Externalities and fiscal policy in a Lucas-type model

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Abstract

This paper devises a fiscal policy capable of decentralizing the optimal growth path in a Lucas-type model when average human capital has an external effect in the goods sector and average learning time has an external effect in human capital accumulation.
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1. Introduction

Several authors have considered externalities in endogenous growth models with human capital accumulation (e.g., Lucas, 1988; Chamley, 1993; Benhabib and Perli, 1994). In this case, optimal growth paths and competitive equilibrium paths may not coincide, although an adequate government intervention can correct this market failure. Actually, García-Castrillo and Sanso (2000) and Gómez (2003) devise fiscal policies that are capable of decentralizing the optimal equilibrium in Lucas' (1988) model when average human capital has an external effect in goods production.

This paper devises a fiscal policy by means of which the optimal equilibrium can be decentralized in a Lucas-type model. We extend the analyses of García-Castrillo and Sanso (2000) and Gómez (2003) to

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allow also for the average learning time to have an external effect on human capital accumulation. This issue is interesting since these external effects are also plausible (e.g., Chamley, 1993; Benhabib and Perli, 1994) and one may conjecture that the optimal policy will depend on the class of externalities considered. We assume diminishing point-in-time returns in human capital accumulation, because they have been shown to be essential for capturing the characteristic life-cycle phases of human capital accumulation (e.g., Rosen, 1976). The optimal policy proposed by García-Castrillo and Sanso (2000) must rely on lump-sum taxation at least in the transitional phase. More realistically, we assume that the government cannot resort to lump-sum taxes. Interestingly enough, we find that a similar policy to that devised by Gómez (2003), conveniently modified to correct for an externality of average learning time as well, can decentralize the optimal equilibrium.

Section 2 describes the market economy and Section 3 the centralized economy. Section 4 devises the optimal policy. Section 5 concludes.

2. The decentralized economy

Consider an economy inhabited by a constant population, normalized to one, of identical infinitely-lived agents who derive utility from consumption, c, according to

$$\int_0^\infty e^{-\rho t}\left(c^{1-\sigma}-1\right)/(1-\sigma)\mathrm{d}t \qquad \rho>0, \ \sigma>0. \tag{1}$$

Time endowment is normalized to unity, which can be allocated to work, u, or learning, $1-u$. Human capital, h, is accumulated according to

$$\dot{h} = B(1-u)^\nu(1-u)_a^\omega h \qquad B>0, \ \nu>0, \ \omega\geq0, \ \nu+\omega\leq1, \tag{2}$$

where $(1-u)_a$ expresses externalities associated to average learning time.

The government taxes capital income at a rate τ_k, labor income at a rate τ_h and subsidizes investment in education at a rate s_h. The sole cost of education is foregone earnings, $w(1-u)h$, a fraction s_h of which is therefore financed by the government. The agent's budget constraint is

$$\dot{k} = (1-\tau_k)rk + (1-\tau_h)wuh - c + s_hw(1-u)h, \tag{3}$$

where r is the interest rate, and w, the wage rate. We shall express (3) equivalently as

$$\dot{k} = (1-\tau_k)rk + (1-\hat{\tau}_h)wuh - c + s_hwh, \tag{4}$$

where

$$\hat{\tau}_h = \tau_h + s_h. \tag{5}$$

Output, y, is produced with Cobb–Douglas technology:

$$y = Ak^\beta(uh)^{1-\beta}h_a^\psi \qquad A>0, \ 0<\beta<1, \ \psi\geq0,$$

where h_a expresses externalities associated to average human capital. Profit maximization by competitive firms implies that $r=\beta y/k$ and $w=(1-\beta)y/(uh)$.

The government runs a balanced-budget and cannot resort to lump-sum taxation:

$$\tau_k rk + \tau_h wuh = s_h w(1 - u)h,$$

or using (5),

$$\tau_k rk + \hat{\tau}_h wuh = s_h wh. \tag{6}$$

Let J be the current-value Hamiltonian of the agent's problem,

$$J = (c^{1-\sigma} - 1)/(1 - \sigma) + \lambda((1 - \tau_k)rk + (1 - \hat{\tau}_h)wuh - c + s_h wh) + \mu B(1 - u)^v(1 - u)_a^\omega h.$$

The first-order conditions are

$$c^{-\sigma} = \lambda, \tag{7a}$$

$$\lambda(1 - \hat{\tau}_h)wh = \mu v B(1 - u)^{v-1}(1 - u)_a^\omega h, \tag{7b}$$

$$\dot{\lambda}/\lambda = \rho - (1 - \tau_k)r, \tag{7c}$$

$$\dot{\mu}/\mu = \rho - B(1 - u)^v(1 - u)_a^\omega - (\lambda/\mu)((1 - \hat{\tau}_h)wu + s_h w), \tag{7d}$$

$$\lim_{t\to\infty} \lambda k e^{-\rho t} = \lim_{t\to\infty} \mu h e^{-\rho t} = 0. \tag{7e}$$

Hereafter, let $\gamma_z = \dot{z}/z$ denote the growth rate of the variable z. In what follows, the equilibrium conditions $h = h_a$ and $1 - u = (1 - u)_a$ will be imposed.

From (7a) and (7c), we get

$$\gamma_c = ((1 - \tau_k)\beta y/k - \rho)/\sigma. \tag{8}$$

Using (6), (4) can be expressed as

$$\gamma_k = y/k - c/k. \tag{9}$$

Log-differentiating (7b) yields

$$\gamma_\lambda - \dot{\hat{\tau}}_h/(1 - \hat{\tau}_h) + \gamma_y - \gamma_u = \gamma_\mu + (1 - v - \omega)\gamma_u u/(1 - u) + \gamma_h. \tag{10}$$

From (7b) and (7d), we get

$$\gamma_\mu = \rho - B(1 - u)^{v+\omega} - B(1 - u)^{v+\omega-1}(vu + vs_h/(1 - \hat{\tau}_h)). \tag{11}$$

The following system characterizes the dynamics of the economy in terms of the variables u, $x = kh^{(1-\beta+\psi)/(\beta-1)}$ and $q = c/k$, that are constant in the steady state:

$$\gamma_q = ((1 - \tau_k)\beta - \sigma)Ax^{\beta-1}u^{1-\beta}/\sigma + q - \rho/\sigma, \tag{12a}$$

$$\gamma_x = Ax^{\beta-1}u^{1-\beta} - (1 - \beta + \psi)B(1 - u)^{v+\omega}/(1 - \beta) - q, \tag{12b}$$

$$\gamma_u(1-u)/(\beta(1-u)+(1-v-\omega)u)$$

$$\times\left\{\beta\left(\tau_k Ak^{\beta-1}u^{1-\beta}-q\right)+B(1-u)^{v+\omega-1}\left((1-\beta+\psi)(1-u)+vu+\frac{vs_h}{1-\hat{\tau}_h}\right)-\frac{\dot{\hat{\tau}}_h}{1-\hat{\tau}_h}\right\}.$$

$$(12c)$$

Eq. (12a) is obtained from (8) and (9), and (12b) from (2) and (9). Substituting γ_μ from (11), γ_λ from (7c) and γ_y from $\gamma_y=\beta\gamma_k+(1-\beta)\gamma_u+(1-\beta+\psi)\gamma_h$ in (10), and then replacing γ_k from (9) and $\gamma_h=B(1-u)^{v+\omega}$ in the ensuing expression, we get (12c).

3. The centrally planned economy

The central planner takes all the relevant information into account. He maximizes (1) subject to $\dot{k}=Ak^\beta u^{1-\beta}h^{1-\beta+\omega}-c$ and $\dot{h}=B(1-u)^{v+\omega}h$. Let $\bar{J}$ be the current-value Hamiltonian of the planner's problem,

$$\bar{J}=\left(c^{1-\sigma}-1\right)/(1-\sigma)+\bar{\lambda}\left(Ak^\beta u^{1-\beta}h^{1-\beta+\psi}-c\right)+\bar{\mu}B(1-u)^{v+\omega}h.$$

The first-order conditions are

$$c^{-\sigma}=\bar{\lambda},\tag{13a}$$

$$\bar{\lambda}(1-\beta)Ak^\beta u^{-\beta}h^{1-\beta+\psi}=\bar{\mu}(v+\omega)B(1-u)^{v+\omega-1}h,\tag{13b}$$

$$\dot{\bar{\lambda}}/\bar{\lambda}=\rho-\beta Ak^{\beta-1}u^{1-\beta}h^{1-\beta+\psi},\tag{13c}$$

$$\dot{\bar{\mu}}/\bar{\mu}=\rho-B(1-u)^{v+\omega}-\left(\bar{\lambda}/\bar{\mu}\right)\left((1-\beta+\psi)Ak^\beta u^{1-\beta}h^{-\beta+\psi}\right),\tag{13d}$$

$$\lim_{t\to\infty}\bar{\lambda}ke^{-\rho t}=\lim_{t\to\infty}\bar{\mu}he^{-\rho t}=0.\tag{13e}$$

Proceeding as in the case of the market economy, we arrive at

$$\gamma_q=(\beta-\sigma)Ax^{\beta-1}u^{1-\beta}/\sigma+q-\rho/\sigma,\tag{14a}$$

$$\gamma_x=Ax^{\beta-1}u^{1-\beta}-B(1-\beta+\psi)(1-u)^{v+\omega}/(1-\beta)-q,\tag{14b}$$

$$\gamma_u=\frac{(1-\beta+\psi)B(1-u)^{v+\omega}((1-\beta)(1-u)+(v+\omega)u)-(1-\beta)\beta(1-u)q}{(1-\beta)(\beta(1-u)+(1-v-\omega)u)}.\tag{14c}$$

Equating the growth rates to zero yields the steady state. From (14a) and (14b), we get

$$x^* = \left(\frac{A(1-\beta)\beta u^{*1-\beta}}{B(1-\beta+\psi)\sigma(1-u^*)^{\nu+\omega} + (1-\beta)\rho} \right)^{1/(1-\beta)}, \tag{15a}$$

$$q^* = B(1-\beta+\psi)(\sigma-\beta)(1-u^*)^{\nu+\omega}/((1-\beta)\beta) + \rho/\beta. \tag{15b}$$

Substituting (15b) into (14c) and simplifying yields

$$B(1-\beta+\psi)(1-u^*)^{\nu+\omega-1}((1-\sigma)(1-u^*) + (\nu+\omega)u^*) = (1-\beta)\rho. \tag{15c}$$

Now, we analyze when (15c) has a solution $u^* \in (0,1)$. From (15c), we define

$$P(u) = B(1-\beta+\psi)(1-u)^{\nu+\omega-1}((1-\sigma)(1-u) + (\nu+\omega)u) - (1-\beta)\rho,$$

which is defined at least in $[0,1)$. P is twice continuously differentiable and monotonically increasing because

$$P'(u) = B(1-u)^{\nu+\omega-2}(1-\beta+\psi)(\nu+\omega)(\sigma(1-u) + u(1-\nu-\omega)) > 0, \quad \forall u \in (0,1).$$

If $\nu + \omega < 1$, $\lim_{u \to 1} P(u) = +\infty$, and so, (15c) has a solution $u^* \in (0,1)$ if and only if

$$P(0) = B(1-\beta+\psi)(1-\sigma) - (1-\beta)\rho < 0,$$

or equivalently,

$$\rho(1-\beta) > B(1-\beta+\psi)(1-\sigma). \tag{16a}$$

If $\nu + \omega = 1$, $\lim_{u \to 1} P(u) = P(1) = B(1-\beta+\psi) - (1-\beta)\rho$, and so, (15c) has a solution $u^* \in (0,1)$ if and only if

$$P(0) = B(1-\beta+\psi)(1-\sigma) - (1-\beta)\rho < 0,$$

$$P(1) = B(1-\beta+\psi) - (1-\beta)\rho > 0,$$

or equivalently,

$$B(1-\beta+\psi) > \rho(1-\beta) > B(1-\beta+\psi)(1-\sigma). \tag{16b}$$

As (15b) and (15c) imply $q^* > 0$ and $x^* > 0$ if $u^* \in (0,1)$, the steady state is feasible. We can also state the following proposition (see the proof in Appendix A).

Proposition 1. *The steady state of the optimal-growth problem is locally saddle-path stable.*

4. The optimal policy

This section devises a fiscal policy capable of decentralizing the optimal equilibrium. First, note that (12b) and (14b) coincide. Comparing (12a) with (14a), they coincide if $\tau_k=0$. Equating (12c) and (14c), with $\tau_k=0$, yields

$$s_h = \frac{((1-\beta)\omega + \psi(v+\omega))(1-\hat{\tau}_h)u}{(1-\beta)v} + \frac{(1-u)^{1-v-\omega}\dot{\hat{\tau}}_h}{Bv}. \tag{17}$$

We guess a constant optimal $\hat{\tau}_h$, i.e., $\dot{\hat{\tau}}_h=0$. Hence, (17) reduces to

$$s_h = \frac{(1-\beta+\psi)\omega + \psi v}{(1-\beta)v}(1-\hat{\tau})u. \tag{18}$$

Solving (6) and (18), with $\tau_k=0$, we obtain

$$\hat{\tau}_h = \frac{\psi}{1-\beta+\psi} + \frac{\omega}{v+\omega} - \frac{\psi\omega}{(1-\beta+\psi)(v+\omega)}, \tag{19}$$

which is constant as guessed, and $0<\hat{\tau}_h<1$. The optimal subsidy is then

$$s_h = \left(\frac{\psi}{1-\beta+\psi} + \frac{\omega}{v+\omega} - \frac{\psi\omega}{(1-\beta+\psi)(v+\omega)}\right)u, \tag{20a}$$

which satisfies $0<s_h<1$. Now, (5), (19) and (20a) yield

$$\tau_h = \hat{\tau}_h - s_h = \left(\frac{\psi}{1-\beta+\psi} + \frac{\omega}{v+\omega} - \frac{\psi\omega}{(1-\beta+\psi)(v+\omega)}\right)(1-u), \tag{20b}$$

which satisfies $0<\tau_h<1$.

These results are consistent with the Pigouvian tax intuition that a subsidy directed at the source of a positive externality will correct the market failure. The following proposition summarizes these findings.

Proposition 2. *The optimal equilibrium can be decentralized if capital income is not taxed and investment in human capital is subsidized at a rate*

$$s_h = \left(\frac{\psi}{1-\beta+\psi} + \frac{\omega}{v+\omega} - \frac{\psi\omega}{(1-\beta+\psi)(v+\omega)}\right)u.$$

The subsidy can be financed by taxing labor income at a rate

$$\tau_h = \left(\frac{\psi}{1-\beta+\psi} + \frac{\omega}{v+\omega} - \frac{\psi\omega}{(1-\beta+\psi)(v+\omega)}\right)(1-u).$$

Lump-sum taxation is not required to balance the government budget.

Comparing (7a) and (13a), decentralization of the optimal equilibrium requires $\lambda=\bar{\lambda}$. Hence, $\gamma_\lambda=\gamma_{\bar{\lambda}}$ which, using (7c) and (13c), yields $\tau_k=0$. Log-differentiating (7b) and (13b), recalling that a constant $\hat{\tau}_h$ was guessed, decentralization also requires $\gamma_\mu=\gamma_{\bar{\mu}}$; i.e., using (7d) and (13d),

$$(\lambda/\mu)(1-\hat{\tau}_h + s_h/u)(1-\beta)Ak^\beta u^{1-\beta}h^{-\beta+\psi} = (\bar{\lambda}/\bar{\mu})(1-\beta+\psi)Ak^\beta u^{1-\beta}h^{-\beta+\psi}. \tag{21}$$

Hence, the optimal policy must be set so that private and social marginal products of human capital in the goods sector, measured in terms of human capital as numeraire, coincide. This can be accomplished by setting s_h according to (18), which, thus, corrects for the effects of two divergences: the difference between the private and the social marginal product of human capital in goods production because of the externality associated to average human capital, and the difference between the imputed real price of physical capital in the centralized economy, λ/μ, and that in the market economy, λ/μ, because of the externality associated to average learning time (see (7b) and (13b)). The government's budget constraint (6), with $\tau_k=0$, provides the additional condition required to determine $\hat{\tau}_h$ and s_h.

The optimal policy just derived is similar to that devised by Gómez (2003), conveniently adapted to account for the externality of average learning time as well. Eqs. (20a) and (20b) comprise three terms. The first term, $\psi u/(1-\beta+\psi)$ and $\psi(1-u)/(1-\beta+\psi)$, respectively, depends on the relative size of the externality associated to average human capital, and fully corrects for this external effect in the absence of externalities in the educational sector ($\omega=0$). This is the optimal policy derived by Gómez (2003). The second term, $\omega u/(v+\omega)$ and $\omega(1-u)/(v+\omega)$, respectively, depends on the relative size of the externality associated to average learning time, and fully corrects for this external effect in absence of externalities in goods production ($\psi=0$). The third term, which depends on the product of the relative size of both externalities, is nonzero only if both external effects coexist.

As Gómez (2003), let us suppose now that the government subsidizes the stock of human capital at a rate $\hat{s}_h$. The agent's budget constraint is then

$$\dot{k} = (1-\tau_k)rk + (1-\hat{\tau}_h)wuh - c + \hat{s}_h h. \tag{22}$$

Handling the first-order conditions as in Section 2, it can be observed that the relationship $\hat{s}_h = s_h w$ carries over all the calculations, and so, the dynamics of the market economy are also summarized by Eqs. (12a,b,c), after s_h being substituted by $\hat{s}_h/w$. Thus, from (19), (20a) and $\hat{s}_h = s_h w$, we can derive the following result.

Proposition 3. *The optimal equilibrium can be decentralized if capital income is not taxed and the stock of human capital is subsidized at a rate*

$$\hat{s}_h = \left(\frac{\psi}{1-\beta+\psi} + \frac{\omega}{v+\omega} - \frac{\psi\omega}{(1-\beta+\psi)(v+\omega)} \right) \frac{(1-\beta)y}{h}.$$

The subsidy can be financed by taxing labor income at a constant rate

$$\hat{\tau}_h = \frac{\psi}{1-\beta+\psi} + \frac{\omega}{v+\omega} - \frac{\psi\omega}{(1-\beta+\psi)(v+\omega)}.$$

Lump-sum taxes are not required to balance the government budget. The government's size, measured as the subsidy (or taxes) share of output, is constant and equals

$$\phi = \frac{\hat{s}_h h}{y} = \left(\frac{\psi}{1-\beta+\psi} + \frac{\omega}{v+\omega} - \frac{\psi\omega}{(1-\beta+\psi)(v+\omega)} \right) (1-\beta).$$

5. Conclusions

We devise a fiscal policy capable of decentralizing the optimal equilibrium in a Lucas-type model with externalities in goods production and in human capital accumulation. The optimal growth path can be decentralized by keeping capital income untaxed and instituting a subsidy to investment in education or to the stock of human capital, which can be financed by a labor income tax. Lump-sum taxation is not needed to balance the government budget either in the steady state or in the transitory phase.

Acknowledgements

Appendix A

Proof of Proposition 1. Linearizing Eqs.(14a,b,c) around the steady state (x^*,q^*,u^*) yields

$$\begin{pmatrix} \dot{x} \\ \dot{q} \\ \dot{u} \end{pmatrix} = A \begin{pmatrix} x - x^* \\ q - q^* \\ u - u^* \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x - x^* \\ q - q^* \\ u - u^* \end{pmatrix},$$

where

$$a_{11} = -(1 - \beta)Ax^{*\beta-1}u^{*1-\beta},$$

$$a_{12} = -x^*,$$

$$a_{13} = \left((1 - \beta)Ax^{*\beta-1}u^{*-\beta} + B(1 - \beta + \psi)(1 - u^*)^{v+\omega-1}(v + \omega)/(1 - \beta) \right)x^*,$$

$$a_{21} = (1 - \beta)(\sigma - \beta)Ax^{*\beta-2}u^{*1-\beta}q^*/\sigma,$$

$$a_{22} = q^*,$$

$$a_{23} = (1 - \beta)(\sigma - \beta)Ax^{*\beta-1}u^{*-\beta}q^*/\sigma,$$

$$a_{31} = 0,$$

$$a_{32} = -\beta(1 - u^*)u^*/(\beta(1 - u^*) + u^*(1 - v - \omega)),$$

$$a_{33} = B(1 - \beta + \psi)(1 - u^*)^{v+\omega-1}u^*(v + \omega)/(1 - \beta).$$

The determinant of A is negative:

$$det(A) = -B(1-\beta+\psi)(1-u^*)^{v+\omega-1}u^*\beta Ax^{*\beta-1}u^{*1-\beta}q^*(v+\omega)$$

$$\times \frac{\sigma(1-u^*)+u^*(1-v-\omega)}{\sigma(\beta(1-u^*)+u^*(1-v-\omega))} < 0.$$

Using (15a) and (15b), we obtain

$$a_{11}+a_{22} = \rho + \frac{B(1-\beta+\psi)(1-u^*)^{v+\omega}(\sigma-1)}{1-\beta} = \frac{B(1-\beta+\psi)(1-u^*)^{v+\omega-1}u^*(v+\omega)}{1-\beta} > 0,$$

where the last equality follows from (15c). The trace of A is then positive:

$$tr(A) = a_{11}+a_{22}+a_{33} = 2B(1-\beta+\psi)(1-u^*)^{v+\omega-1}u^*(v+\omega)/(1-\beta) > 0.$$

Hence, A must have one stable and two unstable eigenvalues.   □

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