

$$\begin{aligned} L_{\pi} &= \sum_{k=1}^C L(\alpha_{c+1} | y=k) P(y=k|x) \\ &= \lambda_{\pi} \sum_{k=1}^C P(y=k|x) \\ &= \lambda_{\pi} \end{aligned}$$

If we set $y=j$

$$\begin{aligned} L_j &= \sum_{k=1}^C L(\alpha_i | y=k) P(y=k|x) \\ &= P(y=j|x) \sum_{k=1}^C \lambda_s P(y=k|x) \\ &= \lambda_s (1 - P(y=j|x)) \end{aligned}$$

$$\text{If } P(y=j|x) \geq 1 - \frac{\lambda_{\pi}}{\lambda_s}$$

$$\begin{aligned} L_j &\leq \lambda_s \left(1 - \left(1 - \frac{\lambda_{\pi}}{\lambda_s} \right) \right) \\ &= \lambda_{\pi} \\ &= L_{\pi} \end{aligned}$$