POLITECNICO
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Thesis proposals

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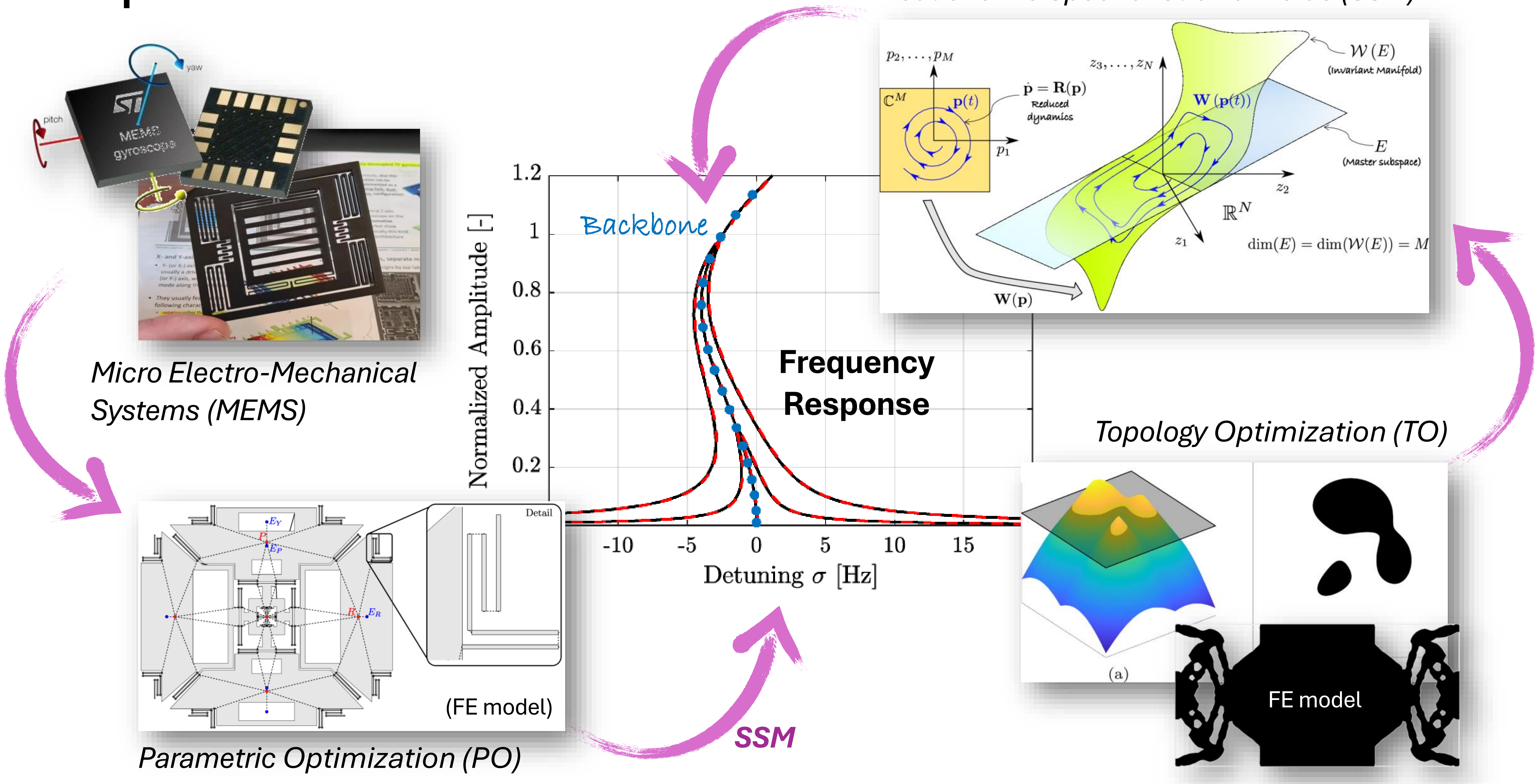
Graphic abstract



Introduction I

on model reduction

In engineering applications, **Finite Element** (**FE**) models are ubiquitous and usually feature hundred of thousands or even millions of unknowns. Solving such high-dimensional system of equations is often demanding, even when using very efficient commercial software. If interested in retrieving the forced **frequency response** (**FR**) of a nonlinear system, however, often the only option is to carry out direct time integration. In most cases, this makes the analysis unfeasible (i.e., months of computational time).

In recent years, a strong research effort has been made to develop **Model Order Reduction** strategies (**MOR**), loosely consisting into constricting the dynamics of a high-dimensional model to a low-dimension subspace/manifold. Modal analysis is an example of a linear MOR, where the motion of a system is described by a subset of vibration modes. Among many available options, reduction to **Spectral Sub-Manifolds** (**SSMs**) have recently emerged as a promising, systematic methodology for nonlinear systems.

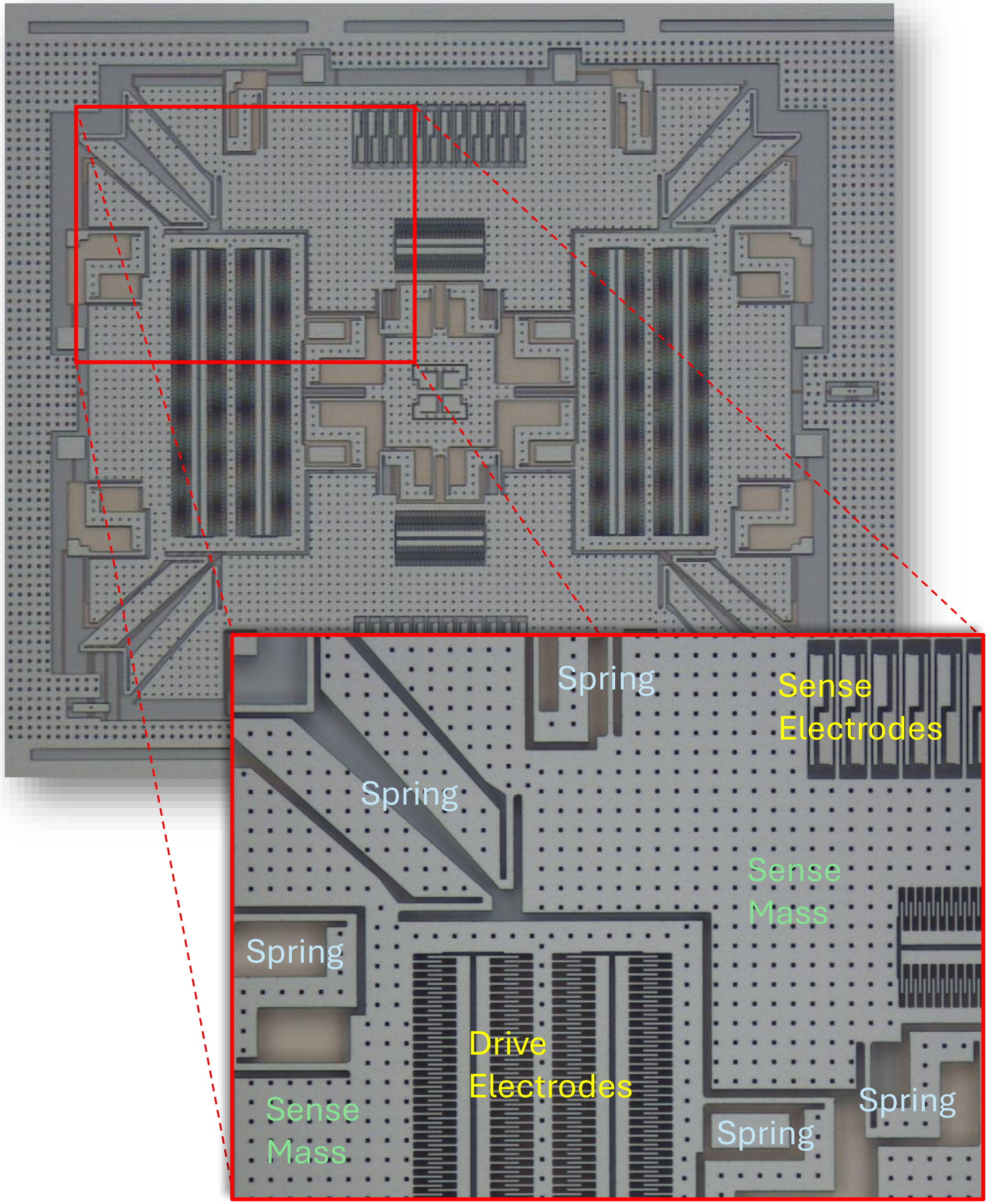
SSMs have already been studied in many research works, dealing with nonlinear frequency responses, backbone curves, internal resonances, multi-body systems and parametric resonances.

Introduction II

on micro sensors

While nonlinear vibration problems can be found in many different mechanical engineering applications (as in lightweight structures, such as plane wings), they are of particular interest in the field of **Micro Electrical-Mechanical Systems** (**MEMS**). These are devices with characteristic dimension of about one micron and that can often be found as sensors in many electronic devices (such as accelerometers and gyroscopes in smartphones).

The unfavorable aspect ratio of their subcomponents, the very low pressures at which they operate, the relatively high displacements they undergo, and the effects introduced by the electro-mechanical coupling, make the MEMS field the ideal playground for the nonlinear dynamics' practitioner.



SEM image of a triaxial MEMS gyroscope

Introduction III

on structural optimization

Even in a context of linear dynamics, the design of MEMS structures can be a long (and tedious) trial and error kind of work. For instance, eigenfrequencies must be manually tuned by changing the suspensions springs accordingly, while ensuring that all the other device specifications are met. To cope with this problem, in the last years optimization strategies have been deployed.

Parametric Optimization (**PO**) consists in finding the best geometric parameters that minimize an objective function while satisfying some constraints. This may be extremely useful to automatically change a pre-defined layout by selecting optimal beam width and lengths so that all eigenfrequencies and eigenmodes are tuned as desired.

In **Topology Optimization** (**TopOpt** or **TO**) instead, no geometry is a priori known, but it is generated starting from a grid of squares (cubes in 3D) representing material (black) or voids (white). The two most popular strategies to perform TO are **density-based** methods, where each square can assume a continuous value between 0 and 1 to allow differentiation in the optimization problem, and **Level-Set** methods, where the profile of the structure is defined (in the 2D case) by the intersection between a 3D function and a plane (the level-set plane).

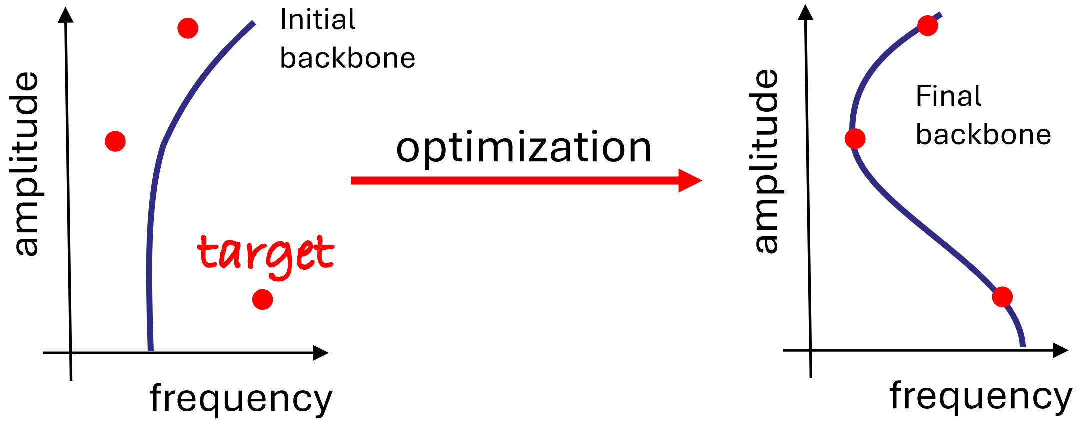
Even in these fields there is a lot of research going on, but results till now have been *restricted to **linear** dynamics*. What would happen in a nonlinear dynamical setting?

Projects

Background: we have recently proposed a method to tailor the nonlinear response of a structure using SSMs (LSM) and an optimization procedure (see Ref. [5]). By defining a set of frequency-amplitude pairs we can fit the backbone curve of the system to these points.

Relevance: if a linear optimization is performed, usually the resulting structure turns out to behave in a (strong) nonlinear manner. This ultimately makes said optimization useless.

Extensions: from this cornerstone, multiple paths can be explored, each one being a potential thesis topic.



Topic 1 – PO: Adjoint formulation at arbitrary order

The computation of the sensitivities of the LSM, required for the optimization procedure, is still very cumbersome and computationally demanding, reason why this approach is still limited to low-dimensional models with a low number of parameters.

To remove this constraint, the adjoint method can be used to compute the sensitivity with respect to the design parameters all together, rather than one by one. This effectively allows to use a very high number of parameters without affecting the computational cost of the optimization.

At the time being, the adjoint formulation has been developed for SSM up to the 3rd order (see Ref. [11]). This entails some limitations. The aim of this project is to extend the adjoint formulation to handle arbitrary expansion order of the SSM and to apply it to a relevant numerical example.

Tools: YetAnotherFEcode, SSMTool.

Projects

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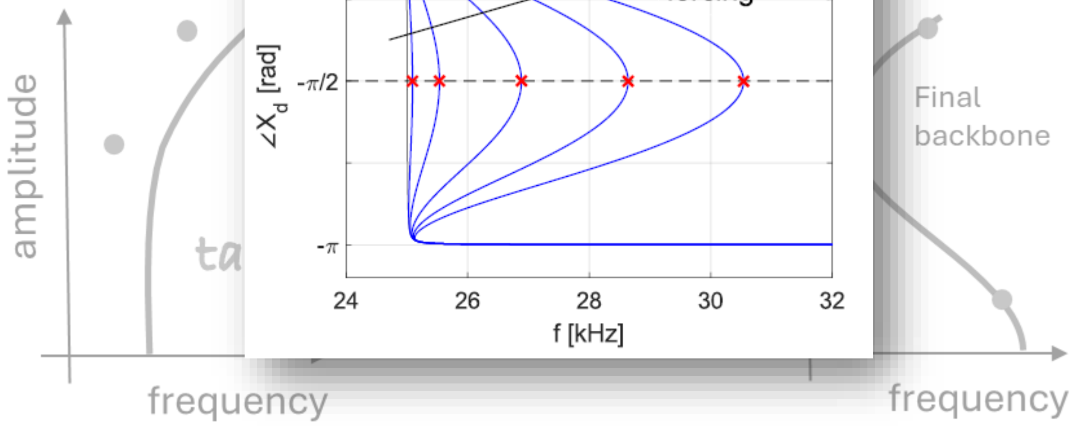

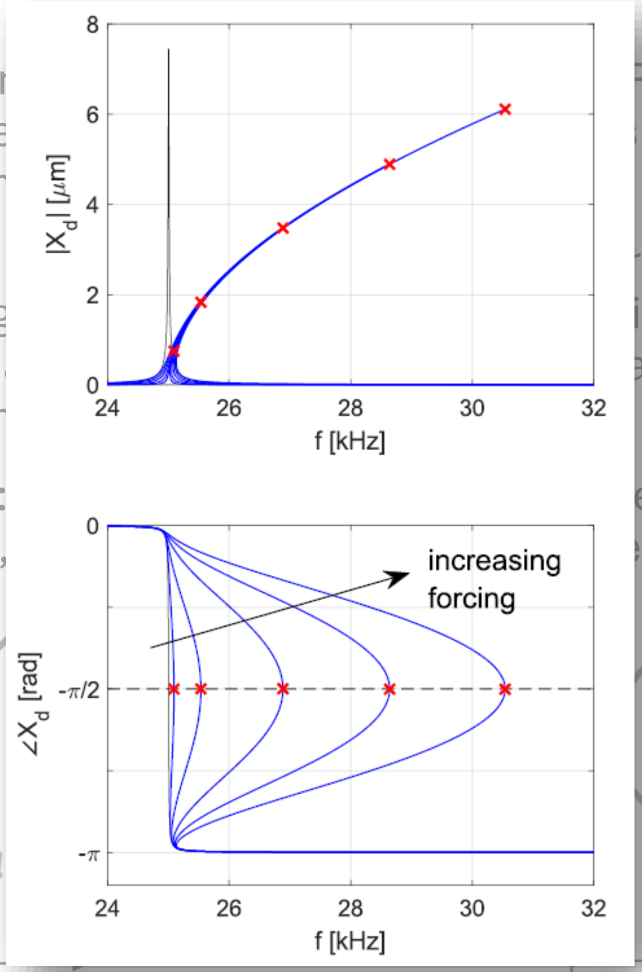

Topic 2 – PO: Frequency Response (with adjoints)

At the time being, only the optimization of the backbone curve of a system has been addressed. However, for practical purposes, it is often relevant to consider the forced Frequency Response (FR), which embeds information about damping, forcing level and stability of possibly multiple solutions, which cannot be inferred from the backbone curve.

The aim of this project is then to extend the computation of the sensitivities to the case of forced and damped systems (i.e., non-autonomous, non-conservative). This can be done using direct differentiation and/or using the adjoint method.

Finally, the findings will be tested on a relevant numerical example, possibly a MEMS gyroscope to showcase the relevance of the method.

Tools: [YetAnotherFEcode](), [SSMTool]().

Projects

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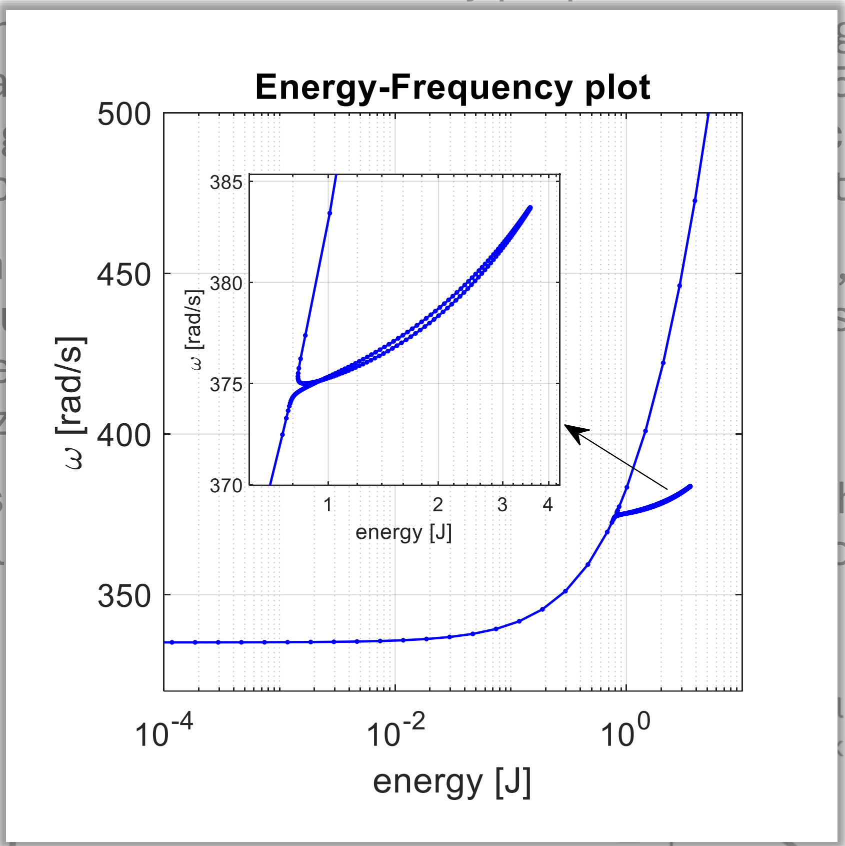

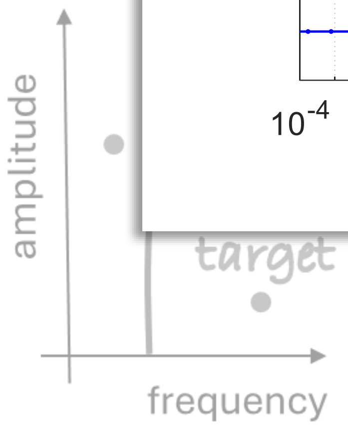

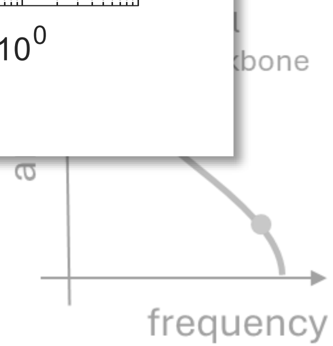

Topic 3 – PO: Internal resonances

As previously discussed, there are multiple objective that can be pursued using the same "core" strategy (SSM + optimization).

As of now, the backbone computation can target only a single mode. A natural extensions of this work is then to find a way to handle multiple modes when they interact. In Ref. [9], the authors have studied the case of SSMs in presence of internal resonances, which is a special type of resonance which occurs when two ("nonlinear") eigenfrequencies are multiples of each other (e.g., 1:2, 1:3, ...)

Internal resonances are particularly difficult to predict and may often lead to unwanted behavior of the system. In MEMS gyroscopes, for instance, the resonance may "fool" the drive control loop and make the device resonate at a wrong frequency and amplitude. The aim of this project is to find and implement a strategy to tailor the modal coupling governing the internal resonances, either to avoid or to exploit them.

Tools: YetAnotherFEcode, SSMTool.

Projects

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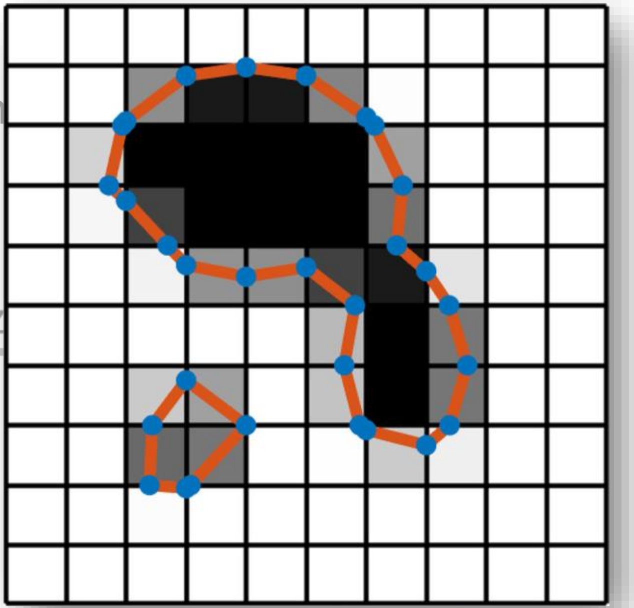
Extensions: from this corn… be explored, each one bein…

In TO each element has a density, each density is a parameter



Backbone expression:

$$\Omega = \omega_0 + \gamma\rho^2$$

γ is computed with the adjoint method

frequency

Topic 4 – Topology Optimization (TO)

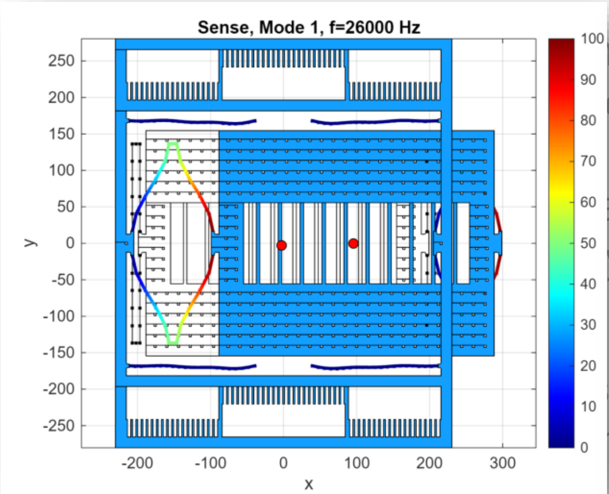
The low number of parameters in PO represents a further implicit constraint to the outcome backbone. Indeed, some set of frequency-amplitude pairs may be physically unfeasible for a given set of parameters, and feasible for another one. Usually, the more the geometry is free to change, the better the outcome of the optimization. In this spirit, in this project we aim to extend the results for PO to TO, where *each element density* corresponds to a parameter (i.e, we end up with $10^5 - 10^6$ parameters).

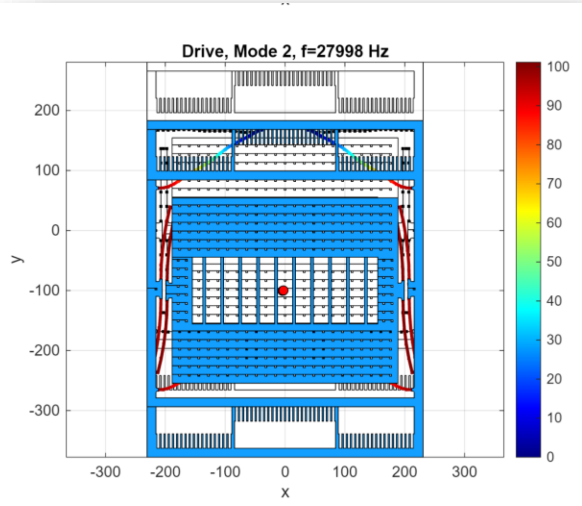
Currently, we have implemented TO to tune the hardening (softening) behavior of the system, by leveraging the 3rd order coefficient of the backbone curve using the adjoint formulation in [11]. The objectives are similar to the ones described in topics 1-3 (namely, adjoint formulation, frequency responses, and/or internal resonances).

However, the candidate will be required to fit the aforementioned topics into the TO setting, which will require the candidate to acquire specific related knowledge and skills.

The candidate will possibly have to use of OpenLSTO, a code for TO in C++, as Matlab can handle only small-size examples.

Projects

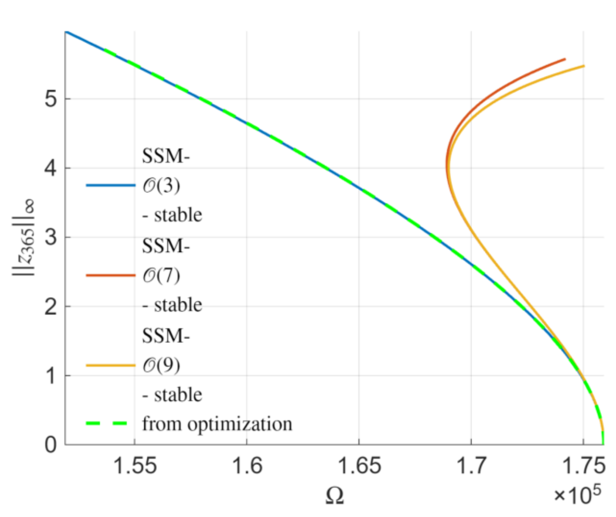




Extensions: from this corners

Backbone expression:

$$\Omega = \omega_o + \gamma\rho^2 + O(\rho^3)$$



$$\min_\mu J$$

$$s.t. \quad \frac{\omega_d}{\omega_{d,\text{target}}} - 1 = 0$$

$$\frac{\omega_s}{\omega_{s,\text{target}}} - 1 = 0$$

$$\frac{\gamma}{|\gamma_{\text{target}}|} - \text{sign}(\gamma_{\text{target}}) < 0$$

Topic 5 – Applications: PO with SSM and MPC

All the projects described so far focus on the method, rather than on the outcomes.

In the present topic, the candidate will be asked to study and use pre-developed optimization tools for real case scenario applications in the MEMS field.

In the figures on the left, a MEMS prototype has been optimized in order to shift its initial hardening behavior into a softening one (using the adjoint formulation in [11]). These results were obtained in YetAnotherFEcode using a Multi-Point Constraint (MPC) strategy, where the masses (in light blue) are modelled as rigid bodies connected to the beam elements through a master-slave approach. Thanks to SSMs, adjoints and MPC, this optimization is extremely fast and allows unprecedented results.

The specific application may include MEMS triaxial gyroscopes, accelerometers and micromirrors, and may be agreed with the candidate.

Depending on the candidate inclinations, also further extensions on the implementation side are available.

Projects

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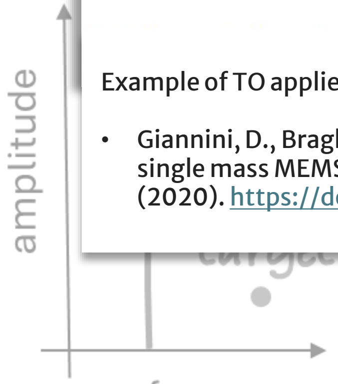
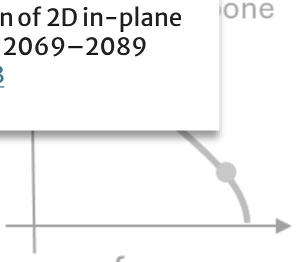
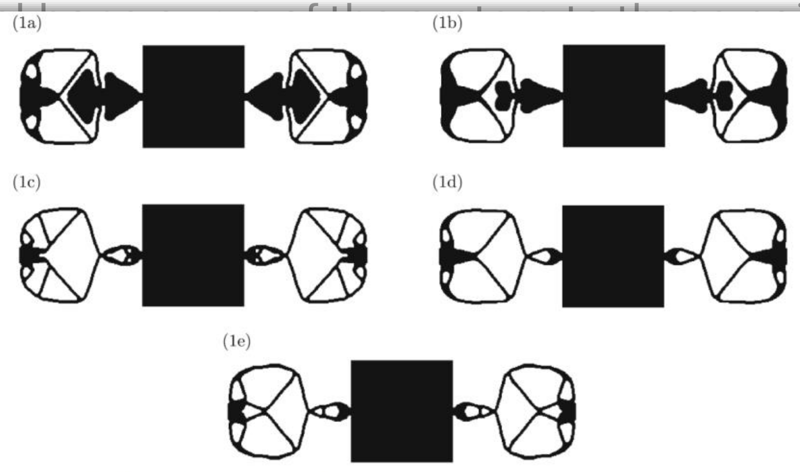






Example of TO applied to MEMS.

- Giannini, D., Braghin, F. & Aage, N. Topology optimization of 2D in-plane single mass MEMS gyroscopes. *Struct Multidisc Optim* **62**, 2069–2089 (2020). https://doi.org/10.1007/s00158-020-02595-3

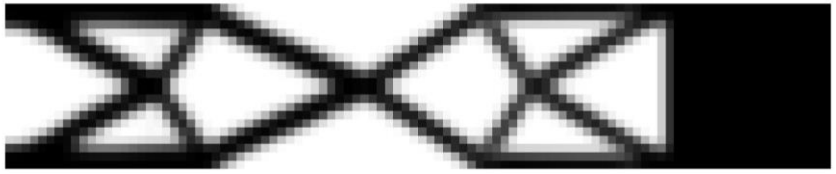
Topic 6 – Applications: TO with SSM

As for topic 5, here the focus will be on deploying pre-developed TO strategies to produce real, manufacturable layouts for MEMS gyroscopes, accelerometers and micromirrors.

This will involve the use of some specialized tools, such as OpenLSTO, a code for TO in C++ developed by Alicia Kim's group at UCSD, or ParaLeSTO-COMSOL, which wraps around the commercial software COMSOL.

Even in this case, for the willing candidate, possible implementation-oriented extensions can be pursued.

TO clamp-clamp beam example (half domain):



Linear optimization

Non-Linear optimization

Projects

Topic 7 – SSM-enhanced Control-Based Continuation (CBC)

In this project we aim to experimentally determine the backbone and the frequency response of a system using Control Based Continuation. CBC consists, loosely speaking, in creating a phase controller (PLL, Phase-Locked Loop) to lock a system at resonance (90°) and then extract the backbone by increasing the forcing with continuation algorithms, typical of numerical analysis.

Recently, techniques to compute an SSM from inexpensive experimental tests have emerged. These data-driven SSMs are extremely interesting from the application point of view, as they rely on the real system and because they do not suffer from the inherent limitations of the numerical method (which is limited, as a Taylor series, to small deviations from the expansion point).

The object of this project is to embed an SSM-based "observer" into a CBC to enhance its capabilities. In particular, we envision that such a configuration would be able to better reject noise, allowing the CBC to converge faster and to handle also equilibrium points with smaller and smaller basins of attraction.

After an initial explorative work using numerical models, the final aim would be to showcase the method on an experimental case, ideally a MEMS structure from our lab.

Tools: Matlab, LabVIEW (for the controller).

Projects

Topic 8 – Parameter continuation of SSM for parametrized defects

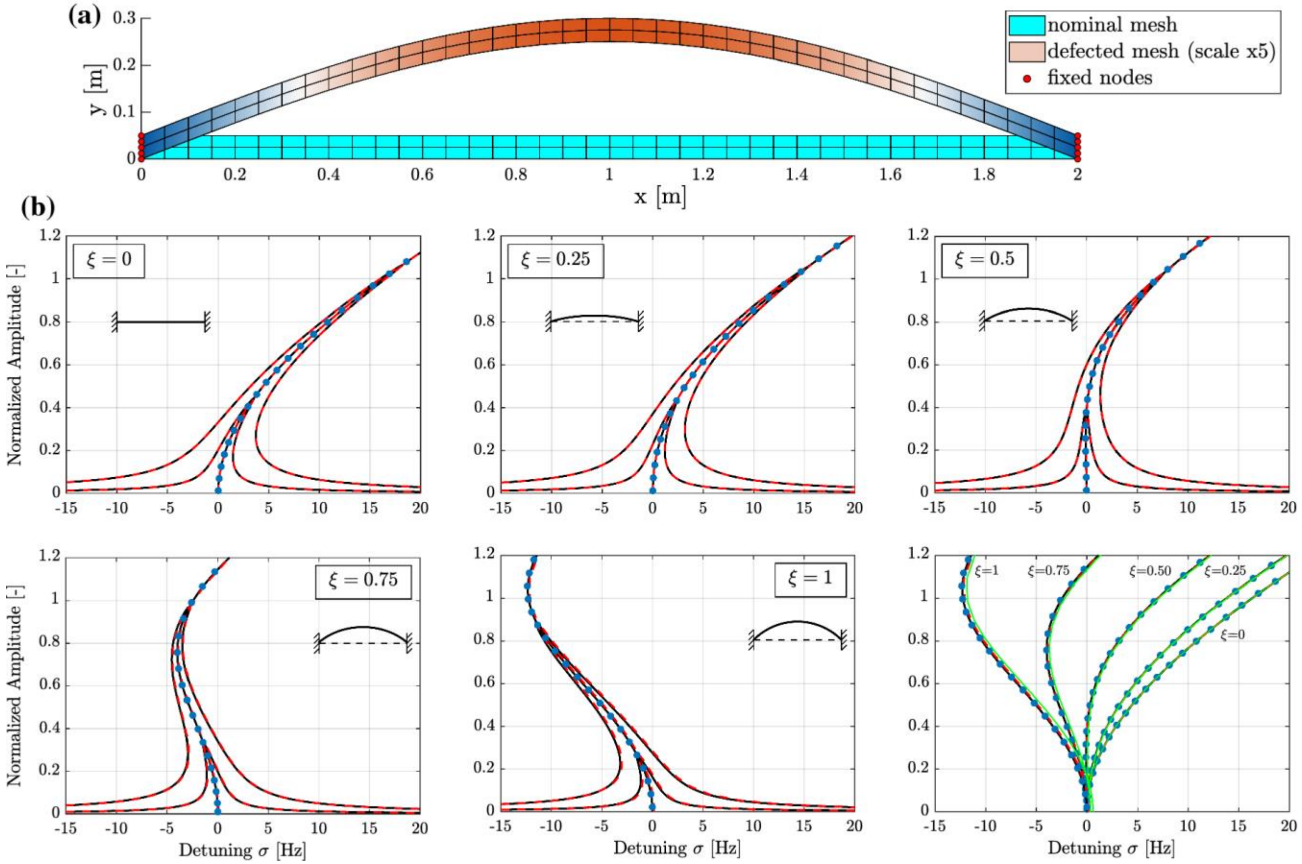
In this project we aim to develop an analytic formulation for SSMs depending on a parameter, much in the fashion of Ref. [11]. In particular, the focus is on a formulation for *shape defects* previously studied in:

- Marconi, J., Tiso, P., Quadrelli, D.E. *et al.* A higher-order parametric nonlinear reduced-order model for imperfect structures using Neumann expansion. *Nonlinear Dyn* **104**, 3039–3063 (2021). https://doi.org/10.1007/s11071-021-06496-y

where a different MOR strategy was used.

Indeed, the problem of shape defects is very important in the MEMS field industry, as defects have a deep impact on the performances of sensors, both at linear and nonlinear level.

A shape-parametric SSM would help in the statistic prediction of the sensor performances and their sensibility to defects.



Example of a clamp-clamp beam. A shape defect, in the form of an arch, is increased from 0 to 1 times the beam thickness, shifting its response from hardening to softening.

Notes

- The listed proposals are indicative. A candidate may come with his/her own idea, which we would be happy to discuss.
- All the proposed topics have large overlaps and deep links between each other. As such, we envision that multiple students may collaborate on similar topics (but they won't be forced to do so).
- With different priority levels, we are actively working on the listed topics. Depending on when you contact us, a topic could already be closed, but other may be become available.
- There are 2 PhD scholarships at Polimi founded by STMicroelectronics on some of the proposed themes.

Possible collaborations and periods abroad (depending on the project)

- TU Delft, NL (prof. Shobhit Jain)
- UCSD, US (prof. Alicia Kim)
- ETH, CH (prof. Paolo Tiso)
- KU Leuven, BE (Dr. Daniele Giannini)

Requirements and application

Requirements

The candidate must be familiar with:

- Linear Algebra and Ordinary Differential Equations
- Linear Dynamics
- Finite Elements
- basic Matlab programming skills

Additionally, it is appreciated a background in:

- Nonlinear dynamics
- Nonlinear Finite Elements
- Object-oriented programming
- C++ (only for TopOpt projects)

Duration

Thesis projects usually last between 6 and 9 months.

Application

To apply for a thesis project, send an e-mail to the indicated contacts attaching:

- your Transcript of Records (ToR) for both bachelor and master degrees
- (optional) your curriculum vitae
- (optional) motivation letter

Contacts

For any further information or to make an appointment, please write to:

Jacopo Marconi: jacopo.marconi@polimi.it

Shobhit Jain: shobhit.jain@tudelft.nl

Matteo Pozzi: matteo1.pozzi@polimi.it

Selected Bibliography

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