Assignment 1 Submission

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11/04/2025

Statement of Contribution

Each member attempted each question, and we combines and discussed our personal findings to be set into this document.

Question 1 Normal distribution. (8 marks)

(a) (1 mark) Find the probability that the user spends more than 15 minutes per month at the site.

Answer: 0.9938

```
1 - pnorm(15, mean = 25, sd = 4)
```

(b) (2 marks) Find the probability that the user spends between 20 and 35 minutes per month at the site.

Answer: 0.8881

```
pnorm(35, mean = 25, sd = 4) - pnorm(20, mean = 25, sd = 4)
```

(c) (2 marks) What is the amount of time per month a user spends on Facebook, if only 1% of users spend this time or longer on Facebook?

Answer: 34.3054

```
qnorm(0.99, mean = 25, sd = 4)
```

(d) (3 marks) Between what values do the time spent of the middle 90% distribution of Facebook users fall?

Answer: (18.42059, 31.5794)

```
qnorm(0.05, mean = 25, sd = 4)
qnorm(0.95, mean = 25, sd = 4)
```

Question 2 Blood fat concentration (11 marks)

(a) (6 marks) Conduct a two-independent sample t-test using R to determine whether the concentration of plasma cholesterol is significantly different between patients with no evidence of heart disease and those with narrowing of the arteries.

1. Hypotheses:
   $H_0 : \mu_1 = \mu_2$ against $H_A : \mu_1 \neq \mu_2$
   where μ_1 and μ_2 are the population means of plasma cholesterol concentrations in patients with no evidence of heart disease and those with narrowing of the arteries respectively.
2. Assuming unequal variances, the test statistic t can be given as:

$$t = \frac{195.2745 - 216.1906}{\sqrt{\frac{1303.9231}{51} + \frac{1850.2488}{320}}} = -3.7357$$

   Similarly, the degrees of freedom df is given by:

$$df = \frac{(\frac{1303.9251}{51} + \frac{1850.2488}{320})^2}{\frac{1}{51-1}(\frac{1303.9231}{51})^2 + \frac{1}{320-1}(\frac{1850.2488}{320})^2} = 74.5745$$

3. the sampling distribution thus is $t_{df=74.5745}$.
4. The calculated p-value = 0.0003641 < 0.01.
5. Decision. Given the p-value is less than the significance level (0.01), we reject the null hypothesis H_0 at the 1% significance level
6. Conclusion. Therefore, we conclude that the population means of plasma cholesterol concentrations in patients with no evidence of heart disease and those with narrowing of the arteries are significantly different.

```
#No Disease
mean1 <- 195.2745
var1 <- 1303.9231
n1 <- 51

#Disease
mean2 <- 216.1906
var2 <- 1850.2488
n2 <- 320
```

```
#calculate t and df
t_stat <- (mean1 - mean2)/sqrt(var1/n1 + var2/n2)
df <- (var1/n1 + var2/n2)^2 / ((var1/n1)^2/(n1-1) + (var2/n2)^2/(n2-1))

#calculate P_val
p_val <- 2 * pt(-abs(t_stat), df)
```

(b) (3 marks) Determine a 99% confidence interval for the mean difference in concentration of plasma cholesterol between the two groups of patients.

A 99% confidene interval for the mean difference in concentration of plasma cholesterol between the two groups of patients can be given as:

(-35.7164, -6.1158)

With the mean_difference = -20.9161, standard_error = 5.5990, critical t_value = 2.6434 and same degrees of freedom from (a), at df = 74.5745 ~ 75.

```
# Calculate the mean difference
mean_diff <- mean1 - mean2

# Calculate the standard error of the difference
```

```r
se_diff <- sqrt(var1/n1 + var2/n2)

# Find critical t-value for 99% confidence interval (two sided)
t_crit <- qt(0.995, df)

# Calculate the margin of error
margin_error <- t_crit * se_diff

# Calculate the confidence interval
lower_ci <- mean_diff - margin_error
upper_ci <- mean_diff + margin_error
```

(c) (2 marks) Explain the correspondence between the confidence interval in (b) and a test of the hypotheses you listed in question (a).

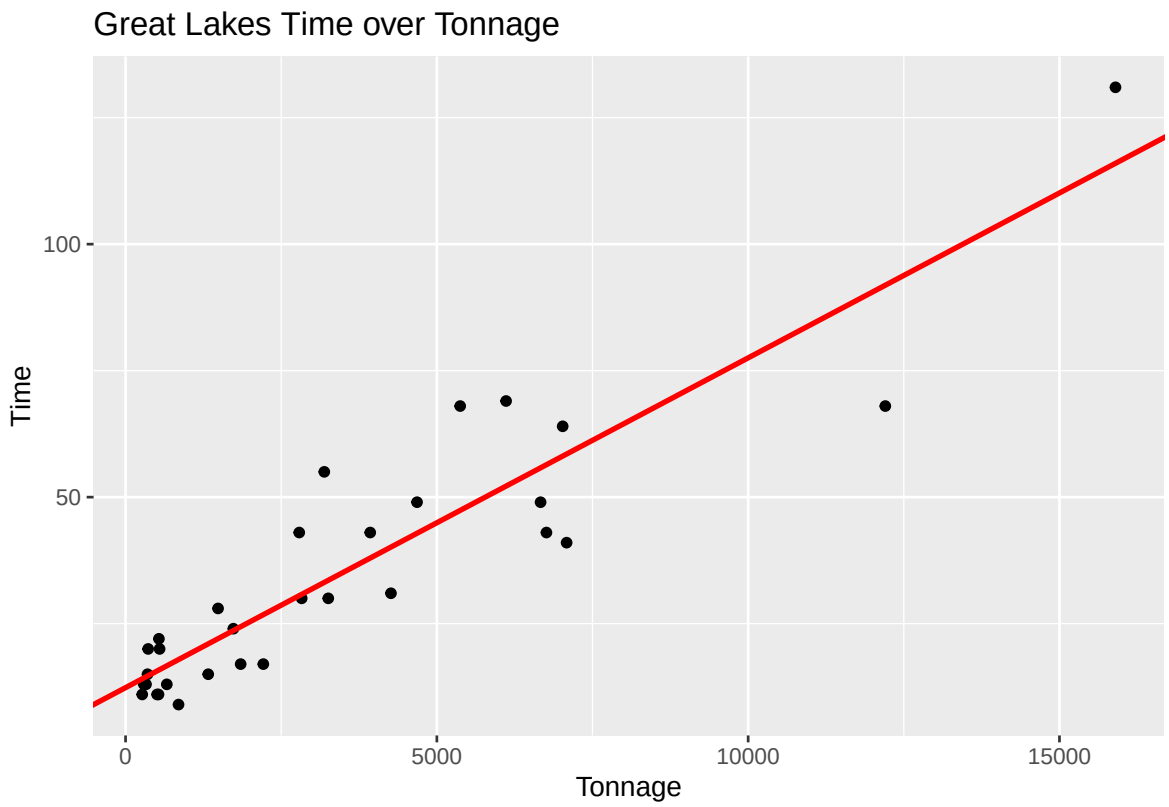
Since the confidence interval does not contain zero, this aligns with rejecting the null hypothesis at the 1% significance level in question (a).

The confidence interval is entirely negative, meaning that the mean plasma cholesterol concentration in the group with no heart disease is lower than the mean in the group with narrowing of arteries. The magnitude of the difference falling between 6.12 and 35.72 units with 99% confidence.

The fact that zero is not in the interval corresponds directly to the p-value (0.0003641) being less than 0.01, both leading to the same conclusion of rejecting H_0.

Question 3 Regression (31 marks)

(a) (2 marks) Fit a simple linear model M_1 to these data. Present the appropriate scatterplot and plot the fitted line onto the scatterplot. Comment about the output in a few concise sentences.



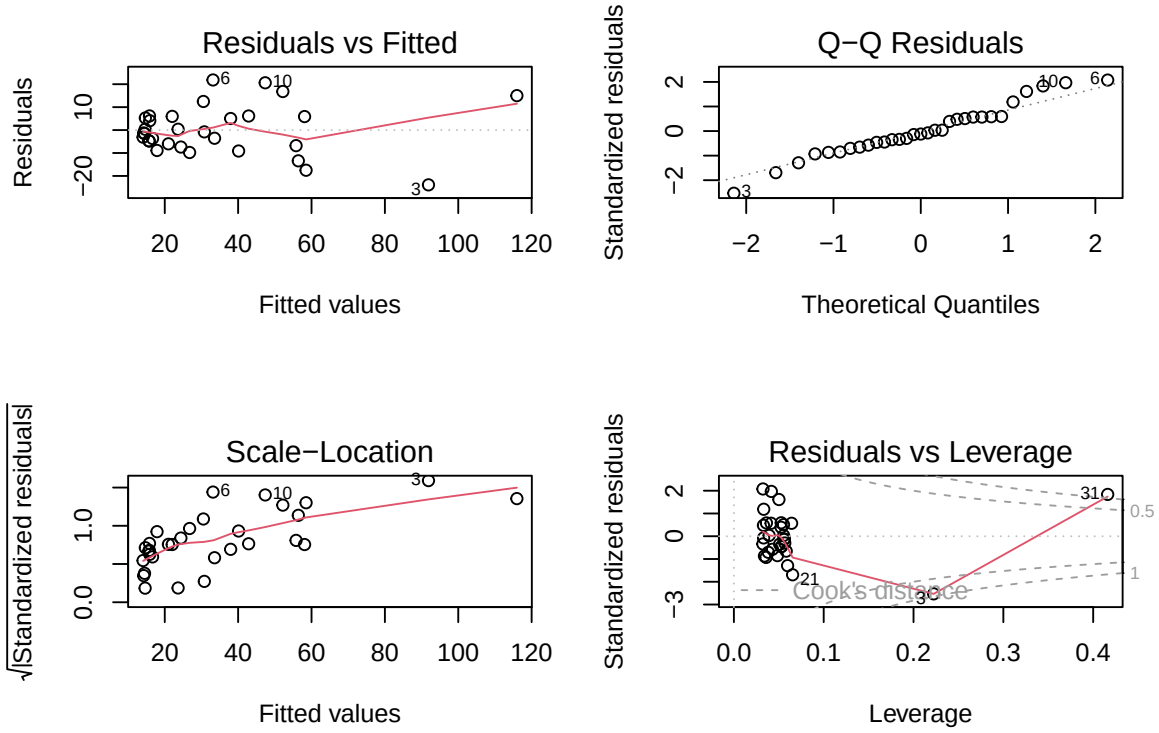
(b) (5 marks) Provide the model summary and diagnostics checking plots for model M_1. Does the straight line regression model M_1 seem to fit the data well? Comment about the output in a few concise sentences.

Answer:

The model summary for model M_1:

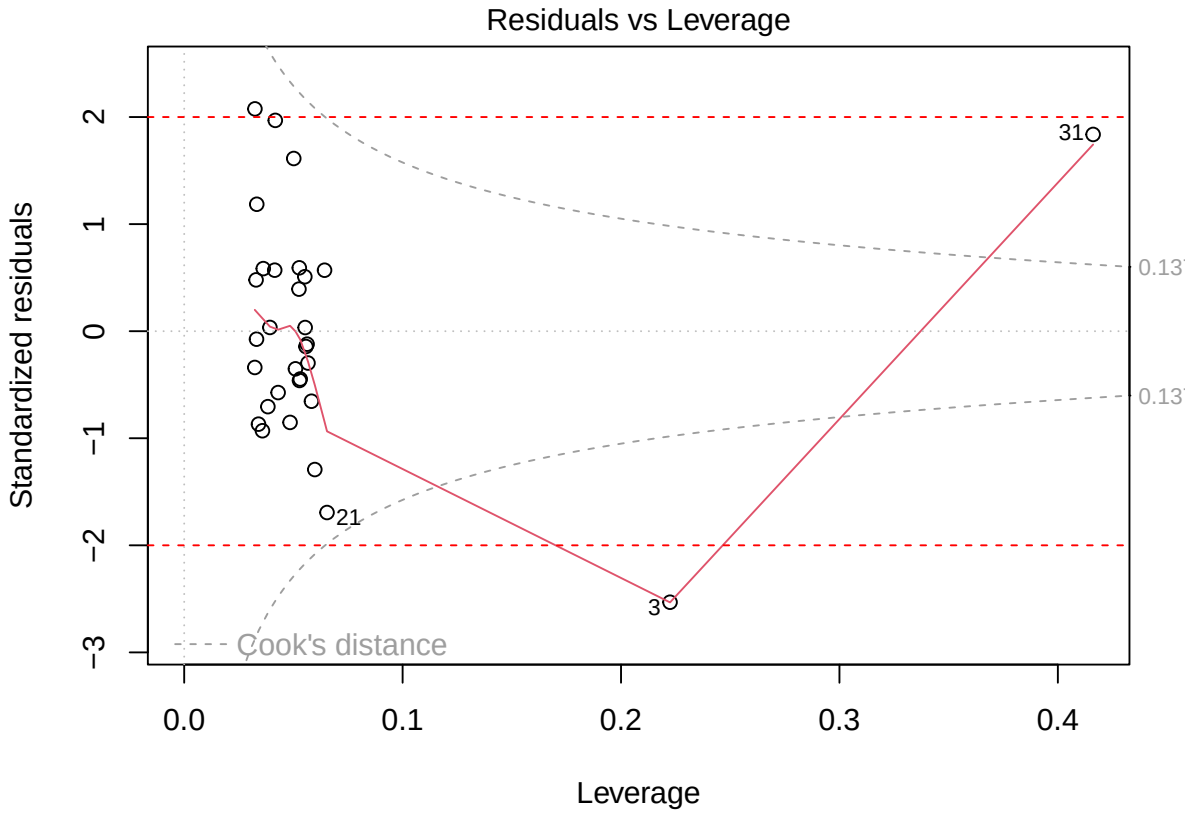
```
##
## Call:
## lm(formula = Time ~ Tonnage, data = glakes)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -23.882  -6.397  -1.261   5.931  21.850
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 12.344707   2.642633   4.671 6.32e-05 ***
## Tonnage      0.006518   0.000531  12.275 5.22e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.7 on 29 degrees of freedom
## Multiple R-squared:  0.8386, Adjusted R-squared:  0.833
## F-statistic: 150.7 on 1 and 29 DF,  p-value: 5.218e-13
```

Diagnostics checking plots for model M_1



(c) (5 marks) Do you think there are outliers or influential points in the data? What influence do these points have on the model fit? Use leverage and Cook's distance for this investigation.

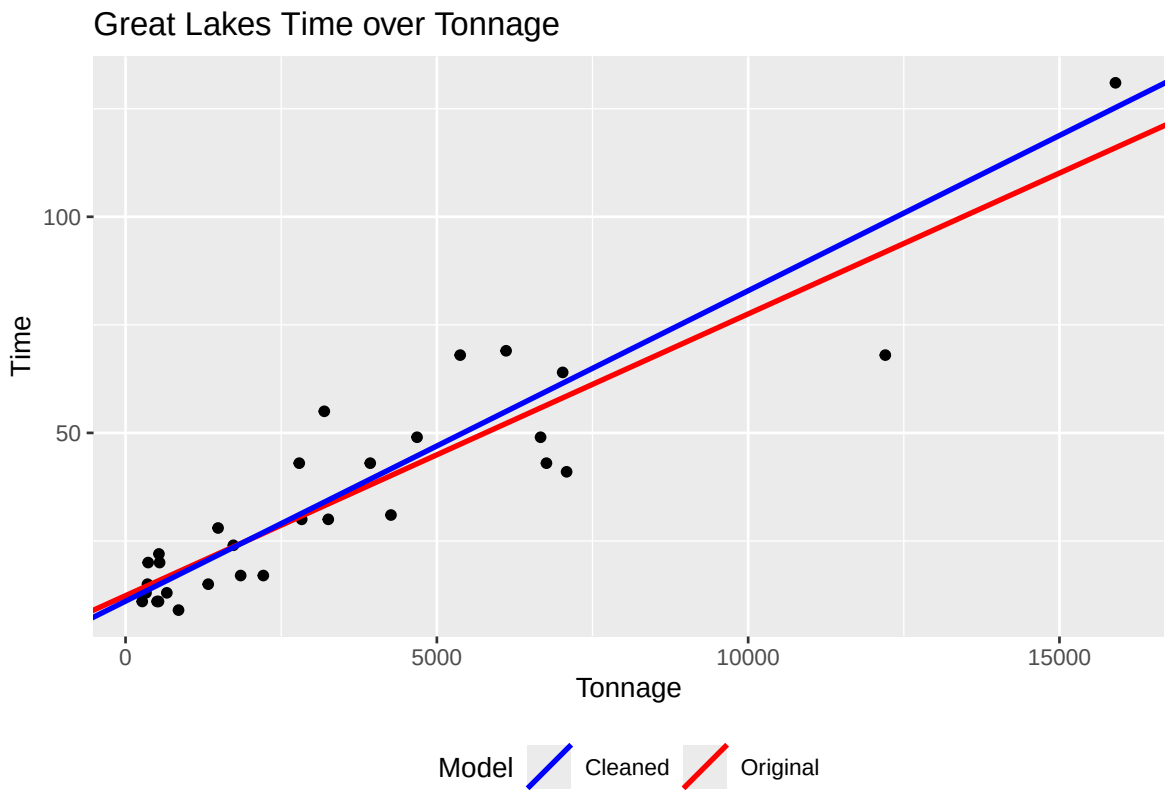
Having a look at the Cook's distance plot:



We say that a point is classified as a leverage point in simple linear regression, if it's leverage $h_{ii} > 4/n$. As $n = 31$, the rule is $h_{ii} > 0.13$. As leverage h_{ii} in the plot above has two points with $h_{ii} > 0.13$, there are two leverage points in the data.

We say that points are outliers if their standardized residuals have an absolute value of greater than 2. From the plot above, we can see that there are 2 points whose standardized residuals have an absolute value of greater than 2, such that there are 2 outliers.

Influential points are observations that have a big influence on the fitted line. This happens when the observations are both high leverage and is unusual (potentially an outlier). We note that the only point which meets both of these criterias is observation 3 and fitting a model after removing this point gives us:

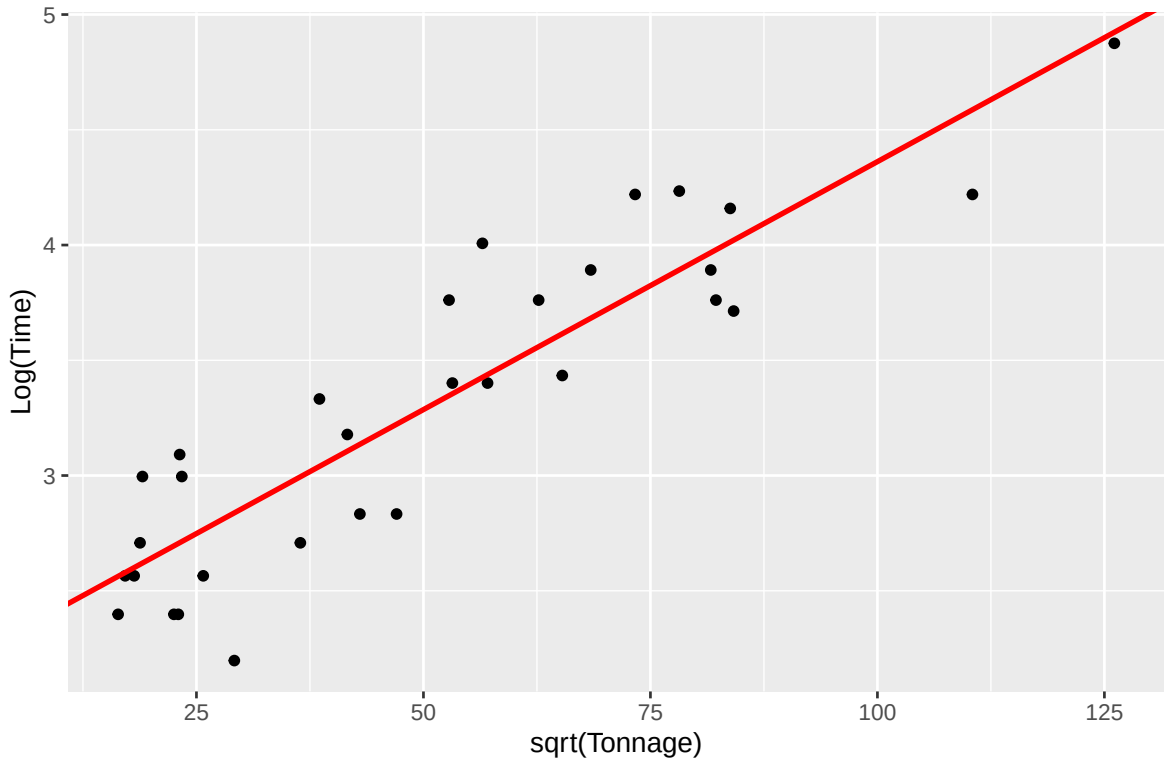


The new model better fits what appear to be a linear relationship

(d) (4 marks) Fit a regression model to the transformed M_2 model. Present the appropriate scatterplot and plot the fitted line onto the scatterplot. Does the transformed line regression model M_2 seem to fit the data well? Comment about the output in a few concise sentences.

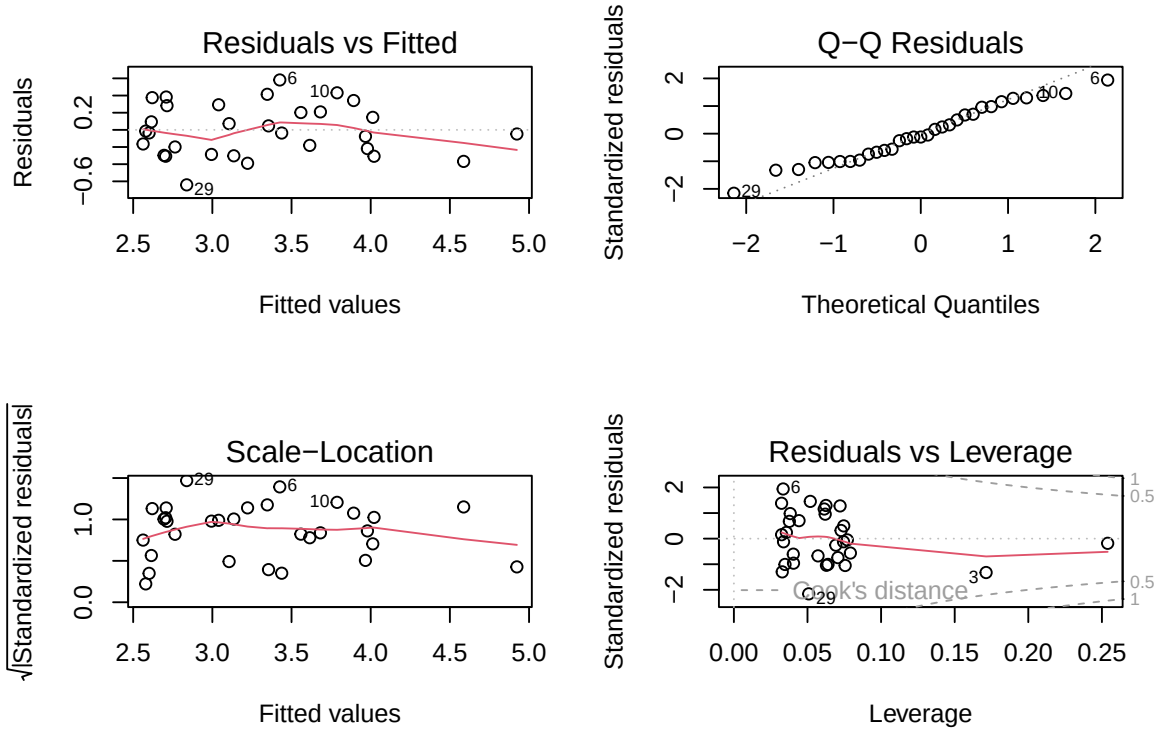
```
#model M_2 data transformations
log_Time = log(glakes$Time)
sqrt_Tonnage = sqrt(glakes$Tonnage)

#linear regression model M_2
glakes_lm_2 <- lm(log_Time~sqrt_Tonnage, data = glakes)
```



(e) (5 marks) Provide the model summary and diagnostics checking plots for model M_2. Does the straight line regression model M_2 seem to fit the data well? Comment about the output in a few concise sentences.

```
##
## Call:
## lm(formula = log_Time ~ sqrt_Tonnage, data = glakes)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.6408 -0.2522 -0.0357  0.2457  0.5814
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.210424   0.111580   19.81  < 2e-16 ***
## sqrt_Tonnage 0.021514   0.001909   11.27  4.1e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3048 on 29 degrees of freedom
## Multiple R-squared:  0.8141, Adjusted R-squared:  0.8077
## F-statistic:   127 on 1 and 29 DF,  p-value: 4.098e-12
```



(4 marks) Perform a hypothesis testing for a positive slope at a significance level of 5% based on model M_2.

1. $H_0 : \beta_1 \leq 0$ against $H_A : \beta_1 > 0$
   where β_1 is the slope of the model M_2. Note that the alternative hypothesis is one sided.
2. The test statistic (from R output as shown in question (e):

$$t = \frac{\hat{\beta} - 0}{se(\hat{\beta})} = \frac{0.021514}{0.001909} = 11.27$$

3. The sampling distribution for the test statistic t is $t_{df=(n-2)}$ that is $t_{df=31-2=29}$.
4. The p-value = $\mathrm{P}(t_{29} > 11.27)$ = 2.048434e-12
5. Decision. Given the p-value is less than the significance level (0.05), we reject the null hypothesis H_0 at the 5% significance level.
6. Conclusion. Therefore, we conclude that the slope is statistically significantly positive.

(f) (6 marks) Compare a 95% confidence interval of the mean response and a 95% prediction interval for a new value when Tonnage = 10,000 using the untransformed model M_1 and transformed model M_2 respectively. Provide two scatterplots that consist the fitted model, the confidence and prediction intervals for each of M_1 and M_2 respectively. Comment about the output in a few concise sentences.

When Tonnage = 10,000, for model M_1, the confidence interval of the mean response is given as (69.3647, 85.6821) and the prediction interval is given as (54.1705, 100.8763).

```
#95% confidence and prediction interval for model 1
newdata_1= data.frame(Tonnage = 10000)
conf_int_1 <- predict(glakes_lm_1, newdata = newdata_1,
                      interval = "confidence", level = 0.95)
```

```r
pred_int_1 <- predict(glakes_lm_1, newdata = newdata_1,
                      interval = "predict", level = 0.95)
```

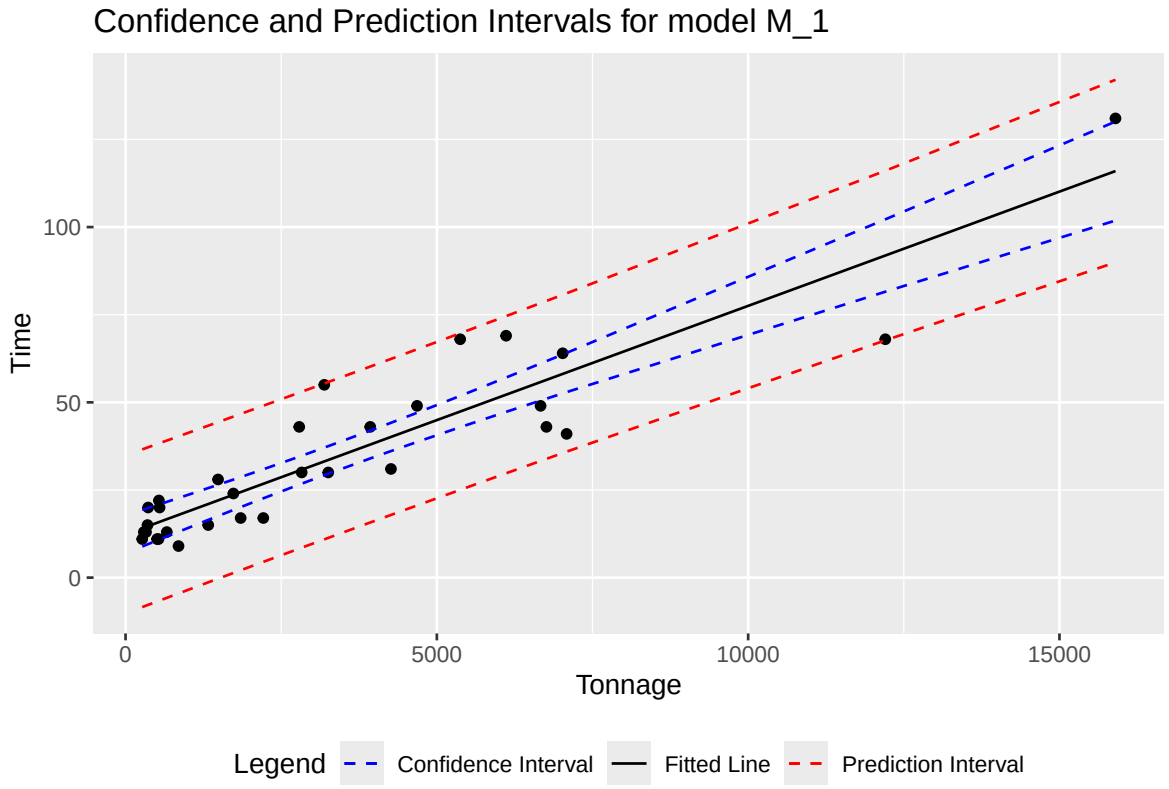
For model M_2, as it is a transformed model, when Tonnage = 10,000, $\sqrt{Tonnage} = 100$. The confidence interval of the mean response is given as (4.1400, 4.5837) and the prediction interval is given as (3.7002, 5.0234).

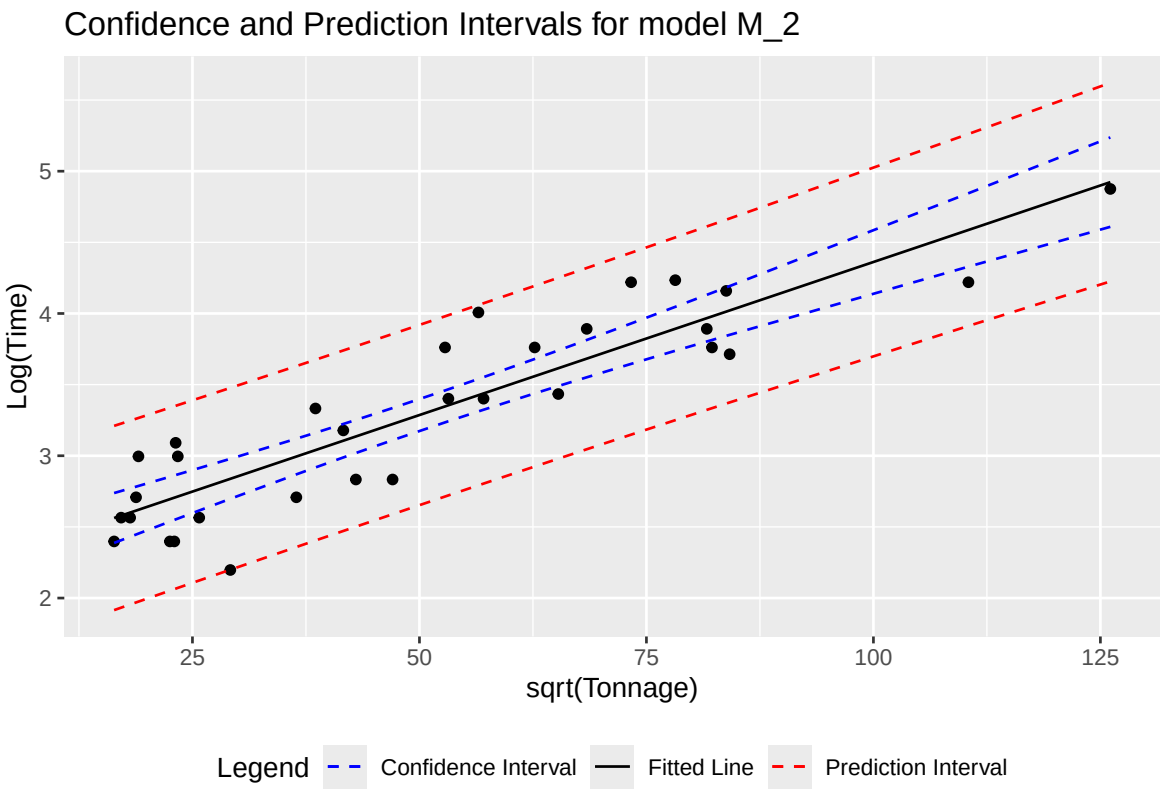
```r
#95% confidence and prediction interval for model 2
newdata_2= data.frame(sqrt_Tonnage = 100)
conf_int_2 <- predict(glakes_lm_2, newdata = newdata_2,
                      interval = "confidence", level = 0.95)
pred_int_2 <- predict(glakes_lm_2, newdata = newdata_2,
                      interval = "predict", level = 0.95)
```

For both of these models, the prediction intervals are wider than the confidence intervals because they include both:

1. Uncertainty about the mean response (as in confidence intervals)
2. Random variation of individual observations around the mean

Taking a look at the plots for the fitted model, the confidence and prediction intervals for each of M_1 and M_2 respectively:





We see that the prediction intervals is slimmer than the confidence interval, and are tightest in the middle of the data, widening at the extremes.

Most data points also falls within the prediction interval, indicating the model captures the overall variability well.