Special Topics in Astrodynamics ae4889

Resit assignment tasks & deliverables

The resit assignment for the course Special Topics in Astrodynamics ae4889 is phrased in the form of a mock research proposal with clear tasks to be performed by you. The research proposal is about designing **a space-based geoengineering mission: using solar-sail technology as sunshades to limit the adverse effects of climate change**. Each student will be given their own case to examine (e.g., unique solar-sail performances) so that the work from all students combined forms a large parametric analysis that will help answering the research questions posed in the research proposal.

Mock research proposal

Title

Sunshades – solar-sail technology for space-based geoengineering

Background

Geoengineering is defined as the "deliberate large-scale manipulation of environmental processes that affect the Earth's climate with the intent to counteract the effects of global warming" [1]. The research community has proposed a wide variety of geoengineering techniques; an overview can be seen in Figure 1. Though controversial, interest in these geoengineering techniques is gradually rising because reports from the Intergovernmental Panel on Climate Change (IPCC) clearly indicate that only with highly ambitious emission reductions will humanity be able to keep temperatures within livable levels[1]. Geoengineering techniques may "buy us time" so that society has a chance to meet those strict reductions. However, note that researchers in the field largely agree that geoengineering techniques are not meant to *replace* emission reductions and would only be implemented until greenhouse gas concentrations fall below a threshold that allows a sustainable global climate.

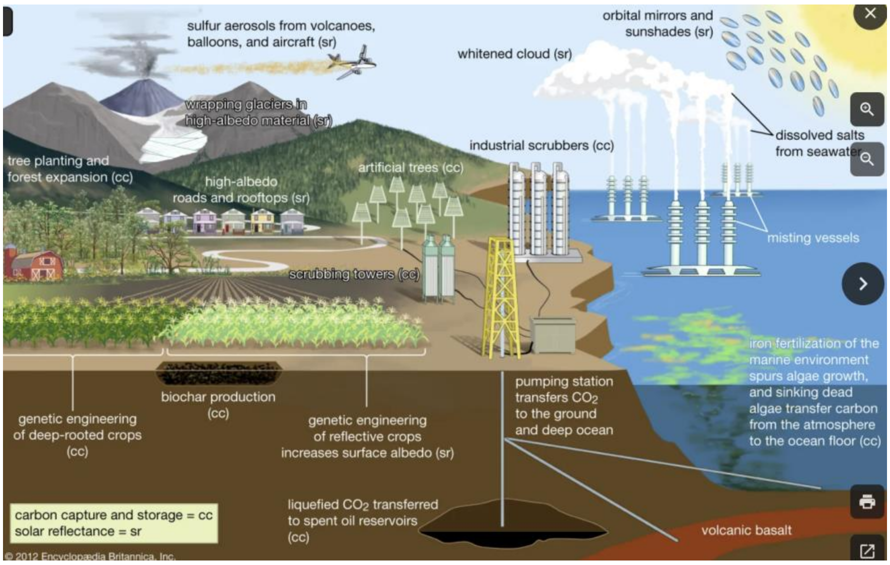


Figure 1 Overview of geoengineering techniques [source: Encyclopædia Britannica].

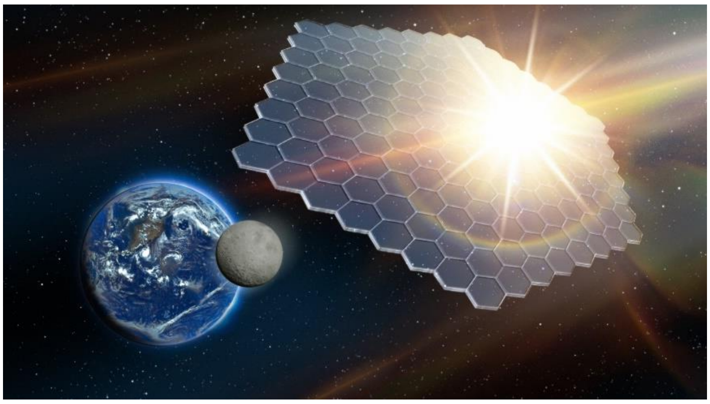
[1] Intergovernmental Panel on Climate Change (IPCC) Reports, https://www.ipcc.ch/reports/, Accessed 1 April 2024

The techniques in Figure 1 can largely be grouped into two categories:

1. **Carbon dioxide removal** where CO_2 is actively removed from the atmosphere to decrease the total atmospheric carbon content.

2. **Solar radiation management** where the aim is to reduce the amount of solar radiation absorbed by the Earth.

Solar radiation management can be achieved by either increasing the fraction of sunlight reflected by the Earth itself (e.g., marine cloud brightening or stratospheric aerosol injection) or by decreasing the total amount of solar radiation reaching the Earth. The latter is achieved from space, leading to the term "space-based geoengineering". Space-based geoengineering involves positioning a sunshade or space reflector between the Sun and Earth, see Figure 2. The sunshade would have to remain stationary between the Earth and Sun. Therefore, the vicinity of the Sun-Earth L_1 point is often considered as the best location. Furthermore, to offset the effects of a 680 ppm CO_2 atmospheric concentration (i.e., a doubling of the CO_2 concentration since the 1980s), a reduction in solar insolation of 1.7% would be required [2], which results in a continuous small solar eclipse when viewed from Earth (see the bottom-right schematic in Figure 2).



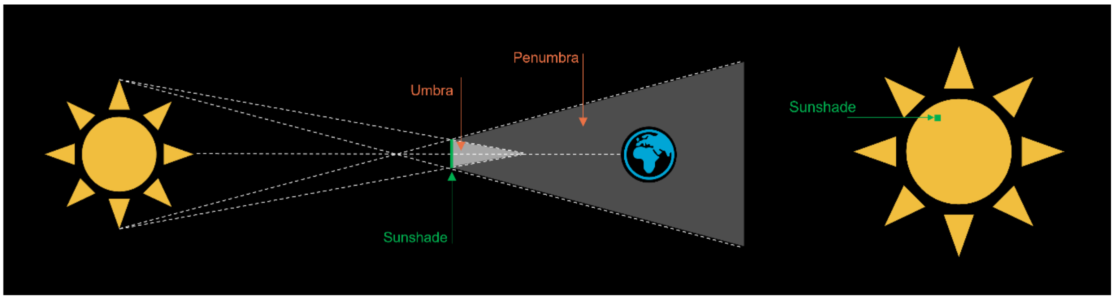


Figure 2 Top: Artist impression of a sunshade [source: The Planetary Society]; Bottom: A schematic illustrating the sunshade concept, including a "view from Earth" on the right.

The technology that lies at the foundation of the concept of sunshades are solar sails. Solar-sail technology is a green and sustainable low-thrust propulsion method that produces thrust purely by reflecting solar photons off a large, highly reflective membrane. Though often thought to be science fiction, four successful technology demonstration missions have been completed to date and NASA's next solar-sail mission, the 9 x 9 m^2 Advanced Composite Solar Sail System (ACS3) mission, see Figure 3, is scheduled for launch in April 2024. The "lightweightness" of the solar sail and its sunlight-blocking capability makes the technology highly suitable for the application of sunshades. Furthermore, the thrusting capabilities of the sail may be exploited for transferring the sunshade from Earth to the Sun-Earth L_1 point as well as for creating a safe end-of-life scenario (i.e., no impact or close encounters with Earth).

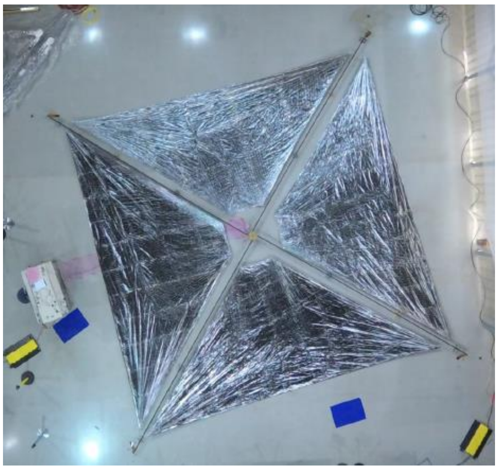


Figure 3 NASA's ACS3 sail after a ground deployment test [source: NASA].

Research objective and research question(s)

While previous studies have looked into the concept of sunshades, see for example References [3-7], little research has gone into the detailed modelling of the motion of the sunshade in the vicinity of the L_1 point. Literature also shows little to no research on how the shade can be deployed (i.e., how to manoeuvre the sunshade from Earth to its final destination) nor does the literature show any end-of-life analyses. The proposed research objective therefore is:

> **To identify suitable orbital motion in the vicinity of the L_1 point for a sunshade mission scenario and analyse the solar-sailing capabilities of the sunshade for mission deployment and a safe end-of-life scenario.**

By achieving this research objective, the following main research question may be answered:

> **Is solar-sail propulsion a viable means for a sunshade mission scenario, thereby contributing to our understanding of the viability of space-based geoengineering?**

More detailed sub-research questions that may be answered are:

1) What is the relation between sail performance, shade size and orbital stability?
2) Based on the relation found under research question 1), can an optimum sail performance be derived?
3) Does the natural motion towards and away from the orbital motion in the vicinity of the L_1 point allow for mission deployment and a safe end-of-life scenario?

Workplan

To answer the research questions posed above and fulfil the stated research objective, the work packages as illustrated in Figure 4 have been defined.

WP1: Locate the (artificial) Lagrange points – the classical L_1 Lagrange point of the Sun-Earth circular restricted three-body problem (CR3BP) is computed, a low-fidelity solar-sail model is implemented, and the artificial Lagrange point that result in the solar-sail augmented CR3BP is investigated.

WP2: Compute the stability, and the (un)stable manifolds, of the classical L_1 Lagrange point – the stability of the Lagrange point obtained under WP1 is computed, and the resulting unstable and stable manifolds emanating from the Lagrange point are evaluated.

WP3: Compute solar-sail periodic orbits for a sunshade mission scenario and the required sunshade size – based on a given initial condition, a family of solar-sail periodic orbits is computed and the "largest" orbit that still allows to cast a shadow on Earth is obtained and the required sunshade size is computed.

WP4: Determine the orbit stability, the resulting (un)stable manifolds, and a possible mission deployment and safe end-of-life scenario – WP2 is repeated for the orbit obtained under WP3 and the evolution of the (un)stable manifolds is considered for mission deployment and a safe end-of-life scenario.

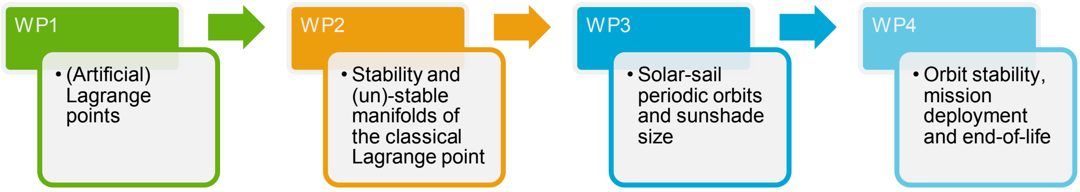


Figure 4 Work-packages for the research proposal "Sunshades – solar-sail technology for space-based geoengineering".

Tasks

The assignment (particularly WP3 and WP4) needs to be conducted using your own problem specifications. These specifications include:

1) The ID of a classical northern halo orbit around the Sun-Earth L_1 point in the JPL Three-Body Periodic Orbit Catalog (for Task 3.1)

2) The solar-sail performance of the sunshade (expressed through the solar-sail lightness number) (for Task 3.2b and onwards)

3) The initial conditions of a solar-sail southern halo orbit around the Sun-Earth L_1 point for the solar-sail performance specified under 2) (for verification purposes in Task 3.2b)

Some important remarks are:

- The values for **your personal problem specifications** can be found on Brightspace under "Assignment".
- Values for common astrodynamics constants and parameters can be found in the Appendix and **need to be used** in the execution of the tasks detailed below.
- **Provide your answer per (sub)task,** i.e., it should be clear what part of your report belongs to which (sub)task.
- Take note of the specified format in which the answers to the tasks need to be provided. **Not complying with the specified format will have an effect on your assessment.**
- The **deadline for submission of the full assignment is set on Friday 10 January 2025 at 23:59 (midnight)**. Submission after this deadline will result in a **one point deduction per 24 hours** that your submission is late.

WP1 (Artificial) Lagrange points

Task 1.1a Compute the coordinates of the Sun-Earth L_1 point using Newton's method.

Format: use a table like the one in Table 1 to provide your coordinates (x, y, z) in the Sun-Earth synodic reference frame $S(x, y, z)$ and in dimensionless form.

Table 1 Dimensionless coordinates (*x*,*y*,*z*) of the Sun-Earth L_1 point in the Sun-Earth synodic reference frame.

Lagrange point	x	y	z
Sun – Earth L_1			

Task 1.1b Verify the correctness of the coordinates obtained under Task 1.1a.

Format: Use a maximum of 50 words (excluding any references used) to describe your method of verification and the outcome of the verification. Indicate the number of words used.

Task 1.2a We now want to shift the Sun-Earth L_1 point **towards** the Sun and **above** the ecliptic plane. To enable the shift, we use solar-sail propulsion and assume an ideal performance of the sail. The amount by which the Lagrange point shall be shifted can be found **in dimensionless form** in Table 2.

- Compute the required solar-sail performance, i.e., the solar-sail lightness number, to achieve this shift.
- Comment on the near-term feasibility of the required solar-sail performance and justify your reasoning.

Format: Use a table like the one in Table 2 to provide the required solar-sail lightness number. For your comment and justification on the near-term feasibility, use a maximum of 50 words (excluding any references used). Indicate the number of words used.

Table 2 Required lightness number to maintain the shifted Lagrange point.

Lagrange point	Required *dimensionless* shift in Lagrange point		Required lightness number, β
	along the Sun-Earth line	**perpendicular** to the ecliptic plane	
Sun – Earth L_1	0.01	0.01	

Task 1.2b Compute, explain, and comment on the required orientation of the sail to maintain the shifted Lagrange point of Task 1.2a by specifying the orientation of the sail normal vector.

Format: Use a table like Table 3 below to provide the components of the normal vector, $\boldsymbol{n} = [n_x \quad n_y \quad n_z]$, in the Sun-Earth synodic reference frame. Use a maximum of 100 words (excluding any references used) to

- *Explain the obtained sail orientation by considering **all types** of accelerations acting on the spacecraft*

- *Considering the limited set of acceleration vectors that a solar sail can produce, discuss whether the newly obtained Lagrange point can be maintained with solar-sail propulsion or not.*

Indicate the number of words used.

Table 3 Required sail orientation to maintain the shifted Lagrange point.

Lagrange point	n_x	n_y	n_z
Sun – Earth L_1			

Task 1.2c Verify the correctness of the lightness number and sail orientation obtained under Tasks 1.2a and 1.2b.

Format: Use a maximum of 50 words (excluding any used references) to describe your method of verification and outcome of the verification. Indicate the number of words used.

WP2 Stability and (un)-stable manifolds of the classical Lagrange point

Task 2.1a Assess and comment on the stability of the **classical** Sun-Earth L_1 point (i.e., the one found under Task 1.1a) through an eigenvalue analysis of the Jacobian of the linearised system.

Format: Use a table like the one in Table 4 to provide the eigenvalues, $\lambda_1 \ldots \lambda_4$ of your Lagrange point. For your comment on the stability, use a maximum of 50 words (excluding any references used). Indicate the number of words used.

Table 4 Eigenvalues of the Lagrange point.

Lagrange point	λ_1	λ_2	λ_3	λ_4
Sun – Earth L_1				

Task 2.1b Verify the correctness of the eigenvalues obtained under Task 2.1a.

Format: Use a maximum of 100 words (excluding any used references) to describe your method of verification and outcome of the verification. Indicate the number of words used.

Task 2.2 Compute and plot the unstable and stable manifolds of the Sun-Earth L_1 point. Use absolute and relative integration tolerances of 10^{-12}, a propagation time of 3 years, and a dimensionless perturbation along the eigenvectors of 10^{-5}.

Format: Plot the resulting manifold trajectories on a suitable grid size with an aspect ratio such that the data units are the same in every direction (i.e., a circular trajectory should look like a circle, not an ellipse). In your plot, use different colours and/or line styles to make a clear distinction between the unstable and stable manifolds. Use axes labels and a legend to make the figure legible.

WP3 Solar-sail periodic orbits and sunshade size

For this work package, assume the size of the Earth to be 150x larger, i.e., use a radius of the Earth of 956,700 km. A justification for using this larger Earth is given in the workshop slides.

Task 3.1 Use the ID number in your personal problem specification to obtain the initial conditions, $\boldsymbol{x}_0 = [x_0 \quad y_0 \quad z_0 \quad v_{x,0} \quad v_{y,0} \quad v_{z,0}]$, of a **classical** (no solar-sail acceleration) northern halo orbit around the Sun-Earth L_1 point from the JPL Three-Body Periodic Orbit Catalog[2]. Use the "Orbit Filter" settings as show in Figure 5. The "Physical Properties of the System" section indicates that these orbits are computed for a mass ratio of $\mu = 3.05420 \times 10^{-6}$.

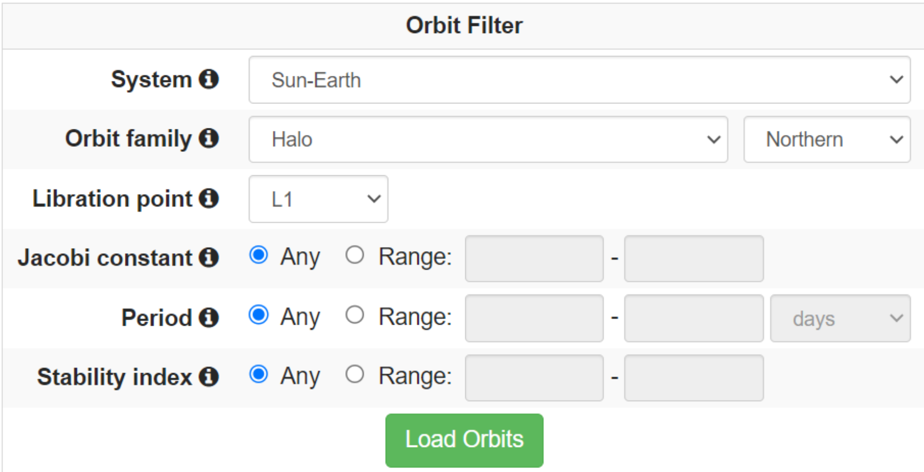


Figure 5 Correct "Orbit Filter" settings in the JPL Three-Body Periodic Orbit Catalog.

Using the mass ratio as specified above, propagate the initial condition. This propagation should lead to a periodic orbit. Plot this periodic orbit.

Format: Plot the periodic orbit on a suitable grid size with an aspect ratio such that the data units are the same in every direction. Provide four subplots: a 3D plot and (x, y)-, (x, z)- and (y, z)-projections, all in the Sun-Earth synodic reference frame $S(x, y, z)$. Add to each subplot:

- *The "inflated" Earth and the Sun-Earth L_1 point*
- *Make sure the "inflated" Earth is scaled correctly*

In your plots, use different colours and/or line/marker styles to make a clear distinction between the different elements in the figure. Use axes labels and a legend to make the figure legible.

Task 3.2a The mass ratio of the periodic orbit computed in Task 3.1a is different from the one in this assignment (see the data in the Appendix). This needs to be corrected. Use the **differential correction method** to do so. Assume that the differential corrector has converged when the norm on the error in the state after half an orbital revolution (so, at the first (x, z)-plane crossing), $\varepsilon = \|[v_{x,1/2} \quad v_{z,1/2}]\|$, is smaller than 10^{-12}. Note that, considering the small difference in value for the mass ratio, the differential corrector should converge within a few iterations (approximately five), if implemented correctly.

Add the resulting halo orbit to the subplots of Task 3.1 and provide the subplots once more, taking account of the format specification provided in Task 3.1.

[2] JPL Three-Body Periodic Orbit Catalog, https://ssd.jpl.nasa.gov/tools/periodic_orbits.html, Accessed 1 May 2024

Task 3.2b The orbits in Tasks 3.1 and 3.2a are **classical** halo orbits. To obtain a halo orbit for the sunshade, the next step is to add the solar-sail acceleration to the dynamics.

Starting from the orbit obtained in Task 3.2a, use the differential corrector and **a continuation on the lightness number**, β, to slowly introduce the solar-sail acceleration to the dynamics. Keep the initial *z*-coordinate constant and only change the initial *x*-coordinate and velocity in *y*-direction in the differential corrector.

1) For the sail attitude, assume a normal vector of $\boldsymbol{n} = [n_x \quad n_y \quad n_z] = [1 \quad 0 \quad 0]$, i.e., the sail normal is always along the Sun-Earth line so that the sail membrane is always perpendicular to the Sun-Earth line.

2) Stop the continuation when the lightness number equals the lightness number in your personal problem specification. Use an adequate step size, $\Delta\beta$, in the continuation so that the differential correct converges within several iterations.

3) As in Task 3.2a, assume that the differential corrector has converged when the norm on the error in the state after half an orbital revolution (so, at the first (x, z)-plane crossing), $\varepsilon = \|[v_{x,1/2} \quad v_{z,1/2}]\|$, is smaller than 10^{-12}.

Add the resulting solar-sail halo orbit (i.e., the final orbit of the continuation, so the one for the lightness number in your personal problem specification) to the subplots of Task 3.2a and provide the subplots once more, taking account of the format specification provided in Task 3.1.

To verify your differential corrector, your personal problem specification includes a unique initial condition for a southern solar-sail halo orbit for your specified lightness number. These initial conditions have been computed using a Runge-Kutta 4(5) integrator with relative and absolute tolerances of 10^{-12}. Your differential corrector should converge within one or two iterations if implemented correctly.

Task 3.2c From the “ground track” of the orbit obtained in Task 3.2b (i.e., the (y, z) projection), it is clear that the sunshade orbits outside the Earth’s disc and will therefore not cast a shadow on Earth. To correct for this, conduct **a second continuation, this time on the initial *z*-coordinate, $\boldsymbol{z_0}$**.

1) Reduce the initial *z*-coordinate until the solar-sail halo orbit falls inside the Earth’s disc, i.e., until the orbit amplitude in the (y, z)-plane (i.e., $\sqrt{y^2 + z^2}$) equals the “inflated” Earth’s radius. You have then obtained the “final sunshade orbit”, i.e., the largest orbit for which the sunshade still casts a continuous shadow on the Earth. Use an adequate step size, Δz_0, in the continuation so that the differential correct converges within several iterations.

2) As in Tasks 3.2a and 3.2b, assume that the differential corrector has converged when the norm on the error in the state after half an orbital revolution (so, at the first (x, z)-plane crossing), $\varepsilon = \|[v_{x,1/2} \quad v_{z,1/2}]\|$, is smaller than 10^{-12}.

Provide the initial conditions of the final sunshade orbit and add the final sunshade orbit to the subplots of Task 3.2b. Provide the subplots once more, taking account of the format specification provided in Task 3.1.

Format: For the initial conditions of the final sunshade orbit, use a table like the one in

Table 5.

Table 5 Initial condition of the final sunshade orbit, i.e., the largest orbit for which the sunshade still casts a continuous shadow on the Earth.

Lagrange point	x_0	y_0	z_0	$v_{x,0}$	$v_{y,0}$	$v_{z,0}$
Sun – Earth L_1						

Task 3.3 For the orbit with initial conditions in Table 5, compute the size in m^2 of the sunshade required to achieve a reduction in insolation of 1.7%. Comment on the feasibility of such a sunshade size considering past and current solar-sail missions. To compute the sunshade size, consider the point along the orbit *farthest* from the Earth and use the following equation [6]:

$$R_{Disk} = R_{Sun} \frac{d_{Disk}}{d_{Sun}} \sqrt{\frac{\Delta S}{S}}$$

where R_{Sun} is the radius of the Sun, d_{Disk} the distance of the sunshade from the Earth, d_{Sun} the Sun-Earth distance and $\frac{\Delta S}{S}$ is the targeted decrease in the solar flux, i.e., 1.7%.

Format: Provide the sunshade size and use a maximum of 50 words (excluding any references used) to comment on the feasibility in light of past and current solar-sail missions. Indicate the number of words used.

WP4 Orbit stability, mission deployment and end-of-life

For this work package, assume the size of the Earth to be its true size, i.e., 6,378 km.

Task 4.1 Assess and comment on the linear stability of the final sunshade orbit (i.e., the orbit with initial conditions in Table 5) through an eigenvalue analysis of the monodromy matrix.

Format: Use a table like the one in Table 6 to provide the eigenvalues, $\lambda_1 \ldots \lambda_6$ for the final sunshade orbit. Use a maximum of 100 words (excluding any used references) to comment on the stability of the orbit. Indicate the number of words used.

Table 6 Eigenvalues of the final sunshade orbit.

Lagrange point	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
Sun – Earth L_1						

Task 4.2 Compute and plot the unstable and stable manifolds of the orbit with initial conditions in Table 5 by integrating 10 perturbed initial conditions along the orbit, where the initial conditions are equally spaced in time. Use absolute and relative integration tolerances of 10^{-12}, a propagation time of 5 years, and a dimensionless perturbation along the eigenvectors of 10^{-8}. Note that, to be true manifolds, **the sail attitude along the manifolds needs to be the same as the sail attitude in the orbit**.

Format: Provide the following two plots on a suitable grid size with an aspect ratio such that the data units are the same in every direction. In your plot, make a clear distinction *between the unstable and stable manifold trajectories. Use axes labels and a legend to make the figure legible.*

1) *(x,y)- and (x,z)-projections of the (un)stable manifolds corresponding only to the initial condition in Table 5 (i.e., 1 set of (un)stable manifolds).*

2) *(x,y)- and (x,z)-projections of the (un)stable manifolds corresponding to all 10 perturbed initial conditions along the orbit (i.e., 10 sets of (un)stable manifolds).*

Task 4.3 Compute the closest distance to Earth along any of the unstable and stable manifolds computed in Task 4.2. Comment on whether the manifolds provide a potential means to deploy the sunshade (i.e., transfer it from Earth to the L_1 region) and whether they provide a means for a safe end-of-life disposal strategy (i.e., no impact or close encounter with Earth).

Format: Use a table like the one in Table 7 to provide the closest distance to Earth ***in km****. Use a maximum of 150 words (excluding any used references) to comment on the transfer feasibility and safe end-of-life disposal strategy. Indicate the number of words used.*

Table 7 Closest distance to Earth along any of the unstable and stable manifolds.

	Closest distance to Earth [km]
Unstable manifolds	
Stable manifolds	

Deliverables

The final deliverables are:

1) **A report**. This report should:

   - Include a title page with your name, student number, and details of the assigned case using tables like the ones below (the example table contains dummy data). **Also indicate on the title page who you collaborated with (if anyone).**

Student name	Jane Doe
Student number	1111111

Case details	
Parameter	**Value**
ID in JPL Three-Body Periodic Orbit Catalog	100
Solar-sail lightness number	0.01
Initial conditions of solar-sail southern halo orbit (up to three digits)	$\mathbf{x}_0$ = [0.911 0 0.199 0 0.199 0]

   - Include only the answers to the tasks outlined in this mock research proposal and clearly indicate which part of your report belongs to which task. I.e., **provide your answer per (sub)task;** it should be clear what part of your report belongs to which (sub)task.

   - **Follow the format instructions provided for each task. Not complying with the specified format will have an effect on your assessment.**

   - Include a list of indexed references

2) **Your code.** Upload your code as a clearly identifiable set of files in a new, *private* GitHub repository and include the link to the repository in the report. Invite the responsible instructor and teaching assistants to this private GitHub repository so they can access your code.

Assessment

For the assessment of your report, a rubric with the following criteria will be used:

1. Scientific soundness
   a. Correctness of the answers to the tasks defined in the research proposal
   b. Verification of answers against test data / validation against scientific literature
   c. Discussion supported by relevant references from scientific literature and publications
      i. Make good use of guidance, materials, and references provided; find additional, relevant references
   d. Free of scientific errors

2. Clarity of discussion and logical, scientific argumentation

3. Scientific writing & reporting. For example:
   a. Declaration and unique use of variables, if applicable
   b. Legibility of figures (axis labels, legend)
   c. Cross-referencing of figures and tables
   d. Impartial and unambiguous language

4. Report includes all elements described under “Deliverables”

Appendix - Table of constants and parameter values

Parameter	Value
Mass of the Sun	1.9891e+30 kg
Mass of Earth[3]	6.0477e+24 kg
Astronomical unit	149,597,870.7 km
Radius of the Sun	695,700 km
Radius of the Earth	6,378 km

[3] Mass of Earth-Moon system.

References

[1] Welch, A., Gaines, S., Marjoram, T., and Fonseca, L. “Climate engineering: The way forward”, *Environmental Development*, Vol.2, 2012, pp. 57-72. doi: 10.1016/j.envdev.2012.02.001

[2] Govindasamy B., Caldeira K., “Geoengineering Earth’s radiation balance to mitigate CO2-induced climate change”, *Geophysical Research Letters*, Vol. 27, No. 14, 2000, pp. 2141-2144, doi: 10.1029/1999GL006086

[3] McInnes, C.R., “Space-based geoengineering: Challenges and requirements”, *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, Vol. 224, No. 3, 2010, pp. 571-580, doi: 10.1243/09544062JMES1439

[4] Vaughan, N.E. and Lenton, T.M., "A review of climate geoengineering proposals," *Climatic Change*; Vol. 109, No. 3, 2011, pp. 745-790. doi: 10.1007/s10584-011-0027-7

[5] Angel, R., "Feasibility of cooling the Earth with a cloud of small spacecraft near the inner Lagrange point (L1)," *Proceedings of the National Academy of Sciences*; Vol. 103, No. 46, 2006, pp. 17184-17189. doi: 10.1073/pnas.0608163103

[6] Sánchez, J.-P. and McInnes, C.R., "Optimal Sunshade Configurations for Space-Based Geoengineering near the Sun-Earth L1 Point," *PLOS ONE*; Vol. 10, No. 8, 2015, pp. e0136648. doi: 10.1371/journal.pone.0136648

[7] Salazar, F.J.T., McInnes, C.R., and Winter, O.C., "Intervening in Earth’s climate system through space-based solar reflectors," *Advances in Space Research*; Vol. 58, No. 1, 2016, pp. 17-29. doi: 10.1016/j.asr.2016.04.007