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Strategic candidacy, monotonicity, and strategy-proofness

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Abstract

We show that any voting rule satisfying unanimity and candidate stability (meaning that no candidate gains by withdrawing from the election) satisfies strategy-proofness. It follows that Gibbard–Satterthwaite theorem implies candidate stability theorem of Dutta, Jackson, and Le Breton.

Keywords: Election; Political economy; Social choice; Axiomatic approach

JEL classification: D71; D72

1. Introduction

In elections, the outcomes are affected by candidates' strategic entry decisions. Consider a voting rule that selects a candidate running in the election, given the set of entering candidates and the preferences of voters. Dutta et al. (2001) show that if the rule satisfies unanimity and candidate stability (which means that no candidate gains by withdrawing from the election), then the rule is

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dictatorial. The present paper proves that any voting rule satisfying unanimity and candidate stability satisfies monotonicity (in the sense that if any voter changes his preference by raising the ranking of the winner of the election, then the same candidate is the winner under the new preference profile). Given the equivalence of monotonicity and strategy-proofness (Muller and Satterthwaite, 1977), our result shows that Gibbard–Satterthwaite theorem can be used to provide another proof of the result due to Dutta, Jackson, and Le Breton.

2. Preliminaries

Let A^* be a finite set of potential candidates with $|A^*| \geq 3$. The collection of all non-empty subsets of A^* is denoted by $\mathcal{A} = 2^{A^*} \setminus \emptyset$. Let V be a finite set of voters. We restrict our attention to the case where no candidate is a voter, i.e., $A^* \cap V = \emptyset$. Denote by $N = A^* \cup V$ the set of agents.

Each agent i has a complete and transitive preference relation R_i defined over A^*. As usual, aR_ib means that a is at least as good as b for agent i. The associated strict preference relation is denoted by P_i. We assume that preferences are strict, i.e., if aR_ib and $a \neq b$ then aP_ib. For all $A \in \mathcal{A}$, top (A, R_i) denotes the most preferred candidate in A for R_i. For all $A \in \mathcal{A}$, let $R_i|_A$ denote the preference relation over A induced by R_i. Let $\mathcal{R}_i$ be the set of admissible preferences of agent i. Each candidate is assumed to rank himself first; for all $a \in A^*$ and $R_a \in \mathcal{R}_a$, we have top $(A^*, R_a) = a$. Each voter may rank candidates in any strict order. A preference profile is denoted by $R = (R_i)_{i \in N}$. Denote by (R'_i, R_{-i}) the preference profile obtained by replacing R_i of R with R'_i. Let $\mathcal{R}$ be the set of admissible preference profiles. Denote by $R|_A$ the preference profile over A induced by R. The strict upper contour set for $R_i \in \mathcal{R}_i$ at $a \in A^*$ is denoted by $U(a, R_i) = \{b \in A^* : bP_ia\}$. Similarly, the strict lower contour set is denoted by $L(a, R_i) = \{b \in A^* : aP_ib\}$.

A voting function φ: $\mathcal{A} \times \mathcal{R} \rightarrow A^*$ is a function with the following properties.

2.1. Feasibility

For all $A \in \mathcal{A}$ and $R \in \mathcal{R}$, we have $\varphi(A, R) \in A$.

2.2. Independence of non-voters' preferences

For all $A \in \mathcal{A}$, $R \in \mathcal{R}$, and $R' \in \mathcal{R}$ such that $R_i = R'_i$ for all $i \in V$, we have $\varphi(A, R) = \varphi(A, R')$.

2.3. Independence of infeasible alternatives

For all $A \in \mathcal{A}$, $R \in \mathcal{R}$, and $R' \in \mathcal{R}$ such that $R|_A = R'|_A$, we have $\varphi(A, R) = \varphi(A, R')$.

The first argument of $\varphi(A, R)$ is the set of candidates who enter the election. The second argument is the preference profile. Feasibility says that φ selects a candidate who is in the race. Independence of non-voters' preferences says that φ depends only on the preferences of voters. Independence of infeasible alternatives says that φ depends only on the preferences over the candidates in the race. The selection by φ is referred to as the winner of the election.

We investigate voting functions with some of the following axioms. The first two axioms are introduced by Dutta et al. (2001). Candidate stability says that no candidate gains by withdrawing from the election. Strong candidate stability says that no losing candidate can affect the outcome of the election by withdrawing from it. The other axioms are standard.

2.4. Candidate stability

For all $a \in A^*$ and $R \in \mathcal{R}$, we have $\varphi(A^*, R)\ R_a \varphi(A^* \setminus \{a\}, R)$.

2.5. Strong candidate stability

For all $a \in A^*$ and $R \in \mathcal{R}$ such that $a \neq \varphi(A^*, R)$, we have $\varphi(A^*, R) = \varphi(A^* \setminus \{a\}, R)$.

2.6. Unanimity

For all $R \in \mathcal{R}$, if top $(A^*, R_i) = a$ for all $i \in V$ then $\varphi(A^*, R) = a$.

2.7. Efficiency

For all $R \in \mathcal{R}$, there does not exist $a \in A^*$ such that $aP_i\ \varphi(A^*, R)$ for all $i \in V$.

2.8. Monotonicity

For all $R \in \mathcal{R}$, $i \in V$, and $R_i' \in \mathcal{R}_i$ such that $L(\varphi(A^*, R), R_i) \subseteq L(\varphi(A^*, R), R_i')$, we have $\varphi(A^*, R) = \varphi(A^*, (R_i', R_{-i}))$.

2.9. Strategy-proofness

For all $R \in \mathcal{R}$, $i \in V$, and $R_i' \in \mathcal{R}_i$, we have $\varphi(A^*, R)\ R_i \varphi(A^*, (R_i', R_{-i}))$.

2.10. Dictatorship

Given $A \in \mathcal{A}$, $\varphi(A, \cdot)$ is dictatorial if there exists a dictator $i \in V$ such that $\varphi(A, R) = \text{top}(A, R_i)$ for all $R \in \mathcal{R}$.

3. Results

To obtain our main result, Proposition 1, we prove series of lemmas first. It should be reminded that no candidate is a voter, and each voter may rank candidates in any strict order. The first lemma is due to Dutta et al. (2001).

Lemma 1. *If φ satisfies candidate stability, then φ satisfies strong candidate stability.*

The proof uses the fact that if $a \neq \varphi(A^*, R) \neq \varphi(A^* \setminus \{a\}, R)$ then we can find $R_a' \in \mathcal{R}_a$ such that $\varphi(A^* \setminus \{a\}, (R_a', R_{-a}))\ P_a' \varphi(A^*, (R_a', R_{-a}))$ since the change from R_a to R_a' does not affect φ. The next lemma says that even if any voter changes his preference, the winner of the election remains unchanged as long as the strict upper contour set for the voter at the winner is unchanged.

Lemma 2. *Suppose that φ satisfies candidate stability. For all $R \in \mathcal{R}$, $i \in V$, and $R_i' \in \mathcal{R}_i$, if $U(\varphi(A^*, R), R_i) = U(\varphi(A^*, R), R_i')$ then $\varphi(A^*, R) = \varphi(A^*, (R_i', R_{-i}))$.*

Proof. Consider two candidates $a, b \in A^* \setminus \{\varphi(A^*, R)\}$ such that R_i ranks a right above b. Suppose that R_i' is obtained from R_i by switching i's preference between a and b.

$$\begin{cases} R_i : \ldots, a, b, \ldots, \varphi(A^*, R), \ldots \\ R_i' : \ldots, b, a, \ldots, \varphi(A^*, R), \ldots \end{cases} \quad \text{or} \quad \begin{cases} R_i : \ldots, \varphi(A^*, R), \ldots, a, b, \ldots \\ R_i' : \ldots, \varphi(A^*, R), \ldots, b, a, \ldots \end{cases}$$

We claim that $\varphi(A^*, R) = \varphi(A^*, (R_i', R_{-i}))$. Choose $x \in \{a, b\}$, $x \neq \varphi(A^*, (R_i', R_{-i}))$. Strong candidate stability implies that $\varphi(A^*, R) = \varphi(A^* \setminus \{x\}, R)$ and $\varphi(A^*, (R_i', R_{-i})) = \varphi(A^* \setminus \{x\}, (R_i', R_{-i}))$. Since $R_i|_{A^*\{x\}} = R_i'|_{A^*\{x\}}$, independence of infeasible alternatives implies that $\varphi(A^* \setminus \{x\}, R) = \varphi(A^* \setminus \{x\}, (R_i', R_{-i}))$. Hence $\varphi(A^*, R) = \varphi(A^*, (R_i', R_{-i}))$.

Consider next any $R_i, R_i' \in \mathcal{R}_i$ such that $U(\varphi(A^*, R), R_i) = U(\varphi(A^*, R), R_i')$. Note that R_i' is obtained from R_i by iterations of switching i's preference between two neighboring candidates within the strict upper or lower contour sets. By the previous claim, $\varphi(A^*, R) = \varphi(A^*, (R_i', R_{-i}))$. □

Lemma 3. *If φ satisfies candidate stability and unanimity, then φ is efficient.*

Proof. Suppose that for some $R \in \mathcal{R}$, there exists $a \in A^*$ such that $aP_i\varphi(A^*, R)$ for all $i \in V$. For each voter i, let $R_i' \in \mathcal{R}_i$ be such that $\mathrm{top}(A^*, R_i) = a$ and $R_i|_{A^* \setminus \{a\}} = R_i'|_{A^* \setminus \{a\}}$.

$$\begin{cases} R_i : \ldots, a, \ldots, \varphi(A^*, R), \ldots \\ R_i' : a, \ldots\ldots, \varphi(A^*, R), \ldots \end{cases}$$

For each candidate c, let $R_c' = R_c$. By Lemma 2, $\varphi(A^*, R) = \varphi(A^*, R')$. By unanimity, $\varphi(A^*, R') = a$. Hence $\varphi(A^*, R) = a$, in contradiction with the fact that $aP_i\varphi(A^*, R)$ for all $i \in V$. □

The next lemma says that if any voter changes his preference by moving the ranking of a candidate from above to below the winner (or from below to above the winner), then either the winner is unchanged or the relocated candidate becomes the new winner.

Lemma 4. *Suppose that φ satisfies candidate stability. For all $R \in \mathcal{R}$, $i \in V$, and $R_i' \in \mathcal{R}_i$, if*

$$U(\varphi(A^*, R), R_i) \cap L(\varphi(A^*, R), R_i') = \{a\} \quad \text{and} \quad R_i|_{A^* \setminus \{a\}} = R_i'|_{A^* \setminus \{a\}}$$
$$\text{or} \quad L(\varphi(A^*, R), R_i) \cap U(\varphi(A^*, R), R_i') = \{a\} \quad \text{and} \quad R_i|_{A^* \setminus \{a\}} = R_i'|_{A^* \setminus \{a\}}$$

then $\varphi(A^*, (R_i', R_{-i})) \in \{\varphi(A^*, R), a\}$.

Proof. Consider R_i and R_i' as described above.

$$\begin{cases} R_i : \ldots, a, \ldots, \varphi(A^*, R), \ldots\ldots \\ R_i' : \ldots\ldots, \varphi(A^*, R), \ldots, a, \ldots \end{cases} \quad \text{or} \quad \begin{cases} R_i : \ldots\ldots, \varphi(A^*, R), \ldots, a, \ldots \\ R_i' : \ldots, a, \ldots, \varphi(A^*, R), \ldots\ldots \end{cases}$$

Suppose that $\varphi(A^*, (R_i', R_{-i})) \neq a$. By strong candidate stability, $\varphi(A^*, R) = \varphi(A^* \setminus \{a\}, R)$ and $\varphi(A^*, (R_i', R_{-i})) = \varphi(A^* \setminus \{a\}, (R_i', R_{-i}))$. By independence of infeasible alternatives, $\varphi(A^* \setminus \{a\}, R) = \varphi(A^* \setminus \{a\}, (R_i', R_{-i}))$. Hence $\varphi(A^*, (R_i', R_{-i})) = \varphi(A^*, R)$. □

The next lemma is a strengthening of Lemma 4; if any voter changes his preference by moving the ranking of a candidate from above to below the winner, then the winner is unchanged.

Lemma 5. *Suppose that φ satisfies candidate stability and unanimity. For all $R \in \mathcal{R}$, $i \in V$, and $R_i' \in \mathcal{R}_i$, if $U(\varphi(A^*, R), R_i) \cap L(\varphi(A^*, R), R_i') = \{a\}$ then $\varphi(A^*, (R_i', R_{-i})) = \varphi(A^*, R)$.*

Proof. Take any $R \in \mathcal{R}$, $i \in V$, and $R_i' \in \mathcal{R}_i$ such that $U(\varphi(A^*, R), R_i) \cap L(\varphi(A^*, R), R_i') = \{a\}$. Let $x = \varphi(A^*, R)$ and choose $b \in A^* \setminus \{a, x\}$. Let V_1, V_2, V_3, and V_4 be the partition of V such that (i) for all $j \in V_1$, aP_jx and bP_jx, (ii) for all $k \in V_2$, aP_kx and xP_kb, (iii) for all $\ell \in V_3$, $xP_\ell a$ and $bP_\ell x$, and (iv) for all $h \in V_4$, xP_ha and xP_hb. Note that either $i \in V_1$ or $i \in V_2$. For each candidate c, let $R_c^1 = R_c^2 = R_c^3 = R_c$. Consider the following profiles R^1, R^2, and R^3 such that $R^1|_{A^* \setminus \{a,b\}} = R^2|_{A^* \setminus \{a,b\}} = R^3|_{A^* \setminus \{a,b\}} = R|_{A^* \setminus \{a,b\}}$.

for all $j \in V_1 \setminus \{i\}$:	$R_j^1 : \ldots, b, a, x, \ldots$	$R_j^2 : \ldots, b, a, x, \ldots$	$R_j^3 : \ldots, b, a, x, \ldots$
for all $k \in V_2 \setminus \{i\}$:	$R_k^1 : \ldots, a, x, b, \ldots$	$R_k^2 : \ldots, a, b, x, \ldots$	$R_k^3 : \ldots, a, x, b, \ldots$
for all $\ell \in V_3$:	$R_\ell^1 : \ldots, b, x, a, \ldots$	$R_\ell^2 : \ldots, b, x, a, \ldots$	$R_\ell^3 : \ldots, b, x, a, \ldots$
for all $h \in V_4$:	$R_h^1 : \ldots, x, b, a, \ldots$	$R_h^2 : \ldots, b, x, a, \ldots$	$R_h^3 : \ldots, x, b, a, \ldots$
if $i \in V_1$:	$R_i^1 : \ldots, b, a, x, \ldots$	$R_i^2 : \ldots, b, a, x, \ldots$	$R_i^3 : \ldots, b, x, a, \ldots$
if $i \in V_2$:	$R_i^1 : \ldots, a, x, b, \ldots$	$R_i^2 : \ldots, a, b, x, \ldots$	$R_i^3 : \ldots, x, a, b, \ldots$

We obtain R^1 from R by adjusting each voter's preference rankings of a and b without changing his strict upper contour set at x. By Lemma 2, $\varphi(A^*, R^1) = \varphi(A^*, R) = x$. Next, we obtain R^2 from R^1 by switching the rankings of x and b for all voters in V_2 and V_4. By Lemma 4, $\varphi(A^*, R^2) \in \{x, b\}$, and efficiency implies that $\varphi(A^*, R^2) = b$ since all voters prefer b to x under R^2. Finally, we obtain R^3 from R^1 by switching i's rankings of a and x. By Lemma 4, $\varphi(A^*, R^3) \in \{x, a\}$. To show that $\varphi(A^*, R^3) = x$, suppose to the contrary that $\varphi(A^*, R^3) = a$. By strong candidate stability, $\varphi(A^* \{x\}, R^3) = a \neq b = \varphi(A^* \setminus \{x\}, R^2)$, which shows that independence of infeasible alternatives is violated because $R^2|_{A^* \setminus \{x\}} = R^3|_{A^* \setminus \{x\}}$. Hence $\varphi(A^*, R^3) = x$. By Lemma 2, $\varphi(A^*, R^3) = \varphi(A^*, (R_i', R_{-i}))$, and hence $\varphi(A^*, (R_i', R_{-i})) = x = \varphi(A^*, R)$. □

Lemmas 2 and 5 imply our main result.

Proposition 1. *If φ satisfies candidate stability and unanimity, then φ satisfies monotonicity.*

Using Proposition 1 together with Muller–Satterthwaite and Gibbard–Satterthwaite theorems, we present an alternative proof of candidate stability theorem due to Dutta, Jackson, and Le Breton.

Theorem 1. *Monotonicity and strategy-proofness are equivalent* (Muller and Satterthwaite, 1977).

Theorem 2. *If $\varphi(A^*, \cdot)$ is strategy-proof and $\varphi(A^*, \cdot)$ attains at least three outcomes, then $\varphi(A^*, \cdot)$ is dictatorial* (Gibbard, 1973, and Satterthwaite, 1975).

Theorem 3. *If φ satisfies candidate stability and unanimity, then for all $A \in \mathcal{A}$ with $|A| \geq |A^*| - 1$, $\varphi(A, \cdot)$ is dictatorial with a common dictator* (Dutta et al., 2001).

Proof. By Proposition 1 and Theorem 1, $\varphi(A^*, \cdot)$ is strategy-proof. By unanimity and the fact that $|A^*| \geq 3$, at least three outcomes are attained by $\varphi(A^*, \cdot)$. By Theorem 2, $\varphi(A^*, \cdot)$ is dictatorial with some dictator $i \in V$.[1]

[1] Instead of Theorems 1 and 2, we may directly apply the following result due to Reny (2001); if φ satisfies unanimity and monotonicity, then $\varphi(A^*, \cdot)$ is dictatorial. His proof of the result is very concise and transparent.

We are left to show that for any $R \in \mathcal{R}$ and $A \in \mathcal{A}$ with $|A|=|A^*|-1$, $\varphi(A, R)=\text{top}(A, R_i)$. Let $\{a\}=A^*\setminus A$. For all $j \in V$, let $R_j' \in \mathcal{R}_j$ be such that $R_j|_A=R_j'|_A$ and a is ranked last by R_j'.

$$\begin{cases} R_j : \ldots, a, \ldots, \varphi(A^*, R), \ldots\ldots \\ R_j' : \ldots\ldots, \varphi(A^*, R), \ldots\ldots, a \end{cases} \quad \text{or} \quad \begin{cases} R_j : \ldots\ldots, \varphi(A^*, R), \ldots, a, \ldots \\ R_j' : \ldots\ldots, \varphi(A^*, R), \ldots\ldots, a \end{cases}$$

For all $c \in A^*$, let $R_c'=R_c$. By independence of infeasible alternatives, $\varphi(A, R)=\varphi(A, R')$. By strong candidate stability, $\varphi(A^*, R') \in \{\varphi(A, R'), a\}$, and efficiency implies that $\varphi(A^*, R')=\varphi(A, R')$ because all voters prefer $\varphi(A, R')$ to a under R'. Since i is the dictator for $\varphi(A^*, \cdot)$, we have $\varphi(A^*, R')=\text{top}(A^*, R_i')=\text{top}(A, R_i)$. Hence $\varphi(A, R)=\text{top}(A, R_i)$. □

4. Conclusion

Our result shows that any voting rule satisfying unanimity and candidate stability satisfies monotonicity, and Gibbard–Satterthwaite theorem can be used to prove candidate stability theorem due to Dutta et al. (2001). Ehlers and Weymark (2003) provide an alternative proof of candidate stability theorem by applying a generalization of Arrow's theorem (Arrow, 1963) due to Grether and Plott (1982).

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