

Since  $\log$  is concave, we can say

$$H(S) = H\left(\frac{p}{p+n}\right) = \frac{p}{p+n} \log_2\left(\frac{p+n}{p}\right) + \frac{n}{p+n} \log_2\left(\frac{p+n}{n}\right)$$

$$\leq \log_2\left(\frac{p}{p+n} \cdot \frac{p+n}{p} + \frac{n}{p+n} \cdot \frac{p+n}{n}\right) = \log_2(2) = 1$$

when  $n=p$ ,

$$H(S) = \frac{1}{2} \log_2(2) + \frac{1}{2} \log_2(2) = 1$$