

3.2 $y \sim \text{Bernoulli}(\gamma)$

$$x_j | y=1 \sim N(\mu_j^1, \sigma_j^2)$$

$$x_j | y=0 \sim N(\mu_j^0, \sigma_j^2)$$

$$P(y=1/x) = \frac{P(x|y=1)P(y=1)}{P(x)}$$

$$= \frac{P(x|y=1)P(y=1)}{P(x|y=0)P(y=0) + P(x|y=1)P(y=1)}$$

$$= \gamma \frac{\prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j}}{e^{-\frac{(x_j - \mu_j^1)^2}{\sigma_j^2}}}$$

$$(1-\gamma) \frac{\prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j}}{e^{-\frac{(x_j - \mu_j^0)^2}{\sigma_j^2}}} + \gamma \frac{\prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j}}{e^{-\frac{(x_j - \mu_j^1)^2}{\sigma_j^2}}}$$

$$P(y=0/x) = \frac{P(x|y=0)P(y=0)}{P(x)}$$

$$= \frac{P(x|y=0)P(y=0)}{P(x|y=0)P(y=0) + P(x|y=1)P(y=1)}$$

$$= \gamma \frac{\prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j}}{e^{-\frac{(x_j - \mu_j^0)^2}{\sigma_j^2}}}$$

$$(1-\gamma) \frac{\prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j}}{e^{-\frac{(x_j - \mu_j^0)^2}{\sigma_j^2}}} + \gamma \frac{\prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j}}{e^{-\frac{(x_j - \mu_j^1)^2}{\sigma_j^2}}}$$