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Wage and price controls in the equilibrium sequential search model

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Abstract

In this paper, we study the effects of wage and price controls on employment, output, and welfare in a simplified version of the Bénabou (J. Econom. Theory 60 (1993) 140) equilibrium sequential search model with bilateral heterogeneity. We show that a price ceiling increases output but the change in welfare depends on three effects: the reduction in aggregate search costs, the increase in surplus due to increased output, and the transfer of production to the least efficient firm. The model is formally identical to a standard equilibrium search model of the labor market so analogous results hold for the minimum wage.

JEL classification: D43; D83

Keywords: Search; Price dispersion; Wage dispersion; Price ceilings; Minimum wage

1. Introduction

A binding price ceiling reduces both output and welfare in a perfectly competitive market, but does the opposite in monopolistic or oligopolistic markets. Analogous results hold for labor markets. These traditional models form the basis for a clear-cut, bipolar perspective on wage and price controls, in both their positive and normative aspects. From a positive viewpoint, the perfectly competitive and monopoly/monopsony models can, in principle, be distinguished empirically by the effect of wage and price controls on employment and output. From a normative viewpoint, these models suggest a positive association between employment and output, on the one hand, and welfare

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on the other. Hence, one can determine if wage and price controls represent good policy from their effects on employment and output. This bipolar perspective is clearly evident in the recent debates on the effects of the minimum wage on employment.[1]

In all of these traditional models, the Law of One Price holds. These models therefore neglect an empirically important source of firms' market power: search frictions due to imperfect information. The consequence of such frictions, wage and price dispersion, has been extensively documented in real-world markets.

In this paper, we investigate the effects of wage and price controls in the standard equilibrium sequential search model studied in Reinganum (1979), Carlson and McAfee (1983), and Rob (1985) as well as many others. Specifically, our model is a simplified version of the Bénabou (1993) model, where buyers have heterogeneous search costs and firms have heterogeneous production costs (bilateral heterogeneity). The simplicity of the model was inspired by Shy (1995, Chapter 16). In our model, a pair of duopolists, one high-cost and the other low-cost, produce an identical good under constant returns to scale. The demand side of the market is modeled by the nonnegative real line, where each $s \in [0, \infty)$ represents a buyer with search cost s. Buyers inelastically demand one unit of the good, which they value at unity, and search sequentially for the lowest price.

We first derive the standard result that the equilibrium without price controls exhibits price dispersion iff the firms have different production costs.[2] We then show that a binding price ceiling induces *both* firms to lower their prices, but this reduction is greater for the high-price, high-cost firm. Hence, price dispersion is reduced, which weakens the incentive to search. The subsequent reduction in search has two effects. First, aggregate search costs are reduced, which is a welfare gain. Second, the reduction in search means that some buyers switch from the low-cost firm to the high-cost firm, which is a welfare loss. In our model, the entry decision of buyers is endogenous, so the third effect is that lower prices induce more buyers into the market and an increase in total output, which is a welfare gain. So the total effect on welfare depends on the size of the second effect, which depends on the difference in production costs across firms. We emphasize that the first and second effects arise from the endogenous nature of buyers' search strategies, and hence may not feature in more recent equilibrium search models characterized by matching functions; see the discussion in Section 4 below.

We then show that by renaming variables, we obtain a standard equilibrium sequential search model of the labor market where firms have different marginal revenue products (MRPs) per worker. So the above results carry over to the minimum wage: a binding minimum wage induces both firms to raise their wages, but the increase is greater for the low-wage, low-MRP firm, so wage dispersion is reduced. This matches (i) and (ii) of the following stylized facts: (i) the minimum wage tends to increase wages, even those which were previously above the minimum; (ii) it compresses the wage distribution, especially at the lower end; and (iii) the existence of a spike in the wage distribution at the value of the minimum wage (see Card and Krueger, 1995;

[1] E.g., see Card and Krueger (1994, 1995, 2000) and Neumark and Wascher (2000).

[2] This is consistent with Rob (1985) since the distribution of search costs is essentially uniform.

DiNardo et al., 1996; Lee, 1999). Of course, we cannot comment on (iii) in a model with just two firms. In our model, the minimum wage increases employment by enticing more workers into the labor force, but its effect on welfare depends on the difference in MRPs across firms. Note that the employment effect of the minimum wage in equilibrium search models depends on the particular assumptions made; see the discussion in Section 3.

Search frictions seem to be an important characteristic of real-world labor and product markets, so we conclude that the neat, bipolar perspective on wage and price controls is too simplistic. As much as we like unambiguous results, the effect on welfare *depends*. Furthermore, one cannot make inferences about welfare on the basis of empirical results about the effects of wage and price controls on employment and output.

The plan for the rest of the paper is as follows. We study product markets in Section 2 and labor markets in Section 3. In Section 4, we discuss related results from other equilibrium search models, different from the one in this paper. Section 5 concludes. All proofs are in the appendix.

2. Price ceilings

We consider a simple equilibrium search model where a pair of duopolists produce an identical good under constant returns to scale. We assume that the low-cost firm produces output at zero cost per unit while the high-cost firm produces output at cost $c_{\mathrm{H}} \geqslant 0$ per unit. The qualitative nature of our results are unaffected.

Each $s \in [0, \infty)$ represents a potential buyer with search cost s which is incurred every time the buyer visits a store.[3] Buyers inelastically demand one unit of the good which they value at unity; i.e., buyers receive one unit of utility if they buy the good and zero otherwise. Consumer's surplus is therefore one minus the purchase price or zero, depending on whether the buyer bought the good or not, minus all search costs.

We now consider the search strategy of buyer s given that the buyer has decided to enter the market. As usual, we assume that buyers and sellers act as if they know the distribution of prices. Let p_{L} denote the price of the low-cost firm and p_{H} the price of the high-cost firm. Assume for the moment that $p_{\mathrm{H}} \geqslant p_{\mathrm{L}}$ (we will show that this is true in equilibrium). The buyer visits one of the firms at random and incurs the cost s.[4] If the buyer is lucky and ends up at the low-cost firm then she buys the good from there. If not, she has two options: buy the good at the high-cost firm with corresponding surplus $1 - p_{\mathrm{H}} - s$ or search again with corresponding surplus $1 - p_{\mathrm{L}} - 2s$. So an unlucky buyer searches again $s \leqslant \hat{s}$ where $\hat{s} = p_{\mathrm{H}} - p_{\mathrm{L}}$ (the assumption that the indifferent buyer searches again is inconsequential).

[3] We do not make the standard assumption that the first search is free because we are interested in the effect of a price ceiling on the extent of the market.

[4] After the first search, s is sunk so the buyer will not exit the market without buying the good as long as $p_{\mathrm{H}} \leqslant 1$ which is true in equilibrium.

Fig. 1. Classification of buyers' search strategies.

We now consider the entry decision of buyer s. If $s > \hat{s}$ the buyer searches at most once and enters if $1 - \bar{p} - s \geqslant 0$ where $\bar{p}$ is the average (expected) purchase price.[5] Fig. 1 summarizes the situation assuming that $\hat{s} \leqslant 1 - \bar{p}$ which is equivalent to[6]

$$p_{\mathrm{H}} \leqslant \frac{2 + p_{\mathrm{L}}}{3}. \tag{2}$$

Buyers in $[0, 1 - \bar{p}]$ enter the market and all others stay out. All of the buyers in $[0, \hat{s}]$ buy from the low-cost firm although half will have to search twice. Each firm gets half of $[\hat{s}, 1 - \bar{p}]$. So the sales of the low-cost firm are given by

$$q_{\mathrm{L}} = \hat{s} + \left(\tfrac{1}{2}\right)(1 - \bar{p} - \hat{s}) = \left(\tfrac{1}{2}\right)(1 - \bar{p} + \hat{s}), \tag{3}$$

while those of the high-cost firm are given by

$$q_{\mathrm{H}} = \left(\tfrac{1}{2}\right)(1 - \bar{p} - \hat{s}). \tag{4}$$

Note that (3) and (4) continue to hold when $p_{\mathrm{L}} > p_{\mathrm{H}}$.

As usual, we assume that firms are Stackelberg leaders vis-à-vis buyers (i.e., firms take as given and exploit buyers' search strategies) but play Nash among themselves. The profit-maximization problem of the low-cost firm is

$$\max_{0 \leqslant p_{\mathrm{L}} \leqslant (2+p_{\mathrm{H}})/3} \Pi_{\mathrm{L}} = p_{\mathrm{L}} q_{\mathrm{L}}, \tag{5}$$

with reaction function

$$p_{\mathrm{L}} = \tfrac{1}{3} + \tfrac{1}{6}\, p_{\mathrm{H}}, \tag{6}$$

while the profit-maximization problem of the high-cost firm is

$$\max_{c_{\mathrm{H}} \leqslant p_{\mathrm{H}} \leqslant (2+p_{\mathrm{L}})/3} \Pi_{\mathrm{H}} = (p_{\mathrm{H}} - c_{\mathrm{H}}) q_{\mathrm{H}}, \tag{7}$$

with reaction function

$$p_{\mathrm{H}} = \tfrac{1}{3} + \tfrac{1}{2}\, c_{\mathrm{H}} + \tfrac{1}{6}\, p_{\mathrm{L}}. \tag{8}$$

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[5] If $s \leqslant \hat{s}$ then expected surplus is

$$\left(\tfrac{1}{2}\right)(1 - p_{\mathrm{L}} - s) + \left(\tfrac{1}{2}\right)(1 - p_{\mathrm{L}} - 2s) \tag{1}$$

so if any buyer $s > \hat{s}$ enters then all buyers $s \leqslant \hat{s}$ enter as well.

[6] We can restrict p_{H} as in (2) without loss of generality because for higher p_{H} the high-cost firm makes no sales.

Solving (6) and (8), we get

$$p_L^e = \frac{3c_H + 14}{35}, \tag{9}$$

$$p_H^e = \frac{18c_H + 14}{35}. \tag{10}$$

This is an equilibrium provided that $c_H \leqslant p_H^e \leqslant (2 + p_L)/3$ or $c_H \leqslant \frac{14}{17}$.

Proposition 1. *If $c_H < \frac{14}{17}$ then the unique Nash equilibrium is given by (9) and (10) where the high-cost firm makes positive sales. Furthermore, the equilibrium exhibits price dispersion iff $c_H > 0$. If $c_H \geqslant \frac{14}{17}$ there is a unique Nash equilibrium in which the high-cost firm makes no sales.*

If $c_H < \frac{14}{17}$ then equilibrium price dispersion

$$\tilde{s}^e = p_H^e - p_L^e = \tfrac{3}{7}\, c_H \tag{11}$$

is increasing in c_H. In particular, the equilibrium exhibits price dispersion iff $c_H > 0$. In these equilibria, the high-cost firm is forced to charge a high price in order to cover its costs. In response, the low-cost firm adopts a price-cutting strategy and captures the bulk of the market. The equilibria associated with $c_H \geqslant \frac{14}{17}$ are uninteresting because the high-cost firm has been priced out of the market and has essentially exited the industry.

We emphasize that the objective of this paper is *not* to generate wage and price dispersion under minimal assumptions. Indeed, Burdett and Judd (1983) accomplish this without any ex ante heterogeneities in an equilibrium nonsequential (or noisy) search model while Bénabou (1988) achieves the same in an equilibrium sequential search model with both search and menu costs.

We now consider the effects of a price ceiling p^* on output and welfare. We assume the interesting case $0 < c_H < \frac{14}{17}$ and consider a price ceiling such that $c_H \leqslant p^* < p_H^e$. (We assume that the government wants to avoid creating a monopolist for the usual reasons not modeled here.) We make the standard definition that *welfare* equals consumers' surplus (which includes search costs) plus industry profits.

Proposition 2. *Let $0 < c_H < \frac{14}{17}$ and p^* be a price ceiling such that $c_H \leqslant p^* < p_H^e$. If $p^* > \frac{2}{5}$ (a moderate price ceiling), the unique equilibrium is*

$$p_L^* = \tfrac{1}{3} + \tfrac{1}{6}\, p^*, \tag{12}$$

$$p_H^* = p^*, \tag{13}$$

which continues to exhibit price dispersion. If $p^* \leqslant \frac{2}{5}$ (a severe price ceiling), the unique equilibrium is

$$p_L^* = p_H^* = p^*. \tag{14}$$

In both equilibria, prices are lower, price dispersion is lower, and output is higher relative to the equilibrium in (9) and (10).

For a moderate price ceiling, the high-cost firm is forced to lower its price to the legal maximum. The low-cost firm continues to follow its reaction function in (6) so it also lowers its price but not by as much as the high-cost firm, so price dispersion persists but is less than before. More rigorously, note that

$$\hat{s}^* = p_{\mathrm{H}}^* - p_{\mathrm{L}}^* = \tfrac{5}{6}\, p^* - \tfrac{1}{3} \tag{15}$$

is positive since $p^* > \frac{2}{5}$ but less than $\hat{s}^e$ since $p^* < p_{\mathrm{H}}^e$. For a severe price ceiling, the low-cost firm finds it more profitable to charge the legal maximum than to undercut. This is because the price ceiling forces the high-cost firm to charge a relatively low price, so the group $[0, \hat{s}]$ of buyers the low-cost firm could capture by undercutting is relatively small. In this case, price dispersion is eliminated. In both cases, prices are lower so output is higher.

Proposition 3. *Let $0 < c_{\mathrm{H}} < \frac{14}{17}$ and p^* be a price ceiling such that $c_{\mathrm{H}} \leqslant p^* < p_{\mathrm{H}}^e$. Let ΔW denote the change in welfare as a result of the price ceiling.*

(i) *If $\frac{98}{149} \leqslant c_{\mathrm{H}} < \frac{14}{17}$ (c_{H} is high), then $\Delta W < 0$.*
(ii) *Let p_{b}^* be the "break-even" price ceiling*

$$p_{\mathrm{b}}^* = \frac{2(893c_{\mathrm{H}} - 161)}{1,155}, \tag{16}$$

where $\Delta W = 0$ and

$$sign\,(p_{\mathrm{b}}^* - c_{\mathrm{H}}) = sign\,(c_{\mathrm{H}} - \tfrac{322}{631}). \tag{17}$$

If $\frac{2}{5} \leqslant c_{\mathrm{H}} < \frac{98}{149}$ (medium c_{H}),
(a) *if $p_{\mathrm{b}}^* < c_{\mathrm{H}}$ then $\Delta W > 0$*
(b) *if $p_{\mathrm{b}}^* \geqslant c_{\mathrm{H}}$ then*

$$sign\,\Delta W = sign\,(p^* - p_{\mathrm{b}}^*). \tag{18}$$

In both cases, the optimal price ceiling is given by

$$\tilde{p} = \frac{2 + 34c_{\mathrm{H}}}{33}. \tag{19}$$

(iii) *If $0 < c_{\mathrm{H}} < \frac{2}{5}$ (c_{H} is low), then $\Delta W > 0$. The optimal price ceiling is $\tilde{p}$ when*

$$\tfrac{4}{289}(4 + 3\sqrt{66}) \leqslant c_{\mathrm{H}} < \tfrac{2}{5} \tag{20}$$

and $p^* = c_{\mathrm{H}}$ otherwise.

Consider Fig. 2 where

$$\bar{p}^e = \tfrac{1}{2}\,(p_{\mathrm{H}}^e + p_{\mathrm{L}}^e) = \tfrac{1}{2}\,\left(\tfrac{21c_{\mathrm{H}}+28}{35}\right), \tag{21}$$

$$\bar{p}^* = \tfrac{1}{2}\,(p_{\mathrm{H}}^* + p_{\mathrm{L}}^*) = \tfrac{1}{2}\,\left(\tfrac{7}{6}\, p^* + \tfrac{1}{3}\right). \tag{22}$$

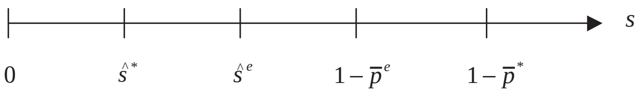

Fig. 2. Search strategies before and after the price ceiling.

We first note that the contribution to welfare of any single transaction is

$$\begin{aligned}&\text{consumer's surplus} + \text{producer's surplus}\\&\quad = (1 - \text{purchase price} - \text{search costs}) + (\text{purchase price} - \text{marginal cost})\\&\quad = 1 - \text{search costs} - \text{marginal cost}.\end{aligned} \tag{23}$$

In other words, a transaction increases welfare if the utility of consuming the good exceeds the production and transaction costs of providing it. So to determine the effects of the price ceiling on welfare, all we need to do is to determine its effect on aggregate search costs and keep track of those buyers who switched firms or did not buy the good in the old equilibrium but now do so in the new equilibrium. In particular, we need only consider the intervals

$$I_1 = [\hat{s}^*, \hat{s}^e], \tag{24}$$

$$I_2 = [1 - \bar{p}^e, 1 - \bar{p}^*]. \tag{25}$$

We first consider I_1. In the old equilibrium, all of these buyers bought from the low-cost firm but half of them searched once while the other half searched twice. In the new equilibrium, all of these buyers search once but half buy from the low-cost firm while the other half buy from the high-cost firm. The reduction in search costs is a welfare gain but the transfer of production to the high-cost firm is a welfare loss.

Next, we consider I_2. These buyers were out of the market in the old equilibrium but now enter and buy in the new equilibrium. Half of them buy from the low-cost firm and the change in welfare associated with them is $1 - s$ which is positive. The other half buy from the high-cost firm and the change in welfare associated with them is $1 - s - c_{\mathrm{H}}$ which could be negative. In the proof of Proposition 3, we show that the net effect is positive so the increase in the extent of the market improves welfare.

Summing up, we have identified three effects of the price ceiling on welfare:

(1) *Search intensity and the allocation of resources*. A binding price ceiling reduces price dispersion which weakens the incentive to search. The subsequent reduction in search leads to a less efficient allocation of resources by transferring production from the low-cost firm to the high-cost firm.

(2) *Aggregate search costs*. At the same time, the reduction in search lowers aggregate search costs which is a welfare gain.

(3) *Extent of the market*. The price ceiling lowers the expected purchase price which increases the extent of the market which is a welfare gain.

Clearly, the net effect of the price ceiling on welfare depends on the relative size of the first effect which depends on c_{H}. In (i), where c_{H} is high, the price ceiling reduces welfare. In (ii), where c_{H} is medium, the result can depend on where the price ceiling

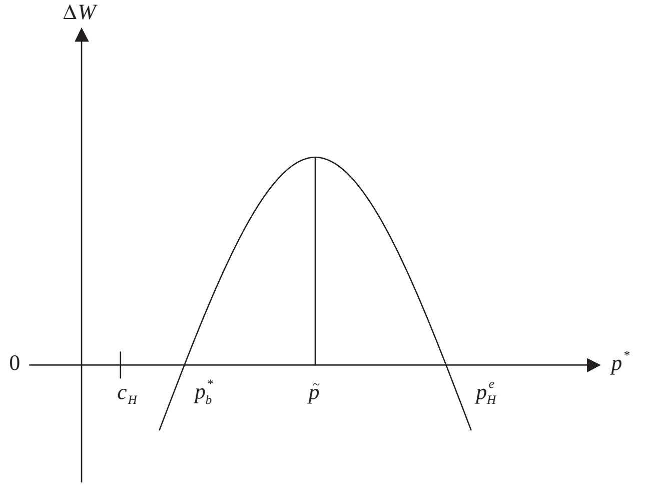

Fig. 3. Change in welfare as a function of the price ceiling.

is set. In the proof, we show that ΔW is a quadratic polynomial in p^*. Clearly, $\Delta W = 0$ at $p^* = p^e_{\mathrm{H}}$ and the break-even price ceiling p^*_{b} is the smaller solution to $\Delta W = 0$. It follows that if $p^*_{\mathrm{b}} > c_{\mathrm{H}}$ then there are two regimes: $\Delta W > 0$ for $p^*_{\mathrm{b}} < p^* < p^e_{\mathrm{H}}$ and $\Delta W < 0$ for $c_{\mathrm{H}} \leqslant p^* < p^*_{\mathrm{b}}$. See Fig. 3.

On the other hand, if $p^*_{\mathrm{b}} \leqslant c_{\mathrm{H}}$ then $\Delta W > 0$ except possibly at $p^* = p^*_{\mathrm{b}}$. In (iii), where c_{H} is low, the price ceiling improves welfare. Since $c_{\mathrm{H}} < \frac{2}{5}$, the government can either set a moderate price ceiling, $p^* > \frac{2}{5}$, or a severe one, $p^* \leqslant \frac{2}{5}$, which eliminates all price dispersion. In this case, ΔW consists of two pieces, both of which are quadratic polynomials. When (20) holds, the optimal price ceiling is $\tilde{p}$ (the maximizer for the upper quadratic portion), otherwise it switches discontinuously to $p^* = c_{\mathrm{H}}$ (the maximizer for the lower quadratic portion).

3. The minimum wage

We now show that the model in the previous section is formally identical to a standard equilibrium sequential search model of the labor market where firms have different MRPs per worker and analogues of Propositions 1–3 continue to hold.

Let p_{H} denote the MRP of the high-MRP firm and p_{L} the MRP of the low-MRP firm where $p_{\mathrm{H}} \geqslant p_{\mathrm{L}}$. Differences in MRPs may reflect differences in output prices (e.g., search frictions or product differentiation) or differences in marginal physical products. Each $s \in [0, \infty)$ now represents a potential worker with search cost s. Unemployed workers receive one unit of utility which reflects the value of leisure and unemployment benefits. Let w_{H} denote the wage of the high-MRP firm and w_{L} the wage of the low-MRP firm (we correctly anticipate that $w_{\mathrm{H}} \geqslant w_{\mathrm{L}}$ in equilibrium). The search strategies and entry decisions of workers are clearly similar to those in the previous section. If $\hat{s} = w_{\mathrm{H}} - w_{\mathrm{L}}$ and $\bar{w}$ is the average wage then the number of workers hired by the high-MRP firm is

$$l_{\mathrm{H}} = \tfrac{1}{2}\,(\bar{w} - 1 + \hat{s}) \tag{26}$$

and the number of workers hired by the low-MRP firm is

$$l_{\mathrm{L}} = \tfrac{1}{2}\,(\bar{w} - 1 - \hat{s}). \tag{27}$$

The profit-maximization problem of the high-MRP firm is given by

$$\max_{(2+w_{\mathrm{L}})/3 \leqslant w_{\mathrm{H}} \leqslant p_{\mathrm{H}}} \Pi_{\mathrm{H}} = (p_{\mathrm{H}} - w_{\mathrm{H}})l_{\mathrm{H}} \tag{28}$$

and for the low-MRP firm

$$\max_{(2+w_{\mathrm{H}})/3 \leqslant w_{\mathrm{L}} \leqslant p_{\mathrm{L}}} \Pi_{\mathrm{L}} = (p_{\mathrm{L}} - w_{\mathrm{L}})l_{\mathrm{L}}. \tag{29}$$

We now substitute l_{H}, l_{L}, $\hat{s} = w_{\mathrm{H}} - w_{\mathrm{L}}$, and $\bar{w} = \left(\frac{1}{2}\right)(w_{\mathrm{H}} + w_{\mathrm{L}})$ into (28) and (29) to obtain

$$\begin{aligned}\Pi_{\mathrm{H}} &= \tfrac{1}{2}\,(p_{\mathrm{H}} - w_{\mathrm{H}})\left(\tfrac{3}{2}\,w_{\mathrm{H}} - \tfrac{1}{2}\,w_{\mathrm{L}} - 1\right)\\ &= \tfrac{1}{2}\,(w_{\mathrm{H}} - p_{\mathrm{H}})\left(1 - \tfrac{3}{2}\,w_{\mathrm{H}} + \tfrac{1}{2}\,w_{\mathrm{L}}\right),\end{aligned} \tag{30}$$

$$\begin{aligned}\Pi_{\mathrm{L}} &= \tfrac{1}{2}\,(p_{\mathrm{L}} - w_{\mathrm{L}})\left(\tfrac{3}{2}\,w_{\mathrm{L}} - \tfrac{1}{2}\,w_{\mathrm{H}} - 1\right)\\ &= \tfrac{1}{2}\,(w_{\mathrm{L}} - p_{\mathrm{L}})\left(1 + \tfrac{1}{2}\,w_{\mathrm{H}} - \tfrac{3}{2}\,w_{\mathrm{L}}\right).\end{aligned} \tag{31}$$

In this model, the variables are wages and the parameters are MRPs. Returning to the model in the previous section, let c_{L} denote the production cost of the low-cost firm, which for simplicity was assumed to be zero. Now substitute q_{H}, q_{L}, $\hat{s} = p_{\mathrm{H}} - p_{\mathrm{L}}$, and $\bar{p} = (1/2)(p_{\mathrm{H}} + p_{\mathrm{L}})$ into (5) and (7) to obtain

$$\Pi_{\mathrm{H}} = \tfrac{1}{2}(p_{\mathrm{H}} - c_{\mathrm{H}})\left(1 - \tfrac{3}{2}\,p_{\mathrm{H}} + \tfrac{1}{2}\,p_{\mathrm{L}}\right), \tag{32}$$

$$\Pi_{\mathrm{L}} = \tfrac{1}{2}(p_{\mathrm{L}} - c_{\mathrm{L}})\left(1 + \tfrac{1}{2}\,p_{\mathrm{H}} - \tfrac{3}{2}\,p_{\mathrm{L}}\right). \tag{33}$$

In this model, the variables are prices and the parameters are production costs. Identifying variables and parameters in the two models, we see that they are formally identical.

So the results in the previous section also apply to labor markets. If p_{L} is high enough (relative to p_{H}), then the low-MRP hires workers in the equilibrium without the minimum wage, which exhibits wage dispersion iff $p_{\mathrm{L}} < p_{\mathrm{H}}$. The minimum wage induces both firms to raise their wages but the increase for the low-MRP firm is greater so wage dispersion falls. Employment increases because higher wages entice more workers into the labor force. A moderate minimum wage reduces wage dispersion but a severe minimum wage eliminates it. The effects of the minimum wage on welfare are: a reduction in aggregate search costs, an increase in surplus due to increased employment, and the transfer of workers to the low-MRP firm. Hence, the total effect on welfare depends on the relative size of p_{L} in the obvious way.

In our model, we included a labor supply curve with positive slope to make the theoretical point that employment effects need not be the same as welfare effects.

In fact, the employment effects of minimum wages are controversial. It is therefore worthwhile to discuss alternative assumptions, which may have different implications for the effects of the minimum wage in equilibrium sequential search models. First, it is often assumed that the interval of potential workers is fixed, the first search is free, and MRPs are constant, although these may differ across firms. In this case, a minimum wage would have no effect on employment, which is always full as long as at least one firm operates. A minimum wage would still reduce aggregate search costs and lead to inefficient switching, so the welfare effect would still depend on the relative sizes of these two effects. An alternative possibility, which does not seem to have attracted much attention in the literature, would be to assume that the first search is not free, but the MRP is decreasing in the quantity of labor. In such a model, employment would be endogenous and an increase in the minimum wage may lead to greater unemployment.

4. Related literature

In this section, we relate the above model and results to others in the literature. Rauh (2001) develops an equilibrium sequential search model similar to the one in this paper except that buyers and sellers make small, heterogeneous mistakes about the distribution of prices. It is shown that a price ceiling can improve welfare without decreasing output. This is because a price ceiling can make beliefs more homogeneous, which reduces price dispersion, search, and aggregate search costs. A similar effect occurs in the present model.

We have studied the effects of wage and price controls in the standard equilibrium sequential search model but there are other equilibrium search frameworks available. In Burdett and Judd (1983), buyers use a *nonsequential* search rule, which entails choosing the number of stores to visit before setting out. The model generates price dispersion without any ex ante heterogeneities. However, it is well known that nonsequential search rules are not sequentially optimal since the first search might reveal a very low price (even the lowest price in the market) so it would make no sense to continue searching. Furthermore, Fershtman and Fishman (1994) have shown that in this model, a price ceiling can *raise* the average market price and the minimum wage can similarly *lower* the average wage. These results are empirically doubtful and may be unique to nonsequential search.

Burdett and Mortensen (1998) have developed another class of equilibrium search models particularly adapted to labor markets. The model has been refined and extended in subsequent papers by Bontemps et al. (1999, 2000) and van den Berg and Ridder (1998) among others. In this model, wage offers arrive according to an exogenous Poisson process with arrival rate λ. A minimum wage can increase the steady-state level of employment by inducing workers to accept higher wages rather than continue searching. This model takes into account search by both the employed and unemployed and can generate wage dispersion without any ex ante heterogeneities. Furthermore, it can match several stylized facts about labor markets, especially those relating to job duration, wages, and firm size. On the other hand, although λ can differ for the employed and unemployed, search is completely exogenous in all other respects. In particular, λ does not depend on wage dispersion so the model cannot account for the effects of the minimum wage on search intensity, aggregate search costs, and the effect of search intensity on the matching of buyers to firms (i.e., the transfer of workers from the high-MRP firm to the low-MRP firm). The model in this paper should therefore be viewed as complementary to the Burdett and Mortensen (1998) class of models, since the two consider different aspects of equilibrium search.

5. Conclusions

In the perfectly competitive model, binding wage and price controls adversely affect employment, output, and welfare. In the monopoly/monopsony model, the opposite is true. These (extreme) models have dominated discussions about wage and price controls in both theoretical and empirical circles. They can, in principle, be distinguished empirically and, once that debate is settled, the welfare effects are then straightforward (positive) corollaries of the effects on employment and output.

In this paper, we studied the effects of wage and price controls in a simplified version of the standard equilibrium sequential search model. In this model, a binding price ceiling increases output but the effect on welfare depends on the relative sizes of three effects: the reduction in aggregate search costs, the increase in surplus due to the increase in output, and the transfer of production to the high-cost firm. We then showed that the model also applies to labor markets, so these results also hold for the minimum wage. The minimum wage increases employment by enticing more workers into the labor force but the effect on welfare depends on three effects: the reduction in aggregate search costs, the increase in surplus due to increased employment, and the transfer of workers to the low-MRP firm.

In our view, the model offers a more balanced and less simplistic view on the effects of wage and price controls. Furthermore, the model is predicated on search frictions, an important source of firms' market power in real-world labor and product markets. At the very least, the paper shows that the welfare effects of wage and price controls cannot be deduced from their effects on employment and output.

Appendix A

Proof of Proposition 1. We first note that

$$0 < \tfrac{1}{3} + \tfrac{1}{6}\, p_{\mathrm{H}} < \frac{2 + p_{\mathrm{H}}}{3}, \tag{A.1}$$

so (6) is the unique best response for the low-cost firm. The high-cost firm has three potential best responses:

(S1) $p_{\mathrm{H}} = c_{\mathrm{H}}$,
(S2) p_{H} equals the expression in (8), or
(S3) $p_{\mathrm{H}} = \frac{(2+p_{\mathrm{L}})}{3}$.

Since (S1) and (S3) both yield zero profits for the high-cost firm we can discard (S1). So the only candidates for equilibrium are the solutions to the pair [(6), (8)] or the pair [(6), (S3)]. The solutions to [(6), (8)] are given in (9) and (10) where we know that (9) is the best response to (10). Given (9), (S2) is less than or equal to (S3) iff $c \leqslant \frac{14}{17}$ so (9), (10) is an equilibrium in that case. The solutions to [(6), (S3)] are given by

$$p_{\mathrm{L}} = \tfrac{8}{17}, \tag{A.2}$$

$$p_{\mathrm{H}} = \tfrac{14}{17}, \tag{A.3}$$

where we know that (A.2) is the best response to (A.3). Given (A.2), (S2) is greater than or equal to (S3) iff $c \geqslant \frac{14}{17}$ so (A.2) and (A.3) is an equilibrium in that case. The two equilibria coincide when $c = \frac{14}{17}$. □

Proof of Proposition 2. In the proof of Proposition 1 we showed that the unique best response for the low-cost firm was (6). With the imposition of the price ceiling, the low-cost firm now has two potential best responses:

(L1) p_{L} equals the expression in (6), or
(L2) $p_{\mathrm{L}} = p^*$.

The high-cost firm now has four potential best responses:

(H1) $p_{\mathrm{H}} = c_{\mathrm{H}}$,
(H2) p_{H} equals the expression in (8),
(H3) $p_{\mathrm{H}} = p^*$, or
(H4) $p_{\mathrm{H}} = (2 + p_{\mathrm{L}})/3$.

We can eliminate (H1) since (H3) or (H4) do at least as well. So an equilibrium candidate is a solution to a pair of equations where the first is either (L1) or (L2) and the second is one of (H2)–(H4).

The price ceiling rules out the old equilibrium corresponding to [(L1), (H2)]. A price ceiling cannot be applied to the solution to [(L1), (H4)] because it would create a monopolist. The solution to [(L1), (H3)] is given by (12) and (13). Given (13), (L1) is less than or equal to (L2) iff $p^* \geqslant \frac{2}{5}$ and is the best response in that case. Given (12) and $p^* < p_{\mathrm{H}}^e$, (H2) is greater than (H3). Given (12) and $p^* < \frac{14}{17}$, (H3) is less than (H4). So p^* is binding and (H3) is the best response. So (12) and (13) constitute an equilibrium iff $p^* \geqslant \frac{2}{5}$. The solution to [(L2), (H3)] is of course (14). Given (H3), (L1) is greater than or equal to (L2) iff $p^* \leqslant \frac{2}{5}$ so (L2) is the best response in that case. Given (L2), (H2) is greater than (H3) since

$$p^* < p_{\mathrm{H}}^e < \frac{3c_{\mathrm{H}} + 2}{5}. \tag{A.4}$$

Note that this rules out the solution to [(L2), (H2)] as an equilibrium. Furthermore, since $p^* < 1$ (H3) is less than (H4). This rules out the solution to [(L2), (H4)]. So (14) is an equilibrium iff $p^* \leqslant \frac{2}{5}$. The two equilibria coincide when $p^* = \frac{2}{5}$. □

Table 1
Welfare accounting

Searches in old equilibrium	Low cost in new equilibrium	High cost in new equilibrium
1	0	c_{H}
2	s	$s - c_{\mathrm{H}}$

Proof of Proposition 3. We first consider (i) and (ii) where $\frac{2}{5} \leqslant c_{\mathrm{H}} \leqslant p^* < p^e_{\mathrm{H}}$. Table 1 summarizes the situation on I_1.

For example, consider the bottom-right corner of the table. These buyers searched twice in the old equilibrium and bought from the low-cost firm but in the new equilibrium they search once and buy from the high-cost firm. So the change in welfare associated with these buyers is $s - c_{\mathrm{H}}$. It follows that the change in welfare on I_1 is

$$\Delta W_1 = \frac{1}{2}\int_{I_1}(s - c_{\mathrm{H}})\,\mathrm{d}s = \frac{1}{4}[(\hat{s}^e)^2 - (\hat{s}^*)^2] - \frac{1}{2}c_{\mathrm{H}}(\hat{s}^e - \hat{s}^*). \tag{A.5}$$

From (11) and (15), we see that given c_{H} the expression in (A.5) is a strictly concave quadratic polynomial in p^* which is zero when $p^* = p^e_{\mathrm{H}}$. Its maximizer $p^* = (6c_{\mathrm{H}}+2)/5$ is greater than p^e_{H} so $\Delta W_1 < 0$ for all $c_{\mathrm{H}} \leqslant p^* < p^e_{\mathrm{H}}$.

We now consider I_2. From the discussion in the text, it follows that the change in welfare on I_2 is given by

$$\Delta W_2 = \int_{I_2}\left(1 - s - \frac{1}{2}c_{\mathrm{H}}\right)\,\mathrm{d}s = \frac{1}{2}[(\bar{p}^e)^2 - (\bar{p}^*)^2 + c_{\mathrm{H}}(\bar{p}^* - \bar{p}^e)]. \tag{A.6}$$

From (21) and (22), we see that given c_{H} the expression in (A.6) is a strictly concave quadratic polynomial in p^* which is zero when $p^* = p^e_{\mathrm{H}}$. Its maximizer $p^* = (6c_{\mathrm{H}}-2)/7$ is less than c_{H} so $\Delta W_2 > 0$ for all $c_{\mathrm{H}} \leqslant p^* < p^e_{\mathrm{H}}$.

We now consider $\Delta W = \Delta W_1 + \Delta W_2$ which for a given c_{H} is again a strictly concave quadratic polynomial in p^* which is zero when $p^* = p^e_{\mathrm{H}}$. Its maximizer p^*_* is given by (19) which is greater than c_{H} and less than p^e_{H} if $c_{\mathrm{H}} < \frac{98}{149}$. So $\Delta W < 0$ for all $c_{\mathrm{H}} \leqslant p^* < p^e_{\mathrm{H}}$ when $c_{\mathrm{H}} \geqslant \frac{98}{149}$ but $\Delta W > 0$ at $p^* = p^*_*$ when $c_{\mathrm{H}} < \frac{98}{149}$. To verify (18) we solve $\Delta W = \Delta W_1 + \Delta W_2 = 0$ for p^* and get p^*_{b} in (16).

We now prove (iii) where $0 < c_{\mathrm{H}} < \frac{2}{5}$. From our previous work, $\Delta W > 0$ for $\frac{2}{5} \leqslant p^* < p^e_{\mathrm{H}}$ and the optimal p^* on that interval is given in (19) provided that

$$p^*_* \geqslant \frac{2}{5} \quad \Leftrightarrow \quad c_{\mathrm{H}} \geqslant \frac{28}{85} \tag{A.7}$$

and $p^* = \frac{2}{5}$ otherwise.

When $c_{\mathrm{H}} \leqslant p^* < \frac{2}{5}$ the equilibrium becomes (14). In that case, $\hat{s}^* = 0$ so

$$\Delta W_1 = -\frac{33}{96}c^e_{\mathrm{H}} < 0. \tag{A.8}$$

Furthermore, $\bar{p}^* = p^*$ so

$$\Delta W_2 = \frac{1}{2}[(\bar{p}^e)^2 - (p^*)^2 + c_{\mathrm{H}}(p^* - \bar{p}^e)]. \tag{A.9}$$

Since $c_{\mathrm{H}} < \frac{8}{7}$, $\Delta W_2 > 0$ at $p^* = \frac{2}{5}$. Its maximizer is $p^* = c_{\mathrm{H}}/2$ so $\Delta W_2 > 0$ for all $c_{\mathrm{H}} \leqslant p^* < \frac{2}{5}$.

We now consider $\Delta W = \Delta W_1 + \Delta W_2$. Since $c_{\mathrm{H}} < \frac{392}{893}$, $\Delta W > 0$ at $p^* = \frac{2}{5}$. Its maximizer is $p^* = c_{\mathrm{H}}/2$ so $\Delta W > 0$ on $c_{\mathrm{H}} \leqslant p^* < \frac{2}{5}$ where the optimal price ceiling is $p^* = c_{\mathrm{H}}$.

Summing up, $\Delta W > 0$ for all $c_{\mathrm{H}} \leqslant p^* < p^e_{\mathrm{H}}$ and $p^* = c_{\mathrm{H}}$ is the optimal price ceiling when $0 < c_{\mathrm{H}} \leqslant \frac{28}{85}$. Comparing $\Delta W_{p^*=p^*_*}$ and $\Delta W_{p^*=c_{\mathrm{H}}}$ on $\frac{28}{85} < c_{\mathrm{H}} < \frac{2}{5}$ we get that

$$\operatorname{sign}(\Delta W_{p^*=p^*_*} - \Delta W_{p^*=c_{\mathrm{H}}}) = \operatorname{sign}\left[c_{\mathrm{H}} - \frac{4}{289}(4 + 3\sqrt{66})\right]. \tag{A.10}$$

This completes the proof. □

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