Problem 6

Provide one example for each of the following cases, where A, B are 2×2 matrices.

1. $(A + B)^2 \neq A^2 + 2AB + B^2$
2. $AB = 0, A \neq 0, B \neq 0$

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Solution

1. For $(A+B)^2 \neq A^2 + 2AB + B^2$, the left side of the equation is $(A+B)^2$:

$$\begin{aligned}(A+B)^2 &= (A+B)(A+B) \\ &= A(A+B) + B(A+B) \\ &= A^2 + AB + BA + B^2\end{aligned}$$

We must find a matrix such that $AB \neq BA$. It is clear with these two matrices:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$$

2. For $AB = 0, A \neq 0, B \neq 0$, we simply need to find a matrix that cancels each other out during matrix multiplication:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$$

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