Model Predictive Control using a Grey Box Model

Temperature control of a building

Summer school 2018 DTU - CITIES and NTNU - ZEN:

Time series analysis - with a focus on modelling and forecasting in energy systems

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Introduction

In this exercise we will illustrate the main concepts of Model Predictive Control (MPC) by using it to control the temperature of a building. As the name suggests MPC relies on models and predictions, and so before designing a controller one needs to model the system and acquire relevant forecasts. In this exercise we will focus on the control part, assuming that we have perfect forecasts and models.

Questions

Question 1

Consider the model for a single room building developed during the previous exercises:

$$dT_i = \frac{1}{C_i}\left(\frac{1}{R_m}(T_m - T_i) + \frac{1}{R_a}(T_a - T_i) + \Phi + \eta * A_w G_v\right) dt + dw_i,$$
$$dT_m = \frac{1}{C_m}\left(\frac{1}{R_m}(T_i - T_m) + (1-\eta) A_w G_v\right) dt + dw_m,$$

where T_i and T_m are the temperatures of the indoor air and floor respectively, while T_a is the temperature of the air outside. C_i and C_m are heat capacities of the inside air and floor, and R_m and R_a are the thermal resistances between air-floor and air-outside. Φ is the effect of a radiator. G_v is global solar radiation with A_w being the effective window area. η describes the fraction of radiation going to the air and to the floor. w_i and w_m are both Wiener processes describing the stochasticity of the model. For this model it is assumed that the heating is through a radiator to the air of the building. How would you change the model, if instead the building was heated by floor heating?

Question 2

Assume that you were to control the temperature of the air in this building. Would you expect it to be equally difficult for floor heating and air heating? If not, which one would be the most difficult and why?

Question 3

Open the script `r/ex_MPC.R`. The first part estimates a grey box model using CTSMR and data from a test facility at DTU (`http://www2.imm.dtu.dk/courses/02427/compEx3_E12.pdf`). The second part discretizes the estimated model, by computing the matrices of the standard state-space formulation (Include the equations for this?):

$$\begin{aligned} X_{t+1} &= AX_t + BU_t + \epsilon_{t+1}, \\ Y_t &= CX_t + DU_t + e_t, \end{aligned}$$

where $X_t = \begin{bmatrix} T_t^i \\ T_t^m \end{bmatrix}$, namely the temperature of the inside air and the thermal mass. As is often the case $D = 0$ and $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$ so that Y_t is the measured indoor air temperature. U_t is the amount of heating at time t. Have a look at the values of A and B, and see they fit with your intuition.

Question 4

In this question we will formulate the equations of the MPC problem. The whole point of the control is to keep the temperature within comfort boundaries, so let us start with this. Assume that we do not care about the exact temperature of the building, as long as it is between 23 and 25 C°. Now look at the data concerning the heating of the building (data$Ph1). What is the maximum amount and minimum amount of heating? Finish the MPC formulation and explain it in words:

$$\begin{aligned} \arg\min_{U_t,\, U_{t+1},\ldots,U_{t+N}} &\sum_{k=0}^{N} U_{t+k} \\ &s.t \\ X_{t+1} &= AX_t + BU_t, \\ &\leq CX_{t+1} \leq \quad, \\ &\leq U_t \leq \quad. \end{aligned}$$

Question 5

Run the section `Perform Control` of the R-script, which simulates the controlled temperature using the same outside temperature and solar radiation as the data used for fitting the grey box model. The heating is optimized by solving the problem formulated in the previous question, with comfort boundaries and constraints on the heating equipment specified initially. By default the noise of the model is turned off, so the controller is able to perfectly align the air temperature with the lower comfort comfort boundary for maximum efficiency.

The control horizon is specified by `N` and is equal to 30 by default, meaning that the controller looks 30 time steps (300 min or 5 hours) ahead in time. What happens as you vary `N`?

Change the variable `Air` to `FALSE`, to simulate the building with floor heating instead of air heating. How does this affect the control?

Question 6

In reality unpredicted disturbances occur, which can be included in the model by increasing `Noise`. What happens when for `Noise` equal to 0.5 or 1? What is the effect of the control horizon and heating medium (air or floor)? Since the noise is quite significant, especially for `Noise` equal to 1 you should try different combinations of the control parameters for a fixed seed, to see their effect, but also change the seed to see several realizations of the noise.

Question 7

In practice it is not possible to guarantee that the temperature stays within comfort boundaries, which is evident from the simulation results. However, it seems like the controller is doing a particularly bad job in this case, with the temperature going below the comfort limits many times. Why is this the case? How could we mend this issue?

Question 8

Sometimes one experiences varying prices or penalties, so that the objective is not to minimize energy consumption, but cost. This can easily be implemented by E-MPC (Economic Model Predictive Control). Mathmatically the problem changes to

$$
\begin{aligned}
\arg\min_{U_t,\, U_{t+1},\ldots,U_{t+N}} & \sum_{k=0}^{N} \lambda_{t+k} U_{t+k} \\
& s.t \\
X_{t+1} &= AX_t + BU_t, \\
& \leq CX_{t+1} \leq \quad, \\
& \leq U_t \leq \quad,
\end{aligned}
$$

where λ_t is the penalty or price at time t. Change the price in the R-script from a constant one to a varying one. This could for example be a PRBS signal which is in the script as a comment. How does the optimization change when the prices are varying? What is the effect of the control horizon on the ability to minimize varying costs? What about the other parameters?

Question 9

In this example the process noise (ϵ_t) was estimated as part of the grey box model, and thus this can be used to estimate the distribution of the air temperature given the current temperature and the control action. If $\epsilon_t \sim N(0, \sigma^2)$, then e.g

$$
(X_{t+1} | X_t, U_t) \sim N(AX_t + BU_t, \sigma^2),
$$

and more generally

$$(X_{t+n}|\{X_s, U_s; t \leq s \leq t+n-1\}) \sim N(A^n X_t + \sum_{k=1}^{n} A^{k-1} B U_t, \sum_{k=1}^{n} A^{k-1}\Sigma).$$

We can use this to add an additional constraint that ensures that the probability of violating the comfort boundaries stays below some value, p:

$$\mathbb{P}(CX_t < T_{min}) < p$$
$$\Updownarrow$$
$$\int_{-\infty}^{T_{min}} \frac{1}{\sqrt{2\pi \sum_{k=1}^{n} A^{k-1}\Sigma}} e^{-\frac{\left(x - A^n X_t + \sum_{k=1}^{n} A^{k-1} B U_t\right)^2}{2\sum_{k=1}^{n} A^{k-1}\Sigma}} dx < p.$$

Since the noise intensity (Σ) is constant this formulation only depends on the difference between the expected temperature ($A^n X_t + \sum_{k=1}^{n} A^{k-1} B U_t$) and temperature limit (T_{min}). This means that we only have to compute the expression once, and the actual control implementation remains linear.

Change the variable `Stochastic` to `TRUE` and rerun the control simulation to see the effect. The variable `ViolationFraction` can be used to adjust how often we are willing to accept comfort violations. Describe the difference compared to the previous case, where the stochasticity was not considered.

Questions and comments: `pbac@dtu.dk` or `rung@dtu.dk`

Appendix

The continous time state space model

$$dX_t = AX_t + BU_t,$$
$$Y_t = CX_t + BU_t,$$

can be discretized with a time step of t_s to a model on the form

$$X_{t+1} = A_d X_t + B_d U_t,$$
$$Y_t = C_d X_t + D_d U_t,$$

where

$$A_d = e^{At_s},$$
$$B_d = A^{-1}(A_d - I)B,$$
$$C_d = C,$$
$$D_d = D.$$

References