Problem 4

Consider the vectors $u = \begin{bmatrix} 1 & 2 \end{bmatrix}^T$ and $v = \begin{bmatrix} 2 & 3 \end{bmatrix}^T$. Define the matrix $M = uv^T$. Compute the eigenvalues and eigenvectors of M.

Solution
Computing M:

$$\begin{aligned} M &= uv^T \\ &= \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \end{aligned}$$

Problem 4 continued on next page...

Getting the eigenvalues from the characteristic equation:

$$\begin{aligned}
det(A - \lambda I) &= 0 \\
det \begin{bmatrix} 2-\lambda & 3 \\ 4 & 6-\lambda \end{bmatrix} &= 0 \\
(2-\lambda)(6-\lambda) - 3 \cdot 4 &= 0 \\
12 - 8\lambda + \lambda^2 - 12 &= 0 \\
\lambda^2 - 8\lambda &= 0 \\
\lambda(\lambda - 8) &= 0 \\
\lambda_1 = 8, \lambda_2 &= 0
\end{aligned}$$

The eigenvalues are then $\lambda_1 = 8$ and $\lambda_2 = 0$. Now, by solving the equation $(A - \lambda I)v = 0$ for each eigenvalue, we find the corresponding eigenvectors:

$$v_{\lambda_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$v_{\lambda_2} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$