

# Morban's thesis

Marco Orban

# 1. Introduction

## 1.1 Background

Osteoarthritis is a very common type of arthritis that causes pain, swelling and stiffness in various body parts such as the hands, hips, back and knees. Over time, it affects bones, cartilage and other tissues, and is common in adults over the age of 45. Knee osteoarthritis, also known as degenerate joint disease of the knee, is a type of osteoarthritis that is predominantly seen in the elderly, and is a progressive disease that results in knee stiffness and swelling and pain after sitting or staying still for a long time. Although this disease can be treated, but not cured, with physical therapy and medications that slow down its progression, severe knee osteoarthritis can only be resolved by means of surgery, in which the whole knee is replaced by a prosthesis. Total knee arthroplasty (TKA), also called total knee replacement (TKR) is a very effective and consistently successful surgery that provides good outcomes for patients suffering from end-stage knee osteoarthritis. TKR results in greatly improved pain relief and better quality of life for patients [1].

Since osteoarthritis is a disease that is most commonly seen in the elderly, as the percentage of elderly people in developed countries around the world increases, the prevention and treatment of diseases like osteoarthritis become more important from a public health perspective. In particular, Taiwan has already exceeded the

threshold (14%) of the definition of aged society established by the United Nations, with 3.983 million citizens over the age of 65, accounting for 17.18% of the population [2]. Additionally, it has been estimated that Taiwan's National Health Insurance already spends 5% of its total expenditure on TKR every year, and the incidence of TKR has already tripled in the period between 1996 and 2010 [3].

There has been a growing interest of using additive manufacturing (AM) for the manufacture of the prosthetics used in TKA, since additive manufacturing is a technology well suited for the creation of custom, lightweight components with complex geometries, while also producing less material compared with other methods of manufacture [4]. Even though additive manufacturing offers many advantages for the production of structures tailored to each individual patient, one of the major obstacles that stands in the way of a more widespread adoption are the high costs of AM. One way to reduce the cost of the total procedure is by reducing the amount of material used, which can also be accomplished by reducing the amount of scrap from the fabricated parts. In additive manufacturing, much of the scrap comes from discarding the support structures that the components require for manufacturing, and thus making smaller support components or using smaller volumes for them would be a valid strategy to further reduce the cost of additive manufacturing components. The total deformation of the part after manufacture is also an important factor to consider, since additional costs can be incurred from the addition or removal of material from the manufactured component by means of machining, in order to meet tolerances [4].

## 2. Literature Review

### 2.1 Introduction

Additive manufacturing, also known as 3D printing, is a manufacturing process in which parts are built by stacking layers of material on top of each other until the desired geometry is created. In particular, powdered-bed fusion is a type of additive manufacturing in which the base material is composed of a metallic powdered. A bed is then filled with this powder, and a high-powered laser or electron beam is utilized to selectively add heat to locations of this powdered metal, thus melting it. As the melted portions cool, it leaves a layer of solid material, and the whole component is then built layer by layer in this manner. If a laser is used for manufacturing, the process is then known as laser powder bed fusion (LPBF).

PBF involves the layer-by-layer addition of material in which a heat source selectively melts and fuses regions of a powder bed to form a solid structure. When the material is melted and fused by using a laser, the process is called selective laser melting (SLM). The starting point of selective laser melting is a 2D computer-assisted design (CAD) model, which is sliced into 2-dimensional pieces, and later uploaded to an SLM machine. The machine then uses the laser to melt layers of powder following the information of the sliced model. At the end of each slice, the layer is left to cool down and solidify, and a new layer of powder is evenly distributed using a coater, and the melting

process begins anew, until the final geometry has been reached [5]. The part is then removed from the powder bed where it might then undergo different post-processes, such as removal of supporting structures, heat treatment, or any other additional or removal of material to achieve the required dimensions and tolerances.

Additive manufacturing in general allows for the creation of parts with complex geometries that would otherwise be difficult, time-consuming or too costly to manufacture with other more traditional means of manufacturing such as machining or casting. This can be compared with the manufacture of prosthesis using traditional manufacturing, which relies on the high-volume production of standardized parts and shapes. Due to the ease of creating parts with complex geometries and with shorter lead times, additive manufacturing has had a growing interest from the medical industry for the creation of custom-made implants tailored to an individual patient's anatomy [6] [4]. Additionally, this benefit of creating customized parts also leads to an improvement of patient comfort and better outcomes in orthopedic and surgical applications, as well as enhancing surgical precision and reducing complications of post-operative care [7] [8].

Despite its advantages in creating components with complex free-form 3D geometry, SLM still faces many challenges that limit its application and development. Some of these challenges are related to the development of excessive deformation, thermal stresses, and other mechanical defects such as delamination, distortion and microcracks that result from the great thermal gradients that occur within the component as it is being built, caused by the high heating and cooling rates inherent in the process [9].

Additionally, even though additive manufacturing is a viable alternative for the production of complex-shaped components, its cost can still remain high due to machine, material and process-level expenses. In particular, as a means of creating medical implants for knee-replacement therapy, the high cost of AM limits its usage for revision procedures, i.e. second or third procedures in which the objective is to replace a failed or uncomfortable component [4]. To solve the problem of high material costs, Laureijs et al. [10] have identified that one of the main driving costs is the price of the powder price. One of the viable options to drive down the price of prosthesis created using

additive manufacturing would be to reduce the amount of scrap material involved in the process. Scrap material in additive manufacturing results from discarding the support structures that the part requires as well as any addition or removal of material to the part in post-processing steps to ensure that the part is maintained within tolerances [10].

The support structures alluded to in the previous paragraph refer to the structures that are built alongside the manufactured component that serve to stabilize it during the printing process. These structures are used to aid with supporting overhang areas, which are defined as areas in the structure that are almost horizontal. Support structures can also facilitate the remove of the part from the base plate and other workpieces, and can also aid with thermal diffusion or prevent residual stresses caused by thermal gradients [11].

The design of support structures in additive manufacturing is thus an important consideration, with many factors affecting how the support structure will be built. A well optimized and design support structure will enable the part to be realized, while at the same time using the least amount of material possible to obtain a specified objective. Thermal and mechanical requirements are usually at the forefront of support structure design as they have a direct effect on the quality of the component, although there might be other considerations for the design, such as ease of removability, build time, and material efficiency. Additionally, large portions of surfaces that are almost horizontal and that are unsupported during manufacturing tend to be heavily distorted after manufacturing [12]. At overhang regions, support structures are also required to be as stiff as possible to withstand the weight of the part itself and prevent it from distorting during the building process [13] [14]. Another important factor that impacts the design of support structure is the structure's performance to transfer heat away from the build component, since high thermal conduction can improve the cooling process during fabrication, which helps prevent issues caused by excessive thermal deformation such as thermal residual stresses, thermal dilation, cracks or warping [12] [15].

Topology optimization is a method that is well-suited for the creation and opti-

mization of shapes that satisfy specific constraints while realizing certain objectives [16], and is a well-known technique that has been obtaining more interest from the additive manufacturing community for both the design of the components themselves and for their support structures in additive manufacturing. Topology optimization has been studied for designing parts that either have as little overhang surfaces as possible, thus requiring less supporting structures, or for designing parts that are self-supporting, thus requiring no supporting structures at all. Although this aid considerably in reducing the amount of material used, these approaches do not take into consideration other effects such as the thermal dissipation of the component, or require changing the geometry of the component, thus impacting its functionality [17]. Changes in the geometry of the component might also impose geometrical constraints that can restrict the part’s performance [18]. Therefore, instead of using topology optimization to change the shape of the part, much research has been made to apply topology optimization to the design of the support structure itself.

Since topology optimization is one of most popular methods for the design of support structures and the central idea used for this thesis, the following section will consist of an overview of the main ideas and implementation of topology optimization.

## 2.2 Topology optimization

### 2.2.1 Formulation

Topology optimization is an optimization technique that seeks to find the optimal shape within a volume that satisfies certain governing equations while at the same time satisfying specific constraints. This technique is usually utilized for the design of structures with no preconceived shape. Mathematically speaking, topology optimization seeks to find the optimal distribution of a design variable  $\rho$  within a design domain  $\Omega$ . The placement of  $\rho$  will also obey certain governing equations that are valid within the domain, and the existence of this distribution of  $\rho$  will also depend on a certain objective

that is wished to be minimized, alongside other constraints that might be imposed in the system. A classical example of topology optimization is the so called binary compliance problem, in which regions of solid and void material are distributed inside a volume, with the intention of designing a structural component that will be able to withstand certain loads applied to its boundaries, but that will have the least amount of deformation possible. Additionally, this structure should at most use a specified fraction of its design volume, or its total weight must be kept within a certain limit. We can express this specific problem mathematically as:

$$\text{minimize} \quad c(\boldsymbol{\rho}) = \mathbf{F}^T \mathbf{U} \quad (2.1)$$

$$\text{subject to} \quad \mathbf{K}(\boldsymbol{\rho}) \mathbf{U} = \mathbf{F} \quad (2.2)$$

$$V = \sum_{i \in \mathbb{N}_e} \rho_i v_i \leq V_c \quad (2.3)$$

$$0 \leq \rho_{min} \leq \rho_i \leq 1, \quad \forall i \in \mathbb{N}_e \quad (2.4)$$

where in equation 2.1 the objective function that is to be minimized is given by  $c(\boldsymbol{\rho}) = \mathbf{F}^T \mathbf{U}$ , where  $\rho$  takes the value 0 or 1 depending on whether our small region in space is empty or contains material,  $\mathbf{U}$  is the displacement of piece of material,  $\mathbf{F}$  is the force applied to it, and  $\mathbb{N}_e$  is a finite set of elements inside the design domain  $\Omega$ . The quantity  $c$  is referred to as compliance, and physically it is the inverse of the stiffness of the structure. The deformation of this material in the design domain is controlled by Hooke's Law, which is shown in equation 2.2.  $\mathbf{K}(\boldsymbol{\rho})$  is defined as a global stiffness matrix [19]

$$\mathbf{K}(\boldsymbol{\rho}) = \sum_{i \in \mathbb{N}_e} \mathbf{K}_i(\rho_i) \quad (2.5)$$

where  $\rho_i$  is the density at each region  $i$ , and we assemble all those densities into the vector  $\boldsymbol{\rho}$ .

More generally, the formulation of topology optimization can be written as a mini-



mization problem over a domain subject to one or more constraints, where the quantity that is being minimized is called the objective function. This objective function in turn could be an assembly of sub-objectives, each with its own relative weight with respect to the final solution. The minimized quantity is also written as a function of the design variables, which in most topology optimization problems refers to the amount of material that is present in a small region of the design domain. Additionally, we might impose an upper limit of the total amount of volume used. This was shown in equation 2.3, where the maximum allowed volume fraction of the design domain  $\Omega$  is denoted as  $V_c$ . Mathematically, the generalized problem of topology optimization can be formulated as follows:

$$\begin{aligned}
& \text{minimize} && X_{obj} = w_1 O_1 + w_2 O_2 + \dots + w_m O_m \\
& \text{subject to} && \text{governing equations} \\
& && V = \sum_{i \in \mathbb{N}_e} \rho_i v_i \leq V_c \\
& && 0 \leq \rho_{min} \leq \rho_i \leq 1, \quad \forall i \in \mathbb{N}_e \\
& && w_1 + w_2 + \dots + w_m = 1
\end{aligned} \tag{2.6}$$

where  $O_1, O_2, \dots, O_m$  are the sub objectives, and  $w_1, w_2, \dots, w_m$  their respective relative weights.

### 2.2.2 Homogenization and SIMP

Unfortunately, the formulation above suffers from a major problem. Stated as is, the problem above is well-known to be ill-posed [20], as it is possible to obtain a chattering design with an infinite number of holes of infinitesimal size, thus rendering this compliance problem to be unbounded [21]. To remedy this situation, several approaches have been proposed in the literature. One approach to control the chattering design is ensure that the total perimeter of the resulting structure has an upper bound [22]

[23], but this method suffers from several complications in implementation, and small variations in the parameters of the algorithm can lead to wildly different designs of the final structure [23].

A different alternative would be the utilization of a homogenization method [24], [25] [26] in which the binary representation of the material within the design domain is relaxed and intermediate values of densities are allowed, instead of just allowing empty and filled-values. One of the difficulties with using the homogenization method is how to interpret the intermediate densities of the material. In topology optimization problem involving the design of fluid flow media, the minimum value of the density could be interpreted as a fluid, the maximum value could be interpreted as a solid, while intermediate values could be interpreted as porous media [27]. In structural problems, intermediate values could be interpreted as periodic composite materials with high-resolution microscopic features [28] [29], materials that are composed of lattice structures [30], or even complex structures consisting of anisotropic fiber-reinforced composite materials [31]. Although the homogenization method is successful in solving the chattering design problem, validation of the resulting topologies using most Finite Element Method (FEM) software is very computationally expensive, since the resulting microstructures requires very fine meshes with high number of elements and nodes [32]. Additionally, the use of microstructures can also lead to stress amplifications which need to be managed appropriately to avoid regions of high stress concentration that might compromise the stability or functionality of the manufactured part [33]. It is also worthy of note that when this method was introduced back in the 1980's, manufacturability of components designed with the homogenization method was not feasible, as it was very difficult to manufacture components with microstructures or lattices. Nevertheless, the recent progress of additive manufacturing has revived the interest for structures with microstructures due to their newfound manufacturability [34].

A simpler method that tries to avoid the complications of homogenization theory is the Solid Isotropic Material Penalization method (SIMP), which utilizes yet another continuous density function. The main point of SIMP is to apply a power-law inter-

polation function to the material density, with the objective to penalize intermediate densities and drive them to their extreme values of void and full material. [35] [36]. This method was beneficial at the time this method was being developed in the late 80's and 90's, since as previously mentioned, it was yet fairly difficult to manufacture parts with complicated microstructures, and thus methods that drove densities to a binary result of void and material were preferred. Additionally, since the implementation of this method is simpler and less computationally intensive than homogenization, many modern FEM software use SIMP for topology optimization [37].

To understand its implementation, let us return to the problem of the design of a support structure by minimizing its compliance (equations 2.1 - 2.4). Since the material density is allowed to take intermediate values, the same applies for the structure's mechanical properties, and therefore Young's modules could be computed using the following power law:

$$E_i = E_i(\rho_i) = \rho_i^p E_i, \quad (2.7)$$

where  $E_i$  is the modulus of elasticity at a region  $i$  of the design domain  $\Omega$ ,  $\rho_i$  is the density field at that region  $i$ ,  $E_0$  is the modulus of elasticity of the solid material and  $p$  is the penalization factor that tries to drive the density towards its binary void and solid values. However, since the SIMP method is usually used in conjunction with FEM to solve for the density distribution, it is required to avoid void material configurations that would result in singularities during the numerical computation steps. Instead, we can rewrite equation 2.7 as:

$$E_i = E_i(\rho_i) = E_{min} + \rho_i^p (E_0 - E_{min}), \quad (2.8)$$

in which the addition of the  $E_{min}$  avoids singularities, and  $E_0$  can be chosen to be small to represent regions of void material.

### 2.2.3 Filtering techniques

Unfortunately, usage of the SIMP method as stated in 2.8 can also result in checkerboard patterns or might add very thin details that are comparable in size to the mesh size, causing the final result to converge to a very different topology depending on the of the length scale of the mesh [38]. One of the possible techniques in the literature used to solve this problem is the use of filtering techniques on the density field. The main idea of these filtering techniques is to modify the density field at a point so that it becomes a weighted average of the densities of its neighboring points. Using this idea, a new filtered density field can be mathematically defined using the following equation [21]:

$$\bar{\rho}_i = \frac{\sum_{j \in N_i} H_{ij} v_j \rho_j}{\sum_{j \in N_i} H_{ij} v_j}, \quad (2.9)$$

where  $\rho_j$  is the density field at a small region  $j$  of the design domain  $\Omega$ ,  $v_j$  is the total volume of the region in consideration, and  $N_i$  is a neighboring region around  $v_j$ . The term  $H_{ij}$  is the weighing function, and it is analogous to a kernel, or convolution matrix, that is used in image processing applications to modify the properties of a pixel based on its surroundings, and that is typically used to blur, sharpen, emboss or perform other transformations to the image used (i.e. an image filtering function).  $H_{ij}$  could be defined as a function of the distance between the element and neighboring elements, and should be chosen so that it is linearly or exponentially decaying away from the element. A possible weighting function could be calculated as

$$H_{ij} = \max(1 - \frac{(x_j - x_i)^2 + (y_j - y_i)^2}{r}, 0), \quad (2.10)$$

where in this example, the design domain is two-dimensional, and  $x_i$  and  $y_i$  is the point in consideration, and  $r$  is the radius of filtration. In other literature, equation 2.10 is also expressed as [21]

$$H_{ij} = r - \text{dist}(i, j), \quad (2.11)$$

Where  $dist(i, j)$  contains all the neighboring points  $j$  within a distance  $r$  of point  $i$ .

Another alternative to using equation 2.15 is to instead define the filter implicitly using a solution to the Helmholtz partial differential equation (PDE) using homogeneous Neumann boundary conditions that are imposed on the boundary of the design domain [39]

$$\begin{aligned} -r^2 \nabla^2 \bar{\rho} + \bar{\rho} &= \rho \\ \frac{\partial \bar{\rho}}{\partial \mathbf{n}} &= 0, \end{aligned} \tag{2.12}$$

where  $r$  is the filter radius. The solution of this equation is a function that decays monotonically as is desired, and also preserves the volume of the design domain after the filtering process is completed. Additionally, equation 2.12 can be solved within the same finite element solver that is utilized for the solution of the whole topological optimization problem, and it requires no extra information beyond the mesh connectivity around each volume or mesh element [40]. It is also important to note that when the Helmholtz filter or any other filtering function is implemented into a finite element solver, the filter radius  $r$  must be bigger than the mesh edge size in order to obtain mesh independent results [41].

## 2.2.4 Threshold projection

Even though filtering is effective at solving the checkerboard design pathology, the structure that is designed can also suffer from intermediate density values. To try and converge these intermediate values into a void/solid configuration, projection functions can be used to map the results of the filtered density field into 0/1. One method proposed by Xu et al. [42] is a volume-preserving, modified continuous Heaviside function which takes the following form

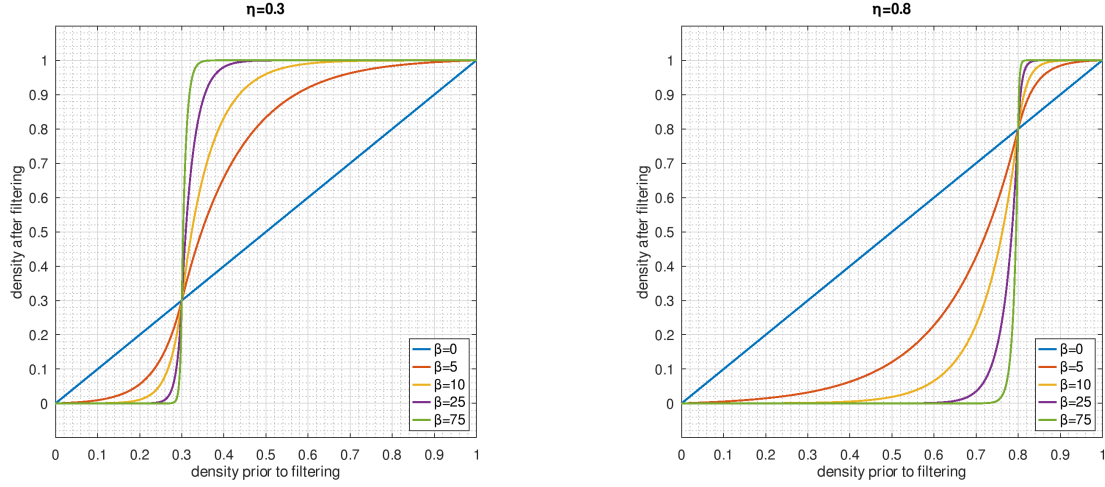


Figure 2.1: Smoothed modified Heaviside filters with  $\eta = 0.3$ ,  $\eta = 0.8$ , and different values of  $\beta$ .

$$\tilde{\rho} = \begin{cases} \eta[e^{-\beta(1-\bar{\rho}/\eta)} - (1-\bar{\rho}/\eta)e^{-\beta}] & 0 \leq \bar{\rho} \leq \eta \\ (1-\eta)[1 - e^{-\beta(\bar{\rho}-\eta)/(1-\eta)} + (\bar{\rho}-\eta)e^{-\beta}/(1-\eta)] + \eta & \eta \leq \bar{\rho} \leq 1. \end{cases} \quad (2.13)$$

In equation 2.13  $\tilde{\rho}$  is the new projected density value,  $\bar{\rho}$  is the density value obtained from the Helmholtz filter function explained in the previous section,  $\eta$  is a parameter that will determine where the cutoff point between the 0/1 values will be made in the x-axis, and  $\beta$  will determine how fast the transition from 0/1 will be. Figure 2.1 has been included to aid the reader in understanding how the parameters affect the shape of the function.

Although the expression in equation 2.13 is successful in projecting the filtered density values, it can be replaced by a simpler and shorter expression utilizing the hyperbolic tangent function [38]

$$\tilde{\rho}_i = \frac{\tanh(\beta\eta) + \tanh(\beta(\bar{\rho}_i - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1 - \eta))} \quad (2.14)$$

Equation 2.14 has the added benefit of computing the 0/1 projection faster, and in

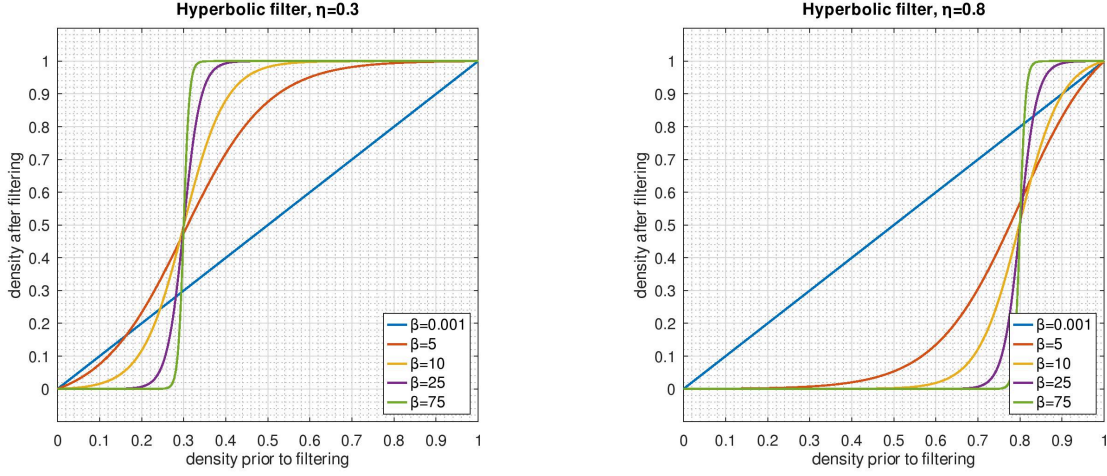


Figure 2.2: Hyperbolic tangent filter function with  $\eta = 0.3$ ,  $\eta = 0.8$ , and different values of  $\beta$ .

the limit as  $\beta \rightarrow \infty$  it yields the same result as equation 2.13. Plots of this equation are shown in figure 2.2.

One final point must be made before finishing this section. When a density filter is applied, the mechanical properties of the structure, such as Young's modulus or thermal conductivity, also become a function of the filtered function. For example, in the context of the structural problem described by equations 2.1 - 2.4, Young's modulus described by equation 2.8 changes into the following:

$$E_i = E_i(\tilde{\rho}_i) = E_{min} + \tilde{\rho}_i^p(E_0 - E_{min}), \quad (2.15)$$

where  $\tilde{\rho}_i$  denotes the filtered density using the hyperbolic tangent filter at a small element  $i$  of the design domain  $\Omega$ .

### 2.2.5 Final formulation of the topology optimization problem

In this work, topology optimization was utilized to design a support structure for components created by additive manufacturing. The design variables are the densities of the support structure throughout its design domain, which is the volume between the component and the base plate. The objective function utilized was a multi-objective

function that sought to simultaneously maximize heat conduction and minimize the compliance of the supporting structure. The material properties of the support structure that depend on the design variables are the thermal conductivity and Young's modulus. A Helmholtz filter was utilized, and the final density was computed using a hyperbolic tangent projection. Additionally, a volume fraction constrain was employed with the intention of reducing the amount of material used. Mathematically, all of these conditions can be represented as follows:

$$\text{minimize} \quad X_{obj} = w_1 \mathbf{F}^T \mathbf{U} + w_2 \mathbf{T}^T \mathbf{Q} \quad (2.16)$$

$$\text{subject to} \quad \boldsymbol{\kappa}(\tilde{\boldsymbol{\rho}}) \mathbf{T} = \mathbf{Q} \quad (2.17)$$

$$\mathbf{K}(\tilde{\boldsymbol{\rho}}) \mathbf{U} = \mathbf{F} \quad (2.18)$$

$$V = \sum_{i \in \mathbb{N}_e} \tilde{\rho}_i v_i \leq V_c \quad (2.19)$$

$$0 \leq \rho_{min} \leq \tilde{\rho}_i \leq 1, \quad \forall i \in \mathbb{N}_e \quad (2.20)$$

$$w_1 + w_2 = 1, \quad (2.21)$$

where  $\tilde{\boldsymbol{\rho}} = [\tilde{\rho}_1, \tilde{\rho}_2, \dots, \tilde{\rho}_n]^T$  and the physical densities  $\tilde{\rho}_i$  are defined by equation 2.14,  $\mathbf{U}$  denotes the nodal displacement vector,  $\mathbf{T}$  denotes the nodal temperature vector,  $\mathbf{F}$  is the vector of nodal forces, and  $\mathbf{Q}$  is the thermal load vector. Equation 2.16 is the objective function, with the first term being mechanical compliance, and the second term being thermal compliance, which is the inverse of thermal conductivity. Equation 2.19 is the volume constraint equation. Equation 2.17 governs the thermal conduction of the material, with the thermal conductivity matrix  $\boldsymbol{\kappa}$  calculated as:

$$\boldsymbol{\kappa}(\tilde{\boldsymbol{\rho}}) = \sum_{i \in \mathbb{N}_e} [k_{min} + \tilde{\rho}_i^p (k_0 - k_{min})] \mathbf{k}_i^0 \quad (2.22)$$

where  $k_{min}$  and  $k_0$  are the element's minimum and maximum thermal conductivities, and  $\mathbf{k}_i^0$  is the element's conductivity matrix. On the other hand, equation 2.18 governs



the deformation of the structure, with the stiffness matrix  $\mathbf{K}$  calculated as

$$\mathbf{K}(\tilde{\rho}) = \sum_{i \in \mathbb{N}_e} [E_{min} + \tilde{\rho}_i^p (E_0 - E_{min})] \mathbf{K}_i^0 \quad (2.23)$$

with  $E_{min}$  and  $E_0$  denoting the limits of Young's modulus, and  $\mathbf{K}_i^0$  is the element's stiffness matrix as defined in [21].

Additionally, the reader must note that in equations 2.22 and 2.23,  $\sum$  does not denote summation, but instead the finite element assembly operator [19].

## 2.3 Summary of literature

Foundational works on the theory behind topology optimization include the work of Sigmund, Bendsoe, Lazarov, Wang and others [43], [35], [24], [44], [16], [28], [39], [38], [25], [34]. All of the previous works explain thoroughly the logic, method, and implementation of topology optimization, most of them focusing on its application for structural design problems.

Topology optimization has been identified as a powerful tool for the creation of parts made by additive manufacturing and SLM. Much focus has been given to the study of distortion of AM components. Miki and Yamada [45] proposed a method utilizing topology optimization that accounts for the distortion of the manufactured part. Misiun et al. [46] studied recoater collision and global distortion of products created by AM in the context of SIMP method based topology optimization, applied to the construction of a bracket. Komini et al. [47] investigate the creation of robust topology optimization algorithms in the distortion of parts under uncertain and variable additive manufacturing processes.

There are also several papers that investigate the design of parts that do not require any support structures, instead using topology optimization to design self-supporting components. Xu et al. [48] utilized a topology optimization framework based on Moving Morphable Components and Moving Morphable Voids, while Zheng et al. [49] used

implicit B-spline representations instead of voxel-shaped representations to expedite the optimisation process significantly. Other methods include placing restriction on overhang angles and orientation to avoid horizontal structures, such as the study by Weihong and Lu [50]. Although this type of method is obviously beneficial for saving efforts in post-processing and reducing material costs, some manufacturers would be unwilling to change their current designs as it would impact the functionality or requirements of their components. For example, Ameen et al. [51] studied the limitations of self-supporting structures in additive manufacturing; parts with convex overhanging structures built without supporting structures result in considerable deformation at these locations. Shuzhi et al. [52] formulated a topology optimization method for support structures that uses the inherent strain method to constrain distortion. Lee and Xie [53] developed an optimization algorithm that detects the best locations for supports in the boundaries of the design space, which can lead to an increase in stiffness and aid in minimizing deformation.

These distortions are usually caused by high thermal gradients that create residual stresses near the molten pool. The component or the support structure could be designed to maximize its thermal conduction and lead away heat as to minimize thermal deformation. Lohan et al. [54] discuss the possible practical objectives for heat conduction topology optimization. Lohan et al. or example, [55] also studies the use of generative design algorithms to create dendritic structures that will conduct heat away from the main component. Support structures could help to dissipate heat away from the component to reduce these residual stresses. The study by Iga et al. [56] considers not only thermal loading and conduction, but also tries to incorporate convection in the topology optimization algorithm. Huang et al. [57] uses lattice support structures to maximize heat conduction but also keeping the total space of the support structure low. Miki et al. [45] and Ogawa et al. [58] incorporate transient analyses to design a support structure with maximal heat conduction. Chung [11] also explores the effect of different lattice support structure geometries on the heat conduction performance of support structures.

## 2.4 Research purpose, originality, and contribution

The aforementioned studies are characterized by a high degree of academic rigor; however, from the perspective of a manufacturer, the existing software solutions already incorporate proprietary algorithms tailored to their specific needs. One of the primary concern for manufacturers lies in the optimization of material usage. Consequently, it is imperative to investigate how the parameters governing the optimization process influence the characteristics of the final product. Additionally, an exploration of the extent to which the volume fraction can be manipulated without compromising product integrity is essential. Therein lies the central focus of the present research.

Therefore, the purpose of this research is twofold: firstly, to investigate a full procedure for the design and validation of supporting structures of components manufactured through SLM utilizing topology optimization, identifying any tricky parts of the process. As mentioned before, Most studies only concern themselves on the study of topological optimization algorithms for the design of support structures, but don't delve into the full process of CAD creation and validation of the impact of the support structure on the final part by means of simulations and experiments. This work takes a look at the whole process, and discusses the challenges and opportunities for improvement of CAD creation and FEM simulation of support structures.

The second objective of this research is to investigate the influence of topology optimization parameters on the total deformation of components manufactured through Selective Laser Melting (SLM). To achieve this, various topologies for support structures were generated by manipulating parameters such as volume fraction, objective function weights, and hyperbolic tangent projection filter settings. Subsequently, simulations of the SLM component building process utilizing these structures were conducted, enabling an analysis of how different topology optimization parameters affect the total deformation and resultant stress of the finished SLM products. Particular emphasis was placed on the effect of volume fraction, with the aim of determining whether a reduction in the volume fraction of the supporting structure would adversely impact

the quality of the final component.

Successfully identifying a set of parameters that yield optimal results in terms of minimizing deformation and stress can provide manufacturers with valuable insights for enhancing their manufacturing processes. Conversely, if the parameters are found to have minimal influence on the final product's quality, manufacturers may choose to adopt parameters that align more closely with their operational requirements without the concern of compromising part integrity. For instance, if adjustments to the volume fraction of the support structure do not significantly affect the quality of the final component, manufacturers could utilize a lower volume fraction to reduce material costs while maintaining confidence in the quality of the produced parts.

Therefore, the contributions of this study to the field are as follows:

1. Employ topology optimization methodologies for the development of Computer-Aided Design (CAD) support structures for components manufactured through Selective Laser Melting (SLM), while validating their effectiveness in mitigating thermal deformation and stress in the final product.
2. Investigate the impact of topology optimization parameters on the thermal deformation and stress experienced by support structures. This investigation will specifically focus on the effects of hyperbolic tangent projection, volume fraction, and the weight ratio of objective functions.
3. Identify the optimal combination of topology optimization parameters that minimizes thermal deformation and thermal stress in components produced via SLM.
4. Identify specific geometric configurations that enhance thermal deformation or stress performance, or determine configurations that maintain deformation or stress within acceptable limits while simultaneously reducing material usage.

## 3. Methodology

### 3.1 Introduction

To analyze the performance of different support structures created using topology optimization, a comparison study was made in which parts created by additive manufacturing were paired with different support structures. This study assumed that different structures will conduct heat energy differently, and thus some topologies might be more effective in removing heat faster from each material layer as it is being processed. This increased thermal conduction would then result in less overall thermal deformation, as the manufactured component would expand less due to the decreased time in high temperatures.

This section will explain the full process taken to run the simulations and analyze the data. An overview of the process can be seen in Figure 3.1. The process starts from the creation of the CAD for the manufactured components, followed by the design and CAD creation of the support structures. The components and the support structures are then merged, and imported into the additive manufacturing software to simulate the results of manufacture. The results from the manufacturing simulation are then analyzed by means of graphs and statistical methods.

The subsequent sections explain in detail each stage of this process.

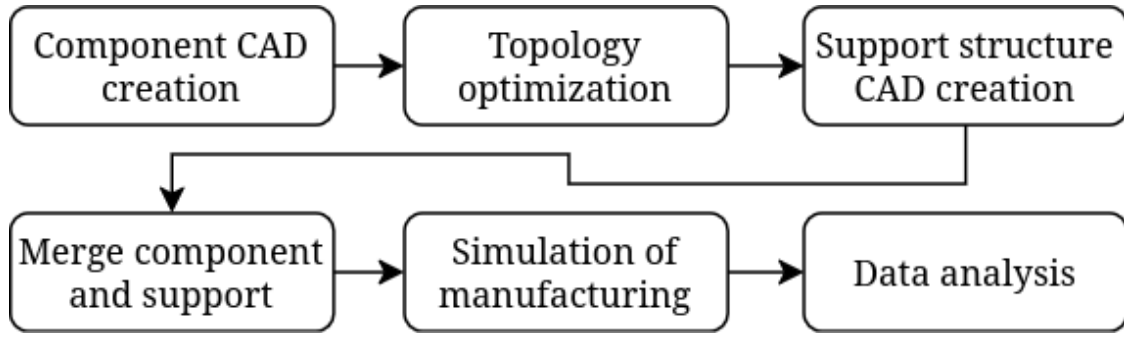


Figure 3.1: Process diagram.

## 3.2 Component CAD creation

### 3.2.1 Simple geometry

The components with simple geometries utilized in this study consist of a cube, three triangular components with different slopes, and three cylindrical components with different values of curvature. To reiterate, these components have the same dimensions that were used in the study of lattice support structure performance by Peishu **peishu\_thesis**. All the CAD models used for the simple geometry study were created using FreeCAD, an open-source CAD software. All the components were exported as .STEP files, and then they were merged with their corresponding support structures using the software nTop.

- A cube with side length of 30 mm (Figure 3.2).
- Three triangular components with varying slopes. All triangular components have a base of  $30 \times 30 \text{ mm}^2$ , with slopes of  $15^\circ$ ,  $30^\circ$  and  $45^\circ$ . The measurements are shown in figure.

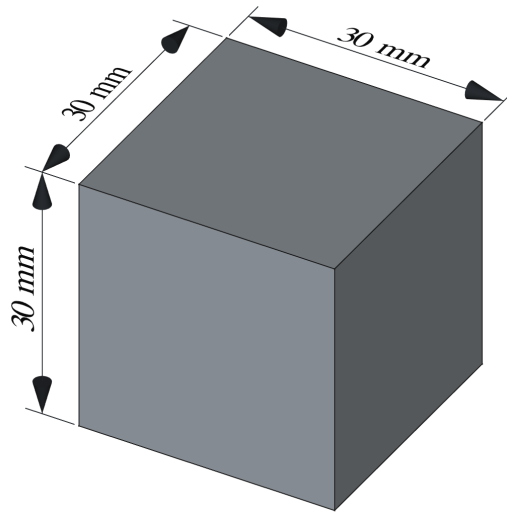
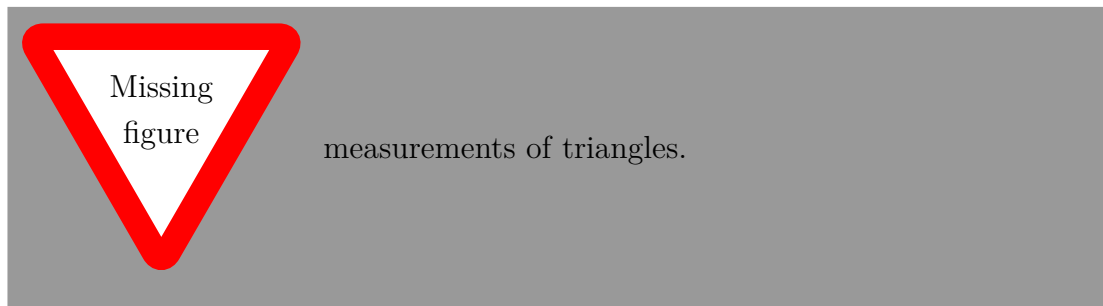


Figure 3.2: Dimensions of cube component.

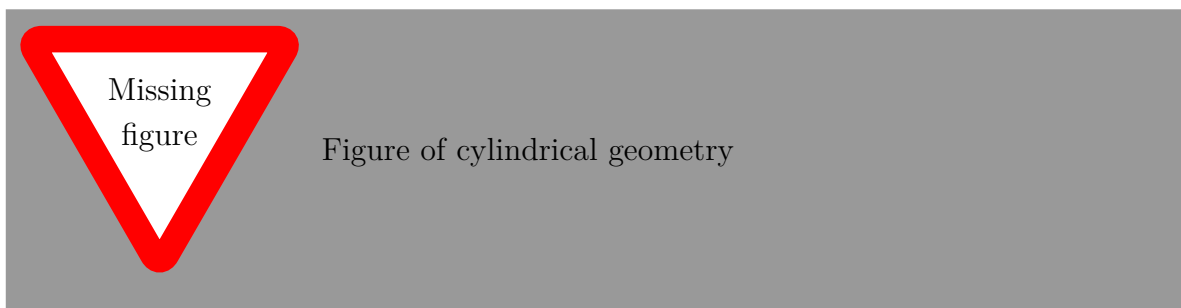


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### 3.2.2 Femoral component

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### 3.3 Support structure creation

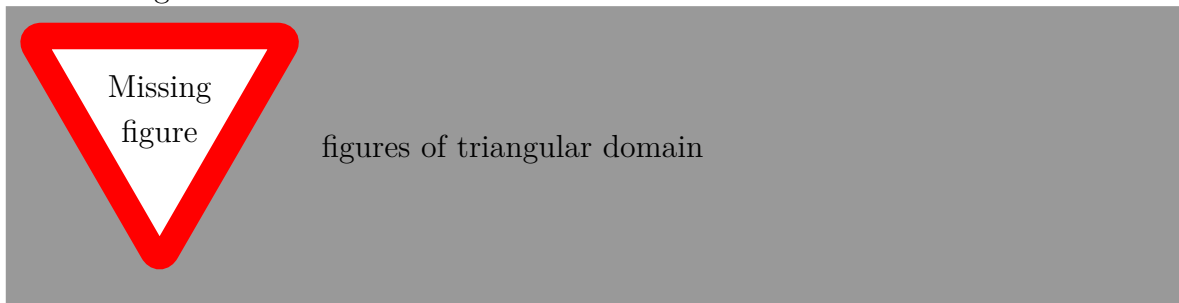
The supporting structures of the components were created using the method of topology optimization. The design was implemented using the topology optimization module of COMSOL Multiphysics 6.2.

The steps for creating a valid topology optimization model are: determine the design volume, choose a suitable mathematical model for the behavior of the material within the design volume, choose the objective functions and decide what constraints need to the material within the volume be applied to the design volume. The following section explains all of these steps in detail.

#### 3.3.1 Design domain

##### Simple geometry design domain

The design domain for the simple geometry components consists of the volume between the bottom face of each component and the base plate. For the cube, the design domain ends up being a rectangular prism with dimensions 10 cm x 30 cm x 30 cm. For the triangles, the design domain consists of different triangular spaces with a square base, shown in figures



Since all of these components can be generated by the drawing their profile on a plane and extruding it in a direction perpendicular to the plane, the design domain utilized in the topology optimization model is also a 2D model. This grants the benefit

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of faster computation, and thus the supporting structure is created by first finding the planar topology and also extruding it in a perpendicular direction.

## Femoral component design domain

### 3.3.2 Mathematical model

The mathematical models consists of the governing equations used to describe the physics of the system and the constraints imposed on the model, as well as the objective function that needs to be minimized.

For this system, the most appropriate objective function would be the maximization of thermal conductivity; in order to minimize the thermal deformation of the manufactured component, it is imperative to transfer heat away from it as fast as possible. The physical equation describing the thermal conduction of a material is given by Fourier's Law, which can be expressed as:

$$q = -k\nabla T \quad (3.1)$$

where  $q$  is the heat conduction through the material,  $k$  is the material's thermal conductivity and  $\nabla T$  is the thermal gradient.

For this study, the only constraint applied on the system was the volume fraction. Volume fraction specifies the maximum amount of volume that the designed topology should take from the design domain. The volume fractions utilized were 50% and 75%.

### 3.3.3 Objective functions

An additional objective function to be considered would be the deformation of the supporting structure as well. If the supporting structure is composed of very thin segments, there is a high possibility that the structure might buckle, which would add significant geometric errors to the component. In the worst case scenario, the structure itself would collapse, causing the entire manufacturing process to fail. In order to

Write down equation of thermal conductivity for topology optimization domain here.

avoid this, it is necessary to limit the amount of deformation that is allowed in the support structure. This can be achieved by utilizing an additional objective function of structural compliance minimization. We define structural compliance as the amount of energy stored in the deformation of material. Minimizing this stored energy is thus equivalent to minimizing the deformation. Therefore, the second objective function of minimization of structural compliance is also used in the topology optimization model.as heat passes into the support structure from the bottom of the manufactured component,

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The next step is applying certain constraints on the system. For this study, we are mostly concerned with the principal constraint is the volume fraction, which is the maximum amount of volume that the topology can cover within the design domain. This criteria is chosen because we seek to use less material for the supporting structure, as long as we can maintain the total deformation of the manufacturing component beneath a threshold. For the model in COMSOL, the values of volume fraction = 0.5 and volume fraction = 0.75 were used.

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### 3.3.4 COMSOL implementation

Mesh

Physical parameters

Simulation itself

Merge them  
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the weights.

As discussed in the previous chapter, Helmholtz filters are frequently used in topology optimization studies to avoid chattering designs.

Talk about  
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filter

## 3.4 Component and support structure merging

### 3.4.1 CAD for support structure

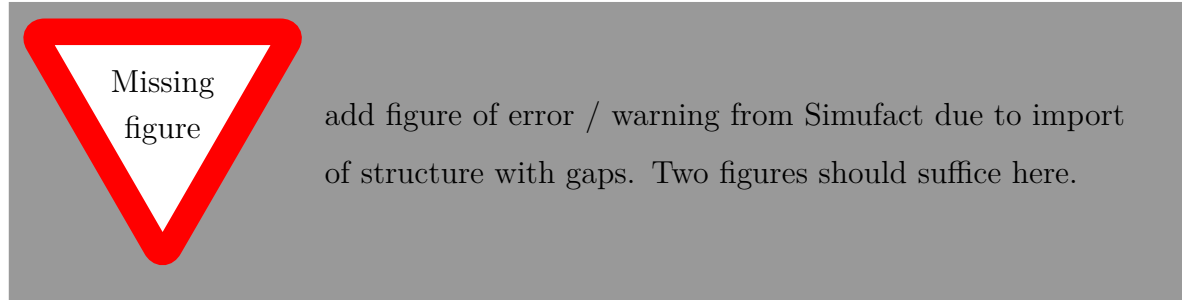
Hyperbolic  
tangent

This section talks about how FreeCAD was used to create the support structure once the pictures of the topologies have been obtained.

### 3.4.2 Merging of part with support structure

Once the CAD file of the component and the support structure has been built, it is necessary to merge them together and import them into Simufact to undergo simulation of the manufacturing process. The software used for blending the component and its support structure is nTop . nTop's interface makes it very easy to merge the part, and also allows to blend the support structure and the component, which effectively creates a fillet between the nodes of both components to allow for a smooth transition between bodies. Of course, blending the component and the support structure in this manner would not give any benefit in a real manufacturing process, as the structure and the component would not be able to be separated easily. NEvertheless, this blend radius is beneficial for the simulation since it was observed that a direct union and import of the support structure + component in Simufact resulted in having very small gaps between the two pieces, resulting in a non manifold geometry that would cause the finite element model to have gaps between some of its nodes.

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## 3.5 Simulation of manufacturing process

The software utilized to simulate the manufacturing process is Simufact Additive version 2023.2. Simufact Additive is capable of simulation building process of additive manufacturing components, and coupling thermal and stress physics to predict the temperature values of the component throughout the building process and the total stresses, strains and deformations resulting from the manufacturing process.

In order to set up Simufact correctly, the building process and the building space

geometry must be specified before each simulation. The building parameters and building geometries used in this study are the same that were used in the analysis of thermal deformation using lattice support structures done by Peihsy and al.

add reference here

### 3.5.1 Simufact simulation parameters and voxelization

After the component and the support structures were merged, they were imported into Simufact. It is during this step that all the factors related to the simulation are set, which include the machine properties, material properties, and build parameters. As mentioned previously, these were chosen to be identical to the study of PeiHsu to ensure that the results of this study could be compared to the results of that one.

add that thermo-mechanical process was used and explain what it is.

The first parameter to be chosen is the process properties, which determines the physics that Simufact takes into consideration to run the simulation. Simufact provides three different types of processes: mechanical, thermal, and thermomechanical. As stated in the Simufact manual , mechanical provides a fast mechanical analysis that only uses inherent strains as the main input. This type of analysis does not take into consideration the temperature fields during the building process. The thermal process on the other hand only considers the thermal behaviour of the components, and the temperature field of the support structures, components and base can be analyzed. The thermomechanical process couples the stress and thermal analyses, and allows for the prediction of temperature, distortions and stresses of the part. This latter process is the one used in this study.

insert reference to manual here

After choosing the process property, the machine parameters must be specified. This includes the machine build plate geometry and the laser parameters. The machine build plate chosen was a circular plate with an 80 mm radius. The build space dimensions consists of a space of 160 mm in all three x-y-z directions. As for the laser parameters, the simulations were carried out with one laser with a maximum laser power of 500 W and a maximum laser speed of 2000 mm / s, an efficiency of 25 percent, and a beam width of 25 mm. All of these parameters are summarized in the table

add the table of building parameters here.

The building parameters for the process need also to be set. These include material layer parameters and any thermal parameters and temperature specifications for the build environment and base plate. The powder layer thickness was chosen to be 0.03 mm, with a recoater time of 10 s. The powder initial temperature was set to 25 °Celcius, with an initial base temperature of 200 degrees.

### 3.5.2 Convergence analysis

To make sure that the results of the simulation would not depend on the voxel density of the

## 3.6 Data collection and organization

*This section will detail how I collected the data from Simufact and what software and methods I used to organize it an analyze it. The actual results will go in the following chapter, aptly named results, duh*

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Figure: measurements of triangles. . . . .	21
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Figure: Figure of cylindrical geometry . . . . .	22
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Figure: figures of triangular domain . . . . .	23
Add here the cylindrical component geometry . . . . .	23
Write down equation of thermal conductivity for top optimiation domain here. .	24
probably want to add a reference to this somewhere. . . . .	25
Write equation of compliance here . . . . .	25
Merge them together to get the final objective function. Explain about the weights.	25
Talk about Helmholtz filter . . . . .	25
Hyperbolic tangent . . . . .	25
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