An Algorithmic Approach to 15-Minute City

Marco Lam

June 9, 2024

Abstract

The 15-Minute City is an urban planning concept introduced in the last decade that promotes accessibility. It emphasizes that residents should be able to meet basic needs—such as groceries, education, healthcare, and leisure—within a 15-minute travel time from their homes. The concept aims to deliver environmental, social, and economic benefits by reducing reliance on automobiles, encouraging active transportation, and enhancing residents' quality of life through easy access to essential services and amenities. The concept has gained traction during the COVID-19 pandemic, which highlighted the significance of local services and amenities in urban settings.

The 15-Minute City concept has been explored across various research fields, including urban planning, transportation, and environmental science. Within the field of Computer Science, although methodologies have been developed for the topic, a generalised purpose algorithmic approach to identify a 15-Minute City is still lacking. Most existing studies are data-driven, focusing on specific cities with solutions that are often neither algorithmic nor generalised.

This thesis aims to develop a general, adaptive, and efficient algorithm to identify city areas that can be classified as a 15-Minute City. It examines several existing algorithms for graph data structures, such as Breadth-First Search, Dijkstra's algorithm, Johnson's algorithm, and their variations. The proposed "15-Minute City algorithm" synthesises ideas and techniques from these algorithms to offer a comprehensive and efficient solution for determining 15-Minute City areas.

Contents

List of Tables

List of Algorithms

Chapter 1

Introduction

The 15-Minute City is an urban design concept that promotes accessibilities to essential urban functions within a 15-minute travel from the homes of residens. The concept was first proposed by Moreno in 2016 as a solution to build safer, more resilient, sustainable and inclusive cities and to harmonise the notion of Smart Cities [MAC$^+$21]. In 2021, Moreno et al. discussed the relationship between the concept of the "15-Minute City" to Urban Planning Pandemic Response, its Emerging Variations (i.e. 20 minute city [CDSKL19]), walkable neighbour [WDL$^+$19] smart cities.

The formal "15-Minute City Concept" proposed by Moreno et al. argues that residents should be able to enjoy a higher quality of life where they will be able to effectively fulfil six essential urban social functions to sustain a decent urban life. These include

1. Living
2. Working
3. Commerce
4. Healthcare
5. Education
6. Entertainment

The authors then proposed the "modified 15-Minute City" framework, depicting the four identified dimensions that could be incorporated with the already existing one proposed.

1. Density
2. Proximity
3. Diversity
4. Digitisation

Since COVID-19 pandemic, this topic has gained a tremendous amount of attention and has growth exponentially in popularity [LC23, ABCM22] among various research fields in literature, with urban design being the most studied [LC23]. The concept of the 15-Minute City has been studied and shown that it brings benefits to the society including environmental, social, and economical impacts [ABCM22]. Although the 15 minutes threshold has attracted the most attention, the notion of the t-Minute City has also been considered [MAC$^+$21]. Some examples are the 20-Minute City in Tempe, Arizona [CDSKL19], the 30-Minute City in Sydney, Australia [SWL20] and Olivari et al. studied 5 to 60 Minute City in Ferrara, Italy [OCNG23].

Most 15-Minute City studies in literature have employed data-driven approaches, these studies focus on a specific city or location and their methodologies are therefore only applicable to the specific location of interest. For example, Weng et al. [WDL$^+$19] and Olivari et al.[OCNG23] relied on census data on population density for their respective studies which may not be available for some countries and regions. The choice of aminuties to be included in the 15-Minute City is subjective and varies from one study to another. There is also a lack of research on the complexities and computational challenges of implementing the 15-Minute City concept in practice.

In 2023, Lima et al. noted that computational approaches to 15-Minute City design presents significant challenges such as “data availability and quality”, “computational cost” and “adaptability” [LC23]. In this thesis, we aim to address the latter two cahllenges mentioned by Lima et al. [LC23]. We propose an algorithmic approach to the 15-Minute City concept that is general, adaptable and efficient. We will study a number of exisitng relevent graph search algorithms and the complexities of the proposed algorithm. We will implement the proposed algorithm on Rust programming language and evaluate its performance on a number of real-world locations. We will also compare the ”15-Minite City” generated by the proposed algorithm with existing solutions in literature.

Chapter 2

Review of literature

As mentioned in the previous section, the concept of the “15-Minute City” has been well-studied in a varity of research fields, especially since the 2021 global pandemic. In this section, a selected number of previous works and studies will be discussed related to the 15-Minute City concept. These works will be grouped according to their methodologies and approaches, which include graph representation, grid tessellation, flow data, Walk Score, and other works.

Table 2.1 provides a summary of these studies discussed in this section excluding other works, specifically the methodologies considered and employed in determining and finding the so-called ”15-Minute City” given a location. Other works will cover studies in other research areas and the general concept of the“15-Minute City”. Finally, the detailed descriptions of each study and their methodologies used will be summerised in the following sections.

Table 2.1: Summary of relevant previous works

Approach	**Work**	**Description**
Graph Representation	[BDMM23], [CCRZ22], [RSRBH23]	Maps represented by graphs as a mathematical structure
Grid Tessellation	[GGZC22], [OCNG23], [PAMC24]	Maps divided according to various shapes and 15-MC calculations are applied to each area independently
Flow Data	[ZZK$^+$23], [SWL20]	Use foot travel data to incorporate human mobility patterns
Walk Score	[Wal], [WDL$^+$19]	Proprietary methodology based on sets of specific factors

2.1 Graph Representation

A graph in the context of Graph Theory, it contains a set of vertices (nodes) and a set of edges that connect paris of vertices. In the settings of city planning, a map can be represented as a graph where the vertices are the discrete locations, such as an interaction, an address etc. The edges can be used to represent the streets, with a weight which could be the length of the particular street, or any relevent measures. The graph representation is a common approach to model and visualise the urban environment and it can be useful to find the 15-Minute City.

2.1.1 Graph Representation of the 15-Minute City: A Comparison between Rome, London, and Paris

Barbieri et al. defined the general t-Minute City (hereafter t-MC) on an urban graph with respect to a given set of services [BDMM23]. The urban graph is represented by a planer graph $G(V, E)$, where an urban graph is a connected graph can be drawn without any edges crossing.

In this urban graph, the nodes of G are the intersections of the roads, and the lengths of the edges are proportional to the travel time with a coefficient, which is the speed of the pedestrians. Services $f \in V$ are then placed by adding an extra node to the graph, or label the nearest junction with the service.

Given a list of N_i services of type i, $C_k^i = (C_1^i, C_2^i, ..., C_{N_i}^i)$ are the nodes that reach f^i in less than 15 minutes. The set of vertices that form a 15-MC with respect to services of type i is given by the union of all these vertices

$$C^i = \bigcup_{k=1}^{N_i} C_k^i \subseteq V$$

If services f_i, f_j of the same type are far enough, it is possible that $C_i \cap C_j = \emptyset$. The authors noted that these "gaps" can be recognised as "the places where it is necessary to intervene to reconnect". Then for K types of services, the set of the 15-minute vertices is

$$C = \bigcap_{i=1}^{K} C^i = \bigcap_{i=1}^{K} \left(\bigcup_{j=1}^{N_i} C_k^j \right) \subseteq V$$

Finally, the authors define G_C as the graph induced on G by the vertices C, the 15-MC graph is the subgraph G_C of the urban graph G. To formalise, the set C depends on the travel time t, the service matrix $\mathbf{s}$, and the travel speed v, the graph of the 15-MC can be defined as

$$G_C = G_C(C, E_C; t, \mathbf{s}, v) \subseteq G$$

A metric/ratio $\gamma(r, x_0; t, \mathbf{s}, v)$ was also defined as a function of the radius r with respect to a given origin x_0, which can be used to characterise the 15-MC and compare different cities or areas of the same city, where

$$\gamma(r, x_0; t, \mathbf{s}, v) \equiv |C(r, x_0; t, \mathbf{s}, v)|/|E|$$

A possible generalisation of the index was then suggested which takes into account the properties or weights $w(e), e \in E$ of each edge, such as the length of the path, the population density or the slope of the streets

$$\gamma = \frac{\sum_{c \in C} w(c)}{\sum_{e \in E} w(e)}$$

If $w(\cdot)$ is the population density, the edges of the parks and archaeological area have $w = 0$, and the index γ is not biased.

The authors then applied the model to the cities of Rome, Paris, and London, they claimed to have used a “shortest path search algorithm” for graph search [Dij59] to calculate for f^i for service types pharmacies, post offices and supermarkets.

This paper transforms map data into planar graphs, the algorithm starts from services rather than address, this reduces computational complexity due to the fact that there are less services than overall loactions in a map. The authors referenced an article about Dijkstra algorithm for graph search algorithm but their implementation was not mentioned.

2.1.2 Exploring the 15-minute neighbourhoods

According to the authors (Caselli et al.), “in the proposed study, the 15-Minute City theme is addressed with an analytical model designed and developed using GIS to assess existing conditions of accessibility to neighbourhood services for all the resident social groups.” [CCRZ22]

The GIS-model is implemented by improving and integrating a Territorial Information system (managed with ArcGIS software). Extracting the pedestrian paths feature class to generate a link-node graph with all walking routes available. The model also considers that users “might choose to walk along road margins or cross in the proximity of road intersections.”

The paper studied the area covered that can be travelled to “neighbour cores” within 15 minutes by the following calculation:

$$\text{Length}(km)/(3km/h \times 60min) + \text{DF}(min)$$

where DF is the delay factor at crossings, with DF = 20 seconds for non-signalised crossings and 40 seconds for signalised.

The authors applied the calculations to the Cittadella District in Parma, Italy. They then compared this area with its population distribution to study the proportion of population covered by the 15-Minute City.

The authors did not explicity define the neighbour cores, only by “urban nodes well served by necessities shops and services, such as supermarkets, grocery stores, bars, drugstores, and banks.” However, using such nodes to calculate for 15-Minute City contributes to faster running time. The actual 15-Minute City search approach was not mentioned, and population data may not be a widely available data source for a generlaised solution.

2.1.3 The inclusive 15-Minute City: Walkability analysis with sidewalk networks

The paper proposed a framework for assessing multi-factor walkability on a sidewalk network model [RSRBH23]. The sidewalk network model is defined as a graph where the nodes are intersections or crosswalks, the edges have 3 types: sidewalks, crosswalks, and pedestrian-only paths and 4 attributes: length, width, slope, and pedestrian hazard. The pedestrian-only paths includes pedestrianised streets, living streets, and paths through parks and plazas. Pedestrian hazard is a metric to describe how dangerous each sidewalk segment by using a fine-grained map of estimated pedestrian safety in Barcelona [BRSR+21] and by exploiting Deep Learning tools. The resulting network is denser than the road network (approximately 4 to 1 in both nodes and edges). Each node of the graph has also been assigned a population according to census data.

This network is then simplified by a percolation analysis according to the sidewalks' properties (i.e., width, slope, or hazard). The authors then noted that an average, 1260 metres can be travelled in 15 minutes at a walking speed of 1.4 m/s, in accordance with literature [BW17]. Using government data, the authors selected a list of services.

To find such a set of links, the authors extended the classic Dijkstra algorithm to

1. explore all nodes within the threshold time from a single source, and
2. record all edges that can be traversed within the threshold, not only the ones that form part of a shortest path.

The implementation of the proposed solution was written in Python, using the igraph library [igr]. The authors then studied the impact of population size of the largest and second largest connected component of Barcelona's sidewalk networks by changing the parameters of percolation analysis.

This model takes hazard as a factor. The percolation analysis is a main focus of the study. The Supplementary notes included the modification of Dijkstra algorithm. However, population data and the list of services originated from government data, which could be a limitation.

2.2 Grid Tessellation

Grid Tessellation methods divide the map into a grid of "cells", where each cell is considered as a separate area. This method is useful to divide the map into smaller areas and calculate the 15-Minute City for each area independently. This method could be useful to reduce computational complexity and to study the 15-Minute City in a more granular level.

2.2.1 Urban accessibility in a 15-Minute City a measure in the city of Naples, Italy

The authors (Gaglione et al.) in this paper proposed a 4 steps methodology through a GIS environment to define the areas accessible in 15 minutes within a given location [GGZC22].

1. With a systemic approach, 17 variables have been identified by
   - The characteristics of the population.
   - The characteristics of urban fabrics, in particular their shape.
   - The physical characteristics relating to safety, amenities and pleasantness of the pedestrian. network.
2. The relationships among different groups of characteristics were identified through a (Pearson) correlation analysis to remove some variables.
3. Relating the demand (users) to the supply (local urban services). The authors used a proximity analysis by calculating the Euclidean distances from the centroids of the census sections to the related closer local urban services, then study how users can move along the pedestrian network by labelling 13 characteristics on each pedestrian path. The authors noted that the urban areas accessible in 15 minutes are defined "on the basis of the travel times defined on each link of the pedestrian network and the distribution and location of all the local services examined."

4. The population density is then compared with the 15-minute accessible areas. Individual age groups are also studied in this context.

The authors then explored the effect of choosing different grid size, shapes etc. in terms of the results of the minute city. In this paper, the authors did not specifically state the walking speeds used by each age group and by the whole population for the model. This paper once again uses the population distribution to study the proportion of population covered by the 15-Minute City. The authors did not mention the algorithm used to calculate the 15-Minute City.

2.2.2 Are Italian cities already 15-minute?

Olivari et al. proposed a data-driven approach solution by defining the NExt proXimity Index (NEXI), which exploits the data to answer the question: "Which parts of your city or town already follow the 15-minute model?" [OCNG23]

A list of service categories is selected, including Education, Entertainment, Grocery, Health, Post Office, Banks, Parks, Restaurants, Cafes, Bars and Shops. The nodes of the road network are the intersection points of the network geometries and the points of interest are the geographical location of the various services. For each node the algorithm computes the time needed to reach at an average walking speed, to the closest point of interest of any given category, being constrained to move only on roads accessible to pedestrians. More specifically, the time needed to reach the points of interest is computed as $t = l/s$, where:

- l is the length of the shortest route to the PoI (Point of Interest), on a road network made only of walkable roads,
- s is the approximate walking speed of an average person, that is 5 km/h.

If all categories can be reached within 15 minutes, the node is then considered to be a 15-minute node. Using a 5 km/h speed, the maximum reachable distance in 15 minutes is 1250 metres. "if the average time to reach all the categories from the nodes in that area is lower or equal to 15 minutes".

The algorithm computes the level of proximity of a given area as the mean of the levels of proximity of the nodes inside that area. Therefore, an area is 15-minute if the average time to reach all the categories from the nodes in that area is lower or equal to 15 minutes. The authors used hexagons with a diameter of 250 metres as the smallest resolution unit.

In this paper, 3 NEXI indices are proposed by the authors:

1. The NEXI-Minutes assigns to each category for each area a value of time which is the average time to reach each category.
2. The NEXI-Global takes inspiration from the Walk Score methodology, measuring the global proximity to all service categories on a scale that goes from $0 - 100$, where 0 means that none of the categories is at least within a 30-minute walk, while 100 means that all categories are within a 15-minute walk and all values in between describe an intermediate situation.
3. A discomfort index which takes population into account, where

$$\text{Discomfort} = (100 - \text{Global}) \times \text{Population}$$

The approach deployed in this paper is data-driven, it can be considered as a Grid Tessellation method built on top of a graph representation method, as the authors first calculated travel times from each node of the graph to each service category, then applied Grid Tessellation to calculate for each NEXI scores. The paper uses popularion cenus data and the graph search algorithm is not mentioned.

2.2.3 Travel-time in a grid: modelling movement dynamics in the "minute city"

Pezzica et al. claimed to "provide initial insights and recommendations for developing more robust 15-Minute City models" and emphasised "the importance of technical modelling steps in determining the mapping outputs which support the assessments of 15-minute cities" [PAMC24]. In the paper, the authors experimented on evaluating grid-based methods and identified four noteworthy variables:

1. Grid tessellation choices
2. Software application pick
3. Speed selection for travel-time calculations
4. Classification rules' adoption for mapping urban functions against mapped amenities

The authors emphasised that the lack of standardised modelling protocols in grid-based t-Minute City assessments can lead to inconsistent planning decisions, which can hinder synchronic comparisons and foster the formulation of exclusive policies and inconsistent planning decisions. Hence, it is crucial to undertake concerted efforts towards standardisation, including by bridging the gap between the planning practice and software development communities, to effectively address the existing challenges in t-Minute City modelling and representation. This entails establishing shared modelling protocols, algorithms (across various software applications), and standardised data inputs. All these can substantially enhance the consistency and reliability of grid-based t-Minute City assessments focused on travel-time estimates. Ultimately, advancing research, fostering collaboration, and promoting knowledge sharing, can elevate the quality of evidence generated through spatial analysis, leading to better-informed decisions in t-Minute City planning.

2.3 Flow Data

2.3.1 Towards a 15-minute city: A network-based evaluation framework.

In the context of 15-Minute City, it is suggested that the allocation of facilities should account for how people access and use local service and amenities rather than merely considering population size [CLZ20].

The authors in this paper proposed a methodological framework for evaluating 15-Minute City based on network science approaches. The paper proposes a network-based approach to evaluating 15-Minute City [ZZK+23]. This approach differs from mostly used accessibility measurements by accounting for human mobility patterns.

The network-based evaluation framework contains 3 parts:

1. Optimal mobility network is estimated based on the spatial distribution of urban amenities and population using a maximum flow algorithm.
2. The actual origin-destination network is obtained using mobile phone signalling data.
3. The differences between actual origin-destination network and the optimal network are measured to provide insights on the extent to which human mobility patterns, as a reflection on the usage patterns of urban amenities, match or do not match the schemes of urban planning and construction.

The authors then applied the framework model to a case study in Nanjing, China. This paper relies on data which may not be available in all cities, including population cenus data and mobile phone signalling data.

2.3.2 Measuring polycentricity via network flows, spatial interaction and percolation

This paper studied polycentricity based on inflow and outflow trip data and considered 3 network-based centricity metrics [SWL20], including

1. Trip-based centricity Index
2. Density-based centricity Index
3. Accessibility-based centricity index

In particular, accessibility-based centricity index computes the total number of jobs available and the number of workers available within a time threshold from a location. The time threshold was set to 30 minutes in the study to align with the polycentricity-inspired masterplan proposed by The Greater Sydney Commission (GSC) in 2018 which applies to the Sydney-GMR (Sydney Greater Metropolitan Region), noting that "access to jobs, goods and services is provided to the community in three largely self-contained regions."

2.4 Walk Score

2.4.1 Walk Score

Walk Score is a proprietary measure of how walkable a location is based on the distance and availability of nearby amenities, such as grocery stores, restaurants, schools, parks, etc. The higher the Walk Score, the more walkable the location is.

According to the Walk Score methodology [Wal], for each address, hundreds of walking routes to nearby amenities are analysed. Points are awarded based on the distance to amenities in each category. Amenities within a 5 minute walk (0.25 miles, about 0.4 kilometres) are given maximum points. A decay function is used to give points to more distant amenities, with no points given after a 30 minute walk. Walk Score also measures pedestrian friendliness by analysing population density and road metrics such as block length and intersection density. Walk Score utilises data sources include Google, Factual, Great Schools, Open Street Map, the U.S. Census, Localise, and places added by the user community.

The Walk Score calculation can be summarised into 4 steps

1. Assigning raw weights for selected amenities
2. Calculating distances from each location (community, data from government) to the selected amenities
3. Computing the total scores based on the distances and modifying the scores according to decay factors (e.g. street intersections and block length)
4. Normalising scores to $0 - 100$

Walk Score ranges from 0 to 100, with the following descriptions:

- $90 - 100$: Walker's Paradise. Daily errands do not require a car.
- $70 - 89$: Very Walkable. Most errands can be accomplished on foot.
- $50 - 69$: Somewhat Walkable. Some errands can be accomplished on foot.
- $25 - 49$: Car-Dependent. Most errands require a car.
- $0 - 24$: Car-Dependent. Almost all errands require a car.

Although the Walk Score algorithm is not open source, it is interesting to note that it considers walking distance between 5 to 30 minutes. Its calculation has been validated [CDM11].

2.4.2 The 15-Minute Walkable Neighbourhoods: Measurement, Social Inequalities and Implications for Building Healthy Communities in Urban China

In this paper, Weng et al. noted some of the limitations of the Walk Score calculation [WDL+19], such as

1. It targets at overall population and the walking demands of different pedestrian groups have not been included in the assessment.
2. The decay effect of amenity varies greatly among population groups and categories of amenities.
3. Actual traffic situation has not been considered when calculating distances based on Euclidean distance.

The authors proposed a modified method to measure 15-minute walkable neighbourhoods based on the Walk Score metric, taking into account pedestrians' characteristics and amenity attributes (scale and category). 6 categories of amenities were studied in the city of Shanghai, China, including education, medical care, municipal administration, finance and telecommunication, commercial services, and elderly care. A questionnaire was conducted to 132 respondents to conclude the parameters considered in the metric. A decay factor was also used to account for different age groups etc in terms of walking speed. The map data of Shanghai was captured from Baidu Map. This paper did not discuss any algorithms used and modification on the Walk Score calculation is heavily based on a questionnaire of a small sample size.

2.5 Other works

2.5.1 A Grammar-Based Optimisation Approach for Designing Urban Fabrics and Locating Amenities for 15-Minute Cities

This paper uses a geometric grammar based approach to explore computation to support decision-making concerning the layout of urban fabrics and the location of amenities in a neighbourhood [LBD22]. The authors (Lima et al.) used an inductive method for qualitative content analysis. However, the authors noted that this solution "does not address irregular or non-orthogonal urban block patterns, and the influence of nearby amenities located outside the studied fabric was not considered."

2.5.2 The Quest for Proximity: A Systematic Review of Computational Approaches towards 15-Minute Cities

Lima and Costa developed a comprehensive overview of the use of computational tools to support the analysis and design of 15-Minute Cities using a systematic literature review [LC23]. They noted that the topic of the 15-Minute City has growth exponentially in popularity and especially in the field of urban design. They concluded that computational approaches to 15-minute city design presents significant challenges such as "Data availability and quality", "Computational cost" and "Adaptability".

2.5.3 The 15-minute city: Urban planning and design efforts toward creating sustainable neighbourhoods

Khavarian-Garmsir et al. collected 103 documents, dealing with underlying principles, sustainability advantages, and critics of the 15-Minute City concept [KGSS23]. The authors defined 7 dimensions which constitute the 15-Minute City:

1. Proximity
2. Density
3. Diversity
4. Digitalisation
5. Human scale urban design
6. Flexibility
7. Connectivity

The authors summarised the sustainability contributions in social, economical and environmental aspects in the society by 15-Minute City and also the barrier of implementations.

2.5.4 The Theoretical, Practical, and Technological Foundations of the 15-Minute City Model: Proximity and Its Environmental, Social and Economic Benefits for Sustainability

15-Minute City has four main cornerstones (proximity, diversity, density, and digitisation). The authors in this paper (Allam et al.) explored the proximity dimensions of the 15-Minute

City and how it could influence mixed land use to yield environmental, social, and economic benefits [ABCM22].

2.5.5 Urban Transition and the Return of Neighbourhood Planning. Questioning the Proximity Syndrome and the 15-Minute City

Marchigiani and Bonfantini developed an evidence-based approach to a deeper analysis of policy design and implementation of the 15-Minute City [MB22]. They concluded that the implementation and effectiveness of the 15-Minute City depend on the concrete and contextual conformation of each city. The authors ended the paper stating that city development "needs some design framework and structure capable of addressing transformations, and their space and time location and sequences" on top of the "15 minute device" and that "it needs a syntax for an urban planning course of action that is incremental and adaptive but not limited to the contingent, blurred, and agnostic appeal of a catchy label."

Chapter 3

Problem Statement

In the previous section 2, serveral studies have discussed the needs for standardisation in the methodogy used in 15-Minute City. In particular, Pezzica et al. argued that "the absence of standardised modelling protocols imposes significant limitations on the application of Minute City models, hinders synchronic comparisons, and can indirectly foster the formulation of exclusive policies and inconsistent planning decisions" [PAMC24]. Lima et al. also noted that "introducing computational approaches to 15-Minute City design presents significant challenges and potential bottlenecks. On the other hand, exploring these challenges as opportunities for inserting new research is also possible since the theme is rising", some of these challenges are "data availability and quality", "computational cost" and "adaptability" [LC23]. Furthermore, Marchigiani et al. found that the approach to 15-Minute City needs to be changed and adapted to each location case by case [MB22].

Furthermore, a number of other literature works discussed in Chapter 2 have mentioned the use of a graph search algorithm to find the 15-Minute City or the travelling time from the source location of interest. Notably, Caselli et al. (Section 2.1.2, [CCRZ22]) defined 15-Minute City from the "neighbour cores" and Rhoads et al. (Section 2.1.3, [RSRBH23]) used a modified Dijkstra's algorithm to comupte a 15-Minute City on side-walk networks with a walkability analysis. The grid tessellatin approaches used by Gaglione et al. (Section 2.2.1, [GGZC22]) and Olivari et al. (Section 2.2.2, [OCNG23]) also searched for travelling time in a spatial space.

With these in mind, the aim of this thesis is to develop a general, adaptable algorithm to identify the 15-Minute City, where a person can travel to all their essential needs within 15 minutes from their home. In Computer Science, Graph Theory is a well-established field and there exist many efficient graph search algorithms. In this thesis, we will develop an algorithm by adapting various techniques from existing algorithms, along with inspirations from a number of approaches explored in literature which we discussed in the previous section 2.

The solution proposed in this thesis should be able to support different types of graphs and service locations. The designed algorith will focus on the 'computational cost" and "adaptability" problems listed by Lima et al. [LC23]. The algorithm should allow for an arbitrary set of service types. The edge weights in the graph use time unit, as this promotes the freedom for the users to incorporate different characterise to the roads which could affect the travelling time, such as the the street width, slope, and/or the mode of transportation. Therefore, the solution to the problem stated in this section can be considered as an adoption or a modifictaion to the existing approaches to the 15-Minute City problem, specifically with an improvment of the computational efficieny in mind.

Formally, the algorithm to the 15-Minute City problem should satisfy the following properties.

Inputs

1. A graph $G(V, E)$ representing the area of which t-Minute City is computed. V is the set of vertices representing locations within the area, and E is the set of weighted, undirected edges such that $E \subseteq V \times V$ and the weights $w : E \to \mathbb{R}_+$ of the edges are proportional to the time required to travel along the corresponding edge, in minutes.
2. For every vertex $v \in V$, v contains the label $v.l \in \{0,1\}^p$, where $v.l[i]$ represents the availability of service type i of the location.
3. A time threshold t in minutes.

Output

A set of vertices $R \subseteq V$ which can reach to at least one location of each service type within t minutes. i.e. denote $d(v, w)$ as the shortest path distance between v and w, we have

$$\forall v \in R : \forall i \in [1, p],\ \ \exists w \in V,\ \ w.l[i] = 1 : d(v, w) \leq t$$

Moreover, denote $v.r[i] \in \{0,1\}^p$ as a binary vector indicating whether v can reach a location of service type i within t minutes, such that

$$v.r[i] = 1 \iff \exists w \in V,\ \ w.l[i] = 1 : d(v, w) \leq t$$

then, we have

$$\forall v \in R,\ \ v.r = \mathbf{1}$$

In this setting, a location $v \in V$ in this graph could be a location of an amenity of interest, or a road junction, such that $v.l \in \{0\}^p$. More specifically, given a list of services of p distinct types and their locations, each location can be inserted into the graph by

- Labelling the closest node in the graph by the service type, or
- Adding a vertex to V along an existing edge in E, in such a way that the distance from the vertex to the nearest road junction is minimised.

Chapter 4

Graphs and Algorithms

With the problem statement formalised in the previous section 3, we will now discuss the graph data structure and the algorithms which could potentially assist in solving the 15-Minute City problem.

4.1 Graphs

Graph is a mathematical data structure in Graph Theory representing pairwise relationships between objects. The earliest scientiifc paper was the “Seven Bridges of Königsberg” by Leonhard Euler published in the 18^{th} centry. The “Seven Bridges of Königsberg” was proven to have no solution to the problem of crossing each of the seven bridges exactly once and returning to the starting point. Euler represented the problem by a graph, where the land masses were represented by vertices and the bridges were represented by edges.

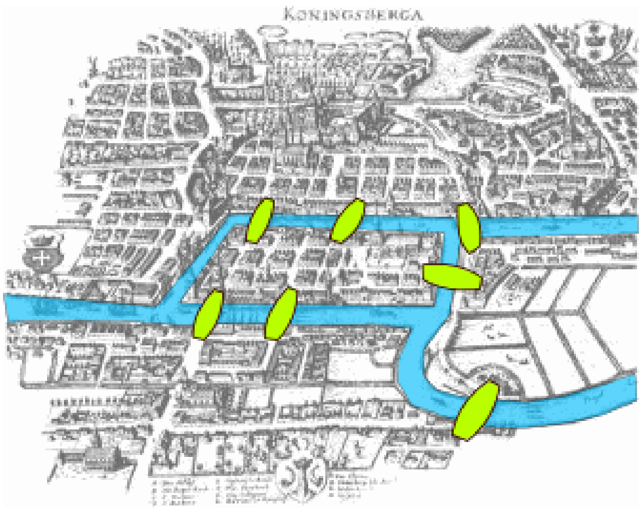


Figure 4.1: Konigsberg Bridges

In general, a graph can be defined as $G(V, E)$, where V is the set of vertices and $E \subseteq V \times V$ is the set of edges. The edges can be directed or undirected, and weighted or unweighted.

Labels can also be assigned to the vertices and edges, which can be used to represent additional information about the graph. For example, in "Seven Bridges of Königsberg", the vertices could be labelled with the names of the islands/areas and the edges could be labelled with the names of bridges. Furthermore, the edges could be weighted with the length of the bridges.

4.2 Graph Search Algorithms

In this section, we will discuss 3 types of graph search algorithms, the graph traversal problem, minimum spanning tree and the shortest path problem. The graph traversal problem is a problem of visiting all the nodes in a graph, while the shortest path problem is a problem of finding the shortest path between two nodes in a graph. The algorithms discussed in this section are fundamental graph algorithms and we will discuss their characteristics, properties, and whether they are suitable for solving the 15-Minute City problem.

4.2.1 Graph Traversal Problem

The Graph Traversal Problem is a problem of visiting all the nodes in a graph. There are two types of graph traversal algorithms, Breadth-First Search (BFS) and Depth-First Search (DFS). Both algorithms can be used to search for connected components of a graph and search for cycles, and have complexity $\mathcal{O}(V + E)$.

Breadth-first Search (BFS)

Breadth-first Search algorithm is a single-source graph search algorithm where the graph contains unweighted, undirected edges. The algorithm searches the graph from the source node level by level, that means the algorithm will search all nodes adjacent to the source node, before moving on to searching all nodes adjacent to these nodes. The visited nodes will be marked so that the algorithm will not be stuck in a cycle. An example of an application of this algorithm is maze solving, where it can be used to find the shortest path through a maze from the source node (i.e. entrance/exit).

Depth-First Search (DFS)

Depth-First Search algorithm is another popular single-source graph search algorithm for graphs with unweighted, undirected edges. Depth-First Search starts from the source node, travels along one of its edges and visits the adjacent node, the algorithm then repeats the process to this adjacent node and so on. Once the algorithm gets to the final node (i.e. there are no edges connected to this node where the adjacent node has not been visited), the algorithm travels to the previous level and checks if that node has an alternate adjacent node through a different edge. This process is then repeated until all nodes have been visited which can be reached to from our source node. Similar to Breadth-first Search algorithm, this algorithm can be used to detect connected components of a graph and search for cycles.

4.2.2 Minimum Spanning Tree

Minimum Spanning Tree (MST) is a Graph Theory problem of finding a tree that connects all the nodes in a graph with the least possible total edge weight and without any cycles.

There are several algorithms that can be used to solve the minimum spanning tree problem, such as Prim's algorithm and Kruskal's algorithm. These algorithms can be used to find the minimum spanning tree in a weighted, undirected graph.

Prim's Algorithm

Prim's algorithm is used for finding the MST within a weighted, undirected graph. Prim's algorithm starts from the source node, it keeps a record of the nodes it has selected. It repeatedly searches for the edge with the smallest weight that connects a node from the selected set of node and another node outside of this set. Prim's algorithm is a greedy algorithm and an example of an application is used in network design problems to find the minimum cost to connect all nodes in a network. The complexity of Prim's algorithm is $\mathcal{O}(|E| + |V| \log |V|)$.

Kruskal's Algorithm

Kruskal's algorithm is similar to Prim algorithm that it is a greedy algorithm which finds a MST in a weighted, undirected graph. The algorithm finds and records the minimum weighted edge, and selects the 2 nodes connected by the edge. It then searches and records for the next smallest weighted edge and the nodes connected by it. The algorithm repeats the process until all nodes have been selected and the edges recorded form the minimum spanning tree. Kruskal's algorithm can be used in clustering problems where the objective is to group similar items together while minimising the total dissimilarity. It has a complexity of $\mathcal{O}(|E| \log |V|)$.

4.2.3 Shortest Path Problem

The Shortest Path Problem is a problem of finding the shortest path between a pair of nodes in a graph. There are several algorithms that can be used to solve the shortest path problem, such as Dijkstra's algorithm, Bellman-Ford algorithm, Floyd-Warshall algorithm, and Johnson's algorithm. These algorithms can be used to find the shortest path in a graph with weighted edges. Two applications for these algorithms would be network routing to find the shortest paths in computer networks and GPS navigation to find the shortest route between two locations.

Dijkstra's Algorithm

Dijkstra's algorithm finds the shortest path from a single source node to all other nodes in a non-negative, weighted graph. It begins from the source node, in every iteration, Dijkstra's algorithm considers all of the current node's neighbours and update their tentative distances through the current node. If this distance is smaller than the previously assigned distance then update the assigned distance to the new one. The current node is then marked as visited and its tentative distance is then fixed. The algorithm then repeat the same steps on each of the neighbour nodes in the ascending order of their temporary tentative distances, until all nodes in the graph have been visited. Dijkstra's algorithm has a complexity of $\mathcal{O}((|V| + |E|) \log |V|)$.

Uniform Cost Search

Uniform Cost Search is a variant of Dijkstra's algorithm which finds the shortest path from a single source node to all other nodes in a non-negative graph. The main difference is that while Dijkstra's algorithm initialises the priority queue with the distance of the source node to 0 and all other nodes to ∞ within the graph, Uniform Cost Search initialises these only when they are needed. This has a benefit of reducing the space complexity of the algorithm, especially in a large graph. The time complexity of Uniform Cost Search is however, the same as Dijkstra's Algorithm at $\mathcal{O}((|V| + |E|) \log |V|)$.

Bellman-Ford Algorithm

Bellman-Ford algorithm is another algorithm which finds the shortest path from a single source node to all other nodes in a graph with negative edge weights and no negative weights cycles. The algorithm relaxes the edges in the graph by updating the distance of the destination node if the distance of any possible source node plus the weight of the edge is less than the current distance of the destination node. The algorithm then repeats the process for all edges in the graph until no more updates can be made. The algorithm then checks for negative cycles in the graph by relaxing the edges one more time. The complexity of Bellman-Ford algorithm is $\mathcal{O}(|V||E|)$.

Floyd-Warshall's Algorithm

Floyd-Warshall's algorithm finds the shortest path from every node in a graph to every other nodes. The algorithm works by considering all possible paths between two nodes and updating the shortest path if a shorter path is found. The algorithm then repeats the process for all pairs of nodes in the graph. The algorithm is able to handle negative edge weights and negative cycles in the graph. The complexity of Floyd-Warshall's algorithm is $\mathcal{O}(|V|^3)$.

Johnson's Algorithm

Johnson's algorithm is similar to Floyd-Warshall's algorithm that it finds the shortest path from every node in a graph to every other nodes with negative edge weights. The algorithm works by first adding a new node to the graph and connecting it to all other nodes with an edge weight of 0. The algorithm then runs the Bellman-Ford algorithm on this new graph to find the shortest path from the new node to all other nodes. The algorithm then reweights the edges in the graph to remove the negative edge weights. The algorithm then runs Dijkstra's algorithm on the reweighted graph to find the shortest path from the source node to all other nodes. This algorithm utilises the benefits of both Bellman-Ford and Dijkstra's algorithm to find the shortest paths in a graph with negative edge weights. The complexity of Johnson's algorithm is $\mathcal{O}(|V||E| + |V|^2 \log |V|)$. Hence, Johnson's algorithm is more efficient than Floyd-Warshall's algorithm in a sparse graph, where the number of edges is less than the number of nodes squared.

4.3 Adaption to the 15-Minute City Problem

In the last sections, we have discussed the graph data structure and the algorithms that could potentially assist in solving the 15-Minute City problem. It can be seen that the Minimum Spanning Tree problem is not directly applicable to the 15-Minute City problem, as the problem is not about connecting all the nodes in a graph with the least possible total edge weight. As for the Graph Traversal algorithms BFS and DFS, these algorithm are designed for unweighted graphs which are not enough for us to represent city maps.

The All-Source-Shortest-Path algorithms such as Floyd-Warshall's algorithm and Johnson's algorithm are not suitable for the 15-Minute City problem, as it is unnecessary to find the shortest path between every pair of nodes in the graph, while 15-Minute City problem is about finding the reachable essential needs within 15 minutes from residence. This leaves us with the Single-Source-Shortest-Path algorithms such as Dijkstra's algorithm, and Bellman-Ford algorithm. While Bellman-Ford algorithm has an advantage of supporting negative edge weights, it is not useful for our problem as the edge weights in our graph have time unit and therefore, non-negative. Finally, Dijkstra's algorithm and Uniform Cost Search are the most suitable algorithms for the 15-Minute City problem. Therefore, our proposed soltuion will be based on Dijkstra's algorithm and Uniform Cost Search.

Chapter 5

Proposed Solutions

5.1 Dijkstra's Algorithm

The first solution proposed in this thesis is a modified version of Dijkstra's algorithm, adapting the methodogies used by Pezzica et al. [CLRS22], which incooperate the Dijkstra's algorithm with a (Minimum) Priority Queue data structure. The data structure maintains a dynamic set Q of elements, each set element in Q has a key and it supports the following dynamic-set operations.

- INSERT$(Q; x; k)$: inserts element x with key k into set Q.
- MINIMUM(Q): returns element of Q with smallest key.
- EXTRACT-MIN(Q): removes and returns element of Q with smallest key.
- DECREASE-KEY$(Q; x; k)$: decreases value of element x's key to k. Assumes $k \leq x$'s current key value.

All operations take $O(\log n)$ time in an n-element heap with the exception of MINIMUM(Q) being $\Theta(1)$.

We extend the algorithm so that the algotithm will stop once all nodes within t minutes have been visited. For each node considered in the algorithm, a new label $v.d$ is created where $v.d \in \mathbb{R}_{\geq 0}$ representing the distance from the current source node of the algorithm, initialised to ∞.

As the 15-Minute City concept is primarily used to study cities' characteristics, the input graph of the algorithm is expected to be far larger than each 15 minute area. Therefore, it is necessary to stop the algorithm once all nodes within t minutes have been visited to prevent the algorithm from running indefinitely. The modified algorithm is shown in Algorithm 1.

Algorithm 1 Modified Dijkstra's Algorithm

Input: A graph $G(V, E)$, weights $w : E \to \mathbb{R}_{\geq 0}$, source vertex s,
time threshold t and i denotes the index of the service type

Output Assign $v.r[i] = 1$ for vertices that can reach to source node s within threshold t

```
for each vertex v ∈ V do
    v.d ← ∞
end for
s.d ← 0
Q ← ∅
for each vertex v ∈ V do
    INSERT(Q, v)
end for
while Q ≠ ∅ do
    v ←EXTRACT-MIN(Q)
    if v.d > t then
        Q ← ∅                                    ▷ Break out of While loop
    else
        v.r[i] ← 1
        for each vertex u ∈ Adj[v] do
            if u.d > v.d + w(u, v) then
                u.d ← v.d + w(u, v)
                DECREASE-KEY(Q, u, u.d)
            end if
        end for
    end if
end while
```

The modified Dijkstra's algorithm shown above only searches for vertices within t minutes from a single source node. For our context of the 15-Minute City, this needs to run for each location of each service type. Taking inspriation from the approach used by Barbieri et al. ([BDMM23], 2.1.1), the 15-Minute City algorithm will use the modified Dijkstra's algorithm to find the 15-Minute City with the service locations of each service type as the source nodes, rather than searching from every vertex in the graph. This approach is expected to be more efficient as the number of service locations is expected to be far smaller than the number of vertices in the graph, i.e. $S \subset V \Rightarrow |S| < |V|$.

The 15-Minute City algorithm as the solution of the problem is shown in Algorithm 2.

Algorithm 2 15-Minute City Algorithm

Input: A graph $G(V, E)$, weights $w : E \to \mathbb{R}_{\geq 0}$, a time threshold t
and a list S of service vertices of p types
Output Set $R \subseteq V$ representing the t-Minute City

```
for all vertex v ∈ V do
    v.r ← {0}^p
    v.l ← {0}^p
end for
for all service v ∈ S do
    v.l[i] ← 1 for each service type i which belongs to vertex v
end for
for each service type i ∈ {1, ..., p} do
    for each vertex s where s.l[i] = 1 do
        Modified_Dijkstra(G, w, s, t, i)
    end for
end for
R ← ∅
for each vertex v ∈ V do
    if v.r = 1 then
        R ← R ∪ {v}
    end if
end for
```

5.1.1 Analysis

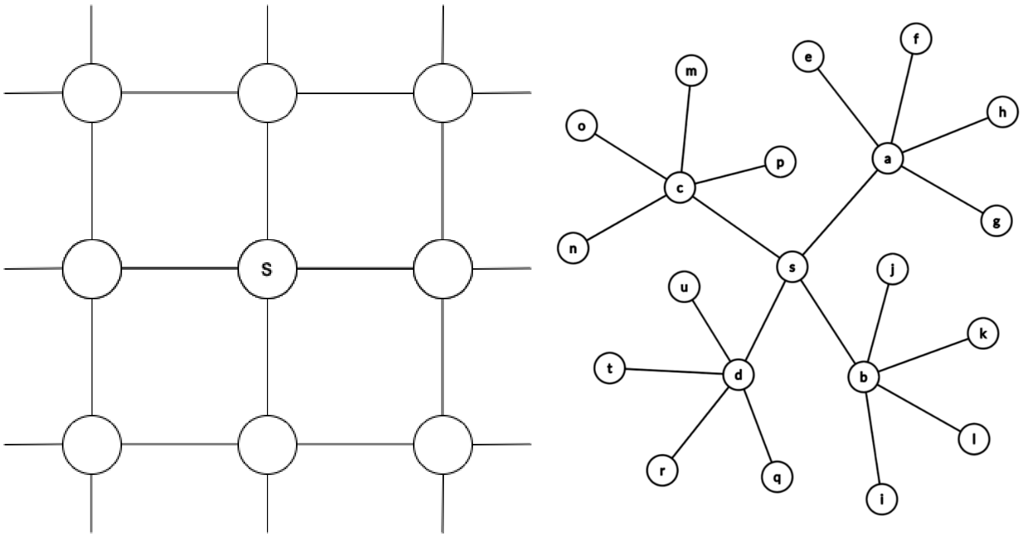
The time complexity of the modified Dijkstra's algorithm depends on the following:

- Initialisation: $O(|V|)$
- INSERT: $|V| \cdot O(|\text{INSERT}|) = O(|V| \cdot |\text{INSERT}|)$
- EXTRACT-MIN: $|V| \cdot O(|\text{EXTRACT-MIN}|) = O(|V| \cdot |\text{EXTRACT-MIN}|)$
- DECREASE-KEY: $|E| \cdot O(|\text{DECREASE-KEY}|) = O(|E| \cdot |\text{DECREASE-KEY}|)$

The time complexity of the algorithm is also affected by the data structure used to implement the priority queue. A binary heap is a common choice for implementing a priority queue, which has a time complexity of $O(\log |V|)$ for INSERT, EXTRACT-MIN and DECREASE-KEY operations. However, if a Fibonacci heap is used instead, the time complexity of the operations is reduced to $\Theta(1)$, $O(\log |V|)$ and $\Theta(1)$ respectively.

As the latter two operations in the algorithm dominates the former two operations, the time complexity of algorithm 1 is $O((|V| + |E|) \log |V|)$ if a binary heap is implemented. This can be reduced to $O(|V| \log |V| + |E|)$ if a Fibonacci heap is considered instead.

For the complete 15-Minute City Algorithm 2, the algorithm is run for each location of each service type. Denote q as the maximum number of locations for any service type, the time complexity of the algorithm is $O(p \cdot q \cdot (|V| \log |V| + |E|))$ if a binary heap is implemented and $O(p \cdot q \cdot (|V| \log |V| + |E|))$ if a Fibonacci heap is implemented. In both cases, the time complexity consider the size of the entire graph, which could be arbitrarily large when a city or a large area is studied. Due to the fact that the modified Dijsktra's algorithm stops once all nodes within weight t are searched, it is important to note that the actual complexity of the algorithm could be potentially much smaller.



(a) Example of a grid city (b) Example of a tree city at 2 levels

Cases where the city is larger than the "15-Minute City"

Example Consider a city with only square grids and each edge has a weight of 1 (figure 5.1a). The algorithm will effectively search a total of $1,024$ edges and 545 nodes for $t = 15$. In general, for a grid city (where each node has a degree of 4) given a time threshold t, the number of nodes and edges searched by the modified dijastra algorithm can be calculated as follows:

$$|\text{Edges}| = (2 \cdot (t+1))^2$$
$$|\text{Vertices}| = 1 + 2 \cdot (t+1) \cdot (t+2)$$

This structure of a grid-like city can be applicable to cities such as New York and Barcelona.

Generalisation However, if the graph of interest has the following characteristics:

- Starting from the source node s.
- Every node has d successors with d edges of weight 1.

An illustration of the graph with 2 levels and $d = 4$ are shown in figure 5.1b. In this arrangment, given a time threshold t and d the number of nodes branching out from each parent node, the number of nodes and edges need to be visited are as follows:

$$|\text{Edges}| = \sum_{l=1}^{t+1} d^l = d \cdot \left(\frac{1 - d^{t+1}}{1 - d} \right)$$
$$|\text{Vertices}| = 1 + |\text{Edges}|$$

For example, setting $t = 15$ and $d = 4$, the graph will have $|E| = 5,726,623,060$ and $|V| = 5,726,623,061$.

By limiting the maximum degree for each node in the graph, this graph structure can be considered as the worst case in time complexity for the algorithm with the maximum possible degree of $d+1$ with time threshold t. Fixing the degree of the nodes, any other arrangment of the graph (such as a clique) will have an equal or smaller time complexity, as the number of nodes visited will be smaller, and the number of edges unchanged. The time complexity of the modified Dijsktra's algorithm is then:

$$O(V) = O(\sum_{l=1}^{t+1} d^l) = O(d^{t+1})$$

$$O(E) = O(1 + \sum_{l=1}^{t+1} d^l) = O(d^{t+1})$$

and

$$O(V \log V + E) = O(d^{t+1} \log d^{t+1} + d^{t+1}) = O(d^{t+1} \log d^{t+1})$$

Therefore, the time complexity of the 15-Minute City algorithm on this graph is:

$$O\left(p \cdot q \cdot d^{t+1} \log d^{t+1}\right)$$

To generalise this notation to edge weights other than 1, define ϵ as the minimum edge weight in the graph. Then the algorithm can travel at most $\lfloor t/\epsilon \rfloor$ edges in t minutes. Therefore, the time complexity of the algorithm can be expressed as:

$$O\left(p \cdot q \cdot d^{1+\lfloor t/\epsilon \rfloor} \log d^{1+\lfloor t/\epsilon \rfloor}\right)$$

The space complexity of Dijkstra's algorithm is $O(|V| + |E|)$. In the modificated version of this algorithm, this is simply $O(d^{1+\lfloor t/\epsilon \rfloor} + d^{1+\lfloor t/\epsilon \rfloor}) = O(d^{1+\lfloor t/\epsilon \rfloor})$. Therefore, the space complexity of the 15-Minute City algorithm on this graph is:

$$O\left(p \cdot q \cdot d^{1+\lfloor t/\epsilon \rfloor}\right)$$

Cases where the city is smaller than the "15-Minute City"

In this case, the city is smaller than the 15-Minute City, the algorithm will search all nodes in the graph. The time complexity of the algorithm is simply just

$$O(|V| \log |V| + |E|)$$

5.2 Uniform Cost Search Adaption

For the space complexity of the propsed algorithm above, it is important to note that the modification algorithm of Dijkstra's algorithm inserts all $|V|$ vertices of the graph into the priority queue set Q, this is repeated for each service location of each type. Therefore, the space complexity would be $O(p \cdot q \cdot |V|)$. Hence the algorithm proposed may not be suitable for large graphs for space complexity.

The problem described here can be solved by adapting a technique from the "Uniform Cost Search" algorithm. The algorithm is an extension of Best-first search, it is similar to Dijkstra's algorithm, as well as some of the modification to Dijkstra's algorithm we propsoed above. However, Uniform Cost Search algorithm does not insert all vertices into the priority queue. Instead, it only inserts the vertices that are reachable within the time threshold t. A modification of this algorithm is shown in Algorithm 3. This algorithm can then be used in Algorithm 2 to replace the modified Dijkstra's algorithm in 1.

Algorithm 3 Modified Dijkstra's Algorith 2

Input: A graph $G(V, E)$, weights $w : E \to \mathbb{R}_{\geq 0}$, source vertex s,
time threshold t and i denotes the index of the service type
Output Assign $v.r[i] = 1$ for vertices that can reach to source node s within threshold t

```
Q ← ∅                                          ▷ Initialise an empty priority queue
INSERT(Q, s)
while Q ≠ ∅ do
    v ←EXTRACT-MIN(Q)
    if v.d > t then
        Q ← ∅                                  ▷ Break out of While loop
    else
        v.r[i] ← 1
        for each vertex u ∈ Adj[v] do
            if u ∉ Q then
                u.d ← v.d + w(u, v)
                INSERT(Q, u)
            else if u.d > v.d + w(u, v) then
                u.d ← v.d + w(u, v)
                DECREASE-KEY(Q, u, u.d)
            end if
        end for
    end if
end while
```

5.2.1 Analysis

In this implementation, the algorithm is more efficient in practice as it only inserts vertices that are reachable within the time threshold t into the priority queue. Hoewver, in the worst case, both time complexity and space complexity remain unchanged.

5.3 Inspiration from Johnson's algorithm

Johnson's algorithm uses both Dijkstra and Bellman-Ford as subroutines and performs better than Floyd-Warshall algorithm in sparse graphs. For the goal of the 15-Minute City algorithm, the set of service vertices can be far smaller than the entire graph in size. Thus, all-pairs shortest path algorithms are not optimal solutions to the problem. However, Johnson's algorithm's approach in connecting multiple nodes with a new node and 0 weight can be applied to the 15-Minute City problem.

This approach can be applied to the 15-Minute City problem by adding a new vertex s to the graph and adding edges from the new vertex to all service vertices of the same type. This allows us to eliminate the inner loop of Algorithm 2 where Modified_Dijkstra(G, w, s, t, i) is called. This alternate approach of the 15-Minute City algorithm is shown in Algorithm 4.

Algorithm 4 15-Minute City Algorithm 2

Input: A graph $G(V, E)$, weights $w : E \to \mathbb{R}_{\geq 0}$, a time threshold t
and a list S of service vertices of p types

Output Set $R \subseteq V$ representing the t-Minute City

```
for all vertex v ∈ V do
    v.r ← {0}^p
    v.l ← {0}^p
end for
for all service v ∈ S do
    v.l[i] ← 1 for each service type i which belongs to vertex v
end for
for each service type i ∈ {1, ..., p} do
    Create a new vertex s
    Add edges from s to all vertices v where v.l[i] = 1 and w(s, v) ← 0
    Modified_Dijkstra_2(G, w, s, t, i)
    Remove s and all edges connected to it
end for
R ← ∅
for each vertex v ∈ V do
    if v.r = 1 then
        R ← R ∪ {v}
    end if
end for
```

5.3.1 Analysis

For each service type, the proposed approach increases the number of vertices by 1 and the number of edges by the number of service vertices of the same type. Denote q_w as the maximum number of vertices of the same service type, then for each service type, Modified_Dijkstra_2 algorithm (Algorithm 3) is run, where the algorithm starts from the newly inserted vertex s, it visits at most q_w vertices and continues its search as before. The time complexity of the Modified_Dijkstra_2 algorith is then:

$$O(q_w \cdot (d^{1+\lfloor t/\epsilon \rfloor} \log d^{1+\lfloor t/\epsilon \rfloor})) = O(d^{1+\lfloor t/\epsilon \rfloor} \log d^{1+\lfloor t/\epsilon \rfloor})$$

and space complexity:

$$O(q_w \cdot d^{1+\lfloor t/\epsilon \rfloor}) = O(d^{1+\lfloor t/\epsilon \rfloor})$$

which are the same as before.

However, the time and space complexity of the 15-Minute City algorithm are now:

$$O\left(p \cdot d^{1+\lfloor t/\epsilon \rfloor} \log d^{1+\lfloor t/\epsilon \rfloor}\right) \text{ and } O\left(p \cdot d^{1+\lfloor t/\epsilon \rfloor}\right)$$

respectively, which are smaller by an order of q_w compared to the previous algorithm.

Chapter 6

Adaption to existing solutions

Similar approach was used by Barbieri et al. [BDMM23], for n types of services. For each service type, the locations of each service can be denoted as $f_j^i, j = 1..., m$, where m denotes the number of services of service type i in the graph.

Subsequently, define $C^i = \bigcup_{j=1}^{m} C_j^i$ the nodes that can be reached by f^i in less than 15 minutes. More specifically, C_j^i is the set of nodes which can be reached by f_j^i in less than 15 minutes.

$$C^i = \bigcup_{j=1}^{m} C_j^i \subseteq V$$

If services f_k^i, f_l^i of the same type i are far enough, it is possible that $C_k \cap C_l = \emptyset$. This represents a gap in the graph where its nodes cannot reach to service type i within 15 minutes.

Finally, we can define the 15-Minute City as the nodes which can reach to all service types within 15 minutes, namely

$$C = \bigcap_{i=1}^{n} \bigcup_{j=1}^{m} C_j^i = \bigcap_{i=1}^{n} C^i \subseteq V$$

Notes: Barbieri et al. proposed to use a planar graph [BDMM23]. However, it is not clear if a planar graph would be able to represent bridges/tunnels etc effectively, as the definition of a planar graph is that it can be drawn on the plane in such a way that its edges intersect only at their endpoints.

Chapter 7

Experiments

Bibliography

[ABCM22] Z. Allam, S. E. Bibri, D. Chabaud, and C. Moreno, "The Theoretical, Practical, and Technological Foundations of the 15-Minute City Model: Proximity and Its Environmental, Social and Economic Benefits for Sustainability," *Energies*, vol. 15, no. 16, p. 6042, Aug. 2022. [Online]. Available: https://www.mdpi.com/1996-1073/15/16/6042

[BDMM23] L. Barbieri, R. D'Autilia, P. Marrone, and I. Montella, "Graph Representation of the 15-Minute City: A Comparison between Rome, London, and Paris," *Sustainability*, vol. 15, no. 4, p. 3772, Feb. 2023. [Online]. Available: https://www.mdpi.com/2071-1050/15/4/3772

[BRSR+21] C. Bustos, D. Rhoads, A. Solé-Ribalta, D. Masip, A. Arenas, A. Lapedriza, and J. Borge-Holthoefer, "Explainable, automated urban interventions to improve pedestrian and vehicle safety," *Transportation Research Part C: Emerging Technologies*, vol. 125, p. 103018, Apr. 2021. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S0968090X21000498

[BW17] E. Bosina and U. Weidmann, "Estimating pedestrian speed using aggregated literature data," *Physica A: Statistical Mechanics and its Applications*, vol. 468, pp. 1–29, Feb. 2017. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S0378437116306604

[CCRZ22] B. Caselli, M. Carra, S. Rossetti, and M. Zazzi, "Exploring the 15-minute neighbourhoods. An evaluation based on the walkability performance to public facilities," *Transportation Research Procedia*, vol. 60, pp. 346–353, Jan. 2022. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S2352146521009455

[CDM11] L. J. Carr, S. I. Dunsiger, and B. H. Marcus, "Validation of Walk Score for estimating access to walkable amenities," *British Journal of Sports Medicine*, vol. 45, no. 14, pp. 1144–1148, Nov. 2011. [Online]. Available: https://bjsm.bmj.com/lookup/doi/10.1136/bjsm.2009.069609

[CDSKL19] D. Capasso Da Silva, D. A. King, and S. Lemar, "Accessibility in Practice: 20-Minute City as a Sustainability Planning Goal," *Sustainability*, vol. 12, no. 1, p. 129, Dec. 2019. [Online]. Available: https://www.mdpi.com/2071-1050/12/1/129

[CLRS22] T. Cormen, C. Leiserson, R. Rivest, and C. Stein, *Introduction to Algorithms, fourth edition*. MIT Press, 2022. [Online]. Available: https://books.google.it/books?id=HOJyzgEACAAJ

[CLZ20] Y. Chai, C. Li, and Y. Zhang, "A new time-geography research framework of community life circle," *Progress in Geography*, vol. 39, no. 12, pp. 1961–1971, 2020. [Online]. Available: http://www.progressingeography.com/CN/10.18306/dlkxjz.2020.12.001

[Dij59] E. W. Dijkstra, "A note on two problems in connexion with graphs," *Numerische Mathematik*, pp. 269–271, Dec. 1959. [Online]. Available: https://doi.org/10.1007/BF01386390

[GGZC22] F. Gaglione, C. Gargiulo, F. Zucaro, and C. Cottrill, "Urban accessibility in a 15-minute city: a measure in the city of Naples, Italy," *Transportation Research Procedia*, vol. 60, pp. 378–385, 2022. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S2352146521009509

[igr] "igraph," http://igraph.org/, accessed: 2024-03-06.

[KGSS23] A. R. Khavarian-Garmsir, A. Sharifi, and A. Sadeghi, "The 15-minute city: Urban planning and design efforts toward creating sustainable neighborhoods," *Cities*, vol. 132, p. 104101, Jan. 2023. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S0264275122005406

[LBD22] F. T. Lima, N. C. Brown, and J. P. Duarte, "A Grammar-Based Optimization Approach for Designing Urban Fabrics and Locating Amenities for 15-Minute Cities," *Buildings*, vol. 12, no. 8, p. 1157, Aug. 2022. [Online]. Available: https://www.mdpi.com/2075-5309/12/8/1157

[LC23] F. T. Lima and F. Costa, "The Quest for Proximity: A Systematic Review of Computational Approaches towards 15-Minute Cities," *Architecture*, vol. 3, no. 3, pp. 393–409, Jul. 2023. [Online]. Available: https://www.mdpi.com/2673-8945/3/3/21

[MAC+21] C. Moreno, Z. Allam, D. Chabaud, C. Gall, and F. Pratlong, "Introducing the "15-Minute City": Sustainability, Resilience and Place Identity in Future Post-Pandemic Cities," *Smart Cities*, vol. 4, no. 1, pp. 93–111, Jan. 2021. [Online]. Available: https://www.mdpi.com/2624-6511/4/1/6

[MB22] E. Marchigiani and B. Bonfantini, "Urban Transition and the Return of Neighbourhood Planning. Questioning the Proximity Syndrome and the 15-Minute City," *Sustainability*, vol. 14, no. 9, p. 5468, May 2022. [Online]. Available: https://www.mdpi.com/2071-1050/14/9/5468

[OCNG23] B. Olivari, P. Cipriano, M. Napolitano, and L. Giovannini, "Are Italian cities already 15-minute? Presenting the Next Proximity Index: A novel and scalable way to measure it, based on open data," *Journal of Urban Mobility*, vol. 4, p. 100057, Dec. 2023. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S2667091723000134

[PAMC24] C. Pezzica, D. Altafini, F. Mara, and C. Chioni, "Travel-time in a grid: Modelling movement dynamics in the "minute city"," in *Innovation in Urban and Regional Planning*, A. Marucci, F. Zullo, L. Fiorini, and L. Saganeiti, Eds. Cham: Springer Nature Switzerland, 2024, pp. 657–668. [Online]. Available: https://doi.org/10.1007/978-3-031-54118-6_58

[RSRBH23] D. Rhoads, A. Solé-Ribalta, and J. Borge-Holthoefer, "The inclusive 15-minute city: Walkability analysis with sidewalk networks," *Computers, Environment and Urban Systems*, vol. 100, p. 101936, Mar. 2023. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S0198971522001806

[SWL20] S. Sarkar, H. Wu, and D. M. Levinson, "Measuring polycentricity via network flows, spatial interaction and percolation," *Urban Studies*, vol. 57, no. 12, pp. 2402–2422, Sep. 2020. [Online]. Available: http://journals.sagepub.com/doi/10.1177/0042098019832517

[Wal] "Walk score," https://www.walkscore.com/methodology.shtml, accessed: 2024-03-06.

[WDL+19] M. Weng, N. Ding, J. Li, X. Jin, H. Xiao, Z. He, and S. Su, "The 15-Minute Walkable Neighborhoods: Measurement, Social Inequalities and Implications for Building Healthy Communities in Urban China," *Journal of Transport & Health*, vol. 13, pp. 259–273, Jun. 2019. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S2214140518305103

[ZZK+23] S. Zhang, F. Zhen, Y. Kong, T. Lobsang, and S. Zou, "Towards a 15-minute city: A network-based evaluation framework," *Environment and Planning B: Urban Analytics and City Science*, vol. 50, no. 2, pp. 500–514, Feb. 2023. [Online]. Available: http://journals.sagepub.com/doi/10.1177/23998083221118570