Excercise 5 - Model Predictive Control using a Grey Box Model

Temperature control of a building

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Introduction

In this exercise we will illustrate the main concepts of **Model Predictive Control (MPC)** by using it to control the temperature of a building. As the name suggests MPC relies on models and predictions, and so before designing a controller one needs to model the system and acquire relevant forecasts. In this exercise we will focus on the control part, assuming that we have perfect forecasts and models.

Question 1

Consider the model for a single room building developed during the previous exercises where T_i and T_m are the temperatures of the indoor air and floor respectively, while T_a is the temperature of the air outside. C_i and C_m are heat capacities of the inside air and floor, and R_m and R_a are the thermal resistances between $air \leftrightarrow floor$ and $air \leftrightarrow outside$. Φ is the effect of a radiator. G_v is global solar radiation with A_w being the effective window area. η describes the fraction of radiation going to the air and to the floor. w_i and w_m are both Wiener processes describing the stochasticity of the model. For this model it is assumed that the heating is through a radiator to the air of the building.

- How would you change the model, if instead the building was heated by floor heating?

The effect of the radiator ϕ has to be set to 0 for dTi equation and added to the dTm. In the code this is done by defining an alternative B mattrix ("altB") where the first element is 0 and the other is the coefficient.

Question 2

Assume that you were to control the temperature of the air in this building.

- Would you expect it to be equally difficult for floor heating and air heating?

No, there is a different in the thermal mass and the time constant of the floor is higher so it takes longer to reach the desired temperature.

- If not, which one would be the most difficult and why?

It is more difficult to control floor heating, specially if you are want sudden/fast changes. The floor heating will hold the temperature longer due to the thermal mass.

Question 3

- Have a look at the values of A and B in the script and see they fit with your intuition.

Depending on the type of heating (air or floor), the parameters in the second column of ***B*** *and* ***altB*** *change to 0 in either of the states (the one that is not being used), making this way possible the control with the selected type of heating.*

```
# Parameters of continous time model
A   # System dynamics, parameters of the states
```

```
##            [,1]      [,2]
## [1,] -4.723932  4.670725
## [2,]  0.529854 -0.529854
```

```
B    # Air heating, parameters of the inputs
```

```
##            [,1]     [,2]      [,3]
## [1,] 0.05320661 1.142277 2.6931338
## [2,] 0.00000000 0.000000 0.3071377
```

```
altB # floor heating
```

```
##            [,1]      [,2]      [,3]
## [1,] 0.05320661 0.0000000 2.6931338
## [2,] 0.00000000 0.1295816 0.3071377
```

Question 4

In this question we will formulate the equations of the MPC problem. The whole point of the control is to keep the temperature within comfort boundaries, so let us start with this. Assume that we do not care about the exact temperature of the building, as long as it is between $23°C$ and $25°C$.
Now look at the data concerning the heating of the building *(data$Ph1)*.

- What is the minimum amount and maximum amount of heating?

```
## [1] 0
```

```
## [1] 1.5
```

- Finish the MPC formulation and explain it in words:

The limits for the set temperature are between $23°C < CX_{t+1} < 25°C$*, the limits for the input heating are, as shown above, between* $0kW < U_t < 1.5kW$.

In this model we are assuming that the heater can be turned instantly on/off but in reality you would normally add a cost to switching it in order to do the optimization.

Question 5

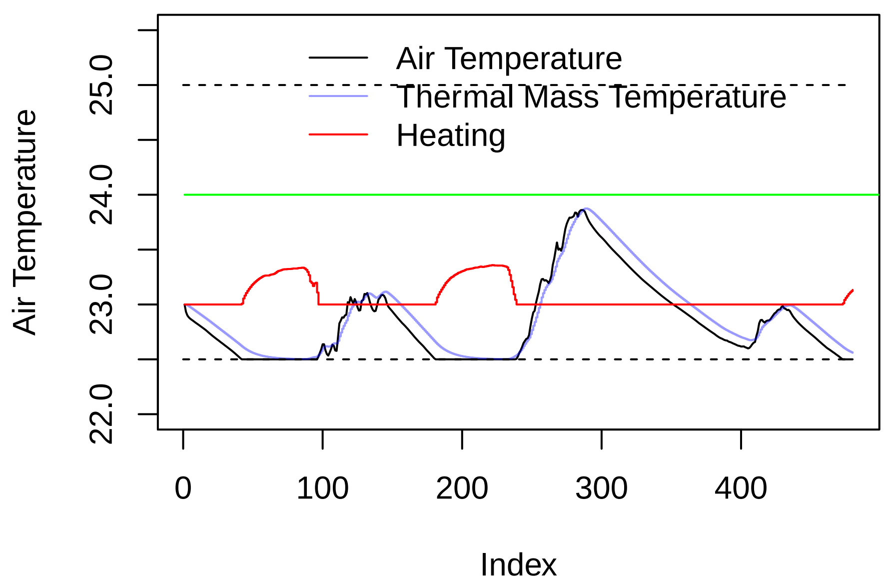
Run the section *Perform Control* of the R-script, which simulates the controlled temperature using the same outside temperature and solar radiation as the data used for fitting the grey box model. The heating is optimized by solving the problem formulated in the previous question, with comfort boundaries and constraints on the heating equipment specified initially. By default the noise of the model is turned off, so the controller is able to perfectly align the air temperature with the lower comfort comfort boundary for maximum efficiency.

The control horizon is specified by N and is equal to 30 by default, meaning that the controller looks 30 time steps (of 10 minutes) ahead in time (so 300 min or 5 hours).

- What happens as you vary N?
  Answer

Change the variable Air to **FALSE**, to simulate the building with floor heating instead of air heating.

- How does this affect the control?
  Answer



Question 6

In reality unpredicted disturbances occur, which can be included in the model by increasing Noise.

- What happens when for Noise equal to 0.5 or 1?
  Answer

- What is the effect of the control horizon and heating medium (air or floor)?
  Answer

Since the noise is quite significant, especially for Noise equal to 1 you should try different combinations of the control parameters for a fixed seed, to see their effect, but also change the seed to see several realizations of the noise.

Question 7

In practice it is not possible to guarantee that the temperature stays within comfort boundaries, which is evident from the simulation results. However, it seems like the controller is doing a particularly bad job in this case, with the temperature going below the comfort limits many times.

- Why is this the case? How could we mend this issue?
  Answer

Question 8

Sometimes one experiences varying prices or penalties, so that the objective is not to minimize energy consumption, but cost. This can easily be implemented by E-MPC (Economic Model Predictive Control). Mathematically the problem changes where lt is the penalty or price at time t. Change the price in the R-script from a constant one to a varying one. This could for example be a PRBS signal which is in the script as a comment.

- How does the optimization change when the prices are varying?
  Answer
- What is the effect of the control horizon on the ability to minimize varying costs?
  Answer
- What about the other parameters?
  Answer

Question 9

In this example the process noise (et) was estimated as part of the grey box model, and thus this can be used to estimate the distribution of the air temperature given the current temperature and the control action. Since the noise intensity (S) is constant this formulation only depends on the difference between the expected temperature and temperature limit (Tmin). This means that we only have to compute the expression once, and the actual control implementation remains linear.

Change the variable Stochastic to TRUE and rerun the control simulation to see the effect. The variable ViolationFraction can be used to adjust how often we are willing to accept comfort violations.

- Describe the difference compared to the previous case, where the stochasticity was not considered.
  Answer

References

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