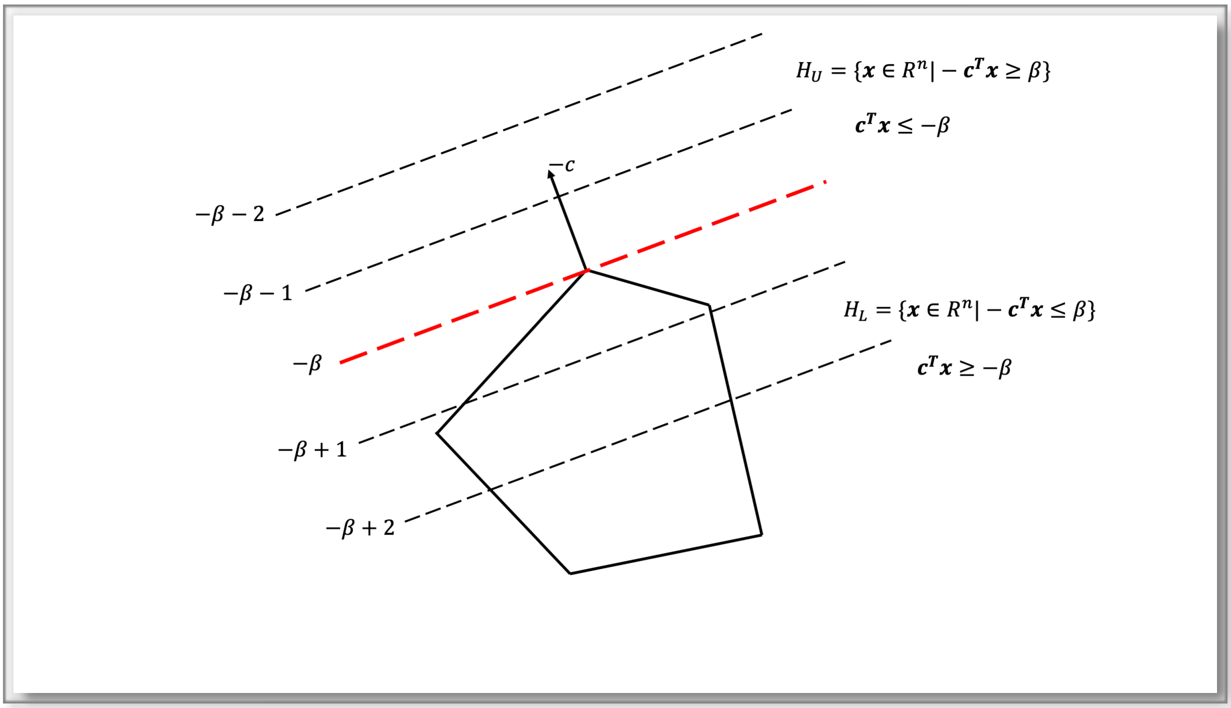
Simplex Method

Matlab Code User Manual



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2019

What problem can be solved?

We have two programs including two-phase simplex and dual simplex. All programs solve standard form linear programming:

$$
\text{(P)} \quad \begin{aligned} \min \quad & \boldsymbol{c}^T \boldsymbol{x} \\ \text{s.t.} \quad & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \\ & \boldsymbol{x} \geq \boldsymbol{0} \end{aligned}
$$

DO NOT solve its dual problem:

$$
\text{(D)} \quad \begin{aligned} \max \quad & \boldsymbol{b}^T \boldsymbol{w} \\ \text{s.t.} \quad & \boldsymbol{A}^T \boldsymbol{w} \leq \boldsymbol{0} \end{aligned}
$$

How to use?

• Two-phase simplex program

It contains two m-file: main.m, simplex.m. You should open main.m to input.

For example, you want to solve the following problem:

$$
\begin{aligned} \min \quad & x_1 - 2x_2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 40 \\ & 2x_1 + x_2 \leq 60 \\ & x_1, x_2 \geq 0 \end{aligned}
$$

You should convert it to standard form:

$$
\begin{aligned} \min \quad & x_1 - 2x_2 + 0x_3 + 0x_4 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 = 40 \\ & 2x_1 + x_2 + x_4 = 60 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}
$$

We can have the input corresponding to the problem:

```
%Input: please make sure b>=0
c=[1;-2;0;0];
A=[1,1,1,0;2,1,0,1];
b=[40;60];
```

NOTE: please make sure vetor $b \geq 0$! If not, times -1 for both sides of the constraint equality to make $b \geq 0$.

- Dual simplex program

It contains three programs: dual_simplex.m, main.m, simplex.m. You should open main.m to input.

Using another example：

$$\begin{aligned} \min \quad & -2x_1 - x_2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 2 \\ & x_1 \leq 1 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Convert to standard form:

$$\begin{aligned} \min \quad & -2x_1 - x_2 + 0x_3 + 0x_4 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 = 2 \\ & x_1 + x_4 = 1 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

We can have the input corresponding to the problem:

```
clc;

c=[-2;-1;0;0];
A=[1,1,1,0;1,0,0,1];
b=[2;1];


[m,n]=size(A);
```

NOTE: this time $b \geq 0$ is not necessary to be satisfied.

A brief introduction of the simplex and dual simplex algorithms

Key step of simplex algorithm:

Step 1: Find a bfs $\boldsymbol{x}$ with $\boldsymbol{A} = \left[\boldsymbol{B} \mid \boldsymbol{N}\right]$.

Step 2: Check for n.b.v.'s $r_q = \boldsymbol{c}^T \boldsymbol{d}_q = c_q - \boldsymbol{c}_q \boldsymbol{B}^{-1} \boldsymbol{A}_q$

If $r_q \geq 0$, $\forall$ nonbasic x_q, then the current bfs is optimal.

Otherwise, pick one(Brand's Rule to prevent cycling) $r_q < 0$.

Go to next step.

Step 3: If $\boldsymbol{d}_q \geq 0$, then LP is unbounded below.

Otherwise, find $\alpha = \min_{i:\text{basic}} \{ \frac{x_i}{-d_{qi}} \mid d_{qi} < 0 \}$

Then $\boldsymbol{x} := \boldsymbol{x} + \alpha \boldsymbol{d}_q$ is a new bfs.

Update $\boldsymbol{B}$ and $\boldsymbol{N}$. Go to Step 2.

In order to get initial bfs, solve the phase one problem:

$$
\begin{array}{lll}
 & \min & \sum_{i=1}^{m} u_i \\
(\text{PhI}) & \text{s.t.} & \boldsymbol{Ax} + \boldsymbol{Iu} = \boldsymbol{b}(\geq \boldsymbol{0}) \\
 & & \boldsymbol{x}, \boldsymbol{u} \geq \boldsymbol{0}
\end{array}
$$

Key step of dual simplex algorithm:

Step 1: Find an initial dual feasible solution with $\boldsymbol{A} = [\boldsymbol{B} \,|\, \boldsymbol{N}]$.
Step 2: Check $\boldsymbol{x}_B$, if $\boldsymbol{x}_B \geq \boldsymbol{0}$, then the current solution is optimal. Otherwise, pick one component of $\boldsymbol{x}_B$ that is negative.
Step 3: If $\tilde{a}_{p,l} \geq 0$, $\forall l \in \bar{N}$, then (P) is infeasible. Otherwise pick nonbasic variable x_j that satisfies $\lambda = \min_{l \in \bar{N}} \{-\frac{r_l}{\tilde{a}_{p,l}} \,|\, \tilde{a}_{p,l} < 0\}$. Step length $\alpha := \frac{\bar{b}_p}{-\tilde{a}_{p,j}}$. Edge direction is $\boldsymbol{d} = -\boldsymbol{B}\boldsymbol{N}_j$. Then $\boldsymbol{x} := \boldsymbol{x} + \alpha\boldsymbol{d}$ is a new solution. Update $\boldsymbol{B}$ and $\boldsymbol{N}$. Go to Step 2.

In order to get initial dual feasible solution, solve the following problem:

$$
\begin{array}{ll}
\min & \boldsymbol{c}^T\boldsymbol{x} \\
\text{s.t.} & \boldsymbol{Ax} = \boldsymbol{Be} \\
 & \boldsymbol{x} \geq \boldsymbol{0}
\end{array}
$$

where $\boldsymbol{B}$ is a basis of (P) and $\boldsymbol{e}$ is a column vector with m components, each component of $\boldsymbol{e}$ is 1.

This introduction may be a little confusing for rookies. You can search the relevant textbooks to get further information.