

Production Externalities, Congruity of Aggregate Signals, and Optimal Task Assignments*

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Abstract

In this paper, we consider the role of production externalities in the task assignment problem. Milgrom and Roberts (1992) suggest that complementarities available when agents are assigned to diverse tasks are necessary to overcome distortions in effort allocations caused by an inability to fine-tune incentives when agents' compensation is based on aggregate imperfect signals. Our analysis formalizes this intuition in a setting that encompasses externalities under both diverse and similar task assignments.

Keywords Complementarities; Congruity; Effort allocations; Incentives; Repetition; Task assignments

JEL Descriptors D82, J41, M54

Externalités de la production, congruité des indicateurs agrégés et attribution optimale des tâches

Condensé

Les externalités de la production soulèvent d'intéressantes questions relatives à l'attribution des tâches dans les cas qui mettent en présence des mandataires multiples à qui sont assignées des tâches multiples. Le point de vue selon lequel la répétition génère des gains est le raisonnement sur lequel se fondait la pratique antérieure consistant à confier aux mandataires des tâches similaires. Toutefois, l'on observe depuis peu une tendance à confier aux mandataires des tâches variées dans le but d'exploiter les complémentarités. Cette tendance se manifeste notamment par la concentration des ventes et du service sous la responsabilité d'une même personne, plutôt que le partage de ces tâches entre plusieurs, l'organisation de l'analyse du crédit et de l'établissement du prix des prêts en fonction des dossiers plutôt qu'en fonction des étapes du processus d'attribution du crédit, le regroupement des activités de production et de mise à l'essai plutôt que la séparation de ces tâches, et la contribution des ingénieurs à la conception des divers composants de produits plutôt que leur spécialisation dans la

* Accepted by Steve Huddart. The authors would like to thank David Aboody and Steve Hansen for many useful discussions and two anonymous referees for helpful comments and suggestions.

conception de composants particuliers. Même si la conviction qu'il importe de varier les tâches assignées paraît prévaloir dans le cas des externalités de la production, les auteurs demeurent perplexes quant à savoir si les externalités relatives à l'un ou l'autre choix d'attribution des tâches sont avantageuses ou dysfonctionnelles.

Malgré les avantages découlant des externalités qui, dit-on, se rattachent à l'attribution de tâches variées, cette attribution suppose également la prévision de coûts plus élevés associés à l'offre d'incitatifs lorsque les indicateurs, aux fins contractuelles, revêtent la forme courante d'indicateurs agrégés brouillés des efforts des mandataires. L'agrégation des efforts des mandataires relativement à l'exécution de diverses tâches restreint la capacité du mandant de bien adapter les incitatifs offerts pour tenir compte des différences dans la contribution marginale de chaque tâche au rendement de la production et du niveau de brouillage dans la mesure des efforts des mandataires. En conséquence, dans le choix entre l'attribution de tâches similaires et l'attribution de tâches variées, le mandant fait face à un compromis possible entre les externalités de la production et les distorsions dans la répartition des efforts, en raison de l'agrégation, du brouillage et des restrictions auxquelles est astreinte la conception des incitatifs.

L'objectif des auteurs est ici d'envisager le problème d'attribution des tâches du mandant en présence d'externalités de la production de la nature décrite ci-dessus. Holmstrom et Milgrom (1991) ont expliqué le coût prévu supérieur de l'offre d'incitatifs dans le cas de l'attribution de tâches variées, mais sans tenir compte de l'incidence des externalités de la production sur cette attribution. Milgrom et Roberts (1992) ont formulé l'hypothèse selon laquelle les complémentarités sont nécessaires pour justifier l'attribution de tâches variées à la lumière des coûts prévus supérieurs des incitatifs, sans définir formellement le compromis. Le travail de Zhang (2003), qui se penche sur un problème d'attribution des tâches dans lequel une externalité de la production est intégrée à la structure de probabilité des rendements de la production conditionnels aux efforts du mandataire, se rapproche de l'analyse des auteurs. Dans son analyse, les rendements peuvent être prévus par contrat plutôt que d'être indicatifs des efforts du mandataire en tant que tels. Le compromis, dans ce contexte, se situe entre les gains issus des complémentarités et la perte d'hétérogénéité que l'on impute aux variations dans la précision des éléments qui brouillent ces rendements.

La principale proposition des auteurs consiste à diviser nettement la règle de décision du mandant relative à l'attribution des tâches en deux composants, l'un dénotant l'effet relatif des externalités sur les rendements prévus du mandant, déduction faite de la rémunération, et l'autre dénotant l'effet des distorsions dans la répartition des efforts. Les complémentarités interviennent dans la règle de décision uniquement dans le premier composant, tandis que les effets de la répétition influent sur les deux composants. En s'appuyant sur cette représentation du choix du mandant, les auteurs démontrent que plus la disparité est grande entre les productivités marginales ou entre la mesure dans laquelle sont brouillés les indicateurs des efforts des mandataires relativement à l'exécution de diverses tâches, plus sont élevés les coûts d'option associés à l'incapacité d'adapter les incitatifs, en situation d'attribution de tâches variées, à la distorsion de la répartition des efforts qui en résulte. Les conséquences d'une modification de la répétition devraient exacerber ou améliorer cette distorsion, selon les productivités marginales et les niveaux de brouillage.

Les auteurs décrivent leur modèle dans la section 2, exposent les résultats de leur analyse dans la section 3 et tirent les conclusions de leur étude dans la section 4.

1. Introduction

Production externalities pose interesting issues regarding task assignments in multitask, multi-agent settings. The rationale for the past practice of assigning agents similar tasks was the view that there are gains to repetition. However, there has been a trend in recent times toward assigning agents to diverse tasks in order to exploit complementarities. Illustrations of this trend include combining sales and service with the same person rather than assigning those duties to different individuals; organizing credit review and loan pricing by case rather than by stage in the credit-granting process; bundling production and testing operations rather than separating those tasks; and having engineers contribute to the design of several different product components rather than having them specialize in particular components.¹ Although the prevailing view seems to favor diverse task assignments, when it comes to production externalities, we remain agnostic whether externalities for either task assignment choice are beneficial or dysfunctional.

Notwithstanding the benefits from externalities that are said to attach to diverse task assignments, such assignments also imply the prospect of higher expected costs in providing incentives when signals for contracting purposes take the common form of noisy aggregate signals of agent efforts. The aggregation of agent efforts across diverse tasks restricts the ability of the principal to fine-tune incentives to take account of differences in the marginal contributions of each task to the payoff from production and levels of noise in measuring agent efforts. Accordingly, in choosing between similar and diverse task assignments, the principal faces a potential trade-off between production externalities and distortions of effort allocations due to aggregation, noise, and restrictions on the design of incentives.

Our purpose in this paper is to consider the principal's task assignment problem in the presence of production externalities of the nature described above. Holmstrom and Milgrom (1991) have characterized the higher expected cost of providing incentives under diverse task assignments, but without considering the impact of production externalities on those assignments. Milgrom and Roberts (1992) offer the conjecture that complementarities are necessary to justify diverse task assignments in light of the higher expected costs of incentives without formally characterizing the trade-off. Close to our analysis, Zhang (2003) considers a task assignment problem in which a production externality is embedded in the probability structure of payoffs from production to be conditional on agent efforts.² In her analysis, the payoffs are contractible rather than signals of agent efforts per se. The trade-off under this structure is between the gains to complementarities and a so-called heterogeneity loss from variations in the precisions of the noise components of the payoffs.

Our main proposition separates the principal's task assignment decision rule neatly into two components: one captures the relative effect of externalities on the principal's expected payoffs net of compensation, and the other captures the effect of distortions in effort allocations. Complementarities enter the rule only through the first component while the effects of repetition influence both components. From this representation of the principal's choice, we show that the greater the disparity in either the marginal productivities or the noisiness of signals of agent

efforts across different types of tasks, then the greater the opportunity costs of being unable to fine-tune incentives under diverse task assignments with consequent distortion in effort allocations. The effects of a change in repetition are to exacerbate or ameliorate this distortion depending on marginal productivities and noise levels.

Section 2 describes our model; section 3 contains the results from our analysis; and section 4 concludes the paper.

2. Model

A risk-neutral principal (the firm) engages two identical risk-averse agents (workers) to perform four tasks indexed by (i, j) ; $i, j \in \{1, 2\}$. Tasks (1, 1) and (1, 2) are equivalent and (2, 1) and (2, 2) are equivalent in the sense of having the same marginal effects on production, $b_1 > 0$ and $b_2 > 0$, respectively.³ The (hidden) effort that is supplied by an agent in performing task (i, j) is denoted a_{ij} . The payoff from the production that results from these tasks depends on whether each agent is assigned similar tasks or diverse tasks.

Under similar task assignments, the payoff from the production is

$$x_S = b_1(a_{11} + a_{12}) + b_2(a_{21} + a_{22}) + \gamma(a_{11}a_{12} + a_{21}a_{22}), \gamma \leq 1 \quad (1).$$

Without loss of generality, we assume that $b_2 > b_1 > 1$. The third term on the right-hand side of (1) reflects gains (losses) due to repetition, depending on whether high effort in one task increases (decreases) the marginal productivity of effort the agent applies to the other similar task. The upper bound on the marginal effects of repetition per unit of effort applied to the other similar task, $\gamma \leq 1$, rules out pathological cases in which an agent's optimal effort may become negative. Whether the effect of repetition is positive or negative depends on the nature of the production process and its behavioral implications. For example, there may be economies to specialization. However, contemporary wisdom appears to be that negative behavioral effects of repetition dominate efficiency gains in modern production environments.⁴

Under diverse task assignments, the payoff from production is

$$x_D = b_1(a_{11} + a_{12}) + b_2(a_{21} + a_{22}) + \alpha(a_{11}a_{21} + a_{12}a_{22}), \alpha \leq 1 \quad (2).$$

The third term on the right-hand side of (2) reflects gains to complementarities or losses when agents are assigned to diverse tasks. As above, the upper bound on the marginal effects of complementarities per unit of effort applied to the other diverse task, $\alpha \leq 1$, rules out pathological cases. The common rationale given for diverse task assignments in the recent literature, that familiarity with one task facilitates marginal productivity of the same agent in performing the other, suggests complementarities are a positive externality (for example, economies of scope).

We assume that the payoff from production is not contractible. Explanations may be that payoffs are affected by too many other factors or surface too late to be of use in contracting with agents. These conditions appear to be commonplace in many work settings where employees are compensated before the full outcomes of

their efforts become observable. Incentives are, we assume, based on imperfect aggregate signals of each agent's total efforts:

$$y_{S1} = a_{11} + a_{12} + \varepsilon_1, \quad y_{S2} = a_{21} + a_{22} + \varepsilon_2 \quad (3),$$

where $\varepsilon_1, \varepsilon_2$ are normal independent mean zero noise terms with variances σ_1^2, σ_2^2 , respectively, if each agent is assigned to similar tasks, and

$$y_{D1} = a_{11} + a_{21} + \varepsilon_{12}, \quad y_{D2} = a_{12} + a_{22} + \varepsilon_{21} \quad (4),$$

where $\varepsilon_{12}, \varepsilon_{21}$ are normal independent mean zero noise terms with variance $\sigma^2 = (\sigma_1^2 + \sigma_2^2)/2$ if each agent is assigned to diverse tasks. While this characterization has a nice interpretation of signals as unbiased estimates of total agent efforts, the restriction to equal (unit) weights on effort components is imposed as a technical convenience in reducing the number of parameters in the analysis that follows.⁵

Next, we assume that agents are identical with private monetary cost of effort

$$C(a_{ij}) = \frac{1}{2} a_{ij}^2 \quad (5),$$

and utility for compensation, W , net of private monetary cost $a^2/2$,

$$U(W, a) = -e^{-\rho(W - a^2/2)} \quad (6).$$

As in Feltham and Xie 1994, we restrict incentive compensation contracts to be linear in signals, implying contract payments for agent k of the form

$$w_k(y_k) = \beta_k + v_k y_{Sk}, \quad w_k(y_k) = \beta_k + v_k y_{Dk}, \quad k = 1, 2 \quad (7),$$

when agents are assigned similar tasks and diverse tasks, respectively.⁶

We assume that the principal seeks to maximize the expected payoff from production less the expected wages paid to the agents. Because the agents require a premium to bear risk of $(1/2)\rho v_k^2 \sigma_k^2$, $k = 1, 2$, the expected wages must cover that premium as well as their private costs of effort and meet their reservation wage that (without loss of generality) we assume to be 0. Accordingly, the principal's problem under similar task assignments (that is, $k = i$) is

$$\text{Max}_{\{a_{ij}\}, \{v_k\}} V_S = x_S - \frac{1}{2} \sum_{i=1,2} \left(\sum_{j=1,2} a_{ij}^2 + \rho v_i^2 \sigma_i^2 \right) \quad (8),$$

subject to agents choosing actions that maximize certainty equivalents:⁷

$$\begin{aligned}
 a_{11}, a_{12} &\in \arg \max v_1 E(y_{S1}) - \frac{1}{2} \left(\sum_{j=1,2} a_{1j}^2 + \rho v_1^2 \sigma_1^2 \right) \\
 a_{21}, a_{22} &\in \arg \max v_2 E(y_{S2}) - \frac{1}{2} \left(\sum_{j=1,2} a_{2j}^2 + \rho v_2^2 \sigma_2^2 \right)
 \end{aligned} \tag{9}.$$

Analogously, the principal's problem under diverse task assignments is

$$\text{Max}_{\{a_{ij}\}, \{v_k\}} V_D = x_D - \frac{1}{2} \sum_{j=1,2} \left(\sum_{i=1,2} a_{ij}^2 + \rho v_j^2 \sigma_j^2 \right) \tag{10},$$

subject to

$$\begin{aligned}
 a_{11}, a_{21} &\in \arg \max v_1 E(y_{D1}) - \frac{1}{2} \left(\sum_{i=1,2} a_{i1}^2 + \rho v_1^2 \sigma_1^2 \right) \\
 a_{21}, a_{22} &\in \arg \max v_2 E(y_{D2}) - \frac{1}{2} \left(\sum_{i=1,2} a_{i2}^2 + \rho v_2^2 \sigma_2^2 \right)
 \end{aligned} \tag{11}.$$

3. Analysis

Having defined the principal's problem for similar and diverse task assignments, we characterize the solution for each before turning to the choice between them.

Similar task assignments

From the constraints given by (9), we can solve for the optimal actions as functions of an agent's marginal wage payment (recall that $k = i$):

$$a_{kj} = v_k, \quad k, j = 1, 2 \tag{12}.$$

Substituting from (12) into (8), differentiating with respect to the marginal wage payments, and solving for those payments,⁸ we obtain

$$v_k = \frac{b_k}{M_k - \gamma}, \quad M_k = 1 + \rho \frac{\sigma_k^2}{2}, \quad k = 1, 2 \tag{13}.$$

Observe that under similar task assignments, the principal can fine-tune incentives to provide stronger incentives for tasks with higher marginal productivity of effort and weaker incentives for tasks involving signals with greater noise in order to reduce the risk premium required. The principal's expected net payoff under similar task assignments is obtained by substituting for effort levels in (8) from (12) and (13):

$$V_S = \frac{b_1^2}{M_1 - \gamma} + \frac{b_2^2}{M_2 - \gamma} \quad (14).$$

Note that the effect of repetition is to scale up expected payoffs if $\gamma > 0$ and scale down those payoffs if $\gamma < 0$. Complementarities are, of course, absent in this case.

Diverse task assignments

From the constraints given by (11), we can solve for the optimal actions as functions of an agent's marginal wage payment (recall that now $k = j$):

$$a_{ik} = v_k, \quad k, i = 1, 2 \quad (15).$$

Substituting from (15) into (10), differentiating with respect to the marginal wage payments, and solving for those payments, we obtain

$$v_k = \frac{b_1 + b_2}{M_1 + M_2 - 2\alpha}, \quad M_k = 1 + \rho \frac{\sigma_k^2}{2}, \quad k = 1, 2 \quad (16).$$

Observe that now the marginal wage is constructively restricted to be the same across different types of tasks, thereby eliminating the principal's ability to fine-tune incentives. Consequently, a larger disparity in marginal productivities or levels of noise across tasks implies a greater distortion of incentives for each type of task.

The principal's expected net payoff under diverse task assignments is obtained by substituting for effort levels in (10) from (15) and (16):

$$V_D = \frac{(b_1 + b_2)^2}{M_1 + M_2 - 2\alpha} \quad (17).$$

Similar to the effects of repetition under similar task assignments, the effects of complementarities on expected net payoffs under diverse task assignments are to scale expected payoffs up or down depending on the sign of α . However, the implications of complementarities and repetition vary with respect to effort allocations across diverse tasks.

Optimal task assignments

Our first proposition characterizes the principal's choice between similar and diverse task assignments:

PROPOSITION 1. *The principal strictly prefers diverse task assignments if and only if the following condition holds:*

$$\left[b_1 \left(\frac{M_2 - \gamma}{M_1 - \gamma} \right)^{1/2} - b_2 \left(\frac{M_1 - \gamma}{M_2 - \gamma} \right)^{1/2} \right]^2 + 2(\gamma - \alpha)V_S < 0 \quad (18).$$

(See the appendix for proof of Proposition 1.)

The squared term in the above condition is a measure of the relative effort allocations across diverse tasks when agents are assigned to similar tasks as compared with diverse tasks.⁹ As we elaborate below, the larger this term, the greater the benefit from an ability to fine-tune incentives under similar task assignments, and the more costly the distortion of effort allocations that ensue from an inability to fine-tune incentives under diverse task assignments. The remaining term on the left-hand side of (18) is a measure of the relative effects of repetition associated with similar task assignments, γ , and complementarities, α , associated with diverse task assignments. The fact that the condition only depends on complementarities through its presence in this term implies a convenient separation of the relative benefits of complementarities from the implicit costs of distortions in effort allocations under diverse task assignments. The effects of repetition on (18) are more complex. Because the benefits or costs of repetition modify the total marginal productivity of effort applied to diverse tasks under similar task assignments, they also affect marginal wages and, hence, effort allocations in that case.

Dealing first with the impact of complementarities on the principal's choice of task assignments, we have the following obvious corollary to Proposition 1:

COROLLARY 1. *For a fixed level of repetition effects, γ , there exists a critical level of complementarities, α^* , such that the principal strictly prefers diverse task assignments to similar task assignments if and only if $1 \geq \alpha > \alpha^*$ where*

$$\alpha^* = \gamma + \left[b_1 \left(\frac{M_2 - \gamma}{M_1 - \gamma} \right)^{1/2} - b_2 \left(\frac{M_1 - \gamma}{M_2 - \gamma} \right)^{1/2} \right]^2 / (2V_S) \quad (19).$$

(See the appendix for proof of Corollary 1.)

Thus, for any given set of parameters other than α such that the right-hand side of (19) is strictly less than 1, there is a non-empty region in which diverse task assignments are optimal. It is readily apparent from (18) that as we allow the benefits of complementarities to become large in comparison to the benefits (if any) of repetition, then, given that the implicit costs of distortions in effort allocations are unaffected by complementarities, we would expect diverse task assignments to dominate. Alternatively, if the benefits of repetition are sufficiently high, there may be no scope for diverse task assignments to dominate.

Alternatively, we can characterize a critical level of repetition effects such that similar task assignments are preferred to diverse task assignments.

COROLLARY 2. *For a fixed level of complementarities, α , there exists a critical level of marginal repetition effects, γ^* , such that the principal strictly*

prefers similar task assignments to diverse task assignments if and only if $1 \geq \gamma > \gamma^$, where*

$$\gamma^* = \alpha + \frac{b_1 b_2}{V_D} - \frac{\sqrt{[(b_1^2 - b_2^2) - V_D(M_1 - M_2)]^2 + 4b_1^2 b_2^2}}{2V_D} < \alpha \tag{20}.$$

(See the appendix for proof of Corollary 2.)

The fact that the third term on the right hand side of (20) exceeds the second implies the same ordering of externalities; that is, benefits from complementarities must exceed those from repetition for diverse task assignments to be preferred.

Incentives and effort allocations

We now consider the effects of task assignments and externalities on the efficient design of incentives and relative allocation of efforts. Table 1 depicts all combinations of effort allocation ratios for similar and diverse task assignments assuming that either agent efforts are publicly observable (the first-best case) or only noisy signals of each agent’s total efforts are contractible (the second-best case).

Table 1 confirms the obvious intuition that effort allocation ratios will always be equal across similar tasks no matter whether efforts per se are publicly observable or only imperfect aggregate signals are contractible or how agents are assigned.

TABLE 1
Effort allocations

Panel A: First-best allocations			
		Agents assigned similar tasks	Agents assigned diverse tasks
Effort allocation between diverse tasks	$\frac{a_{1j}}{a_{2j}}$	$\frac{b_1}{b_2}$	$\frac{b_1 + \alpha b_2}{b_2 + \alpha b_1}$
Effort allocation between similar tasks	$\frac{a_{i1}}{a_{i2}}$	1	1
Panel B: Second-best allocations			
Effort allocation between diverse tasks	$\frac{a_{1j}}{a_{2j}}$	$\frac{\frac{b_1}{M_1 - \gamma}}{\frac{b_2}{M_2 - \gamma}} = \frac{\frac{b_1}{1 + \frac{\rho\sigma_1}{2} - \gamma}}{\frac{b_2}{1 + \frac{\rho\sigma_2}{2} - \gamma}}$	1
Effort allocation between similar tasks	$\frac{a_{i1}}{a_{i2}}$	1	1

Feltham and Xie (1994) provide a useful characterization of performance measures for examining distortions in effort allocations when an agent is assigned to diverse tasks. In particular, they define a measure of noncongruity for an aggregate signal relative to the principal's expected payoff before compensation as $[(b_1 + \alpha b_2) - (b_2 + \alpha b_1)]^2$.¹⁰ They define perfect congruity as the case in which the above measure is zero, equivalently:

$$\frac{b_1 + \alpha b_2}{b_2 + \alpha b_1} = 1 \quad (21).$$

From Table 1 it is evident that when each agent is assigned diverse tasks and aggregate signals of an agent's efforts are employed in setting incentives, second-best effort allocation ratios will correspond to first-best effort allocation ratios if and only if those signals are perfectly congruent; that is, (21) holds. Note further that as α increases from 0 to 1 the distortion in effort allocations from perfect congruity is reduced, implying that less inefficiency from restricting incentives (marginal wages) is the same for diverse tasks when performed by the same agent. In other words, complementarities between efforts applied to diverse tasks tend to reduce differences in their (total) marginal effects on expected payoffs from production, thereby making it less important whether incentives can be fine-tuned.

Also, from Table 1, we can see that given similar task assignments and aggregate signals of each agent's total effort, second-best effort allocations will correspond to first-best allocations when the following condition is met:

$$\frac{b_1}{b_2} = \frac{\frac{b_1}{M_1 - \gamma}}{\frac{b_2}{M_2 - \gamma}} = \frac{b_1}{b_2} \left(\frac{1 + \frac{\rho \sigma_2}{2} - \gamma}{1 + \frac{\rho \sigma_1}{2} - \gamma} \right) \quad (22).$$

Under similar task assignments, signals represent total efforts by each agent, and those efforts are applied to similar tasks. As a result, incentives can be fine-tuned to fit the type of task by setting a different marginal wage for each agent. Equation (22) can be interpreted as perfect congruity with respect to diverse tasks in the case of similar task assignments. Notwithstanding that first-best effort allocations are feasible under similar task assignments, it is evident that second-best effort allocations will correspond to first-best effort allocations if and only if $\sigma_1 = \sigma_2$. As the disparity in noisiness of signals across diverse tasks under similar task assignments becomes larger, so too does the distortion of effort levels from those in the first-best case. Note further that the effects of repetition may either exacerbate or diminish distortions in effort allocations depending on the sign of γ and the orderings of marginal productivity parameters, b_1 , b_2 , and noise levels, σ_1^2 , σ_2^2 . We say more about the effects of repetition later.

In order to focus more closely on the role of congruity, we put aside externalities by assuming for the moment that $\alpha = \gamma = 0$. We can isolate the effects of signal congruity per se by further assuming equivalent noise across the two types of tasks, $\sigma_1 = \sigma_2$. It is apparent from (22) that similar task assignments induce the first-best effort allocations and, from (18), that such assignments dominate diverse task assignments by an amount proportional to Feltham and Xie's 1994 noncongruity measure; that is, $(b_1 - b_2)^2/2M$, $M = M_1 = M_2$. In this case the advantage of similar task assignments lies entirely with the benefits of bundling like tasks so as to avoid under- (over-) production of the more (less) valuable tasks in terms of their contribution to expected payoffs. Alternatively, we can isolate the effects of differential noise by assuming that $b_1 = b_2$, but allow $\sigma_1 \neq \sigma_2$. Although aggregate signals of each agent's total efforts are now congruent under diverse task assignments, the expected payoffs net of compensation are still greater under similar task assignments because of more efficient risk sharing. For example, in the case where $\sigma_2 > \sigma_1 = 0$, it is optimal to have one agent bear all the risk, which is only feasible under similar task assignments. More generally, we observe from the squared term in (22) that, as the distortion in effort allocations across diverse tasks under diverse task assignments from those allocations under similar task assignments becomes greater, the critical level of complementarities required for the principal to prefer diverse task assignments becomes higher as well.

Although comparisons of second-best effort allocation ratios with those in the first-best case are insightful, a different perspective can be obtained by comparing second-best effort allocation ratios across diverse tasks under similar and diverse task assignments. Effort allocation ratios will be the same under diverse and similar task assignments if and only if

$$\frac{\frac{b_1}{M_1 - \gamma}}{\frac{b_2}{M_2 - \gamma}} = \frac{\frac{b_1}{1 + \frac{\rho\sigma_1}{2} - \gamma}}{\frac{b_2}{1 + \frac{\rho\sigma_2}{2} - \gamma}} = 1 \quad (23).$$

Note that the above condition implies that the squared term in (18) equals zero. In other words, given that effort allocations are unaffected by task assignments, the only thing that matters is whether more benefits are derived from complementarities under diverse task assignments than from repetition under similar task assignments. It is evident from (23) that the disparity between effort allocations under diverse and similar task assignments in the second-best case depends the effects of repetition. The following proposition provides a comparative static descriptive of these effects.

PROPOSITION 2. *If the aggregate signal of total efforts for the agent assigned to the task with the higher marginal productivity parameter under similar task assignments is less noisy than the aggregate signal of total efforts for the other agent, $\sigma_2^2 < \sigma_1^2$, then a marginal increase in the repetition parameter, γ , results in effort allocations under diverse task assignments that deviate further from those under similar task assignments¹¹; that is, for $j = 1, 2$*

$$\frac{\partial}{\partial \gamma} [(a_{1j} - a_{2j})^2] = 2 \left[\frac{b_1}{(M_1 - \gamma)} - \frac{b_2}{(M_2 - \gamma)} \right] \left[\frac{b_1}{(M_1 - \gamma)^2} - \frac{b_2}{(M_2 - \gamma)^2} \right] > 0 \quad (24).$$

Intuitively, if the task with the greater marginal productivity is accompanied by a signal with a lower level of noise, then increasing the beneficial effect of repetition will cause the principal to strengthen the incentive placed on that task relative to the incentive placed on the other task. In turn, this change in incentives implies a higher opportunity cost of an inability to fine-tune incentives under diverse task assignments.

4. Conclusion

In this paper, we characterize task assignment decisions in the presence of production externalities. The tension the principal faces in making such decisions is between the relative effects of complementarities and repetition and distortions in effort allocations when each agent is assigned diverse tasks rather than similar tasks.

Summarizing our results, the beneficial effects of complementarities necessary to justify diverse task assignments must exceed the opportunity cost of an inability to fine-tune incentives. These costs become more severe as disparities in marginal productivities or noise levels increase across diverse tasks. When one type of task dominates in the sense of both higher marginal productivity and less noise in signals, then an increase in the beneficial effects of repetition (or reduction in the dysfunctional effects) leads to greater deviations in effort allocations under diverse task assignments from those under similar task assignments and, hence, higher opportunity costs.

Like many models, ours is intended as a metaphor. At the core of our analysis is the efficient resolution of the multitask, multi-agent assignment problem in the presence of production externalities when only noisy aggregate signals of agent efforts are available for contracting purposes. While we streamline the problem, the essential conflict between exploiting complementarities by bundling diverse tasks and tailoring incentives to induce desired actions seems likely to be present in a wide range of settings. A fairly literal illustration of a setting where our model might apply is the assignment of workers to one or more stages of an assembly line where wages are based on total hours worked as a proxy for effort and the effort applied to one task impacts on the marginal productivity of effort applied to

another. Such an application can be likened to the pin factory as a metaphor for economies of specialization. However, the forces in play within our analysis do not appear to depend on restricting either the numbers of tasks to just two types or the signals available for contracting to just one.¹² The key issue is whether the exploitation of positive production externalities more than offsets the opportunity costs of less efficient compensation commonly associated with assigning agents to diverse tasks.

Appendix

Proof of Proposition 1 and Corollary 1

$$V_D = \frac{(b_1 + b_2)^2}{M_1 + M_2 - 2\alpha}, V_S = \frac{b_1^2}{M_1 - \gamma} + \frac{b_2^2}{M_2 - \gamma} \text{ where } M_k = 1 + \rho \frac{\sigma_k^2}{2}, k = 1, 2$$

$$\begin{aligned} V_D < V_S &\Leftrightarrow \frac{(b_1 + b_2)^2}{M_1 + M_2 - 2\alpha} > \frac{b_1^2}{M_1 - \gamma} + \frac{b_2^2}{M_2 - \gamma} \\ &\Leftrightarrow (b_1 + b_2)^2 > b_1^2 \frac{M_1 + M_2 - 2\alpha}{M_1 - \gamma} + b_2^2 \frac{M_1 + M_2 - 2\alpha}{M_2 - \gamma} \\ &\Leftrightarrow (b_1 + b_2)^2 > b_1^2 \frac{M_1 - \gamma + M_2 - \gamma + 2(\gamma - \alpha)}{M_1 - \gamma} \\ &\quad + b_2^2 \frac{M_1 - \gamma + M_2 - \gamma + 2(\gamma - \alpha)}{M_2 - \gamma} \\ &\Leftrightarrow 2b_1b_2 > b_1^2 \frac{M_2 - \gamma + 2(\gamma - \alpha)}{M_1 - \gamma} + b_2^2 \frac{M_1 - \gamma + 2(\gamma - \alpha)}{M_2 - \gamma} \\ &\Leftrightarrow b_1^2 \frac{M_2 - \gamma}{M_1 - \gamma} + b_2^2 \frac{M_1 - \gamma}{M_2 - \gamma} - 2b_1b_2 + 2(\gamma - \alpha) \left(\frac{b_1^2}{M_1 - \gamma} + \frac{b_2^2}{M_2 - \gamma} \right) < 0 \\ &\Leftrightarrow \left(b_1 \sqrt{\frac{M_2 - \gamma}{M_1 - \gamma}} - b_2 \sqrt{\frac{M_1 - \gamma}{M_2 - \gamma}} \right)^2 + 2(\gamma - \alpha)V_S < 0. \end{aligned}$$

The above inequality holds if and only if

$$\alpha > \alpha^* \text{ where } \alpha^* = \gamma + \left(b_1 \sqrt{\frac{M_2 - \gamma}{M_1 - \gamma}} - b_2 \sqrt{\frac{M_1 - \gamma}{M_2 - \gamma}} \right)^2 / (2V_S). \quad \blacksquare$$

Proof of Corollary 2

$$\begin{aligned}
V_D < V_S &\Leftrightarrow V_D < \frac{b_1^2}{M_1 - \gamma} + \frac{b_2^2}{M_2 - \gamma} \\
&\Leftrightarrow V_D (M_1 - \gamma)(M_2 - \gamma) < b_1^2 (M_2 - \gamma) + b_2^2 (M_1 - \gamma) \\
&\Leftrightarrow V_D \gamma^2 + [b_1^2 + b_2^2 - (M_1 + M_2)V_D]\gamma + V_D M_1 M_2 - b_1^2 M_2 - b_2^2 M_1 < 0.
\end{aligned}$$

Observe that the left-hand side of the above inequality is in quadratic form. The corresponding equality has two real roots γ^*, γ^{**} because

$$\begin{aligned}
&[b_1^2 + b_2^2 - (M_1 + M_2)V_D]^2 - 4V_D(V_D M_1 M_2 - b_1^2 M_2 - b_2^2 M_1) \\
&= (b_1^2 + b_2^2)^2 - 2V_D(b_1^2 - b_2^2)(M_1 - M_2) + V_D^2(M_1 - M_2)^2 \\
&= [(b_1^2 - b_2^2) - V_D(M_1 - M_2)]^2 + 4b_1^2 b_2^2 > 0.
\end{aligned}$$

With some algebraic manipulation, we obtain

$$\gamma^{**}, \gamma^* = \alpha + \frac{b_1 b_2}{V_D} \pm \frac{\sqrt{[(b_1^2 - b_2^2) - V_D(M_1 - M_2)]^2 + 4b_1^2 b_2^2}}{2V_D}.$$

Hence, the inequality $V_D \gamma^2 + [b_1^2 + b_2^2 - (M_1 + M_2)V_D]\gamma + V_D M_1 M_2 - b_1^2 M_2 - b_2^2 M_1 < 0$ holds if and only if $\gamma^* < \gamma < \gamma^{**}$. The above inequality holds when $\gamma = 1$, implying $\gamma^* < 1 < \gamma^{**}$. Note that we have set $\gamma \leq 1$ and $\alpha \leq 1$ to keep optimal efforts positive. Therefore, $V_D < V_S$ if and only if $\gamma > \gamma^*$, where

$$\gamma^* = \alpha + \frac{b_1 b_2}{V_D} - \frac{\sqrt{[(b_1^2 - b_2^2) - V_D(M_1 - M_2)]^2 + 4b_1^2 b_2^2}}{2V_D}. \quad \blacksquare$$

Proof of Proposition 2

Recall that the measure of the relative effort allocations across diverse tasks when agents are assigned to similar tasks as compared with diverse tasks is

$$(a_{1j} - a_{2j})^2 = \left(\frac{b_1}{M_1 - \gamma} - \frac{b_2}{M_2 - \gamma} \right)^2.$$

$$\begin{aligned} \text{Hence, } \frac{\partial}{\partial \gamma} [(a_{1j} - a_{2j})^2] &= \frac{\partial}{\partial \gamma} \left[\left(\frac{b_1}{M_1 - \gamma} - \frac{b_2}{M_2 - \gamma} \right)^2 \right] \\ &= 2 \left[\frac{b_1}{(M_1 - \gamma)} - \frac{b_2}{(M_2 - \gamma)} \right] \left[\frac{b_1}{(M_1 - \gamma)^2} - \frac{b_2}{(M_2 - \gamma)^2} \right] \end{aligned}$$

$\sigma_2^2 < \sigma_1^2$ implies $M_2 < M_1$. Because $M_2 > \gamma$ and $b_1 < b_2$, it follows that

$$2 \left[\frac{b_1}{(M_1 - \gamma)} - \frac{b_2}{(M_2 - \gamma)} \right] \left[\frac{b_1}{(M_1 - \gamma)^2} - \frac{b_2}{(M_2 - \gamma)^2} \right] > 0. \quad \blacksquare$$

Endnotes

1. See Brickley, Smith, and Zimmerman 1996 for further descriptions of these illustrations.
2. Zhang's 2003 analysis is predicated on an example by Demski 1994 that depicts inefficiencies in compensation from diverse task assignments.
3. While exactly the same marginal effects for each pair of tasks is convenient, the essential aspect is that tasks within pairs are more similar in those effects than tasks across pairs.
4. See Hackman and Oldham 1980 for further discussion.
5. The assumption of equal marginal effects of efforts on signals is not crucial to our later analysis of the effects of incongruities of signals relative to expected payoffs given that marginal productivities of efforts across diverse tasks are assumed to be arbitrary constants.
6. Linear contracts can be viewed as an approximation of optimal contracts useful in characterizing the strength of incentives as weights applied to signals available for contracting.
7. The risk premiums in (9) below and later in (11) are independent of efforts and can be deleted.
8. The first-order approach to solving the principal's problem is appropriate except in knife-edge cases where the denominator to (13) is zero.
9. To see this, note that under similar task assignments

$$(a_{1j} - a_{2j})^2 = \left(\frac{1}{M_1 - \gamma} \right) \left(\frac{1}{M_2 - \gamma} \right) \left[b_1 \left(\frac{M_2 - \gamma}{M_1 - \gamma} \right)^{1/2} - b_2 \left(\frac{M_1 - \gamma}{M_2 - \gamma} \right)^{1/2} \right]^2, J = 1, 2.$$

10. This expression differs from that in Feltham and Xie 1994 because of our assumption of signal coefficients of 1 and the effect of complementarities on expected payoffs.
11. Recall the measure of the disparity in efforts across different types of tasks under similar task assignments below from endnote 9 following the discussion of Proposition 1.

12. Additional signals may reduce the opportunity costs of an inability to fine-tune incentives under diverse task assignments, but need not recover the properties of similar task assignments in that respect.

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