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Monitored financial equilibria

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Abstract

The financial monitoring system is formalized as an economic primitive in addition to preferences, endowments, production sets and asset spans. New definitions of budget constraints and economic equilibria follow. The concept of a general equilibrium is then appropriately revised to accommodate this perspective on the role of a financial monitoring system. Implications for asset pricing are derived and the relationship of equilibria to risk management considerations is discussed.

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1. Introduction

The standard model of an economy in general equilibrium (see for example Arrow and Debreu, 1954; Arrow and Hahn, 1971; or Cornwall, 1984) takes as primitives the preferences of individuals over consumption possibilities, their endowments and the technology of production sets describing the possibilities for transforming resources into consumption goods. An equilibrium is then seen as a price system for traded goods under which all excess demands, aggregated across maximizing agents, are simultaneously zero. This fundamental paradigm has served well much subsequent research in economic theory, including the general equilibria of asset economies

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(Milne, 1988) the study of financial structure in incomplete markets equilibria (Allen and Gale, 1994) and the modeling of dynamic financial equilibria (Duffie and Huang, 1985).

We note, however, that finance plays a possibly limited role (Arrow, 1964) in this basic model given that no financial constructs contribute to the model primitives. In the final analysis we obtain the results of Modigliani and Miller (1958) on the irrelevance of financial policy and the intuition that financial arrangements redistribute the rewards of economic activity with no effect on the set of possibilities. We offer instead a revised concept of an economy in equilibrium that we term a monitored financial equilibrium (MFE), as opposed to the standard concept of a general equilibrium (GE).

The standard GE model has been extended, see for example Geanakoplos (1990), to the GEI model of a general equilibrium with incomplete markets. In this incomplete markets extension there is a distinction between financial and real assets with the former allowing transfers of funds across time and states of nature to access real assets by market participants. The GEI model has been further extended to permit bankruptcy and default in a general equilibrium context by for example, Dubey et al. (1997) and Sabarwal (2003). Other authors have also noted the dependence of the size of the real economy on financing considerations in the presence of exposure to default. We note here in particular the work of Hellwig (1981), Araujo et al. (2000) and Geanakoplos (2001). The general principle adopted in these extensions is to explicitly specify what happens to each agents final allocation in default with the agent taking explicit note of the personal costs of bankruptcy in making his or her decisions with respect to promises and consumption plans.

We wish to depart from this formulation on the understanding that default is not supposed to occur and is effectively being avoided by the monitoring mechanisms in place. Hence we do not specify post default allocations, nor do our agents endogenize costs of bankruptcy, but act on the conviction that if monitoring specifications have been met then sufficient safety has been attained and agents evaluate utilities on the assumption of no default on their part or on the part of counterparties. We wish to take the view, here, on the one hand that it may be unrealistic to expect market participants to effectively evaluate post default allocations as these are generally the consequences of complicated legal battles that cannot be accurately foreseen. On the other hand we see the costs of defaults when they occur as falling upon the monitoring authority that approved the transactions upon satisfication of monitoring requirements. The maximizing agents are thereby removed from the consequences of defaults.

Additionally, we do not insist on market clearing on the part of demands and supplies coming from maximizing agents. There may be a residual, but if so, it must be acceptable to the monitoring authority that is willing to cover the slack provided it is judged acceptable in a very precise sense that we define. In both these dimensions, that is, allocation of default costs and insistence on market clearing among maximizing agents, we make a departure from many traditional equilibrium specifications. The consequences of this reformulation are then outlined with special emphasis on asset pricing issues.

A MFE in contrast to a GE, or a GEI with or without default, enhances the set of primitives beyond preferences, technology, endowments and financial asset spans for GEIs, and defines in addition the monitoring system of the economy under study. The monitoring system is defined as a precise mathematical construct in the model. To keep matters simple, we restrict attention in this paper to exchange economies and abstract from the production side of the economy. The role of the monitoring system and its impact on the real economy will be transparent even for the exchange economy. It is anticipated that this impact is far greater for production economies. We later comment briefly on the technical issues involved in such extensions.

The monitoring system serves in the MFE we define as a passive counterparty for all transactions among all other participants in the economy, who are maximizing agents. The passivity is reflected by not modeling the monitoring system as a maximizing agent, but as a holder of risks of last resort on predefined terms. The monitoring system accepts all risks that meet certain tests and the net aggregate position of other market participants need not be simultaneously zero (hence the departure from traditional market clearing), but needs only to be acceptable to the monitoring system. Such a formulation effectively redefines the equilibrium.

In defining the set of risks acceptable to the monitoring system we borrow the concept of acceptable opportunity from the recent literature on risk management. The concept was introduced in Artzner et al. (1998) and was further employed in Carr et al. (2001), to refine the concept of arbitrage opportunity and revisit the fundamental theorems of no arbitrage pricing. Here we introduce acceptable opportunities as a primitive in the formulation of an economic equilibrium, for the special case of an asset exchange economy, and revisit the question of the existence of such a revised equilibrium. Given the existence of a passive monitoring system in our economy, we expand the structure of assets that market participants may trade by including in addition to the liquid securities with secondary markets, an over the counter market that trades personalized assets directly with the monitoring system.

The essential idea behind the concept of acceptability is a generalization of the idea of non-negativity. We recognize that any agent, including the monitoring system will accept a zero or non-negative flow at zero cost. Our reformulation of an economic equilibrium is made by redefining what it means to be zero or non-negative. The classic formulation of this idea may be alternatively presented as making an acute angle with all the unit vectors of the dual space. Equivalently we may state that the expectation is zero or non-negative when we evaluate the outcome using the indicator function of any set as the pricing kernel. The collection of all such pricing kernels forms a convex set and the condition that we have positive or zero outcome expectations with respect to such kernels forces the outcomes themselves to be zero or non-negative and hence clearly acceptable. The set of all such kernels is the dual cone of the positive orthant. The generalization of this zero or non-negativity condition is attained by selecting a proper convex subset of kernels in the dual cone of the positive orthant, defined by identifying its extreme points. For practical purposes we take the set of extreme points to be finite and work with a polyhedral dual cone.

The finite set of extreme points defining acceptability may be given the interpretation of multiple beliefs but this is not necessary. In fact the indicator functions of sets probably do not correspond to the beliefs of any one. Likewise, the collection of test functions in the terminology of Carr et al. (2001) are just that. They have meaning as a collection, defining what the monitoring system will regard as close enough to non-negative to take up at zero cost, but individually they need not and probably do not correspond to beliefs. Carr et al. (2001) do however suggest possibilities for constructing these measures by extreme views on probability, risk aversion and agent positions. In this paper, these measures are exogeneous and part of the primitives of the monitored financial equilibrium.

The question arises as to what this monitoring system is, where does it get its funds, and how can it finance shortfalls in the system. Is it infinitely endowed and how so. We view the resources of this monitoring system like those of the Federal Deposit Insurance Corporation in the US as on occasion possibly coming from past contributions of economies that have long gone by (currently there are no premiums collected by the Federal Deposit Insurance Corporation). Taxes, premiums from participants or other mechanisms may have been or may be employed to fund the lenders of last resort, including the use of military conquest. The number of ways in which such a monitoring fund is established is an open question from the perspective of this paper. We recognize that its size and funding is endogeneous to the larger question that in our view necessarily enters the political dimension. Here, we take it as a given primitive supporting the economic equilibrium.

Is the monitoring system infinitely endowed? We believe not, but well enough endowed to combat most flow mismatches, accumulating flows in some states and depleting them in others. It is possible that the magnitude of default is so large that it breaks the fund of the monitoring system and for the purposes of our analysis, the economy closes down in this state. A new economy, unknown to us will presumably result but to pursue analysis of the consequences of such an outcome is beyond the scope of our model, and beyond our scope. Our model is therefore open to this crash outcome, in which we are silent on the result. We proceed on the assumption that it does not happen and the maximizing agents in our model assume the same. We recognize that in principle one could see defaults so large that the fund of the Federal Deposit Insurance Corporation is depleted 10 times over. Were this to occur, the allocation of consequences are unknown to all market participants and they cannot endogenize such matters into their optimal plans.

The financing or absorption of potential defaults on terms defined up front occurs across all economic activities and is not restricted to investments. In most developed economies people finance many consumption items including housing and automobile consumption using loan arrangements with prior approval from the financing system. If they meet certain tests applied to their circumstances the loans are approved and consumption as well as investment activities are financed as desired. The funds backing such loans fall back on the banking system and ultimately on the reserves of insurance funds maintained for such purposes. The presence of funds or reserves for such mismatch purposes is not a sign of market failure in the private sector but a way of enhancing the size of the real sector by permitting transactions that could not otherwise take place. It is more a mechanism of market enhancement than a reflection of a market failure. The Pareto superiority of the resulting economy is its raison d'etre. The limits on the size of the real expansion are placed cautiously by attempting to ensure that the fund supporting the equilibrium is not busted so as to lead to a collapse of the economy.

A primary motivation for our formulation lies in recognizing the economic advantages such a system brings to the production and investment sector. The basic principles are apparent from just a consumption exchange economy. The difficulties posed in extending the model to the production side relate to issues of firm objective functions in incomplete markets equilibria. Additionally, there is continuing work on the appropriate definition of acceptable opportunities, a core idea of this paper, to multiperiod models. These definitional issues need to be settled before such extensions are feasible. In this initial presentation of the core equilibrium ideas in the presence of a supporting and monitoring fund we choose to abstract from the difficulties of modeling investment in incomplete markets with some multiperiod version of acceptability. We note in our defense that even pure consumption economies trading promises require monitoring to clear promises as is evidenced in developed economies where credit card payments are cleared for approval prior to the completion of planned consumption transactions. We view the promises of anonymous market participants in private markets as lacking credibility unless they are approved by a monitoring authority that presumably has sole access to records of personal endowments. We note in this regard the importance of such an authority for the smooth functioning of the private exchange economy.

Having established the existence of a MFE, we address the question of the efficiency of the MFE. A standard equilibrium has the efficiency property of being a Pareto optimum. A similar optimality result is formulated and established for the MFE. We observe that the welfare enhancement capabilities of the monitoring system appear in the form of making a Pareto improvement for the economy relative to the traditional equilibrium. This is an elementary consequence of constraint relaxations offered by the monitored system and we merely mention this result without demonstration.

The asset pricing principles relevant to a MFE are steered by the monitoring system and we next take up an analysis of the resulting equilibrium. In particular we observe that if a particular risk is not priced by the monitoring system then it will not be priced in a MFE. We illustrate the types of equilibria one might be expected to encounter in a MFE. The focus of traditional asset pricing models in understanding the cross-sectional patterns in excess returns has been on studying return generating mechanisms with a view to determining common, systematic or undiversifiable components in the generating mechanisms as the risks priced in equilibrium. For a MFE on the contrary, it is the positions and perspectives of the monitoring system that are the core determinants of risks priced in equilibrium.

In this regard we note that as the monitoring system is not a utility maximizing consumer, correlations with consumption are not expected to be an important concern in asset pricing. The primary concern of the monitoring system is to protect its resource base in covering losses occurring in the economic system with respect to which its payouts are linked. Hence, what would be important are correlations with losses in various important sectors of the economy, with possibly little regard to the consumption of particular economic agents.

The outline for the paper is as follows. Section 2 describes the economic primitives. The monitored financial equilibrium is formulated in Section 3. Section 4 provides existence theorems. Section 5 analyses the asset pricing implications of the new equilibrium and discusses the connections with risk management issues. Section 6 concludes.

2. The economic primitives

We deal with a one period two date model with times $\{0, 1\}$. There is economic uncertainty resolved at time 1 in accordance with the finite probability space (Ω, P) where Ω is the finite set of possible states with elements ω and P is a probability measure defined on the subsets of Ω. The probability of a single event ω is given by p_ω. This probability measure serves as a reference measure used to identify the states with positive probability. Market participants have their own probability assessments that are supposed equivalent to the measure P. For simplicity we identify the states with positive probability for all market participants.

There are n individuals in the economy denoted by $i \in I$, $|I| = n$. All consumption occurs at time 1 and the consumption space is defined by non-negative vectors $c = (c_\omega, \omega \in \Omega)$ and each individual has a preference ordering over consumption defined by the utility function $U^i(c)$ for individual i. We make the usual assumptions and suppose that the utility functions are continuous, increasing and concave. Special cases of the utility function could be of the form of expected utility using a subjective probability. For generality we work initially with the general state preference utility function. Later, when addressing the nature of asset pricing in equilibrium we specialize to the expected utility formulation. In addition each individual has an endowment given by $\xi = (\xi_\omega, \omega \in \Omega)$.

Trading in the economy at time 0 are m_a assets indexed by $j \in J$, $|J| = m_a$, that have claims to time 1 cash flows $a^j(\omega)$ for asset j in state ω. The time zero market prices of these assets are denoted by π_j for $j \in J$ and they are to be determined in equilibrium.

However, in addition to the traded assets of the traditional asset exchange economy as studied in Milne (1988) we also have over the counter (OTC) assets that may be offered by market participants into the financial system in exchange for traded assets or consumption goods. These assets are investor specific or personalized and are indexed by $k^i \in K_i$, $|K_i| = m_{b^i}$ and they have state ω cash flows given by $b^{ik}(\omega)$ for the kth asset specific to investor i.

The important difference between these asset classes is that the former has active liquid markets supplying price discovery while the latter may be taken up by the financial system in exchange for a swap to deliver a promised state contingent cash flow but it may not be bought and sold freely in a secondary market. By way of example consider an education loan in which one promises repayments based on ones labor income projections in exchange for a lump sum to finance a college education. The asset associated with ones specific labor income projections is not readily traded in a market with price discovery. Yet it may be offered in exchange for traded goods into the financial monitoring system. This second class of assets is very much part of the real economy, but given the absence of secondary markets, there are no market prices associated with these assets.

An additional economic primitive provided by the monitoring authority is the definition of acceptable opportunities. These are state contingent cash flows that the financial system will hold as a counterparty, at zero cost. Following CGM we define a state contingent cash flow $x(\omega)$ as acceptable at zero cost if for a finite set of test measures Q^l, $l \in L$, with probability q^l_ω for event ω, it is the case that expected benefits exceed a non-positive floor. Specifically we require that

$$\sum_{\omega \in \Omega} q^l_\omega x(\omega) \geqslant f^l, \quad \text{all } l \in L, \tag{1}$$

where f^l is the non-positive floor associated with the lth measure. We require in addition that there are measures associated with a floor of zero as this rules out strictly negative cash flows or anti-arbitrages from being acceptable. In CGM the class of measures with zero floors were termed the valuation measures while the remaining measures were called stress test measures. The inequalities (1) for the valuation measures alone define a convex cone that contains all non-negative cash flows and may be seen as a generalization of both, non-negativity and arbitrage. Stress test measures further restrict the acceptability to avoid the implications of scaling positions upward without bound.

For the purposes of this paper we shall abstract from scale or size issues and focus on trading at the margin. Hence we take all the test measures to be valuation measures with zero floors. More generally in the presence of stress test measures scaled cash flows may be unacceptable to the monitoring system and this serves as a check on the size of transactions to be considered as a perturbation of the equilibrium. By ignoring these measures for the time being, we restrict attention to pricing at the margin. However, we note that the monitoring authority only limits possible pricing rules in a scale invariant way to be in some prespecified cone. Subject to this broad restriction maximizing agents in the economy continue to make marginal calculations in determining the specific equilibrium that eventuates.

Cash flows are acceptable at zero cost by the monitoring authority provided they have a positive expected value under a finite set of valuation test measures. These test measures effectively describe for the market participants of the economy the degree to which they may allowably diverge from an ability to cover liabilities in all cases. For example, the monitoring system may permit a business plan to engage in laying fiber cables between cities even if there is a scenario in which the information processing productivity of the cable is enhanced 100-fold rendering the asset essentially worthless. This happens as all the test measures place a low probability on this scenario and find the benefits on average across other scenarios positive enough to counteract this bad scenario. Monitoring systems that demand positivity under all scenarios essentially stifle all economic activity. The monitoring system must allow for exposure to risk and must develop the resource base to withstand them.

Investors may take short or long positions in the traded and OTC personalized assets and propose a trade holding w_{ij} units of traded asset j and v_{ik} units of the kth OTC personalized asset with a view to financing a consumption plan $c = (c^i(\omega), \omega \in \Omega)$. The traditional approach to defining the attained consumption plan is to add the trading cash flows to the endowment and define the financed state contingent consumption by

$$c^i(\omega) = \sum_{j \in J} w_{ij} a^j(\omega) + \sum_{k \in K_i} v_{ik} b^k(\omega) + \xi^i(\omega), \quad \text{all } \omega \in \Omega, \ \text{all } i \in I. \tag{2}$$

Such a definition implicitly supposes a complete and exact calculation across all states, many of which may be difficult to define or contemplate up front or at time 0.

Imagine alternatively the situation where the endowment and asset cash flows are offered up front to the monitoring authority that contracts to provide the consumption stream c in a swap. It is then certainly acceptable to payout c provided we are assured that

$$c^i(\omega) \leqslant \sum_{j \in J} w_{ij} a^j(\omega) + \sum_{k \in K_i} v_{ik} b^k(\omega) + \xi^i(\omega), \quad \text{all } \omega \in \Omega, \ \text{all } i \in I.$$

The monitoring authority however does not test this inequality for each individual state. Such a test is literally infeasible and includes scenarios that are not really worth the time spent on testing the positivity, given the low likelihood of the scenario. We propose to regard a consumption plan as financed provided the residual or potential shortfall is judged as acceptable to the monitoring authority or that

$$x^i(\omega) = -c^i(\omega) + \sum_{j \in J} w_{ij} a^j(\omega) + \sum_{k \in K_i} v_{ik} b^k(\omega) + \xi^i(\omega) \tag{3}$$

is an acceptable cash flow in the CGM sense. If there are eventually states where the consumption is greater than the receipts to the assets plus endowment, then the monitoring authority takes the loss, having accepted the swap contract up front.

It is the presence of this monitoring authority with its approval to cover a proposed consumption or other economic plan, that permits maximizing agents to act as if there is no default. Hence these agents do not incorporate personal costs of bankruptcy into their optimization calculations, as they would in the equilibria described in Dubey et al. (1997) or Sabarwal (2003). Default is a concern only of the monitoring authority. Of course, the monitoring authority has to define acceptability tightly enough and may do so as to exclude it altogether. The latter is however, an extreme case.

Apart from defining consumption plans attained by trading strategies each investor faces a budget constraint. Traditionally following Milne (1988), for an asset exchange economy, this constraint requires that the market value of ones final cash flow position be no greater than the value of one's endowment. If only traded assets are transacted and the endowments are in the form of traded assets then this constraint reduces to the self-financing condition for net trades

$$\sum_{j\in J} w_{ij}\pi_j = 0.$$

More generally, it is permissible that the cost of net trades be non-positive.

In the presence of OTC assets being offered in exchange for traded assets the monitoring authority is left holding the cash flow

$$\sum_{j\in J} w_{ij}a^j(\omega) + \sum_{k\in K_i} v_{ik}b^{ik}(\omega),$$

and the resulting value will be non-positive if the cash flows are non-positive. The traded component has the value

$$\sum_{j\in J} w_{ij}\pi_j,$$

and this may be positive, as for our education loan example considered earlier provided the financial system is satisfied with what is offered in the OTC component. The monitoring authority employs its candidate valuation measures to the non-traded component and accepts the proposed trade provided

$$\sum_{j\in J} w_{ij}\pi_j + \sum_{\omega\in\Omega} q^l_\omega \left(\sum_{k=1}^{m_b} v_{ik}b^k(\omega)\right) \leqslant 0, \quad \text{for all } l \in L. \tag{4}$$

Eq. (4) is the revised budget constraint for our monitored financial economy. Except for the allowance of OTC personalized asset trades the constraint is comparable to the traditional constraint requiring that one be able to finance ones purchases in the market place by selling endowments. The OTC assets enhance the trading opportunities available to market participants and they may be used provided they have a sufficiently high value to the monitoring authority under all its test measures. We further note that as Eq. (4) compares undiscounted cash flow expectations with costs at prices π_j for asset $j \in J$, these prices are in fact time *one* delivery futures prices. We shall maintain this perspective in the paper, recognizing that additional discounting considerations are involved if one is to work with immediate delivery spot prices.

Investors are viewed as maximizing the utility of attainable consumption plans defined by the acceptability of the consumption swap (3) using trades satisfying the budget constraint (4). The monitoring authority plays a critical role as a model primitive in both concepts of attainability and budget feasibility. We note that attainability plus budget feasibility do not reduce to the acceptability of swapping endowments for consumption or the acceptability of $\xi^i - c^i$, for though $\xi^i - c^i$ is equal to the difference of two acceptable cash flows under attainability and budget feasibility, acceptability is not closed under differences and the investor may need to come up with an acceptable trade and attain acceptability of endowments plus net trades less consumption. In this regard we mention that acceptability is a generalization of non-negativity, and the latter is also not closed under differencing.

It is useful to consider the two roles in the extreme case when the valuation test measures are expanded to include all the indicator functions of all the elements of the state space. In this case attainability reduces to the traditional definition of an attainable consumption plan for now $c^i(\omega)$ must satisfy Eq. (2). With regard to budget feasibility we have that the cash flow from the OTC assets traded must exceed the value of the traded goods in all states and so the loan by the monitoring authority to the individual to engage in traded asset exchanges as planned is fully repaid with a surplus. The monitoring authority has no problems whatsoever if its test measures include all the indicator functions of all the possible states. But in this case one also does not allow any risks to be taken and one insists that all plans meet all obligations by everyone with probability one. Real world monitored financial equilibria permit risk taking.

Having presented the details of the structure of a monitored financial economy we comment briefly on the status of the monitoring authority and in particular on its relationship to the other maximizing economic agents. One might ask what the monitoring authority is and where did it get its resources. How can the agents of the economy dismiss the monitoring authority and what are the potential political economy conflicts and resolutions between this authority and the economic agents. We view the authority as a repository of capital reserves that are to be used to cover gaps between outcomes and the approved plans and promises of participating economic agents. As mentioned in the introduction, this repository may have been the recipient of funds from agents in economies gone by, as is the case for the Federal Deposit Insurance Corporation in the US today that accumulated its reserve capital from premium contributions in the past. If the monitoring authority is depleted of its resource then contributions would be sought from participants over time via the political mechanism. We recognize the political dimension and do not close the model by making the maximizing agents of the current economy the sole contributors to the reserve fund of the authority on terms that they then optimize over. These political questions do not arise and are not answered until an eventuality leads to a collapse of the authority's resource base. On the occurrence of such an eventuality, the economy we model closes on a system wide default. Our model is only meant to describe equilibria in the absence of such an occurrence.

3. A monitored financial equilibrium

We come now to the definition of a monitored financial equilibrium for our financial economy. In a traditional asset exchange economy an equilibrium requires that market prices be determined by market clearing and in equilibrium we have that

$$\sum_{i\in I} w_{ij} = 0, \quad \text{for all } j. \tag{5}$$

Market clearing occurs separately for each asset. The assets are traded in separate markets and for every purchase there is a counterparty that sells the asset in question. As a consequence, the state contingent cash flows associated with these trades, clear on a state by state basis as under (5),

$$\sum_{i\in I} w_{ij}a^j(\omega) = 0, \quad \text{for all } \omega \in \Omega \text{ and } j \in J. \tag{6}$$

In a monitored financial equilibrium, the monitoring authority may be a counterparty to investor specific asset demands or supplies. The monitoring authority trades all the assets of the economy, the liquid ones and the personalized OTC assets. Hence we do not require Eq. (5) to hold or that excess demands clear across all the individual market participants. The excess supply or demand if any is implicitly either taken up by the monitoring authority or supplied by it.

However, the monitoring authority in its capacity as the last resort counterparty will not take up any arbitrary set of excess demand or supplies. Its aggregate position must be acceptable to the authority. For this we have to consider the aggregate cash flows from all transactions of all participants other than the monitoring authority. We therefore aggregate across all assets and write the aggregate holding of the investing community as

$$W(\omega) = \sum_{i\in I}\left(\sum_{j\in J} w_{ij}a^j(\omega) + \sum_{k\in K_i} v_{ik}b^{ik}(\omega)\right). \tag{7}$$

The monitoring authority holds as a trading counterparty $-W(\omega)$. In addition it holds the swap contracts with each individual of

$$-c^i(\omega) + \sum_{j\in J} w_{ij}a^j(\omega) + \sum_{k\in K_i} v_{ik}b^{ik}(\omega) + \xi^i(\omega).$$

The aggregate exposure or liability for the monitoring authority then amounts to

$$Z(\omega) = \sum_{i\in I}\left(c^i(\omega) - \xi^i(\omega)\right).$$

Certainly, if $Z(\omega)$ is non-positive, the monitoring authority has no real concerns as it is a beneficiary in all states of the world. The monitoring authority will however take all acceptable cash flows, and we relax non-positivity to acceptability in the CGM sense. This requires that in equilibrium we have that

$$\sum_{\omega\in\Omega} q^l_\omega Z(\omega) \leqslant 0, \quad \text{for all } l \in L. \tag{8}$$

A monitored financial equilibrium (MFE) is a set of prices for the traded assets, such that budget feasibility (4), utility maximizing, attainable consumption plans (3), associated with trading plans for traded and OTC personalized assets lead to acceptable aggregate exposures (8) for the monitoring authority.

It is clear that the monitoring authority plays a critical role in determining the size of the real economy for a monitored financial equilibrium. We have commented that an expansion of the test measures to include the indicator functions for all states severely limits risk taking and reduces the real economy in an unreasonable way. The actual monitoring system in the developed world has been expanding the cone of acceptable risks over the years as may be evidenced by the changes in the bankruptcy law over time. After all, the world abolished debtor's prisons and developed limited liability procedures for business incorporation. Yet the system cannot be too lax and subject to frequent failure or payouts of residual shortfalls as the resulting loss of confidence is devastating for the smooth functioning of the economy. The design of balanced risk management systems is therefore critical to the functioning of a monitored financial economy.

4. Existence a monitored financial equilibrium

A monitored financial equilibrium as defined in the previous section permits shortfalls between consumption plans and trading receipts at the individual level. It also permits shortfalls between aggregate demands and supplies at the market level. It relies in each case on the monitoring authority stepping in to take up the slack on terms of acceptability that are predefined and known to all market participants. Yet it is unclear that all these independently asserted conditions can all be simultaneously met and that a monitored financial equilibrium in fact exists, and this for an arbitrary choice of valuation test measures defining acceptability.

An existence proof serves two purposes. First it settles the issue of all conditions being possibly simultaneously met. Second, it is instructive in the steps to be followed in finding the equilibrium and hence consequently in studying its properties. With these motivations in mind we take up the question of proving the existence of a monitored financial equilibrium in this section. The answer is in the affirmative and a slight modification of the original proof by Debreu (1959) suffices. We begin however with an analysis of the optimization problem facing each market participant.

The utility maximization problem facing the individual investor (and we drop the subscript i for the investor for notational convenience) is the problem

$$\max_{c \geqslant 0, w_j, v_k} \quad U(c)$$

s.t.

$$\sum_{\omega \in \Omega} q^l_\omega \left(c(\omega) - \sum_{j \in J} w_j a^j(\omega) - \sum_{k \in K} v_k b^k(\omega) - \xi(\omega) \right) \leqslant 0, \quad \text{all } l \in L, \tag{9a}$$

$$\sum_{j \in J} w_j \pi_j + \sum_{\omega \in \Omega} q^l_\omega \left(\sum_{k \in K} v_k b^k(\omega) \right) \leqslant 0, \quad \text{all } l \in L. \tag{9b}$$

Introducing the Lagrange multipliers $\lambda^l \geqslant 0$, $\gamma^l \geqslant 0$ for the two set of constraints we have, in addition to the constraints, the first order conditions with respect to $c(\omega)$, w_j, v_k that must be satisfied by the optimal solution. These are:

$$\frac{\partial U}{\partial c(\omega)} - \sum_{l \in L} \lambda^l q^l_\omega \leqslant 0, \quad \text{with equality if } c(\omega) > 0, \tag{10}$$

$$\sum_{l\in L}\sum_{\omega\in\Omega}\lambda^l q_\omega^l a^j(\omega) - \pi_j\left(\sum_{l\in L}\gamma^l\right) = 0, \quad \text{for all } j, \tag{11}$$

$$\sum_{l\in L}\lambda^l\sum_{\omega\in\Omega}q_\omega^l b^k(\omega) - \sum_{l\in L}\gamma^l\sum_{\omega\in\Omega}q_\omega^l b^k(\omega) = 0, \quad \text{for all } k. \tag{12}$$

We first employ the conditions (12) to understand the relationship between the two sets of Lagrange multipliers. For this purpose we define for each valuation measure l and each OTC personalized asset with index k, the matrix of asset valuations under the valuation test measures by

$$C_{lk} = \sum_{\omega\in\Omega}q_\omega^l b^k(\omega).$$

Let the difference between the Lagrange multipliers be defined by

$$\zeta = \lambda^l - \gamma^l.$$

Observe that the conditions (12) may be written as

$$\zeta' C = 0.$$

Now suppose the OTC assets are rich enough to include a personal loan of a dollar. We term this condition *OTC loan feasibility*. More precisely, under OTC loan feasibility, there is a portfolio with weights η_k such that

$$\sum_{k\in K}b^k(\omega)\eta_k = 1, \quad \text{for all } \omega\in\Omega.$$

The promise of a dollar may be replaced for our purposes by a small loan of ε dollars. Basically the monitoring authority can verify personal endowment assets of each market participant to approve the promise of delivery of ε dollars by the person. We note further that such a loan will probably not trade in the market as the market is not able to verify personal endowments and trusts that trade patterns approved by the monitoring authority are sound. The verification of personal assets is here left to the monitoring authority. One may view the mechanisms of credit ratings and their access to a variety personal records as part of the monitoring mechanisms by which the market is informed that particular counterparties are good up to certain promise levels. We view the personal promise of a dollar as credible in the market only subject to the approval of the monitoring authority and not a traded asset in the market. It is interesting in this regard to note that when the market is trading promises for future delivery, individuals are not able to buy and sell freely what they wish as their promises lack credibility. We view the task of attaining credibility as enforced by the revised budget constraint of the monitored financial equilibrium. The promises of people in the market are not tenable without the approval of the monitoring authority.

Under OTC loan feasibility we have that

$$C\eta = \mathbf{1}_{|L|},$$

and if $\zeta' C = 0$ then $\zeta' C \eta = 0$ or equivalently

$$\sum_{l \in L} \lambda^l = \sum_{l \in L} \gamma^l.$$

Combining (11) now with (12) under OTC loan feasibility we deduce an important property for the market prices of traded assets. These conditions taken together imply that

$$\pi_j = \sum_{l \in L} \frac{\lambda^l}{\left(\sum_{l \in L} \lambda^l\right)} \sum_{\omega} q^l_{\omega} a^j(\omega). \tag{13}$$

The market prices of the traded assets must be in the convex hull of the valuation measures defining acceptability for the financial system. It is interesting that a similar result was derived by CGM under the hypothesis of the absence of strictly acceptable opportunities. Here it is a consequence of the joint role played by the valuation test measures in both defining attainability and budget feasibility, coupled with the OTC loan feasibility condition.

We also note that the search for the equilibrium market prices of the traded assets may be restricted to the class consistent with the CGM hypothesis of no strictly acceptable opportunities (NSAO). Given such candidates for the market prices of traded assets, each individual must select a personalized convex combination of valuation measures that is consistent with these market prices of the traded assets, in that Eq. (13) holds. We may term such a combination of valuation measures a personalized valuation measure defined by

$$\tilde{q}_{\omega} = \sum_{l \in L} \frac{\lambda^l}{\left(\sum_{l \in L} \lambda^l\right)} q^l_{\omega}.$$

The concept of *acceptable completeness* was introduced in CGM. Completeness is attained when the asset space is large enough to attain all possible cash flows or state contingent claims. Under acceptability the interest is not in the cash flows per se, but in their valuations under the test measures. Hence we say that the market is acceptably complete if the asset space is large enough to attain all possible valuations. A case in point is when we have fewer test measures than we have assets. It was shown in CGM that when the traded asset markets were acceptably complete then these personalized valuation measures would be unique as there is then only one solution possible for the normalized set of Lagrange multipliers λ^l satisfying (13). The uniqueness of measures in the convex hull of the test measures is intimately connected with acceptable completeness or the span of asset valuations being onto the space of valuations. We refer the reader to CGM for further details on this issue.

We come now to the first order conditions with respect to the state contingent consumption $c(\omega)$. We observe that these conditions are consistent with $c(\omega)$ being a solution to a problem of the form

$$\begin{aligned} \max_c \quad & U(c) \\ \text{s.t.} \quad & \sum_{\omega \in \Omega} \tilde{q}_\omega c_\omega \leqslant A, \end{aligned}$$

for some value of the constant A. In fact we learn from multiplying the attainability condition by λ^l and summing and normalizing that

$$\sum_{\omega \in \Omega} \tilde{q}_\omega c_\omega \leqslant \sum_j w_j \pi_j + \sum_{\omega \in \Omega} \tilde{q}_\omega \left(\sum_{k \in K} v_k b^k(\omega) \right) + \sum_{\omega \in \Omega} \tilde{q}_\omega \xi(\omega). \tag{14}$$

Budget feasibility now implies that we may take for A the value of the endowment under the personalized valuation measure $\tilde{q}$. Hence the consumptions are solutions to a classical utility maximization problem using a set of Arrow Debreu state prices to value the state contingent cash flows and define a classical budget constraint.

This realization helps us bound the set of possible solutions. Assuming that $q^l_\omega > 0$ for all ω we have that

$$c(\omega) \leqslant \frac{\max_{i,\omega} \xi^i(\omega)}{\min_{l,\omega} q^l_\omega} = B.$$

Under budget feasibility we deduce that for all $l \in L$

$$\sum_j w_j \pi_j + \sum_{\omega \in \Omega} q^l_\omega \left(\sum_{k \in K} v_k b^k(\omega) \right) \leqslant 0,$$

and hence that

$$\sum_j w_j \pi_j + \sum_{\omega \in \Omega} \tilde{q}_\omega \left(\sum_{k \in K} v_k b^k(\omega) \right) \leqslant 0.$$

It follows from (14) that

$$\sum_{\omega \in \Omega} \tilde{q}_\omega c_\omega \leqslant \sum_{\omega \in \Omega} \tilde{q}_\omega \xi(\omega) \leqslant \max_{i,\omega} \xi_i(\omega).$$

Hence we must have that for all ω

$$c(\omega) \leqslant \frac{\max_{i,\omega} \xi_i(\omega)}{\tilde{q}_\omega} \leqslant \frac{\max_{i,\omega} \xi^i(\omega)}{\min_{l,\omega} q^l_\omega}.$$

A proof of the existence of a monitored financial equilibrium may now be constructed along classical lines as follows.

Define

$$\Theta = \left\{ \theta = (\theta_l, l \in L) \mid \theta_l \geqslant 0, \text{ all } l \in L \text{ and } \sum_{l \in L} \theta_l = 1 \right\}.$$

We have that Θ is a compact convex set. Define the compact convex set of possible consumptions by

$$\mathscr{C} = \{c = (c(\omega), \omega \in \Omega) \mid c(\omega) \leqslant B, \text{ all } \omega \in \Omega\}.$$

Now define the compact convex set of possible aggregate excess demands by

$$\mathscr{Z} = \left\{ z = (z(\omega), \omega \in \Omega) \mid z(\omega) = \sum_{i \in I} \left(c^i(\omega) - \xi^i(\omega)\right), \ c^i \in \mathscr{C} \text{ for all } i \in I \right\}.$$

Let $\mathscr{V}$ be the compact convex set of all potential valuations of aggregate excess demands in $\mathscr{Z}$. Specifically we have

$$\mathscr{V} = \left\{ v = (v_l, l \in L) \mid v_l = \sum_{\omega \in \Omega} q^l_\omega z(\omega), \ \text{all } l \in L, \ z \in \mathscr{Z} \right\}$$

We construct a point to set mapping $\Phi : \Theta \times \mathscr{V} \to \Theta \times \mathscr{V}$ as follows. For $\theta \in \Theta$ and $v \in \mathscr{V}$ we define

$$\Phi(\theta, v) = (\Gamma(v), \Lambda(\theta)),$$

where

$$\Gamma(v) = \left\{ \theta \in \Theta \mid \sum_{l \in L} \theta_l v_l \geqslant \sum_{l \in L} \theta'_l v_l, \ \text{all } \theta' \in \Theta \right\}.$$

For the construction of $\Lambda(\theta)$ we first define for all $j \in J$,

$$\pi_j = \sum_{\omega \in \Omega} \tilde{q}_\omega a^j(\omega), \qquad \tilde{q}_\omega = \sum_{l \in L} \theta_l q^l_\omega.$$

We then solve each market participants problem for this set of prices $(\pi_j, j \in J)$ for the traded assets to find the consumptions $c^i(\omega)$, and the trades w_{ij}, v_{ik} and we define

$$v_l = \sum_{\omega \in \Omega} q^l_\omega \left[\sum_{i \in I} (c^i(\omega) - \xi^i(\omega)) \right], \quad \text{all } l \in L.$$

Finally we set $\Lambda(\theta) = v$, $v = (v_l, l \in L)$.

For a monitored financial equilibrium we need to show that we can find a situation such that $v_l \leqslant 0$, for all $l \in L$. Consider now a fixed point for the mapping Φ that exists by an application of classical fixed point theorems. We denote this fixed point by (θ^*, v^*).

We need to observe that for the mapping under construction we have that

$$\sum_{l \in L} \theta^*_l v^*_l \leqslant 0. \tag{15}$$

It then follows from classical arguments and the definition of $\Gamma(v^*)$ that

$$\sum_{l \in L} \theta^*_l v^*_l \geqslant \sum_{l \in L} \theta_l v^*_l,$$

for all $\theta = (\theta_l, l \in L) \in \Theta$. Taking the unit vectors for the choice of θ we conclude that each $v^*_l \leqslant 0$.

To verify the condition (15) we note that by construction

$$\begin{aligned}
\sum_{l\in L}\theta_l v_l &= \sum_{l\in L}\theta_l \sum_{\omega\in\Omega} q^l_\omega\left(\sum_{i\in I}(c^i(\omega)-\xi^i(\omega))\right)\\
&\leqslant \sum_{l\in L}\theta_l \sum_{\omega\in\Omega} q^l_\omega\left[\sum_{i\in I}\sum_{j\in J} w_{ij}a^j(\omega)+\sum_{i\in I}\sum_{k\in K^i} v_{ik^i}b^{k^i}(\omega)\right]\\
&= \sum_{\omega\in\Omega}\tilde{q}_\omega\left[\sum_{i\in I}\sum_{j\in J} w_{ij}a^j(\omega)+\sum_{i\in I}\sum_{k\in K^i} v_{ik^i}b^{k^i}(\omega)\right]\\
&= \sum_{i\in I}\left[\sum_{j\in J} w_{ij}\pi_j+\sum_{k^i\in K^i} v_{ik^i}\sum_{\omega\in\Omega}\tilde{q}_\omega b^{k^i}(\omega)\right]\leqslant 0.
\end{aligned}$$

The two inequalities above are a consequence of the attainability condition and the budget constraint respectively, applied for each individual i.

We observe therefore that under the two assumptions of OTC loan feasibility and the strict positivity of the valuation test measures $q^l_\omega > 0$, all $l \in L$, and $\omega \in \Omega$ we have the existence of a monitored financial equilibrium. We also note that one would expect the equilibrium weight θ_l on measure $l \in L$ to be high if the values of excess demands under this measure tend to be large and positive. This property mirrors the activity of raising the prices of goods in excess demand, here we raise the weight of valuation measures being positively charged by the investment community.

A general competitive equilibrium for a complete markets economy is known to be Pareto efficient. For acceptably complete markets in the traded assets, a monitored financial equilibrium also has a similar property. However, as the MFE is less constrained with access to the resources of a monitoring authority capable of funding shortfalls and being a relatively wealth unconstrained counterparty to transactions, the Pareto frontier of a MFE will dominate the frontier of a standard general equilibrium.

5. Asset pricing in an MFE

The pricing of state contingent cash flows in a monitored financial equilibrium is directed by the monitoring authority. By defining the primitive of acceptability and insisting on attainability coupled with budget feasibility defined with respect to these measures each market participant is guided to select a measure in the convex hull of the valuation measures defining acceptability. For acceptably complete markets for the traded assets we have seen that the selected measure in the convex hull is common across all market participants and we have a unique selection of the pricing kernel. We now consider the implications of this new perspective for asset pricing more generally. The discussion is partitioned into two subsections. The first comments on traditional cross-sectional asset pricing models. The second comments on the risk management issues facing the monitoring authority of the monitored financial equilibrium.

5.1. Excess return analysis

For the purpose of discussing asset pricing, we need to fix a true probability measure under which we formulate excess returns that must be explained in terms of the monitored financial equilibrium. So far this measure was just a reference measure defining the set of states with positive probability but with little relevance for the actual probabilities. Let us now suppose that the measure P with probabilities p_ω is the true probability measure under which we wish to explain expected returns across assets. Consider also the case of acceptably complete markets for the traded assets. We then have a unique measure Q with probabilities q_ω in the convex hull of the valuation measures with weights $\theta_l > 0$, $l \in L$, for which

$$\pi_j = \sum_{\omega\in\Omega} q_\omega a^j(\omega), \qquad q_\omega = \sum_{l\in L} \theta_l q^l_\omega.$$

We may now write our time one delivery futures markets prices as expectations under P as follows:

$$\pi_j = \sum_{l\in L} \theta_l E^P\left[\frac{q^l}{p} a^j\right] = E^P[a^j] + \sum_{l\in L} \theta_l \operatorname{Cov}^P\left(\left(\frac{q^l}{p} - 1\right), a^j\right),$$

where Cov^P denotes the covariance under the measure P. Switching to returns

$$R^j = \frac{a^j}{\pi_j},$$

with expected excess returns

$$\alpha_j = \frac{E[a^j]}{\pi_j} - 1,$$

we may write that

$$\alpha_j = -\sum_{l\in L} \theta_l \operatorname{Cov}^P\left(R^j, \left(\frac{q^l}{p} - 1\right)\right).$$

The difference between this asset pricing equation and the ones derived in traditional GE models of asset pricing is that in the latter the question is posed as to what are the covariance risk factors priced in the equilibrium whereas here these factors are a not the issue. The factors constitute by design of the valuation test measures of the monitoring authority, the primitives of the economy. The economic equilibrium by selection of the weights (θ_l, $l \in L$) determines the relative importance of each prespecified valuation perspective in the final equilibrium. In fact we define the matrix A of dimension $|J|$ by $|L|$ with entries

$$A_{jl} = -\operatorname{Cov}^P\left(R^j, \left(\frac{q^l}{p} - 1\right)\right)$$

as the excess returns of assets $j \in J$ by the valuation perspectives $l \in L$ as part of the prespecified primitives of the economy. The monitored financial equilibrium then determines the actual excess returns prevailing at any time by

$$\alpha = A\theta, \tag{16}$$

where θ is the vector of equilibrium weights. Eq. (16) is the asset pricing equation of the MFE model, and the issue is what are the prior excess return perspectives prevailing in the economy at any time, i.e. the matrix A, and what are the weights given to each perspective in the equilibrium.

The matrix A is derived from covariation of returns with the extreme points of the convex hull of potential valuation measures entertained by the monitoring authority. If we view the primary interests of this authority as that of protecting its reserves then states constituting a drain on these reserves are potentially heavily weighted. By contrast, the traditional market portfolio of the capital asset pricing model reflects the gain and loss interests of the typical agent in an economy with agents displaying mean variance preferences, and covariation of returns with this portfolio are then paramount. In the latter case risk is measured by the incremental contribution to the variance of the aggregate portfolio. For the monitored financial equilibrium risk could be measured by exposure to loss scenarios of concern to the monitoring authority that may have little to do with the aggregate market portfolio. The monitoring authority could nominate variables of interest to it and define measures adapted to these variables that exaggerate outcomes in certain directions by, for example, exponentially tilting the measure with respect to the outcome.

The final equilibrium is however a balance of the interests of the monitoring authority and the maximizing agents of the economy. This is because the monitoring authority does not determine the pricing kernel of the economy, but only the subset in which the solution must lie. It is the interaction of maximizing agents that determines the nature of the final monitored financial equilibrium. This is clear from the market determination of the weights θ_l to be attached to each of the perspectives nominated by the monitoring authority. There may not be much preferential interest among maximizing agents to take on risks of a particular type and in the absence of substantial excess demands under this valuation measure it will receive a minimal weight in equilibrium. In general we would expect the weights to follow the preferential interests of the maximizing agents.

5.1.1. The risk primitives

We further enquire in this section into the nature of risks priced into the monitoring system primitives or the form of the measure changes q^l/p for $l \in L$. Much traditional research into asset pricing focuses attention onto the common and therefore undiversifiable components of risks in the return generating mechanism. However, what is critical to market participants is the impact on their welfare and this is not directly tied to factors generating asset returns. The monitoring authority when testing cash flows for acceptability could ask how potential counterparties in various situations would evaluate the cash flow, assuming that they had to take it on in addition to their perceived position at the margin. These calculations are relevant, for example, to the monitoring authority of the Federal Deposit Insurance Corporation when they seek to close failed banks by merger. One seeks the best valuations from the relatively healthy in times of crises when many participants may be damaged. An analysis of valuation at the margin by prospective counterparties leads to a standard expression for candidate valuation measures.

More formally, if we employ an expected utility criterion for the potential counterparty taking on the risk with a state preference utility function $U^l(\omega)$ and a subjective probability for state $\omega \in \Omega$ of p^s_ω then we have that

$$y^l_\omega = \frac{q^l_\omega}{p_\omega} = \kappa \frac{\partial U^l(\omega)}{\partial w_\omega} \frac{p^s_\omega}{p_\omega},$$

where κ is a normalizing constant and w_ω is the wealth of the counterparty in state ω.

The subjective probabilities could reflect views of potential counterparties in times of stress where wealth specifications have also been downgraded to reflect potential stress situations. The resulting personalized measure changes y^l_ω are not endogeneous but are exogeneous expectations of what the monitoring authority may hope to encounter in times of stress. The monitoring authority has to decide if it will approve plans that are positive under its stress test measures. It may choose to test these as it likes, but it must make a decision.

We take the view that particularly with respect to economies trading promises, in the absence of a monitoring authority verifying personal endowments and approving trades, the only real equilibrium will be the no trade equilibrium. Such an authority is not unlike systems in place in developed economies where people make credit card payments for goods and services with the prior approval of the credit card company, on the explicit understanding that they are covered and any default is the problem of the credit card company. The latter may eventually choose to write off the loss as a loan well made but irrecoverable given the turn of events. For the private market counterparty delivering goods and services there is no loss or default at all.

In general, if $f(R)$ is the marginal distribution of an asset return one period later, to evaluate the covariation between that asset price and the measure change y^l_ω we have to compute

$$\mathrm{Cov}^P\left(R, \left(\frac{q^l}{p} - 1\right)\right) = \int_{-1}^{\infty} (R - ER) g(R) f(R)\, \mathrm{d}R,$$

$$g(R) = E\left[\kappa \frac{\partial U^l(\omega)}{\partial w_\omega} \frac{p^s_\omega}{p_\omega} - 1 \mid R\right],$$

where $g(R)$ is the conditional expectation of the measure change y^l_ω given the asset return R. The potential counterparties one may be dealing with in assessing a risk may be both long or short the market or asset under consideration. Hence it would be useful to allow for perspectives in which $g'(R)$ was negative as well as positive.

5.2. Risk management

Perhaps the most significant relationship of a MFE is to the role of risk management in modern economies. The fundamental concept of acceptable opportunity on which the monitoring authority system is modeled in the MFE comes from the recent literature on risk management, notably the paper by Artzner et al. (1998). As noted earlier, this idea was used by Carr et al. (2001) to revisit the theorems of no arbitrage and here we revisit the idea of a general equilibrium.

Reserve policies supporting economic activity are the critical concern for risk management and are important for making activities and transactions acceptable to the financial system in a financial equilibrium. They play no role in a traditional general equilibrium as here there is no potential gap between plans and outcomes and in a very fine sense there is then no risk involved. For a monitored financial equilibrium on the other hand, the risk management system has to balance the benefits of relaxing constraints on the real economy and permitting the accompanying growth in activity with the costs of exposure to financial crises and the need to draw on reserves to finance gaps in the final outcomes.

The reserve policies are the primary way of gaining acceptability for a trading position. It is true that one may have shortfalls and not be concerned as one now has the ability to swap with the monitoring authority. However, as explained in the original formulation of coherent risk measures, the primary mechanism to attaining swap acceptability with the monitoring authority is to add cash or capital to the position. Personal reserves are therefore opportunity enhancing in a monitored financial equilibrium and capital is productive in a very precise sense. There are simply more approved actions one may take.

We have supposed full disclosure of plans in this preliminary study of a monitored financial equilibrium. Our monitoring authority faced few problems of moral hazard whereby subsequent actions may alter the nature of approved risks. Monitoring transactions is not without its own difficulties. Finally one has to consider the welfare questions of designing such an authority, its resource base and balance the benefits of opening the constraint set against the costs of utility foregone in building the reserves.

Our objective in the presentation here was to illustrate how such an authority guides the hand of asset pricing in equilibrium, to establish the equilibrium, and to note the necessity in real world transactions of not testing them to the limits of no trust and no trade. We anticipate that the study of monitored financial equilibria from an empirical standpoint in various asset markets will help identify collections of measures that could usefully be adopted in the formulation of risk control systems and help the monitored financial system to better withstand the pressures of failures. We need to better understand the mechanisms for generating test measures or scenarios in the language of modern risk management.

6. Conclusion

This paper proposes a new concept of market equilibrium that enhances the set of primitives of the traditional model (preferences, endowments, technology and the financial asset span of the GEI) to include the monitoring authority as a passive agent of last resort who takes up the slack generated by the rest of the economy on prespecified terms. This new equilibrium is termed a monitored financial equilibrium and it is shown under certain mild conditions that the equilibrium exists. We recognize that by construction the new equilibrium can be Pareto superior to the traditional equilibrium. It does however require the setting up of a reserve fund to cover gaps in outcomes relative to approved plans. The general questions of the costs and benefits of such a design and related issues of moral hazard remain open. We note the necessity of such mechanisms and illustrate how they guide asset pricing in equilibrium.

An analysis of the asset pricing implications suggests new directions for empirical research in asset pricing. Essentially pricing models lie in the convex hull of test measures used by the monitoring authority. We anticipate interesting research on describing the boundaries of these measures and their relationship to on going risk management procedures.

Future research must also build on ongoing research on multiperiod acceptability definitions to consider the case of dynamic equilibria for a monitored financial system. The case of continuous state spaces also poses problems of its own, based on the structure of topologies used in defining the approximations of cash flows.

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