Problem 7

Let u denote a real vector normalized to unit length. That is, $u^T u = 1$. Show that $A = I - 2uu^T$ is orthogonal, i.e., $A^T A = I$.

Solution

$$\begin{aligned} A^T A &= (I - 2uu^T)^T (I - 2uu^T) \\ &= (I^T - (2uu^T)^T)(I - 2uu^T) \\ &= (I - 2uu^T)(I - 2uu^T) \\ &= I^2 - 2Iuu^T - 2uu^T I + 4(uu^T)^2 \\ &= I^2 - 4uu^T + 4(uu^T)^2 \\ &= I^2 - 4 + 4 \\ &= I \end{aligned}$$