Linear Programming

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Lecture 2 Preliminaries

Outline:
1.Standard form LP
2.Embedded assumptions
3.Converting to standard form

● Explicit Form
Minimizing one objective function
Equality constraints
Non-negative variables

● Matrix Form
cost vector solution vector right-hand-side vector

$$\boldsymbol{c} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix} \qquad \boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \qquad \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

constraint matrix

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \qquad \begin{array}{rl} \min & \boldsymbol{c}^T\boldsymbol{x} \\ \text{s.t.} & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \\ & \boldsymbol{x} \geq \boldsymbol{0} \end{array}$$

● Converting to standard form

$$\begin{array}{rl} \min & 3x_1 - 2x_2 - 4|x_3| \\ \text{s.t.} & -x_1 + 2x_2 \leq -5 \\ & 3x_2 - x_3 \geq 6 \\ & 2x_1 + x_3 = 12 \\ & x_1, x_2 \geq 0 \end{array}$$

Rule 1: Unrestricted(free) variables

$$x_i^+ = \begin{cases} x_i & \text{if } x_i \geq 0 \\ 0 & \text{otherwise} \end{cases} \qquad x_i^- = \begin{cases} 0 & \text{otherwise} \\ -x_i & \text{if } x_i \leq 0 \end{cases} \qquad \text{Thus } x_i^+, x_i^- \geq 0$$

$x_i \in R \iff x_i = x_i^+ - x_i^-$
By-product: $|x_i| = x_i^+ + x_i^-$.
Potential problem: the requirement of $x_i^+ \times x_i^- = 0$.

Rule 2: Inequality constraints -> add a slack variable or subtract an excess variable.

Rule 3: Minimization of the objective function.

$$\max\ \boldsymbol{c^T x} = -\min\ (-\boldsymbol{c^T x})$$

● More on free variable and absolute value

Potential problems:

1. One quadratic constraint is missing. (Similar with complementary slackness in form)

$$x_i^+ \times x_i^- = 0$$

2. Increasing dimensionality.
3. One origin solution corresponds to many new solutions.

5=5-0=6-1=8.3-3.3

4. $|x|$ is a convex function while $-|x|$ is a concave function.
5. Maximize $c|x|$ would be problematic with c being positive.