Project for the "Local Volatility Model" course

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You have a file `MarketData_XY.xls` that contains market data for several assets. In particular, for each asset you will find:

- a market volatility surface $\{\sigma_{mkt}(T_i, K_{i,j})\}$ for $i \in \{1, \dots, N\}$, $j \in \{1, \dots, M\}$
- market expiry dates $(T_1, \dots, T_N)$
- market strikes $\{K_{i,j}\}$ (or deltas $\{\Delta_j\}$ for FX assets)
- forwards $(F_0(T_1), \dots, F_0(T_N))$ and discount factors $(D_0(T_1), \dots, D_0(T_N))$ at market expiry dates

so that the market price of a call option with expiry T_i and strike $K_{i,j}$ is obtained by Black formula $Bl(F_0(T_i), K_{i,j}, T_i, \sigma_{mkt}(T_i, K_{i,j}), D_0(T_i))$.

Assume that the local volatility function is parameterized by a local volatility matrix $\{v_{i,j}\}$ at the nodes $\{K_{i,j}\}$ with piecewise constant interpolation in time and spline interpolation with flat extrapolation in strike.

Solve the following exercises:

1 Write a function in Matlab that calibrates a local volatility model using fixed-point algorithm. Code must be optimized so that at each iteration of the algorithm only one Dupire equation is solved

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2.1 Consider market data of asset E CORP, contained in file `MarketData_XY.xls`, and calibrate a local volatility model with maximum calibration error of 10bps (i.e., threshold=0.001).

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2.2 With the model found above, price two call options with expiry $T = 0.5$ and strikes $K = \kappa \cdot S(0)$ with $\kappa = 0.9, 1.1$, by solving Dupire equation or by Monte Carlo simulation. Compute the relevant implied (spot) volatilities.

2.3 Using a Monte Carlo simulation, price two forward starting option with start date $T_1 = 2$, expiry $T_2 = 2.5$ and strikes $\kappa = 0.9, 1.1$. Compute the implied forward volatilities.

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in example forward smile.PROVA

~~2.4~~ Using the model implied volatilities above, compute the skew of the spot and forward smile, namely slope of the spot/forward implied volatility as a function of strike κ. What typical feature of the local volatility model do you observe?

2.5 Now, suppose that you find market quotes for the same forward starting options considered above and realize that are very different from the model prices. Is this difference between model and market prices a symptom of bad calibration?

IN EXAMPLE CALIB FAIL.PROVA

~~3~~ Consider market data of asset FAIL contained in file `MarketData_XY.xls` and try calibrate a local volatility model. Verify that the calibration procedure fails and explain why (hint: compute market prices $C_0(T_i, K_{i,j})$ of call options via Black's formula and observe the convexity of the map $K_{i,j} \mapsto C_0(T_i, K_{i,j})$ for fixed T_i)

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~~4.1~~ Consider market data of the FX asset $Y = EUR/USD$, contained in file `MarketData_XY.xls`. Forwards at the expiry dates are given in terms of the spot $Y(0)$, the domestic discount factors $\{D_0^d(T_i)\}$, the foreign discount factors $\{D_0^f(T_i)\}$ according to the formula $F_0(T_i) = Y(0)\frac{D_0^f(T_i)}{D_0^d(T_i)}$. Find market strikes $\{K_{i,j}\}$ from the quoted deltas and calibrate a local volatility model with maximum calibration error of 10bps (i.e., threshold=0.001).

4.2 Consider a plain vanilla option with expiry T_5 (the fifth market expiry provided) and strike $K_{5,2}$ (the strike corresponding to 25-Delta). Moreover consider a digital option[1] with the same expiry T_5 and strike $K_{5,2}$.
Under the Local Volatility model calibrated in [4.1] compute, for both plain vanilla and digital option, Monte Carlo price and confidence interval of level 95% for the true option price [2]. Use N=100000, M=100

4.3 Compute the Monte Carlo price and confidence interval of the same options of [4.2] under Black dynamics where the model parameter σ is taken as the market volatility $\sigma_{5,2}^{mkt}$ namely the 25-Delta volatility at expiry T_5. Use N=100000

4.4 Consider the confidence intervals of level 95% for the price of the plain vanilla option under Local Volatility and Black model, as computed in [4.2] and [4.3]. Check that they overlap or at least are very close, and explain why both models give the same result (up to numerical errors such as calibration error and Monte Carlo Error)
Do the same analysis for the digital option. Do the confidence intervals overlap?

[1] recall that a digital option is a derivative whose payoff at expiry T is $\Phi(S(T)) = \mathbf{1}_{S(T)>K}$ where K is the strike of the option

[2] see slides, and in particular the definition of the so-called Monte Carlo error $\sqrt{\hat{v}_N/N}$ where $\hat{v}_N$ is the sample variance of the Monte Carlo simulation. Also notice that sample variance can be computed using MATLAB *std* function

The project has to be done in groups of 2-3-4 persons. As soon as the composition of a group is communicated by email, the group will receive the file containing market data.
Each group will have to send all Matlab code used to solve the problems and a document (pdf, word, powerpoint, etc..) discussing the results.
Deadline: 5 Jan 2022 / 26 Jan 2022
Please use the following email address: `riccardo.longoni@intesasanpaolo.com`