Submission for Apoorv Malia and Marco Lagos

A3 2.1

$k(x, x') = ck_1(x, x')$

The gram matrix of k_1 is K_1, which is positive semi-definite

for $\forall u$, $u^T K_1 u > 0$

The gram matrix of k is cK_1

for $\forall u$, $u^T c K_1 u > 0$ since $c > 0$

A3 2.2

$k(x, x') = f(x) k_1(x, x') f(x')$

$k_1(x, x') = \phi_1(x)^T \phi_1(x')$

$k(x, x') = f(x) \phi_1(x)^T \phi_1(x') f(x')$

Let $\phi(x)^T = f(x) \phi_1(x)^T$
and $\phi(x') = \phi_1(x') f(x')$

$\therefore k(x, x') = \phi(x)^T \phi(x')$

Therefore $k(x, x')$ is a valid kernel

A3 2.3

$k(x, x') = k_1(x, x') + k_2(x, x')$

let the gram matrix for k be $K = K_1 + K_2$

Then $\forall u$,

$$u^T K u = u^T (K_1 + K_2) u$$

$$= \underbrace{u^T K_1 u}_{>0} + \underbrace{u^T K_2 u}_{>0}$$

$\therefore u^T K u > 0$

A3 2.4

$K(x, x') = x^T rev(x')$

let $c^T = (1, 0, 0 \ldots\ldots, 0)^T$

$$c^T K c = x_1^T rev(x_1')$$
$$= x_1^T(-x_1)$$
$$= -1$$

K is not a valid kernel