pursuing two alternative approaches.

First, we regress *EFN* on measures of both financial development and investor protection, and collect the predicted values and the residuals of *EFN*. Presumably,

Table 3

Efficiency of capital allocation as a function of measures of investor protection and legal origin

The dependent variable is Wurgler's (2000) measure of the efficiency of capital allocation, the elasticity of industry investment to value-added. In columns (1)–(3), we use the overall elasticity of investment to value-added. In columns (4)–(6), we use the between-year elasticity of investment to value-added, and in columns (7)–(9), we use the within-year elasticity as the measure of the efficiency of capital allocation. The first independent variable, *EFN*, is the aggregate external financing needs summarized in Table 1. The next three independent variables are the number of important shareholder (*share rights*) and creditor (*creditor rights*) rights in the country's legal code, and a measure of the rule of law (*rule of law*). These three measures come from La Porta et al. (1988). The fifth independent variable, *RIGHTS*, is an index of effective investor rights. It is constructed by Wurgler (2000) as the product of *rule of law* and the sum of *share rights* and *creditor rights*. The last three independent variables are dummies for the country's legal origin and come from La Porta et al. (1998). Robust *t*-statistics are in parentheses.

	Dependent variable: η overall			Dependent variable: η between-year			Dependent variable: η within-year		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
<i>EFN</i>	0.723 (1.29)	1.153 (1.55)	2.077*** (4.07)	−0.732 (−0.47)	−0.005 (−0.00)	1.710* (1.69)	0.902** (2.05)	0.686* (1.75)	1.246*** (3.02)
<i>Share rights</i>	−0.016 (−0.72)			−0.057 (−0.98)			0.027 (1.23)		
<i>Creditor rights</i>	−0.021 (−0.84)			0.024 (0.28)			0.010 (0.39)		
<i>Rule of law</i>	0.093*** (3.96)			0.137*** (3.58)			0.012 (0.58)		
<i>RIGHTS</i>		0.035 (1.56)			0.051 (1.29)			0.031* (1.70)	
<i>French origin</i>			0.114 (1.60)			0.068 (0.33)			−0.009 (−0.15)
<i>German origin</i>			0.304*** (3.30)			0.274 (1.50)			0.070 (0.81)
<i>Scandinavian origin</i>			0.244*** (2.98)			0.388** (2.23)			0.030 (0.30)
Intercept	0.069 (0.45)	0.120 (0.79)	−0.126 (−0.96)	0.624** (2.22)	0.720 (1.64)	0.283 (0.78)	−0.093 (−0.66)	0.002 (0.02)	−0.042 (−0.42)
Num. observations	40	40	58	40	40	58	40	40	58
R^2	0.48	0.24	0.40	0.11	0.02	0.10	0.20	0.22	0.21

***, **, * indicate statistical significance at 1%, 5%, and 10% (two-tail) test levels, respectively.

the residuals of this regression contain information on determinants of external financing needs that are unassociated with both financial development and investor protection. We then regress the within-year elasticity of investment to value-added on the predicted values and residuals.¹⁵ In order to maximize the number of observations, we use legal origin instead of the direct measures of investor protection. We use *turnover* and *net interest margin* to measure financial development, but the results are qualitatively identical when we use Wurgler's measure of financial development. For brevity, we report the results in the text. We find the following coefficients and *t*-statistics:

$$\eta_c^{\text{within}} = -0.054 + 1.359(EF_{\text{predicted}}) + 1.251(EF_{\text{residuals}}) \quad (7)$$

(−0.40)
(2.47)
(2.45)

where the coefficients on both $EF_{\text{predicted}}$ and $EF_{\text{residuals}}$ are significant at a 5% confidence level. The coefficient on $EF_{\text{residuals}}$ suggests that the determinants of EFN that are unassociated with both legal origin and financial development have a significant effect on the efficiency of capital allocation.

A more direct approach to tackle the endogeneity issue is to search for exogenous determinants of EFN that can be used as instruments for EFN in the type of regressions reported in Tables 2 and 3. Recent literature has argued that predetermined variables such as geographic endowments and a country's natural proclivity for the production of certain commodities is associated with economic development, perhaps through their effect on long-lasting institutions (Easterly and Levine, 2003). In the context of this paper, a particularly appealing hypothesis is that a country's industrial structure (and thus its external financing needs) is partly determined by that country's commodity endowments. To illustrate this point, consider the case of Chile. Chile has very low external finance dependence (0.120, which is in the 5th percentile), because Chilean industry is heavily concentrated in non-ferrous metals (copper), an industry with low financial dependence. This concentration is clearly historically determined by Chile's natural copper endowment, and thus the low external financing dependence of Chilean manufacturing industry has an arguably exogenous determinant.

To perform a more systematic analysis, we collect data on our sample countries' production of a given set of leading commodities. We follow Easterly and Levine's (2003) approach and construct dummies for whether a country produced any amount of the commodity in 1998–1999. Easterly and Levine (2003) argue that, while the produced quantity of a given commodity is endogenously determined, whether *any* amount of the commodity is produced reflects exogenous characteristics. The set of commodities is the same one used in Easterly and Levine, and includes bananas, coffee, copper, maize, millet, oil, rice, rubber, silver, sugarcane, and wheat. Because their data only include countries that are former colonies, we refer to the original sources to collect data for all the countries in our sample. We obtain data for the production of crops from the Food and Agricultural Organization statistics. Data for metals and for oil come from the World Mineral

¹⁵We thank an anonymous referee for suggesting this test.

Statistics yearbook. Similarly to Easterly and Levine, we use all commodities together in the regressions, and do not attempt to choose a specific subset of commodities.

We use the commodity dummies as instruments for *EFN* and estimate Eq. (5) using a two-stage least squares procedure. We use the within-year elasticity as the measure of allocative efficiency. Easterly and Levine (2003) show that commodity endowments help explain cross-country differences in economic development (through their effect on institutions). Thus, we include *GDP* among the set of control variables to control for the possibility that commodity dummies affect the efficiency of capital allocation through their effect on economic development, and not through their effect on external financing needs. Given that the model predicts an effect of external financing needs on allocative efficiency over and above the effect of investor protection, we also include the country's legal origin among the set of controls to account for cross-country variations in investor protection. We choose not to use the more direct measures of investor protection because of the decrease in the number of observations. Therefore, we estimate the following model:

$$\text{Second stage : } \eta^{\text{within}} = \alpha + \beta \text{EFN} + \gamma \text{GDP} + \delta(\text{Legal origin}) + u \quad (8)$$

$$\begin{aligned} \text{First stage : } \text{EFN} = a &+ \mathbf{b}(\text{Commodity dummies}) + c\text{GDP} \\ &+ \mathbf{d}(\text{Legal origin}) + v. \end{aligned} \quad (9)$$

The results (which we report in the text below) suggest that the cross-country variation in external financing needs that is explained by the commodity dummies is significantly related to the efficiency of capital allocation. The first-stage F-test that the coefficient vector \mathbf{b} is equal to zero suggests that the commodity dummies do explain cross-country variation in external financing needs. The F-statistic is equal to 2.46, with a *p*-value of 0.018.¹⁶ Moreover, the coefficient β on *EFN* in the second-stage regression is equal to 1.877, with a *t*-statistic of 2.06 and a *p*-value of 0.044. The positive and significant β is evidence that exogenous variation in external financing needs is indeed related to the within-year elasticity of investment to value-added.

In sum, the results in this section provide some support for the prediction that external finance and the efficiency of capital allocation are positively related. The positive correlation between external financing needs and Wurgler's (2000) measure of allocative efficiency (the elasticity of industry investment to industry value-added) remains after controlling for several possible determinants of this elasticity, including overall financial development and measures of investor protection. The correlation also seems to be driven by the component of the elasticity that is more directly related to the model's predictions (the within-year component), and it remains significant after accounting for the endogeneity of external finance.

¹⁶We use heteroskedasticity-consistent standard errors to draw all inferences.

4. Conclusion

We develop an equilibrium model to understand how the efficiency of capital allocation depends on outside investor protection and firms' external financing needs. Because of poor investor protection, firms may not be able to raise finance for their high productivity projects while, at the same time, other firms fail to liquidate mediocre projects and to supply their capital to the market. We show that the presence of outside investors improves the allocation of capital by increasing the liquidation of mediocre projects.

We present some empirical evidence that is consistent with one of the novel implications of the model, namely that the aggregate demand for external finance is positively correlated with the efficiency of capital allocation. We interpret these results as suggestive that the specific mechanism in our model is influencing the speed and efficiency with which different systems accomplish capital reallocation across different sectors. However, further empirical research is required to firmly establish this conclusion. For example, while we report some evidence that exogenous variation in external financing needs increases Wurgler's (2000) measure of allocative efficiency, we cannot rule out the possibility that the instruments that we use for external finance (commodity endowments) influence allocative efficiency through other mechanisms that we do not control for. It would also be desirable to experiment with alternative measures of the efficiency of capital allocation, and if possible, explore alternative data sets that allow the researcher to control more effectively for important variables such as investor protection.

On the other hand, we do believe that the mechanism we describe in this paper can help explain cross-country differences in the equilibrium efficiency of investment. Our main argument is that the extent to which firms rely on external finance affects financial arrangements (the allocation of control in corporations between external and internal investors) and this, in turn, influences economic efficiency. Thus, our paper proposes a new channel through which financial arrangements affect economic efficiency. While our model is by nature a static one, this is mostly for theoretical simplicity. Extensions of our theory could try to integrate our arguments into dynamic theories of comparative advantage and into theories of the endogenous evolution of financial arrangements. A dynamic version of our model might also allow us to determine whether the effects we model in this paper can help explain cross-country differences in income growth and in overall economic development.

Appendix A

A.1. Proof of Lemma 1

We explain in the text that, given investors' allocation rule, projects' offers are optimal. We also explain in the text that investors' allocation rule is optimal. We only need to show that, for the proposed equilibrium return, R^* , and allocation rule, r^j , the market clears.

Recall that R^* is defined by the solution to the equation in footnote 6, which can be rearranged as

$$0 \leq K + T - x^{-1}(R^*) - \int_{\{j \in C | \bar{p}^j > R^*\}} dj \leq \int_{\{j \in C | \bar{p}^j = R^*\}} dj. \quad (\text{A.1})$$

Plugging the equilibrium allocation in the market clearing condition, we need to show that

$$x^{-1}(R^*) + \int_{\{j \in C | \bar{p}^j > R^*\}} dj + \bar{r} \int_{\{j \in C | \bar{p}^j = R^*\}} dj = K + T. \quad (\text{A.2})$$

Consider two cases. First, consider the case in which $\int_{\{j \in C | \bar{p}^j = R^*\}} dj = 0$. In this case, Eq. (A.1) reduces to Eq. (A.2) and thus the market clears. The second case is one in which $\int_{\{j \in C | \bar{p}^j = R^*\}} dj > 0$. In this case, any \bar{r} is consistent with investor maximization behavior and thus we are free to choose the value that satisfies Eq. (A.2). However, because \bar{r} is a probability, we need to check that it falls between 0 and 1. This fact can be confirmed using Eq. (A.1).

A.2. Proof of Lemma 2

In the following lemma we cover all cases for $Z < p_L + p_M$; the version of Lemma 2 described in the main text is a special case.

Lemma 2. *There are functions $\lambda_k(Z)$ for $k = 0, 1, 2, 3, 4, 5$ such that equilibrium contracts offered by entrepreneurs at date t_0 are:*

- a. For $Z \leq p_L$, $(q_L, D_{1L}, D_{2L}) = (1, Z/p_L, 0)$; $(q_M, D_{1M}, D_{2M}) = (0, 0, 0)$; $(q_H, D_{1H}, D_{2H}) = (0, 0, 0)$.
- b. For $p_L < Z \leq p_L + p_H \frac{Y_H}{Y_M} - p_H$, $(q_L, D_{1L}, D_{2L}) = (1, 1, 0)$, $D_{1M} = D_{1H} = 1$, $q_H = 0$ and

1. For $\lambda < \lambda_0$, $q_M = \frac{Z - p_L}{p_M}$, $D_{2M} = D_{2H} = 0$.
2. For $\lambda_0 \leq \lambda \leq \lambda_1$, the contract is identical to that in Case b.1.
3. For $\lambda_1 < \lambda \leq \lambda_2$, $q_M = ((Z - p_H D_{2H})/\hat{R} - p_L)/p_M$, $D_{2M} = 0$, and $D_{2H} = \lambda Y_H - \hat{R}$, where \hat{R} is implicitly defined by $x(K + Z - \frac{\lambda Y_H}{\hat{R}} p_H) = \hat{R}$.
4. For $\lambda_2 < \lambda$, $q_M = 0$, $D_{2M} = 0$, $D_{2H} = \frac{\hat{R}(Z - p_L)}{p_H}$, where

$$\hat{R} = \begin{cases} x(K + p_L - p_H) & \text{if } \lambda Y_M < x(K + p_L - p_H) \\ \lambda Y_M & \text{if } x(K + p_L - p_H) \leq \lambda Y_M \leq x(K + p_L - p_H - p_M) \\ x(K + p_L - p_H - p_M) & \text{if } \lambda Y_M > x(K + p_L - p_H - p_M). \end{cases}$$

- c. For $p_L + p_H \frac{Y_H}{Y_M} - p_H < Z \leq p_L + p_M$, $(q_L, D_{1L}, D_{2L}) = (1, 1, 0)$, $D_{1M} = D_{1H} = 1$, $q_H = 0$ and

1. For $\lambda < \lambda_0$, the contract is identical to that in Case b.1.

2. For $\lambda_0 \leq \lambda \leq \lambda_1$, the contract is identical to that in Case b.1.
3. For $\lambda_1 < \lambda \leq \lambda_3$, the contract is identical to that in Case b.3.
4. For $\lambda_3 < \lambda \leq \lambda_4$, $q_M = ((Z - p_H D_{2H})/\hat{R} - p_L)/p_M$
 $D_{2M} = 0$, and $D_{2H} = \lambda Y_H - \hat{R}$, where $\hat{R} = \lambda Y_M$.
5. For $\lambda_4 < \lambda \leq \lambda_5$,

$$q_M = \frac{Z - \frac{\lambda}{\hat{R}}(p_H Y_H + p_M Y_M) - p_L + p_M + p_H}{p_M(2 - \frac{\lambda}{\hat{R}} Y_M)},$$

$D_{2M} = \lambda Y_M - \hat{R}$, and $D_{2H} = \lambda Y_H - \hat{R}$, where \hat{R} satisfies $x(K + 2(p_L + p_M q_M(\hat{R})) - 1) = \hat{R}$.

6. For $\lambda > \lambda_5$, $q_M = 0$, and D_{2M} , D_{2H} can take any values that satisfy $p_M D_{2M} + p_H D_{2H} + p_L \hat{R} = Z \hat{R}$, $D_{2M} \leq \lambda Y_M - \hat{R}$, and $D_{2H} \leq \lambda Y_H - \hat{R}$, where \hat{R} is defined by $\hat{R} = x(K + p_L - p_H - p_M)$.

Proof. We focus on symmetric equilibria in which all entrepreneurs offer the same contract. We denote the proposed equilibrium contract by $\{q_s, D_{1s}, D_{2s}\}_{s=L,M,H}$ (no superscript j). Note that \bar{P}_s is the same for all entrepreneurs. We refer to the common value by \bar{P}_s . Also, because all entrepreneurs are identical and offer the same date- t_0 contract, their probability of getting additional capital as a function of the realized productivity is the same in equilibrium. We denote by r_s the probability that an entrepreneur receives additional funds when the realized productivity of his project is s .

To verify that a date- t_0 contract is an equilibrium contract, we proceed in two steps. First, assuming all entrepreneurs offer the same contract, we use Lemma 1 to find the equilibrium return, R^* , and allocation rule, r , in the date- t_1 reallocation market. The second step is to show that, given the equilibrium R^* and r found in the first step, no entrepreneur can profitably deviate by offering a different date- t_0 contract.

Below we apply these two steps to each of the cases listed in Lemma 2. To simplify those derivations, we state some preliminary results. The date- t_0 contract determines the values of \bar{P}_H and \bar{P}_M and also the aggregate amount of liquidation, T . Using Lemma 1 and the equation in footnote 6, we compute R^* and the allocation of capital to projects as a function of these variables. Table A1 is derived assuming that $\bar{P}_H \geq \bar{P}_M$ (which will always be the case).

Because $Y_L = 0$, projects cannot pledge anything when the realized productivity is low and, as a result, r_L is always 0. Fig. 2 is a graphical representation of Table A.1 in the case in which $\bar{P}_H = \lambda Y_H$. Note that Case 1 of Table A.1 ($\lambda Y_H < x(K + T)$) corresponds to Panel A of Fig. 2. In this case all the capital is allocated to the general technology because the high productivity projects cannot offer a sufficiently high return to attract capital. Case 2 of Table A.1 correspond to Panel B of Fig. 2. In this case, an amount $x^{-1}(\lambda Y_H)$ is allocated to the general technology and the rest of the capital supply, $K + T - x^{-1}(\lambda Y_H)$, is allocated to high productivity projects, each receiving capital with the same probability. The equilibrium return is λY_H and the condition $\lambda Y_H \leq x(K + T - p_H)$, which is equivalent to $x^{-1}(\lambda Y_H) \geq K + T - p_H$, guarantees that $r_H \leq 1$.

Table A.1

Date- t_1 equilibrium: R^* , r_H and r_M as a function of T , \bar{P}_H and \bar{P}_M

Case	Range	R^*	r_H	r_M
1	$\bar{P}_H < x(K + T)$	$x(K + T)$	0	0
2	$x(K + T) \leq \bar{P}_H \leq x(K + T - p_H)$	\bar{P}_H	$\frac{K+T-x^{-1}(\bar{P}_H)}{p_H}$	0
3	$\bar{P}_M < x(K + T - p_H) < \bar{P}_H$	$x(K + T - p_H)$	1	0
4	$x(K + T - p_H) \leq \bar{P}_M \leq x(K + T - p_H - p_M)$	\bar{P}_M	1	$\frac{K+T-p_H-x^{-1}(\bar{P}_M)}{p_M}$
5	$x(K + T - p_H - p_M) < \bar{P}_M$	$x(K + T - p_H - p_M)$	1	1

The second preliminary result regards a deviation by entrepreneur j at date t_0 . To check that the proposed contract is an equilibrium contract, we check that no entrepreneur has an incentive to deviate. Given the equilibrium return R^* and the allocation rule r , the best contract $\{q_s^j, D_{1s}^j, D_{2s}^j\}_{s=L,M,H}$ (we use the superscript j to denote the deviation) the entrepreneur can offer at date t_0 maximizes his date- t_2 payoff

$$U_E^j = \sum_s p_s \{q_s^j (1 - D_{1s}^j) R^* + (1 - q_s^j) [Y_s + r_s^j (Y_s - D_{2s}^j - R^*)]\}, \quad (\text{A.3})$$

subject to the participation constraint of the date- t_0 investor

$$U_I^j = \sum_s p_s \{q_s^j D_{1s}^j R^* + (1 - q_s^j) r_s^j D_{2s}^j\} \geq ZR^*. \quad (\text{A.4})$$

With probability p_s , the realized productivity of the project is s . In that event, the project is liquidated with probability q_s^j , as specified in the contract. The first term inside the braces in Eq. (A.3) is the entrepreneur's payoff conditional on liquidation and the second one is his payoff conditional on the project continuing until date t_2 . When the project is liquidated, the entrepreneur pays D_{1s}^j to the date- t_0 investor and can invest the rest at the market return of R^* . When the project is not liquidated, the unit of capital invested at date- t_0 yields Y_s at date- t_2 . In addition, the entrepreneur raises an additional unit of capital with probability r_s^j (r_s^j can be found by applying the equilibrium allocation rule in Eq. (3) to the contract $\{q_s^j, D_{1s}^j, D_{2s}^j\}_{s=L,M,H}$). The additional unit of capital generates Y_s at date t_2 . In addition, because the returns from the second unit are pledgeable, the entrepreneur must pay the promised D_{2s}^j to the date- t_0 investor and R^* to the date- t_1 investor. Eq. (A.4) simply states that the date- t_2 payoff of the date- t_0 investor should be at least ZR^* , which is the amount he could have gotten by simply storing his Z units of capital from date t_0 date t_1 and then investing at the market return R^* .

It is optimal for the entrepreneur to offer a contract that keeps the date- t_0 investor at his participation constraint. Because utility can be transferred one-to-one from the entrepreneur to the date- t_0 investor, the entrepreneur's problem is equivalent to maximizing the combined payoff

$$U_E^j + U_I^j = \sum_s p_s \{q_s^j R^* + (1 - q_s^j) [Y_s + r_s^j (Y_s - R^*)]\}, \quad (\text{A.5})$$

subject to the date- t_0 investor participation constraint.

In the text we show that $Y_L < R^* < Y_M < Y_H$ (Eq. (4)). With this condition, it is clear that the liquidation probabilities that maximize the combined payoff are $q_L^j = 1, q_M^j = q_H^j = 0$. However, as we explain below, in some cases, to satisfy the participation constraint, the liquidation probability q_M^j needs to be strictly positive.

We now consider each of the cases of Lemma 2. We define

$$\begin{aligned}\lambda_0 &\equiv x(K + Z)/Y_H, \quad \lambda_1 \equiv x(K + Z - p_H)/Y_H, \\ \lambda_2 &\equiv \frac{x(K + p_L - p_H)}{Y_H} \left(1 + \frac{Z - p_L}{p_H}\right), \\ \lambda_3 &\equiv \frac{x(K + Z - p_H \frac{Y_H}{Y_M})}{Y_M}, \\ \lambda_4 &\equiv x \left(K + 2 \left(Z - \frac{p_H Y_H}{Y_M} + p_H \right) - 1 \right) / Y_M\end{aligned}$$

and

$$\lambda_5 \equiv \frac{Z - p_L + p_H + p_M}{p_H Y_H + p_M Y_M} x(K + p_L - p_M - p_H).$$

(a) The proposed equilibrium contract maximizes the combined payoff and leaves the date- t_0 investor with exactly ZR^* at date t_2 . Therefore, no entrepreneur can offer a better contract.

(b.1) and (c.1): We first find R^* using the proposed contract. The aggregate amount of liquidation is $T = \sum p_s q_s = p_L + p_M \frac{Z - p_L}{p_M} = Z$. Also, $\bar{P}_H = \lambda Y_H - D_{2H} = \lambda Y_H$ and $\bar{P}_M = \lambda Y_M - D_{2M} = \lambda Y_M$. The condition that $\lambda < \lambda_0$ implies that $\bar{P}_H < x(K + Z) = x(K + T)$. Therefore, we can use Case 1 of Table A.1 to get $R^* = x(K + Z)$.

Next, we check that no entrepreneur has incentives to deviate. Because $\bar{P}_H^j = \lambda Y_H - D_{2H}^j < R^*$, the entrepreneur cannot offer a sufficiently high return to raise additional capital in the high productivity state even when he sets D_{2H}^j to zero. The same is true in the medium productivity state. Thus, for $s = H$ and M , $p_s^j = 0$ and the value of D_{2s}^j is irrelevant in this case. Because the entrepreneur raises no additional capital at date t_1 and the returns from the unit of capital invested at date t_0 are not pledgeable at date t_2 , the date- t_0 investor needs to be paid out of the date- t_1 liquidation proceeds. As we explained above, liquidating the project in the low productivity state raises the joint surplus, but liquidating in the medium and high productivity states lowers it. Thus, it is optimal to pay the date- t_0 investor all the proceeds of the liquidation of the low productivity project, i.e., $D_{1L}^j = 1$. Because $p_L < Z$, liquidation in the low productivity state does not yield sufficient funds to satisfy the participation constraint. Thus, the entrepreneur needs to liquidate either the medium or high productivity project. Because liquidating any of these projects reduces the joint surplus, it is optimal to set $D_{1M}^j = D_{1H}^j = 1$ to minimize the probability of liquidation. Finally, because liquidating in the medium productivity state reduces total surplus less than liquidating in the high productivity state, the entrepreneur sets $q_M^j = \frac{Z - p_L}{p_M}$ and $q_H^j = 0$ to satisfy the investor participation

constraint. Because the best deviation is just the proposed equilibrium contract itself, we conclude that there are no profitable deviations at date t_0 .

(b.2) and (c.2): As in the proof of Cases b.1 and c.1, we have that the aggregate amount of liquidation is $T = \sum p_s q_s = p_L + p_M \frac{Z - p_L}{p_M} = Z$ and that $\bar{P}_H = \lambda Y_H - D_{2H} = \lambda Y_H$ and $\bar{P}_M = \lambda Y_M - D_{2M} = \lambda Y_M$. The condition that $\lambda_0 \leq \lambda \leq \lambda_1$ implies that $x(K + Z) \leq \lambda Y_H \leq x(K + Z - p_H)$ or $x(K + T) \leq \bar{P}_H \leq x(K + T - p_H)$. Therefore, we can use Case 2 of Table A.1 to get $R^* = \bar{P}_H = \lambda Y_H$.

Next, we check that no entrepreneur has incentives to deviate. First, $D_{2H}^j = 0$. According to the allocation rule in Eq. (3), if the entrepreneur sets $D_{2H}^j > 0$ then $\bar{P}_H^j = \lambda Y_H - D_{2H}^j < R^*$, and the entrepreneur raises no funds in the high productivity state, whereas if he sets $D_{2H}^j = 0$ so that $\bar{P}_H^j = R^*$, he raises capital with positive probability. Next, $\bar{P}_M^j = \lambda Y_M - D_{2M}^j < R^*$ for all D_{2M}^j so the value of D_{2M}^j is irrelevant. Following an identical argument as in the proof of Cases b.1 and c.1, we can show that $q_L^j = 1$, $q_M^j = \frac{Z - p_L}{p_M}$, $q_H^j = 0$, and $D_{1L}^j = D_{1M}^j = D_{1H}^j = 1$. Because the best deviation is just the proposed equilibrium contract itself, we conclude that there are no profitable deviations at date t_0 .

(b.3) and (c.3): One can show with some simple algebra that, in Case b (defined by $Z \leq p_L + p_H \frac{Y_H}{Y_M} - p_H$), it holds that $\lambda_2 \leq \lambda_3$, and that, in Case c, (defined by $Z > p_L + p_H \frac{Y_H}{Y_M} - p_H$) it holds that $\lambda_2 > \lambda_3$. As a result, in Cases b.3 and c.3 it holds that $\lambda_1 < \lambda \leq \min\{\lambda_2, \lambda_3\}$.

First, we find the equilibrium of the date- t_1 market. The aggregate amount of liquidation is $T = p_L + p_M q_M = Z - \frac{p_H D_{2H}}{R} = Z - \frac{\lambda Y_H}{R} p_H + p_H$. Note that $\bar{P}_H = \lambda Y_H - D_{2H} = \hat{R} = x(K + Z - \frac{\lambda Y_H}{R} p_H) = x(K + T - p_H)$, where the first equality is the definition of \bar{P}_H , the second one follows from the value of D_{2H} in the proposed equilibrium contract, the third one follows from the definition of \hat{R} , and the fourth one follows by using the expression for T derived above. Therefore, we can use Case 2 of Table A.1 to find that $R^* = \bar{P}_H = \hat{R}$. Also, note that $r_H = \frac{K + T + x^{-1}(\bar{P}_H)}{p_H} = 1$ and $r_M = 0$.

We now check whether an entrepreneur has incentives to deviate from the date- t_0 contract. Again, it is optimal to set $q_L^j = 1$ and $D_L^j = 1$. However, because $p_L < Z$, the proceeds from the liquidation of the low productivity project are not sufficient to satisfy the investor participation constraint. Because the joint surplus is reduced when the project is liquidated in the medium and high productivity states, the entrepreneur benefits by keeping those probabilities as low as possible. For this reason, instead of pledging funds out of the liquidation proceeds, it is better to pledge from the date- t_2 cash flows by setting D_{2H}^j as high as possible. However, there is an upper bound to D_{2H}^j . If this quantity is so high that $\bar{P}_H^j = \lambda Y_H - D_{2H}^j < R^*$, the entrepreneur does not raise any additional capital at date t_1 . However, by setting $\bar{P}_H^j \geq R^*$, he raises one unit of capital at date t_1 with certainty.¹⁷ Thus, the maximum D_{2H}^j that allows the entrepreneur to raise capital at date t_1 is $\lambda Y_H - R^*$. Below we show that $\lambda Y_M < R^*$ and thus D_{2M}^j is irrelevant. Finally, to minimize q_M^j further,

¹⁷The fact that he raises one unit for sure when $\bar{P}_H^j > R^*$ follows from Eq. (3). The fact that he raises one unit of capital for sure even when $\bar{P}_H^j = R^*$ follows from the derivation of r_H in the previous paragraph.

D_{1M}^j is set to 1. The value of q_M^j that satisfies the participation constraint is

$$q_M^j = \frac{Z - \frac{p_H D_{2H}}{R^*} - p_L}{p_M}.$$

Again, because the best deviation is the proposed contract itself, there are no profitable deviations.

We still need to show two things. First, we need to show that $\bar{P}_H \geq \bar{P}_M$ as assumed in deriving Table A.1. Second, we need to show that the contract is feasible, i.e., $0 \leq q_M \leq 1$ and $D_{2H} \geq 0$.

We define the function

$$F(R, \lambda) = x \left(K + Z - \frac{\lambda Y_H}{R} p_H \right) - R. \quad (\text{A.6})$$

The equation $F(R^*, \lambda) = 0$ implicitly defines R^* as a function of λ .

Since $\bar{P}_H = R^*$ and $D_{2M} = 0$, to show that $\bar{P}_H \geq \bar{P}_M$, it suffices to show that $R^* \geq \lambda Y_M$. The function $F(\lambda Y_M, \lambda) = x(K + Z - \frac{\lambda Y_H}{Y_M} p_H) - \lambda Y_M$ is strictly decreasing in λ and $F(\lambda_3 Y_M, \lambda_3) = 0$. Therefore $F(\lambda Y_M, \lambda) > 0$ for $\lambda < \lambda_3$. But since $F_R < 0$, it must be that $R^*(\lambda) > \lambda Y_M$ for all $\lambda < \lambda_3$.

To show that $D_{2H} \geq 0$, consider the function $F(\lambda Y_H, \lambda) = x(K + Z - p_H) - \lambda Y_H$, which is strictly decreasing in λ . It is also the case that $F(\lambda_1 Y_H, \lambda_1) = 0$ and consequently $F(\lambda Y_H, \lambda) < 0$ for $\lambda > \lambda_1$. But since $F_R < 0$, it must be that $R^*(\lambda) < \lambda Y_H$ and thus $D_{2H} > 0$ for all $\lambda > \lambda_1$.

To show that $q_M \geq 0$ we first compute

$$\frac{\partial R^*}{\partial \lambda} \frac{\lambda}{R^*} = - \frac{F_\lambda}{F_R} \frac{\lambda}{R^*} = 1 + \frac{1}{x(K + Z - \frac{\lambda Y_H}{R^*} p_H) \frac{\lambda Y_H p_H}{R^{*2}} - 1} < 1,$$

where

$$\frac{\partial R^*}{\partial \lambda} = - \frac{F_\lambda}{F_R}$$

by the implicit function theorem and the inequality follows because x' is negative. Now we compute

$$\frac{\partial q_M}{\partial \lambda} = - \frac{p_H Y_H}{R^*} \left(1 - \frac{\partial R^*}{\partial \lambda} \frac{\lambda}{R^*} \right) < 0.$$

Finally, note that $F(x(K + p_L - p_H), \lambda_2) = 0$ or equivalently $R^*(\lambda_2) = x(K + p_L - p_H)$. Plugging the value of λ_2 and R^* in the expression for q_M leads to $q_M = 0$. Since $\frac{\partial q_M}{\partial \lambda} < 0$, then for $\lambda < \lambda_2$, $q_M > 0$. The result that $q_M \leq 1$ follows from $Z \leq p_L + p_H$.

All other cases: These proofs are very similar to the previous cases. \square

A.3. Proof of Proposition 1

For $Z < p_L + p_M$, $q_H = 0$ and, therefore, the aggregate payoff can be written as

$$\Pi = p_H(1 + r_H)Y_H + p_M(1 - q_M)(1 + r_M)Y_M + \omega x(\omega), \quad (\text{A.7})$$

where $\omega = K + p_L + p_M q_M - p_H r_H - p_M(1 - q_M)r_M$.

In Case a, the contract specifies that $q_M = q_H = 0$, and $D_{2H} = D_{2M} = 0$. Thus, $T = p_L$, $\bar{P}_H = \lambda Y_H$, and $\bar{P}_M = \lambda Y_M$. Using these values in Table A.1, it can be readily seen that r_H and r_M are weakly increasing in λ . Since $\frac{d}{d\omega}(\omega x(\omega)) = x(\omega) + \omega x'(\omega) < Y_M < Y_H$, then $\frac{\partial \Pi}{\partial r_H} = p_H[Y_H - \frac{d}{d\omega}(\omega x(\omega))] > 0$, and $\frac{\partial \Pi}{\partial r_M} = p_M(1 - q_M)[Y_M - \frac{d}{d\omega}(\omega x(\omega))] \geq 0$. Therefore, in this region, $\frac{d\Pi}{d\lambda} \geq 0$. Finally, in this region, the equilibrium is independent of Z , and thus $\frac{d\Pi}{dZ} = 0$.

As we explain in the proof of Lemma 2, in Cases b.1 and c.1, it holds that $T = Z$, $\bar{P}_H = \lambda Y_H$, and $\bar{P}_M = \lambda Y_M$. We also show in that proof that $\bar{P}_H < R^*$ and as a result, $r_H = r_M = 0$. Because the equilibrium allocation is not affected by λ , $\frac{d\Pi}{d\lambda} = 0$. Finally, in this region, $\frac{d\Pi}{dZ} = -Y_M + \frac{d}{d\omega}(\omega x(\omega)) < 0$.

We show in the proof of Lemma 2 that Cases b.2 and c.2 correspond to Case 2 of Table A.1, and therefore, $r_H = [K + Z - x^{-1}(\lambda Y_H)]/p_H$, and $r_M = 0$. Because r_H is increasing in λ , and $\frac{\partial \Pi}{\partial r_H} > 0$, it holds that $\frac{d\Pi}{d\lambda} \geq 0$. Finally, in this region, $\frac{d\Pi}{dZ} = Y_H - Y_M > 0$.

Now we analyze Cases b.3 and c.3. In the proof of Lemma 2, we show that, in this region, $r_M = 0$, $r_H = 1$, and $\frac{\partial q_M}{\partial \lambda} < 0$. In this region, an increase in λ or Z does not affect the equilibrium allocation r_H or r_M , but it does affect the amount of liquidation q_M . Since $\frac{\partial q_M}{\partial \lambda} < 0$, then $\frac{d\Pi}{d\lambda} = p_M[-Y_M + \frac{d}{d\omega}(\omega x(\omega))]\frac{\partial q_M}{\partial \lambda} > 0$. In this region,

$$\frac{d\Pi}{dZ} = p_M \left[-Y_M + \frac{d}{d\omega}(\omega x(\omega)) \right] \frac{\partial q_M}{\partial Z}.$$

Differentiating the expression for q_M with respect to Z leads to

$$\frac{\partial q_M}{\partial Z} = \frac{1}{p_M} \left(1 + \frac{p_H \lambda Y_H}{R^{*2}} \frac{\partial R^*}{\partial Z} \right).$$

To obtain the sign of $\partial R^*/\partial Z$, consider the function

$$G(R, Z) = x \left(K + Z - \frac{\lambda Y_H}{R} p_H \right) - R$$

such that $G(R, Z) = 0$ defines R^* as a function of Z . By the implicit function theorem, $\frac{\partial R^*}{\partial Z} = -\frac{G_Z}{G_R} > 0$. Thus, in this region, $\frac{\partial q_M}{\partial Z} > 0$, and consequently, $\frac{d\Pi}{dZ} < 0$.

It is easy to show, using almost identical computations, that in the other cases, the aggregate payoff weakly increases with λ and weakly decreases with Z .

References

- Allen, F., 1993. Stock markets and resource allocation. In: Mayer, C., Vives, X. (Eds.), *Capital Markets and Financial Intermediation*. Cambridge University Press, Cambridge, pp. 81–107.

- Allen, F., Gale, D., 2000. Comparing Financial Systems. MIT Press, Cambridge, MA.
- Beck, T., Levine, R., Loayza, N., 2000a. Financial intermediation and growth: causality and causes. *Journal of Monetary Economics* 46, 31–77.
- Beck, T., Levine, R., Loayza, N., 2000b. Finance and the sources of growth. *Journal of Financial Economics* 58, 261–300.
- Bencivenga, V., Smith, B., Starr, R., 1995. Transaction costs, technological choice, and endogenous growth. *Journal of Economic Theory* 67, 153–177.
- Boyd, J., Smith, B., 1992. Intermediation and the equilibrium allocation of investment capital: implications for economic development. *Journal of Monetary Economics* 30, 409–432.
- Demirguc-Kunt, A., Levine, R., 1996. Stock market development and financial intermediary growth: stylized facts. *World Bank Economic Review*.
- Demirguc-Kunt, A., Levine, R., 2001. Financial Structure and Economic Growth: Cross-Country Comparisons of Banks, Markets and Development. MIT Press, Cambridge, MA.
- Demirguc-Kunt, A., Maksimovic, V., 1998. Law, finance, and firm growth. *Journal of Finance* 53, 2107–2137.
- Diamond, D., 1991. Debt maturity structure and liquidity risk. *Quarterly Journal of Economics* 106, 709–737.
- Dittmar, A., Smith, J., Servaes, H., 2003. International corporate governance and corporate cash holdings. *Journal of Financial and Quantitative Analysis* 38, 111–133.
- Easterly, W., Levine, R., 2003. Tropics, germs, and crops: how endowments influence economic development. *Journal of Monetary Economics* 50, 3–39.
- Galetovic, A., 1996. Specialization, intermediation, and growth. *Journal of Monetary Economics* 38, 549–559.
- Gershenkron, A., 1962. Economic backwardness in historical perspective. *A Book of Essays*. Harvard University Press, Cambridge, MA.
- Goldsmith, R.W., 1969. Financial Structure and Development. Yale University Press, New Haven, CT.
- Hart, O., Moore, J., 1994. A theory of debt based on the inalienability of human capital. *Quarterly Journal of Economics* 109, 841–879.
- Holmstrom, B., Tirole, J., 1997. Financial intermediation, loanable funds and the real sector. *Quarterly Journal of Economics* 112, 663–691.
- Jensen, M., 1986. Agency costs of free cash flow, corporate finance, and takeovers. *American Economic Review* 76, 323–329.
- King, R., Levine, R., 1993a. Finance and growth: Schumpeter might be right. *Quarterly Journal of Economics* 108, 639–671.
- King, R., Levine, R., 1993b. Finance, entrepreneurship and growth. *Journal of Monetary Economics* 32, 513–542.
- La Porta, R., Lopez-De-Silanes, F., Shleifer, A., Vishny, R., 1997. Legal determinants of external finance. *Journal of Finance* 52, 1131–1150.
- La Porta, R., Lopez-De-Silanes, F., Shleifer, A., Vishny, R., 1998. Law and finance. *Journal of Political Economy* 106, 1113–1155.
- Levine, R., 1991. Stock markets, growth and tax policy. *Journal of Finance* 46, 1445–1465.
- Levine, R., 1997. Financial development and economic growth: views and agenda. *Journal of Economic Literature* 35, 688–726.
- Levine, R., Zervos, S., 1998. Stock markets, banks and economic growth. *American Economic Review* 88, 537–558.
- Rajan, R., Zingales, L., 1998. Financial dependence and growth. *American Economic Review* 88, 559–573.
- Shleifer, A., Wolfenzon, D., 2002. Investor protection and equity markets. *Journal of Financial Economics* 66, 3–27.
- Wurgler, J., 2000. Financial markets and the allocation of capital. *Journal of Financial Economics* 58, 187–214.