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LOCALIZED TECHNICAL PROGRESS AND CHOICE OF TECHNIQUE IN A LINEAR PRODUCTION MODEL

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ABSTRACT

The problem of choice of technique in single production linear models has been extensively analysed under the assumption that the set of processes available in the economy is exogenously given and globally known. However, since Atkinson and Stiglitz's 1969 article economists have considered technical change as a cumulative, localized and adaptive process. The aim of this paper is to develop an adaptive model of choice of technique within a classical theoretical framework. Our model provides, although in a very stylized way, an explicit description of the relationship between the currently employed processes of production and the new ones. This allows us to analyse in a rigorous way the 'secular' dynamics of the economy.

1. INTRODUCTION

The problem of choice of technique in single production linear models has been extensively analysed (see, e.g. Pasinetti (1977), Lippi (1979); for a comprehensive treatment see Kurz and Salvadori (1995, Chapters 3 and 5)). One of the main results is that if the set of techniques is compact, then there exists a long-period technique, i.e. a technique at whose prices no existing process pays positive extra-profits. This result has been provided by means of an adaptive process in the case of a finite number of processes, and by non-constructive theorems in the case of an infinite number of processes (see, e.g. Bidard, 1990; Kurz and Salvadori, 1995).

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The literature on choice of technique assumes that the set of processes available in the economy is exogenously given and globally known, and for this reason it exhibits several shortcomings. First, it does not make clear how the new processes of production are made available. Second, it does not make clear the relationship between the new processes and the current ones. Third, the long-period configuration is independent from the initial technique and, moreover, the most efficient technique will eventually always be adopted. Therefore, empirically important facts like path-dependent inefficiencies and lock-in phenomena (see, e.g. David, 1986; Liebowitz and Margolis, 1995) are completely ruled out from the analysis. The way in which new processes are made available concerns mainly the theory of technological discovery and it will not be taken up here. The second and third points have been at the centre of a radical revision of the view of technical progress in the last 30 years. In fact, since Atkinson and Stiglitz's article on localized technical progress (Atkinson and Stiglitz, 1969), economists have conceived technical progress as a cumulative, localized and adaptive process that cannot be identified merely with a shift of the production function. In fact, it is mainly a cumulative and localized phenomenon concerning the blueprint currently employed and, possibly, a few other blueprints 'close to' the one employed. The theory of 'technological paradigm' (or 'dominant design' or 'technological guidepost') (see, e.g. Abernathy and Utterback, 1975; Nelson and Winter, 1977; Sahal, 1981; Dosi, 1982, 1988) can be considered one of the main research outcomes carried out within this new and alternative context. This theory asserts that firms move along certain technological 'trajectories' which represent their technological opportunities and that these opportunities get depleted over time (the so-called 'Wolf's Law'). Moving along trajectories means, therefore, a 'relatively coherent pattern of change of input coefficients' over time (Dosi, 2001, p. 16). Localized technical change, furthermore, implies that the concept of 'best practice technique' is a local concept, and this, in turn, paves the way to deal with lock-in phenomena and path-dependent inefficiencies.

To the best of our knowledge, no attempt has been done to analyse the problem of choice of technique in linear models of production under the assumption that technical progress is an adaptive and localized phenomenon. The aim of this paper is to try to start to fill in this gap by developing a very general deterministic model of multivalued adaptive choice of technique in a linear model of production *à la* Sraffa.[1] Our model provides, although in

[1] Multivalued adaptive models have been widely employed in economics, see for example, Day and Kennedy (1970), Cherene (1978) and the bibliography here quoted. For more recent applications see D'Agata (2000), D'Agata and Santangelo (2003).

a very abstract and stylized way, an explicit description of the relationship between the currently employed processes of production and the new ones, and this allows us to analyse in a rigorous way the dynamics of an economy with localized technical progress. Since our main aim is to provide a rigorous foundation to the theory of localized technological progress, in this initial study we focus only on two basic problems: the existence of a *local secular technique*[2] and the convergence towards it of the adaptive process generated by the technical progress itself. More specifically, we first provide a quite general result concerning the convergence towards a local secular technique of the sequence of technologies generated by the adaptive process. We point out that this result ensures also the existence of the local secular technique. However, we point out that the dynamics generated by the adaptive process may be unsatisfactory on both theoretical and empirical grounds. Therefore, we provide sufficient conditions ensuring that the sequence also satisfies economically reasonable conditions.

The choice of developing our analysis within a theoretical framework *à la* Sraffa is not only due to analytical convenience but also to several theoretical reasons. The first reason is that linear models seem to be particularly suitable to deal with localized technical progress.[3] Second, because localized technical progress, as conceived by the literature on the technological trajectories, should be analysed '(i)n ways that might be to different degrees independent from changes in relative prices and demand patterns' (Dosi, 2000, p. 16). Thus, the classical approach adopted here seems to be the natural theoretical framework for this task. Last but not the least, by using a classical framework to analyse localized technical change we can provide a contribution to Pasinetti's theory of structural change (see Pasinetti, 1981, 1993), although limited only to the price equation side.

The next section introduces the model intuitively and highlights its properties and limits. Section 3 provides some preliminary remarks, while the

[2] Following a terminology close to Marshall's, we distinguish between *long-period* and *secular* configurations. According to this author, the long-period configuration is a situation in which all factors of production can be freely changed and it yields the concept of normal prices. The secular movements of normal prices being the movements 'caused by the gradual growth of knowledge, of population and of capital, and the changing conditions of demand and supply from one generation to another' (Marshall, 1982, p. 315). Since we consider the problem of technical change, our analysis considers secular movements of normal prices. The associated rest situation will be called a secular configuration. The local nature of the secular configuration we consider is due to the fact that knowledge of techniques is localized.

[3] 'A paradigm-based production theory suggests as the general case, in the short term, fixed-coefficient (Leontieff-type) techniques, with respect to both individual firms and industries. . . .' (Cimoli and Dosi, 1995, p. 249) (but see also David, 1975).

model and the results are contained in section 4. Section 5 contains final remarks.

2. SOME PRELIMINARIES

In this section we illustrate the model intuitively. Moreover, by means of numerical examples we motivate some specific results that will be provided in section 4 and highlight how our model can deal with phenomena like path-dependency and lock-in.

2.1 An intuitive illustration of the model

Consider an economy with one produced input (say corn) and one non-produced input (say labour). The price equation for this economy is: $(1 + r) ap + w\ell = p$, where r is the rate of profit, p the price of corn, w the wage rate, a is the production coefficient of corn and ℓ the labour coefficient. We assume that corn is the standard of value, hence $p = 1$. Assuming r and w are given, from the price equation we can obtain the unitary isocost at $(1 + r)$ and w, which is defined by the relation: $\ell = (1 - (1 + r)a)/w$ (see also Opocher, 2002). Figure 1 illustrates this set for $w = 0.5$ and $r = 0$ (curve q) and for $w = 0.5$ and $r = 1$ (curve t). Curve v represents the case when $w = 2$ and $r = 1$. It is natural to assume that the set of all potential techniques, indicated by X, is a subset of the first orthant. Figure 1 also illustrates an example of set X (for a non-convex case).

The adaptive process of localized technical progress we have in mind is illustrated in figure 2. Suppose the economy starts from a given set of known processes of production, described in the figure by points A_0 and B_0; these two processes of production are historically given. Given the wage rate at level w, process A_0 will be employed since it pays the highest rate of profit. Atkinson and Stiglitz (1969) point out that, because of learning-by-doing or investment in Research and Development (investment that is not made explicit in the model but which can be considered included in the current process of production), a technical advance will be generated, and that this progress may have little or no effect on other process B_0, while it improves the current process of production A_0. This can be justified by the fact that only a subset of processes 'around' A_0, $F(A_0)$, will be discovered, hence if $F(A_0)$ contains B_0, then also process B_0 will be affected by technical progress, otherwise not.

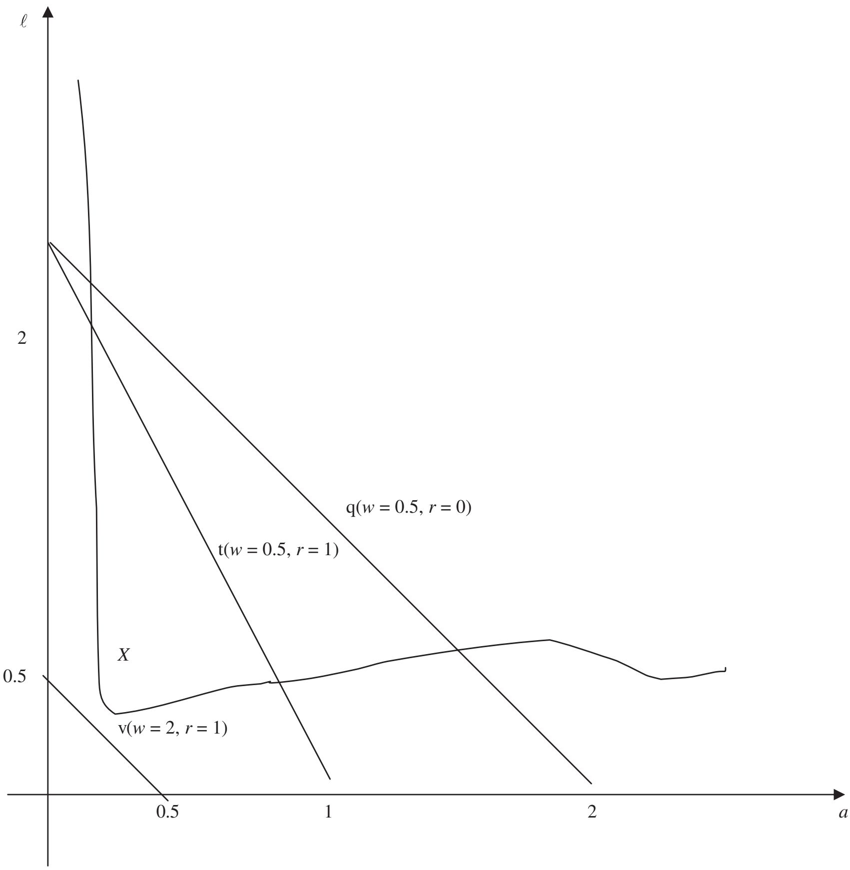

Figure 1. The set of potential techniques X and three a–λ loci each ensuring given wage and profit rates.

In the case illustrated in figure 2, at time 1 producers can choose any process among those in $F(A_0)$, and it is reasonable to assume that from process A_0 the corn producers will move to the process A_1, which is the process, among those known, which maximizes the rate of profit.[4] Once employed process A_1 at time 1, then the subset of processes $F(A_1)$ will be available (notice from the figure that processes in $F(A_0)$ can be disregarded because the processes in this set are not profitable) and a new more profitable

[4] This does not necessarily mean that producers are profit rate maximizers. For more on this point, see footnote 6.

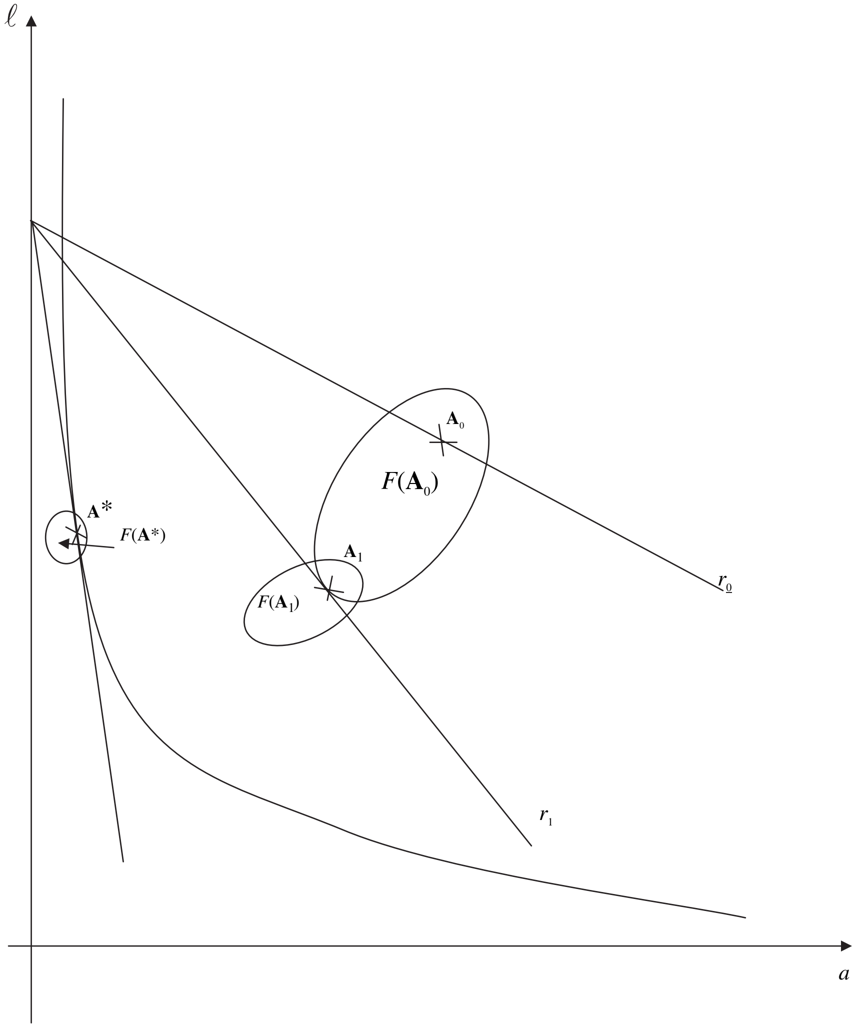

Figure 2. The iterative process activated by the localized technical progress (with wage rate constant). At time 0*, it is assumed that only process* $\boldsymbol{T}_0$ *is known and activated, and the profit rate* r_0 *is earned. At that time the set of methods* $F(\boldsymbol{T}_0)$ *will be discovered and will be available at time* 1*. At time* 1 *the technique yielding the highest profit rate,* $\boldsymbol{T}$*, will be adopted, profit rate* r_1 *will be earned, and so on, until the secular local configuration* $\boldsymbol{T}^*$ *is attained, at which no further improvement in technology is possible.*

process, A_2, will be introduced at time 2, and so on.[5] A local secular technique is a process A^* such that it is the profit-rate maximizing process amongst all process available, $F(A^*)$(again, sets $F(A_0)$, $F(A_1)$, . . . can be ignored) (see figure 2). In the next section we show that under quite general assumptions on sets $F(\cdot)$ a local secular technique exists and that the adaptive process generated by the discovery of new techniques converges towards it.

2.2 Technological change and reasonable dynamics

It should be clear that the dynamics of technology depends upon two facts: the shape and size of the sets $F(A_0)$, $F(A_1)$, . . . , and the rule of choice of the 'best technique' (in the example above we have assumed that at the beginning of each period the profit-rate maximizing technique is chosen). The latter fact will be dealt with in section 4. As far as the former is concerned, we are not able to introduce any natural assumption on the shape and size of sets $F(\cdot)$ without a theory of technical discovery. This generality, however, does not allow us to obtain any specific result concerning the dynamic properties of the adaptive process of technological innovation. In fact, the aim of the following example 1 is to show that the adaptive process may generate a technology dynamic which contradicts empirical facts (see e.g. Nelson and Winter, 1982, p. 216) as well as the established theory of technological trajectories (see the literature referred to in section 1). Because of the possibility of unsatisfactory dynamics as shown by example 1, in the next section, after obtaining the general converging result we focus our attention on sufficient conditions ensuring that the adaptive process generates a sequence of techniques which are satisfactory from both the empirical and the theoretical point of view.

Example 1: Consider a corn economy like the preceding one. Suppose that the set of all possible potential processes is set $X = \{(a, \ell) \mid a \geq 0.25, \ell \geq 1, \ell \geq 2 - 2a\}$ and suppose that $w = 0.5$. The relationship between the coefficients a and ℓ yielding the same rate of profit is described by the following relation: $\ell = 2 - 2(1 + r)a$. Suppose the initial process employed, A_0, is $(a_0, \ell_0) = (0.5, 1.5)$ and that the following rule gives the set of processes known at time t: $F(A_t) = \{(a, \ell) \in R^2_+ \mid a \leq 0.5, \ell \leq 1.5, \ell \geq 2 - (2 - (t + 2)^{-1})a\}$. Figure 3

[5] An approach similar to ours has been adopted by Nelson and Winter (1982, Chapter 9) within an evolutionary context. See also Cimoli and Dosi (1995) and Cimoli and Della Giusta (1998).

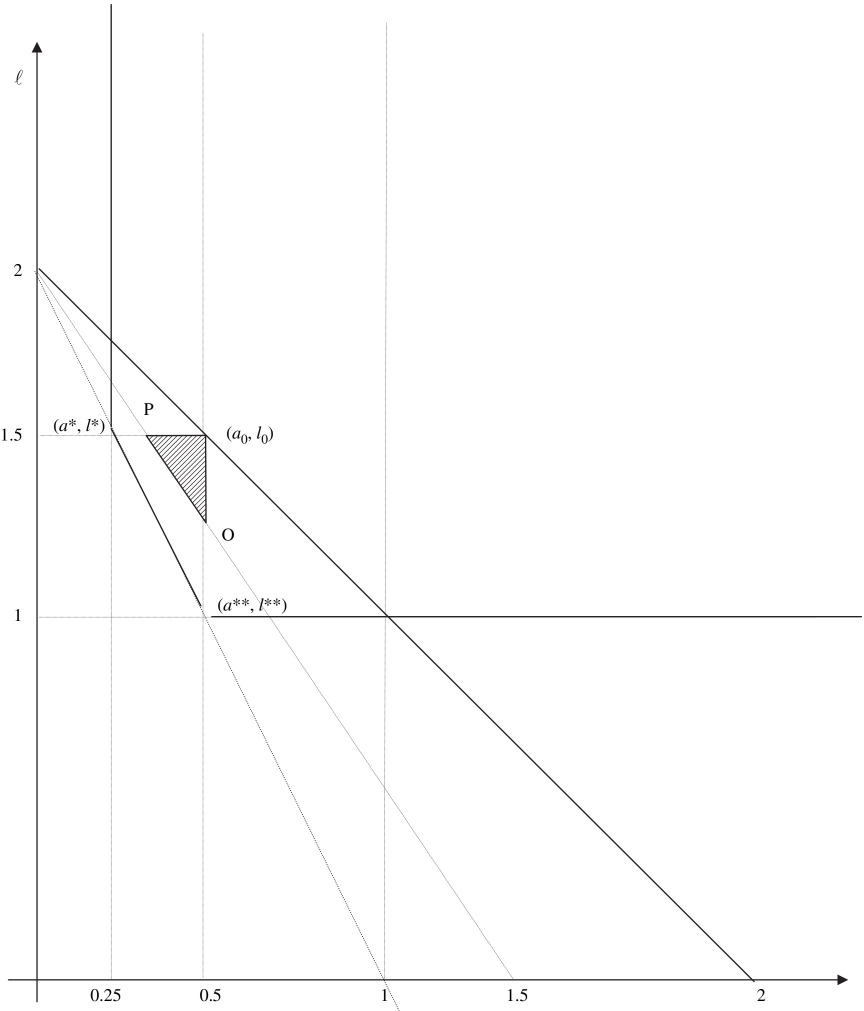

Figure 3. It is possible to generate dynamics which are capital saving at odd periods and labour saving at even periods, contradicting theoretical and empirical findings.

illustrates this case where the shaded area is set $F(A_0)$. It is easy to show that at period t, producers are indifferent among all the processes in set $F(A_t)$ which satisfies the condition $\ell = 2 - (2 - (t + 2)^{-1})a$ (for time 0, segment $P0$ in figure 3) hence the following choice rule may be used: at time $t + 1$ the process (a_{t+1}, ℓ_{t+1}) will be adopted, where:

$(a_{t+1}, \ell_{t+1}) \in F(A_t)$ with $\ell_{t+1} = 1.5$ and $\ell_{t+1} = 2 - \left(2 - (t+2)^{-1}\right)a_{t+1}$ if t is odd and $(a_{t+1}, \ell_{t+1}) \in F(A_t)$ with $\ell_{t+1} = 0.5$ and $\ell_{t+1} = 2 - \left(2 - (t+2)^{-1}\right)a_{t+1}$ if t is even.

It is easy to show that the sequence (a_t, ℓ_t) has two convergent subsequences $\{(a_{2t}, \ell_{2t})\}$ and $\{(a_{2t+1}, \ell_{2t+1})\}$ with $\{(a_{2t}, \ell_{2t})\}$ converging to $(a^*, \ell^*) = (0.25, 1.5)$ and $\{(a_{2t+1}, \ell_{2t+1})\}$ converging to $(a^{**}, \ell^{**}) = (0.5, 1)$ (see figure 3). Subsequence $\{(a_{2t}, \ell_{2t})\}$ lies on the segment $(a^*, \ell^*) - (a_0, \ell_0)$, subsequence $\{(a_{2t+1}, \ell_{2t+1})\}$ lies on the segment $(a^{**}, \ell^{**}) - (a_0, \ell_0)$). Hence the dynamics of technical progress hardly can be said to have any 'coherent pattern of change in input coefficients' because the trajectory $\{(a_{2t}, \ell_{2t})\}$ is purely capital saving, while the trajectory $\{(a_{2t+1}, \ell_{2t+1})\}$ is purely labour saving. As it is clear from this example, the problem lies in the fact that we do not put any restriction on the size and shape of the set of the newly discovered techniques (i.e. of the set $F(\cdot)$). In the next sections, we describe in a formal way the adaptive model here introduced and provide sufficient conditions on set $F(\cdot)$ ensuring that the unsatisfactory dynamics of the kind pointed out in the preceding example does not emerge.

2.3 Path-dependency and lock-in

As already indicated in section 1, one of the most important features of localized technical change is the possibility to explain important empirical phenomena like path-dependency and lock-in. In this section we show, by means of a very simple example, how the model here developed can explain such phenomena.

A process is said to show path-dependence if the dynamics are determined by the initial conditions: 'A *path-dependent* sequence of economic changes is one of which important influences upon the eventual outcome can be exerted by temporally remote events, including happenings dominated by chance elements rather than systematic forces' (David, 1975, p. 332). Localized knowledge is one of the most important causes of path-dependence as the mode of development of a technology is strongly influenced by the initial conditions. Linked with path-dependency there is the possibility of inefficiency, i.e. the possibility of being locked-in to a technology that does not provide maximal payoffs, this because '[t]aking decisions and ... eliminating options in the context of ignorance entail the risk of missing the best route of development' Foray (1997, p. 737). The following example shows how path-dependency and inefficiency can arise in our model.

Example 2: Consider a corn economy like the one introduced in example 1. Suppose that the set of all possible potential processes is set $X = \{(a, \ell) \mid a \geq$

0.025, $\ell \geq 0.1\}$ and suppose that $w = 1$. Hence, the relationship between the coefficients a and ℓ yielding the same rate of profit is described by the following relation: $\ell = 1 - (1 + r)a$. Suppose the two initial processes are available: $A_0 = (a_0, \ell_0) = (0.125, 0.75)$, $A_0' = (a_0', \ell_0') = (0.375, 0.25)$ and that the following rules give the set of processes known at each time and for each process A_0' and A_0', where a_t and a_t' indicates the corn coefficients associated with the method effectively employed at time t starting from process A_0 and A_0', respectively:

$$F(A_{t+1}) = \left\{(a, \ell) \in R_+^2 \middle| \ell = 0.75, 0.025 + 0.1(2+t)^{-1} \leq a \leq a_t\right\},$$
$$F(A_{t+1})' = \left\{(a, \ell) \in R_+^2 \middle| \ell = 0.25, 0.025 + 0.35(2+t)^{-1} \leq a \leq a_t'\right\}.$$

It is easy to check that $a_t = 0.025 + 0.1(2 + (t - 1))^{-1}$ and $a_t' = 0.025 + 0.35 (2 + (t - 1))^{-1}$, hence $1 + r_{t+1} = \dfrac{1 - \ell_{t+1}}{a_{t+1}} = \dfrac{0.75(1+t)}{0.125 + 0.025t} > 1 + r_{t+1'} = \dfrac{1 - \ell'_{t+1}}{a_{t+1'}} = \dfrac{0.25(1+t)}{0.375 + 0.025t}$, i.e. any firm which starts its activity by using process A_0 will get profits higher than any other firm starting by using process A_0'.

3. TECHNICAL PRELIMINARIES

Consider a single production n-good economy *à la* Sraffa (1960). We suppose that the set of all possible techniques *potentially* available in sector $i = 1, 2, \ldots, n$ is given by set X_i and set $X = \Pi_i X_i$. A generic process of sector i is denoted by $\mathbf{b}_i \equiv (\mathbf{a}_i, \ell_i)$ where $\mathbf{a}_i$ is the $1 \times n$-dimensional input vector and ℓ_i the labour coefficient. A generic *technique* is described by a matrix $\mathbf{A} = (\mathbf{a}_1, \ldots, \mathbf{a}_n)^{\mathrm{T}}$ of input coefficients and by a vector $\ell = (\ell_1, \ldots, \ell_n)^{\mathrm{T}}$ of labour coefficients, where $(\mathbf{a}_i, \ell_i) \in X_{\mathrm{i}}$ (the superscript T indicates the transposition operator). A generic technique $(\mathbf{A}, \ell)$ is denoted also by $\mathbf{T}$ *Assumption 1:* For every i, X_i is a compact subset of R_+^{n+1}; moreover, for any $\mathbf{b}_i, \in X_i$, one has that $\mathbf{a}_i \geq 0$ and $\mathbf{a}_i \neq 0$. Finally, for every $\mathbf{T} \equiv (\mathbf{A}, \ell) \in X$, matrix $\mathbf{A}$ is undecomposable.

The compactness of sets X_i and the undecomposability of matrix $\mathbf{A}$ are simplifying assumptions. Undecomposability means that all goods are basics (see Sraffa, 1960). Let us assume that technique $\mathbf{T} \equiv (\mathbf{A}, \ell) \in X$ is used. Hence the following price equation is associated: $(1 + r(\mathbf{T}))\mathbf{Ap}(\mathbf{T}) + w(\mathbf{T})\ell = \mathbf{p}(\mathbf{T})$, where the symbols have the usual meaning. We assume that for every $\mathbf{T} \in X$, $w(\mathbf{T}) = w$ and that $\mathbf{d}^{\mathrm{T}}\mathbf{p}(\mathbf{T}) = 1$; i.e. the wage rate is exogenously given at level w and the bundle indicated by vector $\mathbf{d}$ is used as a standard of value. It is well known that, under particular assumptions on technology (see later), a positive maximum wage rate $W(\mathbf{T})$ can be associated to technique $\mathbf{T} = (\mathbf{A}, \boldsymbol{\ell})$.

Lemma 1: The rate of profit $r(\boldsymbol{T})$ is a continuous function of technique $\boldsymbol{T}$ in X.

Proof: It follows from Assumption 1, which ensures that some elements of matrix **A** are positive. ■

Behavioural assumption: Producers introduce a new process only if it pays positive extra-profits at the current production prices.

The following lemma implies that this behaviour ensures that whenever a new process is introduced the economy experiences an increase of the uniform profit rate. In fact, consider two techniques **T** and **T′**:

Lemma 2: If $(1 + r(\boldsymbol{T}))\boldsymbol{A}'\boldsymbol{p}(\boldsymbol{T}) + w\,\boldsymbol{\ell}' \leq \boldsymbol{p}(\boldsymbol{T})$, with some inequality holding as strict inequality, then $r(\boldsymbol{T}') > r(\boldsymbol{T})$.

Proof: The result is well known as the Okishio Theorem, so the proof will be omitted (see, e.g. Bowles, 1981). ■

Lemma 3: If $\boldsymbol{T}^* \in X$ is such that $r(\boldsymbol{T}^*) \geq r(\boldsymbol{T})$, for every $\boldsymbol{T} = (\boldsymbol{A}, \boldsymbol{\ell}) \in X$, then $(1 + r(\boldsymbol{T}^*))\boldsymbol{A}\,\boldsymbol{p}(\boldsymbol{T}^*) + w\,\boldsymbol{\ell} \;\geq \boldsymbol{p}(\boldsymbol{T}^*)$.

Proof: Suppose not. Then, there exists an industry $i \in \{1, 2, \ldots, n\}$ and a process $(\mathbf{a}_i, \mathbf{l}_i) \in X_i$ such that: $(1 + r(\mathbf{T}^*))\mathbf{a}_i\,\mathbf{p}(\mathbf{T}^*) + w\,\ell_i < p_i(\mathbf{T}^*)$. Denote by $\mathbf{T}'$ the technique $\mathbf{T}^*$ whose i-th process is replaced by process $(\mathbf{a}_i, \mathbf{l}_i)$. Then, $(1 + r(\mathbf{T}^*))\mathbf{A}'\,\mathbf{p}(\mathbf{T}^*) + w\,\boldsymbol{\ell}' \leq\; \mathbf{p}(\mathbf{T}^*)$, where the strict inequality holds true only for the i-th equation. Lemma 2 ensures that $r(\mathbf{T}') > r(\mathbf{T}^*)$ for $\mathbf{T}' \in X$, which is a contradiction. ■

The technique $\mathbf{T}^*$ satisfying the profit rate condition in Lemma 3 is called *dominant* (see, e.g. Bidard, 1990) or *long period position technique* (Kurz and Salvadori, 1995).

Lemmas 2 and 3 ensure that producers, following the behavioural assumption, introduce a process yielding extra profits at the current production prices, and that this process increases the rate of profit; moreover, when the technique yielding the highest rate of profit is attained, there is no incentive to replace active processes with inactive ones. The behavioural assumption, therefore, intuitively justifies the conclusion that if we allow 'enough time' to producers to change processes, then eventually they will activate the dominant technique (i.e. the profit-rate maximizing one).

4. A MODEL OF CHOICE OF TECHNIQUES WITH LOCALIZED TECHNICAL CHANGE

Consider the economy introduced in the preceding section and suppose that a technique $\mathbf{T}_t = (\mathbf{A}_t, \ell_t)$ is adopted at time t. Let us assume that the set of processes available at time $t + 1$ in industry i are the elements of the set $F_i(\mathbf{T}_t) \subset X_i$. This set is also called the set of technological opportunities of industry i at time $t + 1$. Let us adopt the following definitions: A g*lobal secular configuration* (GSC) is a technique $\mathbf{T}^* = (\mathbf{A}^*, \ell^*) \in X$ such that for every i: $(1 + r^*(\mathbf{T}^*))\mathbf{a}_i\, \mathbf{p}(\mathbf{T}^*) + w\, \ell_i \geq p_i(\mathbf{T}^*)$ for every $(\mathbf{a}_i, \ell_i) \in X_i$. A *local secular configuration* (LSC) is a technique $\mathbf{T}^* = (\mathbf{A}^*, \ell^*) \in X$ such that for every i: $(1 + r(\mathbf{T}^*))\mathbf{a}_i\, \mathbf{p}(\mathbf{T}^*) + w\, \ell_\mathrm{I} \geq p_i(\mathbf{T}^*)$ for every $(\mathbf{a}_\mathrm{i}, \ell_\mathrm{i}) \in F_i(\mathbf{a}_i^*, \ell_i^*)$. Finally, suppose that only the subset of processes $Y_i \subset X_i$ is 'temporarily' available to industry i, then *a temporary configuration with respect to the subset of techniques* Y (TC-Y) is a technique $\mathbf{T}^* = (\mathbf{A}^*, \ell^*) \in Y \equiv \Pi_i Y_i$ such that for every i: $(1 + r(\mathbf{T}^*))\mathbf{a}_i\, \mathbf{p}(\mathbf{T}^*) + w\, \ell_i \geq p_i(\mathbf{T}^*)$ for every $(\mathbf{a}_i, \ell_i) \in Y_i$. Intuitively, a TC-$Y$ is a technique which yields non-negative extra costs in all industries at the current production prices and given the set of processes currently available in the economy. Notice that over time a LSC is a self-enforcing configuration, while a TC-Y is not necessarily so if technical progress is taken into account. The following is a standard result and means that a TC-Y is a profit-rate maximising technique among the available ones.

Remark 1: A TC-$\Pi_i F_i$(T) is a solution to the following problem: max $r(\mathbf{T})$ sub $T \in \Pi_i F_i(\mathbf{T})$.

The dynamics of the economy are modelled according to the following adaptive process which should further highlight the difference between LSCs and TC-Ys:

Adaptive process (AP): At time 0 we assume that technique $\mathbf{T}_0 = (\mathbf{A}_0, \ell_0)$ is historically activated, determining a price vector $\mathbf{p}(\mathbf{T}_0)$ and a rate of profit $r(\mathbf{T}_0)$. With reference to industry i, the subset of processes $F_i(\mathbf{T}_0) \subset X_i$ will be 'discovered' at time 0 and we assume that these processes will be available at time 1. Set $F_i(\mathbf{T}_0)$ can be called the technological opportunity set of firms in industry i at time 1. From the interpretative point of view set $F_i(\mathbf{T}_0)$ is obtained because of either an activity of R&D or learning-by-doing. It is worth emphasizing that in our model, this set does not depend only upon the process employed in industry i but it may depend also on the processes used in other sectors. So, the way in which we formalize technological progress is able to deal with the existence of spillovers among sectors. Since we want to characterize knowledge as local, it is reasonable to assume that the newly discovered processes in sector i are discovered 'around' the current one, $\mathbf{b}_{i0}$; i.e. $\mathbf{b}_{i0} \in F_i(\mathbf{T}_0)$ (for further assumptions on $F_i(\mathbf{T}_0)$ see Assumption 3 below and remarks hereafter). At time 1 we assume that a TC-$\Pi_i F_i(\mathbf{T}_0)$ technique, say $\mathbf{T}_1$, is adopted.[6] Hence, at this time a price vector $\mathbf{p}(\mathbf{T}_1)$ and a profit rate $r(\mathbf{T}_1)$ will rule. At time 1, however, a new subset of processes $F_i(\mathbf{T}_1)$ is discovered in industry i and will be available at time 2, and so on.

The following assumptions summarize the properties we adopt on the initial technique $(\mathbf{A}_0, \ell_0)$ and on the set $F_i(\mathbf{T}_t)$ for every t and every i:

Assumption 2: The dominant eigenvalue of matrix $\mathbf{A}_0$ is less then 1. Moreover, $0 \leq w \leq W(\mathbf{T}_0)$.

Assumption 3: For every i, correspondence F_i: $X_i \rightarrow X_i$ defined as above is compact-valued and continuous. Moreover, for every t, $\mathbf{b}_{it} \in F_i(\mathbf{b}_{it})$.

The first part of Assumption 2 ensures that the technique introduced at time 0 is productive; the second part implies that $\boldsymbol{p}(\mathbf{T}_0)$ is positive and $r(\mathbf{T}_0)$ is non-negative. The first part of Assumption 3 is technical, in particular, continuity means that set $F_i(\mathbf{b}_{it})$ changes 'smoothly' as the technique used changes. The fact that $\mathbf{b}_{it} \in F_i(\mathbf{b}_{it})$ ensures that $F_i(\mathbf{b}_{it})$ is always non-empty, moreover, it characterizes the new processes as being localized 'around' the currently employed process (more specific topological assumptions could be introduced to characterize this 'local' property; for example, it may be assumed that set $F_i(b_{it})$ contains a neighbourhood of process b_{it}. These

[6] In the light of the comments made just after the proof of Lemma 3, this assumption should not be interpreted as if producers are profit-rate maximizers, and it could be replaced by more realistic assumptions without affecting the main results and problems considered in this paper. An important implication of this assumption is that at the beginning of each period, input prices are equal to output prices. This is justified if period 0 is 'long enough' to allow full adjustment of prices to production prices associated with the dominant technique. This means that in our analysis each period corresponds to the long period considered by works dealing with technical change (see, e.g. Pasinetti, 1977; Lippi, 1979; Bidard, 1990; Kurz and Salvadori, 1995). This assumption seems to be reasonable here as we are interested mainly in *secular* equilibria, and our dynamics therefore concern changes in long-period position techniques. The assumption that each period is 'long enough' to allow us to consider production prices associated with the dominant technique will be maintained throughout the paper. ■

assumptions, suggestive as they may be, are not necessary for the subsequent analysis).

Lemma 4: Under Assumptions 1 and 2, the AP is well defined. Moreover, it generates a sequence of positive price vectors $\{\boldsymbol{p}(\boldsymbol{T}_t)\}$ *and non-negative profit rates* $\{r(\boldsymbol{T}_t)\}$ *such that for every t,* $r(\boldsymbol{T}_{t+1}) \geq r(\boldsymbol{T}_t)$, *where the equality holds only if the technique* $\boldsymbol{T}_t$ *is a LSC.*

Proof: Given the initial technique $\mathbf{T}_0 = (\mathbf{A}_0, \ell_0)$, at time 1 the technique employed is a TC-$\Pi_i F_i(\mathbf{T}_0)$. Set $\Pi_i F_i(\mathbf{T}_0)$ is closed by Assumption 2 and therefore compact by Assumption 1. By Lemma 1 the problem in Remark 1 is well-defined, so the technique that will be chosen at time 2 is well-defined as well, and so on. The second part of the lemma follows from the Behavioural Assumption and from Lemma 2. ■

Proposition 1: Under Assumptions 1 and 3 and for whatever initial technique $(\mathbf{A}_0, \ell_0)$ satisfying Assumption 2, the AP stops either in a finite or in an infinite number of steps. In the first case, the (finite) sequence of techniques converges to a LSC; in the latter case, the (infinite) sequence of techniques either converges to a LSC or the limit of every convergent subsequence is a LSC.

Proof: If AP stops in a finite number of steps, then the finite sequence must, trivially, converge to only one LSC. If the AP generates an infinite sequence of techniques, by Assumption 3, for every i and for every t correspondence B_i defined by the set $B_i(\mathbf{T}_t)$—where the latter is the set of processes in industry i which belongs to some TC-$\Pi_i F_i(\mathbf{T}_t)$—is upper semi-continuous; moreover, it is closed-valued. Hence, it is closed (Border, 1985, Proposition 11.9.(a)). Notice that $B_i(\mathbf{T}_t) \subset X_i$. Now, the AP together with the correspondence $\Pi_i B_i(\mathbf{T}(t))$ can be modelled as an algorithm *à la* Zangwill (1969), and it is possible to show that all conditions for applying Convergence Theorem A in Zangwill (1969, p. 91) are satisfied. Thus, by this theorem, the assertion holds true. ■

Proposition 1 is a quite general result, since it shows not only that, under the stated assumptions, the economy converges to the LSCs, but it also ensures, thanks to a standard compactness argument, the *existence* of a LSC. A shortcoming of Proposition 1, already emphasized, is that if the AP generates an infinite sequence of techniques, this sequence may converge to two or more radically different LSCs. A straightforward way to exclude this possibility is to assume that there is a unique LSC (and hence a unique GSC). In this case, in fact, an easy argument shows that the whole convergent sequence generated by the AP converges to the GSC. The remaining part of this section is devoted to providing two alternative sufficient conditions ensuring that the whole (infinite) sequence converges to a single LSC.

Assumption 4: There exists a family of disjoint compact neighbourhoods of the LSCs, say $\mathcal{N}$, such that each element of the family contains at most one LSC and, if T is a LSC, then $\Pi_i F_i(\mathbf{T}) \subset N(\mathbf{T})$, where $N(\mathbf{T}) \in \mathcal{N}$.

Assumption 4 intuitively means that each LSC does not belong to the technological opportunity set of any other LSC. In the language of technological paradigm, this assumption might be interpreted in the sense that each trajectory leading to a LSC is 'powerful' (see Dosi, 1982, p. 154). The following proposition states that Assumptions 1, 2, 3 and 4 are sufficient for generating nice dynamics.

Proposition 2: Under Assumptions 1, 2, 3 and 4 the whole (infinite) sequence generated by the AP converges to a LSC.

Proof: First we show that if $\{\mathbf{T}_t\}$ is a sequence generated by the AP, then $d(\mathbf{T}_t, \mathbf{T}_{t+1}) \to 0$ as $t \to \infty$, where d is any metric defined on the set of techniques. Suppose this is not so. Then, there exists a subsequence $\{\mathbf{T}_t\}$ such that $d(\mathbf{T}_t, \mathbf{T}_{t+1}) \to \beta > 0$ as $i \to \infty$. We may assume that $\{\mathbf{T}_t\}$ converges to $\mathbf{T}^*$ and that $\{\mathbf{T}_{t+1}\}$ converges to $\mathbf{T}^{**}$. Clearly, $d(\mathbf{T}^*, \mathbf{T}^{**}) \geq \beta$. By Proposition 1, $\mathbf{T}^*$ and $\mathbf{T}^{**}$ are LSCs; moreover, $\mathbf{T}_{t+1} \in F(\mathbf{T}_t)$, then by Assumption 3, $\mathbf{T}^{**} \in F(\mathbf{T}^*)$ But this contradicts Assumption 4. Suppose now that the assertion of Proposition 2 is not true. Therefore, if $\{\mathbf{T}_t\}$ is the sequence generated by the AP, then there must exist (at least) two subsequences, say $\{\mathbf{T}_{t'}\}$ and $\{\mathbf{T}_{t''}\}$ converging to $\mathbf{T}'$ and $\mathbf{T}''$, respectively. By Proposition 1, every accumulation point of sequence $\{\mathbf{T}_t\}$ is an LSC; therefore, by Assumption 4, it is possible to take two positive numbers ε' and ε'' such that points $\mathbf{T}'$ and $\mathbf{T}''$ are the only accumulation points in $B_{\varepsilon'}(\mathbf{T}')$ and $B_{\varepsilon''}(\mathbf{T}'')$, where $B_{\varepsilon'}(\mathbf{T}') \subset N(\mathbf{T}')$ and $B_{\varepsilon''}(\mathbf{T}'') \subset N(\mathbf{T}'')$. Choose a positive number Z in such a way that $d(\mathbf{T}(z), \mathbf{T}(z + 1)) < \varepsilon'/3$ for $z \geq Z$ (That such a number exists follows from the result at the beginning of this proof). However, $(\mathbf{T}')$ is an accumulation point of sequence $\{\mathbf{T}_t\}$, therefore for infinitely many indices s one has that $d(\mathbf{T}_s, \mathbf{T}') < \varepsilon'/3$. On the other hand, $(\mathbf{T}'')$ is another accumulation point of $\{\mathbf{T}_t\}$, hence, by the fact that $(\mathbf{T}'') \notin B_{\varepsilon'}(\mathbf{T}')$, there must exists infinitely many indices q such that $d(\mathbf{T}_q, \mathbf{T}') \geq 2\varepsilon'/3$. From the way in which Z has been defined, it follows that there exist infinitely many indices $m \geq Z$ such that $(\varepsilon'/3) \leq d(\mathbf{T}_m, \mathbf{T}') \leq (2\varepsilon'/3)$. This implies that there exists an accumulation point in the set $\{\mathbf{T} \in X \mid (\varepsilon'/3) \leq d(\mathbf{T}, \mathbf{T}') \leq (2\varepsilon'/3)\} \subset N(\mathbf{T}')$, which contradicts Assumption 4. ■

Assumption 4 imposes conditions on the 'size' of the sets of technological opportunities of each industry when the economy is at LSCs. However, especially in the case in which technical progress is induced by learning-by-doing, it is natural to assume that learning-by-doing is bounded, i.e. that the incremental discovery of new processes of production tends to zero as time passes (see Young, 1993, especially footnote 3; but refer also to Wolf's Law quoted in the introduction for R&D). The boundedness of learning-by-doing in our approach can be formalized by assuming that the 'size' of set $F_i(\mathbf{T}_t)$ tends to zero as time flows.

Assumption 5: For every i, $\mathrm{diam}(F_i(\mathbf{T}_t)) \to 0$ as $t \to \infty$ where $\mathrm{diam}(F_i(\mathbf{T}_t) = \inf\{n \mid B(n) \subset F_i(\mathbf{T}_t)\}$ and $B(n)$ denotes a ball of radius n in $R_+^{n\times(n+1)}$.

Assumption 5 ensures the following result, whose proof is trivial.

Proposition 3: Under Assumptions 1, 2, 3 and 5 the (infinite) sequence of techniques generated by the AP converges to a LSC.

5. FINAL REMARKS

In this paper we have developed an adaptive linear model with localized technical progress. We have provided sufficient conditions ensuring that the dynamics converge towards local secular techniques and that the sequence is compatible with empirical results and with theoretical assertions concerning the patterns of change of technical coefficients. It may be worthwhile to emphasize that the theory of technological trajectories we have adopted is only a simplified view of it. In particular we have excluded trajectories concerning the evolution of the structure of the produced goods.

We have assumed throughout the paper that the set of technological opportunities is multivalued. This assumption may be justified also on the grounds that local technical progress can have spillovers on nearby processes.

Finally, if we assume that only one new process is discovered at each time, then our analysis is simplified; however, while Proposition 1 still holds true, the problem of 'irregular' dynamics disappears. In fact, under the assumption of continuity, the adaptive process generates a sequence of techniques with only one accumulation point. In this case, moreover, our analysis can be considered the discrete time version of Pasinetti's theory of structural change, although limited only to the price and distribution side of the economy.

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