

# Fresnel integrals

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The Fresnel integrals are defined as

$$S(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right) dt \quad \text{and} \quad C(x) = \int_0^x \cos\left(\frac{\pi t^2}{2}\right) dt \quad (1)$$

which are transcendental functions<sup>1</sup>. They are related to Fresnel diffraction in classical optics. In order to investigate the properties of these integrals, they will in the following be calculated numerically using a numerical integration routine in C# and compared to tabulated values given in [1]. The tabulated values in [1] are calculated using the power series expansion of equation 1 which are given by

$$S(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right) dt = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+3}}{(2n+1)!(4n+3)} \quad (2)$$

and

$$C(x) = \int_0^x \cos\left(\frac{\pi t^2}{2}\right) dt = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{(2n)!(4n+1)}. \quad (3)$$

The result of the numerical integration is presented in figure 1. Analytically, equation 1 may be shown to converge to 1/2 in the limit  $x \rightarrow \infty$ . To test convergence, the numerical integration was made for higher values of  $x$  as depicted in figure 2. Issues of the numerical integration routine are clearly seen in the intervals  $x \in [18, 21]$ ,  $x \in [22, 30]$  and  $x \in [37, 47]$ . Otherwise, the integrals seem to converge for all other values of  $x$  to 1/2. Since the test of convergence is not made for  $x = \text{machine infinity}$ , it is not very indicative.

## References

- [1] A. Van Wijngaarden and W. L. Scheen, *Table of Fresnel integrals*, The Computation Department of the Mathematical Centre, Amsterdam, 1949.

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<sup>1</sup>A transcendental function is an analytic function which does not satisfy a polynomial equation (as opposed to algebraic functions).

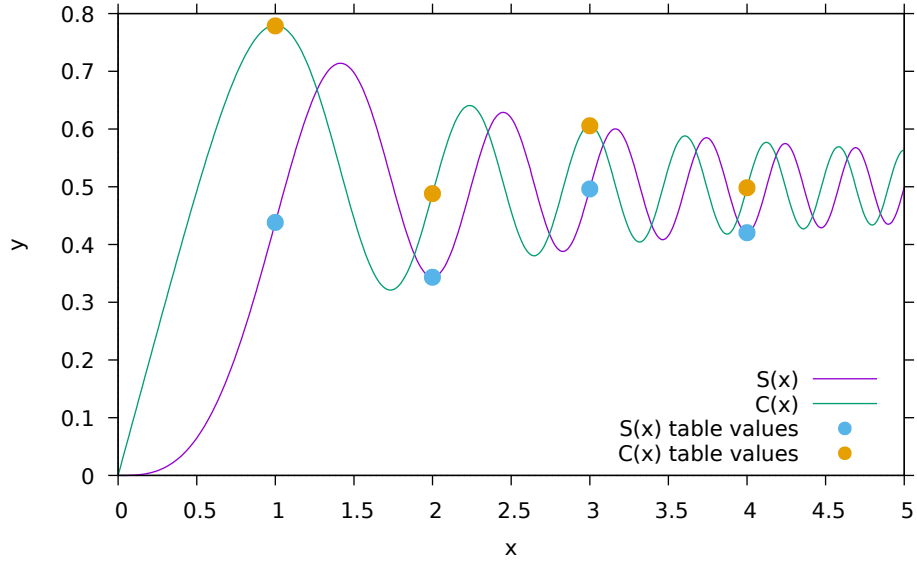


Figure 1: Numerical integration of the Fresnel integrals given by equation 1 along with the tabulated values given by [1].

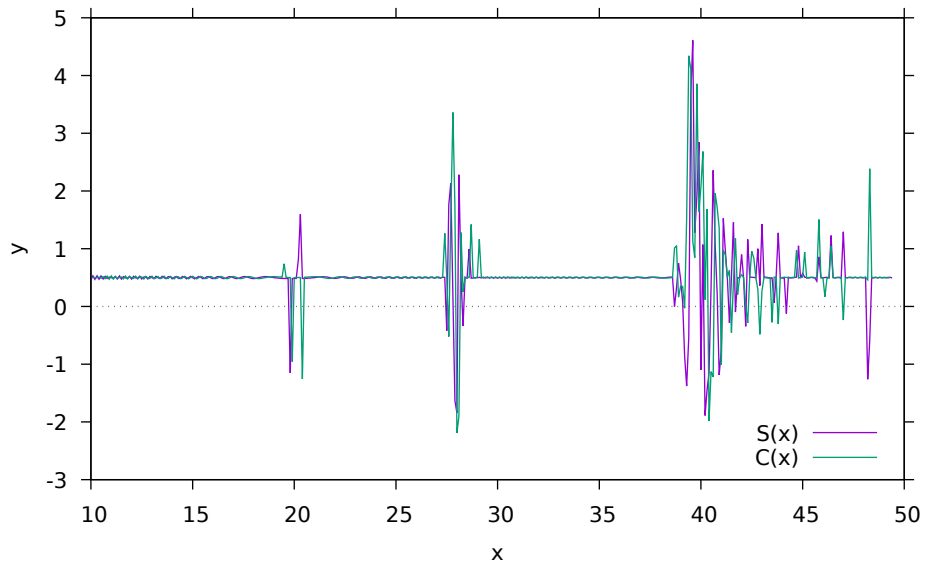


Figure 2: Large values of  $x$  in numerical integration of equation 1 to test convergence toward  $1/2$ .