Submission for Apoorv Walia and Marco Lagos

2.1. We are given that $P(x=1)=\theta$
and $P(x=0)=1-\theta$

$$f(x_i,\theta)=\theta^{x_i}(1-\theta)^{1-x_i}$$

$$L(\theta)=\prod_{i=1}^{m} f(x_i,\theta)$$

$$=\theta^{x_1}(1-\theta)^{1-x_1}\cdot\theta^{x_2}(1-\theta)^{1-x_2}\cdot\ldots\ldots\ldots\ldots\,\theta^{x_m}(1-\theta)^{x_m}$$

$$=\theta^{\sum_{i=1}^{m}x_i}(1-\theta)^{m-\sum_{i=1}^{m}x_i}$$

$$\log(L(\theta))=\sum_{i=1}^{m}x_i\log(\theta)+\left(m-\sum_{i=1}^{m}x_i\right)\log(1-\theta)$$

$$\frac{\partial\log(L(\theta))}{\partial\theta}=\frac{\sum_{i=1}^{m}x_i}{\theta}-\frac{m-\sum_{i=1}^{m}x_i}{1-\theta}$$

Setting the derivative equal to 0, we get

$$(1-\theta)\sum_{i=1}^{m}x_i-\theta\left(m-\sum_{i=1}^{m}x_i\right)=0$$

$$\sum_{i=1}^{m}x_i-\cancel{\theta\sum_{i=1}^{m}x_i}-\theta m+\cancel{\theta\sum_{i=1}^{m}x_i}=0$$

$$\therefore\ \boxed{\hat{\theta}=\sum_{i=1}^{m}\frac{x_i}{m}}$$