#### 145071 - Real time operating systems and middle ware $\,$

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## Chapter 1

## Introduction to the course

### 1.1 Overview of the course

| Real time systems About the topic we will cover:  | Real time<br>- systems            |
|---|-----------------------------------|
| About the topic we will cover:  |                                   |
| - Real-Time Computing and temporal constraints  | Real-Time<br>Computing            |
| Real time systems are software and hardware systems (hence computing systems), that have to comply with temporal constraints.   |                                   |
| <ul> <li>Definitions and task model</li> </ul>  | temporal<br>constraints           |
| We will make things much clearer and better defined by introducing a sequence of definitions and mathematical models that will allow us to given this notion of temporal constraint a well founded meaning .  |                                   |
| - Real Time Scheduling  | Real Time                         |
| We will also study solutions that allow us to enforce these real time constraints and this solution will have much to do on how we schedule shared resources.   | - Scheduling                      |
| POSIX API   | POSIX API                         |
| We will move to a concrete ground and see what is the exact shape that these notions take once they are moved in a computer program.  |                                   |
| Real-Time Scheduling  | Real-Time<br>Scheduling           |
| As regards the Real-Time scheduling we will see many interesting policies, but since this is<br>not a course on Real Time scheduling what we will do is provide the knowledge of real time<br>scheduling so that the reader will be able to understand the mechanism of real time operating | beneduring                        |
| systems and thereby make best use of these technologies in future projects.   |                                   |
| Operating System structure  | Operating Sys                     |
| Since it will be important to keep in check the latencies, we will cover:   | structure                         |
| <ul> <li>Notes about traditional kernel structures.</li> </ul>  |                                   |
| In order to keep latencies in check, we need proper technological solutions that make our   |                                   |
| operating systems differ quite a bit from standard operating systems.  — Sources of kernel latencies.   |                                   |
| <ul> <li>Some approaches to real-time kernels (e.g dual kernel approach, interrupt pipes, micro-<br/>kernels, monolithic kernels and RT).</li> </ul>  |                                   |
| Real-Time Kernels and OSs   | Real-Time                         |
| Developing Real-Time applications   | - Kernels and OS                  |
| 2 Real-Time Systems   | Developing Real-Time applications |

In order to discuss about the Real-Time systems we need to provide some basic definitions:

- Real-Time Operating Systems (RTOS) is an operating system providing support to Real Time applications
- Real-Time Application the correctness of the application does not only depends on the output values/results that the application produces, but also on the time when such values are delivered

Operating
Systems (RTOS)

 Operating System different perspective: an operating systems can be looked at from many

Real-Time Application
Operating System

interrupt

- 1. Set of computer programs, of critical programs to be precise: because they have to be written efficiently, otherwise the hardware resources get disrupted, hence the system cannot operate correctly.
- 2. Interface between applications and hardware.

Whenever an application interacts with an hardware, it is not of the developer interest to directly control the hardware. The Operating System provides an API that enables you to open a connection to a peripheral and takes care of all the low level interactions. On this regard, understanding the notion of interrupt will be of fundamental importance, because it is, essentially, what gave rise to concurrent programming: in the case we would like to interact with a peripheral, rather than continuously check if the peripheral has ended what it is supposed to do, you can tell the peripheral to communicate when it has completed the given task.

1 • 1

Anyway the Operating systems acts as an interface towards the hardware and hides away all these complex details.

- 3. Control the execution of application programs.
- 4. Manage the hardware and software resources.

#### 1.2.1 Real-Time Operating Systems

Since the OS is something that lies in-between the user application and the hardware resources we can summarize the aforementioned interpretation of the OS, as:

- Service Provider for user programs (i.e. exports a programming interface).

  This concept looks at the OS from the perspective of the software application.
- Resource Manager (i.e. implements schedulers)

  This concept looks at the OS from the perspective of the hardware.

\_ Service Provider

\_ Resource Manager

#### OS as a Service Provider

One way of looking at the OS is as a Service Provider, in the sense that it provides:

- Process Synchronization mechanism
- Inter-Process Communication (IPC)
- Process/Thread Scheduling, i.e. ways to create and schedule tasks
- Input/Output
- Virtual Memory

And all these services are accessible through an API.

#### OS as a Resource Manager

If you think at the Operating System as a Resource Manager, then it is something that takes care of many things:

1. Process Management

Process Management

The fact that multiple applications can run at the same time on a PC, even though there is a small amount of processor available to manage these applications. (generally 2,4 or 8).

The number of application that you are likely to create is often on the hundreds, hence it is necessary to make an appropriate sharing of the limited resources that you have in order for all the applications to live correctly.

2. Memory Management

Memory Management

Supposing one is using a 64-bit architecture, what will happen is that a space of memory is addressable with 64 bit. As a consequence we can imagine that the addressable memory is space has  $2^{64} - 1$  memory locations available.

And each application sees, these much space available for its execution. But however large the space can be in a machine, it will never match the aforementioned size. It could potentially for one task, but in the case a machine is hundreds of tasks and each of them wants to use that much memory, there is no way that the hardware can provide enough physical memory to satisfy all of them.

To counteract this problem, it is common practice to schedule the memory as well, because you take advantage of the fact that an application CAN use  $2^{64} - 1$  memory locations, but at a given time it uses a tiny portion of these locations. It is only that tiny portion of memory locations that needs to be made available to the running task.

In this scenario, the OS makes it possible to accommodate within the physical memory of the computer these small slices of the available space that the application uses. So somehow it operates as a resource manager for the memory as well.

3. File Management

File Management

4. Networking, Device Drivers, Graphical Interface

Networking

The important thing is that all of these resources, like the processor, the memory, the drivers etc..., are shared between all the tasks. All these resource managers have to be distributed among all the spectrum of tasks in such a way that the tasks behave properly, i.e. if you do not provide frequently enough these resources they would not be able to deliver the result on time (the OS manages this problem on its own).

\_ Device Drivers

Graphical

In the case we decide to look at the Operating System as a Resource Manager, we need to think of a structure for the OS that makes this resource management effective, effective in the sense that we believe it is the most relevant for our specific range of application.

The way OSs handles devices, interrupt, etc. can be very different (and optimized in very different ways) depending on the type of application one is looking at. However, the type of optimizations we are interested in are those that allow our application to have time-limited execution.

#### 1.2.2 Real-Time Concepts and Definitions

A Real-Time application is an application of which the time when a result is produced matters. In particular:

Real-Time application

- a correct result produced too late is equivalent to a wrong result, or to no result.
- it is characterized by temporal constraints that have to be respected.

Example: let us consider a mobile vehicle with a software module that

- 1. Detects obstacles
- 2. Computes a new trajectory to avoid them
- 3. Computes the commands for engine, breaks, ...
- 4. Sens the commands

If you decide to steer to the left or to the right there is a limited amount of time in which the operation has to be carried out. Hence if one can find an extremely effective strategy for steering the wheels but the strategy amounts to setting the values for the motors after one second, it is completely useless, since the vehicle is most likely to crash.

Hence a time violation in executing a task is a critical problem: it means that the developed application is useless and also dangerous.

But then, what is a reasonable time frame for completing the steering operation? Depends on the speed in which the vehicle is traveling. But no matters if the vehicle is traveling at high or low speed the timing constraint is there, and if it is violated, the vehicle will eventually crash against the obstacle.

As a consequence: when a constraint is set, that constraint needs to be respected. And this is one of the core concept of Real Time: it is not necessarily synonym of fast execution, but rather of predictable execution.

predictable

Real time computing has much more to do with predictability than of being quick.

Examples of temporal constraints could be:

- must react to external events in a predictable time
- must repeat a given activity at a precise rate
- must end an activity before a specified time

In this case, we can clearly notice that the temporal constraints can be either one shot events or periodic events, but in both cases, a common characteristic, there is a need of being predictability.

Temporal constraints are modeled using the concept of deadline.

\_ deadline

#### Real-Time & Determinism

A Real-Time system is not just a "fast system", because the speed is always relative to a specific environment, i.e. the steering commands temporal constraint is set by the velocity of the vehicle.

Running faster is good, but does not guarantee the correct behavior. In fact, it is far more valuable to that temporal constraints are always respected; in other terms Real time systems prefer to run fast enough to respect the deadlines, to be reliable.

Hence, the type of analysis that is necessary to perform is not an analysis based on of average/typical cases but rather an analysis of worst case: I have to prove that even in the worst-case scenario, there is not deadline violation.

#### Throughput vs Real-Time

This predictability creates a wide gap between what a Real Time system is and what a general purpose system is, because general purpose systems are optimised for the average case, but a real time system only cares about the worst case. As a consequence, the way one designs a Real Time system is very different from the way a general purpose system is designed.

In fact:

• When one optimize for the average case, what one would look at is the number of times that an application completes a task every second, and this is called Throughput.

Throughput

Task

• When one have a worst case requirement, the notion of throughput is not relevant anymore, and the analysis focuses in every single instance the maximum delay will be bounded.

#### Processes, Threads and Tasks

Let us introduce some notion and general terms that we will extensively using during the course

• Algorithm logical procedure used to solve a problem Algorithm Program formal description of an algorithm, using a programming language Process Instance of a program (program in execution) Program Thread flow of execution, something that is able to execute using your processor along with other threads. These threads can be part of the same program and they can be executed Process in parallel. Task process or thread Thread

Hence there are two different ways of sharing resources: one are threads in which your share computing resources and memory space, and processes in which you share computer resources but each of the processes has its own memory space.

Unfortunately, there is no common definition of a task: somebody use the terms with the same meaning as a thread and sometimes it is used with the same meaning as a process. In this class we will refer to threads.

Henceforth, when we talk about a task we will refer to a program that it is running and they share the same memory space with other programs.

#### 1.3 Real-Time Tasks

A task can be seen as a sequence of actions and a deadline must be associated to each one of them.

We, therefore, are after is a definition of a formal model that identifies what these tasks or actions are and associate deadlines with them.

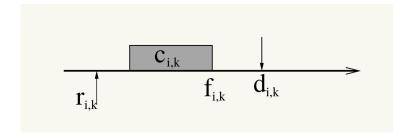
#### 1.3.1 Mathematical Model of a Task

We can define a Real-Time task  $\underline{\tau_i}$  as a stream of jobs or instances  $J_{i,k}$ . In other terms, it is a Real-Time task sequence of activity that is activated periodically or aperiodically.

Each job  $J_{i,k} = (r_{i,k}, c_{i,k}, d_{i,k})$ , that is characterized by a tuple of four values:

- $r_{i,k}$  instant when the job is activated
- $c_{i,k}$  computation time
- $d_{i,k}$  absolute deadline
- $f_{i,k}$  finishing time (of the computation), which has to be smaller than the absolute deadline

Furthermore, since each task i is a sequence of jobs, we need to differentiate between them. That is why each job  $J_{i,k}$  is uniquely identified by its task index i and the k-th activation of the i-th task.



Formally: A job is an abstraction used to associate deadlines (temporal constraints) to activities

- $r_{i,k}$  time when job  $J_{i,k}$  is activated (by an external event, a timer, an explicit activation, etc..)
- $c_{i,k}$  computation time needed by job  $J_{i,k}$  to complete
- $d_{i,k}$  absolute time instant by which job  $J_{i,k}$  must complete job  $J_{i,k}$  respects its deadline if  $f_{i,k} \leq d_{i,k}$
- Response time of job  $J_{i,k}$

 $ho_{i.k} = f_{i.k} - r_{i.k}$ 

#### 1.3.2 Periodic Tasks

A Periodic Task is uniquely represented by a tuple of three values in the form:

\_ Periodic Task

computation time

absolute deadline

$$\tau_i = (C_i, D_i, T_i)$$

and it represents a stream of jobs  $J_{i,k}$  with:

$$r_{i,k+1} = r_{i,k} + T_i$$
 
$$d_{i,k} = r_{i,k} + D_i$$
 
$$C_i = \max_k \{c_{i,k}\}$$

where

•  $T_i$  is the task period

- $D_i$  is the task relative deadline
- $C_i$  is the task worst-case execution time (WCET)
- $R_i$  is the task worst-case response time (WCRT) defined as

sponse time (WCRT) defined as 
$$R_i = \max_k \{\rho_{i,k}\} = \max_k \{f_{i,k} - r_{i,k}\}$$
 worst-case response time (WCRT) (WCRT)

For the task to be correctly scheduled, it must hold that:

$$R_i \leq D_i$$

A periodic task has a regular structure (called cycle), in the sense that:

\_ cycle

scheduling

relațive

- 1. it is activated periodically with a period of  $T_i$
- 2. it executes a computation
- 3. when the computation terminates, it suspends waiting for the next period

Hence, its fundamental implementation can be represented as:

Tasks are graphically represented by using a scheduling diagram. For instance, figure 1.1 shows a schedule of a periodic task

$$\tau_1 = (3, 6, 8)$$
 $WCET_1 = 3 \quad D_1 = 6 \quad T_1 = 8$ 

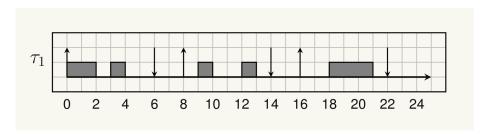


Figure 1.1: Periodic task graphical representation

Notice that, while job  $J_{1,1}$  and  $J_{1,3}$  execute for 3 units of time (WCET), job  $J_{1,2}$  executes for only 2 units of time.

#### 1.3.3 Aperiodic Tasks

Aperiodic Tasks are not characterised by periodic arrivals, meaning that

Aperiodic Task

- A minimum interarrival time between activations does not exist
- Sometimes, aperiodic tasks do not have a particular structure

However, aperiodic tasks are of fundamental importance given that they model:

- 1. Tasks responding to events that occur rarely (e.g. a mode change)
- 2. Tasks responding to events with irregular structure (e.g. bursts of packets from the network, ...)

A possible pattern for an aperiodic task  $\tau_1$  is proposed in figure 1.2

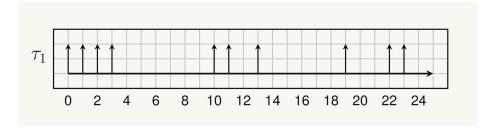


Figure 1.2: Aperiodic task graphical representation

notive that arrivals might be bursty, and there is not a minimum time between them

### 1.3.4 Sporadic Tasks

Sporadic Tasks are aperiodic tasks characterised by a minimum interarrival time between jobs. In \_ sporadic Task this sense, they are similar to periodic tasks, but while a periodic task is activated by a periodic timer, a sporadic task is activated by an external event (e.g. the arrival of a packet from the network) \_ external event Hence, its fundamental implementation can be represented as:

Given its similarity with periodic task, a sporadic task can be represented by a tuple of three values

$$\tau_i = (C_i, D_i, T_i)$$

and it represents a stream of jobs  $J_{i,k}$  with

$$r_{i,k+1} \ge r_{i,k} + T_i$$
$$d_{i,k} = r_{i,k} + D_i$$
$$C_i = \max_k \{c_{i,k}\}$$

where

- $T_i$  is the task minimum interarrival time (MIT)
- $D_i$  is the task relative deadline
- $C_i$  is the task worst-case execution time (WCET)

The task is correctly scheduled if  $R_i \leq D_i$ 

Since sporadic task can be represented by a mathematical model, they can be graphically displayed. Hence, figure 1.3 shows a possible schedule of a sporadic task  $\tau_1 = (2, 5, 9)$ .

minimumm interarrival time (MIT)

Notice that

$$r_{1,2} = 12 > r_{1,1} + T_1 = 9$$
  
 $r_{1,3} = 21 = r_{1,2} + T_1 = 21$ 

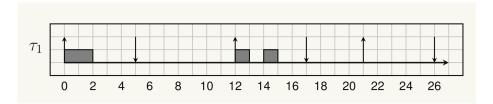


Figure 1.3: Sporadic task graphical representation

### 1.4 Task Criticality

A deadline is said to be hard if deadline miss causes a critical failure in the system, whereas a task hard is said to be a hard real-time task if all its deadlines are hard, which means that all the deadlines must be guaranteed before starting the task, i.e.

hard real-time

$$\forall j, \rho_{i,j} \leq D_i \quad \Rightarrow \quad R_i \leq D_i$$

Example:

The controller of a mobile robot, must detect obstacles and react within a time dependent on the robot speed, otherwise the robot will crash into the obstacles

A deadline is said to be soft if a deadlien miss causes a degradation in the Quality of Service (QoS), but is not a catastrophuc event, whereas a task is said to be a soft real-time task if it has soft deadlines.

In other terms, some deadlines can be missed without compromising the correctess of the system, but the number of missed deadlines must be kept under control, because the *quality* of the results depend on the number of missed deadlines.

Quality of Service (QoS)

soft real-time task

Unline the hard real-time task, soft real-time tasks can be difficult to characterize, particularly:

- What's the tradeoff between "non compromising the system correctness" and not considering missed deadlines?
- Moreover, some way to express the QoS experienced by a soft real-time task is needed

Exmplaes of QoS definitions could be

- no more than X consecutive deadlines can be missed
- no more that X deadlines in an interval of time T can be missed
- the deadline miss probability must be less than a specified value, i.e.

deadline miss

$$P\{f_{i,j} > d_{i,j}\} \leq R_{max}$$

• the deadline miss ratio must be less than a specified value, i.e.

deadline miss

$$\frac{\text{number of missed deadlines}}{\text{total number of deadlines}} \le R_{max}$$

• the maximum tardiness must be less than a specified value, i.e.

\_ tardiness

$$\frac{R_i}{D_i} < L$$

• ...

A common example of implementation of a soft real-time task are the Audio and Video players. Particularly, assuming a framerate of 25 fps, which imply a frame period of 40 ms, if a frame is played a little bit too late, the user might even be unable to notice any degration in the QoS, however, skipped frames can be disturbing.

skipped frames

In fact missing a lot of frames by 5 ms can be better than missing only a few frames by 40 ms. Another common example, can be found in some robotic systems where some actuations can be delayed with little consequences on the control quality.

In any case, soft real-time constraints does not mean no guarantee on dealines, given that tasks can have variable execution times between different jobs.

These execution times might depend on different factors:

- Input data
- HW issues (cache effects, pipeline stalls,  $\ldots)$
- The internal state of the task
- ...

## Chapter 2

## Real-Time scheduling and analysis

In the previous chapter the reader was introduced to some fundamental notions and concepts that will be extensively used throughout the course.

Since, in this chapter, we will focus on the scheduling of real-time tasks and the analysis of each scheduling policy, it is worth mentioning some basic definitions:

- Algorithm logical procedure used to solve a problem
- Program formal description of an algorithm, using a programming language
- Process Instance of a program (program in execution)
  - Program: static entity

- Process: dynamic entity

\_ static entity

\_\_ dynamic entity

- the term task is used to indicate a schedulable entity (either a process or a thread), in particular;
  - A thread reresents a flow of execution (it executes with shared resources, multi thread within the same process)
  - A process represents a flow of execution + private resources (it executes with its own resources), such as address space, file table, etc...

Tasks do not run on bare hardware, but then how can multiple tasks execute on one single CPU? The OS kernel is a piece of the operating system that takes care of multi-programming and somehow it is able to create the illusion that each CPU/processor has its own space, whereas in fact it is sharing the same resources with other processes.

In the end the kernel provides the mechanism that enable multiple tasks to execute in parallel; in a sense tasks have the illusion of executing concurrently on a dedicated CPU per task.

On this regard, with the term concurrency we refer to the simultaneous execution of multiple threads/processes in the same PC. Concurrency is implemented by multiplexing tasks on the same CPU. Tasks are alternated on a real CPU and the task scheduler decides which task executes at a given instant in time. In other terms, in order to implement the concurrency mechanism it is necessary to introduce this new component (i.e. the task scheduler), since it makes sure that the time of your pc is shared between the different processes or tasks that compete for the resources at that time.

Concurrency

 ${\tt multiplexing}$ 

task scheduler

Tasks are associated to temporal constraints (aka deadlines), hence the scheduler must allocate the CPU to tasks so that their deadlines are respected.

## 2.1 Real-Time Scheduling

One of the key components of an OS and a OS kernel is the scheduler. A scheduler generates a schedule from a set of tasks. From a mathematical perspective it is a function:

first consider UP systems (given that it has a simpler definition). A schedule  $\sigma(t)$  is a function mapping (every point in) time t into an executing task

$$\sigma: t \to \mathcal{T} \cup \tau_{idle}$$

where  $\mathcal{T}$  os the taskset and  $\tau_{idle}$  is the idle task.

taskset

At the end of the day what the processor/OS kernel does is to implement a function like the scheduler. Clearly, if you have more that one processor, the function under consideration is multivalued, because at every point in time it decide the allocation of the different processors to different tasks.

idle task

For an SMP system (i.e. m CPUs),  $\sigma(t)$  can be extended to map t in vectors  $\tau \in (\mathcal{T} \cup \tau_{idle})^m$ .

Therefore, formally, the scheduler implements  $\sigma(t)$  and as a consequence the scheduler is responsible for selecting the task to execute at time t. However, a function is a denotational way of describing an algorithm, i.e. we specify mathematically what the function is. The operational way of describing an algorithm on the other hand, is a squence of steps to be taken in order to implement perational

the function. Hence, from an operational/algorithmic point of view:

• Scheduling algorithm is an algorithm used to select for each time instant t a task to be executed on a CPU among the ready task

Scheduling algorithm

• Given a task set  $\mathcal{T}$ , a scheduling algorithm  $\mathcal{A}$  generates the schedule  $\sigma_{\mathcal{A}}(t)$ 

In the frame of reference of real-time systems we are interested in finding conditions on the scheduling choice that allow us to meet all the deadlines. Hence, a task set is schedulable by an algorithm  $\mathcal{A}$ , if by applying that scheduling algorithm,  $\sigma_A$  does not contain missed deadlines.

On this note, the Schedulability test is a way to decide if, given a task set  $\mathcal{T}$  and an algorithm  $\mathcal{A}$ , that task set is going to be schedulable.

Schedulability t.est.

The key point here is that given the fact that by making a proper choice of scheduling algorithm, one can obtain a schedulable task set, how can I find a way to schedule the activities so that all the deadlines are met? And this is the problem of real time scheduling.

#### 2.2 Cyclic Executive Scheduling

One way to find a solution of the schedulability problem is to consider a task set of fixed tasks (i.e. you will always have those tasks activated in the very same way). This is the case of legacy applications and where reliability is fundamental, e.g. military and avionics systems (Air traffic control, Space Shuttle, Boeing 777).

In this scenario, one can obtain a paper and pencil solution, i.e. trying to decide how to schedule things in such a way that the task set is schedulable and stick to that solution.

Of course, there is a more systematic way of solving such problem and such process is called Cyclic Executive Scheduling.

The Cyclic Executive Scheduling (aka timelice scheduling or cyclic scheduling) is a scheduling algorithm with very low overhead (in the sense that scheduling decisions are taken-offline) and with a very simple and well tested idea, that was originally used in avionic systems to schedule periodic tasks.

Cyclic Executive

timelice scheduling

cvclic scheduling

#### 2.2.1The Idea

Cyclic Executive Scheduling is a

• static scheduling algorithm (i.e. it is decided offline and it is always applied repeatedly in the same way).

static scheduling algorithm

- As a consequence, all scheduling decision are taken upfront; it is not something that defers decision to the runtime.
- Jobs are assumed not to be preemptable, i.e. a scheduled job executes until termination, it retains control of the CPU for the entire computation time.

The procedure to follow in order to apply this scheduling algorithm is:

1. Split the time axis into slots, each of which is statically allocated to the tasks (scheduling table).

scheduling table

2. Since we are dealing with periodic tasks, the same sequence of tasks will repeat periodically, hence once the scheduling table is completed, it starts from the beginning and repeats the same sequence over and over again.

A periodic timer activates execution (allocation of a slot).

In summary three components are needed to uniquely define this scheduling algorithm:

1. Major Cycle: least common multiple (lcm) of all the tasks' periods (aka hyperperiod)

2. Minor Cycle: greatest common divison (gcd) of all the tasks' periods (i.e. granularity at which the scheduling decisions will repeat)

3. A timer: that fires every Minor Cycle (Δ)

Minor Cycle

#### 2.2.2 Implementation

As previously mentioned from an execution stand point this scheduling algorithm is extremely simple because every minor cycle:

- 1. A periodic timer is initialized
- 2. Every time the timer fires the scheduler read the scheduling table
- 3. The scheduler calls the approriate function(s) associated to the corresponding task(s)
- 4. The scheduler returns to sleep until the next minor cycle (i.e. the timer fires)

#### 2.2.3 Advantages

From it's radical simplicity, allows to create a total check and a certification of the code that can be shipped to a space shuttle or any other avionic application and execute theoretically forever.

An OS is much more difficult to certify, because asynchronious events can be generated and it is impossible to consider upfront every possible course of events that can happen. On the contrary, once you solved the cyclic executing scheduling problem, you have a global solution that is guaranteed to be replicated.

Furthermore, with its simple implementation there is no need for a real-time operating system, no real task exist (just function calls), and only one single stack is necessary for all the *tasks* (since each task runs until completion, there will never be a situation in which the execution of the function are active at the same time).

Since it involves a non-preemptable scheduling there is no critical races (i.e. there will never be two tasks that shared a variable) and therefore there is no need to protect data (i.e. no need for semaphores, pipes, mutexes, mailboxes, etc...).

non-preemptable scheduling

A timer

Lastly, since there is no need for an OS, all the CPU/execution time is taken by the tasks, hence It has low run-time overhead, and the jitter (delay in which the result is completed, i.e. difference \_\_jitter between when a task start and it finishes ) can be explicitly controlled.

#### 2.2.4 Drawbacks

However, this scheduling algorithm has some important drawbacks:

- it is not robust w.r.t. overloads. This type of solution assumes that all the tasks do not give back control until they are finished (until its allocated slots terminates). However, if the task execute for much more of its WCET or the allocated time slot, what happens is that the schedule gets disrupted with a subsequent domino effect.
- It is difficult to expand the schedule (static schedule): introducing a new task requires that the whole system/schedule must be redesigned.
- Not easy to handle aperiodic/sporadic tasks
- All task periods must be a multiple of the minor cycle time
- Difficult to incorporate processes with long periods (big tables)
- Variable computation time imply that it might be necessary to split tasks into a fixed number
  of fixed size procedures. This requires to actively change the code into smaller parts, and, in
  the case of third party library/code, this is not always allowed.

### 2.3 Fixed Priority Scheduling

In general, we want to have more flexibility than the cyclic executive scheduling (in fact it is applied on very niche and particular situations).

What can be done therefore is utilize preemptive scheduling algorithms, i.e. algorithm that are allowed to suspend the execution of a task and resume it later.

Therefore a task has, two states: ready and executing, and the OS is allowed to enforce a state change.

In the class of preemptive algorithm we will look at different algorithms, but the simplest is the Fixed Priority Scheduling. In this scheduling algorithm:

• Every task  $\tau_i$ , when it is created, is assigned a integer number that is encoding a fixed priority  $p_i$ 

• The active task with the highest priority is scheduled (POSIX convention)

Priorities are integer numbers: the higher the number, the higher the priority (In the research literature, sometimes authors use the opposite convention).

And so, the work of the scheduler at any point in time is to look at the ready queue and pick up the task with the highest priority.

Observations:

- The response time of the task with the highest priority depends only on its computation time, and therefore it is always less than or equal to its WCET.
- The response time of tasks with lower priority depends on its own computation time and the interference of higher priority tasks.

• Clearly, the scheduling priorities can change how the things behave: are we sure that choosing a different priority assignment policy we would obtain a schedulable task set? Yes. So, we have two issues: the scheduling analysis (given a task set and the priorities, is everything schedulable) and syntesis problem (given the task set and computation time, find the assignment of priorities that guarantees schedulability).

interference

preemptive scheduling

algorithm

executing

Fixed Priority Scheduling

ready

scheduling analysis

\_ syntesis problem

#### 2.3.1 Priority Assignment

Given a task set how to assign priorities?

Depends on the different objectives that you would like to achieve:

• Schedulability, i.e. find the priority assignment that makes all tasks schedulable

• Response time (optimization), i.e. find the priority assignment that minimise the response time of a subset of tasks

By now we consider the first objective only, hence we will investigate the optimal priority assignment (Opt).

Schedulability

Response time (optimization)

optimal priority assignment (Opt)

### 2.3.2 Optimal Priority Assignment

A priority assignment is an optimal priority assignment (Opt):

- If the task set is schedulable with another priority assignment, then it is schedulable with priority assignment Opt (i.e. the optimal priority assignment cannot do any worse that any other feasible assignment).
- If the task set is not schedulable with Opt, then it is not schedulable by any other assignment. (If the optimal choice fails then there is no other possible solutions/other alternatives will fail as well).

Formally, given a periodic task set  $\mathcal{T}$  (all tasks are activated periodically) with all tasks having relative deadline  $D_i$  equal to the period  $T_i$  ( $\forall i, D_i = T_i$ ), and with all offsets equal to 0 ( $\forall i, r_{i,0} = 0$ ):

• The best assignment, in this case, is the Rate Monotonic (RM) assignment

Rate Monotonic (RM)

• Shorter period imply higher priority (hence the name rate monotonic: the priority is monotonic w.r.t. the rate. The higher the frequency/rate, the shorter the period, the higher the priority).

Given a periodic task set with deadline different from periods and with all offsets equal to 0  $(\forall i, r_{i,0} = 0)$ 

• The best assignment is the Deadline Monotonic (DM) assignment

Deadline Monotonic (DM)

• Shorter relative deadline imply higher priority (the shorter the relative deadline, the higher the priority).

Note that the Deadline Monotonic assignment refers to relative deadline (interval in time, fixed value) not absolute deadline (instant in time, change over time).

For sporadic tasks, the same rules are valid as for periodic tasks with offsets equal to and by considering maximum activation rate.

## Chapter 3

## Real-Time Analysis

The best possible assignment for fixed prority is the Rate Monotonic and Deadline Monotonic assignment.

But how can I be sure that even using the optimal priority assignment, the schedule will meet all the deadlines?

This is the problem of Real-Time analysis: an activity that given a task set and a scheduling policy, it responds to the query on whether or not the task set is schedulable.

What we have done so far is something very naive, i.e. simulating the schedule and verify that there is no deadline miss, but does not scale well in the case of long observation time of the system, until the schedule repeats itself.

Luckly there is a point/tme horizon, such that if no deadline miss happens withing this time horizon, you can certify none will occur ever after. How long is this time horizon depends:

• If there is no offset, we will find ourselves in the critical case, because if we have simultaneous activation of all the tasks in the task set, the system have an immediate request of computation time that needs to be satisfied.

In this case, it is sufficient to simulate the schedule until the hyperperiod, i.e. the least common \_ hyperperiod multiple of the periods

 $H = lcm\{T_i\}$ 

This is because after the least common multiple of the period the task activation will be repeated.

• If there is an offset, the schedule diagram can be shifted to meet the condition of the previous point.

Anyhow, given an offset

$$\phi_i = r_{i,}$$

it is necessary to simulate the schedule for at least

$$2H + \phi_{max}$$

before being sure that the schedule will repeat itself.

• If tasks periods are prime numbers the hyperperiod can be very large

Fortunately, there are better tests, that we can find, called Utilisation Bound Analysis.

Utilisation
Bound Analysis

### 3.1 Utilisation-Based Analysis

The Utilisation is the fraction of the processor that a task needs for its execution.

Hence, if a task have an activation time of 5 and a computation time of 1, then its utilization is around 1/5/=20%.

So it would be nice to have a test that tells if a scheduling algorithm produces as feasible schedule simply based, on the utilisation bound.

A sufficient test is based on the Utilisation bound:

The utilisation least upper bound for scheduling algorithm  $\mathcal{A}$  is the smallest possible utilisation

Utilisation bound

utilisation least upper bound  $U_{lub}$  such that, for any task set  $\mathcal{T}$ , if the task set's utilisation U is not greater than  $U_{lub}$  ( $U \leq U_{lub}$ ), then the task set is schedulable by algorithm  $\mathcal{A}$ 

Formally:

• Each task uses the processor for a fraction of time, called worst case utilisation

$$U_i = \frac{C_i}{T_i}$$

• The total processor utilisation is

$$U = \sum_{i} \frac{C_i}{T_i}$$

and it is a measure of the processor's load

One thing that we can immediately say is that if the total utilisation exceed 100 % there is no way that the task set is schedulable, i.e.

$$U > 1$$
 taskset is not schedulable

If the sum of all the utilisation is greater than 1/100% we encounter a situation called stochastic instability, meaning that in the average you are asking the system more than it can offer.

In this case we have something similar since we have the worst case utilisation. In case the system requires more that it is available, there is no way the taskset is schedulable. Hence, this analysis yield a worst case guarantee.

Moreover, if  $U < U_{lub}$  the taskset is schedulable. However:

- there is a "gray area" between  $U_{lub}$  and 1, it depends on the specific performance of the algorithm under consideration.
  - So the ability of an algorithm to achieve a good utilisation bound is a measure of how efficient and effective the algorithm is.
- We would like to have  $U_{lub}$  close to 1.
- $U_{lub} = 1$  would be optimal (hard upper bound, after 1 we are sure that the task set is not schedulable), it is physical impossible to outperform 100% utilisation.

#### 3.1.1 Utilisation Bound for RM

The notion of least upper bound can be described considering 2 tasks: one can make many choices of periods and computation times of the two tasks, taking the minimum value of U below which all the task sets that I can choose are schedulable.

It can happen that some task that exceed the  $U_{lub}$  is schedulable, but following the notion of least upper bound, as far as the task set is below this value the task set is guaranteed to be schedulable.

Formally: consider n periodic (or sporadic) tasks with relative deadline equal to periods. The least upper bound for Rate Monotonic assignment can be obtained via the following expression:

$$U_{lub} = n(2^{1/n} - 1)$$

Note:

- $U_{lub}$  is a decreasing function of n
- For large n:  $U_{lub} \approx 0.69$

| n | $\mathbf{U}_{lub}$ | n  | $\mathbf{U}_{lub}$ |
|---|--------------------|----|--------------------|
| 2 | 0.828              | 7  | 0.728              |
| 3 | 0.779              | 8  | 0.724              |
| 4 | 0.756              | 9  | 0.720              |
| 5 | 0.743              | 10 | 0.717              |
| 6 | 0.734              | 11 |                    |

Therefore the schedulability test consists in:

1. Computing the utilisation

$$U = \sum_{i=1}^{n} \frac{C_i}{T_i}$$

- 2. If  $U \leq U_{lub}$ , the task set is schedulable
- 3. If U > 1 the task set is not schedulable
- 4. If  $U_{lub} < U \le 1$ , the task set may or may not be schedulable

#### 3.1.2 Utilisation Bound for DM

In case of deadlines different from periods, one can always consider a conservative approximation in which the period is equal to the deadline.

If relative deadlines are less than or equal to periods, instead of considering

$$U = \sum_{i=1}^{n} \frac{C_i}{T_i}$$

we can consider:

$$U' = \sum_{i=1}^{n} \frac{C_i}{D_i}$$

Then the test is the same as the one for RM, except that we must use U', instead of U.

And consequently:

$$\tau = (C, D, T) \rightarrow \tau' = (C, D, D)$$

If I can guarantee that this conservative approximation is schedulable than I can guarantee that the original task set is schedulable as well.

In summary:

- $\tau'$  is a worst case for  $\tau$
- If  $\tau'$  can be guaranteed,  $\tau$  can be guaranteed too

#### 3.1.3 Pessimism

The previously described condition is an extremely conservative approximation and most often than not the test would yield a non-schedulable result (above the least upper bound, even though the task set is schedulable).

The bound is very pessimistic: most of the times, a task set with  $U > U_{lub}$  is schedulable by RM.

A particular case is when tasks have periods that are harmonic: a task set is harmonic, if for every two tasks  $\tau_i, \tau_j$  either  $T_i$  is multiple of  $T_j$  or  $T_j$  is multiple of  $T_i$ . In this case, and only in this case, the least upper bound of RM grows to 1 (i.e. For a harmonic task set, the utilization bound is  $U_{lub} = 1$ ).

In other words, Rate Monotonic is an optimal algorithm for harmonic task sets.

## 3.2 Response Time Analysis

So far with the utilisation bound we have seen that there is a sufficient schedulability condition: if the total utilisation is below the least upper bound than the system is schedulable. But this test works only if one makes very specific choices, i.e. to have the priority set using the RM assignment, otherwise the least upper bound does not apply.

Moreover, in the case in which the total utilisation is above the least upper bound, no conclusion can be drawn/no guarantees.

Therefore, we would like to utilize a necessary and sufficient test, which won't be analytical and nice as we have seen from an algorithmic point of view, but it will be a little more reliable since it requires to make much less assumptions.

necessary and sufficient

Before introducing this test, let us recall the concept of response time: difference between the activation and the finishing/completion time. If one is able to compute the worst-case response time

worst-case response time

and if one can verify that such quantity remains below the relative deadline, then the system is schedulable.

relative deadline

The best thing about this test is that there is no assumption on the priority assignment, hence it can be applied to all algorithms. However, the periodic or sporadic tasks that we will be consider, are assumed to have no offsets (even though it is possible to have a variant of this analysis that applies).

Formally:

- For every task  $\tau_i$  in the task set
- Compute the worst case response time  $R_i$  for  $\tau_i$ , i.e.

$$R_i = \max_{j} \{\rho_{i,j}\}$$
 with:  $\rho_{i,j} = f_{i,j} - r_{i,j}$ 

- If  $R_i \leq D_i$ , then the task is schedulable
- Otherwise, the task is not schedulable

Consider a number of task set ordered by decreasing priority (i.e.  $i < j \rightarrow p_i > p_j$ ) and consider no assumptions about the tasks offset (i.e. tasks can be activated at any time). We can define the Critical Instant as the most critical condition for the computation of the response time (if you allow \_\_Critical Instant the offset to change).

The obvious result is that if you allow the offset to change, the worst case situation in terms of response time (i.e. situation that produces the longest response time) is one in which all tasks start at the same time. Because if all the tasks start at the same time one would observe an utilisation

Consider:

- a number of task set ordered by decreasing priority (i.e.  $i < j \rightarrow p_i > p_j$ )
- no assumptions about the tasks offset (i.e. tasks can be activated at any time)
- the worst possible offsets combination

A job  $J_{i,j}$  released at the critical instant experiences the maximum response time for  $\tau_i$ :

$$\forall k, \rho_{i,j} \geq \rho_{i,k}$$

**Theorem 1.** The critical instant for task  $\tau_i$  occurs when job  $J_{i,j}$  is released at the same time with a job in every high priority task.

Hence if all the offsets are 0, the first job of every task is released at the critical instant.

How one can compute the worst case response time? The worst case response time  $R_i$  for task  $\tau_i$ depends on:

- Its execution time
- The execution time of higher priority tasks (since the higher priority tasks can preempt task \_ preempt  $\tau_i$  and increase its response time), aka scheduling interference.

This factor is given by the number of activations of the higher priority tasks, in the window from the critical instant to the response time times the execution time/computation time of the higher priority tasks.

scheduling

as a consequence, the worst case response time can be computed as:

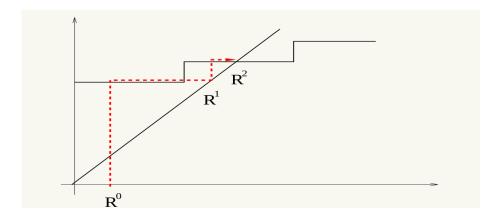
$$R_i = C_i + \sum_{h=1}^{i-1} \left\lceil \frac{R_i}{T_h} \right\rceil C_h$$

Unfortunately, one can notice that the above expression in recursive/implicit equation + nonlinear given the presence of the ceiling operator (i.e.  $R_i = f(R_i)$ ), as a consequence, there is no closed-form expression for computing the worst case response time  $R_i$ .

We need an iterative method to solve the equation.

The point where the bisector intersects the stairway, is the solution to the equation. But how \_\_bisector one can find such intersection?

In terms of algorithms it means that, one have a first guess which is



• Iterative solution:

$$R_i = \lim_{k \to \infty} R_i^{(k)}$$

where  $R_i^{(k)}$  is the worst case response time for  $\tau_i$  at step k• Hence, we can start from a first estimation of the response time:

$$R_i^{(0)} = C_i$$

• Compute the interference in this interval

$$R_i^{(1)} = C_i + \sum_{h=1}^{i-1} \left[ \frac{R_i^{(0)}}{T_h} \right] C_h$$

Since all the tasks starts at the same instant (critical instant), I am sure that I would have at least one activation, hence

$$R_i^{(0)} = C_i + \sum_{h=1}^{i-1} C_h$$

• Repeat the iterative step:

$$R_i^{(1)} = C_i + \sum_{h=1}^{i-1} \left[ \frac{R_i^{(k-1)}}{T_h} \right] C_h$$

• The iteration stops when:

$$-R_i^{(k+1)} = R_i^{(k)}$$
$$-R_i^{(k)} > D_i \text{ (non schedulable)}$$

This is a standard method to solve non-linear equations in an iterative way. If a solution exists (the system is not overloaded, i.e. total utilisation of the task is below 1),  $R_i^{(k)}$  converges to it, otherwise the failing condition avoids infinte iterations.

#### 3.2.1 Considerations

- 1. The response time analysis is an efficient algorithm
- 2. In the worst case the number of steps N for the algorithm to converge is exponential
- 3. depends on the total number of jobs of higher priority tasks in the interval  $[0, D_i]$

$$N \propto \sum_{h=1}^{i-1} \left\lceil \frac{D_h}{T_h} \right\rceil$$

4. If s is the minimum granularity of the time, then in the worst case

$$N = \frac{D_i}{s}$$

5. However, such worst case is very rare: usually, the number of steps is low.

There are a few corner case conditions: Suppose you have three tasks:

$$\tau_1 = (1, 4, 4)$$
  $\tau_2 = (2, 6, 6)$   $\tau_3 = (1, 8, 8)$ 

This task set is schedulable, however, the slack for the last task is too large and too much time is spent being idle.

There is an high discontinuity of the response time, in fact, one can easily construct examples in which, this condition can bring the system to a critical failure. To be precise: a small variability in the computation time can cause and high variability in the response time (due to preemption of low priority tasks) and occasionally cause deadline misses (domino effect: if you deviate too much from the nominal condition this might result in deadline misses).

domino effect

The worst case response time is a metric that is very much used to assess the feasibility of the schedule but it cannot be used to assess the robustness of a schedulability condition.

Consequently, he slack (time it takes to reach the deadline) is by no means a measure of how much robust your system is.

robustness of a scheudulabilit

### 3.3 Time Demand Analysis

It is very similar to response time analysis apart from some critical differences that make it so while the time demand analysis does not give you any information on the delays introduced by a computation, it does give you information about the robustness of a schedule.

time demand analysis

Both time demand analysis and response time analysis equally based on necessary and sufficient conditions, equally well founded, that provides you with a different type of information

robustness of a schedule

- Response time analysis does not only tell you if a task set is schedulable or not but it also tells you what are the delays introduced in the computation
- Time demand analysis does not give you any information about the delays introduced in the computation but it gives you information on the sensibility/robustness of the result

Processor Demand approach has been proposed by Lehoczky and others in 1989 and later refined by Audsley and other in 1993 and by Baruah in 1990. The concept is very simple: in any interval, the computation demanded by all tasks in the set must never exceed the available time.

- First and foremost we need to consider an interval in which we compute the computation demanded by all the tasks in this interval
- If there is an instant before the deadline in which the total computation time demanded by the task is matched by the offer of the computation time of the processor, then we reached a point where we have exhausted all the backlog and we can say that the system is schedulable.
- The problem is: how to compute the time demanded by a task set  $\mathcal{T}$ ?

  We can exploit the key obseration of critical instant and all the offsets are set to 0 (this allows us to only consider the first job of each task).

Given an interval  $[t_1, t_2]$ , let  $\mathcal{J}_{t_1, t_2}$  be the set of jobs started after  $t_1$  and with deadline lower that or equal to  $t_2$ :

$$\mathcal{J}_{\sqcup_{\infty},\sqcup_{\in}} = \{J_{i,j} : r_{i,j} \ge t_1 \land d_{i,j} \le t_2\}$$

In other terms, this subset is the collection of jobs that needs to be offered within the interval  $[t_1, t_2]$ . Moreover we can define the processor demand in  $[t_1, t_2]$  as:

processor demand

$$W(t_1, t_2) = \sum_{J_{i,j} \in \mathcal{J}_{t_1, t_2}} c_{i,j}$$

In other terms, is the sum of the computation times of all the jobs that have been activated and completed within the specified interval.

In this case, the lower case signifies the theoretical possibility of each job to have a different computation time. However, in practice we will consider the WCET  $C_i$  instead of  $c_{i,j}$ .

The computation demanded in every interval  $[t_1, t_2]$ , has to be smaller than the length of the interval (computation time offered by the CPU)

$$W(t_1, t_2) \le t_2 - t_1 \quad \forall (t_1, t_2)$$

In theory, we should consider the hyperperiod and make  $t_1$  and  $t_2$  change and repeat the check for every combinations.

In practice, a first simplification is not considering the whole period given that this analysis can be done for each task. And for each task you can consider only the first job. Because the first job is the one affected by the highest computation time (cfr. critical instant).

Hence, if the first job of a task succeeds in satisfying this constraint we are in a good shape. There will not be another job in which the computation demanded by the old task system would be higher.

Clearly for a task you need to consider in the computation demand not the entire task set but only the computation demand of the task itself + the demand of the tasks having higher priority. Formally:

 $W_i(t_1,t_2)$  is the time demanded in  $[t_1,t_2]$  by all tasks  $\tau_j$  with  $p_j \geq p_i \ (\Rightarrow j \leq i)$ . As mentioned:

- We can consider only  $W_i(0,t)$ , i.e. from the critical instant
- For task  $\tau_i$ , only check  $W_i(0,t)$  for  $0 \le t \le D_i$
- It can be shown that we can change the universal quantifier  $\forall$  into the existential quantifier  $\exists$ . If you consider a task  $\tau_i$ , what is the number of jobs originated by it in the interval [0, t]?

$$\left| \frac{t}{T_i} \right|$$

If instead we use the ceil instead the floor, we are making a conservative approximation (overevaluation), but this overevaluation allows us to swap the two quantifiers.

• We can therefore say if there is an instant in which this overevaluation/overestimation is matched by the offer than we are ok-

$$W_i(0,t) = C_i + \sum_{h=1}^{i-1} \left[ \frac{t}{T_h} \right] C_h$$

We have obtained a function is a stairway and we have to look for a point in which  $W_i(0,t) < t$ .

#### 3.3.1 The Guarantee

The Task  $\tau_i$  is schedulable iff  $\exists t : 0 \le t \le D_i \land W_i(0, t) \le t$ .

A task set  $\mathcal{T}$  is schedulable iff:

$$\forall \tau_i \in \mathcal{T}, \exists t : 0 \le t \le D_i \land W_i(0, t) \le t$$

Sometimes, different notations in literature

$$W_i(0,t) \to W_i(t) = \sum_{h=1}^i \left\lceil \frac{t}{T_h} \right\rceil C_h$$

implying that one of the extreme is zero.

Someone defines

$$L_i(t_1, t_2) = \frac{W_i(t_1, t_2)}{t_2 - t_1}$$
 
$$L_i = \min_{0 \le t \le D_i} L_i(0, t) \qquad L = \max_{\tau_i \in \mathcal{T}} L_i$$

The guarantee tests then becomes: Task  $\tau_i$  is schedulable iff  $L_i \leq 1$ .  $\mathcal{T}$  is schedulable iff  $L \leq 1$ .

#### 3.3.2 Simplifications

The test might still be long (need to check many values of L(0,t) to find the minimum).

The number of points to check for computing  $W_i/L_i$  can be reduced by considering the so called scheduling points: i.e. points in which the stairways changes value.

Formally, the scheduling points are mathematically defined as:

scheduling points

$$S_i = \{k T_h | h \le i; 1 \le k \le \left\lfloor \frac{T_i}{T_h} \right\rfloor \}$$

hence the number of scheduling points is a multiple of  $T_h$  for  $h \leq i$ , and the test can be rewritten in the simplified way:

$$L_i = \min_{t \in S_i} L_i(0, t)$$

To be fair, we have to say that this si even too much, because it can be shown that some of these scheduling points dominate the others: there is a selection of scheduling points to which you can restrict your analysis.

## Chapter 4

## Shared Resources

So far we have assumed that all tasks can run and compete for a processor indipendently (i.e. do not interact with one another).

However there are several occasions in which this assumption can simply not be made: the most evident is when two tasks have to share information and/or exchange variables, another scenario can happen when two or more tasks have to compete for shared resources.

So in this chapter we will look into this aspect and in particular we will start talking about the notion of atomicity.

### 4.1 The notion of atomicity

Atomocity: an hardware instruction is atomic if it cannot be "interleaved" with other instructions. Atomocity In other terms, even if there are instructions/activities/tasks with higher priority, an atomic instruction will carry on its execution without interruption until it has finished.

Moreover:

- atomic operations are always sequentialized
- atomic operations cannot be interrupted

sequentialized

cannot be
interrupted

always

- they are safe operations
- e.g., transferring one word from memory to register or viceversa. It cannot be allowed
  that someone access/modify the word while it is transferring into a register, hence the
  task/instruction responsible for moving into the register the word has to be an atomic
  instruction.
- non atomic operations can be interrupted
  - they are not "safe" operations
  - non elementary operations are not atomic

Usually, you prefer to have, whenever possible, non atomic operations because they are the ones that allow you to exploit in full the possibility to schedule the processor to activities having an higher priority.

#### 4.1.1 Non atomic operations

Consider a simple, increment operation:

$$x = x + 1$$

in assembler, this expression would probably take the form:

```
1 LD R0, x
2 INC R0
3 ST x, R0
```

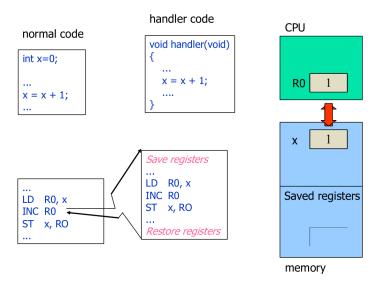
where the variable x is loaded into a register R0, the register will then be incremented and the result will be stored into x.

#### Interrupt on non-atomic operations

A simple operation like incrementing a memory variable, may be composed by three machine

If the same operation is done inside an interrupt handler (routing executed in response to an interrupt), an inconsistency can arise: The normal code starts to execute:

interrupt handler



- 1. The value of x is set to 0 in memory
- 2. The value of x is copied into the register R0
- 3. Before the increments something happens that triggers the execution of the interrupt handler
- 4. The interrupt handler creates a copy of all the registers (that contains the value of x), and then it executes its incrementation of R0.
- 5. The interrupt handles finishes and all the saved registered values are loaded back in/restored. Hence the normal code assumes that x is zero (which is the value of the register before the interrupt) and increments it by one and stores it into x.
- 6. Eventually, x will be equal to 1. From a logical point of view two increments should have taken place, but in fact one of them was not successfully completed, because during the execution fo the increment I was allowed to be interrupted.

The issue with this scenario is that it does not always happen like this: sometimes the normal code is able to complete before the interrupt is fired. Under this circumstance, we would see that the variable x is incremented both times instead of one.

This condition is known as Critical race, because you can have multiple execution of the code \_ critical race that interleave their operations in slightly different way and obtain different results. And in fact, this is one of the nastiest problem in computer science and computer engineering, because sometimes the problem does not materialize for years.

#### Task concurrency in non-atomic operations

The reason why this problem is not easy to debug and notice is that we cannot make any assumption in advance about the relative speed of the processes, hence, we do not know the order of execution of the hardware instructions.

Moreover, you cannot make any assumption on when certain events are triggered, because you can have jitter in the generation of the interrupt.

Recall the example of incrementing variable x.

- Incrementing x is not an atomic operation, that instead should be.
- atomic behaviour can be obtained using interrupt disabling or special atomic instructions. A simple solution then is disabling the interrupt before the execution of the normal code and resuming them after the execution of the code has ended.

Clearly, the same bad interleaving problem can happen in case of concurrent task as displayed in figure ??:

- 1. Consider two concurrent tasks, such that the priority of B is higher than A, and a shared memory approach
- 2. Normally when task B starts it interrupts task A, It could happen that task B starts right after task A has taken its snapshot of the variable x
- 3. What you come up with is exactly the same problem as before: sometimes the variable will be 2 and sometimes it will be 1.



The same situation can happen in more convoluted problems:

- 1. Consider a struct with two fields (a, b), with some functions that operates on the data structure.
- 2. And assume that we have two tasks A and B, which call respectively, two different methods.
- 3. If A start executes and increments a and gets interrupted before incrementing b, resulting in a non consistent state



#### 4.2 Critical sections

At this point, we can provide the reader with some definitions that will be important for the follow up of our discussion.

#### 4.2.1 Definitions

• the shared object where the conflict may happend in a resource.

A resource shared by two or more threads, shared by a thread and an interrupt handler or etc...

shared object

resource

- the parts of the code where the problem may happen are called critical sections.

  A critical section is a sequence of operations that cannot be interleaved with other operations on the same resource.
- Two critical sections on the same resource must be properly sequentialized.
- we say that two critical sections on the same resource must execute in Mutual exclusion. Two critical sections cannot be executed at the same time, but one needs to stand by when the other executes.
- There are three ways to obtain mutual exclusion:
  - 1. implementing the critical section as an atomic operation (i.e. disabling interrupts before executing the operation, and restoring interrupts at the end). This approach is really tough and extreme to implement because if one disables the interrupts, the I/O system of the machine is no longer allowed to work properly.
  - 2. disabling the preemption (system-wide), for instance by raising the priority of the critical section/of the task to the maximum in the system and restoring back the priority at the end. However, this approach has some drawbacks because all the tasks will suffer from this suspension fo the preemption even though they do not use the shared resources.
  - 3. selectively disabling the preemption (using semaphores and mutual exclusion). I.e. preemption will be disabled only for the task that operates on the shared resource.

critical section as an atomic

Mutual exclusion

parts of the

code

disabling the preemption

(system-wide)

operation

selectively disabling the preemption

#### Atomic operations

If we implement a critical section as an atomic operations, implies that the interrupts are disabled suring the execution of such critical section. In order to do so, Assembler language provides the CLI instruction and restore the interrupts right at the end of the execution via STI.

Problems:

- If the critical section is long, no interrupt can arrive during the critical section.
  e.g. consider a timer interrupt that arrives every 1 ms. If a critical section lasts for more than 1 ms, a timer interrupt could be lost.
- Concurrency is disabled during the critical section: we must avoid conlict on the resource, not disabling interrupts!

#### Disabling preemption system-wide

Instead of disabling interrupts, it is ar more safer to disable preemption. It is, in fact, possible to disable preemption for a limited interval of time.

Consider a task that is normally non preemptive (i.e. cannot be interrupted by anyone). Such task can execute the critical section without interruption, then the function schedule is called signaling that some other task having a higher priority can take over.

This is what happens in OSEK (a kind of RTOS, widely used in cars), that use ono-preemptive tasks together with the Schedule() primitive.

Problems: if a high priority critical thread needs to execute, it cannot make preemption and it is delayed, even if the high priority task does not access the resource.

#### Selectively disabling preemption

The best way to implement mutual exclusion is by selectively disable preemption.

There exist some general mechanisms to implement mutual exclusion only between the processes that use a resource: OSEK, for instance, uses the GetResource/ReleaseResource primitive (in POSIX there is mutex\_lock and mutex\_unlock):

#### 4.2.2 Schedulability and critical sections

Up to now, we have considered independent tasks. However tasks do interact between each other: they use shared data, non preemptable resources, etc...

independent tasks

The big issue is that when one disables preemption (even selectively) the priority mechanism is no longer enforced: a task with lower priority for the length of the critical section will execute instead of an higher priority task (it operates as if it had the highest priority).

This issue is commonly known as priority inversion: period in which a high priority task is prevented from executing by a low priority task. If priority inversion is not correctly manages it may result in a violation of timing constraits.

priority inversion

### 4.3 Interacting Tasks

Until now, only independent tasks:

- A job never block or suspends
- A task only block on job termination

In real world, jobs migh bloc for various reasons:

- Tasks exchange data through shared memory (mutual exclusion)
- A task might need to synchronize with other tasks while waiting for some data
- A job might need a hardware resource which is currently not available

Example: Consider a contorl application composed by three periodic tasks:

- $\tau_1$  reads the data from the sensors and applies a filter. The results are stored in memory
- $\tau_2$  reads the filtered data and computes some control law (updating the state and the outputs); both the state and the outputs are stored in memory
- $\tau_3$  reads the outputs and writes on an actuator

All o the three tasks access data in shared memory, hence when they execute concurrently, if we do not schedule/manage the resources properly it may happen that some data structure (in the information exchange) could be corrupted/inconsistent (a task can read a piece of the structure before the task responsible to providing it has finished modifying it).

#### 4.3.1 Task Intraction Paradigms

The possible ways of managing interactions between tasks are:

• Private Resources (Client/Server paradigm)

Private Resources

• Shared Resources

Shared Resources

Resource Manager

#### Private Resources

The resources are privatly handlesd by a task. Hence a Resource Manager (aka server task) per resource is instantiated. This method is based on processes, i.e. they have their own address space (direct access to the resource).

In case some other tasks would like to access and use a resource, it would send a message/request to the relative manager of the resource via Inter-Process Communication .

This is the same mechanism that we use when accessing the web.

#### **Shared Resources**

Another approach is to consider shared resources: resources are no longer privately owned by any task, but it is possible for all the tasks to have access to the resources.

This is a shared memory model, as such it is based on multithreaded programming (multiple thread of execution, but address space is the same).

This interaction happens via mutexes, semaphores, condition variables, ...

#### 4.3.2 Resources and Critical sections

If you look into both approaches, you will quickly realise that at some point there will be a problem of concurrency: especially in reading and writing to its resources.

You can have the impression that when using a multiprocess based on the model of private resources you have evaded the problem, but in reality not.

For sake of simplicity we will analyze the problem for the case of shared resources:

- We will use mutexes (specifically devoted to the issue of implementing mutual exclusion) and not semaphores (can be used a mutually exclusive region, but also for general purpose synchronization).
- Extensions to IPC based communication are possible.

On the frame of reference of the interacting tasks, we can redefine it as a schedulable entity or active entity that can perform operations on private or shared data.

Focusing on shared data, we need to model the mechanism that protects such shared data: as a consequence, the shared data that get protected by mutual exclusive objects (aka mutexes) is called protected object.

In other terms, Protected Objects:

Protected Objects

Resources

- Encapsulate shared information (Resources)
- Passive Objects (data) shared between different tasks
- Operations on protected objects are mutually exclusive (i.e. protected)

Passive Objects

In order to protect the object:

- A mutex variable gets locked at the beginning, so that a task becomes the owner of the associated resource
- At this point when the task has locked the resource and before it releases the lock, the task effectively owns the resource

#### 4.3.3 Shared Resources

A Shared Resource  $S_i$ 

Shared Resourc

- can be used by multiple tasks
- is protected by a mutex (mutual exclusion semaphone)
- has a 1 to 1 relationship between resources and mutexes. Convention:  $S_i$  can be used to indicate either the resource or the mutex

In order to make real time scheduling analysis with this concept in ming we need to extend our system model according to this definition; now the system is not limited to a set of tasks...

#### System Model

A system/Application can be modeled by:

• A set  $\mathcal{T}$  of N periodic (or sporadic) tasks:

$$\mathcal{T} = \{ \tau_i : 1 \le i \le N \}$$

• A set S of M shared resources:

$$\mathcal{S} = \{S_i : 1 \le i \le M\}$$

- Task  $\tau_i$  uses resource  $S_j$  if it accesses the resource (in a critical section)
- The k-th critical section of task  $\tau_i$ , that uses the resource  $S_j$  will be called:

$$cs_{i,j}^k$$

• The length of the longest critical section of  $\tau_i$  on  $S_j$  will be  $\xi_{i,j}$ 

#### **Blocking Time**

The consequences of using this method and model when it comes to evaluating the timing behavior of the task. We will need to introduce the definition of Blocking time, i.e. the time that a task has to wait for a resource to be released.

Blocking time

Formally:

- When task  $\tau_i$  tries to access a resource S already held from task  $\tau_j$ ,  $\tau_i$  blocks
- Blocking time: time between the instant when  $\tau_i$  tries to access S (and blocks) and the instant when  $\tau_j$  releases S (and  $\tau_i$  unblocks)

Note that this is needed for implementing mutual exclusion and cannot be avoided. Notice that when we talk about blocking time we refer to the time a task has to wait when somebody having a priority lower than it owns a resource (not higher).

Clearly if this blocking time becomes too large, then the very mechanism of scheduling priorities that allowed me to guarantee the timely execution of the task is disrupted.

Blocking times can be particularly bad in priority scheduling if a high priority task wants to access a resource that is held by a lower priority task: a lowe priority task executes, while a high priority one is blocked; schedulability guarantees can be compromised.

We need to find means to account for the blocking time: in fact, if this blocking time was limited or bounded than we can account for it in the WCET of the task, but this is possile only if this blocking times are deterministic and not too large.

# Bibliography