

$$3.3 \quad P(y=1|x) = \frac{\gamma \prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{(x_j - \mu_j^1)^2}{\sigma_j^2}}}{(1-\gamma) \prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{(x_j - \mu_j^0)^2}{\sigma_j^2}} + \gamma \prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{(x_j - \mu_j^1)^2}{\sigma_j^2}}}$$

Since we have uniform priors, we can get rid of γ in the expression

$$\begin{aligned} &= \frac{\sqrt{2\pi}^{-d} \prod_{j=1}^d \frac{1}{\sigma_j} e^{-\frac{\sum_{j=1}^d (x_j - \mu_j^1)^2}{\sigma_j^2}}}{\sqrt{2\pi}^{-d} \prod_{j=1}^d \frac{1}{\sigma_j} e^{-\frac{\sum_{j=1}^d (x_j - \mu_j^0)^2}{\sigma_j^2}} + \sqrt{2\pi}^{-d} \prod_{j=1}^d \frac{1}{\sigma_j} e^{-\frac{\sum_{j=1}^d (x_j - \mu_j^1)^2}{\sigma_j^2}}} \\ &= \frac{1}{1 + e^{\frac{\sum_{j=1}^d (x_j - \mu_j^1)^2}{\sigma_j^2} - \frac{\sum_{j=1}^d (x_j - \mu_j^0)^2}{\sigma_j^2}}} \\ &= \frac{1}{1 + e^{\frac{\sum_{j=1}^d (\mu_j^{0^2} - \mu_j^{1^2})}{\sigma_j^2} + 2(\mu_j^0 - \mu_j^1)x_j}} \\ &= \frac{1}{1 + e^{\frac{\sum_{j=1}^d (\mu_j^{0^2} - \mu_j^{1^2})}{\sigma_j^2} + \sum_{j=1}^d 2(\mu_j^0 - \mu_j^1)x_j/\sigma_j^2}} \end{aligned}$$

For logistic regression

$$P(y=1|x) = \frac{1}{1 + e^{-\theta^T x}}$$

$$= \frac{1}{1 + e^{-\theta_0 - \sum_{j=1}^d \theta_j x_j}}$$

If we set $\theta_0 = \frac{\sum_{j=1}^d (\mu_j^{0^2} - \mu_j^{1^2})}{\sigma_j^2}$

and $\theta_j = 2(\mu_j^0 - \mu_j^1)/\sigma_j^2$

Then with above parameterization $P(y=1|x)$ for gaussian naive bayes with uniform priors is equivalent to $P(y=1|x)$ for logistic regression