Report of the Assignment 1

Report of the first assignment of Robot Dynamics and Control

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0. Introduction

0.1. CAD

In this assignment, the fundamental concept of force and torque equilibrium for a robotic manipulator is covered.
First of all, I used **Fusion360** to create all of the individual pieces of the CAD model, based on the parameters in the assignment paper. Here’s the list of the pieces:

- **Basis** : the main fixed component to which the link1 is attached to.
- **Motor1** : the motor simulating the joint between the basis and link1.
- **Link1** : the first of the two links.
- **Motor2** : the motor simulating the joint between the link1 and link2.
- **Link2** : the last link of the assembly.

Because I created a rigid joint between the two, the first motor and link1 both rotate with respect to the basis. Both the second motor and link2 rotate in relation to link1.

I didn’t build the model on Fusion following the *Denavit-Hartenberg* convention, but I have computed it on **MATLAB 2022a**. It allows a geometric transformation to be represented in three-dimensional Euclidean space with the minimum number of parameters, namely four.

For this assignment i had to solve two exercises, each with a different model configuration, as I have an RR *(2 rotational joints)* Planar Manipulator in the first and a PR *(prismatic-rotational joints)* Planar Manipulator in the second.

0.2. Data transfer

After finishing the model, I exported all of the CAD data to Matlab via **Inventor 2021** and used it to perform all of the computational operations required for the exercises.

In order to import the data on matlab i had to insert :

```
smimport('Assembly.xml')
```

I obtained a **SMIdata** file containing various data that I used for the project.

0.3. Matlab algorithm

In order to compute the equilibrium torques τ_1, τ_2 (and force and torque F, τ_2) I made a matlab function called in a matlab script that given the joint angles, the forces and the torques applied on the given points. The function, called *compute_tau,* starts by computing the position of the centers of mass CoM_1 and CoM_2 of the two links, with respect to the origin and depending on the current angles θ_1, θ_2. To compute the centers of mass, I consider the mass of the link and the motor as they are rigidly attached together.

The function then computes the position of the origins ${}^0O_1$, ${}^0O_2$ of the link frames and the position of the application points, also in this case depending on the angles. Given the two origins, the function can then compute the transformation matrices, and the jacobian matrices J_1, J_2.

In order to compute the required informations about torques and forces the function also computes the rigid body jacobian matrices $S_{1,2}, S_{2,2}, S_{3,1}$ and the contribute of the gravity force C.

After all this computations, the function has all the element it needs to compute the results.

1. Exercise 1

This is the configuration of the robot manipulator for the first exercise of the assignment.

The robotic arm simply consists in two rotational joints, the first of the two located in the base frame, the origin of our coordinate system.

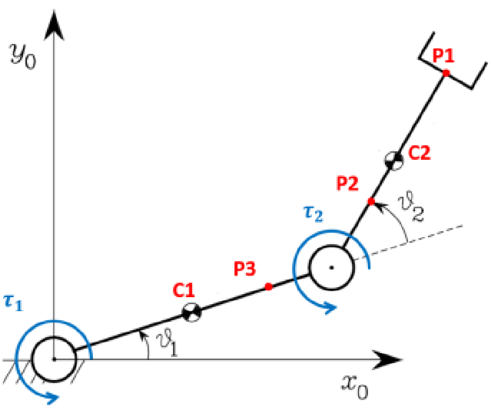

Figure 1: RR Planar Manipulator

The forces are applied at three different points, which are listed below:

- P_1 - at the end of the second link;
- P_2 - 20cm below the CoM_2;
- P_3 - 40cm from the CoM_1.

1.1. Robot equilibrium condition

I want to find the conditions on control actions such that the robot is in equilibrium: that means that the position and orientation of the robot's points remain the same over time.

In general, the robot must satisfy the following conditions when it is in equilibrium:

$$(1.1.1) \qquad \tau + \sum_{i=1}^{n} {}^oJ^T \begin{bmatrix} M_i^{(ext)} \\ F_i^{(ext)} \end{bmatrix} = 0$$

We can keep the robot in **Equilibrium Conditions** using the actuation vector.

To compute the torques in Matlab, I used the following equation:

$$(1.1.2) \qquad \tau + \sum_{i=1}^{n} {}^{o}J^{T}_{Ci/0} S^{T}_{Pi/Ci} \begin{bmatrix} M_i^{(ext)} \\ F_i^{(ext)} \end{bmatrix} + C(q) = 0$$

where

$$S^{T}_{Pi/Ci} \begin{bmatrix} M_i^{(ext)} \\ F_i^{(ext)} \end{bmatrix} = \begin{bmatrix} M_{Ci}^{(ext)} \\ F_{Ci}^{(ext)} \end{bmatrix}$$

that are the set of torques and forces that represent the resultant external forces and torques acting on the body expressed at the center of mass.

1.2. Answers

An important point to clarify is the reference system used in the fusion environment:

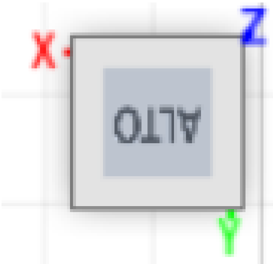

Figure 1: Reference system

1.2.1. Question 1

The geometric configuration of the RR Planar Manipulator for the first question of the exercise 1, considering $\theta_1 = 90^\circ$ and $\theta_2 = -90^\circ$, is :

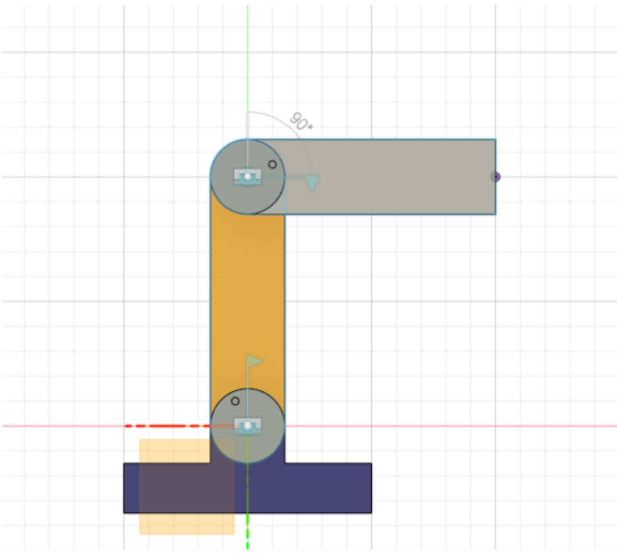

Figure 1.1 : RR Planar Manipulator in 1.1 geometric configuration

Both links' gravitational forces are applied to the center of mass of the two links, so we have two contributors here.

In this case there isn’t any external force or torque acting on the robot, so the resulting formula is formula:

$$(1.2.1) \qquad \tau_{eq} = -\sum_{i=1}^{n} {}^{o}J^{T}_{Ci} \begin{bmatrix} 0 \\ m_i \, {}^{o}g \end{bmatrix}$$

Given the high mass of the model, I expect only two high-intensity contributions of equal amplitude, because the torque does not change depending on the point of application, and the only moment that the CoM_1 has is the same as that caused by the gravitational force on the CoM_2.

1.2.2. Question 2

The geometric configuration of the RR Planar Manipulator for the second question of the exercise 1, considering $\theta_1 = 0^\circ$ and $\theta_2 = 90^\circ$, is :

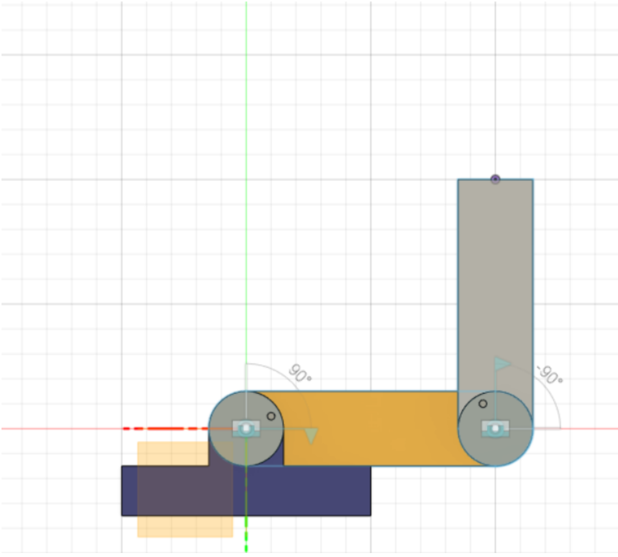

Figure 1.2 : RR Planar Manipulator in 1.2 geometric configuration

As for the previous exercise here I expect to have two contributions given from the gravity force.

The only difference is that τ_2 = 0 because the force applied on CoM_2 is parallel to the distance vector.

1.2.3. Question 3

The geometric configuration of the RR Planar Manipulator for the third question of the exercise 1, considering $\theta_1 = 30^\circ$ and $\theta_2 = 60^\circ$, is :

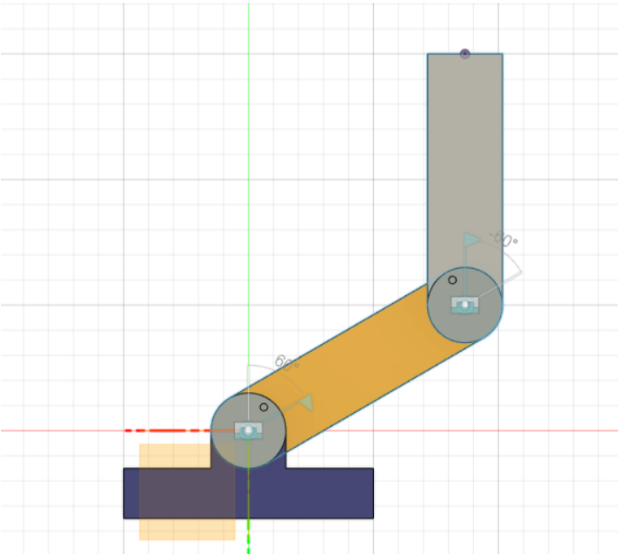

Figure 1.3 : RR Planar Manipulator in 1.3 geometric configuration

In this point of the exercise gravity is null, so I expect that the results are much lower than the two previous exercises. A force $F = [-0.7, -0.5]$ is applied on P_1. The two components of F are negative, but since $|F_x| > |F_y|$, in this particular configuration F push the second link in a counter-clockwise direction. I expect then two negative τ_1, τ_2, with a low module.

The force F is then applied to P_2, with same components. Since the distance of P_2 from the CoM_2 is less than the distance of P_1, I expect a even lower module then before, with the same negative sign.

1.2.4. Question 4

The geometric configuration of the RR Planar Manipulator for the fourth question of the exercise 1, considering $\theta_1 = 30^\circ$ and $\theta_2 = 60^\circ$, is :

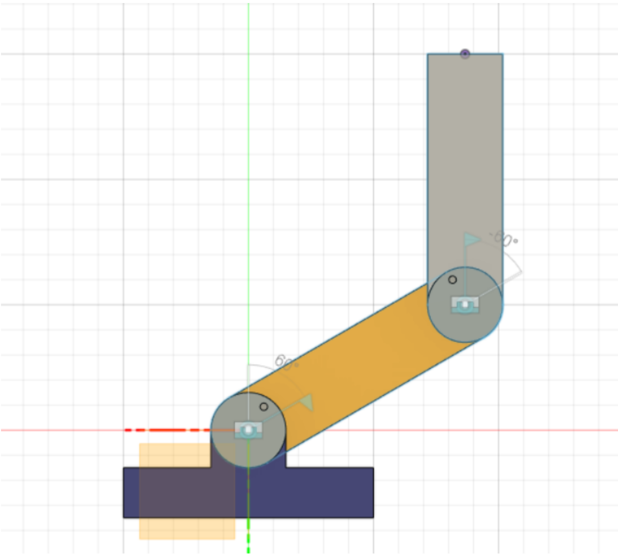

Figure 1.4 : RR Planar Manipulator in 1.4 geometric configuration

Also here gravity is absent, and the value of the force applied in the point P_3 is

$F = [1.5, -0.3]$. Also there's a torque $\tau_{ext} = 1.2$ applied on P_1, so overall I expect to have small values for the equilibrium torques. Since the value of τ_{ext} is positive it should generate a counterclockwise rotation along the *z-axis* so I expect to have a negative value of τ_2 to overcome to this rotation. Concerning τ_1, also in this case I expect to have a negative value since the resultant of the force should again generate a counterclockwise rotation along the *z-axis* .

1.2.5. Question 5

The geometric configuration of the RR Planar Manipulator for the fifth question of the exercise 1, considering $\theta_1 = 30^\circ$ and $\theta_2 = -60^\circ$, is :

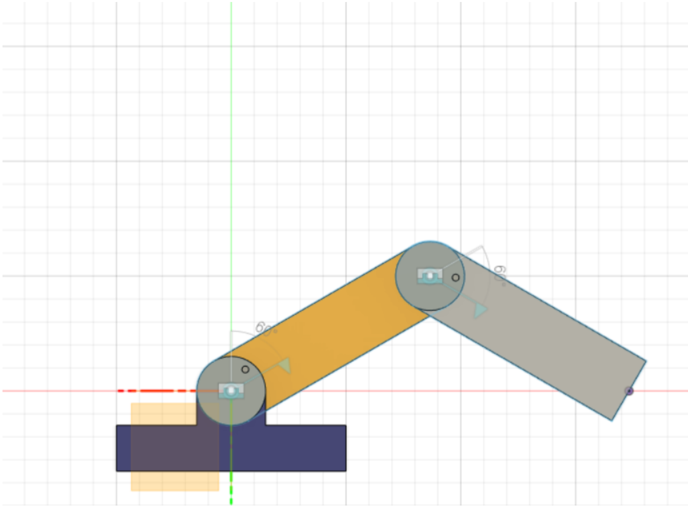

Figure 1.5 : RR Planar Manipulator in 1.5 geometric configuration

Gravity is back again in this last exercise, and there is a force applied in P_3 with a low absolute value, so I expect high values of equilibrium torques due to gravity's effect. Because gravity causes a clockwise rotation along the *z-axis*, the torques should act counterclockwise and have a positive value to counteract this rotation.

1.3. Final results

	1.1	1.2	1.3.1	1.3.2	1.4	1.5
τ_1 (Nm)	4551.2	13306	-0.6170	-0.1148	-0.3422	15464
τ_2 (Nm)	4551.2	0	-0.7000	-0.1978	-1.2000	3941.3

2. Exercise 2

For the second exercise of the assignment, this is the robot configuration.

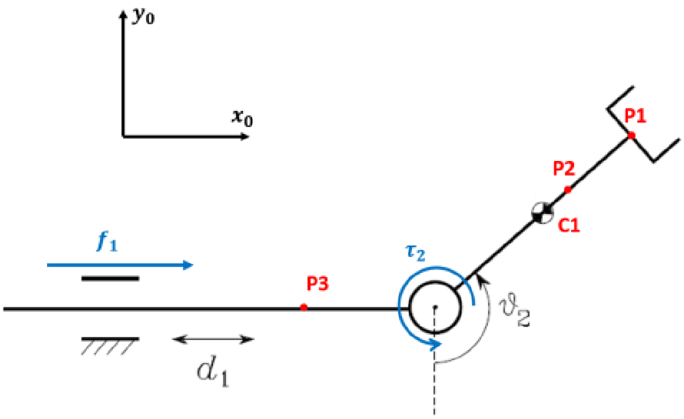

Figure 2: PR Planar Manipulator

Calculate the robot's generalized joint forces required to balance external forces acting on it under the following conditions.

The forces are applied on three points, here specified the specific locations:

- P_1 - at the end of the second link;

- P_2 - 15cm above the CoM_1
- P_3 - 20cm left from the CoM_{joint_1}.

It's important to note that the first link in this exercise is prismatic, so the first angle with respect to the base has to be considered null.

2.1. Answers

An important point to clarify is the reference system used in the fusion environment:

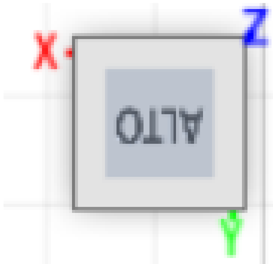

Figure 2: Reference system

2.2.1. Question 1

The geometric configuration of the PR Planar Manipulator for the first question of the exercise 2, considering $\theta_2 = 45^\circ$(since the first joint is a **prismatic** joint) :

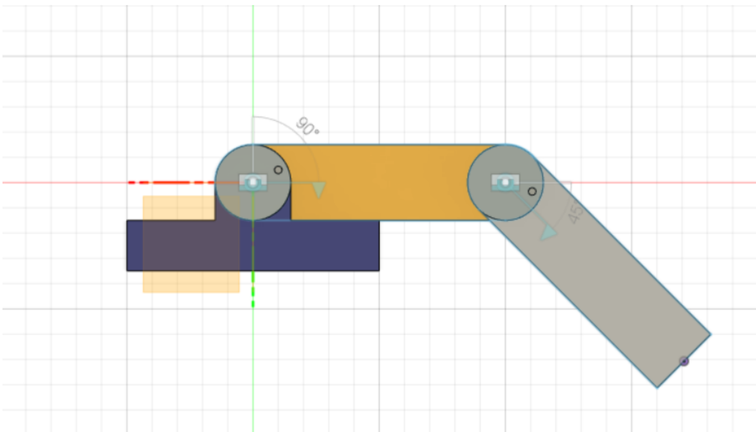

Figure 2.1 : PR Planar Manipulator in 2.1 geometric configuration

The gravity contribution of the first link is ignored due to the reaction force created by the ground. In fact, the equation is:

$$(2.1.1) \qquad -\sum_{i=1}^{n} {}^oJ_{Ci}^T \begin{bmatrix} 0 \\ m_i\,{}^og \end{bmatrix} = -\,{}^oJ_{Ci}^T (m_2 + m_{motor2})\,{}^og$$

2.2.2. Question 2

The geometric configuration of the PR Planar Manipulator for the second question of the exercise 2, considering $\theta_2 = 90^\circ$(since the first joint is a **prismatic** joint) :

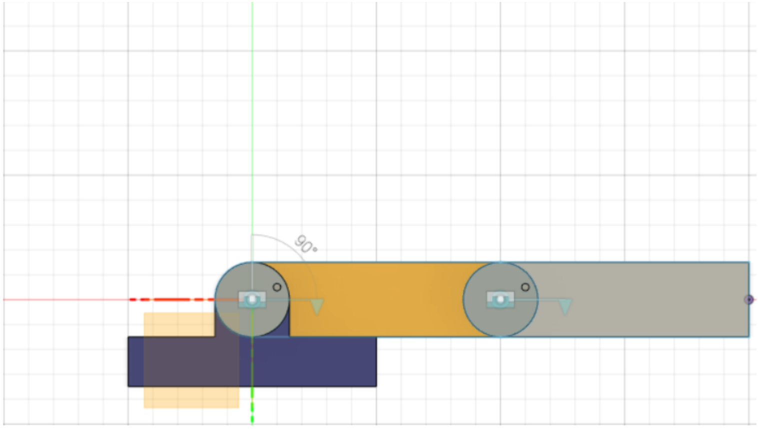

Figure 2.2 : PR Planar Manipulator in 2.2 geometric configuration

As before, the gravity contribution of the first link is ignored.

Gravity will then push down the second link on his center of mass CoM_2, generating a torque pointing to the ground. τ_2 should then have a positive value, in order to counteract the force of gravity.

2.2.3. Question 3

The geometric configuration of the PR Planar Manipulator for the third question of the exercise 2, considering $\theta_2 = 45^\circ$(since the first joint is a **prismatic** joint) :

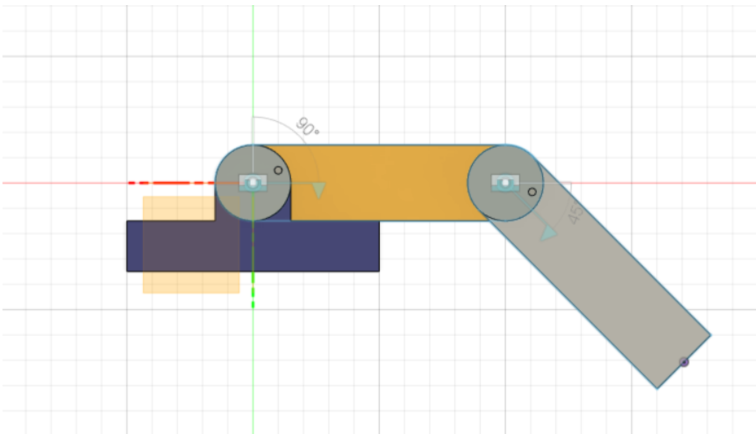

Figure 2.3 : PR Planar Manipulator in 2.1 geometric configuration

On the top of the second link, there is now only one external force $F = [-0.8, -0.8]$operating on P_1. Also the gravity here is absent. To find the equilibrium torques, I must now employ rigid jacobians. As result I expect to have τ_{eq} and F_{eq} with low values and because the external force causes the manipulator to rotate clockwise and traslate to the left, I expect τ_{eq} to try to avoid this rotation by acting counter-clockwise, and F_{eq} to act along the *x-axis.*

2.2.4. Question 4

The geometric configuration of the PR Planar Manipulator for the fourth question of the exercise 2, considering $\theta_2 = 45^\circ$(since the first joint is a **prismatic** joint) :

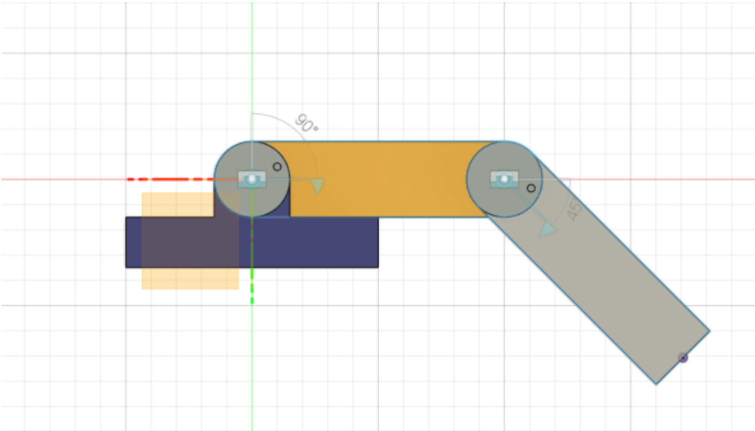

Figure 2.4 : PR Planar Manipulator in 2.1 geometric configuration

P_2 is now only subjected to the external force $F = [-0.8, -0.2]$ and the second link is subjected to a torque $\tau_{ext} = 0.5$ applied on P_1. I'll have two contributions, one due to force and the other due to torque, requiring us to compute the rigid jacobian.

2.2.5. Question 5

The geometric configuration of the PR Planar Manipulator for the fifth question of the exercise 2, considering $\theta_2 = 45^\circ$ (since the first joint is a **prismatic** joint) :

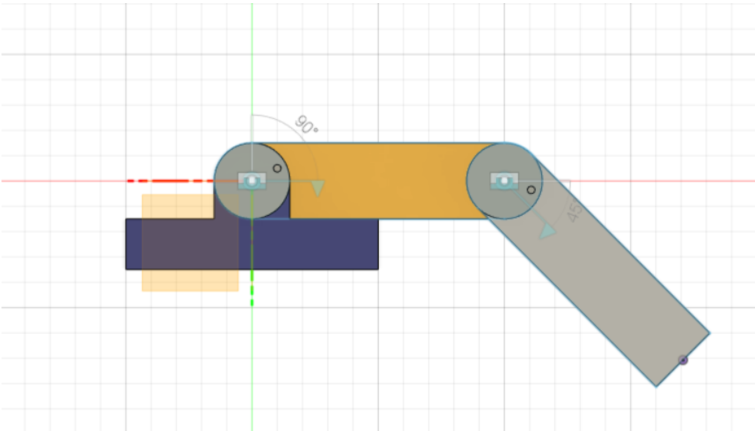

Figure 2.5 : PR Planar Manipulator in 2.1 geometric configuration

I now have two external forces operating on the second joint's center of mass, and the gravity which is again considered. Because all of the forces causes the revolute joint to rotate clockwise, I expect $\tau_2 > 0$.
The required informations can be found using this formula:

$$(2.5.1) \qquad \begin{bmatrix} F_{eq} \\ \tau_{eq} \end{bmatrix} = -\sum_{i=1}^{n} {}^{o}J_{Ci/0}^{T} S_{Pi/Ci}^{T} \begin{bmatrix} M_i^{(ext)} \\ F_i^{(ext)} \end{bmatrix} - C(q)$$

Which looks like this, in the expanded form:

$$(2.5.2) \qquad \begin{bmatrix} F_{eq} \\ \tau_{eq} \end{bmatrix} = -J_{link1}^{T} S_{P1/C2}^{T} W_{ext,1} - J_{link2}^{T} S_{P2/C2}^{T} W_{ext,2} - J_{link2}^{T} W_{grav,2}$$

2.2. Final results

	2.1	2.2	2.3	2.4	2.5
F_1 (N)	0	0	0.800	0.800	-1.500
τ_2 (Nm)	3218	4551	1.131	-0.0527	3218

3. Conclusions

The values of τ_1, τ_2 and the F I discovered were just what I had expected.

An important remark should be made on the total weight of the robot, which is about 2000 Kg. The robot is really heavy, and therefore the values of τ_1 and τ_2, when the gravity is condiered, are very high. Nevertheless, the results are consistent to the robot weight.

It was a bit challenging to import and pre-process all the required data in matlab, since the *datafile* generated by Simscape Multibody isn't easy to understand at a first glance. However, after some time, I became able to use in the correct way that data, in order to simplify my job.

Overall I am proud of the final result, also because anything inside the matlab function isn't hard-coded, but is computed in real time based on the motor angles.