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Brain drain or brain gain: A revisit

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Abstract. Recent literature has turned to the brain gain effect, instead of the brain drain effect, that emigration may bring to a source country. This paper, however, suggests brain drain remains a likely outcome. Suppose that foreign language skill affects an individual productivity when working abroad. A brain drain may occur when the (exogenously or endogenously determined) probability of immigration is large. We also consider the case that the probability of immigration is determined by a signal, and provide a condition under which the individual will under-invest in education, which results in a brain drain for the source country.

JEL classification: F22, O15, J61

Key words: Brain gain, brain drain, probability of immigration

1. Introduction

In development literature, brain drain is referred to as the flows of high skilled immigrants from developing countries to developed countries. Earlier research found that skilled emigration tends to lower the source country employment level and thus has negative welfare implications for the source country; see Bhagwati and Hamada (1974). Kwok and Leland (1984) suggested that brain drain incurs not only the losses of high skilled labor, but due to asymmetric information the immigrants tend to be the best individuals within the high skilled group. Further analyses were provided in Lien (1987a,

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1987b) and Katz and Stark (1987). The results highlight the cost of brain drain in lost talent beyond the number of immigrants. Lien (1988) suggested that brain drain problem is worsened when developing countries emulate the discipline reward systems prevailing in developed countries.

Recent work, however, turn to the possible benefits of brain drain for the source countries. Miyagiwa (1991) considered scale economies in advanced education and demonstrated that brain drain may raise the education and income levels of the source country. Stark et al. (1997, 1998) found that skilled migration can bring the source country to a higher average level of human capital per worker. Mountford (1997) showed that, when migration is not a certainty, brain drain may increase average productivity in the source economy. Assuming the level of human capital formation in the source country is positively correlated with the probability of emigration, Vidal (1998) found, through intergenerational transfer of human capital, an incidental surge in emigration can lead the source country out of an underdevelopment trap. Finally, Stark and Wang (2002) considered production externality of the economy-wide average human capital into the model. They concluded a strictly positive probability of migration to a richer country can enhance welfare and nudge the economy toward the social optimum.

Using the data from 36 OECD countries, Beine et al. (2001) found a significantly positive impact of migration on human capital, in support of a beneficial brain. The sign of the global effect of migration on growth, however, is not confirmed. They explained this by distinguishing an ex-ante brain effect and an ex-post drain effect, and concluded that the sign of the overall effect depends on which effect dominates. Note that their empirical sample excludes developing countries. Although the choice is consistent with perfect human capital transferability assumption within their analytical framework, applications of their empirical findings to developing countries are doubtful. Intuitively, brain gains (or brain circulation) seem to be more plausible for developed countries whereas brain drains prevail in developing countries.

For example, Zhang (2001) estimated the net human capital gain and loss in accordance with China's current emigration pattern. He estimated that, with rising probability of going abroad, China has experienced a dramatic loss of human capital, in amount of 4-5 billion US dollars for the period 1978–1997.

This article presents reservations to the previous brain gain results and explains the ambiguous sign of overall immigration effect, observed by Beine et al. (2003). We show that a migration probability may lead the source country to a lower level of human capital formation and therefore a lower social welfare. The point of departure is human capital transferability across countries and observability of human capital itself. Specifically, while previous literature assumes perfect human capital transfer, we argue that the transferability is affected by the (foreign) language skill of the immigrant. Before immigration, an individual chooses education and language skill simultaneously subject to a budget constraint. Envisioning a great probability of going abroad, the substitution effect between language skill and human capital may prompt the individual to choose a high language skill and a low human capital level. The result is a less educated "Americanized" population with better language skills and lower human capital.

Acknowledging the importance of social capital in production, we can replace the language skill acquisition by host-country social capital formation in the above analysis. That is, an individual may invest in accumulation of host-country social capital (of which language is a major component) to prepare the possible immigration. The substitution between host-country social capital and "general" human capital may generate a near "Americanized" developing country with low human capital.

In a recent paper, Trum and Uebelmesser (2003) examined the relationship between human capital transferability and brain gain. Assuming the degree of human capital transferability is exogenously given, they found brain gain occurs if human capital is at least partially transferable and individuals exhibit a constant relative risk aversion smaller than 1. In contrast, our analysis allows individuals to invest and choose the degree of human capital transferability under the budget constraint.

Another factor we consider in this paper is the determination of immigration probability. Previous literature assumes either the probability is a constant or it is determined by education threshold. While education level is observable, the host country employers are more interested in the unobservable (or partially observable) "ability" of the immigrant workers. As a result, screening device may be applied to determine the probability of immigration. We assume that tests and exams (such as GRE, GMAT, LSAT, and other license certificates) are adopted for this purpose. An individual therefore may invest in these exams to promote his/her chance to go abroad. The budget constraint may lead to an over-investment in the test skills and an under-investment in education. The opportunity to go abroad leads the developing country to lower human capital formation. Finally, we extend our analysis to the case where the immigration probability is determined jointly by an individual's human capital and language skill. The result reinforces the conclusion that a brain drain may occur when the exogenously or endogenously determined probability to immigrate is large.

2. Basic framework

Consider a production economy in which only one single commodity is produced. We normalize the price of the commodity to be 1. There are N identical risk neutral workers in the source country. There is only one input in the production function, the human capital (or the education level), e. Let $f(e)$ denote the "neoclassical" production function such that $f'(e) > 0$ and $f''(e) < 0$. Assume education can be obtained at per unit cost c_1 and an individual incurs a budget constraint, m. An individual will choose the optimal education level, e^*, to maximize the objective function $f(e) - c_1 e$. Thus, we have $f'(e^*) = c_1$, that is, the marginal cost equals the marginal benefit of education.

If the individual goes abroad, he faces a concave production function, $g(e, l)$, where l is the language skill that determines the transferability of human capital. We assume $\partial g(e, l)/\partial e > f'(e)$ for any l. In particular, at $l = 0$, $\partial g(e, 0)/\partial e > f'(e)$; otherwise, no individual will go abroad. Zhang (2001) showed that the rate of return to education in China is lower than that for Chinese abroad. The above assumption, therefore, accounts for the higher productivity in the foreign country. There is also a premium or discount for living in the foreign country, C. Let c_2 denote the unit cost of language skill.

Let $p(0 \leq p \leq 1)$ denote the probability that an individual emigrates. The individual would like to choose education and language skill to maximize $p[g(e,l)+C]+(1-p)f(e)-c_1e-c_2l$. The resulting optimal choice is (e^{**}, l^{**}) such that $p[\partial g(e^{**}, l^{**})/\partial e]+(1-p)f'(e)=c_1$ and $p[\partial g(e^{**}, l^{**})/\partial l] = c_2$.

However, while education is well publicly subsidized, acquisition of language skill is usually self-financed. Due to borrowing restrictions, we assume there is a budget constraint, m and, furthermore, the above solution cannot be sustained under the budget constraint: $c_1e^{**}+c_2l^{**} > m$ (whereas $c_1e^* \leq m$). Thus, the above optimal solution is infeasible when the budget constraint is incorporated. A tradeoff between human capital and language skill takes place.

3. Brain drain or brain gain

Given the budget constraint, the first order condition is as follows:

$$p\left[\partial g\left(\hat{e},\hat{l}\right)/\partial e\right]+(1-p)f'(\hat{e})=(c_1/c_2)p\left[\partial g\left(\hat{e},\hat{l}\right)/\partial l\right], \tag{1}$$

where $(\hat{e},\hat{l})$is the optimal education and language skill for the individual such that $c_1\hat{e}+c_2\hat{l}=m$. Within the current context, a brain gain (or brain drain) occurs if $\hat{e} > e^*$ (or $\hat{e} < e^*$). Suppose that

$$\partial g\left(\hat{e},\hat{l}\right)/\partial l < (c_2/p). \tag{2}$$

The right-hand-side of Eq. (1) is smaller than c_1. Because $\partial g(e,l)/\partial e > f'(e)$, the left-hand-side of Eq. (1) is greater than $f'(\hat{e})$, implying $f'(\hat{e}) < c_1$. Note that $f'(e^*)=c_1$ and $f''(e)<0$, we have $\hat{e} > e^*$. In other words, Eq. (2) is a sufficient condition for a brain gain. Intuitively, when investment in language skill is costly (i.e., c_2 is large) or generates a small payoff (i.e., p or $\partial g(e,l)/\partial l$ is small), an individual will choose to invest more in the human capital. The following proposition provides a necessary and sufficient condition for brain gain (and brain drain).

Proposition 1. *A brain gain, $\hat{e} > e^*$, occurs if and only if the following condition is met:*

$$p < \left[\frac{1}{c_2}\frac{\partial g\left(\hat{e},\hat{l}\right)}{\partial l}-\frac{1}{c_1}\frac{\partial g\left(\hat{e},\hat{l}\right)}{\partial e}+1\right]^{-1}. \tag{3}$$

It can be shown that the right-hand-side of Eq. (3) lies between 0 and 1. Thus, there exists $\hat{p} \in [0,1]$ such that brain gain (or brain drain) occurs whenever the probability of going abroad is less (or greater) than $\hat{p}$.

Thus, brain drain occurs when language skill is highly productive when working in the foreign country such that the individual is willing to take a gamble by investing less in education and more in language acquisition. The tradeoff is required by the binding budget constraint. Assuming human capital is a normal good, an increase in the budget produces an income effect promoting investment in human capital.

Proposition 2. *As budget constraint becomes less stringent, the individual will increase human capital investment:* $\partial\hat{e}/\partial m > 0$.

If the source country is richer, then the individual will invest more in human capital in the presence of immigration possibilities. For a poor source country, the individual invests less in human capital. Therefore, brain drain is more likely to occur for poor source countries.

4. Investment in test skills

In the previous analysis, we follow the literature assuming the probability of immigration is exogenously determined. In reality, the probability is frequently affected by the traits of an individual. Interviews and tests are conducted to determine the preference ordering among possible immigrants. An individual therefore is presented with incentives to improve test skills or to adopt foreign cultural traits. This section evaluates the possibility of brain drains versus brain gains when test skill investment is taken into account.

Suppose that the immigration probability p is a strictly increasing function of test skills, t such that $p = h(t)$with $h'(.) > 0$. The better test skills an individual has (for GRE, TOEFL, and others), the more likely for him to go abroad. Let $c_3$denote the unit cost of test skill. To isolate the effects of test skills from language skills, we abstract language skills from the production function in the foreign country. That is, foreign production function is assumed to be $g(e)$, whereas domestic production function remains to be $f(e)$. The assumption implies that test skill serves only signaling purposes; it has no effect on an individual's productivity.

The individual will choose optimal education and test skill, $\tilde{e}$ and $\tilde{t}$, to maximize $h(t)[g(e) + C] + [1 - h(t)]f(e)$ subject to the budget constraint: $c_1 e + c_3 t \leq m$. The resulting first order condition is:

$$h(\tilde{t})g'(\tilde{e}) + [1 - h(\tilde{t})]f'(\tilde{e}) = (c_1/c_3)h'(\tilde{t})[g(\tilde{e}) + C - f(\tilde{e})]. \tag{4}$$

Let $\tilde{p} = h(\tilde{t})$ and let A denote the right-hand-side of the above equation. We have the following proposition.

Proposition 3. A brain drain, $\tilde{e} < e^*$, occurs if and only if the following condition is met:

$$\tilde{p} < \frac{A - c_1}{g'(\tilde{e}) - c_1}. \tag{5}$$

Note that A is proportional to the marginal effect of the test skill on immigration probability. When the marginal effect is large, an individual will reduce human capital investment and replace by improving his test skill to explore the benefits from immigration. The substitution effect leads to brain drain. On the other hand, when the marginal effect of test skill is small, the individual will invest more in human capital. The possibility of immigration brings a brain gain to the source country.

Alternatively, the test skill is likely a function of language proficiency and human capital. Thus, the probability of immigration is jointly determined by education level and language skills. The individual then chooses e and l to maximize the objective function: $p(e,l)[g(e,l)+C]+(1-p(e,l))f(e)-c_1e-c_2l$ under the same budget constraint.

Proposition 4. *A brain drain, $\hat{e}<e^*$, is more likely to occur if the endogenously determined probability to immigrate is large provided the budget constraint is binding.*

Allowing for endogenous probability of immigration keeps our previous conclusions fundamentally intact. When the budget constraint is not binding, the new opportunity to go abroad always results in a brain gain. However, with a binding budget, a brain drain occurs when the endogenously determined probability of immigration is sufficiently large. Specifically, a binding budget constraint imposes the individual to choose between human capital and other alternatives. A high (exogenous) probability of immigration promotes the individual to acquire language skills in place of human capital when the language acquisition cost is low. On the other hand, a high endogenous probability is supported by abundant test skills, implying low human capital.

5. Cost of acquiring language skill

It may be argued that the cost of acquiring language skills is inversely related to an individual's human capital. An individual with higher education can acquire language skill with less effort. Mathematically, the language acquiring cost is $c_2(e)$ such that $c_2'(e)<0$. The original first-order-condition, i.e., Eq. (1), is modified to:

$$p\left[\partial g\left(\hat{e},\hat{l}\right)/\partial e\right]+(1-p)f'(\hat{e})=\left[\left(c_1+c_2'(\hat{e})l\right)/c_2(\hat{e})\right]p\left[\partial g\left(\hat{e},\hat{l}\right)/\partial l\right] \quad (6)$$

A sufficient condition to ensure $f'(\hat{e})<c_1$ is $\partial g(\hat{e},\hat{l})/\partial l<[c_1c_2(e)]/[p(c_1+c_2'(e)l)]$. Note that $f'(e^*)=c_1$ and $f''(e)<0$, we have $\hat{e}>e^*$(i.e., a brain gain) if p is small. In other words, our qualitative conclusion remains valid with this cost modification: a brain drain is likely to occur when the probability of immigration is sufficiently large.

6. Further considerations

Stark and Wang (2002) included human capital externality into the production function. They showed that absent immigration opportunity an individual would invest less in human capital than the social optimal level. Immigration probability may help correct the under investment. Incorporating the externality, the domestic production function becomes $f(e)+F(e_H)$, where e_H is the average human capital. As the individual does not take the externality into account in his decision-making process, the results presented in the previous sections remain valid.

Mountford (1997) and Stark and Wang (2002) allow for heterogeneity of workforce. Allowing heterogeneous work force does not affect our results. For example, assume there are two groups of individuals. Proposition 1 provides a condition under which brain drain will occur. If this condition holds for both groups, then the brain drain appears in the source country.

Similarly, if neither group satisfies this condition, the source country has a brain gain. In case the condition is valid for one and only one group, further conditions are required to predict whether a brain drain or a brain gain will occur.

7. A specific example

To illustrate our brain drain versus brain gain results, we adopt the logarithmic production function from Stark and Wang (2002) for the source country. That is, we assume $f(e) = \alpha \ln(e+1)$, where $\alpha > c_1$ denotes the positive (private) return to education. Human capital externality is represented by $F(e_H) = \eta \ln(e_H + 1)$, where $\eta > 0$. To account for the interaction between education and language skills in the foreign country production function, we adopt a CES production function $g(e, l) = [\beta e^{0.5} + \gamma l^{0.5}]^2 + C$. We choose the values of α, β, and γ to ensure the private return to education in the foreign country be greater than that in the source country, and thus providing incentives for emigration.

Given the above assumptions, without immigration opportunity, an individual will choose the optimal education level as follows: $e^* = (\alpha/c_1) - 1$. The social optimal education level is $e^{**} = [(\alpha + \eta)/c_1] - 1 > e^*$. With an exogenously given immigration probability, the individual choose his optimal education and language skill as $(\hat{e}, \hat{l})$. We solve the expected utility maximization problem numerically.

Based upon the analysis in Sect. 3, there exists a certain $\hat{p}$ in [0, 1], such that $\hat{e} < e^*$ if $p > \hat{p}$ and vice versa. In Fig. 1, we display the relationship between $\hat{e}$ and the probability of immigration, p, assuming the following parameter values: $\alpha = 0.4$, $\beta = 0.45$, $\gamma = 0.35$, $\eta = 0.3$, $c_1 = 0.1$, $c_2 = 0.05$, and $m = 0.5$. Also C is chosen to be 100 to ensure that immigration is always desirable.

Figure 1 shows that brain gain occurs when the probability of immigration is less than 0.43; otherwise, we observe a brain drain. It also shows that the social optimal level of education is not attainable under immigration. The result is generated by the large human capital externality assumed in this numerical setup. Consequently, the brain gain under immigration is not

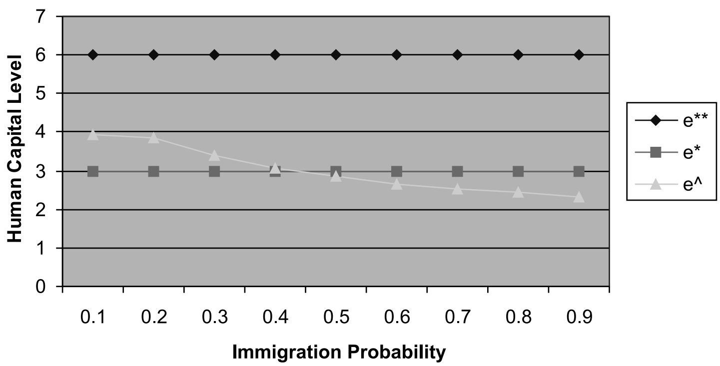

Fig. 1. A numerical illustration ($\eta = 0.3$)

sufficient to overcome the difference between the social optimum and the inefficient market outcome. On the other hand, if the externality parameter η is small, for example, $\eta = 0.05$, the social optimal is achieved or even exceeded when the probability of immigration is below 0.27; see Fig. 2.

8. Conclusions

Instead of concerns for brain drain, recent literature has turned to the brain gain effect that emigration may bring to a source country. This paper, however, suggests brain drain remains a likely outcome. More specifically, we provide conditions to help predict whether a brain drain or a brain gain will occur when a country accommodates for immigration opportunity.

Our results are driven by the interactions of imperfect human capital transferability, investment in improving immigration probability and the budget constraint. When language skill affects the degree of human capital transferability, an individual has to choose between investing in human capital or language skill facing a budget constraint. Similarly, an individual can improve his expected utility by increasing his chance to go abroad instead of human capital accumulation. If the resource is sufficient, neither investment needs to be sacrificed. The budget constraint may lead to reduction in human capital investment.

Suppose that foreign language skill affects an individual productivity when working abroad. A brain drain may occur when the (exogenously determined) probability of immigration is large. It enhances the incentives for an individual to replace human capital investment by language skill acquisition. When the probability is small, we observe a brain gain. As expected, high return to language skill, low cost of language acquisition, tight budget constraint, all contribute to the occurrence of a brain drain.

We also consider the case that the probability of immigration is determined by a signal (such as a test or an interview). Once again we provide a condition under which the individual will under invest in education by improving his test skills. It results in a brain drain for the source country.

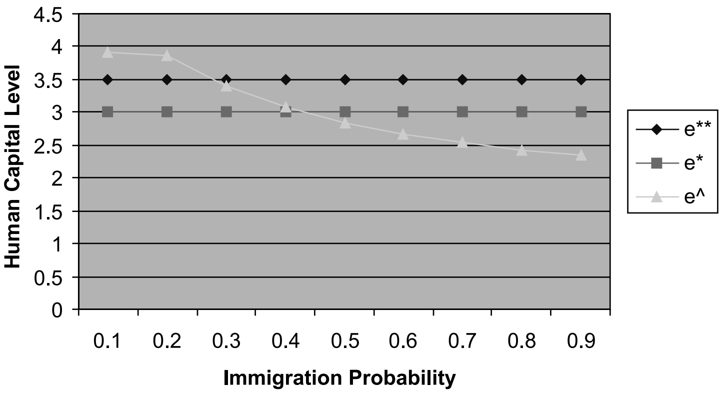

Fig. 2. A numerical illustration ($\eta = 0.05$)

Appendix

Proof for Proposition 1

From the first-order-condition in (1), we can solve for $f'(\hat{e})$:

$$f'(\hat{e}) = [p/(1-p)]\{(c_1/c_2)\left[\partial g\left(\hat{e},\hat{l}\right)/\partial l\right] - \left[\partial g\left(\hat{e},\hat{l}\right)/\partial e\right]\} \tag{A1}$$

For emigration policy to have the brain gain effect, we must have $\hat{e} > e^*$, that is, $f'(\hat{e}) < c_1$. Combining this with (A1), we get the necessary and sufficient condition for brain gain, $\hat{e} > e^*$ as displayed in Eq. (3). From (A1), we have

$$(c_1/c_2)\left[\partial g\left(\hat{e},\hat{l}\right)/\partial l\right] = [(1-p)/p]f'(\hat{e}) + \left[\partial g\left(\hat{e},\hat{l}\right)/\partial e\right], \tag{A2}$$

which in turn implies $[\partial g(\hat{e},\hat{l})/\partial l]/c_2 - [\partial g(\hat{e},\hat{l})/\partial e]/c_1 = [(1-p)/pc_1]$ $f'(\hat{e}) > 0$. It then follows that $[\partial g(\hat{e},\hat{l})/\partial l]/c_2 - [\partial g(\hat{e},\hat{l})/\partial e]/c_1 + 1 > 1$. The right-hand-side of Eq. (3) therefore satisfies $0 < \{[\partial g(\hat{e},\hat{l})/\partial l]/c_2 - [\partial g(\hat{e},\hat{l})/\partial e]/c_1 + 1\}^{-1} < 1$. As a consequence, there exists $\hat{p} = \{[\partial g(\hat{e},\hat{l})/\partial l]/c_2 - [\partial g(\hat{e},\hat{l})/\partial e]/c_1 + 1\}^{-1} \in [0,1]$ such that brain gain (or brain drain) occurs whenever the probability of going abroad is less (or greater) than $\hat{p}$.

Proof for Proposition 2

Using subscripts to denote partial derivatives, the first order condition in (1) can be written as:

$$pg_e\left(\hat{e},\hat{l}\right) + (1-p)f'(\hat{e}) = (c_1/c_2)pg_l\left(\hat{e},\hat{l}\right) \tag{A3}$$

Now, take the partial derivative with respect to m for both $\hat{e}$ and $\hat{l}$ in Eq. (A3) and simplify, we derive

$$[pg_{ee} + (1-p)f'' - (c_1/c_2)pg_{le}](\partial\hat{e}/\partial m) = [(c_1/c_2)pg_{ll} - pg_{el}]\left(\partial\hat{l}/\partial m\right) \tag{A4}$$

Also, take the partial derivative with respect to m in the budget constraint,

$$c_1(\partial\hat{e}/\partial m) + c_2\left(\partial\hat{l}/\partial m\right) = 1 \tag{A5}$$

Combine (A4) and (A5) and solve for $\partial\hat{e}/\partial m$ as follows:

$$\{[pg_{ee} + (1-p)f'' - (c_1/c_2)pg_{le}] + (c_1/c_2)[(c_1/c_2)pg_{ll} - pg_{el}]\}(\partial\hat{e}/\partial m) = [(c_1/c_2)pg_{ll} - pg_{el}]/c_2$$

The production functions satisfy $f' > 0, f'' < 0, g_e > 0, g_l > 0, g_{ee} < 0, g_{ll} < 0$, and $g_{el} = g_{le} > 0$. Thus, $[pg_{ee} + (1-p)f'' - (c_1/c_2)pg_{le}] + (c_1/c_2)[(c_1/c_2)pg_{ll} - pg_{el}] < 0$ and $(c_1/c_2)pg_{ll} - pg_{el} < 0$. We conclude that, as budget constraint becomes less stringent, the individual will increase human capital investment: $\partial\hat{e}/\partial m > 0$.

Proof for Proposition 3

Let $\tilde{p} = h(\tilde{t})$ and let $A = (c_1/c_3)h'(\tilde{t})[g(\tilde{e}) + C - f(\tilde{e})]$. We rewrite the first order condition, i.e., Eq. (4) as follows:

$$\tilde{p}g'(\tilde{e}) + [1-\tilde{p}]f'(\tilde{e}) = A \tag{A6}$$

Note that the first order condition requires A be positive, as the left-hand-side of (A6) is always positive. By assumption, $h' > 0$. In addition, to induce immigration, it requires $g(\tilde{e}) + C > f(\tilde{e})$. From the first order condition and the budget constraint $c_1 e + c_3 t = m$, we solve for the optimal $\tilde{e}$ and $\tilde{t}$. Suppose that $\tilde{e} < e^*$ (i.e., brain drain prevails). From (A6), $f'(\tilde{e}) = [A - \tilde{p}g'(\tilde{e})]/(1-\tilde{p})$. Because $f'(\tilde{e}) > f'(e^*) = c_1$ and $g'(\tilde{e}) > f'(\tilde{e}) > c_1$, we have $f'(\tilde{e}) = [A - \tilde{p}g'(\tilde{e})]/(1-\tilde{p}) > c_1$, implying $[g'(\tilde{e}) - c_1]\tilde{p} < A - c_1$. Moreover, from (A6), we have $A - c_1 = \tilde{p}[g'(\tilde{e}) - f'(\tilde{e})] + f'(\tilde{e}) - c_1 > 0$. Thus,

$$\tilde{p} < \frac{A - c_1}{[g'(\tilde{e}) - c_1]} \tag{A7}$$

Because $A - g'(\tilde{e}) = (1-\tilde{p})[f'(\tilde{e}) - g'(\tilde{e})] < 0$, $A - c_1 < g'(\tilde{e}) - c_1$. There exists a number $\tilde{\tilde{p}} = (A - c_1)/[g'(\tilde{e}) - c_1] \in [0, 1]$ such that emigration will have brain drain effect on the source country.

Proof for Proposition 4

The individual will choose e and l to maximize the following objective function: $p(e,l)[g(e,l) + C] + [1 - p(e,l)]f(e) - c_1 e - c_2 l$,subject to the budget constraint $c_1 e + c_2 l \le m$. Let λ denote the Lagrangian multiplier. The resulting first order conditions are as follows.

$$p_e[g(e,l) + C - f(e)] + p(e,l)[g_e - f_e] + f_e = (1+\lambda)c_1 \tag{A8}$$

$$p_l[g(e,l) + C - f(e)] + p(e,l)g_l = (1+\lambda)c_2 \tag{A9}$$

Let $(\bar{e}, \bar{l})$denote the solution. If the budget constraint is not binding, $\lambda = 0$. In addition, let $p_{\bar{e}} = p_e(\bar{e}, \bar{l}), g_{\bar{e}} = g_e(\bar{e}, \bar{l})$,and let $f_{\bar{e}} = f'(\bar{e})$. From (A8), we derive

$$g(\bar{e}, \bar{l}) + C - f(\bar{e}) = (1/p_{\bar{e}})\left[c_1 - p(\bar{e}, \bar{l})(g_{\bar{e}} - f_{\bar{e}}) - f_{\bar{e}}\right]. \tag{A10}$$

Because $g(\bar{e}, \bar{l}) + C - f(\bar{e}) > 0$ and $g_{\bar{e}} > f_{\bar{e}}$, we have $f_{\bar{e}} < c_1$. Thus, only brain gain occurs.

If, instead, the budget constraint is binding, $\lambda \ne 0$. Dividing (A8) by (A9) leads to

$$c_2 f_{\bar{e}} - p(\bar{e}, \bar{l})(c_1 g_{\bar{l}} - c_2 g_{\bar{e}} + c_2 f_{\bar{e}}) = \left[g(\bar{e}, \bar{l}) + C - f(\bar{e})\right](c_1 p_{\bar{l}} - c_2 p_{\bar{e}}). \tag{A11}$$

Let $\bar{p} = p(\bar{e}, \bar{l})$, we have four possible cases:

Case I. $c_1/c_2 < p_{\bar{e}}/p_{\bar{l}}$ and $c_1/c_2 < g_{\bar{e}}/g_{\bar{l}}$

Herein $f_{\bar{e}} < (\bar{p}/(1-\bar{p}))g_{\bar{l}}[c_1/c_2 - g_{\bar{e}}/g_{\bar{l}}] < 0$, contradicting to the assumptions.

Case II. $g_{\bar{e}}/g_{\bar{l}} < c_1/c_2 < p_{\bar{e}}/p_{\bar{l}}$

In this case, $f_{\bar{e}} < (\bar{p}/(1-\bar{p}))g_{\bar{l}}[c_1/c_2 - g_{\bar{e}}/g_{\bar{l}}] < (\bar{p}/(1-\bar{p}))g_{\bar{l}}[p_{\bar{e}}/p_{\bar{l}} - g_{\bar{e}}/g_{\bar{l}}]$. Thus, $(\bar{p}/(1-\bar{p}))g_{\bar{l}}[p_{\bar{e}}/p_{\bar{l}} - g_{\bar{e}}/g_{\bar{l}}] < c_1$ is a sufficient condition for brain gain.

Case III. $p_{\bar{e}}/p_{\bar{l}} < c_1/c_2 < g_{\bar{e}}/g_{\bar{l}}$

Herein $f_{\bar{e}} > (\bar{p}/(1-\bar{p}))g_{\bar{l}}[c_1/c_2 - g_{\bar{e}}/g_{\bar{l}}] > (\bar{p}/(1-\bar{p}))g_{\bar{l}}[p_{\bar{e}}/p_{\bar{l}} - g_{\bar{e}}/g_{\bar{l}}]$. As a result, $(\bar{p}/(1-\bar{p}))g_{\bar{l}}[p_{\bar{e}}/p_{\bar{l}} - g_{\bar{e}}/g_{\bar{l}}] > c_1$ is a sufficient condition for brain drain.

Case IV. $c_1/c_2 > p_{\bar{e}}/p_{\bar{l}}$ and $c_1/c_2 > g_{\bar{e}}/g_{\bar{l}}$

Now, $f_{\bar{e}} > (\bar{p}/(1-\bar{p}))g_{\bar{l}}[c_1/c_2 - g_{\bar{e}}/g_{\bar{l}}]$. If $p_{\bar{e}}/p_{\bar{l}} < g_{\bar{e}}/g_{\bar{l}} < c_1/c_2$ and $f_{\bar{e}} > [\bar{p}/(1-\bar{p})]g_{\bar{l}}[(c_1/c_2) - (g_{\bar{e}}/g_{\bar{l}})\} > 0$, then both brain drain and brain gain are possible. If, instead, $g_{\bar{e}}/g_{\bar{l}} < p_{\bar{e}}/p_{\bar{l}} < c_1/c_2$, then $(\bar{p}/(1-\bar{p}))g_{\bar{l}}[p_{\bar{e}}/p_{\bar{l}} - g_{\bar{e}}/g_{\bar{l}}] > c_1$ is a sufficient condition for brain drain.

In sum, the higher $\bar{p}$, $g_{\bar{l}}$, and $p_{\bar{e}}/p_{\bar{l}} - g_{\bar{e}}/g_{\bar{l}}$ are, the more likely it is for brain drain to occur.

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