EQUAÇÕES DIFERENCIAIS PARCIAIS

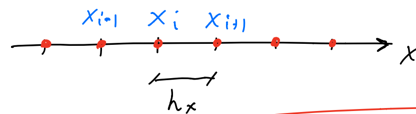
* REVISÃO: EQUAÇÕES PARABÓLICAS

EX: $\dfrac{\partial f}{\partial t} = \alpha^2 \dfrac{\partial^2 f}{\partial x^2} \qquad f = f(x,t)$

$\begin{cases} i \rightarrow x \\ j \rightarrow t \end{cases}$

$$\boxed{\frac{\partial f}{\partial t} \longrightarrow \frac{f(x_i, t_{j+1}) - f(x_i, t_j)}{h_t}}$$

$x_i = h_x \cdot i$
$t_j = h_t \cdot j$

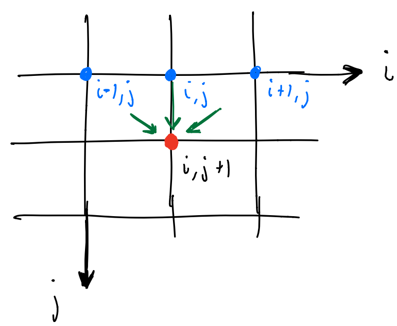


$$\boxed{\frac{\partial^2 f}{\partial x^2} \longrightarrow \frac{f(x_{i+1}, t_j) - 2f(x_i, t_j) + f(x_{i-1}, t_j)}{h_t^2}}$$

$$f(x_i, t_j) \longrightarrow f_{ij}$$

$$\boxed{f_{i,j+1} = (1-2r) f_{ij} + r(f_{i-1,j} + f_{i+1,j})}$$

$$r = \frac{\alpha^2 h_t}{h_x^2} \leq \frac{1}{2}$$

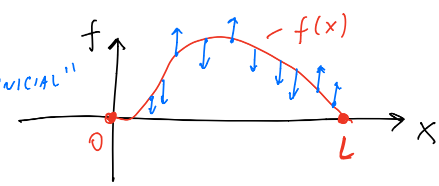


* EDP'S HIPERBÓLICAS

ESTAMOS INTERESSADOS EM RESOLVER O PROBLEMA DE CONTORNO

$$\begin{cases} \dfrac{\partial^2 f}{\partial t^2} = \alpha^2 \dfrac{\partial^2 f}{\partial x^2} \quad \text{(EQ. DA ONDA)} \\ f(0,t) = f(L,t) = f_0 \\ f(x,0) = f(x) \rightarrow \text{"POSIÇÃO INICIAL"} \\ \left.\dfrac{\partial f}{\partial t}(x,t)\right|_{t=0} = g(x) \end{cases}$$

$\hookrightarrow$ "VELOCIDADE INICIAL"



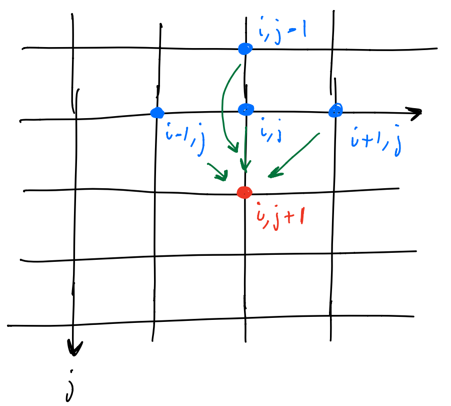
$$\begin{cases} \dfrac{\partial^2 f}{\partial t^2} \longrightarrow \dfrac{1}{h_t^2}(f_{i,j+1} - 2f_{ij} + f_{i,j-1}) \\ \dfrac{\partial^2 f}{\partial x^2} \longrightarrow \dfrac{1}{h_x^2}(f_{i+1,j} - 2f_{ij} + f_{i-1,j}) \end{cases}$$

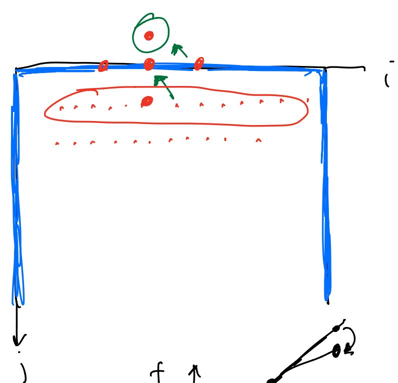
$$\frac{1}{h_t^2}\left(f_{i,j+1} - 2f_{ij} + f_{i,j-1}\right) = \frac{\alpha^2}{h_x^2}\left(f_{i+1,j} - 2f_{ij} + f_{i-1,j}\right)$$

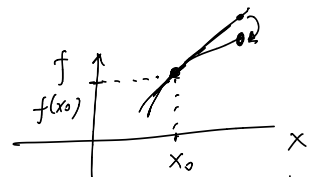
$$f_{i,j+1} - 2f_{ij} + f_{i,j-1} = \underbrace{\frac{\alpha^2 h_t^2}{h_x^2}}_{\tilde{\lambda}}\left(f_{i+1,j} - 2f_{ij} + f_{i-1,j}\right)$$

$$f_{i,j+1} = 2f_{ij} - f_{i,j-1} + \tilde{\lambda}\left(f_{i+1,j} - 2f_{ij} + f_{i-1,j}\right)$$

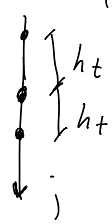
$$\boxed{f_{i,j+1} = -f_{i,j-1} + 2(1-\tilde{\lambda})f_{ij} + \tilde{\lambda}\left(f_{i-1,j} + f_{i+1,j}\right)}$$







$$\left.\frac{\partial f}{\partial t}(x,t)\right|_{t=0} = g(x)$$



$f = ?$ $\quad \frac{df}{dx}$

$$\frac{f_{i,j+1} - f_{ij}}{h_t} = g(x)$$

$$\frac{f_{i,j+1} - f_{i,j-1}}{2h_t} = g(x)$$

$$f_{i,j+1} - f_{i,j-1} = 2h_t\, g(x)$$

$$\boxed{f_{i,j-1} = f_{i,j+1} - 2h_t\, g(x)}$$

$$j = 0 \Rightarrow \rightarrow f_{i,-1} = f_{i,1} - 2h_t\, g(x)$$

MAS

$$f_{i,j+1} = -f_{i,j-1} + 2(1-\tilde{\lambda})f_{ij} + \tilde{\lambda}\left(f_{i-1,j} + f_{i+1,j}\right)$$

$$\Rightarrow$$

$$f_{i,j+1} = -f_{i,j+1} + 2h_t\, g(x) + 2(1-\tilde{\lambda})f_{ij} + \tilde{\lambda}\left(f_{i-1,j} + f_{i+1,j}\right)$$

$$f_{i,j+1} = h_t\, g(x) + (1-\tilde{\lambda}) f_{i,j} + \frac{\tilde{\lambda}}{2}\left(f_{i-1,j} + f_{i+1,j}\right)$$

$j = 0 \implies$

$$\boxed{f_{i,1} = h_t\, g(x) + (1-\tilde{\lambda}) f_{i,0} + \frac{\tilde{\lambda}}{2}\left(f_{i-1,0} + f_{i+1,0}\right)}$$

RESUMINDO:

$$\boxed{\begin{aligned} f_{i,1} &= h_t\, g(x) + (1-\tilde{\lambda}) f_{i,0} + \frac{\tilde{\lambda}}{2}\left(f_{i-1,0} + f_{i+1,0}\right), \quad j=0 \\ f_{i,j+1} &= -f_{i,j-1} + 2(1-\tilde{\lambda}) f_{i,j} + \tilde{\lambda}\left(f_{i-1,j} + f_{i+1,j}\right), \quad j>0 \end{aligned}}$$

$$\tilde{\lambda} \equiv \left(\frac{\alpha h_t}{h_x}\right)^2$$

CONDIÇÃO DE ESTABILIDADE: $\boxed{\tilde{\lambda} \leq 1}$

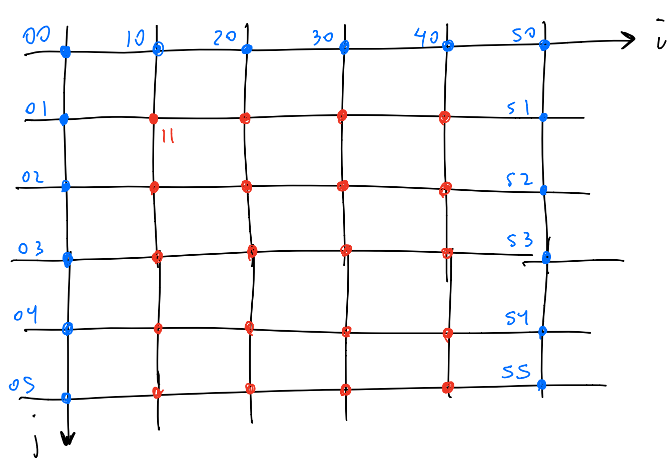
EX: CONSIDERE UMA CORDA ELÁSTICA COM $L=2$ E $\alpha = 1$, COM AS EXTREMIDADES FIXAS EM 0. SUPONHA QUE A CORDA TEM UM FORMATO INICIAL DADO POR

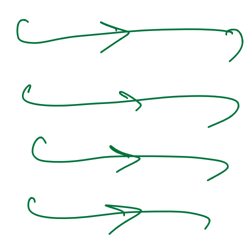
$$f(x) = 5 \operatorname{sen}\left(\frac{\pi x}{2}\right)$$

E VELOCIDADE INICIAL ZERO. USANDO $h_x = h_t = 0{,}4$, CALCULE $f(x,t)$ PARA $0 \leq t \leq 2$. A SOLUÇÃO EXATA DO PROBLEMA É

$$f(x,t) = 5 \operatorname{sen}\left(\frac{\pi x}{2}\right) \cos\left(\frac{\pi t}{2}\right).$$

$$\tilde{\lambda} = \left(\frac{\alpha h_t}{h_x}\right)^2 = \left(\frac{1 \cdot 0{,}4}{0{,}4}\right)^2 = 1 \leq 1. \text{ OK, É ESTÁVEL.}$$





$f \to f[6][6]$

EX: IDEM EXEMPLO ANTERIOR, MAS

$$h_x = 0{,}4$$
$$h_t = 0{,}2 \qquad f(x,0) = x(2-x)$$
$$\alpha = 1$$
$$g(x) = 0.$$

$$\lambda = \left(\frac{\alpha h_t}{h_x}\right)^2 = \left(\frac{1.\,0{,}2}{0{,}4}\right)^2 = 0{,}25 < 1 \quad \text{OK}$$

EX: CONSIDERE UMA ONDA COM FORMATO INICIAL

$$f(x) = 0,$$

QUE OBEDECE AS SEGUINTES CONDIÇÕES DE CONTORNO:

$$f(0,t) = 0$$
$$f(16,t) = 0$$
$$\left.\frac{\partial f}{\partial t}\right|_{t=0} = (12x - 96)\, e^{-2(x-8)^2}$$

A CORDA TEM COMPRIMENTO $L = 16$. CALCULE NUMERICAMENTE $f(x,t)$ COM x VARIANDO DE 0 A 16 E t VARIANDO DE 0 A 10. USE $h_t = h_x = 0{,}2$. CONSIDERE $\alpha = 1$.