Available online at www.sciencedirect.com

SCIENCE DIRECT®

Economics Letters 87 (2005) 337–345

economics letters

www.elsevier.com/locate/econbase

Money as real options in a cash-in-advance economy

Stacie Beck, David R. Stockman*

Department of Economics, University of Delaware, Newark, DE 19716, United States

Received 1 June 2004; received in revised form 29 June 2004; accepted 19 January 2005
Available online 2 April 2005

Abstract

We characterize the optimal exercise point and equilibria in which money as real options has positive option value. We investigate the effect of uncertainty on the option value of money and the optimal exercise strategy.

Keywords: Money; Real options; Cash-in-advance

JEL classification: D5; E4; G1

1. Introduction

In this paper, we show how money can be characterized as options, thus integrating monetary theory into financial theory and leading to a better intuition for the effect of uncertainty on the liquidity demand for money and near-monies. We illustrate our approach using Svensson (1985) version of the cash-in-advance (CIA) model of money demand.

2. The option value of money

The real options literature stresses two important aspects of real investments that add an option component to an investment opportunity: irreversibility and the possibility of delay. In a CIA model, a unit of money is an option on goods. If exercised today, the holder receives $1/P$ goods where P is the

* Corresponding author. Tel.: +1 302 831 1903; fax: +1 302 831 6968.
E-mail address: stockman@udel.edu (D.R. Stockman).

0165-1765/$ - see front matter
doi:10.1016/j.econlet.2005.01.011

price level (with a non-storable perishable good, this is completely irreversible). If not exercised, the option is carried over into the future. Since the time of exercise is determined by the holder and there is no expiration date on the option, a dollar is an infinitely-lived American call option on goods. If the holder exercises all of his options, the CIA constraint is binding. Otherwise, the CIA constraint is not binding. We formalize this intuition by showing that the equilibrium found in the CIA economy is equivalent to an economy with infinitely-lived American call options on goods that are in positive net supply.

2.1. An equivalent option economy

Let Q_t be the quantity of options held by the household at time t. Each option allows the household to S_t units of the non-storable consumption good at time t if exercised. If not exercised at time t, the option remains alive. If the option is exercised at date $t+s$, the household receives S_{t+s} units of the consumption good. Let $0 \leq \alpha_t \leq 1$ be the fraction of options exercised at time t. A household is a seller–shopper pair. The endowment y_t (a stationary Markov process with bounded support) is taken by the seller to the option exercise market where it is exchanged for options. The shopper also goes to the option exercise market where some or all of the options are exercised, i.e., exchanged for goods. Subsequently, there is an asset market where options can be bought or sold and during which the household receives a lump-sum transfer of options T_t from the government. Let V_t be the value of an option at time t that can be exercised no earlier than t. The household's problem is to choose a portfolio and exercise strategy $\{Q_{t+1},\alpha_t\}$ along with a consumption plan $\{c_t\}$ to maximize

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U(c_t)\right], \tag{1}$$

subject to

$$0 \leq c_t \leq \alpha_t Q_t S_t, \tag{2}$$

$$V_t Q_{t+1} = V_t(1-\alpha_t)Q_t + y_t + V_t T_t, \tag{3}$$

$$0 \leq \alpha_t \leq 1, \tag{4}$$

taking as given Q_0 and $\{V_t, S_t, y_t, T_t\}$. The government is controlling the supply of these options on goods according to

$$Q_{t+1} = w_t Q_t, \tag{5}$$

where w_t is a stationary Markov process with compact and connected support that is known at time t in the option exercise market. The government achieves this change in supply by way of lump-sum transfers,

$$T_t = (w_t - 1)Q_t. \tag{6}$$

What is the intuitive interpretation of this option economy? Since the option is an infinitely-lived American option, it must satisfy an option pricing formula:

$$V_t = \max\{S_t, E_t[z_{t+1}V_{t+1}]\}, \tag{7}$$

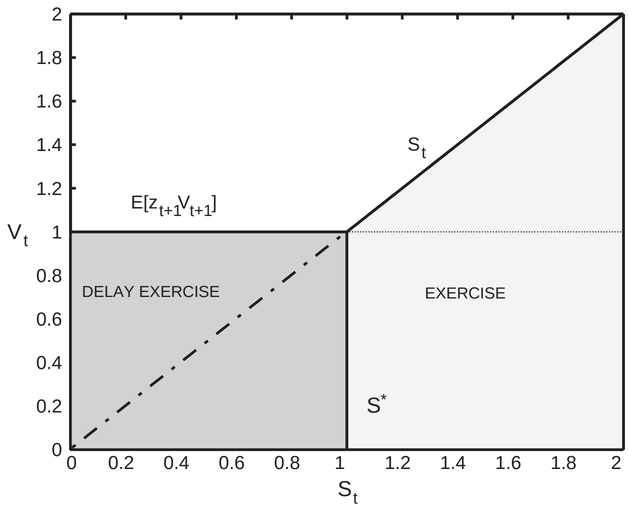

Fig. 1. Graph of option value: $V_t = \max[S_t, E_t[z_{t+1}V_{t+1}]$. If $S_t > S^*$, the household should exercise the option. In equilibrium, the economy is at the kink S^* or to the right.

where V_t is the value of the option, S_t is the number of units of the consumption good that could be obtained per exercised option (the real value of the underlying in units of consumption less the exercise price, which in this case is zero), and z_{t+1} is a stochastic discount factor. The graph of the option pricing formula (as a function of the units of the underlying consumption good) is in Fig. 1.

The household will always optimally exercise its options to be either at the "kink" in the payoff function (call this point S^*) or to the right, i.e., wishing it had more options to exercise or indifferent between exercising the marginal option. This implies that in equilibrium, we will have $S_t = V_t$. The value S^* may be interpreted as the "optimal stopping point." When $S = S^*$, the individual is indifferent between exercising and not exercising [the option is "at the money"]. In the range $(0, S^*)$, the individual strictly prefers not to exercise, [the option is "out of the money"]. In the range $(S^*, +\infty)$, there is strict preference to exercise [the option is "in the money"]. As with real options, an increase in uncertainty raises the value of the option and shifts the optimal stopping point to increase the "no-exercise" range of the state space.

In the Appendix to the paper we prove the following proposition:

Proposition 1. *A stationary rational expectations equilibrium (SREE) in the CIA economy is isomorphic to a stationary rational expectations equilibrium in the real options economy.*

3. A non-binding CIA constraint and positive option value of cash

The options interpretation of money facilitates the understanding of the impact of uncertainty on the value of money. We analyze three models with shocks that are iid across time: (1) nominal shocks only, (2) real shocks only, and (3) both real and nominal shocks.[1]

[1] Since the results and intuition of model (3) combines those of models (1) and (2), we delegate model (3) to an Appendix.

3.1. Money growth shocks

Suppose the endowment process is constant and monetary policy follows an exogenous growth rate as in Eq. (5), where w_t is an iid random variable with compact and connected support $[w_l, w_u]$ and c.d.f. $F(w)$. Next, we characterize conditions under which the option value will be positive.

Proposition 2. *If there exists an equilibrium with states under which the option value is positive (CIA constraint does not bind), then this will occur over a region* $[w_l, \hat{w}]$ *with* $w_l < \hat{w} < w_u$. *Furthermore, we have the following functional forms for real money balances* $q(w)$, *where* $q = SQ$:

$$q(w) = \begin{cases} y\hat{w}/w & \text{for } w_l \leq w \leq \hat{w} \\ y & \text{for } \hat{w} < w \leq w_u. \end{cases} \tag{8}$$

Proof. See appendix. □

Expression (8) indicates that, when $w < \hat{w}$ the option value is positive, whereas when $w > \hat{w}$ the option value is zero. A low realization of w implies a contraction in the money supply, thus a lower price level tomorrow relative to today is probable. This expected drop in the price level leaves the household indifferent between spending and not spending an extra available dollar. Hence, in this region of state space, the option value of money is positive. Likewise, a high realization of w implies an expansion in the money supply and a likely higher price tomorrow relative to today. This encourages the household to spend all of its cash holdings. Later we show how the ranges of these regions are affected by nominal uncertainty.

3.2. Endowment shocks

Now let the endowment process be an iid process with compact and connected support on $[y_l, y_u]$ with c.d.f. $G(y)$. The money growth rate is constant: $w = \bar{w} > \beta$. Equilibria with a non-binding CIA constraint will depend on the agent's attitude toward risk reflected in the curvature of the utility function. In particular what matters is

$$Z(y) := U'(y)y\bar{w}. \tag{9}$$

The function $Z(\cdot)$ is not restricted in terms of slope (increasing, decreasing, constant, etc.).

Proposition 3. *If there exists an equilibrium with states under which the CIA constraint does not bind (the option value is positive), then this will occur over a region*

$$N^{\hat{y}} := \{y : Z(y) < \hat{y}\}, \tag{10}$$

and it will bind over

$$B^{\hat{y}} := \{y : Z(y) \geq \hat{y}\}, \tag{11}$$

where $Z(\cdot)$ *is given by* (9). *Furthermore, we have the following functional form for real money balances* q:

$$q(y) = \begin{cases} (\hat{y}/\bar{w})[1/U'(y)] & \text{for } y \in N^{\hat{y}} \\ y & \text{for } y \in B^{\hat{y}}, \end{cases} \tag{12}$$

where the constant $\hat{y} = E[\beta U'(y')q(y')]$.

Proof. See appendix. □

How does an endowment shock affect the agent's decision to spend cash? Assume for a moment the CIA is always binding and ask how the current level of output affects the desirability of money as an asset. In this case, we have

$$1>E\left[\beta\left(\frac{U'(y')}{U'(y)}\right)\left(\frac{S'}{S}\right)\right].$$

We say that money is more desirable the closer the RHS is to 1 (i.e., the return to money, S'/S, approaches the necessary return to justify carrying money forward). Does a large y today make money a more or less desirable asset? Since the CIA is binding then $y=q=SQ$. A large y today leads to a higher S today and lower return, S'/S, making money *less* desirable (ceteris paribus). However, a large y today lowers $U'(y)$ today raising the stochastic discount factor $\beta U'(y')=U'(y)$, making money *more* desirable. In effect, a higher stochastic discount factor lowers the threshold of return that money needs to yield to leave the household indifferent between spending and not spending an additional available dollar. Since these two mechanisms work in opposite directions, the effect is ambiguous and depends on the slope of $Z(y)$.

4. Numerical examples

When will the option value of cash be positive and result in cash hoarding? How does this option value vary as real and nominal volatility change? To answer these questions, we specify preferences and shocks and solve for the equilibrium numerically. Preferences are given by

$$U(c)=\frac{c^{1-\phi}}{1-\phi}, \tag{13}$$

with $\phi>0$ [when $\phi=1$, we have $U(c)=\log(c)$] and time discount factor $\beta=0.989$. We model both the money growth and endowment shocks as uniformly distributed random variables with $E(w)=1$ and $E(y)=2$. We simulate the model under various levels of real and nominal uncertainty.[2] We show how the region of positive option value is affected and find that money has positive option value for levels of real and nominal volatility that are within the range of US experience.

4.1. Volatility of nominal shocks

First, we vary the volatility of the money growth, holding the endowment fixed. In Table 1, the standard deviation of the growth shock σ_w is increased, and we see $\hat{w}=\hat{w}(\sigma_w)$ with

$$\frac{\partial\hat{w}}{\partial\sigma_w}>0.$$

[2] In the Appendix we include a sensitivity analysis for preference parameter ϕ and allow the endowment and money growth shock to be correlated with each other.

Table 1
Impact of volatility of money growth shocks. Numbers based on 1000 simulations of length 2000. Money growth shock is uniformly distributed on $[w_l, w_u]$. The variable $\hat{w}$ represents the lowest money growth shock for which the CIA constraint is binding

w_u/w_l	σ_w	$\hat{w}$	E(q)	σ_q
1.01/0.99	0.006	0.989024	2	0
1.02/0.98	0.012	0.990345	2.002722	0.005551
1.03/0.97	0.017	0.993757	2.009612	0.014832
1.04/0.96	0.023	0.998342	2.018893	0.025346
1.05/0.95	0.028	1.003777	2.029872	0.036618
1.06/0.94	0.035	1.009919	2.042287	0.048520
1.07/0.93	0.040	1.016703	2.055935	0.060960
1.08/0.92	0.046	1.024114	2.071089	0.074054

This means that the range (as a fraction of the state space) over which the option is not exercised is increasing in the volatility of w. The $E(q)$ is increasing in σ_w because: (1) $q(w)$ is a convex function of w and (2) $\partial\hat{w}/\partial\sigma_w>0$. This also explains why σ_q is increasing in σ_w. Notice also that for fixed w, an increase in nominal uncertainty raises $q(w, \sigma_w)$. What is the intuition for this result? Recall that $q=SQ$ which represents the real value of the option, (i.e. real money balances). Thus an increase in (nominal) uncertainty raises the value of the option as one would expect using the intuition of option pricing. The optimal stopping point $S^*(w)=(y/Q)\hat{w}/w$, which implies $\partial S^*/\partial\sigma_w=(y/Qw)\partial\hat{w}/\partial\sigma_w>0$. However, an increase in uncertainty (a larger σ_w) raises the optimal stopping point and increases the fraction of the state space where there is delayed exercise, i.e., the range of S (or w) increases in which there is cash hoarding (see Fig. 2). Table 1 shows that when the nominal shock is uniformly distributed over a range of $\pm 2\%$, we get cash hoarding when the money growth rate is less than -1%. An increase in the range of the nominal shock to $\pm 8\%$ results in cash hoarding whenever the money growth rate is less than $\pm 2.4\%$, illustrating that cash hoarding can occur even with positive money growth rates, if nominal

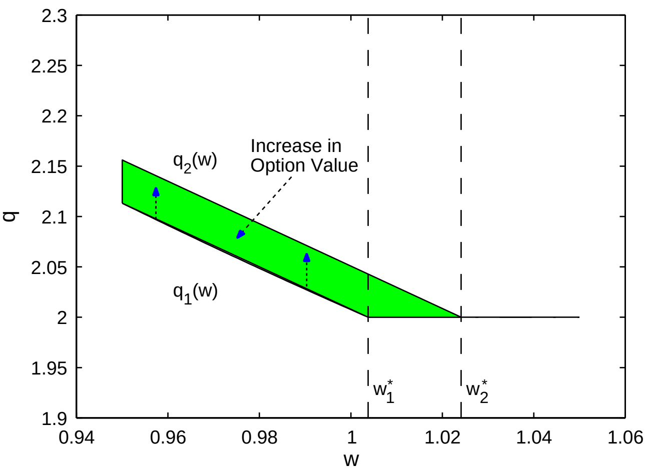

Fig. 2. Increasing nominal uncertainty raises the value of the option and raises the optimal exercise point. Recall that $q=QS=QV$ is the real value of the option and also equals the value of real money balances.

Table 2
Impact of volatility of endowment shocks with $\phi=2$. Since $\phi>1$, we have the CIA constraint binding over $[y_l, \hat{y}]$ and not binding over $[\hat{y}, y_u]$. Numbers based on 1000 simulations of length 2000. Endowment shock is uniformly distributed on $[y_l, y_u]$

y_u/y_l	σ_y	$\hat{y}$	$E(q)$	σ_q
2.02/1.98	0.012	2.022122	1.999981	0.011544
2.04/1.96	0.023	2.019355	2.002701	0.027215
2.06/1.94	0.035	2.012406	2.009593	0.047603
2.08/1.92	0.046	2.003257	2.018859	0.069479
2.10/1.90	0.058	1.992679	2.029756	0.092260
2.12/1.88	0.069	1.981070	2.042078	0.115740
2.14/1.86	0.081	1.968667	2.055585	0.139753
2.16/1.84	0.092	1.955625	2.069903	0.164262

uncertainty is high enough. Money has positive option value for levels of nominal volatility that are within the range of US experience.[3]

4.2. Volatility of endowment shocks

With a constant money growth rate and stochastic endowment, preferences do matter for the qualitative nature of the solution. For the class of preferences used in the simulations, the function $Z(y)$ is monotonic. If $\phi>1$, $Z(y)$ is monotonically decreasing, so the option to purchase will be exercised for low realizations of y and will not be exercised for high realizations of y. If $\phi<1$, $Z(y)$ is monotonically increasing, these regions are reversed, i.e., the option will be exercised for high realization of y and not exercised for low realization of y.[4] We will limit our attention here to the case where $\phi>1$ as is more commonly assumed in the literature, and delegate the case $\phi<1$ to the Appendix.

Using our intuition from real options, an increase in σ_y should: (1) raise the optimal stopping point S^* and (2) increase the value of the option. The optimal stopping point for this class of preferences is $S^*=(1/Q)\hat{y}^{1-\phi}y^{\phi}$. This implies that

$$\frac{\partial S^*}{\partial \hat{y}}<0 \qquad \text{if } \phi>1$$

In order for S^* to be increasing in σ_y we need

$$\frac{\partial \hat{y}}{\partial \sigma_y}<0 \qquad \text{if } \phi>1$$

which is exactly what we see in Table 2.

A higher S^* increases the no-exercise range of the option. When ϕ is large, say $\phi=2$, changes in the discount factor $\beta U'(y')/U'(y)$ due to a change in y dominate changes in money's expected rate of return (S'/S). This reflects a strong desire to smooth consumption. Hence when the current realization

[3] Using US M2 per capita data, we calculate the standard deviation to be 5.99% over the entire sample 1890–1998, with a range of 2.01–8.83% during different decades. The new definition of M2 is used from 1969–1998. Source: *Historical Statistics of the United States*, published by the Bureau of Census for 1890–1968 and the Federal Reserve System for 1969–1998.

[4] Under log-utility, the option will be exercised for all realizations of y (provided an equilibrium exists).

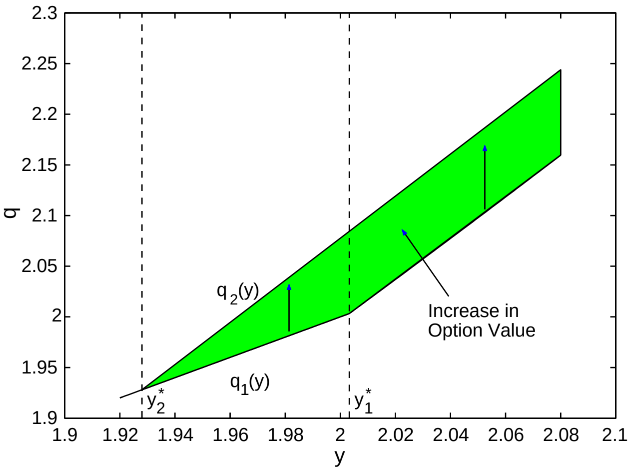

Fig. 3. $\phi = 2.0$ (strong preference for consumption smoothing): Increasing real uncertainty raises the value of the option, the value of real money balances, and increases the no-exercise range.

of y is high relative to expected future y, the large increase in the discount factor leads to cash hoarding despite the expected low return on money. The exercise range of the option is over low realizations of y (and the no-exercise range is over high realizations of y). When uncertainty increases, the value of the option increases and the optimal exercise point S^* increases ($\hat{y}$ decreases). For example, in Table 2 we see that when the endowment shock is uniformly distributed over a range of $\pm 2\%$, we get cash hoarding when the endowment shock is $+1\%$ and higher. If there is an increase in uncertainty to an endowment shock range of $\pm 5\%$, we get cash hoarding over an even wider endowment shock range of -1% and higher.

The increase in option value is illustrated in Fig. 3 for $\phi = 2$. Viewing $q(y,\sigma_y)$, we see that for fixed y, an increase in real uncertainty raises $q(y,\sigma_y)$. This increase in the option value of cash explains why $E(q)$ is increasing in σ_y in Table 2. The σ_q increases with σ_y at a faster rate when $\phi > 1$ because the option has positive value at relatively high levels of y increasing the dispersion of q.

5. Conclusion

We have shown that the CIA model is isomorphic to a model with real options on consumption. Positive option values (the "no-exercise" zone) corresponds to the range where the CIA constraint does not bind. We analyzed the impact of real and nominal uncertainty on the option value of cash and the optimal exercise strategy using numerical simulations revealing how the intuition of real options applies to money. We find cash hoarding and liquidity traps can exist for plausible levels of real and nominal uncertainty.

We expect that this approach could be modified to include a cash-in-advance constraint on asset purchases where money would function as options on both output and assets. The option value of near-monies may be modeled by adding delays and/or costs to the exercise of the option to purchase. The option value concept offers an intuitive approach to understanding the value of cash as liquidity under uncertainty, and its response to real and nominal uncertainty.

Appendix A

Available upon request.

Reference

Svensson, Lars E.O., 1985. Money and asset prices in a cash-in-advance economy. Journal of Political Economy 93, 919–944.