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Public schooling, college subsidies and growth

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Abstract

How does the mix of public education expenditures across primary and secondary (K-12) education and post-secondary (college) education influence economic growth? To address this, I build an overlapping generations endogenous growth model. Human capital is accumulated through compulsory K-12 education and through optional college education. Government uses tax revenue to provide quality in K-12 schooling and to subsidize college tuition. When total expenditures are small, all funds should provide K-12 quality. When expenditures are above a critical value, a positive share should subsidize tuition. The share should increase with total expenditures and with the degree of complementarity of human capital accumulated through the two types of education. Also, increased education spending is more likely to increase growth when a larger share subsidizes tuition.

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1. Introduction

Government is the principal source of funding for primary and secondary (K-12) education in the United States.[1] More than 90% of expenditures are financed by federal, state, and local governments. The role of government in financing post-secondary (college) education is substantial but smaller. Private sources account for nearly half of total funding.[2] Public K-12 expenditures directly affect a large majority of the school-aged population. For those under age 16, K-12 enrollment is near 100% and

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[1] For ease of exposition, I use 'K-12' (kindergarten through twelfth grade) and 'college' to describe mandatory, general education and optional specialized education.

[2] 1999 figures (Education at a Glance (2002), Table B4.2).

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more than 88% of K-12 students enroll in public schools.[3] The impressive enrollment figures reflect mandatory attendance laws in each of the 50 states.[4] In contrast, no state requires schooling beyond the high school level and college enrollment is much lower.[5] To a first approximation the U.S. educational system is one of mandatory, publicly funded K-12 education and discretionary college education with partial public funding.

The choice of public education expenditures, then, has two dimensions. Policy-makers must decide upon both the level of public expenditures and its allocation across K-12 and college education. Previous investigations of the growth effects of public education funding have largely focused on the first decision. This literature has taken two broad approaches to modeling spending. In several papers, expenditures convert to quality of education which is a direct input in the production of human capital. Examples include Glomm and Ravikumar (1992, 1997, 1998), Eckstein and Zilcha (1994) Kaganovich and Zilcha (1999) and Cassou and Lansing (2003). In other work, public expenditures subsidize private spending. This feature is found, for example, in Zhang (1996), Milesi-Ferretti and Roubini (1998), Hendricks (1999) and Brauninger and Vidal (2000).[6]

In the U.S., direct government provision of education resources resembles K-12 education most closely while subsidies are most meaningful in the college education decision. Thus important aspects of each type of public education expenditure are captured in these models. Several papers contrast publicly provided education with subsidies to private education.[7] However, none simultaneously allow public inputs to K-12 education and subsidies of private college education expenditures. Thus the models are silent on how the mix of expenditures across these uses may influence growth.

There are several reasons to suspect the mix may be important. First, since K-12 education is mandatory, expenditures do not directly promote enrollment. If they influence the level of human capital, it is through education quality. In contrast, funding college education in part lowers the private cost to encourage a greater quantity of

[3] The enrollment rate for older high school-aged students is also high and many who drop out eventually finish. In 2001, more than 86% of 18–29-year-olds had completed high school (Digest of Education Statistics (2002), Tables 6 and 107).

[4] 30 states require attendance to age 16. The remainder require attendance to age 17 or 18. (Digest (2002), Table 150). Angrist and Krueger (1991) find that about 25% of potential dropouts remain in school as a result of such laws.

[5] Of those who complete high school, only about 62% enroll in a post-secondary education institution (Digest (2002), Table 183). Many enrollees never complete a degree program. Of those first-time post-secondary students enrolled in 4-year institutions in the 89–90 school year, only 53.3% had finished a bachelor's degree by the Spring of 1994. Nearly a quarter had no degree and were no longer enrolled (Digest (2000), Table 311).

[6] While evidence of the importance of education and human capital in determining growth in observed economies is inconclusive, some evidence suggests it can be quite important. See Temple (1999, 2001) for a discussion of this issue.

[7] Glomm and Ravikumar (1992) find that public funding may lead to lower per capita income than private funding of education. Zhang (1996) confirms this and shows that subsidies to private education can dominate public education in terms of growth and welfare. Brauninger and Vidal (2000) argue that while a marginal increment to subsidies can decrease growth, growth is maximized with pure public education.

education. Secondly, human capital accumulated through K-12 and college education may be imperfect substitutes in production.[8] Thus the effects of increasing the supply of each may not be symmetric.

This paper addresses the growth effects of each decision. In particular, for a given *level* of government education expenditure, I examine how resources should be split across K-12 and college education to maximize growth. I then consider the consequences of increasing the level of expenditure given that a constant *share* subsidizes college.

The framework for analysis is an overlapping generations model where human capital accumulation fuels long-run growth. The model is sufficiently stylized to allow analytical results. Young agents make a compulsory investment in K-12 education resulting in a human capital endowment. They also decide whether to make an additional investment in college education. K-12 education is publicly funded; college education requires private expenditures but may be subsidized.[9] Agents supply labor inputs in proportion to their human capital endowment. Those with college education additionally supply skill as a separate input to production. The growth rate depends upon quality in K-12 education and the level of skill employed. K-12 quality is proportional to spending and college attendance responds to subsidies. Thus government influences growth both by choosing a level of expenditures and by allocating resources across these uses.

I find that when the level of government education spending is sufficiently small, all resources should provide K-12 quality. When the level rises above a critical value, a positive share should subsidize college if skill and labor are substitutes in production. An increase in the share of resources subsidizing college education increases the share of the population earning degrees. This has a positive growth effect. With total expenditures held constant, the requisite decrease in K-12 funding has a negative growth effect. When total education expenditures are small, the negative effect dominates. With a higher level of expenditures, growth is maximized where the two effects offset. In this case, the appropriate share devoted to subsidies is larger when the level of expenditures is larger. Also, subsidies should comprise a smaller share of expenditures when K-12 education is more important in generating human capital and when skill and labor are more substitutable in production.

The growth effect of increasing the level of public education spending depends upon its allocation. Increased spending increases the quality of K-12 education; a positive growth effect. It may also alter the quantity of college education. The direction of the second effect is unclear. Increased spending directly lowers the private cost of skill but the resulting tax increase crowds out savings, increasing the cost of borrowing and consequently increasing the cost of skill. With expenditures allocated in large part to subsidies, the first effect dominates and increased expenditures increase college enrollment. This is an additional positive growth effect so increasing expenditures

[8] It is common to model the labor of agents with different education levels as distinct inputs. For example, see Katz and Murphy (1992), Krusell et al. (2000), and Blankenau and Ingram (2002).

[9] There are several papers in the growth literature where both public and private expenditures are inputs in human capital production. See Glomm and Ravikumar (1992, 1998) and Cassou and Lansing (2003). Often the private investment is a time cost rather than a tuition cost as in this paper.

increases growth. If a small share is devoted to subsidies, the second effect dominates and college education falls, generating a negative growth effect. In this case, there may be an interior growth maximizing level of expenditures.

This analysis shares some features with Kaganovich and Zilcha (1999). They consider the case where government can provide either of two inputs to human capital. Government is the exclusive provider of the first input while the second can be provided either by government through education vouchers or privately through parental expenditure. Government expenditure on the second input discourages private expenditure by providing precisely the input that parents provide. Also, government expenditure does not influence enrollment. They find that vouchers can increase growth when parental expenditure is low due to a low degree of altruism or when both altruism and total government education expenditures are high. In this paper, government expenditure on the privately funded input encourages private investment and influences enrollment. Investment in the private input is made by current learners and altruism is not a factor. Thus while the results in the two papers are complementary, they analyze distinct cases of a government choosing across two types of education expenditures.

The following section describes the environment in which these experiments are conducted. Section 3 describes the possible equilibria in the model and Section 4 looks at policy implications. The final section summarizes and concludes.

2. The environment

The economy consists of overlapping generations of three-period-lived, ex ante homogeneous agents, a government and a single representative firm. The size of each generation is normalized to one. Agents in their first period are learners. Learners are presented sequentially with two education opportunities. The first is compulsory and funded by government. This investment endows each period t learner with h_t units of human capital. The second is optional, may be funded in part by government and augments human capital in a manner made explicit below. For brevity, these investments are referred to as K-12 and college education.

2.1. Private education costs and benefits

Education expenditures for learner i in period t depend upon the education level chosen (e_t^i). As all learners make the first investment, agents are differentiated by whether they have a college education ($e_t^i = c$) or no college education ($e_t^i = n$). Let $C_t^i | e_t^i,\ e_t^i \in \{c, n\}$, be private education expenditures for period t learner i. Then

$$C_t^i | e_t^i = \begin{cases} 0 & \text{if } e_t^i = n, \\ T_t(1 - \psi_t) & \text{if } e_t^i = c, \end{cases}$$

where T_t is the tuition cost of a college education and $\psi_t \in [0, 1]$ is the share paid by government. Learners have neither income nor additional expenses and thus incur debt obligations in this amount.

In the second period, agents are earners. Each inelastically supplies h_t units of human capital in a competitive labor market. The time t education-contingent payment *per unit of human capital* for agent i is $\omega^i_{t+1}|e^i_t$ where

$$\omega^i_{t+1}|e^i_t = \begin{cases} \omega_{n,t+1} & \text{if } e^i_t = n, \\ \omega_{n,t+1} + \omega_{c,t+1} & \text{if } e^i_t = c. \end{cases}$$

Gross labor income, then, is $h_t\omega^i_{t+1}|e^i_t$ and the college education premium is $h_t\omega_{c,t+1}$. Earners use income to repay any college loan obligations and to pay taxes. The remainder is allocated across current consumption and savings. In the third period, agents are old and use income from savings to finance consumption. They have no additional income or expenditures.

Each initial earner is endowed with h_0 units of human capital and a fraction, $\Pi_0 \in (0,1]$, of these earners are endowed with both a college education and loan obligations. If $\Pi_0 \in (0,1)$, loan obligations are such that net income is equal across the original earners.

2.2. The agents' problem

Generations are indexed by the year that members are learners. Members of generation t are learners in period t, earners in period $t+1$ and old in period $t+2$. Each member of each generation makes two decisions: an education decision as a learner and a resource allocation decision as an earner.

Optimal education: Uncertainty regarding education is resolved prior to resource allocation and a bond market operates freely. Thus the optimal education choice maximizes the present discounted value of lifetime income net of taxes and education expenses. Learners choose an education strategy taking the strategy of others as given and correctly evaluating the returns to each possible outcome. Specifically, in period t learner i of generation t chooses a probability with which to earn a college education, $\pi^i_t \in [0,1]$, taking the population proportion of college educated workers, $\Pi_j \in (0,1]$, $\forall j \geqslant 0$, the cost of education, the education premium and taxes as given.[10] Taxes are lump sum and do not directly influence education choices. Optimality in education requires

$$\pi^i_t \begin{cases} = 1 & \text{if } h_t\omega_{c,t+1} \geqslant (1+r_{t+1})(1-\psi_t)T_t, \\ \in [0,1] & \text{if } h_t\omega_{c,t+1} = (1+r_{t+1})(1-\psi_t)T_t, \\ = 0 & \text{if } h_t\omega_{c,t+1} \leqslant (1+r_{t+1})(1-\psi_t)T_t, \end{cases} \tag{1}$$

[10] There is never an equilibrium with $\Pi = 0$. Intuitively, as Π approaches 0, the wage rate for skilled labor in the following period approaches infinity. In this case $\pi^i = 0$ cannot be a best strategy. For brevity, I omit this case in the ensuing discussion.

where r_{t+1} is the interest rate paid in period $t+1$ on period t loan obligations. Eq. (1) states that an individual prefers to acquire a college education if the present value of the college education premium exceeds the private cost of college.[11] If the premium just compensates the cost, the individual is indifferent and randomizes over the choices. If the cost exceeds the premium, the agent will not earn a degree.

Optimal consumption: Preferences are identical across agents and logarithmic over consumption as an earner and while old with discount rate β. Let $x_{t,t+1}$ and $x_{t,t+2}$ be consumption by agent i of generation t in periods $t+1$ and $t+2$. Agent specific indexation is suppressed here to limit notational complexity. A lump sum tax τ_t is levied on earners. An agent saves by purchasing bonds and bond holdings by the agent in period $t+1$ are $b_{t,t+1}$.

Subsequent to the education decision, each agent chooses $b_{t,t+1}$, $x_{t,t+1}$ and $x_{t,t+2}$ to solve

$$\max \quad [\ln x_{t,t+1} + \beta \ln x_{t,t+2}], \tag{2}$$

$$\text{s.t.} \quad \begin{cases} x_{t,t+1} \leqslant h_{t+1}\omega_{t+1}|e_t - (1+r_{t+1})C_t|e_t - b_{t,t+1} - \tau_t, \\ x_{t,t+2} \leqslant (1+r_{t+2})b_{t,t+1}. \end{cases} .$$

Solving Eq. (2) gives the optimal savings for each earner,

$$b_{t,t+1} = \widetilde{\beta}(h_t\omega_{t+1}|e_t - (1+r_{t+1})C_t|e_t - \tau_t), \tag{3}$$

where $\widetilde{\beta} \equiv \beta/(1+\beta)$.

2.3. The firm's problem

A representative firm hires human capital to produce a final output good which is sold in a competitive market. Human capital provides two inputs to production: skill and labor. The quantities of skill and labor employed in period t are S_t and L_t. The firm's production function is

$$Y_t = A[(1-\gamma)S_t^\rho + \gamma L_t^\rho]^{1/\rho},$$

where $A > 0$, $\rho \leqslant 1$ and $0 \leqslant \gamma \leqslant 1$. In the case where $\rho = 0$, $Y_t = AS_t^{(1-\gamma)}L_t^\gamma$. With this specification, $(1-\rho)^{-1}$ is the elasticity of substitution between skill and labor and γ and scales their relative importance. The factor market is competitive so that marginal productivities determine factor prices.

[11] It is common in the growth literature to impose a time cost or both a time cost and a goods cost to education. Galor and Moav (2000) and Brauninger and Vidal (2000), however, have only a goods cost proportional to the cost of skill as in this paper. The goods cost in the current model allows tractability. The type of cost is important in papers considering the implications of different taxing schemes (see Milesi-Ferretti and Roubini (1998)) since a time input goes untaxed. Here taxes are lump sum which diminishes the importance of the distinction.

2.4. Human capital

The cost of college education is proportional to the cost of hiring skill (say, as a tutor). Thus

$$T_t = \theta h_{t-1}\omega_{c,t}, \tag{4}$$

where $\theta > 0$ is a scalar. This is similar to Galor and Moav (2000, p. 492). It reflects that skilled labor is a key input in education.[12] All agents provide labor in proportion to their human capital endowment. Agents with college education also provide skill as an additional input. The amount of skill provided depends upon the human capital endowment. This reflects the notion that an agent with more human capital will be more productive in acquiring skill. For simplicity, I assume that the quantity of skill provided is a scalar of the human capital endowment. To economize on notation, the constant of proportionality in providing both skill and unskilled labor is set to one. Given this, the supply of labor and skill are

$$L_{t+1} = h_t,$$

$$S_{t+1} = \Pi_t h_t. \tag{5}$$

That is, the labor input depends only on the human capital endowment while the skill input depends on both the endowment and the private college education choice.[13] This specification is particularly convenient in that private decisions affect only the supply of skill. Given this, wages for skill and labor will be given by

$$\omega_{c,t} = \frac{Y_t}{h_{t-1}}\frac{(1-\gamma)\Pi_t^{\rho-1}}{(1-\gamma)\Pi_t^{\rho}+\gamma},$$

$$\omega_{n,t} = \frac{Y_t}{h_{t-1}}\frac{\gamma}{(1-\gamma)\Pi_t^{\rho}+\gamma}. \tag{6}$$

Government education expenditures, E_t, convert one for one to units of education quality. Per capita human capital accumulates according to

$$h_{t+1} = H(E_t, h_t, \Pi_t).$$

The human capital endowment to the current generation may increase as more resources are directly devoted to its production (as E_t increases), as the store of human capital increases and as the number of people employed in skilled positions increases.

Including the prior generation's skill level and education level is natural in light of studies that show a strong relationship between education outcomes of children and parents.[14] Including public K-12 expenditures is perhaps more suspect. There is some empirical support for its inclusion. For example Card and Krueger (1992) find

[12] Setting tuition proportional to the total wage of a skilled worker yields similar results.

[13] An alternative to this specification is to have college educated workers provide only skill and k-12 educated provide labor. Numerical exercises show that the qualitative results are similar under two specifications. It is unclear which best matches observed economies. The current specification captures that jobs which require a college education include many tasks for which such training is not needed.

[14] See Coleman et al. (1966) and Hanushek (1986) for example.

evidence that education quality, measured in terms of length of school year, class size and teachers' salaries positively influences wage rates. More recently Krueger (1998, p. 30) argues that "… the widely held belief … that additional resources yield no benefits in the current system … is not supported in the data." However, this assessment is not uncontested. For example, Hoxby (2000) finds no evidence that smaller class size increases student achievement. Hanushek (1998, p. 12) concludes that "… the United States has made steady and large investments in human capital. The resources invested, however, have had little payoff in terms of student performance."[15] In light of this uncertainty, K-12 expenditures are included with a parameter to gauge their relative importance in producing human capital.

It proves convenient to specify a form of H that is constant returns to scale in the sole reproducible input, h_t. I consider here the case where

$$h_{t+1} = h_t + BE_t^{\mu}[(1-\alpha)S_t^{\sigma} + \alpha L_t^{\sigma}]^{(1-\mu)/\sigma}, \quad \sigma \neq 0,$$
$$h_{t+1} = h_t + BE_t^{\mu}(S_t^{1-\alpha}L_t^{\alpha})^{1-\mu}, \quad \sigma = 0, \tag{7}$$

with parameter restrictions $B \geqslant 0$ and $\mu, \alpha \in [0,1]$ and $\sigma \leqslant 1$. Here μ determines the relative importance of K-12 quality and the human capital aggregate in producing human capital. In the case $\alpha = 1$, this is similar to the human capital accumulation equation in Glomm and Ravikumar (1997). If $B = 0$, this is a simplified version of the Lucas (1990) specification.

2.5. Government

Government is subject to a period balanced budget constraint. Revenue in period t comes from lump sum taxes on earners and is a fixed as a share, g, of total output. Specifically

$$\tau_t = gY_t. \tag{8}$$

Revenue is split across two uses: purchases of K-12 education quality and college tuition subsidies. The amount spent on quality is E_t. The amount spent on subsidies is equal to the cost of tuition scaled by government's share and the fraction of learners making the investment; i.e. the amount spent on subsidies in period t is $\psi_t T_t \Pi_t$ where $\psi_t \in [0,1]$. Government spending and revenue must satisfy $E_t + \psi T_t \Pi_t = gY_t$.

I focus on two aspects of government policy: the effects of changes in the quantity of government education expenditures governed by g, and the effects of changing the mix of government education revenue across these two uses. To facilitate this second investigation, define $\phi \in [0,1]$ as the share of total revenue spent on subsidies to higher

[15] These papers are part of a much larger discussion on the expenditure/quality relationship in K-12 education. Some evidence is summarized in the two papers. Two particularly interesting additional studies are: Berliner and Biddle (1995) who argue that demographic changes rather than school failure largely explain unimpressive trends in test scores and Hanushek and Kimko (2000) who argue that evidence supports that education quality matters for growth but that expenditures are not linked to meaningful measures of quality.

education. Government policy then is a choice of ϕ and g subject to

$$E_t = (1-\phi)gY_t, \tag{9}$$

$$\psi_t T_t \Pi_t = \phi g Y_t. \tag{10}$$

3. Equilibria

Definition 1. A competitive Nash equilibrium in this economy is a sequence of probabilities $\{\pi_t^i\}_{t=0}^{\infty}$, allocations $\{x_{t,t}^i, x_{t,t+1}^i\}_{t=0}^{\infty}$ and bond holdings $\{b_{t,t+1}^i\}_{t=0}^{\infty}$ chosen by each agent $i_t \in [0,1]$, $\forall t \geqslant 0$; population proportions $\{\Pi_t\}_{t=0}^{\infty}$, human capital stocks $\{h_t\}_{t=0}^{\infty}$, prices $\{r_t, \omega_{c,t}, \omega_{n,t}\}_{t=0}^{\infty}$, and fiscal policy choices $g, \phi, \{\psi_t, \tau_t, E_t\}_{t=0}^{\infty}$ such that: each agent in each generation chooses π_t^i to satisfy Eq. (1) and $\pi_t^i = \Pi_t \forall i, t$; each agent solves Eq. (2) to determine the optimal consumption pattern, the firm's choice of labor and capital inputs maximizes profits with prices given by Eq. (6); agents supply inputs according to Eq. (5); human capital accumulates according to Eq. (7); government expenditures satisfy Eqs. (8)–(10); the bond market clears (savings equals private education expenditures); factor markets clear; and the good market clears, i.e. $Y_t = x_{t,t+1} + x_{t-1,t+1} + T_t \Pi_t$.

Given an equilibrium sequence of probabilities, $\{\Pi_t\}_{t=0}^{\infty}$, and an initial level of human capital, h_0, other endogenous values are uniquely determined. Because of this, equilibria are discussed in terms of $\{\Pi_t\}_{t=0}^{\infty}$. In any period, there are two potential types of equilibria: a pure strategy equilibrium with $\pi_t^i = \Pi_t = 1$ and a mixed strategy equilibrium with $\pi_t^i = \Pi_t \in (0,1)$. From Eq. (1), a mixed strategy equilibrium in period t requires that the return to college education just equals the cost. Earners with and without college degrees, then, have the same lifetime income net of education costs and consequently save the same amount. From Eq. (3), this amount is $\widetilde{\beta}(h_{t-1}\omega_{n,t} - \tau_t)$.

Bond market clearing requires that savings by earners equals borrowing by learners. With $\Pi_t \in (0,1)$ this is

$$\widetilde{\beta}(h_{t-1}\omega_{n,t} - \tau_t) = \Pi_t \theta (1-\psi_t)\omega_{c,t} h_{t-1}.$$

Substituting Eqs. (8) and (10) into this and simplifying gives

$$\widetilde{\beta}(h_{t-1}\omega_{n,t} - gY_t) + \phi g Y_t = \Pi_t \theta \omega_{c,t} h_{t-1}. \tag{11}$$

The first expression on the left-hand side is period t savings and the second is government college education expenditures. Bond market clearing requires that the sum of these equal total college expenditures (right-hand side). Substituting in for wages, dividing each side by Y_t and solving for Π_t yields

$$\Pi_t = \Pi_{t-1}^{1-\rho} \frac{\gamma}{1-\gamma} \left(\frac{\widetilde{\beta}}{\theta} + \frac{g}{\theta}(\phi - \widetilde{\beta}) \right) + \frac{g}{\theta}(\phi - \widetilde{\beta})\Pi_{t-1}. \tag{12}$$

Definition 2. A balanced growth path exists when output and human capital grow at a common, constant rate and the share of the population earning degrees is constant at some $\Pi \in (0,1]$.
(i) Along a mixed strategy balanced growth path $\Pi \in (0,1)$.
(ii) Along a pure strategy balanced growth path $\Pi = 1$.

Letting $\Pi_t = \Pi_{t+1} = \Pi^*$ in Eq. (12) gives the value of Π along a mixed strategy balanced growth path

$$\Pi^* = \left(\frac{\gamma}{1-\gamma} \frac{\widetilde{\beta} + g(\phi - \widetilde{\beta})}{\theta - g(\phi - \widetilde{\beta})} \right)^{1/\rho}. \tag{13}$$

Proposition 1 gives conditions under which pure and mixed strategy balanced growth paths exist and under which there is convergence to a balanced growth path. All proofs are in the Appendix.

Proposition 1. *Define* $\phi_c \equiv \frac{\theta}{g} + \widetilde{\beta} - \frac{\gamma}{g}\,(\widetilde{\beta} + \theta)$.

(i) *When $\rho > 0$ and $\phi \leqslant \phi_c$, both a pure and mixed strategy balanced growth path exist. If education is sufficiently costly, the economy converges locally to the mixed strategy balanced growth path. The pure strategy balanced growth path is supported only if the economy begins along this path.*
(ii) *When $\rho < 0$, a pure strategy balanced growth path exists if $\phi \leqslant \phi_c$ and a mixed strategy balanced growth path exists if $\phi_c \leqslant \phi < \frac{\theta}{g} + \widetilde{\beta}$. In each case, the balanced growth path is supported only if the economy begins along this path.*
(iii) *When $\rho = 0$, a balanced growth path exists only if $\phi = \phi_c$. The level of Π is indeterminate.*

As empirical evidence suggests $\rho > 0$, I consider this case for the remainder of the discussion.[16] Corollary 1 summarizes conditions under which a mixed strategy balanced growth path exists with $\rho > 0$.

Corollary 1. *If* $g > \min[g_c \equiv \gamma - \frac{\theta}{\widetilde{\beta}}(1-\gamma), 0]$ *a mixed strategy balanced growth path exists for some $\phi > 0$. If in addition* $g < g_a \equiv \frac{\theta - \gamma(\widetilde{\beta}+\theta)}{(1-\widetilde{\beta})}$, *a mixed strategy balanced growth path exists for any ϕ.*

The requirement that g exceed g_c is a requirement that borrowing exceed savings when all agents attend college and tuition is not subsidized; i.e. when $\Pi = 1$ and $\phi = 0$. To see this, divide each side of Eq. (11) by Y_t. Dropping time subscripts and substituting for wages, this gives

$$\widetilde{\beta} \left(\frac{\gamma}{[(1-\gamma)\Pi^\rho + \gamma]} - g \right) + g\phi = \theta(1-\gamma)\,\frac{\Pi^\rho}{[(1-\gamma)\Pi^\rho + \gamma]}. \tag{14}$$

[16] Katz and Murphy (1992) estimate $\rho \approx 0.29$. Blankenau (1999) estimates $\rho \approx 0.414$. The Appendix deals with the case where $\rho < 0$.

Setting $\phi=0$ and $\Pi=1$ in Eq. (14) gives $\widetilde{\beta}(\gamma-g)=\theta(1-\gamma)$ which is equivalent to $g=g_c$. If g decreases from g_c with $\phi=0$, the left-hand side in Eq. (14) is larger. To preserve the equality, the right-hand side must increase or the other left-hand side term must decrease. Either of these requires an increase in Π. But $\Pi>1$ is not possible. Thus $g<g_c$ does not allow bond market clearing. Intuitively, when $\phi=0$ and $\Pi=1$ there are no subsidies and all agents borrow for college. Borrowing is as high as possible. If savings exceed borrowing in this case, the bond market will not clear for any parameterization.

As ϕ increases from 0, the left-hand side of Eq. (14) decreases and an increase in Π is required to preserve the equality. It is straightforward to show that with $g>g_c$ and $\phi=0$, $\Pi^*<1$. Thus over some range of ϕ, an increase in Π is possible. As ϕ increases further, eventually either $\phi=1$ or $\Pi^*=1$. If $\phi=1$, further increases are not possible. If $\Pi^*=1$, further increases in ϕ cannot generate further increases in Π^*. Thus the equality cannot hold and a mixed strategy balanced growth path does not exist. When $g<g_a$, $\Pi^*<1$ even with $\phi=1$ so that such a growth path can always exist.

The parameter space that supports a mixed strategy equilibrium also supports a pure strategy equilibrium. To see why, note that with $\Pi=1$ bond market clearing requires

$$\widetilde{\beta}(\gamma-g+z)=\theta(1-\gamma)-g\phi, \tag{15}$$

where z is the net return to schooling; $z\equiv h_{t-1}\omega_c-(1+r_t)(1-\psi)\theta\omega_c h_{t-2}$. The left-hand side is savings (see Eq. (3)) and the right-hand side is private education expenditures with $\Pi=1$. In order for $\Pi=1$ to be optimal, it must be that $z\geqslant 0$. Solving for z in Eq. (15), it is straightforward to show $z\geqslant 0$ requires $\phi\leqslant\phi_c$. Proposition 1 shows that while both exist, convergence is possible only to the empirically relevant mixed strategy equilibrium.

4. Policy analysis

4.1. The effects of policy on education levels

Consider the effect of government policy on the level of college education along the balanced growth path. Along the pure strategy balanced growth path there is clearly no effect. Corollary 2, which follows from Eq. (13), summarizes the effect along the mixed strategy balanced growth path.

Corollary 2. *Consider the parameter space for which a mixed strategy balanced growth path exists.*

(i) *Π^* is positively related to ϕ.*
(ii) *Π^* is positively related to g if $\phi>\widetilde{\beta}$.*
(iii) *Π^* is negatively related to g if $\phi<\widetilde{\beta}$.*

Increasing the share of funding going to college education increases enrollment (item (i) above). The effect of increasing total expenditures depends on how the funds are used. To understand items (ii) and (iii) it is convenient to rewrite Eq. (14) as

$$g(\phi - \widetilde{\beta}) = \theta \frac{(1-\gamma)\Pi^{\rho}}{[(1-\gamma)\Pi^{\rho}+\gamma]} - \frac{\gamma\widetilde{\beta}}{[(1-\gamma)\Pi^{\rho}+\gamma]}. \tag{16}$$

The left-hand side of this expression is the net *direct* effect of government policy in the bond market. Increasing g by one unit increases public funding of college education by ϕ per unit of output. However, the increase in g is funded by an equal increment to the per capita tax burden of earners. Earners decrease their savings by $\widetilde{\beta}$ per unit of output in response.[17] If $\phi > \widetilde{\beta}$, the increased supply of government funds outweighs the decreased supply of private funds and the net direct impact is an increase in funds available for college education. Available funds increase also if ϕ increases as this requires no change in the tax rate but increases government college education spending.

Bond market clearing requires an *indirect* effect through changes in Π to offset the increase. These indirect effects are captured on the right-hand side of Eq. (16). The first expression is total education expenditures as a fraction of output (derived from $\frac{\Pi\theta\omega_c h_t}{Y_t}$). As Π increases, more agents borrow for education. However, the resulting decrease in skilled wages lowers tuition costs and per student borrowing falls. The first effect dominates and the education expenditures ratio is increasing in Π. The second expression is savings as a fraction of output given zero taxes (derived from $\frac{\widetilde{\beta}\omega_n h_t}{Y_t}$). An increase in skill decreases ω_n relative to output. As savings is proportional to ω_n, the savings rate is decreasing in Π. As this enters negatively in the expression, the right-hand side of Eq. (16) is increasing in Π. Thus a government policy which increases available funds requires an increase in Π to maintain bond market clearing.

4.2. The growth implications of K-12 versus college public education expenditures

The primary purpose of this paper is to evaluate the implications of government policy choices for long run growth. Let $\lambda_{t+1} \equiv \frac{h_{t+1}-h_t}{h_t}$ be the growth rate of human capital in period $t+1$. Then substituting for E_t, S_t and L_t and dividing both sides of Eq. (7) by h_t yields

$$\lambda_{t+1} = ((1-\phi)gA[(1-\gamma)\Pi_{t-1}^{\rho}+\gamma]^{1/\rho})^{\mu}[(1-\alpha)\Pi_{t-1}^{\sigma}+\alpha]^{(1-\mu)/\sigma}.$$

Analytical results are available in the special case where $\sigma = \rho$ and $\alpha = \gamma$. Thus the growth implications of K-12 versus college public education expenditures are first discussed assuming these parameter restrictions and a subsequent numerical exercise shows sensitivity of the findings to changes in σ. With $\sigma = \rho$ and $\alpha = \gamma$, h_t and Π_t enter

[17] In part, this is due to the tax structure. Only earners are savers and only earners are taxed. Thus an increase in taxation, absent general equilibrium adjustment, decreases savings. Collecting some taxes from the old would diminish this effect as it does in Uhlig and Yanagawa (1996) and Blankenau and Ingram (2002). However, taxing only the old is not realistic and the basic argument holds when some tax is imposed on learners.

the final goods and human capital accumulation functions symmetrically. Simplifying shows that the growth rate of human capital along the balanced growth path is

$$\lambda = A^{\mu}((1-\phi)g)^{\mu}[(1-\gamma)\Pi^{\rho}+\gamma]^{1/\rho}. \tag{17}$$

A change in ϕ potentially affects the growth rate in two ways: directly through its influence on the human capital endowment and indirectly through its effect on Π. Along a mixed strategy balanced growth path these are competing effects. An increase in ϕ decreases K-12 quality but increases the share of the population acquiring skill. This opens the possibility that subsidies can increase growth. With $\Pi = 1$ the indirect effect is not operative and subsidies necessarily lower growth. Proposition 2 summarizes the share of resources that should subsidize college education to maximize growth in each of these circumstances.

Proposition 2. *Let $\sigma = \rho$, $\alpha = \gamma$ and define $\phi^* \equiv \frac{1-\mu\rho\left(\frac{\theta}{g}+\widetilde{\beta}\right)}{1-\mu\rho}$. If $0 \leqslant \phi^* \leqslant \phi_c$ the mixed strategy balanced growth rate is maximized at $\phi = \max[\phi^*, 0]$. Otherwise, the balanced growth rate is maximized at $\phi = 0$.*

In the proof to Proposition 2 it is shown that $\phi < \phi_c$ is sufficient for $\frac{\theta}{g} + \widetilde{\beta} > 1$ so that $\phi^* < 1$ is assured. Using the envelope theorem this also assures $\frac{\partial\phi^*}{\partial\rho}, \frac{\partial\phi^*}{\partial\mu} < 0$. Corollary 3 summarizes features of the growth maximizing share of expenditures allocated to college subsidies.

Corollary 3. *It is never growth maximizing to allocate all resources to subsidies. It is growth maximizing to devote no resources to subsidies* (i) *along a pure strategy balanced growth path, and* (ii) *along a mixed strategy balanced growth path with $g < \frac{\theta\mu\rho}{1-\widetilde{\beta}\mu\rho}$.*

Along a mixed strategy balanced growth path with $g > \frac{\theta\mu\rho}{1-\widetilde{\beta}\mu\rho}$ some resources should be devoted to college education. The growth maximizing share is larger when total education expenditures are larger, when skill is more important in producing human capital, and when skill and labor are more complementary in production.

Since growth is necessarily zero when all resources are devoted to subsidies, it is not surprising that this is never growth maximizing. When $\Pi = 1$ subsidies cannot encourage further college enrollment. In this case, taking any resources from education quality to subsidize college education will lower growth.

To understand why it is optimal to not subsidize college when g is small, put Π^* into Eq. (17) and rearrange to obtain

$$\lambda = ((1-\phi)gA)^{\mu}\left[\frac{\gamma(\widetilde{\beta}+\theta)}{\theta - g(\phi-\widetilde{\beta})}\right]^{1/\rho}. \tag{18}$$

This expression highlights that an increase in ϕ has two growth effects. First, it lowers the share of income going to education quality. The growth effects of this are captured in the expression $((1-\phi)gA)^{\mu}$. Secondly, it increases growth through an increase in

Π. The effects of this are captured in the bracketed part of expression (18). Whether an increase in ϕ increases or decreases growth depends on the relative size of these effects. It is straightforward to show that the elasticity of growth with respect to a change in ϕ ($\varepsilon_{\lambda\phi} \equiv \frac{\partial\lambda}{\partial\phi}\frac{\phi}{\lambda}$) is equal to

$$\varepsilon_{\lambda\phi} = -\frac{\mu\phi}{(1-\phi)} + \frac{g\phi}{\rho(\theta - g(\phi - \widetilde{\beta}))}.$$

The first right-hand side expression is negative and measures the responsiveness of growth from changes in the share of output devoted to quality when ϕ increases. This is referred to as the 'share effect' for brevity. The second is positive and measures the responsiveness of growth from changes in Π when ϕ increases.[18] This is referred to as the 'skill effect'. Growth can be increased by increasing ϕ so long as the share effect is smaller than the skill effect in absolute value; i.e. so long as

$$\frac{g}{(\theta - g(\phi - \widetilde{\beta}))}\frac{1}{\rho} \geqslant \frac{\mu}{(1-\phi)}. \tag{19}$$

At $\phi = 0$, this requires $g > \frac{\theta\mu\rho}{1-\widetilde{\beta}\mu\rho}$. Thus when g is sufficiently small, growth cannot be increased with subsidies.

Expression (19) as an equality is also useful in understanding why the growth maximizing value of ϕ, when not equal to zero, is increasing in g. Note that the share effect is unchanging in g while the skill effect is increasing in g. Thus ϕ must increase with g to preserve the equality. More intuitively, as g increases and education quality becomes large, the growth effect of additional skill becomes large relative to the effect of additional quality.

An increase in μ increases the importance of quality in generating growth. As a consequence, the share effect increases in absolute value with μ. The more important is quality in generating growth, the larger the effect of taking resources from K-12 quality for subsidies. Thus to maximize growth, a greater share of resources should finance K-12 quality; college subsidies should be lower.

Note also that an increase in ρ decreases the skill effect. When ρ is larger, labor is a better substitute for skill in production. This has two complementary consequences for the effect of subsidies on growth. First, the responsiveness of Π^* to a change in ϕ is smaller. Secondly, growth is less responsive to an increase in Π.[19] With skill less responsive to ϕ and growth less responsive to skill, the growth effect of increasing subsidies is smaller; college subsidies should be lower.

While this model is stylized to allow analytical solutions, it suggests a method of evaluating whether the United States is currently allocating its education expenditures to maximize growth. To provide an example, I roughly calibrate the model. Following Blankenau (1999), I set $\rho = 0.414$ and $\gamma = 0.47$.[20] In 1999 government education expenditures on primary, secondary and post-secondary, non-tertiary education accounted

[18] $\theta - g(\phi - \widetilde{\beta}) > 0$ when $\phi < \phi_c$ as required.

[19] Straightforward calculations show that the elasticities $\frac{\partial\Pi^*}{\partial\phi}\frac{\phi}{\Pi^*}$ and $\frac{\partial\lambda}{\partial\Pi}\frac{\Pi}{\lambda}$ are decreasing in ρ.

[20] In Blankenau (1999) γ is increasing in response to skill-biased technological change. This is the predicted 1992 value. Evidence is mixed on continued skill-biased technological change.

for 3.5% of GDP while government spending on tertiary education was 1.4% of GDP (Education at a Glance (2002), Table B3.1). This gives $g = 0.049$ and $\phi = 0.29$. I set $\tilde{\beta} = 0.264$, corresponding to $\beta = 0.95$ annually over 20 years. I put these values and $\Pi = 0.62$ into Eq. (13) to get an estimate of $\theta = 0.29$. This value of Π is in line with recent post-secondary enrollment figures (see Digest of Economic Statistics (2002), Table 183). As discussed in Glomm and Ravikumar (1998), estimates of μ tend to be small. Implied values range from 0 (Coleman et al. (1966)) to 0.12 (Card and Krueger (1992)). As a benchmark, I set $\mu = 0.1$, an intermediate value explored by Glomm and Ravikumar (1998).

These parameters suggest $\phi^* = 0.78$. This is considerably higher than the observed value, $\phi = 0.29$, suggesting that growth in the U.S. could be enhanced by reallocating resources toward college education. However, this implication is highly sensitive to the choice of μ. If the true value of μ is 0.296, the current split is growth maximizing; if μ is larger college is oversubsidized. This suggests that an extended version of the model may be useful in identifying gains to growth from reallocating education resources. The result, however, will depend critically on how important government K-12 expenditures are in generating human capital.

4.3. The growth implications of aggregate public education expenditures

Consider now the effect of increasing g holding ϕ fixed. Recall that g and Π^* are positively related if $\phi > \widetilde{\beta}$. In this case, increasing g will increase growth since both the supply of skill and the quantity of education quality are larger. If instead $\phi < \widetilde{\beta}$, increasing g has competing effects on growth. There is a direct increase in the share of output devoted to education quality but Π falls which, on its own, would decrease growth. In this instance, the growth effects of public education expenditures are not immediately clear. Proposition 3 summarizes these effects.

Proposition 3. *Define* $g^* \equiv -\frac{\theta}{(\phi-\widetilde{\beta})}\frac{\mu\rho}{1-\mu\rho}$. *If* $0 \leqslant \phi \leqslant \min\{\phi_c, \widetilde{\beta}\}$, *the mixed strategy balanced growth rate is maximized at* $g = g^*$. *Otherwise, the balanced growth rate is increasing in* g.

A non-trivial equilibrium with $g = g^*$ can exist only if $g^* > 0$. With $\rho > 0$ this requires $(\phi - \widetilde{\beta}) < 0$. In this case, g^* is increasing in both μ and ρ. The growth maximizing g balances the *direct* effect of increased K-12 expenditures with the *indirect* effect of a decrease in Π. When μ rises, K-12 quality is more important in generating human capital. When ρ rises, labor and skill are more substitutable in production. Thus increases in these parameters diminish the significance of the decrease in Π from crowding out and increase the growth maximizing level of expenditures.

When θ rises, education is more costly and the fall in Π from an increase in g is smaller. Since the indirect effect is less onerous, g^* is larger. Recall that $(\phi - \widetilde{\beta})$ is the net direct impact of government policy in the bond market. An increase in $(\phi - \widetilde{\beta})$ means that the net drain of resources from the bond market from an increase in g is smaller. Thus the negative impact on Π of g is smaller and g^* is larger.

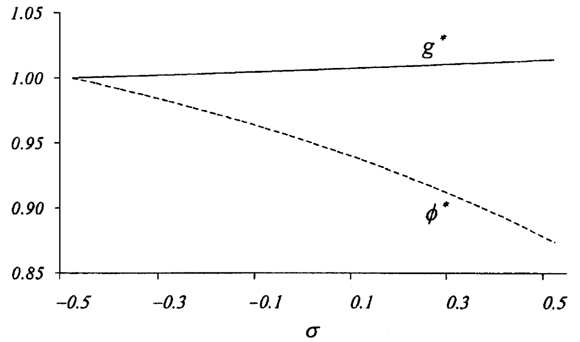

Fig. 1. Sensitivity of g^* and ϕ^* to σ.

4.4. Sensitivity analysis

While $\rho = \sigma$ and $\alpha = \gamma$ are required for analytical results, relaxing these assumptions does not alter the findings above in a significant way. Fig. 1 shows normalized values of g^* and ϕ^* for $\sigma \in [-0.5, 0.5]$ holding ρ and other values constant with $\alpha = \gamma$. Note that as skill and labor become more substitutable in producing human capital (as σ increases) the growth maximizing share of expenditures devoted to college education falls and the growth maximizing level of government education expenditures rises. This is consistent with the effects of increasing ρ in the case where $\rho = \sigma$ (see above expressions for g^* and ϕ^*). Parameters here are chosen such that $g^*, \phi^* \in (0,1)$ but are otherwise arbitrary. Conducting the experiment for a wide variety of parameters shows this relationship to be robust. Thus changes in σ alone have the same effect qualitatively as changes in ρ and σ simultaneously; the assumption that $\rho = \sigma$ is apparently innocuous in terms of qualitative results. Results are similar in the case where $\rho = \sigma$, $\alpha \neq \gamma$. In particular g^* is increasing and ϕ^* is decreasing in α.

Note that the above analysis nests the more commonly developed case where private education expenditures simply increase the supply of the single labor input. To see this, note that the labor inputs are perfect substitutes in the special case where $\rho = 1$. This parameter is not an argument in ϕ_c and with $\rho = 1$, $\phi^* = \frac{1-\mu((\theta/g)+\widetilde{\beta})}{1-\mu}$ and $g^* = \frac{\theta}{\widetilde{\beta}-\phi}\frac{\mu}{1-\mu}$ so that nontrivial policy choices may remain so long as $\mu < 1$. Thus the assumption that skill and labor are separate inputs in production is not essential to the results.

5. Conclusion

Government plays an important role in funding both K-12 and college education. Policy-makers must choose both a level of education funding and its allocation across these uses. This paper analyzes the effects of both decisions. In the empirically relevant case of substitutability, if total expenditures are large enough, some share of the revenue should go to subsidize college tuition. The share should be larger when total expenditures are larger and when skill and labor are less substitutable in production. If a large enough share of public expenditures goes to subsidies, increased total expenditures increase growth. If too small a share goes to subsidies, increased spending may decrease both the share of the population attending college and the growth rate. In this case, the growth maximizing level of expenditures is increasing in the share of expenditures devoted to subsidies. Together these results suggest that increased education spending is more likely to increase growth when a larger share is devoted to subsidizing college education.

The model is stylized to allow concise results. As such, additional insights or refinements of the above might be gained from generalizations. The most obvious omission from the model is physical capital. Its inclusion would present an additional tension in each of the main experiments. In the first, if capital is more complementary with skill than labor, increasing subsidies may have a significant secondary effect by also increasing the level of physical capital investment. In the second, increasing total expenditures would have a more complex effect as the required tax increase displaces physical capital. A second meaningful generalization would be a more complex and realistic treatment of taxation. Many papers which consider fiscal policy in endogenous growth models are concerned with the growth implications of distortionary taxes and several authors have shown that the effect of productive expenditures is influenced by how such expenditures are financed.[21] As taxes are lump sum in this paper, such experiments cannot be conducted. With these modifications, the model would lend itself to a careful calibration to gauge the magnitude of the effects of policy changes.

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Appendix

Proof of Proposition 1. A mixed strategy balanced growth path can exist only if $0 \leqslant \Pi^* \leqslant 1$. The numerator of Π^* from Eq. (13) can be written as $\gamma(\widetilde{\beta}(1-g)+g\phi)$ which is positive. The denominator is positive if $\phi < \frac{\theta}{g} + \widetilde{\beta}$. If $\rho > 0$, then $\Pi^* \leqslant 1$ if the denominator exceeds the numerator. This holds when $\phi \leqslant \phi_c$. If $\rho < 0$, $\Pi^* \leqslant 1$ requires that $\phi \geqslant \phi_c$. Since $\phi_c < \frac{\theta}{g} + \widetilde{\beta}$, with $\rho > 0$, $\phi \leqslant \phi_c$ is sufficient for both $\Pi^* \geqslant 0$ and $\Pi^* \leqslant 1$. With $\rho < 0$, $\phi_c \leqslant \phi < \frac{\theta}{g} + \widetilde{\beta}$ is required.

[21] For example, this is a consideration in Glomm and Ravikumar (1998), Baier and Glomm (2001) and Blankenau and Simpson (2004).

Next consider a pure strategy balanced growth path with $\Pi^* = 1$. Substituting in for $\omega^i_{t+1}|c^i_t$ and $C^i_{t+1}|c^i_t$, Eq. (3) gives savings. Since this must equal borrowing, bond market clearing requires

$$\widetilde{\beta}(h_{t-1}\omega_{n,t} + h_{t-1}\omega_{c,t} - (1+r_t)\theta(1-\psi)\omega_{c,t-1}h_{t-2} - \tau_t) = \theta(1-\psi)\omega_{c,t}h_{t-1}.$$

As wages and the interest rates are constant with Π constant, these time subscripts are dropped. Note from Eqs. (10) and (4) that with $\Pi_t = 1$, $\psi = \frac{\theta\phi g Y_t}{\widetilde{\beta}\omega_{c,t}}$. Using this and Eq. (8), the above can be rearranged as

$$(h_{t-1}\omega_{c,t} - (1+r_t)\theta(1-\psi)\omega_{c,t-1}h_{t-2}) = \frac{\theta\omega_{c,t}h_{t-1}}{\widetilde{\beta}} - \frac{\phi g Y_t}{\widetilde{\beta}} - \omega_{n,t}h_{t-1} + gY_t. \tag{A.1}$$

Substituting in for the T_t, Eq. (1) states that $\Pi_t = 1$ is an equilibrium only if $h_{t-1}\omega_c - (1+r_t)(1-\psi)\theta\omega_{c,t}h_{t-2} \geqslant 0$. Using Eq. (A.1) this reduces to $\phi \leqslant \phi_c$. From Eq. (12) it is clear that when $\rho = 0$, a balanced growth path exists only if $\phi = \phi_c$ and the level of Π is indeterminate.

Now consider convergence locally to Π^* when $\Pi_0 \neq \Pi^*$. Write Eq. (12) as $\Pi_t = f(\Pi_{t-1})$ where $f(\Pi_{t-1}) = a_1\Pi_{t-1}^{1-\rho} + a_2\Pi_{t-1}$ with $a_1 = \frac{\gamma}{1-\gamma}\left(\frac{\widetilde{\beta}}{\theta} + \frac{g}{\theta}(\phi - \widetilde{\beta})\right)$ and $a_2 = \frac{g}{\theta}(\phi - \widetilde{\beta})$. It is straightforward to show that $a_2 \leqslant 1$ when $\phi \leqslant \frac{\theta}{g} + \widetilde{\beta}$ as it must when $\Pi^* \geqslant 0$. A first-order Taylor series expansion gives

$$\Pi_t \approx f(\Pi_{t-1})|_{\Pi_{t-1}=\Pi^*} + f'(\Pi_{t-1})|_{\Pi_{t-1}=\Pi^*}(\Pi_{t-1} - \Pi^*).$$

Note that $f(\Pi_{t-1})|_{\Pi_{t-1}=\Pi^*} = \Pi^*$ and $f'(\Pi_{t-1})|_{\Pi_{t-1}=\Pi^*} = (1-\rho)a_1(\Pi^*)^{-\rho} + a_2$. Inspection of Eq. (13) reveals that $(\Pi^*)^{-\rho} = \frac{1-a_2}{a_1}$. Thus

$$\Pi_t \approx \Pi^* + ((1-\rho)(1-a_2) + a_2)(\Pi_{t-1} - \Pi^*).$$

The behavior of the difference equation is governed by $((1-\rho)(1-a_2)+a_2)$ and there is convergence if $|((1-\rho)(1-a_2)+a_2)| < 1$. Note $((1-\rho)(1-a_2)+a_2) < 1$ holds if and only if $\rho > 0$. $((1-\rho)(1-a_2)+a_2) > -1$ holds if and only if $a_2 > \frac{\rho-2}{\rho}$ or $\phi > \frac{\rho-2}{\rho}\frac{\theta}{g} + \widetilde{\beta}$. If $\theta > g\widetilde{\beta}$ this holds for any ρ, ϕ. When $\rho = 0$, Eq. (12) reduces to $\Pi_t = \frac{\widetilde{\beta}\gamma + g(\phi - \widetilde{\beta})}{\theta(1-\gamma)}\Pi_{t-1}$. This will go to zero or infinity unless $\phi = \phi_c$. When this holds, any level of Π can be supported. □

Proof of Proposition 2. Define $Z \equiv [(1-\gamma)\Pi^\rho + \gamma]^{1/\rho}$. From Eq. (17), then,

$$\frac{\partial\lambda}{\partial\phi} = -\mu g A((1-\phi)gA)^{\mu-1}Z + ((1-\phi)gA)^{\mu}\frac{\partial Z}{\partial\phi}. \tag{A.2}$$

In a pure strategy equilibrium with $\Pi = Z = 1$, $\frac{\partial\lambda}{\partial\phi} < 0$ and $\phi = 0$ maximizes growth. Note from Eq. (13) that $(\Pi^*)^\rho(1-\gamma) = \gamma\frac{\widetilde{\beta} + g(\phi - \widetilde{\beta})}{\theta - g(\phi - \widetilde{\beta})}$. Adding γ to each side, and raising each to the power ρ^{-1} gives

$$Z = \left(\frac{\gamma(\widetilde{\beta} + \theta)}{\theta - g(\phi - \widetilde{\beta})}\right)^{1/\rho}$$

so that

$$\frac{\partial Z}{\partial \phi} = \frac{\rho^{-1} Z g}{\theta - g(\phi - \widetilde{\beta})}. \tag{A.3}$$

If $\rho < 0$, $\frac{\partial Z}{\partial \phi} < 0$, $\frac{\partial \lambda}{\partial \phi} < 0$ and $\phi = 0$ maximizes growth. When $\rho > 0$, $\frac{\partial Z}{\partial \phi} > 0$ and growth is maximized where $\frac{\partial \lambda}{\partial \phi} = 0$. Putting Eq. (A.3) into $\frac{\partial \lambda}{\partial \phi} = 0$ and simplifying shows that growth is maximized where $\phi = \phi^*$. To check second-order conditions, note $\frac{\partial \lambda}{\partial \phi}|_{\phi=0} = Z(gA)^\mu(-\mu + \frac{\rho^{-1} g}{\theta + \widetilde{\beta} g})$ which is positive if $g > \frac{\theta \mu \rho}{(1 - \mu \rho \widetilde{\beta})}$ as required for $\phi^* > 0$. Furthermore $\lim_{\phi \to 1} \frac{\partial \lambda}{\partial \phi} = -\infty$. Since $\frac{\partial \lambda}{\partial \phi} = 0$ only at $\phi = \phi^*$, $\frac{\partial \lambda}{\partial \phi}$ is decreasing in ϕ and second order conditions are met. □

Proof of Proposition 3. From Eq. (17)

$$\frac{\partial \lambda}{\partial g} = \mu A(1-\phi)(g(1-\phi)A)^{\mu-1} Z + ((1-\phi)gA)^\mu \frac{\partial Z}{\partial g}. \tag{A.4}$$

In a pure strategy equilibrium with $\Pi = Z = 1$, $\frac{\partial \lambda}{\partial g} > 0$ and the growth rate is increasing in g. When $\Pi = \Pi^*$

$$\frac{\partial Z}{\partial g} = \frac{\rho^{-1} Z(\phi - \widetilde{\beta})}{\theta - g(\phi - \widetilde{\beta})}. \tag{A.5}$$

When $\rho > 0$ and $\phi \geqslant \widetilde{\beta}$, $\frac{\partial Z}{\partial g} > 0$ and the balanced growth rate is increasing in g. When $\rho > 0$ and $\phi < \widetilde{\beta}$, $\frac{\partial Z}{\partial g} < 0$ and growth is maximized where $\frac{\partial \lambda}{\partial g} = 0$. Putting Eq. (A.5) into $\frac{\partial \lambda}{\partial g}$ gives

$$\frac{\partial \lambda}{\partial g} = Z(A(1-\phi))^\mu \left(\mu g^{\mu-1} + g^\mu \frac{\rho^{-1}(\phi - \widetilde{\beta})}{\theta - g(\phi - \widetilde{\beta})} \right).$$

Setting this to 0 and simplifying shows that growth is maximized where $g = g^*$. Since $g^* > 0$ requires $\phi \leqslant \widetilde{\beta}$ and $\phi \leqslant \phi_c$ is required for existence, $g = g^*$ is growth maximizing only if $0 \leqslant \phi \leqslant \min\{\phi_c, \widetilde{\beta}\}$.

If $\rho < 0$ and $\max\{\phi_c, \widetilde{\beta}\} \leqslant \phi \leqslant \frac{\theta}{g} + \widetilde{\beta}$, the mixed strategy balanced growth rate is again maximized at $g = g^*$. With $\rho < 0$ and $\phi \leqslant \widetilde{\beta}$, $\frac{\partial Z}{\partial g} > 0$ and the balanced growth rate is increasing in g. When $\phi > \widetilde{\beta}$, $\frac{\partial Z}{\partial g} < 0$ and growth is maximized where $\frac{\partial \lambda}{\partial g} = 0$. Putting the expression for $\frac{\partial Z}{\partial g}$ into $\frac{\partial \lambda}{\partial g} = 0$ and simplifying shows that growth is maximized where $g = g^*$. Since $\phi_c \leqslant \phi < \frac{\theta}{g} + \widetilde{\beta}$ is required for existence, $g = g^*$ is growth maximizing only if $\max\{\phi_c, \widetilde{\beta}\} \leqslant \phi < \frac{\theta}{g} + \widetilde{\beta}$.

Consider now the second-order conditions.

$$\left.\frac{\partial^2 \lambda}{\partial g}\right|_{g=g^*} = \frac{\partial Z}{\partial g} \widetilde{A} \left[\mu (g^*)^{\mu-1} + (g^*)^\mu \frac{\rho^{-1}(\phi - \widetilde{\beta})}{\theta - g^*(\phi - \widetilde{\beta})} \right]$$
$$+ Z\widetilde{A}(g^*)^\mu \left[\frac{\mu}{g^*} \frac{\rho^{-1}(\phi - \widetilde{\beta})}{\theta - g^*(\phi - \widetilde{\beta})} + \frac{\rho^{-1}(\phi - \widetilde{\beta})^2}{(\theta - g^*(\phi - \widetilde{\beta}))^2} + \frac{(\mu - 1)\mu}{(g^*)^2} \right],$$

where $\widetilde{A} \equiv (A(1-\phi))^{\mu}$. Letting $g = g^*$, the first bracketed expression is 0 and the second is < 0 if $\mu(\rho - 1) < 1 - \mu$ which always hold. Since $Z\widetilde{A}(g^*)^{\mu} > 0$, $\frac{\partial^2 \lambda}{\partial g}|_{g=g^*} < 0$; second-order conditions are met. $\square$

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