ALGORITHMIC TRADING:
AN ANALYSIS OF 'A MEAN-FIELD GAME OF MARKET-MAKING AGAINST STRATEGIC TRADERS'

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1 Introduction

We start this analysis by briefly motivating our choice behind the selection of this paper and by stating the objectives of the researchers who wrote the article. Firstly, when considering the list of papers, we decided to work on something that was, to some extent, new for us. We were aware of the existence of Mean-Field Game (from now on, MFG) Theory, however, we never formalised our knowledge of the topic. We understand the validity of such approach when dealing with financial markets modelling and, knowing that especially in Paris there are various esteemed researchers working on this, we were naturally driven by the name of the paper and their authors. Given these reasons, we intended to use this paper as an introduction to the field which is very dynamic and active at the moment.

'A Mean-Field-Game of Market-Making against Strategic Traders' [3] has been co-written by three authors, namely Bastien Baldacci (formally from École Polytechnique, Paris), Philippe Bergault (Dauphine-PSL, Paris) and Dylan Possamaï (ETH, Zurich) and it has been published in the SIAM Journal on Financial Mathematics in December 2023 [1], so it is a very recent work, which has not yet seen a great number of direct involvement in terms of future citations. However, it enters a very active environment, adding a valuable contribution to the field as, to the knowledge of the authors and from the research that we did ourselves, it is indeed the first paper that considers optimal market-making taking into account the strategic behaviour of market-takers. In fact, the objective of the three authors is to propose a major-minor MFG in which the market maker is the major player, while the market-takers are minor strategic agents. The reason behind the involvement of MFG Theory is justified by the nature of the problem which can involve N number of strategic market takers, which become increasingly intractable when N grows. This is well explained in **Section 2.4**, where the authors underline that a system of $N + 1$ Hamilton-Jacobi-Bellman (from now on, HJB) equations of dimension $N + 2$ becomes rapidly difficult to deal with. Furthermore, it is believed that such approach will also hold an improvement with respect to current optimal market-making models.

A last notice to mention is the use of the free version of the paper from Arxiv [2], fully aware that it still has few insignificant typos which we will not report in this analysis. We believe this version of the paper is definitive and we will consider it as such throughout this review.

2 Summary of the Paper

Before delving deeper into the findings of the paper, we briefly delineate its structure in order to provide the reader with a proper context and how the three authors decided to plan their work. Overall, we don't discuss **Section 1** about the introductory setting since it is addressed at a later stage of this analysis and **Section 6** which delineates only the natural conclusions.

Outline

Section 1 is dedicated to a general introduction of the problem and the academic environment in which it is developed, providing a vast and vibrant context that involves three main domains of research: algorithmic trading, market making, and MFG Theory. We speak vastly about the literature in a later section of this analysis, since there are many valuable references to other researchers' work. Furthermore, the introduction provides many non-technical details and notions for a better understanding of the following pages, parallel to issuing an overall presentation of some of the current challenges and novelties facing the academic activity. **Section 2** introduces a first setting, which is later revealed as non-tractable, since it is a finite players model, numerically unstable given the elevated number of (HJB) equations to treat the issue of multiple market-takers. This problem gives the reason to develop a virtually infinite players model, using Mean-Field-Game Theory which is detailed in **Section 3**, modifying the previous finite version. **Section 4** changes the setting to a Markovian one, motivated by **Carmona and Wang [9]** in order to facilitate the whole problem, by slightly changing the admissible controls and, consequently, the two optimisation problems for the market-maker and the market-takers. In this section, the authors present the two important theorems which are both proved in the appendix of the paper, and a final corollary. All of these results are discussed more extensively in a later paragraph. The numerical part is left to **Section 5**, where the paper shows some results with varying parameters such as the inventory q, the price signal b and the intensities λ. Finally, it concludes with **Section 6**, where a summary is presented with both achievements and limitations, hinting at new developments in the future.

[1] For the website reference, we add here the link: https://epubs.siam.org/doi/10.1137/22M1486492

[2] https://arxiv.org/abs/2203.13053

Section 2

This first explicative section serves more as a tool to later justify the introduction of its MFG version. Overall, it does not present a completely formal framework, avoiding a proper formulation of the problem, but eventually, some of the elements introduced in this section are useful for a swift transition to the MFG approach. Here, they establish all the main ideas that they carry on for the rest of the paper. Before delving into this section, we explain a practical choice that we do, in order to make the report compact: in terms of notation, every time the reader finds "$\cdot$", we intend it as a substitute for both 0 (market-maker) and $i = \{1, ..., N\}$ (market-takers). This choice is purely intended as a way to not define everything present in the original paper which would eventually cause this summary to be too long. Firstly, they introduce $N \in \mathbb{N}$ traders, each one asking for a sell (resp. buy) price around a mid-price S_t with an ask spread of δ^a (resp. bid spread δ^b), defined by the function $P^a(S, \delta^{\cdot,a})$ (resp. with b). The number of orders is modelled with a counting process whose intensities are dependent on the spread and the two coefficients: the average number of transactions A^m, the sensitivity of the intensity to mid-to-ask (resp. mid-to-bid) quotes proposed k^m and the volatility of the asset σ. Similarly, without entering all the details of the paper, we have the same for the market-maker, with a clear difference regarding the intensity function, depending on both the ask spread of the market-maker and the average of the bid spreads of the market-takers (resp. bid spread and average of ask spreads). To complete the initial setting, the set of admissible controls for all the actors are defined. Then, the dynamic of the price process is the following

$$\mathrm{d}S_t = \sigma \mathrm{d}W_t + \frac{\kappa}{N} \sum_{i=1}^{N} (\mathrm{d}N_t^{i,b} - \mathrm{d}N_t^{i,a}) \tag{1}$$

with κ and σ as positive constants, with the Brownian motion independent of the market-maker's counting processes. Finally, a signal on the price is defined for each market-taker, $b_t^i := N_t^{i,+} - N_t^{i,-}$ where the two new processes are Poisson processes independent of $(W, N^{0,b}, N^{0,a}, N^{i,b}, N^{i,a})$.

Now, we have all the elements to define the two problems, respectively for the market-maker and for the market-takers. Finally, defining the cash process

$$X_t^{\cdot} := X_0^{\cdot} + \int_0^t P^a(S_s, \delta_s^{\cdot,a}) \mathrm{d}N_s^{\cdot,a} - \int_0^t P^a(S_s, \delta_s^{\cdot,b}) \mathrm{d}N_s^{\cdot,b}, \quad X_0^{\cdot} \in \mathbb{R}, \tag{2}$$

the inventory process

$$\mathrm{d}q_t^{\cdot} = \mathrm{d}N_t^{\cdot,b} - \mathrm{d}N_t^{\cdot,a}, \quad q_0^{\cdot} \in \mathbb{R} \tag{3}$$

and the proper probability measure, we can express the value functions. At this point we realise that the setting we introduced is not feasible numerically as the number of market-takers N grows.

Section 3

In this section, the MFG problem is introduced taking a representative market-taker as the new counterpart. The setting requires some adjustments to the previously defined intensities, overall impacting the rest of the equations, however, for brevity reasons, we don't detail them here. More interesting is the new definition of the admissible controls

$$\mathcal{A}_\infty = \left\{ (\delta^b, \delta^a) \in \mathbb{L}^0(\mathbb{R}^2, \mathbb{F}) : (\delta^b, \delta^a) \text{ bounded by } \delta_\infty > 0 \right\}, \tag{4}$$

with $\mathbb{F} := (\mathcal{F})_{t\in[0,T]}$ and with the market-maker choosing the bid and ask prices $(\delta^b, \delta^a) \in \mathcal{A}_\infty$ and, similarly, the market-maker having $(\delta^{m,b}, \delta^{m,a}) \in \mathcal{A}_\infty$. They also introduce a probability flow $\mathcal{V}$ of the joint distribution of the controls of the mean-field of the market-takers, their inventories and their signals which eventually leads to a needed change of probability measure $\frac{\mathrm{d}\mathbb{P}^{\delta,\delta^m,\mathcal{V}}}{\mathrm{d}\mathbb{P}}$. Consequently, we have the corresponding expectation $\mathbb{E}^{\delta,\delta^m,\mathcal{V}}$ under the probability $\mathbb{P}^{\delta,\delta^m,\mathcal{V}}$. At this point, after defining the function $\mathcal{L} : [\![-\tilde{q}^m, \tilde{q}^m]\!] \times \mathbb{R}^2 \longrightarrow \mathbb{R}$ by $\mathcal{L}(y, x^b, x^a) := \mathbb{1}_{\{y>-\tilde{q}^m\}}\Lambda^m(x^b) - \mathbb{1}_{\{y<\tilde{q}^m\}}\Lambda^m(x^a)$, we can state the two optimisation problem, respectively for the market-maker and the market-taker

$$V(\mathcal{V}) := \sup_{\delta \in \mathcal{A}_\infty} \mathbb{E}^{\delta,\delta^m,\mathcal{V}}\left[\int_0^T \left(\delta_s^b \mathbb{1}_{\{q_s < \tilde{q}\}} \Lambda\left(\int_{\mathbb{R}} x\nu_s^a(\mathrm{d}x), \delta_s^b\right) + \delta_s^a \mathbb{1}_{\{q_s > -\tilde{q}\}} \Lambda\left(\int_{\mathbb{R}} x\nu_s^b(\mathrm{d}x), \delta_s^a\right)\right.\right.$$
$$\left.\left. - \phi\sigma^2(q_s)^2 + q_s\kappa \int_{\mathbb{R}^4} \mathcal{L}(y, x^b, x^a)\mathcal{V}_s(\mathrm{d}x^b, \mathrm{d}x^a, \mathrm{d}y, \mathrm{d}z)\right)\mathrm{d}s\right], \tag{5}$$

$$V^m(\mathcal{V}) := \sup_{\delta^m \in \mathcal{A}_\infty} \mathbb{E}^{\delta,\delta^m,\mathcal{V}}\left[\int_0^T \left(\delta_s^{m,b} \mathbb{1}_{\{q_s^m < \tilde{q}^m\}} \Lambda^m(\delta_t^{m,b}) + \delta_s^{m,a} \mathbb{1}_{\{q_s^m > -\tilde{q}^m\}} \Lambda^m(\delta_t^{m,a})\right.\right.$$
$$\left.\left. + b_s^m q_s^m - \gamma\sigma^2(q_s^m)^2 + q_s^m\kappa \int_{\mathbb{R}^4} \mathcal{L}(y, x^b, x^a)\mathcal{V}_s(\mathrm{d}x^b, \mathrm{d}x^a, \mathrm{d}y, \mathrm{d}z)\right)\mathrm{d}s\right] \tag{6}$$

with:

- initial inventory conditions are respectively $q_0 \in [\![-\tilde{q}, \tilde{q}]\!]$ and $q_0^m \in [\![-\tilde{q}^m, \tilde{q}^m]\!]$;
- v_s^a and v_s^b the first two marginal laws of $\mathcal{V}_s$ (distribution of the controls at the ask and bid);
- $\phi > 0$ and $\gamma > 0$ the risk aversion parameters of the market-maker and the market-taker respectively.

The authors take the simpler Markovian setting to derive their results in the next section.

Section 4

Finally, this section is dedicated to a Markovian setting where the authors state the two main proved results about the equilibrium of the MFG. Most of the setting has already being described in the previous sections, hence we don't outline once again the newly modified optimisation problems and admissible controls, nor the master equation which would take too much space. Here, the authors finalize their work by defining the Markovian optimisation problems and, consequently the master equation to find a solution. That is a HJB-Fokker-Planck PDEs system with initial and terminal condition. This long, though elegant, master equation allows the authors to conclude the theoretical dissertation by stating the two main theorems, the first one guaranteeing the existence of a solution and the second being a verification theorem for the optimality of the solution. The first proof is quite extensive and it follows various steps involving the introduction of appropriate functional spaces, finding unique solution for the first two equations of the HJB-Fokker-Planck system (coupled HJB eq.), repeating the procedure for the last equation (Fokker-Planck eq.) and conclude with Schauder's fixed point theorem, which imply the existence of a solution, but not the uniqueness. For the procedure, the contribute of **Proposition 2 from Bergault and Guéant [5]** is essential for the bound of the controls. The second proof is more standard as it is a better known verification theorem. Conclusively, this permits to obtain **Corollary 4.5** which grants the Markovian equilibrium. However, uniqueness of the equilibrium is mentioned only as a reasonable result coming from the findings of the numerical experiments' section. The authors leave the uniqueness part given the difficult nature of proof.

Section 5

Finally, the authors implement their mean-field game model in a discretized setting to simulate the interaction between a market-maker and the strategic market-takers. The experiments are calibrated with realistic market parameters, including a fixed time horizon of 5 days, discrete inventory and signal spaces, and a trade size of 10 assets. The numerical solution to the master equation is obtained via explicit Euler schemes, with the objective of finding the fixed-point. The simulations reveal several important behaviours. First, the joint distribution of the market-takers' inventories and signals shows that traders tend to hold positions that match the direction of their signal. When the signal is neutral, the distribution of inventories is symmetric around zero, but when the signal becomes strongly positive or negative, the distribution shifts accordingly. The trader's optimal bid and ask quotes also depend on both inventory and signal, adjusting in a way that lets them take advantage of favourable signals while still controlling risk. On the other hand, the market-maker, who does not observe the signal and is more risk-averse, sets quotes symmetrically around zero inventory. The spreads widen as the inventory moves away from zero, reflecting the increasing cost of holding risk. The value function is highest when the inventory is neutral, highlighting the trade-off between earning profits from providing liquidity and managing inventory exposure. Finally, increasing the volatility of the private signal, by accelerating the rate of signal switching, disrupts the inventory dynamics of traders: unable to adjust quickly enough, they accumulate positions that do not match the signal direction, producing a bimodal inventory distribution and reducing overall market activity.

3 Critical Analysis

In this section we consider three main aspects fundamental for our analysis. Firstly, we address the novelties that the paper proposes and their overall importance in the context of both MFG and market-making. Then we move to the study of the literature (essentially completing our summary of the paper from the previous section) surrounding the work proposed, with both a thorough examination of the references with their motivation of their significance and a glance at the following papers which mentioned **Baldacci et al. [3]**. Finally, we state some limitations that this work presents, proposing a glimpse of possible extension, without entering the technical details, for the sake of brevity and complexity.

Novelties and Relevance

The authors state very clearly the problem the market-maker is facing when dealing with a high number of market-takers whose trading signals, strategies and inventories are all different from each other. Then, as well as their counterparts, the market-maker proposes quotes based on his own inventory, signals and behaviour of the market-takers. Finally, the approach considered, a major-minor MFG, makes sense economically given the usual nature of market-making where there are very few actors providing liquidity and, on the other side, many clients. In fact, as an extreme case, essentially a monopoly, the authors mention Citadel Securities which holds 4000 U.S. listed-options names, meaning 99% of the traded volume.

As mentioned before, the paper proposes a never seen approach regarding the strategical nature of market-takers. That is obviously an important milestone to add in the context of a MFG approach to financial markets modelling. This development constitutes an enhancement of **Avellaneda and Stoikov [2]**, where the behaviour of market-takers is considered only as a function of the quotes of the market-maker, rather than a function of their view on the price process and their current inventory. As the authors mention in their **Introduction**, this additive layer of complexity allows to consider a more accurate estimation of the PnL of the market-maker's strategy.

The relevance of the paper is undeniable for the context of major-minor MFG problems, however, in the literature, there is also an evident shift towards a more realistic setting, contextualised to the presence of informed players. We give more details on the topic in the next paragraph about future works.

The literature

The literature in which this paper is placed is very vast, with many researchers and some professionals connected with each other, either via a professor-student relationship or by a general interest on the topic concentrated especially in the academic environment in Paris, involving universities such as Pantheon-Sorbonne, École Polytechnique, Dauphine-PSL and Collège de France. As we can see for some of the references that the authors add in their paper, most of the researchers come from one of these previously mentioned institutions. We also remind that MFG has been mostly developed by academics based in France such as Pierre-Louis Lion, Pierre Cardaliaguet and Jean-Michel Lasry.

Overall, the literature they refer to is a mix of more theoretical and fundamental papers, others that tackle specific matters, namely linear-quadratic problems, and some are more contextual to the financial application to market making. We report here some of them which we considered essential for a complete understanding of this paper, especially **Lasry and Lions [15]**, **Cardaliaguet, Cirant and Porretta [7]** and **Carmona and Wang [10]**, which delve deeper into the technical aspect of MFG with major players. Afterwards, we mention some other important authors who reduced some MFG problems to explicitly solvable systems of ODE such as **Huang [14]** or the work of **Firoozi and Caines [12] [11]** and **Jaimungal and Caines [13]**. Finally, there are various papers related to the financial markets applications and we name just a few of them, we consider the main ones such as **Almgren and Chriss [1]**, the previously mentioned **Avellaneda and Stoikov [2]** and many previous works mainly by **Guéant**, **Bergault** and **Baldacci** collaborating between each other or with other researchers. All of these citations were made in the **Introduction** with the intent to provide the reader with all the necessary context needed for the understanding of the following posed problem and how they tackled it. However, throughout the paper, the need to refer to others' work decreases rapidly, as most of the research is indeed original. In fact, most of the time, later citations serve the purpose of synthesising part of the context which otherwise would require much more details that the authors leave to more specialised works, if the reader is interested. Nonetheless, the paper remains very fluent and every result obtained, section by section, is easily understandable by a reader that possesses most of the mathematical tools to understand the complex field of MFG, such as Stochastic Analysis and Control Theory.

There are not many later published works addressing this paper, to be exact, only four. In **Mastrolia and Xu [16]**, the paper has been referred to for its technical validity regarding its stochastic problems, but without providing any further details for specificities. All the other papers published after, recognise the paper as an important milestone in the MFG setting in finance. However, they all tend to develop what seems to be a more interesting research problem at this moment. In fact, while **Baldacci et al. [3]** does not make any assumptions on either informed brokers or traders, the new papers try to address this new matter. Namely, we have: **Shelton and Veiga [17]** from Bank of America which shows how to derive fast, closed-form expressions for setting stop-loss, take-profit, and maximum holding time thresholds in a market-making framework, explicitly addressing the risks posed by informed players; then there are **Bergault and Sánchez-Betancourt [6]** and **Cartea, Jaimungal and Sánchez-Betancourt [18]** dealing with Nash equilibrium and informed traders and brokers. The last two are actually closely connected, having Sánchez-Betancourt working on both and quoting the first one in the second.

Finally, it is very likely to have new papers on their way which build more on this initial idea, developing more this trending topic of informed players on the market, which makes the model more realistic. To our knowledge, there is indeed a work-in-progress co-written by **Cardaliaguet, Bergault and Yan** which we cannot quote since it is not yet finished. Nonetheless, we are aware that it revolves around the optimal hedging of an informed broker facing many traders utilising Stackelberg's equilibrium. Eventually, it recognises the chosen paper as an important milestone in the MFG setting for finance.

Limitations and Improvements

In the paper, there are two fundamental theorems, respectively **Theorem 4.3** and **Theorem 4.4** which are both proved rigorously in the appendix, as you would expect. However, the conclusive result in **Corollary 4.5** only provides a Markovian equilibrium of the MFG. As the authors also mention in **Remark 4.6**, they do not prove the uniqueness of the equilibrium. The main argument that they give for the difficulty of the proof regards the presence of problematic controlled point processes.

The second intent of this section is to provide some possible ideas for the proof, which, after some careful reading of other materials, we realised is indeed complicated. Considering our previous knowledge, we acted in the best of our abilities to find any useful tool to deal with such matter. To find a feasible solution, we initially consulted the initial material on MFG by **Lasry and Lions** or in **Bensoussan et al. [4]**, looking for a more general understanding of the conditions required. We grasped the general idea of the monotonicity method for the proof. However, it is not easily understandable from our master equation if monotonicity can be directly applied. Besides, it is also clear that at the time of their first publications on the topic, even though Lasry and Lions considered a comparable kind of setting, involving such PDEs system, the main results were introduced at a later stage of research. For this reason, we checked **Cardialaguet et al. [8]** comprehensive work on the existence and uniqueness of the solution of the master equation, in our case a coupled HJB-Fokker-Planck system. At this point, we tried to clarify the nature of our master equation, whether it falls under first or second master equation order, and our findings seem to confirm it is first order, for the absence of 'common noise', which would eventually trigger the second order. However, the uniqueness proofs in Cardaliaguet's framework rely on continuous dynamics with diffusive terms, these assumptions are hardly applicable in a jump-driven, discrete setting. Consequently, it is not straightforward to extend those results to prove the uniqueness of our equilibrium. Furthermore, even though the authors believe in the possibility of a unique solution, justified by their numerical results, we point out that, generally, it is not always attained in MFG.

Overall, these famous approaches cannot be directly applied to solve the uniqueness problem stated in the paper. We would need more time and expertise about MFG Theory to effectively conclude or involve viable ideas for a possible extension.

4 Conclusion

The paper gives a very comprehensive introduction to the application of MFG in finance, especially when dealing with a major market-maker with multiple market-takers who behave strategically. Even though the discussions and some results are taken for granted, as the paper does not have the initial purpose to introduce the reader to MFG, it serves as an extensive work based on fundamental results from other active researchers. In the light of this, we think that the paper is well written, it properly guides the reader as the structure is logical and well defined, adding all the necessary external references, without sacrificing immediate clarity from the authors.

The results obtained are very important and interesting for the field of MFG in finance, possibly opening new chapters and studies based on the same idea of having strategic market-takers. A marginal note regards the missing uniqueness of the equilibrium proof that is just mentioned in **Remark 4.6** as hard and possibly long. We understand the argument and it makes sense for the authors to leave it to a more general work, given their finding on the absence of numerical instability in the experiments. Overall, this seems to suggest the uniqueness of the equilibrium in the MFG. In the end, we tried to provide some ideas to prove it, mentioning some arguments required by the problem, such as monotonicity, contraction or convexity, trying to add a personal contribution to this work.

Even though the paper is not yet fully established in the current environment of MFG, mostly because of its recent appearance, we believe that it could inspire future works that build on it, either from the idea of strategic market-takers or from a technical point of view such as the proof of a unique equilibrium of the MFG.

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