

Morban's thesis

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Chapter 1

Introduction

1.1 Background

Osteoarthritis is a very common type of arthritis that causes pain, swelling and stiffness in various body parts such as the hands, hips, back and knees. Over time, it affects bones, cartilage and other tissues, and is common in adults over the age of 45. Knee osteoarthritis, also known as degenerate joint disease of the knee, is a type of osteoarthritis that is predominantly seen in the elderly, and is a progressive disease that results in knee stiffness and swelling and pain after sitting or staying still for a long time. Although this disease can be treated, but not cured, with physical therapy and medications that slow down its progression, severe knee osteoarthritis can only be resolved by means of surgery, in which the whole knee is replaced by a prosthesis. Total knee arthroplasty (TKA), also called total knee replacement (TKR) is a very effective and consistently successful surgery that provides good outcomes for patients suffering from end-stage knee osteoarthritis. TKR results in greatly improved pain relief and better quality of life for patients [1].

Since osteoarthritis is a disease that is most commonly seen in the elderly, as the percentage of elderly people in developed countries around the world increases, the prevention and treatment of diseases like osteoarthritis become more important

from a public health perspective. In particular, Taiwan has already exceeded the threshold (14%) of the definition of aged society established by the United Nations, with 3.983 million citizens over the age of 65, accounting for 17.18% of the population [2]. Additionally, it has been estimated that Taiwan's National Health Insurance already spends 5% of its total expenditure on TKR every year, and the incidence of TKR has already tripled in the period between 1996 and 2010 [3].

There has been a growing interest of using additive manufacturing (AM) for the manufacture of the prosthetics used in TKA, since additive manufacturing is a technology well suited for the creation of custom, lightweight components with complex geometries, while also producing less material compared with other methods of manufacture [4]. Even though additive manufacturing offers many advantages for the production of structures tailored to each individual patient, one of the major obstacles that stands in the way of a more widespread adoption are the high costs of AM. One way to reduce the cost of the total procedure is by reducing the amount of material used, which can also be accomplished by reducing the amount of scrap from the fabricated parts. In additive manufacturing, much of the scrap comes from discarding the support structures that the components require for manufacturing, and thus making smaller support components or using smaller volumes for them would be a valid strategy to further reduce the cost of additive manufacturing components. The total deformation of the part after manufacture is also an important factor to consider, since additional costs can be incurred from the addition or removal of material from the manufactured component by means of machining, in order to meet tolerances [4] [5].

Chapter 2

Literature Review

What I need to do in the literature review is an introduction, body and conclusion. The introduction should be an overview of the topics that I am going to talk about, and probably introduce the research motivation.

2.1 Introduction

Additive manufacturing, also known as 3D printing, is a manufacturing process in which parts are built by stacking layers of material on top of each other until the desired geometry is created. In particular, powdered-bed fusion is a type of additive manufacturing in which the base material is composed of a metallic powder. A bed is then filled with this powder, and a high-powered laser or electron beam is utilized to selectively add heat to locations of this powdered metal, thus melting it. As the melted portions cool, it leaves a layer of solid material, and the whole component is then built layer by layer in this manner. If a laser is used for manufacturing, the process is then known as laser powder bed fusion (LPBF).

Additive manufacturing in general allows for the creation of parts with complex geometries that would otherwise be difficult, time-consuming or too costly to manufacture with other more traditional means of manufacturing such as machining or casting. This can be contrasted with the manufacture of prothesis using traditional manufacturing,

which relies on the high-volume production of standardized parts and shapes. Due to the ease of creating parts with complex geometries and with shorter lead times, additive manufacturing has had a growing interest from the medical industry for the creation of custom made implants tailored to an individual patient's anatomy [6] citenarraAdditiveManufacturingTotal2019. Additionally, this benefit of creating customized parts also leads to an improvement of patient comfort and better outcomes in orthopedic and surgical applications, as well as enhancing surgical precision and reducing complications of post-operative care [7] [8].

Nevertheless, even though additive manufacturing holds promise for the production of customized medical implants, its cost can still remain high due to machine, material and process-level expenses, and thus its usage can still be limited only for revision procedures, i.e. second or third procedures in which the objective is to replace a failed or uncomfortable component [4]. To solve the problem of high material costs, Laureijs et. al. propose that, in the case of powdered base additive manufacturing, one of the main driving costs is the price of the powder price. One of the viable options to drive down the price of additive manufacturing then would be to reduce the amount of scrap material involved in the process. Scrap material in additive manufacturing results from discarding the support structures that the part requires as well as any addition or removal of material to the part in post-processing steps to ensure that the part is maintained withing tolerances [5].

The design of support structures in additive manufacturing is thus an important consideration, with many factors affecting how the support structure will be built. A well optimized and design support structure will enable the part to be realized, while at the same time using the least amount of material possible to obtain a specified objective. Thermal and mechanical requirements are usually at the forefront of support structure design as they have a direct effect on the quality of the component, although there might be other considerations for the design, such as ease of removability, build time, and material efficiency.

Thermal conductivity is an important factor since good thermal conduction in the

support structures can improve the cooling process during fabrication, which helps prevent issues caused by excessive thermal deformation such as thermal residual stresses, thermal dilation, cracks or warping [9] [10]. Additionally, large portions of surfaces that are almost horizontal and that are unsupported during manufacturing tend to be heavily distorted after manufacturing [9]. These regions, which are called overhang structures, typically require support structures that will reduce the deformations and prevent warpage of overhangs. At these regions, it is crucial that support these support structures also require to be as stiff as possible to prevent any distortion in the built part and to be able to withstand the weight of the part itself [11]. The support structures must then also be robust enough to withstand the weight in the part, especially in these overhang areas, to prevent distortion during the build process [12].

Topology optimization is a method that is suitable for the creation and optimization of shapes that satisfy specific constraints while realizing certain objectives [13], and is a well-known technique that has been obtaining more interest from the additive manufacturing community for both the design of the components themselves and for their support structures in additive manufacturing. . Topology optimization has been studied for designing parts that either have as little overhang surfaces as possible, thus requiring less supporting structures, or for designing parts that are self-supporting, thus requiring no supporting structures at all. Although this aid considerably in reducing the amount of material used, these approaches do not take into consideration other effects such as the thermal dissipation of the component, require changing the geometry of the component thus impacting its functionality [14], or impose geometrical constraints that can restrict the component's performance [15].

Topology optimization can be used for the design of the desired component itself, as to minimize and avoid the amount of overhang structures, or to make components that are self supporting.

Lee and Xie [16] developed an optimization algorithm that detects the best locations for supports in the boundaries of the design space, which can lead to an increase in stiffness and aid in minimizing deformation.

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maybe talk more about topology optimization here

this paragraph talks about two uses for topology optimization: for the part itself, and for support structures.

2.2 Additive manufacturing

This section needs to be more specific about how additive manufacturing works.¹

2.3 Support structures

2.4 Topology optimization

Topology optimization is an optimization technique that seeks to find the optimal shape within a volume that satisfies certain governing equations, while at the same time satisfying specific constraints. This technique is usually utilized for the design of structures with no preconceived shape. When topology optimization is used for the purpose of designing structures, the optimal shape resulting from the analysis can be interpreted as a material distribution within the specified volume, which is called the design domain. A classical application of topology optimization is the binary compliance problem, in which regions of solid and void materials are distributed in order to minimize the work done by external forces (this is called compliance), also subject to a volume constraint [17]. The problem of topology optimization can be expressed mathematically as follows [18]

$$\min : c(\rho) = f^T u \text{ s.t. : } K(\rho)u = fV = \Sigma i \in \Omega \rho_i v_i \quad (2.1)$$

In the above formulation, the design domain

This binary compliance problem suffers from a serious drawback; the problem is ill-posed and the solution will converge to a material distribution in the shape of a checkerboard pattern, with infinitesimal holes. To alleviate this, several restrictions must be made and the equations used to generate the structure must be modified.

Talk here about density-based approach and SIMP

Talk here about the filters, density filters and Helmholtz filters

Techniques such as perimeter control methods [] density gradient control and mesh-

independent density filters could be used to solve the problem of a chattering design [reference goes here, filter in topology optimization. A filtering approach implements an algorithm that iterates over all the points in the design domain, and uses weighted values of its surroundings to determine the final density value of the point in question, effectively computing the weighted average of a point and its surroundings.

Talk here about hyperbolic tangent and what it does to the filter

Here, the full equation of topology optimization using all of the above should be shown.

2.5 Heat transfer in supporting structures

2.6 Compliance in support structures

2.7 Research Purpose

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2.8 Research originality and contribution

Chapter 3

Methodology

3.1 Introduction

To analyze the performance of different support structures created using topology optimization, a comparison study was made in which parts created by additive manufacturing were paired with different support structures. This study assumed that different structures will conduct heat energy differently, and thus some topologies might be more effective in removing heat faster from each material layer as it is being processed. This increased thermal conduction would then result in less overall thermal deformation, as the manufactured component would expand less due to the decreased time in high temperatures.

This section will explain the full process taken to run the simulations and analyze the data. An overview of the process can be seen in Figure 3.1. The process starts from the creation of the CAD for the manufactured components, followed by the design and CAD creation of the support structures. The components and the support structures are then merged, and imported into the additive manufacturing software to simulate the results of manufacture. The results from the manufacturing simulation are then analyzed by means of graphs and statistical methods.

The subsequent sections explain in detail each stage of this process.

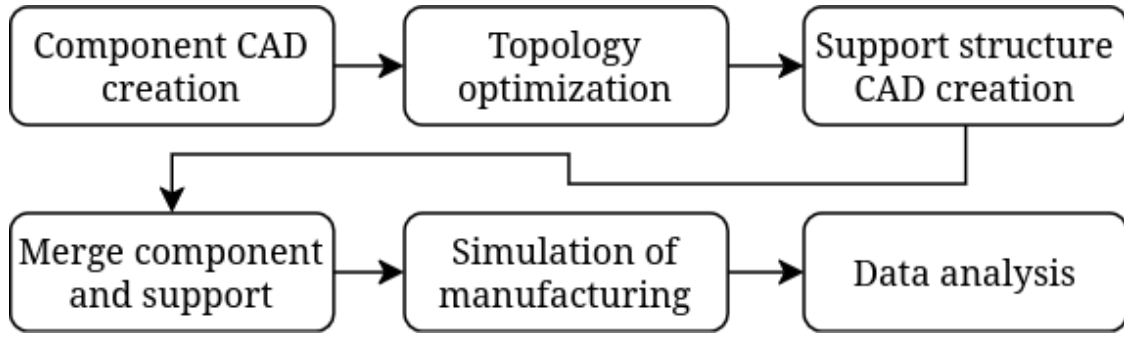


Figure 3.1: Process diagram.

3.2 Component CAD creation

3.2.1 Simple geometry

The components with simple geometries utilized in this study consist of a cube, three triangular components with different slopes, and three cylindrical components with different values of curvature. To reiterate, these components have the same dimensions that were used in the study of lattice support structure performance by Peishu **peishu_thesis**. All the CAD models used for the simple geometry study were created using FreeCAD, an open-source CAD software. All the components were exported as .STEP files, and then they were merged with their corresponding support structures using the software nTop.

- A cube with side length of 30 mm (Figure 3.2).
- Three triangular components with varying slopes. All triangular components have a base of $30 \times 30 \text{ mm}^2$, with slopes of 15° , 30° and 45° . The measurements are shown in figure.

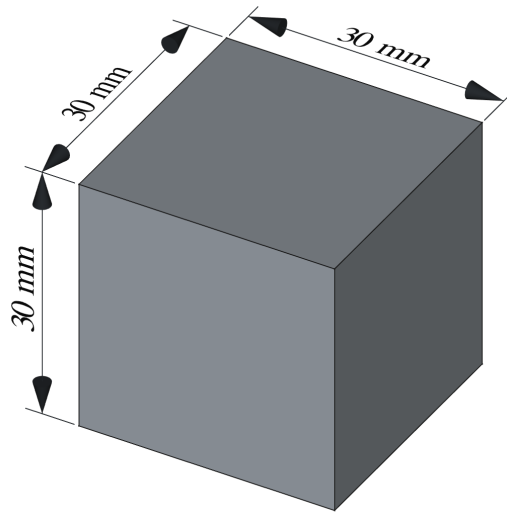
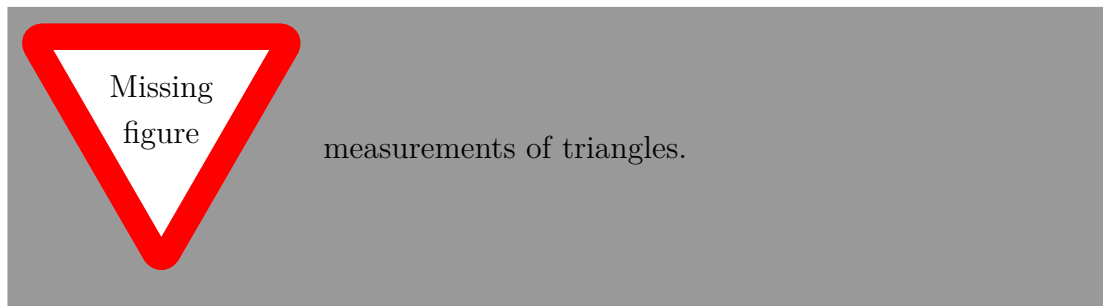


Figure 3.2: Dimensions of cube component.

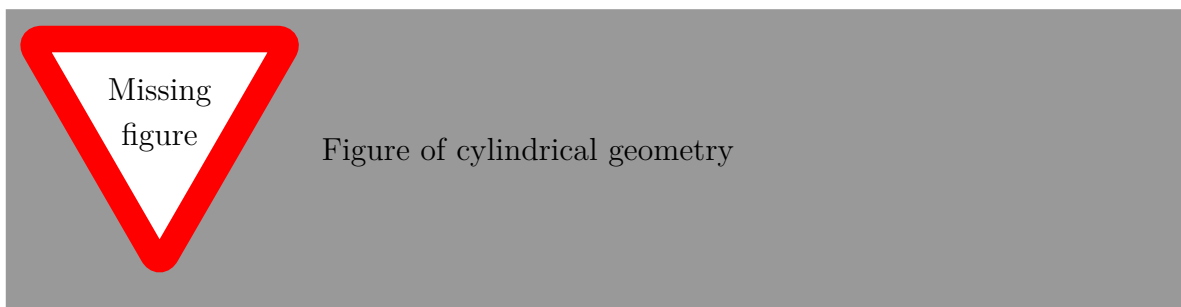


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3.2.2 Femoral component

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3.3 Support structure creation

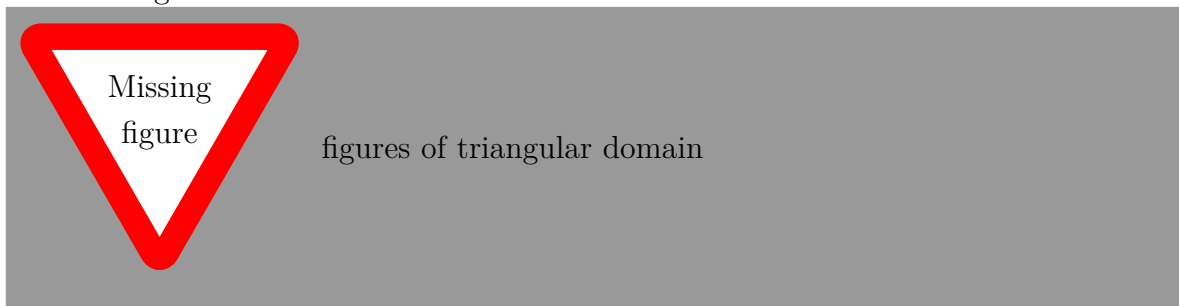
The supporting structures of the components were created using the method of topology optimization. The design was implemented using the topology optimization module of COMSOL Multiphysics 6.2.

The steps for creating a valid topology optimization model are: determine the design volume, choose a suitable mathematical model for the behavior of the material within the design volume, choose the objective functions and decide what constraints need to the material within the volume be applied to the design volume. The following section explains all of these steps in detail.

3.3.1 Design domain

Simple geometry design domain

The design domain for the simple geometry components consists of the volume between the bottom face of each component and the base plate. For the cube, the design domain ends up being a rectangular prism with dimensions 10 cm x 30 cm x 30 cm. For the triangles, the design domain consists of different triangular spaces with a square base, shown in figures



Since all of these components can be generated by the drawing their profile on a plane and extruding it in a direction perpendicular to the plane, the design domain utilized in the topology optimization model is also a 2D model. This grants the benefit

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of faster computation, and thus the supporting structure is created by first finding the planar topology and also extruding it in a perpendicular direction.

Femoral component design domain

3.3.2 Mathematical model

The mathematical models consists of the governing equations used to describe the physics of the system and the constraints imposed on the model, as well as the objective function that needs to be minimized.

For this system, the most appropriate objective function would be the maximization of thermal conductivity; in order to minimize the thermal deformation of the manufactured component, it is imperative to transfer heat away from it as fast as possible. The physical equation describing the thermal conduction of a material is given by Fourier's Law, which can be expressed as:

$$q = -k\nabla T \quad (3.1)$$

where q is the heat conduction through the material, k is the material's thermal conductivity and ∇T is the thermal gradient.

For this study, the only constraint applied on the system was the volume fraction. Volume fraction specifies the maximum amount of volume that the designed topology should take from the design domain. The volume fractions utilized were 50% and 75%.

3.3.3 Objective functions

An additional objective function to be considered would be the deformation of the supporting structure as well. If the supporting structure is composed of very thin segments, there is a high possibility that the structure might buckle, which would add significant geometric errors to the component. In the worst case scenario, the structure itself would collapse, causing the entire manufacturing process to fail. In order to

Write down equation of thermal conductivity for topology optimization domain here.

avoid this, it is necessary to limit the amount of deformation that is allowed in the support structure. This can be achieved by utilizing an additional objective function of structural compliance minimization. We define structural compliance as the amount of energy stored in the deformation of material. Minimizing this stored energy is thus equivalent to minimizing the deformation. Therefore, the second objective function of minimization of structural compliance is also used in the topology optimization model.as heat passes into the support structure from the bottom of the manufactured component,

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The next step is applying certain constraints on the system. For this study, we are mostly concerned with the principal constraint is the volume fraction, which is the maximum amount of volume that the topology can cover within the design domain. This criteria is chosen because we seek to use less material for the supporting structure, as long as we can maintain the total deformation of the manufacturing component beneath a threshold. For the model in COMSOL, the values of volume fraction = 0.5 and volume fraction = 0.75 were used.

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3.3.4 COMSOL implementation

Mesh

Physical parameters

Simulation itself

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As discussed in the previous chapter, Helmholtz filters are frequently used in topology optimization studies to avoid chattering designs.

Talk about
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3.4 Component and support structure merging

3.4.1 CAD for support structure

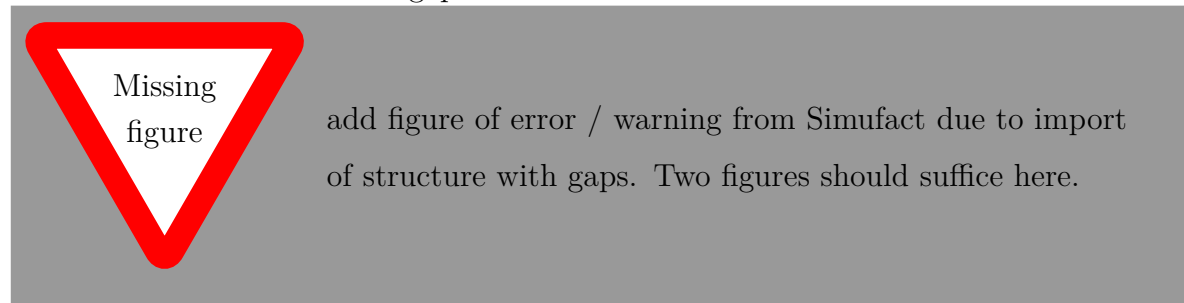
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This section talks about how FreeCAD was used to create the support structure once the pictures of the topologies have been obtained.

3.4.2 Merging of part with support structure

Once the CAD file of the component and the support structure has been built, it is necessary to merge them together and import them into Simufact to undergo simulation of the manufacturing process. The software used for blending the component and its support structure is nTop . nTop's interface makes it very easy to merge the part, and also allows to blend the support structure and the component, which effectively creates a fillet between the nodes of both components to allow for a smooth transition between bodies. Of course, blending the component and the support structure in this manner would not give any benefit in a real manufacturing process, as the structure and the component would not be able to be separated easily. NEvertheless, this blend radius is beneficial for the simulation since it was observed that a direct union and import of the support structure + component in Simufact resulted in having very small gaps between the two pieces, resulting in a non manifold geometry that would cause the finite element model to have gaps between some of its nodes.

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3.5 Simulation of manufacturing process

The software utilized to simulate the manufacturing process is Simufact Additive version 2023.2. Simufact Additive is capable of simulation building process of additive manufacturing components, and coupling thermal and stress physics to predict the temperature values of the component throughout the building process and the total stresses, strains and deformations resulting from the manufacturing process.

In order to set up Simufact correctly, the building process and the building space

geometry must be specified before each simulation. The building parameters and building geometries used in this study are the same that were used in the analysis of thermal deformation using lattice support structures done by Peihsy and al.

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3.5.1 Simufact simulation parameters and voxelization

After the component and the support structures were merged, they were imported into Simufact. It is during this step that all the factors related to the simulation are set, which include the machine properties, material properties, and build parameters. As mentioned previously, these were chosen to be identical to the study of PeiHsu to ensure that the results of this study could be compared to the results of that one.

add that thermo-mechanical process was used and explain what it is.

The first parameter to be chosen is the process properties, which determines the physics that Simufact takes into consideration to run the simulation. Simufact provides three different types of processes: mechanical, thermal, and thermomechanical. As stated in the Simufact manual, mechanical provides a fast mechanical analysis that only uses inherent strains as the main input. This type of analysis does not take into consideration the temperature fields during the building process. The thermal process on the other hand only considers the thermal behaviour of the components, and the temperature field of the support structures, components and base can be analyzed. The thermomechanical process couples the stress and thermal analyses, and allows for the prediction of temperature, distortions and stresses of the part. This latter process is the one used in this study.

insert reference to manual here

After choosing the process property, the machine parameters must be specified. This includes the machine build plate geometry and the laser parameters. The machine build plate chosen was a circular plate with an 80 mm radius. The build space dimensions consists of a space of 160 mm in all three x-y-z directions. As for the laser parameters, the simulations were carried out with one laser with a maximum laser power of 500 W and a maximum laser speed of 2000 mm / s, an efficiency of 25 percent, and a beam width of 25 mm. All of these parameters are summarized in the table

add the table of building parameters here.

The building parameters for the process need also to be set. These include material layer parameters and any thermal parameters and temperature specifications for the build environment and base plate. The powder layer thickness was chosen to be 0.03 mm, with a recoater time of 10 s. The powder initial temperature was set to 25 °Celcius, with an initial base temperature of 200 degrees.

3.5.2 Convergence analysis

To make sure that the results of the simulation would not depend on the voxel density of the

3.6 Data collection and organization

This section will detail how I collected the data from Simufact and what software and methods I used to organize it an analyze it. The actual results will go in the following chapter, aptly named results, duh

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Merge them together to get the final objective function. Explain about the weights.	13
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