Project

Calibration and Monte Carlo for Additive Processes

Financial Engineering

Consider the Additive Normal Tempered Stable Process (ATS) introduced in [2] with α=0.5 (ATS NIG). You are asked to calibrate the model from European options on the S&P 500 on the 7th of June 2019 and simulate the process using the FFT based method described in [1].

The S&P 500 2019-06-07.cvs file contains all calls and puts for every liquid maturity. The spot price S_0 is 2873.3.

i) Calibrate the Forward and Discount Factors following the technique described in [3]. Hereinafter, based on these calibrated values, consider zero-rates and dividends as a deterministic function of time.

ii) Calibrate the ATS NIG slice by slice (i.e. at each step consider options with the same maturity) as described in subsection 3.2 of [2]. Impose the additivity conditions of Theorem 2.1 when calibrating the model. Fit a power law ATS as described in [2]. Does the ATS satisfies Assumptions 1 and 2 of the method described in [1]?

iii) Write a function that simulates one ATS forward increment (between two times: s, t s.t. $t > s > 0$) using the Lewis-FFT algorithm (the steps are described in Appendix B of [1]). As described in [1] the CDF $\hat{P}$ should be estimated using the FFT. How one should simulate the underlying price S (based on point i) ?

iv) Consider 30 European call options with log-moneyness in a regular grid $\sqrt{t}\,(-0.2, 0.2)$. Write a function that prices options using the closed Lewis formula (eq. 6 in [2]) for a power scaling ATS. Price the 30 European call options with the Lewis-FFT-S simulation method. With expiry=19/06/2019, M=12, and 10^6 simulations. Compute the errors (RMSE, maximum error, and MAPE) w.r.t. the closed Lewis formula prices.

v) [Facultative] Write a function that, given a vector of times $(0 = t_0, t_1 \ldots t_i, \ldots t_Q)$ with $t_i > t_{i-1}$, simulates a path of the underlying S_{t_i} (not the forward) with monitoring dates $t_0, \ldots, t_Q$. Price a sprint autocallable put with $K = 2500$. This derivative pays (once and then the product is called) on 12 monthly monitoring dates t_i

$$\frac{1}{S_0} max\left(K - S_{t_i}, 0\right) * \frac{1}{S_0} \max_{j<i} S_{t_i}$$

Payment dates (two days after monitoring dates) go from t_1^p=9 Jul 2019 to t_{12}^p=9 Jun 2020.

Realize a library in Matlab.

Hints:

a. Calibrate the ATS using the most liquid instruments; i.e. OTM-forward calls and puts.
b. Calibrate the ATS starting from the first maturity.
c. Once you have estimated the CDF $\hat{P}$ and simulated the random vector of uniform U, we suggest using Matlab interp1($\hat{P}, x, U)$ to perform the numerical inversion of the CDF (i.e. interp1 implements all the algorithm steps in [1, Appendix B] after the simulation of U). Notice that Matlab's interp1 has the option 'spline'.
d. The first time the autocallable derivative is ITM is called (it ends) and it expires at t_{12}; it pays nothing if it is never called.
e. Use the modified following convention for business days.
f. When pricing the sprint autocallable, interpolate the ATS parameters between the available maturities. Choose a reasonable interpolation algorithm and justify it.

[1] Azzone, M., and R. Baviera. A fast Monte Carlo scheme for additive processes and option pricing. arXiv preprint arXiv:2112.08291 (2021).

[2] Azzone, M. and R. Baviera. Additive normal tempered stable processes for equity derivatives and power-law scaling. *Quantitative Finance* 22.3 (2022): 501-518.

[3] Azzone, M., and R. Baviera. Synthetic forwards and cost of funding in the equity derivative market. *Finance Research Letters* 41 (2021): 101841.

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Vocabulary

RMSE	Root Mean Square-Error
MAPE	Mean Absolute Percentage Error