Building a basic simulation for Proton Therapy Imaging

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Abstract

The following work focuses on building an approximate model in Python to simulate the production of an image of an object through the use of a beam of protons, with the future goal of effectively simulating the process via which protons lose energy during a proton therapy session.

1 Introduction

The thesis project on which this work is based consists of building a phantom of a rat's skull, in order to take measurements that would be a reliable reproduction of an actual proton irradiation performed on a real animal test subject. The steps taken during the realization of this work follow those that are going to be taken during the thesis project:

- **Building a basic model**: Approximating the phantom to a sphere
- **Adding depth to the model**: Using a less simplified model for the protons' energy loss (simplified Bethe-Bloch formula) and adding varying density to the sphere
- **Reaching a good compromising model**: Implementing an even less simplified version of the Bethe-Bloch[1] formula and simulating the presence of eyes in the phantom.

2 Building the simulation

In this section the steps taken during this work are going to be described.

2.1 Basic Model

The base version of this simulation approximates the shape of the rat's skull to a sphere with uniform density; the protons are assumed to be produced from an uniform source and parallel to each other, with an undefined initial energy.
The energy loss is assumed to be linear, depending on the length of the protons' path inside the sphere.

2.2 Adding depth to the model

The next step to make the simulation more realistic was to implement varying density for the sphere, by simply making the inner section denser (but still keeping the "material" the same).
In addition, the following simplified version of the Bethe-Bloch formula was used to compute the protons' energy loss:

$$E_{loss} = \frac{k\rho L}{E_{in}} \tag{1}$$

Where $L =$*path length,* $\rho =$*density,* $k =$*arbitrary constant depending on the material,* $E_{in} =$ *initial energy of the proton.*

2.3 Reaching a good compromising model

The final simulation adds the presence of eyes in the "skull", which can be approximated to two small spheres made of a different material, located in the northern hemisphere of the bigger one.
To compute the energy loss of the protons, the Bethe-Bloch formula was used:

$$E_{loss} = \frac{KZ\rho L}{A\beta^2}(\log(\frac{2 \times 0.511\beta^2}{I}) - \beta^2) \tag{2}$$

Where $K = 0.307$ *(constant)*, Z =*atomic number*, ρ=*density* $[g/cm^3]$, A = *atomic mass number*, β *is the relativistic factor* , $I = 10 \times Z$ *mean excitation potential (approximation in* eV*)*, L =*path length*.
The skull is assumed to be uniformly made of bone, which consists for the most part of bone mineral ($Ca_{10}(PO_4)_6(HO)_2$) with an average density of $\rho = 1.5g/cm^3$, while the eyes' constituents can be approximated to water (H_2O), with a density of $\rho = 1g/cm^3$.
Since for both these materials the structure consists of molecules, the effective value of Z must be computed; the following formula[2] was used:

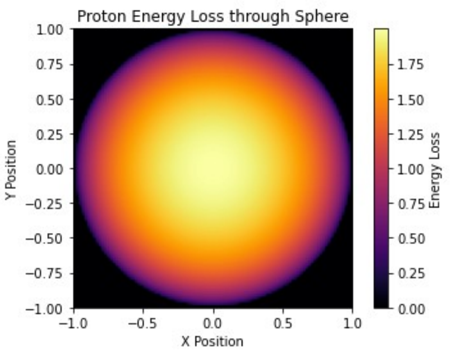
$$Z_{eff}(compound) = \sqrt[2.94]{\sum_i f_i Z_i^{2.94}} \tag{3}$$

Where f_i is the fraction of the total number of electrons associated with each element and Z_i is the atomic number of each element.
Regarding the atomic mass number A, a weighted average is used.
This gives:

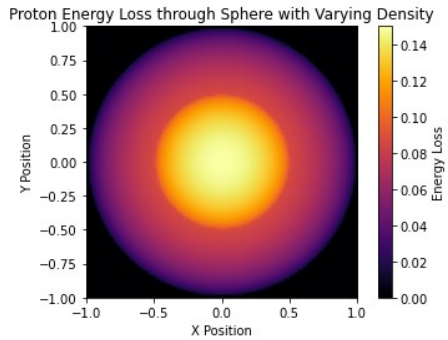
- $Z_{eff}(Ca_{10}(PO_4)_6(HO)_2) = 15.86$ and $A_w(Ca_{10}(PO_4)_6(HO)_2) = 56$
- $Z_{eff}(H_2O) = 7.42$ and $A(H_2O) = 18$

3 Results

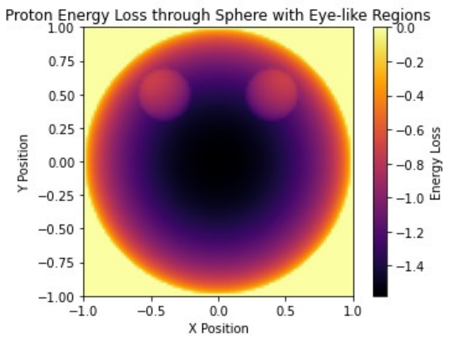
In this sections are shown the images obtained as a result from running the three different simulation models.



(a) Basic Model: uniform density sphere



(b) Intermediate Model: varying density sphere



(c) Final Model: sphere with eye-like regions

Figure 1: Images produced from the 3 different models; starting from the upper-left and going clockwise: a)Basic Model, b)Intermediate Model, c)Final Model

4 Conclusions

This work aims to show how the process of building a model should look: starting from a very simple version, adding layers to it step by step allows to reach results that are closer and closer to reality, until those results reach a degree of fidelity that satisfies the initial goals of a research/project; in the case of the work shown here, the results obtained represent themselves an intermediate step towards what is going to be an even more complex simulation model.

References

[1] J. Bethe, H.; Ashkin. *Experimental Nuclear Physics*, New York:(J. Wiley.):253, 1953.

[2] R. C Murty. Effective atomic numbers of heterogeneous materials. *Nature*, 207(4995):398—-399, 1993.