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Biproportional Methods of Structural Change Analysis: a Typological Survey

LOUIS DE MESNARD

ABSTRACT *Analysts often are interested in learning how much an exchange system has changed over time or how two different exchange systems differ. Identifying structural difference in exchange matrices can be performed using either 'directed' or 'undirected' methods. Directed methods are based on the computation and comparison of column- or row-normalizations of the matrices. The choice of row or column for the normalization implies a specific direction of the exchanges, so that the column-wise normalized results should not be compared to the row-wise normalized results. In this category fall the simple comparison of coefficient matrices and the causative method. Undirected methods do not impose such underlying constraints on exchanges. Hence, I present a set of undirected methods that can be used to compare structural matrices: the biproportional ordinary filter, the biproportional mean filter and the bi-Markovian filter. While doing so, I recall why the bicausative method must be dismissed. I then classify the methods according to their orientation and data needs, and illustrate how the results can differ from one method to the next using French tables for 1980 and 1997.*

KEYWORDS: *Biproportion; RAS; causative matrices; structural change*

1. Introduction

Determining how structures of exchange have changed is a necessity in many fields. Input–output analysts are interested in differences in production structure (i.e. the structure of exchange between producing sectors) both temporally and internationally. Spatial economic analysts and regional scientists want to analyse how exchanges among regions differ, sometimes over time. Finance theorists are concerned with the evolution of cross-shareholding. And sociologists want to understand how communications vary among individuals. Not all these applications are economic, but they share a common desire—to compare structures of exchange.

This paper surveys some methods for evaluating the temporal evolution of exchange structure or the cross-sectional differences between exchange structures. Typically, just two structures are compared. I denote these by two matrices $\mathbf{Z}$ and $\mathbf{Z}^{\star}$. There are two major categories of methods for comparing exchange structures: *directed methods*, where the economy is assumed to be either demand- or supply-

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driven, and *undirected methods*, mainly based on biproportional techniques, where such hypotheses are not assumed.

Consider two structures of exchange (or production) $\mathbf{Z}$ and $\mathbf{Z}^\star$ over time or space. To measure the difference between structures by computing the difference matrix $\mathbf{Z}—\mathbf{Z}^\star$, a function of this difference as $|\mathbf{Z}—\mathbf{Z}^\star|$ or any more sophisticated norm $\|\mathbf{Z}—\mathbf{Z}^\star\|_p$, is incorrect since the two matrices are quite unlikely to have the same column or row margins: differences in the growth of agents (the sectors in an input–output context) exist. The resulting 'size effect' confounds any analysis, since any measured change also includes a true technology difference a 'structural change effect' that is the focus of structural analysis. So, both matrices must have at least the same column margins to eliminate the column-wise size effect or the same row margins to eliminate the row-wise size effect (for cases using directed methods). To remove completely size effects for the case of undirected methods, any such structural analysis should compare matrices with the same column *and* row margins, which implies the use of projection techniques.

2. Recalling 'Directed Methods'

'Directed methods' of structural analysis are used to compare matrices of coefficients that are normalized either column-wise or row-wise.

2.1. Comparing Matrices Column-wise or Row-wise

In this paper, 'additive methods' of structural analysis use the mathematical operation of addition or subtraction to compare matrices. Alternatively, multiplicative methods use multiplication (or division) for the same purpose. With this in mind, we compare structures of linear models of exchange and of linear models of production.

Let us examine a structure of exchange where an agent i exchanges an amount $z_{ij} \geqslant 0$ of a single commodity with an agent j. Quantity z_{ij} is expressed in a homogeneous unit, for example in terms of money. The system is closed; that is, the aggregate wealth of agents i is equal to x_i before and after the exchange. Further, let us assume that each sender and each receiver can exchange all resources available to him/her at the outset, but no more than that:

$$z_{i\bullet} = x_i \text{ for all } i, \text{ or } \mathbf{Zs} = \mathbf{x} \tag{1}$$

where $z_{i\bullet} \equiv \Sigma_j z_{ij}$ and $\mathbf{s}$ is the summation vector: $\mathbf{s}' = (1 \ldots 1)$ while prime (′) indicates array transposition. In the same way we have

$$z_{\bullet j} = x_j \text{ for all } j, \text{ or } \mathbf{s}'\mathbf{Z} = \mathbf{x}' \tag{2}$$

where $z_{\bullet j} \equiv \Sigma_i z_{ij}$. A peculiarity of these tables is that they can be meaningfully summed by rows and by columns; x_i is the sum of row i, while x_j is the sum of column j. While simple, such a structure can describe the propagation of an idea among the people or even the spread of a disease; it can also describe an input–output structure where only the exchanges between sectors are of interest. In the following, a formal exchange structure is considered to simplify the exposé. This structure can be a social structure, where agents are individuals who exchange messages, ideas, etc; or it can be a matrix describing the structure of the financial sector with its crossed-sharing (Dietzenbacher *et al.*, 2000). It also can be an inter-industry trade matrix open or closed with respect to households (with a supplementary row for value added and a supplementary column for final demand) or alternatively the inter-industry trade matrix closed with respect to households and trade with the rest of the world (i.e. imports and exports).

One can compare two matrices by eliminating differences between their respective column margins. It can be done with column coefficients that are formed by the ratio $a_{ij}=z_{ij}/x_j$, or in matrix terms, $\mathbf{A}=\mathbf{Z}\hat{\mathbf{x}}^{-1}$. In economics parlance, a_{ij} indicates how much (in monetary terms) sector j buys of sector i's good to produce one unit of its own good; in the Leontief framework the use of a_{ij}'s assumes that the implied technological relationship is stable and that the economy is demand driven. In this context, two column coefficient matrices typically are compared—an initial matrix $\mathbf{A}$ and a final matrix $\mathbf{A}^\star$, usually belonging to two different periods or countries. In a pure exchange model, $\mathbf{A}$ and $\mathbf{A}^\star$ are Markovian, so one can evaluate the difference between the two matrices: $\mathbf{D}=\mathbf{A}^\star-\mathbf{A}$; this classifies it as an 'additive' method. Matrix $\mathbf{D}$ must be compared to the null matrix: if structural change is null, then $\mathbf{D}$ must be equal to $\mathbf{0}$. A natural indicator of coefficient change at two different dates could be the absolute difference between them: $\sigma_{ij}=|a_{ij}-a^\star_{ij}|$. Each of these measures is informationally inefficient, however. One must evaluate and analyse n^2 terms, while the matrices have themselves terms each. It clearly would be better to reduce the number of indicators. For example, one could compute n indicators by measuring column differences: since we have defined $\mathbf{A}$ and $\mathbf{A}^\star$ to have the same column margins, one could do this by computing each column's Frobenius norm of the columns of the difference matrix, an indicator of change in the composition of the column computed in percentages: $\sigma_j=[\Sigma_i(a^\star_{ij}-a_{ij})^2]^{1/2}$. Using this measure, the greater is the value for a particular column, the greater is the variation in the implied technology coefficients within the column.

The same thing can be performed on row coefficient matrices when Ghosh's (1958) point of view is adopted: in this case, row-wise coefficients likewise must be assumed stable. A row coefficient $b_{ij}=z_{ij}/x_i$ (or in matrix terms, $\mathbf{B}=\hat{\mathbf{x}}^{-1}\mathbf{Z}$) indicates how much of a unit of output from sector i is allocated to another sector j. Again, one can compare the relative variability in the distribution across sectors of two allocation matrices $\mathbf{B}$ and $\mathbf{B}^\star$ by $\sigma_i=[\Sigma_j(b^\star_{ij}-b_{ij})^2]^{1/2}$.

The difficulty with directed methods is that the results obtained for column coefficients are not comparable to the results obtained for row coefficients. If row coefficients are stable, column coefficients cannot be, and vice versa; that is, assuming $\mathbf{B}^\star=\mathbf{B}$ then $\mathbf{A}^\star=\hat{\mathbf{x}}^\star\hat{\mathbf{x}}^{-1}\mathbf{A}\hat{\mathbf{x}}(\hat{\mathbf{x}}^\star)^{-1}\neq\mathbf{A}$ (the circumflex over a vector denotes its transformation into a diagonal matrix). This works except in the case of absolute joint stability (Chen & Rose, 1986, 1991). In that case, if $\mathbf{B}^\star=\mathbf{B}$ and $\mathbf{x}^\star=k\mathbf{x}$, then $\mathbf{A}^\star=\hat{\mathbf{x}}^\star\hat{\mathbf{x}}^{-1}\mathbf{A}\hat{\mathbf{x}}(\hat{\mathbf{x}}^\star)^{-1}=\mathbf{A}$.

2.2. Causative Matrix Approach

First, using multiplicative rather than additive operations, one can compute $a^\star_{ij}a^{-1}_{ij}$ for all i, j instead of $|a^\star_{ij}-a_{ij}|$ for all i, j. Interestingly, moving from an additive to a multiplicative measure does not change the basic properties of the comparative measure, however. Interestingly multiplicative methods are more powerful: the causative matrix approach belongs to this category.

The causative matrix approach (Rogerson & Plane, 1984; Plane & Rogerson, 1986, Jackson, *et al.*, 1990) consists of computing two Markovian matrices from the allocation coefficient matrices[1] $\mathbf{A}^{\mathrm{M}}=\mathbf{A}\mathbf{M}^{-1}$ and $\mathbf{A}^{\star\mathrm{M}}=\mathbf{A}^\star(\mathbf{M}^\star)^{-1}$. These resulting matrices are called transition matrices, where the matrix $\mathbf{A}^{\star M}$ is assumed to be linked to the matrix $\mathbf{A}^{M}$ by the formula:

$$\mathbf{A}^{\star M} = \mathbf{C}\mathbf{A}^{M} \tag{3}$$

Matrix $\mathbf{C}$ is called a '*left causative matrix*'. It explains the change between $\mathbf{A}^{M}$ and $\mathbf{A}^{\star M}$ and is found by inverting $\mathbf{A}^{M}$:

$$\mathbf{C} = \mathbf{A}^{\star M}(\mathbf{A}^{M})^{-1} \tag{4}$$

Matrix $\mathbf{C}$ has n^2 terms, and has a rather complex economic interpretation. Typically, matrix $\mathbf{C}$ is compared to the identity matrix. That is, all diagonal elements are compared to 1, while all off-diagonal elements of each row are compared to 0. As formula (3) implies, each coefficient $a_{ij}^{\star M}$ of $\mathbf{A}^{\star M}$ is not linked simply to the corresponding coefficient a_{ij}^{M} of $\mathbf{A}^{M}$ but rather to all coefficients of a column of $\mathbf{A}^{M}$, i.e. $a_{ij}^{\star M} = \Sigma_k c_{ik} a_{kj}^{M}$. Hence, an interpretation of differences from the identity matrix can be difficult to develop.

Equations (3) or (4) allow one to describe this approach as 'multiplicative'. An alternative solution would have been computing $\mathbf{D} = \mathbf{A}^{\star M} - \mathbf{A}^{M}$, which is the same as the above comparison of column or row coefficients, referred to as 'additive' (except that direct matrices were considered). In this case, the difference matrix $\mathbf{D}$ would be compared to the null matrix and the interpretation is simpler because $\mathbf{D}$ contains no mix of coefficients from $\mathbf{A}^{M}$ and $\mathbf{A}^{\star M}$ as in $\mathbf{C}$ and each $a_{ij}^{\star M}$ is linked to the corresponding coefficient a_{ij}^{M} by the simple formula $a_{ij}^{\star M} = d_{ij} + a_{ij}^{M}$, which yields a simpler interpretation of the differences.

Observe that a reverse comparison can be done by posing $\mathbf{A}^{M} = \tilde{\mathbf{C}}\mathbf{A}^{\star M}$, where $\tilde{\mathbf{C}}$ is the causative matrix for this reverse analysis. $\tilde{\mathbf{C}}$ is the inverse of $\mathbf{C}$, i.e. $\tilde{\mathbf{C}} = \mathbf{A}^{M}(\mathbf{A}^{\star M})^{-1} = \mathbf{C}^{-1}$.

Note that Jackson *et al.* (1990) also define a right causative matrix: $\mathbf{A}^{\star M} = \mathbf{A}^{\star M}\mathbf{R} \Rightarrow \mathbf{R} = (\mathbf{A}^{M})^{-1}\mathbf{A}^{\star M}$. A right causative matrix conflicts with the assumptions of the demand-driven model because the rows of $\mathbf{A}^{M}$ are being multiplied by the terms of $\mathbf{R}$, which have no meaning when the Leontief model is demand-driven (while they have one from Ghosh's perspective).

3. 'Undirected' Methods

In this group of methods, no hypotheses are made about what drives the economy, demand or supply. Both hypotheses are incompatible: however, even if the demand-driven hypothesis might seem more plausible, epistemologically it is generally preferable not to pose a hypothesis when it can be avoided, provided that the hypothesis can be disproved later. So, while under directed methods we analysed either column or row coefficients after row or column normalization, in the case of undirected methods we compare coefficients without a normalization that carries with it all of the baggage of an underlying demand- or supply-driven model. There are many tools to perform this operation of projection of a matrix and the problem is to choose one of these tools, or, in other words, there are an infinite number of matrices that can have the same margins and the problem is to choose one of these matrices. The resulting matrix may vary depending on the tool chosen to perform the projection, and consequently the results of the methods may vary.

3.1. Methods that Generalize the Comparison of Column and Row Coefficients

Above, the idea was to give to each sector at year 0 the same size as the corresponding sector at year 1. This can be performed by giving the same column total to both sectors (computing column coefficients, i.e. a proportion; this was the more usually done) or by giving the same row total to both sectors (computing row coefficients, a proportion). The first method detects the matrix structural change only of column vectors, the second detects matrix structural change of row vectors. If one chooses to favour column coefficients, one considers that the norm is: column coefficients must be normally fixed and all variation must be declared as 'structural change', but one excludes that row coefficients are fixed and one cannot say anything about their structural change (remember that column and row coefficients cannot be simultaneously fixed except in the case of absolute joint stability. Conversely, if one chooses to favour row coefficients, one considers that the norm is: row coefficients must be normally fixed, all variation being declared as 'structural change' but one cannot say anything about structural change of column coefficients (for the same reason).

Now, I give up the idea of choosing between the two directions, column or row, to generalize the comparisons of both column and row coefficients by giving the same column and row total to both sectors, in order to detect a matrix structural change of both column and row vectors at the same time. If one follows this generalization, the norm becomes: both column and row coefficients must be fixed, all variation being declared as 'structural change'. This family of methods will be called, in what follows, 'undirected methods'. The question is: how to perform this if column and row coefficients cannot be stable at the same time?

In a general way, the simplest principle of this type of undirected method consists of projecting one matrix using the margins (the column and row totals) of another matrix, something close to normalizing both column and rows. Nevertheless, while for directed methods it was equivalent to compare $\mathbf{Z}\hat{\mathbf{x}}^{-1}\hat{\mathbf{x}}^{\star}$ with $\mathbf{Z}^{\star}$, or $\mathbf{A}$ with $\mathbf{A}^{\star}$, this is no longer the case. Nonetheless, it is generally preferable to work with flow matrices instead of column coefficient matrices because flow matrices are not directed.

Starting from an initial flow matrix $\mathbf{Z}$ and a final flow matrix $\mathbf{Z}^{\star}$, the principle consists of computing a matrix $\hat{\mathbf{Z}}$ that is as close as possible to $\mathbf{Z}$ but with the row and column margins of $\mathbf{Z}^{\star}$, see Figure 1. Since they have the same margins, it is possible to compare $\hat{\mathbf{Z}}$ and $\mathbf{Z}^{\star}$: the effect of the differential productivity (columns) or final demand (rows) growth of sectors is removed in this way, leaving only the structural change effect in the comparison. As said in the introduction, projections techniques are absolutely necessary at this step because both row-wise and column-wise size effects must be removed to keep only the structural effect, which is impossible to do if one compares directly $\mathbf{Z}$ and $\mathbf{Z}^{\star}$.

Many tools can be used to project a matrix, so the problem is selecting one. Alternatively, one can view the problem as choosing one matrix from among the infinite number of matrices with the same set of margins. The matrix that results depends on the tool performing the projection. The methods can be categorized based on the way they project. Moreover, in evaluating the distance from the target, some methods compute the difference and others the ratio between the projection and the target. This marks a clear dimension to a typology of projection methods.

Distance-minimization methods One way to compare $\hat{\mathbf{Z}}$ and $\mathbf{Z}^{\star}$ is to find the matrix $\hat{\mathbf{Z}}$ that is closest to a matrix $\mathbf{Z}$, which respects the margins of another matrix $\mathbf{Z}^{\star}$

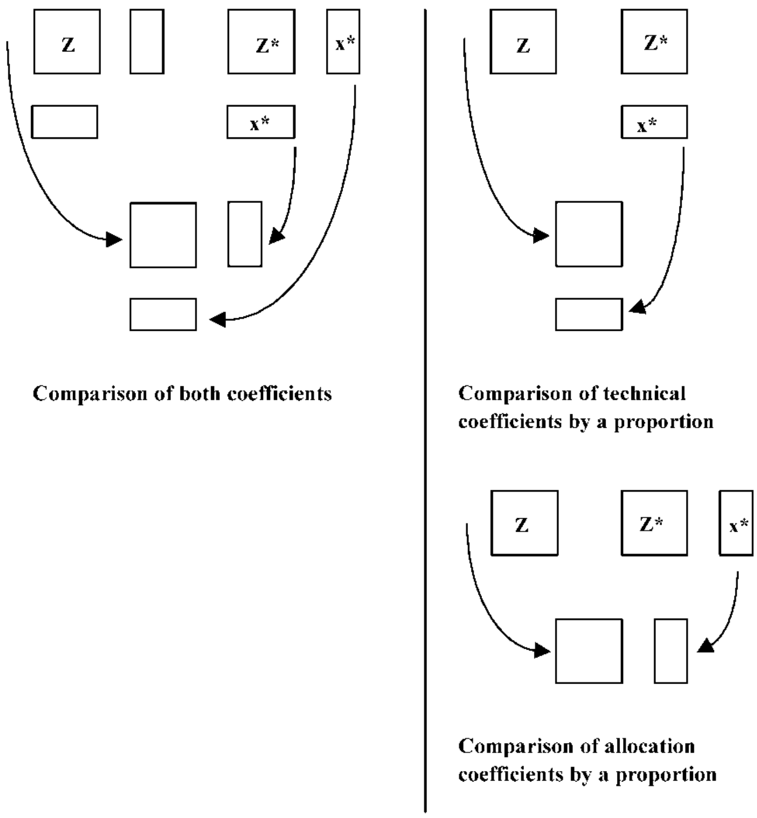

Figure 1. Principle of matrix comparisons over time.

(i.e. under constraints of margins: $\Sigma_i \hat{z}_{ij} = \Sigma_i z^\star_{ij}$ and $\Sigma_j \hat{z}_{ij} = \Sigma_j z^\star_{ij}$). Unfortunately, negative terms can arise in the projection $\hat{\mathbf{Z}}$ because of its additive form. For example, the orthogonal projection, i.e. the minimization of the quadratic deviation, that is min $\Sigma_i \Sigma_j (\hat{z}_{ij} - z_{ij})^2$ yields $\hat{\mathbf{Z}} = \mathbf{P} + \mathbf{Z} + \mathbf{Q}$, where $\mathbf{P}$ and $\mathbf{Q}$ are as follows

$$\mathbf{P} = \begin{bmatrix} p_1 & \cdots & p_1 \\ \dots & & \dots \\ p_n & \cdots & p_n \end{bmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{bmatrix} q_1 & \cdots & q_n \\ \dots & & \dots \\ q_1 & \cdots & q_n \end{bmatrix}$$

Hence, non-negativity of terms in the projected matrix is not guaranteed (de Mesnard, 1990a).

Remember that negative terms are impossible to explain in an input–output context. In graph theory, negative terms are interpreted as reverse flows; so in input–output analysis, a $\hat{z}_{ij} < 0$ means that it is the flow of commodity i sold by j to i (while z_{ij} is the flow of commodity i sold by i to j). This is nonsensical, except for items like recycled commodities, e.g. scrap steel sold by the automotive industry to the steel industry.[2] In such cases, j (automobiles) likely continues to buy commodity i (steel) from sector i (steel sector), so that $\hat{z}_{ij}$ is the net balance of the normal (primary steel) and reverse (scrap steel) flows. Nevertheless, it is always possible to remove negative terms by posing $\hat{z}_{ij} \geqslant 0$ for all i, j, as a set of n^2 constraints. In most cases, however, solutions will go to the bounds, so the negative

$\hat{z}_{ij}$ will become zero, and a null term, $\hat{z}_{ij}=0$, will be compared to a positive term, $z^{\star}_{ij}>0$.

Among the possible methods are:

- The above *minimization of the quadratic deviation* (the square of the Frobenius norm of the difference matrix): $\min \Sigma_i \Sigma_j (\hat{z}_{ij}-z_{ij})^2$.
- The *minimization of the absolute differences*: $\min \Sigma_i \Sigma_j |\hat{z}_{ij}-z_{ij}|$, which is not continuously derivable.
- The *minimization of the Hölder norm of power* p: $\min \Sigma_i \Sigma_j |\hat{z}_{ij}-z_{ij}|^p$ knowing that the Hölder norm (Rotella & Borne, 1995, p. 78) is $\|\hat{\mathbf{Z}}-\mathbf{Z}\|_p=[\Sigma_i \Sigma_j |\hat{z}_{ij}-z_{ij}|^p]^{1/p}$, a generalization of the two preceding measures.
- *Pearson's* χ^2: $\min \Sigma_i \Sigma_j [(\hat{z}_{ij}-z_{ij})^2/z_{ij}]$.
- *Neyman's* χ^2: $\min \Sigma_i \Sigma_j [(\hat{z}_{ij}-z_{ij})^2/\hat{z}_{ij}]$.

Besides the negative terms, these methods often lead to various problems since some non-linearities or non-differentiabilities can be found in the system of equations (it is the case for Neyman's χ^2 and for the absolute differences).

Biproportional methods I have proposed (de Mesnard, 1990a, 1990b, 1996, 1997) a biproportional filter to analyse structural change. Remember that a biproportional solution $\hat{\mathbf{Z}}$ places upon $\mathbf{Z}$ the same margins as $\mathbf{Z}^{\star}$, $\hat{\mathbf{Z}}=K(\mathbf{Z},\mathbf{Z}^{\star})$, which equivalently can be expressed **PZQ**, where **P** and **Q** are diagonal matrices that respect two conditions. First, $\hat{\mathbf{Z}}$ must have the same row and column margins as $\mathbf{Z}^{\star}$. That is

$$\begin{cases} \Sigma_j \hat{z}_{ij}=\Sigma_j z^{\star}_{ij} & \text{for all } i \\ \Sigma_i \hat{z}_{ij}=\Sigma_i z^{\star}_{ij} & \text{for all } j \end{cases} \tag{5}$$

Second, $\hat{\mathbf{Z}}$ is the matrix nearest to $\mathbf{Z}$ following a certain criterion. This criterion can be, among others:

- *The maximization of entropy criterion* (Wilson, 1970): max H, with $H=-\Sigma_i \Sigma_j \hat{z}_{ij} \log \hat{z}_{ij}$, under the constraint that $C=\Sigma_i \Sigma_j \hat{z}_{ij} c_{ij}$ where C is the total cost and c_{ij} is a cost that can be considered as representative of $\mathbf{Z}^0$. If no cost constraint is considered, max H alone, the result is $\hat{z}_{ij}=z^{\star}_{i\bullet} z^{\star}_{\bullet j}/z^{\star}_{\bullet\bullet}$, which follows the Bernoulli–Laplace principle. If $C=C_{\min}$, the result of the transportation problem is obtained: the entropy maximization cannot give another result than that of the cost minimization.
- *Kullback-Liebler minimum information discrimination criterion* (Kullback & Liebler, 1951; Kullback, 1959; Snickars & Weibull, 1977), which follows Shannon's theory: $\Sigma_i \Sigma_j \hat{z}_{ij} \log(\hat{z}_{ij}/z_{ij})$.
- *Minimum interactions criterion* (Watanabe, 1969; Guiasu, 1979).

Several algorithms respect these two conditions. Among them is RAS. For example, the terms **P** and **Q** can be of the following form. For any i, j:

$$p_i=z^{\star}_{i\bullet}/\Sigma^m_{j=1} q_j z_{ij} \quad \text{and} \quad q_j=z^{\star}_{\bullet j}/\Sigma^n_{i=1} p_i z_{ij} \tag{6}$$

I showed that all algorithms that respect these two conditions of a biproportional method provide the same results (de Mesnard, 1994). These algorithms have a unique solution as demonstrated for RAS by Bacharach (1970). They cannot be solved analytically, only iteratively. Generally, their convergence is obtained quickly, although the time consumed in getting convergence depends on the choice of algorithm (Bachem & Korte, 1979). In addition, note that if all $p^{(k)}_i$ at iteration k

are positive, all $q_j^{(k)}$ will also be positive, as long as all terms of $\mathbf{Z}$ are positive. This avoids the possibility of obtaining negative terms, which are always hard to interpret.[3]

Biproportional ordinary filter With an *ordinary biproportional filter*, the flow matrix $\mathbf{Z}$ is projected such that it obtains the same margins as the flow matrix $\mathbf{Z}^\star$, i.e. $\hat{\mathbf{Z}}=K(\mathbf{Z},\mathbf{Z}^\star)$. In this case, the result $\hat{\mathbf{Z}}$ is compared with $\mathbf{Z}^\star$ by calculating the difference matrix $\mathbf{Z}^\star - K(\mathbf{Z},\mathbf{Z}^\star)$ (de Mesnard, 1990a, 1990b). The idea is to perform calculations that parallel those used in the shift-share method.[4] Since $K(\mathbf{Z},\mathbf{Z}^\star)$ has the same row and column margins as $\mathbf{Z}^\star$, computing $K(\mathbf{Z},\mathbf{Z}^\star)$ removes the shift (growth) effect (positive or negative) between $\mathbf{Z}$ and $\mathbf{Z}^\star$. Therefore, what remains in $\mathbf{Z}^\star - K(\mathbf{Z},\mathbf{Z}^\star)$ is the share (structural) effect since $\mathbf{Z}^\star - \mathbf{Z}$ is the combination of both effects.

Then the relative variations in columns, rows or over the whole array are obtained by dividing the norm by the sector size (i.e. the margin) to obtain comparable percentages. For column j, row i, and overall, we—respectively—have

$$\sigma_j^C = \frac{\sqrt{\sum_i [z_{ij}^\star - K(\mathbf{Z},\mathbf{Z}^\star)_{ij}]^2}}{z_{\bullet j}^\star}, \tag{7}$$

$$\sigma_i^R = \frac{\sqrt{\sum_j [z_{ij}^\star - K(\mathbf{Z},\mathbf{Z}^\star)_{ij}]^2}}{z_{i\bullet}^\star}, \quad \sigma = \frac{\sqrt{\sum_i \sum_j [z_{ij}^\star - K(\mathbf{Z},\mathbf{Z}^\star)_{ij}]^2}}{z_{\bullet\bullet}^\star}$$

Van der Linden & Dietzenbacher (1995, 2000) developed similar concepts. To compare two matrices, they projected the first on the margins of the second. Instead of operating on flows, they operated on technology,[5] i.e. $\hat{\mathbf{A}}=K(\mathbf{A},\mathbf{A}^\star)$. As a result, they did not evaluate the difference in a sector over time row-wise or column-wise, only the differences in specific matrix cells: they computed d_{ij} by dividing $\hat{a}_{ij}$ by $a_{ij}^\star$, i.e. $d_{ij}=\hat{a}_{ij}/a_{ij}^\star$ for all i, j.

Other indicators of distance are possible for the numerator of the relative variations σ, as the Hölder norm; for example,

$$\sigma = \frac{\left[\sum_i \sum_j |z_{ij}^\star - K(\mathbf{Z},\mathbf{Z}^\star)_{ij}|^p\right]^{1/p}}{z_{\bullet\bullet}^\star} \tag{8}$$

The larger is p, the higher is the weight of large gaps between $z_{ij}^\star$ and $K(\mathbf{Z},\mathbf{Z}^\star)_{ij}$. Note that, in Jackson & Murray (2004) the gap between the final matrix and the estimated matrix—obtained by projection with one of the models listed by these authors (Section 3)—measured by one of the above measures, is equal to the structural change as defined in this article.

When more than two years are considered, it is also possible to conduct two sets of analyses: one 'sliding' and one cumulative (de Mesnard, 1990a, 1990b). Denoting years by 0, 1, 2, ..., t, ... T, and the matrices by $\mathbf{Z}^t$, the sliding projections allow detecting structural change from the current years by the following: $\mathbf{Z}^{t+1} - K(\mathbf{Z}^t,\mathbf{Z}^{t+1})$ for $t=0$ to $T-1$.

Considering that two base years are possible, year 0 and year T, cumulative projections allows one to detect structural change from the current years to base year T, or the structural change from the base year 0 to the current years: $\mathbf{Z}^T - K(\mathbf{Z}^t,\mathbf{Z}^T)$ for $t=0$ to $T-1$ or $\mathbf{Z}^t - K(\mathbf{Z}^0,\mathbf{Z}^t)$ for $t=0$ to $T-1$.

Both results are not necessarily identical. Overall, for $T+1$ years—that is, T intervals of time—one must compute T sliding projections and comparisons, plus $2T$ cumulative projections and comparisons, for a total of $3T$ projections!

Note that in biproportional techniques, operating on flow matrices and on column coefficient matrices does not yield the same results even when $\mathbf{A}$ is derived from $\mathbf{Z}$ by a diagonal matrix multiplication (de Mesnard, 1994):

$$K(\mathbf{A},\mathbf{A}^\star) = K(\mathbf{A},\mathbf{Z}^\star <\mathbf{x}^\star>^{-1}) \neq K(\mathbf{A},\mathbf{Z}^\star) \qquad \text{but}$$

$$K(\mathbf{A},\mathbf{A}^\star) = K(\mathbf{Z}<\mathbf{x}>^{-1},\mathbf{A}^\star) = K(\mathbf{Z},\mathbf{A}^\star).$$

A difficulty is that it is also possible to project $\mathbf{Z}^\star$ on the margins of $\mathbf{Z}$ and then to compare the result to $\mathbf{Z}$ itself (de Mesnard, 1990a, 1990b). This can be done in two ways: by direct computation from $\mathbf{Z}$ to $\mathbf{Z}^\star$, $\mathbf{Z}^\star - K(\mathbf{Z},\mathbf{Z}^\star)$ as above and by reverse computation from $\mathbf{Z}^\star$ to $\mathbf{Z}$, $\mathbf{Z} - K(\mathbf{Z}^\star,\mathbf{Z})$. For column j, row i, and overall, we—respectively—have

$$\sigma_j^C = \frac{\sqrt{\sum_i [z_{ij} - K(\mathbf{Z}^\star,\mathbf{Z})_{ij}]^2}}{z_{\bullet j}},$$

$$\sigma_i^R = \frac{\sqrt{\sum_j [z_{ij} - K(\mathbf{Z},\mathbf{Z}^\star)_{ij}]^2}}{z_{i\bullet}}, \quad \sigma = \frac{\sqrt{\sum_i \sum_j [z_{ij} - K(\mathbf{Z}^\star,\mathbf{Z})_{ij}]^2}}{z_{\bullet\bullet}}$$

In any case, the number of computations is again doubled and provides two different sets of results, with no criterion declaring one superior to the other.[6] This leads us to the following section.

Biproportional mean filter Two sets of results must be computed: a direct and a reverse. To do so is costly, especially since the two sets of results are not comparable. This is troubling since it remains difficult to identify the structural change between $\mathbf{Z}$ and $\mathbf{Z}^\star$. Understanding that the basic idea of biproportional filtering consists of providing the same margins to flow matrices $\mathbf{Z}$ and $\mathbf{Z}^\star$, one can try to find a third matrix $\mathbf{Z}^B$ with the requisite margins to remove these difficulties. If $\mathbf{Z}^B$ has the same margins as $\mathbf{Z}$, or is equal to $\mathbf{Z}$, then $K(\mathbf{Z}, \mathbf{Z}^B) = \mathbf{Z}$ and $K(\mathbf{Z}^\star,\mathbf{Z}^B) = K(\mathbf{Z}^\star,\mathbf{Z})$, the reverse of the ordinary biproportional projector. If $\mathbf{Z}^B$ has the same margins as $\mathbf{Z}^\star$, then $K(\mathbf{Z}^\star,\mathbf{Z}^B) = \mathbf{Z}^\star$ and $K(\mathbf{Z},\mathbf{Z}^B) = K(\mathbf{Z},\mathbf{Z}^\star)$, the direct projection of the ordinary biproportional projector. Since there are many possible alternatives between these two 'polar' matrices, a wide range of possible results exists. One possibility consists of choosing $\mathbf{Z}^B$ in such a way that the variance of the difference is minimized. That is, min $\| K(\mathbf{Z},\mathbf{Z}^B) - K(\mathbf{Z}^\star,\mathbf{Z}^B) \|^2$.

Unfortunately, this expression is not linear in $\mathbf{Z}$, $\mathbf{Z}^\star$ or to the margins of $\mathbf{Z}^B$. It has no analytical solution because biproportion is a transcendent operator. Such a problem can be solved only iteratively and to this day remains very computationally intensive even for small matrices.

However, one type of matrix is a good candidate for $\mathbf{Z}^B$, namely a function of

Z and **Z***. For example, $\bar{\mathbf{Z}}$, the mean of **Z** and **Z***, with $\bar{\mathbf{Z}}=\frac{1}{2}(\mathbf{Z}+\mathbf{Z}^{\star})$. I (de Mesnard, 1998) call this the *biproportional mean filter*. Note that $\mathbf{Z}^B$ could be also a third matrix of an intermediate year, e.g. 1988 if **Z** is 1980 and **Z*** is 1996. But remember that only the margins of this matrix are important.

In the biproportional mean filter, each matrix **Z** and **Z*** is projected to the margins of $\bar{\mathbf{Z}}$, the mean of **Z** and **Z***, to give $K(\mathbf{Z},\bar{\mathbf{Z}})$ and $K(\mathbf{Z}^{\star},\bar{\mathbf{Z}})$; then, $K(\mathbf{Z},\bar{\mathbf{Z}})$ is compared with $K(\mathbf{Z}^{\star},\bar{\mathbf{Z}})$ by calculating the Frobenius norm of the difference matrix $K(\mathbf{Z}^{\star},\bar{\mathbf{Z}})-K(\mathbf{Z},\bar{\mathbf{Z}})$, as in the case of the ordinary biproportional filter. Here, however, there is only one set of computations, not two. This allows removal of the effects of differential sectoral growth, but not the effect of differences in the sectoral size, which again obliges the analyst to distinguish by sector size. For column j, row i, and overall, we—respectively—have

$$\sigma_j^C=\frac{\sqrt{\sum_i \theta_{ij}^2}}{\bar{z}_{\bullet j}},\quad \sigma_i^R=\frac{\sqrt{\sum_j \theta_{ij}^2}}{\bar{z}_{i\bullet}},\quad \sigma=\frac{\sqrt{\sum_i\sum_j \theta_{ij}^2}}{\bar{z}_{\bullet\bullet}}\quad \text{with } \theta_{ij}=K(\mathbf{Z}^{\star},\bar{\mathbf{Z}})_{ij}-K(\mathbf{Z},\bar{\mathbf{Z}})_{ij}$$

It remains possible to conduct both sliding and cumulative analyses if necessary.

A figure based on an Edgeworth box illustrates the method (see Figure 2). Consider the matrices:

$$\mathbf{Z}=\begin{array}{cc}\begin{bmatrix}5 & 5\\ 4 & 1\end{bmatrix} & \begin{array}{c}10\\ 5\end{array}\\ \begin{array}{cc}9 & 6\end{array} & \end{array}\quad\text{and}\quad \mathbf{Z}^{\star}=\begin{array}{cc}\begin{bmatrix}3 & 1\\ 6 & 5\end{bmatrix} & \begin{array}{c}4\\ 11\end{array}\\ \begin{array}{cc}9 & 6\end{array} & \end{array}$$

so, that

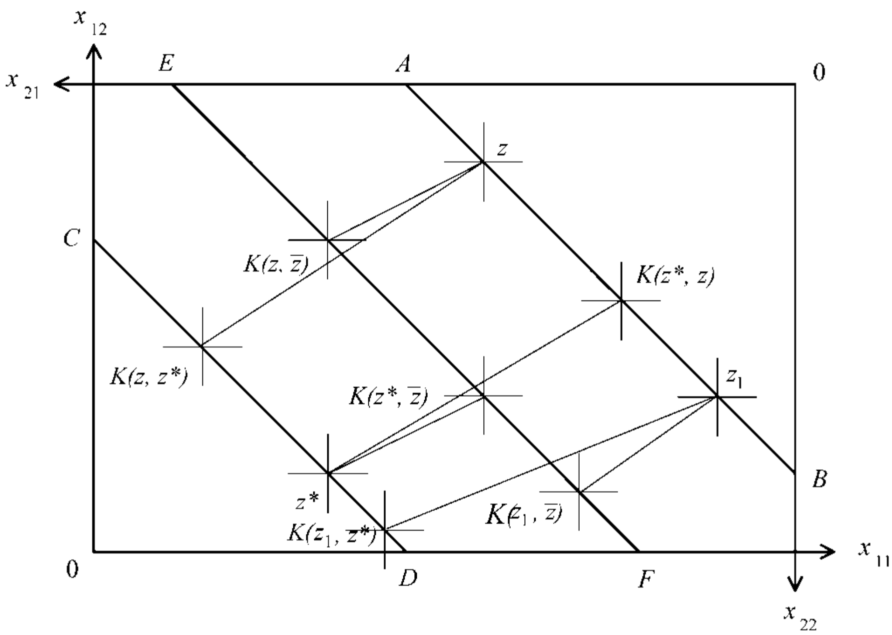

Figure 2. Edgeworth box for the ordinary biproportional projector and the mean biproportional projector.

$$K(\mathbf{Z},\mathbf{Z}^\star)=\begin{bmatrix}1.42 & 2.58\\ 7.58 & 3.42\end{bmatrix} \quad \text{and} \quad K(\mathbf{Z},\mathbf{Z}^\star)=\begin{bmatrix}6.74 & 3.26\\ 2.26 & 2.74\end{bmatrix}$$

This matrix is represented by the following Edgeworth box, where the sides of the box correspond to the column constraints of matrix $\mathbf{Z}$, the line AB corresponds to the row constraints of $\mathbf{Z}$ and the point z corresponds to $\mathbf{Z}$. With matrix $\mathbf{Z}^\star$, column constraints are the same, and row constraints become the line CD, while $\mathbf{Z}^\star$ is represented by point $z^\star$. The length of segment $\{K(z,z^\star), z^\star\}$, which corresponds to the variation found by the direct projection, is closed to the length of segment $\{K(z^\star,z), z\}$, which corresponds to the variation by the reverse projection. Consider another matrix $\mathbf{Z}_1$ with the same margins as $\mathbf{Z}$:

$$\mathbf{Z}_1=\begin{array}{cc} \begin{bmatrix}8 & 2\\ 1 & 4\end{bmatrix} & \begin{array}{c}10\\ 5\end{array}\\ \begin{array}{cc}9 & 6\end{array} & \end{array} \quad \text{so that} \quad K(\mathbf{Z}_1,\mathbf{Z}^\star)=\begin{bmatrix}3.74 & 0.26\\ 5.26 & 5.74\end{bmatrix}$$

Since $\mathbf{Z}$ and $\mathbf{Z}_1$ have the same margins, $K(z^\star,z_1)$ is confused with $K(z^\star,z)$. The segment $\{K(z_1,z^\star), z^\star\}$ is clearly shorter than the segment $\{K(z^\star,z_1), z_1\}$. This is because the projection of z_1 is near the limit of the box: the orthogonal projection of z_1, found by an additive method, is even outside the limit of the box (it corresponds to negative terms in the projected matrix) and the ordinary biproportional projection corrects it.

Consider the matrix $\bar{\mathbf{Z}}$ represented by the segment EF, that is

$$\bar{\mathbf{Z}}=\frac{1}{2}(\mathbf{Z}+\mathbf{Z}^\star)=\begin{array}{cc} \begin{bmatrix}4 & 3\\ 5 & 3\end{bmatrix} & \begin{array}{c}7\\ 8\end{array}\\ \begin{array}{cc}9 & 6\end{array} & \end{array}$$

Then,

$$K(\mathbf{Z},\bar{\mathbf{Z}})=\begin{bmatrix}3.00 & 4.00\\ 6.00 & 2.00\end{bmatrix}, \quad K(\mathbf{Z}^\star,\bar{\mathbf{Z}})=\begin{bmatrix}5.00 & 2.00\\ 4.00 & 4.00\end{bmatrix},$$

$$K(\mathbf{Z}_1,\bar{\mathbf{Z}})=\begin{bmatrix}6.25 & 0.75\\ 2.75 & 5.25\end{bmatrix}$$

The second variant is a generalization of the normalization of the column or row coefficients: for this reason, it constitutes a special category.

Biproportional bi-Markovian filter The biproportional bi-Markovian filter could appear as a particular case of the biproportional mean filter because the matrix $\bar{\mathbf{Z}}$ is replaced by the bi-Markovian matrix $\mathbf{1}^B$. Not only will the effect of the differential growth of sectors be removed without predetermining if the economy is demand or supply—driven as in the ordinary biproportional filter, and not only will the problem of the double result be removed as in the biproportional mean filter, but the effect of differential size of sectors will be removed: after projection all sectors in a column will have the same margin, i.e. the same size, and all sectors in a column will have the same margin.

In the *biproportional bi-Markovian filter*, both flow matrices are binormalized, that is to say normalized by columns and by rows simultaneously: each matrix $\mathbf{Z}$ and $\mathbf{Z}^{\star}$ is transformed into a bi-Markovian matrix, $\mathbf{Z}^{B}$ and $\mathbf{Z}^{\star B}$. A bi-Markovian or binormalized matrix is a matrix where all margins, both columns and rows, are equal to 1: this is possible only for square matrices. Naturally, any other number can be chosen; the important thing is that all margins must be equal:

$$\mathbf{1}^{B}=\begin{array}{c c}\left[\begin{array}{ccc} & & \\ & & \\ & & \end{array}\right] & \begin{array}{c}1\\ \cdots\\ 1\end{array}\\ \begin{array}{ccc}1 & \cdots & 1\end{array} & \end{array}$$

As said before, there are many tools to project a matrix and the results may vary depending on the tool chosen to perform it; here, biproportion is chosen to avoid negative terms in the projected matrics. Two biproportional projections are performed: $\mathbf{Z}^{B}=K(\mathbf{Z},\mathbf{1}^{B})$ and $\mathbf{Z}^{\star B}=K(\mathbf{Z}^{\star},\mathbf{1}^{B})$. For example, for $K(\mathbf{Z},\mathbf{1}^{B})=\mathbf{PZQ}$ we have $p_i=1/\Sigma_{j=1}^{m} z_{ij}\,q_j$, for all i, and $q_j=1/\Sigma_{i=1}^{n} p_i\,z_{ij}$, for all j.

The same reasoning holds for $\mathbf{Z}^{\star}$. Then, $\mathbf{Z}^{B}$ is compared to $\mathbf{Z}^{\star B}$ calculating the Frobenius norm of the difference matrix $\mathbf{Z}^{\star B}-\mathbf{Z}^{B}$. The rest of the method is similar to the biproportional mean filter, replacing matrix $\bar{\mathbf{Z}}$ by the bi-Markovian matrix $\mathbf{1}^{B}$ when calculating relative variations. For column j, row i, and overall, we—respectively—have

$$\sigma_j^{C}=\sqrt{\Sigma_i\,\theta_{ij}^{2}},\ \ \sigma_i^{R}=\sqrt{\Sigma_j\,\theta_{ij}^{2}},\ \text{ and }\ \sigma=\sqrt{\Sigma_i\,\Sigma_j\,\theta_{ij}^{2}},$$
$$\text{with }\ \theta_{ij}=z_{ij}^{\star B}-z_{ij}^{B}$$

Again both sliding and cumulative analyses can be conducted.

For rectangular matrices of dimension (n, m), the margins of the n rows are equal to μ and the margins of the m columns are equal to λ: $\mathbf{1}^{B}\mathbf{s}=\mu\mathbf{s}$ and $\mathbf{s}'\mathbf{1}^{B}=\lambda\mathbf{s}'$. For example, one can take:

$$\mathbf{1}^{B}=\begin{array}{c c}\left[\begin{array}{ccc} & & \\ & & \\ & & \end{array}\right] & \begin{array}{c}1\\ \cdots\\ 1\end{array}\\ \begin{array}{ccc}\frac{n}{m} & \cdots & \frac{n}{m}\end{array} & \end{array} \tag{9}$$

or,

$$\mathbf{1}^{B}=\begin{array}{c c}\left[\begin{array}{ccc} & & \\ & & \\ & & \end{array}\right] & \begin{array}{c}m\\ \cdots\\ m\end{array}\\ \begin{array}{ccc}n & \cdots & n\end{array} & \end{array} \tag{10}$$

For the matrix $\mathbf{1}^{B}$ defined in equation (9) we have $p_i=m/\Sigma_{j=1}^{m} z_{ij}\,q_j$, for all i, and $q_j=n/\Sigma_{i=1}^{n} p_i\,z_{ij}$, for all j. One divides by m for columns and by n for rows when calculating relative variations. That is, for column j, row i, and overall, we—respectively—have

$$\sigma_j^C=\frac{1}{n}\sqrt{\Sigma_i\,\theta_{ij}^2},\ \sigma_i^R=\frac{1}{m}\sqrt{\Sigma_j\,\theta_{ij}^2},\ \text{and}\ \sigma=\frac{1}{nm}\sqrt{\Sigma_i\,\Sigma_j\,\theta_{ij}^2},$$

with $\theta_{ij}=z_{ij}^{\star B}-z_{ij}^{B}$

Note that we should be indifferent between forms (9) or (10) for matrices $\mathbf{1}^B$ since they are equivalent. A separable modification of $\mathbf{Z}$ (or $\mathbf{Z}^\star$) is ineffective (de Mesnard, 1994), i.e. if $\mathbf{Z}$ is replaced by $\tilde{\mathbf{Z}}=\hat{\psi}\mathbf{Z}\hat{\omega}$, then $K(\tilde{\mathbf{Z}},\mathbf{1}^B)=K(\mathbf{Z},\mathbf{1}^B)$.

This method might strongly distort the underlying structure of the data, but it is only a generalization of the normalization procedure applied to column or row coefficient matrices, discussed earlier. On the other hand, not only is the effect of the differential sectoral growth removed without any predetermination of the economy's orientation—as required by an ordinary biproportional filter—and the problem of the double result removed—as required in the biproportional mean filter—but the effect of differential sectoral size is removed. As mentioned previously, this is because post-projection columns all total to the same number.

For example, take

$$\mathbf{Z}=\begin{array}{c}\begin{bmatrix}5&5&6\\4&1&3\\3&4&5\end{bmatrix}\begin{array}{c}16\\8\\12\end{array}\\ \begin{array}{ccc}12&10&14\end{array}\end{array}\quad\text{and}\quad \mathbf{Z}^\star=\begin{array}{c}\begin{bmatrix}2&3&8\\6&1&4\\1&2&6\end{bmatrix}\begin{array}{c}13\\11\\9\end{array}\\ \begin{array}{ccc}9&6&18\end{array}\end{array}$$

Ordinary biproportional filter:

$$K(\mathbf{Z},\mathbf{Z}^\star)=\begin{bmatrix}3.124&2.920&6.956\\4.190&0.979&5.831\\1.686&2.100&5.213\end{bmatrix}\quad\text{and}$$

$$K(\mathbf{Z}^\star,\mathbf{Z})=\begin{bmatrix}4.056&5.228&6.716\\5.637&0.807&1.555\\2.307&3.964&5.729\end{bmatrix}$$

Biproportional mean filter:

$$\bar{\mathbf{Z}}=\begin{array}{c}\begin{bmatrix}3.5&4&7\\5&1&3.5\\2&3&5.5\end{bmatrix}\begin{array}{c}14.5\\9.5\\10.5\end{array}\\ \begin{array}{ccc}10.5&8&16\end{array}\end{array}$$

$$K(\mathbf{Z},\bar{\mathbf{Z}})=\begin{bmatrix}4.011&3.953&6.536\\4.195&1.033&4.272\\2.294&3.014&5.192\end{bmatrix}\quad\text{and}\quad K(\bar{\mathbf{Z}},\mathbf{Z})=\begin{bmatrix}2.925&4.119&7.456\\6.008&0.940&2.552\\1.567&2.942&5.991\end{bmatrix}$$

Biproportional bi-Markovian filter:

$$\mathbf{Z}^M=\begin{array}{c}\begin{bmatrix}0.284&0.401&0.314\\0.489&0.173&0.338\\0.226&0.426&0.348\end{bmatrix}\begin{array}{c}1\\1\\1\end{array}\\ \begin{array}{ccc}1&1&1\end{array}\end{array}\quad\text{and}$$

Table 1. Example for columns

Columns	Biprop. Direct (%)	Biprop. Reverse (%)	Average direct + reverse (%)	Biprop. Mean (%)	Bi-Mark. (%)
1	24.87	16.77	20.82	21.29	19.37
2	2.17	3.01	2.59	2.55	3.08
3	12.50	12.64	12.57	13.17	16.80
Overall	9.63	7.49	8.56	8.92	8.61

Table 2. Example for rows

Rows	Biprop. Direct (%)	Biprop. Reverse (%)	Average direct + reverse (%)	Biprop. Mean (%)	Bi-Mark (%)
1	11.82	7.54	9.68	9.88	10.54
2	23.41	27.39	25.40	26.32	21.02
3	11.65	8.39	10.02	10.32	10.67
Overall	9.63	7.49	8.56	8.92	8.61

$$\mathbf{Z}^{\star M} = \begin{array}{cc} \begin{bmatrix} 0.201 & 0.424 & 0.375 \\ 0.647 & 0.151 & 0.201 \\ 0.182 & 0.425 & 0.424 \end{bmatrix} & \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \\ \begin{array}{ccc} \quad 1 & \quad\quad 1 & \quad\quad 1 \end{array} & \end{array}$$

The results are synthesized in Tables 1 and 2. As it can be seen, the bi-Markovian filter may heavily distort the data. For example, it might seem to violate common sense if z_{13} is twice z_{23} when z_{13}^M is approximately equal toz_{23}^M. But one must keep in mind that (1) both $\mathbf{Z}$ and $\mathbf{Z}^\star$ are projected simultaneously and eventually distorted and (2) it is almost the case for $K(\mathbf{Z},\mathbf{Z}^\star)$ and $K(\mathbf{Z},\bar{\mathbf{Z}})$ which are very close to each other. Remember also that the closer the points ($\mathbf{Z}$ or $\mathbf{Z}^\star$) are to the limits of the Edgeworth box, the more the data are distorted. This distortion is the cost of avoiding negative points.

3.2. The Bicausative Method

The bicausative approach uses causative matrices but abandons their directed character; this method was simultaneously 'conceived' and 'killed' by de Mesnard (2000). It nonetheless remains interesting to investigate because its drawbacks are not all that simple. The bicausative method starts from the *double causative* method proposed by Jackson *et al.* (1990, p. 268): $\mathbf{\Pi}^{\star M} = \mathbf{C}_L \mathbf{\Pi}^M \mathbf{C}_R$. The double causative method could be identified as being undirected since it replaces Markovian matrices with flow matrices, but it does not permit the estimation of $\mathbf{C}$ matrices and is quite computationally intensive, requiring estimation of $2n^2$ parameters. Note that it remains oriented because matrices $\mathbf{\Pi}^M$ and $\mathbf{\Pi}^{\star M}$ derive from technology coefficient matrices.

Here, the two diagonal matrices, $\mathbf{L}$ and $\mathbf{R}$, replace $\mathbf{C}_L$ and $\mathbf{C}_R$, so that the number of parameters falls to $2n$. Change between flow matrices is assumed to follow the form $\mathbf{LZR}$, which is *not* biproportional because $\mathbf{L}$ and $\mathbf{R}$ are diagonal matrices. Since **LZR** is generally not equal to $\mathbf{Z}^\star$, the choice is made to find the matrices $\mathbf{L}(n,n)$ and $\mathbf{R}(m,m)$ by minimizing the sum of squares of the differences between $z^\star_{ij}$ and $l_i z_{ij} r_j$. That is, minimize SS subject to equation (5) with $SS=\Sigma_{i=1}^{n}\Sigma_{j=1}^{m}(z^\star_{ij}-l_i z_{ij} r_j)^2$. For any i and j, this yields

$$l_i=\Sigma_{j=1}^{m} z^\star_{ij} z_{ij} r_j/\Sigma_{j=1}^{m} z_{ij}^2 r_j^2 \quad \text{and} \quad r_j=\Sigma_{i=1}^{n} z^\star_{ij} z_{ij} l_i/\Sigma_{i=1}^{n} z_{ij}^2 l_i^2$$

which is solved iteratively and is denoted $LS(\mathbf{Z},\mathbf{Z}^\star)=\mathbf{LZR}$. The diagonal matrix **R** affects all terms of a column equally, and the diagonal matrix **L** affects all terms in a row equally.

Note that in reverse form, from $\mathbf{Z}^\star$ to $\mathbf{Z}$, one obtains $LS(\mathbf{Z}^\star,\mathbf{Z})=\tilde{\mathbf{L}}^\star\mathbf{Z}^\star\tilde{\mathbf{R}}^\star$ with, $\tilde{l}^\star_i=\Sigma_{j=1}^{m} z^\star_{ij} z_{ij} \tilde{r}^\star_j/\Sigma_{j=1}^{m}(z^\star_{ij})^2(\tilde{r}^\star_j)^2$ for all i, and $\tilde{r}^\star_j=\Sigma_{i=1}^{n} z^\star_{ij} z_{ij}\tilde{l}^\star_i/\Sigma_{i=1}^{n}(z^\star_{ij})^2(\tilde{l}^\star_i)^2$ for all j.

However, unlike the case of causative matrices, but as for the biproportional filter, results for direct and reverse forms are not the same. To compare the results of the direct and reverse methods, one should compute $LS(\mathbf{Z},\mathbf{Z}^\star)=\mathbf{LZR}$ and $(\tilde{\mathbf{L}}^\star)^{-1}LS(\mathbf{Z}^\star,\mathbf{Z})(\tilde{\mathbf{R}}^\star)^{-1}=\mathbf{Z}^\star$ for year 1—while $LS(\mathbf{Z}^\star,\mathbf{Z})=\tilde{\mathbf{L}}^\star\mathbf{Z}^\star\tilde{\mathbf{R}}^\star$ is calculated for year 0—so matrix **R** should be compared to matrix $(\tilde{\mathbf{R}}^\star)^{-1}$ and matrix $(\tilde{\mathbf{L}}^\star)^{-1}$ should be compared to matrix **L**.

As mentioned above, the bicausative-matrix method has some drawbacks. First, the estimators **L** and **R** are not 'identified' (in the econometric sense) because they are specified at a coefficient of proportionality. That is, there remains one degree of freedom when **L** and **R** are derived; in other terms, they are hyperbolically homogeneous. Initialization at $r_j(0)=\lambda$, for all j, yields $l_i^\infty=\bar{l}_i^\infty/\lambda$, for all i, and $r_j^\infty=\lambda\,\bar{r}_j^\infty$, for all j, where $\bar{l}_i^\infty$ and $\bar{r}_j^\infty$ denote the values obtained at equilibrium after initialization at $r_j(0)=1$, for all j, and the product $l_i^\infty x_{ij} r_j^\infty$ remains unchanged: $l_i^\infty x_{ij} r_j^\infty=\bar{l}_i^\infty x_{ij}\bar{r}_j^\infty$, for all i and j. Similar results are obtained if one initializes $l_i(0)=\lambda$ instead of $l_i(0)=1$ for all i. The results depend on the initial values, i.e. $\exists j_1, j_2$ with $r_{j_1}(0)\neq r_{j_2}(0)$, for all j. This is problematic because non-identification concerns the coefficients of **L** and **R** that are being sought. This is not the case with other methods such as the biproportional filter, where only the identified products **PZQ** are sought. Hence, problems of convergence of the iterative algorithm arise due to local equilibria.

Moreover, interpretation of the bias between $\mathbf{Z}^\star$ and $LS(\mathbf{Z},\mathbf{Z}^\star)$ is problematic: $\mathbf{Z}^\star-LS(\mathbf{Z},\mathbf{Z}^\star)$can be slightly different from **0** but the quality of the analysis depends on the size of this bias. One can discuss how this manner of calculating bias mixes two phenomena (de Mesnard, 2000)—the bias caused by the differences in the agent-sector size and the bias caused by the true structural effect because both matrices **Z** and $\mathbf{Z}^\star$ do not have the same margins. Thus, one could control for the differences in the margins by giving the same margins to both matrices, for example by performing a biproportional adjustment: $K[LS(\mathbf{Z},\mathbf{Z}^\star),\mathbf{Z}^\star]$.

The bias of the bicausative method is, however, similar to the difference matrix of the biproportional filtering method. Thus, in computing the structural bias of the bicausative method, one must compute the same structural change found by the biproportional filtering method. This can be proven. Since **L** and **R** are diagonal in the expression $LS(\mathbf{Z},\mathbf{Z}^\star)=\mathbf{LZR}$, one has (de Mesnard, 1994): $K[LS(\mathbf{Z},\mathbf{Z}^\star),\mathbf{Z}^\star]=K[\mathbf{LZR},\mathbf{Z}^\star]=K[\mathbf{Z},\mathbf{Z}^\star]$, and the bias is: $\mathbf{Z}^\star-K[LS(\mathbf{Z},\mathbf{Z}^\star),\mathbf{Z}^\star]=\mathbf{Z}^\star-K[\mathbf{Z},\mathbf{Z}^\star]$, which is the solution obtained with an ordinary biproportional filter.

Note that in the biproportional filtering method, the difference $\mathbf{Z}^\star-K(\mathbf{Z},\mathbf{Z}^\star)$ is not biased since it is the subject of the analysis.

4. A Simple Application to France 1980–1997

The application provided here only illustrates how results among the various methods discussed in this paper can differ. To keep things simple, only the exchange matrix is used to compute structural change. However, it remains true that, to have complete results, it would be necessary to do two more sets of computations: first, with the household sector and then with the household sector and trade with the rest of the world.

This application is based on two input–output tables for France: a benchmark table for 1980 and a preliminary table for 1997 (INSEE, various years) both aggregated to nine major sectors. The data are in 1980 monetary units deflated by INSEE at a detailed industry level. Only the intermediate block of tables is used. In this simple application, in order to obtain a square table that can be analysed by the causative method, the Trade and the Non-Marketable Services sectors have been removed (they have only a column but not a row in the French accounting system: that is, they are 'industries' without a product). In order to compute technology coefficients and allocation coefficients, imports, customs duty, commercial margins and taxes form the value added vector and the gross formation of fixed capital, stock variations and exports form the final demand vector. The French data used in my tests are presented in Tables 3 and 4.

The results of the different methods are not always directly comparable. For example, the results of the causative method are not comparable to those of other methods: they are not relative percentage differences. For the causative method, the first column indicates the value of the diagonal terms of the left causative matrix, the second column the sum of the off-diagonal terms of each row. The results of the bicausative method are not provided because they are the same as those for the biproportional filter, as discussed above.

For other methods, only the percent difference can be compared, especially in the case of comparisons between results for technology or allocation coefficients and other methods. Also remember, with directed methods the results for columns are not comparable to those for rows, while they can be compared with undirected methods. There are two ways to project with an ordinary biproportional filter. Hence, in order to produce a single measure one is obliged to combine the two results, direct and reverse, by averaging the two. The causative matrix and various matrices obtained by biproportion are shown in Tables 5–12.

Tables 13 and 14 and Figures 3 and 4, present the results comparing technology or allocation coefficients in a first column. In the second column, they display the results of the causative method. Their third and fourth columns present the results for the ordinary biproportional filter for direct and reverse computations, while the fourth column indicates the average of columns three and four. The sixth column of each table contains the results of the biproportional mean filter, and the last column gives the results of the biproportional bi-Markovian filter.

First, note that, except for the comparison of normalized coefficients and for the causative method (where overall results are nonsensical), overall results for columns and rows are the same: the biproportional filters are not directed. The direct ordinary biproportional filter gives overall results higher than the reverse ordinary biproportional filter: 8.8% versus 1.7%, with an average of 5.2%. The biproportional mean filter is higher at 6.9%, while the bi-Markovian filter yielded values at 7.6%. These results can be compared to those of normalized coefficients: 10.8% for columns and 10.0% for rows.

Table 3. Input–output table for 1980

1980	Agriculture . . .	Energy	Minerals	Manu-facturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services	Final Demand	Output
Agriculture . . .	270 732	196	63	24 955	0	25 520	233	2 305	0	468 699	792 703
Energy	18 603	167 784	23 722	48 846	8 091	6 285	28 118	7 129	877	221 557	531 012
Minerals	1 962	2 303	83 346	72 775	60 063	1 880	493	810	0	71 271	294 903
Manufacturing	50 722	13 485	10 610	439 871	74 100	11 480	13 867	59 304	3 437	1 136 942	1 813 818
Buildings	1 033	6 042	381	2 050	231	406	627	2 917	5 891	431 123	450 701
Trade	831	263	1 401	2 627	813	3 524	1 866	8 703	823	136 133	156 984
Transport and Telecomm.	5 632	5 985	10 125	36 106	13 034	4 026	24 126	21 715	4 407	143 731	268 887
Services	18 792	12 857	9 866	83 142	48 570	12 646	15 907	103 334	12 802	476 609	794 525
Financial Services	1 038	568	829	5 826	5 940	790	636	1 796	3 812	115 447	136 682
Added-value	423 358	321 529	154 560	1 097 620	239 859	90 427	183 014	586 512	104 633	3 201 512	5 240 215
Output	792 703	531 012	294 903	1 813 818	450 701	156 984	268 887	794 525	136 682	5 240 215	

Table 4. Input–output table for 1997

1997	Agriculture . . .	Energy	Minerals	Manu – facturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services	Final Demand	Output
Agriculture . . .	322 195	82	18	26 579	0	29 155	262	3 793	0	652 127	1 034 211
Energy	21 967	131 572	17 340	57 330	9 039	7 886	37 493	11 455	1 511	278 729	574 322
Minerals	1 897	13 704	73 056	75 138	52 009	2 019	294	1 147	0	86 226	305 490
Manufacturing	65 350	13 689	9 949	643 225	77 183	14 998	22 418	110 662	3 360	1 876 975	2 837 809
Buildings	1 308	7 462	311	2 567	205	450	779	5 147	11 980	435 214	465 423
Trade	902	283	908	2 756	595	3 834	2 524	12 399	420	168 423	193 044
Transport and Telecomm.	8 304	7 026	9 786	66 975	15 001	7 352	53 145	59 148	8 055	253 161	487 953
Services	34 278	26 771	13 246	160 772	65 040	21 598	25 851	224 065	34 205	838 362	1 444 188
Financial Services	3 168	1 791	1 616	18 459	12 291	1 341	2 107	5 507	987 446	145 990	1 179 716
Added-value	574 842	371 942	179 260	1 784 008	234 060	104 411	343 080	1 010 865	132 739	4 735 207	8 522 156
Output	1 034 211	574 322	305 490	2 837 809	465 423	193 044	487 953	1 444 188	1 179 716	8522156	

Table 5. Causative matrix 1980/1997, based on inverse matrices

Matrix C based on inverse matrices	Agriculture	Energy	Minerals	Manu-facturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services
Agriculture	0.978020	−0.000188	−0.000588	−0.007009	−0.000213	−0.032388	−0.000909	−0.001059	−0.002325
Energy	−0.004692	0.928007	−0.035239	−0.011832	−0.005093	−0.003089	−0.039038	−0.000944	−0.013546
Minerals	−0.001302	0.023335	1.004467	−0.022213	−0.053228	−0.003719	−0.003390	−0.000441	−0.006258
Manufacturing	−0.002428	−0.000301	−0.004832	0.988599	−0.017293	0.001931	−0.007881	−0.000600	−0.045208
Buildings	0.000198	0.001970	−0.000297	0.000086	0.914776	0.000280	−0.000781	0.000314	−0.027662
Trade	−0.000040	0.000308	−0.001498	−0.000352	−0.000421	0.952228	−0.001363	−0.002084	−0.006364
Transport and Telecommunications	0.001252	0.002198	−0.002823	0.003449	0.000187	0.013095	1.033929	0.013584	−0.033719
Services	0.012133	0.026504	0.012507	0.014496	0.031391	0.036846	−0.003537	0.971099	−0.074636
Financial Services	0.016858	0.018167	0.028304	0.034777	0.129894	0.034817	0.022970	0.020131	1.209719

Table 6. Causative matrix 1980/1997, based on direct matrices

Matrix C based on direct matrices	Agriculture	Energy	Minerals	Manufacturing	Buildings	Trade	Buildings	Services	Financial Services
Agriculture	0.958951	−0.011314	0.009073	−0.022090	0.210353	−0.906155	0.047631	0.077316	−0.421638
Energy	0.009650	0.826050	−0.002153	−0.003934	−0.384087	0.158514	−0.052877	−0.000919	0.449191
Minerals	0.003914	0.042692	0.979570	−0.033269	1.035944	0.176445	−0.046811	0.007051	−1.588412
Manufacturing	0.010407	0.018109	0.009308	1.006449	−0.246185	0.134790	−0.003001	−0.063338	−0.317095
Buildings	0.000078	0.041925	−0.004515	−0.000760	0.109691	−0.018828	−0.048532	0.032454	−0.131094
Trade	0.000586	0.006263	−0.002530	0.000722	−0.145419	0.839969	−0.001829	−0.010946	0.083538
Transport and Telecommunications	−0.001645	0.023506	−0.030048	0.006167	−0.714651	0.202697	1.281932	0.010606	−0.403282
Services	0.017805	0.180426	0.023596	0.042680	−2.619437	0.599590	−0.311563	1.077434	0.855066
Financial Services	0.000253	−0.127658	0.017698	0.004034	3.753791	−0.187022	0.135049	−0.129658	2.473725

Table 7. $K(\mathbf{Z}, \mathbf{Z}^{\star})$

K(1980, 1997)	Agriculture	Energy	Minerals	Manufacturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services
Agriculture	318 681.59	210.33	50.96	30 271.13	0.00	28 527.08	350.48	3 992.43	0.00
Energy	18 057.72	148 475.97	15 822.34	48 861.03	4 855.38	5 793.55	34 878.55	10 182.58	8 665.87
Minerals	2 429.58	2 599.87	70 918.02	92 868.44	45 981.21	2 210.80	780.14	1 475.93	0.00
Manufacturing	67 661.55	16 399.18	9 725.24	604 678.58	61 108.96	14 542.76	23 638.62	116 406.95	46 672.16
Buildings	418.84	2 233.33	106.15	856.55	57.90	156.33	324.87	1 740.33	24 314.69
Trade	636.23	183.57	737.04	2 072.65	384.81	2 562.17	1 825.65	9 804.63	6 414.25
Transport and Telecommunications	7 565.83	7 329.65	9 346.05	49 983.50	10 824.61	5 136.02	41 416.47	42 924.26	60 265.60
Services	24 341.50	15 182.38	8 781.22	110 980.98	38 894.09	15 555.60	26 330.36	196 954.98	168 804.89
Financial Services	19 576.16	9 765.72	10 742.97	113 228.13	69 256.05	14 148.70	15 327.85	49 840.90	731 839.53

Table 8. $K(\mathbf{Z}^{\star}, \mathbf{Z})$

K(1997, 1980)	Agriculture ...	Energy	Minerals	Manufacturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services
Agriculture ...	273 889.90	84.50	21.83	22 082.10	0.00	25 267.38	190.00	2 468.30	0.00
Energy	20 982.61	152 345.43	23 633.24	53 519.88	11 860.75	7 679.54	30 552.36	8 376.08	505.10
Minerals	1 566.47	13 717.60	86 078.54	60 639.78	58 997.72	1 699.73	207.11	725.06	0.00
Manufacturing	46 507.00	11 809.23	10 102.70	447 383.68	75 456.67	10 881.68	13 610.55	60 287.65	836.83
Buildings	1 121.01	7 752.34	380.32	2 150.16	241.36	393.19	569.57	3 376.85	3 593.21
Trade	864.63	328.84	1 241.92	2 581.94	783.51	3 746.83	2 064.04	9 098.41	140.90
Transport and Telecommunications	4 772.20	4 894.60	8 024.58	37 617.41	11 842.82	4 307.52	26 055.54	26 021.29	1 620.03
Services	19 471.11	18 433.89	10 736.07	89 254.48	50 752.67	12 507.73	12 527.33	97 433.03	6 799.70
Financial Services	170.09	116.56	123.80	968.58	906.51	73.40	96.51	226.34	18 553.23

Table 9. $K\lfloor \mathbf{Z}, \text{mean}(\mathbf{Z}, \mathbf{Z}^\star) \rfloor$

K(1997, 1980)	Agriculture	Energy	Minerals	Manufacturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services
Agriculture	443 611.27	252.94	69.75	34 575.44	0.00	31 133.74	372.38	4 125.98	0.00
Energy	22 559.34	160 249.48	19 436.02	50 086.37	6 054.57	5 674.61	33 257.97	9 444.19	6 744.94
Minerals	2 691.08	2 487.85	77 236.96	84 402.84	50 836.16	1 919.88	659.54	1 213.68	0.00
Manufacturing	69 824.67	14 620.63	9 868.26	512 017.82	62 946.10	11 766.36	18 619.28	89 184.53	30 007.34
Buildings	520.68	2 398.60	129.75	873.72	71.85	152.37	308.25	1 606.21	18 832.06
Trade	791.24	197.23	901.28	2 115.02	477.68	2 498.22	1 732.96	9 052.53	4 969.85
Transport and Telecommunications	7 566.51	6 332.86	9 190.53	41 016.57	10 805.59	4 027.12	31 614.47	31 870.26	37 550.11
Services	24 458.55	13 179.53	8 675.84	91 500.90	39 008.92	12 254.58	20 193.61	146 924.40	105 674.67
Financial Services	14 414.66	6 212.38	7 778.11	68 410.81	50 901.62	8 168.12	8 614.54	27 246.23	335 734.03

Table 10. $K\lfloor \mathbf{Z}, \text{mean}(\mathbf{Z}, \mathbf{Z}^\star) \rfloor$

K(1997, mean)	Agriculture	Energy	Minerals	Manufacturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services
Agriculture	447 695.56	102.47	24.61	30 584.01	0.00	31 405.51	284.76	4 044.59	0.00
Energy	26 607.69	143 317.39	20 668.93	57 505.61	10 530.94	7 404.95	35 521.77	10 647.78	1 302.44
Minerals	2 089.59	13 575.02	79 192.21	68 540.24	55 103.96	1 724.09	253.31	969.58	0.00
Manufacturing	65 996.65	12 432.17	9 887.54	537 936.50	74 973.55	11 741.90	17 708.47	85 763.47	2 414.75
Buildings	1 352.14	6 936.93	316.38	2 197.51	203.83	360.62	629.88	4 083.15	8 813.06
Trade	1 042.86	294.24	1 033.09	2 638.69	661.68	3 436.36	2 282.52	11 001.00	345.56
Transport and Telecommunications	7 762.20	5 906.15	9 001.92	51 844.42	13 487.39	5 327.60	38 856.87	42 429.23	5 358.21
Services	31 642.52	22 223.84	12 032.97	122 901.65	57 749.23	15 456.03	18 665.55	158 729.30	22 469.93
Financial Services	2 248.79	1 143.30	1 128.85	10 850.86	8 391.93	737.94	1 169.87	2 999.90	498 809.05

Table 11. 1980 bi-Markovian $K(\mathbf{Z}, 1^M)$

K(1997, mean)	Agriculture	Energy	Minerals	Manufacturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services
Agriculture	0.716452	0.000834	0.000347	0.036431	0.000000	0.236817	0.001885	0.007235	0.000000
Energy	0.037608	0.545333	0.099677	0.054474	0.022610	0.044555	0.173760	0.017095	0.004888
Minerals	0.006306	0.011901	0.556792	0.129035	0.266846	0.021189	0.004844	0.003088	0.000000
Manufacturing	0.084256	0.036014	0.036632	0.403081	0.170143	0.066870	0.070413	0.116849	0.015742
Buildings	0.028669	0.269591	0.021978	0.031386	0.008862	0.039512	0.053192	0.096026	0.450786
Trade	0.022224	0.011308	0.077876	0.038756	0.030054	0.330478	0.152544	0.276074	0.060686
Transport and Telecommunications	0.028159	0.048109	0.105219	0.099586	0.090079	0.070586	0.368727	0.128781	0.060753
Services	0.044341	0.048773	0.048386	0.108222	0.158413	0.104634	0.114732	0.289210	0.083287
Financial Services	0.031984	0.028138	0.053093	0.099030	0.252994	0.085359	0.059904	0.065641	0.323858

Table 12. 1997 bi-Markovian $K(\mathbf{Z}^\star, 1^M)$

K(1997, 1980)	Agriculture	Energy	Minerals	Manufacturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services
Agriculture	0.720838	0.000270	0.000134	0.034507	0.000000	0.235614	0.001456	0.007180	0.000000
Energy	0.047732	0.421356	0.125007	0.072290	0.047219	0.061896	0.202379	0.021060	0.001060
Minerals	0.004291	0.045684	0.548240	0.098624	0.282818	0.016496	0.001652	0.002195	0.000000
Manufacturing	0.074098	0.022876	0.037427	0.423232	0.210399	0.061428	0.063144	0.106166	0.001230
Buildings	0.048258	0.405756	0.038069	0.054960	0.018184	0.059972	0.071396	0.160674	0.142732
Trade	0.023670	0.010945	0.079055	0.041969	0.037538	0.363426	0.164535	0.275302	0.003559
Transport and Telecommunications	0.024620	0.030702	0.096263	0.115233	0.106928	0.078738	0.391423	0.148380	0.007712
Services	0.048265	0.055555	0.061880	0.131365	0.220170	0.109850	0.090421	0.266942	0.015553
Financial Services	0.008228	0.006855	0.013925	0.027820	0.076744	0.012580	0.013594	0.012101	0.828153

Table 13. Comparison of methods for France, column vectors, in %

Column sectors	Norm. tech. coeff.	Causat. diag direct	Causat. off direct	Biprop. direct	Biprop. reverse	Average direct + reverse	Biprop. mean	Bi-Mark.
Agriculture	4.025	0.959	−1.017	4.368	1.614	2.991	2.690	3.474
Energy	17.647	0.826	0.173	12.517	9.629	11.073	11.313	18.981
Minerals	5.025	0.980	−0.402	8.325	2.611	5.468	5.856	5.265
Manufacturing	5.300	1.006	−0.457	11.082	2.407	6.745	8.313	8.983
Buildings	8.449	0.110	−0.129	28.205	3.310	15.757	21.918	19.439
Trade	8.081	0.840	−0.070	16.434	2.629	9.532	10.873	8.514
Transport and Telecomm.	11.076	1.282	−0.907	12.383	5.406	8.894	9.365	6.788
Services	5.090	1.077	−1.212	12.675	3.685	8.180	9.155	8.976
Financial Services	93.409	2.474	3.466	28.381	51.625	40.003	34.906	60.008
Overall	10.768			8.794	1.670	5.232	6.886	7.584

Note: the results of the causative method are not at all comparable with those of other methods.

Table 14. Comparison of methods for France, row vectors, in %

Row sectors	Norm. alloc. coeff.	Biprop. direct	Biprop. reverse	Average direct + reverse	Biprop. mean	Bi-Mark.
Agriculture	1.134	1.346	1.321	1.333	1.113	0.499
Energy	11.340	7.229	5.500	6.364	6.528	13.486
Minerals	7.465	9.984	7.568	8.776	9.000	4.931
Manufacturing	4.597	6.303	1.380	3.841	4.900	5.206
Buildings	11.860	46.245	14.837	30.541	45.759	34.591
Trade	9.812	27.363	4.137	15.750	22.753	6.755
Transport and Telecomm.	10.421	24.975	5.063	15.019	20.238	6.858
Services	6.726	24.652	3.938	14.295	20.028	10.146
Financial Services	86.703	27.410	77.517	52.463	34.245	55.073
Overall	10.009	8.794	1.670	5.232	6.886	7.584

Note: no results for row coefficients with the causative method.

It may not be appropriate to compare overall change, however. It could be better to examine structure at the sectoral level. Here, all methods converge toward similar results, although certain differences do crop up. Normalized coefficients give larger overall results due to Financial Services, and reverse projections yield lower results for many sectors. On the other hand, *Financial Services* may have created anomalies.[7] Interestingly, the bi-Markovian filter did not yield the most extreme results. Moreover, for column structures it provided results similar to those from the biproportional mean filter and the average ordinary biproportional filter. For rows, the biproportional mean filter and the average ordinary biproportional filter yielded similar results but those of the bi-Markovian filter diverged for many sectors (*Energy*, *Trade*, *Transport and Telecommunications* and *Services*). The causative method remained difficult to compare to other methods.[8]

5. Conclusions

It is interesting to learn how much exchange structures have changed over time and how two comparable exchange structures differ. Such an analysis can be

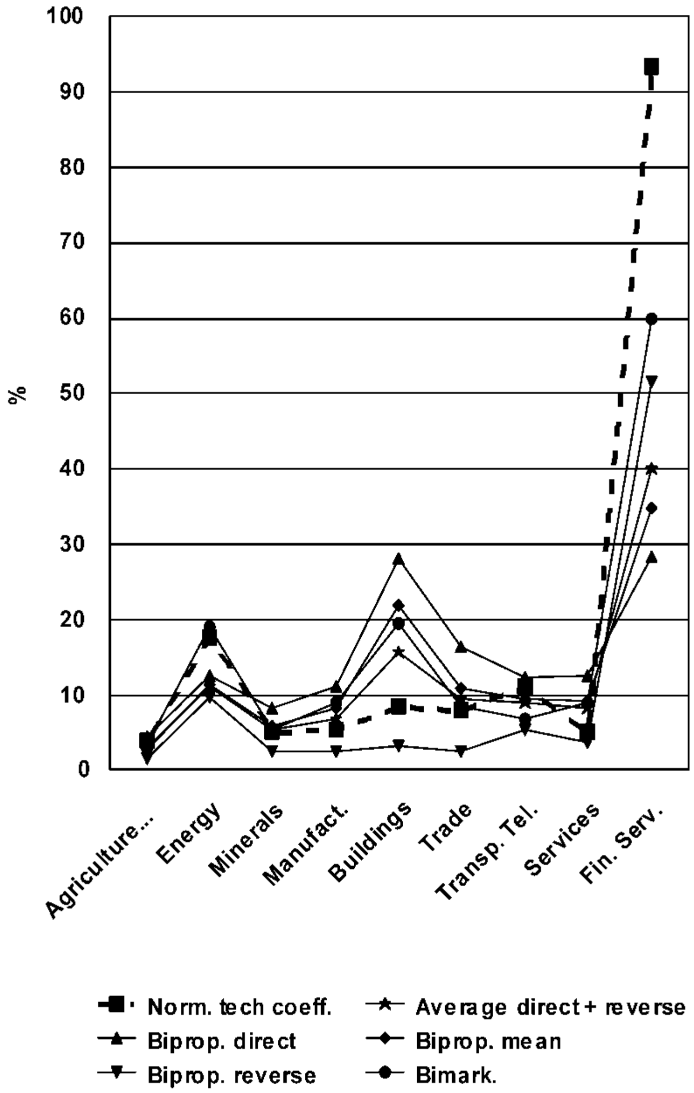

Figure 3. Comparison of methods for France, column vectors, in % (the results of the causative method are not figured).

performed either with directed or undirected methods. This paper shows that the results from such an analysis can depend rather heavily upon the method of analysis that is used. Further research could try to develop a set of axioms to discover which method is best from a theoretical perspective. It is at least clear from results presented here that the biproportional mean filter and the bi-Markovian filter yield only one set of results. In contrast, the ordinary biproportional filter requires two different sets of comparisons. Biproportional methods are superior to the simple comparison of coefficients since they combine the results for column and row coefficients.

Which method is best? This question is hard to answer. What is certain is that it is always preferable to avoid choosing one that cannot verify or disprove the

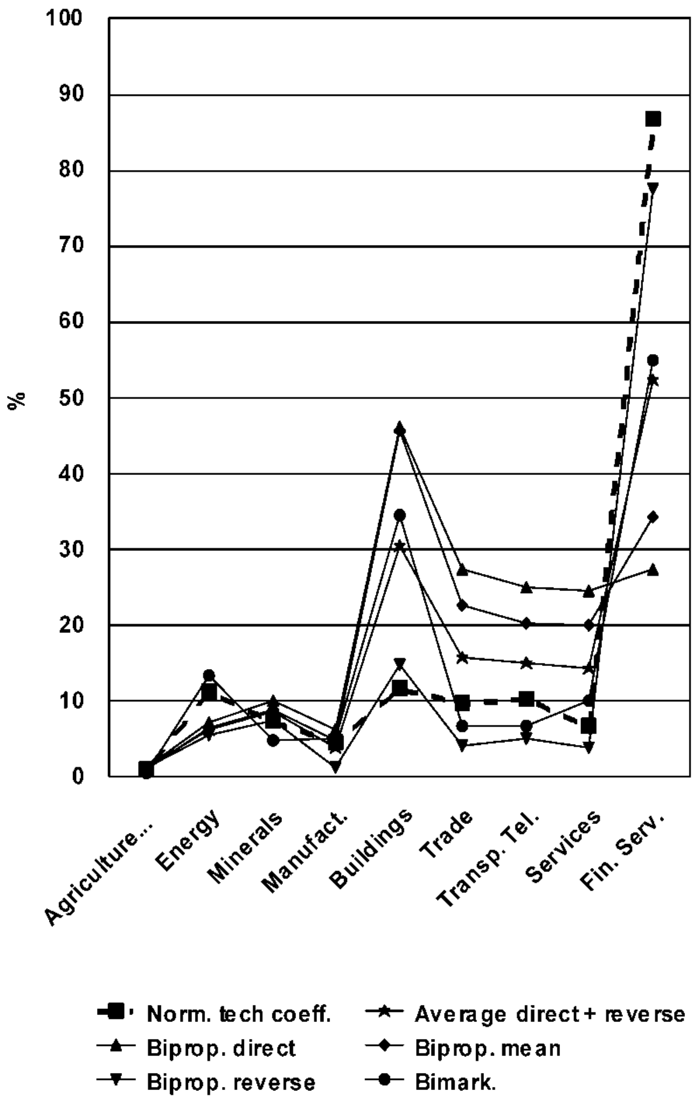

Figure 4. Comparison of methods for France, row vectors, in % (no results for row coefficients with the causative method).

hypothesis it is being used to test. Directed methods rely upon column- or row-normalized coefficients. Thus, in order to use them, the direction of the exchange (demand- or supply-driven) must be pre-specified. Undirected methods do not demand such a pre-specification but their results are not always as conclusive as those from directed methods. The bicausative method must be set aside because of convergence problems. Of what remains, 'multiplicative' methods are preferred because, unlike 'additive methods', they do not generate negative terms; among them, there are the various biproportional filtering methods. The biproportional mean filter and the bi-Markovian filter are practically superior to the ordinary biproportional filter because they avoid computing both direct and reverse results; all three are two-dimensional generalizations of the shift-share method; that is, they remove the effect of sectoral growth. The bi-Markovian filter is theoretically superior to the biproportional mean filter since it is also a two-dimensional generalization of the simple comparison of (row or column) coefficients: moreover, it removes directly the effect of sector size, without requiring dividing the absolute variations by the size of sectors to obtain relative variations in percentages.

Notes

1. Actually, Jackson *et al.* (1990) consider the inverse Leontief matrices $(\mathbf{I}-\mathbf{A})^{-1}$ instead of the technology matrices **A** to compute the transition matrices. This does not change the principle of the method much, only the interpretation of the results.
2. While we discuss this interpretation, generally a special service sector recycles such material.
3. Recently, Junius & Oosterhaven (2003) proposed a version of RAS able to handle negative terms.
4. The shift-share method also compares a region to a country, not just a region or country with itself over time. Hence, the biproportional filter can be interpreted as a two-dimensional generalization of the shift-share method.
5. Since both margins are projected, even though it is a column coefficient matrix, the approach is also relevant to undirected methods since it could also be applied to flow matrices. Perhaps it is somewhat contradictory to start from a directed model (demand-driven) and then to apply the same technique to a more-general undirected method. Moreover, it might seem curious to project using both margins, since only the column margin is important to the value of the column coefficients.
6. For $T+1$ years, this yields six T projections to the total if both sliding and cumulative projections are done! However, often only two periods are considered, so cumulative projection is unnecessary. It remains that two sets of results must be computed: a direct and a reverse.
7. With all methods, the main result is the overwhelming dynamism of Financial Services, for both columns and rows. This is caused by the strong recent growth in the exchange among financial institutions. French accounts include many rather artificial movements, both positive and negative, of finances among banks, while only the balances are really exchanged each month; for example, bank A pays 10 billion to bank B, while bank B pays 9.5 billion to bank A: the French national accounting system accounts two movements, of 10 billion from A to B and 9.5 from B to A, while only the balance, here 0.5 billion, is really paid by A to B. A proposed reform of the French national accounting system will ensure that only these balances will be included.
8. Financial Services, Transport and Telecommunications, Services and Manufacturing have diagonal elements that exceed one, while that for Buildings is near zero. The off-diagonal element for Financial Services is highly positive, while those of Agriculture, Services, and Transport and Telecommunications are strongly negative.

References

Bacharach, M. (1970) *Biproportional Matrices and Input–Output Change* (Cambridge, UK, Cambridge University Press).

Bachem, A. & Korte, B. (1979) On the RAS-algorithm, *Computing*, 23, pp. 189–198.

Chen, C.-Y. & Rose, A. (1986) The joint stability of input–output production and allocation coefficients, *Modeling and Simulation*, 17, pp. 251–255.

Chen, C.-Y. & Rose, A. (1991) The absolute and relative joint stability of input–output production and allocation coefficients, in A. W. A. Peterson (ed) *Advances in Input–Output Analysis* (New York, Oxford University Press), pp. 25–36.

Dietzenbacher, E., Smid, B. & Volkerink, B. (2000) Horizontal integration in the Dutch financial sector, *International Journal of Industrial Organization*, 18, pp. 1223–1242.

Ghosh, A. (1958) Input–output approach to an allocative system, *Economica*, 25, pp. 58–64.

Guiasu, S. (1979) An entropic measure of connection and interdependence between the subsystems of a given large system, in: *Proceedings of the 3rd Formation Symposium on Mathematical Methods for the Analysis of Large-scale Systems*, 16–19 May 1978 (Prague, Academia), pp. 113–124.

INSEE (various years) Comptes et indicateurs economiques, Rapport sur les comptes de la nation, *INSEE Résultats*, Séries générales.

Jackson, R. W. & Murray, A. T. (2004) Alternative input–output matrix updating formulations, *Economic Systems Research*, 16, pp. 135–148 (this issue).

Jackson, R. W., Rogerson, P., Plane, D. & Ó hUallacháin, B. (1990) A causative matrix approach to interpreting structural change, *Economic Systems Research*, 2, pp. 259–269.

Junius, T. & Oosterhaven, J. (2003) The solution of updating or regionalizing a matrix with both positive and negative entries, *Economic Systems Research*, 15, pp. 87–96.

Kullback, S. (1959) *Information, Theory and Statistics* (New York, Wiley).

Kullback, S. & Liebler, R. A. (1951) On information and sufficiency, *Annals of Mathematics and Statistics*, 22, pp. 79–86.

van der Linden, J. A. & Dietzenbacher, E. (1995) The nature of changes in the EU cost structure of production 1965–85: an RAS approach, in: H. W. Armstrong & R. W. Vickerman (eds) *Convergence and Divergence among European Regions* (London, Pion), pp. 124–139.

van der Linden, J. A. & Dietzenbacher, E. (2000) The determinants of structural change in the European Union: a new application of RAS, *Environment and Planning A*, 32, 2205–2229.

de Mesnard, L. (1990a) Dynamique de la structure industrielle française (Paris, Economica).

de Mesnard, L. (1990b) Biproportional method for analyzing interindustry dynamics: the case of France, *Economic Systems Research*, 2, pp. 271–293.

de Mesnard, L. (1994) Unicity of biproportion, *SIAM Journal on Matrix Analysis and Applications*, 15, pp. 490–495.

de Mesnard, L. (1996) Biproportional methods and interindustry dynamics : the case of energy in France, in: J.-B. Lesourd, J. Percebois & F. Valette (eds) *Models for Energy Policy* (London, Routledge).

de Mesnard, L. (1997) A biproportional filter to compare technical and allocation coefficient variations, *Journal of Regional Science*, 37, pp. 541–564.

de Mesnard, L. (1998) Analyzing structural change: the biproportional mean filter and the biproportional bi-Markovian filter, Paper presented at the *Twelfth International Conference on Input–Output Techniques*, New York, 18–22 May.

de Mesnard, L. (2000) Bicausative matrices to measure structural change: are they a good tool?, *Annals of Regional Science*, 34, pp. 421–449.

Plane, D. & Rogerson, P. (1986) Dynamic flow modeling with interregional dependency effects, *Demography*, 23, pp. 91–104.

Rogerson, P. & Plane, D. (1984) Modeling temporal change in flow matrices, *Papers of the Regional Science Association*, 54, pp. 147–164.

Rotella, F. & Borne, P. (1995) *Théorie et pratique du calcul matriciel* (Editions Technip).

Snickars, F. & Weibull, J. W. (1977) A minimum information principle: theory and practice, *Regional Science and Urban Economics*, 7, pp. 137–168.

Watanabe, S. (1969) *Knowing and Guessing. A Quantitative Study of Inference and Information* (New York, Wiley).

Wilson, A. G. (1970) *Entropy in Urban and Regional Modeling* (London, Pion).