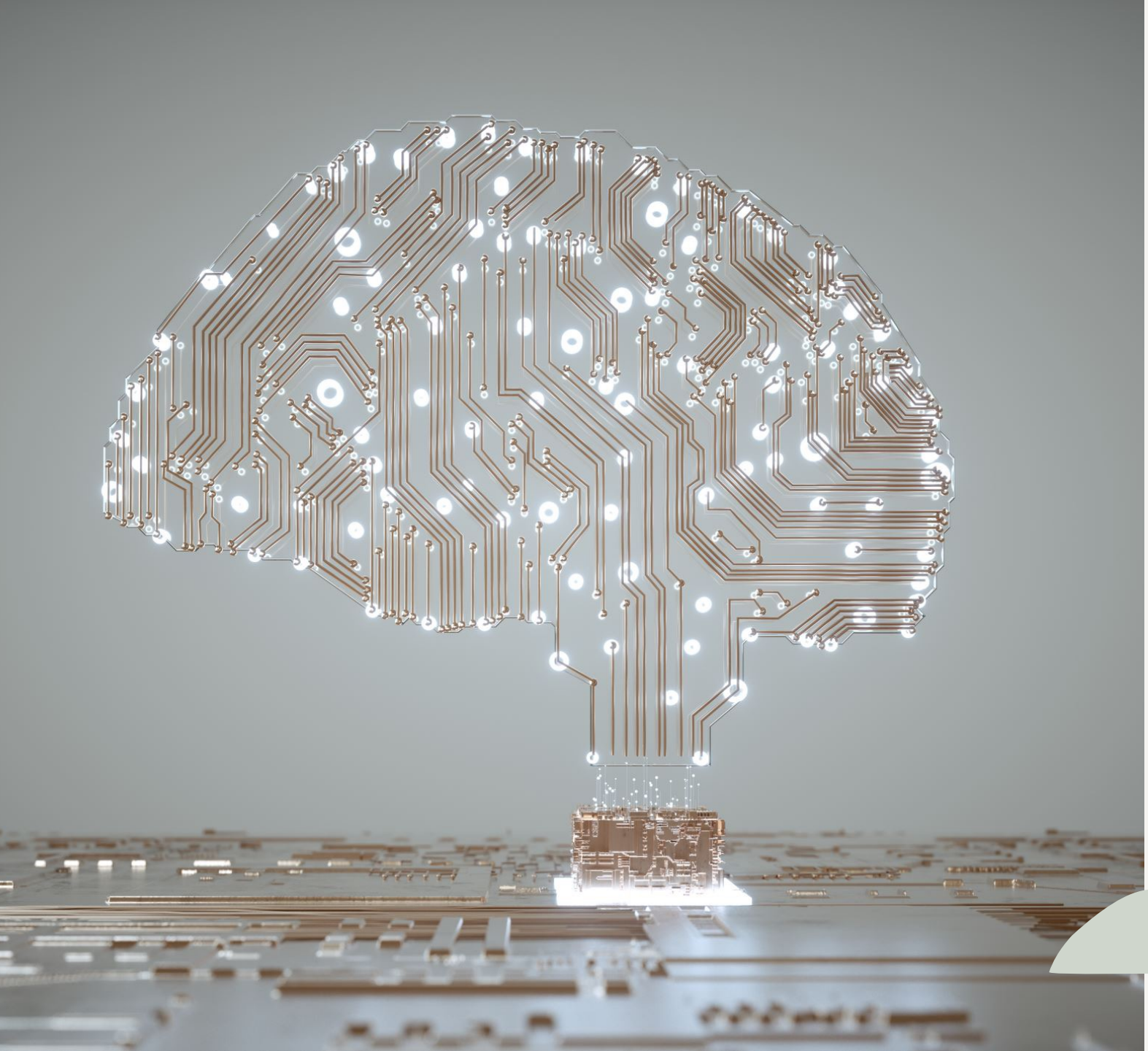


LOGISTIC REGRESSION *for Image Classification*

CREATIVE PROGRAMMING AND COMPUTING

Course

Prof. Massimiliano Zanoni

Slides and Presentation by

Student Marco Muraro

POLITECNICO MILANO 1863

MAE MASTER OF SCIENCE MUSIC AND ACOUSTIC ENGINEERING

POLITECNICO
MILANO 1863

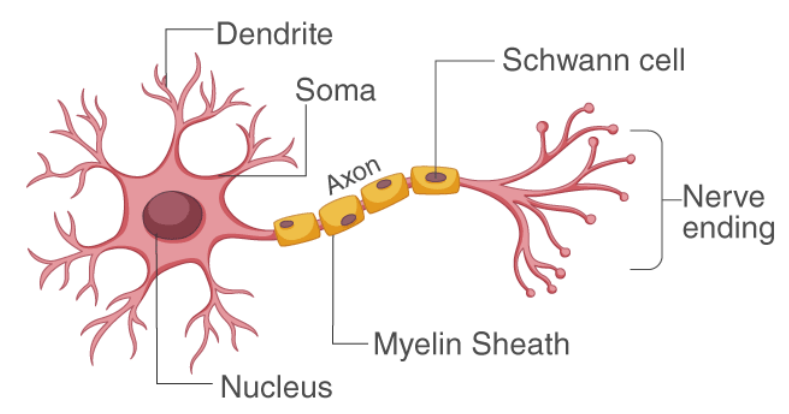
NEURON

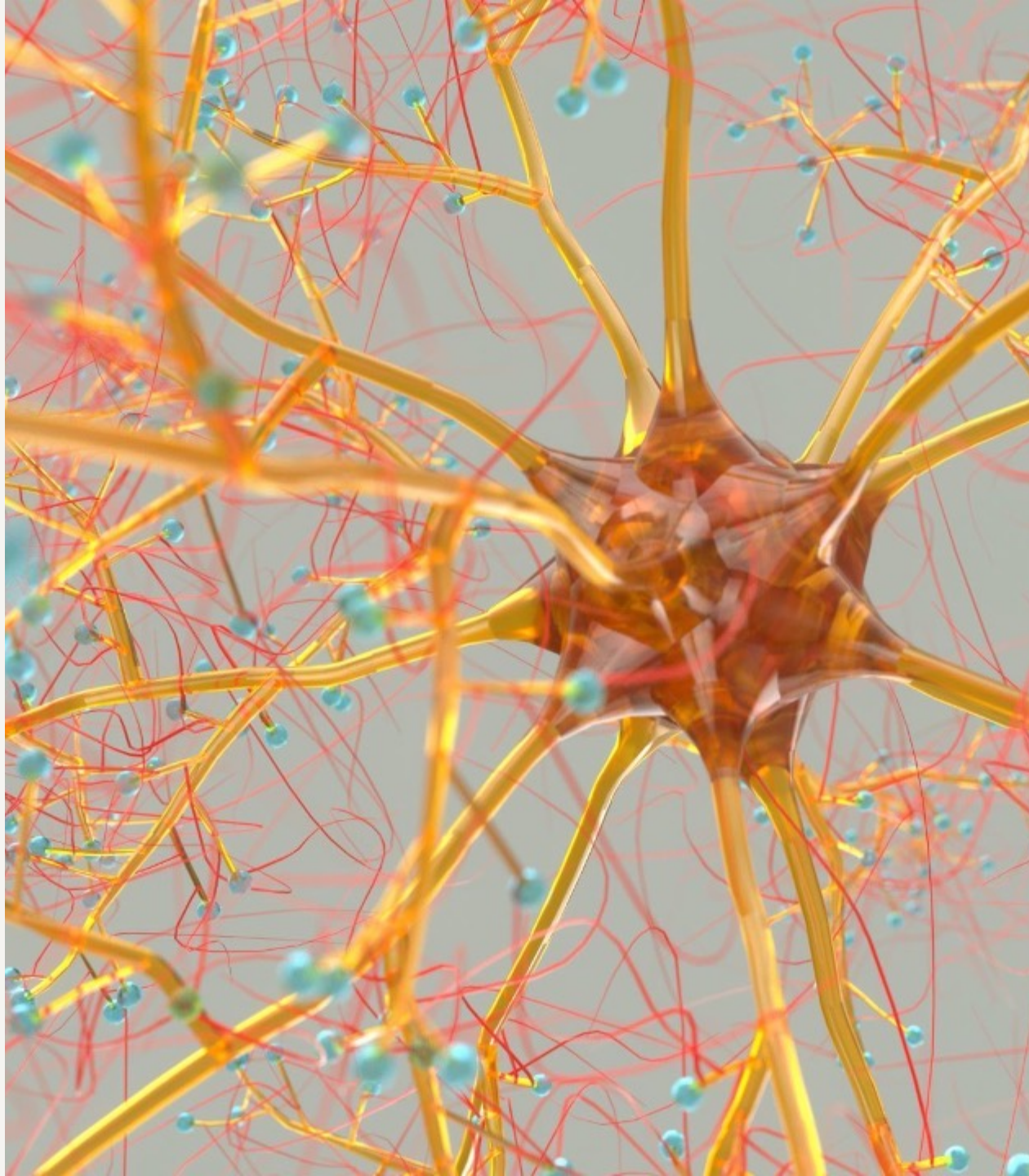
The **brain** is a **network of neurons**

Dendrites collect electrical stimuli coming from other neurons and accumulate input charges

Activation Process

The neuron fires a new electrical impulse when the total charge exceeds a certain threshold

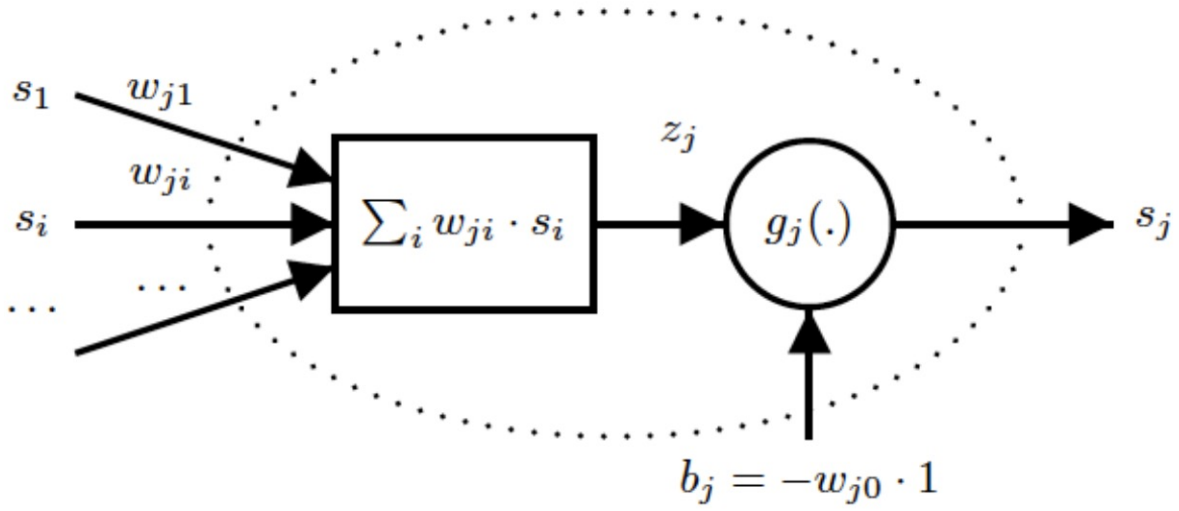




PERCEPTRON

Feed-Forward Model

$$s_j = g_j(z_j - b_j) = g_j\left(\sum_{i=1}^{N} w_{ij}s_i + w_{j0}s_0\right) \quad \rightarrow \quad \boldsymbol{s_j = g_j\left(\sum_{i=0}^{N} w_{ij}s_i\right)} \quad \textbf{Output}$$



Activation Value $z_j = \sum_{i=1}^{N} w_{ij}s_i$

Activation Function $g_j(\cdot)$

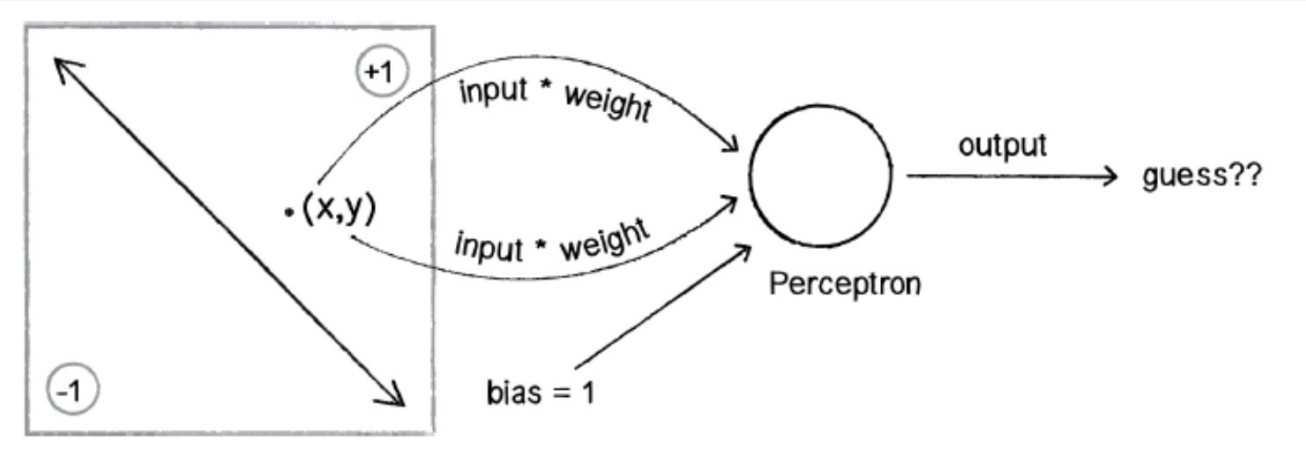
Activation Threshold $b_j = -w_{j0}s_0 = -w_{j0}$

Linear Separation

Decision boundary is defined as a straight line

$$x_1 w_1 + x_2 w_2 + w_0 = 0$$

$$x_2 = -\frac{w_1}{w_2} x_1 - \frac{w_0}{w_2}$$



BINARY CLASSIFICATION

The **learning process** consists in **learning the weights giving the best linear decision boundary**

The perceptron keeps learning by mistakes and updates the weights to find the best ones

The **output** is a **class** in the classification problem

POLITECNICO
MILANO 1863

BINARY CLASSIFICATION

Learning Process

- Start with a **random guess for the weights**
- The perceptron **guesses an answer** $$s_j = g_j\left(\sum_{i=1}^{N} w_{ij}s_i - b_j\right) = g_j\left(\sum_{i=0}^{N} w_{ij}s_i\right)$$
- Compute the **error function**

  $\epsilon = \hat{s_j} - s_j$ where $\hat{s_j}$ is the **desired output** and s_j is the **estimated output** (guess)

- **Adjust the weights**

  HEBB LEARNING RULE $w_i^{n+1} = w_i^n + \Delta w_i$ $\Delta w_i = \eta \cdot \hat{s_j} \cdot s_i$

- **Repeat the process** until **stop condition**

  η is the **learning rate**

  The **number of iterations** is denoted as the **number of epochs**

POLITECNICO
MILANO 1863

LOGISTIC REGRESSION

for Binary Classification

Logistic Regression is the process of modelling the probability of a discrete outcome given an input variable.

The most common application of logistic regression is **binary classification**:
the **binary outcome** is TRUE/FALSE, YES/NO, -1/1, 0/1, etc.

Given a feature vector x,
logistic regression aims to find

$$\hat{y} = P(y = 1|x)$$

Learning Process

Learning the weights w and the bias b
giving the best estimate $\hat{y}$ for the
probability $P(y = 1|x)$

$$\hat{y} = g(w^T x + b)$$

LOGISTIC REGRESSION

for Binary Classification

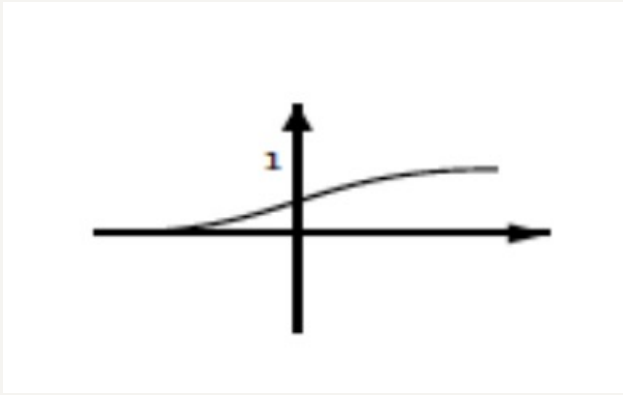
Activation function $g(\cdot)$ is applied

$$\hat{y} = g(w^T x + b)$$

We need a function mapping input data into a probability, i.e. within the range $[0,1]$

Sigmoid Function

$$g(z_j) = \frac{1}{1 + e^{Kz_j}} \quad K < 0$$



LOGISTIC REGRESSION

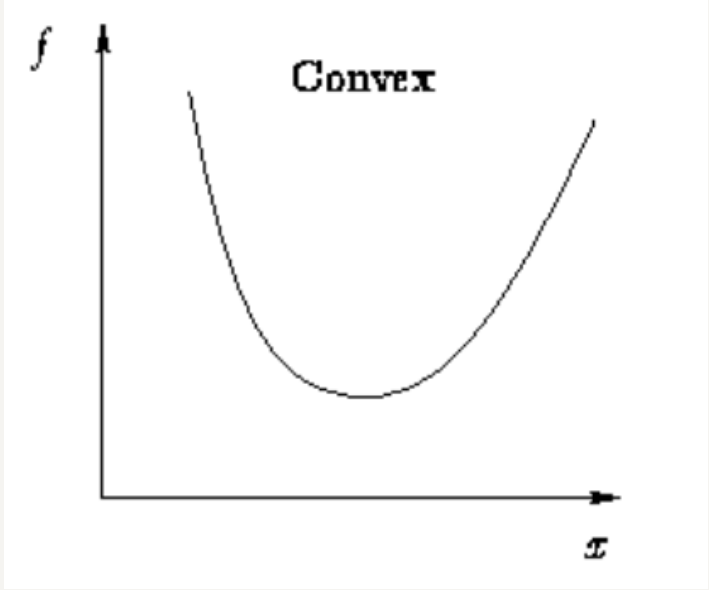
for Binary Classification

Loss Function

In order to learn the best weights w and the bias b, we need a **loss function** $L(\hat{y}, y)$ to be minimized

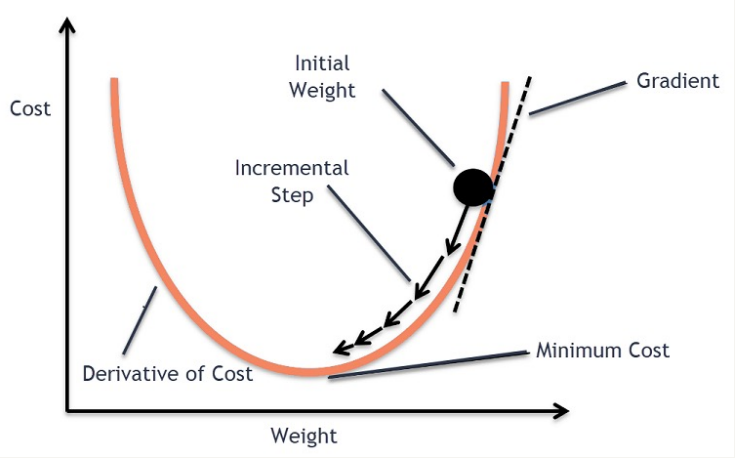
$$L(\hat{y}, y) = -\log(y\ log\hat{y} + (1-y)\log(1-\hat{y}))$$

for a **single feature vector**



LOGISTIC REGRESSION

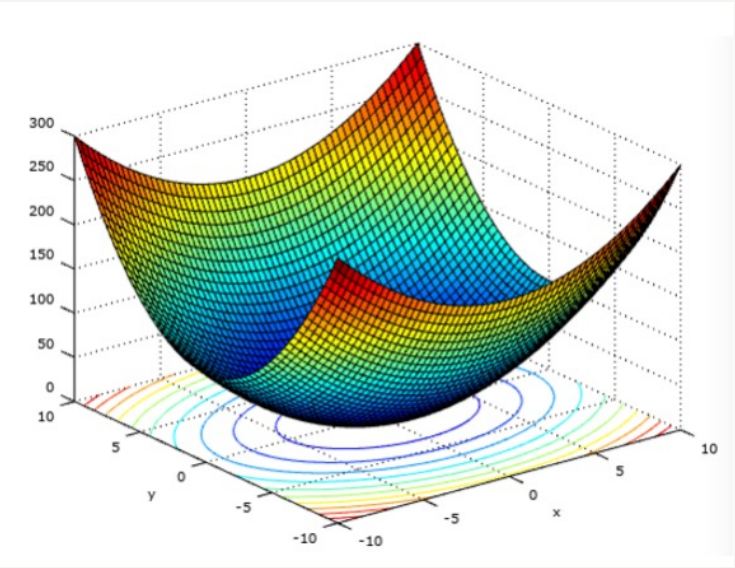
for Binary Classification



Cost Function

For the **whole dataset** (multiple feature vectors)

$$J(w,b) = \frac{1}{m}\sum_{I=1}^{m} L\left(\hat{y}^{(i)}, y^{(i)}\right) = -\frac{1}{m}\sum_{I=1}^{m}\left(y^{(i)}\, log\hat{y}^{(i)} + \left(1-y^{(i)}\right)\log\left(1-\hat{y}^{(i)}\right)\right)$$



Therefore, the weights w and the bias b keep being adjusted **minimizing** $J(w,b)$ until the local minimum (global optimum) has been reached

$$w = w - \alpha\frac{dJ(w)}{dw} \qquad b = b - \alpha\frac{dJ(b)}{db}$$

α is the **learning rate**

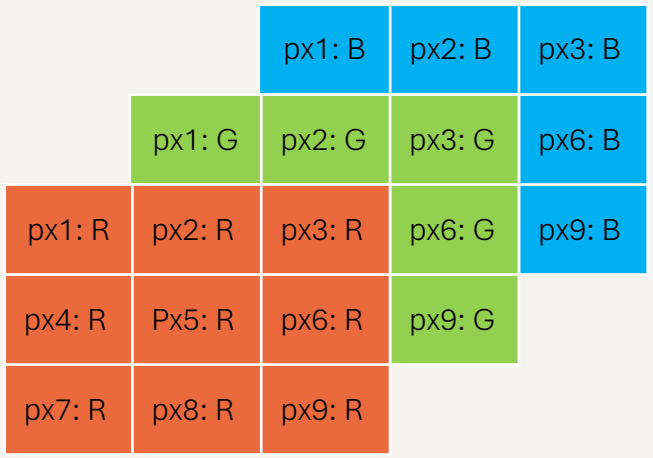
The adjustment procedure is called **Back-Propagation** and the algorithm employed for optimization process is called **Gradient Descent**

POLITECNICO
MILANO 1863

LOGISTIC REGRESSION

for Image Classification

LOGISTIC REGRESSION



px1	px2	px3
px4	px5	px6
px7	px8	px9

Dimensions: (width, height, channels)

(R, G, B) colour

Flattening Process

px1	R
px2	G
px3	B
px4	
px5	
px6	
px7	
px8	
px9	

Feature Vector

px1: R
px1: G
px1: B
px2: R
px2: G
px2: B
px3: R
px3: G
px3: B
...

POLITECNICO MILANO 1863

Let's jump to the code!

LOGISTIC REGRESSION *for Image Classification*

CREATIVE PROGRAMMING AND COMPUTING

Course

THANK YOU!

References

- Lecture Slides by Prof. Massimiliano Zanoni
- Edgar, T.W. & Manz, D.O.. (2017). Research Methods for Cyber Security. Chapter 4.

POLITECNICO MILANO 1863

MAE MASTER OF SCIENCE MUSIC AND ACOUSTIC ENGINEERING

Prof. Massimiliano Zanoni

Slides and Presentation
by
Student Marco Muraro