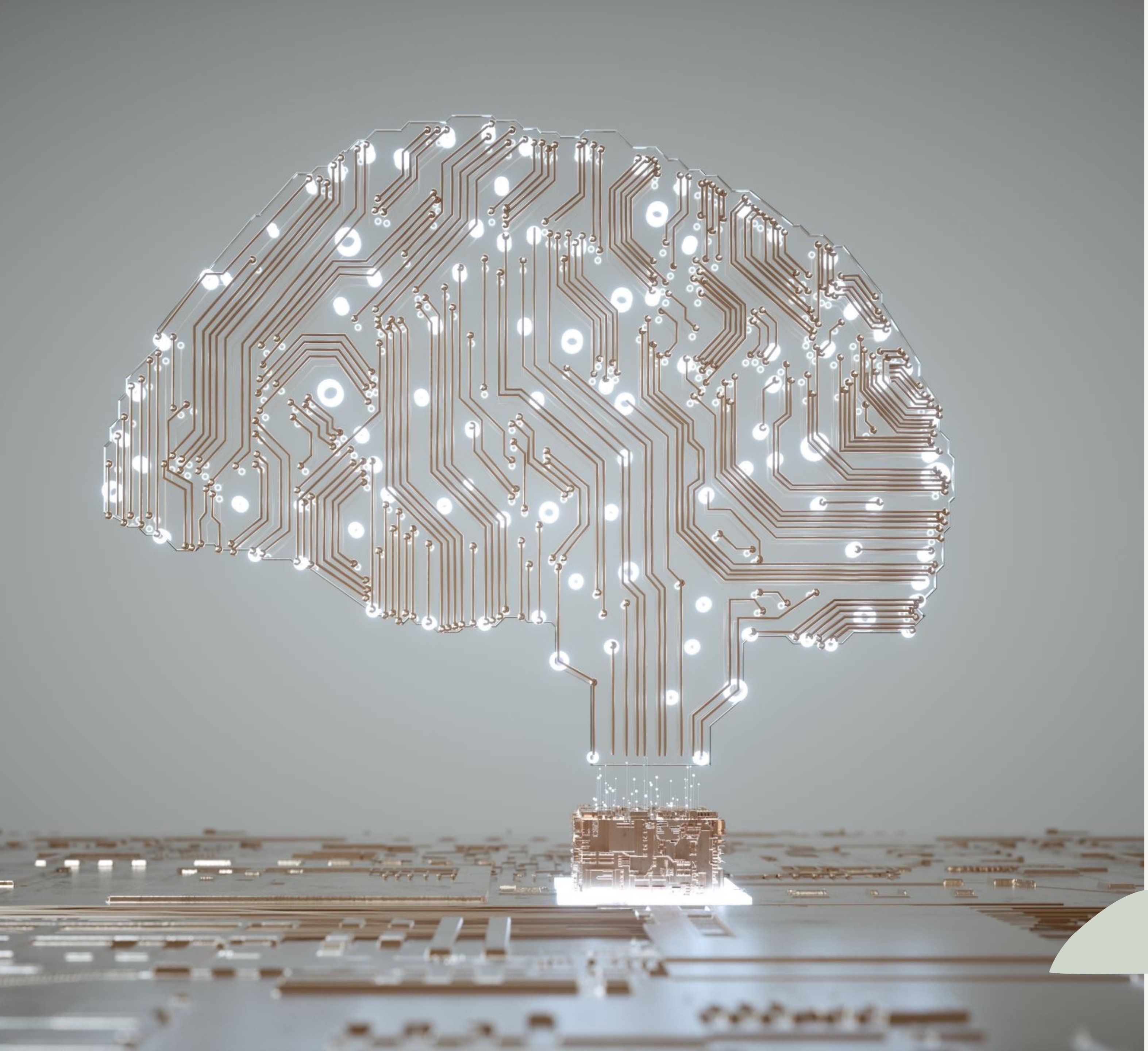

LOGISTIC REGRESSION *for Image Classification*

CREATIVE PROGRAMMING AND COMPUTING

Course

Prof. Massimiliano Zanoni

Slides and Presentation
by
Student Marco Muraro

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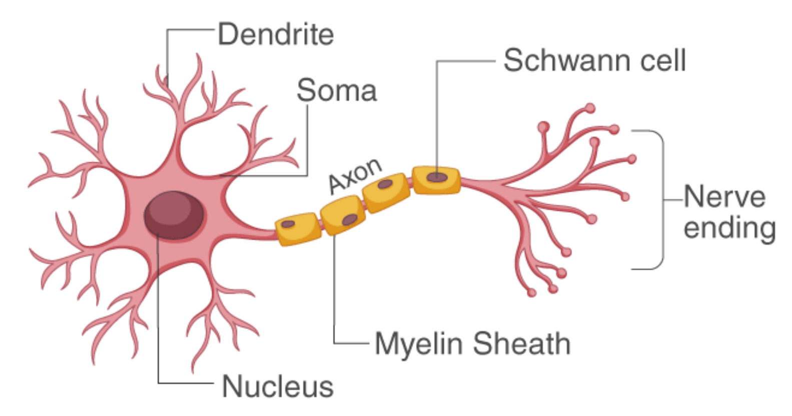
NEURON

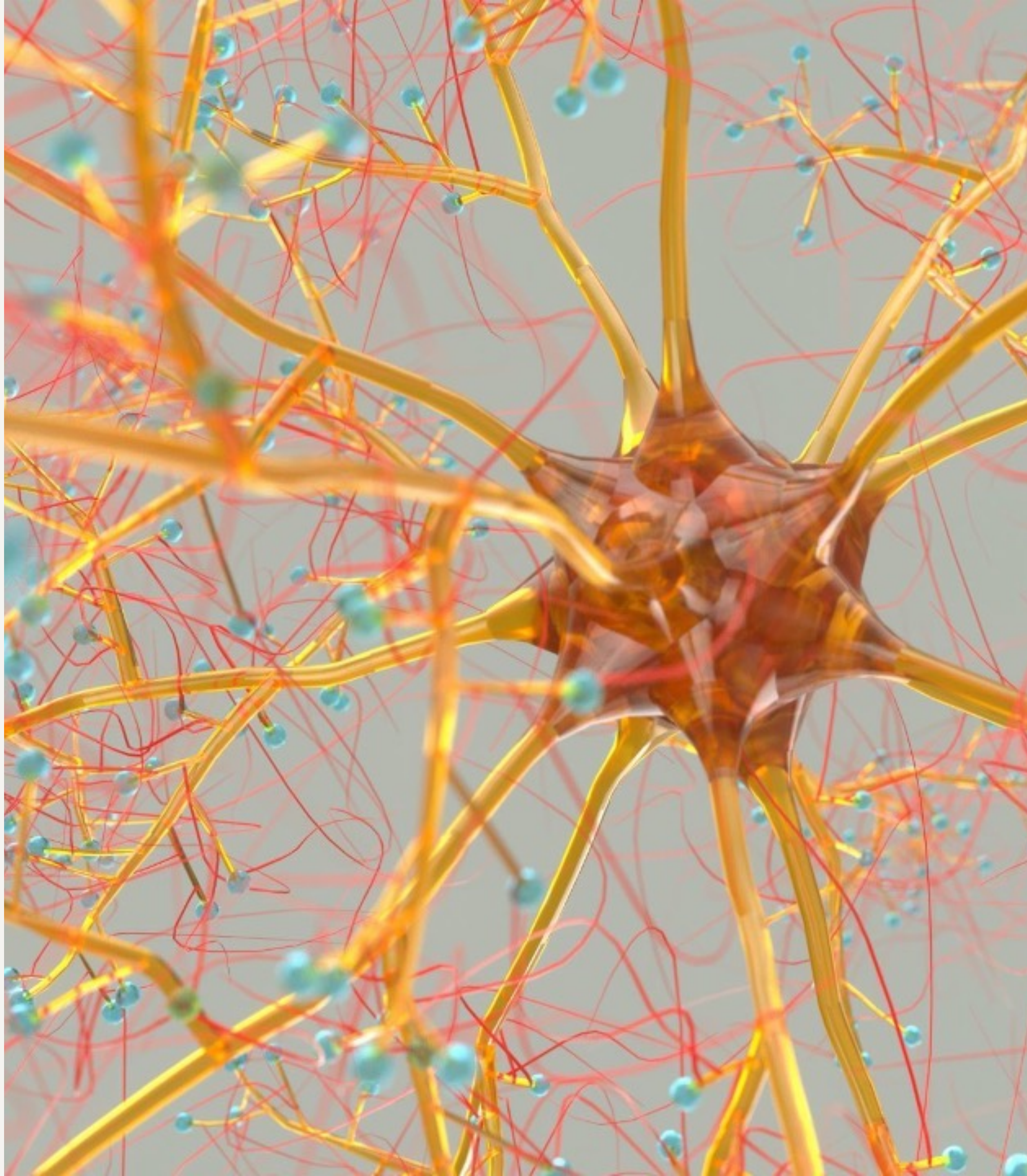
The **brain** is a **network of neurons**

Dendrites collect electrical stimuli coming from other neurons and accumulate input charges

Activation Process

The neuron fires an electrical impulse
when the total charge exceeds a certain threshold



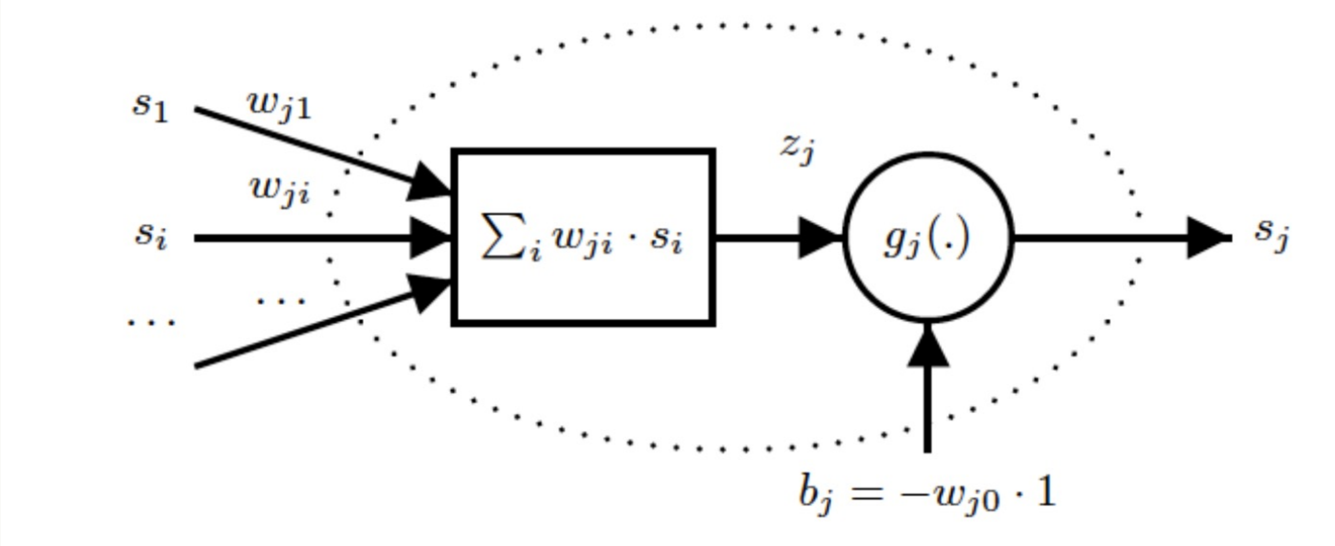


PERCEPTRON

Feed-Forward Model

$$s_j = g_j(z_{ij} - b_j) = g_j\left(\sum_{i=1}^{N} w_{ij}s_i + w_{j0}s_0\right) \quad \longrightarrow \quad \boldsymbol{s_j = g_j\left(\sum_{i=0}^{N} w_{ij}s_i\right)}$$

Output



Activation Value $z_{ij} = \sum_{i=1}^{N} w_{ij}s_i$

Activation Function $g_j(\cdot)$

Activation Threshold $b_j = -w_{j0}s_0 = -w_{j0}$

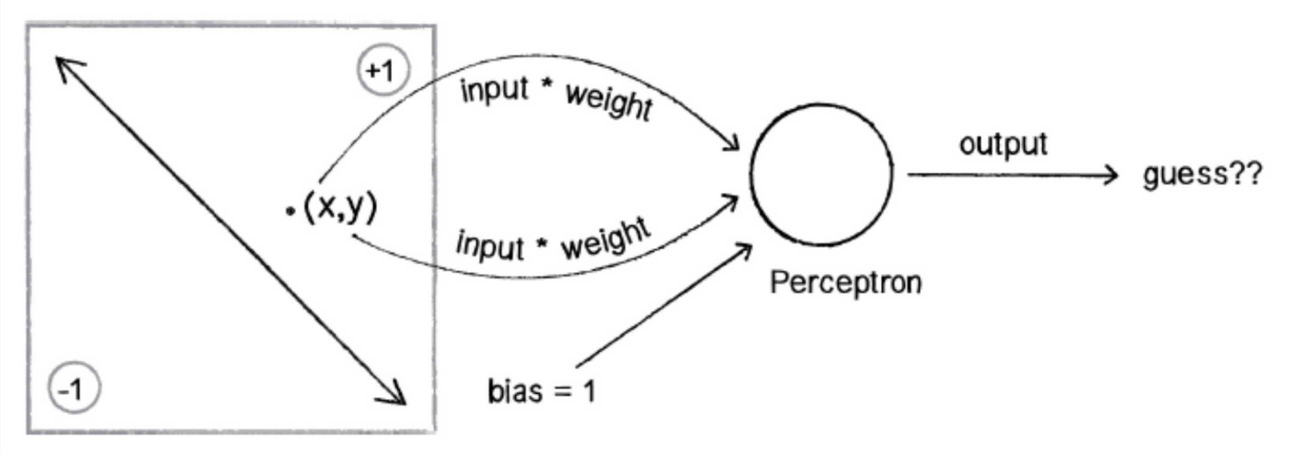
BINARY CLASSIFICATION

Linear separation

Decision boundary is described as a line

$$x_1 w_1 + x_2 w_2 + w_0 = 0$$

$$x_2 = -\frac{w_1}{w_2} x_1 - \frac{w_0}{w_2}$$



The **learning process** consists into **learning the weights giving the best linear decision boundary**

The perceptron keeps learning by mistakes and updates the weights to find the best ones

The **output** is a **class** in the classification problem

BINARY CLASSIFICATION

Learning Process

- Start with a **random guess for the weights**

- The perceptron **guesses an answer** $$\boldsymbol{s_j = g_j\left(\sum_{i=0}^{N} w_{ij}s_i - b_j\right) = g_j\left(\sum_{i=0}^{N} w_{ij}s_i\right)}$$

- Compute the **error function**

$\boldsymbol{\epsilon = \hat{s}_j - s_j}$ where $\boldsymbol{\hat{s}_j}$ is the estimated output and $\boldsymbol{s_j}$ is the desired output

- **Adjust the weights**

HEBB LEARNING RULE $w_i^{n+1} = w_i^n + \Delta w_i$ $\Delta w_i = \eta \cdot \hat{s}_j \cdot s_i$

- **Repeat the process** until **stop condition** η is the **learning rate**

The **number of iterations** is denoted as the **number of epochs**

LOGISTIC REGRESSION

for Binary Classification

Logistic Regression is a process of modelling the probability of a discrete outcome given an input variable.

The most common logistic regression models a binary outcome: TRUE/FALSE, YES/NO, -1/1, 0/1

Given a feature vector x,
logistic regression aims to find

$$\hat{\boldsymbol{y}} = \boldsymbol{P}(\boldsymbol{y} = \mathbf{1}|\boldsymbol{x})$$

→

Learning Process

Learning the weights w and the bias b giving the best estimate $\hat{y}$ for the probability $P(y = 1|x)$

$$\hat{\boldsymbol{y}} = \boldsymbol{g}(\boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{x} + \boldsymbol{b})$$

LOGISTIC REGRESSION

for Binary Classification

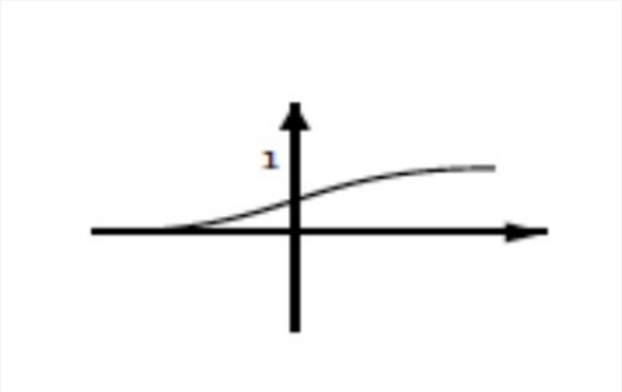
Activation function $g(\cdot)$ is applied

$$\hat{y} = g(w^T x + b)$$

We need a function mapping input data into a probability, i.e. within the range $[0,1]$

Sigmoid Function

$$g(z_j) = \frac{1}{1 + e^{K z_j}} K < 0$$



LOGISTIC REGRESSION

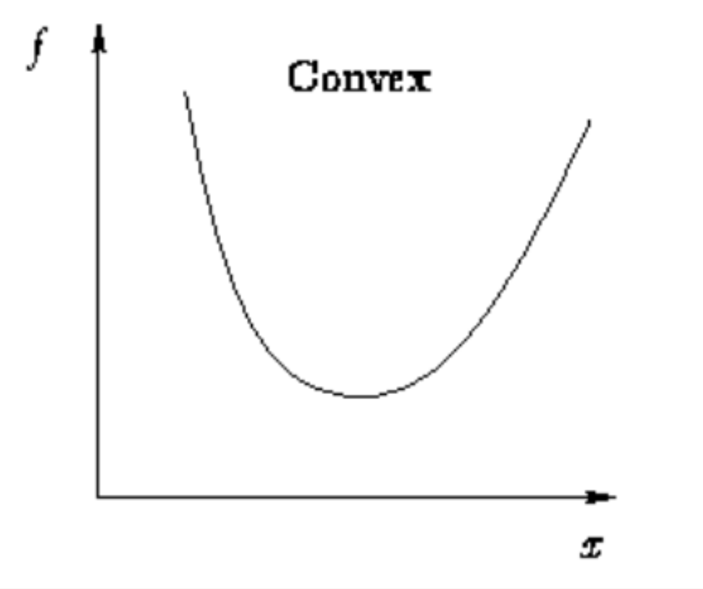
for Binary Classification

Loss Function

In order to learn the best weights w
and the bias b we need a loss function
$L(\hat{y}, y)$ to be minimized

$$L(\hat{y}, y) = -\log(y\ log\hat{y} + (1 - y)\log(1 - \hat{y}))$$

for a single feature vector



LOGISTIC REGRESSION

for Binary Classification

Cost Function

For the whole dataset (multiple feature vectors)

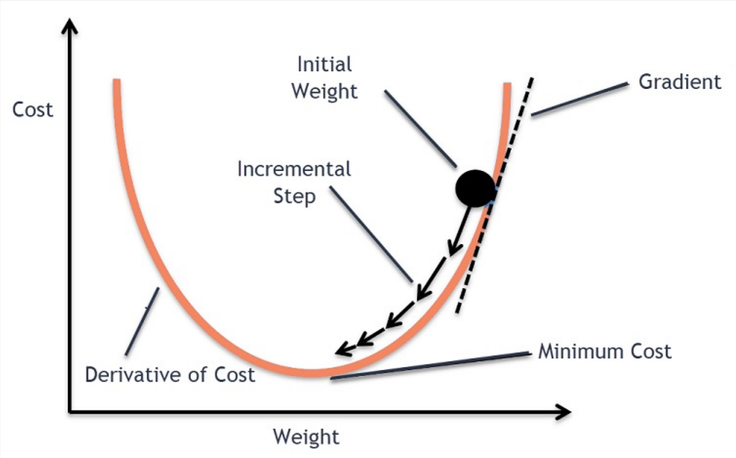
$$J(w,b) = \frac{1}{m}\sum_{I=1}^{m} L\left(\hat{y}^{(i)}, y^{(i)}\right) = -\frac{1}{m}\sum_{I=1}^{m}\left(y^{(i)}\, log\hat{y}^{(i)} + \left(1-y^{(i)}\right)\log\left(1-\hat{y}^{(i)}\right)\right)$$

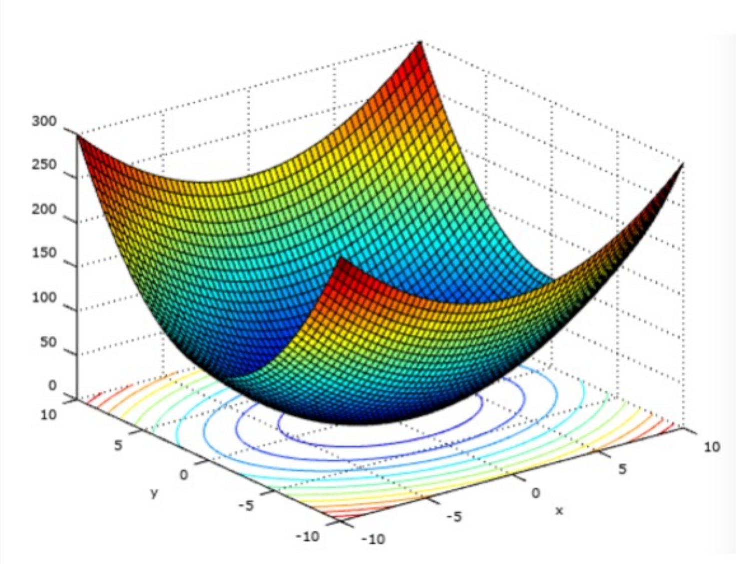
Therefore, weights w and b keep being adjusted minimizing $J(w,b)$ until the local minimum (global optimum) has been reached

$$w = w - \alpha\frac{dJ(w)}{dw} \qquad b = b - \alpha\frac{dJ(b)}{db}$$

α is the **learning rate**

The adjustment procedure is called **Back-Propagation** and the algorithm employed for optimization process is called **Gradient Descent**

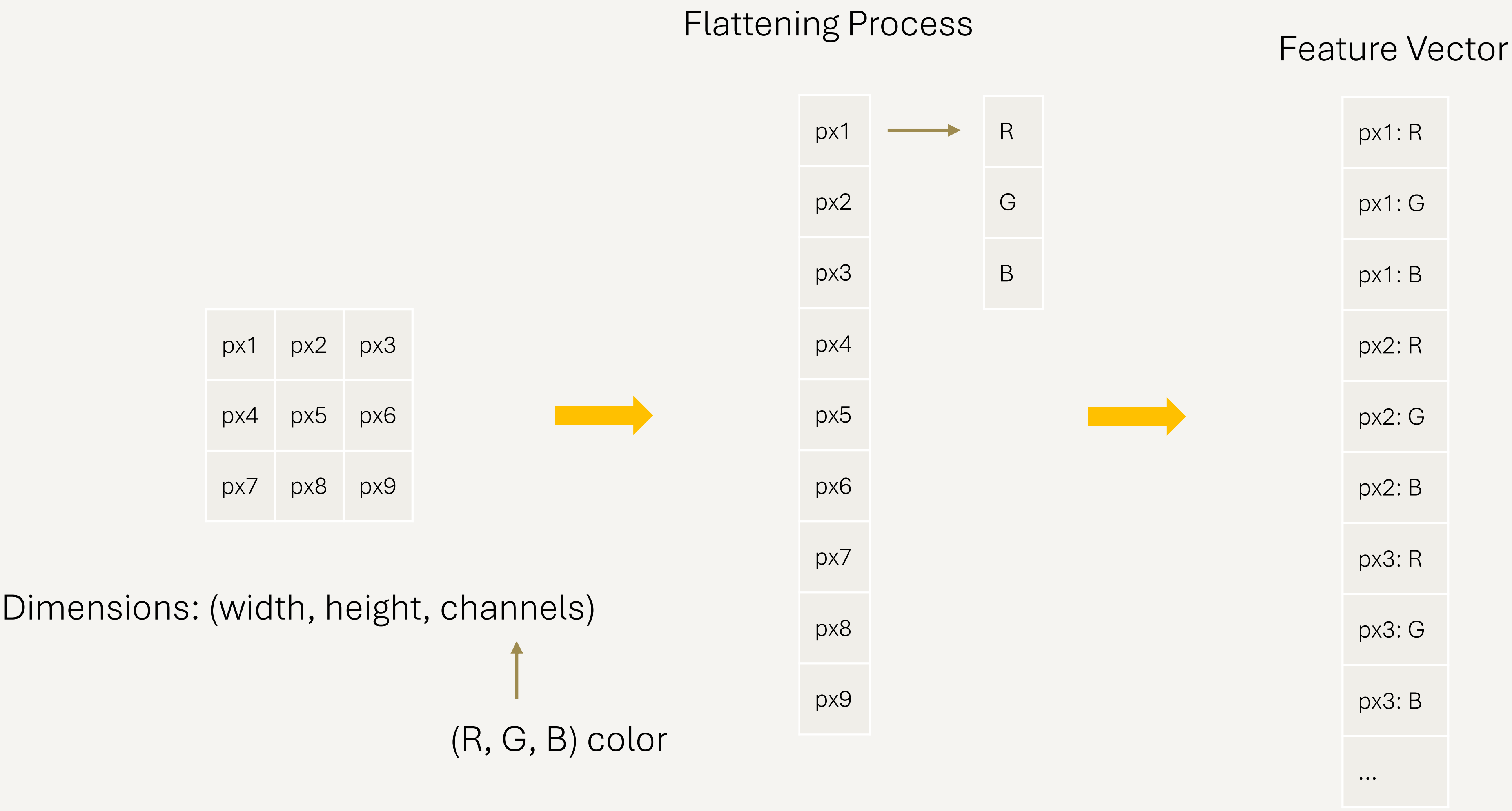




LOGISTIC REGRESSION

for Image Classification

LOGISTIC REGRESSION



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Let’s jump to the code!

THANK YOU!

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References

- Lecture Slides by Prof. Massimiliano Zanoni
- Edgar, T.W. & Manz, D.O.. (2017). Research Methods for Cyber Security. Chapter 4.

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